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Private Equity and Diversified Portfolios

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INTRODUCTION

Private Equity is an alternative investment that provides capital to non-listed companies. It is widely used by privately owned small and medium firms in order to fund new projects and growth opportunities that would not be equally supported by banks. The financial institutions, called Private Equity firms, in charge of the management of these investments, gather the capital from investors and provide the target company with the additional capital that has been required.

The typical form used by these Private Equity firms to gather the capital is the Private Equity fund, a closed-end fund built as a limited partnership that is managed by General Partners from the Private Equity firm. The General Partners deal with the investors, namely, the Limited Partners, calling capital and distributing the profits, and work with the target companies, providing them with support and advisory services in order to monitor and make more efficient the use of the invested capital.

Thus, Private Equity has several side benefits, in particular for the target companies. It supports good governance, innovation of technologies, efficient managers; it allows companies to expand, creating new jobs opportunities and enriching the firm's local area; it encourages the sharing of ideas and methods among the GPs, the LPs and the target companies, together with the creation of a network of services among them. Side benefits are the reason why the investor's objective is not necessarily the maximization of profit, since Private Equity can represent a solution for those institutional investors that have social duties to fulfill as well.

Institutional investors are the main category of investors in Private Equity. This is due to the investment's characteristics that do not allow all types of investors to get exposure to Private Equity. Investments are illiquid, long-term and expensive, so that only institutional investors and a small group of individuals can afford to invest directly in private companies or in Private Equity funds. The investment is long-term because

the typical time horizon of a fund is ten years on average and investors receive their capital back once the fund has exited its investments. It is illiquid because there is not an efficient secondary market for Private Equity investments. It is expensive because, very often, to become a Limited Partner an investor has to commit a minimum amount of capital to the fund and has to meet all the capital calls, which are usually done at the beginning of the fund life. Therefore, not every investor can afford to be a Limited Partner for the entire life of the fund. However, the profit that the Limited Partner is entitled to should be significant, given that the GPs work in order to produce returns that should be higher than those of the market, in order to compensate the investors for the illiquidity of the investment. The top PE funds can even boast IRRs (which is the preferred measure of performance of these funds) between 20% and 50%.

Given the high expectations on the PE returns, Private Equity is an attractive investment for retail investors as well, who unfortunately cannot easily enter a classic Private Equity fund. But the PE market has opened specific means of investment to retail investors, such as listed PE funds, ETFs on PE indices and listed PE firms, who are then able to get exposure to this asset class avoiding issues like illiquidity and the minimum capital requirement.

Given that the possibility to invest in Private Equity is now open to every kind of investor, it is important to understand whether this asset class actually improves the return of the investor. As a matter of fact, there are several papers on the drivers of the PE returns and on the comparison of the PE return with the market. Talking about the PE return which is expected to be higher than the return on the market, unfortunately, the previous literature has not come to a unique result, creating new doubts on the actual benefits that this asset class can bring to an investor. However, the analysis of the benefits of PE on a portfolio has delivered consistent results over the years. It means that diversifying through PE can improve the risk-return profile of the investor's portfolio.

This thesis is aimed at proving that a diversified portfolio benefits from including Private Equity among its assets. In order to implement this analysis, three optimization methods are used: the Mean-Variance, the Mean-MAD and the Mean-CVaR approaches. The second and the third methods are proposed since they do not need the assumption of normal returns. Moreover, the third method should take into consideration the greater risk associated with the losses that Private Equity has compared to other asset classes, while the Markowitz approach neglects this detail.

Chapter 1 deals with the main characteristics of Private Equity: which forms of Private Equity exist, who the investors are and their characteristics, and how a single PE transaction works. Chapter 2 is about the Private Equity funds. The focus is on the organizational structure: how the terms of the fund are set, which are the main events of the stages of the life of the fund, the roles of General Partners and of Limited Partners, how the compensation of the managers works. Chapter 3 is about the performance of Private Equity: here the methods for measuring the return of a PE fund are explained, together with the methods for benchmarking a PE investment. In addition, an overview on the previous literature about the comparison between the return of PE and the return of the market is presented. Chapter 4 presents the theory behind the approaches used in the experimental part, namely the Mean-Variance, the Mean-MAD and the Mean-CVaR portfolio optimization methods. Lastly, Chapter 5 shows how the implementation of the analysis has been conducted, focusing on the comparison between the efficient frontiers obtained with and without Private Equity and on the comparison of the results obtained using the three approaches.

CHAPTER 1

PRIVATE EQUITY

1.1 INTRODUCTION TO PRIVATE EQUITY

Private Equity (PE) is gaining increasing visibility on the international economic system. Together with hedge funds and real estate, it is one of the alternative asset classes, thus different from the traditional asset classes like stocks and derivatives. It is an investment means that provides capital, both equity and debt, to companies not traded on the public market. This capital is used for several purposes, like develop new products, strengthen the capital structure of the company willing to enter a public market, and so on.

As it is common knowledge, after the financial crisis in 2008, small companies and private firms experienced increasing difficulty in finding liquidity to implement their projects in order to grow or even to survive. In particular in Italy during 2015, these micro and small firms, often family businesses, reached the number of 4.222.442 units and represented the 98.3% of businesses in the country, thus generating a significant part of the Italian GDP (according to the Istat's data reported by ANSA, December 10, 2015). On the one hand, these economic activities allow a great flexibility in terms of products and decision-making efficiency; on the other hand, because of their limited dimension, not all of them can survive the generational turnover and have a stable growth. These firms are generally undercapitalized and their entrepreneurs are not open to the idea of another institution entering the capital because they are afraid of losing the business' control and the decisional power independence. This is one of the reasons why

entrepreneurs are used to addressing their financing needs to banks, at least before the credit crunch, result of the financial crisis occurred. Neglecting the potential risk of limiting their own development, companies very often tried to self-finance their projects, whereas Private Equity would have been a better alternative. In fact, Private Equity provides the required liquidity without having to recover the investment in the short term, thus allowing the company to implement research and innovation projects that, because of their nature, deliver results in the long term.

The institution providing capital is called Private Equity Firm, and, along with the capital, the firm provides other services in order to enhance the chances that the companies entering its portfolio achieve their goals. Specifically, the PE firm acts as an advisor for both the capital structure optimization and the business innovation. Entrepreneurs might be afraid of the intromission of the Private Equity firm's managers into their businesses because they fear that this fact can slow down decision-making, tighten their freedom and cause a drastic cultural change in the company. Moreover, the investors usually require a person on the Board of Directors and the entrepreneurs who have to get used to new bureaucracy, new policies and new governance structures can see this fact as an independence limitation. This could translate into new social and economic costs but Private Equity firm's specialists can really help these companies to grow and to improve their profit generation potential. However, the Private Equity Firm does not invest just its own funds: the PE process starts with a fundraising activity aiming to raise large pools of equity from specific types of investors. In fact, it gathers capital from investors who have investment knowledge and significant amounts of available money, such as pension funds, banks, insurance companies but also wealthy individuals able to understand the risk of the product. Investing in Private Equity is a tough decision indeed, forasmuch as the investment's characteristics do not fit every investor's availability. First of all, Private Equity is a long-term investment since it can last ten years; secondly, it is extremely illiquid since there is not a secondary market for PE investments; lastly, PE is able to produce high potential returns for investors who unfortunately have to bear high risk as well. Moreover, it is not as transparent as other asset classes publicly traded, thus PE is difficult to monitor.

Once created the PE fund, the fund's managers, who are called General Partners (GP), move forward to the acquisition of the selected operating firms and companies, which are called target companies, that provide the best value creation opportunities.

The acquisition is done using both equity and borrowings and their relative proportions depend on which kind of target companies the fund is focusing on, as it will be explained later in the chapter. For example, private equity invests in firms at different life stages, from start-ups to mature companies, and also in firms with different development needs, from technology development to product innovation, from geographical expansion to financial structure strengthening. Each case provides a challenge for the PE firm and a specific capital structure would suit the investment. PE firms achieve the planned renovation of target companies with a series of interventions implemented through the influence of the company's governance and management. Then, GPs control and optimize the cash flow generation, realizing the potential value they recognized in the selected targets before acquisition. As a consequence, the selling price the PE firm will ask future acquirers will be greater than the acquisition price: in this way, the PE firm realizes the return expected by the PE fund investors.

As previously stated, investors in private equity expect high rates of return given the high risk levels. The possibility to profit through the company's additional value created by PE firms attracts more and more investors who consider this alternative asset class as a new means of diversification for their investments portfolio, since PE is believed to be characterized by a low correlation with traditional assets. But, given the risk of this asset class, the evaluation process is a central activity for both Private Equity managers and investors. Fund's managers need capital to invest in target companies, therefore they have to convince investors that their performance is going to be a top quartile one. On the other side, investors require a high hurdle rate to take PE into consideration. Furthermore, PE investors are passive partners: the fund is organized as a limited liability corporation where just the fund's managers are the active partners allowed to deal with the portfolio companies. Hence, PE investors have to trust the PE firm since, once the capital is committed, Limited Partners (LP) do not have power over the GP's activity. The methodology that managers use to guarantee the Private Equity performance is based on the adoption of an active approach to the management of their investment, becoming an involved business partner in the target company. As previously explained, the involvement is firstly implemented through representation on the Board of Directors. As a consequence, they are then able to monitor and influence the target company by advising the owners and entering the decision-making process regarding the implementation of financial and business strategies. Their advice and support can help the target

company's growth in many ways: Private Equity managers usually have sufficient expertise in developing corporate culture, in improving the corporate governance and in selecting winning strategies to enter a new market, both through innovative production ideas and through the creation of strategic alliances. Very often the target company can exploit the investors' partnership for services supply. However, Private Equity firms are not involved in the target company's daily production operations: they rather advise and influence the target company's managers who will have to sustain the company itself at the investment exit moment.

Private Equity's benefits are several: first of all, as previously described, PE creates value, as this is the means to realize the profit goal. Value creation is achieved through costs reduction (agency costs as well), operating efficiency improvement, existing asset exploitation and finally innovation: all these inefficiencies elimination techniques should stimulate the business' growth. Therefore the financial performance enhancement should persist also after the target company's exit from the portfolio.

A second consideration is whether Private Equity creates value for Limited Partners, also known as the PE fund's investors. Their primary objective is to realize a significant return, but not all the researchers agree on the actual ability of PE to meet the high expectations of investors. Ljungqvist and Richardson (2003) stated that risk-adjusted excess returns of PE funds raised between 1981 and 2001 were about 5% annually. On the other side, Kaplan and Schoar (2005) documented that funds raised during the same period underperformed the S&P 500 index, after fees deduction. This mixed evidence is due not only to different approaches and assumptions applied to the returns analysis but also to the lack of available data. Since researchers use different databases, their results are often surprisingly conflicting. Harris, Jenkinson, and Kaplan (2014) discovered that the results obtained from available data in databases understate the real PE returns: they received data directly from institutional investors and conducted a double analysis, one using the verified data and one using a PE data provider, and the result was a consistent outperformance of the market coming from the verified data analysis and an opposite response coming from a commercial database.

Another benefit of Private Equity is given by a better governance structure in the target companies. In particular, the presence of General Partners on the Board of Directors mitigates the agency problems. These are the consequences of the misalignment of the managers' and the owners' interests, that result in a communication gap between

the two categories and in an underperformance of the company. In mature companies, an example of an agency problem arises from dividends. As it is known, growth companies do not pay high dividends, because their growth opportunities are worthier than other opportunities offered by the market. But mature companies sometimes do not distribute dividends as well, preferring to retain earnings. Thus, managers have control over additional cash: this is positive if they have profitable investment opportunities, but it is negative in the opposite case. Agency theory states that the managers' control over retained earnings in the absence of good investments prospects does not maximize the shareholders' value: in fact, managers do not feel the pressure to perform and might undertake value-decreasing projects. This resulting underperformance is called Agency Cost. General Partners can overcome the information asymmetries between shareholders and managers through the introduction of incentives and a monitoring system of the managers' behavior. Alternatively, another way to solve agency problems is to fund the acquisition with debt: managers will have to optimize their efforts, adopting a profitable investment policy and exploiting the existing assets in order to generate enough cash flows to meet debt obligations. Managers, who are then exposed to a series of new pressures, are advised by the PE firm that brings new expertise in the target company with the aim of improving the operating efficiency of managers.

Finally, side benefits generated by PE deserve to be mentioned. Private Equity supports start-ups and small firms, thus improving the economic conditions of local regions. By stimulating the growth of these firms, it contributes to the creation of new jobs and to the introduction of new competing technologies in areas that could be under-developed. Similar results are achieved when PE intervenes in support of mature firms with unexplored potential. Furthermore, since Socially Responsible Investments are gaining increasing attention in this sector as well, even the PE investments are selected with a particular interest towards the environment, the social engagement, and good corporate governance and culture.

1.2 Types of Private Equity

Private Equity has a broad definition: it is an asset class consisting in capital investment into companies not listed on a public stock exchange. There are several ways in which Private Equity firms implement these investments, using both equity securities and debt

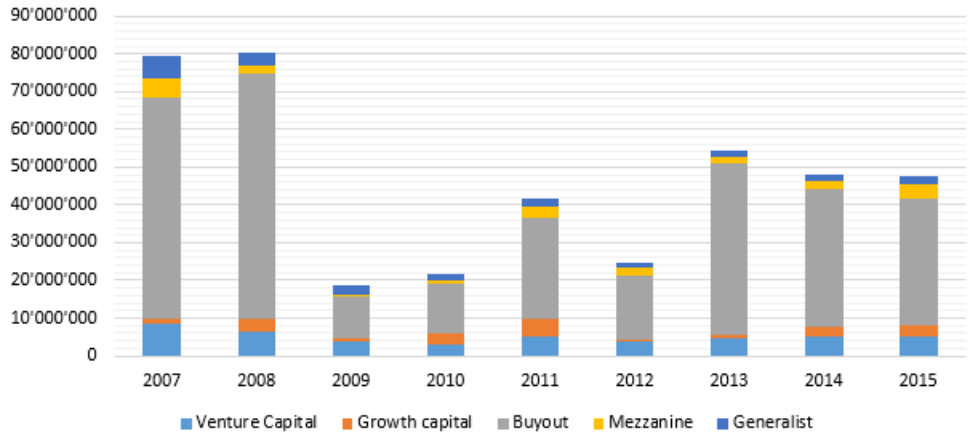

Figure 1.1: Amounts raised per PE type between 2007 and 2015 in Europe; source of data: *Invest Europe*

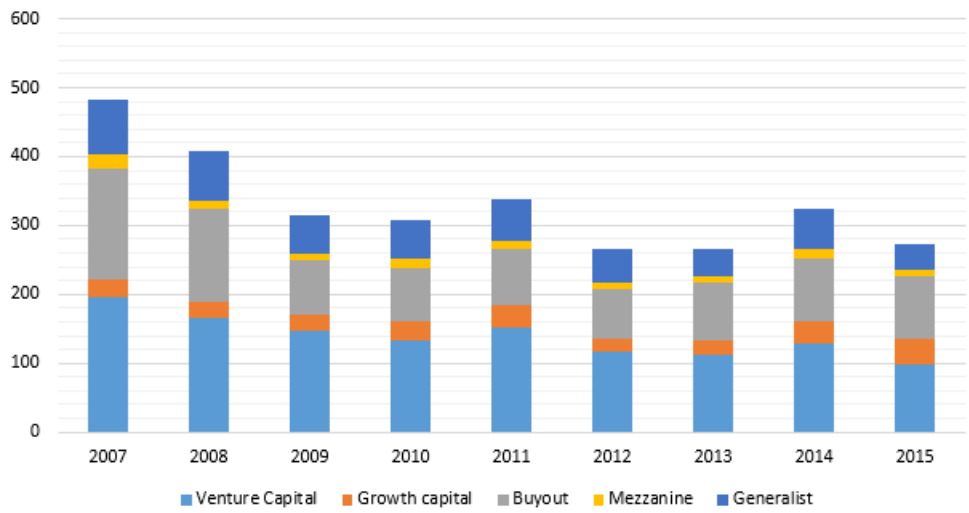

Figure 1.2: Number of funds raised per PE type between 2007 and 2015 in Europe; source of data: *Invest Europe*

and focusing on different types of companies. Major types of Private Equity are *Leveraged Buyouts*, *Venture Capital*, *Mezzanine Capital* and *Growth Capital*.

As *Figure 1.1* displays, the category consistently raising the most significant capital amounts is Leveraged Buyouts, even in the post-crisis period. The situation drastically changes considering the number of funds raised: each year the majority of new raised

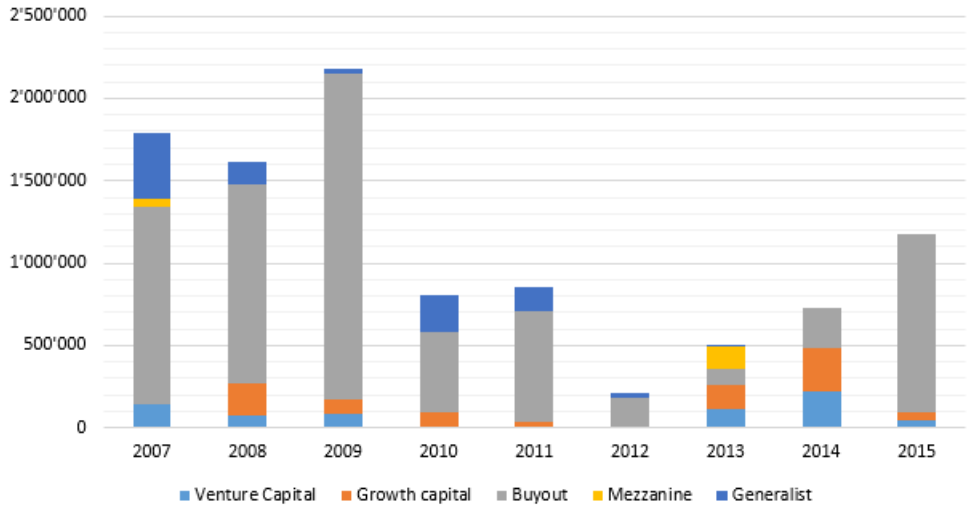

Figure 1.3: Amounts raised per PE type between 2007 and 2015 in Italy; source of data: *Invest Europe*

funds is represented by Venture Capital funds. The reason of this difference is due to the operative nature of the two PE types. Leveraged Buyouts funds, as it will be explained later, acquire mature firms while Venture Capital funds focus their interest on young firms. Therefore, Leveraged Buyout requires greater fund sizes than Venture Capital.

The Italian case is slightly different: not only Leveraged Buyouts maintain their superiority in terms of total raised amounts but a similar situation exists in terms of numbers of funds raised per PE type, too.

The most interesting data is given by the comparison of *Figure 1.1* and *1.3*, that result in an unexpected growth of the total Italian PE investments in 2009, a countertrend with respect to the European tendency. What is even more curious is that this growth has been dragged by Leveraged Buyouts which are mostly financed by debt. As it will be suggested later, a reason for this banks behavior is that banks prefer opening credit lines in favor of PE firms instead of entrepreneurs because of their superior expertise. A possible explanation of the Italian singularity could be related to this credit shift from single individuals to PE managers as a consequence of the credit crunch.

Leveraged Buyout *Leveraged Buyout* (LBO) is simply the acquisition of a company by a Private Equity fund from existing owners or shareholders. In particular, this fund is

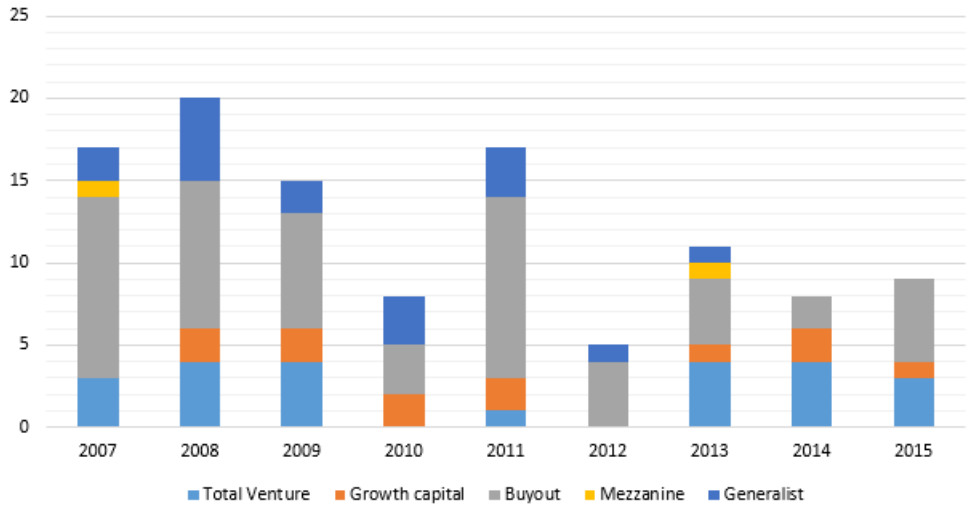

Figure 1.4: Number of funds raised per PE type between 2007 and 2015 in Italy; source of data: *Invest Europe*

set up by a Private Equity firm, which serves as General Partner. The LBO acquisition's main characteristic is that it is mainly funded through debt capital, using the target company's assets as collaterals: therefore the new capital structure of the target company is composed of the amount of debt used in the acquisition process as liabilities and the fund's capital as equity.

The LBO investors' returns are generated in two ways: first through the reconstruction of the company's business and second through the use of debt capital in the acquisition process. On the one hand, at the investment exit, the selling price of the target company should be greater than the acquisition price, since the PE firm improved the business in order to erase all the inefficiencies. On the other hand, the PE firm exploits the leverage effect: firstly the interests on debt capital are tax deductible, thus the free cash flows to the company increase. Secondly, since during the acquisition, thanks to the recapitalization process, the equity has been substituted by the fund capital, when the debt is gradually paid back, the equity stake of the fund and the target company itself increase in value.

As debt capital plays an important role in LBO operations, the leverage ratios change according to the economic conditions, especially with respect to the debt capital supply. In fact, during late 1980 the leverage ratios were about 90% while in credit crisis periods

they dropped to 50%-60%, as in 2008 (Baker, Filbeck and Kiymaz, 2015).

An LBO operation usually invests in mature companies that do not produce the desired net profit because of inefficiencies. The PE firm starts screening all the potential target companies with a relatively well-established business and that are facing a stable demand, selecting those which underperform the peer group and has unused debt capacity. Once the target company has been identified, it is purchased by the PE firm who heavily finances the acquisition with debt. The PE fund's managers then move forward to improve the company through the elimination of the inefficiencies: the resulting increased cash flows are firstly allocated to debt repayment.

Since the company's transformation could not satisfy the PE firm's expectations, this kind of Private Equity is risky and the fundraising activity could be problematic. Several LPs prefer to commit to the fund once the first investment has been accomplished, in order to judge whether the other investors' money is well managed or not, considering that the final profit will result only at the investment exit and will follow debt repayment. Furthermore, investors require high hurdle rates from the investment since the presence of debt in the acquisition process makes the deal riskier. The General Partners' skills and past performances represent an important factor in the fundraising activity, which is successful just if investors trust the fund's managers on the basis of their reputation.

GPs' contribution is particularly relevant in the pre-investment phase, in which they have to decide whether a company represents a good choice for an LBO transaction: they have to decide between companies with real opportunities for economic value creation or increment and that also present sufficient potential cash flows to pay back the great amount of debt used in the acquisition. During the investment phase, the PE managers bring their know-how in the target company, enhancing its profitability through the reduction of costs and the improvement of the operational efficiency, achieved also by motivating the target company's managers through different types of incentives. As a consequence, cash flows increase along with the target company's value. At the exit moment, the return is finally realized: the return can be higher or lower according to the GPs' ability to negotiate the price and to choose the best timing and type of exit channels.

Venture Capital The Venture Capital is a kind of Private Equity investment focused on rapidly expanding companies. It is appreciated specifically for its predisposition to

develop local economies since it is able to turn a simple firm into a structured company or organization. This form of Private Equity benefitted of a great expansion in the last decades thanks to the significant development of the technological sector, especially in the forms of information technology, digital communication and biotechnology. Venture Capital funds extensively invested in these sectors because they recognized the huge potential of this industry still at an early development stadium and had the intuition of financing the most innovative ideas to contribute to the acceleration of the whole industry growth.

Venture Capital investors have a competitive advantage over traditional funding institutions, such as banks and public markets. In fact, they are able to manage in a more efficient way the informational asymmetries and the uncertainty characterizing the investments in young companies. In particular, Venture Capital firms collect information about entrepreneurs, about the management team, about technologies, about the market place and the project's business plan: the analysis is then conducted using careful due diligence in order to evaluate whether the company under examination can serve the fund's purpose. On the other hand, banks focus their analysis on the financial history and on the assets of the company to use as collaterals: unfortunately, since the company is usually very young and its main business is generally based on intangible assets, none of the two is available (Ueda 2004). Moreover, in order to align entrepreneurs' and investors' interests, Venture Capital firms stipulates exhaustive contracts dealing also with the entrepreneur's activities and duties, therefore reducing agency problems (Kaplan and Stromberg, 2004).

Again, the General Partners' activity is a major challenge during the portfolio selection and fundraising process. The reason is that once invested, the ultimate return is realized once the investment is exited. Thus, since Venture Capital funds' investment phase lasts from five to eight years, they classify as long-term investments and the underlying assets are illiquid. As a consequence, the GPs have to select the target companies with due diligence in order to meet the return expectation of the fund's investors. Furthermore, the GPs play a special role in the investment phase, acting as strategic advisors and entering the company's Board of Directors. This is a difficult task to accomplish in a portfolio where there are several companies in the needs of assistance, but this element remains critical for the investment's success.

Mezzanine Capital Mezzanine Capital is a hybrid of debt and equity: these two financing means are opposite, and in between lies an entire series of financial instruments which can be more debt or equity-like according to their characterization. It refers to instruments like preferred equity and subordinated debt, instruments which, depending on their specific characteristics, position themselves between senior debt and equity. Therefore, mezzanine investments can take various forms but all types have in common the ranking position compared to equity and debt. Firstly, mezzanine capital is senior in rank to common stock, given the superiority of the mezzanine investor's right to recover the invested capital with respect to equity holders. On the other side, senior debt holds priority of payment with respect to mezzanine capital. Moreover, debt providers' investment are usually unsecured. All these features make mezzanine capital more expensive than senior debt but less expensive than equity, namely, the most expensive form of capital. For this reason, mezzanine capital is usually present in companies' capital structure since it allows shareholders to provide less equity capital: given the lowest company's cost of capital, the return on equity is maximized (Silbernagel, Vaitkunas and Giddy, 2010).

Mezzanine capital is particularly useful in private equity funds because it enables PE firms to take money out of a fund's single investment in order to have more money for investing in other targets, it facilitates some exit channels like management acquisition and lastly its characteristics fit growth opportunities.

Growth Capital Growth Capital is a particular kind of Private Equity used in specific situations. The PE firm usually invests in relatively mature companies about to undertake restructuring operations or to expand through entering new markets or acquiring other companies, without having significant ownership changes. For this reason, the name growth capital is also replaceable by the expression expansion capital. PE firms form growth capital funds' portfolios searching for target companies that are more mature than start-ups or young firms, which are the preferred targets of Venture Capital, not in the need of major transformations so that the expansion project requires fewer funds. Growth capital targets are mature companies that have a stable revenue generation history but that are unable to generate sufficient additional cash flows to be reinvested in profitable transformational projects (Stowell, D. 2012). Generally, growth capital candidates cannot easily access debt because of their existing debt levels (as mature

companies already have debt in their capital structures) and because of their incapacity of generating increasing earnings. Therefore, companies looking for this minority amount can become Private Equity targets, hence accepting all the other supplementary services the General Partners have to offer. Structurally, growth capital is usually an equity investment in the form of preferred equity, even though other mezzanine types are possible through the introduction of debt features like interest payments.

1.3 INVESTORS

The Private Equity system is based on the activity of intermediaries who permit the interaction between private companies and investors. They make the allocation of non-listed capital possible: therefore, on the one hand, they have to fundraise from the investors who are willing to invest in securities with a risk profile in line with the Private Equity's one; on the other hand, they have to find a basket of potential targets where they will select the final investments from.

As it has already been highlighted, investments in Private Equity are characterized by a risk level that is quite high on average. In fact, given the different components of the broad asset class, every PE kind has its own risk profile. It is sufficient to recall the characteristics of Venture Capital and of Leverage Buyout: the first invest in a risky start-up, the second in a less risky well-established company. Nonetheless, the asset class' risk level remains high and investors are consequently fewer than those acting on public markets.

Firstly, the major group is composed by institutional investors, thus specialists of financial portfolio management with precise skills and competencies. This is the case of banks, that can actually act both as intermediaries and as investors, thanks to their field knowledge. Sovereign wealth funds, which are state-owned funds managing a pool of money derived from the country reserves, also have the necessary know-how to invest in Private Equity. Funds of funds are a major category of investors. These usually are Private Equity funds that do not take positions directly in the companies but indirectly take equity positions in other funds. Moreover, among institutional investors, there are some that are characterized by an open contrast between the investment's riskiness and their social nature, as foundations, which are non-profit organizations that employ private wealth for public or charitable purposes. Other institutional investors which are

warmly expected to invest with the aim of hitting the hurdle rate are pension funds: for social reasons, they are recommended to invest without taking too much risk. Their investment has to be reconsidered taking in mind their investment policy, based on long-term horizon and on diversification, that should lower the whole portfolio risk even if PE is included.

The second category of investors is referred to wealthy individuals. There are two significant cases: the first is represented by *business angels* who are investors with excellent investing techniques, personal capitals and therefore able to select the target companies they judge the most promising or the PE firms they trust the most when they do not invest directly. The second case is represented by *High Net Worth Individuals*, who own significant wealth and are willing to commit a fraction in Private Equity securities as part of a diversified portfolio.

Institutional investors are interested in this asset class as a consequence of the extension of the basket of investment opportunities offered by the market, caused in particular by the birth of alternative investments asset classes composed of Private Equity, Hedge Funds and Real Estate. The reason why alternative investments are gaining growing importance in institutional portfolios is due to the low correlation with traditional asset classes. The portfolio including alternative investments better diversifies its risk.

As it appears from *Figure 1.7* the greater PE investor groups are banks, pension funds, funds of funds and private investors. Between the years 2007 and 2015, the average investors base has been as displayed in *Table 1.1*.

The Government Agencies' activity is an interesting element, because it increased after the financial crisis in 2008 from a 3% to a 13% in 2009 and then it maintained a significant role among the PE investors. This behaviour suggests that Government Agencies intervenes in order to sustain the PE industry in difficult moments when other investor types do not dare to enter these risky investments. Considering that the total fundraised amounts dropped in 2009 from 80 to 19 billions, Government Agencies are the only investor type to actually increase their positions by the 8%, while all the other investor types decrease their invested amounts by 70%-80%, as it is displayed in *Figure 1.5*. A possible reason for this behavior could be found in the PE side benefits: PE is an efficient means for local development, therefore the Government delegates some of its function to PE.

