Final Thesis

A dynamic Core-Satellite Portfolio with ETFs

The evolution of portfolio insurance strategy and its application

Supervisor
Ch. Prof. Marco Corazza

Graduand
Luca Dell’Andrea
Matriculation Number 827333

Academic Year
2015 / 2016
To my Family,
INDEX

Introduction .......................................................................................... 5

Chapter 1: Portfolio management strategies ...................... 7
1.1: Passive management strategy ......................................................... 10
1.2: Active management strategy ......................................................... 12
1.3: “Core-Satellite” strategy .............................................................. 15

Chapter 2: Characteristics and History of ETFs .......... 18
2.1: ETPs family .................................................................................. 18
2.2: Historical background ................................................................. 20
2.3: Main ETF actors in the European market ..................................... 22
2.4: A brief regulatory framework ...................................................... 23
2.5: Index constructions ...................................................................... 27
2.5.1: General view ................................................................................. 27
2.5.2: Some ETF strategies ................................................................. 29
2.6: Different ETF replications strategies ............................................. 30
2.6.1: Physical replication ................................................................. 31
2.6.1.1: Full replication ................................................................. 32
2.6.1.2: Replication by sampling ......................................................... 33
2.6.2: Synthetic Replication ........................................................... 34
2.6.2.1: The unfunded swap model ..................................................... 35
2.6.2.2: The funded swap model ....................................................... 38

Chapter 3: Portfolio insurance strategies ................. 39
3.1: Buy and hold strategy ................................................................. 40
3.2: Costant mix strategy ................................................................. 41
3.3: Option based portfolio insurance strategy (OBPI)………………………………43
3.4: Costant proportion Portfolio insurance strategy and its variation……………47
3.4.1: Variable proportion portfolio insurance strategy……………………………50
3.4.2: Exponential proportion portfolio insurance strategy…………………….. 55

Chapter 4: Empirical Analysis………………………………………………57

Conclusion……………………………………………………………………..83

Matlab Code…………………………………………………………………85

Appendix…………………………………………………………………91

References……………………………………………………………………109
Introduction

The purpose of this paper is to develop a particular portfolio investment strategy limiting its downside risk, assuming low risk and maximizing its performance. This kind of portfolio is suggested for Pension Funds or Insurance companies where their objective is to ensure a minimum annual return with a limited risk assumed or build portfolios with low risk assumed. The core financial asset of a pension fund and of an Insurance company is represented by fix income asset class. The idea is to develop a Dynamic Core-Satellite Portfolio using particular financial instruments called ETFs. The Core component of this strategy is constituted Fix Income asset, instead the Satellite component might be constituting by equity, commodities or by fix income asset (but riskier and with different period of time). Building this portfolio, I will impose some constraint to the strategy that permit to safeguard the performance of the investor.

In the first chapter will be analysed the principle strategies used by portfolio managers and which are the characteristics of each strategy. It will describe which is the financial planning that every investor must be follow and it will be analysed deeply the two different style largely used by the investor: passive management style and active management style. At the end of this chapter will be describe a new concept of mix strategy between active and passive management styles: the Core-Satellite portfolio strategy and its features.

The second chapter will be more concentrated in the ETFs world (Exchange Trade Funds). ETFs are financial instruments largely used by investors and portfolio managers. Their manly characteristic is to replicate a particular index market and they are less expensive compared to the mutual funds. ETFs instrument apply the basic concept of modern portfolio theory where it is important build a diversified portfolio to diminish the overall risk. The ETFs market is grown exponentially during last 30 years and today they can cover any investor necessity. The objective of this chapter is to give some basic concept to the reader of what are ETFs, how they can be used
from the investors with a brief European regulatory framework and which are the
different replication methods.
The third chapter is more focused on the Portfolio Insurance Strategies largely used
and known by the portfolio managers and investors. They are: “Buy and Hold”
strategy, “Costant Mix” strategy, “Option Based Portfolio Insurance” strategy (OBPI)
and “Costant Proportion Portfolio Insurance” strategy (CPPI). In this chapter is
explained how these strategies works and how they are used by managers. In
particular, I will be more focused in the application of CPPI strategy, analysing some
new variations that have recently been developed by some academy researcher.
Finally, in the last chapter, there will be an empirical application of a Dynamic Core-
Satellite portfolio strategy with the method of Exponential Proportion Portfolio
Insurance (EPPI) strategy using ETFs instruments as component of both Core and
Satellite. The study will analyse four different portfolios with four different Satellite
components, assuming some initial constraints imposed by the manager. To evaluate
the performance of each portfolio compared to its benchmark I used some widely
used financial ratio such as: Sharpe ratio, Information ratio, Sortino ratio, Omega
ratio, Maximum drawdown.
Chapter 1: Portfolio Management

Since the last decade, technological progress, globalization, evolution of legislation and the continuous innovation of the financial world have allowed at a more informed investment decision process. So, managers are able to base their decisions on much more complex things. The recent financial market transformation process has expanded the investment opportunities. The constant increment of competitiveness has defined new rules about investment strategies and asset management.

In the last few years we have seen a constant growth of private asset management and many people consulting financial advisors to manage their savings. Financial advisors are required to find an equilibrium between return and risk, so a good knowledge of market structures and a good asset allocation based on the risk aversion of the client is essential.

Every advisor needs a good knowledge which are the market risks, regardless its constant variation, in particular the stock market, which are more vulnerable to the emotions of the investors.

Portfolio management consists of all the techniques and strategies that an advisor can use to reach a better asset allocation by reaching the desired return, given a particular risk aversion.

In particular, “asset allocation” is the process in which we want to distribute our resources in different asset classes or investments. The principal categories of investments are financial assets (equity, bonds or liquidity) or real assets (commodities, real estate, private equity).

It is very important where you invest your money and in which asset class but the timing, is also important. Every investor tries to buy assets when they are cheaper and sell when they are expensive, but this kind of process is very hard for every investor, professional or not.
There are two essential fundamentals that an investor have to follow when he build its investment: trying to maximize the expected return and minimizing the risks. They are:

- “start-up”, which is the asset allocation process;
- “control process”, which is the periodical portfolio rebalancing.

In the first phase, the asset allocation process consist in which is the strategy that an investor wants to follow. Firstly, he needs to define which is the style strategies, where investors can make portfolio allocation decisions, choosing between a broad categories of securities, such as:

- “large-cap style”: the investors invest in the large capitalization company, with a company value greater than 10$ billion;
- “growth style”: this kind of strategy looks to the future earnings potential of the companies, trying to identify the stocks that offer a potential growth earning at above-average rates.
- “value style”: here managers look for stocks that are incorrectly priced compared to the intrinsic value of the company. In this case the manager style is more focus on stocks where there is a low price/earnings ratios or where they pay higher dividends.
- “index strategy”: permits to the investor to invest in a index at all, following the market. It is an example of passive strategy.

These kind of strategies listed above are more focused about the fundamentals of a company or about the macroeconomic system (for example the economic cycle), but exist other kind of strategies where the manager is focused more on the diversification\(^1\), trying to maximize the expected return and, at the same time, 

\(^1\)It Is a risk management technique that mixes a wide variety of investments within a portfolio. The objective of the diversification is to minimize or eliminate the unsystematic risk events in a portfolio. A better diversification is given in portfolio where the asset are not perfectly correlated.
minimizing the related risk. In the last case there are some different application related to the modern approach of portfolio theory\(^2\).

There are different ways of diversification: for asset class, for geographic area, for sectors or based on rating valuation.

Different investment styles may make the asset allocation more balanced, allowing a choice between the better opportunity from different phases of the market in a contest of high level of globalization. For example in the case of a recession period is suggested to invest in high-quality companies with long business history, in companies with strong balance sheets not leveraged and with strong cash flows. In other hand, in case of recovery is suggested to invest in growth companies, where it is expected higher future dividend. These strategy, together, can represent a perfect combination for a portfolio style manager.

The asset allocation activity is part of a more general process: the “financial planning”, in which the investor defines his/her risk aversion, the goal of the investment, and the structure of asset allocation.

A good financial planning is given considering several factors, such as:

- time horizon;
- risk aversion;
- investment goals;
- financial situation;
- income;
- fiscal situation;
- scenario evolution.

These parameters constitute the asset allocation framework of every investor and must be monitored and eventually rebalanced. In addition, the asset management process is formed by three separate and connected phases: each phase permit to find the better combination between risk and expected return. The three phases are:

\(^2\) For example Harry Markowitz, “Portfolio selection: efficient diversification of investments”, 1959.
• “Strategic” asset allocation: has the objective to choose the investment with middle-long term view. It looks at the perfect mix of asset class that permits to reach, with higher probability, a correct risk premium.\(^3\)

• “Tactic” asset allocation: underlying the revision and rebalanced process of the strategic portfolio composition, finalized to pick up the variation between asset class from the market movement of the short or middle term.

• “Operative” asset allocation: identifies the activity of portfolio construction following the directive of “tactic” and “Strategic” asset allocation.

Usually, every investor uses a benchmark to monitor their investment choices and it is a good instrument to monitor the management activism on the composition of the portfolio asset classes.

There are two different styles of management: passive management and active management.

1.1. Passive management

Passive management is a particular investment strategy, by which the management tries to maximize its expected return and minimize his portfolio decision, minimizing the transfer fees and the fiscal imposition on capital gains.

It is, usually, tracking\(^4\) the path of a preselected market index, we can define it as “benchmarks” or with a composition of several market indexes.

Passive management is based on a famous principle based on a prefixed assumption: the market is “efficient”, explaining that the equilibrium price totally reflects the available information and it is not possible to obtain a better performance in the market.

---

\(^3\) A risk premium is the return in excess of the risk-free rate of return that an investment is expected to yield. It can be described such as a compensation for the investor for the greater risk assumed (compared to a risk-free asset) in a given investment.

\(^4\) It is a term used in the financial sector to follow the same path of a preselected benchmark index.
This kind of strategy has the following particularity: to create a portfolio of assets that try to replicate a defined benchmark where passive investments managers do not take positive or negative views on the market. This strategy could be characterized by a minimization of management fees, because the operations are made only when there are changes on the benchmark composition. A “buy and hold strategy” is a passive investment strategy, the investor buys assets with the intention to hold them for a long period of time, having no regards of the fluctuations on the market. The expected extra-performance of a passive portfolio could be given only from systematic risk exposure.

The first step for every account manager is the purchase of a market portfolio, for example, a market index such as FTSE MIB index, DAX index, DOWN JONS index, which tries to replicate the same weighting index structure\(^5\). These portfolios must be held from middle to long periods, without trading activity and having the same benchmark risk exposure\(^6\).

For small investor this strategy has an important constraint: sometimes the benchmark index could be composed by a huge number of assets, for example Russel 3000 Index is a stock market index that measures the performance of the 3.000 largest US public company. For a small investor may be very expensive buy the exact composition of Russel 3000 index.

Regarding the passive style, the portfolio composition change’s are given by benchmark index changes, for example if there is a M&A operation\(^7\) or a new entry in the market index.

\(^5\) There are two most important weighting index structure: “Price-Weighted Index”, where each stock influences the index in proportion to its price per share, and “Weighted Average Market Capitalization”, is a stock market index weighted by the market capitalization of each stock in the index (in this case larger companies have a greater proportion of the index).

\(^6\) It represents, for the investor, the same risk exposition of the index tracked. The risk of any investment is given by its standard deviation.

\(^7\) Merger and Acquisition operations
There are three different methodologies on the passive management:

- buy and hold;
- constant mix;
- constant proportion.

Firstly, the “buy and hold” is a passive and static investment strategy: after the portfolio creation there is not any change of its composition, any trading movement. Its performance is given only from the benchmark index structure.

Then, the “constant mix” is a dynamic and passive investment strategy: for a specific period of time a fixed percentage of the investment is invested in a particular mix of financial assets, but sometimes there are some periodically rebalanced to adjust the portfolio to the market changes. This strategy creates value and is better than “buy and hold” strategy in presence of volatility without trend.

Finally, “constant proportion” is a strategy that permits to control the value of total asset over a variable allocation between risky and non-risky assets over time. The rebalancing must be continuous, but the transaction costs and technical constraints suggest rebalancing the portfolio only after significant market movement, accepting a limiting risk to get losses.

In conclusion, one of the advantages of passive management is a lesser number of purchase/sell operations compared to other kinds of portfolio management allowing a minimization of the transaction costs.

1.2 Active management

Active management is an investment strategy based on valuation and market analyses with the purpose to beat the benchmark, obtaining a greater performance compared to the benchmark performance. This kind of strategy is for investors that think the
market is not efficient, where there are overvalued or undervalued asset and it is possible to “predict” the future price paths.

In this strategy there are more operations than in the passive management that try to anticipate the market movements.

Active management consists of a different portfolio weighting compare to the passive management. The difference is connected to projections and future estimations. The strategy is based on the concept that the market prices don’t reflect the best intrinsic value estimation, and in consequence, careful research of mispriced stocks and an investment/disinvestment strategy could beat the market.

There are two important concepts of the active management strategy: the “market timing” and the “security selection”.

Firstly, “market timing” refers to all techniques that permit the investor to identify the best moment to enter and the best moment to exit from an investment, for example, enter when prices are low and go away when prices are high. The management based on market timing assume a dynamic portfolio composition in harmony with market previsions and future prices developments.

Then, “security selection” or “stock picking”. Investors increase the weight of the portfolio securities considered undervalued with equal risk exposure.

In this way the intention is to generate a greater expected return compared to the average performance of the market.

In this strategy there are higher transaction and analysis costs.

Moreover, there is another stock selection in active management, where the investor invests in security considered “value” or “growth”.

In the “growth” selection, the manager invests in companies that he believes will growth faster than average in cash flow, revenues or earnings. Usually, many growth-oriented companies reinvest earnings in expansion projects or acquisition, instead of using them to pay shareholder dividends. The prices of these companies shows a higher volatility.
On the “value” selection, the manager focuses on companies with lower than average price-to-book (P/B) ratios and price-to-earnings (P/E) ratios, but with a high dividend yields. These companies are considered undervalued from the market, giving an opportunity for value investors to profit by buying when the price is deflated. Furthermore, there are two more approaches that may bring the creation of a portfolio: “top-down” approach and “bottom-up” approach.

If the investment allocation process follows a “top-down” approach, the investor decides in the following order: in which asset class or market to invest (equity, bond, liquidity or other), then in which country the portfolio must be partitioned and at the end which are the securities to bet on. The basic idea of this strategy lies in the conviction that portfolio risk depends mostly on the market in which the investor wishes to operate and regard to country risk. The manager must find the best mix between geographic area and markets. In this strategy the manager is asked to make an assessment about the economic cycle of each country or geographic area analysing real and financial variable such as Gross Domestic Product (GDP), Consumer Price Index (ICP), Producer Price Index (PPI), unemployment rate, work force cost and so on. For every economic cycle trend the investor will decide which are the sectors to over-weight and which are the security with higher potentiality to increase.

If the investment allocation process follows a “bottom-up”, the manager will instead invest in companies that only follow the fundamental principles, with any concern for market condition, geographic area or macroeconomic factors. With this approach the investors assume that individual companies can perform better in an industry that is not performing very well. This approach is more tied with company’s products or services and its financial stability, is not important the geographic area where it is incorporated. The optimal asset allocation must follow the return maximization invested in securities that are considered undervalued whilst also, try to minimize the risk through a wide sectorial diversification.
1.3 “Core-Satellite” Strategy

Actually, there are combinations of active and passive portfolio management strategies. This combination is done to obtain a better return trying to put a small part of portfolio value in a riskier asset class.

If a manager assumes only an active strategy on a portfolio, he assumes a greater risk compared to a passive management and with more transaction costs.

A possible combination of these two strategies generates a new kind of portfolio with the objective to actively manage a risky portfolio, trying to limit the negative performance.

This strategy is called “Core – Satellite”.

The approach “Core – Satellite” consists to split the portfolio in two components:

- “Core”: is passively managed and its goal is to replicate the benchmark or the market index. This component usually is greater than 50% of overall portfolio value, because represent the core part invested in the portfolio, for example sovereign bonds;

- “Satellite”: represent the actively managed part of the portfolio and it shows the manager bet’s. Usually, the satellite shows a low correlation to the Core part, gaining a higher positive tracking error\(^8\). In this case the satellite will overperform the portfolio when the core part, instead, will underperform. It has a composition between 0% to 50% of overall portfolio and it can invest in more than one asset class or market.

The role of the “core” is to contain both risks and management costs. However, for the “satellite”, it is to provide a greater diversification and generate greater income compared to the benchmark.

\(^8\) It is a measure of how closely a portfolio follows the index to which it is benchmarked. It is computed across the difference of standard deviation between the portfolio and the index returns. It is considered as a measure of the deviation from the benchmark. The formula is: \(TE = \omega = \sqrt{Var(r_p - r_b)}\) where \(r_p - r_b\) is the difference between the portfolio return and the benchmark return.
The risk of this portfolio can be shown as the combination of the market risk (given by the “core”) and the active risk component (given by the “satellite”), generated from the manager.

If the benchmark, of the core part, is global it can represent the systematic risk. If the purpose of the manager is to reduce the systematic risk, he need to invest in assets where their beta\(^9\) is lesser than 1, it means less systematic risk than the market.

Active risk is tied to the investment research that produces positive alpha\(^{10}\) and is controlled by the portfolio manager.

In other words, core-satellite portfolio is a management method of an investment designed to minimize volatility, costs and tax liability where the “core” portfolio component refers to the beta of portfolio, instead the “satellite” portfolio component refers to the alpha coefficient.

An example of core-satellite approach could be the following:

- in the figure 1 we have a portfolio with a “core” component that invest in U.S. large-cap or Ex\(^{11}\) U.S. large-cap and three “satellite” components invest in U.S. small-cap, Emerging markets and Ex U.S. small-cap. This strategy permits to the investor to invest in the American stock market and a little on the Emerging markets, US Large-cap and Europe small cap.

---

\(^9\) Beta shows the systematic risk of a security or a portfolio compare to the market. It is used in the CAPM model (Capital Asset Price Model) to calculate the expected return of an asset. Beta is computed as the covariance between stock A and Market divided the variance of the market. A beta of 1 indicates that the security price will move with the market. A beta > 1 means that the security’s price will be more volatile than the market, instead a beta < 1 means the opposite.

\(^{10}\) Usually, alpha is considered as the active return on an investment. The excess returns of a fund relative to the performance of the benchmark index is the fund’s alpha. When an investor use alpha as a measurement of performance he assumes that the portfolio is sufficiently diversified so as to eliminate unsystematic risk. If the alpha has a value equal to 0 it would indicate that the portfolio is tracking perfectly with the benchmark index.

\(^{11}\) Not including
• In the figure 2 the “core” part invests on beta exposure to broad markets and the “satellite” has the strategy to invest in companies that produce positive alpha.

Figure 1

Figure 2

Usually, “core-satellite” strategy use Exchange Trade Fund\textsuperscript{12} (ETF) because permits to the manager to invest in a benchmark index or in a market index without buying the underlying assets.

