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Volatility as an Emerging Asset Class

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Notation

AR- Autoregressive model

ARCH- Autoregressive Conditional Heteroskedasticity

BOE-Bank of England

BOJ- Bank of Japan

B/S- Balance Sheet

CBOE- Chicago Board of Options Exchange

CFE- Chicago Futures Exchange

ECB- European Central Bank

ETP- Exchange Traded Product

ETF-Exchange Traded Fund

ETN- Exchange Traded Note

EWMA- Exponentially Weighted Moving Average

DJIA- Dow Jones Industrial Average

Fed- Federal Reserve Bank

FOMC- Federal Open Market Committee

IASB- International Accounting Standards Board

IMF- International Monetary Fund

GARCH- Generalized Autoregressive Heteroschedasticity

GDP- Gross Domestic Product

GFC- Global Financial Crisis, 2008

LLC- Limited Liability Company

LSE - London Stock Exchange

LTRO-Long Term Refinancing Operation

M&A- Merger and Acquisition

MBS- Mortgage-Backed Security

NAV- Net Asset Value

NYSE- New York Stock Exchange

OEX- S&P 100 Index

OTC- Over-the-Counter Market

PBOC- People's Bank of China

SIFI- Systemically Important Financial Institutions

SV- Stochastic Volatility model

SSC- South Sea Company

SPX- S&P 500 Index

SX5E- Euro Stoxx 50

TGARCH- Threshold Generalized Autoregressive Heteroschedasticity

QE- Quantitative Easing

VaR- Value at Risk

VIX- Volatility S&P 500 Index

Vols.- Implied Volatility

VOV- Volatility-of-volatility

VRO- VIX Options Settlement Index

VVIX- Volatility of VIX index

VXO- Volatility S&P 100 Index

VXX- iPath S&P 500 VIX Short-Term Futures ETN

VXZ- S&P 500 VIX Mid-Term Futures ETN

V2TX-VSTOXX-EURO STOXX 50 Volatility

Introduction

The underlying idea for this paper sprang up from an interesting academic literature on the volatility as a self-standing asset class. Traditionally, the volatility has been perceived as a risk measure of an investment, nonetheless in the last two decades financial engineering has offered individual and institutional investors a new asset on which to trade, speculate, hedge or diversify their portfolio. In fact, an investor can trade on volatility through a wide range of financial products both traded on exchange boards and OTC. Since the origins of stock exchanges, the volatility of an asset represented a huge risk, thus the investors continued to diversify their portfolio through a selection of different asset classes to minimise the underlying correlation among them. This traditional paradigm weakens as soon as the correlations among different asset classes reduce, which very often happens during the market crashes and produce simultaneous losses. Since volatility is the only parameter which high spikes during markets collapses, a long position in the volatility itself would reduce such losses by generating profits, and thus qualifying the pure volatility exposure as an efficient asset to hedge and diversify the portfolio.

Moreover, post-GFC financial markets have been characterised by bull market behaviour resulting in great returns over the last 5 years until May 2015 and the lowest historical volatility levels, nonetheless the volatility of volatility has remained consistently high. At the same time, the main central banks have increased their monetary supply to the highest levels ever seen in the history. As a result, more economists have sustained the idea that Fed monetary policies had significantly contributed to the stock market blooming in US and consequently to artificially lower the spot volatility. However, the investors' fear of a possible future collapse did not disappear, on the contrary the implied volatility premiums increased significantly. To prove this intuition the author of this paper has analysed different aspects of volatility and its derivative products to produce some interesting results.

The first chapter of this paper opens with a brief historical excursus paragraph on stock market exchanges as the first place, where markets' volatility

represented an important source of risk. Afterwards, some different volatility estimation methodologies are analysed and compared. The core of this chapter consists in the introduction of two main topics on volatility, respectively discussed in the next two chapters. Firstly, the central banks monetary responses to the global crisis are analysed and simultaneously compared to the stock market/spot volatility changes. Secondly, traded SPX volatility index, VIX as well STOXX 50 volatility index VSTOXX are presented and the negative correlation between stocks' and their volatility indexes is investigated. Thus, in order to understand the "fear gauge" VIX reputation, the mechanics of CBOE's new and old VIX derivations are presented. As it will be explained, the VIX represents 30-days future S&P 500 implied volatility. The derivation of such volatility indexes required a deep analysis of the existing OTC variance and volatility swaps.

In the second chapter the global economic situation from 2008 as well the responses of the main central banks to GFC are discussed. In fact, the central banks have decreased interest rates to zero and afterwards expanded their balance sheets to stimulate the economic recovery from recession. From the author's analysis two different situations delineated: one in United States and the second in European Union. After a huge QEs monetary injection, the United States showed a significant bloom in financial markets as well historically lowest VIX levels, but at the same time GDP and even more unemployment rate presented significantly lagging recovery. In EU sovereign debt crises in 2011 and once again Greek sovereign issue in 2015 truncated the economic recovery, whereas the unemployment rate is still far from being improved to pre-crises levels, nonetheless the ECB intervention decreased significantly sovereign bonds yields. The research shows that central banks policies have synthetically decreased the volatility indexes level and boosted financial markets performance rather then real economy; much more significant results emerged in US, since in Europe QE will last at least until September 2016. In the last paragraph the VIX performance is compared to VSTOXX and some interesting observations are presented.

As previously mentioned, after having proved that the spot volatility has been artificially kept low by monetary policies, in the last and final chapter some other topics on volatility indexes are presented. Firstly, one might have thought that investing in volatility could be an easy subject, however it is much more complex than trading on a stock index. The exposure on VIX can be obtained through VIX futures, options and other more structured traded products such as ETNs and ETFs. The main difference is that a stock index ETF can replicate its underlying index, whereas VIX cannot be replicated in the same way. Secondly, the cost of hedging through the ETPs on VIX is too expansive because of VIX mean reversion characteristics, term structure and high volatility of volatility. In fact, as a result of the lowest VIX levels, futures market has been trading continuously in contango, which produced significant losses in value for VIX replicating short-term structured products. In the last paragraph, modern and some possible future scenarios for volatility are explained. Nowadays the investors are nourishing a profound fear of a possible market collapse, which emerges from their willingness to pay the highest risk premiums in the last two decades. Lastly, a perspective on hyperinflation as a possible future scenario is presented.

Chapter 1

Historical Excursus

Volatility of financial markets is a concept that historically was strictly connected to the stock exchange: in our imagination the stock exchange is NYSE par excellence. Obviously, the risk/return trade-off theory would not exist without a physical place where the investors could exchange assets. However, the modern and complex stock exchange we know today was far different from the first stock exchanges, which were prospering without a single stock de facto being traded.

In the 1300's, the Venetian moneylenders were experts in such a field. These lenders traded loans with different risk/interest profiles and government debts (not only Venetian but also other governments' securities) among each other. As this activity advanced, the lenders commenced to sell debt securities to the first well-being investors as brokers nowadays do.

As far as 1531, the first stock exchange "Sans the Stock" was established in Antwerp[1]. The reason for such a name is straightforward; no stock[2] was physically exchanged, still it represented a marketplace where brokers and lenders were meeting to trade and conclude deals.

In the 1600's, the imperial governments of France, Britain and Netherlands chartered East India Companies to travel East Indies and Asia[3]. The sea voyages were excessively risky because of Barbary pirates, bad weather and poor navigation. To limit these risks investors were investing in different Limited Liability Companies labelled with East India denomination. These LLCs were regularly created for a single voyage and at the end dissolved, while a new LLC was normally instituted for the next voyage. Every investor owned a share on the trades profit proportional to the investment made. As this business further developed, the East India Companies issued stocks that produced dividends on the revenues from all the voyages the companies undertaken, no more on a single voyage. Thus, the size of the companies further grew assisted by the

[1] Belgium.

[2] They dealt in promissory notes (individual debt issues) and government bonds. Source: http://www.investopedia.com/articles/07/stock-exchange-history.asp

[3] As well Portugal and Spain dominated in this field.

royal charters forbidding competition, which insured investors with colossal profits. From 1602 to 1650 the annual average dividends received by the investors fluctuated slightly over 16%.

The Dutch East India Company is recognised as a first multinational corporation to issue stocks and to establish the Amsterdam Stock exchange in 1602 in order to trade its securities[4]. Despite the fact that the Amsterdam Stock Exchange is considered to be the first official stock exchange, some historians argue that it was first only in terms of volume, liquidity, publicity and "speculative freedom of transactions"[5]. In 1688, the anticipatory book *Confusion of Confusions* on stock trading of Joseph de la Vega[6] has been published. From the description emerges a sophisticated but at the same time inclined to excesses market, where market volatility is unpredictable[7].

A further diffusion and trade of East India companies' stock papers bloomed in London. Especially popular, the trades were conducted in Coffee Houses along the Exchange Alley[8]. Nonetheless, the first Royal Exchange was established in 1571 by Thomas Gresham, by following the Antwerp Bourse model, the stock traders were not allowed to deal in there because of their rude manners. Only in 1669, the new Royal Exchange was rebuilt after a Great Fire of London to move trades from coffee houses to a modern stock exchange, which did not success because of a great number of unlicensed brokers. The trading business in Coffee shops further grew during the King William's Nine Years' War (1688-1697)[9], when the Kingdom issued government bonds to finance wars. Since most of the newly established joint-stock companies were designed through public-private partnerships to reduce the cost of sovereign debt, there was a total lack of regulations on the shares issuance. The demand for British East India Companies stocks increased exponentially such that the stocks were selling for a fortune: indeed everyone was eager for a piece of paper paying

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[4] J. Penso de la Vega (1668), *Confusión de Confusiones*, reprint Wiley, 1996.

[5] F. Braudel (1983), *Civilization and capitalism 15th–18th century: The wheels of commerce*, New York, Harper & Row.

[6] Known as Joseph Penso.

[7] Retrieved from https://en.wikipedia.org/wiki/Stock_exchange

[8] Situated in an old neighbourhood of London, connecting coffe shops and shops.

[9] Also known as the First Intercontinental War, Second Indian War or War of the Grand Alliance frightened in the North America.

huge dividends. Sooner or later this “irrational exuberance”[10] would end up with a financial bubble.

In 1711 South Sea Company (SSC) was established with a granted monopoly to trade with South America. As soon as the company was listed the stocks were sold and re-issued again. Nonetheless there was no reasonable chance that the trade would turn up or that the profit from the monopoly would be realized[11], the company used the newly funded investments to open exclusive offices in the finest parts of London. Obviously, the price for these stocks rose considerably and even more when the company enlarged its activities trading in government debt. The peak was reached in 1720 before the company missed to pay dividends of its meager profit compared to other lucrative British East India companies and the South Sea Bubble burst: stock price crashed slightly above its original trading price. To worsen the investors’ ruin, there were other “businessmen”[12] inspired by the success of SSC offering new shares in their own fraudulent projects. They were even promising shares in ventures of such colossal importance that it could not be acknowledged to the investor, which increased their expectations. Undoubtedly, all of these “blind pools” of stocks were sold and the following collapse forced the Parliament of Great Britain to ban all not authorized joint-stock companies by royal charter to issue stocks through the Bubble Act 1720, which was held in place until 1825. These first lucrative and fraudulent activities represented only the beginning of the speculation, bubbles and the successive collapses in the financial markets as it still exists today and the history continuously repeats itself.

As a consequence to the Bubble Act, the stock trading became narrower and was repressed in the next decades. However, the market withstand with the shares being traded in New York on the 11 Wall Street, NYSE. For two next centuries NYSE maintained its primate until the Great Depression left its scars on it too.

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[10] Shiller R.J.

[11] The only profitable trade involved slaves trade, but it still resulted in being unprofitable.

[12] Retrieved from: http://www.investopedia.com/articles/07/stock-exchange-history.asp

During 20th century London Stock Exchange (LSE) emerged in Europe, although many international corporations preferred to list in New York. Many other countries such as Japan, Hong Kong, Australia, Canada, Germany, Switzerland, Holland and France established their own stock exchanges mainly for the domestic companies. Nowadays, NYSE Euronext as well Nasdaq represents the benchmark to the worldwide indexes performance.

As financial engineering evolved not only equity stocks have been actively exchange traded but also a wide range of the underlying assets broadened. Nowadays you can trade on stock indexes, precious metals, commodities, currencies, bonds, interest rates, even volatility, moreover you can use derivatives such as futures and options on these wide ranges of the underlying assets.

Financial Volatility Estimates

Volatility is defined as a measure of risk or uncertainty about the magnitude of the variation of an asset's value. A higher volatility means that the securities price might fluctuate over a wider range of values over a short period in either direction, instead a lower suggests a gradual variability over a given period of time without any sudden price changes.

According to some papers, the stock markets volatility is associated with time-varying volatilities of the paramount economic variables such as consumption, money supply, business cycle and capital investments. Thus, it might be inferred that the stock market volatility mirrors the uncertainty about the future economic development and at the same time, in reverse, the expectation of future macroeconomic policies influence stock volatility changes[13].

That is the reason why in this paper the central banks' economic monetary policies and economic performances are analysed to determine their effects on the financial market performance and its derived volatility. As a risk measure, volatility reflects uncertainty in the market (e.g. Tech and Real Estate bubbles, GFC), which is expressed through extremely high volatility. As well the information arrival is essential, the news about the expected profits or losses

[13] Liu J. (May 15, 2013), *Essays on Return and Volatility on World Stock Markets*.

might have more or less immediate impact on volatility: the news effect is higher for small stocks, but it is smaller for instance on stock indexes. Moreover, it is worth mention that different asset classes might present different variation in percentage terms: the riskier countries present higher volatility on their currencies and sovereign bonds, but still it is much lower compared to individual stocks. All of this is perfectly explained through risk/return tradeoff theory.

In financial statistics volatility is the standard deviation of the asset returns: given an expected return on the asset (its mean), the volatility measures the deviation from that expected return. In order to measure market risk, VaR or simply to derive Black Sholes option price it is necessary to estimate or forecast volatility parameters. In the literature has been discussed various volatility models, each appropriate to capture stylised volatility features. Each of these models will be briefly analysed as following:

Historical volatility is the simplest model, which involves variance or volatility estimation of the historical returns to apply to future periods. This variance model was traditionally used for evaluation of option prices, albeit more and more financial practitioners and academics prefer more sophisticated time series volatility models to evaluate options price. Nonetheless, this method is still robust as a benchmark to compare the forecasting power of the other more complex models.

Implied volatility model is used to estimate or forecast the intrinsic volatility of an option. In a traded option the volatility of the underlying asset is determined from the available contract and market data such as option price, the time to maturity, the strike price, the risk free interest rate, spot price of the underlying asset[14]. Normally, the numerical procedure applied to derive the implied volatility is the Newton-Raphson bisections method[15]. The market forecast of the underlying asset returns volatility is expressed by the implied volatility over the lifetime of the option[16].

[14] Canina L., and S. Figlewski (1993). *The Information Content of Implied Volatility*. Review of Financial Studies, No .6, pp. 659-681.

[15] Watscham and Parramore, 2004.

[16] Brooks, C. (2014), *Introductory Econometrics for Finance*, 3rd Edition.

Exponentially weighted moving average (EWMA) model basically extends the average historical volatility by allowing the recent observations to have a more substantial contribution to the volatility forecast compared to the old data. The weights assigned to the previous observation declines exponentially over time, which reflects the empirical evidence that the old volatility might be less relevant than more recent data to explain present or future volatility. This approach improves distortions the simple historical volatility model generates. For instance, when a volatility shock falls out of the measurement sample, the volatility estimate might suddenly change. On the other hand, if the volatility shock is still included in a large sample, the future forecast might remain artificially high even though the market has calmed down. The EWMA variance model can be expressed through the following notation:

$\sigma_t^2 = (1-\lambda)\sum_{j=0}^{\infty}\lambda^j\,(r_{t-j}-\bar{r})^2,$

where $\bar{r}$ is the average estimated return and λ is the "decay factor" indicating the weight assigned to the recent observation versus old ones. From the Risk Metrics studies λ is set to be 0.94, nonetheless it could be estimated. Since the underlying assumption assumes the average return to be equal to 0, which is a reasonable assumption on a daily basis frequency, but for higher frequency data it might lead to the significant loss of accuracy. Obviously, this model gives the highest weight to the last observation of the time series. As well EWMA model has its drawbacks: firstly, the infinite sum in the equation will be truncated at some fixed lag with a finite sum of the available historical data and the summation will not sum to one, which will require a correction for small data samples. Secondly, the EWMA does not own the property most of the time series models such as GARCH do, namely the tendency to the unconditional variance as the prediction horizon of the series increases.

From stochastic volatility specifications class, the *autoregressive volatility models* present the simplest examples. The *ARCH (q)*, the *autoregressive conditional heteroschedasticity* model[17], is a non-linear model, which accounts for "volatility clusters" and "volatility pools". The main idea behind is that volatility normally occurs through clusters, a small volatility is followed by small

[17] The errors variance is not constant over time.

fluctuations and high volatility outbreaks are pursued by high price changes. Thus, the present volatility level is positively correlated directly to the previous observations. The ARCH and GARCH models own a good property that the long run volatility forecasts tend to the unconditional variance, while most of the volatility series are "mean-reverting". The model is formulated in a general form as following:

$y_t = \beta_1 + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4} + u_t$, with $u_t \sim N(0, h_t)$

$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \cdots + \alpha_q u_{t-q}^2 + e_t.$

Where the first equation is the AR (4) mean equation and the second represents the conditional variance equation. Obviously, h_t has a strictly positive value since variance cannot take negative values[18]. Nonetheless in the last decade this model has rarely been used as ARMA presents several drawbacks. The question arises on the number of the squared error lags q to be chosen; if the number of q required is too large the model might be not parsimonious and produce large conditional variance. Moreover, the more parameters are estimated into the model, the more likely the non-negative constrains might be violated. Some of these problems are overcome in GARCH model, which is universally applied in financial practice.

In *GARCH (p,q)*[19] model, *generalized autoregressive heteroschedasticity*, the present conditional variance equation depends upon the previous observations variance. The generalized form of the model is expressed as following:

$y_t = \beta_1 + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4} + u_t$, with $u_t \sim N(0, h_t)$

$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \cdots + \alpha_q u_{t-q}^2 + \beta_1 h_{t-1} + \cdots + \beta_p h_{t-q}.$

Provided all the relevant and available past information the conditional variance is estimated one-period ahead. The interpretation of h_t is straightforward: is a weighted function of a long-term average (α_0 dependent), previous period squared errors volatility $\alpha_1 u_{t-1}^2$ and fitted conditional variance from the previous lag $\beta_1 h_{t-1}$. In the literature GARCH is often told to be ARMA model for conditional variance. After several mathematical rearrangements GARCH in fact

[18] Non-negativity constraints must be satisfied: $\alpha_i \geq 0,\ \forall i = 0,1,2 \ldots, q$.

[19] Bollerslev, Taylor, 1986.

can be expressed as a restricted infinite order ARCH model[20]. However, in practice the only GARCH (1,1) is widely employed. With only three parameters to estimate this model is extremely parsimonious and most of the times it is sufficient to capture the "volatility clustering". As well in this case the non-negativity constrains must be satisfied: the unconditional variance is only defined as long as $\alpha_1 + \beta < 1$.

Many extensions to the basic GARCH model have been developed. Since the GARCH implements a symmetric volatility feedback to positive or negative shock, it does not take into account behavioral finance as the strong magnitude of negative shocks is linked to falling markets, whereas low volatility is linked to gradually rising markets. The main models that account for this asymmetry are GJR[21], TGARCH[22], EGARCH,[23] GARCH-in-mean[24] models. GJR and TGARCH models introduce a dummy variable in the conditional variance equation to account for possible asymmetries. EGARCH offers a few advantages over the original GARCH: since the $ln\ (h_t)$ is modeled, the non-negativity constrains will be always satisfied. GARCH-in-mean model includes the conditional variance of asset returns into the conditional mean equation. The GARCH-type models are mostly employed to forecast future volatility of the underlying asset to price plain vanilla options over its lifetime.

The univariate nature of the above examined models pose limitations to the so called "volatility spillovers" effect. In fact, by observing different markets or assets it might be inferred that volatility changes tend to spread from one market or asset to another. Moreover, very often it might be interesting to determine covariances between series: most of the models such as portfolio hedging, VaR estimates, CAPM betas, hedge funds alfas require covarinces as input. But multivariate extensions to GARCH models can overcome all these complications and can be used in joined volatility modeling and forecasting as well in the conditional covariance estimation. To this regard, in the literature the

[20] Brooks, C. (2014), *Introductory Econometrics for Finance*, 3rd Edition.
[21] Glosten, Jagannathan and Runkle, 1993.
[22] Threshold GARCH.
[23] Exponencial GARCH model, Nelson, 1991.
[24] Engle, Lilien and Robins, 1987.

most popular multivariate GARCH (MGARCH) models are VECH[25], the diagonal VECH and the BEKK[26] models.

Stochastic volatility (SV) models differ from GARCH-type models by enclosing a second error term in the conditional variance equation and are largely used in financial literature option pricing. The Black and Scholes constant variance assumptions are hardly realistic since deep "in-the-money and far out-of-the-money options are relatively underpriced to the actual traded prices"[27]. As a result of this empirical observation, the stochastic volatility models have been developed, where "the logarithm of an observable variance is modeled by a linear stochastic specification such as an autoregressive model"[28]. On one hand these models can be considered as a discrete time approximation of the continuous time models, which are used in option pricing framework. On the other, these models are complex to estimate, especially in case of multivariate SV models, thus GARCH models are preferred since MLE[29] estimation is relatively simple. Among these models it is worth to recall the basic model with geometric Brownian motion and the solution of its differential equation with constant volatility, which present a starting point for Black Sholes and Cox-Ross-Rubinstein model. Moreover, Heston, CEV[30], SABR[31], 3/2 and Chen[32] models are widely used in the option pricing literature.

The mathematician Mandelbrot B. in its bestseller *Misbehavior of Markets, A Fractal View of Risk, Ruin and Reward* has presented an interesting intuition on volatility estimation. He proposes an innovative intuition on the volatility estimation and market risk evaluation through the fractal geometrical approach.

Moreover, he proposed an interesting critical overview by proving the misbehavior of the modern financial market.

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[25] Bollerslev, Engle and Wooldridge, 1988.
[26] Engle and Kroner, 1995.
[27] Brooks, C. (2014), *Introductory Econometrics for Finance*, 3rd Edition, pp.462.
[28] Ibid.
[29] Maximum Likelihood Estimation.
[30] Constant Elasticity of variance model, which is properly a local volatility model.
[31] Stochastic Alpha, Beta, Rho model, which reproduce volatility smile effects.
[32] Lin Chen, 1994.

Volatility and Economic Policies

Do really the financial markets behave as an "impossible object", which play along with investors' perception? Is it really a mysterious object, which limit our naïve perception and make us perceive something possible or true when it is definitely not? According to Christopher Cole's view, the "financial markets are a game of impossible objects" and their volatility is nothing else than our own changing perception[33]. From his critical attitude emerges the modern paradox of an economy where cost of the risk is distorted and manipulated by global central banks. He states that the traditional efficient market frontier theory does not exist anymore and that financial markets we see are not what we think. Nonetheless, why volatility is such an important issue? Firstly, volatility measures risk by displaying investors' reliance on markets' representation of the future economy. Secondly, the financial markets have acquired a high level of abstraction by becoming itself an economic reality, which often might be disconnected from the real world: the reason why volatility can be itself an asset class, which is somehow untied from the market fluctuations.

If you try to believe that an impossible object can be compared to the financial markets, remember not to trust to your common sense because risk-free assets are not risk-free, fear might be the best reason to buy, volatility may be cheap and expensive at the same time and lastly but not least importantly, remember that "knowledge is not what we know but certainty in what we do not"[34].

From 2007, 16 of the central banks have performed quantitative easing to support the economic instabilities. In this way the central banks have artificially supported the performance of risk assets and volatility of the financial markets. The main concern is not about how well the markets are performing during the monetary policy, but what happens immediately thereafter. The investors perceive the central bank message in a very simple way: when central banks are printing money volatility declines, risk assets returns increase and when they stop printing, it is better to get out from there.

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[33] Artemis Capital Management, LLC (2012), *Volatility of Impossible Object- Risk, Fear, and Safety in Games of Perception*, pp. 2.

[34] Ibid.

The money creation machine has reached its astonishing level of US $9 trillion in the first quarter 2012[35]. Nonetheless, from the past in perspective it was only an inception of the global monetary stimulus. After QE1, QE2 and Operational Twist in US and 2012 ECB's LTRO, the balance sheets of the Central Banks have further exponentially increased. The total assets for major 5 central banks accounted for approximately $US 16.5 trillion in July 2015[36]. In the following graph the nominal assets amount of Fed, ECB, PBOC (People's Bank of China), BOE (Bank of England), and BOJ (Bank of Japan) are presented. The nominal amount has been converted from the original currency into $US. In order to do so an annual average exchange rate has been applied to each year of the Balance Sheet historical data[37].

