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Mutual fund performance in slump periods: negative data DEA approaches

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Introduction

Mutual funds are widely used nowadays, they provide different strategies and risk/return target and allow investors to choose among different categories in order to find the type of fund that better fits their goals. However, the wide choice and the increasing number of funds lead to the need to use several different indicators that allow the investor to determine which fund has achieved the best performance, and that provide a coherent classification taking into account the risk-return trade-off of each fund.

Nonetheless, the traditional performance measures require strong assumptions, in particular they require positive returns in order to justify the risk exposure. Moreover, when we apply these indices to the performance assessment of mutual funds, the presence of negative returns brings to misleading results and to a wrong classification.

Hence, the necessity to find another tool able to overcome these limitations and that allows us to make a reasonable comparison among each fund.

So we decide to apply the Data Envelopment Analysis for the evaluation of equity mutual fund performance. The DEA is a methodology that allows us to compute the efficiency of the production units or decision units (Decision Making Units), defined as entities that, in a production process, use the inputs to obtain a certain number of outputs. In particular, we decide to concentrate the analysis in times of crisis since that some of the key performance measures, such as the Sharpe ratio, may present problems in the interpretation, if used when the market is in a negative phase, or has just passed one. However, the main DEA models require the non-negativity assumption for the inputs and outputs in order to provide a proper efficiency measure, but usually, in negative market phases some of the variables used are negative (for example the average return of the fund). Accordingly, with some adjustment we will be able to handle negative data and to use the Data Envelopment Analysis even in presence of negative values, avoiding any distortions in the results obtained.

In the first chapter, we briefly present the mutual funds and the models traditionally used for the performance evaluation. The interpretation of some of the traditional performance measures might be difficult in time of crisis, but in order to deal with this problem, in chapter two we will show how the Sharpe index behaves when the average return of the fund is negative, and the adjustments proposed in literature that aim to modify the index in order to make it suitable to be used in time of crisis.

The third chapter introduces the Data Envelopment Analysis as a measure that evaluates the efficiency of the inputs with a given output; we will present the constant and the variable return to scale models, the Additive model and, finally, the Slack-Based Measure model. Then, in the fourth chapter we will present the adjustments to the main DEA models in order to obtain a correct measure able to handle negative data.

In chapter five we will describe some of the main models that have been developed in order to analyse the performances of mutual funds using the DEA approaches, in particular we will describe the model proposed by Murthi, Choi and Desai (1997) that represent the first attempt to apply the DEA model in this field, then we will focus on the model proposed by Wilkens and Zhu (2001) and on that by Basso and Funari (2014).

In chapter six we will apply the DEA models explained in the previous chapters with the purpose of evaluating the performance of 222 European equity mutual funds randomly chosen and extracted from the Bloomberg database. The data ranges from 30/11/2016 to 30/11/2013 and includes: fund domicile, commissions (front loads and back loads), Net Asset Value expressed as per-share amount in euros. Finally, we will compare the results obtained from the application of the DEA models with the traditional performance indices.

Chapter. 1
THE MUTUAL FUNDS AND THE PERFORMANCE MEASURES

1.1 Introduction.

Over the last sixty years mutual fund industry has captured the investing public's interest, thanks to the flexibility and adaptability to the specific request of each investor. Despite some recent developments in the financial field, that led to the creation of other competitive products, this sector has seen a strong grown. According to the International Investment Funds Association, the assets in mutual fund have increased more than seven times from the 1993.

According to the Security and Exchange Commission, a mutual fund is: "a type of investment company that pools money from many investors and invests the money in stocks, bonds, money-market instruments, other securities, or even cash". The definition is very strict and highlights some characteristics of this financial instrument. First, the fund is an asset that is collected from a multitude of investors who, however, do not buy directly financial instruments but fund shares. The value of the shares is quantified daily and is given by the Net Asset Value per shares plus any fees that the fund may charge. Fund share has the characteristic of being redeemable, meaning that the investor will sell his shares directly back to the fund at their current NAV.

1.2 Advantages and Disadvantages

Mutual funds are widely used nowadays, and there are many categories of mutual fund, each with different strategies and risk/return target. Thanks to the regulatory environment in which they are collocated, that ensures they operate in the best interest of the customer, they provide several advantages:

- Diversification: mutual funds have the possibility to invest in a variety of securities and financial instruments (typically they own hundreds of different stocks in different sectors and markets) otherwise hardly accessible by a private investor with a small amount of money. The goal is to minimize a possible loss in a particular investment by gains in others.
- Professional Management: managers have a duty to manage the Fund in a transparent and professional way, assisting and advising a client on the basis of his personal needs.
- Costs: normally, a well-diversified portfolio would require high costs for the purchase of each single security; the investor, through the underwriting of shares has the possibility of reduce this type of costs. Furthermore, for a mutual fund the transaction costs are lower because it buys and sells large amount of securities at the same time.
- Liquidity: a large part of mutual fund assets is used for securities trading, but a certain level of liquidity is always guaranteed. This means that the investor has always the right to exit the fund: he may request redemption of shares at any time.

On the other hand, there are also some disadvantages such as:

- Dilution: usually diversification assumes a positive meaning (the goal is to reduce specific risk) and is one of the keys for a successful investment, but when a fund has too many small investments in too many different companies, it might happen that even a high return from a small position, will not affect significantly the overall return. What is more, some funds focus their investment in many companies with a high correlation, for example companies that belong to a particular sector or geographic area, accordingly they are not able to achieve a risk reduction.

- Price uncertainty: typically, the price at which an investor purchases or sells fund shares is calculated once a business day and depends of the Net Asset value of the fund, hence the investor is not able to check the pricing value several times in a day (as it could be done for example with a stock).
- Cost: all mutual funds bear costs (the measure of the total costs is defined as Total expense ratio composed by management fees and additional expenses like trading, legal and auditor fees, it is expressed by a percentage amount and it is given by the ratio between the total costs and the fund total assets value), more or less onerous, and they are charged directly on investors:
    - Annual operating expenses: management and administrative cost, they represent the remuneration of fund managers for their work and the operating expenses. They are counted daily and taken directly from the fund assets periodically. Generally, they increase as the degree of the fund's risk increase. In addition, investors may face the so called “Performance fee” that is a payment made to a fund manager for generating a positive return.
    - Sales charge: these are fees that the investors pay when they purchase (or sell) a fund share. We can distinguish them in:
        1. Front-end load: initial sales charge, the investor pays this fee when he buys the shares, usually this is a percentage, up to 5%, of the amount invested;
        2. Back-end load: also called deferred sales charge, these fees are payed when the share are sold (normally the longer the investor hold the share, the less he will pay).
    - Other costs: costs associated with: the account maintenance, the trading of securities, legal expanses etc.

1.3 Mutual fund classification

There are different types of fund, the choice is very wide and investors can choose different strategies and different risk levels that better fit their goals. There are three main categories, each with different risk levels, however they are all linked by the same rule: the higher the potential return, the higher the risk involved.

Money Market funds

Money market funds can invest only in short-term debt instrument with high credit quality, mostly treasury bills and certificates of deposit with a residual maturity not greater than 397 days. These type of funds can be differentiated by different terms of investment policy: Short term funds with a weighted average maturity of 60 days, Regular funds with a weighted average modified duration up to 0.5 and Enhanced funds with a weighted average modified duration between 0 and 1. The objective of money market funds is to provide investors with a cash-equivalent asset and are characterized as a low risk and return investment with high liquidity.

Bond funds

This type of fund invests primary in bonds or other fixed income securities; the goal is to provide investors with a steady cash flow. Bond funds can be classified by their weighted average modified duration: short term, intermediate and long term, with maturities of 3 or less years, 3-10 years and more than 10 years respectively. Usually the shorter a fund’s duration, the lower its yield, however a longer duration brings to a greater impact on the bond value from changing in interest rate (they have the opposite direction).

Modified duration expresses the change the in value of the bonds affected by the change in value of the interest rate, the formula is:

$$Modified\ duration = \frac{Macauley\ Duration}{(1+\frac{YTM}{n})} \tag{1.1}$$

Where YTM is the Yield to maturity of the bond and n is the number of coupons periods per year.

An investor can choose among different types of bond funds classified by the credit quality or by the sectors in which they operate:

- Investment grade: these are funds that invest in high quality bonds, this category includes: government bonds funds, inflation-protected funds and corporate bond funds;
- High-yield/low-grade: generally, a high-yield fund invests in bonds with a lower investment grade (usually called junk bonds). Such bonds usually have a higher return in order to remunerate the investor for their greater risk;
- Other bonds fund: such as multisector bond funds (with a higher flexibility), international and global bond funds and convertible bond funds.

Equity funds

Equity fund invest mainly in equities; we can classify equity funds with three criteria:

1. Market capitalization: it depends on the size of the companies in which they invest. Small and mid-cap equity funds invest in stocks of mid-size (or small) companies usually considered as developing companies or with a strong growth in earnings and cash flows, while large-cap equity funds invest a considerable portion of their assets in companies with large market capitalization;
2. Country/region: these funds invest their assets mostly (more or less 80%) in shares of companies established in a determined country;
3. Sector: these funds invests in companies of a specific sector (for example pharmaceutical or energy), their performance is correlated with the performance of the respective sector.

Other types of funds

Socially-responsible funds: also called ethical funds, they invest only in companies that meet specific ethical parameters and determined corporate governance codes (most of these fund don’t invest in industries as tobacco, alcoholic and weapons).

Balanced funds: these funds invest both in stocks and bonds with different proportions, for example they can be equity oriented (60% equity and 40% bonds) or bonds oriented. Flexible funds: they do not follow a rigid investment strategy, instead they may change the weight of their assets invested according to market expectation.

1.4 Performance Indexes

In literature we can find many different indicator of performance that may help investors in determining the risk-reward of their investment if applied to the analysis of stocks, bonds and mutual fund portfolio. Generally speaking, the evaluation of the performance of a mutual fund requires an indicator which takes into account both the expected return and the risk and that allows to compare different generic portfolios. There are different statistical measurements that take into account the risk-return trade-off, since the performance must be related to a "fair" risk in order to avoid a not justified exposure.

1.4.1 Sharpe index

The Sharpe ratio, formulated by the economist William F. Sharpe (1966) and initially defined as the reward to variability ratio, measures the excess return of a fund compared to a not-risky activity per unit of volatility (total risk). Sharpe assumed that, in order to predict the performance of a fund, it is sufficient to know two components: the expected return and the expected risk. In particular, he assumed that all the investors can lend and borrow money at the risk-free rate and that they all share the same expectation regarding the performance of the same fund.
The best fund is the one with the higher Sharpe ratio, that is the one with the higher value for unit of risk and therefore with the best risk-return trade-off.

$$Sharpe\ Ratio\ = \frac{r_p - r_f}{\sigma_p} \tag{1.2}$$

Where:

- r_f is the risk free rate, theoretically on a risk free asset the expected return should be equal to the actual return (there is no variance around the expected return);
- r_p is the average return of the fund in the sample period considered: $r_1 r_2, r_3, \ldots, r_t$ are the return of the fund in the period t=1,2,3,...,T and it is calculated according to the following formula:

$$r_p = \frac{1}{T}\sum_{t=1}^{T} r_t \tag{1.3}$$

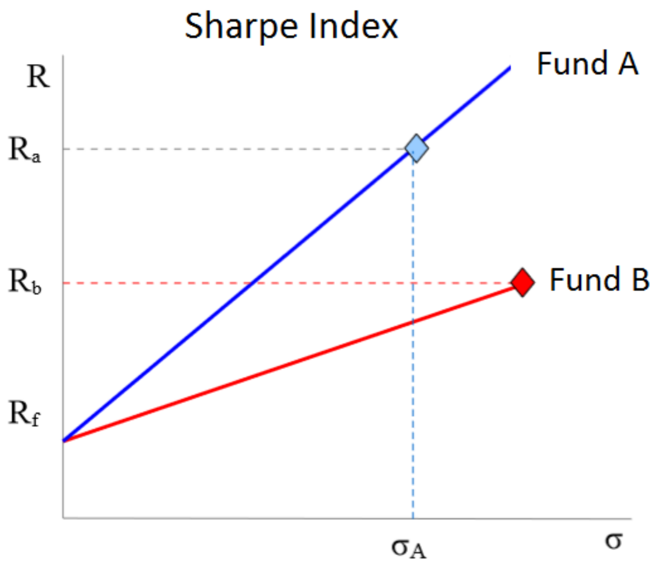
- σ_p is the standard deviation of the fund, usually used as measure of an investment volatility: a relevant dispersion of returns from their mean indicates a high degree of uncertainty about the expected return. Consequently, the greater the dispersion of returns, the greater the value assumed by the standard deviation. On the other hand, a lower value means that there is a greater probability to obtain a return close to the average. However, this measure of risk has some limitations. Indeed, it is not able to capture the difference between positive and negative values, then also a high yields compared to the mean is an index of an increase in the investment risk. Moreover, it suffers the asymmetry of the distribution and mislead data with a skewed distribution.
  It is computed as follow:

$$\sigma_p = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - r_p)^2} \tag{1.4}$$

Therefore, the sharpe index represents a trade-off between risk and performance and measure the excess return per unit of risk; this principle leads to a choice of funds (or other investment activities), based on the identification of the “best fund”.
The representation in Figure 1.1 allows us to better understand the meaning and implications of this index. The blue and red lines are drawn starting from the risk-free

assets to the investment fund (characterized by a specific risk-return profile), the sharpe index is represented by the slope of the two lines.

Figure 1.1. Sharpe index representation of Fund A and B



Source: SHARPE W. F. (1966).

Let us say that the Sharpe ratio is not used to locate the fund in which to invest the totality of the amount available to the investor, but to select the fund that, in an investment strategy in which you can invest and borrow to the same risk-free rate, yields the higher return per unit of risk (Sharpe, 1994). An investor could for example distribute his amount between the Fund A and the risk free asset, taking a position at any point along the line that connects them, or, he might get a higher expected return by investing in the fund all its amount and the money borrowed. If we compare fund A with fund B we can assert that the fund A is able to guarantee an average yield higher than the fund B at the same level of risk.

Let us say that this comparison assumes that the investor puts all his amount in one fund and he borrow/invest on the risk-free rate, therefore the overall risk of the investment strategy is represented only by the risk taken from the fund.

The Sharpe index can also be used in order to assess the efficiency compared to a determined benchmark. By replacing the parameter of the risk-free assets return with the performance of a benchmark, we get the excess return compared to the latter. Then

this quantity will be divided by the standard deviation, defining a measure having the same relationship as the original one but with a different reference.
If an investor would rather allocate his resources among different funds, it should take into account both the risk and the correlation between the returns of each fund with those of the other funds in his portfolio. As a result, the Sharpe ratio is best suited to investors who wish to invest in a single risky activity.

1.4.2 Treynor index

This index was derived by Treynor in 1966, its structure is very similar to the Sharpe ratio, since it also expresses the excess return per unit of risk produced by the fund.
Let us say that the Sharpe ratio is a performance measure more appropriate for an investor who does not diversify, in this scenario the fund represents the only risky assets in the portfolio. Otherwise, if the fund is only one among several risky assets in the portfolio, a more appropriate performance measure is the Treynor index.
The difference with the Sharpe index is the definition of the risk, in the Treynor index the risk is represented by the beta that measures only the systematic risk, the higher this value, the greater excess return is generated by the portfolio on average (Bodie Z., Kane A., Marcus A.J.,2011).
The Treynor index is computed as follow:

$$Treynor\ index\ = \frac{r_i - r_f}{\beta_i} \tag{1.5}$$

The index is a measure of efficiency, expressed by a coefficient as the Sharpe ratio: the higher its value, the greater the excess return for that investment. However, the use of the parameter beta as risk factor leads to some criticism against this measure.
Firstly, β_i is an appropriate risk indicator in presence of well-diversified portfolios: the Treynor index "captures" only the part of systemic risk (not diversifiable) but is not able to evaluate the impact of the residual component of risk.
Secondly, it could lead to misleading results even in the presence of negative β, as we will explain later.

The parameter β_i is computed as follow:

$$\beta_i = \frac{Cov\ (r_i, r_m)}{\sigma_m^2} \tag{1.6}$$

where:

- r_i indicates the fund return;
- r_m indicates the market return;
- σ_m^2 indicates the variance of the market return.

The beta describes how the fund's returns change in relation to market trends, it can take different values, in particular:

- If $\beta_i > 1$, then the fund changes following the market direction and it is more volatile than the market (so it amplifies the movement);
- If $0 < \beta_i < 1$ then the fund changes with the same direction of the market;
- If $\beta_i < 0$ then the fund is going in the opposite direction with respect to the market.

As we have seen, the parameter beta can take negative value, this could bring to a misleading result since the final value of the index would be influenced by the sign of the parameter. Also we should keep in mind that the index is negative if:

1. Beta is negative (as we said), even if the risk-free rate is less than the expected return: this scenario implies that the fund has performed well, betting against the market.
2. Beta is positive, but the expected return is less than the risk-free rate: this would mean that the fund has performed not well.

1.4.3 Sortino Index

The complexity and the different criticism regarding the use of the standard deviation for the definition of risk, have prompted Frank Sortino to abandon the measure of standard deviation and to define an index that uses the Downside risk.

The use of the standard deviation, in order to describe the riskiness of a fund, does not discriminate between positive and negative deviations, because it is computed by the changes from the average return elevated to the square.
In addition, it is not realistic to assume that the returns are always distributed as the Gaussian distribution. Therefore, in order to have a better representation of the returns distribution, we should also consider the kurtosis and skewness.
The kurtosis indicates the extent to which the probability is concentrated in the center and especially in the tails of the distribution rather than in the "*shoulders*," which are the regions between the center and the tails. On one hand, a kurtosis greater than zero indicates that there is a greater probability of a distribution in the tails compared to the one described by the normal distribution (that is, data with high kurtosis tend to have heavy tails); on the other hand, a low kurtosis tend to have light tails.
Skewness measures the degree of asymmetry, with symmetry implying zero skewness, positive skewness indicating a relatively long right tail compared to the left tail, and negative skewness indicating the opposite.
The skewness can take positive or negative values: if we are in presence of positive skewness the standard deviation is said to overestimate the risk, because high positive returns, that the investor should not worry about, increase the value of the risk measure that we are using. On the contrary, in presence of negative Skewness the risk is underestimated.
Let us say that the standard deviation, does not take into account the different attitude that investors have towards the upper or lower deviations from the mean value. However, there are asymmetrical risk measures, which focus only on the left side of the distribution of returns. These measures, on one hand tackle the problem of the non-Gaussian distribution, and on the other hand focus on investor returns that can be lower than the average yield, or those below a minimum acceptable performance.
The Sortino ratio is a performance indicator that measures the over-performance of the fund compared to a minimum return, compared to an asymmetrical risk measure.

This index is defined as follows:

$$Sortino\ index = \frac{r_i - r_{min}}{TTD} \tag{1.7}$$

Where:

- r_i is the average return of the fund;
- r_{min} is usually called the target rate of return, also known as MAR (the minimum acceptable return), it may be zero or any other performance chosen by the investor, but very often we refer to the risk-free rate, the choice of the target depends on the investment objective of the portfolio under consideration;
- TDD is the target downside deviation, the TTD uses the observed returns below the target rate of return (r_{min}), in order to consider only the negative deviations from the target return; is computed as follow:

$$TDD = \sqrt{\frac{1}{N} * \sum_{t=1}^{N} \min(0, r_t - r_{min})^2} \tag{1.8}$$

1.4.4 Jensen's Alpha

In 1967 Jensen, implementing the studies on the CAPM, developed its own model, commonly referred to as Alpha Jensen. The model is based on the CAPM but tries to overcome some stiffness, in particular it considers the possibility that the fund performance could be influenced by the ability of a manager. So the goal was to identify a measure that allow us to determine the ability of the operator to select undervalued stocks on the market.

The Jensen measure, commonly known as Jensen's Alfa, represents the excess return offered by the fund compared to the return that would have been offered at the same level of risk.

Analytically the Alfa Jensen can be determined as follows:

$$Alfa\ Jensen: \alpha_p = (r_P - r_f) - \beta_P(r_M - r_f) + \varepsilon \qquad (1.9)$$

Where:

- $r_P$is the average return of the fund;
- r_f is the risk free rate;
- r_M is the return of the market (or benchmark);
- β_P refers to the systematic risk of the fund;
- ε is the residual vector, it expresses the deviations of the linear regression line from the actual return.

Jensen asserts that: if alpha has a positive value then the fund manager has a good selection skills of undervalued stocks, since he is able to obtain a higher return than expected, according to the systematic risk assumed, otherwise, if alpha has a negative value then the fund manager has failed in selecting the best performing securities, and accordingly, the return is lower compared to the risk assumed.

1.4.5 Information ratio

The Information ratio is a performance indicator that measures the excess return of the fund in comparison with a benchmark.
This index is built starting from the excess return, expressed by the difference between:

$$Er_t = r_{Pt} - r_{Bt} \qquad (1.10)$$

where r_{Pt} is defined as the return of the fund in the period t and r_{Bt} is the return of a benchmark.
This expression helps us to compare different funds that refer to the same benchmark. Funds with an active strategy, will try to have a higher excess return, as the manager aims to achieve a higher return than the benchmark, on the other hand, funds with a

passive strategy will try to replicate the benchmark having and excess return equal or close to zero.

Then we compute the average of excess return, defined in the period starting from t=1 to T, as follow:

$$\overline{Er} = \frac{1}{T}\sum_{t=1}^{T} Er_t \tag{1.11}$$

Now we define $\hat{\sigma}_{ER}$ as the standard deviation of the excess return from the benchmark; it is computed as follow:

$$\hat{\sigma}_{ER} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(Er_t - \overline{Er})^2} \tag{1.12}$$

The standard deviation indicates the additional risk taken by the operator in order to obtain a return higher than the benchmark. Therefore, the higher the value of the standard deviation of the excess return, the greater the risk assumed by the operator in order to achieve a higher return.

Finally, given the average excess return and the standard deviation, we compute the information ratio:

$$Information\,ratio = \frac{\overline{ER}}{\hat{\sigma}_{ER}} \tag{1.13}$$

In few words: the Information ratio is the average excess return per unit of volatility. The Information Ratio then allows us to sum in a single value both the measure of the excess return, and the extra risk of the fund compared to the benchmark. Consequently, it is a performance indicator that shows the manager's ability to generate excess return, so it would not make sense to calculate this indicator for a passively managed fund because, as explained above, if the manager's objective is to replicate the performance of the benchmark, the ER is expected to get close to zero (i.e. the information ratio of any passively managed fund should be zero).

Starting from a different point of view, we can express the Information Ratio in a different way. Let us recall the equation:

$$(r_P - r_f) = \alpha + \beta(r_M - r_f) + \varepsilon \tag{1.14}$$

Where $Var(\varepsilon) = \omega^2$, and the coefficient α expresses the value obtained by an active management. Now, if we set the hypothesis: $\beta = 1$ we can represent the relation among portfolio return (r_P), the benchmark (r_M) and the risk free (r_f).

$$(r_P - r_f) - (r_M - r_f) = (r_P - r_M) \tag{1.15}$$

The equality on the right expresses the excess return (Er_t), that we know being equal to α+ε. Let us say that an active management will be able to create value only by over/under-weighting the securities, in relation to a benchmark, maintaining the same level of risk.

Now, going back to the information ratio formula, we can re-write it as:

$$Information\ ratio = \frac{\alpha}{\omega} \tag{1.16}$$

This representation of the formula avoids misleading result when choosing the beta: for example, some managers could get advantages from choosing a risk (β) greater than the risk taken from the benchmark. Finally, if β is greater than one the coefficient α will be smaller than it would have been if β was equal to one, this will decrease the related information ratio. On the contrary, a β smaller than one will lead to a greater value of α, increasing the Information Ratio (in this case, the estimated α will be greater but the estimated ω will be smaller).

The estimation of the Information Ratio determined by the linear regression is to be preferred in the case in which there is no assurance that the benchmark is appropriate replication of the risk of a particular fund; on the contrary, if the benchmark was chosen with particular accuracy, it is better to use, for simplicity, the first equation.

1.4.6 Modigliani risk-adjusted performance

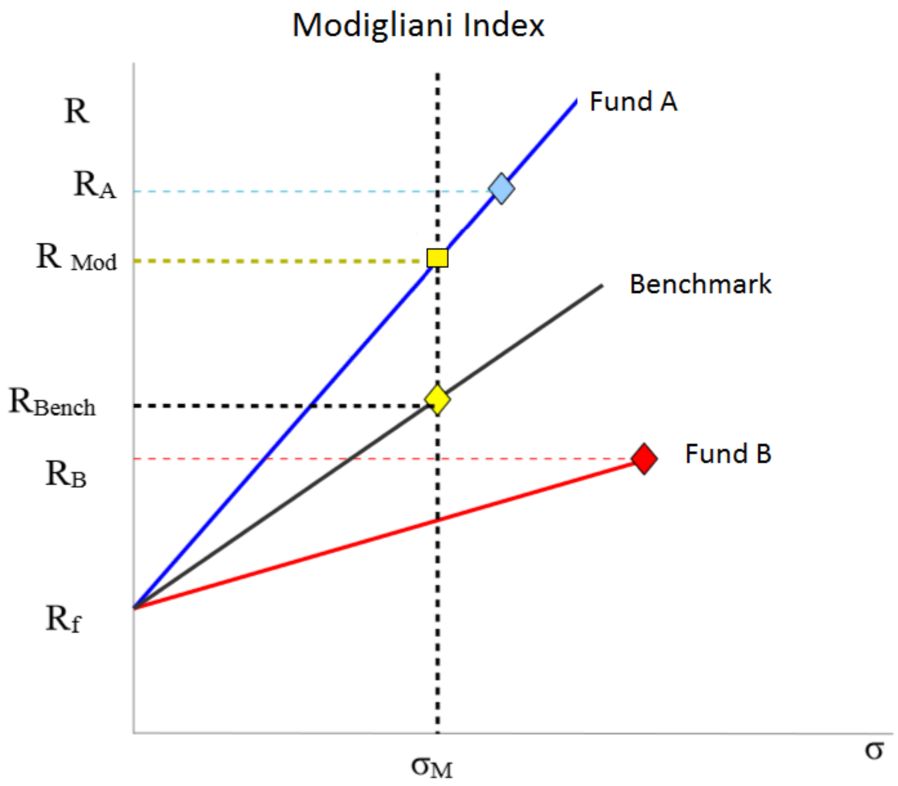
In 1997 F. Modigliani and L. Modigliani proposed a performance measure that takes into account the risk-return trade-off, such as the Sharpe ratio, but that was more familiar to less experienced investors and, moreover, made explicit reference to the market benchmark. This measure compares the performance of one or more funds with the benchmark (market) return. Consequently, for each investment fund with a certain risk and return profile, the Modigliani measure indicates the return that the fund would have had if he had taken the same market risk. So, assuming that an investor places his resources in a single fund, and that he can borrow and invest at the same risk-free rate, then we modify the riskiness of the fund until it equals the market one, finally we look at the performance of the "modified fund". The Modigliani measure, initially defined as: risk-adjusted performance (RAP), is determined according to the following expression:

$$Rap = r_f + \frac{(r_i - r_f) * \sigma_m}{\sigma_i} \tag{1.17}$$

where:

- r_f is the risk-free rate;
- r_i is the average return of the fund;
- σ_m is the standard deviation of the market (or benchmark);
- σ_i is the standard deviation of the fund.

Figure 1.2. Modigliani index representation for fund A and B



As we can see in figure 1.2, we have a representation of two funds, A and B, according to their risk and return values, the line starting from the risk-free in the y-axis to the point determined by the blue and red diamond, determines the Modigliani index.

Drawing a line from the point σ_m in the x-axis (standard deviation of the market), we are able to identify, by the intersection between this line and the line representing the fund, the return that each fund would offer if it had the same level of risk of the market (or benchmark). Based on the analysis of the figure 1.2 we can observe that the fund A is better than fund B because R(B)> R(A).

Modigliani and Modigliani pointed out that: a classification of the funds based on their performance measure or by the Sharpe index does not involve differences and, consequently, the best fund according to the Modigliani index is the best even according to the Sharpe index. In fact, in figure 1.2 we can notice that the line with the highest slope is the one that connects the fund A to the risk-free rate. Consequently, also according to the graphic interpretation of the Sharpe index, fund A is preferable to fund B.

However, this measure is simpler in the interpretation than the Sharpe index, since for each fund "k" a measure called Rap_k can be calculated and compared to the return offered by the market (r_m).

Furthermore, let us call the measure M^2 excess-return as the performance measure of Modigliani and Modigliani, computed as:

$$M^2 excess\ return = \frac{(1 + Rap_k)}{(1 + r_m)} \tag{1.18}$$

Or, arithmetically:

$$M^2 excess\ return = Rap_k - r_m \tag{1.19}$$

Here we refer to the difference between the return offered by the fund at the same level of market risk (Rap_k) and the market return (r_m). It can be stated that:

• if $M^2 excess\ return$ > 0 then a fund k offers, at the same risk, a higher return than that offered by the market;

• if $M^2 excess\ return$ < 0 then a fund k offers a lower yield than the market, always considering the same level of risk.

In few words: we will prefer the fund with the highest value of $M^2 excess\ return$.

Chapter 2.
PERFORMANCE INDICES DURING THE CRISIS

2.1 Introduction

The interpretation of some of the performance measures described in chapter 1 might be difficult in time of crisis. The following chapter will expose the problem linked to the use of ex post information for the evaluation of future performance of the funds.
In particular, we will show how the use of certain indices might bring to misleading result when using historical data, this because the market is in a negative phase, or has just passed one, and so non-positive returns are present. Also we will present the different opinions explained in literature regarding the use of Sharpe index in times of crisis. Finally, we will present different adjustments (one of which also apply to the Information Ratio) that aim to change the Sharpe ratio, so we can use it even in the presence of negative returns, without involving complication in the interpretation of these measures.

2.2 Criticism to the Sharpe ratio

Usually, in the evaluation of a mutual fund performance we use the ex post information. Therefore, investors choose a fund based on the expectations about the future risk-return value; in order to do that, we assume that it is correct to refer to past data, even if we are not sure that the historical performance will be repeated in future periods.

This could bring to some problems in the interpretation of some performance measures, when we use them for evaluating the mutual fund during a negative phase of the market. This because if the market is in a negative phase, or has just passed one, these indices could be negative.

Anyway, there are also some indices that do not bring to a misleading interpretation when taking negative value: the first is the Jensen's Alpha, when this measure takes negative value, it means that the fund has been able to offer a lower return than the return that could be expected in relation risk assumed; the second is the Modigliani and Modigliani index, a negative value means the fund offers a return lower than the benchmark return (or market return) at the same level of risk.

When we are in presence of negative return, we might face some problems in the interpretation of the Sharpe index. For example, if we have two funds with the same return, but with negative value, and we just look to the one with the highest Sharpe ratio, our evaluation will result in considering the fund with the greater standard deviation (higher risk) as the best between them. Moreover, the sharpe ratio can take negative value even when the return, offered by the fund, is positive but less than the risk-free.

Here below, in order to better understand what we said above, we will present three different cases in which we compare different funds with different return/risk value.

In the first example we have two funds, called fund J and fund K, with different returns and different risk values:

Table 2.1 Sharpe ratio of fund with positive excess returns

FUNDS	H	J
RETURN	0.11	0.08
ST. DEVIATION	0.15	0.08
RISK-FREE	0.02	
SHARPE RATIO	0.6	0.75

As we can see from the table 2.1, fund H offers a higher return than fund J, but if we consider also the risk taken by the first, we can observe that SR(J) > SR(H). It is important to notice that if in the comparison we had considered only the performance offered by each fund, we would have come to the opposite conclusion. Fund H would be (not correctly) considered the best, because this fund has achieved better returns, but taking a position at a higher level of risk than fund J.

This observation highlights the importance of not classify the funds considering only their expected return, but to use, for the comparison, a measure such as the Sharpe ratio (or others) that takes into account also the risk.