What is unexpected is the position of banks in the years immediately following the

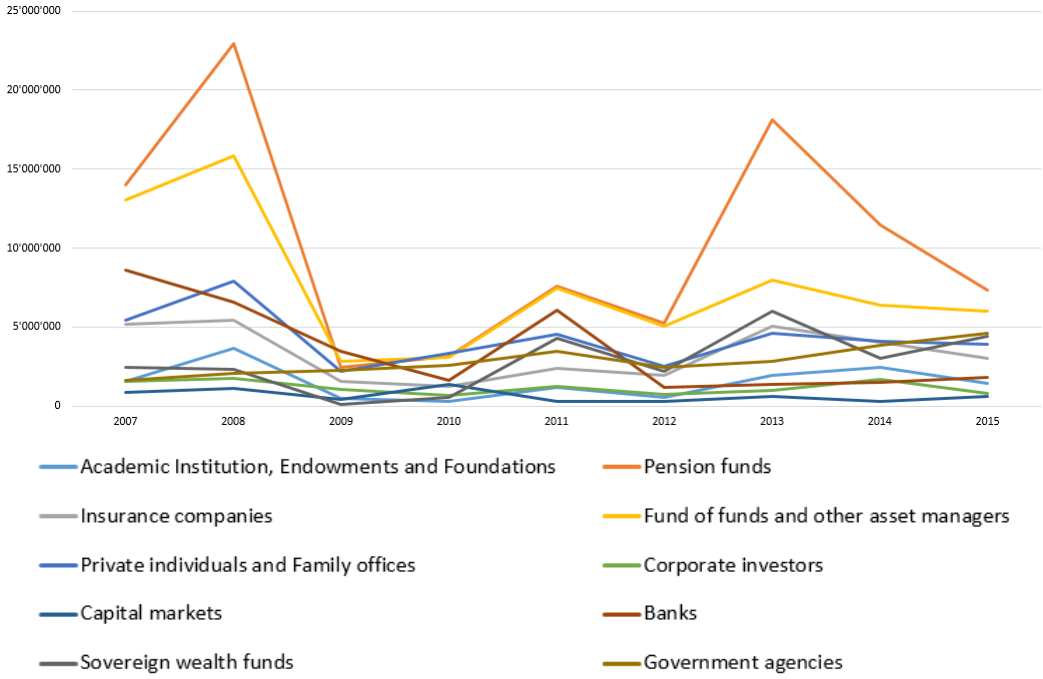

Figure 1.5: Trend of the composition of PE funds' investors in Europe based on *Invest Europe*'s data

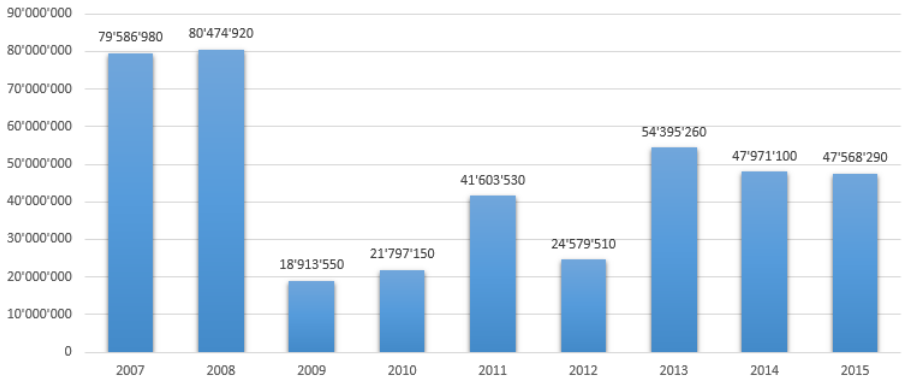

Figure 1.6: New Raised Funds (amounts in € thousands) based on *Invest Europe*'s data

crisis. The total amount invested in PE in 2009 was funded to a considerable extent by banks (21%), while banks were expected, given the credit crunch of those years, to not take into consideration very risky investment opportunities as Private Equity. Generally, it is riskier to enter the capital of companies rather than opening a line of credit. The reason why banks are more oriented to enter a Private Equity fund is that they will have

Pension Fund	24.7%
Fund of funds and other asset managers	19.2%
Private individuals and Family offices	12%
Banks	9.8%
Government Agencies	9.2%
Insurance Companies	8.7%
Sovereign Wealth funds	7.3%
Academic Institution, Endowments and Foundations	3.6%
Corporate investors	3.4%
Capital Markets	2.2%
New Funds Raised	*100%*

Table 1.1: PE funds average investor composition

to face General Partners, managers with the necessary expertise to succeed in the investment. When a firm or a start-up asks the bank in order to access credit, the bank knows it will have to deal with an owner who does not always have the sufficient knowledge about the market and the current economic conditions, who looks firstly to short-term returns and who does not have skills and innovative ideas to survive the crisis. On the other side, PE managers have the essential *know-how* for working in challenging times and situations. Moreover they implement established methodologies to approach the most important decisions the target company has to take. Also fund diversification explains the bank's activity: the PE portfolio risk is lower than a single target company's risk.

1.4 Target Company

The main focus of a Private Equity investment is the target company. PE firms set up funds acquiring companies and firms, both young and mature, with potential growth opportunities represented by unsolved inefficiencies in the business, in the capital structure and in the corporate governance. The target company is then improved in its weaknesses and reinforced in its strengths by the PE managers, the General Partners, whose final objective is to generate profit from the investment exit. Exits opportunities are several: among others, an Initial Public Offering (IPO) or the resale of the company. At the investment exit, the PE firm sells the target company at a price which is higher than the

acquisition price, as a consequence of the increased value generated by the GP's intervention in the company.

GPs select target companies from a basket of possibilities characterized by specific factors. The pre-investment phase is critical and PE firms take years to find the most suitable options among the eligible companies. The subsequent investment period takes up to six or seven years and during this GPs' activity time the capital stays invested without any redemption option. Therefore, the initial choice of the target companies is crucial for the fund's investors since the selection of a wrong fund's portfolio can damage the LPs' interest.

Pre-investment phase Many factors enter the investment decision process: the company sector, the technologies in use, the geographical allocation and the future market prospects. Usually, companies must have three characteristics to meet the choice criteria of the PE firms. First of all, the company in order to be a PE target has to present economic value creation opportunities. Generally, PE managers work to reduce the inefficiencies that decrease the company's value: unexploited assets, flawed strategies, insufficient expertise, wrong capital structure and agency problems. Fund managers can easily solve these problems through the implementation of tested action plans. For example, they can replace those managers whose interest conflicts with the investors' one; another solution is the introduction of incentives for the management who are then pressured to perform; an alternative way is the tightening of the governance structure, increasing the control of owners and investors over the management activity. Agency costs are therefore reduced. GPs also perform an efficiency analysis of the company business divisions, searching for improvement chances. In some cases, GPs proceed with the ultimate divestiture of underperforming units.

A second factor taken into consideration in the investment decision is represented by Free Cash Flows (FCF). FCF must be sufficient to meet debt obligations (in particular in Leveraged Buyouts deals) or to pay dividends in the case of mature firms. When GPs work with young target companies, FCFs are reinvested in the business or used in new profitable projects.

Lastly, PE firms negotiate the terms of the targets' acquisition. An important factor in the selection of companies is the low acquisition price. This means that the current market value should be reasonably low and as a consequence the PE firms can move

forward to a relatively cheap takeover.

Investment phase Once selected the target companies, there are three main restructuring intervention categories to use to change the firm during the investment phase: financial engineering, operational engineering, governance engineering (Kaplan and Stromberg, 2009). Financial engineering is related to capital structure restructuring efforts. Speaking in particular of LBO, the target company acquisition is financed with the objective of replacing the old capital structure with the acquisition financing structure. LBO are then financed with little equity and large debt amounts in order to exploit the leverage effect, which provides good reasons to use debt: tax-deductible interests on debt and increase of equity stake while debt is gradually paid back.

The introduction of management incentives is part of the governance engineering but also of the financial engineering. In fact, incentives can be the giveaway to managers of a part of the target company's equity stake in order to increase their concern for the company and consequently their efforts. Leverage is a management incentive as well, since it makes the company riskier and create pressure on managers because they have to meet debt obligations. Pressure on management, incentives and a better monitoring system of managers' activity strengthen corporate governance, as part of the governance engineering efforts. These are mainly directed to the agency costs reduction. Aligning managers' and investors' interests allows the company to improve its weak governance, eliminating the communication gap and reducing asymmetric information. In the worst case, poorly performing management can be replaced. Moreover, governance engineering operates changes to the Boards of Directors. GPs have a seat on these boards for the investment period and are actively involved in the company's decisions. They also introduce new bureaucracy in order to create fixed efficient processes for companies chasing IPO for example.

Lastly, operational engineering is referred to the improvement of operating expertise aimed at the addition of value to the target company. Obviously, GPs must have the necessary know-how for being effective: this is the reason why sometimes PE firms focus on industries and geographical areas. GPs' operating skills are important for both the investment and pre-investment phases, for identifying and developing potential economic value.

Exit strategies After a successful investment phase in which the restructuring efforts increased the target company's value, GPs have to decide for an exit strategy. First, the PE firm sells the target company, realizing the profit, and secondly it distributes the sale proceeds to limited partners. The first step is completed in three primary ways which have in common a selling price higher than the initial acquisition price: trade sale to a strategic investor, to a new financial sponsor, IPO. Exit strategies are influenced by macroeconomic conditions: GPs' effort toward recognizing the best strategy and the perfect timing is a critical element to generate high returns. For example, IPOs are facilitated by growing public equity markets and low interest rates facilitate trade sales.

The first exit channel is a trade sale to a strategic buyer: this acquirer is interested in the purchase in order to increase its own value thanks to patents, innovative products or synergies, intending to hold the acquisition in the long term. By buying the target company, the acquirer expects a market share and a competitive advantage increase. Expecting high future cash inflows from the acquired company, the acquirer is generally willing to pay a higher price. The target company itself can be the buyer, purchasing back its own shares from the PE fund. This last solution is preferred by the target's managers since they might be replaced after the acquisition from an external strategic buyer.

The second exit strategy is a trade sale to another financial investor, also called secondary. In this case, the PE fund believes that a secondary investor can generate additional value to the company as it is not ready for an IPO but needs to enter a second development stage. The purchase terms are more flexible and the transaction time shorter since both counterparties are M&A experts.

An alternative exit strategy is represented by the Initial Public Offering: the target company enters a public market and its shares get listed on a stock exchange. IPO are very popular among PE firms because, in the case of success, their visibility and prestige grow and the realized returns are the highest possible. However, since IPO involves transaction costs, regulatory requirements, and restrictions, this exit strategy is common for big PE funds and for portfolio companies which were already large before the PE acquisition. IPO is also the longest exit strategy: its preparation can take from six to twelve months because there are more than two parties involved as in the trade sale. Moreover, there are market authorities monitoring the transaction.

Single investment valuation As it has been stressed throughout the chapter, a PE firm generates profit by buying an asset (the target company) at its fair price and by selling it after six or seven years as its value has increased. During the investment period, PE firms must employ valuation methods that monitor both the current and the potential value of the target company, hence estimating future cash flows on the basis of the achieved and the planned improvements, both operational and financial. The generation of cash flows is crucial to the success of the investment, therefore, before investing, the PE firm has to come up with an asset value estimation that combines the actual and the potential cash flow paths, their riskiness and the cost of capital to apply. This estimation must be the most accurate possible, considering that the estimated purchased value can be incorrect due to many factors like the exclusion of macroeconomic factors and the inclusion of wrong cash flow projections, inexact understanding of risk and incorrect cost of capital calculation.

There are several methods employed for evaluating PE target companies. The most famous technique is the evaluation of cash flows, considered as incoming and outcoming amounts generated from operations, investments and financing activities. PE firms generally perform both the measurement of equity cash flows and of enterprise cash flows separately, given the different purpose of the two cash flows types: while the first is destined to equity holders, the latter refers to money available to both debt and equity providers. The most familiar equity cash flow definition is given by equation 1.1 obtained through the sum of Net Income (NI) and non cash costs[1].

$$CF = NI + \text{Amortization} + \text{Depreciation} \tag{1.1}$$

Equation 1.1 can be furtherly improved by adding the change in net working capital between two consecutive accounting periods.

The corresponding definition for enterprise cash flows is given by equation 1.2, called Unlevered Net Income since it is obtained by adding back the interest payments net of their tax shields to the Net Income (NI).

$$UNI = NI + (1 - \text{tax rate}) \times (\text{Net interest expense}) \tag{1.2}$$

[1]Formulas from Damodaran, A. *Applied Corporate Finance*

A necessary correction that has to be pointed out is referred to deferred taxes. In balance sheets, deferred taxes are long-term liabilities that indicate future tax obligations for the company arisen in the current accounting period:

$$NOPAT = UNI + \Delta\text{Deferred taxes} \tag{1.3}$$

where Δ indicates the difference between two consecutive accounting periods. In order to get the free cash flow to the firm (FCF) it is necessary to add back depreciation and to subtract capital expenditures (CAPEX) and the change in net working capital:

$$FCF = NOPAT + \text{Depreciation} - CAPEX - \Delta\text{Net Working Capital} \tag{1.4}$$

Given these formulas, PE firms need to estimate future cash flows deriving from their restructuring interventions. This first step is then implemented using the PE firm financial modeling skills and the expectations it has for the target company: future growth rate, future ability in managing working capital and future capital structure. The latter is especially important, considering that leverage has a great impact on cash flows. Not only debt makes the investment riskier, but interest payments reduce FCFs and money available to equity owners. On the other hand, leverage increases the return on equity, hence the PE firm has to set a tradeoff that resolves in the optimal capital structure. Once completed the estimation and evaluation part, the final step involves the decision on how much the PE firm is ready to pay for the acquisition. This evaluation includes the investment's present value along with other factors determined by the fund characteristics: its size and its legal and contractual provision. The more previous experience the PE firm has and the more knowledge it has on the target company's industry, the more accurate the final evaluation is. Given the estimation of the present value of the target company, a comparison with the acquisition price gives the return on the single investment.

A crucial element is the choice of the number of years to include in the cash flows forecast. Cash flow projection has to last at least until all the PE firm's major activities have ended. Then, each analyst chooses the forecast extension according to his opinion. Including the cash flows that the target company is expected to generate after disinvestment is an additional element that improves the target's evaluation. Generally, in order

to simplify the process, cash flows generated beyond the forecast period are represented by the Terminal Value, a measure of the firm value assuming stable growth.

$$TV = \frac{(\text{Cash Flow}_n)(1+g)}{(r-g)} \tag{1.5}$$

where TV is the terminal value, g is the estimated growth rate after the end of forecasts, r is the discount rate and *Cash Flow*$_n$ is the cash flow expected at time n (the end of the forecast period).

Given these details, the PV present value formula for the target company is:

$$PV = \sum_{t=1}^{n} \frac{\text{Cash Flow}_t}{(1+r)^t} + \frac{TV}{(1+r)^n} \tag{1.6}$$

Generally the discount rate is the Cost of Capital (WACC), obtained by weighting the costs of debt and equity by their relative proportions in the capital structure:

$$WACC = \left(\frac{E}{V} \times k_e\right) + \left(\frac{D}{V} \times k_d \times (1-T)\right) \tag{1.7}$$

where k_e is the cost of equity, k_d is the cost of debt, E is the market value of equity, D is the market value of debt, $V = E + D$ and T is the tax rate. k_d is measured as the paid debt interest. There are several ways to calculate k_e, but the most employed is the CAPM (Capital Asset Pricing Model).

$$k_e = r_f + \beta(r_M - r_f) \tag{1.8}$$

where r_f is the risk free rate, r_M is the return on the market porfolio and β is a mesure of the company's idiosyncratic risk. Clearly, if the target company is not publicly traded, the regression of its returns on a market index is not implementable. In this case, β has to be approximated by comparable publicly traded companies' βs.

The Discounted Cash Flow method has several flaws, since it relies on too many uncertain variables and assumptions, for example the unchanged capital structure. A second possible approach is called *Multiples-Based* valuation. It consists in the comparison between similar firms' indicators. The most used indicator is the EBITDA, an

earnings measure:

$$EBITDA = EBIT + \text{Depreciation} + \text{Amortization} \tag{1.9}$$

where EBIT derives from the subtraction of operating costs from operating revenues. On the basis of the current year and of reasonable assumptions, PE firms forecast a future EBITDA and then multiplies this estimate by an earnings-multiple set by the PE firm itself, considering that earnings-multiples should be set higher if the company is expected to rapidly grow and to generate more profits than the peer group. These multiples are also determined on the basis of other multiples chosen by investors for similar investments or of the average multiple applied to similar assets in the market (Stowell, 2012). The strongest criticism to this approach is that it does not estimate a series of future earnings, preferring the focus on a single year.

1.5 Sustainable Responsible Investment and Private Equity

Sustainable Responsible Investing (SRI) is an investment trend that focus on those investments that provide other benefits than the maximization of profit. In particular, it is an investment strategy that looks for a compromise between the financial analysis and a qualitative analysis that takes into account Environmental, Social and good Governance aspects (ESG). Environmental themes are, for example, waste, water, renewable energy, transportation fuel and toxic chemicals; social themes are diversity and discrimination, human rights, ethical and social practices in the supply chain, employee engagement, improvement of working conditions; governance is about management structure, policies and board-level control (PitchBook 2015). While the general aim of an investment is to create value for both the investor and the company itself, a sustainable responsible investment commits to create value for the community as well, preferring to invest in those companies with remarkable ESG characteristics and which have a potential positive impact on the community or on the environment. There are six main strategies to select the company to invest in (Forum per la Finanza Sostenibile and AIFI, 2015):

- exclusion of certain industries, countries and even single companies according to

their involvement with the production or promotion of damaging goods (such as tobacco, fast food and weapons) and of customs characterized by a bad reputation (pornography, gambling and animal testing);

- selection of companies, industries and countries based on the application of international rules and standards;
- among the investment universe, selection of those companies whose potential positive impacts are higher;
- selection of the investment universe according to a pre-specified focus on a certain theme (environment, good governance, social interest);
- selection of companies that do not have ESG policies but seem willing to collaborate in developing ESG awareness through the interaction with the board of directors;
- impact investing, that is defined as an investment strategy aimed at generating both financial return and social and/or environmental impact through the investment in organizations with ESG policies.

Responsible investing is applicable to every asset class, but Private Equity is particularly fit for it. In fact, the PE firms put their efforts on the improvement of the target companies, and considering all the side effects that their improvements can have on the local area, the GPs can influence the managers to take into consideration the ESG factors in their decisions. Moreover, the consideration that PE investors (namely, the Limited Partners) have a growing interest in the ESG themes and take them into account when selecting the PE firm is a factor that stimulates the GPs to select the target companies according to ESG criterions. This is suggested by recent surveys on the integration of ESG criterions in PE that investigate the reasons that pushes the GPs to implement the practices of responsible investing. According to the ESG survey of PitchBook (2015) conducted on the policies adopted by GPs, 71% of interviewed GPs indicate that the expectations and preferences of LPs influence their decision to apply the ESG practices. Then, 63% indicate the environmental and social impact, 61% the risk management, 47% the corporate governance, 41% the reputation. Among the other factors there are

the fact that the competitors are starting to be interested in ESG together with the government regulation and the employees' interest (in particular concerning the social aspects about the labor policy).

As just mentioned, a factor that push GPs to adopt ESG practices is represented by competitors. The fact that in the whole industry the ESG awareness is increasing means that the majority of PE firms have already taken at least into consideration the sustainability of their investments. Therefore, sustainable investment is suggested firstly because LPs are likely to invest in those PE firms openly considering ESG factors and secondly because a PE firm can see in the sustainable investment an opportunity for innovation and differentiation from other competitors. The GPs should then get informed on the subject and set an original investment strategy, which should include the ESG criterions. The goal they are seeking should be clear for both investors and target companies and the annual reports should include informations on the ESG monitoring activity in order to judge whether the strategy has been followed and if it has been successful, especially compared to the strategies of other GPs.

It is interesting to explain why and how risk management drives the GPs' decision of adopting the ESG initiatives. Risk management is implemented in the form of risk identification and risk mitigation. Therefore, the GPs firstly apply the ESG criterions, especially during the selection of target companies: at this point of the process, the GPs apply the due diligence, which implies the identification of the potential risks. Secondly, the GPs, when monitoring and advising the portfolio companies suggesting policies aimed at the risk mitigation, can prevent the insurgence of damages in terms of ESG. Every target company is in fact exposed to some kind of ESG risk: the most basic ESG risk is associated with the labor policies applied by the company itself. When an ESG risk materializes, the company exposed to it suffers many consequences: firstly a potential legal trial and consequent fines; secondly, a potential financial loss due to the negative image caused by the risk that has materialized. In fact, the stakeholders are several (investors, costumers and even the general public) and are easily reachable by the negative news and advertisement. This type of risk is called reputational risk and involves all the possible consequences of the damage of the reputation of a brand. Furthermore, if the company is part of the portfolio of a PE fund, the GPs share the responsibility with the company's managers, as a result of their controlling and advising function. Thus, GPs as well suffer from negative advertising: the relationship with the

LPs can be damaged as well as their potential success in a following fundraising activity (Malk Sustainability Partners, 2015).

The most common selection method applied by GPs is the exclusion: when forming the portfolio of the PE fund, they avoid those companies that clearly present a significant exposure to the ESG risk. Those companies that show small ESG risk can still enter the portfolio but in this case the GPs will work with the company in order to reach an acceptable level of risk. However, the practice that involves the selection of a company aiming at introducing it to the ESG policies is less frequent, also because in practice it could be harder than expected or even impossible (for example, a weapons manufacturer). Anyway, every GP has to stimulate the target companies to raise awareness on the ESG themes. A common action among PE funds is to mention, during the meetings with the managers and the board of directors of the target companies, the ESG monitoring process that the fund is conducting and that the target company should plan to implement in the future, in order to ensure that the managers are aware of the issue and that they are working in order to protect the business from this type of risk. Moreover, the managers should be aware that the GPs have an ESG strategy and that consequently have expectations on the ESG improvement of the company. The company management is suggested not only to develop ESG policies or to build a monitoring system for the ESG factors, but to create a corporate culture that is aware of the ESG issues.

Concluding, a list of general rules for GPs willing to commit to a sustainable investment strategy is presented. This guide is provided by the six Principles for Responsible Investment (PRI), which have been developed by the United Nations in 2006. These principles are intended to ensure the sustainability of the investment process and, even though they are very general, they give the GP the freedom to design his own investment strategy within a specific framework. The six principles are:

- The GPs should incorporate the ESG risks and opportunities into their selection process aimed at forming the PE fund;
- The GPs should implement an active ownership that is aware of the ESG issues in the companies they invested in;
- The GPs should ask the target companies to disclose data on the ESG issues that they are facing;

- The GPs should promote the acknowledgement of ESG practices among other actors of the PE industry;
- The GPs should collaborate with the United Nations and with their peers in order to improve the effectiveness of the principles;
- The GPs should disclose the details of their activity which involves the ESG issues.

Therefore, the focus is on the investment process and on the reporting activity. What is then suggested is to apply the ESG criterions to the fund creation process and to incorporate them into the due diligence that is required for evaluating the investment decisions. Then, the GPs should seek the commitment of target companies to the ESG policies by sharing with them their ESG objectives. Lastly, they should promote the full disclosure of data, practices and achievements (Spring Associates and PRI Secretariat 2014).

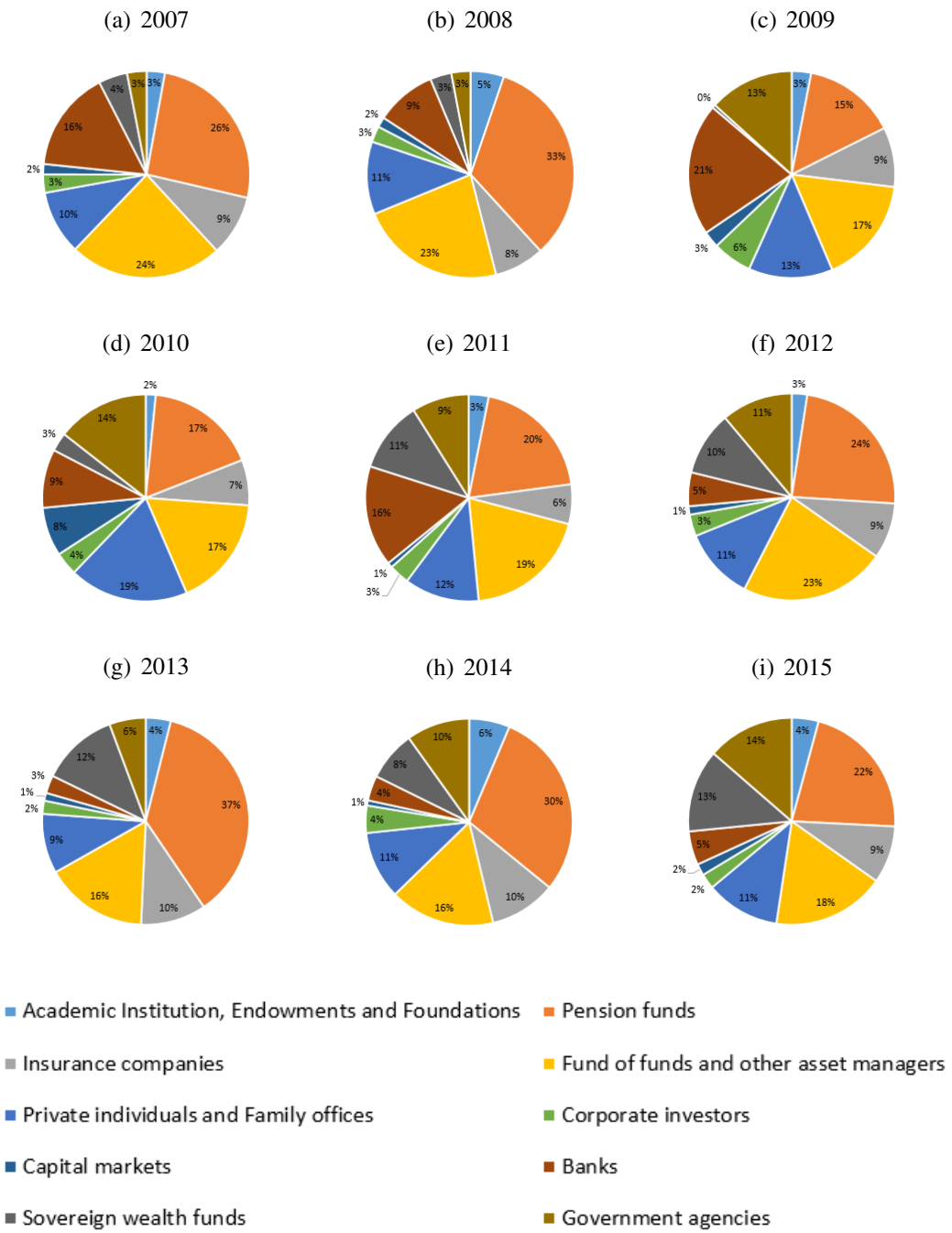

Figure 1.7: Investor Type per year based on *Invest Europe*'s data

CHAPTER 2

PRIVATE EQUITY FUNDS

2.1 ORGANIZATIONAL STRUCTURE OF A PRIVATE EQUITY FUND

2.1.1 Terms of a Private Equity fund

Private Equity funds are limited partnerships built by PE firms to raise funds from investors, the Limited Partners, to pay for investments in companies. The fund's managers, the General Partners, establish the terms of the fund in collaboration with Limited Partners: negotiations are very accurate since both groups have become aware of their risks and intend to preserve their respective stakes.

The fund terms should be set with regard to the objectives of investors. Given that their ultimate goal is making a profit from their investment, fund terms should be set aiming at aligning the GPs' and the LPs' interests, in order to increase the possibility of a higher fund's performance. In other words, measures should be taken to avoid agency problems: GPs should focus their efforts on maximizing the rate of return of the fund. A way to achieve this involves the requirement that a share of the fund capital is committed by the GPs (commonly it is about 1%-5% of the total committed capital, but the percentage can be higher), in order to strenghten their interest in the rate of return of the PE fund (Kocis, et al. 2009). Other methods involve higher fees paid to active managers, the possibility of clawing back the excess carried interest and the inclusion of removal clauses for underperforming management. If the LPs are really unsatisfied

with the GPs' activity, the ultimate safety provisions go beyond the removal of the manager up to the approval of rights on investment decisions for LPs, who can then prevent new investments chosen by the GPs. At the very end, LPs are entitled to declare the dissolution of the fund.

The process aimed at setting the fund terms should ensure that the investment activity conforms to expectations. For this reason, GPs and LPs agree on provisions clearly defining the ultimate fund's purpose, its size, risk profile, the timing of capital calls and distributions, the number of target companies and the intended geographic range. Moreover, GPs have to provide LPs with periodic reports on the fund's results in order to give the investors the possibility to monitor their investments. Also, LPs require provisions that ensure the fund's management continuity: investors should have the possibility to terminate their commitment to the fund if the management team changes. Terms can include provisions reinforcing the LPs' expected return such as the avoidance of disinvestments at less than fair value. Furthermore, LPs, at the commitment moment, agree to meet the capital calls decided by GPs. Given that they are obliged to commit the required money amount in a very limited time window (usually two weeks), they need a provision imposing a limit to the capital commitment in a certain period. Finally, terms usually include the settlement of a deadline for the entrance of late investors and finally the design of a bonus in favor of early investors, since late investors change the stake proportions of other existing partners who were the first to trust the GPs.

Lastly, given the finite life of PE funds, terms should be set with regard to the timing of GPs actions. The typical length is ten years but it is occasionally extended on a yearly basis if GPs believe the investment success depends significantly on liquidation timing. This extension has to be approved by investors. But generally, the different fund activities respect their dedicated time windows.

The Private Placement Memorandum

The terms of a PE fund should include a definition of the investment strategy that the GPs intend to follow in order to make the LPs able to judge whether their objectives can be achieved through that PE fund. The investment strategy should at least define which PE type has been chosen, the level of diversification of investment sought (both geographical and industrial) and the level of leverage employed. These factors identify

different negotiation terms, different expectations on the active management of investments and different levels of risk. On the basis of these details, the LP chooses the fund that satisfies his expectations.