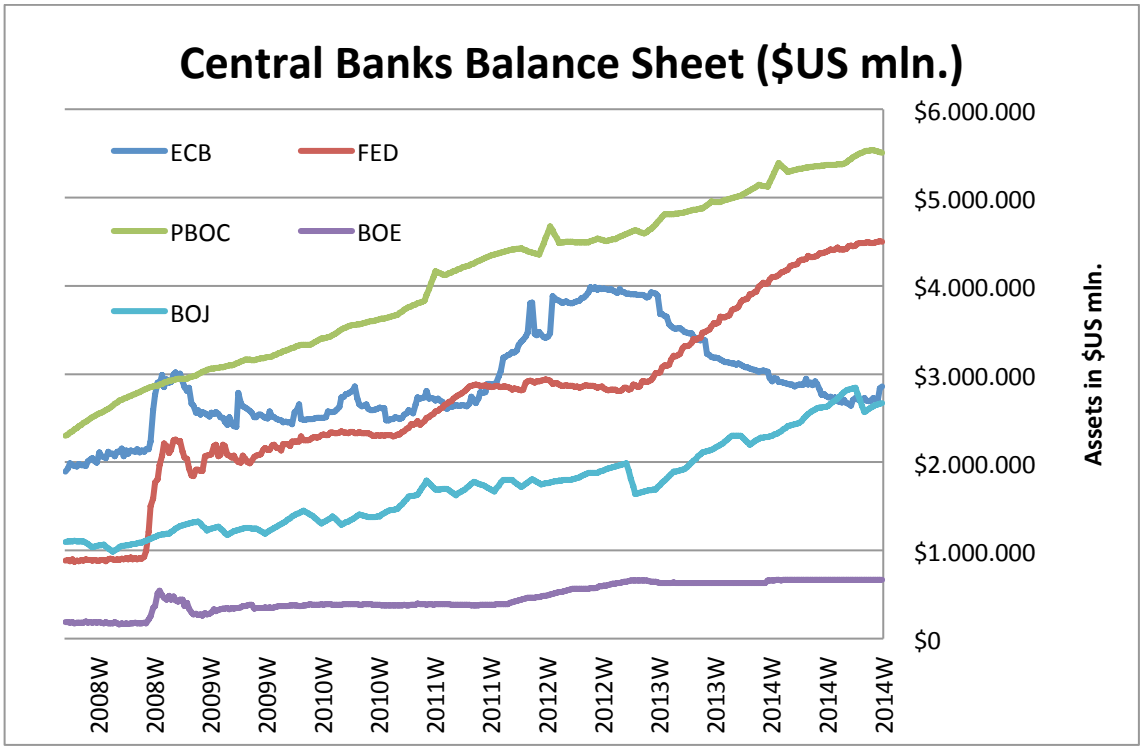

Figure 1. Central Banks Balance Sheets expansion.

The evolution of monetary stimulus in last 6 years is impressive, not only because all of them doubled or triplicated their balance sheets, but also the interest rates have been decreased to the lowest historical level, almost near

[35] Hilsenrath J., Blackstone B., *Central Banks Diversify their Arsenals: Deploying Unconventional Balance-Sheet Techniques from Bank Loans to Securities Purchases*, Wall Street Journal, January 26, 2012.
[36] Fed, ECB, Bank of Japan, Bank of England and People's Bank of China.
[37] Source: Fed, ECB, BOJ, PBOC and BOE.

zero. Fed Balance Sheet increased by more than 400%, ECB first doubled assets and then decreased to 140% of 2008 in last two years (before QE in 2015 has been announced[38]), PBOC has doubled its assets, BOJ has tripled its Balance Sheet and finally BOE quadrupled its initial Balance Sheet.

The reaction of the financial markets to the monetary stimulus is astonishing; nonetheless it is even more astonishing when it finishes. Therefore, during each monetary expansion of the Fed Balance Sheet the performance of SPX improved significantly and VIX became smoother and calmer. The main purpose of QE1 in July 2009 and QE2 in September 2010 was to recover the flooding US economy and boost economic growth after the recession. But as soon as the monetary stimulus was ending the markets were becoming nervous and VIX was flash spiking each time. In order to understand the cumulative effect of such a stimulus Fed has conducted a research concluding that since 1984 an astonishing 80% of the premium earned from domestic equity markets was accomplished during the periods leading up to FOMC announcements[39]. Fed has continued with QE program until 29 October 2014 after having purchased $4.5 trillion in assets. Nowadays, the uncertainty in the US market is still high: the Fed is going to increase interests rates after a period of the lowest near zero federal funds rates. In the following graph, Fed expansion Balance Sheet, QE1, QE2, Operational Twist (OT) as well QE3 are summarised in contraposition to VIX. After Lehman Brothers bankruptcy, VIX spiked by 400% as subprime securities contagion seemed to be able to hit systemically other financial institutions. After QE1 has been announced, the investors calmed down although the future uncertainty was still too high. As soon as the first stimulus finished, without Fed BS support it can be observed a flash crash of SPX and immediately VIX spikes. The same behaviour has been observed as well during 11 August 2011 crash, after the second QE (second yellow area in the graph) round has been terminated. Once again, the volatility increased in October 2014 when the Fed purchases were halted.

[38] More updated analysis to 2015 will be conducted in Chapter 2 of this paper.

[39] Lucca D., Moench E. (September 2011 and revised June 2012), *The Pre-FOMC Announcement Drift*, Federal Reserve Board of New York Staff Reports no. 512.

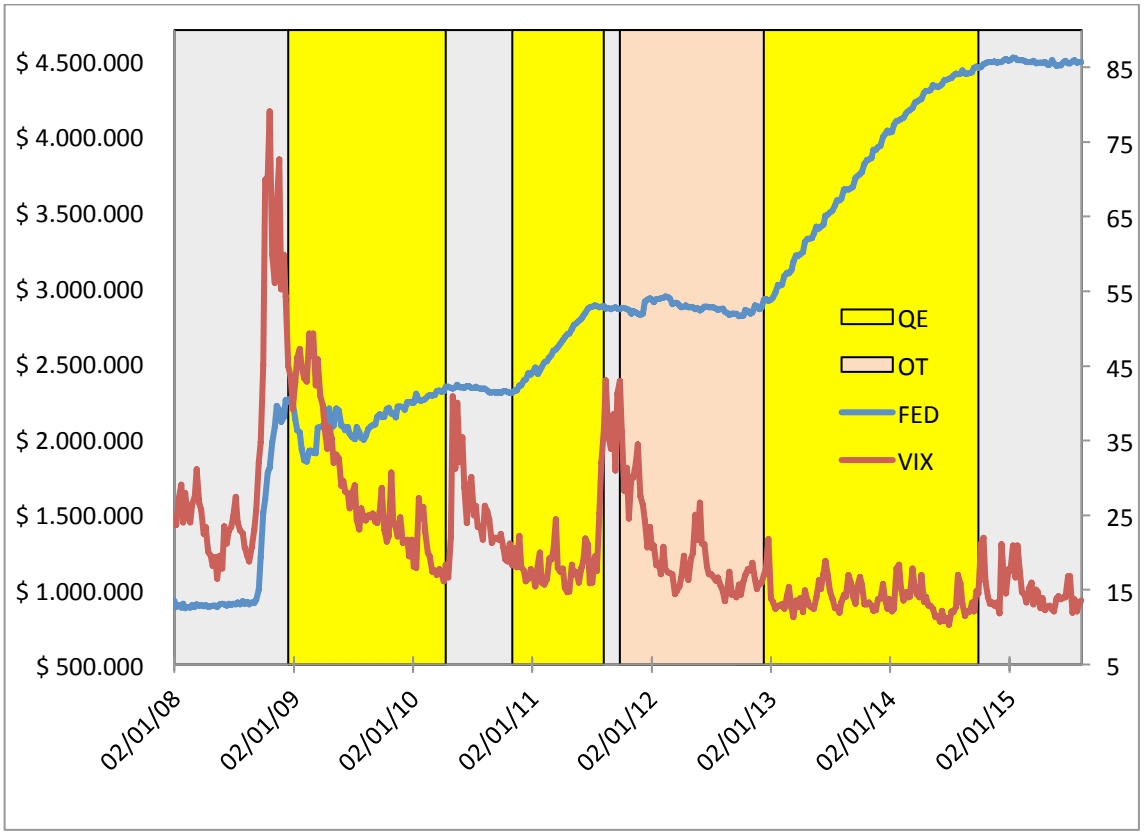

Figure 2. Fed expansion trough QEs and OT versus VIX index.

The financial markets seem really having lost their reliance on fundamental economic growth by reclining on the illusion of central banks "backed-prosperity"[40].

The same irrational behaviour of the financial markets is observed in Europe. During GFC in late 2009, the European countries had to face high-level deficits and growing debt level. The investors' concern on the sustainability of such high level of sovereign debt sharply rose day by day. Although countries such as Greece, Portugal, Ireland, Spain and Cyprus presented the biggest burden, the investors panicked and fear increased concerns over the solvency issue as well of the other European countries. The result was almost apocalyptical: interest rate spread on the sovereign bonds rose to the unbelievable high levels. Greece was on the edge of default with interest rates touching a surreal 29.2% average interest rate[41] on February 2012 and a historical record of 37.1% on

[40] Artemis Capital Management, LLC (March 30, 2012), *Volatility at World's End: Deflation, Hyperinflation and the Alchemy of Risk*, Letter to Investors from Artemis Capital Management, LLC.

[41] Source: ECB.

10-year traded sovereign Greek bonds[42]. The solution obviously arrived with a bailout programme for 4 European countries such as Greece, Portugal, Ireland and Spain financed jointly by so called "Troika"[43]. Through these emergency measures Greek bailout reached €245.6 billion between May 2010 and June 2015[44]. Still, Greece did not reach the economic targets pointed out by its creditors: for the first time a developed country missed a payment deadline to IMF in June 2015. The new commitment of bailout programme has been negotiated and approved on 17 July 2015. As previously mentioned, the ECB has undertaken the same policy line as conducted by Fed by expanding its Balance sheet: it introduced a generous three year LTRO in order to reduce high sovereign yields in the Euro-zone. The amount of the funds allocated through this refinancing operation touched €1 trillion since December 2011. In fact, in 2012 the ECB Balance Sheet has reached an impressive 40% of the Euro-zone GDP[45].

In the European case a question remains: why the ECB solution of extra leveraging Euro-Banking system to the same sovereign debt that originated the crisis, should solve anything in the long-term? Nonetheless, it has undeniably succeeded in soothing the markets and the sovereign debt crisis in the short-run, at least until 2015 when the worst seemed to be over, once again the Troika should intervene with a third round of Greek bailout.

The situation in the emerging economies is not significantly better than in US and EU. The situation in China and its financial market significantly worsened after announcement of economic slowdown.

The reputation and credibility are the only tools the Central Banks can claim to exercise the monetary policy. The challenging question is how long these "Gods of Risk"[46] can operate their monetary policies without being thrown down their thrones? Do really the monetary policy is the solution to calm down financial

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[42] On March 2, 2012.

[43] Sovereign debt bailout creditors: International Monetary Fund, European Commission and European Central Bank.

[44] € 215.9 out of 245.6 billion has been paid; the remainder amount is frozen depending on successful settlement of the third program negotiation.

[45] Euro-zone GDP in 2012 amounted €9.5 billion, whereas ECB Balance Sheet €3.9 billion.

[46] Artemis Capital Management, LLC (2012), *Volatility of Impossible Object- Risk, Fear, and Safety in Games of Perception*, pp.4.

markets, to boost full employment and bailout insolvent countries? The answer is probably not in the long run, although as long as the investors believe in central banks credibility and the illusory reduction of market risk, the conflicting reality of bull markets and investors' fear will coexist.

Traded Volatility Indexes

In the last few decades, financial engineering indulged its fantasy and abstraction in creation of a wide range of financial derivatives on a new asset class such as volatility itself. Nowadays, the volatility is not only a simple statistical measure, but also an asset on which you can speculate, trade and hedge your portfolio. An investor having some insights on the direction of future level of volatility might take a certain position on pure volatility. The reason why he/she cannot simply invest in options is that they provide the exposure not only to the volatility but also to the direction of the underlying price. Obviously, this can be adjusted through a proper Black-Scholes delta hedging; nonetheless in the real market many of the underlying assumptions are violated. Thus, the easiest way to trade volatility is presented by OTC volatility swaps or volatility derivatives or ETPs on volatility indexes.

In 1993, the Chicago Board of Options Exchange (CBOE) introduced the Volatility Index (VIX). This "fear gauge" index is considered by many to be the world's premier benchmark of investors' sentiment and market volatility. The original construction of VIX used at-the-money options implied volatility[47](vols.) on the S&P 100 Index (OEX) with strike prices close to the current spot index level and maturities interpolated at 30-days. Implied volatility is regarded as forecast of consequent realised volatility and an indicator of market stress level[48].

In September 2003, CBOE together with Goldman Sachs renewed the original VIX Index and back-calculated to 1990 based on historical option prices. The new index replaced OEX to S&P 500 (SPX) index as underlying stock index. Furthermore, the expected volatility was calculated by averaging the weighted

[47] Computed as an average of the Black and Scholes (1973) options implied volatility.
[48] Whaley, 2000.

prices of SPX puts and calls over a wide range of strike prices[49]. Thus, CBOE supplied a new methodology to the investors to replicate volatility exposure with a portfolio of SPX options, which transformed VIX from an abstract concept into a practical standard for trading and hedging volatility[50]. The new development for VIX arrived in 2014, when SPX Weeklys series were included.

Weekly options were first introduced in 2005 and now are accessible on hundreds of indexes, equities, ETF, ETNs and have become an actively traded risk management tool. Today, SPX Weeklys account for one-third of all SPX options traded, and average over a quarter of a million contracts traded per day[51]. Thanks to SPX Weeklys, the VIX Index is calculated with S&P 500 Index option series more precisely than VIX Index is intended to present. In fact, VIX is matching the 30-day timeframe, but if SPX options with more than 23 days and less than 37 days to expiration are used, this secures that the VIX Index will always mirror an interpolation of two points (23 and 37 in between 30-days) along the S&P 500 volatility term structure.

On March 24, 2004, CBOE introduced VIX Futures trading by launching a new exchange, the Chicago Futures Exchange (CFE). Thereafter, in February 2006, CBOE introduced also VIX options, the most successful new product in CBOE history. In just ten years, combined trading activity in VIX options and futures has grown to over 800,000 contracts per day[52].

Nonetheless, a question arises: what is the relationship between the underlying index SPX and its volatility index VIX? In the figure 3, the blu line on the left scale shows daily closing prices for the S&P 500, while the red line shows the VIX during 2015 up to 12th August. It is clearly evident that the S&P 500 and VIX move opposite to each other on a daily scale.

[49] Retrieved from: https://www.cboe.com/micro/vix/vixwhite.pdf
[50] Retrieved from: https://www.cboe.com/micro/vix/vixwhite.pdf
[51] Ibid.
[52] Ibid.

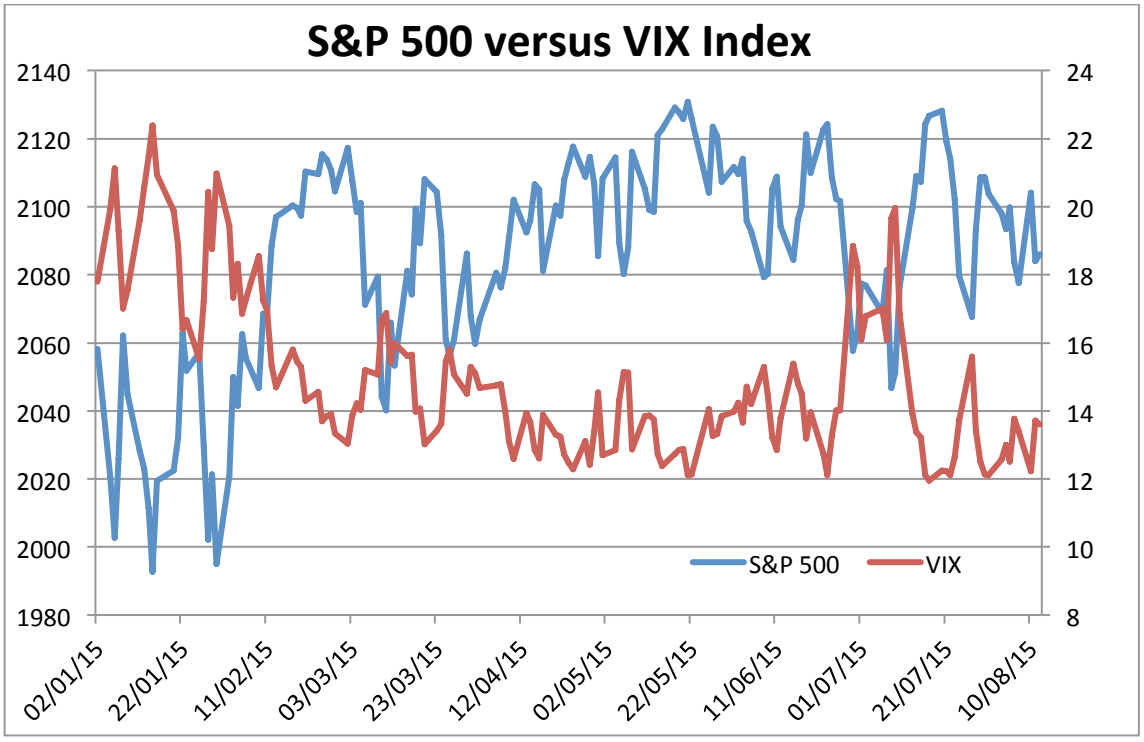

Figure 3. S&P 500 vs VIX in 2015.

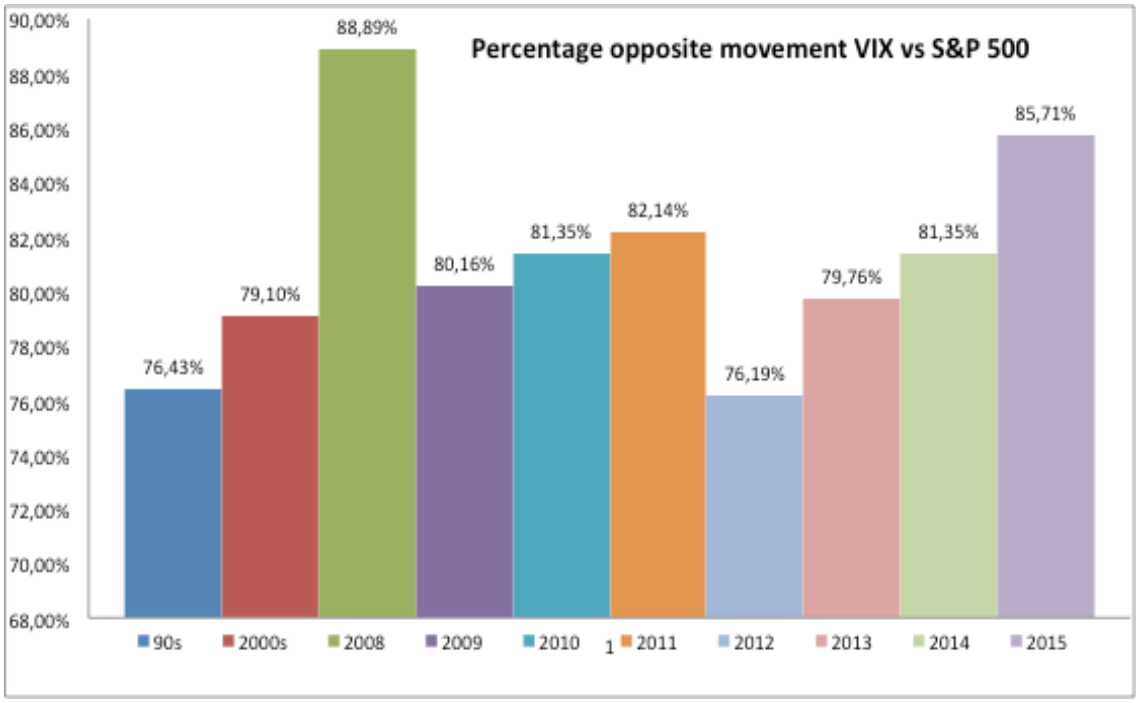

Figure 4. Estimated Percentage of the opposite movement between S&P 500 and VIX.

Moreover, there seems to be evidence that a higher opposite correlation appears during the financial crises (high monetary/economical uncertainty) periods as well as during the end of flourishing economy. To this regard, figure 4 investigates the return behaviour between SPX and VIX as a percentage of their opposite movement from 90s to 2015.

It is interesting to notice that during GFC in 2008 the opposite movement percentage touched almost 90%, which means that 90 out of 100 trading days VIX moved completely opposite to SPX. In 2015 the opposite correlation exhibits a high percentage of 86% as well. This can be explained by future uncertainty investors are nourishing as a collateral effect of three years of historical high returns in stock markets, generous QE cash flows and "cheap money" as Fed decreased interest rates to historical minimum. The uncertainty became a thread as soon as Fed announced "tapering"[53] and the interest rates increase, nonetheless the financial markets continued to fluctuate stable with bated breath and both eyes on Fed action. The coup the grace in 2015 arrived surprisingly not from Fed as expected but from emerging markets: China and Hong Kong. In 2012 the percentage of the opposite movement between SPX and VIX was only at 76%, which means that 24% of the times the both indexes moved in the same direction. By the definition VIX has been created, if both move in the same direction the expectations on the future should be opposite to the present SPX index movement. Thus, in 2012 the US economy gave its positive recovery signals thanks to QE monetary policies and the financial markets calm down definitely post GFC, which might confirm the hypothesis that during calmer periods the opposite relationship between SPX and VIX might weaken.

A further analysis has been conducted through the correlation estimation on the simple returns between VIX and S&P 500 Index from 1990 to 12th August 2015. The results confirmed the author's intuition of highly negative correlation: -0.7049, which has a huge implication in the portfolio hedging strategies. The obtained correlation result (is provided in Appendix 1) as well t-statistics and p-

[53] This term entered in financial terminology after Fed chairman Ben Bernanke announced to Congress that Fed may "taper", that is reduce, the size of QE.

value prove the significance of the presented correlation. This negative correlation is further increased to -0.7438 in the last six and a half years. Moreover a simple regression analysis is provided in Appendix 2, whereas a better estimation will be presented in chapter 2. The R-squared is 50%, while the regression coefficients are significant.

Given the high negative correlation of volatility to stock market returns, the inclusion of volatility in an investment portfolio should offer a diversification effect. The creation of VIX futures and options offers a "pure volatility exposure" standardised product: with its major liquidity and transparency compared to variance derivatives traded on the OTC market. Thus, VIX products became easily accessible to all investors from the smallest retail trader to the hedge funds and largest institutional money managers.

On February 26, 1998, the European counterparties created the EUROSTOXX 50 Index, which became a leading blue-chip corporations index, covering 50 stocks from 12 Eurozone countries[54]. As for SPX also for EURO STOXX 50 (SX5E) a Volatility Index (VSTOXX)[55] was introduced as well. From 2005, Eurex Exchange introduced VSTOXX futures and at a later stage options as new asset classes. These listed volatility derivatives provide investors with targeted and leveraged means to overview the European volatility. The main reason to access volatility via VSTOXX derivatives is to diversify portfolio, optimize and hedge the volatility exposure of equity market positions.

Today, there is a much wider range of volatility derivatives beyond VIX or VSTOXX, including listed volatility futures and options on main indexes (Nikkei, Nasdaq 100, DJIA, Russel 2000, DAX), emerging markets, interest rates, main currencies[56], gold, crude oil, single stocks equity indexes and even volatility of VIX index[57]. The main advantage for these products is to be easily accessible to retail investors, however the liquidity for many of these new products remains challenging. In the following table, the CBOE volatility indexes are summarised.

[54] Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain.

[55] Is based on the implied volatility derived from EURO STOXX 50 Index Options.

[56] CBOE trades volatility indexes on currencies' (Euro, Yen and British Pound futures).

[57] CBOE VIX of VIX Index (VVIX).

At a first glance it can be noticed that futures and options are non-traded on all volatility indexes, only on the main ones.