In Table 2.2 we compare two funds that offer the same performances, both of them are less than the risk-free rate, with the result that the excess returns of both funds are negative, and so are the Sharpe ratios. As we will see, in this case the application of this index will bring to some problem in the interpretation.

Table 2.2 Sharpe ratio of fund with negative excess returns

FUNDS	K	L
RETURN	0.03	0.03
ST. DEVIATION	0.15	0.10
RISK-FREE	0.05	
SHARPE RATIO	-0.13	-0.2

In the example in table 2.2, it may be noted that SR(K)> SR(L), therefore, according to the definition of the Sharpe Ratio, fund K offers a higher return per unit of risk than the fund L. However, we might notice that both funds offer the same returns, then why an investor should consider fund K better than the fund L if the last offer the same return but at a lower level of risk?

In addition, we might wonder: why an investor should invest in a fund that offers a negative excess return instead of investing only in the risk-free asset?

In the last example in Table 2.3, we will compare two funds with negative returns.

Table 2.3 Sharpe ratio of fund with negative returns

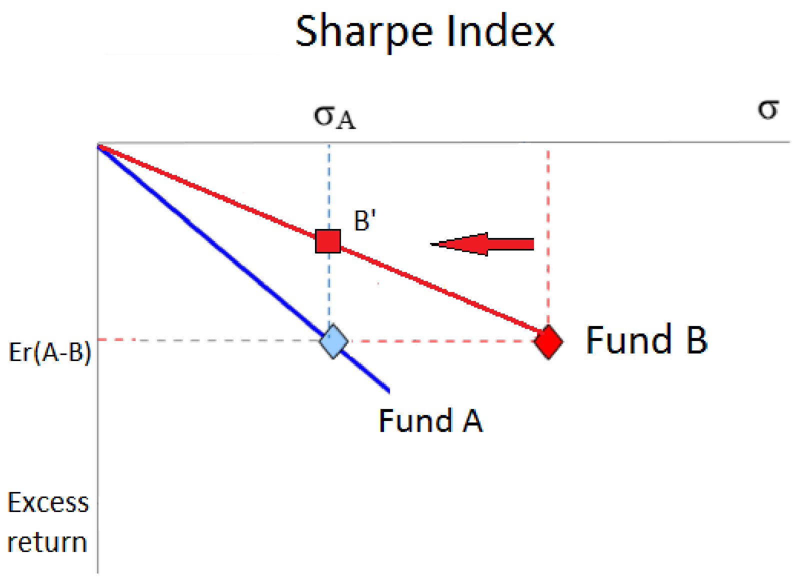
FUNDS	M	N
RETURN	-0.1	-0.08
ST.DEVIATION	0.15	0.11
RISK-FREE	0.03	
SHARPE RATIO	-0.86	-1.0

In this case the problems in the interpretation are even more evident, fund M offers a lower (more negative) excess return but with a higher risk than the fund N and, if we compute the Sharpe Ratio we notice that: SR(M)>SR(N). At this point, we should wonder how to interpret the negative values of the Sharpe ratio and if the selection of a fund, when there are negative excess returns, does make sense. The sharpe ratio would suggest us to invest in the fund that takes the higher risk and gets the lower return, but, as we will see, there are many suggestions in the literature that modify this performance measure in order to interpret the negative values of the index and indicate how to use them for the evaluation of the performance.
Some authors as Ferruz, Irealsen, Scholz and Wilkens proposed some adjustments for the index when used in times of crisis, other authors asserted that the Sharpe index should be used, without changes, regardless the market trend.

2.2.1 A Sharpe ratio modification

Sharpe himself argues that, even if we have negative returns, when comparing two funds we should still prefer the one with the higher Sharpe ratio.

Figure 2.1. Sharpe index of fund with negative excess returns



In figure 2.1 we can notice that both the funds A and B have a Sharpe ratio less than zero, because both the straight lines have a negative slope. In this graph we put the standard deviation in the x-axis as usual, but the Excess return in the x-axis (instead of the return as usual). Fund B, is riskier than fund A, and offers the same excess return, however, the Sharpe ratio associated with it is higher (less negative) than the one of fund A, because we have negative data. Here, Sharpe justifies the choice of fund B by introducing fund B', which has the same Sharpe ratio of the fund B since they are on the same line. As we can notice from Figure 2.1, the fund B' offers an excess-return higher than the fund A, with the same level of risk of the latter.

As a result, we should choose fund B because, according to Sharpe, we can invest and borrow at the risk-free rate and we can take a position on any point on the line that connects the fund to the risk-free activity. So fund B, which has a higher Sharpe ratio (less negative) than the fund A, is the better because, if used in an investment strategy in which you can borrow and invest at the risk-free rate, it allows us to obtain a higher excess return for any level of risk chosen by the investor.

2.2.2 Another interpretation of Sharpe Ratio

Yoshiaki Akeda, in his paper "Another interpretation of Negative Sharpe Ratio", proposed a different point of view, asserting that, when we are in presence of negative returns it is no more correct to consider, for equal performances, the fund with the lower risk value as the best (Akeda Y., 2003).

Therefore, he proposed an alternative way to compare two funds that can be used regardless of the sign taken by the Sharpe index. In particular, Akeda used the expression of the Capital Market Line to build the Z-score of two funds J and K and showed that the difference between the Z-score of the two funds is equal to the difference between the Sharpe ratios of the same funds. The Capital Market Line is the line that represents the expected return of the fund compared to its standard deviation in relation to the risk-free and the market.

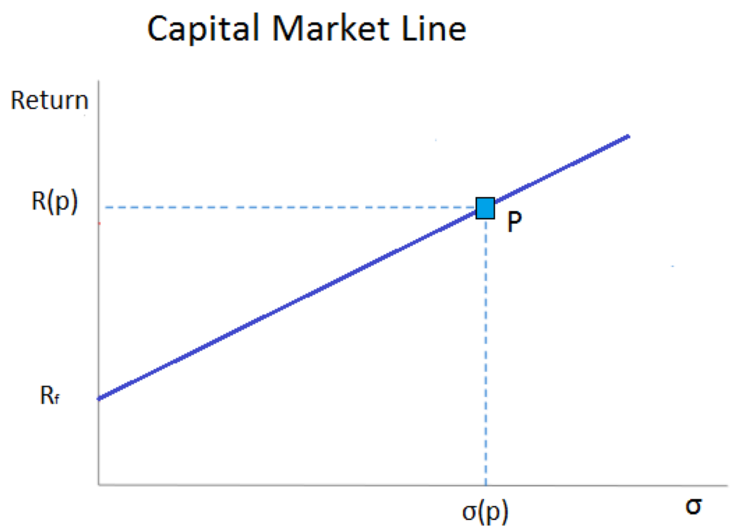
The CML equation is the computed as:

$$r_p = r_f + \sigma_p \left(\frac{r_m - r_f}{\sigma_m} \right) \tag{2.1}$$

Where:

- r_p is the average return of the fund;
- r_m is the average return of the market;
- r_f is the risk free rate;
- σ_p represents the standard deviation of the fund returns;
- σ_m represents the standard deviation of the market returns.

Figure 2.2. Capital Market Line of a generic fund P



The slope of this line is described by the value “h”, that in the ex-ante Capital Market Line assumes a positive value because, if the market is efficient, at a higher risk corresponds a higher expected return.

Now, we define the slope h equal to $\frac{r_m - r_f}{\sigma_m}$, and with a simple computation we define:

$$r_m - r_f = h * \sigma_m \tag{2.2}$$

This equation represents the ex-ante line and if we consider a portfolio J with a risk σ_j, we can define the expected excess return of portfolio J as follows:

$$E[r_j - r_f] = h * \sigma_j \tag{2.3}$$

At this point, Akeda defines the actual excess return as $r_j - r_f$ and, using the standard deviation of fund J, he normalizes the difference between the actual and the expected return. Afterwards, Akeda defines the Z-scores expressed as follows:

$$Z_j = \frac{\{(r_j - r_f) - E[r_j - r_f]\}}{\sigma_j} \tag{2.4}$$

And using the definition of the expected excess return, we define the Z*score of fund J as:

$$Z_j = \frac{\{(r_j - r_f) - h * \sigma_j\}}{\sigma_j} \tag{2.5}$$

In the same way, we determine the Z-score of K as:

$$Z_k = \frac{\{(r_k - r_f) - h * \sigma_k\}}{\sigma_k} \tag{2.6}$$

Generally speaking, the Z_j and Z_k values can be considered as the realizations of a normalized random variable with expected value equal to zero and standard deviation equal to one, the higher the Z-score is the best the performance of the fund. We can observe that in the traditional determination of the Z-score values we use the parameter *h*, while when we make the difference between the Z-scores of two funds, the parameter *h* disappears. Hence, the difference between two Z-scores becomes equal to the difference between the Sharpe ratios of the two funds, calculated using only ex post values:

$$Z_j - Z_k = \frac{r_j - r_f}{\sigma_j} - \frac{r_k - r_f}{\sigma_k} = SR_j - SR_k \tag{2.7}$$

Accordingly, if we assume that a higher Z-score is an indication of a better performance, then we can also conclude that the best fund is the one with the higher Sharpe ratio (Akeda Y., 2003). Even if we consider a negative Sharpe ratio because, as we have seen, we made no assumption about the sign.

2.2.3 W. McLeod and G. van Vuuren approach

As we have seen, the original formulation of the Sharpe ratio is appropriate when we are in presence of positive excess returns, but if we work with negative data, we might face some problems in the interpretation. Anyway, McLeod and van Vuuren claim that it is correct to use the Sharpe ratio, without changing its formula, even when the market is in a negative phase, but in order to avoid the problems linked to the presence of values less than zero, they suggest a further implementation (McLeod W., van Vuuren B., 2004). The authors prove that selecting the fund with the higher Sharpe ratio is equivalent to choose the fund that has the greatest chance to offer a higher return than the risk-free rate. Generally speaking, we are seeking a broader definition of the Sharpe Ration that can be used with positive or negative data without bringing to misleading results. Using the probability:

$$Max\ \Pr(R_p \geq R_f) \Leftrightarrow Max\ Pr\left(\frac{R_p - E(R_p)}{\sigma_p} \geq \frac{R_f - E(R_p)}{\sigma_p}\right) \tag{2.8}$$

Now, if we assume a coefficient z, distributed as z~N(0,1), we have:

$$\Leftrightarrow Max\ Pr\left(z \geq \frac{R_f - E(R_p)}{\sigma_p}\right) \tag{2.9}$$

$$\Leftrightarrow\ Min\ \left(\frac{R_f - E(R_p)}{\sigma_p}\right) \text{ or, in another way: } \Leftrightarrow Max\left(\frac{E(R_p) - R_f}{\sigma_p}\right) \tag{2.10}$$

The final expression is equivalent to a maximization of the Sharp ratio.

Hereafter, we propose a numerical example using the following parameter:

Table 2.4 Sharpe ratio of fund with negative returns

FUNDS	M	N
RETURN	-0.13	-0.075
ST.DEVIATION	0.145	0.09
RISK-FREE	0.09	
SHARPE RATIO	1.466	1.65

Then we compute the probability for each fund, using the equation (2.9):

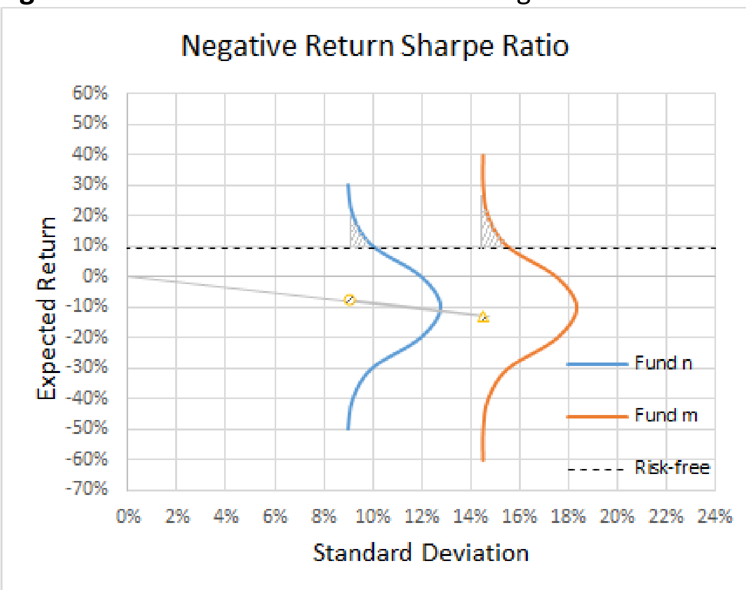
$$Probability\ (R_m > R_f) = \Pr\left(z \geq \frac{0.09 - (-0.13)}{0.145}\right) \tag{2.11}$$
$$= \Pr(z \geq 1.517) = 0.065$$

On the other hand, for the fund n, the probability is:

$$Probability\ (R_n > R_f) = \Pr\left(z \geq \frac{0.09 - (-0.075)}{0.09}\right) \tag{2.12}$$
$$= \Pr(z \geq 1.83) = 0.0335.$$

Therefore, fund M, is more likely to offer a higher return than the risk-free rate, if compared to fund N.

Figure 2.3. Return distributions of two generic fund n and m



Source: McLeod W., van Vuuren B. (2004).

The figure 2.3 graphically show the return distributions of two funds M and N. The highlighted area on the upper tail of each distribution represents the probability that the fund performance exceeds the risk-free return (dashed line). We can also notice that the fund with a higher probability of exceeding the risk-free rate is also associated with a higher probability of getting higher losses.

2.2.4 Ferruz and Sarto: an alternative proposal.

The correction that Ferruz and Sarto (2004) have suggested in order to modify the Sharpe ratio, is to calculate the excess return of the fund compared to the risk-free rate, putting in a ratio the return premium of the fund relative to its standard deviation.

$$S_p(1) = \frac{E(r_p)/r_f}{\sigma_p} \tag{2.13}$$

where:

- $E(r_p)$ is the average return of the fund and r_f is the risk free rate;
- σ_p is the standard deviation of the fund.

The use of equation (2.13) allows us to get a correct performance evaluation even when the average return of the fund is less than the risk-free assets. Nonetheless, this alternative cannot be used when the average return of the fund is less than zero so when $E(r_p) < 0$.

2.2.5 Irealsen: a refinement to the Sharpe ratio

As we have seen above, the Sharpe index in its original formula describes the relation between the excess return and the standard deviation of the return (Irealsen, 2004). The author proposed to modify the denominator in the following way:

$$Sharpe\ ratio = \frac{ER}{SD^{(ER/|ER|)}} \tag{2.14}$$

Where:

- ER is the excess return of the fund (fund return – risk-free asset);
- SD is the standard deviation of the excess return;
- $|ER|$ is the absolute value of the excess return.

Irealsen proposed to modify in the same way the Information Ratio, the modified IR is the following:

$$IR = \frac{ER}{SD^{(ER/|ER|)}} \qquad (2.15)$$

The Sharpe ratio and the Information Ratio are very similar, but in the second one the excess return is computed in relation to a benchmark (and not the risk-free rate as the Sharpe ratio).

The standard Sharpe ratio is modified by an exponent to the denominator that is: the excess return divided by its absolute value. If we have positive data and a positive excess return, the Sharpe ratio will give the same result whether modified or not. Otherwise, when ER is less than zero, the modified Sharpe Ratio and the standard Sharpe ratio can bring to different results.

Finally, Irealsen argues that using the two modified indices in times of crisis, we can get a proper ranking of the funds, where the high excess return and the lower standard deviation are rewarded.

2.2.6 The normalized Sharpe index.

When evaluating a fund, we usually use ex-post data, in particular, the Sharpe Ratio uses ex-post estimated parameters of the fund excess return (Scholz and Wilkens, 2006). The authors have shown that when we have to choose a fund for a future period, we should consider how is the market climate during the evaluation period. This because, regardless the skill of the fund manager, in assessing the performance of a fund, using the traditional Sharpe ratio, we might find values that are often distorted by short-term market sentiment. Consequently, in order to assess the funds regardless of how the market sentiment is, Scholz and Wilkens suggested to use a normalized Sharpe ratio.

Assuming that the excess return of the fund “I” for the period t depends from the excess return of the market, we consider the following linear regression model:

$$er_{it} = JA_i + \beta_i er_{Mt} + \varepsilon_{it} \quad \text{where } \varepsilon_{it} \sim N(0, \sigma_{\varepsilon i}^2) \tag{2.16}$$

Where:

- $er_{it} = r_{it} - r_{ft}$ is the excess return of the fund i in time t;
- JA_i is the Jensen Alpha of the fund;
- β_i represents the systematic risk of the fund;
- $er_{Mt} = r_{Mt} - r_{ft}$ is the excess return of the market portfolio.

Starting from this equation, we can determine the mean and standard deviation of the excess return of the fund that after will be used to determine the normalized Sharpe ratio. The average (mean) return of the fund excess return for a defined period is defined as:

$$\overline{er_i} = JA_i + \beta_i \overline{er_m} \tag{2.17}$$

Where $\overline{er_m}$ is the average excess return of the market in relation to the risk-free asset. The standard deviation of the excess return of the fund is:

$$s_i = \sqrt{\beta_i^2 s_M^2 + s_{\varepsilon i}^2} \tag{2.18}$$

Where $s_{\varepsilon i}$ is the standard deviation of the fund specific residual return. Putting together the two equations, we can now define the Sharpe ratio as:

$$SR_i = \frac{JA_i + \beta_i \overline{er_m}}{\sqrt{\beta_i^2 s_M^2 + s_{\varepsilon i}^2}} \tag{2.19}$$

In order to evaluate the real fund performance, we need to modify the Sharpe Ratio, Scholz & Wilkens suggest to determine separately the parameter of the distribution of the market excess return and the to estimate the fund-specific characteristic JA_i, β_i and $\overline{er_m}$ using a longer data range. If we are able to use a greater time frame, for example more than five years, we will obtain the long-term mean $\widehat{\mu_{\iota M}}$ and the standard deviation $\widehat{\sigma_{\iota M}^2}$.

The normalized Sharpe ratio considers the long-run values, so it is not influenced by the market in short term and therefore allows a more accurate assessment of the fund's management. For this reason, we can always use this index (and not just in times of crisis), regardless of the market trend.

$$nSR_i = \frac{JA_i + \beta_i \widehat{\mu_{\iota M}}}{\sqrt{\beta_i^2 \widehat{\sigma_{\iota M}^2}. + \widehat{\sigma_{\epsilon\iota}^2}.}} \tag{2.20}$$

Chapter 3.
DATA ENVELOPMENT ANALYSIS

3.1 Introduction

In this chapter we will present the data envelopment analysis (DEA), a methodology that allows us to compute the efficiency of the production units or decision units (Decision Making Units), defined as entities that, in a production process, use the inputs for obtaining a certain number of outputs (Cooper et al., 2007). In the following paragraphs we will explain the main DEA models, starting from the CCR model introduced by Charnes, Cooper and Rhodes (1978), followed an extension of the CRR model: the BCC model formulated by Banker, Charnes and Cooper (1984). Then, we will introduce the Additive model (on Additive DEA models see Cooper, Seiford and Tone, 2000) and finally we will present the Slacks-Based Measures model (Cooper, Seiford and Tone, 2007).

The DEA is not only used for the evaluation of the investment funds but, thanks to its general nature, at the beginning was used for any evaluation of efficiency. For this reason, we can apply the DEA to different fields, from industrial to hospital, in order to evaluate different parameters such as the cost per unit or the profit per unit. As we know, in order to indicate if the fund is performing well, the performance measures described in the previous chapter usually use the form of output over input. Even at the fundamental of the DEA there is this setting, however, the measure goes beyond this simple form, overcoming some limitations that we can find in the choice of input and output parameters. The DEA index allows us to compare the efficiency of each unit with the most efficient units among those available. These units, with the greatest efficiency, will be taken as landmark. Regarding the set of units, we make the assumption that they share the same inputs and outputs in order to make a reasonable comparison.

The advantages of using the DEA are the follows:

- The analysis with the DEA method uses a non-parametric approach that does not require the use of an external benchmark, as other performance measures usually do.
- It is not necessary that the input and output units are congruent with each other. The comparison may include various data (one or more outputs, one or more inputs). For example, if we want to analyse an investment fund, we may put in the nominator (output) the performance, while in the denominator (input) there are the entry and exit commissions and a measure that quantifies the risk (usually expressed by the variance).

3.2 The CCR model

As we have seen, the DEA is a measure that evaluates the efficiency of the inputs, in particular: how they are used to obtain the output level of each DMU (Decision Making Unit). We can express inputs and outputs in the following matrix forms:

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{1n} \\ x_{21} & x_{22} & x_{2n} \\ x_{m1} & x_{m2} & x_{mn} \end{pmatrix} \qquad Y = \begin{pmatrix} y_{11} & y_{12} & y_{1n} \\ y_{21} & y_{22} & y_{2n} \\ y_{s1} & y_{s2} & y_{sn} \end{pmatrix}$$

where X is the (*m x n*) matrix for the inputs and Y is the (*s x n*) matrix relative to the outputs. Consequently, x_{mn} is the m-th input, related to the n-th DMU, on the other side, y_{sn} is the s-th output, related to the n-th DMU (Cooper, Seiford and Tone, 2007). As we said before, the DEA goal is to maximize the output / input ratio. Specifically, in the form:

$$\frac{Virtual\ output}{Virtual\ input}$$

In order to achieve this maximization, we associate some weights to each input and output, which will be determined not ex-ante but through an optimization process. Now we can associate to each input and output of the DMU considered, denoted by the DMU_o (where o takes value 1,2, …, n), their weights with the following specifications:

$$Virtual\ output = u_1 y_{1o} + u_2 y_{2o} + \ldots + u_s y_{so} \tag{3.1}$$

$$Virtual\ input = v_1 x_{1o} + v_2 x_{2o} + \ldots + v_m x_{mo} \tag{3.2}$$

Where u is the weight associated to each output and v is the weight associated to each input. Therefore, the ratio to maximize will be:

$$\max \theta = \frac{u_1 y_{1o} + u_2 y_{2o} + \ldots + u_s y_{so}}{v_1 x_{1o} + v_2 x_{2o} + \ldots + v_m x_{mo}} \tag{3.3}$$

with this constraints:

$$\frac{u_1 y_{1j} + \ldots + u_s y_{sj}}{v_1 x_{1j} + \ldots + v_m x_{mj}} \leq 1 \quad \text{(j=1, . . ., n)} \tag{3.4}$$

$$v_1, v_2, \ldots, v_m \geq 0 \tag{3.5}$$

$$u_1, u_2, \ldots, u_s \geq 0 \tag{3.6}$$

The constraints ensure that the ratio between input and output does not exceed the value 1 for each DMU. The weights, determined by the optimization process, will maximize the ratio of the analysed DMU.

Mathematically constraints (3.5) and (3.6) are not sufficient to ensure a positive value in the inequality (3.4). According to Cooper, Seiford and Tone (2007), all the outputs and inputs are assumed to be positive and that, consequently, is reflected in the value of the relative weights. Since, if inputs and outputs are considered within the efficiency analysis, they must necessarily have positive value. Computationally, the solution to the maximization problem is solved through a specification in a linear program (LP), expressed in the following terms:

$$(LP_o) = max\theta = u_1 y_{1o} + \ldots + u_s y_{so} \tag{3.7}$$

with the constrains:

$$v_1 x_{1o} + \ldots + v_m x_{mo} = 1 \tag{3.8}$$

$$u_1 y_{1j} + \ldots + u_s y_{sj} \leq v_1 x_{1j} + \ldots + v_m x_{mj} \qquad (j=1, \ldots, n) \tag{3.9}$$

$$v_1, v_2, \ldots, v_m \geq 0 \tag{3.10}$$

$$u_1, u_2, \ldots, u_s \geq 0 \tag{3.11}$$

Both approaches lead to the same result. We can notice that constraint (3.9) is computed by multiplying constraint (3.4) for its denominator. The solution of the problem has μ* and v* as weights and θ* as optimal value. If θ* is equal to 1, the DMU_o is said to be CCR efficient and there is at least one optimal solution μ* and v*, with μ* and v* greater than zero. Otherwise, a value θ* <1 (or θ* = 1 and at least one element between v* and u* is zero) indicates that the DMU_o is CCR inefficient. If we obtain a value θ* <1 there must be at least a parameter in the inequality (3.9) such that the weights μ* and v* equal the left term with the right one. Formally:

$$E'_o = \left\{ j: \sum_{r=1}^{s} u_r^* y_{rj} = \sum_{i=1}^{m} v_i^* x_{ij} \right\} \tag{3.12}$$

The set of DMU_j that satisfies the equation (3.12) is called the reference set or the peer set of the DMU_o, since, being in the efficient frontier, it allows to obtain the same efficiency of the DMU in the efficient frontier. By combining this information with the optimal weights, we can express θ* as the following ratio:

$$\theta^* = \frac{\sum_{r=1}^{s} u_r^* y_{ro}}{\sum_{i=1}^{m} v_i^* x_{io}} \tag{3.13}$$

Recalling that constraint (3.8.) imposes the condition that the denominator is equal to one:

$$\sum_{i=1}^{m} v_i^* x_{io} = 1 \tag{3.14}$$

it is possible to rewrite θ* as:

$$\theta^* = \sum_{r=1}^{s} u_r^* y_{ro} \tag{3.15}$$

Which allows us to define the single contribution of each output in relation with the total efficiency.

Hereafter, we will show two examples taken from Cooper, Seiford and Tone (2007) in which we use the input-oriented CCR model for the evaluation of a defined number of production units.

3.2.1 One input and one output model

Suppose we have to assess the relative efficiency of eight production units listed in Table 3.1. In this case, the production process is characterized by the use of a single input to produce a single output.

Table 3.1. One input and One output example, CRR model

DMU	***A***	***B***	***C***	***D***	***E***	***F***	***G***	***H***
INPUT	2	3	3	4	5	5	6	8
OUTPUT	1	3	2	3	4	2	3	5

Source: Cooper, Seiford and Tone (2007).

As we sad earlier, it is possible to calculate the optimal solution for each DMU by resolving a linear problem set in the following way:

DMU A

$$\max \theta^* = 1u \tag{3.16}$$
(the output is equal to 1)

With the constraints:

2v = 1

u ≤ 2v	(A)	3u ≤ 3v	(B)
2u ≤ 3v	(C)	3u ≤ 4v	(D)
4u ≤ 5v	(E)	3u ≤ 5v	(F)
3u ≤ 6v	(G)	5u ≤ 8v	(H)

Now, we can find the optimal solution for the DMU A, we know that v*=0.5 and u*=0.5, so recalling $\max \theta^* = 1u$, we have $\theta^* = 0.5$. Here we can notice that the decision unit A is inefficient because is <1. Now we proceed computing the DMU B.

DMU B

$$\max \theta^* = 3u \tag{3.17}$$
(the output is equal to 3)

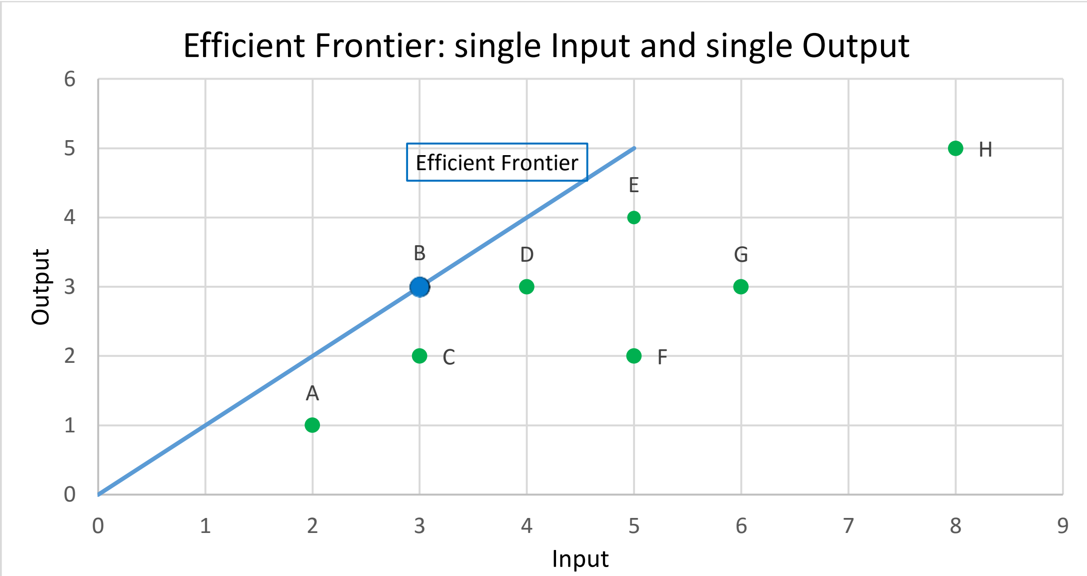
With the following constraints:

3v = 1

u ≤ 2v	(A)	3u ≤ 3v	(B)
2u ≤ 3v	(C)	3u ≤ 4v	(D)
4u ≤ 5v	(E)	3u ≤ 5v	(F)
3u ≤ 6v	(G)	5u ≤ 8v	(H)

The optimal solution for the DMU B is given by v*=0.333 and u*=0.333 so, recalling $\max \theta^* = 3u$, we have $\theta^* = 1$. The DMU B is CCR efficient and is our reference set for all the DMUs. We will compute the efficiency of all the other DMUs and we will represent them in figure 3.1, where the x-axis represents the inputs and the y-axis the outputs. Each DMU is represented by a different point, according to the combination of inputs and outputs. The efficient frontier is represented by the straight line passing through the origin to the point with $\theta^* = 1$, that identify the point of maximum efficiency. In this example we can notice that only the DMU B is on the efficient frontier, so it is the only efficient DMU, while the others are inefficient.

Figure 3.1. Efficient Frontier representation, one input and one output (CCR model)

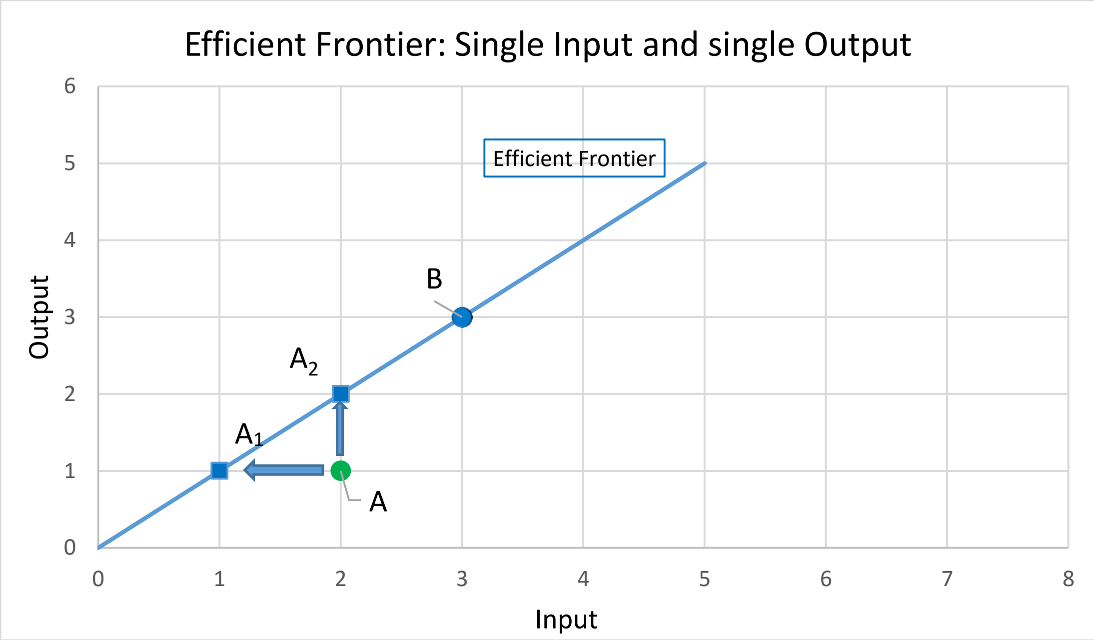


Source: Cooper, Seiford and Tone (2007).