When the GPs promote their fund among investors, they reach the potential LPs with documents that must specify every important factor that will characterize the management and the performance of the fund. The fundamental and most explanatory document describing the fund is called Offering Memorandum or Private Placement Memorandum (PPM). In order to give an understanding of what this kind of document includes, the main sections of the PPM of the *AMG Pantheon Private Equity Fund* (2014) are broadly described. Firstly, the PPM defines the main features of the fund (the applicable jurisdiction, the fact that it is a closed-end fund) and provides an indication of which types of investor this fund is suitable for. Then, the PPM moves forward to explain the investment strategy, defining not only which kind of companies the fund will invest in and in which geographical area it will operate, but also which kind of companies will be excluded. It also anticipates if investments in other PE fund will be allowed. Furthermore, it identifies which PE type has been chosen and describes the general approach that would be applied by GPs when investing, as well as the objectives that are supposed to be achieved through this investment process. Right after the investments selection method, the due diligence is defined. This part of the document is particularly important because it is aimed at ensuring the LPs that their interests in the fund will be respected by GPs. Then, the document moves forward to explain the risks that the LPs will face and how the GPs intend to protect the fund from these risks. For example, the investment, currency and liquidity risks are defined and the leverage and hedging limits are set. At this point, a description of the PE firm and of the GPs (together with their specific roles in the fund and their past performances) is provided. Lastly, the fund's expenses, its dividend policy, the return calculation method and the tax regulation are defined. The final provisions indicate how the fund can be dissolved and liquidated and how often the managers are obliged to report to investors the progress of the fund.

It is important to understand that these provisions in the PPM are not conclusive. In fact, even if the document is legally binding, this document aims at providing the investors with a general view of the fund's objective and strategy, in order to allow potential LPs to understand whether the fund meets their expectations. Other operational binding restrictions to the activity of GPs will be agreed with LPs.

2.1.2 Fund size

Fund size is the value of the assets of the fund plus the un-invested capital. It is important to distinguish between fund size and committed capital, which is another important feature of PE funds and is fixed by a contract signed by both parties. Committed capital is defined as the amount of money that LPs agree to invest in the fund. However, LPs do not give their entire share upfront. GPs, when they need capital to close deals, call capital from LPs to invest in the selected assets, namely, the target companies. As a consequence, the actual fund size, which depends on the value of the assets, is not a fixed figure.

Fund size is particularly important since it classifies super funds and small funds and for this reason fund terms should provide an indication of the agreed fund size. The first group is composed of very large funds ruled by meticulous investor terms: given the great number and diversity of investors, GPs elaborate a administrative structure to meet the needs of every investor type and nationality. For example, foundations, government agencies and foreign investors need special considerations with regard to tax regimes. However, the PE industry is largely represented by smaller funds, where GPs deal with fewer investors, deal with a single fund and regulatory conditions are simpler and faster to set. The lack of negotiation can benefit GPs, who have more freedom of action and do not require extremely developed organizational skills and deep knowledge of tax regulation. Anyway, when more investors commit to the fund, GPs have the chance to impress a greater number of future potential LPs with the purpose of building a larger PE fund in the future.

Moreover, PE funds usually provide first and final closes. The first close occurs when a specific threshold has been hit by the raised capital: at this point, drawdowns can start. The final close means that the committed capital reached a higher threshold, the maximum agreed in the fund terms. At this point, new investors can no longer enter the PE fund. But while the first close is necessary to start the investment phase and to ensure LPs that a certain minimum size has been achieved, the final close does not always occur. Anyway, the final close is useful to LPs concerned that the fund size might exceed the size agreed in the terms. In fact, according to Breslow and Schwartz (2009), both excessively small and excessively large funds cause several worries. Starting from excessively small funds, if the first close did not exist, a small fund size might preclude

the fund managers to apply the approved investment strategy. For example, an the size of an LBO fund is likely to be greater than a VC fund size, given that the latter is more focused on small local targets while the first invest in established companies, thus more expensive to acquire. Moreover, considering that some LP types need specific treatments, if the fund was too small the efforts put on the design of the fund terms would be fewer than what would be appropriate. Lastly, it is easier for a single big investor to exercise his power over the management team of a small fund. For this reason, some institutional investors agree to commit capital when the fund has reached a certain minimum size. On the other hand, excessively large funds present different issues: for example, the PE firm might not have a sufficient number of experts to efficiently employ the huge amount of available capital and to productively deal with the portfolio companies. A second fear is that there will be an inadequate number of investment opportunities that are actually profitable so that less attractive investments are made to deploy available money. Therefore, all these issues related to excessively large funds cause a delay in the investment process; this fact concerns LPs since, as long as the capital is not called, they have to keep their money invested in short-term securities which are very liquid and usually generate returns that are lower than those expected from the PE fund.

Concluding, there is not a single ideal size for a PE fund: depending on the pursued strategy of portfolio managers and the investors' objectives the PE firm will select the appropriate fund size, setting a size cap and a first close threshold.

2.1.3 Life of a Private Equity fund

The lifecycle of a PE fund follows the phases of the development process of target companies, from the selection of targets to the exit from the investment. But other additional stages are necessary as the PE fund also includes fundraising as one major activity. The first period precedes the so-called first closing: GPs offer to investors the participation in the fund until a specific threshold is hit by the committed capital of existing LPs. At this point GPs can potentially start investing. Right after this moment, other investors can still enter the fund within a supplementary time window that ends once the final closing has occurred or because a specific end date to the offering period has been set in the fund terms. When the first closing occurs, the investment phase begins: GPs can

start investing in those target companies that they had been monitoring and had already selected during the offering phase. The investment period, as it should be specified in the fund terms, last from three to five years. Portfolio companies are managed and developed in a subsequent phase lasting from five to seven years ending with the exits from single investments. After the realization of return, the fund's managers will distribute it to the LPs. The fund life should last between eight and ten years.

According to Talmor and Vasvari (2011), during the offering period the GPs have two main tasks: the fundraising activity and the search for investments. The GPs are in charge of promoting the fund among investors: in order to convince the potential LPs, they have to prepare the PPM and the other required documents upfront, in order to give an overview of the fund's expected performance, management and objectives. Especially for first-time fund, this activity can be both expensive and time-consuming: in fact, the GPs cannot exploit previous expertise and network, thus they often have to rely on external legal experts and agents in order to write the documents and reach a sufficient number of investors to raise the minimum capital size. For skilled and successful GPs, the desired amount of capital can be reached in a few months. The second activity of the GPs consists in selecting the potential target companies among a huge number of investment opportunities. They start the analysis of those companies that belong to the industries and geographical areas that have been specified in the PPM. However, it is also possible that the GPs firstly analyse the investable universe, then they decide the characteristics of the future portfolio companies and only later they write the PPM. Again, first-time funds will need more time to select the basket of potential investment opportunities. In general, both activities are expensive: in particular, since the fund's primary source of capital derives from investors, during the offering period the expenses of the fund are faced by the PE firm, since before the first closing, capital calls cannot occur. All the expenses that the PE firm bears during the offering period cause the return of the PE fund to be negative at the beginning of its lifecycle (Diller et al. 2009). However, the monitoring activity and the first selection of future targets during the offering period is justified by the fact that the GPs can ensure investors that committed capital will be immediately invested, therefore showing personal commitment and proactive thinking. On the other hand, this activity should not distract GPs from the other main activity, fundraising, which is necessary to actually invest in the selected companies.

Once committed capital has reached the first threshold, the investment period begins

and GPs can start investing, after calling capital from existing investors, in the targets identified in the previous phase. As mentioned earlier, the investment period can last from three to five years: the length is due to the fact that, in this period, the GPs have to decide which companies will ultimately enter the portfolio of the fund and have to negotiate the terms of the entrance. In order to make the final choice among those companies that have been already selected in the previous phase, the GPs have to apply extensive due diligence to ensure the LPs that the fund has invested in profitable investments. There are different kinds of due diligence that take into account several factors: the impact on the environment, the adequacy of the equipment, the taxation, legal and accounting records. Once, through due diligence, the target companies that will actually enter the portfolio have been identified, the managers draw down capital, the transactions occur and the fund starts to form. Then, since the GPs start to work with these companies aiming at making them improve, the fund will receive part of the proceeds generated by the operations of the companies. According to Breslow and Schwartz (2009) the GPs can reinvest these proceeds in new investment opportunities if this practice is allowed by the terms agreed with the LPs. Similarly, also disposition proceeds can be retained in order to be reinvested, always according to the fund's provisions and if the GPs do not decide to distribute the profit first. However, fund managers usually have the right to recall the distributed capital within a specific time window after the distribution. This compromise is due to the preference of LPs to have the capital immediately available in order to invest it in other investment means. When the fund is allowed to retain the proceeds, it forms reserves (usually investing in short-term and liquid securities) that can be used for buying new assets and facing operating expenses.

As Kocis et al. (2009) state, during the investment period investors can still enter the limited partnership, as long as the threshold of the final closing is not reached by committed capital. However, problems arise when investments have already been made. For this reason, late investors have a different treatment: they have to pay a share of the past capital calls to the fund, which transfers this amount to early LPs. The ratio of this behavior is that both early and late investors should participate in investments according to their weight in the fund's committed capital. Moreover, new investors also have to pay an interest to existing LPs (that the latter would have earned if they had invested the returned amount of money elsewhere).

After the end of the investment period, a new phase called holding period starts,

which lasts five or seven years. During this time window the GPs have again two main tasks: improving the portfolio companies and exiting the investments. Therefore, in this period, the GPs should not make additional investments; instead, they should focus on the development of current portfolio companies. Their purpose is to improve target companies and make them saleable at a higher value than the acquisition one, realizing the return that has been promised to LPs. GPs need to generate the highest return possible with the purpose of satisfying the investors and with the hope that the LPs will be willing to invest again in funds managed by the same GPs. Single disinvestments of targets occur during the holding period, which ends when the last target is sold and the fund is liquidated, preferably before the tenth year. The timing of the exits determines the end of the fund lifecycle: it has to be carefully decided according to the type of exit selected and the market conditions. In fact, the profit realized with the investment determines the success of the fund. For this reason, very often GPs agree with LPs to extend the fund life on a yearly basis if, after the official fund's liquidation date, there are still some targets to exit because the GPs prefer to wait for more favourable market conditions. If the GPs fail to sell these residual assets during this additional time window, these assets can be distributed "in kind" to investors, which means that securities are directly transferred to LPs instead of their value in cash.

Another activity of the holding period is the distribution of net cash inflows. As mentioned earlier, cash inflows are disposition and operational proceeds. The first group is generated by the disinvestment of target companies, which the fund sells at a higher price than the acquisition price. The second group includes all those proceeds arisen before the exit from investments, such as dividends from target companies, interests from corporate bonds, and the like. While the first type is distributed all at once at the end of the single investment, the second type is usually distributed on a periodic basis during the investment holding period. However, for PE funds like LBO and VC, the distribution of disposition proceeds will likely happen at the end of the fund's life, as these fund's types have a buy and hold strategy. Moreover, VC funds have fewer operational proceeds than LBO funds, since the ideal targets are growing companies, which traditionally reinvest their profits instead of distributing them; on the other side, LBO funds invest in mature companies, which are likely to distribute dividends before the LBO acquisition as well.

2.1.4 Commitments to the fund

Particular attention has to be given to capital commitments and capital contributions. As explained earlier talking of fund size, capital commitments are the promised contributions to the fund that the investor subscribed in exchange for a claim in the fund's expected return. Managers draw down committed capital once they require liquidity mainly for purchasing identified targets but also for administrative activities. These are the only two purposes that the capital should be called for; moreover, GPs should draw down capital only if necessary (in other words, they should not create reserves with contributed capital).

Once the capital is called by GPs, the investor is legally bound to make the capital contribution, which is proportional to the investor's commitment to the fund: in this way each LP participates in each investment for the same weight he has at the fund level. The GPs, when they notify the investors of the capital call, should also disclose the contribution deadline, the instructions for transferring the money and the purpose of the contribution (International Private Equity Valuation, 2012). PE funds generally require capital within ten days on average after the call notice. Only in the case of emergency, such as the default of a relevant investor, GPs pressure LPs to provide them with the capital in a shorter amount of time. But when there is no immediate crisis, negotiations with the defaulting investor will take longer until both parties agree on a solution. According to Braendel and Chertok (2010), the GPs have a series of remedies and penalties when a single LP fails to meet the capital call. However, they cannot apply more than one penalty to an investor, since the objective that is pursued is not the punishment of the defaulting investor but rather it is to enable the managers to accomplish the investments no matter what the economic situation of LPs is. The first penalty is the charge of an interest on late payments. The second penalty is the reduction of the stake of the investor in the fund or the reduction of future distributions to the defaulting investor. The ultimate penalty is the redemption of the investor's stake in the fund. In this case, the investor loses his voting rights and is forced to sell his positions in the fund at a considerable discount. The main remedy that GPs have for replacing the capital is, as mentioned earlier, asking the other LPs to cover the defaulted contribution. In order to avoid conflicts of interest raising because a single investor can afford the entire amount, it is preferable that every investor pays according to their weight in the fund's

committed capital.

Since entering a PE fund has as consequence some rigid obligations that must be fulfilled, LPs have to be aware of their default risk and consider whether they will be able to contribute to the needs of the fund. Given the short notice of capital calls, the capital contribution to the fund is not an easy task for every limited partner type: institutional investors commonly do not have problems, but high-net-worth individuals might face several obstacles. In fact, not only they might face issues in selling short-term securities in which the needed money has been invested waiting for the call, but also they might be physically unavailable when managers call the capital. To solve this problem, several investors ask for a longer notice or require at least the existence of a curing period before default provisions become applicable. Another method is the agreement between GPs and LPs that sets a limit to the size of capital calls, in order to make them affordable for every kind of investor. Anyway, in this case, investors are obliged to save a portion of their committed capital in an immediately available source.

Another aspect that investors have to evaluate when analyzing their default risk is if they have enough liquidity to meet the fund's obligations. Sometimes, PE firms give an indication of the size of future capital calls through the imposition of a minimum capital commitment. The presence of this minimum commitment size is particularly observed in the case of heavily subscribed funds so that smaller investors are excluded from the fund as their interest would easily be overcome by bigger investors. However, smaller investors can gain access to these PE funds through the investment in a fund of PE funds, which does not have a capital commitment threshold. Nevertheless, the PE firm has the right to waive this limitation if the fundraising activity proves to be unsuccessful. Anyway, setting a minimum subscription amount gives an indication of future capital calls. If a potential investor is aware of his difficulty in facing onerous contributions, then the decision of entering the fund has to be carefully evaluated: a minimum committed capital that is too high might indicate that the first capital call would be onerous enough to make the investor default.

A particular type of capital commitment is the recalled capital, which is regulated by the Global Investment Performance Standards (CFA Institute, 2010). The general rule is that reinvestment (or recycling) should be prohibited, unless there is a provision in the fund terms allowing for it. Therefore, if GPs and LPs agreed on the possibility of recycling the investment proceeds (both from the operations of the targets and from

an early investment exit), the managers of the fund can retain the proceeds or recall the distributed capital in order to purchase new assets. If the GPs choose to retain the capital, then they form reserves that must be used for the acquisition of new investments or for administrative expenses. If the GPs choose to first distribute the capital, they can recall it within a short period of time. Thus, the fund can draw down a total amount that is greater than the committed capital, increasing the fund size. As a consequence, the investor's position in the fund is greater than what has been established at the beginning of the fund life. Moreover, the investors are obliged to keep their distributed capital invested in liquid low-yielding securities for facing potential capital recalls, while they might prefer to invest the distributed capital in other investment opportunities. This is the reason why the distributed capital can be recalled within a small time window. On the other side, especially in case of early disinvestment from portfolio companies, reinvestment can be a sign of commitment, because it means that GPs actively manage the fund aiming at increasing the potential return.

2.1.5 Fees and Compensation of General Partners

General Partners are responsible for the Private Equity fund and for money of the Limited Partners for a period of time that commonly lasts ten years on average. LPs have limited liability and cannot participate in the management of the fund, which they invested their capital in, while GPs are liable for the debt and the obligations of the fund and are actively in charge of the daily activities. This difference between responsibilities is the reason for the fee structure in favor of managers, which is supposed to motivate GPs and align their interests with those of investors. The two main types of fees are management fees and the carried interest. Basically, PE funds set management fees to a level between 1%-3% of committed capital and carried interest to a level of 20% of profits (Phalippou, 2010). Nevertheless, the actual structure is more complex since it is open to modification achieved through the negotiation between GPs and LPs.

Management Fees Management fees are percentages of the capital committed in a fund. Fees include all the operating expenses associated with investment activities and with the partnership management. Fee payments occur quarterly, semi-annually or annually according to the terms of the fund. According to agreed terms once again, the

percentage can either be fixed or can change throughout the fund life. Also, the way it is calculated can be modified: if it is allowed, usually, at the beginning, the percentage is higher and is referred to committed capital but later it can decrease and might be referred to the fund size. Even if the fund size is expected to grow along with the increased value of portfolio companies, the fee is instead expected to decrease because GPs exploit economies of scale in the fund management process and because the activities of the fund diminish when GPs start to exit investments. As mentioned above, management fees cover all the operating expenses of the fund. In order to estimate their amount, managers have to plan a budget projection and get it approved by investors who then agree to pay these budget-based management fees.

Moreover, there might be differences between fees paid by investors in the same fund: an investor, which owns a larger share of the fund, is likely to have more favorable fees by virtue of his larger negotiation power. A second difference is due to the variety of efforts required by each fund type: commonly, an LBO fund sets a lower fee than a VC fund since the latter requires more operating activities, given that the expected transformation of the target company is more significant. (Baker et al. 2015)

The mechanism that makes the fee basis change is very important because of the rationale behind it. Fees can be calculated on committed capital, on the summation of uncalled capital and cost of existing investments, on the net invested capital (cost of existing investments) or on the Net Asset Value (NAV). The first two fee basis are common during the first five years of the fund, then the fund moves on to the second couple of fee basis. This switch happens because, after the end of the investment period, all committed capital should be invested and the fund expenses should be associated with the actual activities of the fund and with its actual capital composition. For example, when managers sell a target company, they interrupt their monitoring activity and their company-improving efforts: therefore, it is fair for an investor to no longer pay for the senseless management of a liquidated investment. On the other hand, at the beginning of the fund life, managers cannot use net invested capital as fee basis because the existing investment might not be sufficiently large to provide the necessary capital to run the fund (Robinson and Sensoy, 2013).

Carried Interest Carried Interest is the profit share of managers. Therefore, it is calculated as a percentage of the performance of the fund. It is a measure intended to

solve agency problems between GPs and LPs since the right level of carried interest can motivate the GPs to increase their efforts in the fund in order to drive the fund to success, as this becomes an interest of both parties. As mentioned above, the most common level of carried interest is 20% of profits. However, each year, carried interest is payable to GPs only if investors are expected to receive more than a threshold called preferred return, which is generally set at 8%. In other words, if one year the rate of return of a fund is less than 8%, then GPs are not paid but all of that year's profit is distributed to investors. The calculation of the carried interest is complex since the method, the size, and the timing have to be negotiated between GPs and LPs. These negotiations are connected with those involving profit management: how profit is calculated and how it is split between investors. For example, organizational expenses arisen during the formation of the fund represent an obstacle to the identification of the profit. The participants to the fund have to decide whether these expenses are part of the committed capital or if LPs pay for them separately. A similar situation occurs with management fees. Then, the partners have to decide if the profits are calculated as a percentage of the return that accrues to the invested capital net of fees or to the total contributed capital, which includes all of these expenses.

Another issue involves the method for distributing profits. The first method suggests that profits are distributed once a single investment is exited. The second method prefers to set a time window and to aggregate all the profits from single disinvestments during this period in order to distribute the return at the end. Another version of this method set the carried interest payment at the end of the entire fund life and as a percentage of the total IRR. However, the first solution is more favorable for GPs since, in the case of negative results, they do not bear the loss but they still share the return with the LPs when a profit occurs. Therefore, they have an incentive to increase the risk of the portfolio. The second solution, through the aggregation of all the results of the period, mitigates the tendency of GPs to add more risk at the expense of LPs because losses and profits are summed up and, consequently, the total carried interest decreases. (Baker et al. 2015)

2.2 PORTFOLIO MANAGEMENT

General Partners have to build and manage the portfolio with regard to the interest that Limited Partners have in the fund. Therefore, since there is not a secondary market where to sell an underperforming investment, the first selection of target companies is really important for the success of the fund. The decisions on the choice of portfolio companies play a crucial role because they are part of a long term strategy that cannot easily change: the selected investments have to be retained until the fund expiration and the task of GPs involves the active management of these companies.

The traditional approach to the formation of a portfolio follows the Modern Portfolio Theory proposed by Markowitz in 1952, which delivers a mean-variance efficient frontier using the concepts of return, risk, and correlation calculated from periodical asset prices. The problem of Private Equity is that it does not fit the required risk-return framework since an efficient secondary market for PE investments does not exist and, consequently, daily observable prices are unavailable. The only information on the fund value is given by cash flows and those performance measures that can be obtained from them. Additionally, PE targets generally are firms not listed on a public market and consequently the list of daily prices is missing and portfolio choices have to be based on other criteria. Another issue regards diversification in PE portfolios: managers have to find a compromise between diversification, which decreases the firm-specific risk, and specialization, which is necessary for the active management of target companies.

Investing in PE cannot be easily compared to a science because there is not an exact investment theory to follow (Baker et al. 2015). Thus, managers apply other rules to recognize good investments and have to rely on their expertise. General Partners have a list of criteria useful to choose from a basket of potential target companies. The first selection of good deals will help the managers to deliver the promised return to investors. Ideal targets are run by motivated and competent managers, willing to collaborate with GPs and able to efficiently operate in the firm; otherwise, they are likely to be substituted by new managers hired by the PE firm. Depending on the chosen PE type, in particular LBO funds, the firm should be able to bear new leverage on its balance sheet. In order to be a target, a firm should have assets to use as collaterals and must not already have significant or inefficiently structured debt. Moreover, the firm should present high and stable potential cash flows to afford to pay interest and principal. Cash flows increase

if the firm has low capital expenditures and if it owns assets that can be sold if they are not necessary to the cash flow production process. In the case of a VC fund, the firm has to present growth opportunities and usually operates in specific markets that showed great and fast development in the last decades, such as the technology sector. Since a VC fund does not use to leverage its investments, targets do not need to have tangible assets on their balance sheet. For both fund types, the target must have quality tangible and intangible assets that retain some unrealized potential for several reasons: among others, because the management has been inefficient (LBO) or because of the early development stage (VC).

Depending on the chosen fund type, the GPs decide a portfolio strategy to apply to the fund, choosing the right compromise between specialization and diversification across countries, industries, and financing stages. LPs have to be aware of the investment strategy that GPs intend to apply: usually in fund terms there are provisions limiting the maximum amount investible in single industries, countries and financing stages in order to protect the expectations of LPs. The purpose of GPs in selecting the portfolio is to benefit both from specialized knowledge and from unsystematic risk reduction.

The level and kind of diversification depend on several factors such as the characteristics of the fund, the expectations on the future economic development of a certain region or of a specific industry, the number of GPs and their specialization field, the PE industry condition, and finally the general economic conditions. The PE firm and its managers have to set up an investment and management strategy that maximizes their expected utility taking into consideration the previous factors, the target companies' characteristics, and the risk-return framework they are pursuing. Investors interested in the PE industry choose the PE fund that most suits their preferences.

Several researchers addressed their efforts to give a guidance to GPs and LPs in assessing whether PE funds can benefit from diversification or not. Diversification is one of the main features of the Modern Portfolio Theory since it can erase the firm-specific risk from the portfolio. However, as mentioned earlier, Private Equity does not properly fit the Modern Portfolio Theory framework. A huge problem is that several assumptions of the theory are violated: in particular, there are information asymmetries between the actors of a PE transaction, assigning to the GPs a superior knowledge of the deals (Amit et al. 1998). This particular violation explains the results found by Norton and Tenebaum (1993): their main result is that VC funds that are focused on seed and

early stage financing are less diversified than VC funds that are focused on later stage financing. Therefore, the funds that are supposed to be riskier are actually less diversified. Specialization seems to be a strategy to lower the risk of a PE portfolio. Consequently, the four authors argue that GPs who are specialized in a particular industry or local area can make better investment decisions than diversified GPs due to their specific knowledge of the considered business that allows the GPs to recognize successful investments and to provide additional value through an active engagement in the target company. This superior selecting and managing process based on specialization should increase the expected rate of return of the PE portfolio.

This hypothesis on specialization found evidence in an analysis proposed by Ljungqvist and Richardson (2003) who regressed the PE fund returns on four measures of portfolio diversification: the number of assets in a fund, the fraction of companies of the dominant industry, the fraction of invested capital in the dominant industry and lastly a measure of concentration, the Herfindahl-Hirschman Index. They found that when the number of portfolio companies increases, the rate of return of the fund decreases while when the concentration of the fund increases also its rate of return increases. But the regression coefficients are not statistically significant and a fault in the dataset has to be taken into account since the authors did not have all the required information on all the target companies.

On the contrary, other authors found opposite results stating that diversification has a positive impact on PE rates of return. Lossen (2006) distringuished three levels of diversification: naïve, dynamic and systematic. The naïve diversification across the number of assets in the portfolio, the dynamic diversification across time while the systematic diversification is achieved investing across industries, financing stages and countries. He conducted an analysis aimed to discover whether diversification has a negative impact on a PE fund's rate of return. His study conducted on a sample of 100 European PE funds found that the impact of the diversification achieved incrementing the number of portfolio companies is significant and positive. A similar result is obtained for diversification across industries. These two results are in contrast with the thesis that specialization should increase the PE fund return. However, on the contrary, he discovered that diversification across financing stages declines the portfolio return and the analysis on diversification across time and across countries deliver insignificant coefficients that change sign if the model specification varies.

Another later study from Humphery-Jenner (2013) based on a sample of 1505 U.S. PE funds documented a positive impact of diversification on rates of return. In particular, he found that joint diversification across industries and countries increases the IRR, but excess joint diversification decreases the IRR. Additionally, he found that naïve diversification increases the IRR. He investigated the reasons for this positive relationship, if it is simply due to idiosyncratic risk reduction or to additional benefits such as an improved knowledge shared among the fund's target companies and GPs. This knowledge transfer between companies belonging to different industries as well, might foster innovation and growth. Furthermore, GPs investing outside of their specialization field might be more selective and consequently, they might build a superior fund. Additionally, a fund can benefit from diversification because agency costs are avoided. GPs tend to diversify in order to reduce the overall fund risk exposure implying an increment in the IRR that is likely to be higher than the preferred return: this fact means that carried interest is payable to GPs.

However, diversification has a price, which is represented by the fund's set up and managerial costs. The fund costs are proportional to the fund size, taking into consideration the amount of negotiation needed for building a bigger partnership and a portfolio including several companies that might operate in different industries and countries requiring knowledge, skills, and capital not only for negotiating the acquisition but for managing the portfolio companies as well. Schmidt (2004) found that when the number of portfolio companies increases the fund will face only the non-diversifiable risk which he called "PE market" risk. Diversifiable risk reduces by 80% in a portfolio of 15 investments, by 90% in a portfolio of 50 investments while almost complete diversifiable risk reduction is achieved by a portfolio of 200 companies. Therefore, the marginal risk reduction decreases as new investments enter the fund. As a consequence, managers have to exploit diversification as long as its marginal benefits exceed its marginal costs (Statman 1987).

CHAPTER 3

THE PERFORMANCE OF PRIVATE EQUITY

3.1 THE PERFORMANCE AND THE RISK OF A PRIVATE EQUITY FUND

Private Equity funds are organized as limited partnerships where investors do not have active managerial duties. Investors commit their capital, GPs use it to fund the purchase of target companies which are then efficiently improved by GPs until they are sold once their value has increased. Only at this point, investors receive their capital back as well as additional profits generated by the investment sale. Thus, LPs give custody of their capital to GPs and they have to rely on the skills of the managers of the fund to earn a rate of return that is worth the long investment horizon, which can last ten years. These premises do not motivate the interest of LPs in the PE industry. If investors are willing to invest in such an asset class, it means that its past performance has been at least higher than the performances of public markets. Several authors tried to discover which factors drive the performance of PE funds, in order to give investors some criteria to jugde whether the considered fund is likely to earn a high rate of return.

Authors usually agree on highly volatile and right skewed returns and on a concave downward relationship between the fund size and the fund return. Also, they agree that the availability of investment opportunities has a positive impact on the PE fund return and they agree on the cyclicality of the PE industry and on the superior performance of

US funds over EU funds. However, different studies on the PE performance over public markets delivered mixed results and a similar situation occurs with the comparison between LBO and VC funds. Also, authors disagree on the positive relationship between the fund's systematic and/or unsystematic risk and the rate of return of the fund itself. Unfortunately, these issues might be explained by the use of different data sets, by the lack of time series and generally by the low transparency of the PE industry.

3.1.1 Private Equity performance patterns

The past literature reported different points of view about the patterns of Private Equity funds. With regard to external investors, their incomplete knowledge of the industry and their weak control over the way their money is used by managers stimulated several researchers to put their efforts to assess whether the PE asset class outperforms the public stock markets and which kind of PE is the most successful.

In chronological order, Ljungqvist and Richardson (2003) delivered an important study on this subject. Pointing out the illiquidity of the asset class caused by the lack of a secondary market, the long investment horizon, and the impossibility for LPs to influence the fund management, the two authors expected a return which should be higher than the return gained by public markets. Firstly, they empirically found that the IRR of a PE fund is greater the faster the investments are done and the later they are liquidated. Therefore, the authors focused on the cash outflows and inflows patterns throughout the fund life, identifying two factors that influence the investment process: competition between PE firms chasing the same deals, which slows the investing activity, and availability of investment opportunities, which on the contrary fosters the investing activity. Additionally, the reasearch reported that on average the IRR of a PE fund does not turn positive until the eighth year, after a significant amount of disinvestments has occurred. Nevertheless, only the final IRR is the real informative performance measure. They documented an average net-of-fees IRR of 19.81% for funds raised between 1981 and 1993, resulting in an excess return of about 6% over the S&P 500, which is consistent with the hypothesis of the additional premium due to the asset illiquidity.

Two years later, Kaplan and Schoar (2005), following their own previous study of 2003, focused on the comparison of PE with the performance of the S&P 500 and on the comparison of performances of VC and LBO funds. The sample they used comes

from Venture Economics and covers the years from 1980 to 2001. Their analysis has been implemented using equal-weighted and size-weighted avergages of fund performance measures such as IRR and PME (Public Market Equivalent)[1], where the size is the fund's committed capital. The equally-weighted average of IRR delivered slightly higher returns for LBO with respect to VC funds, but, contrary to the authors' expectations, both of the fund types underperformed the S&P 500 with an average PME smaller than one (0.96 for VC and 0.97 for LBO funds). On the other hand, considering size-weighted averages, even if the comparison between IRRs of VC and LBO funds delivered the same output, PME provided a different result for the comparison with the public index: VC funds outperformed the public market with an average PME of 1.21 while LBO funds underperformed the public market with an average PME of 0.93. The researchers explained this change with the analysis of VC and LBO funds characteristics across time. VC funds in the 1990s were bigger and more successful than those of the 1980s, while the opposite happened with LBO funds: the bigger funds were also less successful. Therefore, bigger VC funds with exceptional return cause the weighted average PME to be higher than 1. However, the authors suggested that PE performance gross of fees, which is not part of their study, might outperform the S&P 500 for both PE types.