\textsuperscript{12}ETF are particular funds that have the purpose to track a specific benchmark. They are negotiated on the stock exchange as a share.
Chapter 2: Characteristics and history of ETFs

In this chapter will be analyzed the structure of the ETFs, how they have been developed, how they are used in the financial sector and which are their pros and cons. There will be a short regulatory European framework analysis. Finally, will be analyzed different composition structures for an ETF, if they have a full replication index structure or they have a synthetic replication index structure constituted by derivatives.

2.1 ETPs Family

Over the last 20 years, Exchange-trade products (ETP) have become the most popular investment instrument for either individual and institutional investors. They are considered cheaper and better than mutual funds. ETP can offer diversification at lower cost than mutual funds, and they can provide at the same time arbitrage and trading opportunity for investors.

The constant rise of the ETP market after the dot-com crises has permit to a range of new products emerging. Taking in consideration that an investor is always interested in a unique exposition, risk and tax implications before invest his money, these products are allowed always more suitable because they gather to all these characteristics in a unique product.

We can split ETFs in two different constructions: Physical ETFs and swap-based ETFs. In addition, ETPs family is composed by ETFs, Exchange Traded Commodities (ETCs) and Exchange Traded Notes (ETNs). A physical ETF has the simple function to replicate physically the benchmark index buying its constituents and hold them for the all life of ETF. In some cases, the ETF issuer could decide to replicate the benchmark index only hold a subset of the underlying (this process is knowing as “optimization”). Figure 3 shows the ETPs family for each kind of asset class with the possible strategies.
Considering all kinds of ETF, the ETP industry is grown fast during last 12 years. In the 2003 there were only 220 products with an Asset Under Management (AUM) of only € 139 billion compared to € 2.742 billion at the 30th of October 2015 and with more of 6800 of ETF worldwide. The ETP industry increased rapidly with an annual rate of 28.60% from 2003 to October of 2015. The most important global issuer is the US market with almost € 2 trillion of AUM. If we look at the figure 4, we can see, from 2003 the ETP AUM is increased every year except in 2008 where is diminished by 3%. In 2015 we have seen a stabilization of the number of products descend a reduce momentum on investment after the China Crisis. However assets under management are still surging to new record highs. Anyway, the recently global monetary policies from most famous bank institutions surely have helped the increased of the AUM.

Figure 4: Global ETP

![Global ETP and ETF AUM in Euro](source: Bloomberg)
An important characteristic of ETFs is the comparison with stocks, in effect they can be traded as stocks on the market. Everyone can buy and sell ETF “shares” at any time when the market is open during trading hours. This gives to ETF products much more flexibility than mutual funds. It is possible to know the exact composition of an ETF at any time during regular trading hours. Furthermore, the so-called indicative net asset value (iNAV) for ETFs is constantly computed when the market is open. These characteristics makes ETFs one of the most transparent types of investment around the financial world. Institutional as well as private and retail investors have had a great interest in these products, where their scope is tracking benchmark indexes at a relative low costs.

However, ETFs are noticed by their transparency and some comprehensive due diligence procedures. Undertakings for Collective Investment in Transferable Securities Directives\textsuperscript{13} (UCITS) for the Europe and the improvement of the due diligence procedures from the ETF providers, has helped the expansion of ETF products. Nevertheless, there are several factors to see when an investor wants to use an ETF in its portfolio, they need to take into account underlying investment objective, universe of the investment, horizon and compare them with other ETFs according to their structure, performance, risk and tradability.

\textbf{2.2 Historical background}

In 1965 there was a famous Economist and Nobel prize winner Eugene Fama, that set the ground for the passive management with his study on market efficiency. He provided that all the information available are fully priced and in the long term is impossible to beat the market. He showed that the stock are traded always at their fair value.

\textsuperscript{13} UCITS are a set of European Union Directives that aim to allow collective investment schemes to operate freely throughout the EU on the basis of a single authorization from one-member state.
value and it is impossible for an investor purchase undervalued stocks and/or sell stock at their peak.

“It should be impossible to outperform the overall market through expert stock selection or market timing, and that the only way an investor can possibly obtain higher return is by purchasing”\textsuperscript{14}.

All this study is based on three different hypotheses:

- “Weak”: if we look at the price on the traded assets, they already incorporate all past publicly available information. In this case excess return can’t be realized on strategies based on analysing historical data or historical share prices. In this case is difficult build a trading strategy that provide a higher expected return compared to the market return.

- “Semi-Strong”: taking in consideration the first hypo E. Fama says else the continuous changing prices reflect the new public information. In this case all information available are already reflected on the share prices and it is difficult build a trading strategy that provide a higher expected return compared to the market return based on public information;

- “Strong”: in case market prices reflects information already incorporated in the historical prices series and all public information available and as well as private “hidden” information, he said that it is not possible build a trading strategy that provide a higher expected return compared to the market return based on any private information.

Finally, “strong” assumption take into account both “semi-strong” and “weak” assumptions, as well as “semi-strong” assumption incorporate “weak” assumption, but not vice versa. An investor on average, with a long run view, can reach a performance as the same as the market. The perfect and simple way is buying and holding a well-diversified basket of assets, trying to minimizing fees and taxes. The first company on all the world has tried to follow the Fama’s idea was Wells Fargo

Inc., an American bank. It adopted for the first time a passive strategy and in 1976 started with the first institutional index fund. Instead, the first index fund for retail investors was implemented in 1981 by Vanguard\textsuperscript{15}. Only in 1993 ETF began an available investment opportunity. Under the name of SPDR (referring to the name of “spider”), State Street Global Advisor has launched for the first time the first registered index-tracker, the objective is tracking the US-Stock Index S&P 500. Today the AUM of ETF SPY US is about $178 bln\textsuperscript{16}. The methodology of this ETF is the fully physical replication of the basket index.

2.3 Main factors in the European market

In Europe there are more than ten ETF issuers but Ishares is the largest ETF provider (AUM of 175 bn of euro) in terms of both market shares (48%) and number of products. The second largest European issuers we find db x-trackers with an AUM of EUR 44 bn, representing more than 12% market share. In third position we find Lyxor Asset Management with a market share of 11% and an AUM of EUR 38 bn at the end of 2014.

Form the first time that ETFs were introduced in Europe (2000), the total investment volume of these products has grown at a very fast rate. The ETFs industry in Europe has grown fast and in April 2015 it had 1.429 ETFs and assets for an amount of EUR 476 bn, from more than 30 providers on 23 exchanges. This must be compares to 1.053 ETFs and assets for an amount of EUR 229 bn at the end of 2010.

The majority of the funds are still issued in Luxemburg and Ireland, due to easier inception process. At the end of June 2014 Eur 95 bn were invested in synthetic products representing a $\frac{1}{4}$ of European AUM in ETFs, instead EUR 225 bn were

\textsuperscript{16} Refer to 31/10/15
invested in full / optimized replicating ETFs (almost 71% of all European AUM in ETFs, but globally reach a level of 91% AUM).

In figure 5 are shown the European market shares per ETF issuer in AUM. First of all, we can notice the greater ETF issuer in Europe is Ishares with a market share of 48,1%.

**Figure 5**: European market share per ETF issuer

![European market share per ETF issuer (AUM)](image)

Source: Deutsche Bank and Blackrock 2015

### 2.4 A brief regulatory framework

UCITS directives (Undertakings for Collective Investment in Transferable Securities) governs the regulation of ETFs investment funds such as every kind of investment fund. However, EU member are free to impose additional rules to asset managers with the intention to cover the interests of local asset managers.

ETFs are considered Index tracking UCITS funds with the objective to replicate the composition of an Eligible Index. An Eligible Index must be:
• “sufficiently diversified in relation to its invested underlying strategies (i.e. underlying holdings must not be overly correlated to one another);
• An adequately representative benchmark for the market which it refers to;
• Liquid and subject to regular rebalancing through a detailed re-balancing methodology;
• Calculated in an appropriate manner and published in the public domain;
• Managed independently from the Management Company of the UCITS fund.”

Diversification is important if an investor wants to reduce the risk, from the biggest pension schemes to individuals putting their saving into funds. UCITS funds are designed to be suitable to the retail investors, their rules build in certain levels of diversification with the aim of reducing their vulnerability to the performance of a small number of assets. Usually, in the financial market govern a rule: more different assets are hold in a fund, less will be the risk for an investor to lose a substantial portion of its portfolio in the case a particular asset falls in value.

In figure 6 are shown some constraints imposed by UCITS framework regarding the composition of every UCITS fund. For example, there is a limit of 10% of a fund’s net asset may be invested in securities from a single issuer, but it could rise to 20% in case the fund replicates a stock index or other index; however, with some exception this percentage could rise to 25% or 35% in the case of special supervision.

With the previous UCITS III Directive there was need for every UCITS funds to have a Simplified Prospectus where all important information about the investment had to be contained. Some information such as Total Expenses Ratio\textsuperscript{18} (“TER”), the performance history of UCITS fund and the turnover rate of the portfolio had to be

\textsuperscript{17} ESMA, (2012), ESMA’s guidelines on ETFs and other UTICS issues, Consultation Paper, Reference 2012/474.

\textsuperscript{18} TER is a measure of the total cost of a fund to the investor. These costs may include various fees such as purchase, redemption, auditing. It is computed by dividing the total annual cost by the fund’s total asset averaged over that year, and it is denoted as a percentage.
included on the Simplified Prospectus. At the end, EU Member State regulator had the assignment to review and approve this document. With the last directive UCITS IV introduced an obligation for UCITS funds to create a brief document containing key investor information (“KIID”). The KIID usually is no longer than two pages and provides information to investors in an easy-to-understand format. The consultation of the KIID for each investor must be via website or in paper from at their request.

**Figure 6: UCITS FRAMEWORK**

| “10% per issuer as limit for investments. This may be expanded to 20% in case of index funds.” | “In case of special supervision or guarantees up to 25%, 35% respectively, can be invested.” |
| “Total exposure to one issuer must not exceed 20%.” | “20% as maximum investment volume in bank deposit” |
| “Limits always need to be checked at group level.” | “Up to level of 35% in case shares held are composed of at least six different issues.” |
| “Acquisition of shares issued by one debtor are limited to 10% of voting rights.” | “25% as maximum investment in any other foreign or domestic fund” |

Source: Unicredit 2011

With the previous UCITS III Directive there was need for every UCITS funds to have a Simplified Prospectus where all important information about the investment had to be contained. Some information such as Total Expenses Ratio\(^\text{19}\) (“TER”), the performance history of UCITS fund and the turnover rate of the portfolio had to be

---

\(^\text{19}\) TER is a measure of the total cost of a fund to the investor. These costs may include various fees such as purchase, redemption, auditing. It is computed by dividing the total annual cost by the fund’s total asset averaged over that year, and it is denoted as a percentage.
included on the Simplified Prospectus. At the end, EU Member State regulator had the assignment to review and approve this document.

In a clear and non-misleading format, the following information must be included on the KIID prospectus:

- “The name of the UCITS fund;
- A brief description regarding investment policy and its objectives;
- Past performance presentation, costs and fees (trading costs and performance bonus);
- A clear risk scale profile showing the relating risk for that fund. Must be including in the prospect relating warnings in relation to the risk associated”\(^{20}\) (for example if there is a derivatives exposure or currency risk, ecc).

On 25 July 2013 the European Securities and Markets Authority (ESMA) issued an extensive set of guidelines regarding to UCITS compliant ETFs and relating products. These guidelines effectively made stricter the regulation that govern UCITS ETFs and UCITS fund generally, and therefore have introduced a number of new procedures and requirements for UCITS ETF providers.

The main point is that these requirements will not alter the use of ETFs. They will continue to be index-tracking products. The main changes for UCITS compliant will be:

- “UCITS that fall under the definition of UCITS ETFs will have to carry the identifier “UCITS ETF” in their name;
- UCITS ETFs will have to ensure appropriate redemption conditions for secondary market investors by opening the fund for direct redemptions when the liquidity in the secondary market is not satisfactory.
- UCITS entering into efficient portfolio management techniques (EPM) like securities lending activities will have to inform investors clearly about these activities and the related risks. All revenues net of operating costs generated

by these activities should be returned to the UCITS. When a UCITS enters into securities lending arrangements, it should be able at any time to recall any securities lent or terminate any agreement into which it has entered.

• UCITS receiving collateral to mitigate counterparty risk from OTC financial derivatives transactions or for EPM techniques should ensure that the collateral complies with very strict qualitative criteria and specific limits in relation to the diversification.

• UCITS investing in financial indices will have to ensure that investors are provided with the full calculation methodology of such financial indices, their composition and rebalancing frequency»21.

2.5 Index constructions

In this sub-chapter, firstly, it will be described what are ETFs, which are their objectives and it will explain the different composition methods used. Secondly, it will be analyzed a brief description of some ETF strategies more used.

2.5.1. General view

From a statistical point of view, an index, measures the changes of a specific amount of analyze data during a specific period of time. By the way, for a financial markets point of views an index is a description of an imaginary portfolio of assets describing a particular market or a portion of it. The calculation methodology is different from index to index and is an expression of base value. Thus, for statisticians and financial analysts is more important the percentage than actual numeric value. Stock and bond market indexes are used to construct index mutual funds and ETF whose portfolio mirror the components of the index.

All major index providers have developed their own criteria to analyze securities for inclusion in its broadest global market index. There are some technical differences but providers use also a number of common factors. For example, all indexes are market-cap-weighted and adjusted for free float. In addition, for each security that compose an index a predetermined liquidity requirement must be guaranteed (for example a minimum daily percentage of traded shares) and free-float doorsill to ensure the completely investability and accessibility of the indexes to all investors. Finally, providers may arrange stocks identified a particular sector differently (example mid-cap) when defining their broadest geographic coverage. For example, the S&P Global Broad Market Index (or S&P Global BMI) includes large-, mid-, and small-cap securities, while the FTSE All-World-Index includes only large- and mid-cap securities. Another example are emerging markets countries constituent between FTSE Emerging Index and MSCI Emerging Markets index, MSCI considers South Korea an Emerging Market but FTSE not. They are perfect examples that shows the different exposure for an investor on the same asset class and the same asset allocation. Different ETFs are built to reflected these indices in different ways. For several price indices one or two Total return indices are calculated and disseminated at the same frequency:

- the Gross Total Return: Is obtained by reinvesting in the index the ordinary gross dividends declared by the index constituents. In figure 7 is shown has the price Index plus dividends;

---

22 It is a stock market index weighted by the market capitalization of each stock in the index. In this scheme large companies have a large portion on its composition and small companies have a small portion and a small impact in the index.

23 This market capitalization methodology is used taking the equity’s price and multiplying it by the number of shares available in the market, instead of using the full-market capitalization method where it takes all shares outstanding.

24 The Total Return Indices is a kind of equity index that tracks the capital gains of a group of stocks and assumes that any cash distributions (for example dividends in case of equity or coupon in case of fixed income assets) are reinvested back into the index.
• The Net Total Return: is obtained by reinvesting the net dividend, which is equal to the ordinary gross dividend minus the amount of withholding tax. In Figure 7 is shown as the Gross Total Return minus the withholding taxes (for example in Italy the dividends are taxed by a 26%).

**Figure 7:** Market cap weighted index types

\[
\text{Price Index} + \text{Dividends} = \text{Gross Total Return} - \text{Withholding Taxes} = \text{Net Total Return}
\]

Usually ETFs are tracking Net Total Return such as Euro Stoxx 50 Net Total Return Index for Euro Stoxx 50 ETFs. In fact most ETfs are able to outperform this specific benchmark due to taxation advantages.

### 2.5.2 Some ETF strategies

Innovative tools based on alternative composition scheme instead than free float market capitalization are “Smart Beta” indices. These kinds of indices try to implement different strategies and are available for different regions and countries. In the same way of all other indices, “Smart Beta” Indices are set up on clear rules.

The new strategies used to capture return premiums from all asset classes are called “Smart Beta”. Usually, beta may be explained in a number of different ways, or as a measure of the risk of a portfolio or a stock relative to the market as a whole.

The scope of “Smart Beta” strategies search to escape the limitation of capital-weighting in portfolio-construction, they are viewed as more transparent than active management from investors.

Some authors compare “Smart Beta” such as enhanced indexing. This comparison could be reasonable because either strategies are used by systematic asset manager as lower-cost exposition in a particular and specific market, reducing the management fee costs and gaining much more net performance.
Anyway, this comparison is also misleading in the sense that these kind of strategies ignore market capitalization, instead the usual strategies such as passive and active are based on market capitalization indices.

We can divide smart beta strategies in two categories, on the one hand there are strategies with the goal to add return from one or more characteristics (for example dividends or cash flow, these strategies are called “Fundamental Weighted”), on the other hand there are strategies that try to concentrate on managing risk, trying to reach a lowest volatility in their portfolio or give a greater diversification.

During the last 20 years has been built Smart Beta Strategies using ETF. ETF “Smart Beta” can invests in high dividend stock or in company with low capitalization or in different style such as “growth” or “value”. Moreover, they can follow an alternative strategy investing on volatility Vix index.

Other “Smart Beta” strategies are strategies that try to follow a strategy called “minimum variance”, seeking to minimize the risk portfolio exposition. It tray to overweight the security of an index with low volatility and underweight the security with high volatility.

2.6 Different ETF replications strategies

First of all, there are two different replication strategies: Physical or Synthetic replication. As is shown in figure 8 Physical replication is based on two methods: Full replication or Sampling replication methods where it follows two different sampling methods: or representative sampling or optimized sampling. On the other part Synthetic replication is based on two models: Unfunded swap model and Fully funded swap model.

Usually, Synthetic replication is riskier for an investor, because the investor is more expose to the counterparty default risk and doesn’t know which are the constituent of the swap agreement, but this fact it is compensated by a lower TER requested to the investor.
The style of the replication is very important for an ETF because it may create an over-performance or an underperformance compared to the index, influencing the ETF tracking ability.

In this subchapter will be shown every kind of ETF replication, underlined which are the pros and cons of everyone.

**Figure 8**: ETF replications strategies

### 2.6.1 Physical replication

When an ETF strategy adopt the physical replication method it means that it has as constituent totally or partially the constituent of the tracking index. If an ETFs contain all elements of the benchmark index is said to be fully replicating. Instead, if the strategy has the objective to replicate the benchmark index using an optimization method is said to be partially replicating. There are some cases, in physically replicating ETF method, it is allowed the security lending and in this case the investor is exposed to a counter party risk (this is compensated by a reduction of fee). For
example, SPY US ETF and VOO US ETF are both ETF that track S&P500 Index using the method of fully replication, but on the first it is not allowed the security lending with a TER of 0.095%, instead in the second is allowed with a TER of only 0.045%. This reduction of the total expenses ratio on VOO US ETF compare to SPY US ETF is compensated by a greater risk deriving by security lending. As is shown in figure 9 this difference of the TER between two ETF is of 0.58% over a period of 5 years.

**Figure 9:** SPY US ETF and VOO US ETF vs SPX Index (SPTR total return)

Source: Bloomberg

### 2.6.1.1. Full replication

ETFs that use full replication strategy must invest in the component securities of the underlying benchmark index trying to replicate in the same proportion the benchmark index itself. This kind of replication has a particular consequence, because is necessary to provide every day a list of the ETF constituents. Another problem could arise from dividend distributions because ETFs distributes dividends each quarter or
semester, whereas on the benchmark dividend distribution are computed on a daily basis. There are some broad indices that are costly such as MSCI World or Barclays or JP Morgan indices (for government and corporate bonds respectively) where the constituents are sizable, and it leads a frequent rebalancing transaction costs. Last but not less important if there are illiquid constituents on the index, the spread between bid and ask could be wider than usual and in the case of moment of high volatility it may generate high tracking error in the ETF compare to its benchmarks.