We can notice in figure 3.1 that the DMU A is located under the efficient frontier, but if the DMU A was able to decrease the input (with the output remain equal) by the 50%, it would be on the efficient frontier. Practically, the level of input should be: 0.5*2=1.

If we were considering the CCR output-oriented model, then in order to make the DMU A an efficient making unit, we would increase the output level from 1 to 2 (maintaining the same level of input). The figure 3.2 is a graphic presentation of the problem described above.

Figure 3.2. Improvement of DMU A, one input and one output case (CCR model)



Source: Cooper, Seiford and Tone (2007).

3.2.2 Two Inputs and one Output

In the following example the production process is composed by two inputs and one output for each DMU. In table 3.2 we can find the value taken by each DMU:

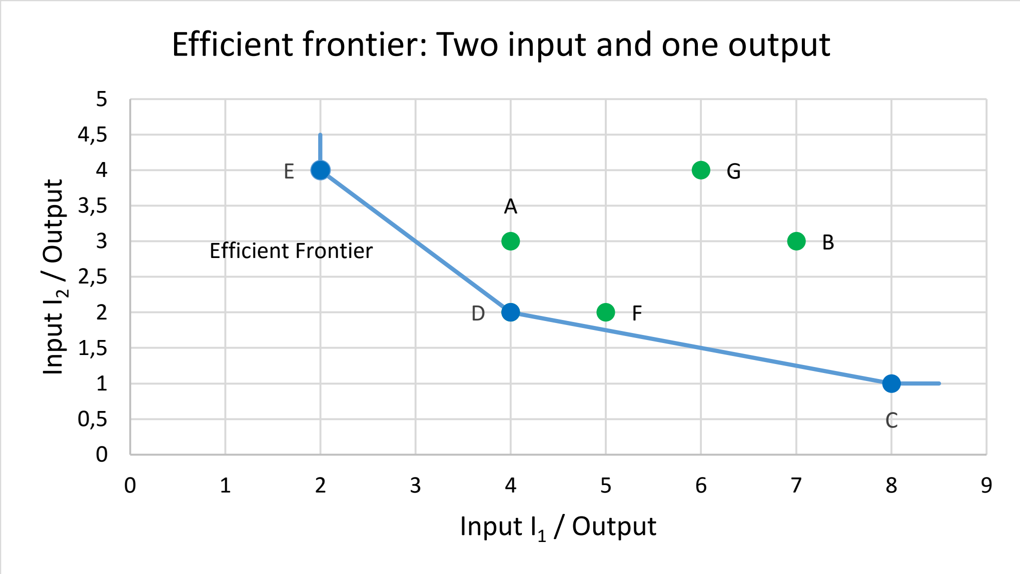
Table 3.2. Two inputs and one output example, CRR model

DMU		***A***	***B***	***C***	***D***	***E***	***F***	***G***
Input	I_1	4	7	8	4	2	5	6
	I_2	3	3	1	2	4	2	4
Output		1	1	1	1	1	1	1

Source: Cooper, Seiford and Tone (2007).

Figure 3.3 shows the combination of Input I_1/Output (x-axis) and Input I_2/Output (y-axis), we identify the efficient frontier as the blue line passing through the DMUs E, D and C.

Figure 3.3. Efficient Frontier representation, two inputs and one output



Source: Cooper, Seiford and Tone (2007).

Using the same method used in "one input and one output" example, we compute the solution for the first DMU:

DMU A

$$max\ \theta^* = u \tag{3.18}$$

With the constraints:

$$4v_1 + 3v_2 = 1$$

$u \leq 4v_1 + 3v_2$	(A)	$u \leq 7v_1 + 3v_2$	(B)
$u \leq 8v_1 + v_2$	(C)	$u \leq 4v_1 + 2v_2$	(D)
$u \leq 2v_1 + 4v_2$	(E)	$u \leq 5v_1 + 2v_2$	(F)
$u \leq 6v_1 + 4v_2$	(G)	All variables are non-negative	

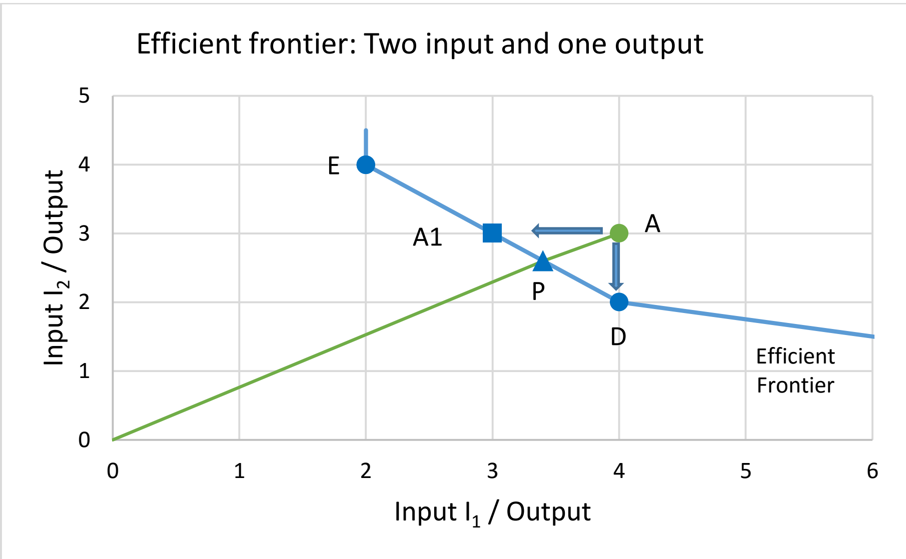
In order to measure the inefficiency of DMU A, we compute the solutions for the weights $v_1 and v_2$, then we find the value of θ^* that is 0.8571. This value can also be expressed as the distance from the origin to the point A, passing through the efficient frontier in P, of the graph in figure 3.4.

Arithmetically:

$$\frac{OP}{OA} = 0.8571 \tag{3.19}$$

Graphically, figure 3.4 is a representation of the optimization problem explained above.

Figure 3.4. Improvement of DMU A, two input and one output case (CCR model)



Source: Cooper, Seiford and Tone (2007).

The point P (blue triangle) is on the line connecting the points D and E, these points are called the reference set for the point A (the inefficient DMU). The reference set changes for different DMUs, for example the reference set for the DMU F (figure 3.3) is composed by the points C and D.

In order to achieve the efficient frontier, for the DMU A we identify the point A_1 that reduce the input I_1 by one, otherwise we might reduce the input I_2 from 3 to 2 and get to the point D.

Recalling the linear equation (3.7): $(LP_o) = max\theta = \; u_1y_{1o}+\ldots+u_sy_{so}$, we can now re-write the equation in a vector-matrix form, being v the row vector for the inputs and u the row vector used as multiplier:

$$(LP_o) \quad \max_{\mathrm{v,u}} \quad uy_o \tag{3.20}$$

With the constraints:

$$vx_o = 1 \tag{3.21}$$

$$-vX + uY \; \leq 0 \tag{3.22}$$

$$v \geq 0, \;\; u \geq 0 \tag{3.23}$$

Let us define ($\mathrm{DLP_o}$) as the dual linear problem expressed by a real variable θ and a non-negative vector of variables: $\lambda = (\lambda_1, \ldots, \lambda_n)^T$:

$$(DLP_o) \quad \min_{\theta,\lambda} \quad \theta \tag{3.24}$$

With the constraints:

$$\theta x_0 - X\lambda \; \geq 0 \tag{3.25}$$

$$Y\lambda \; \geq y_0 \tag{3.26}$$

$$\lambda \geq 0 \tag{3.27}$$

The optimal value θ^* takes value between zero and one: the non-negative assumption comes from constraint (3.23).

In order to give a solution to the dual problem, we will introduce the Production possibility set. Here, the data are not assumed to be non-negative, but at least one component of every input and output vector is positive. Cooper, Seiford and Tone (2007) refer to a "relaxed" positive data assumptions: the data have at least one positive component in both input and output vectors. Now, if we set the matrix $\mathrm{X} = (x_j)$ and the matrix $Y = (y_j)$, with x_j and y_j greater than zero, we can define P as the production possibility set:

$$P = \{(x, y) | x \geq X\boldsymbol{\lambda}, y \geq Y\boldsymbol{\lambda}, \ \boldsymbol{\lambda} \geq 0\} \quad (3.28)$$

Where $\boldsymbol{\lambda}$ is a semi positive vector in R.

We want an input that belongs to the set of production possibilities P which permits at least the same level of output y_0 of DMU_0, with the reduction of the vector x_0 proportional to the smallest possible value. Basically, in an Input-oriented model, the output value is given and we try to minimize the value of the inputs in order to achieve efficiency.

When $\theta^* \leq 1$ we can say that $(X\boldsymbol{\lambda}, Y\boldsymbol{\lambda})$ outperform $(\theta x_o, y_o)$, we can use this property to define the "slack" vectors:

- Input excesses:

$$s^- = \theta x_o - X\boldsymbol{\lambda} \quad (3.29)$$

  Where $s^- \in R^m$ and $s^- \geq 0$;

- Output shortfalls:

$$s^+ = Y\lambda - \ y_o \quad (3.30)$$

  Where $s^+ \in R^s$ and $s^+ \geq 0$.

The definition of the input excesses and output shortfalls is very important and is achieved by solving a two-phase problem. In Phase 1, we solve the DLP_o and we find the value of θ^* that will be used in the second phase. θ^* is equal to the optimal value found in LP_o (the CCR efficiency value). In Phase 2, we solve the linear problem using the values of $\boldsymbol{\lambda}$ and the input excesses and output shortfalls as variables:

$$\max_{\lambda, s^-, s^+} \quad \omega = \ es^- + es^+ \quad (3.31)$$

Where e= (1, . . ., 1) is a vector of ones. With the constraints:

$$s^- = \theta x_o - X\boldsymbol{\lambda} \tag{3.32}$$

$$s^+ = Y\boldsymbol{\lambda} - y_o \tag{3.33}$$

$$\lambda \geq 0,\ \ s^- \geq 0,\ \ s^+ \geq 0 \tag{3.34}$$

Recalling: $es^-0= \sum_{i=1}^{m} s_i^-$ and $es^+ = \sum_{r=1}^{s} s_r^+$.

We want to keep the same level of $\theta = \theta^*$ by finding a solution that maximizes the sum of the input excesses and output. The max-slack solution is achieved by an optimal solution given by $\lambda^*, s^{-*}, s^{+*}$, furthermore, if the optimal solution is zero slack (so the value of s^{-*}, s^{+*} are equal to zero) and the radial efficiency ($\theta^* = 1$) is satisfied, then the DMU_0 is defined as CCR-efficient, otherwise it is called CCR-inefficient because both constraints must be satisfied in order to achieve the efficiency.

3.2.3 The reference set.

We now define E_0 be the reference set for an inefficient DMU_0, computed as follow:

$$E_0 = \left\{ j \middle| \lambda_j^* > 0 \right\}\ \ (j \in \{1, \ldots, n\}) \tag{3.35}$$

and we compute the optimal solution as:

$$\theta^* x_0 = \sum_{j \epsilon E_0} x_j \lambda_j^* + s^{-*}\ \ and\ \ y_0 = \sum_{j \epsilon E_0} y_j \lambda_j^* + s^{+*} \qquad (3.36)\text{– } (3.37)$$

We have defined an optimal solution for the DMU_0 that was inefficient, now we can interpret this optimal solution as the difference between a technical part and a mix inefficiency that will result in a positive combination of observed input value:

$$x_0 \geq \theta^* x_0 - s^{-*} = \sum_{j \epsilon E_0} x_j \lambda_j^* \tag{3.38}$$

In the same way we can write y_0 as a combination of observed output values defined by the positive sum of the observed outputs and the shortfalls:

$$y_0 \leq y_0 s^{+*} = \sum_{j \epsilon E_0} y_j \lambda_j^* \tag{3.39}$$

We are evaluating the efficiency with an Input-oriented model, the efficiency of a DMU can be increased by radially reducing the input values by the parameter θ^* and by eliminating the input excesses (indicated by s^{-*}). Another method for improving an inefficient DMU is based on the increase of the output values using the output shortfalls (s^{+*}).

Then, we can calculate the gross improvement of the inputs Δx_o, and output improvement Δy_o, which will allow us to define the CCR projection: $\widehat{x_{o,}}\widehat{y_o}$ the new levels of input and output:

$$\widehat{x_o} = x_o - \Delta x_o = \theta^* x_o - s^{-*} \leq x_o \tag{3.40}$$

$$\widehat{y_o} = y_o + \Delta y_o = y_o + s^{+*} \geq y_o \tag{3.41}$$

Where:

$$\Delta x_o = x_o - (\theta^* x_o - s^{-*}) = (1 - \theta^*) x_o + s^{-*} \tag{3.42}$$

$$\Delta y_o = s^{+*} \tag{3.43}$$

3.2.4 The output-oriented model

So far, we have tried to obtain an efficient DMU by minimizing the input at a given output level (input-oriented model). This paragraph presents a different model that attempts to maximize the output while the level of input remains the same, this is called the output-oriented model. We now define:

$$DLPO_o \quad \max_{\eta,\mu} \quad \eta \tag{3.44}$$

With the constraints:

$$x_o - X\mu \ \geq 0 \tag{3.45}$$

$$\eta y_o - Y\mu \leq 0 \tag{3.46}$$

$$\mu \geq 0 \tag{3.47}$$

The optimal solution of the Output-oriented model is connected to the Input-oriented model through these relationships:

$$\lambda = \frac{\mu}{\eta}, \quad \theta = \frac{1}{\eta} \tag{3.48}$$

Recalling the input-oriented CCR model described in the equation 3.24 – 2.27, the optimal solution of the output-oriented model is related to the input-oriented model in the following way:

$$\eta^* = \frac{1}{\theta^*} \quad \text{and} \quad \mu^* = \frac{\lambda^*}{\theta^*} \tag{3.49}$$

We define t^- and t^+as the slack of the output-oriented model:

$$t^- = \ x_o - \ X\mu \tag{3.50}$$

$$t^+ = Y\mu - \eta y_o \tag{3.51}$$

Given a value $\theta^* \leq 1$, the relation with the input-oriented model is given by:

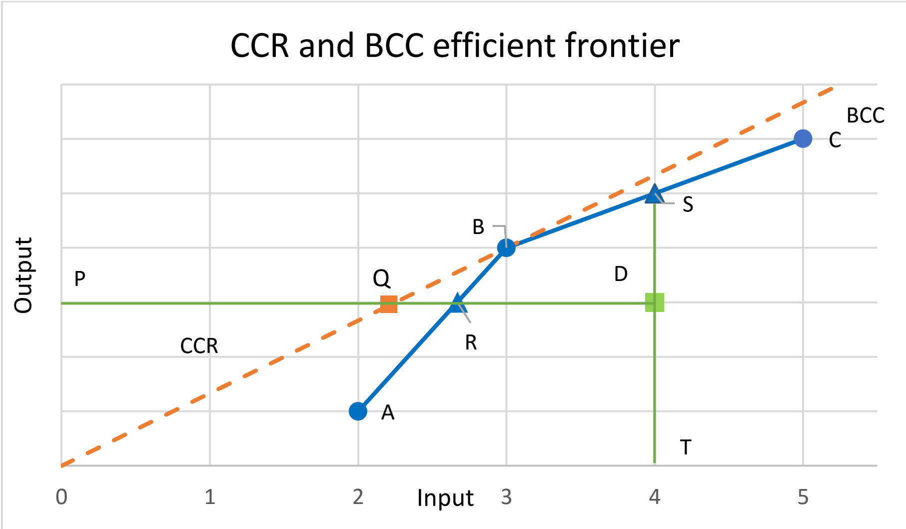
$$t^{-*} = \frac{s^{-*}}{\theta^*} \quad and \quad t^{+*} = \frac{s^{+*}}{\theta^*} \tag{3.52}$$

Where θ^* defines the input reduction rate and η^* expresses the output enlargement rate: the higher this value, the less efficient is the DMU.

3.3 The BCC models

The BCC model, introduced by Banker, Charnes and Cooper (1978), is based on the CCR model. The returns to scale activities are no longer assumed as constants, but as variables. Consequently, the efficient frontier will no longer be characterized by a straight line passing through the origin but, for example, it might assume a convex structure. Graphically, a comparison between the two models is presented in the figure (3.5), as we will see there are clear differences in the efficiency assessment.
We consider a process with one input and one output with 4 DMUs (A, B, C, D), figure 3.5 shows the efficient frontier in the CCR model (described by the orange dotted line that passes from the origin through the point B) and the efficient frontier (blue line) in the BCC model passing through the DMUs A, B and C.
We can notice that for the CCR model only the production unit B is efficient while, according to the model BCC, the efficient production units are A, B, and C.

Figure 3.5. Comparison between CCR model and BCC model



Source: Cooper, Seiford and Tone (2007).

The green square identifies the DMU (D) which, as we can clearly see, it is not efficient. Considering the BCC model, the efficiency of the DMU D is computed as follows:

$$E(D)_{BCC} = \frac{PR}{PD} \tag{3.52}$$

While, the efficiency computed using the CCR model is:

$$E(D)_{CCR} = \frac{PQ}{PD} \tag{3.53}$$

If we had used an output oriented model, the efficiency would have been calculated in this way:

$$E(D_{out})_{BCC} = \frac{DT}{ST} \tag{3.54}$$

Arithmetically, the BCC model is different from the CCR model for the addition of the condition: $\sum_{j=1}^{n} \lambda_j = 1$, also written as $e\lambda = 1$, where e is a row vector of one and λ is a column vector with all non-negative elements (Banker, Charnes and Cooper, 1984).
The production possibility set is defined as:

$$P_{BCC} = \{(x, y) | x \geq X\lambda, y \leq Y\lambda, e\lambda = 1, \lambda \geq 0\} \tag{3.55}$$

3.3.1 Input-oriented model

As we said before, we want to evaluate the efficiency of a general DMU$_o$. Given the scalar θ_b, the BCC model try to solve the following linear problem:

$$\min_{\theta_b, \lambda} \theta_b \tag{3.56}$$

with the constraints:

$$\theta_b x_o - X\lambda \geq 0 \tag{3.57}$$

and $Y\lambda \geq y_o$, $e\lambda = 1$ and $\lambda \geq 0$ as we said before.
The dual problem of the BCC will be expressed as:

$$\max_{v, u, u_o} \; z = uy_o - u_o \tag{3.58}$$

with the constraints:

$$vx_o = 1 \tag{3.59}$$

$$-vX + uY - u_o e \leq 0 \tag{3.60}$$

Where $v$and $u$are vectors both greater than zero while $z$and $u_o$are scalars. The variable $u_o$can take positive or negative value and this represent a clear difference with the CCR model.

For the optimal solution we define the following parameters: θ_b^*, λ^*, s^{-*}, s^{+*}. As we have seen for the CCR model, the points s^{-*}, s^{+*} represent, respectively, the maximal input excesses and the output shortfalls. A general DMU_o is called BCC-efficient if it has no slack (it means that s^{-*}, s^{+*} are equal to zero) and the constraints $\theta_b^* = 1$ is satisfied. Otherwise, the DMU will be considered inefficient.

According to the model: a DMU that is BCC-inefficient will present the following reference set:

$$E_o = \{j|\lambda_j^* > 0\} \quad (j \in \{1, \dots, n\}) \tag{3.61}$$

Banker, Charnes and Cooper (1984) state that the optimal solutions comply with the following condition:

$$\theta_b^* x_o = \sum_{j \epsilon E_0} x_j \lambda_j^* + s^{-*} \tag{3.62}$$

$$y_o = \sum_{j \epsilon E_0} y_j \lambda_j^* - s^{+*} \tag{3.63}$$

According to the BCC-projection, we can define the improvement with:

$$\widehat{x_o} \Leftarrow \theta_b^* x_o - s^{-*} \tag{3.64}$$

$$\widehat{y_o} \Leftarrow y_o + s^{+*} \tag{3.65}$$

The value of $\widehat{x_o}$ and $\widehat{y_o}$ describe the point on the efficient frontier used to assess the DMU_o.

3.3.2 Output-oriented BCC model

The Output-oriented BCC model uses the same formulation of the CRR model, with the addition of the constraint seen in the previous input-oriented BCC model.
The linear program is expressed as:

$$\max_{\eta_b, \boldsymbol{\lambda}} \eta_b \tag{3.66}$$

With the constraints:

$$X\lambda \leq x_o \tag{3.67}$$

$$\eta_b y_o - Y\lambda \leq 0 \tag{3.68}$$

$$e\lambda = 1 \qquad \lambda \geq 0 \tag{3.69}$$

Once defined the linear program, we can define the associated dual form:

$$\min_{v, u, v_o} \quad z = vx_o - v_o \tag{3.70}$$

With the constraints:

$$uy_o = 1 \tag{3.71}$$

$$vX - uY - \ v_o e \geq 0 \tag{3.72}$$

Recalling that v and u are both greater or equal than zero, while v_o is free in sign and is the scalar associated with $e\lambda = 1$ in the envelopment model.

3.4 The additive model

In CCR and BCC models we made a division between the Input-oriented model and the Output-oriented model. This distinction is necessary because such models are constructed on the basis of the radial projections. In particular, in the input-oriented models, the inputs are reduced while maintaining the output constant; on the other hand, in the Output-oriented models, the outputs are reduced by keeping the inputs constant. The Additive model, however, differs from the models described above because it combines both orientations in one model (Cooper, Seiford and Tone, 2007). The maximization process can be defined as:

$$(ADD_0) \qquad \max_{\lambda, s^-, s^+} \quad z = es^- + es^+ \tag{3.73}$$

With the constraints:

$$X\lambda + s^- = x_o \tag{3.74}$$

$$Y\lambda - s^+ = y_o \tag{3.75}$$

$$e\lambda = 1 \tag{3.76}$$

$$\lambda \geq 0, \qquad s^- \geq 0, \qquad s^+ \geq 0 \tag{3.77}$$

There are different Additive models, here we consider a model that takes in to account the constraint (3.76) (other additive models omit the condition $e\lambda = 1$). The dual problem related to this model is described in equation (3.78):

$$\min_{v, u, u_0} \quad w = vx_0 - uy_0 + u_0 \tag{3.78}$$

with the constraints:

$$vX - uY + u_0 e \geq 0 \tag{3.79}$$

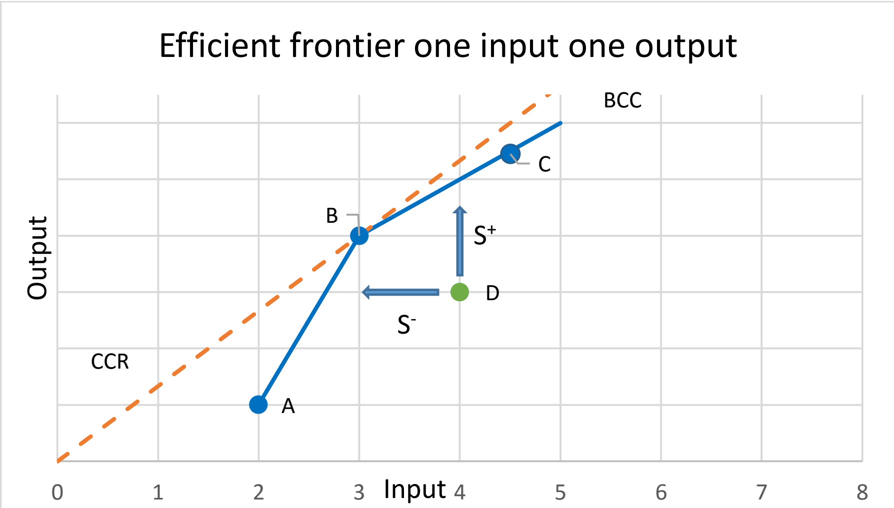
$$v \geq e \tag{3.80}$$

$$u \geq e \tag{3.81}$$

$$u_0 \; free \tag{3.82}$$

Figure 3.6 represents and efficient frontier computed with the CCR and the BCC model. We want to maximize the efficiency of DMU D (green point) using the input excess and the output shortfall at the same time, in order to arrive to the efficient frontier (the blue line).

Figure 3.6. Improvement of DMU D using the Additive model



Source: Cooper, Seiford and Tone (2007).

A possible replacement of DMU D is indicated by the arrow $s^{-}$and s^{+}; according to Cooper, Seiford and Tone (2007) a DMU_0 is ADD-efficient if and only if it is BCC-efficient, moreover, we can define as optimal solution the parameter: λ^{*}, s^{-*}, s^{+*}.
If and only if $s^{-*}$and s^{+*} are equal to zero a DMU_0 is ADD-efficient.

3.5 The Slack-Based Measure

The SBM model, introduced by Tone (2001), measures the efficiency of a generic DMU (x_0, y_0) by solving the following fractional program:

$$\min_{\lambda, s^{-}, s^{+}} \quad p = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^{-}}{x_{i0}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^{+}}{y_{r0}}} \tag{3.83}$$

with the constraints:

$$x_0 = X\lambda + s^- \quad (3.84)$$

$$y_0 = Y\lambda - s^+ \quad (3.85)$$

$$\lambda \geq 0, s^- \geq 0, s^+ \geq 0. \quad (3.86)$$

The objective function p takes values between zero and one, furthermore it satisfies the unit invariant property: the measure is invariant with respect to the unit of measurement of each input and output item (Cooper, Seiford and Tone, 2007). This is because in the numerator the variables s_i^- are dived by x_{i0} and the variables $s_r^+$are divided by y_{i0}. We can easily verify that if the variables s_i^- and s_r^+ increase, the objective value would decrease in a strictly monotone manner (all else held constant).
We now set SBM(P) being an interpretation of the ratio between mean input and output mix inefficiencies and we transform equation (3.83) into:

$$\mathrm{SBM(P)} = \left(\frac{1}{m}\sum_{i=1}^{m}\frac{x_{i0} - s_i^-}{x_{i0}}\right)\left(\frac{1}{s}\sum_{r=1}^{s}\frac{y_{r0} + s_r^+}{y_{r0}}\right)^{-1} \quad (3.87)$$

The first term represents the mean proportional reduction rate of input, the second term is the inverse of the mean proportional rate of output expansion and measures the output mix inefficiency. Given a generic DMU A and a generic DMU B, if $x_A \leq x_B and\ y_A \geq y_{B,} then\ p_A^* \geq p_B^*$and DMU A dominates DMU B.
The fractional problem presented in equation (3.83) – (3.86) can be transformed in the following program by introducing t(a positive scalar variable):

$$\min_{t, \lambda, s^-, s^+} \quad r = t - \frac{1}{m}\sum_{i=1}^{m}\frac{ts_i^-}{x_{i0}} \quad (3.88)$$

with the constraints:

$$1 = t + \frac{1}{s}\sum_{r=1}^{s}\frac{ts_r^+}{y_{r0}} \tag{3.89}$$

$$x_0 = X\,\lambda + s^- \tag{3.90}$$

$$y_0 = Y\lambda - s^+ \tag{3.91}$$

$$\lambda \geq 0, s^- \geq 0, s^+ \geq 0, t > 0. \tag{3.92}$$

Now we represent equation (3.88) in a linear form, defining:

$$S^- = ts^-, S^+ = ts^+ \; and \; \Lambda = t\lambda$$

The linear program in t, S^-, S^+ and Λ is set as follow:

$$(LP) \min \quad r = t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_i^-}{x_{i0}} \tag{3.93}$$

With the constraints:

$$1 = t + \frac{1}{s}\sum_{r=1}^{s}\frac{S_r^+}{y_{r0}} \tag{3.94}$$

$$tx_0 = X\,\Lambda + S^- \tag{3.95}$$

$$ty_0 = Y\Lambda - S^+ \tag{3.96}$$

$$\Lambda \geq 0, S^- \geq 0, S^+ \geq 0, t > 0. \tag{3.97}$$

The last constraint $t > 0$ ensures that the transformation is reversible, we can define the optimal solution according to the Slack-Based Measure model:

$$p^* = r^*, \qquad \lambda^* = \frac{\Lambda^*}{t^*}, \qquad s^{-*} = \frac{S^{-*}}{t^*}, \qquad s^{+*} = \frac{S^{+*}}{t^*}. \tag{3.98}$$

where $r^*, t^*, \Lambda^*, S^{-*}, S^{+*}$are the optimal solutions of the linear program.

Thanks to the optimal solution we are able to assess if a DMU_0 is SBM efficient or not. A generic DMU_0 is efficient if $p^* = 1$, we obtain this result if there are no input excess and no output shortfall ($s^{-*} = 0$ and $s^{+*} = 0$). If a DMU (x_0, y_0) is inefficient and we define:

$$x_0 = X\,\lambda^* + s^{-*} \qquad y_0 = Y\,\lambda^* - s^{+*} \tag{3.99}$$

We can improve DMU (x_0, y_0) by eliminating the input excesses or augmenting the output shortfalls, then we define the value $(\widehat{x_0}\,, \widehat{y_0})$ that represents the point on the efficient frontier achieved by an efficient DMU:

$$\widehat{x_0} \Leftarrow\ x_0 - s^{-*}\ and\ \ \widehat{y_0} \Leftarrow\ y_0 + s^{+*} \tag{3.100}$$

In case of multiple optimal solutions, we know that the reference set could not be unique.

According to Cooper, Seiford and Tone (2007), the set of indices corresponding to positive $\lambda_j^* s$ is called the reference set (R_0) for (x_0, y_0) and is expressed in the following way:

$$R_0 = \{ j | \lambda_j^* > 0 \} \quad (j \in \{1, \ldots, n\}) \tag{3.101}$$

We can now define $(\widehat{x_0}\,, \widehat{y_0})$ using the expression of the reference set R_0:

$$\widehat{x_0} = \sum_{j \in R_0} x_j \lambda_j^* \tag{3.102}$$

$$\widehat{y_0} = \sum_{j \in R_0} y_j \lambda_j^* \tag{3.103}$$

So, through a proper combination of the DMU_j (where j is in the reference set) we can achieve the point $(\widehat{x_0}\,, \widehat{y_0})$on the efficient frontier.

Chapter. 4
DEA APPROACHES WITH NEGATIVA DATA

4.1 Introduction

In the previous chapter we described the main DEA models and we assumed the non-negativity for the inputs and outputs. However, when dealing with real data, the presence of negative values might bring to some interpretation problem due to input and output affected by value less than zero, therefore the measure of the efficiency achieved by a DEA methodology might not be correct. However, with some adjustment we will be able to handle negative data and to use the Data Envelopment Analysis even in presence of negative value.

In particular, in this chapter we will present the translation invariance model, that allows us to transform negative data into positive data without involving a variation in the efficiency measure of the original model. Then we will present other models proposed by different authors, formulated with the aim to apply the DEA even when some values in the inputs or outputs are negative: the Semi-Oriented Radial Measure proposed by Emrouznejad, Anoue and Thanassoulis (2010), then we will describe the Range Directional Model with its variations, the Modified Slack-Based Measure model proposed by Sharpe, Meng and Liu (2007) and finally the Variant of Radial Measure (Gang, Panagiotis and Zhenhua, 2011).