VOLATILITY INDEXES AT CBOE

CBOE INDEXES THAT MEASURE EXPECTED VOLATILITY

Index	Ticker	Webpage	Futures and Options?*
Volatility Indexes (Stock Indexes)			
CBOE Volatility Index®	VIX®	www.cboe.com/VIX	F O
CBOE S&P 100 Volatility Index	VXO	www.cboe.com/VXO	
CBOE Nasdaq-100 Volatility Index	VXN	www.cboe.com/VXN	F
CBOE DJIA Volatility Index	VXD	www.cboe.com/VXD	
CBOE Russell 2000 Volatility Index	RVX	www.cboe.com/RVX	F O
CBOE Short-Term Volatility Index	VXST	www.cboe.com/VXST	F O
CBOE 3-Month Volatility Index	VXV	www.cboe.com/VXV	
CBOE Mid-Term Volatility Index	VXMT	www.cboe.com/VXMT	
Volatility Indexes (Interest Rates)			
CBOE/CBOT 10-year U.S. Treasury Note Volatility Index	VXTYN	www.cboe.com/VXTYN	F
CBOE Interest Rate Swap Volatility Index	SRVX	www.cboe.com/SRVX	
Volatility Indexes (Currency Futures)			
CBOE/CME FX Euro Volatility Index	EUVIX	www.cboe.com/FX	
CBOE/CME FX Yen Volatility Index	JYVIX	www.cboe.com/FX	
CBOE/CME FX British Pound Volatility Index	BPVIX	www.cboe.com/FX	
Volatility Indexes (ETFs)			
CBOE Crude Oil Volatility Index	OVX	www.cboe.com/OVX	F O
CBOE Gold Volatility Index	GVZ	www.cboe.com/GVZ	F O
CBOE EuroCurrency Volatility Index	EVZ	www.cboe.com/EVZ	
CBOE EFA ETF Volatility Index	VXEFA	www.cboe.com/VXEFA	
CBOE Emerging Markets ETF Volatility Index	VXEEM	www.cboe.com/VXEEM	F O
CBOE China ETF Volatility Index	VXFXI	www.cboe.com/VXFXI	
CBOE Brazil ETF Volatility Index	VXEWZ	www.cboe.com/VXEWZ	F O
CBOE Silver ETF Volatility Index	VXSLV	www.cboe.com/VXSLV	
CBOE Gold Miners ETF Volatility Index	VXGDX	www.cboe.com/VXGDX	
CBOE Energy Sector ETF Volatility Index	VXXLE	www.cboe.com/VXXLE	
Volatility Indexes (Single Stocks)			
CBOE Equity VIX® on Amazon	VXAZN	www.cboe.com/VXAZN	
CBOE Equity VIX® on Apple	VXAPL	www.cboe.com/VXAPL	
CBOE Equity VIX® on Goldman Sachs	VXGS	www.cboe.com/VXGS	
CBOE Equity VIX® on Google	VXGOG	www.cboe.com/VXGOG	
CBOE Equity VIX® on IBM	VXIBM	www.cboe.com/VXIBM	
Volatility of VIX			
CBOE VIX of VIX Index	VVIX	www.cboe.com/VVIX	

** CBOE offers futures and options on a number of volatility indexes, with unique pricing and settlement on Wednesdays. The lists above are subject to change.*

Table 1. CBOE volatility indexes[58].

[58] Retrieved from: http://www.cboe.com/framed/pdfframed.aspx?content=/micro/vix/pdf/cboe30c7-volindex_qrg.pdf&section=SECT_MINI_SITE&title=Volatility+Indexes+at+CBOE

VIX Calculation

The most valuable attribute of the VIX Index is the existence of the extensive data set of 25 years of historical data. This provides a functional perspective on how option prices react in response to changing market conditions. As previously mentioned, on September 22, 2003, the CBOE renamed the old VIX into VXO and maintained its quotation until nowadays[59]. In fact, on one hand, we have the historical data on VXO calculated from OEX options with old methodology from 1986 to 2003 and from 2004 until nowadays. On the other, the new VIX calculated from SPX options, introduced in 2003 and back calculated to 1990. Since the SPX market has exponentially increased over years, it sounded extremely logic to increase the number of underlying assets from 100 to 500 to price markets' volatility. Except for this numerical change, the derivation has changed as well as following:

The old VXO

This index is based on OEX options and is derived as an average of the Black-Scholes implied volatilities on eight near-the-money options at the two nearest maturities. When the time to the nearest maturity is within eight calendar days, the next two nearest maturities are used instead[60]. For each of the two maturities (t=1, t=2) two call and two put options are chosen at two strike prices (K=1, K=2) that are nearest to the spot level (S) and straddle it[61]. The two implied volatilities from the call and the put at each strike price are averaged and then linearly interpolated between the two averages of implied volatility at two strike prices to derive the at-the-money spot implied volatility. Or more synthetically: at t=1, $Call_1$ at K_1 and Put_1 at K_1 implied volatilities are averaged and subsequently are linearly interpolated with $Call_2$ at K_2 and Put_2 at K_2 implied volatilities averages[62]. The interpolated at-the-money implied volatilities are then once again interpolated along the maturity dimension to create a 22-day volatility, which compose the VXO.

The first drawback of this method was that the Black-Scholes implied volatility is

[59] The new ticker symbol VXO (prior to September 2003 it was the "original" VIX Index).
[60] Carr P., Wu L. (spring 2006), *A Tale of Two Indices*, The Journal of Derivatives, p.14.
[61] This implies $K_1<S<K_2$.
[62] At t=2, the same mechanism is applied.

an annualised volatility that is equated to the options market quote. Normally, the annualisation is derived as actual over 365 days convention, but instead of using the implied volatility directly, an artificial "trading day conversion" was introduced[63]. Let $\sigma_M^2(t,T)$ be a Black-Scholes at-the-money implied volatility at time t as an annualised percentage with option expiry date at T. This percentage is converted by CBOE into "trading day" volatility $\sigma_{T\_D}^2(t,T)$ as $\sigma_{T\_D}^2(t,T) = \sigma_M^2(t,T)\sqrt{NC}/\sqrt{NT}$,

where *NC* is a number of actual calendar days and *NT* the number of trading days between t and maturity date T. As previously explained, the VXO represents an interpolated (at 22 trading days) trading day volatility, which is based on two trading day volatility at the nearest maturities: $\sigma_{T_D}^2(t_1,T_1)$ and $\sigma_{T_D}^2(t_2,T_2)$. Thus, VXO at a certain time t is calculated as $VXO_t = \sigma_{T_D}^2(t_1,T_1)\frac{NT_2-22}{NT_2-NT_1} + \sigma_{T_D}^2(t_2,T_2)\frac{22-NT_1}{NT_2-NT_1}$,

where NT_1 and NT_2 are the number of trading days between t and the two options expiry dates T_1 and T_2 respectively. Nonetheless, the first conversion $\sigma_{T\_D}^2(t,T) = \sigma_M^2(t,T)\sqrt{NC}/\sqrt{NT}$ raises the level of VXO and makes it no longer comparable to annualized realised volatilities computed from index returns[64]. For this reason, the old computational methodology has raised a lot of criticism from institutional investors and academics for artificially inducing upward bias.

At-the-money implied volatility is often regarded as an approximate forecast for realised volatility. However, this might seem not to have any economic explanation since Black-Scholes model assumes constant volatility. Nevertheless, some academics and practitioners proved with their empirical researches that at-the-money implied volatility is a good proxy to forecast subsequent realised volatility, although biased.

[63] Carr P., Wu L. (spring 2006), *A Tale of Two Indices*, The Journal of Derivatives, p.14.
[64] *Ibid.*

The new VIX

The new formula was intended to improve VIX index for investors, who manage risks linked to the growing markets for volatility and variance swaps. As mentioned so far, VIX index is created on S&P 500 options basis, where each option reflects the market expectation of future volatility.

The general formula used in VIX calculation is as following:

$$\sigma^2(t,c) = \frac{2}{T-t}\sum_i \frac{\Delta K_i}{K_i^2} e^{R(T-t)} Q(K_i,T) - \frac{1}{T-t}\left[\frac{F_t}{K_0} - 1\right]^2,$$

where *T* is options expiry date, *F* is the forward index level derived from index options prices, K_i is the strike price of the i-th out-of-the-money option[65],K_0 denotes the first strike below the forward index level, *R* is risk-free rate to maturity[66], ΔK_i denotes the interval between strike prices as $\Delta K_i = \frac{K_{i+1}-K_{i-1}}{2}$, half the difference between the strike on either side of K_i[67], and finally $Q(K_i,T)$ is the midpoint of the bid-ask spread for each option with strike K_i. In the equation, only out-of-the-money options are used, except for K_0, where $Q(K_0,T)$ presents the average of the call and put option on this strike price. Given that $K_0 < F_t$, the average at K_0 implies that one unit of in-the-money call at K_0 is used. The squared term in the equation convert this in-the-money call into out-of-the-money put using put call parity. The calculations involved account for a broader range of call options with strikes greater than F_t and put strikes lower than F_t, rather than account for only near-the-money strikes as it was in the original index formula. In order to determine the forward index level F_t, a pair of put and call options with prices that are closest to each other are chosen. Then, the forward price is derived through the put-call parity relationship as following:[68]

$$F_t = e^{r(T-t)}\big(C_t(K,T) - P_t(K,T)\big) + K,$$

which is then substituted into the previous variance formula.

The general formula calculates $\sigma^2(t,T)$at two of the nearest maturities T_1 and $T_2$of the available options. An interpolation between $\sigma^2(t,T_1)$ and $\sigma^2(t,T_2)$ is

[65] A call if $K_i > K_0$; and a put if $K_i < K_0$; both put and call if $K_i = K_0$.

[66] The risk-free interest rates are the bond equivalent yields on the US T-bill maturing close to the expiration dates of SPX options.

[67] $\Delta K_i$for the lowest strike is calculated as the difference between the lowest strike and the next higher strike. The same is applied to $\Delta K_i$for the highest strike.

[68] $F = e^{RT}(Call\ Price - Put\ Price) + Strike\ Price.$

performed to obtain $\sigma^2(t,T)$ estimate at 30 days to maturity. Since VIX represent an annualised volatility percentage of this 30 days $\sigma^2(t,T)$, an actual/365 day counting convention is applied:

$$VIX_t = 100\sqrt{\frac{365}{30}\left[T_1\sigma^2(t,T_1)\frac{NC_2-30}{NC_2-NC_1} + T_2\sigma^2(t,T_2)\frac{30-NC_1}{NC_2-NC_1}\right]},$$

where NC1 and NC2 denote the number of actual days to expiration of the two maturities. In this paper, the settlement calculations are daily; nonetheless in the real word trading, in order to replicate the precision that is commonly used in professional trading, the calendar days are translated into minutes. The annualization in the above formula does not experience the artificial upward bias incurred in the VXO calculation.

Some more practical motivations moved to the swap from the old index to the new one. As mentioned at the end of the previous paragraph, VXO economic meaning was not so clear[69], except for Black-Scholes Framework: it simply represented a non-linear but monotonic transformation of at-the-money option prices. Conversely, the meaning for VIX squared is much more concrete: it is a price of a linear portfolio of options[70].

Other reasons that forced CBOE to change the old methodology are based on the concept of variance swaps theory and the replication strategies, which will be deducted in the following chapter.

[69] More details on new VXO economic meaning see next chapter.

[70] Carr P., Wu L. (spring 2006), *A Tale of Two Indices*, The Journal of Derivatives, p.16.

Variance and Volatility Swaps

The volatility swaps present some extremely attractive characteristics for being traded: mean reversion and negative correlation with the market. Thus, the volatility swap might be used to trade on: volatility levels depending on the expected directional movement, spreads between realised and implied volatility, as well to hedge some implicitly short volatility exposures in financial businesses. Normally the options traders speak about volatility, although it is the variance or volatility squared, which has a paramount theoretical significance. In order to correctly evaluate a swap, it is necessary to value the portfolio that replicates the swap, which can be most reliably done through a variance swap.

A variance swap is an over-the-counter financial derivative, which allows investors to exchange future realised variance[71] of an asset between time 0 and time T against current prefixed implied variance at time 0[72]. Thus, in a variance swap contract, the long side of the contract receives the realized return variance and pay a variance swap rate (pre-fixed variance, so called strike, agreed at the contract stipulation). The volatility swap expresses the same underlying concept of a variance swap, although the calculation method is more difficult. As in all swap contracts, a contract costs zero to enter because it is fairly priced: the fixed variance swap rate equals the risk-neutral expected value of the realised volatility. The volatility difference between the two legs must be taken to convert the payoff from volatility percentage points to notional dollar amount[73]. Normally the settlement is made in cash at the expiration of the contract, nonetheless, the counterparties are likely to agree to make some payments in between to maintain a maintenance margin. Given an underlying asset[74] on which the swap is written (e.g. SPX index), on maturity the equity amount is calculated and paid as

$Final\ Equity\ payment = Variance\ Notional * (Final\ Realized\ Variance - Strike\ Price)$. Or more synthetically: $Payoff = N_{\sigma^2} * ({\sigma^2}_{realised} - {\sigma^2}_{strike})$.

[71] Expressed as annualised variance.

[72] Christensen B.J., N.R. Prabhala (1998). *The Relation between Implied and Realized Volatility*. Journal of Financial Economics, No. 50, pp. 125-150.

[73] The notional amount is never exchanged.

[74] This might be a currency, interest rate or stock index.

The swap holder at expiration will receive N_{σ^2} dollars for each variance difference point. If the payoff is positive the variance seller will pay the buyer the underlying asset amount (in our case is SPX index). On the contrary, if the payoff is negative the variance buyer will pay the variance seller a dollar amount equal to the absolute value of the underlying asset amount[75].

The $Final\ Realised\ Volatilty = \sqrt{\frac{252\times\sum_{t=1}^{t=N}(ln\frac{P_t}{P_{t-1}})^2}{Expected\_N}}\times 100,$

where $Expected\_N$ is the expected number of trading days during the observation period. The returns are calculated on a logarithmic basis: $(ln\frac{P_t}{P_{t-1}})$.

In case of variance swap, the market practise is to express the variance notional amount in terms of Vega[76], which will be converted into dollar terms. Vega is the rate of the change in the value of the portfolio with respect to the volatility if the underlying asset[77]:

$\mathcal{V} = \frac{\partial\Pi}{\partial\sigma}$.

Thus, $Variance\ Notional\ Principle\ (N_{\sigma^2}) = \frac{Vega\ Notional^{78}}{2\times\sigma_{Strike}}$,

Such a conventional adjustment is made in case "the realized volatility is 1 Vega point above the strike at maturity, the payoff will be equal to the Vega Notional"[79].

In order to statically replicate variance swap it is necessary to create a "portfolio of options, whose variance sensitivity is independent of stock prices"[80]. For a given level of spot stocks price *S*, the options exposure to Vega *V* varies with strikes *K*, where the largest exposure is when the option is at-the-money and falls rapidly as *S* moves in or out of the money. In order to create a variance independent portfolio, the options with many strike prices have to be combined. Nonetheless, in order to achieve a constant *V* a special strikes weighing is

[75] Bossu S., Strasser E., Guichard R. (February 2005), *Just what you need to know about variance swaps, Quantitative Research & Development*, JPMorgan-London, p.3.

[76] A "Greek letter" in option derivative pricing.

[77] Hull J. (2012), Options, Futures and other Derivatives, 8th edition, p.589.

[78] Or volatility notional.

[79] Bossu S., Strasser E., Guichard R. (February 2005), *Just what you need to know about variance swaps, Quantitative Research & Development*, JPMorgan-London, p.4.

[80] Demeterfi K., Derman E., Kamal M., Zou J., (March 1999), *More than you ever wanted to know about Volatility Swaps*, Quantitative Strategies Research Notes, Goldman Sachs, p.6.

required: options have to be inversely weighted to K^2. This is intuitive as when S moves up, an option with higher strike price will contribute to higher V exposure. Therefore, to offset S-dependence, the amount of higher-strike options is diminished proportionally by K^2. Thus, holding a portfolio of all strikes, weighed by inverse proportion to the square of the strike level, erases the stock dependence. The result is presented in the following graph:

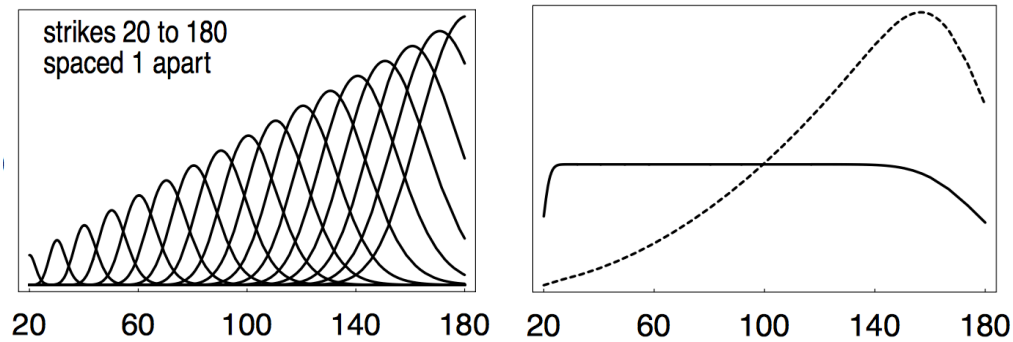

Figure 5. Strikes on different spot prices versus the corresponding sum of their contributions[81].

The left figure shows the contributions of each option with K_i, whereas on the right figure the sum of options equally and inversely proportional to K^2 are weighted. Nonetheless, in practise this approach is often violated as not all strike prices are available and in the presence of implied volatility skew the arguments are not very clear.

Another more general and more rigorous approach of a dynamic hedging captures the fair value of the variance swap. This method consists in observing the implied volatilities as risk-neutral market's expectation of future realised volatility $\sigma^2{}_{strike} = E(\sigma^2{}_{realised})$. The final results for fair value of future variance are presented as following:

$$E(\sigma^2{}_{realised}) = \frac{2}{T}\left[\ln\frac{F_0}{S^*} - (\frac{F_0}{S^*} - 1) + \left[\int_{K=0}^{S^*}\frac{1}{K^2}e^{rT}p(K)dK + \int_{K=S^*}^{\infty}\frac{1}{K^2}e^{rT}c(K)dK\right]\right],$$

where F_0 is the forward price at time 0 of an asset maturing at T, $p(K)$ $c(K)$ are respectively European put and call prices at given strike K with time to maturity T, S^* represents any value of the underlying asset. If the strike prices of the

[81] Graph source: Demeterfi K., Derman E., Kamal M., Zou J., (March 1999), *More than you ever wanted to know about Volatility Swaps*, Quantitative Strategies Research Notes, Goldman Sachs, p.8.

options are known and S^* is set to be equal to the first strike under F_0, the integrals are solved as $\sum_{i=1}^{n} \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i)$,

where $Q(K_i)$ is the price of call option for $K_i < S^*$, for $K_i > S^*$ is the price of put options and for $K_i = S^*$, $Q(K_i)$ is equal to the average of the prices of call and put with strike price K_i. This approach provides "a direct connection between the market cost of option and the strategy to capture future realised variance"[82] and it is valid even implied volatility skew is present and Black-Scholes assumptions are violated.

Some more analytical results so called "rule of thumb" for the fair strike price has been derived in case the skew is linear in strike[83] as: $\sigma^2{}_{strike} = \sigma_{ATMF}\sqrt{1 + 3T * skew^2}$,

Where σ_{ATMF} is at-the-money-forward volatility, T time to maturity and skew is the slope of the skew curve. A similar result has been obtained for log-linear skew: $\sigma^2{}_{strike} \approx \sigma^2{}_{ATMF} + \beta\sigma^3{}_{ATMF}T + \frac{\beta^2}{4}(12\sigma^2{}_{ATMF}T + 5\sigma^4{}_{ATMF}T^2)$. Both analytical results present a good approximation for fair strike price only for non-steep skew.

Unfortunately, there is no replication strategy to simply synthesise a volatility swap as only variance turns up consistently from hedged options trading. In fact, the only necessary condition in variance swap replication strategy is that the stock prices evolves continuously without jumps, but no assumptions are made on about future level of volatility and it is not affected by changes in volatility. In case of volatility swap, the strategy is affected by future changes in volatility as its value depends on the volatility of future realised volatility (VOV), as a result of non-linearity of volatility. From derivative point of view the volatility swaps are observed as a derivative on the variance as underlying asset. It is important to notice that the payoff of a variance swap is not linear but convex in volatility: this has strong consequences on the investor with long position. In fact, the investor who receives realized variance and pay a prefixed strike at the maturity will be subject to boosted gains and minor losses. In

[82] Demeterfi K., Derman E., Kamal M., Zou J., (March 1999), *More than you ever wanted to know about Volatility Swaps*, Quantitative Strategies Research Notes, Goldman Sachs, p.19.

[83] Ibid.

general, the variance swap will over perform a volatility swap and a correction is required. In the following graph this convex relationship of the variance swap and volatility swaps as a function of realised volatility are drawn:

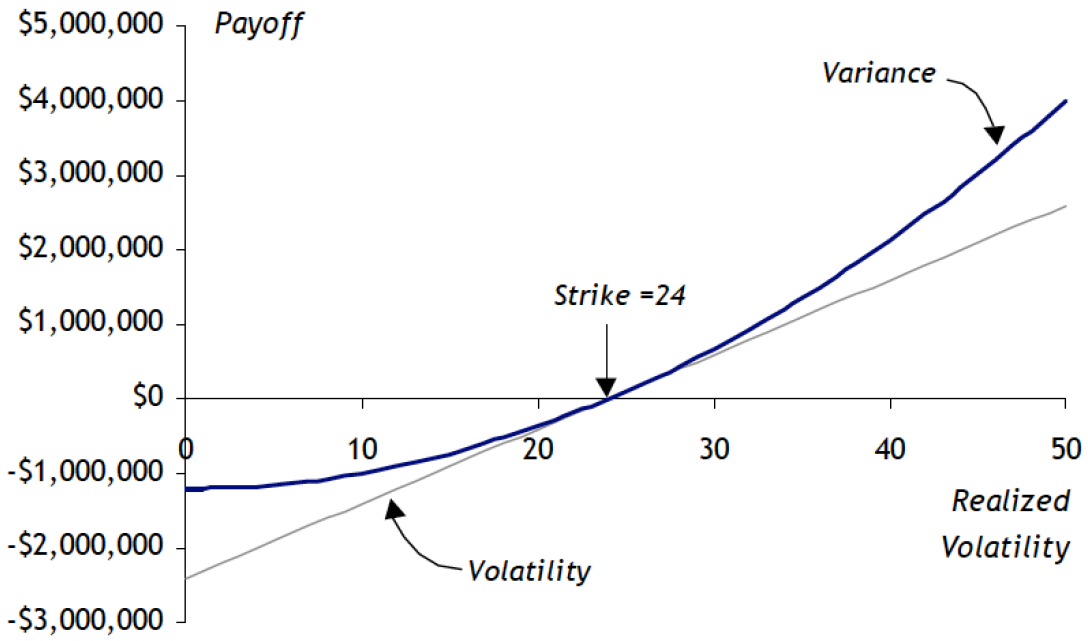

Figure 6. Variance swaps are convex in volatility[84].

Obviously, this bias has a cost, which is mirrored in a slightly higher strike price than "fair" volatility. In practise the fair strike of the variance swap is in line with the 90% put implied volatility[85]. Or by recalling the Jensen inequality, $\sigma^2{}_{strike} < \sqrt{\sigma^2{}_{strike}}$, fair volatility swap strike price has to be slightly lower, thus the straight line will shift left and will not always lie under the parabola.

As a result, it is possible to observe that VIX can be explained in terms of volatility swap on SPX. The new VIX squared approximates the conditional risk-neutral expectation of the annualised 30-day realised return variance: $VIX^2 \cong E_t^Q(\sigma^2{}_{realised,t,t+30})$, which has a very concrete economic interpretation. It can be observed as an approximation of a variance swap or as a price of a portfolio of options to create the underlying strategy.

[84] Source: Bossu S., Strasser E., Guichard R. (February 2005), *Just what you need to know about variance swaps, Quantitative Research & Development*, JPMorgan-London, p.4.

[85] Ibid.

As the author previously explained VIX seems to be good proxy for market volatility, nonetheless from some research immerged that VIX (implied volatility proxy) is approximately 5 percentage points higher than the realised volatility on the SPX and its movements seem not to predict future index returns[86].

From the Carr P. and Wu L. research emerged an interesting observation that SPX options implied volatility increases prior FOMC meetings and drops afterwards, which would create an optimal shortening strategy opportunity. As a result of this observation, the author of this paper in the next chapter has analysed the long-term possible effects of the central banks monetary policies on financial markets, volatility indexes and economy.

[86] Carr P., Wu L. (spring 2006), *A Tale of Two Indices*, The Journal of Derivatives, p.18.

Chapter 2

Economic Overview

In 2008 the worldwide economy has been threatened by a harsh decrease in the amount of world trade, a decrease in the industrial production (especially in US, EU, Japan), a strong increase of unemployment rate as well the spread of financial crisis from the US financial institutions to others worldwide. From 2008-2010 only in Europe 4 millions have lost their jobs, whereas in the following years the unemployment rate has further increased. The main causes of the worldwide trade and investment contraction is due to some important structural changes in the international and American financial system. The GFC has spread over other countries mainly through five fundamental mechanisms:

1. The non-US banks and other financial intermediaries have accumulated losses on sub prime derivatives and/or on US banks stocks.
2. The reduction of the worldwide interbank loans, as a consequence of mistrust among banks, resulted in decreasing level of available loans.
3. The previous caused a significant credit freeze, that is the amount of the consumers' and production loans newly issued by banks decreased, which resulted in a drastic drop of industrial production and consumption.
4. As a consequence of worldwide confidence crises in financial system, the investments and consumptions dropped.
5. The exportation to US and other countries, which reduced their consumption, decreased as well.