In figure 10 is shown briefly how a full replication is built: an authorised Participant (typically a large financial institutions) submits an order for one or more “creation unites” (consist of a specified number of ETF shares, generally the range size is from 25.000 to 200.000 shares.). The ETF shares are delivered to the AP when the specified creation basket is transferred to the ETF. In the case of Full replication, the basket is composed by all index’s constituent.

**Figure 10**: full replicating ETF

2.6.1.2 Replication by sampling

In the same way, with the sampling replication the ETF bring some of the problems mentioned before. The objective of this strategy is buying only selected sample of the underlying index trying to replicate the index itself. This strategy could have some problems. It may overperformance the benchmark with a higher tracking error
compared to the full replication method. This fact is compensated by less trading costs deriving from less constituents incorporated on the ETF basket. Which securities are selected and which are omitted depend from the portfolio management strategy. Usually depend from geographical exposition, market capital weight, benchmark weighting, liquidity or industry. In figure 11 is shown a simple case of ETF sampling replication. The constituents of ETF are not all the index’s constituents for example stock A, B, C, D and E but it is composed only by stock A, B, C and E.

2.6.2 Synthetic replication

In addition to physical replication methods there is another method: Synthetic replication ("swap\textsuperscript{25}-based replication"). These kinds of derivative instruments have the objective to replicate the performance of the benchmark index. Firstly, we have to see if the index is a bond or stock index, the issuer holds a basket of bonds or shares and swaps its performance for the index. For example, if an ETF issuer wants to create a Synthetic ETF on FTSEMIB index without buying the underlying assets, he

\textsuperscript{25} A swap is a derivative in which two counterparties exchange cash flows of one party’s financial instrument for those of the other party’s financial instrument. The most know swap contract is the interest rate swap where the two parties agree to exchange the fix interest rate of a part with the variable interest rate of the other part and vice versa.
needs to have a swap agreement with another counterparty, where this agreement is based on an exchange performance over a substitute basket decided by the counterpart (usually, it has an high correlation with the Index return). However, there is a limit of 10% for every swap’s counterparty over the total volume of the ETF (this limit has been imposed by UCITS directive allowing a greater risk diversification). ETF that use this replication are riskier than physical replication for the following facts: it is not possible to know the basket of the other counterparty and the investor is exposed to the counterparty risk. For these reasons the International Monetary Fund (IMF), the Financial Stability Board (FSB) and the bank for International Settlements (BIS) are worried for synthetic replication and they are trying to build a new regulation trying to give more transparency for the investors and each fund can open more than one swap agreement with more counterparties. However, for synthetic European ETFs, UCITS regulation has imposed a limit of 10% exposure to a swap counterparty. In practice the assets or collateral that are holds by providers must be either near, greater or equal to than their fund’s NAV. It is common that provider interact multiple swap counterparty diversifying the risk exposure of the fund. The greater synthetic ETFs provider are: Lixor AM, dbx-trackers and Comstage.

There are two different methods about synthetic swap replication:

- The unfunded swap model
- The funded swap model

### 2.6.2.1. The Unfunded Swap Model

The unfunded swap model is built using cash from investors buying and holding the basket of securities from a swap counterparty. The swap counterparty promises to give the performance of the index (minus the fees if they are applicable) in exchange for the performance of the securities held by the fund. These securities constitute the “substitute basket” and often is not composed by the same securities of the index the ETF is tracking but, usually, it need to have a high correlation with the index. UCITS
regulation gives some constraints (asset type, liquidity and diversification) about which kind of securities can constitute the swap index. The ETF provider holds the securities in a segregated account at a custodian. The basic concept is shown in fig. 12.

**Figure 12:** Unfunded Swap Model

ETF providers, usually, engage multiple swap counterparties trying to minimize the exposition to any one swap counterparty. The difference between the ETF’s NAV and the value of the substitute basket shows the counterparty risk exposition and it may not exceed 10% of the fund’s NAV. The agreement of the swap is marked-to-market on a daily basis and every time that the counterparty exposure reach the limit of 10% it must be rest. In some cases the reset could be daily, depend from the agreement of the swap.

In the figure 13 is shown an example of a daily counterparty exposure. In the case the counterparty exposure exceeds the UCITS limitation of +/- 10% there will be a reset of the Swap agreement and a compensation from the other part.

Let’s assume at day 1 we have an initial investment of 100 and an index value equal to 100. The swap value is 0. The following day, we assume the index is increase by 5% but the substitute basket value didn’t change. The final swap value is equal to 6, given the difference by Index value minus the substitute basket value (106 - 100 = 6).

In this case the counterparty exposure is below the 10% (only 5.66%) and we don’t have adjustment between the parties. Instead, in day 4, we have a counterparty exposure greater than 10% (10.34%) and there is the swap resetting. In this case the swap counterparty must pay the difference value between two basket, because the index value is greater than the substitute basket value.
Figure 13: example of daily counterparty exposure

<table>
<thead>
<tr>
<th>Day</th>
<th>Index</th>
<th>Subst. Basket Value</th>
<th>Swap Value</th>
<th>ETF NAV</th>
<th>Counterparty Exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 1</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>100</td>
<td>0/100 = 0%</td>
</tr>
<tr>
<td>Day 2</td>
<td>106</td>
<td>100</td>
<td>6</td>
<td>106</td>
<td>6/106 = 5,66%</td>
</tr>
<tr>
<td>Day 3</td>
<td>111</td>
<td>108</td>
<td>3</td>
<td>111</td>
<td>3/111 = 2,70%</td>
</tr>
<tr>
<td>Day 4 before reset</td>
<td>116</td>
<td>104</td>
<td>12</td>
<td>116</td>
<td>12/116 = 10,34%</td>
</tr>
<tr>
<td>Day 4 after reset</td>
<td>116</td>
<td>116</td>
<td>0</td>
<td>116</td>
<td>0/116 = 0%</td>
</tr>
<tr>
<td>Day 5 Before Reset</td>
<td>103</td>
<td>115</td>
<td>-12</td>
<td>103</td>
<td>-12/103 = -11,65%</td>
</tr>
<tr>
<td>Day 5 after Reset</td>
<td>103</td>
<td>103</td>
<td>0</td>
<td>103</td>
<td>0/103 = 0%</td>
</tr>
</tbody>
</table>

Initial investment of 100, starting level of the index 100, swap value is 0.

The index increases where instead the basket remains flat: swap value is 6.

Both the basket and the index rise: the value of the Swap is 3.

Following UCITS rules, there is a limitation of the counterparty exposure of +/- 10%, in this case the swap is reset. Resetting to zero involves a payment of 12 from the swap counterparty to the ETF (reinvestment in the substitute basket).

In this case the value of the Swap falls below the limit of -10%, so the swap is reset. Resetting involves a payment of 11 from the ETF to the counterparty (securities from the substitute basket are sold).

2.6.2.2. The Funded Swap Model

In this case, the cash from the fund are invested into a single or a multiple swap counterparty/ies in exchange for the index performance and the counterparty posts collateral\(^\text{26}\) assets in a segregated account with a third party custodian. Compare to the unfunded swap model here the issuer basically directly gives to the counterparty the cash from its investors in exchange for the returns of the underlying index (figure

\(^{26}\) The collateral used, usually, are securities that come from the swap counterparty’s inventory (typically Organisation for Economic Co-operation and Development (OCED) equities, bonds, cash and funds) and is diversified in accordance with UCITS requirements.
In this case the swap underwriter (the counterparty) uses the cash to purchase a basket of securities, and posts this substitute basket as collateral in a separate account that is committed to the fund. Obviously, the collateral is the property of the fund. However, the custodian account can be opened by both ETF provider and the swap counterparty and the collateral is traded as the property of the fund. In the case of swap counterparty defaults the ETF provider should be able to gain access to the asset without prior approval and dispose of them.

In the case of positive swap counterparty exposition, the ETF provider requests to deliver additional collateral to the swap counterparty. This is required to ensure to maintain a level of collateralization and to have a net counterparty risk exposure to be zero or negative.

**Figure 14**: Funded swap model

![Diagram of funded swap model](image)
Chapter 3: Portfolio insurance strategies

In this chapter will be analyzed some of the most important Portfolio Insurance Strategies more known in the financial world. During the last 20 years, after the dotcom crises of 2001 and the subprime crises of 2007-09, investment managers have been more focused in strategies where it was possible to minimize the risk and maximizing the profit. These strategies have been developed with this objective, with the purpose to give to a particular investor classes, such as pension funds, insurance funds or individual investors (in particular, these strategies are suggested for investor with high risk aversion) new investment product with the goal to limit the downside risk and at the same time increase their wealth over time.

At the end of this chapter will be analyzed deeply a particular portfolio insurance strategy, the CPPI strategy with its variation, which will be the base of this research.

First of all, portfolio insurance is a financial product that has the objective to protect the portfolio, betting on the small risky part of it and limiting the losses.

It is based on the dynamic management characterized by a narrow set of strict and pre-defined trading rules, which define the portfolio adjustment during a period of time. It is asked to the manager to define ex-ante which are the trading rules and how frequently are the portfolio rebalancing.

Portfolio insurance was very important in the USA and some analysts said that it was one of the main causes of the crises of 1987, despite this strategy is still largely used by institutional investors. In Italy this investment technique has not still been largely used by financial operators, but during the last years and after the big financial American crises of 2008 some financial institution and financial advisor started to promote this kind of strategy with some variations.

There are some parameters on the portfolio insurance need to be followed:

- The underlying portfolio: it is the monetary amount in which the investor wants to invest, it could be a portfolio or a market index.
- The Time Horizon: it means the duration of the protection;
• The Benchmark;
• The minimum floor level;
• The objective of portfolio insurance.

There are four different family of strategies about insurance portfolio. They are:
• “Buy and Hold” Strategy;
• “Constant mix” strategy;
• “Option based portfolio insurance” strategy (OBPI);
• “Constant proportion” strategy and its variations.

3.1 Buy and hold Strategy

“Buy and Hold” strategy is characterized by an initial mix (for example 40% of bonds and 60% of equity) that remain fixed for the length of the investment. This kind of strategy grant a cover in the case of equity market crash with the bond exposition. The best performance of this strategy is obtained when the equity market overperformance the bond market and the worst performance is obtained in the opposite.

The payoff function of this strategy is:

\[ A_t = E_t + B_t = Q_0S_t + Q_0^B e^{rt} \]  \hspace{1cm} (1)

where

- \( A_t \): is the market value of my portfolio at time \( t \);
- \( E_t \): is the equity value invested at time \( t \);
- \( B_t \): is the bond value invested at time \( t \);
- \( Q_0 \): quantity of shares invested on the stock market;
- \( Q_0^B \): quantity of bond part invested on bond market;
- \( S_t \): value of equity market at time \( t \);
r: is the risk free rate;
t: is the time horizon of the strategy.
This kind of strategy is good for long term investment like 15-20 years. It is proved that the equity market overperformance the bond market over long periods and there aren’t multiple transaction costs.

3.2 Costant mix strategy

“Constant mix” strategy keep constant the total investment exposition on equity during the life of the investment. The manager objective of a constant mix strategy is to maintain unaltered the strategic mix\(^\text{27}\) in case of high market volatility that would tend a portfolio rebalancing. This would mean, to keep unchanged the risky part of the portfolio. If the value of the stock increase, the manager has to sell equity allowing the rebalance of the portfolio. Every time the value of stock changes the manager has to buy or sell the quote with the purpose to reach the desired mix.

Investor who like constant-mix strategies have tolerances for risk that vary proportionately with their wealth. They will hold stocks at all wealth levels.

Consider an individual investor who has put 70€ in stocks and 30€ in bonds and wishes to maintain in 70/30 constant mix. Now assume that the stock market declines by 5% (from 100 to 95). The stocks of the investor are now worth 63€, giving a total portfolio value of 93€. Now, the stock proportion is 63€/93€, or 67.74% (below to the desired 70%). To achieve the desired level, the portfolio must have 65,10€ in stocks that represent the 70% of 93€. Thus the investor must purchase 2,10€ (65,10€-63€) of stocks, obtaining the money by selling a comparable amount of bonds.

Following the same notations given before, in this case the portfolio quotes are constants and we have:

\(^{27}\) For example, if the initial proportion between risky and riskless assets is 40/60, this proportion will be maintain for all the life of the strategy.
\[ a_t = \frac{E_t}{A_t} = a_0 \in [0,1] \] (2)

\[ 1 - a_t = \frac{B_t}{A_t} = 1 - a_0 \] (3)

In this case \( a_t \) and \( a_0 \), that is the proportion, are always equal over time because they are fixed at the beginning and are constant.

From the previous equation we can compute the portfolio value

\[ A_t = E_t + B_t = a_0 A_t + (1-a_0)A_t = Q_t S_t + Q^B_t e^{rt} \] (4)

and the quantity of risky assets and bonds

\[ Q_t = \frac{a_0 A_t}{S_t} \] (5)

\[ Q^B_t = \frac{(1-a_0)A_t}{e^{rt}} \] (6)

The payoff function is given considering the total variation of \( A_t \), rather \( dA = [SdQ + e^{rt}dQ^B] + QdS + Q^B d e^{rt} \), and assuming that the strategies are auto financed\(^{28}\). This assumption impose the following constraint \( SdQ + e^{rt}dQ^B = 0 \)\(^{29}\) and \( dA = \frac{a_0A_t}{S_t} dS + \frac{(1-a_0)A_t}{e^{rt}r dt} \). A general solution of the previous equation gives the formula of the payoff function, that from a graphical point of view is a concave curve:

\[ A_t = k S_t a_0 e^{(1-a_0)rt} \] (7)

\(^{28}\) A portfolio is auto or self-financed if there is no withdrawal of money; the purchase of a new asset must be financed by the sale of old one or the other assets holds in the portfolio.

\(^{29}\) This constrain is valid because the sum of the two quantity must be 0. If there is a purchase of on part, on the other part is necessary have a sell for the same quantity.
The exposition function is linear and imply a constant risk aversion explained in the following formulas:

\[ E_t = a_0 A_t \]  

(8)

Moreover, the relationship between price and quantity, assuming for simplicity that \( r = 0 \), is

\[ Q_t = \frac{a_0 A_t}{S_t} = a_0 k s_t^{a_0 - 1} \]  

(9)

Deriving the previous formula (9), the price and the demand are inversely proportional: if the price increases the demand decreases and vice versa:

\[ \frac{dQ}{dS} = ka_0 (a_0 - 1)S_t^{a_0 - 2} < 0 \]  

(10)

In the case of fluctuating market this strategy permits to capitalize the gains. Instead, in the case where the direction of the market is constant, during the bear market, this strategy is not the best choice, because the investor could lose than the “buy and hold” strategy.

### 3.3 Option Based Portfolio Insurance (OBPI) strategy

On the “Option based portfolio insurance” strategy the investor must specify the time horizon of the investment and a floor to get at the end of the contract. These strategies imply a floor before the expiration date of the contract, that is computed as present value of the final floor discounted by the floor growth interest rate (usually is the risk free rate). For example, if the horizon is one year and the floor at year-end is 85€, then the floor at any prior time is the present value of 85€ discounted using the
riskless rate of interest. At a 6.25% bond rate, the initial floor is €80. The floor value increase at the riskless rate. After the floor is chosen and its present value calculated, the OBPI strategy consists of a set of rules implemented to bring the same payoff at the horizon as would a portfolio composed of bonds and call options. Now will be shown some computational methods, regarding how the floor in OBPI strategy is fixed and how the portfolio has been built.

Given $C(S,K,r,\sigma,t,T)$ Black-Scholes formula at the time $t$ of an European call option with strike price $K$ (it represent the present value of the floor) and end time $T$:

$$C = S_t N(d) - K e^{-rt} N(d - \sigma t^{0.5})$$ \hspace{1cm} (11)

where:

$$d = \frac{\ln S_t K + (r + 0.5 \sigma^2 t)}{\sigma t^{0.5}}$$ \hspace{1cm} (12)

and $N( )$ is the partition function of a normal distribution and $\sigma$ is the market volatility.

Option based portfolio insurance involves an initial investment $F_0$ in bond and the purchase of $n$ call options, where $n$ and $K^{30}$ are computed by the following equation system:

$$n * C(S,K,r,\sigma,O,T) = A_0 - F_0$$ \hspace{1cm} (13)

where $A_0$ is the market value of my portfolio at time 0, and

$$nK = F_T$$ \hspace{1cm} (14)

$^{30}$ $K$ is the strike price of the option.
where \( F_T \) is the floor at time \( T \), instead the floor at initial time \( 0 \) is its present value:

\[
F_0 = F_T e^{-rT} \tag{15}
\]

The equation 13 shows the total value of the call option need be purchased at the same value of initial cushion. The second shows that the strike price is equal to the final value of the floor. For this reason, there is always enough money for exercising the call options.

At every time \( t \) between \( 0 \) and \( T \) the payoff function is the following

\[
A_t = F_t + n \ast C(S,K,r,\sigma,t,T) \tag{16}
\]

while the floor value is

\[
F_t = F_T e^{-r(T-t)} \tag{17}
\]

At the maturity the portfolio value contained the option based portfolio insurance strategy will be:

\[
A_T = F_T + n \ast \max (S_T - K, 0) \tag{18}
\]

The exposure will be instead:

\[
E_t = n \ast N(d) \tag{19}
\]

where \( d \) may be expressed as function of the cushion \( A_t - F_t \) solving the equation by \( S_t \) we have

\[
C(S,K,r,\sigma,t,T) = \frac{(A_t - F_t)}{n} \tag{20}
\]
On OBPI strategies, for every positive cushion, the equity portfolio component increase as time goes by, up to 100% at the expiration time.

In the following figure 15 is shown a simple payoff function (red line) of European call option in the OBPI Strategy. The cushion has been fixed at 10 and it is the value of the European call option. The blue line represents the underlying asset. In the case the underlying asset fall down more than 10% (consider the value of the asset at time 0 is 100), the investor will lose no more than the cost of the option, but in the case the underlying asset increase by 20% the investor will gain 10 (100*0.20 – 10 the cost of the option).

Figure 15: payoff functions of OBPI

We can create another option based portfolio insurance using, instead the European call option, the European put option.

This kind of strategy was called the synthetic put approach of Rubinstein and Leland (1981). It consists to hold a risky asset, for example stocks, and buy one at-the-money put option on that asset. The resulting portfolio value will be not less than the exercise price net of the premium at expiration. It is a way to hedge the portfolio.
against the downside risk. We need to remember that the actual option is more expensive than synthetic option, because the implied volatility is higher than the historical one in option markets. This kind of strategy is based on the assumption of option pricing theory and it requires the volatility estimation of the asset whose values has to be secured.