Kaserer and Diller (2004) found that the PE industry underperformed the public stock market as well. Their data originates from TVE and icludes information on 190 European PE funds. They computed an average IRR net of fees and of carried interest for LBO funds of 13.4% and of 12% for VC funds. However, they calculated the PME using the MSCI Europe index net of costs and found PME values that are smaller than one, suggesting an industry underperformance with respect to public markets. Also, they found a positive relationship between the committed capital in the vintage year of a fund and its future IRR.

3.1.2 Issues on assessing Private Equity performance and risk

Several researchers tried to find an explanation to the diversity of conclusions of the performance assessments implemented by numerous authors. A possible reason con-

[1]PME will be explained later in this chapter; it is a measure built to compare the performance of a PE fund and of a public index based on the PE fund cash flows. A PME greater than 1 indicates overperformance of the fund, a PME smaller than 1 indicates underperformance of the fund.

siders the quality of available data used in literature. The main problem of PE is the nontransparency: no public datas about PE funds are available. Researchers, analysts and investors buys information from private data providers but even these private companies do not cover the entire PE industry. Therefore, according to the data provider the researcher addresses to, the resulting study will be affected by the given sample of funds. For example, Ljungqvist and Richardson (2003), aware that the result stating the overperformance of PE against the public market was in contrast with other authors' papers, evaluated whether their available sample was representative of the PE industry. They recognized that their sample was less heterogeneous than those used in previous studies, but that sample also provided the information on cash flows timing, which is fundamental for the IRR calculation, while previous literature has been able to compute only absolute return measures.

External investors usually incur mispricing. Investors do not have the expertise of GPs and thus it is possible that they misvalued the asset because of their lack of knowledge or insufficient skills. They might be influenced by the poor data disclosure, since only successful investments such as Microsoft Corporation, Google and Instagram are publicly celebrated. PE returns are in fact highly skewed and only a few good examples reach the news. Moreover, the industry is relatively new and there is scarce information about its past performance. Finally, performances in GPs reports are sometimes gross of fees, in order to make the higher value attract capital. GPs, during the fundraising period, offer a certain expected multiple but they do not give information on how long the investor has to wait for obtain such a multiple. Inexperienced investors can then overvalue the asset. For example, Gottschalg, Phalippou and Zollo (2004) analysed the net-of-fees performances of 500 PE funds covering the years from 1980 to 1995. The result they got was that the PE industry underperformed the public stock markets. The main questions they try to address is why investors still decide to invest in PE and why PE earns low returns if it is an illiquid asset that does not even have hedging properties since, as their research points out, it is pro-cyclical. They proposed mispricing as an explanation to LPs' overestimation of the PE industry: in particular, investors might have underestimated the impact of fees and carried interest or they might have been confused by the low transparency of PE funds or by reports prepared by GPs. For example, GPs can wait until the fund expiration to sell a poor investment, therefore masking the overall bad performance of the fund.

However, the authors argued that the PE underperformance is not just a matter of mispricing the asset; instead, they proposed four explanations. First of all, they believe LPs participate in early funds in order to establish a relationship with GPs and be part of future more profitable funds. Secondly, the public stock markets appeared to be overvalued from the late 1980s to the 1990s. Consequently, PE might not have underperformed the public market. Moreover, the choice of the market index to compare to the PE fund has to be carefully considered: a European fund should be compared with a European stock index. Thirdly, investors are usually large, diversified and preferably institutional. The liquidity premium should be high, but PE funds can afford to keep it low since the share of capital LPs have invested in the PE industry is small. Lastly, LPs might have other objectives than maximizing investment return. In fact, investors can profit from PE through commercial deals established between them and GPs or target companies. Furthermore, with regard to geographically focused funds, investors might benefit from the improvement of the local region.

3.1.3 The drivers of the performance of Private Equity funds

Given an average performance that is not as rewarding as expected, LPs need to select the best funds to invest their capital. Researchers then tried to identify which factors drive the PE fund to success.

Kaplan and Schoar (2003) focused on the expansion and contraction periods of the PE industry. The explanation they provided for this cyclicality involves the attractiveness that PE has in a certain period. When several new funds run by inexperienced GPs enter the PE market after an expansion period for PE, their poor performance causes the overall industry performance to diminish, even though the success of established funds is less affected. Performance persistence of the same GPs across time is a consequence of this industry diluition: new fund managers cannot compete with skilled and renowned GPs who are then able to easily outperform the diluted PE industry. Additionally, they analyzed the likelihood of building new partnerhips. They found that in periods of superior performance of PE, new funds are likely to be started but it is unlikely that there will be follow-on funds later on. A consistent reason can be a poor performance of the first-time funds, which fail to compete with established and high sequence number funds.

Two years later, Kaplan and Schoar produced a second important paper on PE performance. This time they focused more on the drivers of PE return, in particular, they gave an insight on how the fund size, past performance, the public market during investment phase and the fund sequence number have an impact on the fund performance. The regression delivered positive significant coefficient for all variables. In particular, they highlighted that the bigger the fund the more likely it will outperform the S&P 500, but that the fund size marginal impact decreases as the size increases. Their most important contribution is about persistence: they found a strong evidence of the persistence in the returns of PE funds managed by the same GP in sequence. Therefore, GPs whose funds outperform the PE market once are likely to replicate the outperformance in the next fund. This is due to the learning process of GPs and to the possibility that the same asset might be present in two consecutive funds.

Lastly, Gottschalg, Phalippou and Zollo (2004) focused on the relationship with economic conditions and tried to determine the impact of business cycles and of stock markets on PE performance. The PE returns increase if investments are liquidated when stock markets have high valuation levels. In fact, GPs will choose IPO as the ideal exit strategy in that specific moment. Also, credit supply and GDP are two factors positively influencing the PE investments, namely creating a favorable environment for capital availability and providing a larger basket of investment opportunities where to select from. Thus, the PE performance seems to be pro-cyclical: in particular, they pointed out that PE suffers from high losses during a general recessions but is not as sensitive in growth periods. In addition, they investigated the common factors of underperforming funds and the result was that failed funds were mainly European, small and run by inexperienced managers (first-time funds).

Kaplan and Strömberg (2009) addressed again the ciclicality of the PE industry. At the beginning of their paper, the authors argued about the value creation process implemented in PE funds, which is composed of three main interventions: financial, governance and operational engineering. But at the same time, they investigated the reasons for such a low return for investors when empirical evidence about value creation at the target company level should imply the opposite. First, they suggested that the target company acquisition price might not be the lowest possible as wished: targets can be acquired in competitive auctions or from existing shareholders who require a higher price to sell their stake in the company. Secondly, they took into consideration

management fees that are likely to be underestimated by LPs when they decide to invest in the PE fund.

In another section of the paper, the authors investigated which factors drive the PE performance and in particular if the PE industry has boom and bust cycles. According to the hypothesis that credit markets affect PE transactions, they empirically found that PE funds take advantage of the arbitrage opportunity given by a relatively low cost of debt and a relatively high cost of equity. Another explanation for this positive relationship with credit supply has been proposed by Axelson, Stromberg, and Weisbach (2009): since leverage can solve agency problems because lenders are likely to fund only profitable investment, there will be an expansion of PE industry when credit markets are experimenting a favorable period, followed by a contraction when credit markets turn unfavorable. In fact, the PE industry expands because particularly favorable credit markets provide capital to less profitable investments as well. At the fund liquidation moment, the poor performance of PE will cause the future industry contraction, which is worsened by lenders who will undertake deals more carefully. Then they investigated how past performance affects future capital commitments: the regression delivered a positive relationship between lagged return and committed capital and this behavior confirms the industry ciclicality. In fact, high rates of return attract more investors and managers to enter the industry, therefore more capital is committed to PE as an effect of successful past performance; then, inexperienced industry actors pick unprofitable investment causing a contraction phase. Consequently, a greater amount of committed capital is connected to current lower rates of return and to past higher rates of return.

3.1.4 The risk exposure of Private Equity fund

As explained earlier, it is impossible to estimate a risk-return profile for a PE fund that is acceptable for the Markowitz portfolio model. Nevertheless, it is important for investors to have a guide in the risk assessment process of a PE fund.

First of all, it is important to understand which is the risk exposure of a Private Equity fund. The appropriate measure is the interest an investor has in the fund itself, in other words what the investor has committed to the fund: the share in the fund's Net Asset Value and the undrawn commitments. There are three main types of risk the fund

is exposed to, namely the funding, liquidity and capital risk.

- Funding risk: LPs are exposed to funding risk because they have to meet capital calls over the life of a fund. The timing of these calls is unpredictable and investors have just a short notice period to meet their commitments. Investors failing to meet their obligations, since commitments are contractually binding, lose a substantial portion of their share.

- Liquidity risk: it is not impossible for LPs to sell their stakes in the partnership, but it is very difficult because of the substantial lack of an efficient secondary market for PE. The existing market is small and highly inefficient since investors have to accept to sell their stakes at a discounted price rather than the reported NAV.

- Capital risk: this is the risk of losing the capital. Investors can lose capital because of lack of liquidity and because at investment liquidation there is a risk of not recovering all the invested capital. The latter is a long-term risk affected by a number of factors:

  - Manager quality: as stressed so far, the ability of GPs in selecting, managing and selling target companies is of primary importance for the success of the PE fund. Throughout the investment process the duty of GPs is to create value that will justify an exiting price that is higher than the acquisition price.
  - Equity market: PE funds are exposed to Equity markets since at the liquidation of the fund exits from investments depend on the equity valuations: high valuation levels make it easier for GPs to exit from investments at high prices. The timing of disinvestments has to be chosen according to the conditions of the market within the fund life.
  - Interest rates and refinancing terms: LBO funds in particular are highly leveraged and GPs are likely to renegotiate the liabilities maturing before the liquidation of investments. Changes in interest rates influence the future distributions of fund's proceeds to LPs and the value creation process.
  - Foreign exchange: PE funds are exposed to currency risk when managers diversify across countries or when foreign LPs invest in a fund. Currency

risk may cause unpredictable profits and losses because of the mismatch between the target company's or the investor's currencies and the fund's functional currency.

3.1.5 The relationship between risk and performance of Private Equity funds

Also, it is important to investigate what are the impacts of systematic and unsystematic risk on the performance of a PE fund. Since GPs find a compromise between diversification and specialization, firm-specific risk is not completely erased by the PE fund. Moreover, systematic risk has to be estimated since PE performance is pro-cyclical.

A first research, conducted by Ljungqvist and Richardson in 2003, found that systematic risk does not entirely explain the variation of returns of PE funds. They performed an estimation of the systematic risk faced by each PE fund. They identified the portfolio companies and then they assigned to each one the relative β of the industry they belong to, using the several industry βs estimates identified by Fama and French (1997) for the same time window. Finally, they computed the weighted average β of the fund. They identified an average PE industry β of 1.08 for LBO and 1.12 for VC. Consequently, the PE industry seems to be slightly riskier than public markets. Nevertheless, taking into consideration the fact that especially VC targets are young firms with high technical and market risk that still needs assessment, they aknowledged that the computed βs are likely to underestimate the real βs. In addition, they searched for a relationship between IRRs and the portfolio βs in order to point out how much of the variation of the return of the funds is explained by systematic risk. Unfortunately, they found an overall correlation of 0.01. This result suggests that variation in the portfolio systematic risk does not explain the PE fund return variation.

A second important study is about the effect of idiosyncratic risk and has been conducted by Jones and Rhodes-Kropf in 2003. The main focus of their paper was the price of diversifiable risk of VC funds. As they suggested, idiosyncratic risk arises from the principal-agent problem: when a principal hires an agent, he or she commits to a deal stating that the agent is compensated with a percentage of an observable output (in this case, the return of the fund). The agent (GP) invest a significant amount of time and efforts in the selection and management process of investments and sometimes, as re-

quired by fund terms, managers are required to invest their own capital in the fund to show they believe in the success of the investments. Therefore, GPs are entitled to a portion of the fund's returns. Anyway, as part of the incentives wanted by LPs to solve agency problems, the compensation of managers is calculated as a percentage of the fund returns, namely, the carried interest. Idiosyncratic risk arises exactly because of the GP's stake in the fund. A single manager can actively follow a limited number of portfolio companies operating in specific industries. Thus, even if managers try to diversify as much as possible, taking into consideration the limits of their skills, the diversifiable risk does not disappear because of correlated idiosyncratic risk components due to the existing level of specialization, conflicting with diversification. Consequently, the fund and in particular the undiversified managers suffer from significant idiosyncratic risk exposure. Since external investors are usually well-diversified across different asset classes and markets outside the PE fund, only GPs require a premium for the diversifiable risk they are forced to bear by the limited partnership contract design.

In addition, the authors developed a model to estimate the idiosyncratic risk and formulated four theorems on the risk profile of PE funds. Fistly, VC returns which are gross of fees have positive αs, while net-of-fees returns have zero αs on average. This is consistent with the fact that diversified investors require compensation for the systematic risk, therefore there should not be excess return when considering net-of-fees profits, which are what institutional investors receive. Excess returns are paid to managers as compensation of the non-diversifiable risk they bear, consequently, gross-of-fees returns display a positive α. Secondly, GPs do not invest in positive NPV projects whose total risk is large and LPs do not invest if the expected fund alpha, estimated before the fund formation, is not large enough even if positive. Thirdly, gross-of-fees returns are positively correlated to the fund actual idiosyncratic risk. This means that funds with greater unsystematic risk earn a higher alpha, while funds with lower unsystematic risk earn a smaller alpha. Lastly, *actual* net-of-fees VC returns are positively correlated with related idiosyncratic risk, even if on average the α for well-diversified investors is zero: if a GP select an investment opportunity with higher than average risk, the GP ask the entrepreneur of the target a higher return to compensate the GP for the risk undertaken. But since the fund terms fix the compensation percentage of GPs on the basis of the *expected* risk of the portfolio, LPs benefit from the higher risk level because they receive a fraction of the *realized* α.

3.2 Measures of the performance of Private Equity funds

The return on a Private Equity fund is not typically measureable as other traditional asset classes. The reason is to be found in the absence of an efficient secondary market and, as a consequence, in the lack of a stream of fair prices of the asset. Instead, investors have to carefully consider the characteristics of the PE asset class, such as illiquidity and uncertainty of cash flows, when they measure the PE fund performance and when they benchmark their PE holdings against other investment opportunities. There are several metrics that give the investors an understanding of their PE investment that is as broad and complete as possible. Limited Partners have to take into consideration each one of these metrics.

The most important performance measures are the Internal Rate of Return (IRR) and the Public Market Equivalent (PME). Then Multiples are another popular method of analysis. The GIPS standards[2] requires the calculation of the IRR rather than the TWRR (Time Weighted Rate of Return) for PE funds since securities are less marketable and cash flows are less frequent. For the same reason, the GIPS requires a fair valuation of the PE fund at least once a year and recommend a quaterly valuation practice. Moreover, the GIPS requires the use of daily cash flows for periods after the 1 January 2011. Daily cash flows mean that cash flows should be dated on the exact date the capital call or the profit distribution to investors occurred.

3.2.1 Internal Rate of Return (IRR)

The IRR is a metric built to measure returns on an investment and to compare the returns of different investment opportunities. It is calculated as the discount rate that makes the Net Present Value (NPV) of the cash flows of a project or of an investment equal to zero. In the PE case, IRR takes into account drawdowns, distributions to investors and the residual value of the fund. The IRR allows the comparison between different funds characterized by irregular cash flows in terms of size and timing. It permits to rank the funds, showing which fund provides the best rate of return taking into consideration

[2]The Global Investment Performance Standard (GIPS) are a list of rules aimed to give a clear picture of the performance of the fund.

the time value of money as well, which is important given that initially the LPs suffer from cash outflows that will be followed by cash inflows only at the end of the fund life. Moreover, the IRR is straightforward to calculate and to interpret.

Anyway, it is important to make a distinction when the IRR is gross or net of fees and carried interest. LPs prefer to evaluate their investments on the net IRR because it is a more transparent measure of their actual return. Moreover, the GIPS recommends the use of net IRR.

A limitation of the IRR is the assumption of reinvestment of cash flows at a rate of return equal to the IRR itself. In addition, it does not consider the scale of the projects, therefore it can be difficult to evaluate two funds of different sizes without the help of the NPV. In fact, it can happen that, comparing two projects, the smaller one has a higher IRR but a smaller NPV. IRR is highly influenced by the timing of cash flows, which can be planned by GPs in order to improve the performance measure. For example, early and significant wins are used to boost the IRR, while underperforming investments are sold at the end of the fund life to have a minor impact on the IRR. The GPs should be rewarded or penalized for the result of the timing of drawdowns and distributions of proceeds that they decided. A major pitfall of the IRR is the impossibility to calculate this metric when the cash flows follow an unconventional stream, involving inflows and outflows over the course of the fund life, which is typical in PE funds where there commonly are deferred rounds of investment. In this case, there are multiple IRRs that can set the NPV equal to zero, in particular, the maximum number of IRRs is equal to the number of times cash flows change the sign.

3.2.2 Multiples of Invested Capital

Other metrics giving indications on the return of a PE investment are Multiples of the invested money[3]. They give an insight on the cash that investors receive and they represent an alternative to IRR due to the expression of returns as a proportion rather than an absolute figure. Still, they do not consider the scale of the returns on the investments nor they account for the sizes of the funds so that mutually exclusive projects are not comparable. A combination of IRR and Multiples offers a clear picture of the performance of the PE fund since mutiples clearly describe the return from different points of view.

[3]Formulas from Kocis et al. 2009

Multiples are obtained from the ratio of the value of returns to the invested amount of capital as a general rule, then single multiples differ in terms of what they include. A limitation of multiples is that they ignore the time value of money: in particular, they do not provide an indication of the time needed to realize the multiple, or the time needed to the fund to break-even. The timing of cash flow is a factor of the attractiveness of PE fund: the earlier returns are realized the more satisfied the investors are.

Distributions to Paid-in Capital (DPI)

$$PI = \sum \text{Drawdowns} \tag{3.1}$$

$$DPI = \frac{\sum \text{Distributions}}{PI} \tag{3.2}$$

Given that the Paid in Capital (PI) is the summation of the drawdowns, the DPI, also called *Realization Multiple*, is calculated as the ratio of the money distributed to investors to the paid in capital. This metric can be calculated as soon as the first distribution occurs. The break-even point of the fund is represented by the moment that DPI is equal to one since the money returned to LPs reaches the capital they have given to the fund. Therefore, any DPI greater than one is a positive sign for the investors of the fund.

Total Value to Paid-in Capital (TVPI)

$$TVPI = \frac{\sum \text{Distribution} + \text{Residual Value}}{PI} \tag{3.3}$$

TVPI, also called *Investment Multiple*, differs from the DPI because of the consideration of the residual value of the fund, which is the market value of the remaining equity that LPs have in the fund. Therefore it divides the summation of the cumulative distributions and of the residual value by the paid in capital. It represents what the LPs would receive if the distributions already occurred were accrued by the proceeds generated by the liquidation of the unrealized investments at the current market valuation. It surely provides a better understanding of the performance of the fund but it suffers from the uncertain valuation of unrealized assets remaining in the portfolio of the fund. As the fund approach to liquidation and investments are realised, the impact of the resid-

ual value reduces. The different interpretation of the two metrics defines the DPI as the actual multiple of invested capital returned to LPs while TVPI considers potential returns represented by the fair value of the remaining portfolio companies. Therefore, TVPI is a figure which is more significant at the beginning of the fund life since it also reflects the achievements of GPs with regard to target companies. Towards the end of the fund, when almost all the investments have been sold, LPs rely on the DPI as the preferred measure of performance. Both multiples are easily interpretable since any number above one indicates that the fund has returned to investors more than the initial investment.

Residual Value to Paid-in Capital (RVPI)

$$RVPI = \frac{\text{Residual Value}}{PI} \tag{3.4}$$

This multiple is calculated through the division of the residual value for the paid-in capital. The combination of the RVPI and of the TVPI indicates what portion of the return of the fund is still unrealized and therefore depends on the market valuation at the current time.

Paid-in to Committed Capital (PIC)

$$PIC = \frac{PI}{CommittedCapital} \tag{3.5}$$

This ratio, calculated as the ratio of the paid-in capital to the committed capital, provides investors with an indication of the portion of the cumulative drawdowns they have already made. When PIC is equal to one, LPs have paid to the fund all the capital they committed. But PIC can also be greater than one, when for example investors are required to return part of the received distributions (if allowed by the terms of the fund).

3.2.3 Public Market Equivalent

Long-Nickels Public Market Equivalent (LN PME) The LN PME is a return measure that makes an immediate comparison with the public market. In fact, it is calculated as the performance of the public market that would have been generated over the same

time period of the fund considering the fund cash flows. In particular, the cash inflows (drawdowns) and outflows (distributions) of the fund are represented by the purchases and sales of shares of the reference public market index, respecting the timing and size of cash flows. For instance, when a capital call occurs, the same amount is hypothetically invested into the market, buying shares. On the contrary, when GPs exit an investment, the same amount distributed to investors is hypothetically disinvested from the public market index, selling shares. Therefore, the result is a stream of cash flows with the same sizes and timing of the PE fund but compounded at the realized rate of return of the market. The two investments will differ in end values, consequently giving two different and comparable IRR.

Formally, the stream of contributions to the Private Equity fund is defined as $C = \{c_0, c_1, ..., c_n\}$ where n is the final time step. The stream of distributions from the fund is $D = \{d_0, d_1, ..., d_n\}$. The reference index chosen as benchmark is $M = \{m_0, m_1, ..., m_n\}$. The IRR of the PE fund is a function of D, C and NAV_{PE} which is the residual value of the PE fund at time n. In order to compute the PME, it is necessary to compute a stream of investments and disinvestments in the public market and calculate the final NAV_{PME}. Therefore, the future value at time n of the stream of investments is $FV(C) = \{c_0 \times \frac{m_n}{m_0}, c_1 \times \frac{m_n}{m_1}, ..., c_n\}$. The same has to be computed for the disinvestments: $FV(D) = \{d_0 \times \frac{m_n}{m_0}, d_1 \times \frac{m_n}{m_1}, ..., d_n\}$. The final Net Asset Value is given by $NAV_{PME} = \sum FV(C) - \sum FV(D)$. The IRR is then computed as a function of C, D and NAV_{PME}. The higher IRR means that the relative market (PE or public) has outperformed the other.

The challenge with LN PME is the choice of the public market index to use in the calculation. In fact, this index must be relevant: an American PE fund should not be analysed against a European public market, otherwise, the delivered result could be misleading for investors.

Again, net cash flows provide a more transparent value of the return of the PE fund. Moreover, since IRR has as an assumption the reinvestment of proceeds, the selected index should provide for the reinvestment of the income received through dividends.

Arguments against this metric involve the fact that the LN PME will obviously face the same limitations of the IRR (multiple results and the non-consideration of the scale of projects). In addition, the LN PME force the hypothetical investment in the public market to follow a timing that might be unsuggestable for a real investor in the public

market. Therefore, the LN PME would not fairly compare the investments in the two markets because the performance of the public market would be forced and the performance of PE overstated. Lastly, LN PME does not adjust for the relative market risk nor for the different tax impact.

Public Market Equivalent + (PME+) The LN PME has another limitation: the NAV of the public market could become negative. This can happen when the PE fund consistently outperforms the benchmark and simultaneously makes large and frequent distributions, causing the hypothetical investor to take a short position in the public market. In order to solve this problem, the PME+, introduced by Christophe Rouvinez (2003) in the paper "Private Equity Benchmarking with PME+", accounts for the short exposure. PME+ addresses this issue scaling the PE cash outflows (namely, the distributions) so that the closing public market NAV is equal to the residual value of the PE fund. In other words, instead of selling an equal amount, the PME+ suggests selling a fixed proportion of the corresponding PE cash flows.
Formally, the only difference with the previous approach is the following:

$$NAV_{PME+} = \sum FV(C) - s \times \sum FV(D) \tag{3.6}$$

where s is the scaling factor. Then, like the LN PME, the IRRs computed using NAV_{PE} and NAV_{PME+}, have to be compared: an IRR_{PE} greater than IRR_{PME+} means that the PE fund outperformed the public market index. PME+ faces the remaining issues of the LN PME.

Kaplan and Schoar Public Market Equivalent (KS PME) Aiming at making the PE and the public markets comparable, Kaplan and Schoar (2005), developed an alternative PME that, rather than creating a separate hypothetical investment in the public market, discounts the cash flows of the fund by the realized market return over a time period starting at the fund's inception. In particular, it considers separately distributions and drawdowns. The KS PME discounts (or actualizes) every cash flow using the realized market return over the same time period of the considered cash flow, then it sums separately the values for distributions and drawdowns. Lastly, the KS PME divides the sum of distributions for the sum of contributions to the fund. A final result greater than

one is positive because it means that the PE fund outperformed the reference index.

$$PME_{KS} = \frac{\sum_{t=0}^{n} \frac{d_t}{\frac{m_t}{m_0}}}{\sum_{t=0}^{n} \frac{c_t}{\frac{m_t}{m_0}}} \tag{3.7}$$

3.3 Benchmarking Private Equity

Benchmarking Private Equity can be difficult for several reasons. First of all, the benchmarking method should be tailored to each investor, considering the goal and the strategy the investor uses in the approach to this asset class. Each investor has a specific investment strategy that is supposed to meet the goal and the expectations the investors has. Therefore the chosen benchmark should reflect whether the implemented strategy is suitable for the investor. There are two main factors that have to be considered, whose performance is evaluated and the implementation method applied in order to get exposure to PE. First of all, it is important to point out that different benchmarking techniques are applied to General Partners and to Limited Partners since they have different goals. Moreover, there are three ways to invest in PE: investing directly into companies, into PE funds and lastly investing indirectly through funds of PE funds. Generally, in order to benchmark the performance of a PE investment, different approaches improve the understanding of the performance: investors and managers should learn how to use and aggregate the benchmarking methods.

According to Robinson, Schneeweis, Yau, and Weiss (2007) a good benchmark must have some specific characteristics that are not easily applicable to Private Equity and this is one of the reasons why finding a good benchmark is difficult. These characteristics are:

- Unambiguity: there must be a clear understanding of the securities constituting the benchmark in terms of identities, weights, and factor exposure. The composition of the Private Equity universe, for its private nature, is instead often incomplete and undefinable.

- Investibility: it should be possible to hold the benchmark. It is impossible for investors to take a position in the whole PE market. The most viable solution

is holding listed private equity indices but they cannot perfectly reflect the PE universe.

- Measurability: it is possible to measure on a frequent basis the return on the benchmark. The valuation of Private Equity is infrequent and considers different components of a PE investment, according to the objective of the valuation, so that the return of PE cannot be represented by a single figure that should be used in the comparison with the benchmark.

- Appropriateness: the chosen benchmark has to be consistent with the investment characteristics and the area of expertise of the manager or of the investor. Also with PE, the benchmark has to be selected according to the investment implementation method and whose success is valuated.

- Reflection of current investment opinion: the chosen benchmark must be composed of securities the manager or the investor actually knows. Investors, in particular, have limited knowledge of the PE industry and cannot have full details of single PE investments.

- Specification in advance: the benchmark is selected prior to the start of the investment. With PE investment the benchmark cannot be specified in advance because many factors are uncertain, such as fundraising, future investments, etc.

- Ownership: the benchmark should be integral to the investment process and the procedures of the manager who must be aware of the details of the investment needed for the calculation. Given all the issues with the other characteristics, PE benchmarks are not easily embedded into the investment process and procedures of the GPs.

Benchmarks owning all these characteristics are effectively useful to managers and investors in order to monitor investments. In fact, they give an indication of the size of the opportunity cost of investing in the asset under consideration instead of investing in the benchmark.

As shortly explained in the list of characteristics, benchmarking Private Equity face several complications due to the nature of PE. Firstly, the unpredictability of cash flows and the lack of available data on PE transactions and on PE funds make difficult the

creation of a replicable index capturing the whole opportunity set. Secondly, PE investments are exposed to the J curve effect, which is caused by fees and other setup costs and is likely to result in an initial negative performance. The J curve effect must be considered during the benchmarking process. According to what the investor is mostly interested in monitoring, different benchmarks are applied also at an aggregated level, in order to improve the understanding of the PE investment.

3.3.1 Types of benchmarks

Absolute Return Benchmark

There are several types of benchmarks that are just acceptable because they fail to meet all the seven characteristics. An example is the Absolute Return Benchmark, which is based on the return that an asset achieves over a specific period of time. The return of a PE investment can be compared to a minimum hurdle rate of return. This hurdle rate reflects the fairly expected outperformance of PE against other asset classes, in particular, the equity asset class. It is calculated as the expectations of the performance of equity markets in a long-term horizon, increased by a premium intended to compensate the challenges of investing in PE. This premium should include the lack of liquidity and marketability, the compromise between diversification and specialization, the use of leverage and the lack of control over the management of the target company. This benchmark fails to meet the investibility characteristic and moreover it is not applicable to every type of investors, such as business angels who invest directly into sinlge companies. Who invests directly into companies should consider the cost of capital approach, which is useful to judge the attractiveness of the available investment opportunities.

But absolute return benchmarks present some issues. As mentioned earlier, the benchmark is not investable and so the opportunity cost is not easy to compute nor clearly interpretable. Moreover, it focuses on long-term expectations of equity markets and consensus on the return of public equity is hard to find. On the other side, absolute return benchmarks can be appropriate because they are expressed as a single figure which is less sensible to the volatility of the equity markets. Moreover, they are preferable for investors who establish a hurdle rate for their investments.

Public Market Indices

A second benchmark is represented by Broad Market Indices: they are easy to understand and satisfy many entries of the list describing a good benchmark. However, when a PE manager applies an investment style that considerably differs by the benchmark's style, then the benchmark is not appropriate. Therefore, a Style Index can be used as a benchmark, since there are public indices built in order to represent different categories within an asset class. Similarly to broad market indices, style indices meet all the good benchmark characteristics. For this reason, it is one of the most common benchmarks.

In particular, with PE, it is common to perform a comparison with equity markets in light of the fact that a typical exit strategy for target companies is the IPO. Anyway, it has to be pointed out that the drivers of performance of public markets and of private equity are not the same. Moreover, it is very difficult to make a comparison in the short-term given that GPs provide investors with little information (expressed using a metric that is different from what is applied to public equity markets) about the performance of the investment. Lastly, the J-curve effect has to be considered, since it easily causes the PE investment to underperform the equity market index. The reason why public indices are used as benchmarks is due to the goal investors seek in PE, namely the expected superior performance of PE over public equity. Moreover, taking as an example a PE fund specialized in a particular industry or a single PE investment, this method permits the comparison with listed companies. Therefore, in the long-term, public equity benchmarks are acceptable. The implementation involves the PME approach (or one of its variations) since it immediately gives an indication of the performance of PE against the chosen index. However, for applying the PME, the knowledge of the cash flows structure is required. The choice of the index used in the comparison is an important factor. The index should be a total return index that provides the reinvestment of dividends. Then, according to the level and type of diversification applied by the GPs, the appropriate index has to be selected.