As a consequence of this economic situation, the financial markets dropped exponentially and the volatility indexes rocketed to their highest levels, as future economic perspective appeared to be a significant recession, as it really was in the retrospective, and a risk of possible deflationary spiral. Thus, to face the recession and revive the economy, the worldwide central banks have taken steps to decrease interest rates and expand the monetary supply to stimulate the flooding economy, boost investments and consumption. Furthermore, as an emergency response to the crises, the governments authorised large fiscal

stimulus packages through borrowing and spending to balance the drop of private sector demand. In this paper the Fed, ECB, BOJ and PBOC monetary policies during the last 7 years, as the response to the GFC and sovereign debt crises, will be analysed. Most of the banks have duplicated or triplicated their Balance Sheets through QEs, Operational Twist, LTROs and other monetary expansive policies, the result summarized in the Table 1. Nonetheless, yet in 2015, in some countries (such as Italy, Spain, Japan) the economic recovery is still latent and the stagnation continues.

In the following chapters US and EU economical background and Central Banks responses to the crises will be described. Thereafter, the reaction of the real economy as well of the financial markets and volatility indexes to this monetary stimulus will be presented. From the following analyses is emerged that expansive monetary policies have significantly improved the financial markets performance and have artificially reduced the volatility on the financial markets, whereas the real economy has only relatively improved. In this paper the real economy performance is investigated through the GDP and unemployment rate variables analysis.

The main author's intuition consists in affirming that main central banks policies had major impact on financial markets rather than on real economy. This relationship is explained in the following table:

Table 2. Relationships among Central Banks, Real Economy and Financial Markets.

The crossed relations imply the weakened dependencies created in the economy. As long as real economy is relatively stable, although still stagnant, and monetary stimulus continues, the financial markets flourish and volatility

indexes are at their lowest. But as soon as the economy fluctuates, the effect of the real economical shocks hit intensely financial markets as it happened in August 2015 as China's economy slowed down.

United States

In the early 2008 more chief economists have observed the beginning of the recession in US. In the mid-2007 the housing bubble burst, after having reached the unsustainable valuation peak in 2005-2006, and in 2008 the housing market price correction as well the subprime mortgage crisis commenced. As early as October 2007, the US Treasury Secretary Paulson H. has recognised the bursting housing bubble as the "most significant risk to the US economy". The housing mortgages securitization machine and the reckless clients risk assessment, poor performance of credit rating agencies, federal government policies such as deregulation permitting M&As and allowing unemployment rate to grow to control inflation, liquidity crisis are the main causes of GFC. On December 1, 2008, the NBER[87] has declared US having formally entered the recession in December 2007. The following year the unemployment rate jumped to 6.1%, the highest level since five previous years; most of the financial institutions have faced liquidity shocks, which forced government to inject liquidity to bailout AIG, Bear Stearns, to grant some special credit conditions to Citigroup and others, promoted banks M&As in order to avoid further bankruptcies[88] spiral and a plausible financial systemic collapse.

The first step the Central Banks usually undertake is to adjust overnight borrowing interest rates. Thus, from September 18, 2007 to December 16, 2008, Fed Federal Funds rates were pushed down from 4.75% to 0.25%. This "open market operation" is normally undertaken during the "normal" times to move short-term interest rates through securities acquisition or sale to manoeuvre the banking system reserves. This obliges banks to buy new securities, which increases their prices and leads to the desired goal level of interest rates. But during the severe economic distress such as GFC, the interest rates were already lowered down to almost 0 by the end of 2008, by

[87] National Bureau of Economic Research.
[88] Lehman Brothers Failure.

which the Fed concluded any possible further measure on interest rates. Thus, as BOJ anticipated QE in 2001, in November 25, 2008, Fed announced its QE program, which consisted in acquisition of securities to generate the desired level of reserves[89]. The main aim of QE is to increase assets prices, decrease inflation and encourage investments and spending. An updated timeline of the QEs and Operational Twist will be briefly presented, afterwards the reaction of GDP, unemployment rate, SPX as well VIX to these monetary policies will be analysed.

As previously mentioned, from 25th November 2008 to 31st March 2010 through QE1 has been purchased: $100 billion GSE[90] direct obligations, $1.25 trillion MBS, $300 billion of Long-Term Treasury securities. This "Credit Easing" programme was intended to sustain economy through reduction of interest rates and increase of liquidity when credit channels are blocked. In fact, the acquisition of MBS entered into a category of securities, for which the demand during the GFC dropped.

During the second round of QE from 3rd November 2010 to 30th June 2011, $600 billion treasury bonds have been purchased.

From 21st September 2011 to 12th December 2012, the "Operational Twist" has been performed. The FOMC originally declared to purchase of $400 long-term Treasury Bonds between 6 to 30 years and sell an equivalent amount of Treasury securities with maturities of 3 years or less, which extended the average maturity of securities Fed hold. This measure aimed to provide investors with cash from long-term securities to incentivise them to invest in other assets without creation of new money in the economy.

From 13th September 2012 to 30 September 2014, the longest period, QE3 has been implemented. From original statement, the stimulus should consist in $40 billion per month, nonetheless simultaneously to the announcement of "Operational Twist" end, on 12th December 2012, Fed declared the expansion of QE3 from $40 to $85 billion per month. Finally on 18th December 2013 FOMC

[89] Thus, "Quantitative Easing" denomination.

[90] Government-Sponsored Enterprise, such as Freddie Mac, Fannie Mae, Sallie Mae and the Federal Home Loan Banks.

announced QE3 “tapering”, the reduction of the original amount by roughly $10 billion each month.

By the way a question arises, how much QE exactly should boost economic recovery? Theoretically, QEs should help the economy through “portfolio rebalancing”[91]: the investors receive the proceedings from selling their securities to the central bank and invest in other assets, which increase their prices. In this way, lower bond yields should boost borrowing, the higher equity prices should raise consumption level and both of them improve demand and investments. Moreover, as investors start investing in foreign assets, portfolio rebalancing might weaken the domestic currency and increase exports.

The effect of Fed measures on economy seems to be much weaker compared to the effect on SPX and VIX. In the following graphs (Figure 6 and 7) Fed monetary policies are graphically confronted to the unemployment rate and GDP annual variation. The effect of the monetary stimulus might be observed in mid-2009, when the nominal GDP start improving, whereas unemployment rate dramatically increased by reaching its highest peak at the beginning of 2010 with 10.6%.

From the following graphs (figures 7, 8) it might be inferred that the effect of the monetary stimulus on GDP is much more reactive compared to unemployment rate: this means that while GDP was giving signs of recovery, the people were dangerously loosing their jobs. The reason for such a slow repercussion of the real employment to the monetary stimulus might be explained by GDP composition (as 7.2% of real GDP is represented by Financial and Insurance Sector[92]). Since the financial markets and financial institutions have most profited from such monetary policies, as it will be briefly demonstrated, GDP might be biased upward compared to the real economic situation. Some complications in this analysis might arise regarding the availability of historical data on both GDP and unemployment rate, as the first is quarterly and the second are monthly announced. In order to estimate the relationships, a couple of simple regression analyses have been conducted.

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[91] July 14th 2012, *Quantitative Easing: QE, or not QE?*, The Economist, Washington.

[92] Retrieved from: http://selectusa.commerce.gov/industry-snapshots/financial-services-industry-united-states.

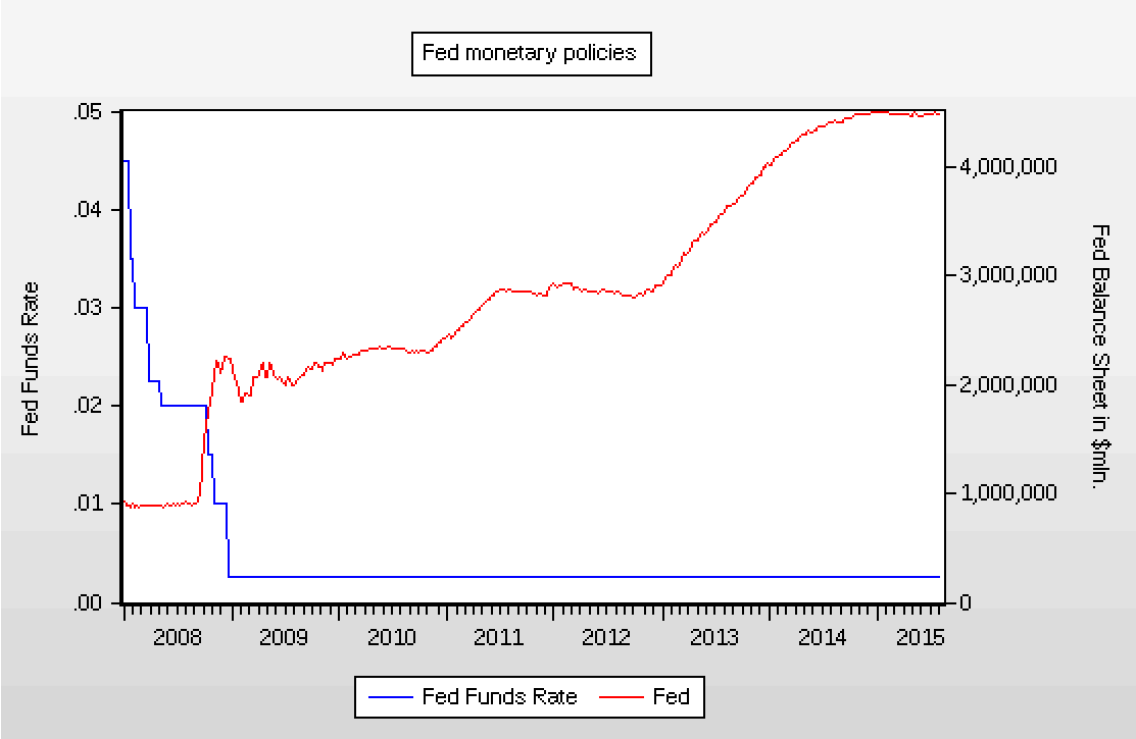

Figure 7. Fed B/S expansion and interest rates policies.

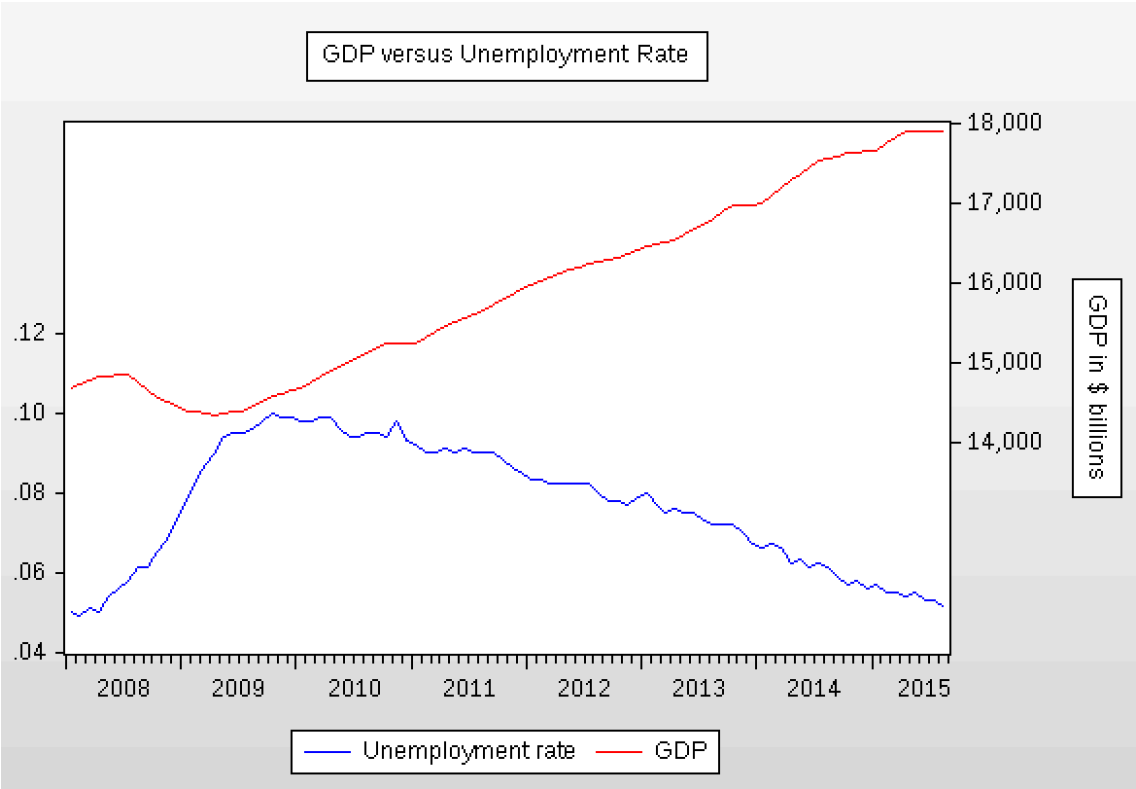

Figure 8. Unemployment rate and nominal GDP[93].

[93] Data source: http://www.bea.gov.

GDP as well unemployment rate data are non-stationary, thus they have been transformed into stationary[94]. From the correlation analysis (Appendix 3) it emerged that Fed B/S is 46.6% positively correlated to GDP, whereas -25% correlation between Fed B/S and unemployment rate has been estimated. These correlations give an "average" correlation coefficient across the time series, which holds for the time period taken into consideration[95]. Some estimation models have been estimated on GDP and unemployment rate in relation to Fed B/S expansion with the following results. The adjusted Fed B/S data seem insignificant to explain the adjusted unemployment rate. In fact, the unemployment rate movements might be better explained by an ARMA (1,1) model, whereas the conditional variance seems to present a white noise behaviour. The regression results are presented in Appendix 5, and they explain 52% of the differentiated unemployment rate variation, whereas the significance of the adjusted Fed B/s variation is rejected. It seems that the monetary stimulus might benefit the employment only in the long run, as it was graphically observed; while in the short-term the effects appear to be extremely marginal. The same result might be observed also for GDP estimation as Fed B/S variation seems not to be significant and the mean estimation might be explained only by a simple AR (1) model with 28% adjusted R-squared: the result is presented in the appendix 6. Nonetheless, Fed B/S increase did have some nominal linear effects on the level of GDP, as the underlying deterministic trend of the two series seems to be the same.

These results, on one hand, confirm that both variables were partially influenced by Fed stimulus although they are mostly related to the previous lagged observations (autoregressive models). On the other hand, the response of the unemployment rate to Fed B/S expansion is much slower compared to GDP response, as financial markets blooming response and not real economy improvement can explain an immediate nominal GDP improvement.

More economists quote the effect of QE as monetary policy working through the "financial sector", which increases wealth inequality through boosting prices of

[94] The adjusted data graph is proposed in the appendix 4.

[95] Theoretically, the correlations are different at each and every point of the considered sample.

financial assets that are principally retained by wealthier individuals and possibly might create asset bubbles[96]. A former Fed board member, Kevin Warsh sustains that QE works as an “asset price channel” benefiting securities owners and not majority of US workers (96%), who receive their income through labour. Whereas, a conventional monetary policy cuts such as interest rates reduction works as a “credit channel” for investments as well mortgage borrowers, which reduces the size of interest payments and increase people’s consumption power, it did not have a desired effect because of “credit crunch”, that is credit availability tightening. Although, Fed B/S expansion generated new recourses, the banks still preferred not to use liquidity and to invest in other more profitable assets such as stocks and/or other less risky activities compared to money borrowing.

After having analysed the effect of the monetary policies on the real economy, it is necessary to deep into the reaction of SPX and VIX. In order to proceed with the regression analysis it is worth underlining some important relationships: from the following graph it can be inferred that the weekly variations of VIX are much more volatile compared to the SPX variation, and at the same time the negative SPX returns imply much higher volatility spikes, which might reflect the pessimistic future reflection. This is the reason for which VIX volatility spikes cannot be modelled through symmetrical models, but through models accounting for positive asymmetrical effects such as swapped TGARCH model. This implication will be extremely important in the portfolio hedging through volatility derivatives in case of negative movements in the underlying assets.

[96] Hartley J. (June 25, 2015), *How Federal Reserve Quantitative Easing Expanded Wealth Inequality*, Forbes Economics & Finance.

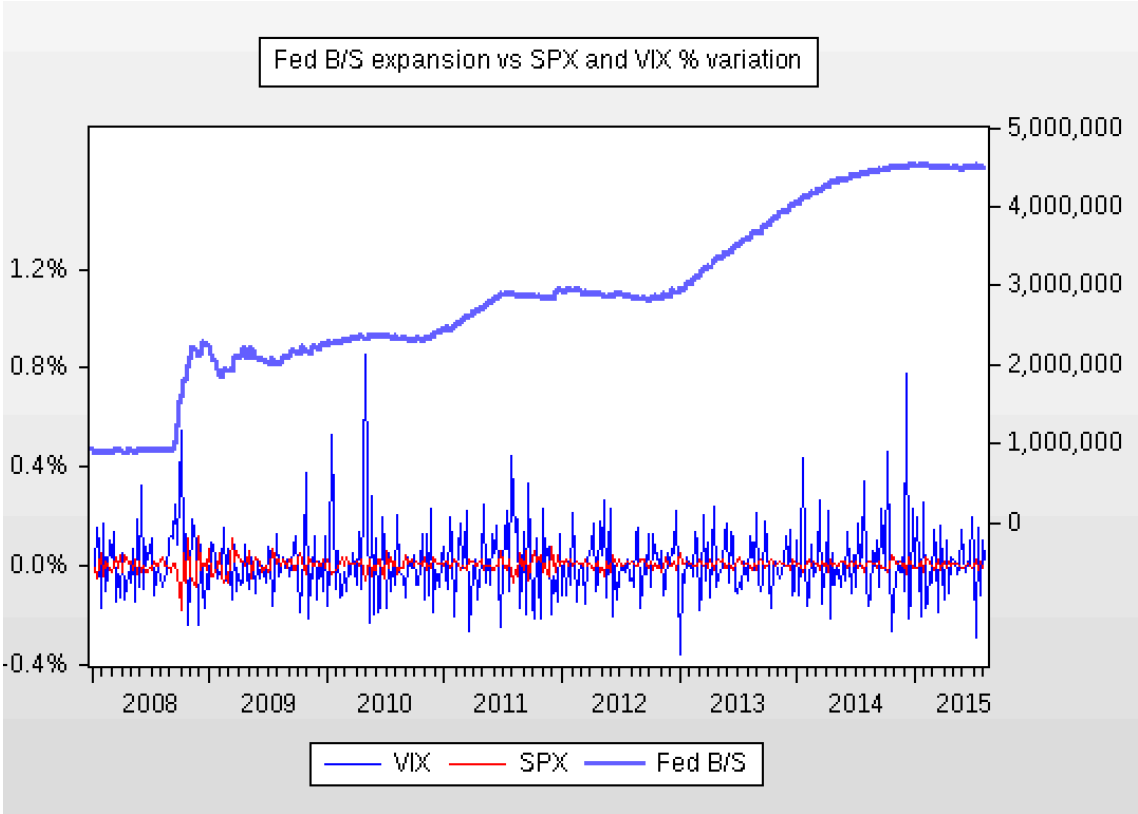

Figure 9. Fed B/S expansion expressed in $ mln. versus SPX and VIX weekly simple return variation.

As it was previously discussed in chapter 1, SPX and VIX are both strictly affected by Fed B/S increase, the first reflects the nominal increase whereas the second calm down to a long run average level. Moreover, it is worth recall that in the last 7 years the correlation between SPX and VIX has been -0.74. A further analysis will be conducted as following: firstly, the relationship between Fed B/S expansion and SPX as well VIX separately will be proposed. Afterwards some more relationships and inferences will be advanced.

The following table summarize the correlations among Fed B/S, VIX and SPX and the significance T-test:

Covariance Analysis: Ordinary
Sample: 12/31/2007 8/10/2015
Included observations: 398

Correlation t-Statistic Probability	FED_B_S	SPX	VIX
FED_B_S	1.000000		
SPX	0.817112	1.000000	
	28.20629	-----	
	0.0000	-----	
VIX	-0.499101	-0.687054	1.000000
	-11.46160	-18.81652	-----
	0.0000	0.0000	-----

Table 3. Correlations among Fed B/S, SPX and VIX.

It can be noticed that SPX weekly closing prices are 81% positively correlated to Fed B/S expansion, whereas VIX moves opposite only 50% times. The last statement is easy to understand and it was graphically presented in chapter 1. In figure 2 (previous chapter) it can be observed that during the periods the B/S expansion interrupted in between two consecutive monetary stimuli, VIX quick spiked more times as it reflected the expected future uncertainty. At this point, the effect of monetary stimulus on SPX is impressive: in the following graph, the SPX growth is specularly mirrored through Fed expansion. As happened for VIX, which reflects SPX future expectation, in between two subsequent monetary policies (grey areas) in 2010 and 2011 there are two significant index drops.

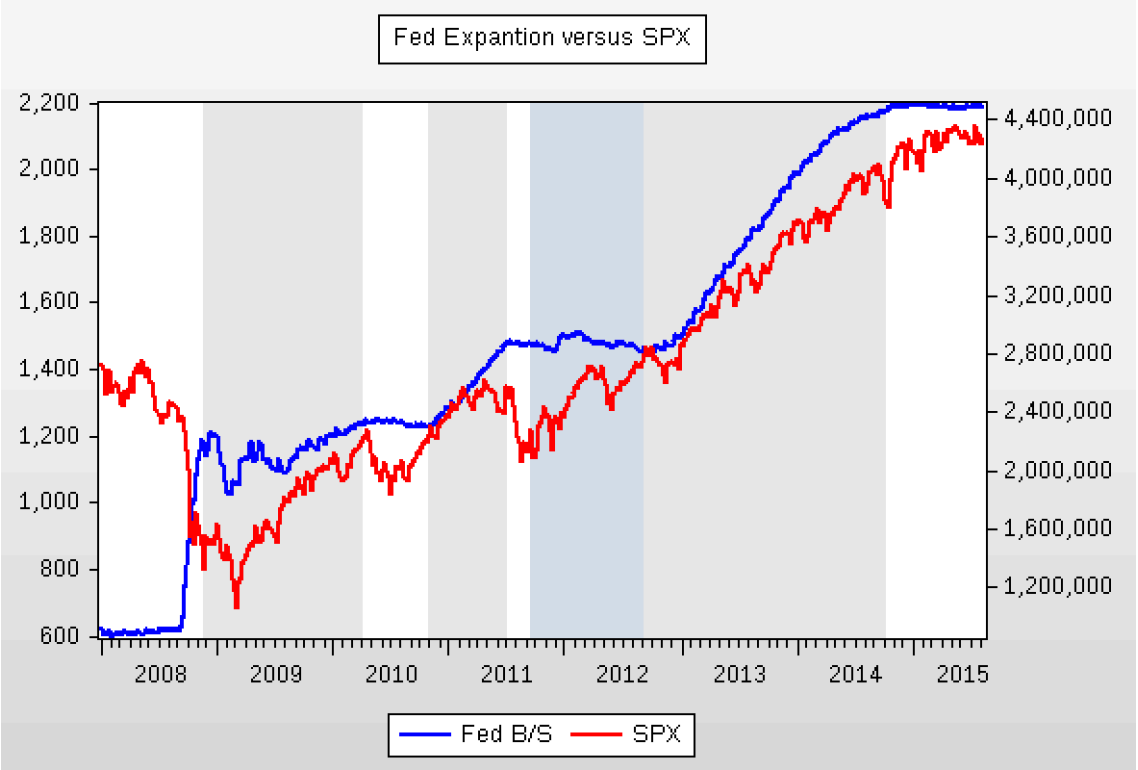

Figure 10. Fed nominal expansion versus SPX weekly closing prices.