3.4 The Costant Proportion Portfolio Insurance (CPPI) strategy

From recent studies on family portfolio insurance strategies, there is a particular strategy that has got a particular popularity in the asset management industry and it is the constant proportion portfolio insurance (CPPI) strategy. This strategy permits to provide a capital guarantee and, for a small part, it allows a dynamic trading strategy gaining from a participation on a certain underlying asset.

This strategy is very simple to implement because it doesn’t require any specific assumption and estimation of any specific input parameters. However, there is a negative characteristic of this strategy, because it suffers from a low participation rate in upward market movements compare to the other insurance strategies, because have difficulties to overweight the upward movements and holds possible gains.

At the beginning, the CPPI was introduced by Perold and Sharpe (1986)\textsuperscript{31} for fixed income securities and Black and Jones (1987)\textsuperscript{32} for equity instruments and during recent years has been implemented some variations of this strategy.

Some research reveals that investor are less sensitive to gains than losses (loss aversion) and they tend to overweight extreme events with a small probability of that they may happen. Some advisors suggest to limit the downside potential risk in their investment strategies and trying, at the same time, preserve the upside gain.


This strategy only requires the investor to select a multiplier, which is represent its risk aversion (a higher multiple mean that the investor has a low risk aversion) and a floor below which he does not want the portfolio value to fall.

At most, the CPPI procedure dynamically allocates a portion of the total investment in a risky asset as a multiple of the cushion, for example, may be the difference between the desired protection and the current wealth. This system can have the same effect of owning a put option. Following this strategy, the total exposition of the risky part of the portfolio tends to zero as the cushion approaches zero. If the cushion is zero, the portfolio is fully invested in cash or, in our case of core-satellite approach, is invested fully in core part. This system, in theory, hold the guarantee to not exceed the cushion (the maximum amount of risk part). The CPPI strategy ensure to the portfolio do not drop below the floor level. In the rare case that it touches the floor, the fund doesn’t exist more, the only performance that remain is from the core component of the portfolio.

This method is easier to understand with an illustration (figure 16). Let’s suppose a portfolio manager try to safeguard an investor a floor equal to 85% of an initial investment, for our specific and simple case is equal to 100m and the cushion is given by the difference from portfolio value and the floor, that is equal to 15 at time T=0. Let’s assume, for simplicity, also the risk free rate is equal to 0. I showed two different cases with two different multiples (the strategy require that the multiple must be greater than 1), on the first is equal to 5 and on the second is equal to 2.

In the first case the risky part at time 0 is equal to 75 (15 X 5, 15 is the value of the cushion at time T=0 and 5 is the multiple chosen for the first case), which leaves 100 – 75 = 25 in cash. Let’s assume that at time T=1 the value of the risky part is increased by 5%, and the cushion now is equal to 18,75. In this case the part invested in the risky asset has increased as well at a new value of 93,75 leaving only 10 on riskless part (cash).

Assume that, at time T=2, the value of the risky asset has declined by -4.76% with a new value of 89,29 from 93,75. The total portfolio value now is equal to 99,29 and
the cushion is now 14.29 compared to 18.75 at time T=1. Moreover, the position in the risky asset is equal to 71.43 given by the cushion (14.29) multiple by 5, which leaves 27.86 in cash. At time T=3 the new value of the risky asset has increased again by another 5% but the final portfolio value is not equal to the value at time T=1, this fact is given by the multiple factor. Bigger the multiple is, the greater the erosion effect on the performance is large in case of stationary market trend (compound effect). For example, with a multiple of 5 the difference of our portfolio value between T=3 and T=1 is 0.94, instead with a multiple of 2 the portfolio value has a difference of only 0.08.

**Figure 16:** two examples of CPPI strategy with different multiple

<table>
<thead>
<tr>
<th>Time</th>
<th>T0</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market index</td>
<td>100</td>
<td>103.8</td>
<td>101.5</td>
<td>101.42</td>
</tr>
<tr>
<td>Portfolio value</td>
<td>100</td>
<td>101.5</td>
<td>99.93</td>
<td>101.42</td>
</tr>
<tr>
<td>Risky asset</td>
<td>75</td>
<td>78.75</td>
<td>93.75</td>
<td>71.43</td>
</tr>
<tr>
<td>Cash</td>
<td>25</td>
<td>25</td>
<td>10</td>
<td>27.86</td>
</tr>
<tr>
<td>Floor</td>
<td>85</td>
<td>85</td>
<td>85.00</td>
<td>85.00</td>
</tr>
<tr>
<td>Cushion</td>
<td>15</td>
<td>18.75</td>
<td>14.29</td>
<td>17.86</td>
</tr>
<tr>
<td>Max exposure</td>
<td>75</td>
<td>93.75</td>
<td>71.43</td>
<td>89.29</td>
</tr>
</tbody>
</table>

However, we should to take into account the transaction cost effects after each operation, these costs have a negative effect in the performance in our portfolio.

If we compare this strategy with both “buy and hold” and “constant mix” strategy, we can notice that: in the case the multiplier is equal to 1 there is a strict correspondence between CPPI and buy and hold (the protection in this case coincide with the riskless portfolio quota); in the case the floor is 0 and 0 < m < 1 we are in the case of constant...
mix strategy where the multiple is the percentage invested in stock while the residual is invested in the riskless part.

CPPI strategy for some particular underlying asset is not the best one to apply, because on the long run and in the case of stationary trend there will be an erosion of the performance.

Some researcher has studied new implementation and improvement of CPPI model, trying to keep the upside capture and downside protection.

In this case we will discuss the new mechanism of both Variable Proportion Portfolio Insurance and Exponential Proportion Portfolio Insurance.

### 3.4.1. The Variable Proportion Portfolio Insurance (VPPI) strategy

Some authors suggest to choose, before conducting a dynamic portfolio insurance, a discipline of rebalance. If there are too many rebalance during the life of the investment the performance may be eroded by transaction costs. Etzioni\(^{33}\) proposed three different methods of rebalancing: lag discipline, time discipline and market move discipline. In our case we will use the market move discipline. A rebalance will be trigger every time when \( \frac{|S_{t+1} - S_t|}{S_t} \geq \alpha \), where \( \alpha \) is the threshold (a specified percentage, \( \frac{|E_{t+1} - S_{t+1}|}{S_t} < \alpha \) there isn’t a rebalance) set by the investor and \( S_t \) is the price of our stock at time \( t \).

A particular statement is given by the following relationship\(^{34}\):

\[
\frac{\partial A}{\partial S} = n_0 
\]  


\(^{34}\) Huai-I L, Min-Hsien Chiang and Hsinian Hsu, (2008), A new choice of dynamic asset management: the variable proportion portfolio insurance, Applied economics, n. 40
which describe the ratio of the instantaneous changes on the value of asset prices over the instantaneous changes of stock prices represents the holding shares at time 0. Recalling that the main purpose of portfolio insurance is the downside protection and to provide upside capture, the holding shares need to increase as the stock prices goes up. In this way as the stock prices goes up, the payoff will increase more rapidly than the stock price if the holding shares are increasing ($dn > 0$); instead, in the case shares holding remain constant (unchanged) the payoff will increase proportionally to the stock prices, in this case $dn = 0$; finally, in the worst case, the payoff may decrease if the holding shares are declining, in this case $dn < 0$. If we need to protect from downside risks, we need to decrease the holding shares as stock price goes down and if the holding shares decrease as stock price goes down our payoff will decrease slowly than stock price decline. For this reason, the dynamic portfolio insurance must be set trying to capture the upside and to limit the downside movements.

In this situation, where the value of the risky assets changes as times changes, we need to understand which are the factor that could affect the changes in holding shares.

Before to go head we need to define some variables:

- $n_0$ is the number of stock holds at time $T = 0$;
- $S_0$ is the value of stock at time $T = 0$;
- $m_0$ is the value of multiple at time $T = 0$;
- $U_0$ is the value of the cushion at time $T = 0$;

35 The upside capture in finance is defined as the manager capability to capture the upside movement in the market. Usually is defined as Up-market capture Ratio that is the statistical measure of an investment manager’s overall performance in up-markets. It is usefull to evaluate how well an investment manager performed relative to an index in periods when that index has risen.

36 In finance the downside risk is an estimation of security’s potential to suffer a decline in value if the market conditions change. This estimation explains how much the investor stands to lose. There are several technic and fundamental metrics used by traders and analysts to estimate the likelihood that an investment’s value will decline (for example standard deviation, historical performance, etc.)
Let's assume a portfolio with initial value $A_0$ at time $t = 0$ and $F_0$ is the fixed proportion of the portfolio set as a floor and the remainder is $U_0$, the cushion. Moreover, we assume that the portfolio is composed by a risky asset and a riskless asset. The total value of the portfolio at time $t = 0$ is given by the sum of the risky asset ($E_0$) and the riskless asset ($B_0$),

$$A_0 = E_0 + B_0 \quad (22)$$

As we have explained before the relationship between $E_0$ and $U_0$ is:

$$E_0 = m_0 \times U_0 \quad (23)$$

where $m$ is the multiple. Let's assume that the risk asset is a stock, we have

$$E_0 = S_0 \times n_0 \quad (24)$$

Where $S_0$ is the stock price at time $t = 0$, and $n_0$ is the number of shares at time $t = 0$. If we substitute equation 24 into equation 22 we obtain:

$$A_0 = S_0 \times n_0 + B_0 \quad (25)$$

The mechanism of dynamic portfolio insurance strategy is to increase the shares hold in the portfolio when the stock prices goes up and diminish the shares hold in the portfolio when the stocks goes down.
As time changes, the value of risky assets change as well. If we differentiate the equation 23 we have the change in value of the risky assets and this change can be achieved by the change in stock price and adjusting the number of shares.

Summarizing, we differentiate the change of value of risky asset (given by risky asset part $R_3$ times the multiplier $m_3$) and the differential of value of risky part is given again as the change in stock price ($dS$) and the adjustment of the number of shares ($dn$) given by the equation 24. Thus, we can obtain the following expression:

$$ n_0 dS + S_0 dn = m_0 dU + U_0 dm $$(26)

we must take into account the condition of changes of the floor is fixed and equal to zero ($dF=0$).

In the following step, we need to remember how the portfolio has been built:

$$ A_0 = U_0 + F_0 $$ (27)

where $A_0$ is the initial value of portfolio at time $t=0$, $F_0$ is our fixed floor proportion and $U_0$ is our cushion. This equation indicates that $dU = dA$ and if we analyze the changes of total assets before conducting a rebalance, where $dn=0$, we have:

$$ dA = n_0 dS + dB = dU $$ (28)

if we substitute last equation into the previous one, we can write the changes of the holding shares as:

$$ dn = \frac{(m - 1)n_0 dS}{S_0} + \frac{m_0 dB}{S_0} + \frac{U_0 dm}{S_0} $$ (29)

Last equation shows that $dn$ which represent the changes of holding shares is given by three terms: stock price changes ($dS$), safe asset changes ($dB$) and from the multiple changes ($dm$).

It is important, now, to analyze the impact of multiple $m$ in three cases:
1. Case 1: \( m=1 \)
   In this case \( dm=0 \) because the multiple is constant during all time the investment (equal to 1) and the change in number of holding shares is (computed by eq. 29):
   \[
dn = \frac{dB}{S_0}
   \tag{30}
   \]
   The value of \( dB \) is always positive because it is a function of the interest rate and time and for this reason \( dn \) is always positive.

2. Case 2: \( m \neq 1 \) and \( m \) is constant
   In this second case \( dm \) is always equal to 0 and equation 29 will be:
   \[
dn = \frac{(m - 1)n_0 dS}{S_0} + \frac{m_0 dB}{S_0}
   \tag{31}
   \]
   The change in holding shares in this case is given by the change value of the bond asset plus the change of the risky asset. However, it is important to analyze the effect of \( m \) when it is greater and lesser than 1.
   \begin{itemize}
   \item \( m>1 \)
     it has the same characteristic of the CPPI strategy. In the case of an increase of stock prices \((dS>0)\), we need to buy more shares, \( dn>0 \). Whereas, in the case of a decrease in stock prices \((dS<0)\), the value of the first term is negative and its contribute to diminish the number of holding shares.
   \item \( m<1 \)
     in this second case, the strategy has the same characteristic of a constant-mix strategy, where there is an inverse consequence that results in a “buy low, sell high” strategy.
   \end{itemize}

3. Case 3: \( m>1 \) and \( m \) is dynamic
We assume that our cushion is set always positive and then $U_0 > 0$. In the case of dynamic multiple ($dm \neq 0$), the number of holding shares is given by three factors: the change of multiple ($dm$), the stock price change’s ($dS$) and from the value of the asset change’s ($dB$). This case satisfies the conditions of portfolio insurance with a $dm>0$ when there is an increase of stock prices. Similarly, $dm<0$ is the other condition for portfolio insurance when there is a decrease of stock prices.

In conclusion, if we compare CPPI strategy with the VPPI strategy and we look the differences between equation 29 and equation 31, in the last strategy there is a greater upside capture when stock price goes up (given by extra-term $\frac{U_0 dm}{S_0}$). Whereas, when there is a decrease in stock prices the decreased multiple can produce more downside protection.

3.4.2 The Exponential Proportion Portfolio Insurance (EPPI) strategy

A new variation of CPPI strategy was conducted in a research by Huai-I. Lee, Min-Hsien Chiang and Hsinan Hsu. They proposed a new constraint for the multiple in the VPPI model, called EPPI.

We need to consider some assumptions in this strategy: self-financing must be satisfied and an initial floor must be fixed.

The multiple is built in the following way:

$$m = \eta + e^{\alpha ln(S_1/S_0)} \quad (32)$$

where $\eta > 1$ is supposed to be an arbitrary constant, and the exponential term is the dynamic multiple adjustment factor (DMAF), where $a > 1$ is a parameter acting to amplify the effect of DMAF. If $a$ is greater than 1 the effect on the number holding shares in the situation when stock price goes up will be amplify, and there will be a shrink effect when the stock price goes down.
It is important to set the reference price in advance because the EPPI strategy uses the stock price before the rebalance as the reference price.

We will analyze the effect on the change in the number of holding shares in this strategy as we have done with VPPI strategy.

If we compute the first derivative of \( m \) with respect to \( S \) in previously equation 32, we can obtain the multiple change’s (\( dm \)) as follows:

\[
\frac{dm}{S} = \frac{a}{S} e^{\alpha ln(S_1/S_0)} dS
\] (33)

If we substitute equation 33 into equation 29 we obtain:

\[
\frac{dn}{S} = \left[ \eta + e^{\alpha ln(S_1/S_0)} \right] \frac{dB}{S_0} + [(\eta - 1)\eta_0 + (\eta_0 + U_0 a) e^{\alpha ln(S_1/S_0)}] \frac{dS}{S_0}
\] (34)

The equation 31 shows the change in the number of holding shares under the EPPI. Every time the stock price goes up there is a rebalance (\( dS > 0 \)). By definition dB is always positive because it is a function of time and interest rare and \( \eta \) is greater than 1. Then, the first term on the right side in the equation 34 is always positive. As long as \( \eta_0 > 0, S_1 > S_0, a > 1 \) and \( U_0 > 0 \), the second term of the right part of equation 31 is positive. Moreover, in the case of an increase of stock prices the, EPPI works trying to capture a greater upside movement compared to the other two strategies.

In the case of a decreasing of stock prices (\( dS < 0 \)), the first term of right hand side of equation 34 is always positive because the exponential term of equation 33 is a number between 0 and 1\(^{37} \). However, on the second term of the right-hand side of equation 33 is negative.

In conclusion, EPPI strategy could create a convex nature portfolio insurance strategy, because when there is an increase of stock price, the portfolio increase exponentially the holding shares to perform an upside capture; whereas, when stock price goes down, DMAF mechanism works to give a more downside protection.

\(^{37} \) Since \( dS < 0 \) and \( a > 1 \), we have \( 0 < e^{\alpha ln(S_1/S_0)} < 1 \).
Chapter 4: Empirical analysis

In this chapter will be applied and analyzed a Dynamic Core-Satellite Portfolio with the EPPI method. Core-satellite Portfolio is a particular strategy where the manager defines which is the Core component (usually is the benchmark), and the Satellite component which it is the manager’s bet in the portfolio, usually this part is riskier than the Core part and it isn’t greater than the 50% of the overall portfolio value. As a Core and Satellite components I choose to use ETFs instruments.

This strategy will be dynamic because I assume that every month there will be a new allocation in both Core and Satellite components, which depend from the performances of the respective assets. In other words, although the weights allocated to the core and to the satellite can be static (for example in the the case of a “Buy and Hold” portfolio strategy where the weights are hold equal for all time of the investment), the proportion invested in the performance-seeking portfolio (in this case the satellite component) can also fluctuate as a function of the current cumulative outperformance of the overall portfolio, thus making the approach dynamic.

Originally, the dynamic core-satellite (DCS) concept has been built on constant proportion portfolio insurance. This principle allows the production of option-like position through systematic trading rules. In other words, the CPPI dynamically allocates assets to a risky asset in proportion to a multiple of a cushion defined as the difference between the current portfolio value and the desired protective floor. If the difference between the portfolio value and the protective floor increase, increase as well the value of the cushion, and finally the value allocated in the risky asset will be greater. Instead, if the difference between the current portfolio value and the desired protective floor decrease, there will be an opposite effect. This effect is similar of owning a put written on the same underlying asset. In this strategy, the exposition of the portfolio tends to zero as the cushion approach to zero. If the cushion is zero, the portfolio is fully invested in cash.
Usually, Core-Satellite portfolios are built by putting assets that are supposed to outperform the core in the satellite. This, usually is given if there is no strict positive correlation between the two components. On the other side, if there is a temporary worsening of the economy, the satellite may in fact underperform the core. The DCS approach makes it possible to diminish the impact of the satellite on the performance during periods of relative underperformance, while it tries to maximize the benefits in periods of outperformance.

Now, the idea is to apply the EPPI approach instead the CPPI into the DCS portfolio strategy, with the intent to capture more upside movements and undervalued downside satellite movements. The particularity of the EPPI is its multiple, where it is composed by two parts as we have defined before in the formula 32: a fixed part \( \eta \) and a variable part where \( \alpha \) is a parameter with the intent to amplifying the monthly performance of the satellite components.

The most important part of the DCS management is setting the floor, ensuring an asymmetric risk management of the overall portfolio. The cushion will increase only if increase as well the difference between the floor and the total portfolio value, giving a more allocation in the risky satellite. Instead, if the cushion decrease, investment in the satellite becomes smaller.

In the previous explanation of how CPPI and EPPI have been built, the floor is a constant fraction of the benchmark value or the Core component in our case, in other words:

\[
F_t = kB_t
\]

By the way, different floors might be used to improve the benefits from the core-satellite management.

The idea now is to impose some constraints to our DCS portfolio to limit the overall risk of the portfolio. The extensions are the following:

- A Benchmark protection floor: it has the purpose to protect k% of the value of any given benchmark (see the equation 35). A benchmark in asset
management is represent as any predetermine target that the asset management must to follow (for example a stock or bond index). In this study I chose a Benchmark protection floor equal to 90% of the portfolio value. Lower this percentage is and higher will be the part allocated in the risky satellite and obviously higher will be the overall risk of the DCS portfolio. For example, if the protection floor is equal to the 80%, the initial cushion will be greater and there will be a greater risk component allocated in the portfolio. If we allocate a greater number of riskier assets in the portfolio, the overall standard deviation will be greater and, obviously, the portfolio will be riskier.