Peer Group Indices

A third popular benchmarking technique, which develops from the concept underlying the public index benchmark, applies Peer Group Indices. This method involves the comparison between a portfolio or a single PE investment against the result of a spe-

cific industry obtained from an analysis of the PE portfolio's peers. However, this index (which is not public) might be imprecise due to the quality of data. Managers building the index cannot have access to the entire PE universe and have to deal with issues like the difference in reporting procedures and different valuation methods. In addition, investors cannot invest in this index. The major problem is the access to the source of data: data on PE funds must be reliable, sustainable and should ensure sufficient coverage of funds of the considered category.

However, this benchmark can be appropriate because potential peer groups can be very specific in terms of investment styles (buyouts, venture capital etc.) and geographies, allowing for a better comparison for every single PE portfolio. The comparison is more precise in terms of the drivers of return of both the index and the asset under consideration. Furthermore, peer group indices represent the true opportunity set available to an investor. Lastly, the peer group comparison already takes into consideration the J curve effect. The application of this methodology involves numerous factors such as fund size, strategy, geography etc. that permits to state whether the PE investment is outperforming or not the specific PE industry and gives a way to judge the job of the GPs.

CHAPTER 4

PORTFOLIO THEORY

Portfolio theory is the branch of finance that analyses the allocation of a certain budget among *n* assets. It supports the investors in their financial choices, taking into account their risk aversion and their desired expected return. In general, a portfolio is a financial tool used in order to transfer wealth from a period to the next one (Ingersoll, 1987). Given the wealth *W* and a set of *N* financial stochastic investment choices $X = \{X_1, ..., X_N\}$, a portfolio is an *N*-vector **x'** $= (x_1, ..., x_N)$ such that x_i is the percentage of *W* invested in X_i, with $\sum_{i=1}^{N} x_i = 1$.

4.1 MEAN VARIANCE PORTFOLIO

The Modern Portfolio Theory (MPT) has been introduced by Harry Markowitz (1952) and is a mathematical framework aimed at the creation of efficient portfolios for a rational investor, who is interested in minimazing the risk and maximizing the return.

The model has five main assumptions:

- the investor is risk averse and maximizes his expected utility;
- all returns are normally distributed, so that the investor cares about their expected returns and volatilities;
- there are no transaction costs or taxes, and all securities are perfectly divisible (Frictionless Markets assumption);
- the portfolio is a one-period investment;

- there is perfect competition in the markets, a single investor cannot affect the probability distribution of returns of the available securities (Price-Taker assumption).

From the first and the second assumption the Mean-Variance criterion is defined: among two alternative investment strategies, the investors prefer the one providing the greater expected return and the smaller standard deviation. Formally, if $E(r_x) \geq E(r_y)$ and $\sigma_x \leq \sigma_y$ with at least one of the strong inequalities holding, then portfolio x will be preferred to portfolio y.

The expected return of a portfolio $E(R_P)$ is the weighted average of the returns of the assets included in the portfolio:

$$E(R_P) = \sum_{i=1}^{N} x_i \times R_i \tag{4.1}$$

where w_i and R_i are the weight and the expected return of the asset i, respectively. Defining $E(R_P) = r_P$, the vectorial notation is $r_P = \mathbf{x}'\mathbf{r}$, where $\mathbf{r}$ is the vector of expected return of the assets.

The variance of the return of the portfolio instead is given by the following formula:

$$\sigma_P^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_i \sigma_j \rho_{i,j} = \sum_{i,j=1}^{N} x_i x_j \sigma_{i,j} \tag{4.2}$$

where σ_i is the standard deviation of asset i, $\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j}$ is the correlation coefficient between assets i and j and $\sigma_{i,j}$ is the covariance between the same assets. The standard deviation of the portfolio, called volatility as well, is the square root of the variance, $\sigma_P = \sqrt{\sigma_P^2}$. Using the vectorial notation, σ_P^2 is the variance-covariance matrix: $\sigma_P^2 = \mathbf{x}'\mathbf{V}\mathbf{x}$, where

$$\mathbf{V} = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \dots & \sigma_{1,N-1} & \sigma_{1,N} \\ \sigma_{2,1} & \sigma_2^2 & \dots & \sigma_{2,N-1} & \sigma_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sigma_{N-1,1} & \sigma_{N-1,2} & \dots & \sigma_{N-1}^2 & \sigma_{N-1,N} \\ \sigma_{N,1} & \sigma_{N,2} & \dots & \sigma_{N,N-1} & \sigma_N^2 \end{bmatrix}$$

In fact, in order to evaluate the risk and return of a portfolio including *N* securities, the joint distribution of returns has to be considered.

r_P and σ_P^2 are the only two metrics necessary to apply the MPT, therefore, this approach considers only the first two moments of the distributions of the returns of the portfolio, and therefore of its components as well, together with their covariances that detect linear dependencies between assets.

The Markowitz Optimization problem maximizes the expected utility of the investor, who seeks a tradeoff between risk and return delivering the highest return and the lowest variance possible. The investor's preferences, in terms of importance and usefulness of a certain outcome, are described by a *Von Neumann-Morgenstern* utility function $U(X)$ which is a concave increasing function. Since the optimization techniques maximize the expected utility of a portfolio in terms of its expected return and variance, the expected utility can be expressed as $E\left[U(X)\right] = f(r_P, \sigma_P^2)$, such that the investor's expected utility increases if the expected return increases:

$$\frac{\partial f(r_P, \sigma_P^2)}{\partial r_P} > 0 \tag{4.3}$$

and decreases if the variance increases:

$$\frac{\partial f(r_P, \sigma_P^2)}{\partial \sigma_P^2} < 0 \tag{4.4}$$

Therefore, the optimization problem aimed at finding the optimal portfolio that maximizes the expected utility can be solved starting from the desired level of expected return π: the portfolio that delivers the rate of return π with the lowest variance possible is the optimal portfolio. The optimal portfolio can be expressed as the portfolio that provides the investor with the highest rate of return possible for a given level of risk. Thus, the portfolio selection problem can be formulated as follow:

$$min_{x_1,\dots,x_N} \sigma_P^2$$

$$s.t. \begin{cases} r_P = \pi \\ \sum_{i=1}^{N} x_i = 1 \end{cases} \tag{4.5}$$

which, in vectorial notation, can be rewritten as:

$$min_{x_1,...,x_N} \mathbf{x}'\mathbf{V}\mathbf{x}$$

$$s.t. \begin{cases} \mathbf{x}'\mathbf{r} = \pi \\ \mathbf{x}'\mathbf{e} = 1 \end{cases} \tag{4.6}$$

The result of this problem is the optimal portfolio characterized by the pair (π, σ) where σ is the smallest possible value for that level of return. The problem can be applied to a set of values for π. The result is a set of pairs that is called *minimum variance set*. In the risk-return plane, this set is the *Efficient Frontier*, whose far left point corresponds to the *minimum variance* portfolio.

4.2 Mean CVaR Portfolio

One of the critics to the Mean-Variance approach involves the assumption on the normality of returns. Researchers tried to overcome this issue by taking into consideration other measures of risk. An example is the method considering the VaR and the Expected Shortfall. These metrics measure the risk associated with the negative tail of the loss distribution of an investment and have two main advantages. First of all, the optimization process does not require the normal distribution of returns. Secondly, while variance, given its symmetry, equally penalizes gains and losses, VaR penalizes negatively skewed distributions that provide a greater downside risk than upside risk (Wang, 2000).

The approach that is going to be presented, that has been firstly introduced by Rockafellar and Uryasev (2000), minimize the CVaR instead of the VaR because the first is a coherent risk measure (since it satisfies the properties of sub-additivity, monotonicity, positive homogeneity and transitional invariance), while the VaR is not subadditive. Defining $\sum$ as a measure of risk, the properties that it has to satisfies are listed and explained as follow:

- Sub-additivity: $\sum(X_i + X_j) \leq \sum(X_i) + \sum(X_j)$ for all X_i and X_j with $i, j = 1, ..., N$; adding a risky investment to another one permits to decrease the overall risk;

- Monotonicity: if $X_i \preccurlyeq X_j$ in terms of riskyness, then $\sum(X_i) \leq \sum(X_j)$ for every couple of X_i and X_j with $i, j = 1, ..., N$;
- Positive homogeneity: $\sum(\lambda X_i) = \lambda \sum(X_i)$ for every positive real number λ and for all X_i with $i = 1, ..., N$, the size of a risky investment linearly affects the risk of the underlying investment;
- Transitional invariance: $\sum(X_i + X_{N+1}) = \sum(X_i) - \gamma$ where X_{N+1} is a riskless investment and $\gamma > 0$, for all X_i with $i, j = 1, ..., N$; adding a riskless investment to a risky one decreases the risk of the overall underlying investment.

The VaR is a coherent risk measure when the returns have an elliptical distribution: in particular, when the returns are normally distributed, the minimization of the VaR would lead to the same results obtained minimizing the variance. Anyway, minimizing the CVaR is closely related to minimize the VaR.

In order to calculate the VaR and CVaR, the loss function is defined as $f(\mathbf{x}, \mathbf{r})$ where $\mathbf{x}$ is the vector describing the portfolio weights and $\mathbf{r}$ is the vector describing the future returns of the assets. Therefore, $f(\mathbf{x}, \mathbf{r})$ is a random variable describing the loss associated with the portfolio vector $\mathbf{x}$; thus, for each $\mathbf{x}$, its distribution depends on the distribution of the returns $\mathbf{r}$, whose density is $p(\mathbf{r})$. The probability that the loss does not exceed a specific threshold α is

$$\Psi(\mathbf{x}, \alpha) = \int_{f(\mathbf{x},\mathbf{r}) \leq \alpha} p(\mathbf{r}) d\mathbf{r} \tag{4.7}$$

Fixing $\mathbf{x}$ and keeping α as a variable of the function Ψ, Ψ is the cumulative distribution function of the loss associated to $\mathbf{x}$. The assumption that has to be made for simplifying the approach is that $\Psi(\mathbf{r}, \alpha)$ is continuous with respect to α; in other words, no jumps occur. Moreover, Ψ is non-decreasing with respect to α. Having defined $\Psi(\mathbf{r}, \alpha)$ and given a probability level $\beta \in (0, 1)$, VaR_β and $CVaR_\beta$ of the loss associated with $\mathbf{x}$ are

$$VaR_\beta \rightarrow \alpha_\beta(\mathbf{x}) = \min\{\alpha \in R : \Psi(\mathbf{x}, \alpha) \geq \beta\} \tag{4.8}$$

$$CVaR_\beta \rightarrow \phi_\beta(\mathbf{x}) = (1 - \beta)^{-1} \int_{f(\mathbf{x},\mathbf{r}) \geq \alpha_\beta(\mathbf{x})} f(\mathbf{x}, \mathbf{r}) p(\mathbf{r}) d\mathbf{r} \tag{4.9}$$

$\alpha_\beta(\mathbf{x})$ is the minimum α value for which $\Psi(\mathbf{x}, \alpha)$ is equal to or greater than β. This means that the probability that the loss is greater than the VaR is $(1-\beta)$. Therefore, the CVaR $\phi_\beta(\mathbf{x})$ is the conditional expectation of the loss given that the loss itself is greater than $\alpha_\beta(\mathbf{x})$.

Rockafellar and Uryasev then introduced a function F_β which is a function of α, convex and continouosly differentiable with respect to α as it is proved in Rockafellar (1970). This function permits to define the CVaR independently from the VaR, otherwise, using $\phi_\beta(\mathbf{x})$, the VaR should be first calculated.

$$F_b(\mathbf{x}, \alpha) = \alpha + (1-\beta)^{-1} \int [f(\mathbf{x}, \mathbf{r}) - \alpha]^+ \, p(\mathbf{r}) d\mathbf{r}$$

$$\text{where } [t]^+ = \begin{cases} t \text{ when } t > 0 \\ 0 \text{ when } t \leq 0 \end{cases} \tag{4.10}$$

The two researchers showed that the CVaR $\phi_\beta(\mathbf{x})$ can be obtained from the minimization of $F_\beta(\mathbf{x}, \alpha)$ with respect to α:

$$\phi_\beta(\mathbf{x}) = \min_{\alpha \in R} F_\beta(\mathbf{x}, \alpha) \tag{4.11}$$

Then they proved that minimizing the CVaR over all $\mathbf{x}$ is equivalent to minimize $F_\beta(\mathbf{x}, \alpha)$ over all $(\mathbf{x}, \alpha)$:

$$\min_{\mathbf{x} \in X} \phi_\beta(\mathbf{x}) = \min_{(\mathbf{x}, \alpha) \in X \times R} F_\beta(\mathbf{x}, \alpha) \tag{4.12}$$

This means that the pair $(\mathbf{x}*, \alpha*)$ contains the minimum CVaR portfolio and the corresponding VaR. Therefore the VaR can be computed as a side effect of the CVaR approach. It can be proved that the portfolios obtained through the Mean-CVaR approach are usually optimal in terms of VaR as well; in other words, the VaR is likely to be the lowest possible, since the VaR cannot exceed the CVaR.

The Mean-CVaR problem is then formulated as follow:

$$\min_{(\mathbf{x}, \alpha) \in X \times R} F_\beta(\mathbf{x}, \alpha)$$

$$s.t. \begin{cases} \mu(\mathbf{x}) \leq -\pi \\ x_j \geq 0 \text{ for } j = 1, ..., N \text{ with } \sum_{i=1}^{N} x_i = 1 \end{cases} \tag{4.13}$$

where $\mu(\mathbf{x}) \leq -\pi$ is the requirement that only those portfolios whose expected return is greater than π are admitted, where $\mu(\mathbf{x})$ is the mean of the loss function.

4.3 Mean MAD Portfolio Optimization

Konno and Yamazaki (1991) proposed another optimization model aimed at overcoming some issues associated with the Mean-Variance approach proposed by Markowitz. In particular, they argued that Markowitz's model suffers from computational difficulties when several assets are taken into consideration, causing the variance-covariance matrix to be dense and consequently making the quadratic programming problem difficult to solve. However, thanks to the advances in technology, this is not an issue anymore. The criticism of the Markowitz model that has been mentioned in the previous section and that justifies the use of the M-MAD method at the expense of the M-V method, is the assumption of normally distributed returns, made in order to allow the use of the standard deviation as the only risk measure. As the authors specified, the investors are risk averse, therefore they do not only prefer less volatility, but they perceive differently profit and loss. The standard deviation is unable to catch this preference. Moreover, since most of the assets' return are not normally distributed, the assumption is not satisfied.

The authors searched for a risk measure able to remove these computational issues and to avoid the assumption of normality. In particular, they found that the Mean Absolute Deviation (MAD), even though it is a central measure of risk, allows to build a model which does not have the assumption of normal return and which requires less computations.

Considering x_i with $i = 1, ..., N$ as the amount of money invested in asset i (the position, not just the percentage), ρ the hurdle rate of return, M_0 the initial capital and u_i the upper bound for the investment in the asset i, the authors formulated the following problem:

$$\min_{x_1,...,x_N} E\left(\left|\sum_{i=1}^{N} R_i x_i - E\left(\sum_{i=1}^{N} R_i x_i\right)\right|\right)$$

$$s.t. \begin{cases} \sum_{i=1}^{N} E(R_i)x_i \geq \rho M_0 \\ \sum_{i=1}^{N} x_i = M_0 \\ 0 \leq x_i \leq u_i, i = 1, ..., N \end{cases} \tag{4.14}$$

where the MAD of the portfolio is minimized taking into account the constraints. The model can be furtherly reformulated in order to obtain a linear mathematical programming problem. The main advantage of this approach is that it does not require the calculation of the variance-covariance matrix.

It is noteworthy that when the returns are normally distributed, the Mean-Variance approach and the Mean-MAD approach deliver the same result, given the theorem that Konno and Yamazaki (1991) proved.

Theorem: *if* $(R_1, ..., R_N)$ *are multivariate normally distributed, then*

$$MAD(X) = \sqrt{\frac{2}{\pi}}\sigma(X)$$

If this relation holds, then minimizing the MAD is equivalent to minimizing the variance.

CHAPTER 5

THE ROLE OF PRIVATE EQUITY IN STRATEGIC ASSET ALLOCATION

Private Equity as an asset class is believed to have low correlation with other asset classes, because, due to its "private" nature, it is not traded as other common assets. Therefore, it is believed to be an asset that can improve the performance of investment portfolios.

As a matter of fact, there are several papers that confirm the positive role of PE in diversified portfolios. The first paper to be considered is Idzorek (2007), which builds the efficient frontiers with and without PE and compares the weights of the different portfolios. The analysis is implemented using the Mean-Variance Optimization approach. This approach will be implemented also in this thesis. Idzorek starts from a diversified portfolio weighted primarly in listed equities of different markets, US bonds and Non-US bonds. The PE asset class is represented by the Red Rocks Listed Private Equity Index, which represents the US PE market, and the Red Rocks International Listed Private Equity Index, which in turn represents the Non-US PE market. The result obtained is that PE improves the efficient frontier and even achieves the 100% allocation in the riskiest portfolio.

Milner and Vos (2003) tested two hypothesis: the low correlation with other asset classes and the positive role in an investment portfolio. They wanted to study why this alternative investment is so attractive. Therefore they took into consideration just PE and listed equities, where the PE asset class is represented by quarterly performance

data on different categories of PE funds obtained from the VentureXpert database. They found mixed evidence about the low correlation with other asset classes: in particular, only Venture Seed Funds and Mezzanine Funds have low correlations, while all other kinds of PE have moderate or even strong correlation with equity markets. However, they found that investors benefit from the inclusion of PE in their portfolios.

Bekkers, Doeswijk and Lam (2009), determined the optimal portfolio including ten different asset classes using a Mean-Variance analysis. Their attempt was to find which alternative asset classes add more value to an investment portfolio. For what concerns PE, they chose to use listed indices because, since they are based on transaction prices, they easily adjust in case of changes in the market or in the PE industry. They found that PE enters the riskier portfolios and that to PE is allocated 100% of the riskier portfolio.

This thesis attempts to reproduce the approach chosen by Idzorek (2007) and Milner and Vos (2003) with a further analysis of the efficient frontier using a two alternative optimization methods that do not require the normality of returns of the assets. In fact, it will be analyzed whether the returns of the chosen asset classes, including PE, follow a normal distribution.

5.1 Choice of assets

According to the approach used by Idzorek (2007), in order to establish whether PE has a positive role within a strategic asset allocation setting, the selected opportunity set must represent the asset classes that a typical investor is exposed to. The selected securities that are going to form the traditional diversified portfolio should represent the exposure to stock markets and to bond markets around the world. Therefore, the chosen stock indices IEV (*iShares Europe ETF*) and URTH (*iShares MSCI World ETF*) cover territories like Australia, North America and the most developed part of the European Union. FM (*iShares MSCI Frontier 100 ETF*) represents the exposure to emerging markets, such as the eastern part of Europe, South America and the Middle East. Then, SHY (*iShares 1-3 Year Treasury Bond ETF*) represents the exposure to U.S. bonds and IGOV (*iShares International Treasury Bond ETF*) to international bonds. SHY has been chosen for the low volatility: it will approximate the risk-free asset.

5.1.1 Listed Private Equity

There are several ways to get exposure to Private Equity. Firstly, it is possible to invest directly in the target company, in a PE fund or in a fund of PE funds. Unfortunately, these investment means are usually not listed and their main investor type is institutional. However, there are three main ways available to get exposure to listed PE. The listed PE allows retail investors to invest in this asset class, because they are not obliged to commit a certain initial amount and they can sell the shares in a public market, solving the problems of liquidity and of long-term investment horizon (Baker et al. 2015).

- *Listed PE Fund*: when a PE fund is listed, it means that the PE firm creates an entity that collects the capital from the listing on a stock exchange that is going to be invested directly in target companies.

- *Listed Funds of PE Funds*: these funds collect capital from both institutional and retail investors through the listing in a stock exchange in order to invest in PE funds. Therefore, the investor indirectly becomes investor in a high number of limited partnerships, thus benefitting from a well defined diversification built by the managers of the fund of funds.

- *Listed Private Equity Firms*: in this case, investors buy shares of the General Partner. Investors profit of the capital gain of the PE firm and of the fees paid by target companies to the GPs. Since a PE firm can manage several PE funds, the investor is still well diversified.

From these Listed PE securities it is possible to build indices that are a good proxy for the PE industry. Generally, a Listed PE index comprises and tracks the performances of listed PE firms, which are publicly traded companies (Bilo et al. 2005). In general, an index can consider those companies whose majority of revenues comes from investing into privately held businesses. Since every PE firm manage several PE funds, the concept of the PE index is similar to that of the fund of PE funds, even though the object is different: the fund should maximize the profit, the index should give information on a certain industry.

An alternative investment means is the Exchange Traded Fund. An ETF is a financial instrument listed on a stock exchange that replicates the performance of the index

that the ETF is meant to track. Hence, this index is taken as a benchmark. The managers of the ETF must minimize the Tracking Error, the difference of the performances of the index and of the ETF. Thus, the ETF is a passively managed fund: the managers are not interested in rebalancing the fund to maximize the return but only in replicating the index performance. Through the ETF it is possible to invest in the index with only one transaction.

5.1.2 Choice of the proxies for Private Equity

This thesis involves two PE proxies: PSP and PEX. They have been selected because they are both listed in the same stock exchange (NYSE Arca), therefore there is consistency in the trading days and the currency. The consistency in the trading days is necessary for creating matrices in Matlab. Other ETFs on PE (such as the *iShares S&P Listed Private Equity ETF*, the *Db x-trackers LPX Mm Private Equity ETF* and the *Lyxor Etf Privex*) are listed on different stock exchange and it is impossible to get the same number of prices over a specific time window.

PSP (*PowerShares Global Listed Private Equity Portfolio ETF*) is an ETF that tracks the *Red Rocks Global Listed Private Equity Index*, while PEX (ProShares Global Listed Private Equity ETF) tracks the *LPX Direct Listed Private Equity Index*. Given that PEX was first instituted on February 28, 2013 the analysis in this thesis will comprise data from this date to December 31, 2016.

The choice of using ETFs on PE instead of Funds of PE Funds is justified by the fact that an ETF better represents the PE industry. An open-end fund usually has other objectives than replicating an index, such as maximizing the expected return for the investors. Therefore, they can be used to reduce risk and to generate a positive α in an investor's portfolio. However, for the scope of this thesis, in order to evaluate whether the PE as an asset class improves the performance of an investment portfolio, the ETFs better represent the dynamics of the entire industry. Moreover, for a retail investor, investing in an ETF is more efficient because it already provides exposure to several PE firms, is diversified and less volatile than other listed PE means.

5.2 Analysis of assets

As mentioned in the previous section, the time window that is considered is 28/02/2013-31/12/2016. From *https://it.finance.yahoo.com/*, daily prices of the selected securities have been downloaded.

First of all, a visual analysis of the movements of the prices of securities over the specified time period is displayed in *Figure 5.1*. For the sake of clarity, all securities' prices have been transformed so that they all start at the hypothetical value of 100, in order to make them comparable. As it can be observed, in general all the stock and PE proxies increase over time and appear to move together, except for the Emerging Market proxy FM that is not able to recover from the general loss experienced at the end of 2015, followed by the performance of the European stock market proxy IEV that is able at least to achieve an overall positive performance over the considered time period. The bond proxy SHY, which is the proxy for the US bonds, is characterized by a stable price, while IGOV, which represents the international bonds, is more volatile and has an overall negative performance.

There is not a single security clearly outperforming the others, but, being the focus of this thesis the PE asset class, it is noteworthy how the proxies for the PE industry are part of the top indices group. This leads to the thought that the correlation between PE and other asset classes is not low, but high and positive, contrary to the common belief. A dedicated analysis to correlations will be provided later in the chapter, but this consideration has already created doubts on the attractiveness of this asset class. It will be important to evaluate if and how PE affects an investment portfolio.

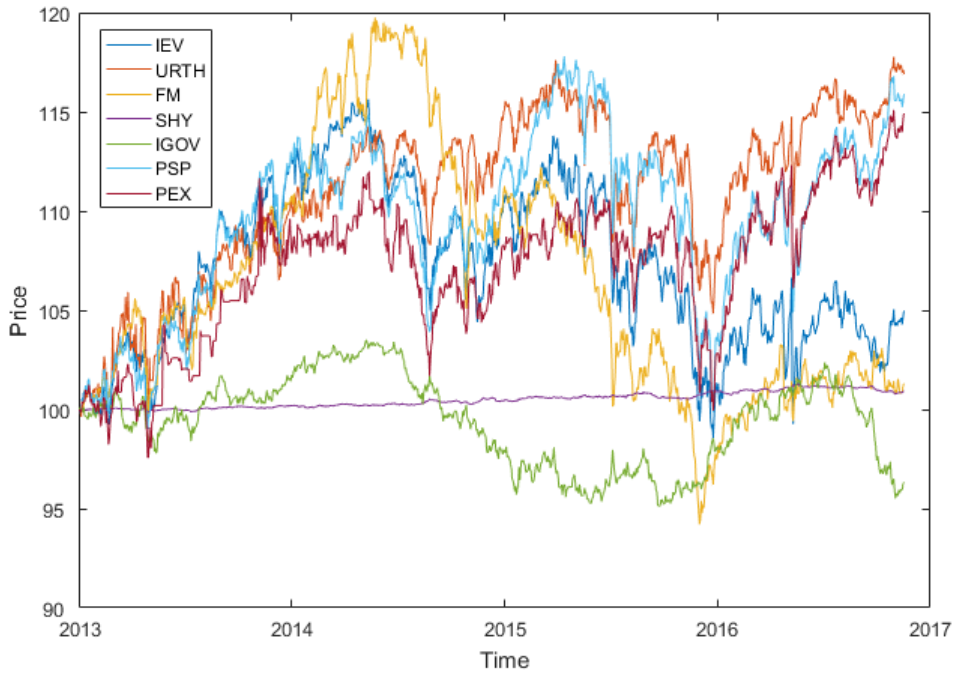

Figure 5.1: Time Series of Prices of the ETFs

Secondly, in order to understand and create expectations on which assets will enter the final portfolio, a descriptive analysis of the variables chosen is performed. It is then necessary to compute the logarithmic returns from the daily prices, according to the following formula:

$$r = ln\left(\frac{P_t}{P_{t-1}}\right) \tag{5.1}$$

where P_t is the price of the security at time *t* and *r* is its logarithmic return. Then, *Table 5.1* displays the mean and standard deviation of each asset, computed according to the formulas (N is the number of time steps):

$$\text{Mean} \rightarrow \overline{r} = \frac{\sum_{t=1}^{N} r_t}{N} \tag{5.2}$$

$$\text{Standard Deviation} \rightarrow s = \sqrt{\frac{\sum_{t=1}^{N}(r_t - \overline{r})^2}{N}} \tag{5.3}$$

Table 5.1: Annualized Mean and Standard Deviation of the ETFs

	Mean	Standard Deviation
IEV	0.0247	0.1695
URTH	0.0758	0.1671
FM	0.0067	0.1411
SHY	0.0048	0.0084
IGOV	-0.0213	0.0804
PSP	0.0718	0.1539
PEX	0.0680	0.1754

Connected to *Table 5.1*, *Figure 5.2* displays the position of each asset on the risk-return plane. As a consequence, the combination of mean and standard deviation is explained in a more intuitive way and the comparison between assets, aimed to identify the efficient assets in the mean-variance sense, is easier. In fact applying the Mean-Variance dominance rule, it is immediately clear which assets dominate the other. Among the bond proxies, SHY dominates IGOV in the mean-variance sense. Among the stock proxies, both URTH and FM are efficient (because $E(R_{URTH}) > E(R_{FM})$ and $\sigma_{URTH} < \sigma_{FM}$) and URTH dominates IEV in the mean-variance sense. Among the PE proxies, PEX is dominated by PSP. However, the efficient securities that are not dominated by any other asset are SHY, PSP and URTH, thus these assets are very likely to enter the efficient portfolios.

Another metric to measure the return is the Compounded Annual Growth Rate, also known as the Annual Return, which expresses the value increment of a security over a specific period of time. In particular, it measures the growth of an investment as growing (or decreasing) consistently over the specified time window on an yearly basis. It is calculated according to the following formula (where y is the number of years):

$$CAGR = \left(\frac{P(N)}{P(1)}\right)^{\frac{1}{y}} - 1 \tag{5.4}$$

According to *Table 5.2*, URTH has the greatest rate of annual return over the period

Table 5.2: Compounded Annual Growth Rate of the ETFs

	IEV	URTH	FM	SHY	IGOV	PSP	PEX
CAGR	2.25%	7.63%	0.59%	0.45%	-2.06%	7.00%	6.62%

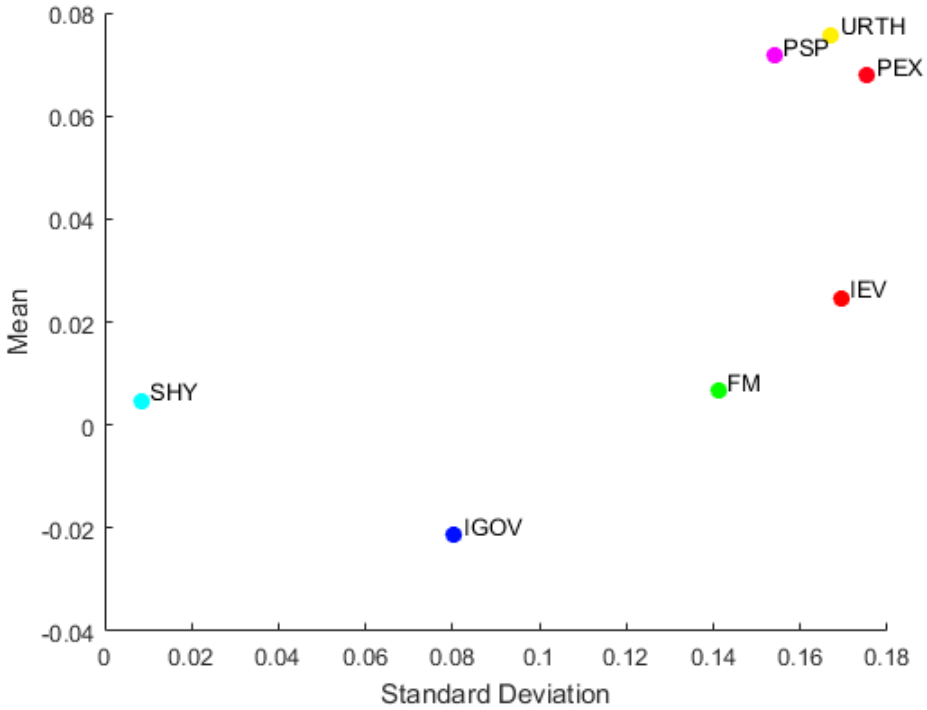

Figure 5.2: Annualized Mean and Standard Deviation of the returns of the ETFs

of time that has been considered, immediately followed by the PE proxies. The less performing asset is the international bond, which is the only asset with a negative return. However, the ranking of the different securities does not change moving from the mean to the CAGR.