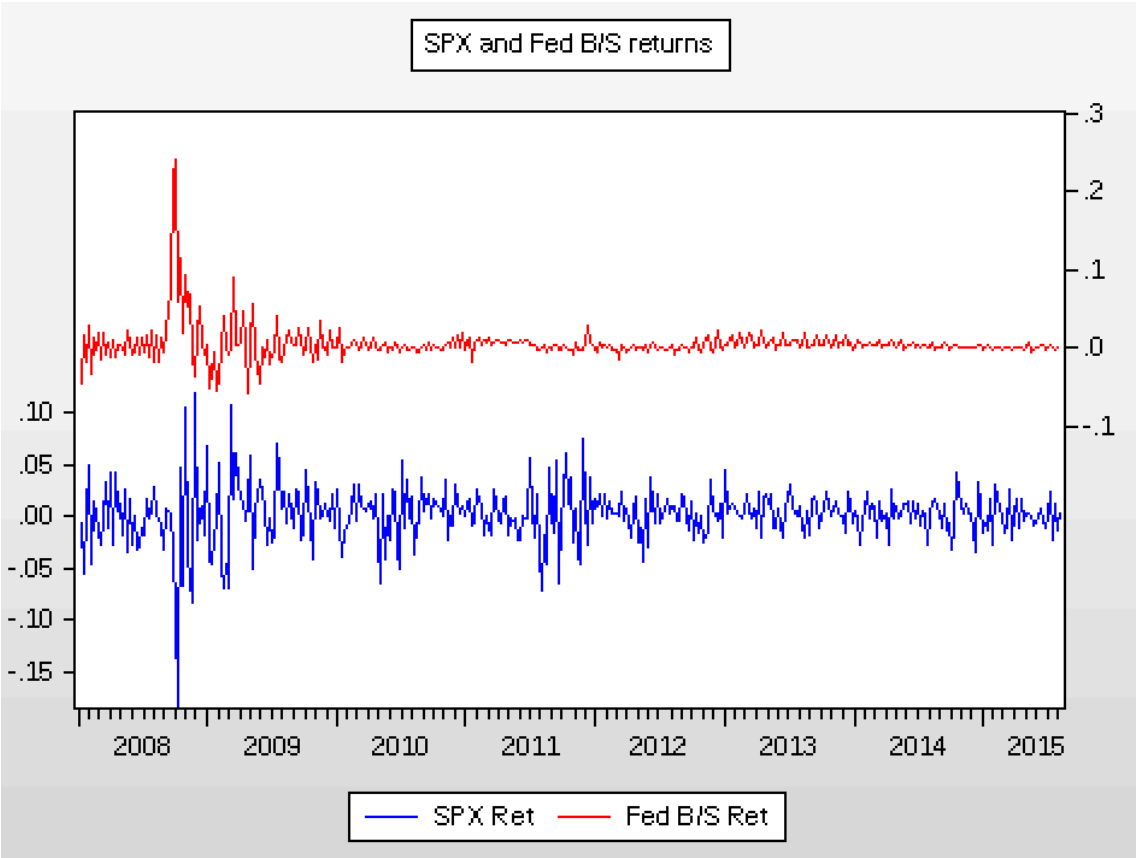

Figure 11. Comparable data between SPX and Fed B/S expressed in simple returns terms.

In order to proof the straightforward Fed B/S effect on SPX performance, a simple AR (1) might have been performed with some astonishing results are as Fed B/S lagged to previous period and SPX auto regression would explain 99% of the SPX movement. Nonetheless, this approach is erroneous since the both data present a strong upward trend. In fact, augmented Dickey-Fuller unit root test has been performed to understand whether the data were stationary: the data were proved to be not stationary. Also graphically it can be easily noticed that the mean presents a changing behaviour. This is the reason for which the simple returns have been calculated to create stationary data, which can be modelled[97]. In the figure 11, the transformed stationary data patterns are drawn.

Dependent Variable: SPX_RET
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
GARCH = C(4) + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
FED_B_S_RET(-1)	-0.209229	0.042820	-4.886241	0.0000
C	0.002469	0.000849	2.908182	0.0036
AR(1)	-0.106820	0.049001	-2.179984	0.0293
Variance Equation				
C	4.39E-05	1.24E-05	3.537685	0.0004
RESID(-1)^2*(RESID(-1)<0)	0.472913	0.085159	5.553305	0.0000
GARCH(-1)	0.696619	0.040607	17.15499	0.0000

R-squared	0.100623	Mean dependent var	0.001529
Adjusted R-squared	0.096034	S.D. dependent var	0.027431
S.E. of regression	0.026080	Akaike info criterion	-4.856323
Sum squared resid	0.266631	Schwarz criterion	-4.795884
Log likelihood	965.1238	Hannan-Quinn criter.	-4.832377
Durbin-Watson stat	2.008837		
Inverted AR Roots	-.11		

Table 4. SPX regression output.

Among the different models explaining the percentage of SPX variation, the author retained the AR (1) for mean with TGARCH (1,1) model for conditional

[97] The Augmented Dickey-Fuller unit root test rejected unit root hypothesis for both return series.

variance as the most appropriate (table 4), based on BIC and AIC[98] comparison. The standardized residuals deriving from Jarque-Bera normality test seems be to normally distributed at 1% confidence level[99]. Although the adjusted R-squared (10%) is not very high, for the explanatory purpose the author is conducting, the results are more than satisfactory. Since the underlying upward deterministic trend has been removed; only the underlying weekly error variations are mainly modelled. Moreover TGARCH (1,1) conditional standard deviation graph is presented in the Appendix 7, as it plots the one-step ahead standard deviation for each observation in the sample[100]. In the model the bad news is reflected in the negative SPX returns, which have a higher impact on volatility and creates a leverage effect. Obviously, a partial explanation of some variations does not depend solely on Fed B/S directly, but also on some psychological behaviour, political and worldwide news influencing SPX quotation, where Fed might indirectly influence the cycle.

The relationship between Fed B/S expansion and VIX has been already presented; nonetheless some more analysis has been conducted. It is important to recall that VIX represents a "fear gauge", and has a contrarian relationship with its underlying SPX index. It might be a bit harder to prove that the last years VIX shrink is directly related to the monetary policies through some analytical tools. In the author's analysis, VIX as well as SPX present non-stationary data, thus a simple percentage variation has been calculated[101]. Moreover, it is logical to think that as soon as the monetary policy run out each round, the uncertainty in the markets increase, thus VIX spikes up: this is the reason why a swapped TGARCH model would be the most appropriate, which might account for these asymmetrical effects in the volatility estimation. Among the different classes of regressions the author analysed, the most appropriate seems to be the following:

[98] Schwarz and Akaike information criterions.
[99] P-value equal to 0.0139.
[100] Retrieved from: http://www.eviews.com/online_help
[101] The Augmented Dickey-Fuller unit root test rejected unit root hypothesis for both return series.

Dependent Variable: VIX_RET
Method: ML ARCH - Normal distribution (OPG - BHHH / Marquardt steps)
GARCH = C(3) + C(4)*RESID(-1)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
FED_B_S_RET(-1)	1.276131	0.214588	5.946898	0.0000
AR(1)	-0.228267	0.067784	-3.367583	0.0008
Variance Equation				
C	0.016893	0.001006	16.79955	0.0000
RESID(-1)^2	0.162341	0.039768	4.082206	0.0000

R-squared	0.073105	Mean dependent var	0.008025
Adjusted R-squared	0.070747	S.D. dependent var	0.146311
S.E. of regression	0.141040	Akaike info criterion	-1.085626
Sum squared resid	7.817705	Schwarz criterion	-1.045334
Log likelihood	218.4112	Hannan-Quinn criter.	-1.069662
Durbin-Watson stat	1.961893		
Inverted AR Roots	-.23		

Table 5. Eviews regression output of VIX returns.

This AR (1) model with ARCH (1) explains 7% of the VIX returns, which is high enough to infer that Fed B/S might have an influence on VIX. Provided that VIX reflects the price of calls and puts on SPX, it is deeply affected by its quotation. Thus, VIX variation regression on SPX not only will reflect the future market expectation, but also the monetary policies incorporated in SPX reaction. By observing the SPX and VIX distributions in the Figure 12[102], it becomes evident that at the same time volatility clusters are mirrored in both indexes. Although from 2012 SPX seems to present a homoscedastic behaviour, VIX spiked up several times. The regression analysis of VIX on SPX produced very good results in the Table 6: 50% of the VIX variation is explained by the first order auto regression AR (1) and by SPX returns, as well the conditional variance might be modelled by a Threshold component of an ARCH model[103]. Although the regression fits the VIX data fairly well, the standardized residuals from the

[102] The Augmented Dickey-Fuller unit root test rejected unit root hypothesis for both return series.
[103] For practical reasons, the original VIX_ret series has been inverted to estimate TGARCH, notheless such transformation has no influence on the estimation output.

regression are non-normal[104] and heavy-tailed on the right side. This precludes us from statistical inference since the standard errors might be biased; nonetheless the explanatory power of the model continues to be significant. The actual empirical distribution, fitted model as well residuals from the regression are presented in the Appendix 8. It can be noticed that some extreme VIX fluctuations seems to be unpredictable and cannot be explained by SPX quotation prices: this reflects the "bull market in fear" reaction, which suppose that the investors are fearful about the future market moves, although today the market might be relatively stable. Moreover, some more relationships and trading mechanics on VIX will be proposed in the next chapter.

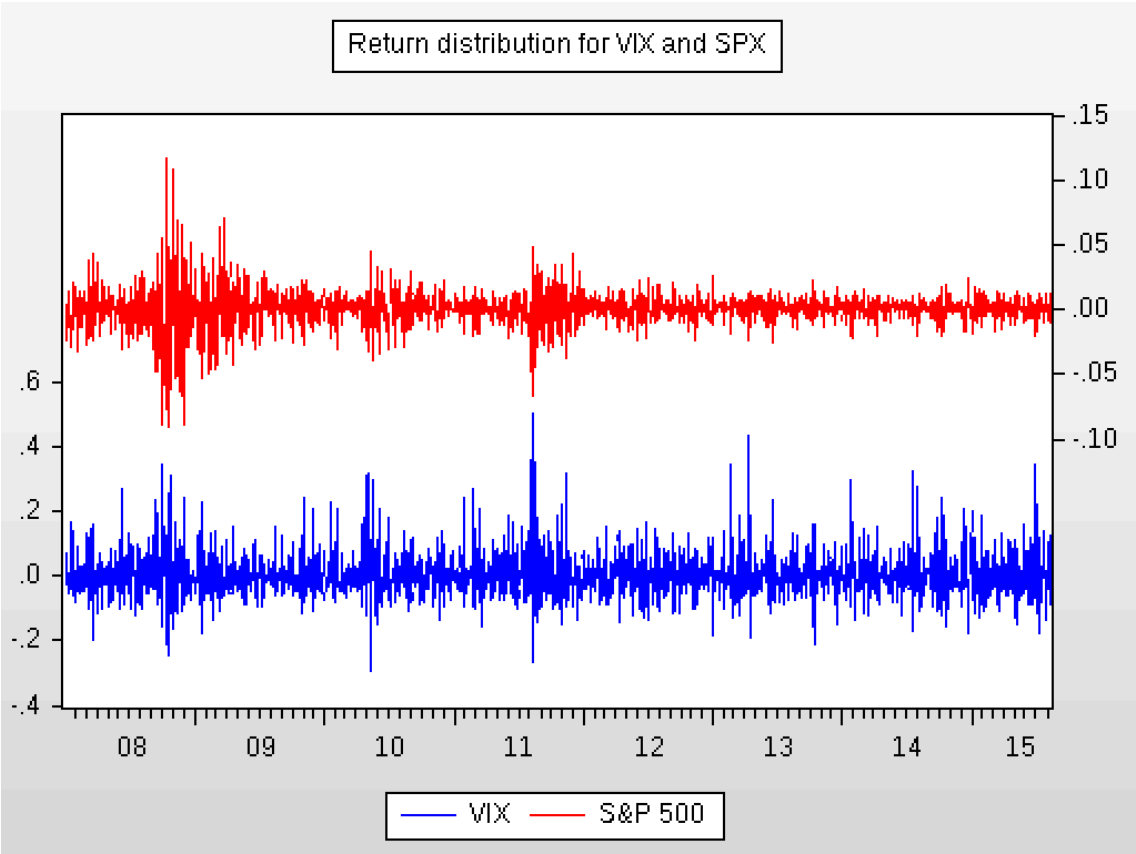

Figure 12. VIX and SPX simple returns distribution.

[104] The Jarque-Bera normality assumption on stardardized residuals is rejected.

Dependent Variable: INVVIX_RET
Method: ML ARCH - Student's t distribution (OPG - BHHH / Marquardt steps)
GARCH = C(3) + C(4)*RESID(-1)^2*(RESID(-1)<0)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
SPX_RET	3.374730	0.133838	25.21506	0.0000
AR(1)	-0.251564	0.027358	-9.195122	0.0000
Variance Equation				
C	0.010275	0.001419	7.240490	0.0000
RESID(-1)^2*(RESID(-1)<0)	-0.020825	0.002981	-6.986902	0.0000
T-DIST. DOF	4.039089	0.720564	5.605453	0.0000

R-squared	0.505168	Mean dependent var	-0.008378
Adjusted R-squared	0.503912	S.D. dependent var	0.146294
S.E. of regression	0.103040	Akaike info criterion	-1.916387
Sum squared resid	4.183206	Schwarz criterion	-1.866116
Log likelihood	384.4446	Hannan-Quinn criter.	-1.896471
Durbin-Watson stat	1.852405		
Inverted AR Roots	-.25		

Table 6. VIX regression output.

In the conclusion, it might be inferred that Fed B/S expansive monetary policy did gradually improve the real US economical performance indicators such as unemployment rate and GDP; nonetheless GDP might have been significantly improved by financial sector performance. As result, SPX prices have increased, whereas volatility index VIX decreased to its lowest levels in absolute value.

European Union[105]

As Fed in US, as well the European Central Bank in for the Eurozone defines and implements the monetary policy, allows euro banknotes' issues, maintains price stability (through inflation stabilization) and manages foreign exchange operations, although some objectives and methodologies used differ. Unlike Fed in the US, which purchases Treasury securities, the Euro system use a different methodology to inject liquidity into the economy through repos[106] contracts.

As previously mentioned in chapter 1, in late 2009, the European debt crises began as some Eurozone member states become unable to repay their national government debt and/or to support their distressed domestic financial sector, as a consequence of GFC diffusion, without foreign parties support. As soon as the Greece openly admitted the insolvency issue and high budget deficit, the forthcoming probability of sovereign default spread panic among general public. A wave of credit ratings downgrades, soaring borrowing costs and interest rates spreads advanced on those Eurozone members considered to be financially unsustainable. Greece, Ireland, Portugal, Cyprus and Spain have received rescue funds to bailout the financial sector and to repay their sovereign debt, whereas to a lesser extent, Italy and France might have entered into serious troubles too. Among different reasons for such crises the main are as following:

1. The currency union has been introduced without homogeneous stability and financial convergence. Moreover, after having entered into monetary union some states did not put attention on public debt and budget deficits reduction to healthier level.
2. As in US, as well in Europe, the relaxed financial sector supervision allowed banks to invest in projects with inflated collateral and exorbitant growth rates, which resulted in some expensive bailouts. Moreover in

[105] In this chapter mainly Eurozone, that is euro monetary union will be presented, accounting for 19 out of 28 European Union Member States.
[106] Repurchase Agreements.

Spain and Ireland the real estate bubbles collapsed by producing colossal shocks to those economies.

3. Generally, the investors provided the Eurozone members with easy credit conditions: states' public deficits and private economy has been financed with "cheap money" independently of its current situation.
4. After having joined the Euro, some countries increased average wages and costs without necessarily having improved competitiveness and efficiency.
5. No strategy or exit mechanism has been foreseen in case of crisis or insolvency issue of any of the Eurozone member states. Thus, no one has been prepared to handle with the possible unforeseen events.

In the following years different funding mechanisms have been created to deal with debt crises such as EFSM[107], EFSF[108] together with IMF has injected funds to bailout Greece, Ireland and Portugal in 2010-2011. As well in the following years other lending mechanism have been created such as ESM[109] (2012) with its €500 billion to financially assist member states in difficulty: Spain and Cyprus have used these funds to recapitalize the financial sector. Moreover, in 2013 the Fiscal Stability Treaty has been introduces as a contract among member states that binds the states to introduce domestic mechanisms to safeguard public budgets and to maintain sustainable public debt levels. In 2014 to the ECB has been committed to supervise the financial stability of EU states. Despite many controversies, ECB has tried to answer to a variety of problems in the following ways:

- As previously mentioned ECB manages the money supply through the refinancing facilities, in which bonds are used in reverse transactions such repos[110] and collateralised loans. During the sovereign debt crisis, in 2011 EBS did purchased bonds issued Italy and Spain to reduce international speculation. This move reflects the Fed move on buying subprime MBS in 2008, although

[107] European Financial Stabilization Mechanism.
[108] European Financial Stability Facility.
[109] European Stability Mechanism, 2012.
[110] In the repos, the bonds ownership swaps to the ECB until the loan is repaid.

here the underlying assets are member sovereign bonds. Since May 2010 to 18 June 2012, as a part of Securities Markets Programme (SMP), ECB bought €212 billion bonds[111]. Since ECB buys bonds from other creditors, it does not reveal the trading price, thus creditors might have profited from selling bonds at higher than market price. Immediately after SMP, EBC announced Outright Monetary Transaction (OMT)[112], which had no limits on size and time of the Eurozone members sovereign bond purchases under certain conditions. In 2014, EBC announced further acquisitions of other debt instruments from banks to boost credit availability for businesses. As soon as the Greek situation worsened in 2015, the Emergency Lending Assistance (ELA) has been established to support banks liquidity crises in cases of massive deposit flight. And finally, on March 2015 the EBC commenced QE programme to relax sovereign stress of its member states with €60 billion purchases of Eurozone government, agencies and European Institutions bonds per month, which will be carried out at least until September 2016[113].

- The main refinancing operations regards repos auctions with very short-term (two weeks and one-month maturity), nonetheless nowadays EBC mainly conducts Long-Term Refinancing Operations (LTRO) with higher maturities (3, 6, 12 and 36 months). In 2003, this kind of financing amounted for nearly €45 billion, which is 20% of overall liquidity produced by ECB. From March 2008, some additional LTROs have been settled through 2 rounds. The LTRO1, with 3 years maturity and low interest (1%) loans, began on 21 December 2011 amounting for €489.2 billion[114]. The collateral required by ECB widened from European government securities, MBS and secure commercial papers. The banks of Italy, Ireland, Spain and Greece obtained the biggest amount of LTRO (€325 billion), thus ECB made sure that banks would be able to pay €200 billion of their maturing debts and hoped that a part of extra cash would be employed to fund businesses or would buy sovereign securities. The LTRO2 began on 29 February 2012 with further €529.5 billion allocated to 800 banks. Moreover,

[111] SMP had a temporary duration.
[112] ECB press conference, 6 September 2012.
[113] Retrieved from: https://www.ecb.europa.eu/press/pr/date/2015/html/pr150122_1.en.html
[114] 523 Banks have taken part in the LTRO1.

some rollovers over the preceding loans have been made. In the following graph the ECB B/S and REFI[115] rates are presented. It can be observed the refinancing rates decreased to 0.05% in September 2014, whereas deposit facility rates[116] decreased to -0.2%. The shaded areas present LTRO's and QE.

As a result of such monetary policies (graph 13), ECB doubled its capitalisation. Nonetheless, the economy did not improve: unemployment rate is still above 10%, inflation still too low, slow GDP growth and new Greek sovereign debt issue. Since interest rates were already set to zero, QE might be the last resort measure ECB has got to boost the economy recovery. In the graph 14, the red line represents unemployment rate, whereas blue line draws the GDP[117] seasonally adjusted growth rate. The unemployment rate increased exponentially from 2009 by reaching its 11.9% peak in 2013, while GDP has firstly recovered after GFC in 2010 and decreased again after sovereign debt crises. The full economic recovery to the pre-GFC level seems too far from being achieved yet.

Nonetheless, it had a significant impact on suppressing high spreads on sovereign bond yields. Provided that ECB bought bonds from other creditors, such as financial institutions, it did not reveal the trading price, which might have profited creditors, who might sell bonds at higher than market price. If fact, when the demand is high, the prices go up and interest rates goes down, as it happened. In the following graph Cypriot, German, Spanish, Italian, French and Greek bond yields are presented in the figure 15:

[115] Refinancing operations rates.
[116] Overnight deposit rates.
[117] Data source: Eurostat, 2015.

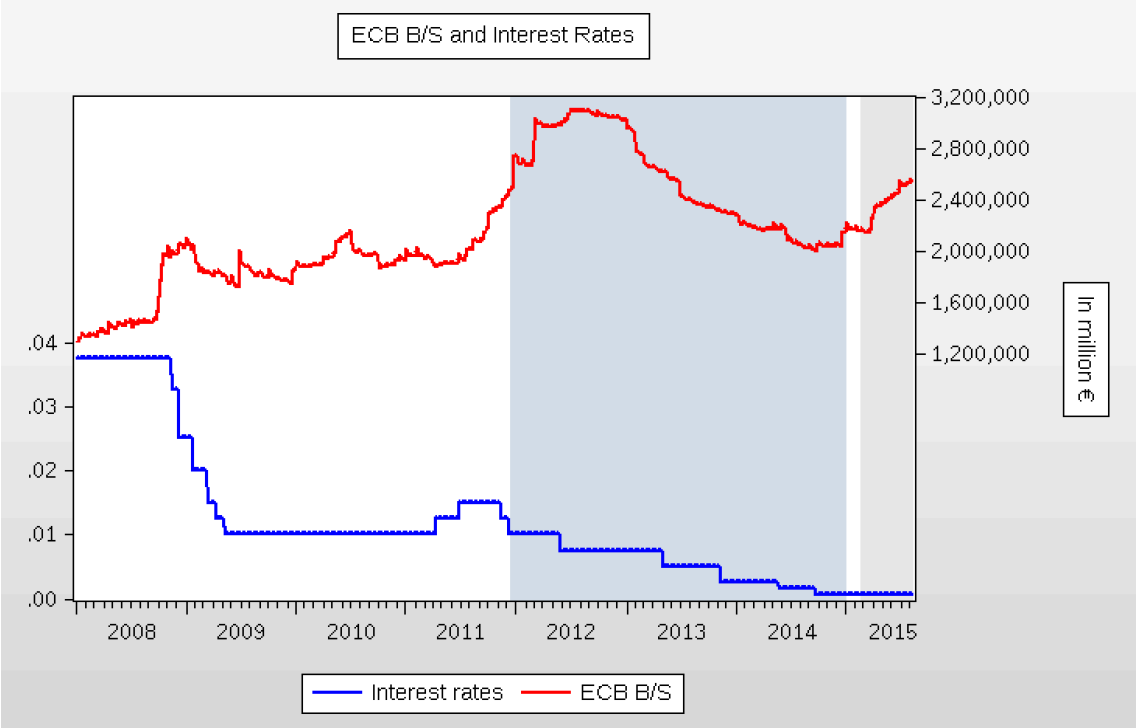

Figure 13. ECB monetary policy[118].

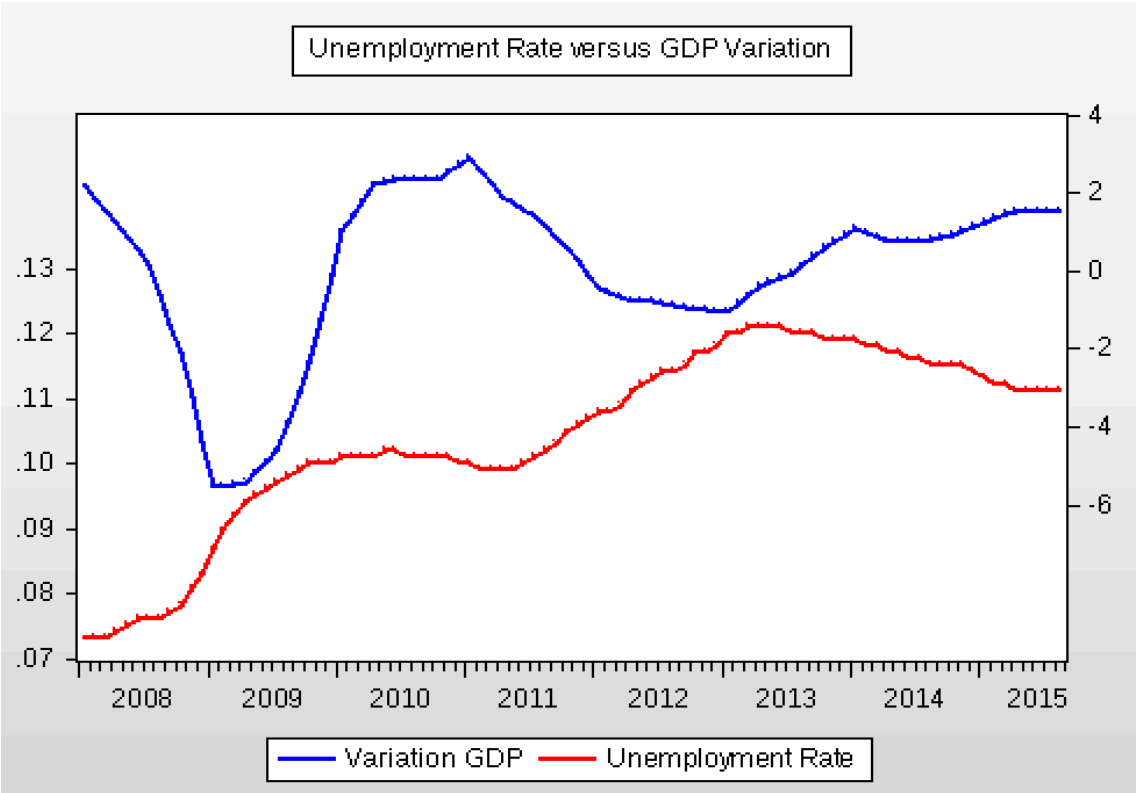

Figure 14. GDP variation versus Unemployment rate.