- A maximum drawdown floor: in finance drawdown is explained as a financial measure of risk of an asset, it is the peak-to-trough decline in a specific historical period of time of a specific investment and it is showed as a percentage from the peak and the trough. Firstly, I imposed a maximum drawdown floor for the Core component (not more than 10%), which it represents our benchmark index. Secondly, I imposed another “drawdown constraint” that requires the overall DCS portfolio must not exceed at all time a 10% of its value. In the case the limit is reached, the portfolio is fully invested in the riskless asset (Core component) for the rest of its life;

- Trailing performance floor: this floor prevents a portfolio from negative performance over a 12-month trailing horizon. In any event:

\[ F_t = P_{t-12} \quad (36) \]

where \( P_{t-12} \) is the value of the portfolio 12 months earlier. The idea is to hold the performance gained 12 months earlier by the portfolio;

- A limit of the allocation in the satellite component: the limit of the maximum allocation in the satellite during all life of the investment it has been fixed at 50% of the value of the portfolio. If after the new allocation the satellite
component exceed the overall portfolio value of 50% there will be a new reallocation to permit to maintain this constraint.

All these constraints will be imposed in our DCS portfolio strategy, with the purpose to limit the overall risk and trying to improve its performance compared to the benchmark.

In this case I used as a benchmark and, obviously, as a Core component a medium-term maturity government bond ETF (Ishares euro gov bnd 3-5Y). It is composed by 3-5 years Euro-bond issued by Eurozone countries. This ETF is fully invested and it doesn’t contain derivatives, it is a UCITS fund and it has an AUM greater than 1,5 billion of euro. Instead, as Satellite component I replicate the strategy four times using four different risky ETFs with different correlation with the Core component as is shown in figure 12. The four risky ETF are the following:

- SPDR S&P500 ETF is the most known ETF. It is an ETF that has as benchmark the S&P500 stock index, and the index contain the 500 largest US companies;
- LYXOR ETF DAX is an ETF that has the purpose to track the performance of the DAX Stock index;
- ISHARES USD Treasury bnd 7-10Y is an ETF that seeks to track the investment results of an index composed of U.S. Treasury bonds with remaining maturities between 7 and 10 years (intermediate maturity);
- GOLD BULLION SECURITIES LTD is an ETF designed to offer for the investors access to the gold market by providing a return equivalent to the movements in the gold spot price.

Every ETF chosen is physical invested (there are no derivatives), have an AUM not lower than 1 billion to guarantee the liquidity of the security and dividend or coupon are reinvested in the asset. All ETF prices are expressed in EURO currency.

---

38 The spot price is the current price at which a particular security can be bought or sold at a specified time and place.
The data used consists of monthly log returns for the period from 31st October 2007 to 31st March 2016. The starting period is chosen trying to include the last economy recession in our data. The overall length of our portfolios is of 8.5 years. I decided to use monthly returns allowing less portfolio rebalancing, a lower volatility and a lower impact of transaction costs.

Figure 17: correlation and max drawdown between assets

<table>
<thead>
<tr>
<th>Correlation with European Gov bond 3-5Y</th>
<th>European Gov bond 3-5Y</th>
<th>SPDR S&amp;P500</th>
<th>Dax ETF</th>
<th>Isares Usd Treasury bond 7-10Y</th>
<th>Gold ETF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Drawdown</td>
<td>-5.26%</td>
<td>-45.62%</td>
<td>-52.52%</td>
<td>-18.75%</td>
<td>-37.04%</td>
</tr>
</tbody>
</table>

In the figure 17 are shown the correlations between the riskless asset and risky assets. We can notice there is a negative correlation with the two ETFs that have a Stock index as a benchmark, this reason is proved by the fact that usually there is a negative correlation between bonds and stocks. Instead, the other two assets have a positive correlation with the Core component. Obviously, I expect that two bond indexes of two different developed countries, with same monetary policies and same financial condition have a positive correlation. In our case we have Europe sovereign bonds and US sovereign bonds, they are both developed countries and during the last decade they have had the same monetary policy applied by their central bank. Finally, the correlation with the commodities is positive with a value of 0.1675.

In the second raw of the figure 17 is shown the max drawdown reached during the all period of the index. Firstly, the core component to be eligible must have a max drawdown not below the level of -10% and in our case the ETF Eurozone 3-5 years have had a max drawdown of -5.26%. We didn’t not impose constraint in the satellite component and we can notice that both two stock index ETFs have had a huge drawdown greater than -45% in each two cases. This fact can be explained because I have taken in consideration the big world recession between 2007 and 2009.
In all this study I assumed that all prices are log normal distributed\textsuperscript{39}.

However, the idea of this study is to build a portfolio less risky for the investor, applying the knowledge of the EPPI strategy in a dynamic portfolio. To evaluate the performance of these news portfolios I have compared it with the strategy of “buy and hold” and I used some financial indicators such as Sharpe ratio, the Information ratio, the Sortino ratio and the Omega ratio to compare the risk and performance between the portfolio simulations.

Sharpe ratio is defined as a measure to compute risk-adjusted return. It was developed by Nobel prize winner William F. Sharpe. The ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk. If, from the mean return we subtract the risk-free rate, we can isolate the performance associate with risk-taking activities. Obviously, a portfolio invested in “zero risk” assets, such as the purchase of U.S. treasury bills, has a Sharpe ratio of zero. Normally, greater is the value of the Sharpe ratio, more attractive will be the risk-adjusted return. The simple formula is the following (equation 37):

\[
\text{Sharpe ratio} = \frac{\bar{r}_p - r_f}{\sigma_p}
\]

where:

- $\bar{r}_p$ is the average return of the portfolio or its expected return;
- $r_f$ is the risk-free rate;
- $\sigma_p$ is the deviation standard of the portfolio.

\textsuperscript{39} Assuming that prices are distributed log normally, the log(1+r) is conveniently normally distributed, because $1 + r_i = \frac{p_i}{p_j} = \exp \left\{ \log \frac{p_i}{p_j} \right\}$. This is handy given much of classic statistics presumes normality. In this case the sum of normally-distributed variables is normal, which is useful when we recall the following logarithmic identity: $\log(1 + r_i) = \log \left( \frac{p_i}{p_j} \right) = \log(p_i) - \log(p_j)$. Thus, compounding returns are normally distributed. There is another benefit: $\sum_i \log(1 + r_i) = \log(p_n) - \log(p_0)$. It describes that the compound return over n periods is merely the difference in log between intial and final periods.
The information ratio (IR) is widely used from investors to evaluate the ability of the manager to generate excess return relative to its benchmark. In other words, the IR is a ratio of portfolio returns above the returns of a benchmark to the volatility of those returns. It is useful to understand if the manager has had the capacity to beat the benchmark or not, and which has been this capacity. The IR is represented by the following formula (equation 38):

$$\text{Information ratio} = \frac{E[R_p - R_b]}{\sqrt{\text{var}[R_p - R_b]}}$$  \hspace{1cm} (38)

where:
- $R_p$ is the portfolio return;
- $R_b$ is the benchmark return.

A high ratio is important because explain that the manager can achieve higher returns more efficiently than one with a low ratio by taking addiction risk. A value of IR greater than 0 show the capacity for the manager to capture more performance, but a value greater than 0.5 shows the 75 percentile of portfolio managers to produce excess performance compared to its benchmark.

Sortino ratio measures the risk-adjusted return of an investment asset, strategy or a portfolio. It has been built as a modification of Sharpe ratio but it penalizes only those returns that fall below a user-specified target or required rate of return defined as MAR (it can be the risk-free rate, an index return or zero). This is because a fund can generate performance that may be higher than the average and if we include these positive returns will make the standard deviation higher. However, investors are more concentrated about the downside returns as these are associated with losses. Normally, the Sortino ratio is used as a way to compare the risk-adjusted performance differing risk and return profiles. In detail, risk-adjusted returns seek to normalize the risk across investments and then see which has the higher return unit per risk. As for the Sharpe ratio, a large Sortino ratio indicates a better risk-adjusted return. The ratio is the following:
Sortino ratio = \frac{r_p - MAR}{DR} \tag{39}

where:
- \( r_p \) is the portfolio return;
- MAR is the minimum acceptable return;
- DR is the downside risk (the target semi-deviation).

The DR is defined as defined in the following formula:

\[ DR = \left( \int_{-\infty}^{\infty} (MAR - x)^2 f(x) dx \right)^{1/2} \tag{40} \]

where:
- \( x \) is the random variable representing the return for the distribution of annual/monthly returns \( f(x) \);
- \( f(x) \) the distribution for the annual/monthly returns.

Sortino ratio is used to analyze portfolios that have low volatility because the Sortino ratio won’t have enough data points to calculate downside deviation. For this reason, the Sortino ratio is better when we want analyze highly volatile portfolios.

Finally, the Omega ratio is a relative measure of the likelihood of achieving a given return, such as a MAR (minimum acceptable return) defined by the investor. The higher the omega value, the greater the probability that a given return will be met or exceed. We can define Omega ratio as the cumulative probability of an investment’s outcome above a threshold level, to the cumulative probability of an investment’s outcome below an investor’s threshold level. The objective of the omega ratio is “captures the effects of all higher moments fully and which may be used to rank and evaluate manager performance. No assumptions about risk preference or utility are necessary”\(^{40}\). This ratio split expected returns into two parts, on the one hand gains or

returns above the expected rate (the upside) and on the other hand losses or returns below the expected rate (the downside). The Omega ratio is computed as in the following formula:

\[
\Omega(r) = \frac{\int_{r}^{b} (1 - F(x)) dx}{\int_{a}^{r} F(x) dx}
\]

(41)

where:
- \( r \) is the threshold return;
- \( F \) is the cumulative density function of returns.

In the case we set as the threshold the mean of the distribution, the omega ratio is equal to 1.

Firstly, I applied the EPPI strategy into the DCS portfolio holding fixed the fix part of the multiplier \( \eta = 1.1 \) and I changed the amplifier performance value \( a \) for every simulation. Secondly, I changed the fixed component value with \( \eta = 1.05 \) and I simulated again the DCS portfolio with the same amplifier performance value \( a \). This process has been conducted in all four cases. For every simulation I took in consideration a transaction commission of five basis points\(^{41}\) (0.05%).

The study has been conducted analysing the portfolio performance after the 5\(^{th}\) year, 6\(^{th}\) year, 7\(^{th}\) year, 8\(^{th}\) year and the final term after 8 years and half since inception. I decided to analysed the performance in each of these years assuming to give the possibility to the investor to disinvest its position in each of these years.

The next figures show the portfolio performance after 8 years and half for each risky asset that has been chosen.

Firstly, the table 1 shows the portfolio performance with Dax ETF and with \( \eta = 1.1 \).

We can see as the performance multiplier increase, the overall portfolio performance increase as well since \( a \) is equal to 6. In the same way is increased the standard

\(^{41}\) It is the most widely transaction commission applied to Pension funds or Insurance funds.
deviation of the portfolio with a value equal to 5,08%. The Sharpe ratio of this strategy is equal to 0,302 that means for every unit risk added in the portfolio the performance increase only of 0,302%. Analysing the Sortino ratio and Omega ratios we have positive values closely to 0 in both ratios. With \( a = 6 \) there is a greater probability to have a return greater than its MAR compared to the other values. By the way if we compare this strategy with the performance of the core component that is our benchmark we can say that:

- the overall performance is increased at least of 30% (from 20,37% of the Core component to 26%) but is augmented also the risk of the portfolio (from 3,24% to 5,08%);
- for every value of \( a \) the portfolio performance is always greater than the performance of the benchmark;
- the capability to perform better than the benchmark is explained by the Information ratio where it is always positive for every value of \( a \), but it is always close to 0.
- If we compare these results with a simple “Buy and Hold” passive strategy (where transaction costs are minimized) we notice that the “Buy and Hold” performance is always been below the level of the DCS strategy, with 17,90% of performance since inception and a standard deviation equal to 3,17% (lower than the benchmark standard deviation).

In figure 18 is shown the performance of each portfolio component, how the floor is changed over time and the monthly risky asset allocation during the lifetime of the investment.

We can see that there was a lower allocation in the risky component during the first year of the investment, where the risky asset performed a negative return at least -50%. After the 2009 the allocation in the risky component has increased reaching the peak of 50% between 2013 and 2014. Furthermore, the DCS portfolio performance has been close to the performance of the riskless asset until 2013 and always above the floor level.
Table 1: portfolio results with Dax ETF and $\eta=1.1$

<table>
<thead>
<tr>
<th></th>
<th>a=1</th>
<th>a=2</th>
<th>a=3</th>
<th>a=4</th>
<th>a=5</th>
<th>a=6</th>
<th>a=7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum drawdown</td>
<td>6.77%</td>
<td>6.80%</td>
<td>6.83%</td>
<td>6.88%</td>
<td>6.94%</td>
<td>7.01%</td>
<td>7.10%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>23.21%</td>
<td>23.78%</td>
<td>24.33%</td>
<td>24.88%</td>
<td>25.43%</td>
<td>26.00%</td>
<td>24.43%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>2.73%</td>
<td>2.80%</td>
<td>2.86%</td>
<td>2.93%</td>
<td>2.99%</td>
<td>3.06%</td>
<td>2.87%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>4.91%</td>
<td>4.93%</td>
<td>4.96%</td>
<td>4.99%</td>
<td>5.04%</td>
<td>5.08%</td>
<td>5.27%</td>
</tr>
<tr>
<td>Sharpe ratio 42</td>
<td>0.246</td>
<td>0.259</td>
<td>0.270</td>
<td>0.281</td>
<td>0.292</td>
<td>0.302</td>
<td>0.256</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0.021</td>
<td>0.025</td>
<td>0.029</td>
<td>0.032</td>
<td>0.036</td>
<td>0.040</td>
<td>0.028</td>
</tr>
<tr>
<td>Sortino ratio 43</td>
<td>0.105</td>
<td>0.110</td>
<td>0.115</td>
<td>0.120</td>
<td>0.125</td>
<td>0.130</td>
<td>0.107</td>
</tr>
<tr>
<td>Omega ratio 44</td>
<td>1.052</td>
<td>1.063</td>
<td>1.072</td>
<td>1.082</td>
<td>1.091</td>
<td>1.10</td>
<td>1.071</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 17.90% and 3.17% respectively.

Figure 18: Portfolio perform. and the monthly risky allocation with $\alpha=6$ and $\eta=1.1$ (with Dax ETF).

---

42 I considered a risk-free rate equal to 1.52%. It has been computed as a geometric average of the overall annual Euribor starting from 31/10/2007.
43 I used as MAR the monthly risk free rate equal to 0.1259%. The ratio is computed on monthly portfolio performance.
44 I used as MAR the monthly return of the Core component equal to 0.20% in all simulations. The ratio is computed on monthly portfolio performance.
Finally, the DCS portfolio with the EPPI method has brought a higher performance compare to the benchmark, but with more standard deviation.

In the case $\eta=1.05$ (table 2) the best $a$ reachable is with a value equal to 7, because if we diminish the value of the fixed part $\eta$ we can use a greater value of the performance amplifier $a$. In this case, the annual performance, the Sharpe ratio and the Information ratio are slightly greater than the case with $\eta = 1.1$. Moreover, if we compare the two strategies with the same $a = 6$, we get that the best strategy is with $\eta = 1.05$ because we have had a greater performance with lower risk (only 5,00% of deviation standard compared to 5,08% of the other strategy). Sortino ratio with a value equal to 0,133 in the best case underline a better return risk-adjusted compared to the other cases with different $a$ value. Omega ratio are same for both $a$ equal to 6 and 7, they are closely to zero but positive.

**Table 2: portfolio results with Dax ETF and $\eta=1.05$**

<table>
<thead>
<tr>
<th>Portfolio analysis in case of the risky asset is ETF Dax and with $\eta=1.05$</th>
<th>$a=1$</th>
<th>$a=2$</th>
<th>$a=3$</th>
<th>$a=4$</th>
<th>$a=5$</th>
<th>$a=6$</th>
<th>$a=7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum drawdown</td>
<td>6,60%</td>
<td>6,63%</td>
<td>6,67%</td>
<td>6,71%</td>
<td>6,77%</td>
<td>6,84%</td>
<td>6,92%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>23,25%</td>
<td>23,83%</td>
<td>24,39%</td>
<td>24,94%</td>
<td>25,50%</td>
<td>26,07%</td>
<td>26,16%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>2,74%</td>
<td>2,80%</td>
<td>2,87%</td>
<td>2,93%</td>
<td>3,00%</td>
<td>3,07%</td>
<td>3,08%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>4,83%</td>
<td>4,85%</td>
<td>4,88%</td>
<td>4,91%</td>
<td>4,95%</td>
<td>5,00%</td>
<td>5,09%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0,252</td>
<td>0,264</td>
<td>0,276</td>
<td>0,288</td>
<td>0,299</td>
<td>0,309</td>
<td>0,306</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0,021</td>
<td>0,026</td>
<td>0,030</td>
<td>0,033</td>
<td>0,037</td>
<td>0,041</td>
<td>0,041</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0,107</td>
<td>0,113</td>
<td>0,1184</td>
<td>0,123</td>
<td>0,128</td>
<td>0,133</td>
<td>0,131</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>1,06</td>
<td>1,07</td>
<td>1,08</td>
<td>1,08</td>
<td>1,09</td>
<td>1,10</td>
<td>1,10</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 17,99% and 3,15% respectively. The maximum drawdown of the Buy and Hold portfolio is 5,33%.

Secondly, in the case of risky asset is S&P500 ETF we get the following results shown in tables 3 and 17.
The overall performance of the portfolio with S&P500 ETF is 36.57% (when $\alpha$ is equal to 1), almost twice the benchmark performance and with a standard deviation of only 4.34%. The annual performance is 4.30% and if we take in consideration the European Union inflation during the last 8.5 years we get a real annual performance of 2.85%. The Sharpe ratio is equal to 0.641 and underline a good value of risk-adjusted return compared to the Core component. The Sortino ratio is equal to 29.20%, but the best Omega ratio is reached when $\alpha = 2$. Omega ratio is largely greater than 1 and indicates a greater probability to get a return bigger than the benchmark.

If we compare this case with the strategy of Dax ETF we can notice that as the $\alpha$ value increase, the performance of the overall portfolio diminishes, increasing its maximum drawdown and the annual standard deviation.