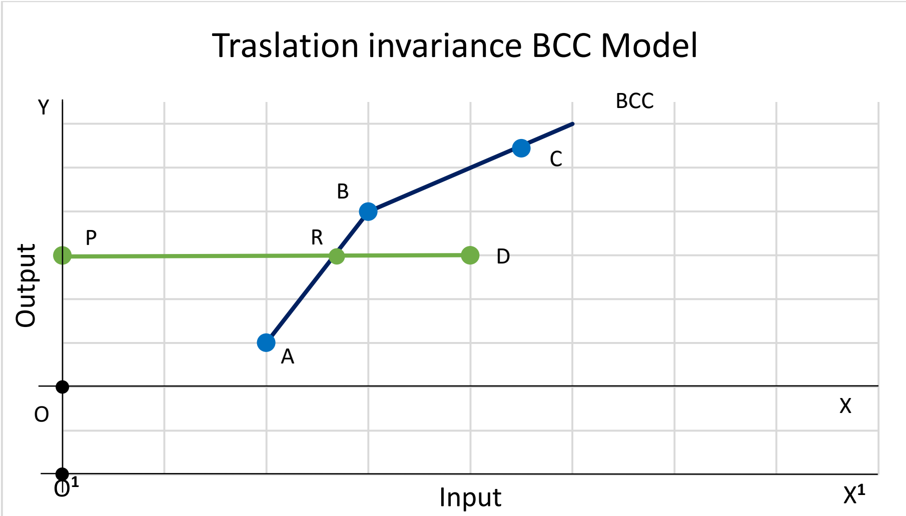
4.2 Translation invariant DEA model

When evaluating the relative efficiency of a sample of DMUs, it might be necessary to be able to handle the presence of negative data in inputs and/or outputs; in order to solve this problem, the translation invariance model is frequently used.

We are defining a model that translates the original input/output values (for instance adding a constant to the original data) without changing the optimal solution computed with the original model (Cooper, Seiford and Tone, 2007).

In figure 4.1 we represent the BCC model, we can consider this model as translation invariant but only partially, how we will see, according to the model, the DMU D efficiency can be computed using the ratio PR/PD. We can observe that this ratio does not vary if the output value is changed by shifting the origin of the axes from O to O^1.

Figure 4.1. Improvement of DMU D using the translation invariance in BCC model



Source: Cooper, Seiford and Tone (2007).

Consequently, it can be stated that in the input-oriented BCC model, the efficiency measure of a decision unit is invariant if the output values are translated (but not those of the inputs). If we consider the BCC output-oriented model, then the efficiency measure of a DMU does not change if the inputs are translated (but not the outputs).

In the third chapter we presented the additive model that is often used when input and output take negative values, because the efficiency evaluation achieved by this model does not depend on the origin of the coordinate system.

To show this property, we consider a data set of inputs (X) outputs (Y) and we translate them by introducing two sets of arbitrary constants $(\alpha_i : i = 1, \dots, m)$ and $(\beta_r : r = 1, \dots, s)$ and obtaining:

$$x'_{ij} = x_{ij} + \alpha_i \Rightarrow x_{ij} = x'_{ij} - \alpha_i \qquad (i = 1, \dots, m : j = 1, \dots, n) \qquad (4.1)$$

$$y'_{rj} = y_{rj} + \beta_r \Rightarrow y_{rj} = y'_{rj} - \beta_r \quad (r = 1, \dots, s : j = 1, \dots, n) \qquad (4.2)$$

Now, recalling the equation (3.74): $X\lambda + s^- = x_o$ (where $\sum_{j=1}^{n} \lambda_j = 1$) of the additive model described in chapter 3, we show the validity of the translation property by replacing X with the equation (4.1):

$$\sum_{j=1}^{n} (x'_{ij} - \alpha_i)\lambda_j + s_i^- = \sum_{j=1}^{n} x'_{ij}\lambda_j + s_i^- - \alpha_i = x'_{i0} - \alpha_i \qquad (4.3)$$

So, the equation (4.3) becomes:

$$\sum_{j=1}^{n} x'_{ij}\lambda_j + s_i^- = x'_{i0} \quad (i = 1, \dots, m) \qquad (4.4)$$

Where λ_j and s_i^- are the same values that satisfy:

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{i0} \quad (i = 1, \dots, m) \qquad (4.5)$$

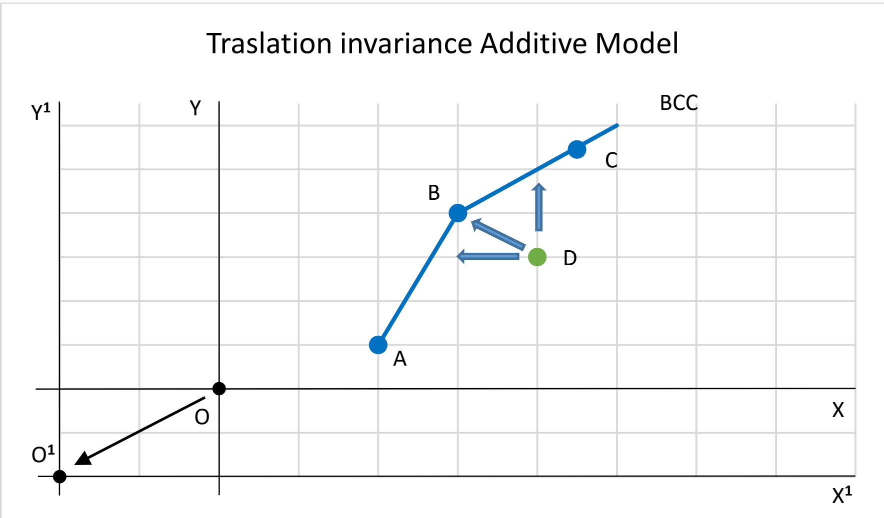
We now proceed with the same computation for Y, similarly we get the value λ_j and s_r^+ that satisfy:

$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{r0} \quad (r = 1, \dots, s) \qquad (4.6)$$

$$\sum_{j=1}^{n} y'_{rj}\lambda_j - s_r^+ = y'_{r0} \quad (r = 1,\ldots,s) \tag{4.7}$$

The equations above show that the optimal values of the objective function do not change if inputs and outputs data are modified by a constant. Therefore, the optimal solution for a generic DMU_0 in the original problem $(\lambda_j^*, s_i^{-*}, s_r^{+*})$ is also the optimal solution for the same DMU_0 in which the data have been translated by adding a constant. In figure 4.2 we can see that the additive model is invariant in translation for both inputs and outputs, because if we evaluate the efficiency of the decision unit considered, it does not change when we translate inputs and outputs, so that the origin of the system is modified.

Figure 4.2. Improvement of DMU D using the translation invariance method



Source: Cooper, Seiford and Tone (2007).

4.2. Semi-oriented Radial Measure

Usually, in the application of the DEA models we might face some difficulties when negative inputs or outputs appear. Let us say that if we consider this type of data, it is relevant to highlight the sign of the variables when we have to determine whether we should increase or decrease an input/output in order to improve the efficiency of a DMU. Emrouznejad, Anoue and Thanassoulis (2010) propose a model with the purpose

of handle the variables that take positive values for some DMU and negative values for others, treating each input/output variable as being the sum of two variables. One variable will take the negative value and the other the positive value, while the sum will lead to the initial value of the variable. Then, we can take the absolute value of the negative part without changing the origin (as the method exposed in paragraph 4.1).
We know recall the standard DEA model under the variable returns to scale assumption: firstly, we will consider the input-oriented BCC model and secondly the output-oriented. In order to better understand the variation from the original model, this time we will call h the optimal solution, we will use the notation $\sum_{j=1}^{n} \lambda_j = 1$ instead of $e\lambda = 1$, X and Y will take the notation X_{ij} and Y_{rj} and (x_0,y_0) will take the notation X_{ij0} and X_{ij0}.
We now compute the standard BCC input-oriented model:

$$\min \quad h \tag{4.8}$$

with the constraints:

$$\sum_j \lambda_j X_{ij} \leq h X_{ij0} \qquad \forall i \tag{4.9}$$

$$\sum_j \lambda_j Y_{rj} \geq Y_{rj0} \qquad \forall r \tag{4.10}$$

$$\sum_j \lambda_j = 1 \tag{4.11}$$

$$\lambda_j \geq 0, \qquad \forall j, \qquad h \text{ free} \tag{4.12}$$

For the standard BCC output-oriented model:

$$\max \quad h \tag{4.13}$$

with the constraints:

$$\sum_j \lambda_j X_{ij} \leq X_{ij0} \qquad \forall i \tag{4.14}$$

$$\sum_j \lambda_j Y_{rj} \geq h Y_{rj0} \qquad \forall r \tag{4.15}$$

$$\sum_j \lambda_j = 1 \tag{4.16}$$

$$\lambda_j \geq 0, \qquad \forall j, \qquad h \text{ free} \tag{4.17}$$

A DMU_0 is efficient for the optimal solution *h* in the input-oriented model, otherwise in the output-oriented model the optimal solution is 1/ *h*.

Emrouznejad, Anoue and Thanassoulis (2010) defined an output Y_k that is positive for some DMUs and negative for others and two variables Y_k^1 and Y_k^2 that take values Y_{kj}^1 and Y_{kj}^2 for the j-th DMU as follows:

$$Y_{kj}^1 = \begin{cases} Y_{kj} & if\ Y_{kj} \geq 0 \\ 0 & if\ Y_{kj} < 0 \end{cases} \tag{4.18}$$

$$Y_{kj}^2 = \begin{cases} 0 & if\ Y_{kj} \geq 0 \\ -Y_{kj} & if\ Y_{kj} < 0 \end{cases} \tag{4.19}$$

For all the values of *j* we have Y_{kj}^1 and Y_{kj}^2 greater than or equal to zero and:

$$Y_{kj} = Y_{kj}^1 - Y_{kj}^2 \tag{4.20}$$

The authors suggest to evaluate a DMUs with positive and negative values in output variables with the following variable returns to scale input-oriented model:

$$\min \ h \tag{4.21}$$

with the constraints:

$$\sum_j \lambda_j X_{ij} \leq h X_{ij0} \qquad \forall i \tag{4.22}$$

$$\sum_j \lambda_j Y_{rj} \geq Y_{rj0} \qquad \forall r \neq k \tag{4.23}$$

$$\sum_j \lambda_j Y_{kj}^1 \geq Y_{kj0}^1 \tag{4.24}$$

$$\sum_j \lambda_j Y_{kj}^2 \leq Y_{kj0}^2 \tag{4.25}$$

$$\sum_j \lambda_j = 1 \tag{4.26}$$

$$\lambda_j \geq 0, \qquad \forall j, \qquad h \text{ free} \tag{4.27}$$

The output variable Y_k is composed by two positive variables: Y_k^1 and Y_k^2, the variable Y_k^1 can be treated as an output and the variable Y_k^2 represents the input. Hence, we can treat the negative output values as inputs (the model aims to reduce the negative output absolute value), while positive values of the output variables are treated as normal outputs.

Using this process, the production possibilities set, when we introduce the variables Y_k^1 and Y_k^2 , is the same that we can get without disaggregating Y_k. We can show this property by multiplying the constraint (4.25) for -1, obtaining:

$$\sum_j \lambda_j (-Y_{kj}^2) \geq (-Y_{kj0}^2) \tag{4.28}$$

Consequently, adding (4.28) to the constraint (4.24):

$$\sum_j \lambda_j (Y_{kj}^1 - Y_{kj}^2) \geq Y_{kj0}^1 - Y_{kj0}^2 \tag{4.29}$$

We recall that $Y_{kj} = Y_{kj}^1 - Y_{kj}^2$ so we achieved the original model presented in (4.10) before disaggregating Y_k. Therefore, any possible solution in the SORM model will be also feasible in the original model, however the converse is not true (Emrouznejad, Anoue and Thanassoulis, 2010).

We can now define the variable X_{lj}^{1} and X_{lj}^{2} from X_{lj}in the following way:

$$X_{lj}^{1} = \begin{cases} X_{lj} & if\ X_{lj} \geq 0 \\ 0 & if\ \ X_{lj} < 0 \end{cases} \tag{4.30}$$

$$X_{lj}^{2} = \begin{cases} 0 & if\ X_{lj} \geq 0 \\ -X_{lj} & if\ \ X_{lj} < 0 \end{cases} \tag{4.31}$$

For all the values of *j* we have X_{lj}^{1} and X_{lj}^{2} both greater than zero, and:

$$X_{lj} = X_{lj}^{1} - X_{lj}^{2} \tag{4.32}$$

Now, we want to evaluate the efficiency when the DMUs take positive and negative values in input and output variables using the input-oriented SORM model. The aim of the following model is to improve the DMU and to yields a measure of efficiency that is the optimal value *h*. Specifically, those DMUs that have a positive value are treated as normal inputs, while the same variable with negative input values are considered as outputs.

$$\min\ \ h \tag{4.33}$$

with the constraints:

$$\sum_{j} \lambda_j X_{ij} \leq hX_{ij0} \qquad \forall i \in I \tag{4.34}$$

$$\sum_{j} \lambda_j X_{lj}^{1} \leq hX_{lj0}^{1} \qquad \forall l \in L \tag{4.35}$$

$$\sum_{j} \lambda_j X_{lj}^{2} \leq hX_{lj0}^{2} \qquad \forall l \in L \tag{4.36}$$

$$\sum_j \lambda_j Y_{rj} \geq Y_{rj0} \qquad \forall r \in R \tag{4.37}$$

$$\sum_j \lambda_j Y_{kj}^1 \geq Y_{kj0}^1 \qquad \forall k \in K \tag{4.38}$$

$$\sum_j \lambda_j Y_{kj}^2 \leq Y_{kj0}^2 \qquad \forall k \in K \tag{4.39}$$

$$\sum_j \lambda_j = 1 \tag{4.40}$$

$$\lambda_j \geq 0, \qquad \forall j \tag{4.41}$$

For this model Emrouznejad, Anoue and Thanassoulis (2010) assumed the input variables X_i (with $i \in I$) and Y_r (with $r \in R$) as output variables, these variables are positive for all DMUs. On the other hand, X_l (with $l \in L$) and Y_k (with $k \in K$) are respectively input and output variables that take positive values for some DMUs and negative values for others. Furthermore: $I \cup L = \{1, \ldots, m\}, I \cap L = \emptyset, R \cup K = \{1, \ldots, s\}\ and\ R \cap K = \emptyset$.

The model presented in (4.33) -(4.41) can be used for evaluating the efficiency of a DMU with positive and negative values in inputs and outputs using an output orientation:

$$\max \ h \tag{4.42}$$

with the constraints:

$$\sum_j \lambda_j X_{ij} \leq X_{ij0} \qquad \forall i \in I \tag{4.43}$$

$$\sum_j \lambda_j X_{lj}^1 \leq X_{lj0}^1 \qquad \forall l \in L \tag{4.44}$$

$$\sum_j \lambda_j X_{lj}^2 \leq X_{lj0}^2 \qquad \forall l \in L \tag{4.45}$$

$$\sum_j \lambda_j Y_{rj} \geq hY_{rj0} \qquad \forall r \in R \tag{4.46}$$

$$\sum_j \lambda_j Y^1_{kj} \geq hY^1_{kj0} \qquad \forall k \in K \tag{4.47}$$

$$\sum_j \lambda_j Y^2_{kj} \leq hY^2_{kj0} \qquad \forall k \in K \tag{4.48}$$

$$\sum_j \lambda_j = 1 \tag{4.49}$$

$$\lambda_j \geq 0, \qquad \forall j \tag{4.50}$$

The model presented in (4.42)-(4.50) is defined as *"output augmentation semi-oriented radial measure"* and brings to an efficiency value *1/h* * where *h** is the optimal value of the model. Emrouznejad, Anoue and Thanassoulis (2010) state that the reasoning done for model (4.33)-(4.41) can be applied also to this model, and the feasible solution for the *output augmentation semi-oriented radial measure* is a subset of the solution for the original output-oriented model.

4.3. A Directional distance approach for negative data

Usually, in DEA models, negative data are treated through some data transformation that may bring to some implications when considering the efficiency results.
Portela, Thanassoulis and Simpson (2004) proposed a solution to this problem that requires no transformation of the data and achieves an efficiency measure that can be used to compare and rank different DMU even in presence of negative data. In presence of data that can assume negative values, it becomes difficult to use a model with constant returns to scale and, at the same time, a variable return to scale model may be problematic when we apply the radial measure of efficiency for the evaluation of a DMU. Therefore, Silva Portela, Thanassoulis, Simpson (2004) proposed to use a model called Range Directional Measure (RDM), which is based on the directional distance model initially formulated by Chung, Färe and Grosskopf (1997), but modified in order determine a proper measure of efficiency even in presence of negative data.

The generic directional distance model formulated by Chambres, Chung and Färe (1996, 1998), in presence of variable returns to scale, is computed as follows:

$$\text{Max } \beta_0 \tag{4.51}$$

With the constraints:

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0} + \beta_0 g_{yr}, \quad (r = 1, \ldots, s) \tag{4.52}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{i0} - \beta_0 g_{xi}, \quad (i = 1, \ldots, m) \tag{4.53}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{4.54}$$

$$(\lambda_j, \beta_0, g_{yr}, g_{xi}) \geq 0 \tag{4.55}$$

In this model we considered a set of units $J = \{1, \ldots, m\}$ with inputs x_{ij} $(i = 1, \ldots, m)$, outputs y_{rj} $(r = 1, \ldots, s)$ and unit $o \in J$ to assess. It represents a non-oriented case and it considers simultaneously an input contraction and an output expansion. However, we are able to choose the orientation by setting respectively $g_{yr} = 0 \; or \; g_{xi} = 0$. Usually we define the directional vectors (g_{xi}, g_{yr}) being the observed input and output levels, this is true when dealing with positive data, nevertheless in presence of negative data the use of observed input/output would bring to a violation of the last constraints (4.55). Portela, Thanassoulis and Simpson (2004) proposed to modify the model presented in (4.51)-(4.55) by introducing an ideal point:

$$I = \left(\max_j (y_j), \min_j (x_j) \right) \tag{4.56}$$

Now we can define the range of possible improvements of unit *o* through the vectors R_{ro} and R_{io}, computed as follows:

$$R_{r0} = \max_{j}\{y_{rj}\} - y_{r0} \quad (r = 1, \dots, s) \tag{4.57}$$

$$R_{i0} = x_{i0} - \min_{j}\{x_{ij}\} \quad (i = 1, \dots, m) \tag{4.58}$$

Vectors R_{ro} and R_{io}, describe the range of possible improvements for the DMU$_o$ considered. On the one hand, R_{ro} defines the best improvement that the production unit considered could reach with each output, on the other hand R_{io} defines the best improvement that the production unit considered could reach with each input.

These improvements will never be negative and, consequently, the two vectors satisfy the constraint of non-negativity on the direction vectors. Accordingly, we define the Range Directional Measure (based on the improvement (4.57) and (4.58)) formulated by Portela, Thanassoulis and Simpson (2004) as follows:

$$\text{Max } \beta_0 \tag{4.59}$$

With the constraints:

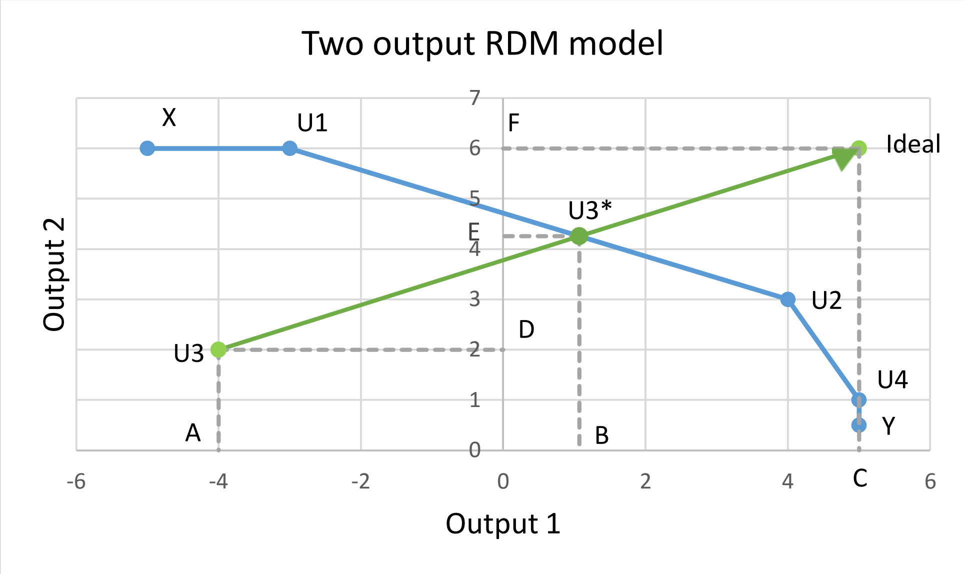
$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0} + \beta_0 R_{ro}, \quad (r = 1, \dots, s) \tag{4.60}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{i0} - \beta_0 R_{io}, \quad (i = 1, \dots, m) \tag{4.61}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \qquad \lambda_j \geq 0 \tag{4.62}$$

The Range Directional Measure (RDM) efficiency measure can be better explained graphically in figure 4.3, where we are assuming an output-oriented model.

Figure 4.3. Efficiency measure of DMU U3 using an output-oriented RDM model



Source: Portela, Thanassoulis and Simpson (2004)

The ratio $\overline{CB}/\overline{CA}$ (which is equivalent to the $\overline{FE}/\overline{FD}$ ratio) of the DMU U3, determines the efficiency measure $(1-\beta)$.

The ratio $\overline{CB}/\overline{CA}$ measures the distance between the level of output 1 at the observed point U3 and its target point defined as U3*. In a similar way we can interpret the $\overline{FE}/\overline{FD}$ ratio with respect to the level of output 2. Therefore, the efficiency of DMU3, which represents the relative distance between the points U3 and U3*, is determined as follows:

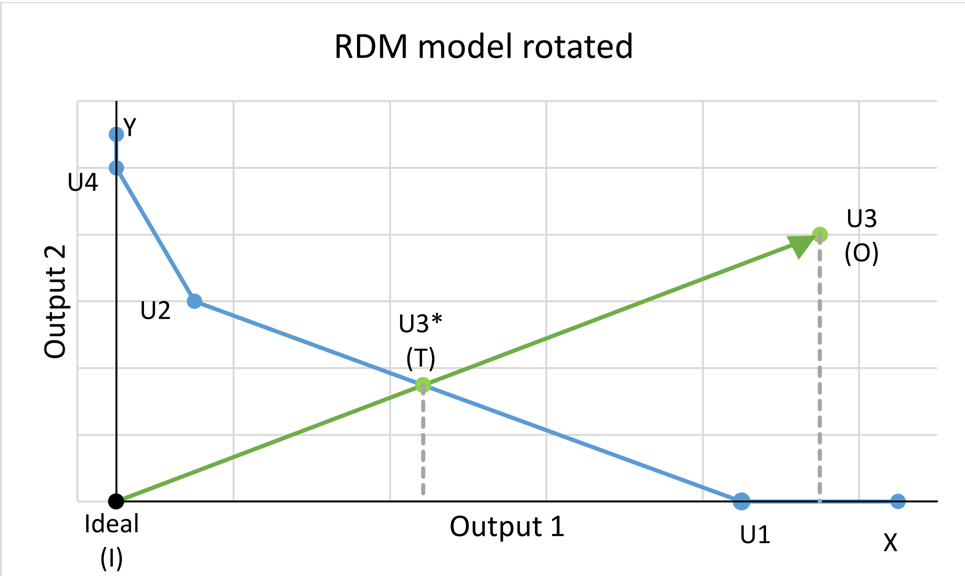
$$\frac{(5-1.07273)}{(5-(-4))}=\frac{(6-4{,}25455)}{(6-2)} = 43.36\% \qquad (4.63)$$

For an efficient DMU the value $1\text{-}\beta$ is equal to 1, and this occurs when $\beta = 0$. However, Portela, Thanassoulis and Simpson (2004) state that: even if the measure of efficiency achieved by the RDM model does not assure projection on Pareto-efficient targets, it may correctly identify weak efficiency. Consequently, the efficient DMU, according to the RDM model, may not meet the conditions of the Pareto efficiency, since in this case for the DMU_0 it is required that both β and all the slack variables are equal to zero.

We can notice that there is a similarity between the measure of efficiency achieved in the RDM model and the radial efficiency measure commonly used in the DEA methodology.

The difference is the use of an ideal point as reference point, we can show this by rotating figure 4.3 and notice how the ideal point is located in the position of the origin as in the traditional DEA models. In figure 4.4 we consider the vector $\overrightarrow{IT}$ defined as the distance between I and T (so from the ideal point to the target point) and the vector $\overrightarrow{IO}$ defined by the distance between I and O (the latter is the observed point). The ratio between the length of $\overrightarrow{IT}$ and $\overrightarrow{IO}$ represent the measure of efficiency 1-β with the point I being the origin.

Figure 4.4. Range Directional model (Figure 4.3) after rotation



Source: Portela, Thanassoulis and Simpson (2004)

Hence, the RDM model behaves in a similar way to a radial measure of efficiency traditionally used in DEA and allows to obtain a DMU efficiency measure even when some variables are negative. However, the efficiency measure obtained with this model, despite the fact that it does not consider all the inefficiency, can, in some cases, identify a lack of efficiency. For example, the decision units X and Y represented in figures 4.3 and 4.4 are not located on the efficient frontier, since their efficiency measure (1- β_X and 1- β_Y), calculated using the RDM model, are different from 1. We can notice that the efficiency measure computed with the RDM model (1-β) is the distance measure between the observed point and the target point, so the lower the distance the more efficient the unit will be (with a higher value of (1-β)).

4.3.1 The inverse Range Directional Model and a RDM variation

This paragraph aims to present some alternative directions of improvement of inputs and outputs. Portela, Thanassoulis and Simpson (2004) proposed a model that identifies the factors on which the production units perform better and it tries to improve them. The model is called the Inverse Range Directional Measure (IRDM) and is defined as follows:

$$\text{Max } \beta_0 \tag{4.64}$$

With the constraints:

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0} + \beta_0 \frac{1}{R_{ro}}, \quad (r = 1, \ldots, s) \tag{4.65}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{i0} - \beta_0 \frac{1}{R_{io}}, \quad (i = 1, \ldots, m) \tag{4.66}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \qquad \lambda_j \geq 0 \tag{4.67}$$

The main difference with the other models previously presented is the use of the directional vectors $1/R_{r0}$ in (4.66) and$1/R_{i0}$ in (4.67) instead of R_{r0} and R_{i0} (in few words: the directional vectors are the inverse of the range of possible improvements). In case of
$R_{r0} = 0$ or $R_{i0} = 0$, the division by zero is avoided using zero instead of $1/R_{r0}$ and $1/R_{i0}$.
The authors suggest to use this model only for target setting, because the efficiency measure, identified by the IRDM, may bring to some interpretation problems when we

try to compare different production units. This because the efficiency measure (1-β) represents the distance from the observed point to the target point, the latter is based on the relative ideal point that is different for each production unit.

In order to handle negative data, we now present an alternative model to the range directional model obtained by replacing the directional vectors R_{i0} and R_{r0}. Given a DMU $z_0 = (x_0, y_0)$, Kerstens and Van de Woestyne (2009) proposed the directional vectors $|x_0|$ and $|y_0|$ in which $|x_{i0}|$ and $|y_{r0}|$ are respectively the input vector components and the output vector components.

$$\text{Max } \beta_0 \tag{4.68}$$

with the constraints:

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0} + \beta_0 |y_{r0}|, \quad (r = 1, \ldots, s) \tag{4.69}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{i0} - \beta_0 |x_{i0}|, \quad (i = 1, \ldots, m) \tag{4.70}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \qquad \lambda_j \geq 0 \tag{4.71}$$

We notice that, the more the value of $\beta_0$is close to zero, the more efficient is the DMU observed.

4.4 The Modified Slacks-Based Model

When dealing with real data, negative inputs and outputs may occur in different situations: in chapter 3 we presented the Slack-Based Model (SBM) proposed by Tone (2001); this model is able to handle negative data but can bring to negative efficiencies and is not translation invariant. The efficiency measure *p,* computed in (3.83) takes values between zero and one (a value of one is obtained only if there are no inefficiencies); this can be easily proved by considering the numerator, which takes values between zero and one, and the denominator, that is greater or equal than one (Tone, 2001). In case of negative data, the measure *p* could not lie in the interval [0,1] and this could bring to some problems in the interpretation of the efficiency measure. In order to overcome these problems, Sharpe, Meng and Liu (2007) proposed a modification of the Slack-Based Model described as follows:

$$\min \quad p = \frac{1 - \sum_{i=1}^{m} \frac{w_i s_i^-}{x_{i0}}}{1 + \sum_{r=1}^{s} \frac{v_r s_r^+}{y_{r0}}} \tag{4.72}$$

with the constraints:

$$\sum_{r=1}^{s} y_{rj}\lambda_j - s^+ = y_{r0} \quad (r = 1,\dots,s) \tag{4.73}$$

$$\sum_{i=1}^{m} x_{ij}\lambda_j + s^- = x_{i0} \quad (i = 1,\dots,m) \tag{4.74}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{4.75}$$

$$\sum_{i=1}^{m} w_i = 1, \quad \sum_{r=1}^{s} v_r = 1 \tag{4.76}$$

$$s^+, s^-, \lambda_j, w_i, v_r \geq 0 \tag{4.77}$$

Recalling the definition of R_{ro} and R_{io} in (4.57) and (4.58) used in the Range Directional Model, the measure *p* in (4.72) lies between zero and one. We can show this process in the following way:

$$\begin{aligned} s^- = x_0 - X\lambda \leq \max_{\lambda}(x_0 - X\lambda) = x_0 + \max_{\lambda}(-X\lambda) \\ = x_0 - \min_{\lambda} X\lambda \end{aligned} \tag{4.78}$$

$$s^{+} = Y\lambda - y_0 \leq \max_{\lambda}(Y\lambda - y_0) = \max_{\lambda} Y\lambda - y_0 \tag{4.79}$$

Now we set $Z_i = \min_j(x_{ij})$, we know that $\min_\lambda X\lambda \geq Z$, so we can re-write (4.58) as $R_{i0} = x_{i0} - Z$, therefore $s^{-} \leq R_{i0}$. We can use the same process with s^{+}, by setting $M_r = \max_j(y_{rj})$, we know that $\max_\lambda Y\lambda \geq M$, so we can state that $s^{+} \leq R_{r0}$.

Then we transform the fractional problem (4.72) into a linear programming problem in the following way:

$$\min \quad r = t - \sum_{i=1}^{m} \frac{w_i s_i^{-}}{R_{i0}} \tag{4.80}$$

with the constraints:

$$1 = t + \sum_{r=1}^{s} \frac{v_r s_r^{+}}{R_{ro}} \tag{4.81}$$

$$t y_0 = Y\Lambda - S^{+} \tag{4.82}$$

$$t x_0 = X\,\Lambda + S^{-} \tag{4.83}$$

$$\sum_{i=1}^{m} w_i = 1, \quad \sum_{r=1}^{s} v_r = 1 \tag{4.84}$$

$$\Lambda \geq 0, S^{-} \geq 0, S^{+} \geq 0, t \geq 0. \tag{4.85}$$

We now introduce the positive scalar variable t as we did in (3.93) and we define the optimal solution according to the Slack-Based Measure model as: $r^{*}, \Lambda^{*}, t^{*}, S^{-*}, S^{+*}$. Finally, we get an optimal solution for the Modified Slack-Based Measure model:

$$p^{*} = r^{*}, \qquad \lambda^{*} = \frac{\Lambda^{*}}{t^{*}}, \qquad s^{-*} = \frac{S^{-*}}{t^{*}}, \qquad s^{+*} = \frac{S^{+*}}{t^{*}}. \tag{4.86}$$

4.5 A Variant of Radial Measure

Traditional DEA models are based on the non-negativity assumption for the input and output values, and this represents a weakness when non-positive values are present in the observed data. So far we have seen different approaches for dealing with this problem, from the Range Directional Measure that uses the difference between the initial evaluated value and the best observed value of a variable proposed by Portela, Thanassoulis and Simpson (2004), to the Semi-Oriented Radial Measure that treats each input-output variable as being the sum of two variables proposed by Emrouznejad, Anoue and Thanassoulis (2010).

Here we want to show the Variant of Radial Measure (VRM) proposed by Cheng, Zervopoulos and Qian (2011), that is able to handle variables with positive values for some and negative values for other DMUs.