Another important aspect to evaluate is the variance-covariance matrix, which is then connected to the correlation matrix, composed of the correlations between the returns of the securities.

$$\text{Variance} \rightarrow \sigma_X^2 = E[(X - \mu_X)^2] \tag{5.5}$$

$$\text{Covariance} \rightarrow \sigma_{X,Y} = E[(X - \mu_X)(Y - \mu_Y)] \tag{5.6}$$

$$\text{Correlation} \rightarrow \rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \times \sigma_Y} \tag{5.7}$$

As it is known, the diversification is able to reduce the portfolio risk if the financial

instruments that the investor is including are not perfectly positively correlated, thus the correlation should respect the condition $-1 \leq \rho < 1$. *Table 5.4* gives an indication of how similar or different the movements of the assets are, useful to understand which combinations have a greater impact on the risk of the portfolio.

There is only one asset, SHY that represents the US bonds, that has a negative

Table 5.3: Variance-Covariance Matrix of the Returns of the ETFs

	IEV	URTH	FM	SHY	IGOV	PSP	PEX
IEV	0.0287	0.0196	0.0124	-0.0002	0.0016	0.0224	0.0135
URTH	0.0196	0.0279	0.0102	-0.0002	-0.0006	0.0175	0.0113
FM	0.0124	0.0102	0.0199	-0.0002	0.0003	0.0116	0.0070
SHY	-0.0002	-0.0002	-0.0002	0.0001	0.0004	-0.0003	-0.0002
IGOV	0.0016	-0.0006	0.0003	0.0004	0.0065	0.0000	0.0003
PSP	0.0224	0.0175	0.0116	-0.0003	0.0000	0.0237	0.0127
PEX	0.0135	0.0113	0.0070	-0.0002	0.0003	0.0127	0.0308

Table 5.4: Correlation Matrix of the Returns of the ETFs

	IEV	URTH	FM	SHY	IGOV	PSP	PEX
IEV	1	0.6934	0.5196	-0.143	0.1169	0.8591	0.4543
URTH	0.6934	1	0.4321	-0.163	-0.049	0.6804	0.3853
FM	0.5196	0.4321	1	-0.136	0.0273	0.5357	0.2836
SHY	-0.143	-0.163	-0.136	1	0.5447	-0.196	-0.108
IGOV	0.1169	-0.049	0.0273	0.5447	1	0.0005	0.0193
PSP	0.8591	0.6804	0.5357	-0.196	0.0005	1	0.4701
PEX	0.4543	0.3853	0.2836	-0.108	0.0193	0.4701	1

correlation with the other securities, except for the other bond proxy, IGOV, which represents the international bonds. Again, SHY is likely to enter the diversified portfolio. Generally, both proxies for bonds are less correlated with the other securities.

Considering specifically the PE asset class, the highest correlation that the two PE proxies have is with IEV (in particular the correlation with PSP is the highest in the matrix), the proxy for the European stock market, followed by the correlation with URTH, the proxy for the world stock market. Therefore, IEV is likely to be left out of the portfolio since it moves similarly to PE and it is also dominated in the mean variance sense by PSP. Among them, the PE proxies have correlation 0.4701, therefore posi-

tive but different from zero. It is then possible that both proxies will enter the efficient portfolios.

5.3 Analysis of the Efficient Frontier

The Efficient Frontier represents the set of efficient portfolios, which are those combinations of assets that maximize the expected return given the risk level. The risk-return characteristics of the assets and the variance-covariance matrix ($\sum_{ij} = COV(X_i, X_j) = E[(X_i - \mu_i)(X_j - \mu_j)]$) are the main inputs of the model. Two constraints have been applied: the weights are positive and sum up to one. The most famous and widely implemented approach is the Mean-Variance Optimization, which maximizes the mean of the return of the portfolio given the level of its standard deviation. However, this approach requires the test of certain assumptions, which, in case they are not verified, will lead the resulting efficient portfolios to underestimate the real level of risk. Therefore, there are other optimization approaches that consider different measures of risk. One of these methods involves the Expected Shortfall.

The aim of this thesis is to determine whether the inclusion of PE improves the risk-return characteristics of an investment portfolio weighted in listed equities and bonds.

5.3.1 Mean-Variance Portfolio Optimization

Mean-Variance Portfolio Optimization combines the available assets applying the quantitative techniques of the Modern Portfolio Theory in order to create portfolios that are efficient in the Mean-Variance sense. If PE positively affects the performance of an investment portfolio which is primarly invested in listed equities and bonds, then the efficient frontier that is composed of those portfolios combining the three asset classes should provide a better risk-return profile than the efficient frontier composed of those portfolios which combine only bonds and equities. In other words, for a given level of standard deviation, the efficient portfolios with PE should deliver an expected return that is greater than the efficient portfolios without PE (Milner and Vos, 2003)

This test has been implemented using the Matlab Financial Toolbox. The result of this comparison is displayed in *Figure 5.3*. Clearly, the Efficient Frontier with PE dominates the Efficient Frontier without PE in the mean-variance sense. Including the PE

asset class dramatically affects in a positive way the risk-return profile of the efficient portfolios. The two efficient frontiers have two points in common, corresponding to the riskiest and the less riskiest portfolios. These portfolios are then expected to be 100% composed of, respectively, the stock proxy URTH and the bond proxy SHY (since they are the efficient assets in the mean variance sense with the higher and lowest variance respectively). PE enters the portfolios characterized by an intermediate level of risk, clearly improving their risk-return profile.

The next step involves the analysis of the allocations of the assets. With a focus on

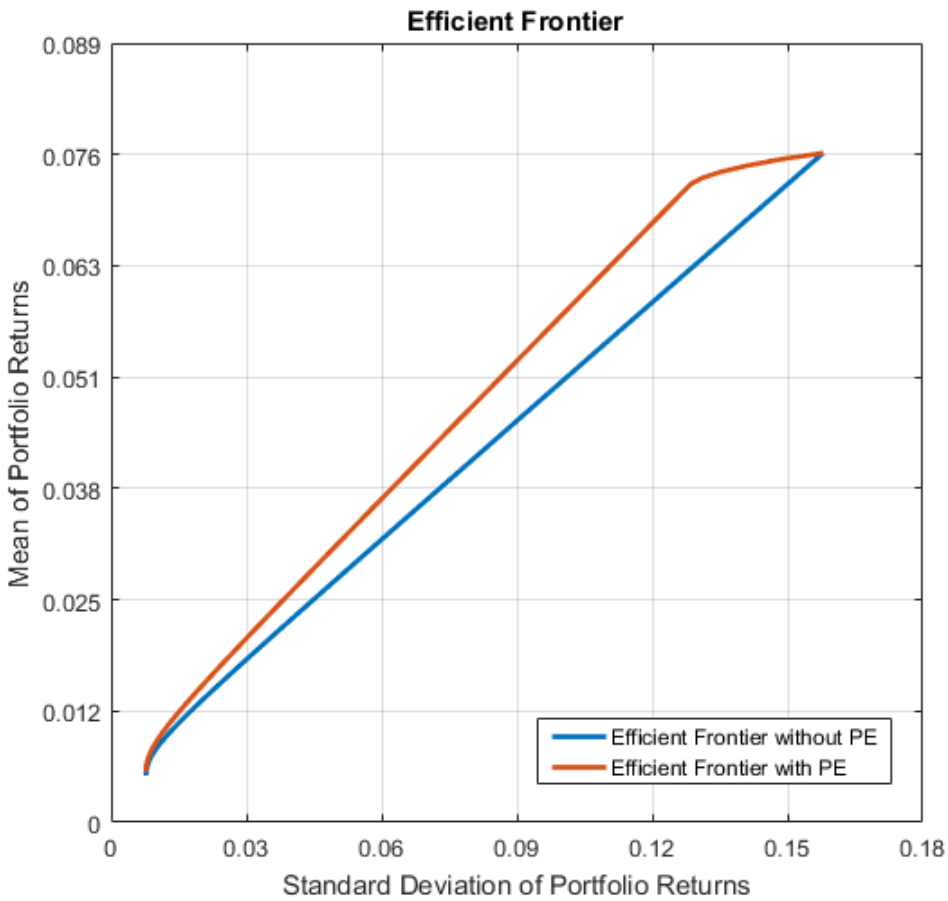

Figure 5.3: Efficient Frontiers: with and without PE

PE, it is interesting to see which proxies actually enter the efficient portfolios. The result of the comparison between the efficient portfolios with and without PE is displayed

in *Figures 5.4* and *5.5*. These figures shows the composition of every portfolio on the efficient frontier. Each vertical cross-section represents a portfolio, whose standard deviation is provided on the x axis and whose composition can be derived comparing the weights of the ETFs entering that specific portfolio with the percentages on the y axis.

As expected from the risk-return analysis, only SHY and URTH enter the diversified portfolios without PE. In fact, recalling the previous dominance analysis, SHY dominates IGOV among the bond proxies and is close to be the risk-free asset given the low volatility. URTH, instead, while it dominates IEV, it does not dominate FM: however its contribution to the portfolio is more significant than the one that FM can bring. In fact, the Sharpe ratios (which indicates how well the risk is compensated by the return) for URTH and FM are respectively 0.4536 and 0.0475.

Examining *Figure 5.5* which shows the asset allocation of the portfolios including PE, it is possible to state that PE takes the place of equity in particular. To equity is again allocated the majority of the riskier portfolios, that are composed only of equity and PE. Both PE proxies enter the efficient portfolios for a growing allocation. To PSP is allocated a greater portion than PEX, consistently with the fact that PSP dominates PEX in the mean-variance sense. Then, in the riskier portfolios at the right of the graphic, the allocation to PE decreases drastically, because URTH provides a greater expected return and standard deviation than PSP and dominates PEX in the mean-variance sense. The same conclusions are achieved by observing *Table 5.5*. This table shows the compositions of ten efficient portfolios obtained combining the three asset classes. As mentioned earlier, only four ETFs are present in the efficient portfolios. To PSP, on average, is allocated the 16% of the portfolio and to PEX the 11%. Overall, to PE is allocated the 27% of the portfolio on average, while to URTH is allocated the 26% and to SHY the 47% on average.

On the contrary, the vertical cross-section at the far left represents the minimum-variance asset allocation, that is 100% composed of SHY in both efficient sets (as a matter of fact, this portfolio is one of the common points among the two frontiers).

Concluding, the presence of PE positively affects the performance of the portfolios combining the other assets. Evidently, these efficient frontiers have been built from historical returns, therefore the fact that PE prevails in the efficient portfolios is limited to the time window considered in this analysis (February 28, 2013 - December 31, 2016).

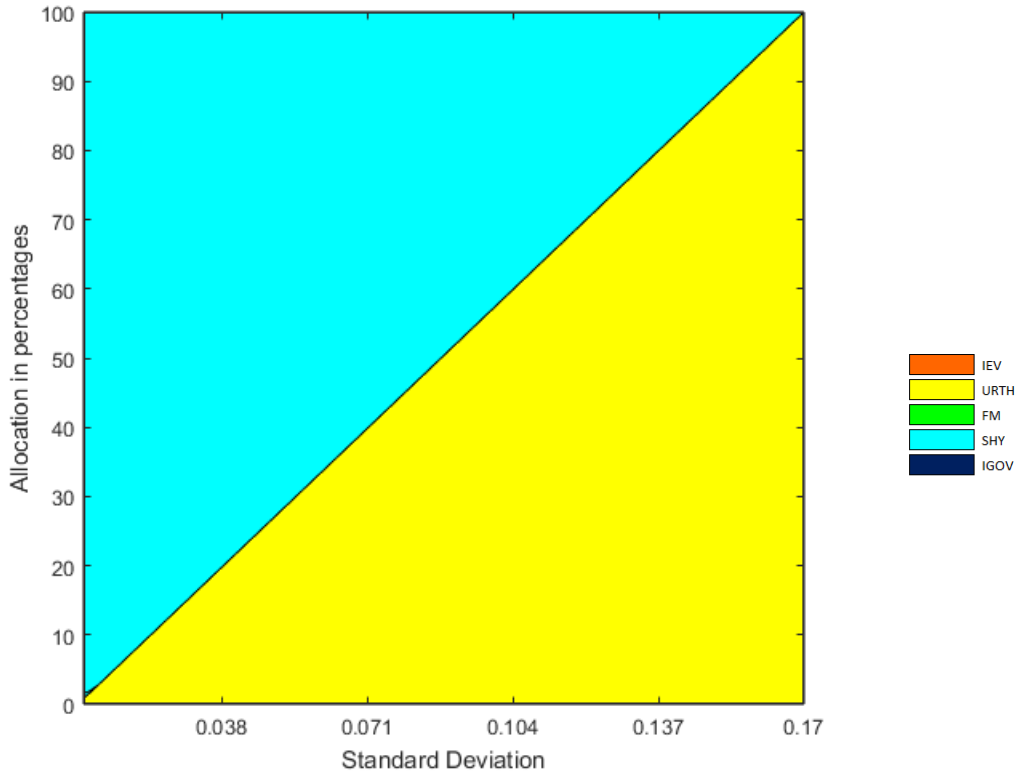

Figure 5.4: M-V asset allocation of efficient portfolios without PE

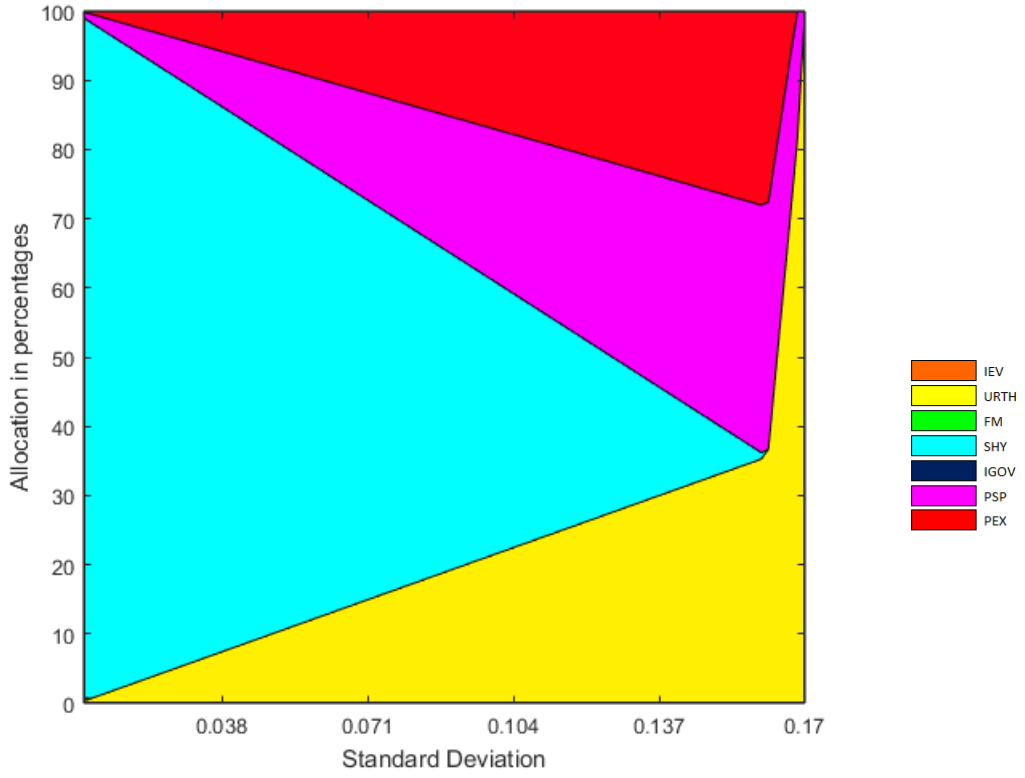

Figure 5.5: M-V asset allocation of efficient portfolios with PE

Table 5.5: Sample of ten efficient portfolios according to the M-V approach

	PE - 27%		URTH	SHY
	PSP - 16%	PEX - 11%	- 26%	- 47%
1	0.78%	0.19%	0.32%	98.71%
2	5.24%	3.16%	4.55%	87.05%
3	9.61%	6.18%	8.82%	75.40%
4	13.97%	9.20%	13.08%	63.75%
5	18.34%	12.21%	17.35%	52.10%
6	22.71%	15.23%	21.61%	40.45%
7	27.07%	18.24%	25.88%	28.80%
8	31.44%	21.26%	30.15%	17.15%
9	35.80%	24.28%	34.41%	5.51%
10	0.00%	0.00%	100%	0.00%

5.3.2 Normality assumption

The Markowitz Efficient Frontier is still one of the most widely implemented methods used to identify the efficient portfolios and then the optimal portfolio. Unfortunately, the model requires that specific assumptions are verified. In particular, a very important assumption that is often not verified involves the distribution of the returns, which has to be normal. A portfolio built from non-normal returns is likely to underestimate the real risk exposure of the portfolio.

However, it is possible to check whether the returns follow a normal distribution in several ways. Then, investors should choose whether to use another approach or build the portfolio using a Mean-Variance Optimization approach being aware that the risk might be underestimated.

First of all, a graphical analysis will set the basis for further investigations. *Figure 5.6* to *Figure 5.12* display the Q-Qplot and the empirical distribution for every security that is part of the current analysis. The Q-QPlot plots the sample quantiles against the theoretical quantiles that, in this case, come from a Normal distribution with the same mean and standard deviation of the sample returns. If the sample and theoretical quantiles come from the same distribution, then the blue points form a straight line. The red line, which passes through the first and third quartiles of the empirical distribution, indicates where the blue points should be if they were from the same distribution whose

quantiles are on the x axis. The fact that, for every ETF, the plots show deviations from a straight line, with special regard to the tails of the distributions, suggests that the normality assumption is not verified.

A second test involves the comparison between the empirical distribution of returns with the theoretical distribution. In this case, the theoretical distribution is normal with the same mean and standard deviation of the sample returns: in the graph, it represents how the distribution of returns should look like if it were normal. Again, the empirical distributions deviate from the theoretical ones. In general, all empirical distributions have heavy tails, thus they are leptokurtic. Moreover, all securities have a more severe peakedness than the normal distribution. Furthermore, IEV and PSP appear to be negatively skewed, since they have a left tail that is longer than the right one. URTH, FM, SHY, IGOV and PEX seems to be symmetric, but a more precise analysis will be implemented later. Anyway, SHY and IGOV are the assets whose returns approach the normal distribution the most.

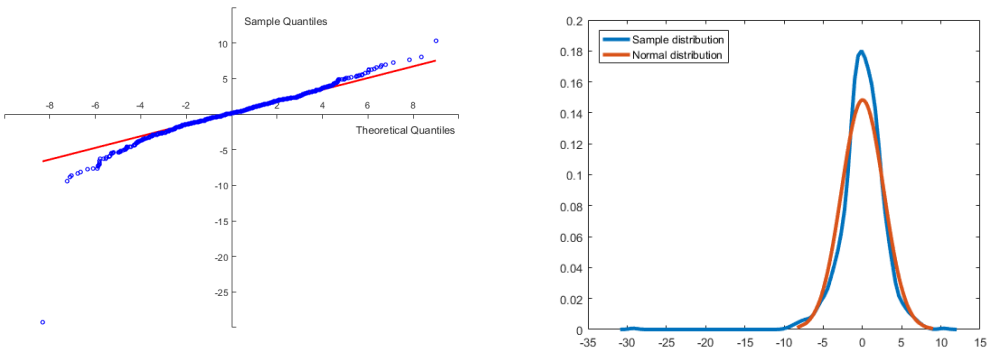

Figure 5.6: IEV: graphical normality test through the QQplot and the distribution of returns

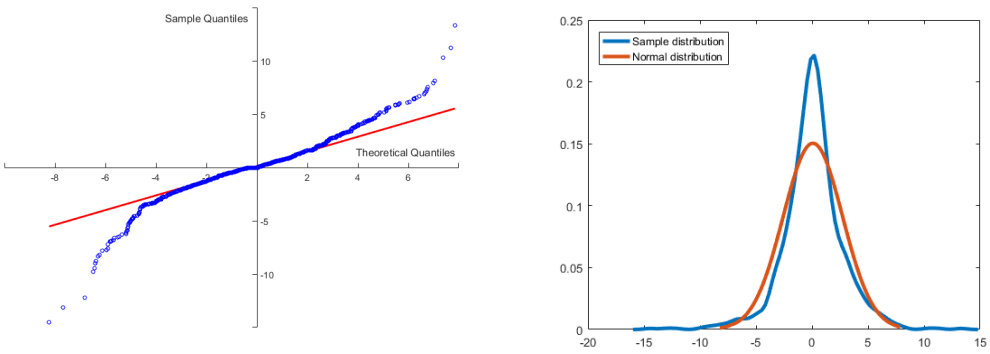

Figure 5.7: URTH: graphical normality test through the QQplot and the distribution of returns

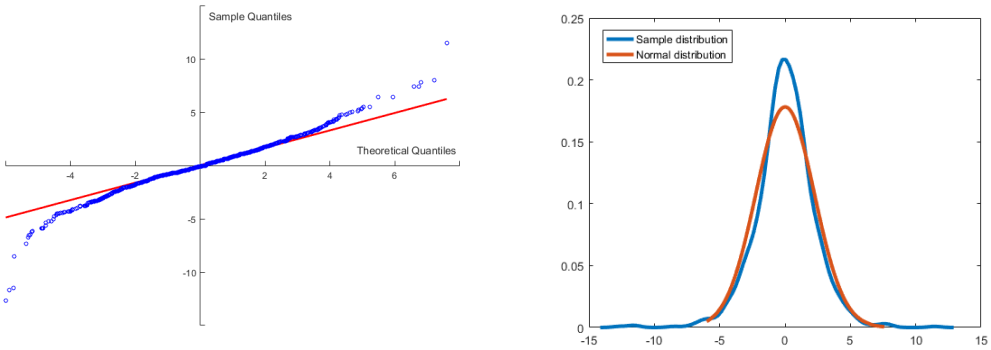

Figure 5.8: FM: graphical normality test through the QQplot and the distribution of returns

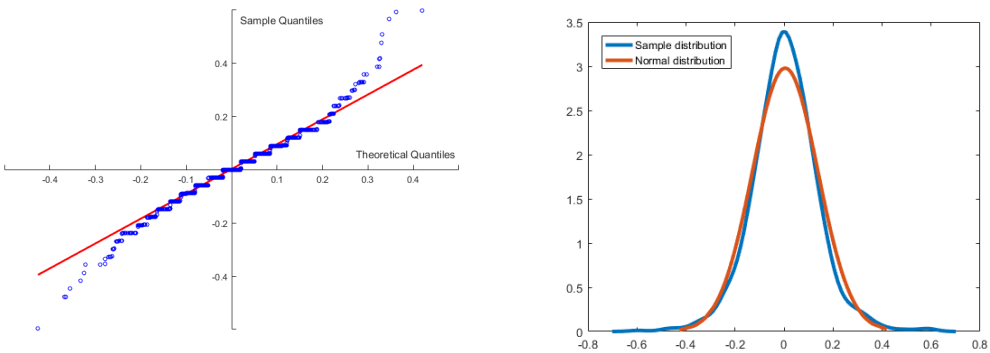

Figure 5.9: SHY: graphical normality test through the QQplot and the distribution of returns

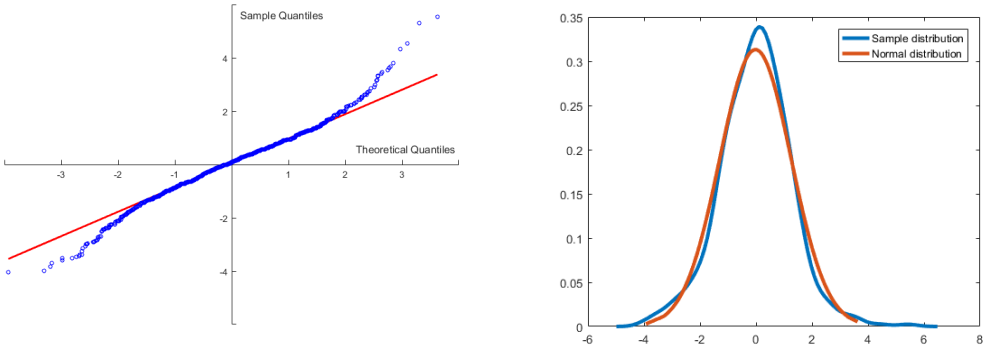

Figure 5.10: IGOV: graphical normality test through the QQplot and the distribution of returns

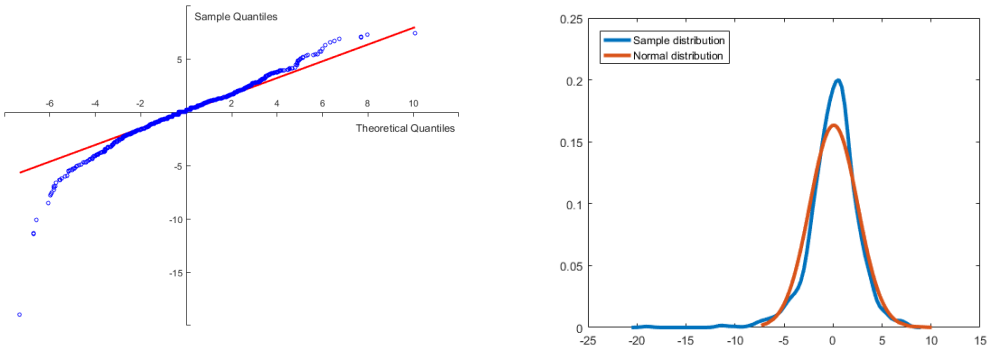

Figure 5.11: PSP: graphical normality test through the QQplot and the distribution of returns

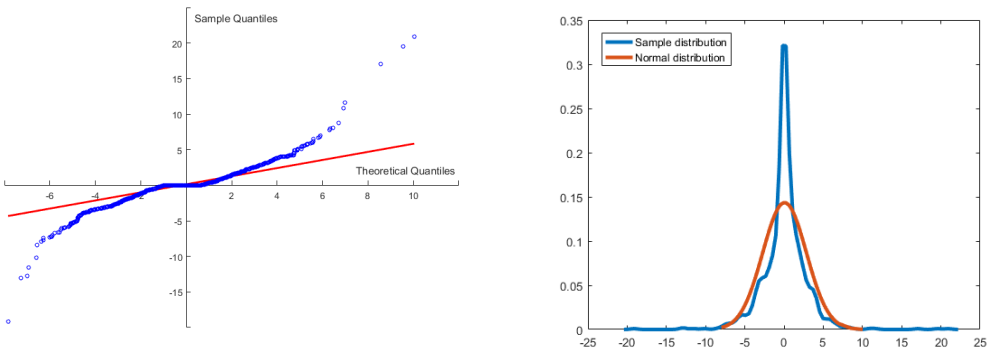

Figure 5.12: PEX: graphical normality test through the QQplot and the distribution of returns

In order to formally prove the non-normality of the available data, the Jarque-Bera Test will be implemented. This test measures the deviation of the available sample from a normal distribution in terms of skewness and kurtosis. The test statistic $JB \sim \chi^2(n)$ is

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \tag{5.8}$$

where *n* is the number of observations, *S* is the skewness and *K* is the kurtosis of the sample. The null hypothesis is that the data belong to a normal distribution, the alternative hypothesis is that they do not come from such a distribution. Matlab returns 1 if the test rejects the null hypothesis at the 5% significance level, and 0 otherwise. And these are the result obtained for the considered assets:

Table 5.6: Jarque-Bera test

	IEV	URTH	FM	SHY	PICB	PSP	PEX
Matlab output	1	1	1	1	1	1	1
p-value	0	0	0	0	0	0	0
JB stat	9031.7	555.2	515.5	166.6	50.3	1256.9	4658.9

The critical value for the test is 5.9257 and clearly all the test statistics are greater than it, so the null hypothesis is rejected. The same conclusion is delivered by the p-values, which are all equal to zero. Therefore, the test reject the null hypothesis at the 5% significance level and it can be stated that the selected securities do not have normally distributed returns.

Table 5.7 displays the values of the skewness and of the kurtosis of the empirical distributions. When the skewness approaches the value 0, then the distribution is symmetric, when the kurtosis approaches the value 3 then the distribution is mesokurtic: these values are peculiar of the normal distribution.

Table 5.7: Skewness and Kurtosis

	Skewness	Kurtosis
IEV	-1.501988	17.627
URTH	-0.2838851	6.6663
FM	-0.351715	6.505168
SHY	0.14132	5.012716
IGOV	0.05698	4.1115
PSP	-0.93775	8.257815
PEX	0.3919726	13.718937

According to the results reported in *Table 5.7* all the distributions are leptokurtic. The kurtosis value indicates the deviation of the shape of the distribution from the normal case. In particular, when the distribution is leptokurtic ($kurtosis > 3$) then the distribution has heavy tails. IGOV is almost normal with skewness 0.05 and kurtosis 4, but the Jarque-Bera test rejected the hypothesis of normality.

5.3.3 VaR and CVaR

The negative skewness is severe for IEV and PSP. URTH, SHY and IGOV are approximately symmetric. FM is moderately left skewed while PEX is moderately right skewed. All the distributions have heavy tails, in particular IEV and the PE proxies.

Especially when there are high values for the standard deviations involved, negatively skewed and heavy tailed distributions are matter of concern for investors, who are interested in minimazing the downside risk they are exposed to. There are two measures for monitoring the downside risk: the Value at Risk and the Expected Shortfall.

The Value at Risk is the worst potential loss (in other words, the maximum negative return) that a security can experience over a given period of time with a given confidence level $\alpha \in (0, 1)$. It is defined as the quantile of order $1-\alpha$ of the Profit/Loss distribution L, or as the smallest number l at the confidence level α such that the probability that the loss L exceeds l is at its maximum $(1 - \alpha)$.

$$VaR_\alpha = min\{l \in R : P(L > l) \leq 1 - \alpha\} \tag{5.9}$$

In this case, the chosen confidence levels are 1% and 5% and the Profit/Loss distribution L is calculated as

$$L_{t+1} = -Q \times (P_{t+1} - P_t) \tag{5.10}$$

where Q is the amount invested, equal to 1 in this example, and P_t is the price of the security at time t.

A second measure that is useful to estimate the downside risk is the Expected Shortfall, called Conditional Value at Risk as well. It is computed as the mean of the values of the Profit/Loss distribution which are greater than the VaR. In other words, it is the expected loss given that the loss is greater than the VaR.

$$ES_\alpha = E[L_{t+1}|L_{t+1} > VaR_\alpha] \tag{5.11}$$

According to the values for these downside risk measures in *Table 5.8*, PEX is the asset that is most exposed to downside risk, followed by IEV, URTH and PSP. Overall, SHY has the lowest values for both VaR and CVaR.