<table>
<thead>
<tr>
<th>Portfolio analysis in case of the risky asset is ETF S&amp;P500 and with $\eta=1.1$</th>
<th>a=1</th>
<th>a=2</th>
<th>a=3</th>
<th>a=4</th>
<th>a=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum drawdown</td>
<td>6.46%</td>
<td>6.43%</td>
<td>6.40%</td>
<td>8.08%</td>
<td>9.90%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>36.57%</td>
<td>36.51%</td>
<td>35.36%</td>
<td>34.08%</td>
<td>32.65%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>4.30%</td>
<td>4.30%</td>
<td>4.16%</td>
<td>4.01%</td>
<td>3.84%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>4.34%</td>
<td>4.33%</td>
<td>4.32%</td>
<td>4.44%</td>
<td>4.72%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.641</td>
<td>0.641</td>
<td>0.611</td>
<td>0.560</td>
<td>0.492</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0.147</td>
<td>0.147</td>
<td>0.136</td>
<td>0.120</td>
<td>0.099</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0.292</td>
<td>0.292</td>
<td>0.278</td>
<td>0.247</td>
<td>0.204</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>1.391</td>
<td>1.392</td>
<td>1.357</td>
<td>1.317</td>
<td>1.276</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 22.75% and 3.22% respectively. The maximum drawdown of the Buy and Hold portfolio is 5.60%.

45 The geometric average inflation of EU during the last 8.5 years has been of 1.54%.
In the figure 19 is shown the monthly risky asset allocation in the portfolio, where during the first months of the strategy, the allocation in the satellite component has been below of the 20% of the portfolio total value, underweighted the downtrend of the S&P 500 during the recession period. Instead, after the mid of 2009 the strategy started to increase automatically the risky asset allocation in the portfolio trying to capture its uptrend.

The chart of the figure 19 shows the portfolio performance and the floor value in these 8 years and half. The portfolio value has started to gain a greater performance compared to the Core component from the last months of 2011 and between 2007 and 2010 its maximum drawdown has been of 6.46% (the maximum drawdown of the S&P500 in the same period was of -45.62%).

**Figure 19**: performance and the monthly risky allocation with \( a = 1 \) and \( \eta = 1.1 \) (with S&P500 ETF).
Analysing the case where $\eta = 1.05$ the maximum performance approachable is 36.70% with a standard deviation of 4.30%. As the case with $\eta = 1.10$, if we increase the value of $a$ the portfolio performance diminish, increasing the portfolio risk.

Thirdly, I used an ETC gold as a satellite component in our DCS portfolio and the results are showed in the following figures 20 and 21, it is important to remember that in this case we had a positive correlation with the benchmark (0.1675).

Table 5 shows the portfolio performance with different value of $a$ for every simulation. The best performance approachable is with a $a = 1$ and $\eta = 1.05$. In all simulations the performance is higher than the Benchmark performance but this strategy has a higher risk, the annual standard deviation is equal to 6.37%. In this case there is a high maximum drawdown in all variations and with a $a = 4$ exceed the limit imposed to the portfolio, making automatically triggering the portfolio fully invested in the riskless component (figure 20) as it happened after the first month of 2011.
Table 5: portfolio results with gold ETF

<table>
<thead>
<tr>
<th>Portfolio analysis in case of the risky asset is ETF gold</th>
<th>Case with η=1.1</th>
<th>Case with η=1.05</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a=1</td>
<td>a=2</td>
</tr>
<tr>
<td>Maximum drawdown</td>
<td>8.96%</td>
<td>8.88%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>30.58%</td>
<td>28.32%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>3.60%</td>
<td>3.33%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>6.53%</td>
<td>6.44%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.318</td>
<td>0.281</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0.060</td>
<td>0.048</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0.148</td>
<td>0.132</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>1,162</td>
<td>1,1249</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 23.48% and 4.27% respectively for η=1.1 and 23.32% and 4.19% for η=1.05. The maximum drawdown of the Buy and Hold portfolio is 6.61% for η=1.1 and 6.34% for η=1.05.

Figure 20: performance and the monthly risky allocation with α = 4 and η=1.1 (with gold ETF).
The Information ratio is close to zero in any situation, that means there is no a good capability from the investor to outperform the benchmark without assuming more risk. As \( \alpha \) value increase, Sortino ratio decrease in both two cases. The greater value equal to 15.8% is when \( \alpha =1 \) and \( \eta=1.05 \), it is the same for the Omega ratio (with a positive value equal to 1.167).

The figure 21 shows the portfolio performance in the case \( \alpha = 1 \) and \( \eta=1.05 \) that represents the best choice in our case. The risky asset has over-performed the portfolio and the benchmark until the end of 2012 and it was very volatile during all period. In this case was difficult for the portfolio to reduce the risk from the risky asset, because if there is a positive correlation between the assets, the possibility of a contemporaneously decreasing (or increasing) of both asset is higher.

In this case the portfolio value (blue line) started to be higher than the benchmark value (yellow line) from the end of 2009 as is shown in the figure 21.

**Figure 21**: performance and the monthly risky allocation with \( \alpha = 1 \) and \( \eta=1.05 \) (with gold ETF)
Finally, the last study has been conducted with the US 7-10Y Bond ETF. The correlation between the benchmark and the US Bond ETF is positive and equal to 0.3291 affecting the construction of the DCS portfolio. In table 6 are shown the portfolio performance. Again, the best choice is represented by a $a$ components equal to 1 and the difference between the two strategies with different fix component is very small, only three basis points. The total portfolio performance in case $\eta=1.05$ is equal to 21.98%, it is 8% greater than the benchmark performance equal to 20.37%. Instead, the standard deviation is increased from 3.24% of the benchmark to 4.94% of the new portfolio created.

The information ratio of the two strategies are very close to 0 or negative in some cases. The Omega ratio is closely to 1 that indicate there is a low probability to get a return greater than the MAR. As the performance multiplier increase the Sortino ratio decrease, becoming negative. However, this is the only case where a “Buy and Hold” portfolio invested for the 89% of portfolio value in the riskless asset and the remaining 11% is invested in the risky asset, has a total performance greater than the DCS portfolio and with a lower standard deviation.

**Table 6: portfolio results with US 7-10Y Bond ETF**

<table>
<thead>
<tr>
<th>Portfolio analysis in the case of the risky asset is ETF US 7-10Y Bond</th>
<th>Case with $\eta=1.1$</th>
<th>Case with $\eta=1.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a=1$</td>
<td>$a=2$</td>
<td>$a=3$</td>
</tr>
<tr>
<td>Maximum drawdown</td>
<td>8.26%</td>
<td>8.34%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>21.95%</td>
<td>21.58%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>2.58%</td>
<td>2.54%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>5.01%</td>
<td>5.03%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.212</td>
<td>0.202</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0.015</td>
<td>0.011</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0.097</td>
<td>0.092</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>1.0322</td>
<td>1.0252</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 22.65% and 3.73% respectively for $\eta=1.1$ and 22.53% and 3.69% for $\eta=1.05$. The maximum drawdown of the Buy and Hold portfolio is 6.46% for $\eta=1.1$ and 6.37% for $\eta=1.05$. 
Figure 22: performance and the monthly risky allocation with $\alpha=1$ and $\eta=1.05$ of US 7-10Y Bond ETF

In figure 22 is shown the portfolio performance of a portfolio with $\alpha = 1$ and $\eta=1.05$. We can see that the floor value is always below the portfolio value and the risky asset has the same behaviour of the benchmark index (riskless asset in this case).

Now, let’s assume to give the possibility to the investor to disinvest earlier his portfolio, for example after the 5th year\(^{46}\), the 6th year\(^{47}\), the 7th year\(^{48}\) and the 8th year\(^{49}\) and analyze if the consideration explained before could be changed\(^{50}\).

\(^{46}\) After the 5th year the total return of the Core component was of 11,92% (2,38% annual), with a standard deviation equal to 3,90%. Sharpe ratio and Sortino ratio are equal to 0,013 and 0,004 respectively. The risk free rate applied is equal to 2,33%.
\(^{47}\) After the 6th year the total return of the Core component was of 14,15% (2,36% annual), with a standard deviation equal to 3,65%. Sharpe ratio and Sortino ratio are equal to 0,0893 and 0,0386 respectively. The risk free rate applied is equal to 2,03%.
\(^{48}\) After the 7th year the total return of the Core component was of 18,14% (2,59% annual), with a standard deviation equal to 3,47%. Sharpe ratio and Sortino ratio are equal to 0,2246 and 0,0956 respectively. The risk free rate applied is equal to 1,81%.
\(^{49}\) After the 8th year the total return of the Core component was of 19,67% (2,46% annual), with a standard deviation equal to 3,30%. Sharpe ratio and Sortino ratio are equal to 0,2564 and 0,1096 respectively. The risk free rate applied is equal to 1,61%.
After the fifth year the situation of each DCS portfolio are the followings:

- **Dax ETF (table 7):** the optimal portfolio is given by a $\alpha$ value equal to 7. If the portfolio has an $\alpha$ value greater than 7, the portfolio starts to be riskier losing performance. The annual portfolio total return is equal to 2.38% and it has a Sharpe ratio equal to 0.03 greater than the Sharpe ratio of the Core component where it is equal to 0.013. If the $\alpha$ value is equal to 5 or 6 or 7 or 8 the portfolio has a greater return risk-adjusted compared to its. Sortino ratio start to be positive only when $\alpha$ is greater than 5 but it is always close to 0. The probability to get return greater than the benchmark is very close to 1 in all simulations.

- **S&P500 ETF (table 8):** the optimal portfolio is got with a $\alpha$ value equal to 10. It is interesting to notice that in the previous case where the portfolio is invested for 8.5 years the optimal portfolio is given by a lower $\alpha$ value, instead after only 5 years of investment the greater performance is reached with a high value of $\alpha$. Here, in all simulations conducted we have a Sharpe ratio that is greater than the Sharpe ratio of the Core component. The annual portfolio total return, in the case $\alpha = 10$, is equal to 3.68% with a standard deviation equal to 4.17%. There is a greater probability to get a monthly return compared to the Benchmark when the portfolio has S&P500 ETF as a satellite component. The Sortino ratio in the best case is equal to 14.20%.

- **Gold ETF (table 9):** here the greater performance is reached with a $\alpha$ value equal to 1. The annual portfolio total return is equal to 5.95% with a standard deviation equal to 7.49%. The Sharpe ratio is equal to 0.49 which mean a good investment compared to the performance of the Core component. However, in this strategy is not suggested to use a $\alpha$ multiplier greater than 3 because the portfolio will gain a performance lower than the Core component. The Omega ratio is equal to 1.454, well above to 1, indicating a great

---

All descriptive tables are contained in the appendix.
probability to get a greater performance compared to the MAR and Sortino ratio is equal to 22.14% well above the value of the benchmark.

- US 7-10Y Bond ETF (table 9): in this case there is no good result from the simulation, because all Sharpe ratio are lower than the value of Core component. This strategy brings more risk without a greater performance. The greater annual performance approachable is equal to 2.22% with a standard deviation equal to 4.31%. In all simulations Omega ratio is below 1 and Sortino ratio is negative. In this case don’t represent a good strategy for the investor because all performance indicators are negative.

If we go to analyze the situation after the sixth year, the DCS portfolios are the followings:

- Dax ETF (table 10): the optimal portfolio is given by a $\alpha$ value equal to 6. If the portfolio has an $\alpha$ value greater than 6, the portfolio starts to be riskier and with a lower performance. The annual portfolio total return is equal to 3.36% and it has a Sharpe ratio equal to 0.27 greater than the Sharpe ratio of the Core component where it is equal to 0.089. For every value of $\alpha$ from 1 to 7, the strategy bring a better risk-adjusted return compared to the benchmark and its IR is slightly above to 0, that describe the manager capability to outperform the benchmark. Sortino ratios is above the value got by the benchmark in all simulations and the Omega ratio is greater than 1. If we compare the DCS strategy with a “buy and hold” strategy, we get a greater performance in the first case.

- S&P500 ETF (table 10): the optimal portfolio is got with a $\alpha$ value equal to 10. Here, in all simulations conducted we have a Sharpe ratio that is greater than the Sharpe ratio of the Core component. The annual portfolio total return, in the case $\alpha = 10$, is equal to 4.15% with a standard deviation equal to 4.11% and a Sharpe ratio equal to 0.52. Omega ratios, in all cases, are bigger than 1, that means there is a great probability to get greater return compared to the benchmark. If we compare this portfolio with the portfolio after 5 years of
investment, holding the same Satellite, we notice that the value of Sortino ratio is increased. That means the variability of my monthly returns are less concentrated below the minimum acceptable return compared to the case the portfolio is invested only in five years.

- **Gold ETF (table 12):** here the greater performance is reached with a $\alpha$ value equal to 1. The annual portfolio total return is equal to 4.02% with a standard deviation equal to 6.98%. The Sharpe ratio is equal to 0.28 which mean a good investment compared to the performance of the Core component. However, in this strategy is not suggested to use a $\alpha$ multiplier greater than 3 because the portfolio will gain a performance lower than the Core component. There is a great omega ratio when the performance amplifier is equal to 1 and the Sortino ratio is 0.122.

- **US 7-10Y Bond ETF (table 12):** still, after 6 years, there are no good results from the simulation, because all Sharpe ratio are lower than the value of Core component. This strategy brings more risk without a greater performance. The greater annual performance approachable is equal to 1.70% with a standard deviation equal to 5.03% and a negative Sharpe ratio (-0.07). Omega ratio has a value below to 1 in all simulations and Sortino ratio is always negative. Only in this case, if we have had applied for a “buy and hold” passive strategy we could get a greater performance (2.39% annual perf. and a standard deviation equal to 4.07%) than the DCS strategy and the benchmark.

If we give the possibility for the investor to disinvest its portfolio after the seventh year we have the following results:

- **Dax ETF (table 13):** the optimal portfolio is given by a $\alpha$ value equal to 6. If the portfolio has an $\alpha$ value greater than 6, the portfolio starts to be riskier and losing performance. The annual portfolio total return is equal to 3.35% and it has a Sharpe ratio equal to 0.33 greater than the Sharpe ratio of the Core component where it is equal to 0.225. For every value of $\alpha$ from 1 to 7, the strategy bring a better risk-adjusted return compared to the benchmark and its
IR is slightly above to 0, that describe the manager capability to outperform the benchmark. If we compare the DCS strategy with a “buy and hold” strategy, we get a greater performance in the first case. The performance of the portfolio after 7 years seem to add a greater value to the portfolio performance without adding a great part of risk. It is explained by an increase of the Sharpe ratio from the 6th to the 7th years. Compared to the value of the same portfolio after 6 years of investments we have better Omega ratio and Sortino ratio. Moreover, in all simulations we have had a greater Sortino ration than the benchmark.

- S&P500 ETF (table 14): the optimal portfolio is got with a $a$ value equal to 7. Here, in all simulations conducted we have a Sharpe ratio that is greater than the Sharpe ratio of the Core component. The annual portfolio total return, in the case $a = 7$, is equal to 4.88% with a standard deviation equal to 3.86% and a Sharpe ratio equal to 0.79. If we compare the performance with the portfolio after the 6th year, the optimal $a$ value is diminished from 10 to 7. However, Sortino ratio and Omega ratio has increased, underlining the greater performance given by this portfolio compared to the benchmark and the risk free rate. For every unit of risk added to this portfolio, we have 0.38% of more return in the case $a = 7$ (instead of 0.095% of the benchmark).

- Gold ETF (table 15): still, the greater performance is reached with a $a$ value equal to 1. The annual portfolio total return is equal to 3.81% with a standard deviation equal to 6.57%. The Sharpe ratio is equal to 0.30 always greater than the value of the benchmark. However, in this strategy is not suggested to use a $a$ multiplier greater than 2 because the portfolio will gain a performance lower than the Core component. If we compare this portfolio with the same portfolio but at the end of the 5th and 6th years we get worst Omega ratio and Sortino ratio.

- US 7-10Y Bond ETF (table 15): after 7 years the Sharpe ratio of this portfolio is became positive but still lower than the benchmark. The greater
performance is reached with a $\alpha$ value equal to 1 with an annual performance and a standard deviation equal to 2.28% and 4.79% respectively. Again, we have a greater performance and a lower standard deviation in the case of a “buy and hold” strategy. For all simulations Omega ratio is always below to 1, explaining that is difficult to have a performance greater than the MAR. Sortino ratio are closely to zero in all cases.

Finally, assuming to give the possibility to the investor to disinvest its portfolio after the 8th year, the DCS performance portfolio are the following:

- Dax ETF (table 16): we have an optimal portfolio with performance multiplier $\alpha$ equal to 6. If the portfolio has an $\alpha$ value greater than 6, the portfolio starts to be riskier and losing performance. The annual portfolio total return is equal to 3.42% and it has a Sharpe ratio equal to 0.357 greater than the Sharpe ratio of the Core component where it is equal to 0.256. For every value of $\alpha$ from 1 to 8, the strategy bring a better risk-adjusted return compared to the benchmark and its IR is slightly above to 0, that describe the manager capability to outperform the benchmark. The best Omega ratio and Sortino ratio, 1.146 and 0.151 respectively, are reached when $\alpha = 6$. If we compare the DCS strategy with a “buy and hold” strategy, we get again a greater performance in the first case.

- S&P500 ETF (table 17): the optimal portfolio is got with a $\alpha$ value equal to 1. It is interesting to notice that, between the 7th and the 8th year the optimal $\alpha$ value is decreased, reaching the minimum value equal to 1. This could be explained by the increasing of unexpected volatility on the market after the spring of 2015 and it started to have a no defined direction till today, as it is shown in the figure 23. For 15 months the prices have been between a value of 190 and 212. The annual portfolio total return, in the case $\alpha = 1$, is equal to 4.67% with a standard deviation equal to 4.32% and a Sharpe ratio equal to 0.708. The IR is equal to 0.171. Compared to the same DCS portfolio but at
the end of 7th year, the Omega ratio and Sortino ratio worsened but they are always greater the Sortino ratio of the Benchmark (0,1095).

Figure 23: candlestick chart of the SPY Total Return ETF

- Gold ETF (table 18): still, the greater performance is reached with a $\alpha$ value equal to 1. The annual portfolio total return is equal to 3,76% with a standard deviation equal to 6,46%. The Sharpe ratio is equal to 0,333 always greater than the value of the benchmark. However, in this strategy is not suggested to use a $\alpha$ multiplier greater than 2 because the portfolio will gain a lower performance than the Core component. Omega ratio is greater than 1 in all simulations and we have a better Sortino only when $\alpha$ is equal to 1 or 2.

- US 7-10Y Bond ETF (table 18): after 8 years the Sharpe ratio of this portfolio is became positive but still lower than the benchmark. The greater performance is reached with a $\alpha$ value equal to 1 with an annual performance and a standard deviation equal to 2,70% and 4,97% respectively. There is no better Sortino in this case, compared to reached by the benchmark and Omega.
ratios are closely to 0. Again, we have a greater performance and a lower standard deviation in the case of a “buy and hold” strategy.

In conclusion, the best performances from the DCS portfolio with the EPPI method is reached only when there are negative correlation with the Benchmark. The best performance is reached with the portfolio that use SPY ETF as Satellite component. Instead, the worst performance is given in the case we use US bond 7-10T ETF as Satellite component.

There is no specific amplifier performance value ($a$) for every kind of portfolio, but it depends from the correlation between Core and Satellite components, and from the duration of the investments. If we have positive correlation it is suggested to use a lower $a$ value for every period decided by the investor. Instead, if there is negative correlation and a middle holding period (5-7 years) we can use an higher $a$ value, between 5 to 8. However, if the investor has intention to hold its portfolio active for a long period of time is suggested to use a lower performance amplifier.
Conclusion

In all four strategies we have had a greater performance compared to the benchmark, in particular if the satellite component has a negative correlation with the Core component, a DCS portfolio strategy with EPPI succeed to increase the total return risk-adjusted of the portfolio.