The traditional input-oriented radial model in computed as follows:

$$min\theta \tag{4.86}$$

with the constraints:

$$X\lambda \leq \theta x_0 \tag{4.87}$$

$$Y\lambda \geq y_0 \tag{4.88}$$

$$\left[\sum_{j=1}^{n} \lambda_j = 1\right] \tag{4.89}$$

$$\lambda \geq 0 \tag{4.90}$$

While the output-oriented model is:

$$max\phi \tag{4.91}$$

with the constraints:

$$X\lambda \leq x_0 \tag{4.92}$$

$$Y\lambda \geq \phi y_0 \tag{4.93}$$

$$\left[\sum_{j=1}^{n} \lambda_j = 1\right] \tag{4.94}$$

$$\lambda \geq 0 \tag{4.95}$$

The constraint $\sum_j \lambda_j = 1$, is in square brackets because it is dropped in the CRR model (so under constant returns to scale) while is present if we consider the BCC model (so with variable returns to scale). The efficiency measure is identified respectively in the optimal values θ^* and $1/\phi^*$ for the input and output oriented model. Consequently, we transform both models by replacing θ and ϕ with 1-β and 1+ β in (4.86) and (4.91). The input-oriented model becomes:

$$max\beta \tag{4.96}$$

with the constraints:

$$X\lambda + \beta x_0 \leq x_0 \tag{4.97}$$

$$Y\lambda \geq y_0 \tag{4.98}$$

$$\left[\sum_{j=1}^{n} \lambda_j = 1\right] \tag{4.99}$$

$$\lambda \geq 0 \tag{4.100}$$

While the output-oriented model is:

$$max\beta \tag{4.101}$$

with the constraints:

$$X\lambda \leq x_0 \tag{4.102}$$

$$Y\lambda - \beta y_0 \geq y_0 \tag{4.103}$$

$$\left[\sum_{j=1}^{n} \lambda_j = 1\right] \tag{4.104}$$

$$\lambda \geq 0 \tag{4.105}$$

Due to the transformation, the efficiency measures are defined respectively by $1-\beta^*$ for the input-oriented model and by $1/(1+\beta^*)$ for the output-oriented model. We can notice that β measures how much an observed DMU should improve in order to reach the efficient frontier, in other words it represent the inefficiency measure. A generic DMU is able to improve its efficiency, with respect to the input/output observed, by applying the proportionate input decrease in the input-oriented model and by applying the proportionate output increase in the output-oriented model.
However, in both models the presence of negative values could bring to a wrong efficiency improvement (for instance by setting a faulty frontier). In order to avoid this problem Cheng, Zervopoulos and Qian (2011) proposed to modify the traditional models by substituting the original value of the input/output in the left side of the constraints (4.97) and (4.103) with the absolute value of inputs/outputs. The input-oriented VRM model becomes:

$$max\beta \tag{4.106}$$

with the constraints:

$$X\lambda + \beta|x_0| \leq x_0 \tag{4.107}$$

$$Y\lambda \geq y_0 \tag{4.108}$$

$$\left[\sum_{j=1}^{n} \lambda_j = 1\right] \tag{4.109}$$

$$\lambda \geq 0 \tag{4.110}$$

While the output-oriented VRM model is:

$$max\beta \tag{4.111}$$

with the constraints:

$$X\lambda \leq x_0 \tag{4.112}$$

$$Y\lambda - \beta|y_0| \geq y_0 \tag{4.113}$$

$$\left[\sum_{j=1}^{n} \lambda_j = 1\right] \tag{4.114}$$

$$\lambda \geq 0 \tag{4.115}$$

Then, the input/output-oriented VRM models (4.106) - (4.110) and (4.12) - (4.15) can be used when considering variables that may have both positive and negative values in the observed DMU. Accordingly, these models ensure that the efficiency improvement process of a generic DMU is always achieved and therefore, we are able to improve an inefficient DMU through input reduction or output increase.

Chapter. 5
DEA MODELS FOR THE PERFORMANCE ASSESSMENT OF MUTUAL FUNDS

In this chapter we will describe some of the main models that have been developed in order to analyse the performances of mutual funds through the DEA approaches.
The different performance measures described in chapters 1 and 2 provide on one side, simple results that can be used for comparing different funds, but on the other side, they do not take into account all the variables that are present when dealing with real data (for example fees and costs incurred by investors). For this reason, in this chapter we will present some of the DEA methods for the analysis of mutual funds performance, that allow to take into account different variables in order to achieve a better efficiency measure.
In particular, we will describe the first attempt to implement the DEA models for the mutual fund performance assessment: the DPEI model, then we will focus on the proposals for handling the negative data problems.

5.1 DPEI Model for mutual fund performance

The proposal of Murthi, Choi and Desai (1997) represents the first attempt to apply a DEA model for calculating the performance of mutual funds.
The DEA portfolio efficiency index does not require the presence of a benchmark, but measures only the performance of a fund with respect to the other funds observed.
The model considers, simultaneously, the performance of a fund and the transaction costs as the expense ratio, the turnover index, the commissions and is computed by considering four input and one output. The output is represented by the fund return,

while the inputs are: the transaction costs and a risk measure σ (the standard deviation of the fund). The transaction costs are:

- Front and back loads: they are the commissions applied to the investor when he purchases or sells fund shares;
- turnover: the costs related to the buy/sell activity of the fund manager;
- the expense ratio: they are the costs related to legal, administration, management and advertisement costs.

The model is developed in the following way:

$$DPEI = \frac{R_0}{\sum_{i=1}^{I} w_i x_{i0} + v\sigma_0} \tag{5.1}$$

with the constraints:

$$\frac{R_j}{\sum_{i=1}^{I} w_i x_{ij} + v\sigma_j} \leq 1 \quad j = 1, \dots, J \tag{5.2}$$

$$w_i \geq \varepsilon \tag{5.3}$$

$$v \geq \varepsilon \tag{5.4}$$

where J is the number of the funds observed, I is the number of inputs and R_j represent the average return of fund j. X_{ij} represents the transaction cost i of fund j, while ε is a convenient small positive number, $w_j$and $v_j$are the weights.

With the DPEI model we want to find the weights that maximize the return of the fund with the given outputs, the value of the index will be in the interval $[0,1]$ and a fund will be efficient if the value given by the model is equal to one and all the slacks are equal to zero.

Nonetheless, we know that in the common DEA models all the inputs and output are assumed to be non-negative and the presence of negative data might bring misleading results. When dealing with real data, a negative value of the fund return or excess return may occur and this could prevent us from obtaining a correct measure of the mutual fund performance when applying the DEA methodology. In the following paragraph we will present different models proposed with the aim to deal with negative data in the evaluation of the mutual fund performance.

5.2 Wilkens and Zhu: a BCC input-oriented model

In order to deal with negative data during slump periods, Wilkens and Zhu (2001) developed a methodology that uses the translation invariance property with the purpose of transforming negative outputs into positive ones by adding a constant to the original outputs.
In particular, in this model inputs and outputs are defined as follows:

- Inputs:
    1. Standard deviation: a risk measure for the funds observed;
    2. PropNeg: the proportion of negative monthly returns during the period considered.
- Outputs:
    1. Fund return: the average monthly return;
    2. Min: the minimum monthly return achieved by the fund during the period under observation;
    3. Skewness: the measure of asymmetry of the probability distribution in relation to its mean.

As we can see, the input values are always positive (PropNeg assumes value between zero and one and the standard deviation is always positive) while the outputs can take negative values and we might need to transform them.
The authors set the fractional program problem in the following way:

$$h_j = \frac{\alpha + \sum_{j=1}^{m} v_i x_{ij}}{\sum_{r=1}^{s} u_r y_{rj}} \tag{5.5}$$

where x_i $(i = 1,\dots,m)$ are the performance risk measures of a set of n units, while y_r $(r = 1,\dots,s)$ are the performance measures (e.g. the return of the fund), the unknown variables are $\alpha, v_i \geq 0$ and $u_r \geq 0$.

Let us transform the model represented in (5.5) into a linear programming problem by setting the denominator equal to 1:

$$h_0^* = \min_{\alpha, v_i, u_r} \alpha + \sum_{j=1}^{m} v_i x_{i0} \tag{5.6}$$

with the constraints:

$$\alpha + \sum_{j=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0, \quad j = 1,\dots,n \tag{5.7}$$

$$\sum_{r=1}^{s} u_r y_{r0} = 1 \tag{5.8}$$

The optimal solution is achieved with the following input-oriented model:

$$\theta_0^* = min\theta_0 \tag{5.10}$$

with the constraints:

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_0 x_{i0} \qquad i = 1,2,\dots,m \tag{5.11}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0} \qquad r = 1,2,\dots,s \tag{5.12}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{5.13}$$

$$\lambda_j \geq 0 \qquad j = 1, \dots, n \tag{5.14}$$

Wilkens and Zhu (2001) proposed a BCC input-oriented model because the translation process for the outputs is allowed only if the model has an input orientation (on the contrary in the output oriented model we can translate the inputs), without changing the efficiency measure.

A fund j is considered efficient if the optimal value $\theta_0^* = 1$ and all the slack variables are null, while if $\theta_0^* \neq 1$ (so $\theta_0^* < 1$) then a fund j is inefficient.

5.3 Basso and Funari: DEA-C and DEA-V

Basso and Funari developed different models, based on the Data Envelopment Analysis, in order to assess the performance of mutual funds, in particular, in this chapter we will present two models: DEA-C and DEA-V, which are an implementation able to deal with negative returns achieved during slump periods.

We now define the inputs and output used in the models:

- Inputs:
    1. K_j: Is the capital required to the investor (net of the initial commissions) to gain an initial capital of one and is computed as:

    $$K_j = \frac{1}{1 - c_{Ij}} \qquad j = 1, 2, \dots, n \tag{5.15}$$

    where c_{Ij} are the initial fee.

    2. Risk measure: the model considers one or more risk measures, such as the beta coefficient, the standard deviation or the downside risk;
- Output: we define M_j as the final value of the investment computed in the following way:

$$M_j = \left(1 - R_j\right)^T (1 - C_{Ej}) \tag{5.16}$$

The final value formula considers the mean rate of return R_j and the exit commissions C_{Ej}, we can notice that M_j is always equal or greater than zero because the value $\left(1 - R_j\right)^T$ is non-negative regardless the phase of business cycle (Basso and Funari, 2016).

Therefore, we obtain a performance measure that is always non-negative, even in slump periods, when the average performance of many funds is negative. So considering the outputs and inputs described above, the DEA-C model with constant returns to scale, for evaluating a generic found j, is:

$$\max_{\{u, v_i\}} \frac{uM_0}{v_1 K_0 + \sum_{i=2}^{h+1} v_i q_{i0}} \tag{5.17}$$

with the constraints:

$$\frac{uM_j}{v_1 K_j + \sum_{i=2}^{h+1} v_i q_{ij}} \leq 1 \qquad j = 1,2,\ldots,n \tag{5.18}$$

$$u \geq \varepsilon \tag{5.19}$$

$$v_i \geq \varepsilon \quad i = 1,2,\ldots,h+1 \tag{5.20}$$

where *u* and *v* are the weights assigned to each output and inputs, ε is a non-Archimedean constant. As we have seen for other models, we can transform (5.17) in a linear programming problem with a single output:

$$\min_{\{u, v_i\}} v_1 K_0 + \sum_{i=2}^{h+1} v_i q_{i0} \tag{5.21}$$

with the constraints:

$$uM_0 = 1 \tag{5.22}$$

$$-uM_j + v_1 K_j + \sum_{i=2}^{h+1} v_i q_{ij} \geq 0 \quad j = 1,2,\ldots,n \tag{5.23}$$

$$u \geq \varepsilon \tag{5.24}$$

$$v_i \geq \varepsilon \quad i = 1,2,\ldots,h+1 \tag{5.25}$$

Consequently, we define the dual problem where $z_0$is the dual variable under the constraints (5.22) and the dual variables related to the input and output constraints are $s_1^+$and s_i^-:

$$\max z_0 + \varepsilon s^+ + \sum_{i=1}^{h+1} \varepsilon s_i^- \tag{5.26}$$

with the constraints:

$$M_0 z_0 - \sum_{j=1}^{n} M_j \lambda_j + s^+ = 0 \tag{5.27}$$

$$\sum_{j=1}^{n} K_j \lambda_j + s_1^- = K_0 \tag{5.28}$$

$$\sum_{j=1}^{n} q_{ij} \lambda_j + s_i^- = q_{i0} \quad i = 2,3,\ldots,h+1 \tag{5.29}$$

$$\lambda_j \geq 0 \quad j = 1,2,\ldots,h+1 \tag{5.30}$$

$$s^+ \geq 0 \tag{5.31}$$

$$s_i^- \geq 0 \quad i = 1,2,\ldots,h+1 \tag{5.32}$$

The efficiency measure given by this model allows to define a virtual unit (a combination of efficient units) that may be considered as an efficient benchmark portfolio that we might attempt to achieve. Let us now add the convexity constraint of the following equation:

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{5.33}$$

We obtain an output-oriented model with variable returns to scale that can be used for performance evaluation of mutual funds. This model is called DEA-V and is computed as follow:

$$\max z_0 + \varepsilon s^+ + \sum_{i=1}^{h+1} \varepsilon s_i^- \tag{5.34}$$

with the constraints:

$$M_0 z_0 - \sum_{j=1}^{n} M_j \lambda_j + s^+ = 0 \tag{5.35}$$

$$\sum_{j=1}^{n} K_j \lambda_j + s_1^- = K_0 \tag{5.36}$$

$$\sum_{j=1}^{n} q_{ij} \lambda_j + s_i^- = q_{i0} \quad i = 2,3,\ldots,h+1 \tag{5.37}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{5.38}$$

$$\lambda_j \geq 0 \quad j = 1,2,\ldots,n \tag{5.39}$$

$$s^+ \geq 0 \tag{5.40}$$

$$s_i^- \geq 0 \quad i = 1,2,\ldots,h+1 \tag{5.41}$$

The constraints in (5.33) allows to obtain an output-oriented returns to scale model (5.34) -(5.41) where the efficiency measure ranges in [0,1].

Chapter. 6
DEA EMPIRICAL APPLICATION TO EQUITY MUTUAL FUND

In chapter 5 we presented the proposals for the evaluation of the mutual fund performance, in particular we focused on the models that were more suitable for an application in presence of negative data. In this chapter we will analyse the performance of 222 European equity mutual funds with a holding period of 7 years.

Initially, we will give a description of the funds observed, then we will present the inputs and outputs used in the application of the different DEA models and finally, we will compare the results with the traditional performance measures presented in chapter 1.

6.1 Data description

In order to investigate how the DEA models behave in presence of negative returns, we analysed the performance of 222 European equity mutual funds during the financial crisis (we also include a period of financial recovery), the fund have been randomly chosen and extracted from the Bloomberg database.

The data ranges from 30/11/2016 to 30/11/2013 and includes: fund domicile, commissions (front loads and back loads), Net Asset Value expressed as per-share amount in euros.

Table 6.1 (see Appendix A) shows in column 2 the Ticker related to each fund and the fund domicile in column 4. Columns 5 and 6 report the initial fees and the exit fees: for each fund we extracted Front loads and Back loads, these fees are paid by the investors indirectly because they affect the fund returns.

In order to assess the performance of each mutual fund, we compute the monthly return using the logarithmic formula:

$$R_{mt} = ln\left(\frac{P_t}{P_{t-1}}\right) \tag{6.1}$$

where R_{mt} is the monthly return, P_t and P_{t-1} are the net asset values at the end of month *t* and month *t-1,* the data take into account the dividends paid during the observed period.

The mean returns in column 7 are the average monthly return computed from the logarithmic returns.

6.2 Inputs and Output description of DEA-V, DEA-WZ and DEA-SBM models

The first model used in the analysis is the DEA-V model proposed by Basso and Funari (2014), table 6.2 (in Appendix B) describes the input and output variables used in the model: the first input in column 4 is the initial payout K_j (for the computation see eq. (5.15)), the second and the third inputs are risk measures.

The risk measures adopted are the standard deviation and the beta (table 6.2, columns 5 and 6). The annual standard deviation of each fund is computed by multiplying the monthly standard deviation for $\sqrt{12}$, while the beta is the ratio between the covariance of the fund return with the market return and the variance of the market return, it is computed in the following way:

$$\beta = \left(\frac{Cov\left(R_j, R_m\right)}{Var(R_m)}\right) \tag{6.2}$$

where R_m is the market return and R_j is the fund return.

The standard deviation and the beta are both included in the model in order to have a coherent risk measure for one risky asset and for a diversified investment.

For the output in column 5 we used M_j, defined as the final value of the investment (see eq. (5.16)). Inputs and outputs are shown in table 6.2 (Appendix B).

The second model applied for the performance analysis is the model (denominated DEA-WZ) formulated by Wilkens and Zhu (2001). DEA-WZ is a BCC input-oriented model that uses the translation invariance property with the purpose of transforming negative outputs into positive ones by adding a constant to the original outputs.

In this model the inputs are defined by the standard deviation and the proportion of negative monthly returns during the period considered, the outputs are: the average monthly return, the minimum monthly return achieved by each fund during the period under observation and the skewness.

From column 7 to column 9 of Table 6.3 (in Appendix C), the modified outputs are computed by adding a constant for which the lowest negative values (highlighted in bold) are equal to zero, in particular:

- Fund number 206 achieved the lowest mean return in the period considered, hence we add a constant $\alpha_1 = mean\,return * (-1) = 0.0178$;
- Fund number 171 achieved the lowest skewness in the period considered, hence we add a constant $\alpha_2 = skewness * (-1) = 2.6754$;
- Fund number 2 achieved the lowest min. return in the period considered, hence we add a constant $\alpha_3 = min.return * (-1) = 0.5482$.

The third model is the DEA-SBM model, it is based on the Slack-Based Measure non-oriented model (see eq. 3.83). In this model we used the same inputs and outputs of DEA-V model.

6.3 DEA models results of the empirical analysis

In the following paragraph we present the results obtained from the application of the three DEA models (DEA-V, DEA-WZ and DEA-SBM models), the ranking of each fund are computed using the MaxDEA basic software, the efficient DMUs have a score equal to one, while the inefficient DMUs have a score <1.

In columns 3 and 4 of Table 6.4 are presented the results obtained from the application of the BCC model proposed by Basso and Funari (2014), described in paragraph 5.3, in column 5 and 6 are presented the score and the ranking of each DMU according to the BCC input-oriented model proposed by Wilkens and Zhu (2001) and in the last 2 columns the results obtained from the application of the Slack-Based Measure.

Table 6.4 Result of the empirical application, a score of 1 indicates an efficient DMU

N.	DMU	DEA-V			DEA-WZ			DEA-SBM	
		Score	Ranking		Score	Ranking		Score	Ranking
1	EVLIGRB FH Equity	0,4268	208		0,7778	101		0,1997	212
2	CAPIRST AV Equity	0,6148	136		0,7179	157		0,2875	176
3	NORRUSA FH Equity	0,5189	185		0,6667	206		0,2467	201
4	MANRUSS FH Equity	0,6299	126		0,7778	102		0,2976	168
5	MANRSCK FH Equity	0,5317	179		0,7778	103		0,2501	197
6	NOREEUA FH Equity	0,5148	188		0,6667	207		0,2502	196
7	MANLAMK FH Equity	0,7741	56		0,7778	104		0,3926	84
8	MANUSSK FH Equity	0,7963	42		0,7241	151		0,4244	59
9	EUBOPSE GA Equity	0,5487	172		0,7179	158		0,2747	185
10	RENGAKA AV Equity	0,5148	187		0,7000	179		0,2623	189
11	EVLJQIA FH Equity	0,6608	102		0,6541	213		0,4114	69
12	OSTEUSA AV Equity	0,4811	196		0,6667	208		0,2366	205
13	AVFVALK FH Equity	0,8713	22		0,7924	94		0,4866	32
14	SPEVUQB FH Equity	0,7956	43		0,7179	159		0,4621	34
15	KFEAEUR FH Equity	0,5304	180		0,7000	180		0,2612	190
16	OPSUOMA FH Equity	0,6195	133		0,8235	47		0,3126	156
17	BRANUSA ID Equity	0,5449	174		0,7179	160		0,2816	182
18	METEAEI ID Equity	0,5226	182		0,6829	195		0,2544	193
19	ASELSEE GA Equity	0,3390	218		0,6829	196		0,1686	217
20	RTECAKA AV Equity	0,6567	109		0,7368	138		0,3676	104
21	DBBSR1C LX Equity	0,8702	23		0,8235	48		0,5093	27
22	NOREMEA FH Equity	0,7833	48		0,8235	49		0,4094	73
23	CELHRSU FH Equity	0,7860	47		0,8880	18		0,4076	75
24	IBERBUS SM Equity	0,7570	65		0,7179	161		0,4473	45
25	BPBAZUS IM Equity	0,7808	51		0,8022	71		0,4536	38
26	NTNAEQ2 ID Equity	0,8659	25		0,8000	73		0,5366	20
27	ICEUSSI FH Equity	0,8380	34		0,7778	100		0,4962	29
28	SELPHOA FH Equity	0,8578	27		0,7833	96		0,5049	28
29	CIENGST AV Equity	0,5419	175		0,7179	162		0,2896	174
30	POSTAZI IM Equity	0,7391	73		0,8485	26		0,4535	39
31	TAPIUSA FH Equity	0,8442	30		0,7190	155		0,5207	25
32	BAMBLCU AV Equity	0,6864	95		0,7386	137		0,3998	79
33	ERGRWTH GA Equity	0,3040	220		0,6512	214		0,1462	220
34	UOBKPCA ID Equity	0,6882	93		0,8485	27		0,3409	134
35	NORAMPB FH Equity	0,7768	54		0,7179	163		0,4437	47

N.	DMU	DEA-V			DEA-WZ			DEA-SBM	
		Score	Ranking		Score	Ranking		Score	Ranking
36	ROBIGEI NA Equity	0,7829	49		0,8235	50		0,4525	41
37	GENDVSM AV Equity	0,6578	107		0,8235	51		0,3819	92
38	HAF NA Equity	0,7421	72		0,7568	120		0,4344	53
39	POPFINL FH Equity	0,8418	33		0,7829	97		0,4504	42
40	AKTAMEA FH Equity	0,7894	45		0,8235	52		0,4454	46
41	BKSECEN SM Equity	0,5061	192		0,6829	197		0,2861	177
42	CIBOUSA SM Equity	0,6958	86		0,7786	99		0,3908	88
43	ALLAUSP AV Equity	0,5867	150		0,7179	164		0,2915	172
44	BIMAUSA IM Equity	0,8434	31		0,8267	42		0,4881	31
45	FOURSTA FH Equity	0,9371	10		0,9197	10		0,6464	9
46	CAPIN15 AV Equity	0,5089	191		0,7568	121		0,2818	181
47	DANCAFA FH Equity	0,7649	61		0,8840	19		0,3946	81
48	CRMGGCH ID Equity	0,9228	12		0,8083	69		0,6122	10
49	BPBAZME IM Equity	0,7813	50		0,6829	198		0,4228	61
50	BESTEMA AV Equity	0,5713	158		0,7778	105		0,2943	171
51	VERGLOB GR Equity	0,3829	212		0,7179	165		0,2098	209
52	MFSGEEB LX Equity	0,9066	15		0,8264	43		0,6103	12
53	HIDIVPS GR Equity	0,6085	139		0,7197	154		0,3192	152
54	BPBAZGL IM Equity	0,6866	94		0,7778	106		0,3994	80
55	IFOSVHC LE Equity	0,9114	14		0,8026	70		0,5932	16
56	ICEGUTB FH Equity	0,5974	144		0,7368	139		0,3343	139
57	ASNMILA NA Equity	0,8157	36		0,8000	74		0,4497	43
58	FOURODY FH Equity	0,8987	16		0,8000	75		0,5333	23
59	MANCREI SM Equity	0,7800	52		0,8485	28		0,3921	85
60	DWSTOPD GR Equity	0,8427	32		0,8485	29		0,5338	22
61	MILGDFX PL Equity	0,6411	118		0,7568	122		0,3767	95
62	SAAKOTA FH Equity	0,8078	37		0,7853	95		0,4244	58
63	FIMEMMA FH Equity	0,5648	164		0,7368	140		0,2890	175
64	KEPETKA AV Equity	0,6444	115		0,8235	53		0,3574	112
65	ALLDWAP GR Equity	0,7516	67		0,8485	30		0,4338	54
66	OPFOCUA FH Equity	0,7888	46		0,8344	39		0,4100	72
67	MANDCHK FH Equity	0,9442	8		0,7368	141		0,6075	14
68	TAP2035 FH Equity	0,7715	58		0,9032	13		0,4527	40
69	GUTUSPO AV Equity	0,6903	91		0,8485	31		0,4215	63
70	ESXTJAP AV Equity	0,4653	202		0,7007	177		0,2709	186
71	NORFAEA FH Equity	0,8056	38		0,8000	76		0,4371	51
72	ICEJPSI FH Equity	0,5906	148		0,6858	190		0,3470	123
73	SYMPAEM IM Equity	0,7195	78		0,8235	54		0,4002	77
74	RAIETAA AV Equity	0,5520	171		0,7179	166		0,2984	167
75	TAPREST FH Equity	0,4247	209		0,6667	209		0,2309	208
76	DWSSTRN GR Equity	0,8953	17		0,9367	7		0,6078	13
77	DEKXTCF GR Equity	0,7512	68		0,9333	8		0,4008	76
78	ROINWAF NA Equity	0,7649	60		0,8000	77		0,4251	57

N.	DMU	DEA-V		DEA-WZ		DEA-SBM	
		Score	Ranking	Score	Ranking	Score	Ranking
79	CELGREI FH Equity	0,6946	88	0,9032	14	0,3468	124
80	CIEMERG SM Equity	0,6605	103	0,7778	107	0,3476	122
81	TAPHYVI FH Equity	0,9161	13	0,8343	40	0,6122	11
82	INTVAEU SM Equity	0,5552	168	0,6667	210	0,3073	158
83	WARMPFG GR Equity	0,7635	63	0,8235	55	0,4604	35
84	MONNMKT GR Equity	0,8727	19	0,8485	32	0,4953	30
85	JANRMAE ID Equity	0,7636	62	0,9032	15	0,4166	68
86	KEPGRAK AV Equity	0,5754	156	0,8000	78	0,3139	154
87	CPBFRSA AV Equity	0,7509	69	0,8750	20	0,4204	64
88	TAPKEMA FH Equity	0,6506	111	0,7568	123	0,3340	140
89	MEDCONA ID Equity	0,5763	154	0,8000	79	0,3294	145
90	3BKOEST AV Equity	0,5231	181	0,7778	108	0,2544	194
91	AMGLETS GR Equity	0,7787	53	0,8235	46	0,4351	52
92	KEPSMAA AV Equity	0,6590	106	0,8000	80	0,3711	100
93	DWSVEBI LX Equity	0,6594	105	0,8485	33	0,3868	91
94	AKTWELT GR Equity	0,6426	117	0,7368	142	0,3919	86
95	ALSPMEQ AV Equity	0,7143	82	0,9655	6	0,4090	74
96	AGFCRE2 FP Equity	0,6287	128	0,8485	34	0,3543	116
97	AAHIP NA Equity	0,5155	186	0,8235	56	0,2606	191
98	BGHEF NA Equity	0,6691	100	0,7778	109	0,3770	94
99	CONGLOB IM Equity	0,6245	131	0,8235	57	0,3655	106
100	RTOSTAA AV Equity	0,4242	210	0,7000	181	0,2077	210
101	HIDIVPE GR Equity	0,6426	116	0,7010	176	0,3380	136
102	LIGOESB GR Equity	0,6601	104	0,8235	58	0,3521	117
103	FNTRNSB FH Equity	0,3315	219	0,7778	110	0,1609	219
104	HELEQFO GA Equity	0,3815	213	0,6024	221	0,2057	211
105	ZENECGI LX Equity	0,8778	18	0,7778	111	0,5661	18
106	ALAEVAB FH Equity	0,6701	99	0,8000	81	0,3716	99
107	RTOSAKA AV Equity	0,3744	214	0,7179	167	0,1887	214
108	NORCHIA FH Equity	0,7717	57	0,8485	35	0,4495	44
109	CHAGPSA ID Equity	0,5909	147	0,8000	82	0,3478	121
110	CAGGRSA ID Equity	0,5799	152	0,8235	59	0,3434	128
111	MONGRMY GR Equity	0,8697	24	0,9032	16	0,5350	21
112	GESCCA2 SM Equity	0,6058	140	0,8000	83	0,3463	125
113	ZKBAKTE SW Equity	0,6662	101	0,8235	60	0,3679	103
114	MEDCOUA ID Equity	0,5616	165	0,7778	112	0,3266	148
115	VANESII ID Equity	0,6344	121	0,7000	182	0,3430	129
116	ALIMISC GA Equity	0,3488	217	0,6222	217	0,1676	218
117	MIRFRAE LX Equity	0,6309	123	0,7568	124	0,3585	111
118	BHWMGFT GR Equity	0,7766	55	0,8235	61	0,3998	78
119	TAPTULE FH Equity	0,8045	40	0,8750	21	0,4297	56
120	RRATSUN GR Equity	0,6216	132	0,7568	125	0,3544	115
121	SCHSCES SM Equity	0,5328	178	0,7568	126	0,2796	183

N.	DMU	DEA-V		DEA-WZ		DEA-SBM	
		Score	Ranking	Score	Ranking	Score	Ranking
122	CSIFEZI SW Equity	0,6480	112	0,7000	183	0,3489	119
123	POPEURO FH Equity	0,6272	130	0,7778	113	0,3367	138
124	GESCOSE SM Equity	0,7292	74	0,6958	188	0,4225	62
125	EDEUROI ID Equity	0,8525	28	0,7399	135	0,5294	24
126	DWSAVER AV Equity	0,6989	85	0,8235	62	0,4108	70
127	BADVANS AV Equity	0,7256	77	0,8004	72	0,4305	55
128	METSCFR FP Equity	0,8655	26	0,7791	98	0,5136	26
129	METREUR FP Equity	0,6179	134	0,7025	174	0,3325	142
130	MELLEEA ID Equity	0,7155	81	0,7778	114	0,3737	98
131	A0BLTJ GR Equity	0,8723	21	0,9333	9	0,5572	19
132	CVPVOPW LE Equity	0,6480	113	0,8235	63	0,3641	107
133	INKAFUG GR Equity	0,9287	11	0,9127	11	0,7516	5
134	IBESCAP SM Equity	0,5774	153	0,7000	184	0,3052	162
135	UBSESGA SM Equity	0,5656	162	0,7368	143	0,2961	170
136	UNIVRAP GR Equity	0,9567	6	0,8270	41	0,7172	6
137	RESMCPA AV Equity	0,5711	159	0,8000	84	0,2908	173
138	IDEUR50 SM Equity	0,5762	155	0,7179	168	0,3146	153
139	ETHIKAT AV Equity	0,6094	138	0,8485	36	0,3444	127
140	ALSTDOM GA Equity	0,3734	215	0,7020	175	0,1799	216
141	INVFINV SM Equity	0,4810	197	0,7387	136	0,3110	157
142	AMCEUEQ SW Equity	0,6297	127	0,7368	144	0,3400	135
143	GUTEUPO AV Equity	0,4538	206	0,7000	185	0,2526	195
144	SCHPEIN SM Equity	0,5608	166	0,7001	178	0,3043	164
145	**CARINVL FP Equity**	1,0000	1	1,0000	1	1,0000	1
146	SUSTNFD AV Equity	0,7674	59	0,8750	22	0,4422	48
147	DWSEINN GR Equity	0,8486	29	0,7190	156	0,4550	36
148	SCHGRVA SM Equity	0,5980	141	0,6700	202	0,3424	133
149	BPAZETI IM Equity	0,5688	161	0,6851	193	0,3235	149
150	ICEEUSI FH Equity	0,6933	89	0,8000	85	0,3789	93
151	MAIIBEX SM Equity	0,5655	163	0,7032	172	0,3198	151
152	CIA109F AV Equity	0,6407	119	0,7368	145	0,3368	137
153	MSGLESM SM Equity	0,5215	183	0,7000	186	0,2844	178
154	MPCEAMI GR Equity	0,6282	129	0,7778	115	0,3281	147
155	NTEUEQ2 ID Equity	0,7122	83	0,8235	64	0,3898	90
156	MEINDGR AV Equity	0,5411	176	0,6703	201	0,2704	187
157	DLSDF NA Equity	0,8053	39	0,7212	153	0,4746	33
158	NTUKEQB ID Equity	0,7071	84	0,9032	17	0,3907	89
159	EUVDIVI SM Equity	0,5694	160	0,6680	205	0,3322	143
160	FTHIDVD GR Equity	0,5976	143	0,7368	146	0,3480	120
161	SCHAKWA AV Equity	0,6563	110	0,8750	23	0,3690	102
162	JIFAVEQ DC Equity	0,6715	97	0,8000	86	0,3738	97
163	OPJAPAN FH Equity	0,4798	198	0,6122	218	0,2837	179
164	FININTN SM Equity	0,5112	189	0,6856	191	0,2975	169