Table 5.8: VaR and ES

	VaR 5%	CVaR 5%	VaR 1%	CVaR 1%
IEV	0.0173	0.0257	0.029	0.0408
URTH	0.0146	0.0256	0.0309	0.0401
FM	0.0137	0.0208	0.0243	0.0333
SHY	0.0008	0.0012	0.0013	0.0017
IGOV	0.0087	0.0114	0.0134	0.0145
PSP	0.0162	0.0242	0.0280	0.0388
PEX	0.0165	0.0259	0.0289	0.0418

However, the single VaR and CVaR figures are not sufficient to state whether a security is likely to be left out from the efficient portfolio, since its role in the optimization is that of the standard deviation. Again, the CVaR has to be plotted against the mean of returns and then the risk-return profile of each asset can be judged. To this purpose, *Figure 5.13* shows the positions of the assets in the Mean-Expected Shortfall plane.

The disposition of the assets is quite similar to the Mean-Variance plane, but some small differences can be spotted among the riskier assets PSP, URTH and PEX. Anyway, from this graph, the same conclusions can be obtained: SHY, PSP, URTH and PEX

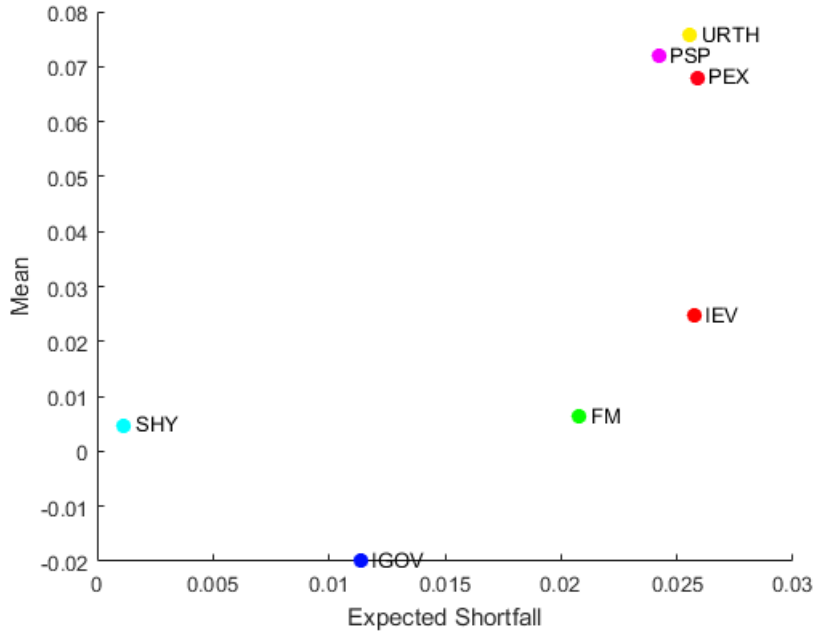

Figure 5.13: Mean and Expected Shortfall; confidence level 5%

are likely to enter the efficient portfolios. Thus, their weights in the portfolios are what is likely to change.

5.3.4 Mean-CVaR Portfolio Optimization

The Mean-CVaR Optimization approach relies on a concept similar to the Mean-Variance Optimization approach: maximizing the expected return for a given level of risk. In this case, the risk measure is the Expected Shortfall or Conditional VaR. The benefit of using the ES is that it does not equally penalize positive and negative returns and, in this sense, it is a better option that the Mean-Variance Optimization technique since the latter takes into consideration only the first two moments of a distribution. Moreover, the CVaR approach does not need the assumption that the return must be normally distributed.

As in the Mean-Variance Portfolio analysis, the comparison between the efficient frontiers built with and without PE is displayed in *Figure 5.14* and *5.15*. Again, PE increases the performance of efficient portfolios with respect to those portfolios that do not include PE among their asset classes.

It is interesting to see if there are differences between the efficient frontiers obtained with the two methods. In order to implement this analysis, the standard deviation of the Mean-CVaR efficient portfolios are estimated from the weights of these portfolios and the standard deviations of the assets. Then, the resulting efficient frontier is plotted together with the Mean-Variance efficient frontier. Anyway, from *Figure 5.16* and *5.17*, there are no significant differences. Nevertheless, it is possible to spot a small difference for the portfolios characterized by a higher standard deviation, in particular when the confidence level is 1%. The risk-return profile of these portfolios seems to be better for the Mean-Variance Portfolios. It is noteworthy that, obviously, the Mean-CVaR efficient portfolios are not efficient in the Mean-Variance sense. A more informative comparison is between the expected returns of the two efficient sets. And, as it can be observed from *Table 5.9*, the expected returns of the two sets are similar but the M-V portfolios have higher expected returns for a given level of standard deviation. This result is obvious considering that the M-V approach maximizes the expected return fixing the volatility. Therefore, the M-CVaR approach cannot deliver a higher expected return. A similar reasoning can be done for the risk measure. *Table 5.10* and *5.11* shows the standard deviations and the expected shortfalls of ten efficient portfolios obtained using the two approaches and by fixing the expected return. The volatility is slightly smaller for the M-V portfolios and the CVaR is slightly smaller for the M-CVaR portfolios. The differences are expected to increase when using a confidence level of 1%, as suggested by *Figure 5.17*. The preferred approach is selected depending on what the investor is interested in minimizing (volatility or expected shortfall) or maximizing (return).

Table 5.9: Comparison of the returns of the portfolios obtained using the two approaches and fixing the standard deviation.

	M-V	M-CVaR 5%
1	0.0062	0.0059
2	0.0136	0.0136
3	0.0216	0.0214
4	0.0294	0.0292
5	0.0372	0.0369
6	0.0450	0.0447
7	0.0528	0.0525
8	0.0607	0.0602
9	0.0685	0.0680
10	0.0758	0.0758

Table 5.10: Comparison of the standard deviation of the portfolios obtained using the two approaches and fixing the target return.

	M-V	M-CVaR 5%
1	0.0081	0.0082
2	0.0178	0.0178
3	0.0329	0.0332
4	0.0486	0.0492
5	0.0645	0.0651
6	0.0805	0.0812
7	0.0965	0.0973
8	0.1126	0.1135
9	0.1286	0.1297
10	0.1671	0.1671

Table 5.11: Comparison of the CVaR of the portfolios obtained using the two approaches and fixing the target return with 5% of confidence.

	M-V	M-CVaR 5%
1	0.00112	0.00111
2	0.002554	0.00253
3	0.005002	0.00496
4	0.007539	0.00749
5	0.010085	0.01002
6	0.01264	0.01256
7	0.015196	0.0151
8	0.017752	0.017645
9	0.020308	0.020192
10	0.02533	0.02533

Table 5.12: Sample of ten efficient portfolios according to the M-CVaR approach - 5% confidence level

	PE - 26%		URTH	SHY
	PSP - 11%	PEX - 15%	- 28%	- 47%
1	0.58%	0.00%	0.97%	98.44%
2	5.13%	3.27%	4.82%	86.78%
3	6.60%	8.99%	9.48%	74.94%
4	8.93%	13.32%	14.51%	63.24%
5	11.76%	16.64%	19.93%	51.67%
6	14.61%	19.98%	25.32%	40.09%
7	17.62%	24.11%	29.88%	28.38%
8	20.04%	28.20%	35.04%	16.72%
9	22.19%%	32.36%	40.39%	5.06%
10	0.00%	0.00%	100%	0.00%

Table 5.13: Sample of ten efficient portfolios according to the M-CVaR approach - 1% confidence level

	PE - 21%		URTH	SHY
	PSP - 6%	PEX - 15%	- 32%	- 47%
1	0.00%	0.00%	1.57%	98.43%
2	1.90%	3.90%	7.37%	86.82%
3	2.77%	8.22%	13.79%	75.22%
4	3.76%	12.57%	20.07%	63.60%
5	6.06%	16.91%	25.11%	51.91%
6	8.69%	21.25%	29.85%	40.21%
7	9.95%	25.60%	35.87%	28.58%
8	11.08%	29.94%	42.03%	16.95%
9	12.20%%	34.29%	48.18%	5.33%
10	0.00%	0.00%	100%	0.00%

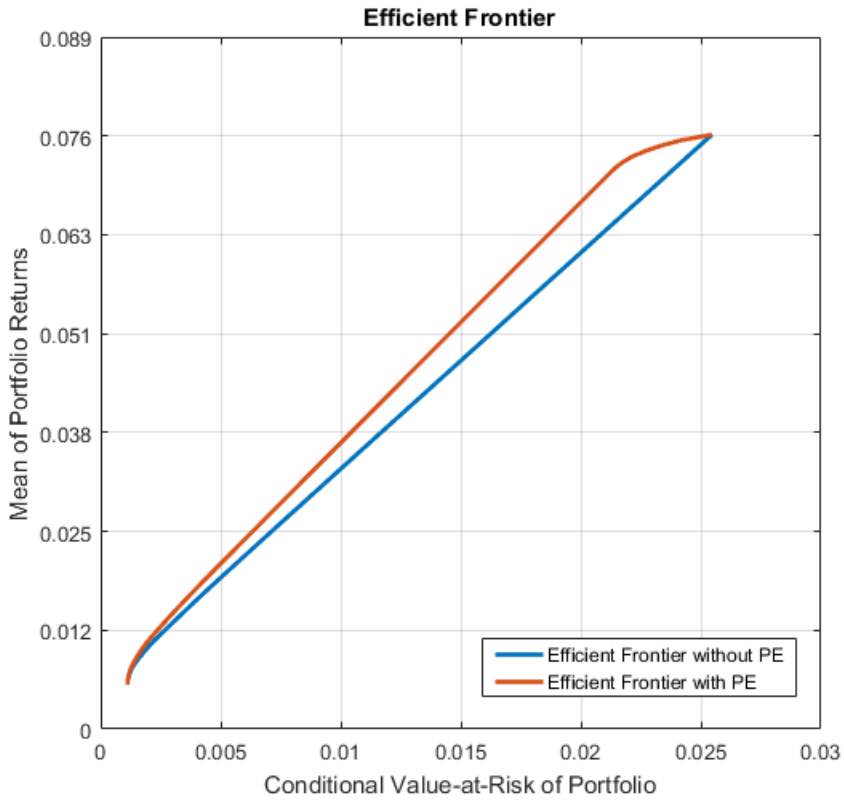

Figure 5.14: Mean-CVaR Efficient Frontiers with and without PE; confidence level 5%

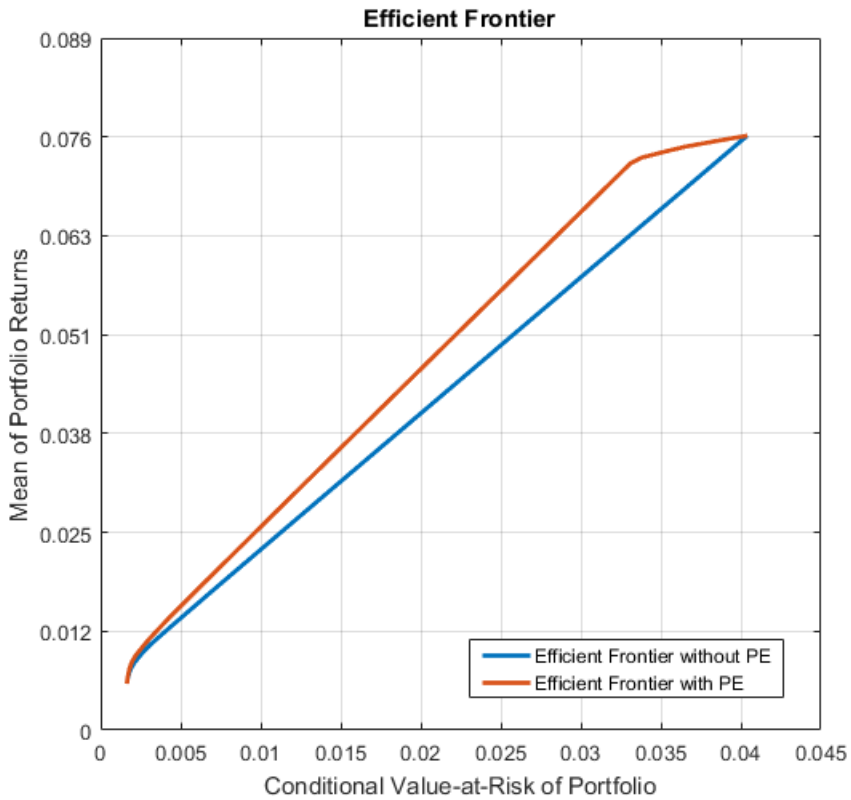

Figure 5.15: Mean-CVaR Efficient Frontiers: with and without PE; confidence level 1%

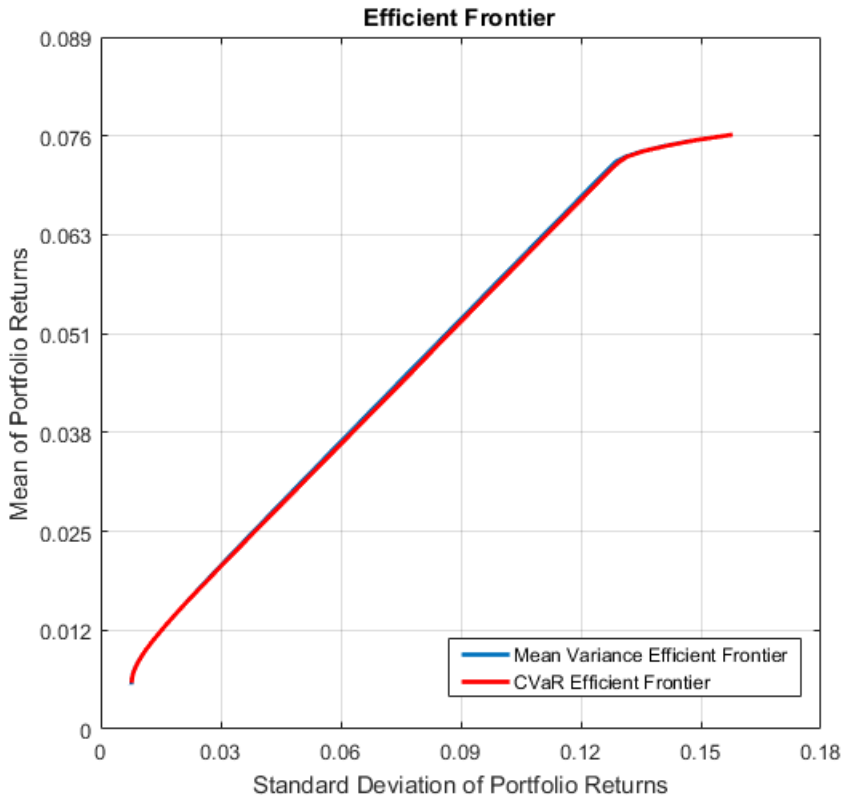

Figure 5.16: Comparison between Efficient Frontiers; confidence level 5%

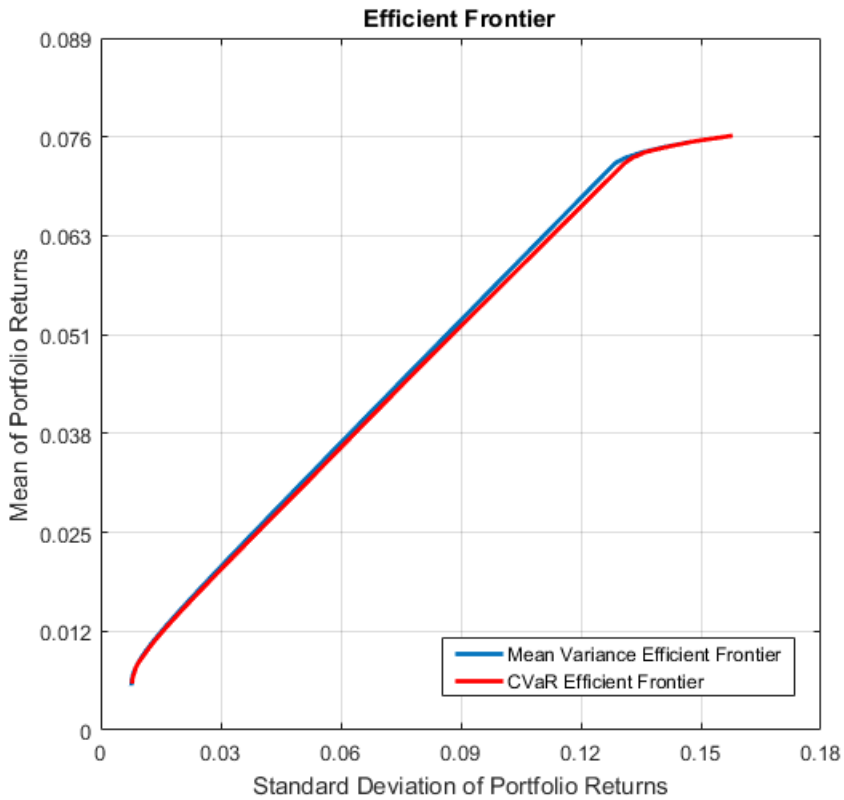

Figure 5.17: Comparison between Efficient Frontiers; confidence level 1%

Another way to compare the results, is to look at the weights of the efficient portfolios, to see if and where there are differences in the asset allocation between the efficient portfolios, in particular with regard to the role of PE. Recalling the analysis of kurtosis and skewness, PSP is negatively skewed and leptokurtic, therefore its weight is expected to decreases in favor of PEX and URTH. In fact, PEX is leptokurtic but positively skewed, while URTH is approximately symmetric and less leptokurtic than the other two securities. For its characteristics, to URTH is expected to be allocated a greater portion of portfolio.

Figure 5.19, *5.20* and *5.21* compare the weights of the assets including PE of the efficient portfolios obtained using the Mean-Variance Optimization and the Mean-CVaR Optimization approaches, with confidence level 1% and 5%. The relevant difference, as suggested above, is the decreasing allocation to PSP and the growing allocation to URTH and PEX. This difference with the M-V Portfolio weights is due to the fact that the returns are not normally distributed and the CVaR technique penalizes negative skewness and excess kurtosis.

Table 5.5, *5.13* and *5.12* compare the different allocations obtained using the two approaches and using two confidence levels. As suggested by the graphical analysis, the average allocation to URTH increases from a 26% in the M-V portfolio, to a 28% in the M-CVaR portfolio (5% confidence level) to a final 32% in the M-CVaR portfolio (1% confidence level). The average percentage allocated to PEX grows from 11% in the M-V portfolio to a 15% in both the M-CVaR portfolios. On the contrary, and as mentioned earlier, the allocation to PSP decreases from a 16% in the M-V portfolio to 11% and to 6% in the M-CVaR portfolios with 5% and 1% confidence level respectively. Overall, the allocation to PE decreases on average.

From the point of view of an investor, the CVaR portfolio can represent a better option since the investor commonly looks positively to positive returns and negatively to negative returns (which are losses), while the M-V approach does not make any difference between losses and profits.

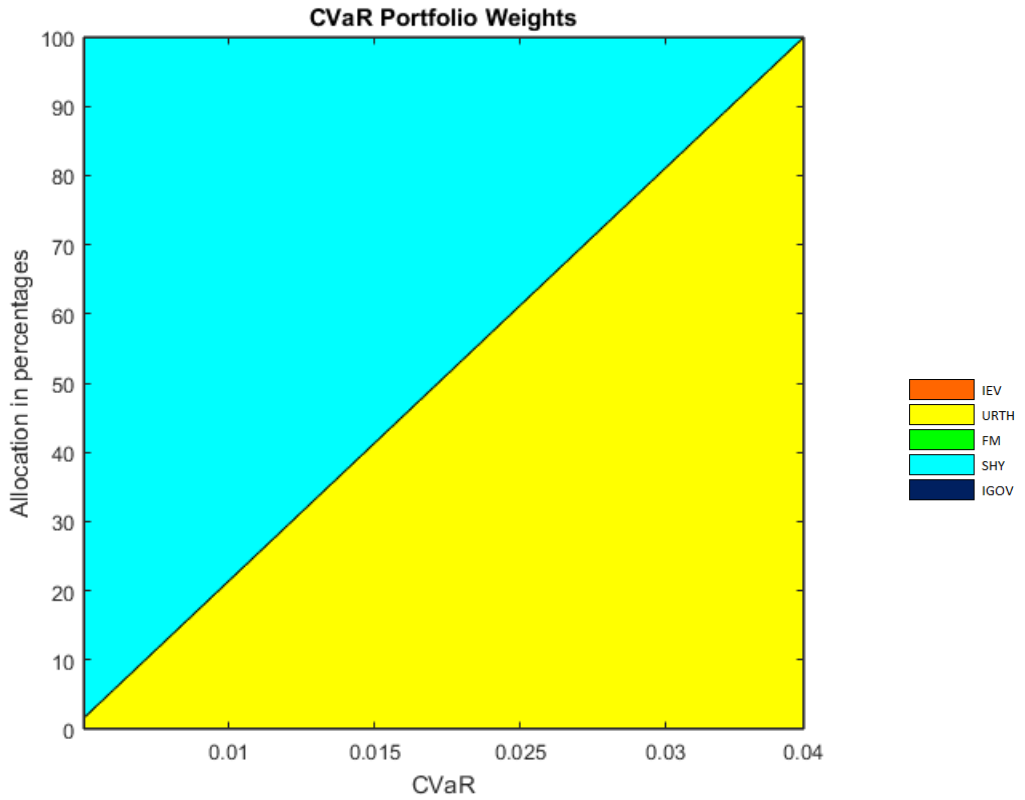

Figure 5.18: M-CVaR asset allocation of the efficient portfolios without PE; confidence level 5% and 1%

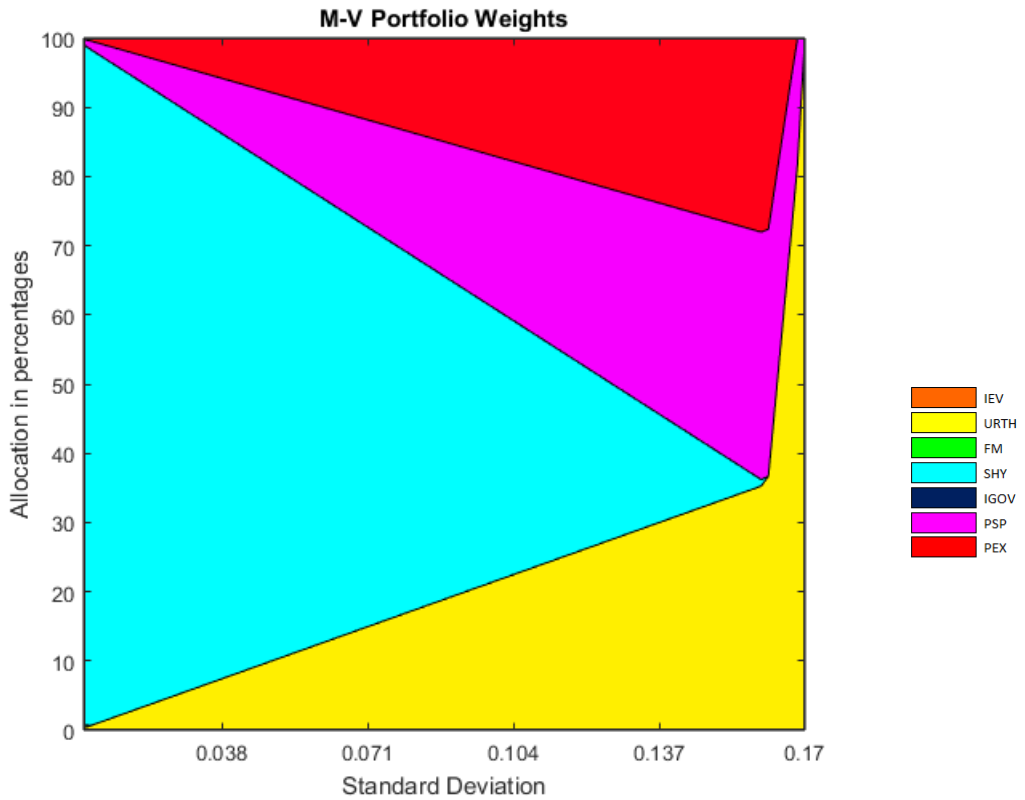

Figure 5.19: M-V asset allocation of the efficient portfolios with PE

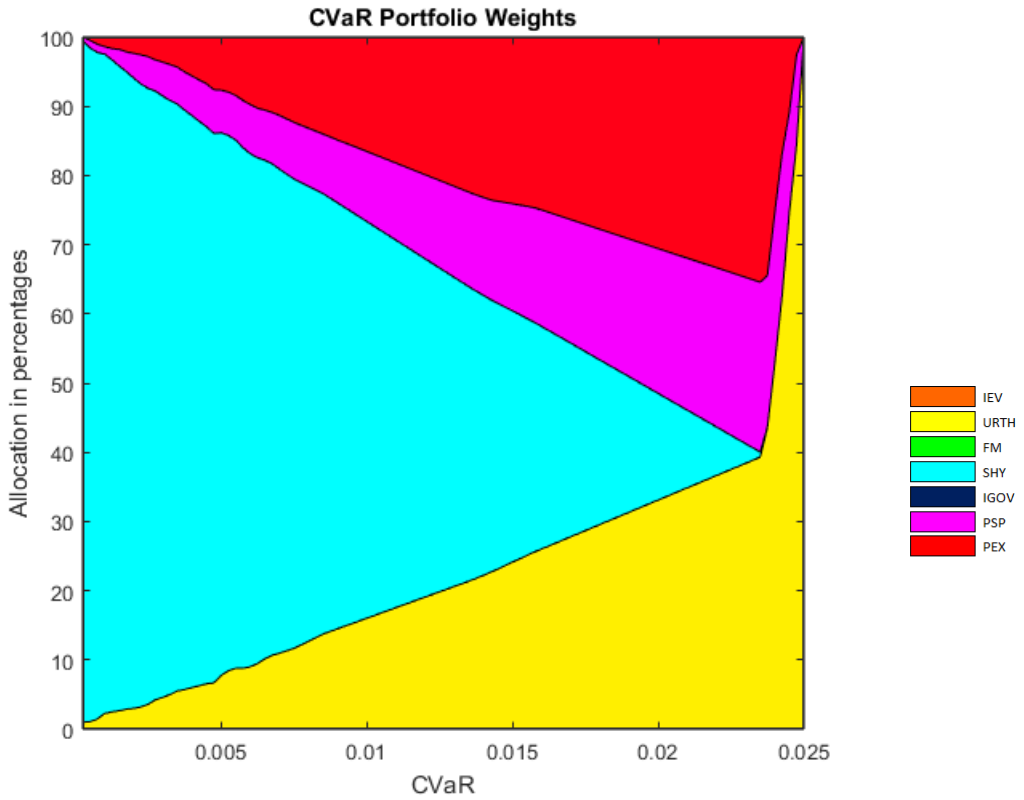

Figure 5.20: M-CVaR asset allocation of efficient portfolios with PE; confidence level 5%

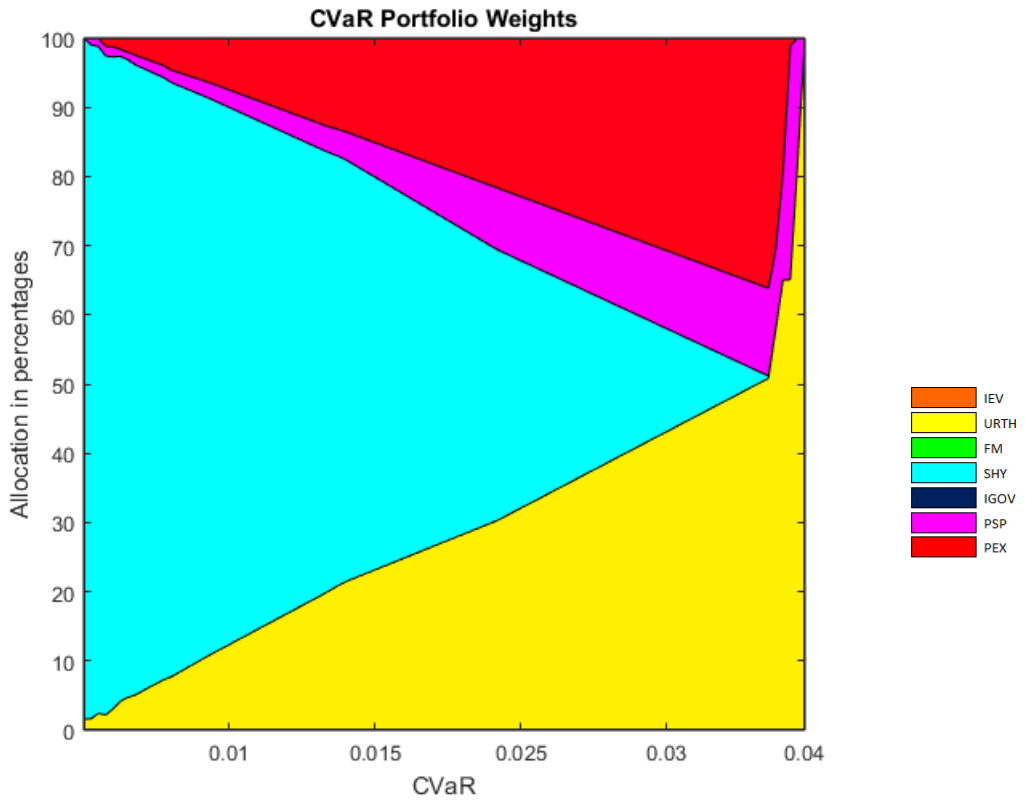

Figure 5.21: M-CVaR asset allocation of the efficient portfolios with PE; confidence level 1%

5.3.5 Mean-MAD Portfolio Optimization

In Matlab it is possible to build the efficient frontier also using the Mean Absolute Deviation (MAD) of the portfolio. The approach exploits again the concept of the Markowitz model: minimizing the risk for a given level of expected return.

Firstly, the MAD values of the selected ETFs are computed according to the following equation:

$$MAD = \frac{1}{N} \sum_{i=1}^{N} |r_i - \overline{r}| \tag{5.12}$$

where $\overline{r}$ is the mean of the N returns of the considered ETF. Therefore, the MAD is the arithmetic mean of the absolute values of the differences between the observed returns and their mean. The MAD values for the selected ETFs are displayed in *Table 5.14*.

Since ranking the ETFs according to the MAD delivers different placements in

Table 5.14: Mean Absolute Deviation

	MAD
IEV	0.1214
URTH	0.1166
FM	0.1029
SHY	0.0063
IGOV	0.0614
PSP	0.1119
PEX	0.1081

terms of risk-adjusted return, the expected results should differ from those of the Mean Variance optimization in terms of efficient frontier and asset allocation. From *Figure 5.22*, it is noteworthy that both PEX and PSP are efficient, therefore the allocation to PEX is expected to be greater than the one obtained in the Mean-Variance framework.