[118] Data source: ECB.

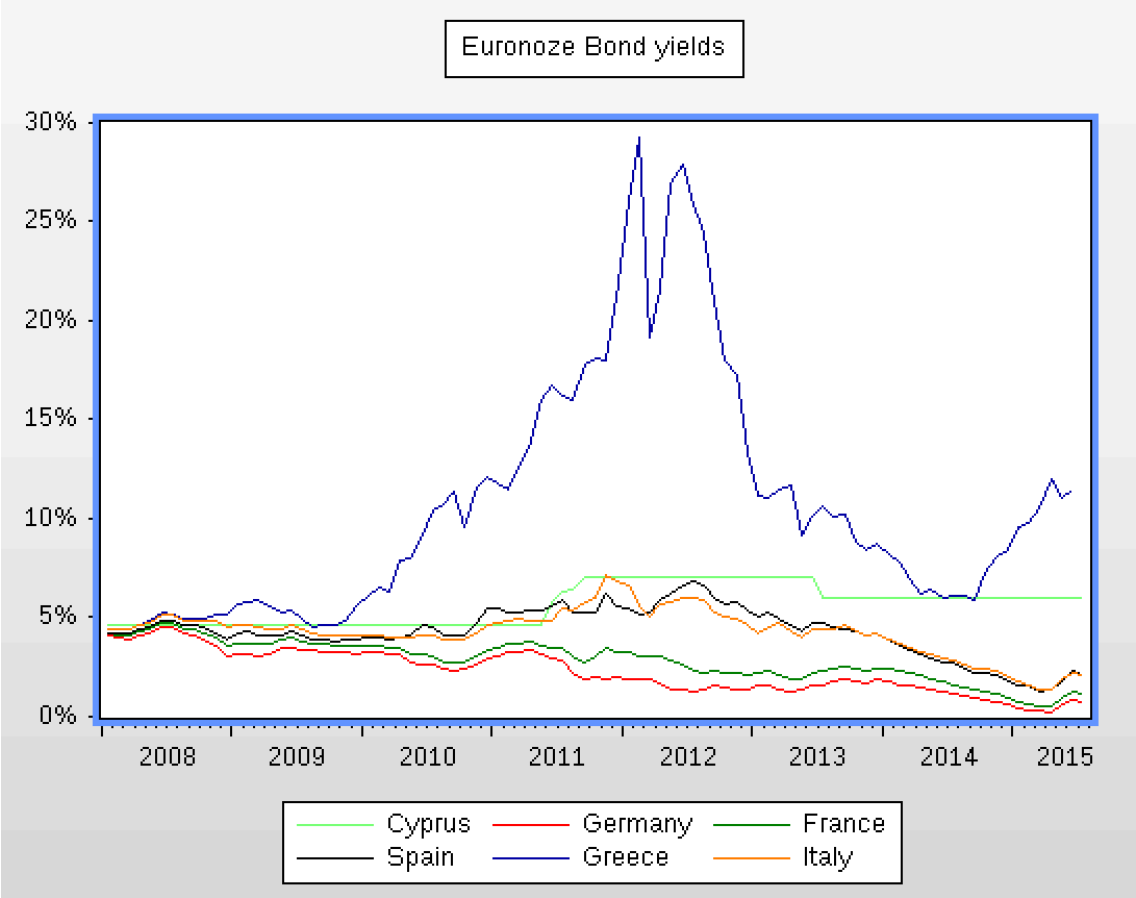

Figure 15. Sovereign Bong Yields.

From the graph, it is evident how sovereign yield deceased to their lower levels since 2008 thanks to ECB support. Although debt-restructuring agreements with creditors and liquidity injections, Greek situation is still difficult as its debt/GDP ratio rages between 177-180%.

After having mentioned monetary policies so far, the effects on the stock index as well on volatility index are going to be presented. As previously mentioned in chapter 1, EURO STOXX 50 (SX5E)[119] is Eurozone Blue-chip index, introduced on 26th February 1998 and traded on Eurex[120], which serves as underlying asset and benchmark for a broad range of financial instruments such as ETF, futures and options. The VSTOXX indexes are based on SX5E real-time options prices (implied volatility) and reflect market expectation on future index volatility. The VSTOXX as VIX are often used as benchmark to determine the predominant level of uncertainty, "fear gauges" in financial market. In the

[119] Made of 50 most liquid and largest corporations.
[120] Eurex Exchange under Deutsche Borse AG.

following graph both indexes are plotted and in the grey areas the ECB monetary policies are shaded: it can be easily noticed that during the sovereign debt crises as soon as the ECB provided support the volatility index lowered, whereas in 2011 in between two monetary supports, the volatility spiked to 50 percentage points.

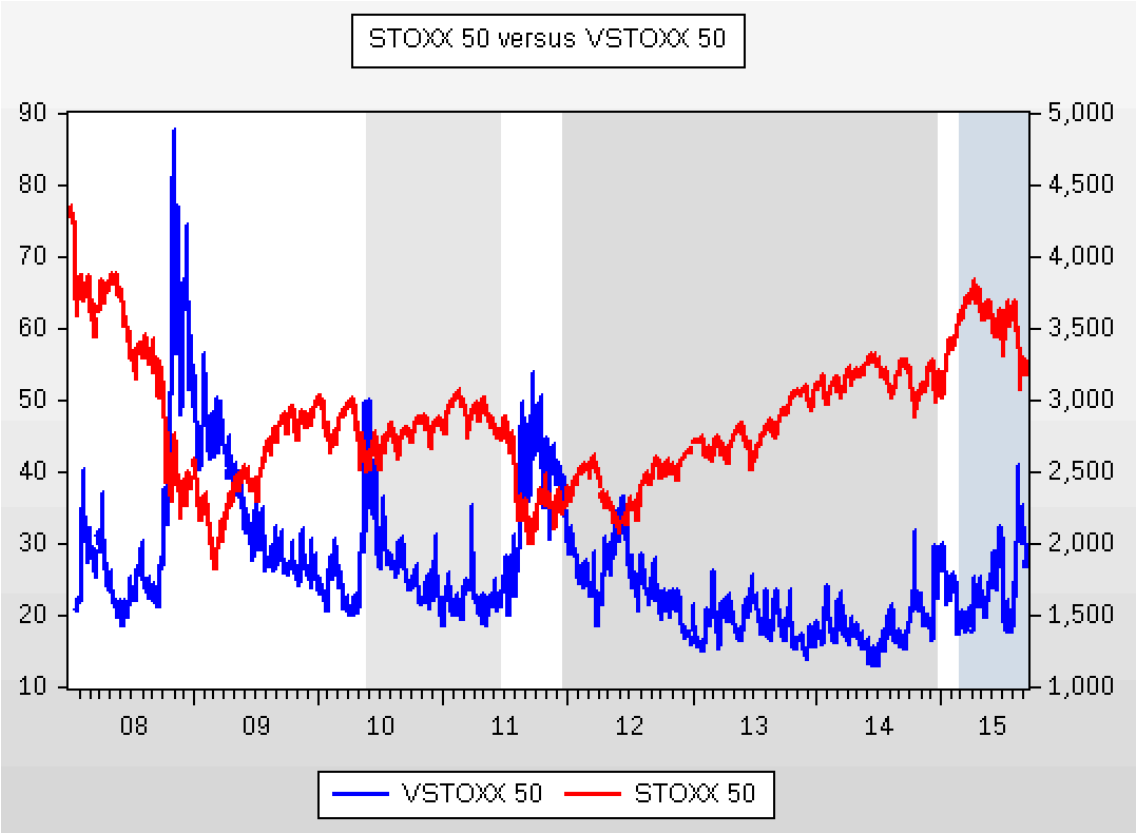

Figure 16. STOXX versus VSTOXX.

It really seems that monetary policies has significantly contributed to volatility flattening: the question is how long the central banks will be able to generate money flows. This measure works as long as ECB measures are credible and there are some margins for new monetary measures, but as soon as the stimulus will saturate with no other possible measures to be undertaken, the markets and volatility will move on its own fate.

The underlying negative correlation between volatility index and equity market is generally explained during market turmoil, when many investors are buying protection for their portfolios, which increases option prices and pushes implied volatility upwards. Moreover, it might worth to recall that the negative correlation might be as well explained by the so-called “leverage effect”. When company’s

equity value declines, the companies B/S become more leveraged, thus the equity is riskier and volatility of the share price increases.

A simple correlation between SX5E and VSTOXX has been estimated being equal to -0.49 from 2008 to September 2015, although it presents an "average" correlation as both series present trends and the correlation changes at any point[121]. Since, the authors evaluation was based on closing prices, some other authors claim a lower correlation of returns being equal to -0.72[122]. Some more inferences can be done to further estimate the relationship between two indexes as it was presented in the US case, nonetheless the same approach might be redundant as both markets' indexes present the same characteristics.

In conclusion, it might be inferred that from 2008 to 2015, the Eurozone performance has been affected twice: firstly by GFC, then by sovereign debt crises. In fact, GDP began to improve, when it was hit once again, whereas unemployment rate never recovered after GFC shock, but even worsened after 2011 sovereign debt crises. The same behaviour might be observed in stock index as after GFC the recovery was truncated by sovereign debt crises and only from mid-2012 the prices gradually increased. As well VIX flash spiked heavily three times over 40 percentage points.

[121] The results are presented in the Appendix 9.

[122] Giese G. (2010), *Volatility as an Asset Class*, Stoxx Limited.

VIX versus VSTOXX

Both indexes are perceived as "fear gauge" and are the best contrarian indexes in the business. Since these indexes track the underlying SPX or STOXX 50 options trading, which employs a wide spectrum of option strike prices to accurately estimate implied volatilities, they reflect investors' expectation on the market moves over the next 30 days. For instance, when investors panic, they buy portfolio insurance through put options, which increase in their prices and VIX spikes up. Contrariwise, when investors sell options, their price become much cheaper and VIX decreases. Moreover, these indexes are structured in such a way to reflect skewed (smiling) characteristic of the volatility surface. As VIX as well VSTOXX presents some main and sub indexes covering different maturities of the STOXX 50 Index options contracts[123].

Since both of them are quoted in percentage points and not in price terms, they might be roughly translated into the expected annualised movement of the underlying index over the next 30 days. A VIX quotation of 15 indicates that the underlying index might fluctuate by 15% over the next year; equivalent to 4.33% monthly volatility, this is because volatility is not linear in time. A VIX and VSTOXX greater than 30 are associated with high uncertainty and stress level on the markets: VIX 20-year average is equal to 20.43[124], whereas 16-year VSTOXX average is equal to 25.22. In the following graph, it can be observed the relationship between the two indexes: they are specularly mirrored, although in some periods they might trade relatively very close. Moreover, the correlation between VSTOXX spot and VIX spot ranges between 0.2 and 0.8 on 20 day rolling correlation[125].

[123] Main indexes ranges from 30 to 360 days, whereas sub indexes cover next 1,2,3,6,9,12,18,24 months.

[124] *How to trade VIX Stock Market Fear Index* (7 October, 2013), The Market Oracle. Retrieved from: http://www.marketoracle.co.uk/Article42579.html.

[125] Shore M. (5 November, 2013). *Forward curves of European and US volatility index futures*. Retrieved from http://eurexgroup.com.

Figure 17. VIX (blue line) versus VSTOXX (black line).

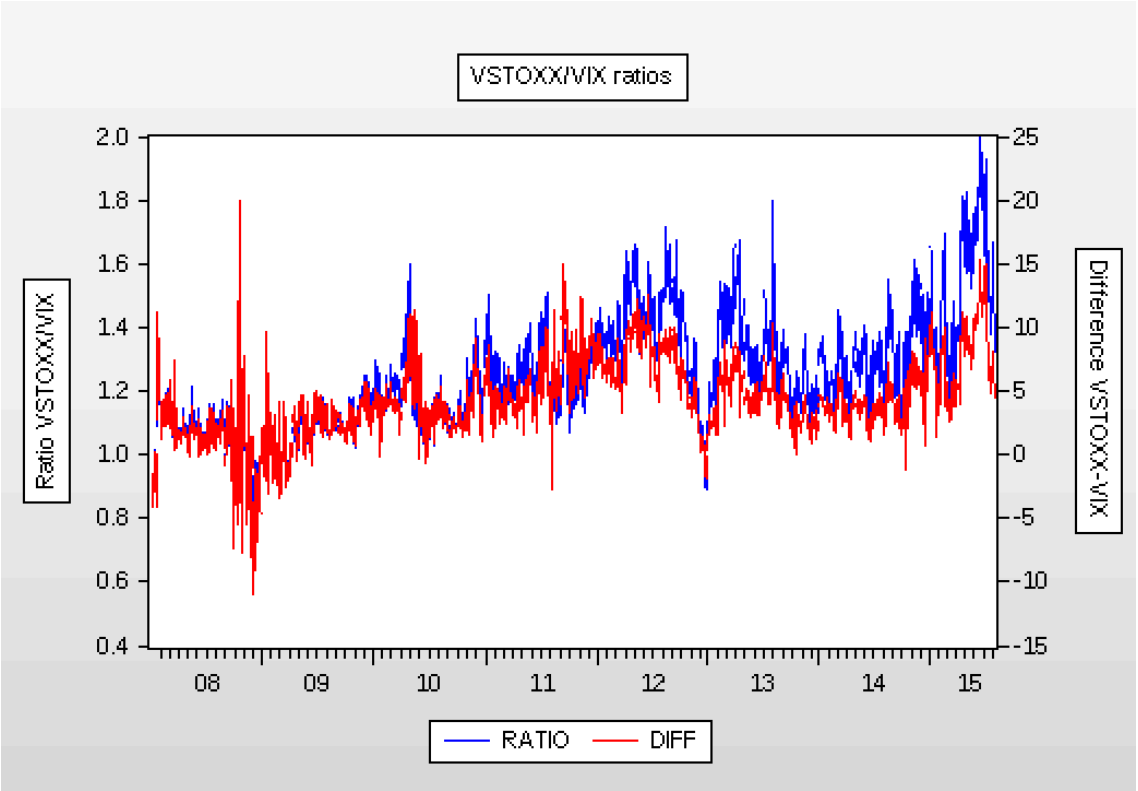

Figure 18. VSTOXX to VIX ratios.

The macro pictures of both countries influences indexes, but some bigger spreads might happen when European or Us counterparty have more negative news, whereas the other over performs. To these regard some ratios might be observed such as VSTOXX/VIX and VSTOXX-VIX difference. The difference in

absolute value between two indexes depends also on the level of quotations: for instance, 8 points difference between 25 and 18 rather 60 and 52 are different. Thus, both indexes should be compared at the same time. From 2012 the ratio index significantly increased: in the figure 18, the ratio growth is evident.

Obviously, VSTOXX is higher compared to VIX, but sometimes happens that when VIX is extremely low, the VSTOXX/VIX ratio goes beyond 1.50 and returns back after a couple of days. This might create a place for arbitrage: go long with VIX and short VSTOXX, nonetheless it is important to keep in mind that these indexes measure implied volatility of short-term options, nonetheless neglect longer terms. Although it is theoretically sure that the difference between two indexes will reduce again, the question is exactly when. And if it will happen later, then such a strategy will produce a loss. In fact, the term structure of VIX and VSTOXX derivatives reflects such mean reversion: the difference between the futures curves of both is less than 5 points[126], which is the difference between long-run averages. Moreover, the average VSTOXX/VIX ratio from 2008 to August 2015 has been equal to 1.24, whereas VSTOXX-VIX ratio to 4.32 percentage points. In order to trade properly VIX against VSTOXX, the forward term structures of both should be deeply analysed. To this regard, in the following chapter the way VIX can be traded and used in portfolio insurance will be discussed, afterwards the same analysis might be conducted on VSTOXX to evaluate possible spread strategies, nonetheless this goes beyond the purpose of this paper.

In the conclusion, in this chapter an analysis on the behaviour of the financial markets and volatility indexes in response to central banks monetary policies has been conducted. As a result, it has been inferred that in US, the monetary policies had major impact on the financial markets and volatility index flattered to its lowest levels; whereas in Europe the monetary stimulus had a significant effect to decrease sovereign bond yields, nonetheless financial markets, economy and volatility indexes only partially improved and the QE will continue at least until September 2016.

[126] Retrieved from http://www.macroption.com/vstoxx-vix-ratio-high-1303.

Chapter 3

How to trade and hedge with VIX

In the previous chapters, volatility has been analysed from different perspectives, and some important results have been produced. It has been demonstrated that VIX significantly smoothed by central bank intervention, and this will work until Fed margins and credibility on monetary support are trustworthy; nonetheless, when investors will realise that Fed has no further tools to support the economy, the markets will be subject to all future shocks. In this chapter, VIX trading strategies will be analysed and some considerations on the volatility as an emerging asset class will be presented.

The VIX reputation as "fear gauge" derives from its negative correlation with SPX; the more terrifying the vast market decline, the higher the VIX tends to rise. A VIX higher than 30 implies a high volatility, instead the values below 24 express calmer times. During the 2008-2009 stock market collapse, VIX spiked as high as 80. The daily VIX percentage move is about 4[127] times SPX percentage move, and the intraday historical maximum and minimum values ranged from 9.39[128] and 89.53[129]. When there is fear in the market, traders might both sell calls and buy puts on SPX, thus why the two positions do not balance out and the VIX does not react less dangerously? When a call is sold against a position hold in the portfolio, a "premium" (option price) is collected; thus when the underlying asset goes down the downside risk is hedged only by the collected amount. Whereas, when a put is sold, the profit or hedge will be higher the sharper the drop of the underlying asset. During the times of general sell-offs, the investors buy insurance for their portfolio and traders invest in speculative positions without much apprehension on the put price they might pay.

[127] Harwood V. (17 March, 2015). *How Much Should We Expect the VIX to Move*, retrieved from http://sixfigureinvesting.com/2012/11/how-much-should-we-expect-the-vix-to-move/
[128] 15 December 2006.
[129] 24 October 2008.

Any bad news such as Fed interest rates increase, awareness about the end of bull market in stocks, China's or European shocks might cause stock prices to fall and increase sooner or later investors' fear level. As a consequence, one might think that trade on VIX is an easy game: go long on VIX at the lowest values and short it when VIX levels increase. Nonetheless, a complication arises: it is not possible to trade directly on VIX because it is a derived index from SPX options. Here again, one might think that by replicating the options from which VIX is calculated, it could be possible to statically replicate VIX; however VIX is not a simple weighted sum of the underlying options, but presents a non-linear transformation because the options sum up to the square of VIX, not VIX itself. Moreover, depending on the kind of VIX derivatives you might prefer, it is necessary to forecast exact timing the VIX will soar. This is the reason why VIX is a better tool for market insurance rather than speculation, still volatility is itself an asset class, which can be traded to lock in profit: the question is how and when to use it. An investor willing to hedge his/her stock portfolio against market crash, might think of long position on VIX as insurance: when the underlying asset is damaged, a reimbursement is received. Nowadays, after GFC the realised volatilities significantly increased, likewise the implied volatilities (by option prices) measured by volatility indexes reflect increased options price during market downturns. This is the reason why the investors looking for portfolio diversification, might find this complex asset class attractive and accessible through a wide range of traded products.

Before analysing the products and the way they are traded, it might be worth to recall some volatility characteristic features:

1. It does not produce any explicit returns.
2. In the long run, it presents mean reversion characteristic tending towards an average level.
3. Typically, it does not present long term upward movements, which is typical in equity markets, but high volatility spikes through "jumps" in short term, tending afterwards to the long run mean level; the equity

returns are typically heteroskedastic, thus volatility indexes present this characteristic as well.

4. As previously analysed, there is significant negative correlation between volatility index and the underlying asset.
5. Typically, the implied volatilities (vols) are higher than realised volatilities, because of the premium paid to the protection seller.

The easiest VIX traded products are offered on the CBOE: options as well futures. Options are European style and present some specific characteristic regarding expiration dates, settlement, margins, multiplier ($100) and others, which can be found on CBOE page. VIX Futures present a contract multiplier of $1000 and all the necessary information on expiration, trades, executions, settlements and margin requirements are available on CBOE web page as well. Nonetheless, an investor willing to trade the VIX, can invest in the existing 25 volatility ETPs, such as VIX futures-based ETNs or ETFs[130], which supply retail demand for vega on the front of the term structure. Among ETPs the most popular are iPath S&P 500 VIX Short-Term Futures ETN (NYSE: VXX) and S&P 500 VIX Mid-Term Futures ETN (NYSE: VXZ)[131]. The main difference between the two categories of products is that ETFs are backed by an appropriate futures or swap, whereas backing of ETNs depends on the credit worthiness of the financial institution (provider). The VXX and VXZ are widely traded because of their simplicity and relatively good short-term performance. However, since VIX is usually observed at spot basis, none of the ETFs or ETNs, which select a range of futures on VIX, performs exactly spot VIX volatility.

With an average 50 million daily shares volume, *VXX* is highly liquid and simple product as it trades like a stock: it can be bought or sold as well short sold with extremely low bid/ask spreads (penny spread). Unlike stocks, the market sets the value of VXX and every 15 seconds an “intraday indicative” is published. If trades diverge too much from this value the trades are adjusted through

[130] The list of most VIX oriented ETNs and ETFs in the US markets are listed on http://sixfigureinvesting.com/2010/12/volatility-tickers.

[131] In February 2009, both have been launched by Barclays iPath

Authorized Participants in such way that VXX ideally tracks CBOE's VIX. VXX holds a long position in first and second month VIX futures contracts, which roll daily[132]. Provided that in long-dated contracts there is an implicit insurance premium, the VXX realises a negative roll yield, which is a great drawback of the VXX. In fact, during the periods of low volatility VXX often trades higher than it should, reflecting the expectation of increasing volatility as a consequence of mean reversion, while during high volatility periods VXX trades lower, reflecting the expectation of the return to lower VIX values. However this roll generates value decay over time or so called contango loss in case of bullish market, or backwardation loss in case of bearish market. To this regard some traders[133] consider VXX to be a good contrarian investment to short-hedge the portfolio, but a relatively poor investment if the timing is not good enough.

The iPath S&P 500 VIX Mid-Term Futures ETN (VXZ) is similar to VXX by the structure, but it holds with longer positions such as 4,5,6 and 7-month VIX futures with an average duration of around 5 months. Thus, it presents a better measure of future volatility and obviously is much less volatile. Obviously, in case of stable and low volatility, this futures index will lose money.

For more expert and higher risks bearer investors, more leveraged alternatives are present such as VelocityShares Daily two-times VIX Short-Term ETN (TVIX)[134], which moves as VXX but with higher returns when VIX spikes up. As well TVIX resents the same contango loss issue as other VIX proxies; it is not a position to buy and hold. Credit Suisse on its TVIX description specifies: "If you hold your ETN as a long-term investment, it is likely that you will lose all or a substantial portion of your investment."

The financial engineering for VIX proxies' products offers also the other side of the volatility as well. Such indexes as XXV[135] replicate the performance of

[132] Retrieved from http://www.investopedia.com/stock-analysis/2012/4-ways-to-trade-the-vix-vxx-vxz-tvix-xxv-xiv0216.aspx
[133] Harwood V.
[134] TVIX is two times leveraged.
[135] Ipath Inverse S&P 500 VIX SHprt-Term ETN.

shorting VXX, while XIV[136] try to replicate the performance of going short on a weighted average maturity of one-month VIX futures. Nonetheless, more expert financial traders claim that most of these structured products fail to capture more than 50% of the daily VIX moves, whereas the longer-term products do even worse[137]. On one hand, the simplest ETN and ETF products are simple to trade, thus acceptable for short-term but not reliable in long-term, as most of VIX products head to zero, that means lose their value[138]. The investors willing to invest in volatility should consider VIX futures and options, as well structured options strategies.

To summarize, there is no way to directly invest on VIX. Firstly, an investor might use CBOE offered products such as futures and VIX options, which are easier and much more intuitive for hedging purposes. Futures present a special term structure, which will be presented in the next chapter as they provide a straightforward explanation of cyclical performance of volatility ETPs. VIX Options are based on VIX futures and not the VIX itself, which might seem trivial. Moreover, there are various type ETPs on which an investor might go short or long, invest in their options (where available), but none of them present a good performance in long run. To this regard, an aggressive strategy on VIX might be presented by leveraged ETPs, which might be very risky if the direction of movement is wrong. On the other side, non-leveraged volatility ETNs such as VXX present roll loss when futures are in contango, which produce 5-10% loss a month. In the figure 17 the split adjusted VXX since it creation versus VIX is presented. It can be noticed that at the beginning VXX in 2009, this index value was about 7000, whereas VIX around 45. In December of the same year VXX had a 63% drop to 2432 compared to only 50% VIX decrease to 22; in September 2015, VXX traded at around 25 and VIX 23. From its inception (01/30/3009) the index prise decreased by 99.6%, while VIX only by 33.6% and SPX icreased by 138.80%[139]. This significant divergence present

[136] VelocityShares Daily Inverse VIX Short-Term ETN.
[137] http://www.investopedia.com/stock-analysis/2012/4-ways-to-trade-the-vix-vxx-vxz-tvix-xxv-xiv0216.aspx
[138] Warner A. (20 September, 2010). A VIX Product That's Headed to Zero, retrieved from http://investorplace.com/2010/09/vix-etn-vxx-losing-value/#.VgZpXlwqhFg
[139] Source: Bloomberg database.

a substantial tracking error, nonetheless with its high volumes VXX is still reliable for short-term because of its negative correlation with the SPX. The reason for this significant loss will be much more clear after VIX futures term structure will be presented. Principally, VXX will always lose value as VIX futures are in contango and the strategy of VXX consists in selling front-month futures and buy second- month futures to maintain 30 days distance between the two, but obviously for the next month futures a higher premium is paid and a lower premium is received from the sold ones.