N.	DMU	DEA-V		DEA-WZ		DEA-SBM	
		Score	Ranking	Score	Ranking	Score	Ranking
165	ESXTEUR AV Equity	0,6303	124	0,7368	147	0,3496	118
166	BLUVEQL SW Equity	0,9459	7	0,9051	12	0,5894	17
167	MSGINGD SM Equity	0,5978	142	0,6543	212	0,3426	130
168	SEITEMU AV Equity	0,6448	114	0,7568	127	0,3566	113
169	MSGLSSM SM Equity	0,5352	177	0,6698	203	0,3061	161
170	BPBAZIT IM Equity	0,4630	204	0,6557	211	0,2437	203
171	AESFUND AV Equity	0,1915	221	0,6087	220	0,0963	221
172	DESEEUS GR Equity	0,5003	194	0,6829	199	0,2668	188
173	BBKDIVI SM Equity	0,6899	92	0,7031	173	0,4101	71
174	TAPTREN FH Equity	0,8725	20	0,8244	45	0,5949	15
175	SAAEURA FH Equity	0,5590	167	0,7368	148	0,3069	160
176	METFRON FP Equity	0,4713	200	0,7778	116	0,2402	204
177	AXAPEEA ID Equity	0,6782	96	0,7778	117	0,3757	96
178	AMEVEAD ID Equity	0,7435	71	0,8000	87	0,4201	65
179	3BKOEKG AV Equity	0,6570	108	0,8000	88	0,3632	108
180	**SGAIMPC FP Equity**	1,0000	1	1,0000	1	1,0000	1
181	CELKIIN FH Equity	0,6299	125	0,6899	189	0,3339	141
182	GWE NA Equity	0,7286	75	0,7568	128	0,4375	49
183	AMAREAE ID Equity	0,7925	44	0,7222	152	0,4232	60
184	FBES150 SM Equity	0,3934	211	0,6688	204	0,1993	213
185	LEGCTEA ID Equity	0,5465	173	0,7179	169	0,3049	163
186	BPBAZEU IM Equity	0,6143	137	0,7000	187	0,3459	126
187	CSWEPOR AV Equity	0,6930	90	0,8485	37	0,3913	87
188	FTEURHD GR Equity	0,5199	184	0,7568	129	0,2833	180
189	UIAAEAI GR Equity	0,7185	80	0,7442	134	0,3926	83
190	CBPVGES LE Equity	0,5716	157	0,8000	89	0,3214	150
191	RTDIVAA AV Equity	0,5931	145	0,8000	90	0,3285	146
192	GLGPDDN ID Equity	0,5803	151	0,8235	65	0,3069	159
193	ESUMWSA AV Equity	0,5895	149	0,8485	38	0,3137	155
194	SZUKAK1 AV Equity	0,5092	190	0,4568	222	0,3426	131
195	OPINTIA FH Equity	0,5909	146	0,6833	194	0,3037	165
196	ESPAESA AV Equity	0,5540	169	0,8750	24	0,3013	166
197	LIGPCAT LX Equity	0,7265	76	0,8000	91	0,4375	50
198	SWLTDVE LX Equity	0,6348	120	0,7568	130	0,3424	132
199	MANMUSG FH Equity	0,7623	64	0,7368	149	0,3706	101
200	**PLEABRE ID Equity**	1,0000	1	1,0000	1	1,0000	1
201	ODALGEU ID Equity	0,9421	9	0,8165	68	0,6501	8
202	BIPINIZ IM Equity	0,6708	98	0,7368	150	0,3630	109
203	FIMNORA FH Equity	0,7192	79	0,7778	118	0,3557	114
204	STERNEU GR Equity	0,5532	170	0,8000	92	0,2780	184
205	MANKING FH Equity	0,4686	201	0,6855	192	0,2440	202
206	ALGFT20 GA Equity	0,1382	222	0,6247	216	0,0647	222
207	LAPLACE GR Equity	0,5016	193	0,7568	131	0,2550	192

N.	DMU	DEA-V		DEA-WZ		DEA-SBM	
		Score	Ranking	Score	Ranking	Score	Ranking
208	VPKEURS AV Equity	0,4502	207	0,8750	25	0,2331	207
209	DWSTURK LX Equity	0,8311	35	0,7179	170	0,4188	67
210	ICEPROB FH Equity	0,4917	195	0,7568	132	0,2497	198
211	MANBORS SM Equity	0,6159	135	0,7641	119	0,3316	144
212	OPKINTA FH Equity	0,4754	199	0,6111	219	0,2471	200
213	MELUGAE ID Equity	0,6333	122	0,8235	66	0,3624	110
214	BNYPNDN GR Equity	0,6951	87	0,8000	93	0,3943	82
215	**OPKIINA FH Equity**	0,9654	5	1,0000	1	0,6734	7
216	POHEUEA FH Equity	0,8017	41	0,8235	67	0,4542	37
217	F129DBE SM Equity	0,4639	203	0,7179	171	0,2492	199
218	WPRVEPA AV Equity	0,3557	216	0,6272	215	0,1853	215
219	EUPROST AV Equity	0,4556	205	0,6829	200	0,2363	206
220	ESISTSA AV Equity	0,7547	66	0,7568	133	0,3664	105
221	**ODPANEA ID Equity**	1,0000	1	1,0000	1	1,0000	1
222	BRAEAIA ID Equity	0,7452	70	0,8256	44	0,4191	66

The efficient DMUs are highlighted in bold, however, for a better interpretation of the results Table 6.5 and Table 6.6 put in evidence, respectively, the efficient funds and the worst funds.

Table 6.5 Efficient funds with a score of 1

DEA-V		DEA-WZ		DEA-SBM	
N.	**Fund Name**	**N.**	**Fund Name**	**N.**	**Fund Name**
145	CARMIGNAC INVESTISS LT-AEACC	145	CARMIGNAC INVESTISS LT-AEACC	145	CARMIGNAC INVESTISS LT-AEACC
180	SG PRUDENCE PEA-C	180	SG PRUDENCE PEA-C	180	SG PRUDENCE PEA-C
200	PLURIMA EUROPEAN ABS RET-INS	200	PLURIMA EUROPEAN ABS RET-INS	200	PLURIMA EUROPEAN ABS RET-INS
221	ODEY-PAN EUROPEAN-EURO R	221	ODEY-PAN EUROPEAN-EURO R	221	ODEY-PAN EUROPEAN-EURO R
		215	OP-KIINA-A		

Table 6.6 Funds with the lower scores

DEA-V		DEA-WZ		DEA-SBM	
N.	**Fund Name**	**N.**	**Fund Name**	**N.**	**Fund Name**
19	ALPHA SEL SOUTH EAST EUR EQY	163	OP-JAPANI-A	116	METLIFE MID & SMALL CAP FUND
103	DANSKE INVEST TRANS-BALKAN G	212	OP-KIINTEISTO-A	103	DANSKE INVEST TRANS-BALKAN G
33	EUROPEAN RELIANCE GROWTH	171	ACTIVE EQUITY SELECT	33	EUROPEAN RELIANCE GROWTH
171	ACTIVE EQUITY SELECT	104	METLIFE EMERGING MARKETS EQUITY	171	ACTIVE EQUITY SELECT
206	METLIFE GRREK LARGE CAP	194	S-ZUKUNFT AKTIEN 1	206	METLIFE GRREK LARGE CAP

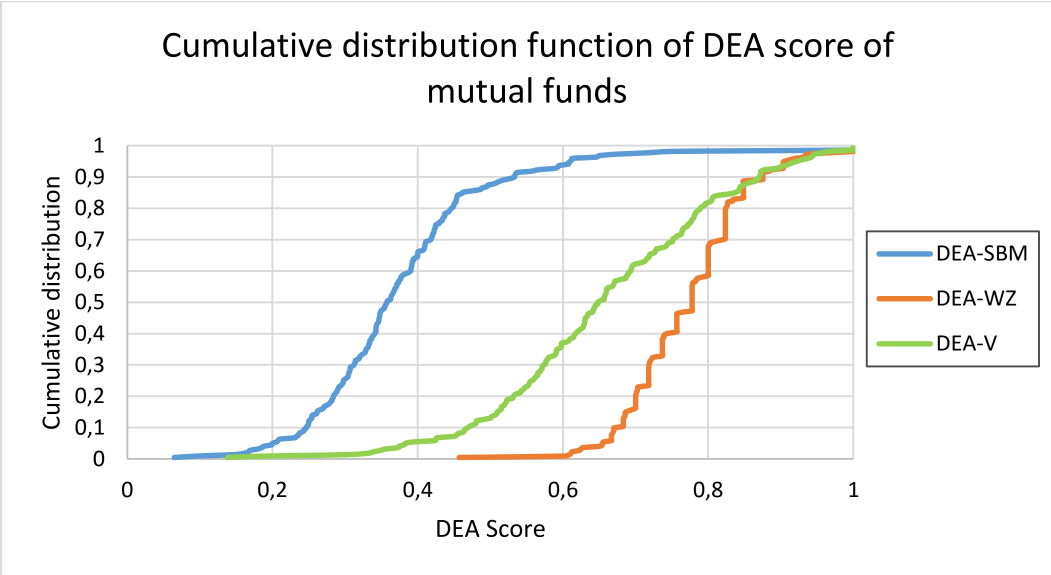
Fund 145, 180, 200 and 221 are efficient according to all models, furthermore the model proposed by Wilkens and Zhu (2001) identifies also fund 215 as efficient. On the other side, fund 171 is located within the last three worst fund in all models and fund 206 is indicated as the worst fund for the DEA-V model and the SBM model. As we can see these two model give similar result in our empirical analysis while in the application of the DEA-WZ model we notice some differences when looking to the score results, furthermore, these considerations are pointed out in table 6.7 where we compute the correlation coefficients between the different DEA models.

Table 6.7 Correlation coefficients of the scores obtained from the application of the DEA-V, DEA-WZ and DEA-SBM models.

	DEA-V	DEA-WZ	DEA-SBM
DEA-V	1	0,640	0,901
DEA-WZ	0,640	1	0,645
DEA-SBM	0,901	0,645	1

In the following figure 6.1, we compute the empirical cumulative distribution function by putting into relation the DEA-V, DEA-WZ and DEA-SBM models, previously applied for the performance evalution.

Figure 6.1. Empirical cumulative distribution function of the DEA-V, DEA-WZ and DEA-SBM score for the European equity mutual funds.



With the purpose to analyse how the efficiency score obtained with the different DEA models changes, we compare the empirical cumulative distribution function in figure 6.1: the blue line is the cumulative distribution function of the Slack-Base Measure model, the green line represent the model proposed by Basso and Funari (2014) and the orange line is the distribution of the DEA-WZ model. We can see how the DEA-WZ model (orange line) achieved, on average, a greater score compared with the other 2 models, this is noticeable from the shift rightwards in the distribution function, however we can notice how the DEA-V model takes greater value from point 0.9066 onwards.

If we focus on the last two model, we can notice that the average DEA-V score is equal to 0.66 which is lower than the average score of the DEA-WZ model (0.7724) where the fees are not taken into account.

6.4 Comparison with the traditional performance indices

In this paragraph we will present the results obtained from the application of the traditional performance indices presented in chapter 1: Sharpe index, Sortino index, Treynor index and Alpha index.

In this analysis the benchmark is represented by a market portfolio for the Western Europe region, namely the STOXX Europe Total Market index (TMI), used for the computation of the beta used in the Treynor and Alpha indices. The Sharpe index and the Sortino index are computed with respect to the 12-Month Euribor rate as risk-free rate.

The ranking of each fund, according to the indices used for the performance evaluation, is given by selecting those funds with the higher value, but let us recall that (as we said in chapter 2) during a period with negative data, the traditional performance indices might give misleading results.

In table 6.8 and table 6.9 we show respectively the funds that have achieved the highest and the lowest score according to the indices.

Table 6.8 Funds with the highest value obtained with the traditional performance indices

Sharpe index		Sortino index		Treynor index		Alpha index	
N.	**Fund Name**	**N.**	**Fund Name**	**N.**	**Fund Name**	**N.**	**Fund Name**
180	SG PRUDENCE PEA-C	180	SG PRUDENCE PEA-C	180	SG PRUDENCE PEA-C	145	CARMIGNAC INVESTISS LT-AEACC
200	PLURIMA EUROPEAN ABS RET-INS	200	PLURIMA EUROPEAN ABS RET-INS	200	PLURIMA EUROPEAN ABS RET-INS	221	ODEY-PAN EUROPEAN-EURO R
136	R + P UNIVERSAL FONDS	136	R + P UNIVERSAL FONDS	67	DANSKE INVEST CHINA-K	200	PLURIMA EUROPEAN ABS RET-INS
133	FLOSSBACH STORCH FUNDAMENT	145	CARMIGNAC INVESTISS LT-AEACC	194	S-ZUKUNFT AKTIEN 1	67	DANSKE INVEST CHINA-K
145	CARMIGNAC INVESTISS LT-AEACC	157	DELTA LLOYD SELECT DIVIDEND	124	CATALANA OCCIDENTE BOLSA ESP	133	FLOSSBACH STORCH FUNDAMENT

Let us notice that the funds 180 and 221 are considered as efficient in each DEA model, the same fund are respectively the first and the second best fund for the Sharpe, Sortino and Treynor indices while, according to the Alpha index, the best fund are those considered efficient in the three DEA models. Fund 145, indicated as efficient in the DEA analysis is present in table 6.8 only if we consider the Sharpe index and the Sortino index. Finally, fund 215 indicated as efficient by the DEA-WZ analysis is not even in the first 15 funds according to the traditional performance measures.

Table 6.9 indicates the worst funds according to the different performance indices: funds 171 and 206 achieved the lowest score even in the DEA analysis, let us notice that fund 1, which is present in table 6.9 under the Sortino, Treynor and Alpha indices, is placed in the 101th position in the ranking achieved by DEA model proposed by Wilkens and Zhu (2001).

Table 6.9 Funds with the lowest value obtained with the traditional performance indices

Sharpe index		Sortino index		Treynor index		Alpha index	
N.	**Fund Name**	**N.**	**Fund Name**	**N.**	**Fund Name**	**N.**	**Fund Name**
140	ALPHA AGG STRAT GREEK EQUITY	33	EUROPEAN RELIANCE GROWTH DOM	103	DANSKE INVEST TRANS-BALKAN G	103	DANSKE INVEST TRANS-BALKAN G
116	METLIFE MID & SMALL CAP FUND	103	DANSKE INVEST TRANS-BALKAN G	5	DANSKE INV-RUSSIA SM-CAP-K	5	DANSKE INV-RUSSIA SM-CAP-K
33	EUROPEAN RELIANCE GROWTH DOM	1	EVLI RUSSIA-B	171	ACTIVE EQUITY SELECT	171	ACTIVE EQUITY SELECT
171	ACTIVE EQUITY SELECT	171	ACTIVE EQUITY SELECT	1	EVLI RUSSIA-B	1	EVLI RUSSIA-B
206	METLIFE GRREK LRG CAP IND F	206	METLIFE GRREK LRG CAP IND F	206	METLIFE GRREK LRG CAP IND F	206	METLIFE GRREK LRG CAP IND F

Finally, table 6.10 shows the correlation coefficients of the score obtained with the DEA models compare to the traditional performance indices.

Table 6.10 Correlation coefficients of Sharpe, Sortino, Treynor and Alpha indices with DEA-V, DEA-WZ and DEA-SBM models.

	Sharpe	**Sortino**	**Treynor**	**Alpha**	**DEA-V**	**DEA-WZ**	**DEA-SBM**
Sharpe	1,000	0,940	0,718	0,799	0,753	0,556	0,777
Sortino		1,000	0,751	0,808	0,738	0,513	0,759
Treynor			1,000	0,847	0,597	0,487	0,631
Alpha				1,000	0,841	0,593	0,926
DEA-V					1,000	0,640	0,901
DEA-WZ						1,000	0,645
DEA-SBM							1,000

In table 6.10 we highlight the different correlations coefficient in order to achieve a clearer comparison among the models and the indices. The blue area presents the correlation among the traditional indices, we notice the strong correlation between the Sharpe index and the Sortino index, we expected this result because the two performance measures are both based on the risk premium and the Sortino index penalizes the returns that fall below a specific rate of return. In our case the target rate of return is the risk-free rate represented by the 12-Month Euribor rate and, as we know, in the period considered most of the risk premia are negative.

From table 6.10, in the green area, we can see that the correlation coefficient between the DEA scores obtained by applying the Slack-Based Measure model and the performance indices is slightly greater than the correlation coefficient computed by putting in relation the DEA-V model and the traditional indices. It is interesting to notice that the correlation coefficients obtained with the DEA-WZ model are lower than the other two models. The greatest correlation coefficients are achieved when we put in relation the DEA model with the Alpha index, in the other cases the lower correlation coefficient between the DEA models applied in the empirical analysis and the traditional performance indices may be due to the fact that the presence of negative returns brings to misleading results when we apply the traditional performance measures and furthermore the DEA models take into account also the transaction costs (front and back loads).

Conclusion

Some traditional performance measures can present interpretation problems if used when the market is in a negative phase, or has just passed one. This problem is related to the fact that in financial practice it is usual to use ex post information for the future performance assessment of mutual funds. As a result, it can happen that some of the data (for example the average return) present negative values when computed from the time series. In chapter two we noticed how the Sharpe ratio considers the fund with the higher standard deviation and with the worse negative return as better, this leads to the need to find another way of assessing the performance.

Thus, we tested the Data Envelopment Analysis in order to find a useful tool for our purpose: we presented the main models, we pointed out the difficulty in using the DPEI index in slump periods (when the outputs could be negative and then, they would not allow us to obtain a correct value of efficiency) and we highlighted the solutions that allow us to apply the DEA technique also in presence of negative data.

In the final chapter we looked at the results obtained by applying the DEA models in the performance evaluation of 222 European equity funds in the period that ranges from December 2006 to November 2013. The first model used in the analysis is the DEA-V model proposed by Basso and Funari (2014) that uses as output the final value of the investment that is always positive, the second model is the DEA-WZ model proposed by Wilkens and Zhu (2001), which is a BCC input-oriented model that translates the data from negative to positive by adding a constant to the original output and finally we applied the Slack-Based Measure model. We want to highlight that the problem of negative data was only in the outputs, since all the inputs used in this analysis have values greater than zero.

Data Envelopment Analysis is a useful tool that help us to easily compare the funds observed without involving interpretation problems, even in times of crisis, because it gives an efficiency measure that is in the range between zero and one, where one indicates the efficient fund.

If we look at the results obtained from the application of these three models, we observe that four funds are considered as efficient according to each model, however the DEA-WZ model identified one more fund as efficient. Regarding the fund with the lower scores we observed a strong correlation among the three different models: in particular, the DEA-V and DEA-SBM models seem to identify the same funds as worse, while the DEA-WZ model gave some different results probably due to the different inputs and outputs used. These considerations are pointed out in the correlation coefficient table that shows us how the first two models have a high correlation, while the one with the last model is around 64%.

Comparing the results obtained with the DEA models and the values calculated by applying the traditional performance measures we noticed a higher correlation between the values of DEA-V and DEA-SBM models and the values of traditional performance measures (in particular with the alpha index). Instead, we observed the lowest correlation coefficient when we compared the results of the DEA-WZ model and those of all the other performance measures used in this analysis.

Finally, by considering the results obtained and the criticisms made to the traditional performance indices, we can state that the DEA models provides a coherent performance measure for mutual funds in times of crisis and so, they can be used as an alternative measures to the traditional ones. It is interesting to note that the performance values obtained with the DEA models consider not only the performance and the risk of the fund, but also the transaction costs (front and back loads), which we should take into account in our investment decisions.

Furthermore, unlike the most conventional measures, the results obtained with DEA models are easy to interpret, even when some funds present negative average yields in the period of analysis, because the efficiency measure is defined in the range between zero and one and this allow us to compare the performance of different funds easily.

Appendix A

Table 6.1 Mutual fund data used in the DEA model applications

N.	Ticker	Fund Name	Country	Front Load	Back Load	Mean return
1	EVLIGRB FH Equity	EVLI RUSSIA-B	FI	0	0,02	-0,0043
2	CAPIRST AV Equity	PIONEER INVEST RUSSIA STK-T	AS	0,05	0	0,0004
3	NORRUSA FH Equity	NORDEA RUSSIA FUND-ACC	FI	0,008	0,01	-0,0020
4	MANRUSS FH Equity	DANSKE INV-RUSSIA	FI	0,01	0,02	0,0005
5	MANRSCK FH Equity	DANSKE INV-RUSSIA SM-CAP-K	FI	0,01	0,02	-0,0015
6	NOREEUA FH Equity	NORDEA EASTERN EUROPE-GROWTH	FI	0,008	0,01	-0,0021
7	MANLAMK FH Equity	DANSKE INV LATIN AMERICA-K	FI	0,01	0,02	0,0029
8	MANUSSK FH Equity	DANSK INV-US SMALL-CAP VAL-G	FI	0,01	0,01	0,0031
9	EUBOPSE GA Equity	EUROBANK EMER EUROP FR EQF-E	GR	0	0,01	-0,0014
10	RENGAKA AV Equity	RAIFFEISEN-ENERGIE-AKTIEN-A	AS	0,05	0	-0,0017
11	EVLJQIA FH Equity	EVLI JAPAN-A	FI	0	0,01	-0,0003
12	OSTEUSA AV Equity	AMUNDI OSTEUROPA STOCK-A	AS	0,04	0	-0,0027
13	AVFVALK FH Equity	ARVO FINLAND VALUE K	FI	0,01	0,01	0,0042
14	SPEVUQB FH Equity	EVLI NORTH AMERICA-B	FI	0	0,01	0,0030
15	KFEAEUR FH Equity	NORDEA OST-EUROPA	FI	0	0,005	-0,0019
16	OPSUOMA FH Equity	OP-SUOMI PIENYHTIOT-A	FI	0,01	0,01	0,0002
17	BRANUSA ID Equity	BRANDES US VALUE FD-A-EUR	IR	0	0	-0,0016
18	METEAEI ID Equity	METZLER EASTERN EUROPE-A	IR	0,05	0	-0,0016
19	ASELSEE GA Equity	ALPHA SEL SOUTH EAST EUR EQY	GR	0,01	0,01	-0,0070
20	RTECAKA AV Equity	RAIFFEISEN-TECHNOLOGIE-AKT-A	AS	0,05	0	0,0011
21	DBBSR1C LX Equity	DB PL C-BRANCHEN STARS-R1C	LX	0	0	0,0040
22	NOREMEA FH Equity	NORDEA EMERG MRKT EQYS-ACC	FI	0,008	0,01	0,0029
23	CELHRSU FH Equity	UB HR SUOMI-ACC	FI	0	0	0,0028
24	IBERBUS SM Equity	IBERCAJA BOLSA USA	SP	0	0,01	0,0024
25	BPBAZUS IM Equity	UBI PRAMERICA AZIONI USA	IT	0,025	0	0,0029
26	NTNAEQ2 ID Equity	NT INV NRTH AMER EQUITY IN-2	IR	0	0	0,0039
27	ICEUSSI FH Equity	EQ USA INDEKSI-1 K	FI	0	0	0,0035
28	SELPHOA FH Equity	SELIGSON PHOEBUS-A	FI	0	0	0,0038
29	CIENGST AV Equity	PIONEER-ENERGY STOCK VT-T	AS	0,05	0	-0,0011
30	POSTAZI IM Equity	BANCOPOSTA AZIONARIO INTER	IT	0	0	0,0020
31	TAPIUSA FH Equity	TAPIOLA USA	FI	0,008	0,01	0,0038
32	BAMBLCU AV Equity	AMUNDI AMRK BLU CP UNHDG A	AS	0,04	0	0,0016
33	ERGRWTH GA Equity	EUROPEAN RELIANCE GROWTH DOM	GR	0,03	0,01	-0,0081
34	UOBKPCA ID Equity	UOB-PARADIGM-A	IR	0,06	0	0,0017
35	NORAMPB FH Equity	NORDEA AMERICA PLUS-GROWTH	FI	0,008	0,01	0,0028
36	ROBIGEI NA Equity	ROBECO QUANT DVLPD MKT EQ IN	NE	0	0	0,0027
37	GENDVSM AV Equity	3 BANKEN DIVIDEND STOCK-MIX	AS	0,035	0	0,0008
38	HAF NA Equity	ALLIANZ AMR EQ 1	NE	0	0,005	0,0021

N.	Ticker	Fund Name	Country	Front Load	Back Load	Mean return
39	POPFINL FH Equity	POP FINLAND	FI	0,01	0,01	0,0038
40	AKTAMEA FH Equity	AKTIA AMERICA-A	FI	0,01	0,01	0,0030
41	BKSECEN SM Equity	BANKINTER SECTOR ENERGIA FI	SP	0	0,01	-0,0024
42	CIBOUSA SM Equity	CAJA INGENIEROS BOLSA USA FI	SP	0	0	0,0013
43	ALLAUSP AV Equity	ALLIANZ INVEST AUSTRIA PLS-A	AS	0,04	0	-0,0003
44	BIMAUSA IM Equity	SYMPHONIA AZIONARIO USA	IT	0	0	0,0036
45	FOURSTA FH Equity	FOURTON STAMINA UCITS	FI	0,01	0,01	0,0051
46	CAPIN15 AV Equity	VBV EMERGING MARKETS EQU	AS	0,05	0	-0,0019
47	DANCAFA FH Equity	DANSKE INVEST FIN HI DIV-ACC	FI	0,01	0,01	0,0027
48	CRMGGCH ID Equity	COMGEST GROW GREATR CH-EUR A	IR	0,04	0	0,0051
49	BPBAZME IM Equity	UBI PRAMERICA AZI MERC EMERG	IT	0,025	0	0,0029
50	BESTEMA AV Equity	ESPA STOCK GLBL-EMRG MARKT-A	AS	0,04	0	-0,0006
51	VERGLOB GR Equity	VE-RI LISTED INFRASTRUCTURE	GE	0,05	0	-0,0053
52	MFSGEEB LX Equity	MFS-GL EQY EURO HEDGED-Q1EUR	LX	0	0	0,0045
53	HIDIVPS GR Equity	HI-DIVIDENDENPLUS-FONDS	GE	0	0,015	-0,0001
54	BPBAZGL IM Equity	UBI PRAMERICA AZIONI GLOBALI	IT	0,0225	0	0,0014
55	IFOSVHC LE Equity	VALUE-HOLDINGS CAPITAL PRNTR	LC	0	0	0,0045
56	ICEGUTB FH Equity	EQ CO2 1- K	FI	0,01	0,01	-0,0003
57	ASNMILA NA Equity	ASN MILIEU WATERFONDS	NE	0,0015	0,0015	0,0032
58	FOURODY FH Equity	FOURTON ODYSSEUS UCITS	FI	0,01	0,005	0,0045
59	MANCREI SM Equity	CX CREIXEMENT	SP	0	0,01	0,0028
60	DWSTOPD GR Equity	DWS TOP DIVIDENDE LD	GE	0,05	0	0,0037
61	MILGDFX PL Equity	IMGA GLOBAL EQUITIES SELECTI	PO	0	0,02	0,0006
62	SAAKOTA FH Equity	SAASTOPANKKI KOTIMAA-A	FI	0,01	0,01	0,0033
63	FIMEMMA FH Equity	FIM BRIC+	FI	0,01	0,02	-0,0008
64	KEPETKA AV Equity	KEPLER ETHIK AKTIENFD-BV A	AS	0,04	0	0,0008
65	ALLDWAP GR Equity	ALLIANZ STRAT WACHSTUM +-A	GE	0	0	0,0022
66	OPFOCUA FH Equity	OP-FOCUS-A	FI	0,01	0,01	0,0030
67	MANDCHK FH Equity	DANSKE INVEST CHINA-K	FI	0,01	0,02	0,0032
68	TAP2035 FH Equity	TAPIOLA 2035	FI	0,008	0,005	0,0027
69	GUTUSPO AV Equity	GUTMANN US-PORTFOLIO-EUR	AS	0,04	0	0,0012
70	ESXTJAP AV Equity	ESPA STOCK JAPAN-T	AS	0,05	0	-0,0030
71	NORFAEA FH Equity	NORDEA FAR EAST FUND.FI-GROW	FI	0,008	0,01	0,0033
72	ICEJPSI FH Equity	EQ JAPANI INDEKSI-1 K	FI	0	0	-0,0006
73	SYMPAEM IM Equity	SYMPHONIA MULTIM EMERG FLESS	IT	0	0	0,0017
74	RAIETAA AV Equity	RAIFFEISEN-NCHHLT-AKTIEN-A	AS	0,05	0	-0,0009
75	TAPREST FH Equity	TAPIOLA INFRA-A	FI	0,008	0,01	-0,0044
76	DWSSTRN GR Equity	DWS GLOBAL GROWTH	GE	0,05	0	0,0046
77	DEKXTCF GR Equity	DEKA-MEGATRENDS CF	GE	0	0	0,0022
78	ROINWAF NA Equity	Robeco Inst Wereldw Aand Fds	NE	0	0	0,0024
79	CELGREI FH Equity	UB GLOBAL REIT FUND-ACC	FI	0	0	0,0013