It is interesting to notice that there isn’t a fix right $a$ value for every asset, for example the DCS strategy with Dax ETF the best $a$ value approachable is equal to 6, instead in the case the satellite is the Gold ETF the best $a$ value is equal to 1. Moreover, if we analyse how the portfolio performance change every year, we have that for the same investment but with different time horizon there could be exist different performance multiplier ($a$) for the same portfolio. For example, in the case that the DCS portfolio use the SPY ETF as the satellite component, the best $a$ value applied may be change if we consider different time horizon, until the 7th year of investment the best performance amplifier may be between 6 to 10, but if we consider the portfolio after 8 years and half we have a value $a$ equal to 1. This fact could be explained that, if we are in a situation of uncertainty, without a clear movement of the prices and a sudden increase of volatility the strategy requires a lower performance amplifier ($a$).

In the remaining cases, Gold ETF and US 7-10Y Bond ETF, where the correlation is positive in both cases, we have a best $a$ value approachable equal to 1, and in the case of US bond ETF as a satellite is difficult to creat a DCS with a better performance compared to the benchmark. This is explained by a positive correlation between two assets.

I noticed that, the DCS portfolio strategy works efficiently when the satellite has a negative correlation with the benchmark or slightly positive, because the task required to this strategy is to outperform the Core component. In other words, if the Core component starts to decrease and the satellite starts to increase is required to the strategy to overweight the satellite component selling the Core asset. This is a great
effect on the performance and in the standard deviation of the portfolio. If we have
two assets that have negative correlation, we can create a portfolio with a higher
Sharpe ratio compared to the benchmark one.
In conclusion, there is no right \( a \) value to apply for this strategy. It is required to the
investor to consider the following objectives:

- its time horizon;
- which is the correlation between the Core and Satellite components;
- its risk.

In the case of negative correlation, we can use a higher performance amplifier value,
by the way if there is positive correlation between two components is suggested to
use a lower fix component \( \eta \) (for example equal to 1.05) and a lower performance
amplifier \( a \) (for example equal to 1).

The primary purpose of this strategy is to increase the investor wealth without
assuming a great risk. For this reason, this strategy has been built for long period
investments, with holding period greater than 5 years and is suggested for pension
plans.

This study has been conducted considering only the case there is only one satellite in
the portfolio, could be interesting implement an optimization of DCS portfolio
strategy with EPPI method in the case there are more than one satellite components in
the portfolio.
Matlab Code

% initial part with the implementation of the dataset (import data as a numeric vector and column matrix)
[n,ns]=size(datanew)
T=n-1;
R=zeros(T,ns);
for i=1:T;
    for j=1:ns;
        R(i,j)=log(datanew(i+1,j)/datanew(i,j));
    end;
end;
% overall log performance of every asset
for i=1:ns;
p(i)=log(datanew(n,i)/datanew(1,i));
end
p=p'
% creation of the vectors for every kind of asset
eub=R(:,1);
gold=R(:,5);
dax=R(:,3);
sp=R(:,2);
usb=R(:,4);
% computation of the correlation between riskless asset and risky asset
corrgold=corr(eub, gold)*100
corrdax=corr(eub,dax)*100
corrsp=corr(eub,sp)*100
corrusb=corr(eub,usb)*100
% computation of the annual standard deviation of the assets
stdevgold=std(gold)*sqrt(12)*100
stdeveub=std(eub)*sqrt(12)*100
stdevdax=std(dax)*sqrt(12)*100
stdevsp=std(sp)*sqrt(12)*100
stdevusb=std(usb)*sqrt(12)*100
% computation of the skewness and kurtosis
sk=skewness(tab)
kur=kurtosis(tab)
clear risky norisky port multiple cushion floor allocation riskyshares eubond risklessshares stock portinv shares stock port
portinv down
format bank
rasset=sp; % risky asset monthly performance
unasset=eub; % riskless asset monthly performance
prasset=US710; % monthly price risky asset
punasset=EU35; % monthly price riskless asset
portinv=1000000; % value of the portfolio assuming we want to invest $1 mln
port=100; % value of the portfolio at time t=0, i start with 100 as an index
taxcommission=0.0005; % I suppose a transaction commission of 5 basis points
points
nu=1.1; % a constant greater than 1 used in the multiplier
multiple=nu;
a=1; % amplification of the performance
k=0.90; % percentage of the floor protection
% Here i start to define some variable useful for the construction of the portfolio
floor=zeros(n,1);
floor(1)=k*port;
cushion=port-floor(1);
risky=multiple*cushion;
norisky=port-risky;
allocation=risky/port;
stock=zeros(n,1);
stock(1)=portinv*allocation;
stcommission=stock(1)*taxcommission;
stock(1)=stock(1)-stcommission;
riskyshares=zeros(n,1);
riskyshares(1)=stock(1)/prasset(1);
eubond=zeros(n,1);
eubond(1)=portinv-stock(1);
bcommission=eubond(1)*taxcommission;
eubond(1)=eubond(1)-bcommission;
risklessshares=zeros(n,1);
risklessshares(1)=eubond(1)/punasset(1);
bond=100;
equity=100;
shares=0;
shareb=0;

% implementation of the variable for a Buy and Hold Portfolio
equityBH=risky;
bondBH=norisky;
portfolio=equityBH+bondBH;
portfolio(i)=port;
kBH=1-equityBH/port;
i=1;
m=1;
while m>0
% here i defined the performance of the risky and riskless assets at time t+1
i=i+1;
norisky(i,1)=norisky(i-1,1)*(1+unasset(i-1,1));
bond(i,1)=bond(i-1,1)*(1+unasset(i-1,1));
eubond(i,1)=risklesssshares(i-1,1)*punasset(i,1);
risky(i,1)=risky(i-1,1)*(1+rasset(i-1,1));
equity(i,1)=equity(i-1,1)*(1+rasset(i-1,1));
stock(i,1)=riskyshares(i-1,1)*prasset(i,1);
port(i,1)=risky(i)+norisky(i);
potinv(i,1)=stock(i)+eubond(i);
% here i want to implement an ipotetic drawdown that must be lower than 10%, if the portfolio lose value more than 10% the portfolio is fully
% invested in the riskless asset
if port(i)>portfolio(i-1)
portfolio(i)=port(i);
else
portafolio(i)=portfolio(i-1);
end
if port(i)/portfolio(i-1)<0.9
allocation(i)=0;
norisky(i,1)=port(i)-(port(i)-norisky(i))*taxcommission;
port(i)=norisky(i);
risky(i)=0;
equity(i)=risk(i);
floor(i)=floor(i-1);
i=i+1;
for s=i:n
allocation(s)=0;
port(s)=port(s-1,1)*(1+unasset(s-1,1));
equity(s)=0;
bond(s)=port(s);
floor(s)=floor(s-1);
end
m=-1;
else
% the floor must be always above or at least equal to 90 to permit to don't lose more than 10% of the initial investment. Moreover i consider a trailing performance floor, this floor prevents a portfolio from posting negative performance over 12-month trailing horizon, regardless of the performance of the portfolio. We assume the floor at time t=12 is the value of the portfolio at time t=1
if i<13
floor(i,1)=k*port(i);
else
floor(i,1)=k*port(i-12);
end
if floor(i)<90
floor(i)=90;
end
% implementation of the EPPI strategy
cushion(i,1)=port(i)-floor(i);
multiple(i)=nu+exp(a*rasset(i-1,1));
risky(i,1)=multiple(i)*cushion(i);
norisky(i,1)=port(i)-risky(i);
allocation(i,1)=risky(i)/port(i);
% here there is a control that the satellite allocation must not exceed % the 50% percent of the portfolio value at the same time
if allocation(i,1)>0.5
risky(i,1)=risky(i,1)-(risky(i,1)-port(i)/2);
norisky(i,1)=norisky(i,1)+(risky(i,1)-port(i)/2);
end
% rebalancing of the portfolio after the new allocation, we consider the transaction cost effect
if allocation(i)>allocation(i-1)
risky(i,1)=risky(i)-(risky(i)-risk(i-1))*taxcommission;
% number of shares need to buy for the new allocation
\[ \text{shares}(i,1) = \text{portinv}(i) \times \text{allocation}(i) / \text{prasset}(i) - \text{riskyshares}(i-1) \]

\[ \text{stcommission}(i) = \text{shares}(i) \times \text{prasset}(i) \times \text{taxcommission}; \]

\[ \text{stock}(i,1) = (\text{shares}(i) + \text{riskyshares}(i-1)) \times \text{prasset}(i) - \text{stcommission}(i); \]

\[ \text{riskyshares}(i,1) = \text{stock}(i) / \text{prasset}(i); \]

\[ \text{norisky}(i,1) = \text{norisky}(i) - (\text{norisky}(i-1) - \text{norisky}(i)) \times \text{taxcommission}; \]

% \text{number of shares need to sell for the new allocation}
\[ \text{shares}(i,1) = \text{riskyshares}(i-1) - \text{portinv}(i) \times (1 - \text{allocation}(i)) / \text{prasset}(i) - \text{stcommission}(i); \]

\[ \text{stock}(i,1) = (\text{riskyshares}(i-1) - \text{shares}(i)) \times \text{prasset}(i) - \text{stcommission}(i); \]

\[ \text{riskyshares}(i,1) = \text{stock}(i) / \text{prasset}(i); \]

\[ \text{norisky}(i,1) = \text{norisky}(i) - (\text{norisky}(i) - \text{norisky}(i-1)) \times \text{taxcommission}; \]

\[ \text{number of shares need to buy for the new allocation}
\[ \text{shares}(i,1) = \text{riskyshares}(i-1) - \text{portinv}(i) \times (1 - \text{allocation}(i)) / \text{prasset}(i) - \text{stcommission}(i); \]

\[ \text{stock}(i,1) = (\text{risklessshares}(i-1) - \text{shares}(i)) \times \text{prasset}(i) - \text{stcommission}(i); \]

\[ \text{risklessshares}(i,1) = \text{stock}(i) / \text{prasset}(i); \]

\[ \text{norisky}(i,1) = \text{norisky}(i) - (\text{norisky}(i) - \text{norisky}(i-1)) \times \text{taxcommission}; \]

\[ \text{number of shares need to buy for the new allocation}
\[ \text{shares}(i,1) = \text{risklessshares}(i-1) - \text{portinv}(i) \times (1 - \text{allocation}(i)) / \text{prasset}(i) - \text{stcommission}(i); \]

\[ \text{stock}(i,1) = (\text{shareb}(i) + \text{risklessshares}(i-1)) \times \text{punasset}(i) - \text{bcommission}(i); \]

\[ \text{risklessshares}(i,1) = \text{eubond}(i)/\text{punasset}(i); \]

\[ \text{else} \]

\[ \text{risky}(i,1) = \text{risky}(i) - (\text{risklessshares}(i-1) - \text{risky}(i)) \times \text{taxcommission}; \]

\[ \text{number of shares need to sell for the new allocation}
\[ \text{shares}(i,1) = \text{riskyshares}(i-1) - \text{portinv}(i) \times (1 - \text{allocation}(i)) / \text{prasset}(i) - \text{stcommission}(i); \]

\[ \text{stock}(i,1) = (\text{risklessshares}(i-1) - \text{shares}(i)) \times \text{prasset}(i) - \text{stcommission}(i); \]

\[ \text{riskyshares}(i,1) = \text{stock}(i) / \text{prasset}(i); \]

\[ \text{norisky}(i,1) = \text{norisky}(i) - (\text{norisky}(i) - \text{norisky}(i-1)) \times \text{taxcommission}; \]

\[ \text{number of shares need to buy for the new allocation}
\[ \text{shares}(i,1) = \text{riskyshares}(i-1) - \text{portinv}(i) \times (1 - \text{allocation}(i)) / \text{prasset}(i) - \text{stcommission}(i); \]

\[ \text{stock}(i,1) = (\text{shareb}(i) + \text{risklessshares}(i-1)) \times \text{punasset}(i) - \text{bcommission}(i); \]

\[ \text{risklessshares}(i,1) = \text{eubond}(i)/\text{punasset}(i); \]

\[ \text{end} \]

\[ \text{else} \]

\[ \text{end} \]

\[ \text{end} \]

% \text{computation of the maxdrawdown of the portafolio and its overall log performance}
\[ \text{maxd} = \text{maxdrawdown}(\text{port}) \times 100; \]

\[ \text{msg} = \text{['the max drawdown of the portfolio is: ' num2str(maxd)]}; \]

\[ \text{disp}(\text{msg}) \]

\[ \text{portperf} = \text{log}(\text{port}(n) / \text{port}(1)) \times 100; \]

\[ \text{msg} = \text{['the overall log performance of the portfolio is: ' num2str(portperf)]}; \]

\[ \text{disp}(\text{msg}) \]

\[ \text{annperfport} = \text{portperf} / 8.5; \]

\[ \text{msg} = \text{['the annual log portfolio performance is: ' num2str(annperfport)]}; \]

\[ \text{disp}(\text{msg}) \]
% computation of a Buy and Hold portfolio with the same initial weights
for i=2:n
bondBH(i,1)=bondBH(i-1,1)*(1+unasset(i-1,1));
equityBH(i,1)=equityBH(i-1,1)*(1+rasset(i-1,1));
portBH(i,1)=bondBH(i)+equityBH(i);
end
% computation of the monthly performance of the portfolios
for i=1:n-1
rendBH(i)=log(portBH(i+1)/portBH(i));
rendport(i)=log(port(i+1)/port(i));
end
maxdBH=maxdrawdown(portBH)*100;
msg= ['the max drawdown of the Buy and Hold portfolio is: ' num2str(maxdBH)];
disp(msg)
portperfbH=log(portBH(n)/portBH(1))*100;
msg= ['the overall log performance of the portfolio buy and hold is: ' num2str(portperfbH)];
disp(msg)
% computation standard deviation of the portfolios
stportBH=sqrt(((std(rasset)*sqrt(12))^2*(1-kBH)^2+(std(unasset)*sqrt(12))^2*kBH*2+corr(rasset,unasset)*kBH*(1-kBH)*2*(std(rasset)*sqrt(12))*(std(unasset)*sqrt(12))))*100;
stport=std(rendport)*sqrt(12)*100;
msg= ['the annual standard deviation of the portafolio is: ' num2str(stport)];
disp(msg)
msg= ['the annual standard deviation of the portfolio buy and hold is: ' num2str(stportBH)];
disp(msg)
% plot
x = linspace(datenum(2007,10,31),datenum(2016,03,31),n);
y = rand(1,n);
dateFormat = 10;
figure
subplot(2,1,1);
plot(x,port,x,equity,'--',x,bond,x,floor);
datetick('x',dateFormat);
xlim([733340 736430]);
ylabel('Index value')
title('Performance')
legend('portfolio','risky asset','riskless asset','floor','Location','northwest')
subplot(2,1,2);
bar(x,allocation,1.2)
datetick('x',dateFormat);
xlim([733340 736430])
ylabel('Percentage of the risky asset allocated')
xlabel('Time')
title('Monthly Allocation')
% information ratio
for i=1:T;
    perfport(i)=log(port(i+1)/port(i));
    excessret(i)=perfport(i)-unasset(i);
end
perfport=perfport';
meanexcessret=mean(excessret);
deveexcessret=std(excessret);
IR=meanexcessret/devexcessret;
msg= ['the information ratio is: ',num2str(IR)];
disp(msg)
% sortino ratio
clear exreturn
mar=log(EU35(n)/EU35(1))/n;
sortino=(mean(perfport')-mar)/sqrt(lpm(perfport',mar,2))*100;
msg= ['the sortino ratio is: ',num2str(sortino)];
disp(msg)
% omega ratio
omega=lpm(-perfport',-mar,1)/lpm(perfport',mar,1)*100;
msg= ['the omega ratio is: ',num2str(omega)];
disp(msg)
Appendix

Table 7: portfolio results with Dax ETF and $\eta=1.1$ (after 5 years).

<table>
<thead>
<tr>
<th>Portfolio analysis in case of the risky asset is ETF Dax and with $\eta=1.1$</th>
<th>a=1</th>
<th>a=2</th>
<th>a=3</th>
<th>a=4</th>
<th>a=5</th>
<th>a=6</th>
<th>a=7</th>
<th>a=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum drawdown</td>
<td>6.72%</td>
<td>6.62%</td>
<td>6.55%</td>
<td>6.49%</td>
<td>6.46%</td>
<td>6.43%</td>
<td>6.43%</td>
<td>6.47%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>9.43%</td>
<td>9.95%</td>
<td>10.44%</td>
<td>10.93%</td>
<td>11.41%</td>
<td>11.90%</td>
<td>12.40%</td>
<td>12.07%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>1.89%</td>
<td>1.99%</td>
<td>2.09%</td>
<td>2.19%</td>
<td>2.28%</td>
<td>2.38%</td>
<td>2.48%</td>
<td>2.41%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>4.71%</td>
<td>4.72%</td>
<td>4.74%</td>
<td>4.77%</td>
<td>4.80%</td>
<td>4.84%</td>
<td>4.88%</td>
<td>4.95%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>-0.09</td>
<td>-0.07</td>
<td>-0.05</td>
<td>-0.03</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Information ratio</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.00</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>-0.035</td>
<td>-0.027</td>
<td>-0.018</td>
<td>-0.010</td>
<td>-0.002</td>
<td>0.006</td>
<td>0.014</td>
<td>0.009</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>0.934</td>
<td>0.949</td>
<td>0.963</td>
<td>0.98</td>
<td>0.991</td>
<td>1.005</td>
<td>1.019</td>
<td>1.010</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 8.04% and 3.53% respectively. The maximum drawdown of the Buy and Hold portfolio is 5.33%.

Table 8: portfolio results with S&P500 ETF and $\eta=1.1$ (after 5 years).

<table>
<thead>
<tr>
<th>Portfolio analysis in case of the risky asset is ETF S&amp;P500 and with $\eta=1.1$</th>
<th>a=4</th>
<th>a=5</th>
<th>a=6</th>
<th>a=7</th>
<th>a=8</th>
<th>a=9</th>
<th>a=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum drawdown</td>
<td>6.37%</td>
<td>6.35%</td>
<td>6.33%</td>
<td>6.31%</td>
<td>6.28%</td>
<td>6.26%</td>
<td>6.24%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>15.79%</td>
<td>16.21%</td>
<td>16.63%</td>
<td>17.06%</td>
<td>17.50%</td>
<td>17.94%</td>
<td>18.40%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>3.16%</td>
<td>3.24%</td>
<td>3.33%</td>
<td>3.41%</td>
<td>3.50%</td>
<td>3.59%</td>
<td>3.68%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>3.60%</td>
<td>4.01%</td>
<td>4.04%</td>
<td>4.07%</td>
<td>4.10%</td>
<td>4.14%</td>
<td>4.17%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.23</td>
<td>0.23</td>
<td>0.25</td>
<td>0.26</td>
<td>0.28</td>
<td>0.30</td>
<td>0.32</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0.092</td>
<td>0.010</td>
<td>0.110</td>
<td>0.120</td>
<td>0.128</td>
<td>0.137</td>
<td>0.146</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>1.161</td>
<td>1.178</td>
<td>1.194</td>
<td>1.211</td>
<td>1.227</td>
<td>1.244</td>
<td>1.261</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 9.88% and 3.60% respectively. The maximum drawdown of the Buy and Hold portfolio is 5.60%.