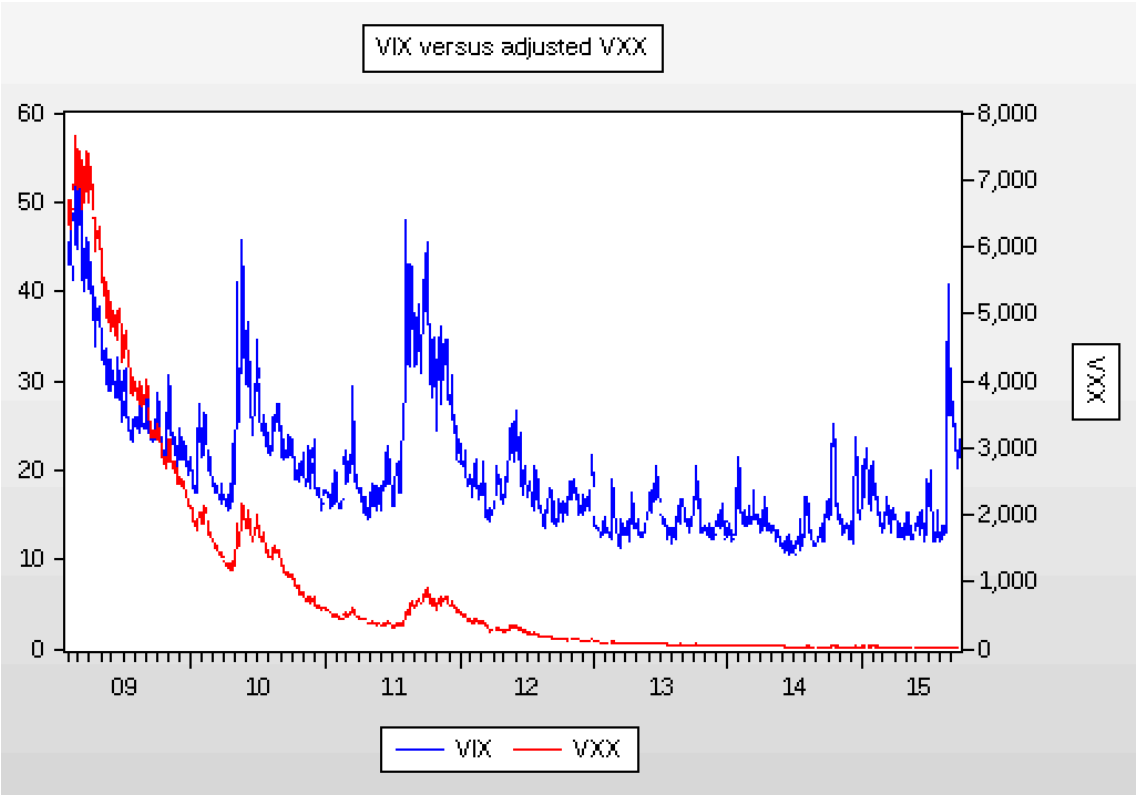

Figure 19. VIX versus VXX (short term note).

Nonetheless, it is worth underline that VIX is not useful to predict the market future value, rather it mirror pretty well the current investors' emotional state on the market moves.

Cost of Hedging

Futures contracts in general, but also VIX futures have been designed to allow businesses to limit assets price risks by locking in the future price of the underlying asset. Nonetheless, nowadays these instruments are widely used for speculation because of low margin requirements on these products. In order to trade futures many factors should be considered. Firstly, here the fundamental cost of carry link that relates futures price to the underlying index, is missing because the underlying VIX is not tradable. To price these futures a statistical method based on the VIX distribution, trend strength, volatility as well mean-reversion features should be applied. Thus, VIX futures are more similar to the options with their own Greeks rather than ordinary futures.

The price between VIX and its futures price might be significantly different; ultimately the two will converge closer to each other as the futures expiration date is approached. Still at expiration the futures could settle up to -+5% different from VIX level. The futures "term structure" reflects different future price expectation of the underlying asset. The term structure can be normal or inverted or both changing in time depending on the kind of securities are traded. Nonetheless, VIX futures curve (a crude proxy for SPX forward volatility markets) mean reverts to its long run mean, thus the term structure depends on the current VIX level. If the VIX spiked above its long-term mean, the futures expectation will be in backwardation, and when the VIX is low, the futures more likely to price in contango. Thus, the futures term structure might be flat, concave or convex. However, in the last 5 years, before August 2015 spikes, the volatility decreased to its lowest values and futures term structure was very often in contango, meaning that the term structure trades at premium to the spot. This is the reason why most of the volatility funds such as VXX were erasing NAV: the funds by rolling the futures curve, were replacing the existing assets with more expensive ones. To this regard, in the following graph are presented VIX futures term structures for 31 July 2015, 24 August 2015 and 25 September 2015. On 31th July the term structure was in contango as the spot VIX was 12.12, which is under the long-run mean, whereas on 24th August,

during 2015 stock market selloff, the VIX jumped to 40, thus the futures where trading in backwardation. On September VIX futures term structure was flat tending to long run mean value, whereas VIX quoted at 23.

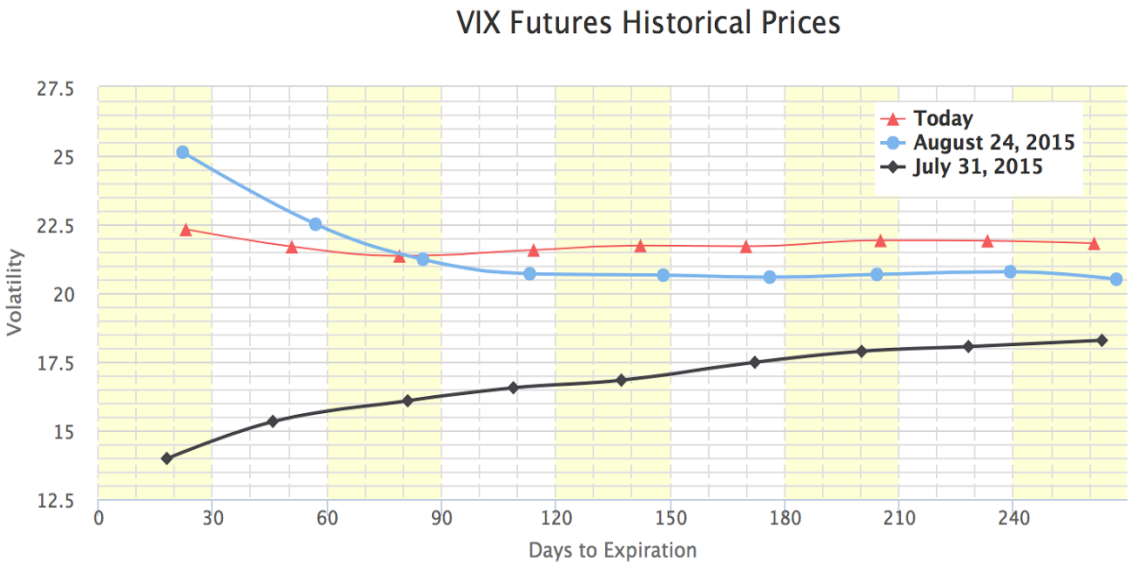

Figure 20. VIX Futures Term Structure[140].

As it has been explained, the futures term structure changing behaviour has a huge impact on the performance of the ETPs that try to replicate VIX. For instance VXX, by rolling 30-days futures, loses continuously value when futures present contango and spot VIX level is low or stable, as the fund is buying more expensive futures compared to the ones sold (near spot VIX value)[141]. On the contrary, when VIX unexpectedly spikes up, the difference between new and old futures is significant, thus the ETN makes money by receiving the difference between high VIX and spot futures value (entered into the contract some time ago). In fact, VXX in last 3 months earned 44.5%, whereas an investor that would hold the position for the last 5 years would lose 54.7%. If other Barclay's volatility notes are observed, the performance significantly changes depending on the term composition as well underlying futures strategies. Moreover, Citigroup, ProShares, Credit Suisse and UBS Group offer some other VIX

[140] Source: http://vixcentral.com
[141] This portfolio is "expected to suffer from roll loss due to term stracture decay", Cit. Standard & Poor's (2009).

tracking as well leveraged ETPs. A perfect time picking is essential in order to invest in volatility products as the long-term buy and hold strategy is extremely losing, whereas short-term hedging is advisable during the periods the investor might feel uncertain about the future market movement.

FUND NAME	TICKER	CATEGORY	FUND FAMILY	3-MO RETURN (NAV)	YTD RETURN (NAV)	1-YR RETURN (NAV)	5-YR RETURN (NAV)
iPath® S&P 500 VIX ST Futures™ ETN	VXX	Volatility	Barclays Funds	+44.05%	-13.34%	-3.62%	-54.47%
iPath® S&P 500 VIX MT Futures™ ETN	VXZ	Volatility	Barclays Funds	+18.3%	+1.31%	+7.23%	-32.18%
iPath® S&P 500 Dynamic VIX ETN	XVZ	Volatility	Barclays Funds	+3.71%	+5.31%	+0.07%	0.0%
iPath® Inverse S&P 500 VIX ST Fut™ ETN	XXV	Volatility	Barclays Funds	-0.49%	-0.45%	-0.85%	+9.78%
iPath® Inverse S&P 500 VIX ST Fut™ETN II	IVOP	Volatility	Barclays Funds	-0.91%	-0.25%	-0.81%	0.0%

Table 7. VIX oriented Barclays ETNs performance[142].

In the last couple of years during the period of calm and growing markets, the VIX futures term structure became much more linear, which increased contango to the mid-term futures, whereas the inverse position performed relatively better. Moreover, it might be interesting to notice how futures term structure flattens after 150 futures days and presents a seasonal pattern in December, when volatility become low and VIX futures slightly cheaper.

Between VIX index and futures there exist a relationship that depends on how fast the VIX moves toward its long-run mean, the volatility of VIX and time to maturity. As it was previously mentioned, the long-term futures mirror the future VIX expectations plus risk premium, while only near the expiration date the futures move near the index level. The correlation has been estimated to be around 87-92%[143], though this relationship is dynamic and changes over time depending on the futures maturity: futures expiring over 4 months might have no correlation to the VIX. This means that delta, that is the futures price variation relative to the index variation, becomes smaller as index increases and the marginal effect of futures increase will be higher when index is under its long-term average and will be lower when index is already above it. Moreover,

[142] Source: http://finance.yahoo.com/etf/lists
[143] Jacobs and Standard & Poor's.

delta also depends on the mean-reversion strength; with strong mean-reversion delta will be smaller as current level signifies less than long-term mean, while with week mean-reversion the futures will substantially depend on the current index level. This relationship is extremely important in VIX replication strategies as well cross hedging of different futures and options. If a trader is short 1,000 delta in June 2014, he/she might buy 1,000 futures contracts or calculate the VIX exposure in term of near-term November 2013 contracts. If June delta is 0.4, and November delta is 0.9, the trader can buy high liquidity near-term futures with same delta exposure. Thus, 1,000*0.4=400 delta of VIX underlying, which is 400/0.9= 400.44 delta in terms of November futures.

The same observation regards other Greeks for futures such as gamma, theta and vega with respect to VIX volatility. Since there is no arbitrage relationship between futures and index, but only a non-linear dynamic in time relationship, all Greeks are different from zero. Nevertheless, delta prevails as principal factor and the relationship between VIX and its futures is treated as approximately linear. The main consequence of this non-linearity with no arbitrage opportunity is a positive expected cost for rolling futures contract from one month to the other as VIX is positively skewed[144] and a price for convexity is paid. The empirical research has demonstrated that in 82% of cases the second month futures are generally more expensive from the previous, moreover bid-ask spread is significantly important cost.

VIX options traded at CBOE are the second useful tool for hedging, speculation and portfolio diversification. It is worth underline that some strategies in practise are not available since short positions are often restricted to the traders from the brokerage firms. Options strategies will be briefly presented after having analysed some factors, which determine VIX options values. Firstly, the put-call parity is violated and the implied volatilities might change significantly from one expiration cycle to another. The VIX options are derived from VIX futures as underlying (not VIX) and can be priced through the Black-Scholes formula, moreover put-call parity relationship holds. As in futures case as well in options,

[144] VIX distribution is presented in Appendix 10.

when VIX spikes and the expectations are downward calls are cheaper; while when VIX is low and the expectations are upward sloping the calls are more expensive.

As previously has been mentioned, VIX options as futures can be expressed through delta in terms of VIX aggregate risk measure (the previous numerical example is applicable). Normally, when time to expiration is long the implied volatility of options decreases, whereas by getting closer to expiration the implied volatility increases. As well for options implied volatility is higher compared to the historical volatility. VIX options Greeks such as theta contains theta plus a vega component, since the volatility term structure reduces time decay. As time to expiry reduces the options lose their vale, while the implied volatility increases. Generally, out-of-the money options are highly prices even though close to the expiry date. Most of the trading platforms provide incorrect values of Greeks, exception made only for few[145]. Moreover, VIX call options present steep skewed distribution as a result of institutional requirement to hedge short volatility exposure and series hedging risks for call sellers as VIX spikes much more upwards rather than downwards.

In regard to VIX options trading, sell of naked calls are subject to restrictions, normally calendar spreads are not allowed and most importantly VIX options underlying assets are futures contract with corresponding maturities. For the last reason the VIX options might weakly track the index, thus even though VIX spikes an investor might not be able to cash in: only on the expiration the two will match even though a differential of a couple of percentage points might be present. In fact, expiring in-the-money VIX option gives a pay out determined through the difference between VRO (VIX Options Settlement Index) quotation and the strike price. VRO presents an expiration value, not exact VIX value on expiration date morning[146]. As well for options bid-ask spreads are significant: normally 0.5 points.

[145] Harwood V. suggests reliability of LIVEVOL and Fidelity brokers to produce correct results for VIX options Greeks.

[146] The settlement process sets one Wednesday as monthly expiry day.

A trader investing in options has to forecast two variables such as expected direction of the underlying asset and the timing of the expected movement, moreover in VIX trading the price level of futures contracts should be observed. Provided the limitations on VIX options writing, an investor might be directly long on calls and/or puts. Moreover, when volatility is low spread strategies such as bull call/put spreads spreads might be preferred, whereas for high volatility bear put/call spreads might be employed; nonetheless these strategies limit both sides of the exposure. Although losses are limited to the option cost plus commissions and trading fees, the profit will be limited as well. One more strategy consists in a straddle: long call and long put, where the investor benefit from upward or downward movement of the underlying. A drawback of these option based strategies compared to volatility futures ETPs is that their performance not only depend on volatility, but also on the price of the underlying.

The main question an investor may be concerned about: how cheap or expensive the volatility price is? In reality, volatility markets may be simultaneously calm on the surface and anxious underneath. If volatility is observed from one side one might observe nothing else than 5 years of lows in VIX index, but if the perspective is slightly changed you might observe a fierce "bull market in fear". This new paradigm for risk pricing is defined by particularly steep volatility term structure, lower level of spot volatility caused by central bank monetary policies, expensive portfolio insurance, intense spikes in volatility and high correlation. Although the volatility is low today, the future expectations are nourished by emotional fear of a possible deflationary collapse. On July 3th 2014 the VIX index fell to the lowest level since 2007 generating financial media attention affirmations such as "volatility is cheap", "it's perfect time to buy VIX". This sounds trivial: low spot-volatility does not necessarily mean cheap volatility. Volatility may be low compared to historical average, which nonetheless ignores some very important factors. For instance, when looking at the fundamental value the absolute value is not relevant. For instance, in August 2012 it was more expensive to buy 1-year forward volatility with the spot-VIX at only $13.45 rather than one day after Lehman Brothers

went bankrupt in September 2008, when VIX was above $31. Nonetheless spot VIX was -57% lower in August 2012, the VIX futures were priced higher on the back of the curve (hedging over 6 months) than during the start of financial crisis. In this way by following "cheap" volatility media advice you would already lost -12% of its value when VIX increased by +15%. In the following graph the futures term structure for both dates are compared:

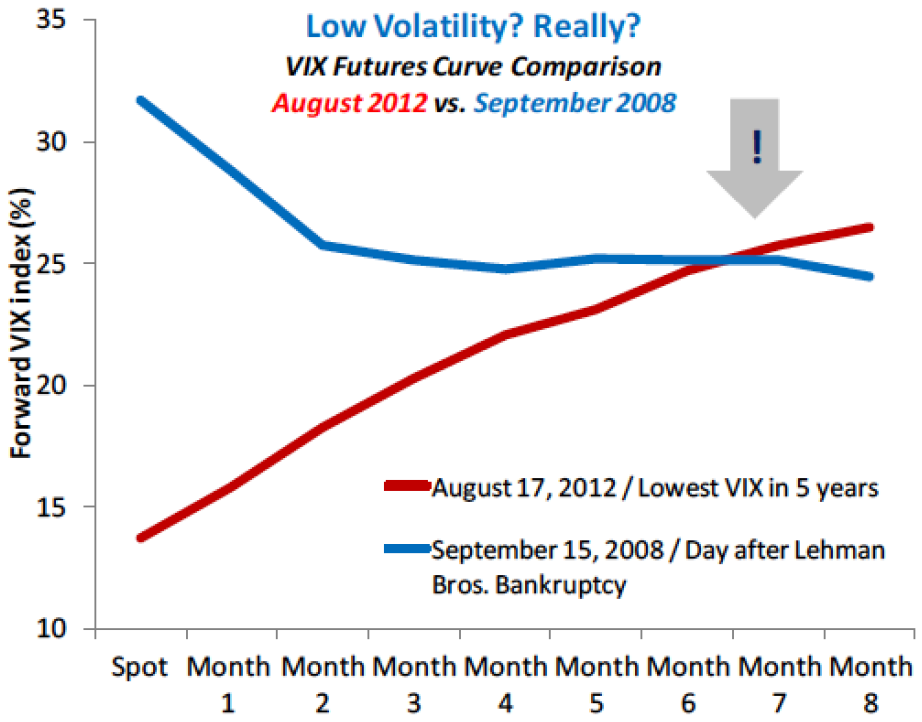

Figure 21. Forward VIX Index Comparison[147].

Given the expected movement of the underlying asset, what really matters is variance premium paid and not absolute price of volatility.

Thus, a smart investor should not hedge his/her portfolio based on a simple heuristic rule of a low absolute VIX price, but follow a relative value approach. In practise, a hedger should buy less expensive volatility on the front of the curve and sell overpriced volatility on the back while dynamically hedging the exposure[148].

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[147] Artemis Capital Management, LLC (2012), *Volatility of Impossible Object- Risk, Fear, and Safety in Games of Perception*, p. 8.

[148] Ibid.

Active managers measure the performance of a fund though the "alpha" performance, which represents the excess returns relative to the benchmark index returns. In volatility environment such strategy is defined as "crises alpha", that is the strategic acquisition of mispriced volatility. The alpha crises fund search for mispricing in the future expectation of uncertainly consisting in balancing long volatility exposure with strategic shorts. The active managers aim to achieve alpha through an efficient portfolio selection based on two components such as asset selection and short volatility/short correlation exposure. Nonetheless, when the markets components become more cross-correlated, asset selections become less significant. Thus, when the asset selection is extracted from the investment procedure, the only exposure that remains is short volatility and short correlation, which is exactly from where the hedge fund managers extract the most of their alpha. For instance, buy-and-hold strategy represents synthetically a short volatility position. Indeed, one segment of a short variance swap consists in promptly timing the market by buying at lows and selling on highs. This is the main reason most of fund managers underperform during the crises, as they are all short volatility traders. In C. Cole's view[149], the accurate source of alpha does not depend on asset class, but rather on the perfect balance on long and short volatility exposure. As a result in 2008 many financial institutions realised that they were asset-class diversified but 100% short volatility.

[149] Cole C. (2014), *Volatility: The Market Price of Uncertainty*, CFA Institute Conference.

Postmodern Economy

Deeper dimension markets like volatility, correlation, and volatility-of-volatility (VOV) are important because they measure our confidence in the financial representation of economic reality[150].

The perfectly efficient market by its definition should be random. The financial markets should mirror the real economic reality. When the market influences too much the economic reality this relationship reverses and the market becomes itself economy, which self-fulfils projection unto itself. In the postmodern economy, market expectations become more important than the real economic development. Thus, financial system markets are trending towards an inexorable disequilibrium. The market pricing mechanism seems being affected by fragile psychology that was progressively leaning on central banks' money creation rather than economic growth. When a system develops into abstraction upon itself, the information encloses less and less information, market randomness decreases and tail events become more likely in either direction. This creates a strong base for systemic risk: the volatility market stability depends mostly on the main central banks such as Fed or ECB balance sheet expansion.

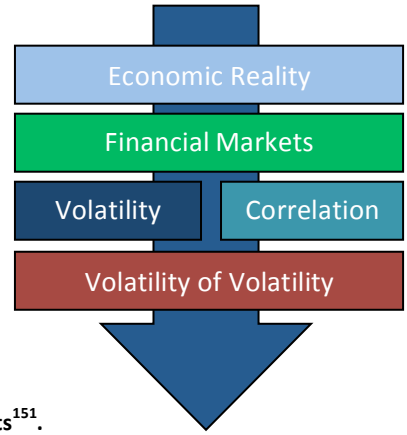


Figure 22. Deeper dimension markets[151].

[150] Artemis Capital Management (2012), LLC, *Volatility of Impossible Object- Risk, Fear, and Safety in Games of Perception*, p. 5.

[151] Ibid.

In Artemis Capital Management Funds'[152] view, third and fourth dimension markets such as volatility, correlation and VOV become highly sensitive (volatile) as soon as central banks decrease monetary stimulus. Nowadays, the tail risk as well as VOV is historically very expensive, which reflects the investors concern on their own uncertainty. To this regard, it is very enlightening the quote: "knowledge is not what you know, but certainty in what you do not". VOV, as previously mentioned, is a fourth dimension derivative that measures investors' confidence in the market as an accurate representation of reality.

The VOV for S&P 500 index is represented by volatility-of-the-VIX through VVIX index, which measures the expected 30-day implied volatility VIX futures contract (not VIX itself). It is calculated through the same method VIX is derived from SPX options. Firstly, from at- and out-of-the-money options the expected variances for different maturities are calculated. Secondly, 30-day expected variance is interpolated. And finally, the square root of the previously interpolated variances is translated into VVIX in percentage terms. Thus, as well for this index there exist a term structure (generally downward sloping) and it mean reverts to its long-term historic mean (88). The VVIX and its term-structure provide investors with some crucial information about:

- The implied volatilities of VIX options, which closely express VIX volatility, or better the volatility of VIX futures and thus their fair value.
- The VVIX and any point on its term structure express the price of VIX options portfolio. By shorting VVIX portfolio an investor can capture the volatility risk premium, and eventually try to profit on the expected mispricing.

During the periods of high VOV, the equity returns are often at their lowest. In fact, for S&P 500 the periods of low VOV over perform periods of high realized volatility-of-the-VIX by 13% annually[153]. Moreover from other research papers, it

[152] Is an US investment fund, which trades on market volatility.

[153] Artemis Capital Management, LLC, (2012), *Volatility of Impossible Object- Risk, Fear, and Safety in Games of Perception*, p. 7.

was found that individual stocks exhibiting high implied VOV underperform low VOV stocks by 10% a year[154].