N.	Ticker	Fund Name	Country	Front Load	Back Load	Mean return
80	CIEMERG SM Equity	CAJA INGENIEROS EMERGENTES	SP	0	0	0,0007
81	TAPHYVI FH Equity	TAPIOLA HYVINVOINTI	FI	0,008	0,01	0,0048
82	INTVAEU SM Equity	INTERMONEY VARIABLE EURO	SP	0	0	-0,0014
83	WARMPFG GR Equity	MPF GLOBAL FONDS-WARBURG	GE	0,03	0	0,0021
84	MONNMKT GR Equity	MONEGA INNOVATION	GE	0,035	0	0,0044
85	JANRMAE ID Equity	JANUS CAPITAL INTECH US-AEAC	IR	0,0625	0	0,0029
86	KEPGRAK AV Equity	KEPLER GROWTH AKTIENFONDS	AS	0,045	0	-0,0005
87	CPBFRSA AV Equity	FRS DYNAMIK	AS	0,05	0	0,0025
88	TAPKEMA FH Equity	TAPIOLA KEHITTYVAT MARKKINAT	FI	0,008	0,01	0,0007
89	MEDCONA ID Equity	CHALLENGE INTER EQUITY-SA	IR	0,05	0	-0,0007
90	3BKOEST AV Equity	3 BANKEN OESTERREICH-FONDS	AS	0,035	0	-0,0017
91	AMGLETS GR Equity	ACC ALPHA SELECT AMI	GE	0,05	0	0,0026
92	KEPSMAA AV Equity	KEPLER SMALL CAP AKTIENFD-A	AS	0,045	0	0,0011
93	DWSVEBI LX Equity	DWS VERMOEGENSBILDUNGSF I LX	LX	0,05	0	0,0009
94	AKTWELT GR Equity	AKTIEN WELT INKA	GE	0,05	0	0,0001
95	ALSPMEQ AV Equity	MONEY&CO EQUITY	AS	0,04	0	0,0018
96	AGFCRE2 FP Equity	ALLIANZ CREACTIONS 2	FR	0,07	0	0,0006
97	AAHIP NA Equity	BNP PARIBAS HIGH INC PROPERT	NE	0	0	-0,0023
98	BGHEF NA Equity	BNP PARIBAS GL HI INCOME EQU	NE	0	0,005	0,0009
99	CONGLOB IM Equity	CONSULTINVEST GLOBAL-C ACC	IT	0	0	0,0000
100	RTOSTAA AV Equity	RT OESTERREICH AKTIENFONDS-A	AS	0,04	0	-0,0042
101	HIDIVPE GR Equity	HI-DIVIDENDENPLUS EUROPA	GE	0	0,015	0,0005
102	LIGOESB GR Equity	LINGOHR-EUROPA-SYTEM-LBB-INV	GE	0,05	0	0,0012
103	FNTRNSB FH Equity	DANSKE INVEST TRANS-BALKAN G	FI	0,03	0,01	-0,0071
104	HELEQFO GA Equity	METLIFE EMERGING MRKTS EQTY	GR	0,01	0,0075	-0,0057
105	ZENECGI LX Equity	WAVERTON-WAV EU CAP GTH-RET	LX	0	0	0,0041
106	ALAEVAB FH Equity	ALANDSBANKEN EUROPE VALUE-B	FI	0,01	0,01	0,0011
107	RTOSAKA AV Equity	RT OSTEUROPA AKTIENFONDS-A	AS	0,04	0	-0,0056
108	NORCHIA FH Equity	NORDEA CHINA FUND-ACC	FI	0,008	0,01	0,0027
109	CHAGPSA ID Equity	MEDIOLANUM-AGGRESSIVE PLS-SA	IR	0	0	-0,0006
110	CAGGRSA ID Equity	MEDIOLANUM-AGGRESSIVE FD-SA	IR	0	0	-0,0009
111	MONGRMY GR Equity	MONEGA GERMANY	GE	0,035	0	0,0043
112	GESCCA2 SM Equity	GESIURIS EURO EQUITIES FI	SP	0	0,01	-0,0002
113	ZKBAKTE SW Equity	SWC CH EF EUROPE II AA	SZ	0	0	0,0008
114	MEDCOUA ID Equity	MEDIOLANUM EQ POWR C COL-SA	IR	0	0	-0,0012
115	VANESII ID Equity	VANGUARD EUROZONE STK-INS-EU	IR	0	0	0,0002
116	ALIMISC GA Equity	METLIFE MID & SMALL CAP FUND	GR	0	0	-0,0069
117	MIRFRAE LX Equity	MIRABAUD-EQUITIES FRAN-A EUR	LX	0	0	0,0001

N.	Ticker	Fund Name	Country	Front Load	Back Load	Mean return
118	BHWMGFT GR Equity	POSTBANK MEGATREND	GE	0,05	0	0,0031
119	TAPTULE FH Equity	TAPIOLA GROWTH	FI	0,008	0,01	0,0032
120	RRATSUN GR Equity	RR ANALYSIS TOPSELECT UNIVER	GE	0,05	0	0,0005
121	SCHSCES SM Equity	S SMALL CAPS ESPANA	SP	0	0	-0,0019
122	CSIFEZI SW Equity	CSIF CH EUROZONE INDEX-D	SZ	0,001	0,0003	0,0005
123	POPEURO FH Equity	POP EUROPE	FI	0,01	0,01	0,0003
124	GESCOSE SM Equity	CATALANA OCCIDENTE BOLSA ESP	SP	0	0	0,0005
125	EDEUROI ID Equity	EDINBURGH PART-EU OP-INC-EI	IR	0,015	0,01	0,0040
126	DWSAVER AV Equity	DWS (AUST) VERMOEGENSBILDNGF	AS	0,05	0	0,0015
127	BADVANS AV Equity	ADVANTAGE STOCK	AS	0,05	0	0,0016
128	METSCFR FP Equity	METROPOLE AVENIR EUROPE	FR	0,04	0	0,0043
129	METREUR FP Equity	METROPOLE EURO-A	FR	0,04	0	0,0003
130	MELLEEA ID Equity	BNY MELLON GL-S/C EUROLND-AE	IR	0,05	0	0,0022
131	A0BLTJ GR Equity	UNIVERSAL-SHARECONCEPT-BC I	GE	0,03	0	0,0042
132	CVPVOPW LE Equity	B&P VISION OPTIMIX WORLD	LC	0,05	0	0,0010
133	INKAFUG GR Equity	FLOSSBACH STORCH FUNDAMENT	GE	0,0005	0	0,0045
134	IBESCAP SM Equity	IBERCAJA SMALL CAPS	SP	0	0	-0,0009
135	UBSESGA SM Equity	UBS ESPANA GESTION ACTIVA	SP	0	0	-0,0012
136	UNIVRAP GR Equity	R + P UNIVERSAL FONDS	GE	0,05	0	0,0043
137	RESMCPA AV Equity	RAIFFEISEN-EUR-SMALLCAP-A	AS	0,05	0	-0,0005
138	IDEUR50 SM Equity	ING DIRECT FN EUROSTOXX 50	SP	0	0	-0,0009
139	ETHIKAT AV Equity	S ETHIKAKTIEN-A	AS	0,05	0	0,0002
140	ALSTDOM GA Equity	ALPHA AGG STRAT GREEK EQUITY	GR	0,01	0,01	-0,0059
141	INVFINV SM Equity	FONDEMAR DE INVERSION	SP	0	0	-0,0031
142	AMCEUEQ SW Equity	BCV EUROPE EQUITY-A	SZ	0,003	0	0,0002
143	GUTEUPO AV Equity	GUTMANN EUROPA PORTFOLIO	AS	0,04	0	-0,0034
144	SCHPEIN SM Equity	SANTANDER INDICE EURO-A	SP	0	0	-0,0013
145	CARINVL FP Equity	CARMIGNAC INVESTISS LT-AEACC	FR	0	0	0,0056
146	SUSTNFD AV Equity	ERSTE RESPONSIBLE ST GLOBAL	AS	0,05	0	0,0026
147	DWSEINN GR Equity	DWS GERMAN SMALL/MID CAP	GE	0,05	0	0,0042
148	SCHGRVA SM Equity	SANTANDER INDIC ESP-OPENBANK	SP	0	0	-0,0005
149	BPAZETI IM Equity	UBI PRAMERICA AZION ETICO	IT	0	0	-0,0011
150	ICEEUSI FH Equity	EQ EUROOPPA INDEKSI-1 K	FI	0	0	0,0013
151	MAIIBEX SM Equity	BANKIA INDICE IBEX	SP	0	0,02	-0,0009
152	CIA109F AV Equity	A 109 - A	AS	0,07	0	0,0009
153	MSGLESM SM Equity	LIBERTY EUROPEAN STOCK MARKE	SP	0	0,02	-0,0019
154	MPCEAMI GR Equity	AMPEGA EUROPA METHOD AKTIENT	GE	0,05	0	0,0006
155	NTEUEQ2 ID Equity	NT INV FND EX UK EQTY INDX-2	IR	0	0	0,0016
156	MEINDGR AV Equity	MEINL INDIA GROWTH	AS	0,05	0	-0,0012
157	DLSDF NA Equity	DELTA LLOYD SELECT DIVIDEND	NE	0,002	0,002	0,0030
158	NTUKEQB ID Equity	NT INV FND UK EQY FND INDX-B	IR	0	0	0,0015
159	EUVDIVI SM Equity	EUROVALOR DIVIDENDO EUROPA	SP	0	0	-0,0011

N.	Ticker	Fund Name	Country	Front Load	Back Load	Mean return
160	FTHIDVD GR Equity	FT GLOBAL HIGHDIVIDEND	GE	0,05	0	-0,0001
161	SCHAKWA AV Equity	SCHOELLERBANK AKT WAHRUNG-A	AS	0,04	0	0,0010
162	JIFAVEQ DC Equity	JYSKE INVEST FAVOURIT EQT FD	DE	0,024	0,006	0,0012
163	OPJAPAN FH Equity	OP-JAPANI-A	FI	0,01	0,01	-0,0029
164	FININTN SM Equity	CAMINOS BOLSA EURO	SP	0	0	-0,0024
165	ESXTEUR AV Equity	XT EUROPA	AS	0,05	0,002	0,0007
166	BLUVEQL SW Equity	BLUEVALOR SU LIFESTYLE BRA-A	SZ	0	0	0,0050
167	MSGINGD SM Equity	ING DIRECT FN IBEX 35	SP	0	0	-0,0005
168	SEITEMU AV Equity	SPAENGLER IQAM EQUITY EUROPE	AS	0,05	0	0,0009
169	MSGLSSM SM Equity	LIBERTY SPAN STOCK MKT INDEX	SP	0	0,02	-0,0016
170	BPBAZIT IM Equity	UBI PRAMERICA AZIONI ITALIA	IT	0,025	0	-0,0033
171	AESFUND AV Equity	ACTIVE EQUITY SELECT	AS	0,05	0	-0,0135
172	DESEEUS GR Equity	DEAM-STRATAV EUROPE STRAT 1	GE	0	0	-0,0026
173	BBKDIVI SM Equity	KUTXABANK DIVIDENDO FI	SP	0	0	0,0012
174	TAPTREN FH Equity	TAPIOLA KULUTTAJA	FI	0,008	0,01	0,0041
175	SAAEURA FH Equity	SAASTOPANKKI EUROOPPA-A	FI	0,01	0,01	-0,0011
176	METFRON FP Equity	METROPOLE FRONTIERE EUROPE	FR	0,04	0	-0,0029
177	AXAPEEA ID Equity	AXA ROSENBERG PAN EUR INDX-A	IR	0	0	0,0010
178	AMEVEAD ID Equity	NEMESIS EURO VAL-EAD A	IR	0	0	0,0021
179	3BKOEKG AV Equity	3 BANKEN NACHHALTIGKEITSFOND	AS	0,05	0	0,0012
180	SGAIMPC FP Equity	SG PRUDENCE PEA-C	FR	0	0	0,0007
181	CELKIIN FH Equity	UB EUR REIT FUND-ACC	FI	0	0	0,0001
182	GWE NA Equity	INSINGER-EQUITY INCOME FUND	NE	0	0	0,0019
183	AMAREAE ID Equity	AMADEUS ASIAN REAL ESTATE-AE	IR	0	0	0,0029
184	FBES150 SM Equity	CAIXABANK BOLSA ESPANA 150	SP	0	0,04	-0,0050
185	LEGCTEA ID Equity	LM-QS MV EUR EQ G&I-AINCA	IR	0,05	0	-0,0010
186	BPBAZEU IM Equity	UBI PRAMERICA AZIONI EURO	IT	0,025	0	0,0001
187	CSWEPOR AV Equity	SPAENGLER PRIVAT: SUBSTANZ	AS	0,05	0	0,0016
188	FTEURHD GR Equity	FT EURO HIGHDIVIDEND	GE	0,05	0	-0,0016
189	UIAAEAI GR Equity	ACATIS AKTIEN EUROPA UI-A1	GE	0,01	0	0,0018
190	CBPVGES LE Equity	B&P VISION - Q-SELECT EUROPE	LC	0,05	0	-0,0005
191	RTDIVAA AV Equity	RAIFFEISEN TOPDIVIDEND AK-A	AS	0,04	0	-0,0002
192	GLGPDDN ID Equity	MAN GLOBAL EQY-D H EUR	IR	0,05	0	-0,0003
193	ESUMWSA AV Equity	ERSTE WWF STOCK ENVRMNT - A	AS	0,04	0	-0,0002
194	SZUKAK1 AV Equity	S-ZUKUNFT AKTIEN 1	AS	0,02	0	-0,0042
195	OPINTIA FH Equity	OP-INTIA-A	FI	0,01	0,01	-0,0004
196	ESPAESA AV Equity	ESPA STOCK EUROPE-A	AS	0,05	0	-0,0009
197	LIGPCAT LX Equity	LIGA-PAX-CATTOLICO-UNION	LX	0,0175	0	0,0020
198	SWLTDVE LX Equity	SWISSCANTO LU TOP DIV EUR-AA	LX	0	0	0,0002
199	MANMUSG FH Equity	DANSKE INV-BLACK-SEA-GROWTH	FI	0,01	0,02	0,0027
200	PLEABRE ID Equity	PLURIMA EUROPEAN ABS RET-INS	IR	0	0	0,0033
201	ODALGEU ID Equity	ODEY INVEST-ALLE EURO -EUR 0	IR	0,02	0	0,0051
202	BIPINIZ IM Equity	ANIMA INIZIATIVA EUROPA-A	IT	0,04	0	0,0013
203	FIMNORA FH Equity	FIM NORDIC	FI	0,01	0,01	0,0019

N.	Ticker	Fund Name	Country	Front Load	Back Load	Mean return
204	STERNEU GR Equity	DWS EUROPE DYNAMIC	GE	0,05	0	-0,0009
205	MANKING FH Equity	DANSKE INV-REAL ESTATE-GROWT	FI	0,01	0,01	-0,0032
206	ALGFT20 GA Equity	METLIFE GRREK LRG CAP IND F	GR	0	0,01	-0,0178
207	LAPLACE GR Equity	HSBC LAPLACE EUROLND EQ INKA	GE	0,05	0	-0,0021
208	VPKEURS AV Equity	ERSTE RESPONSIBLE STOCK EURP	AS	0,05	0	-0,0033
209	DWSTURK LX Equity	DWS TUERKEI	LX	0,05	0	0,0040
210	ICEPROB FH Equity	EQ EUROOPPA KIINTEISTO-1 K	FI	0,01	0,01	-0,0026
211	MANBORS SM Equity	CX BORSA ESPANYA	SP	0	0,01	0,0000
212	OPKINTA FH Equity	OP-KIINTEISTO-A	FI	0,01	0,01	-0,0030
213	MELUGAE ID Equity	BNY MELLON GL-PAN EURO EQ-EA	IR	0,05	0	0,0007
214	BNYPNDN GR Equity	ODDO SUSTAINABILITY F	GE	0,05	0	0,0018
215	OPKIINA FH Equity	OP-KIINA-A	FI	0,01	0,01	0,0054
216	POHEUEA FH Equity	OP-EUROOPPA OSAKE-A	FI	0,01	0,01	0,0032
217	F129DBE SM Equity	CAIXABANK BOLSA DIV EUROPA	SP	0	0,04	-0,0030
218	WPRVEPA AV Equity	WIENER PRIVATBANK EURO PR-A	AS	0,05	0	-0,0061
219	EUPROST AV Equity	ESPA STOCK EUROPE-PROPERTY-T	AS	0,05	0	-0,0032
220	ESISTSA AV Equity	ESPA STOCK ISTANBUL-A	AS	0,04	0	0,0027
221	ODPANEA ID Equity	ODEY-PAN EUROPEAN-EURO R	IR	0,05	0	0,0062
222	BRAEAIA ID Equity	ARG-PAN EURO ALP-EUR I-A	IR	0,05	0	0,0027

Appendix B

Table 6.2. Inputs and Outputs for DEA-V model

N.	Fund ticker	Fees		Inputs			Outputs
		F. Load	B. Load	K_j	St. dev.	β-coeff.	M_j
1	EVLIGRB FH Equity	0	0,02	1,0000	0,3540	0,9630	0,6848
2	CAPIRST AV Equity	0,05	0	1,0526	0,3754	0,7715	1,0311
3	NORRUSA FH Equity	0,008	0,01	1,0081	0,3265	0,7387	0,8384
4	MANRUSS FH Equity	0,01	0,02	1,0101	0,3349	0,8308	1,0194
5	MANRSCK FH Equity	0,01	0,02	1,0101	0,3392	0,9360	0,8605
6	NOREEUA FH Equity	0,008	0,01	1,0081	0,2871	0,6096	0,8317
7	MANLAMK FH Equity	0,01	0,02	1,0101	0,2582	0,3322	1,2529
8	MANUSSK FH Equity	0,01	0,01	1,0101	0,2102	0,2518	1,2887
9	EUBOPSE GA Equity	0	0,01	1,0000	0,2475	0,4738	0,8804
10	RENGAKA AV Equity	0,05	0	1,0526	0,2420	0,3763	0,8634
11	EVLJQIA FH Equity	0	0,01	1,0000	0,1646	0,0614	0,9687
12	OSTEUSA AV Equity	0,04	0	1,0417	0,2722	0,5533	0,7996
13	AVFVALK FH Equity	0,01	0,01	1,0101	0,2227	0,3307	1,4102
14	SPEVUQB FH Equity	0	0,01	1,0000	0,1576	0,1686	1,2766
15	KFEAEUR FH Equity	0	0,005	1,0000	0,2640	0,5701	0,8510
16	OPSUOMA FH Equity	0,01	0,01	1,0101	0,2399	0,4458	1,0027
17	BRANUSA ID Equity	0	0	1,0000	0,2195	0,3587	0,8743
18	METEAEI ID Equity	0,05	0	1,0526	0,2902	0,5814	0,8764
19	ASELSEE GA Equity	0,01	0,01	1,0101	0,2716	0,4181	0,5486
20	RTECAKA AV Equity	0,05	0	1,0526	0,2008	0,1705	1,1014
21	DBBSR1C LX Equity	0	0	1,0000	0,1835	0,2860	1,3963
22	NOREMEA FH Equity	0,008	0,01	1,0081	0,2244	0,2855	1,2655
23	CELHRSU FH Equity	0	0	1,0000	0,2192	0,3389	1,2611
24	IBERBUS SM Equity	0	0,01	1,0000	0,1415	0,1766	1,2146
25	BPBAZUS IM Equity	0,025	0	1,0256	0,1441	0,2063	1,2767
26	NTNAEQ2 ID Equity	0	0	1,0000	0,1554	0,2338	1,3894
27	ICEUSSI FH Equity	0	0	1,0000	0,1533	0,2255	1,3446
28	SELPHOA FH Equity	0	0	1,0000	0,1723	0,2612	1,3764
29	CIENGST AV Equity	0,05	0	1,0526	0,2019	0,2807	0,9087
30	POSTAZI IM Equity	0	0	1,0000	0,1327	0,1416	1,1859
31	TAPIUSA FH Equity	0,008	0,01	1,0081	0,1635	0,1750	1,3640
32	BAMBLCU AV Equity	0,04	0	1,0417	0,1635	0,1636	1,1408
33	ERGRWTH GA Equity	0,03	0,01	1,0309	0,3438	0,4681	0,5008
34	UOBKPCA ID Equity	0,06	0	1,0638	0,2492	0,5161	1,1542
35	NORAMPB FH Equity	0,008	0,01	1,0081	0,1629	0,1890	1,2550
36	ROBIGEI NA Equity	0	0	1,0000	0,1545	0,1839	1,2561
37	GENDVSM AV Equity	0,035	0	1,0363	0,1386	0,1946	1,0678

N.	Fund ticker	Fees		Inputs			Outputs
		F. Load	B. Load	K_j	St. dev.	β-coeff.	M_j
38	HAF NA Equity	0	0,005	1,0000	0,1599	0,1533	1,1908
39	POPFINL FH Equity	0,01	0,01	1,0101	0,2219	0,3699	1,3624
40	AKTAMEA FH Equity	0,01	0,01	1,0101	0,1772	0,1832	1,2776
41	BKSECEN SM Equity	0	0,01	1,0000	0,1805	0,1717	0,8120
42	CIBOUSA SM Equity	0	0	1,0000	0,1600	0,2365	1,1165
43	ALLAUSP AV Equity	0,04	0	1,0417	0,2597	0,5029	0,9751
44	BIMAUSA IM Equity	0	0	1,0000	0,1721	0,2427	1,3533
45	FOURSTA FH Equity	0,01	0,01	1,0101	0,1438	0,3238	1,5167
46	CAPIN15 AV Equity	0,05	0	1,0526	0,1779	0,2384	0,8534
47	DANCAFA FH Equity	0,01	0,01	1,0101	0,2228	0,3581	1,2379
48	CRMGGCH ID Equity	0,04	0	1,0417	0,1940	0,2456	1,5337
49	BPBAZME IM Equity	0,025	0	1,0256	0,1882	0,2700	1,2812
50	BESTEMA AV Equity	0,04	0	1,0417	0,2329	0,3400	0,9494
51	VERGLOB GR Equity	0,05	0	1,0526	0,1726	0,2977	0,6421
52	MFSGEEB LX Equity	0	0	1,0000	0,1551	0,2096	1,4546
53	HIDIVPS GR Equity	0	0,015	1,0000	0,2089	0,3172	0,9763
54	BPBAZGL IM Equity	0,0225	0	1,0230	0,1501	0,1899	1,1235
55	IFOSVHC LE Equity	0	0	1,0000	0,1670	0,2381	1,4624
56	ICEGUTB FH Equity	0,01	0,01	1,0101	0,1627	0,2421	0,9668
57	ASNMILA NA Equity	0,0015	0,0015	1,0015	0,1723	0,2511	1,3105
58	FOURODY FH Equity	0,01	0,005	1,0101	0,1976	0,3297	1,4545
59	MANCREI SM Equity	0	0,01	1,0000	0,2456	0,4433	1,2516
60	DWSTOPD GR Equity	0,05	0	1,0526	0,1342	0,1612	1,3599
61	MILGDFX PL Equity	0	0,02	1,0000	0,1528	0,1599	1,0286
62	SAAKOTA FH Equity	0,01	0,01	1,0101	0,2080	0,3171	1,3074
63	FIMEMMA FH Equity	0,01	0,02	1,0101	0,2288	0,3886	0,9142
64	KEPETKA AV Equity	0,04	0	1,0417	0,1673	0,2675	1,0709
65	ALLDWAP GR Equity	0	0	1,0000	0,1550	0,1856	1,2059
66	OPFOCUA FH Equity	0,01	0,01	1,0101	0,2230	0,3157	1,2767
67	MANDCHK FH Equity	0,01	0,02	1,0101	0,2606	0,0351	1,2798
68	TAP2035 FH Equity	0,008	0,005	1,0081	0,1336	0,2188	1,2440
69	GUTUSPO AV Equity	0,04	0	1,0417	0,1598	0,0937	1,1059
70	ESXTJAP AV Equity	0,05	0	1,0526	0,1682	0,1587	0,7803
71	NORFAEA FH Equity	0,008	0,01	1,0081	0,2186	0,1913	1,3015
72	ICEJPSI FH Equity	0	0	1,0000	0,1571	0,1529	0,9477
73	SYMPAEM IM Equity	0	0	1,0000	0,1654	0,2468	1,1545
74	RAIETAA AV Equity	0,05	0	1,0526	0,1875	0,2882	0,9257
75	TAPREST FH Equity	0,008	0,01	1,0081	0,1855	0,2560	0,6862
76	DWSSTRN GR Equity	0,05	0	1,0526	0,1482	0,2391	1,4718
77	DEKXTCF GR Equity	0	0	1,0000	0,1972	0,2832	1,2053
78	ROINWAF NA Equity	0	0	1,0000	0,1704	0,2320	1,2273
79	CELGREI FH Equity	0	0	1,0000	0,2535	0,4672	1,1146

N.	Fund ticker	Fees		Inputs			Outputs
		F. Load	B. Load	K_j	St. dev.	β-coeff.	M_j
80	CIEMERG SM Equity	0	0	1,0000	0,2119	0,2910	1,0598
81	TAPHYVI FH Equity	0,008	0,01	1,0081	0,1498	0,2892	1,4801
82	INTVAEU SM Equity	0	0	1,0000	0,1943	0,1870	0,8909
83	WARMPFG GR Equity	0,03	0	1,0309	0,1158	0,1408	1,1888
84	MONNMKT GR Equity	0,035	0	1,0363	0,2311	0,3404	1,4440
85	JANRMAE ID Equity	0,0625	0	1,0667	0,1791	0,2503	1,2806
86	KEPGRAK AV Equity	0,045	0	1,0471	0,1757	0,3000	0,9606
87	CPBFRSA AV Equity	0,05	0	1,0526	0,1506	0,2574	1,2382
88	TAPKEMA FH Equity	0,008	0,01	1,0081	0,2333	0,3457	1,0512
89	MEDCONA ID Equity	0,05	0	1,0526	0,1438	0,2027	0,9420
90	3BKOEST AV Equity	0,035	0	1,0363	0,2929	0,5689	0,8654
91	AMGLETS GR Equity	0,05	0	1,0526	0,1262	0,2897	1,2396
92	KEPSMAA AV Equity	0,045	0	1,0471	0,1621	0,2404	1,1002
93	DWSVEBI LX Equity	0,05	0	1,0526	0,1430	0,1571	1,0766
94	AKTWELT GR Equity	0,05	0	1,0526	0,1206	0,1156	1,0109
95	ALSPMEQ AV Equity	0,04	0	1,0417	0,1409	0,2175	1,1630
96	AGFCRE2 FP Equity	0,07	0	1,0753	0,1651	0,1944	1,0543
97	AAHIP NA Equity	0	0	1,0000	0,2377	0,4337	0,8272
98	BGHEF NA Equity	0	0,005	1,0000	0,1561	0,2422	1,0736
99	CONGLOB IM Equity	0	0	1,0000	0,1470	0,1798	1,0021
100	RTOSTAA AV Equity	0,04	0	1,0417	0,2794	0,5653	0,7051
101	HIDIVPE GR Equity	0	0,015	1,0000	0,2051	0,3184	1,0310
102	LIGOESB GR Equity	0,05	0	1,0526	0,2036	0,2830	1,1071
103	FNTRNSB FH Equity	0,03	0,01	1,0309	0,2886	0,6284	0,5461
104	HELEQFO GA Equity	0,01	0,0075	1,0101	0,1969	0,2490	0,6174
105	ZENECGI LX Equity	0	0	1,0000	0,1750	0,1656	1,4085
106	ALAEVAB FH Equity	0,01	0,01	1,0101	0,1728	0,2340	1,0845
107	RTOSAKA AV Equity	0,04	0	1,0417	0,2528	0,4016	0,6222
108	NORCHIA FH Equity	0,008	0,01	1,0081	0,2117	0,1134	1,2468
109	CHAGPSA ID Equity	0	0	1,0000	0,1433	0,1791	0,9481
110	CAGGRSA ID Equity	0	0	1,0000	0,1397	0,1776	0,9304
111	MONGRMY GR Equity	0,035	0	1,0363	0,2136	0,2099	1,4389
112	GESCCA2 SM Equity	0	0,01	1,0000	0,2056	0,1316	0,9720
113	ZKBAKTE SW Equity	0	0	1,0000	0,1829	0,2145	1,0690
114	MEDCOUA ID Equity	0	0	1,0000	0,1407	0,2141	0,9012
115	VANESII ID Equity	0	0	1,0000	0,2010	0,2274	1,0179
116	ALIMISC GA Equity	0	0	1,0000	0,3525	0,4486	0,5596
117	MIRFRAE LX Equity	0	0	1,0000	0,1766	0,1702	1,0124
118	BHWMGFT GR Equity	0,05	0	1,0526	0,2208	0,4055	1,3023
119	TAPTULE FH Equity	0,008	0,01	1,0081	0,1890	0,3156	1,2998
120	RRATSUN GR Equity	0,05	0	1,0526	0,1702	0,1894	1,0426
121	SCHSCES SM Equity	0	0	1,0000	0,2030	0,3408	0,8548

N.	Fund ticker	Fees		Inputs			Outputs
		F. Load	B. Load	K_j	St. dev.	β-coeff.	M_j
122	CSIFEZI SW Equity	0,001	0,0003	1,0010	0,2003	0,2415	1,0406
123	POPEURO FH Equity	0,01	0,01	1,0101	0,1861	0,3060	1,0152
124	GESCOSE SM Equity	0	0	1,0000	0,3512	0,0539	1,0462
125	EDEUROI ID Equity	0,015	0,01	1,0152	0,1699	0,1919	1,3858
126	DWSAVER AV Equity	0,05	0	1,0526	0,1371	0,1562	1,1322
127	BADVANS AV Equity	0,05	0	1,0526	0,1196	0,1447	1,1391
128	METSCFR FP Equity	0,04	0	1,0417	0,1923	0,3392	1,4384
129	METREUR FP Equity	0,04	0	1,0417	0,2059	0,2426	1,0270
130	MELLEEA ID Equity	0,05	0	1,0526	0,2166	0,3331	1,2000
131	A0BLTJ GR Equity	0,03	0	1,0309	0,1443	0,3117	1,4266
132	CVPVOPW LE Equity	0,05	0	1,0526	0,1766	0,2048	1,0867
133	INKAFUG GR Equity	0,0005	0	1,0005	0,1194	0,1114	1,4574
134	IBESCAP SM Equity	0	0	1,0000	0,2012	0,3098	0,9265
135	UBSESGA SM Equity	0	0	1,0000	0,2311	0,2604	0,9075
136	UNIVRAP GR Equity	0,05	0	1,0526	0,0987	0,1358	1,4344
137	RESMCPA AV Equity	0,05	0	1,0526	0,2304	0,4407	0,9578
138	IDEUR50 SM Equity	0	0	1,0000	0,2056	0,1959	0,9246
139	ETHIKAT AV Equity	0,05	0	1,0526	0,1608	0,2261	1,0183
140	ALSTDOM GA Equity	0,01	0,01	1,0101	0,3537	0,4234	0,6044
141	INVFINV SM Equity	0	0	1,0000	0,1357	0,0935	0,7702
142	AMCEUEQ SW Equity	0,003	0	1,0030	0,1917	0,2575	1,0130
143	GUTEUPO AV Equity	0,04	0	1,0417	0,1688	0,2490	0,7542
144	SCHPEIN SM Equity	0	0	1,0000	0,2085	0,2043	0,8999
145	CARINVL FP Equity	0	0	1,0000	0,1299	0,0943	1,6045
146	SUSTNFD AV Equity	0,05	0	1,0526	0,1395	0,1877	1,2472
147	DWSEINN GR Equity	0,05	0	1,0526	0,2561	0,4334	1,4231
148	SCHGRVA SM Equity	0	0	1,0000	0,2355	0,1141	0,9595
149	BPAZETI IM Equity	0	0	1,0000	0,1731	0,1744	0,9127
150	ICEEUSI FH Equity	0	0	1,0000	0,1684	0,3035	1,1125
151	MAIIBEX SM Equity	0	0,02	1,0000	0,2347	0,1241	0,9073
152	CIA109F AV Equity	0,07	0	1,0753	0,2020	0,2831	1,0745
153	MSGLESM SM Equity	0	0,02	1,0000	0,2053	0,1992	0,8368
154	MPCEAMI GR Equity	0,05	0	1,0526	0,2124	0,3503	1,0535
155	NTEUEQ2 ID Equity	0	0	1,0000	0,1841	0,2354	1,1427
156	MEINDGR AV Equity	0,05	0	1,0526	0,2882	0,3464	0,9075
157	DLSDF NA Equity	0,002	0,002	1,0020	0,1286	0,2216	1,2886
158	NTUKEQB ID Equity	0	0	1,0000	0,1644	0,2754	1,1346
159	EUVDIVI SM Equity	0	0	1,0000	0,1657	0,1488	0,9136
160	FTHIDVD GR Equity	0,05	0	1,0526	0,1566	0,1659	0,9931
161	SCHAKWA AV Equity	0,04	0	1,0417	0,1681	0,2222	1,0908
162	JIFAVEQ DC Equity	0,024	0,006	1,0246	0,1727	0,2294	1,1003
163	OPJAPAN FH Equity	0,01	0,01	1,0101	0,1581	0,1451	0,7766