---

$^{51}$ I used as MAR the monthly free risk rate equal to 0.1924% for 5 years of investments.
Table 9: portfolio results with gold ETF and US Bond 7-10Y ETF and \(\eta=1.1\) (after 5 years).

<table>
<thead>
<tr>
<th>Case of the risky asset is ETF gold</th>
<th>Case of the risky asset is ETF US Bond 7-10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\eta=1.1)</td>
<td>(\eta=1.1)</td>
</tr>
<tr>
<td>a=1</td>
<td>a=2</td>
</tr>
<tr>
<td>Maximum drawdown</td>
<td>6.06%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>29.77%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>5.95%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>7.46%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.49</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0.15</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0.224</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>1.454</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 21.06\% and 4.69\% respectively. The maximum drawdown of the Buy and Hold portfolio is 5.77\%.

Table 10: portfolio results with Dax ETF and \(\eta=1.1\) (after 6 years).

| Portfolio analysis in case of the risky asset is ETF Dax and with \(\eta=1.1\) |
|-------------------------------------|---------------------------------------------|
| a=2 | a=3 | a=4 | a=5 | a=6 | a=7 |
| Maximum drawdown | 6.62\% | 6.55\% | 6.49\% | 6.46\% | 6.43\% | 6.43\% |
| Performance since inception | 18.27\% | 18.75\% | 19.22\% | 19.68\% | 20.16\% | 18.59\% |
| Annual performance | 3.04\% | 3.12\% | 3.20\% | 3.28\% | 3.36\% | 3.10\% |
| Annual standard deviation | 4.75\% | 4.74\% | 4.81\% | 4.84\% | 4.88\% | 5.15\% |
| Sharpe ratio | 0.21 | 0.23 | 0.24 | 0.26 | 0.27 | 0.21 |
| Information ratio | 0.04 | 0.05 | 0.05 | 0.06 | 0.06 | 0.04 |
| Sortino ratio | 0.083 | 0.095 | 0.101 | 0.108 | 0.114 | 0.083 |
| Omega ratio | 1.111 | 1.123 | 1.135 | 1.146 | 1.158 | 1.112 |

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 11.83\% and 3.36\% respectively. The maximum drawdown of the Buy and Hold portfolio is 5.37\%.

\[^{52}\] I used as MAR the monthly free risk rate equal to 0.1678\% for 6 years of investments.
Table 11: portfolio results with S&P500 ETF and $\eta=1.1$ (after 6 years).

| Portfolio analysis in case of the risky asset is ETF S&P500 and with $\eta=1.1$ |
|-------------------------------------------------|------------------|------------------|------------------|------------------|
| Maximum drawdown                                 | a=5  | a=6  | a=7  | a=8  | a=9  | a=10 |
| Performance since inception                      | 6.35% | 6.33% | 6.31% | 6.28% | 6.26% | 6.24% |
| Annual performance                               | 22.63% | 23.07% | 23.52% | 23.97% | 24.43% | 24.91% |
| Annual standard deviation                        | 3.93% | 3.96% | 4.00% | 4.03% | 4.07% | 4.11% |
| Sharpe ratio                                     | 0.44  | 0.46  | 0.47  | 0.49  | 0.50  | 0.52  |
| Information ratio                                | 0.12  | 0.12  | 0.13  | 0.13  | 0.14  | 0.14  |
| Sortino ratio                                    | 0.202 | 0.209 | 0.217 | 0.224 | 0.232 | 0.239 |
| Omega ratio                                      | 1,306 | 1,320 | 1,334 | 1,349 | 1,363 | 1,378 |

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 13.54% and 3.39% respectively. The maximum drawdown of the Buy and Hold portfolio is 5.60%.

Table 12: portfolio results with gold ETF and US Bond 7-10Y ETF (after 6 years).

<table>
<thead>
<tr>
<th>Case of the risky asset is ETF gold</th>
<th>Case of the risky asset is ETF US Bond 7-10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta=1.1$</td>
<td>$\eta=1.05$</td>
</tr>
<tr>
<td>a=1</td>
<td>a=2</td>
</tr>
<tr>
<td>Maximum drawdown</td>
<td>8.96%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>23.95%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>3.99%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>7.21%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.27</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0.122</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0.101</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>1.2</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 17.21% and 4.69% respectively for $\eta=1.1$ and 17.06% and 4.58% for $\eta=1.05$. The maximum drawdown of the Buy and Hold portfolio is 6.61% for $\eta=1.1$ and 6.34% for $\eta=1.05$. In case of a Buy and Hold portfolio its performance and its annual standard deviation are 14.43% and 4.07% respectively for $\eta=1.1$ and 14.34% and 4.07% for $\eta=1.05$. The maximum drawdown of the Buy and Hold portfolio is 6.46% for $\eta=1.1$ and 6.37% for $\eta=1.05$. 
Table 13: portfolio results with ETF Dax (after 7 years).

| Portfolio analysis in case of the risky asset is ETF Dax and with $\eta=1.1$ |
|-----------------------------|---|---|---|---|---|---|
|                           | a=1 | a=2 | a=3 | a=4 | a=5 | a=6 |
| Maximum drawdown          | 6,72% | 6,62% | 6,55% | 6,49% | 6,46% | 6,43% |
| Performance since inception | 21,08% | 21,57% | 22,05% | 22,52% | 22,99% | 23,47% |
| Annual performance        | 3,01% | 3,08% | 3,15% | 3,22% | 3,28% | 3,35% |
| Annual standard deviation | 4,55% | 4,57% | 4,59% | 4,62% | 4,65% | 4,69% |
| Sharpe ratio              | 0,26 | 0,28 | 0,29 | 0,30 | 0,32 | 0,33 |
| Information ratio         | 0,03 | 0,03 | 0,04 | 0,04 | 0,04 | 0,05 |
| Sortino ratio$^{53}$      | 0,110 | 0,116 | 0,122 | 0,128 | 0,133 | 0,139 |
| Omega ratio               | 1,073 | 1,084 | 1,095 | 1,105 | 1,116 | 1,126 |

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 15.65% and 3.20% respectively. The maximum drawdown of the Buy and Hold portfolio is 5.33%.

Table 14: Portfolio results with ETF S&P500 (after 7 years).

| Portfolio analysis in case of the risky asset is ETF S&P500 and with $\eta=1.1$ |
|-----------------------------|---|---|---|---|---|---|
|                           | a=4 | a=5 | a=6 | a=7 | a=8 | a=9 |
| Maximum drawdown          | 6,37% | 6,35% | 6,33% | 6,31% | 6,28% | 6,26% |
| Performance since inception | 32,63% | 33,14% | 33,65% | 34,18% | 33,90% | 32,44% |
| Annual performance        | 4,66% | 4,73% | 4,81% | 4,88% | 4,84% | 4,63% |
| Annual standard deviation | 3,78% | 3,80% | 3,83% | 3,86% | 3,88% | 3,97% |
| Sharpe ratio              | 0,75 | 0,77 | 0,78 | 0,79 | 0,78 | 0,71 |
| Information ratio         | 0,18 | 0,18 | 0,19 | 0,19 | 0,18 | 0,00 |
| Sortino ratio$^{53}$      | 0,360 | 0,368 | 0,375 | 0,383 | 0,376 | 0,338 |
| Omega ratio               | 1,495 | 1,511 | 1,536 | 1,542 | 1,531 | 1,456 |

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 19,57% and 3,24% respectively. The maximum drawdown of the Buy and Hold portfolio is 5,60%.

$^{53}$ I used as MAR the monthly free risk rate equal to 0.1499% for 7 years of investments.
Table 15: portfolio results with gold ETF and US Bond 7-10Y ETF (after 7 years).

<table>
<thead>
<tr>
<th></th>
<th>Case of the risky asset is ETF gold</th>
<th></th>
<th>Case of the risky asset is ETF US Bond 7-10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>η=1.1 a=1</td>
<td>8,96%</td>
<td>η=1.1 a=1</td>
<td>8,26%</td>
</tr>
<tr>
<td>η=1.05 a=2</td>
<td>8,88%</td>
<td>η=1.05 a=2</td>
<td>8,34%</td>
</tr>
<tr>
<td>η=1.1 a=3</td>
<td>8,80%</td>
<td>η=1.05 a=3</td>
<td>8,43%</td>
</tr>
<tr>
<td>Maximum drawdown</td>
<td></td>
<td>Maximum drawdown</td>
<td></td>
</tr>
<tr>
<td>Performance since inception</td>
<td>26,47%</td>
<td>Performance since inception</td>
<td>15,86%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>3,78%</td>
<td>η=1.1 a=1</td>
<td>8,80%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>6,74%</td>
<td>η=1.05 a=2</td>
<td>4,85%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0,29</td>
<td>η=1.1 a=3</td>
<td>4,87%</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0,06</td>
<td>η=1.05 a=1</td>
<td>4,89%</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0,130</td>
<td>η=1.05 a=2</td>
<td>4,79%</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>1,154</td>
<td>η=1.05 a=3</td>
<td></td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 19,89% and 4,47% respectively for η=1.1 and 19,79% and 4,39% for η=1.05. The maximum drawdown of the Buy and Hold portfolio is 6,61% for η=1.1 and 6,34% for η=1.05.

Table 16: portfolio results with ETF Dax (after 8 years).

| Portfolio analysis in case of the risky asset is ETF Dax and with η=1.1 |
|---------------------------------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| a=1      | a=2      | a=3      | a=4      | a=5      | a=6      | a=7      |
| Maximum drawdown    | 6,77%    | 6,80%    | 6,83%    | 6,88%    | 6,94%    | 7,01%    | 7,10%    |
| Performance since inception | 25,09%    | 25,55%    | 26,00%    | 26,44%    | 26,89%    | 27,34%    | 25,66%    |
| Annual performance  | 3,14%    | 3,19%    | 3,25%    | 3,31%    | 3,36%    | 3,42%    | 3,21%    |
| Annual standard deviation | 4,90%    | 4,92%    | 4,95%    | 4,98%    | 5,02%    | 5,06%    | 5,25%    |
| Sharpe ratio        | 0,310    | 0,321    | 0,331    | 0,340    | 0,348    | 0,357    | 0,303    |
| Information ratio   | 0,042    | 0,045    | 0,048    | 0,051    | 0,054    | 0,057    | 0,043    |
| Sortino ratio       | 0,130    | 0,135    | 0,139    | 0,143    | 0,147    | 0,151    | 0,124    |
| Omega ratio         | 1,106    | 1,115    | 1,123    | 1,131    | 1,138    | 1,146    | 1,111    |

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 18,18% and 3,21% respectively. The maximum drawdown of the Buy and Hold portfolio is 5,33%.

I used as MAR the monthly free risk rate equal to 0,1335% for 5 years of investments.

---

54 I used as MAR the monthly free risk rate equal to 0,1335% for 5 years of investments.
Table 17: Portfolio analysis in case of the risky asset is ETF S&P500 and with $\eta=1.1$.

<table>
<thead>
<tr>
<th>Portfolio analysis in case of the risky asset is ETF S&amp;P500 and with $\eta=1.1$</th>
<th>a=1</th>
<th>a=2</th>
<th>a=3</th>
<th>a=4</th>
<th>a=5</th>
<th>a=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum drawdown</td>
<td>6.46%</td>
<td>6.43%</td>
<td>6.40%</td>
<td>9.08%</td>
<td>9.90%</td>
<td>&gt;10%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>37.40%</td>
<td>37.20%</td>
<td>35.81%</td>
<td>34.29%</td>
<td>32.61%</td>
<td>30.59%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>4.67%</td>
<td>4.65%</td>
<td>4.48%</td>
<td>4.29%</td>
<td>4.08%</td>
<td>3.82%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>4.32%</td>
<td>4.32%</td>
<td>4.33%</td>
<td>4.48%</td>
<td>4.78%</td>
<td>4.95%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.708</td>
<td>0.703</td>
<td>0.661</td>
<td>0.597</td>
<td>0.515</td>
<td>0.447</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0.171</td>
<td>0.169</td>
<td>0.154</td>
<td>0.134</td>
<td>0.109</td>
<td>0.087</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0.320</td>
<td>0.318</td>
<td>0.298</td>
<td>0.259</td>
<td>0.209</td>
<td>0.173</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>1.463</td>
<td>1.46</td>
<td>1.41</td>
<td>1.357</td>
<td>1.303</td>
<td>1.252</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 22.81% and 3.26% respectively. The maximum drawdown of the Buy and Hold portfolio is 5.60%.

Table 18: Portfolio analysis with gold ETF and US bond 7-10Y ETF (after 8 years).

<table>
<thead>
<tr>
<th>Portfolio analysis</th>
<th>Case of the risky asset is ETF gold</th>
<th>Case of the risky asset is ETF US Bond 7-10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>η=1.1</td>
<td>η=1.05</td>
<td>η=1.1</td>
</tr>
<tr>
<td>a=1</td>
<td>a=2</td>
<td>a=3</td>
</tr>
<tr>
<td>Maximum drawdown</td>
<td>8.96%</td>
<td>8.88%</td>
</tr>
<tr>
<td>Performance since inception</td>
<td>29.95%</td>
<td>27.75%</td>
</tr>
<tr>
<td>Annual performance</td>
<td>3.74%</td>
<td>3.47%</td>
</tr>
<tr>
<td>Annual standard deviation</td>
<td>6.62%</td>
<td>6.53%</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.322</td>
<td>0.284</td>
</tr>
<tr>
<td>Information ratio</td>
<td>0.063</td>
<td>0.051</td>
</tr>
<tr>
<td>Sortino ratio</td>
<td>0.146</td>
<td>0.131</td>
</tr>
<tr>
<td>Omega ratio</td>
<td>1.171</td>
<td>1.133</td>
</tr>
</tbody>
</table>

In case of a Buy and Hold portfolio its performance and its annual standard deviation are 22.17% and 3.77% respectively for η=1.1 and 22.04% and 3.77% for η=1.05. The maximum drawdown of the Buy and Hold portfolio is 6.61% for η=1.1 and 6.34% for η=1.05.
Figure 24: portfolio performance with Dax ETF $\alpha = 1$ and $\eta=1.1$ after 8 years and half of investment.

Figure 25: portfolio performance with Dax ETF $\alpha = 2$ and $\eta=1.1$ after 8 years and half of investment.

Figure 26: portfolio performance with Dax ETF $\alpha = 3$ and $\eta=1.1$ after 8 years and half of investment.
**Figure 27**: portfolio performance with Dax ETF $\alpha = 4$ and $\eta=1.1$ after 8 years and half of investment.

**Figure 28**: portfolio performance with Dax ETF $\alpha = 5$ and $\eta=1.1$ after 8 years and half of investment.

**Figure 29**: portfolio performance with Dax ETF $\alpha = 7$ and $\eta=1.1$ after 8 years and half of investment.
Figure 30: portfolio performance with Dax ETF $\alpha = 1$ and $\eta=1.05$ after 8 years and half of investment.

Figure 31: portfolio performance with Dax ETF $\alpha = 2$ and $\eta=1.05$ after 8 years and half of investment.

Figure 32: portfolio performance with Dax ETF $\alpha = 3$ and $\eta=1.05$ after 8 years and half of investment.
**Figure 33**: portfolio performance with Dax ETF $a = 4$ and $\eta=1.05$ after 8 years and half of investment.

![Figure 33](image1.png)

**Figure 34**: portfolio performance with Dax ETF $a = 5$ and $\eta=1.05$ after 8 years and half of investment.

![Figure 34](image2.png)

**Figure 35**: portfolio performance with Dax ETF $a = 6$ and $\eta=1.05$ after 8 years and half of investment.

![Figure 35](image3.png)
Figure 36: portfolio perf. with S&P500 ETF $a = 2$ and $\eta=1.1$ after 8 years and half of investment.

![Performance Graph](image1)

Figure 37: portfolio perf. with S&P500 ETF $a = 3$ and $\eta=1.1$ after 8 years and half of investment.

![Performance Graph](image2)

Figure 38: portfolio perf. with S&P500 ETF $a = 4$ and $\eta=1.1$ after 8 years and half of investment.

![Performance Graph](image3)
Figure 39: portfolio perf. with S&P500 ETF $\alpha = 5$ and $\eta=1.1$ after 8 years and half of investment.

Figure 40: portfolio perf. with S&P500 ETF $\alpha = 1$ and $\eta=1.05$ after 8 years and half of investment.

Figure 41: portfolio perf. with S&P500 ETF $\alpha = 2$ and $\eta=1.05$ after 8 years and half of investment.
Figure 42: Portfolio perf. with S&P500 ETF $\alpha = 3$ and $\eta=1.05$ after 8 years and half of investment.

Figure 43: Portfolio perf. with S&P500 ETF $\alpha = 4$ and $\eta=1.05$ after 8 years and half of investment.

Figure 44: Portfolio perf. with S&P500 ETF $\alpha = 2$ and $\eta=1.05$ after 8 years and half of investment.
Figure 45: portfolioperf. with Gold ETF $\alpha = 1$ and $\eta = 1.1$ after 8 years and half of investment.

Figure 46: portfolio perf. with Gold ETF $\alpha = 2$ and $\eta = 1.1$ after 8 years and half of investment.

Figure 47: portfolio perf. with Gold ETF $\alpha = 3$ and $\eta = 1.1$ after 8 years and half of investment.
Figure 48: Portfolio perf. with Gold ETF $\alpha = 2$ and $\eta = 1.05$ after 8 years and half of investment.

Figure 49: Portfolio perf. with Gold ETF $\alpha = 3$ and $\eta = 1.05$ after 8 years and half of investment.

Figure 50: Portfolio perf. with Gold ETF $\alpha = 4$ and $\eta = 1.05$ after 8 years and half of investment.
**Figure 51**: portf. perf. with US 7-10Y Bond ETF $\alpha = 1$ and $\eta = 1.1$ after 8 years and half of investment.

![Figure 51](image)

**Figure 52**: portf. perf. with US 7-10Y Bond ETF $\alpha = 2$ and $\eta = 1.1$ after 8 years and half of investment.

![Figure 52](image)

**Figure 53**: portf. perf. with US 7-10Y Bond ETF $\alpha = 3$ and $\eta = 1.1$ after 8 years and half of investment.

![Figure 53](image)
**Figure 54:** portf. perf. with US 7-10Y Bond ETF $a = 4$ and $\eta=1.1$ after 8 years and half of investment.

**Figure 55:** portfolio perf. with US 7-10Y Bond ETF $a = 2$ and $\eta=1.05$ after 8.5 years of investment.

**Figure 56:** portfolio perf. with US 7-10Y Bond ETF $a = 2$ and $\eta=1.05$ after 8.5 years of investment.
Figure 57: portfolio perf. with US 7-10Y Bond ETF $a = 2$ and $\eta=1.05$ after 8.5 years of investment.
References


[www.bloomberg.com](http://www.bloomberg.com)

[www.investopedia.com](http://www.investopedia.com)