Anyway, the inclusion of PE should improve the risk-adjusted return of the efficient portfolios. In order to test it, the comparison of the efficient frontiers of the portfolios including and not including PE will answer the question. As can be observed in *Figure 5.23*, the inclusion of PE improves the efficient frontier.

It is again interesting to compare the efficient frontiers obtained with the Mean-Variance and with the Mean-MAD portfolio optimization methods on the same plane, in order to judge whether there actually is a difference between the two approaches.

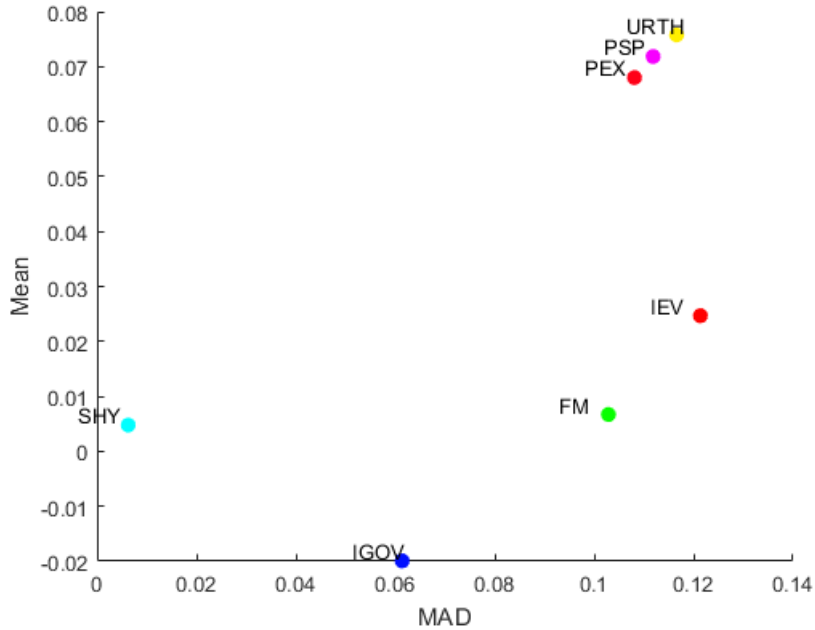

Figure 5.22: Graphical comparison of the mean and MAD of the ETFs

Since the returns are not normal, the expectation is that the efficient portfolios obtained with the two methods are different. However, *Figure 5.24* shows that the efficient frontiers appear to overlap. From the comparison between the asset allocation of the M-V portfolios in *Figure 5.5* and the asset allocation of the M-MAD portfolios in *Figure 5.26*, there are some differences in the weights allocated to PE: specifically, the portion allocated to PEX increases. This result can be obtained comparing *Table 5.5* and *5.16*, which report the allocation of ten efficient portfolios obtained using the two approaches: while the average allocations to PE, URTH and SHY do not change, the average allocations to PSP and PEX are different. PSP decreases from 16% (M-V) to 14% (M-MAD) while PEX increases from 11% (M-V) to 13% (M-MAD). Therefore, the expected returns of the portfolios belonging to the two efficient frontiers should differ at least to a small extent. Through a comparison of the portfolios returns implemented in Matlab, the MAD portfolios deliver a smaller expected return, as observable in *Table 5.15*.

Table 5.15: Comparison of the returns of the portfolios obtained using the two approaches and fixing the standard deviation

	M-V	M-MAD
1	0.0062	0.0057
2	0.0136	0.0135
3	0.0216	0.0213
4	0.0294	0.0291
5	0.0372	0.0368
6	0.0450	0.0446
7	0.0528	0.0524
8	0.0607	0.0602
9	0.0685	0.0680
10	0.0758	0.0758

Table 5.16: Sample of ten efficient portfolios according to the M-MAD approach

	PE - 27%		URTH	SHY
	PSP - 14%	PEX - 13%	- 26%	- 47%
1	1.00%	0.09%	0.26%	98.65%
2	4.22%	3.89%	4.94%	86.95%
3	8.70%	7.00%	9.02%	75.29%
4	12.60%	10.61%	13.20%	63.58%
5	16.17%	14.37%	17.58%	51.88%
6	20.01%	17.97%	21.84%	40.18%
7	23.44%	21.65%	26.42%	28.50%
8	26.81%	25.26%	31.11%	16.82%
9	30.40%%	29.13%	35.37%	5.10%
10	0.00%	0.00%	100%	0.00%

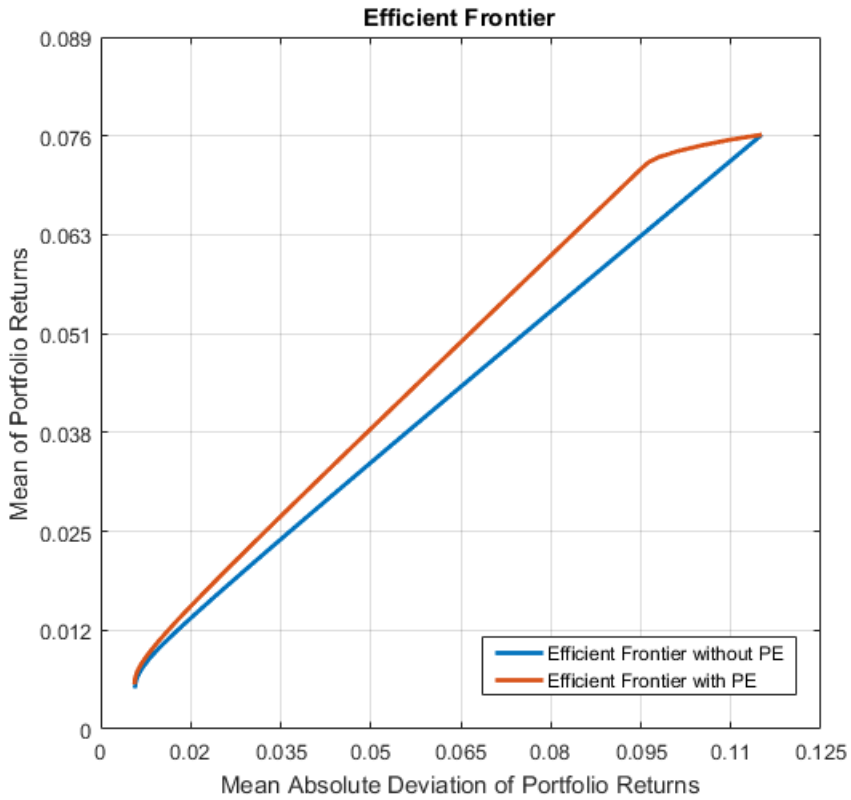

Figure 5.23: M-MAD: Efficient Frontiers with and without PE

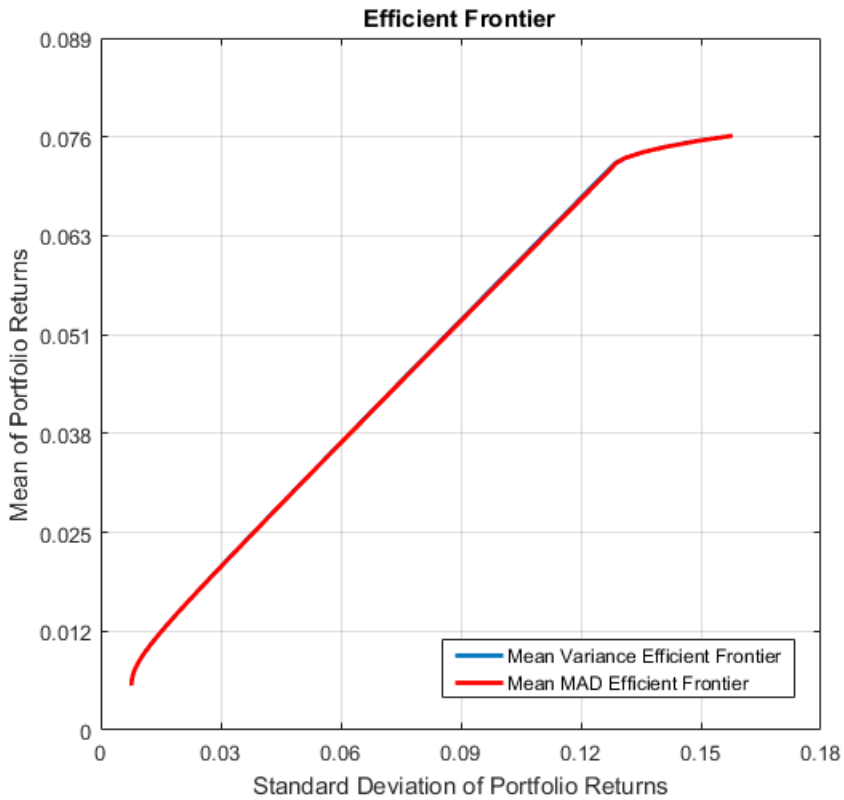

Figure 5.24: Comparison of the efficient frontiers obtained with the M-V and the M-MAD portfolio optimization approaches

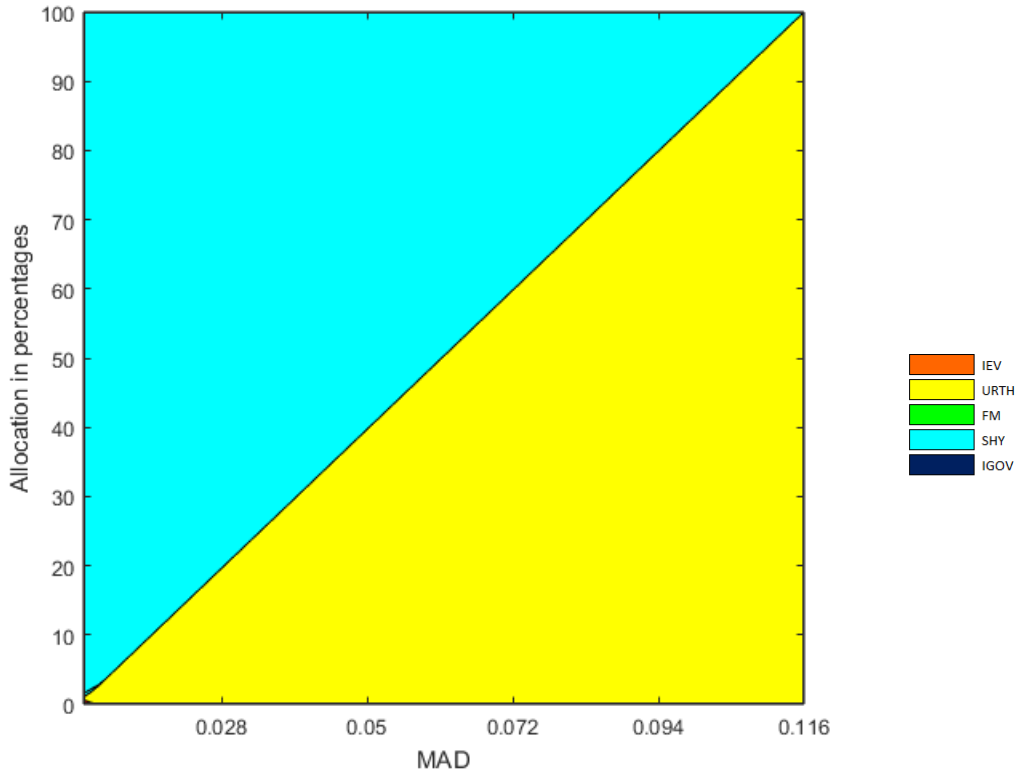

Figure 5.25: Mean-MAD asset allocation without PE

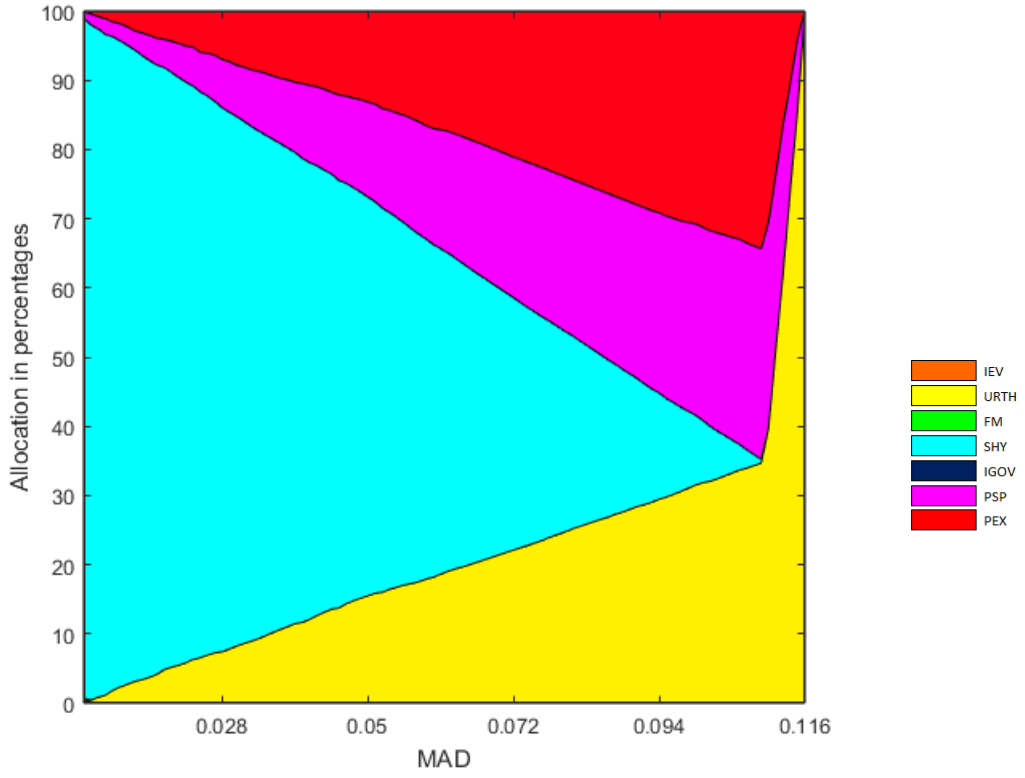

Figure 5.26: Mean-MAD asset allocation with PE

5.4 Final Remarks

At the end of the empirical analysis, the results that have been obtained confirm the thesis that including PE in a diversified portfolio is porfitable. As in the papers of Idzorek (2007) and Milner and Vos (2003), whose concept has been followed, the results are as follows:

- the correlations of PE with other asset classes is positive but lower than one, suggesting that it can increase the effect of diversification;
- the M-V efficient frontier of portfolios that includes PE is better than the M-V efficient frontier of portfolios that does not include PE.

After having assessed that the asset classes' proxies do not have normally distributed returns, other optimization techniques that do not make this assumption have been employed. In particular, these alternative methods are the M-CVaR and the M-MAD optimization approaches. The purpose was to evaluate whether, using an approach that is not based on the normality of returns, the PE asset class enters a diversified portfolio by the same proportion. To this end, the tables, one for each approach, reporting the compositions of ten efficient portfolios are displayed in *Table 5.17*, *5.18* and *5.19*. The portfolios across the three tables deliver the same rate of return. The M-CVaR portfolios have been created using a 5% confidence level, since, as it has been observable in *Section 3*, a confidence level of 1% drastically reduces the portion of PE in the efficient portfolios.

Anyway, it is clear from these tables that the M-CVaR approach is the one that penalizes the most the PE asset class: the allocation to PSP decreases by a maximum of 13% in the ninth portfolio with respect to the M-V approach and PEX, with a maximum gain of 8%, is not able to entirely recover the allocation lost by PSP. On the contrary, the M-MAD portfolios allocate to PE almost the same portions as the M-V portfolios: despite the different allocations to PSP and PEX, the total allocation to PE is almost the same of the M-V portfolios. Overall, the M-V portfolio optimization approach is the one that less penalizes the PE asset class.

Table 5.17: Sample of ten efficient portfolios according to the M-V approach

	PSP	*PEX*	PE	URTH	SHY
1	*0.78%*	*0.19%*	0.97%	0.32%	98.71%
2	*5.24%*	*3.16%*	8.40%	4.55%	87.05%
3	*9.61%*	*6.18%*	15.79%	8.82%	75.40%
4	*13.97%*	*9.20%*	23.17%	13.08%	63.75%
5	*18.34%*	*12.21%*	30.55%	17.35%	52.10%
6	*22.71%*	*15.23%*	37.93%	21.61%	40.45%
7	*27.07%*	*18.24%*	45.32%	25.88%	28.80%
8	*31.44%*	*21.26%*	52.70%	30.15%	17.15%
9	*35.80%*	*24.28%*	60.08%	34.41%	5.51%
10	*0%*	*0%*	0%	100%	0%

Table 5.18: Sample of ten efficient portfolios according to the M-CVaR approach - 5% confidence level

	PSP	*PEX*	PE	URTH	SHY
1	*0.58%*	*0.00%*	0.58%	0.97%	98.44%
2	*5.13%*	*3.27%*	8.40%	4.82%	86.78%
3	*6.60%*	*8.99%*	15.59%	9.48%	74.94%
4	*8.93%*	*13.32%*	22.25%	14.51%	63.24%
5	*11.76%*	*16.64%*	28.40%	19.93%	51.67%
6	*14.61%*	*19.98%*	34.59%	25.32%	40.09%
7	*17.62%*	*24.11%*	41.73%	29.88%	28.38%
8	*20.04%*	*28.20%*	48.24%	35.04%	16.72%
9	*22.19%*	*32.36%*	54.55%	40.39%	5.06%
10	*0%*	*0%*	0%	100%	0%

Table 5.19: Sample of ten efficient portfolios according to the M-MAD approach

	PSP	*PEX*	PE	URTH	SHY
1	*1.00%*	*0.09%*	1.09%	0.26%	98.65%
2	*4.22%*	*3.89%*	8.11%	4.94%	86.95%
3	*8.70%*	*7.00%*	15.70%	9.02%	75.29%
4	*12.60%*	*10.61%*	23.21%	13.20%	63.58%
5	*16.17%*	*14.37%*	30.54%	17.58%	51.88%
6	*20.01%*	*17.97%*	37.98%	21.84%	40.18%
7	*23.44%*	*21.65%*	45.09%	26.42%	28.50%
8	*26.81%*	*25.26%*	52.07%	31.11%	16.82%
9	*30.40%*	*29.13%*	59.53%	35.37%	5.10%
10	*0%*	*0%*	0%	100%	0%

5.5 APPENDIX

5.5.1 Matlab Codes

Here are the codes to get the prices of the ETFs from Yahoo Finance and the codes to get the returns from the prices. Firstly, the PE asset class is not considered in order to estimate the efficient frontier without PE.

```
c=yahoo;
startDate='02/28/13';
endDate='12/31/16';
tickers={'IEV','URTH','FM','SHY','IGOV'};

for i=1:length(tickers)
    Price.(tickers{i})=fetch(c,tickers{i},'Adj Close',startDate,
    endDate,'d');
    temp=Price.(tickers{i});
    ClosePrice(:,i)=flipud(temp(:,2));
end
close(c);

Returns=price2ret(ClosePrice); %Matrix of returns of the ETFs
```

Once created the matrix of returns of the ETFs, it is possible to estimate the efficient frontier according to the Markowitz's model.

```
%Build the efficient portfolios in the M-V framework
pmv=Portfolio;
pmv=pmv.setAssetList(tickers);
pmv=pmv.estimateAssetMoments(Returns);
pmv=pmv.setDefaultConstraints;
pmvwgts=pmv.estimateFrontier(100);

figure; %Asset allocation without PE
colormap hsv
area(pmvwgts');
legend(pmv.AssetList);
xlabel('Standard Deviation')
ylabel('Allocation in percentages')

figure; %Efficient frontier without PE
pmv.plotFrontier(100);
hold on %keep the figure open to plot the efficient
%frontier with PE on the same graph
```

Now, the matrix of returns has to be computed again but including PE.

```
%Build again the matrix of returns with PE this time
tickers={'IEV','URTH','FM','SHY','IGOV','PSP','PEX'};

for i=1:length(tickers)
    Price.(tickers{i})=fetch(c,tickers{i},'Adj Close',startDate,
    endDate,'d');
    temp=Price.(tickers{i});
    ClosePrice(:,i)=flipud(temp(:,2));
end
close(c);
```

```
Returns=price2ret(ClosePrice);
```

Then, the Mean-Variance optimization has to be implemented again using the new returns matrix. The efficient frontier of portfolios including PE is plotted on the same graph of the efficient frontier of portfolios without PE.

```
%Estimate the new efficient portfolios with PE
pmv=Portfolio;
pmv=pmv.setAssetList(tickers);
pmv=pmv.estimateAssetMoments(Returns);
pmv=pmv.setDefaultConstraints;

pmv.plotFrontier(100) %plot the efficient frontier
legend('Efficient Frontier without PE','Efficient Frontier with PE')
hold off

pmvwgts=pmv.estimateFrontier(100);
figure; %asset allocation with PE
colormap hsv
area(pmvwgts');
legend(pmv.AssetList);
xlabel('Standard Deviation')
ylabel('Allocation in percentages')
```

This thesis aims at comparing the efficient frontiers obtained with and without PE using different risk measures. The steps of the M-V analysis have to be followed with the other approaches. The following codes refer to the M-MAD optimization.

```
%M-MAD portfolio
%Again, to see the how the portfolios with and without PE differ,
%these codes have to be run twice with a dataset without PE and
%then with PE
pmm=PortfolioMAD;
pmm=pmm.setAssetList(tickers);
```

```
pmm=pmm.setScenarios(Returns);
pmm=pmm.setDefaultConstraints;

figure; %efficient frontier
pmm.plotFrontier(100);

pmmwgts=pmm.estimateFrontier(100);
figure; %asset allocation
colormap hsv
area(pmmwgts');
legend(pmm.AssetList);
xlabel('MAD')
ylabel('Allocation in percentages')
```

The following codes refer to the M-CVaR optimization using a confidence level of 5%.

```
%M-CVaR portfolio
%Again, to see the how the portfolios with and without PE differ,
%these codes have to be run twice with a dataset without PE and
%then with PE
pmc=PortfolioCVaR;
pmc=pmc.setAssetList(tickers);
pmc=pmc.setScenarios(Returns);
pmc=pmc.setDefaultConstraints;
pmc=pmc.setProbabilityLevel(0.95);
pmcwgts=pmc.estimateFrontier(100);

figure; %asset allocation
colormap hsv
area(pmcwgts');
title('CVaR Portfolio Weights');
legend(pmc.AssetList);
xlabel('CVaR')
ylabel('Allocation in percentages')
```

```
figure; %efficient frontier
pmc.plotFrontier(100);
```

The following codes are used to get the allocations of the efficient frontiers

```
pmvwgts=pmv.estimateFrontier(10);
pmcwgts=pmc.estimateFrontier(10);
pmmwgts=pmm.estimateFrontier(10);
```

Lastly, the following codes are used to compare the returns and the risk measures obtained with the three approaches. Here the codes for the comparison between M-V and M-CVaR is provided.

```
%fix the return, compare SD and CVaR
pmvret=pmv.estimatePortReturn(pmvwgts);
pmcwgts1=pmc.estimateFrontierByReturn(pmvret);

pmvsd1=pmv.estimatePortRisk(pmvwgts);
pmcsd1=pmv.estimatePortRisk(pmcwgts1);

pmvcvar=pmc.estimatePortRisk(pmvwgts);
pmccvar=pmc.estimatePortRisk(pmcwgts1);

%fix the sd, compare the return
pmcsd=pmv.estimatePortRisk(pmcwgts);
pmvwgts1=pmv.estimateFrontierByRisk(pmcsd);

pmvret1=pmv.estimatePortReturn(pmvwgts1);
pmcret=pmv.estimatePortReturn(pmcwgts);
```

Another important analysis is the analysis of the ETFs, in particular of their annualized means, standard deviations and correlations.

```
%Plot the prices of the ETFs over time
[row1,col1]=size(ClosePrice);
```

```
alternative=zeros(row1,col1);
alternative(1,:)=100;
for i=1:col1
    for l=1:(row1-1)
        alternative(l+1,i)=alternative(l,i)*exp(Returns(l,i));
    end
end
plot(alternative);

%Calculations of the mean, standard deviation, correlations,
%covariances, skewness, kurtosis and CAGR
[row,col]=size(Returns);
CAGR=zeros(length(tickers),0);
for i=1:length(tickers)
    CAGR(i)=(ClosePrice(row,i)/ClosePrice(1,i))^(1/4)-1;
end
Mean_ret=mean(Returns)*252;
SD_ret=sqrt(var(Returns)*252);
corr_matrix=corr(Returns);
skew_ret=skewness(Returns);
kurt_ret=kurtosis(Returns);
cov_matrix=cov(Returns)*(252);
```

The MAD and CVaR are estimated as follows:

```
MAD_ret=mad(Returns)*sqrt(252);

%Calculations of VaR and CVaR
L=zeros(row,col);
conflev=0.01;
for i=1:col
    for n=1:(row)
        L(n,i)=-(ClosePrice(n+1,i)-ClosePrice(n,i));
    end
```

```
end
VaR=quantile(L,1-conflev);

CVaR=zeros(1,length(tickers));
for k=1:length(tickers)
    singleasset=L(:,k);
    CVaR(k)=mean(singleasset(singleasset>VaR(k)));
end
```

When analysing the contribution that each ETF gives to a portfolio, a more informative way of comparing the ETFs is given by plotting the means against the risk measures:

```
%Mean-Standard Deviation graph
figure;
colormap hsv
z1=linspace(0,1,length(tickers));
scatter(SD_ret,Mean_ret,50,z1,'filled');
xlabel('Standard Deviation');
ylabel('Mean');
for m=1:length(tickers)
    txt=tickers{m};
    text(SD_ret(m)+0.002,Mean_ret(m)+0.002,txt)
end
```

For the other methods, it is sufficient to substitute the standard deviation with the relative risk measure.

Another section of this thesis aims at proving that the returns of the ETFs are not normally distributed. For this purpose, a graphical analysis based on qqplots and a comparison between the empirical and theoretical distributions is proposed. The theoretical distribution is represented by a normal sample with the same mean and variance of the sample.

```
%qqplots and distribution plots
for n=1:col
    Random_N=zeros(row,1);
```

```
    for s=1:row
        Random_N(s)=normrnd(Mean_ret(n),SD_ret(n));
    end
    figure;
    qqplot(Returns(:,n),Random_N)
    xlabel('Sample Quantiles');
    ylabel('Normal Quantiles');
    [f,xi]=ksdensity(Returns(:,n));
    figure;
    plot(xi,f,'LineWidth',3);
    hold on
    norm=normpdf(sort(Random_N),Mean_ret(n),SD_ret(n));
    plot(sort(Random_N),norm,'LineWidth',3);
    legend('Sample distribution','Normal distribution');
    hold off
end
```

Moreover, in order to formally prove the non-normality of returns, the Jarque-Bera test is implemented. It returns 1 if the nul hypothesis of normality is rejected:

```
%Jarque-Bera test, normality of returns
JBtest_ret=zeros(length(tickers),1);
for i=1:length(tickers)
    JBtest_ret(i)=jbtest(Returns(:,i));
end
```

CONCLUSION

The purpose of this thesis is to present the industry of Private Equity and to investigate whether an investor can profit by getting exposure to it, taking into consideration the opportunities and the challenges that this asset class offers.

Firstly, a Private Equity investment has been defined and an understanding of the industry has been given, especially in terms of the characteristics of the investors, of the attributes that a company has to present in order to be a target, and of the moments of a PE transaction. It has been highlighted how a PE transaction can create value: firstly for the company itself, through the removal of inefficiencies; secondly for the investors, in terms of profit and network; thirdly for the community and the environment, thanks to several side benefits.

Then, the focus has moved on PE funds, which are the most common means for direct investing in Private Equity. Even if every fund has its own peculiarities, a general description of the structure and the operation of the fund has been given. Specifically, the roles of GPs and LPs have been explained, the terms that rule a fund have been broadly described and finally how a fund creates value has been illustrated. The GPs have a prime importance in the success of the fund, which depends on every choice they make starting with the selection of the target companies. Their collaboration with the target companies and the strategies they choose to exit the investments are aimed at increasing the value of these companies. The greater the value created is, the more successful the fund is.

After having explained that Private Equity has the potential to be profitable for investors, the next step was establishing whether the performance of the asset class is greater than that of public markets or not. Considering some issues typical of PE, such as illiquidity and long-term commitment, a PE investment should deliver a return that compensates the investor for these issues. However, since the return of a PE fund can-

not be computed in the same way of that of stocks and therefore it is hardly comparable with the return of other securities, the benchmarking techniques and the performance measures for this asset class have been presented.

Lastly, the role of PE in a diversified portfolio has been evaluated. Beginning with an analysis of the ETFs that will enter the diversified portfolio, PE proved to be risky, but able to deliver high performances as well. An analysis of the correlation of PE with other asset classes outlined a positive correlation smaller than one with public markets indices, suggesting that diversifying through PE should improve the risk-return profile of a portfolio. Furthermore, since none of the selected ETFs is normally distributed, the skewness and the kurtosis of the ETFs have been compared, pointing out that PE can be widely affected by severe losses. The same conclusion is deduced by the Value at Risk and by the Expected Shortfall analysis. For this reason, three different portfolio selection methods have been proposed, in order to get different results according to the risk measure employed. At the end of the empirical analysis that has been conducted, it is possible to state that getting exposure to Private Equity is profitable. According to all the three portfolio optimization methods applied, the PE asset class has, in general, a positive effect on a diversified portfolio, increasing the risk-adjusted expected return of the efficient portfolios. However, as expected, the dimension of the efficient exposure to PE depends on the risk measure that the investor is mostly concerned about. Since the PE asset class suffers from the risk associated with the negative tails in a more severe way than public market indices or government bonds do, an investor interested in minimizing the volatility of the investment will allocate a greater portion of his portfolio to PE than an investor who is more concerned about high losses. But, in general, no matter which risk measure is adopted, an investor will benefit from investing in Private Equity.

Since this thesis uses the ETFs as proxies for the PE industry, this conclusion is true especially for retail investors, who do not have any other possibility than investing in ETFs replicating the PE industry's movements. However, since the PE indices that are replicated by the ETFs are based on the performance of the PE firms' stocks, this conclusion can apply to a wider group of PE investors, which comprises potential Limited Partners. In fact, the success of a PE firm depends on the success of the PE fund that they manage, and this is why the role of GPs is so important. Moreover, the conclusions of this thesis are based on the performance of the ETFs over a specific time window, due to the limited existence of the ETFs on PE. Therefore, the conclusions hold for this

time window.

Further research could include an analysis similar to the one implemented in this thesis but conducted using a different and longer time window, in order to evaluate whether the results are persistent, and using Private Equity funds' data. To this purpose, the access to private databases is necessary, but then the researcher would be able to build indices not only for the overall PE industry but for single categories of PE as well, in order to study which kind of PE is more profitable to invest in. Moreover, non-listed PE is likely to be less correlated to public markets, because of its private nature, therefore a greater diversification could be exploited in order to get higher risk-adjusted returns.

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