N.	Fund ticker	Fees		Inputs			Outputs
		F. Load	B. Load	K_j	St. dev.	β-coeff.	M_j
164	FININTN SM Equity	0	0	1,0000	0,1737	0,1416	0,8202
165	ESXTEUR AV Equity	0,05	0,002	1,0526	0,1722	0,2526	1,0571
166	BLUVEQL SW Equity	0	0	1,0000	0,1905	0,3327	1,5177
167	MSGINGD SM Equity	0	0	1,0000	0,2355	0,1134	0,9591
168	SEITEMU AV Equity	0,05	0	1,0526	0,1711	0,2667	1,0814
169	MSGLSSM SM Equity	0	0,02	1,0000	0,2354	0,1149	0,8587
170	BPBAZIT IM Equity	0,025	0	1,0256	0,2171	0,2844	0,7592
171	AESFUND AV Equity	0,05	0	1,0526	0,2491	0,4495	0,3212
172	DESEEUS GR Equity	0	0	1,0000	0,1902	0,3141	0,8027
173	BBKDIVI SM Equity	0	0	1,0000	0,1714	0,1228	1,1070
174	TAPTREN FH Equity	0,008	0,01	1,0081	0,1282	0,1682	1,3949
175	SAAEURA FH Equity	0,01	0,01	1,0101	0,1759	0,2563	0,9048
176	METFRON FP Equity	0,04	0	1,0417	0,2379	0,3870	0,7833
177	AXAPEEA ID Equity	0	0	1,0000	0,1722	0,2358	1,0881
178	AMEVEAD ID Equity	0	0	1,0000	0,1576	0,2276	1,1930
179	3BKOEKG AV Equity	0,05	0	1,0526	0,1674	0,2853	1,1018
180	SGAIMPC FP Equity	0	0	1,0000	0,0195	0,0146	1,0641
181	CELKIIN FH Equity	0	0	1,0000	0,2023	0,2931	1,0107
182	GWE NA Equity	0	0	1,0000	0,1315	0,1766	1,1691
183	AMAREAE ID Equity	0	0	1,0000	0,2124	0,2376	1,2716
184	FBES150 SM Equity	0	0,04	1,0000	0,3601	0,2045	0,6312
185	LEGCTEA ID Equity	0,05	0	1,0526	0,1677	0,2501	0,9165
186	BPBAZEU IM Equity	0,025	0	1,0256	0,1654	0,2203	1,0073
187	CSWEPOR AV Equity	0,05	0	1,0526	0,1501	0,2303	1,1421
188	FTEURHD GR Equity	0,05	0	1,0526	0,1851	0,2666	0,8720
189	UIAAEAI GR Equity	0,01	0	1,0101	0,1830	0,2471	1,1629
190	CBPVGES LE Equity	0,05	0	1,0526	0,1706	0,2188	0,9586
191	RTDIVAA AV Equity	0,04	0	1,0417	0,1704	0,2597	0,9857
192	GLGPDDN ID Equity	0,05	0	1,0526	0,1980	0,3487	0,9733
193	ESUMWSA AV Equity	0,04	0	1,0417	0,2034	0,2892	0,9798
194	SZUKAK1 AV Equity	0,02	0	1,0204	0,0871	0,0419	0,7047
195	OPINTIA FH Equity	0,01	0,01	1,0101	0,2791	0,2403	0,9564
196	ESPAESA AV Equity	0,05	0	1,0526	0,1813	0,2895	0,9291
197	LIGPCAT LX Equity	0,0175	0	1,0178	0,1382	0,1535	1,1787
198	SWLTDVE LX Equity	0	0	1,0000	0,1876	0,2740	1,0186
199	MANMUSG FH Equity	0,01	0,02	1,0101	0,3723	0,3285	1,2337
200	PLEABRE ID Equity	0	0	1,0000	0,0437	0,0252	1,3139
201	ODALGEU ID Equity	0,02	0	1,0204	0,1575	0,2907	1,5382
202	BIPINIZ IM Equity	0,04	0	1,0417	0,1801	0,3121	1,1149
203	FIMNORA FH Equity	0,01	0,01	1,0101	0,2730	0,4637	1,1639
204	STERNEU GR Equity	0,05	0	1,0526	0,2498	0,4470	0,9277
205	MANKING FH Equity	0,01	0,01	1,0101	0,2150	0,3370	0,7584

N.	Fund ticker	Fees		Inputs			Outputs
		F. Load	B. Load	K_j	St. dev.	β-coeff.	M_j
206	ALGFT20 GA Equity	0	0,01	1,0000	0,4022	0,5979	0,2218
207	LAPLACE GR Equity	0,05	0	1,0526	0,2415	0,3947	0,8412
208	VPKEURS AV Equity	0,05	0	1,0526	0,2180	0,3804	0,7550
209	DWSTURK LX Equity	0,05	0	1,0526	0,3751	0,3496	1,3939
210	ICEPROB FH Equity	0,01	0,01	1,0101	0,2276	0,4620	0,7958
211	MANBORS SM Equity	0	0,01	1,0000	0,2170	0,2082	0,9882
212	OPKINTA FH Equity	0,01	0,01	1,0101	0,2145	0,3488	0,7693
213	MELUGAE ID Equity	0,05	0	1,0526	0,1638	0,1965	1,0621
214	BNYPNDN GR Equity	0,05	0	1,0526	0,1612	0,2163	1,1620
215	OPKIINA FH Equity	0,01	0,01	1,0101	0,2152	0,1740	1,5625
216	POHEUEA FH Equity	0,01	0,01	1,0101	0,1591	0,2186	1,2975
217	F129DBE SM Equity	0	0,04	1,0000	0,1912	0,2773	0,7443
218	WPRVEPA AV Equity	0,05	0	1,0526	0,2195	0,3366	0,5966
219	EUPROST AV Equity	0,05	0	1,0526	0,2221	0,3506	0,7641
220	ESISTSA AV Equity	0,04	0	1,0417	0,3639	0,3541	1,2543
221	ODPANEA ID Equity	0,05	0	1,0526	0,1637	0,1495	1,6771
222	BRAEAIA ID Equity	0,05	0	1,0526	0,1640	0,2392	1,2498

Appendix C

Table 6.3. Inputs, Outputs and modified Outputs for DEA-WZ model

N.	Inputs		Outputs			Modified outputs		
	St. Dev	Prop.Neg	Mean Return	Skew	Min. Return	Mean Return*	Skew*	Min. Return*
1	0,1022	0,4286	-0,0043	-1,6770	-0,4857	-0,0043	-1,6770	-0,4857
2	0,1084	0,4643	0,0004	-1,9262	**-0,5482**	0,0004	-1,9262	-0,5482
3	0,0942	0,5000	-0,0020	-0,8300	-0,3513	-0,0020	-0,8300	-0,3513
4	0,0967	0,4286	0,0005	-1,4894	-0,4300	0,0005	-1,4894	-0,4300
5	0,0979	0,4286	-0,0015	-1,6507	-0,4205	-0,0015	-1,6507	-0,4205
6	0,0829	0,5000	-0,0021	-0,8773	-0,3333	-0,0021	-0,8773	-0,3333
7	0,0745	0,4286	0,0029	-0,7878	-0,2580	0,0029	-0,7878	-0,2580
8	0,0607	0,4643	0,0031	-0,1284	-0,1502	0,0031	-0,1284	-0,1502
9	0,0714	0,4643	-0,0014	-0,9452	-0,2748	-0,0014	-0,9452	-0,2748
10	0,0698	0,4762	-0,0017	-0,7830	-0,2116	-0,0017	-0,7830	-0,2116
11	0,0475	0,5119	-0,0003	-0,3525	-0,1227	-0,0003	-0,3525	-0,1227
12	0,0786	0,5000	-0,0027	-1,4147	-0,3712	-0,0027	-1,4147	-0,3712
13	0,0643	0,4286	0,0042	0,4201	-0,1373	0,0042	0,4201	-0,1373
14	0,0455	0,4643	0,0030	-0,6003	-0,1285	0,0030	-0,6003	-0,1285
15	0,0762	0,4762	-0,0019	-0,9140	-0,3052	-0,0019	-0,9140	-0,3052
16	0,0693	0,4048	0,0002	-0,9544	-0,2628	0,0002	-0,9544	-0,2628
17	0,0634	0,4643	-0,0016	-0,7964	-0,2033	-0,0016	-0,7964	-0,2033
18	0,0838	0,4881	-0,0016	-0,6825	-0,3495	-0,0016	-0,6825	-0,3495
19	0,0784	0,4881	-0,0070	-0,6466	-0,3033	-0,0070	-0,6466	-0,3033
20	0,0580	0,4524	0,0011	-0,8497	-0,2188	0,0011	-0,8497	-0,2188
21	0,0530	0,4048	0,0040	-1,1631	-0,2207	0,0040	-1,1631	-0,2207
22	0,0648	0,4048	0,0029	-1,3357	-0,2611	0,0029	-1,3357	-0,2611
23	0,0633	0,3810	0,0028	0,2134	-0,1544	0,0028	0,2134	-0,1544
24	0,0409	0,4643	0,0024	-0,6237	-0,1049	0,0024	-0,6237	-0,1049
25	0,0416	0,4167	0,0029	-0,4457	-0,1055	0,0029	-0,4457	-0,1055
26	0,0449	0,4167	0,0039	-0,6392	-0,1228	0,0039	-0,6392	-0,1228
27	0,0442	0,4286	0,0035	-0,5973	-0,1101	0,0035	-0,5973	-0,1101
28	0,0497	0,4286	0,0038	-0,2110	-0,1570	0,0038	-0,2110	-0,1570
29	0,0583	0,4643	-0,0011	-1,6437	-0,2405	-0,0011	-1,6437	-0,2405
30	0,0383	0,3929	0,0020	-0,8027	-0,1058	0,0020	-0,8027	-0,1058
31	0,0472	0,4643	0,0038	-0,5193	-0,1295	0,0038	-0,5193	-0,1295
32	0,0472	0,4524	0,0016	-0,4668	-0,1221	0,0016	-0,4668	-0,1221
33	0,0992	0,5119	-0,0081	-0,6000	-0,3310	-0,0081	-0,6000	-0,3310
34	0,0719	0,3929	0,0017	-0,9485	-0,2912	0,0017	-0,9485	-0,2912
35	0,0470	0,4643	0,0028	-0,6313	-0,1277	0,0028	-0,6313	-0,1277
36	0,0446	0,4048	0,0027	-0,8714	-0,1302	0,0027	-0,8714	-0,1302

N.	Inputs		Outputs			Modified outputs		
	St. Dev	Prop.Neg	Mean Return	Skew	Min. Return	Mean Return*	Skew*	Min. Return*
37	0,0400	0,4048	0,0008	-1,1462	-0,1362	0,0008	-1,1462	-0,1362
38	0,0462	0,4405	0,0021	-0,6481	-0,1252	0,0021	-0,6481	-0,1252
39	0,0641	0,4286	0,0038	-0,2416	-0,1645	0,0038	-0,2416	-0,1645
40	0,0512	0,4048	0,0030	-0,8229	-0,1521	0,0030	-0,8229	-0,1521
41	0,0521	0,4881	-0,0024	-1,3932	-0,2295	-0,0024	-1,3932	-0,2295
42	0,0462	0,4286	0,0013	-0,5409	-0,1103	0,0013	-0,5409	-0,1103
43	0,0750	0,4643	-0,0003	-1,9539	-0,3760	-0,0003	-1,9539	-0,3760
44	0,0497	0,4048	0,0036	-0,3883	-0,1540	0,0036	-0,3883	-0,1540
45	0,0415	0,3690	0,0051	-0,9343	-0,1434	0,0051	-0,9343	-0,1434
46	0,0514	0,4405	-0,0019	-0,9385	-0,1597	-0,0019	-0,9385	-0,1597
47	0,0643	0,3810	0,0027	-0,0351	-0,1652	0,0027	-0,0351	-0,1652
48	0,0560	0,4167	0,0051	-0,9689	-0,1839	0,0051	-0,9689	-0,1839
49	0,0543	0,4881	0,0029	-0,8570	-0,2035	0,0029	-0,8570	-0,2035
50	0,0672	0,4286	-0,0006	-1,7176	-0,3149	-0,0006	-1,7176	-0,3149
51	0,0498	0,4643	-0,0053	-1,1251	-0,1849	-0,0053	-1,1251	-0,1849
52	0,0448	0,4048	0,0045	-0,6737	-0,1305	0,0045	-0,6737	-0,1305
53	0,0603	0,4643	-0,0001	-0,4615	-0,1859	-0,0001	-0,4615	-0,1859
54	0,0433	0,4286	0,0014	-0,6006	-0,1229	0,0014	-0,6006	-0,1229
55	0,0482	0,4167	0,0045	-1,3364	-0,1873	0,0045	-1,3364	-0,1873
56	0,0470	0,4524	-0,0003	-0,7737	-0,1251	-0,0003	-0,7737	-0,1251
57	0,0497	0,4167	0,0032	-1,0734	-0,1984	0,0032	-1,0734	-0,1984
58	0,0570	0,4167	0,0045	-0,7355	-0,1848	0,0045	-0,7355	-0,1848
59	0,0709	0,3929	0,0028	-1,8036	-0,3286	0,0028	-1,8036	-0,3286
60	0,0387	0,3929	0,0037	-1,1704	-0,1410	0,0037	-1,1704	-0,1410
61	0,0441	0,4405	0,0006	-1,5033	-0,1823	0,0006	-1,5033	-0,1823
62	0,0601	0,4286	0,0033	-0,0734	-0,1267	0,0033	-0,0734	-0,1267
63	0,0660	0,4524	-0,0008	-0,7352	-0,2473	-0,0008	-0,7352	-0,2473
64	0,0483	0,4048	0,0008	-1,2718	-0,1967	0,0008	-1,2718	-0,1967
65	0,0447	0,3929	0,0022	-0,8755	-0,1337	0,0022	-0,8755	-0,1337
66	0,0644	0,4048	0,0030	0,1257	-0,1457	0,0030	0,1257	-0,1457
67	0,0752	0,4524	0,0032	-0,6261	-0,2164	0,0032	-0,6261	-0,2164
68	0,0386	0,3690	0,0027	-1,3087	-0,1603	0,0027	-1,3087	-0,1603
69	0,0461	0,3929	0,0012	-0,8826	-0,1444	0,0012	-0,8826	-0,1444
70	0,0486	0,4762	-0,0030	-0,5448	-0,1562	-0,0030	-0,5448	-0,1562
71	0,0631	0,4167	0,0033	-0,8542	-0,1891	0,0033	-0,8542	-0,1891
72	0,0453	0,4881	-0,0006	-0,3674	-0,1244	-0,0006	-0,3674	-0,1244
73	0,0478	0,4048	0,0017	-1,1958	-0,1953	0,0017	-1,1958	-0,1953
74	0,0541	0,4643	-0,0009	-1,1499	-0,1976	-0,0009	-1,1499	-0,1976
75	0,0536	0,5000	-0,0044	-0,8629	-0,2318	-0,0044	-0,8629	-0,2318
76	0,0428	0,3571	0,0046	-0,8510	-0,1253	0,0046	-0,8510	-0,1253
77	0,0569	0,3571	0,0022	-1,5963	-0,2264	0,0022	-1,5963	-0,2264
78	0,0492	0,4167	0,0024	-0,7006	-0,1448	0,0024	-0,7006	-0,1448

N.	Inputs		Outputs			Modified outputs		
	St. Dev	Prop.Neg	Mean Return	Skew	Min. Return	Mean Return*	Skew*	Min. Return*
79	0,0732	0,3690	0,0013	-1,5718	-0,3833	0,0013	-1,5718	-0,3833
80	0,0612	0,4286	0,0007	-1,2788	-0,2470	0,0007	-1,2788	-0,2470
81	0,0432	0,4048	0,0048	-1,4597	-0,1901	0,0048	-1,4597	-0,1901
82	0,0561	0,5000	-0,0014	-0,6461	-0,1822	-0,0014	-0,6461	-0,1822
83	0,0334	0,4048	0,0021	-1,0302	-0,1158	0,0021	-1,0302	-0,1158
84	0,0667	0,3929	0,0044	-1,4793	-0,2729	0,0044	-1,4793	-0,2729
85	0,0517	0,3690	0,0029	-1,0238	-0,2068	0,0029	-1,0238	-0,2068
86	0,0507	0,4167	-0,0005	-1,7346	-0,2174	-0,0005	-1,7346	-0,2174
87	0,0435	0,3810	0,0025	-1,1719	-0,1457	0,0025	-1,1719	-0,1457
88	0,0674	0,4405	0,0007	-1,0499	-0,2714	0,0007	-1,0499	-0,2714
89	0,0415	0,4167	-0,0007	-0,8207	-0,1323	-0,0007	-0,8207	-0,1323
90	0,0845	0,4286	-0,0017	-2,0930	-0,4428	-0,0017	-2,0930	-0,4428
91	0,0364	0,4048	0,0026	-0,5979	-0,1052	0,0026	-0,5979	-0,1052
92	0,0468	0,4167	0,0011	-1,2095	-0,1865	0,0011	-1,2095	-0,1865
93	0,0413	0,3929	0,0009	-0,7858	-0,1214	0,0009	-0,7858	-0,1214
94	0,0348	0,4524	0,0001	-0,9221	-0,1179	0,0001	-0,9221	-0,1179
95	0,0407	0,3452	0,0018	-1,2356	-0,1349	0,0018	-1,2356	-0,1349
96	0,0477	0,3929	0,0006	-0,7235	-0,1414	0,0006	-0,7235	-0,1414
97	0,0686	0,4048	-0,0023	-1,9384	-0,3737	-0,0023	-1,9384	-0,3737
98	0,0451	0,4286	0,0009	-1,2318	-0,1933	0,0009	-1,2318	-0,1933
99	0,0424	0,4048	0,0000	-0,6677	-0,1265	0,0000	-0,6677	-0,1265
100	0,0807	0,4762	-0,0042	-1,9891	-0,4319	-0,0042	-1,9891	-0,4319
101	0,0592	0,4762	0,0005	-0,5241	-0,2077	0,0005	-0,5241	-0,2077
102	0,0588	0,4048	0,0012	-0,6309	-0,1849	0,0012	-0,6309	-0,1849
103	0,0833	0,4286	-0,0071	-1,9025	-0,4283	-0,0071	-1,9025	-0,4283
104	0,0569	0,5595	-0,0057	0,0098	-0,1943	-0,0057	0,0098	-0,1943
105	0,0505	0,4286	0,0041	-0,8627	-0,1510	0,0041	-0,8627	-0,1510
106	0,0499	0,4167	0,0011	-1,1186	-0,1880	0,0011	-1,1186	-0,1880
107	0,0730	0,4643	-0,0056	-0,7695	-0,3039	-0,0056	-0,7695	-0,3039
108	0,0611	0,3929	0,0027	-0,8999	-0,2053	0,0027	-0,8999	-0,2053
109	0,0414	0,4167	-0,0006	-0,9900	-0,1335	-0,0006	-0,9900	-0,1335
110	0,0403	0,4048	-0,0009	-0,8871	-0,1194	-0,0009	-0,8871	-0,1194
111	0,0616	0,3690	0,0043	-0,7983	-0,2243	0,0043	-0,7983	-0,2243
112	0,0594	0,4167	-0,0002	-1,3705	-0,2437	-0,0002	-1,3705	-0,2437
113	0,0528	0,4048	0,0008	-1,1329	-0,1993	0,0008	-1,1329	-0,1993
114	0,0406	0,4286	-0,0012	-1,5023	-0,1824	-0,0012	-1,5023	-0,1824
115	0,0580	0,4762	0,0002	-0,7530	-0,1980	0,0002	-0,7530	-0,1980
116	0,1018	0,5357	-0,0069	-0,6288	-0,3779	-0,0069	-0,6288	-0,3779
117	0,0510	0,4405	0,0001	-0,6872	-0,1388	0,0001	-0,6872	-0,1388
118	0,0637	0,4048	0,0031	-1,4818	-0,2566	0,0031	-1,4818	-0,2566
119	0,0546	0,3810	0,0032	-1,3929	-0,2324	0,0032	-1,3929	-0,2324
120	0,0491	0,4405	0,0005	-0,9350	-0,2163	0,0005	-0,9350	-0,2163

N.	Inputs		Outputs			Modified outputs		
	St. Dev	Prop.Neg	Mean Return	Skew	Min. Return	Mean Return*	Skew*	Min. Return*
121	0,0586	0,4405	-0,0019	-0,6952	-0,1661	-0,0019	-0,6952	-0,1661
122	0,0578	0,4762	0,0005	-0,7330	-0,1963	0,0005	-0,7330	-0,1963
123	0,0537	0,4286	0,0003	-1,0135	-0,1856	0,0003	-1,0135	-0,1856
124	0,1014	0,4881	0,0005	0,4332	-0,4532	0,0005	0,4332	-0,4532
125	0,0490	0,4524	0,0040	-0,3725	-0,1296	0,0040	-0,3725	-0,1296
126	0,0396	0,4048	0,0015	-0,7715	-0,1145	0,0015	-0,7715	-0,1145
127	0,0345	0,4167	0,0016	-0,5692	-0,1124	0,0016	-0,5692	-0,1124
128	0,0555	0,4286	0,0043	-0,5128	-0,1626	0,0043	-0,5128	-0,1626
129	0,0594	0,4762	0,0003	-0,4045	-0,1899	0,0003	-0,4045	-0,1899
130	0,0625	0,4286	0,0022	-0,9892	-0,2166	0,0022	-0,9892	-0,2166
131	0,0417	0,3571	0,0042	-1,2062	-0,1457	0,0042	-1,2062	-0,1457
132	0,0510	0,4048	0,0010	-0,7168	-0,1592	0,0010	-0,7168	-0,1592
133	0,0345	0,3690	0,0045	-1,2630	-0,1166	0,0045	-1,2630	-0,1166
134	0,0581	0,4762	-0,0009	-0,9019	-0,1862	-0,0009	-0,9019	-0,1862
135	0,0667	0,4524	-0,0012	-0,8015	-0,2477	-0,0012	-0,8015	-0,2477
136	0,0285	0,4405	0,0043	-0,5779	-0,0826	0,0043	-0,5779	-0,0826
137	0,0665	0,4167	-0,0005	-1,7317	-0,3169	-0,0005	-1,7317	-0,3169
138	0,0593	0,4643	-0,0009	-0,6280	-0,1870	-0,0009	-0,6280	-0,1870
139	0,0464	0,3929	0,0002	-1,4011	-0,1682	0,0002	-1,4011	-0,1682
140	0,1021	0,4762	-0,0059	-0,4416	-0,3054	-0,0059	-0,4416	-0,3054
141	0,0392	0,4524	-0,0031	-0,4574	-0,1002	-0,0031	-0,4574	-0,1002
142	0,0553	0,4524	0,0002	-0,6686	-0,1922	0,0002	-0,6686	-0,1922
143	0,0487	0,4762	-0,0034	-0,9801	-0,1749	-0,0034	-0,9801	-0,1749
144	0,0602	0,4762	-0,0013	-0,5900	-0,1922	-0,0013	-0,5900	-0,1922
145	0,0375	0,4048	0,0056	-0,2012	-0,1016	0,0056	-0,2012	-0,1016
146	0,0403	0,3810	0,0026	-1,1999	-0,1342	0,0026	-1,1999	-0,1342
147	0,0739	0,4643	0,0042	-0,5218	-0,2863	0,0042	-0,5218	-0,2863
148	0,0680	0,5000	-0,0005	-0,3220	-0,2111	-0,0005	-0,3220	-0,2111
149	0,0500	0,4881	-0,0011	-0,4253	-0,1356	-0,0011	-0,4253	-0,1356
150	0,0486	0,4167	0,0013	-0,9264	-0,1649	0,0013	-0,9264	-0,1649
151	0,0678	0,4762	-0,0009	-0,3494	-0,2162	-0,0009	-0,3494	-0,2162
152	0,0583	0,4524	0,0009	-0,9721	-0,2026	0,0009	-0,9721	-0,2026
153	0,0593	0,4762	-0,0019	-0,6335	-0,1898	-0,0019	-0,6335	-0,1898
154	0,0613	0,4286	0,0006	-0,7180	-0,2285	0,0006	-0,7180	-0,2285
155	0,0531	0,4048	0,0016	-0,7834	-0,1789	0,0016	-0,7834	-0,1789
156	0,0832	0,5000	-0,0012	-0,3025	-0,2789	-0,0012	-0,3025	-0,2789
157	0,0371	0,4643	0,0030	-0,3526	-0,1057	0,0030	-0,3526	-0,1057
158	0,0474	0,3690	0,0015	-0,7767	-0,1344	0,0015	-0,7767	-0,1344
159	0,0478	0,5000	-0,0011	-0,4875	-0,1339	-0,0011	-0,4875	-0,1339
160	0,0452	0,4524	-0,0001	-1,0910	-0,1574	-0,0001	-1,0910	-0,1574
161	0,0485	0,3810	0,0010	-1,9145	-0,2357	0,0010	-1,9145	-0,2357
162	0,0498	0,4167	0,0012	-1,4022	-0,1855	0,0012	-1,4022	-0,1855

N.	Inputs		Outputs			Modified outputs		
	St. Dev	Prop.Neg	Mean Return	Skew	Min. Return	Mean Return*	Skew*	Min. Return*
163	0,0457	0,5476	-0,0029	-0,2859	-0,1402	-0,0029	-0,2859	-0,1402
164	0,0501	0,4881	-0,0024	-0,3880	-0,1403	-0,0024	-0,3880	-0,1403
165	0,0497	0,4524	0,0007	-0,6555	-0,1413	0,0007	-0,6555	-0,1413
166	0,0550	0,3690	0,0050	-0,6074	-0,2097	0,0050	-0,6074	-0,2097
167	0,0680	0,5119	-0,0005	-0,3351	-0,2122	-0,0005	-0,3351	-0,2122
168	0,0494	0,4405	0,0009	-0,9741	-0,1713	0,0009	-0,9741	-0,1713
169	0,0679	0,5000	-0,0016	-0,3419	-0,2156	-0,0016	-0,3419	-0,2156
170	0,0627	0,5119	-0,0033	-0,2130	-0,1824	-0,0033	-0,2130	-0,1824
171	0,0719	0,5476	-0,0135	**-2,6754**	-0,4278	-0,0135	-2,6754	-0,4278
172	0,0549	0,4881	-0,0026	-1,3459	-0,2155	-0,0026	-1,3459	-0,2155
173	0,0495	0,4762	0,0012	-0,3571	-0,1261	0,0012	-0,3571	-0,1261
174	0,0370	0,4048	0,0041	-1,3852	-0,1502	0,0041	-1,3852	-0,1502
175	0,0508	0,4524	-0,0011	-1,0582	-0,1879	-0,0011	-1,0582	-0,1879
176	0,0687	0,4286	-0,0029	-1,1593	-0,3112	-0,0029	-1,1593	-0,3112
177	0,0497	0,4286	0,0010	-0,8605	-0,1643	0,0010	-0,8605	-0,1643
178	0,0455	0,4167	0,0021	-0,6527	-0,1539	0,0021	-0,6527	-0,1539
179	0,0483	0,4167	0,0012	-1,6127	-0,2184	0,0012	-1,6127	-0,2184
180	0,0056	0,3571	0,0007	3,3174	-0,0209	0,0007	3,3174	-0,0209
181	0,0584	0,4881	0,0001	-0,0411	-0,2042	0,0001	-0,0411	-0,2042
182	0,0380	0,4405	0,0019	-1,0533	-0,1519	0,0019	-1,0533	-0,1519
183	0,0613	0,4643	0,0029	-0,2714	-0,1603	0,0029	-0,2714	-0,1603
184	0,1039	0,5000	-0,0050	-0,4203	-0,3470	-0,0050	-0,4203	-0,3470
185	0,0484	0,4643	-0,0010	-0,8661	-0,1693	-0,0010	-0,8661	-0,1693
186	0,0478	0,4762	0,0001	-0,6108	-0,1667	0,0001	-0,6108	-0,1667
187	0,0433	0,3929	0,0016	-1,6486	-0,1862	0,0016	-1,6486	-0,1862
188	0,0534	0,4405	-0,0016	-0,9247	-0,1847	-0,0016	-0,9247	-0,1847
189	0,0528	0,4524	0,0018	-0,0478	-0,1645	0,0018	-0,0478	-0,1645
190	0,0492	0,4167	-0,0005	-0,8805	-0,1638	-0,0005	-0,8805	-0,1638
191	0,0492	0,4167	-0,0002	-0,8547	-0,1663	-0,0002	-0,8547	-0,1663
192	0,0571	0,4048	-0,0003	-1,3926	-0,2451	-0,0003	-1,3926	-0,2451
193	0,0587	0,3929	-0,0002	-2,0064	-0,2837	-0,0002	-2,0064	-0,2837
194	0,0251	0,7381	-0,0042	-1,2181	-0,1054	-0,0042	-1,2181	-0,1054
195	0,0806	0,4881	-0,0004	-0,5693	-0,2743	-0,0004	-0,5693	-0,2743
196	0,0523	0,3810	-0,0009	-1,1445	-0,1872	-0,0009	-1,1445	-0,1872
197	0,0399	0,4167	0,0020	-0,6834	-0,1224	0,0020	-0,6834	-0,1224
198	0,0541	0,4405	0,0002	-0,7339	-0,1992	0,0002	-0,7339	-0,1992
199	0,1075	0,4524	0,0027	-0,8583	-0,4185	0,0027	-0,8583	-0,4185
200	0,0126	0,3333	0,0033	-0,5987	-0,0434	0,0033	-0,5987	-0,0434
201	0,0455	0,4167	0,0051	-0,4257	-0,1176	0,0051	-0,4257	-0,1176
202	0,0520	0,4524	0,0013	-0,9184	-0,1950	0,0013	-0,9184	-0,1950
203	0,0788	0,4286	0,0019	-0,9815	-0,2927	0,0019	-0,9815	-0,2927
204	0,0721	0,4167	-0,0009	-2,1201	-0,3519	-0,0009	-2,1201	-0,3519

N.	Inputs		Outputs			Modified outputs		
	St. Dev	Prop.Neg	Mean Return	Skew	Min. Return	Mean Return*	Skew*	Min. Return*
205	0,0621	0,4881	-0,0032	-0,3931	-0,2227	-0,0032	-0,3931	-0,2227
206	0,1161	0,5357	**-0,0178**	-0,3763	-0,3520	-0,0178	-0,3763	-0,3520
207	0,0697	0,4405	-0,0021	-1,1393	-0,2258	-0,0021	-1,1393	-0,2258
208	0,0629	0,3810	-0,0033	-1,8916	-0,3061	-0,0033	-1,8916	-0,3061
209	0,1083	0,4643	0,0040	-0,8346	-0,4166	0,0040	-0,8346	-0,4166
210	0,0657	0,4405	-0,0026	-0,8288	-0,2772	-0,0026	-0,8288	-0,2772
211	0,0626	0,4405	0,0000	-0,0641	-0,1471	0,0000	-0,0641	-0,1471
212	0,0619	0,5476	-0,0030	-0,3792	-0,2437	-0,0030	-0,3792	-0,2437
213	0,0473	0,4048	0,0007	-1,0892	-0,1659	0,0007	-1,0892	-0,1659
214	0,0465	0,4167	0,0018	-0,8857	-0,1503	0,0018	-0,8857	-0,1503
215	0,0621	0,3333	0,0054	-0,6095	-0,1583	0,0054	-0,6095	-0,1583
216	0,0459	0,4048	0,0032	-0,8790	-0,1402	0,0032	-0,8790	-0,1402
217	0,0552	0,4643	-0,0030	-1,0134	-0,2123	-0,0030	-1,0134	-0,2123
218	0,0634	0,5357	-0,0061	-0,1560	-0,2239	-0,0061	-0,1560	-0,2239
219	0,0641	0,4881	-0,0032	-0,6022	-0,2834	-0,0032	-0,6022	-0,2834
220	0,1051	0,4405	0,0027	-1,1401	-0,4560	0,0027	-1,1401	-0,4560
221	0,0473	0,3452	0,0062	0,3421	-0,1144	0,0062	0,3421	-0,1144
222	0,0473	0,4048	0,0027	-0,4610	-0,1435	0,0027	-0,4610	-0,1435

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