In the last years the VOV significantly increased as shown in the following graph:

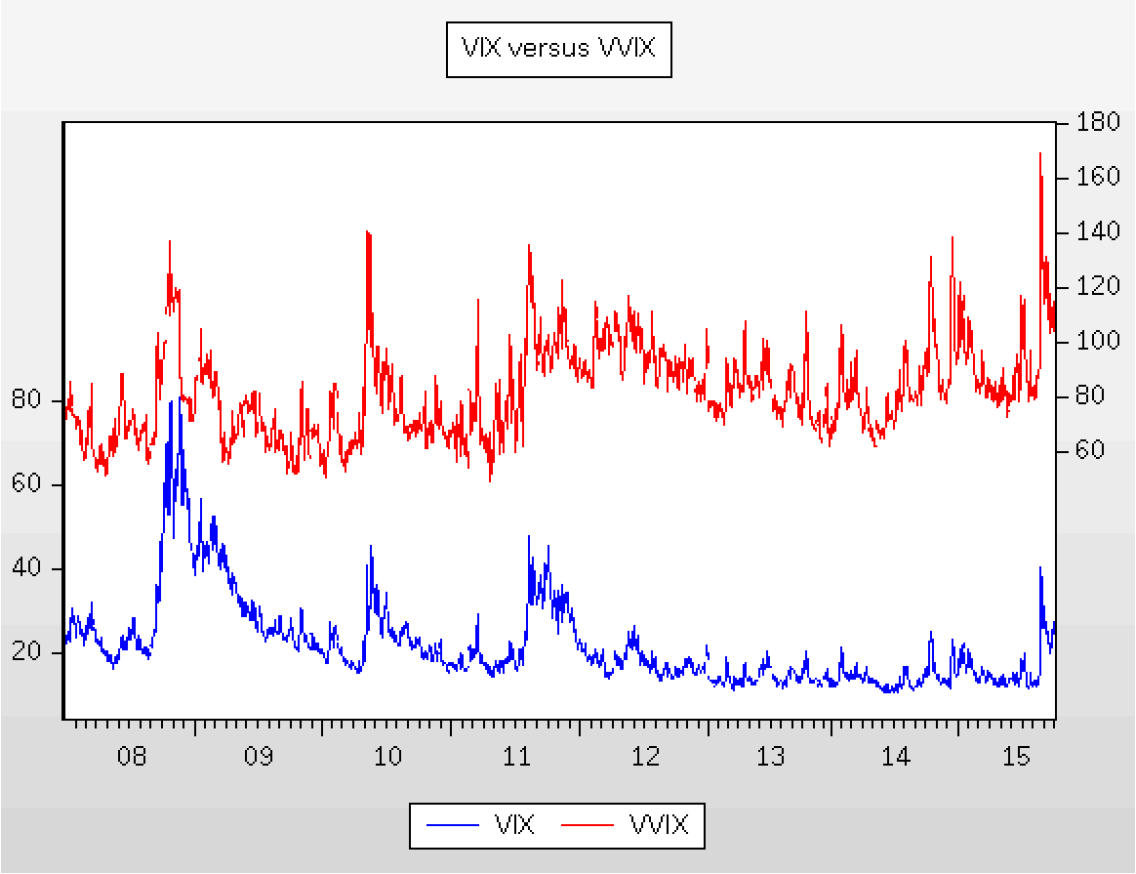


Figure 23. VIX versus implied VIX futures volatility index (VVIX).

This increased VOV skew for VIX options implies higher risk premiums for uncertainty: the steepest the VIX skew, the more investors are willing to pay to hedge against market collapse through VIX options. VVIX can be observed as 1-month variance swap estimated from VIX options, or better an investor investing VVIX portfolio obtains the implied volatility exposure and will get a profit or loss on the difference between implied and realised volatility of VIX futures. The range of future 1-month VIX value can be predicted for almost 40% by VVIX, or more simply this explains how well VIX options predict the future VIX level. Somehow these volatility curves can predict what an investor already

[154]Baltussen G., Van Bekkum S. and Van Der Grient (Erasmus School of Economics & Robeco Quantitative Strategies) (July 30, 2012). *Unknown Unknowns: Vol-of-Vol and the Cross Section of Stock Returns*.

know will happen in the next near future, the so called “known unknowns”: Fed B/S tapering, expected increase of interest rates, sovereign debt crises, wars in middle east, China’s hard landing and similar. The probability of these shock events is already priced in high volatility premiums; obviously these events may undermine returns, but not as much as it would come out of the blue. In this systemic known unknown crashes the volatility may remain high for long periods, whereas the VOV reaches an equilibrium. At this point, Artemis Capital Management LLC manager Christopher Cole would question what is expected as the next “unknown unknowns”? By the definition it is something unknown and completely ignored. A probable answer would be the VOV itself, as the more traders use “uncertainty” as “market timing indicator”, the more unstable and cross-correlated the markets might become. The unknown unknown super speed crashes such as Black Monday 1987, 2010 Flash crash, 2015 stock market Selloff are unpredictable in retrospect and their cause is never exactly understood.

Fear of Deflation

During pre-crisis (2008) period option markets implied probability distribution was centred and/or positively right tailed depending on the asset class. Post-GFC options markets have radically changed nature by shifting the expected return distribution to left tail side, to “bull market in fear” of the left tail risk. This distribution swap is especially evident in the commodities markets, high yielding bonds, US, EU and emerging markets equities. As a result, tail risk hedging against declines in equity market is priced nearly multiple decade highs, which will be briefly discussed in the next chapter. Moreover, as previously mentioned Central Banks interventions had the effect of artificially suppressing spot volatility by monetary expansion, nonetheless the risk perception was not destroyed, instead only shifted to the left tail. In the following graph the cross assets implied return probability distributions are presented to evidence the shift of the risk premium from the centre to left side.

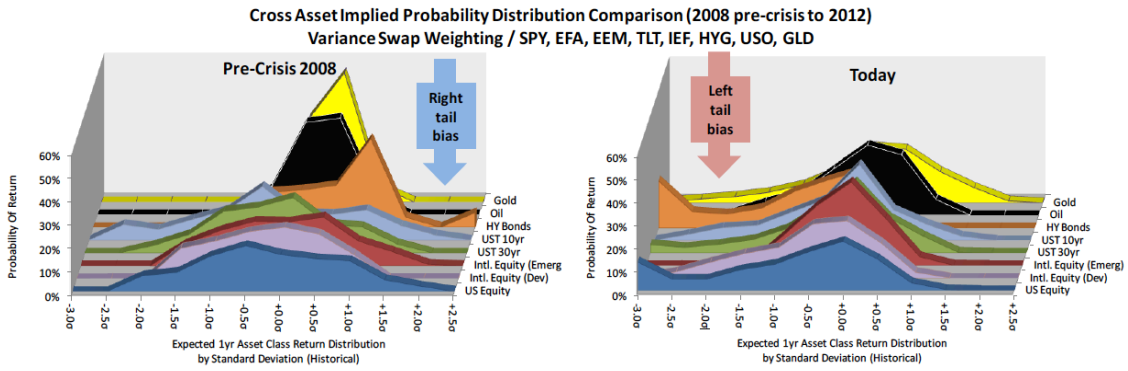


Figure 24. Pre- and post-GFC cross asset implied probability distribution[155].

It is interesting to notice that the worst crashes occurred when the investors were not expecting it or were excessively leveraged as in 2008. In fact, the crash wiped out weak portfolio insurance following by wiping out equity portfolio. Therefore, after GFC most of the investors and SIFIs have already hedged themselves against the market collapse and bought portfolio insurance. Thus, it seemed very unlikely that the market would decline uncontrollably when everyone is ready for it. The reason an investor should not have panicked is that there was a crowd of hedged investors in the market and below them a Central Bank that provided buffers to support the economy. Ironically, when investors are scared they buy insurance and markets are at their best. To this regard, is enough to observe the US markets under Fed expansive regime until March 2015, when with stocks back at records, people saw less need to pay for the insurance, but the coup the grace arrived through China's slowdown.

The volatility behavior during the deflation periods is well known from the historical data back to Great Depression to nowadays. Duding the periods of deflationary crashes the market participants underestimate the length of time volatility spikes. In the following table SPX and 1-month SPX realized volatility is presented. The duration over which volatility remained high is significant. For instance, during the 1987 Black Monday crash for 29 days the volatility over 50% has been registered.

[155] Graph retrieved from Artemis Capital Management (2012), LLC, *Volatility of Impossible Object- Risk, Fear, and Safety in Games of Perception*, p. 9.

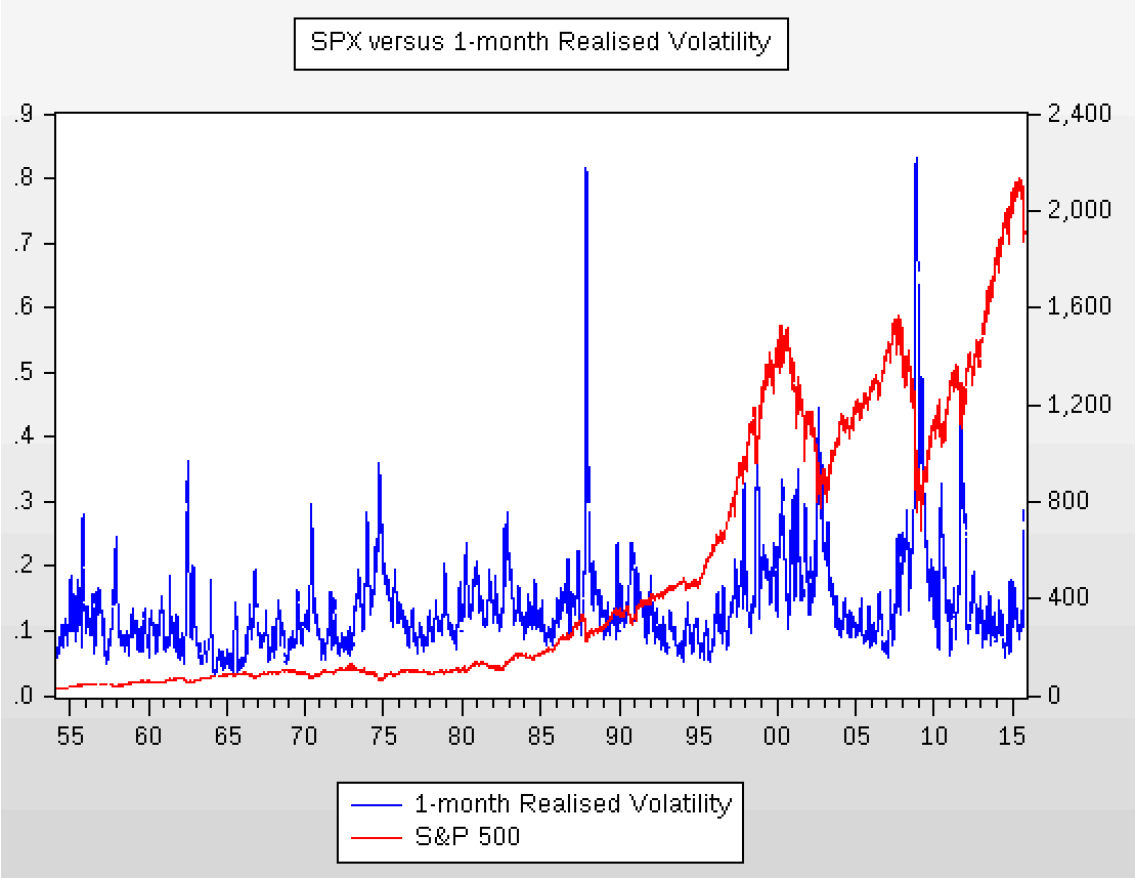


Figure 25. SPX versus one-month realised volatility.

Before 2008 many traders were shorting volatility as VIX were over crossing the "magic" number of 30. This simple heuristic rule was sufficient to provide them sufficient reward, although they were trading on the basis of an anchoring bias, since data from 1929 and 1987 market crashes has been neglected. During 2008 the volatility remained high for the longest time in the stock market history. For such severe deflationary spirals, the historical implied volatility has a limited predictive ability, especially as soon as VIX reached new high. During the last 6 years the investor faced an opposite bias, although realized VIX touched its lowest values the implied volatility was significantly expensive. In fact, very often investors' psychological anchors such as post-traumatic-deflation disorder can explain this irrational behavior[156]. As a result of these painful deflationary collapse memories, the investors were willing to pay high premiums to hedge their portfolios against the next deflationary collapse, which is quantified through

[156] Artemis Capital Management LLC (March 30, 2012), *Volatility at World's End: Deflation, Hyperinflation and the Alchemy of Risk*, Letter to Investors from Artemis Capital Management, LLC.

exceptionally steep term structures curves, overpriced tail risk, high implied VOV and underperformance of portfolio insurance.

Irrational Exuberance for Fear

The last three decades paradigm of debt expansion and lower interest rates cannot be assumed being relevant in the future anymore. Today the markets could be compared to an "impossible object'': as soon as investors' perception shifts, the risk price can change radically. After GFC the investors are living a collective existential fear of a possible deflation that forces them to pay the highest premiums for portfolio insurance in two decades. The "irrational exuberance for fear" the investors are nourishing of course is not misplaced but seems to be mispriced. Tail-events are now being priced as if they were standard risks. Ironically an investor get much cheaper hedging for small declines of higher probabilities (e.g. -5%) rather than rare but extreme crashes. To this regard an important computational research has been conducted by Artemis Capital Management LLC Fund, which analysed the evolution of portfolio insurance premium (thus, tail risk price) through three different methodologies.

The first consists in observing the evolution of portfolio insurance premium through implied probability distributions for returns derived from SPX out-of-the money put and call options. The figure 24, Artemis Capital Management has compared 1-year implied probability distribution of SPX options from 1990 to September 2008 and from 2008 to March 2012. The results are extremely significant to assert a hypothesis of new regime of tail risk. In fact, since 2008 to SPX options has been assigned 21% probability of a 50% or more market collapse for a given year. Nonetheless, the realised historical probability of a -50% or greater crash is only 2.93%[157].

[157] Artemis Capital Management LLC (March 30, 2012), *Volatility at World's End: Deflation, Hyperinflation and the Alchemy of Risk*, Letter to Investors from Artemis Capital Management, LLC.

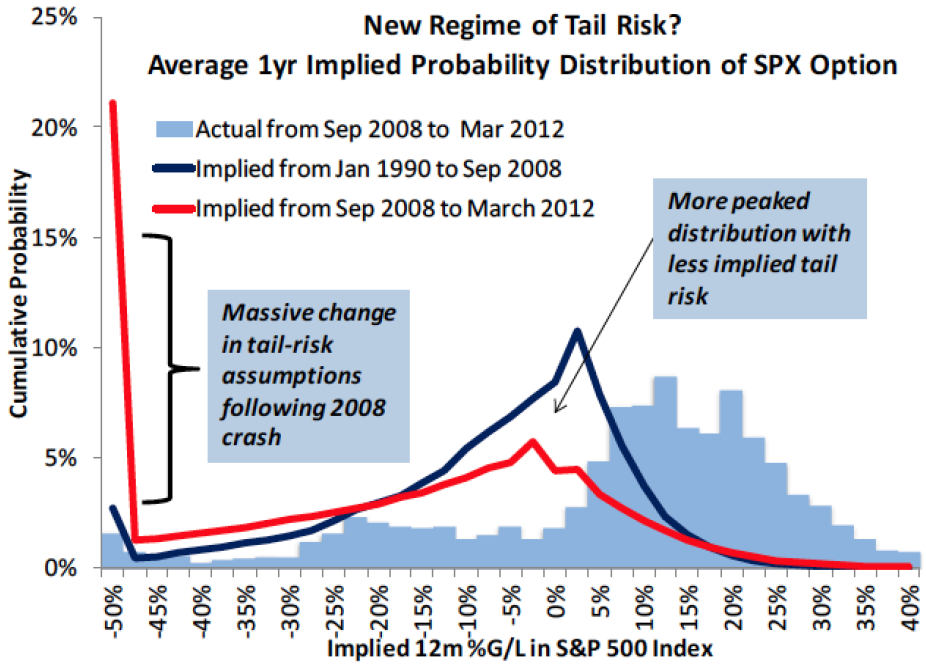


Figure 26. Implied probability distribution from SPX options[158].

The second and third methodologies involve the comparison of implied and realised kurtosis as well Greeks (theta and gamma). The second method consists in extrapolation of implied excess kurtosis from the strips o SPX options through Jiang and Tian model-free movement spline interpolation technique. The excess kurtosis measures how fat are the tails compared to the normal distribution. Thus, a comparison between implied from SPX options and realised excess kurtosis of SPX returns has confirmed the highest levels over a decade. The graph is provided in the Appendix 11.

In the last and less intuitive metrics of the tail risk is expressed through theta to gamma ratio of 30% out-the-money SPX put option. This method measures how much "time decay" an investor is paying for each unit of obtained "hedge". As well in this case the cost of tail risk insurance seems to be overpriced as presented in the Appendix 12.

Non-surprisingly, the metric result validates the highest cost of tail risk protection.

[158] Graph source: Artemis Capital Management LLC (March 30, 2012), *Volatility at World's End: Deflation, Hyperinflation and the Alchemy of Risk*, Letter to Investors from Artemis Capital Management, LLC. p.7.

Hyperinflation

The global economy is tied by trades and investments, which strictly depend on a degree of countries' innovations and productivity. On this, every single central bank decides on the amount of money need to be printed to sustain such development. If the economy is on the edge of the crises and a central bank prints too little money the risk is a new Great Depression of the 1930s, but if it prints too much as during the Weimar Republic Germany in the 1920s the risk is hyperinflation. To this regard, Christopher Cole has proposed an interesting interpretation on this possible future event. Since our perception is biased by 100+ years of deflationary emotional fear, it might completely ignore the possibility of the opposite scenario such as hyperinflation reality.

The IASB defines hyperinflation as price increase of +26% a year or +100% cumulative increase over three years, which is less radical compared to +50% inflation registered in Weimar Republic or Zimbabwe and more reasonable for modern developed economies with a global currency reserves such as US or EU. Nowadays such perspective might seem bizarre and that is exactly the reason why in the Cole's view it might happen. Provided that there is no direct experience on the consequences such a monetary stimulus might generate, the long-term inflation could be the next possible force to be withheld. The main point regarding this scenario is that the right tail is not priced into the option markets and that during the last 30 years the interest rates decreased from 16% to zero (in US). Moreover, one more factor is analysed as essential factor for high inflation such as velocity of money, which nowadays is near close to zero, thus the hyperinflation has no change to happen. Nonetheless, this approach might neglect economic history and the fact that the velocity of money in first instance is more psychological concept rather than economic and can be highly volatile. The reason why central banks believe that too much stimulus can be withdrawn later through the corrective tightening policies represents a strong presumption. As well in Weimar Germany when the government doubled its money supply there was no inflation, the country had one of the healthiest

economies, the stock market was blooming and the deutsche mark was the strongest currency in the world[159].

Unfortunately, there is no precedent to figure out how derivatives market would behave under inflationary spiral. But theoretically it is possible to imagine that the volatility should have been observed as through the mirror. An average investor associates volatility spikes to the turbulent market crashes, although volatility measure is indifferent to the market direction and volatility increases only because the prices drop much faster then they rise. To this regard, by decomposing 1-month SPX realised volatility into percentage variance determined from positive and negative market movements, it emerged that 54% volatility is derived from an increase in market price. However, during the market collapses the variance might be driven up to 99% only by negative changes. Since 1987 the negative volatility skew[160] went along with the investors' deflationary fear and even increased after 2008 market crash; in fact, the VIX moves up and down SPX volatility smile (skew curve). This typical volatility pattern would be completely unsound in hyperinflationary scenario, where the prices rise faster than they fall. In fact, the volatility skew would reverse itself from negative to positive. Obviously, there were no derivatives In Weimar Republic, nonetheless by estimating the realised volatility of German stock market as a proxy for a hypothetical VIX level. As a result, the VIX would spike to a monthly-annualised volatility of 2000% by October 1923, exclusively by an increase in stock prices. In case of such a dramatic hyperinflationary scenario, the volatility spikes are 80% higher compared to volatility realised during 2008 market collapse. Only to give an idea, a hypothetical four-year stock market variance swap (17.5% strike) on $1 million notional amount stipulated in 1919 before inflation would produce $417 billion at maturity[161]. Nowadays, that kind of volatility spikes would wipe out the balance sheet of any

[159] Parsson J. (1974), *Dying of Money: Lessons of the Great German and American Inflations*, Wellspring Press.

[160] Meaning that out-of-the-money put options are traded at the higher volatility level than out-of-the-money call options.

[161] Artemis Capital Management LLC (March 30, 2012), *Volatility at World's End: Deflation, Hyperinflation and the Alchemy of Risk*, Letter to Investors from Artemis Capital Management, LLC.

financial institution, as no bank would be able to absorb that kind of derivatives losses.

In the following graph the performance of Weimar stock market is presented:

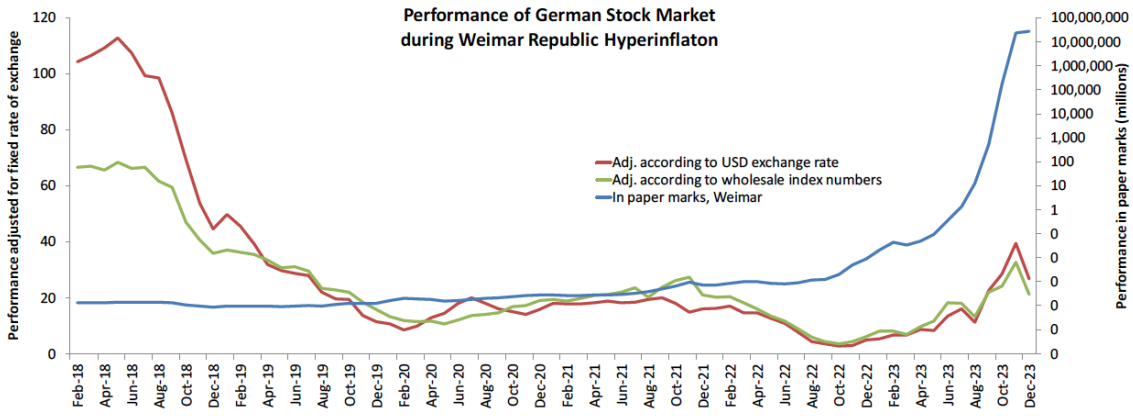


Figure 27. German stock market performance during Hyperinflation[162].

For a developed economy with reserve currency this approach might seem unreasonable, but as well for wide housing collapse it seemed unbelievable it could produce a recession and systemic risk. To understand how the modern market would behave in a hyperinflationary dynamic Artemis has simulated a stochastic-based model presented in its paper work "Volatility at World's End: Deflation, Hyperinflation and the Alchemy of Risk". For the purpose of this paper, the author limits to present the opposite scenario for volatility trading and tail risk insurance on the presupposition that long-term deflation might not be a primary risk. As long as the central banks will provide support through monetary expansion, such risk might rise. As a result, an investor should keep in mind the possible implications of such a switch on volatility term structure, call options premiums, VIX new structure and higher volatility. The single asset that nowadays would be able to hedge against hyperinflation is volatility itself.

[162] Graph source: Artemis Capital Management LLC (March 30, 2012), *Volatility at World's End: Deflation, Hyperinflation and the Alchemy of Risk*, Letter to Investors from Artemis Capital Management, LLC. p. 11.

Conclusion

This paper aimed to investigate on volatility from different perspectives. In fact, as a results of its negative correlation with the market returns, volatility is seen as an asset class on which to invest, trade and hedge portfolio. As it has been proved as soon as the financial markets drawdown, the volatility assets high spike by generating profits. This main intuition encouraged the author to further investigate on possible ways to replicate VIX and to include the volatility exposure in a portfolio. As it has been explained, VIX replicates 30-days implied volatility of S&P 500 by deriving the implied volatilities from options.

In the last 5 years, VIX decreased to the historically lowest values, nonetheless the greatest contribution to calm down the markets has been provided by Fed balance sheet expansion. At the same time, the investors were nourishing a profound fear of a possible future deflation, which is demonstrated by historically highest insurance premiums paid to protect their portfolio. The author tried to demonstrate the effects of QEs on real economy, financial markets and volatility. In United States the financial markets have performed significantly well by following Fed expansion, the volatility has artificially decreased, however the real economy variables has improved relatively slowly. The most straightforward improvement has been observed in GDP, however it might be biased by financial sector component, as it was the first sector supported by financial stimulus; in comparison the unemployment rate presented much lagging recovery. In Eurozone the recovery of financial markets and economy has been much weaker as sovereign debt crisis issue arise. By comparison of the two “fear gauges” VIX and VSTOXX emerged that the investors’ fear level is slightly higher in Europe, which is explained by higher instability level and lower monetary stimulus.

As it has been explained to better understand volatility indexes, the OTC variance and volatility swaps have been analysed. In fact, a long position in a volatility swap pays a fixed strike (implied volatility) and receives realised volatility. Thus by analogy, the long position on VIX based instruments as underlying, would pay the cost of implied volatility and at 30-days expiration

would receive the payoff on the difference between realised and implied volatilities. But some important practical and technical issues arise, which create some imperfect and very often-biased VIX replication strategies. Firstly, the volatility swap pricing consists in impossibility of replication: only a variance swap can be replicated through a portfolio of options with wide range of strike prices. Since the volatility swap presents a non-linear payoff to variance swap (volatility is the squared root of variance) the replication is imprecise and produces some important under pricings. Secondly, no ETP can exactly replicate VIX exposure and VIX futures are employed. Provided that in long-term VIX mean reverts and that in the last 5 years the VIX level presented the lowest values, the VIX futures were continuously traded in contango, which caused a huge loss in value of short-term VIX replicating funds. Thus, very often such structured products require a perfect timing in order to produce a desired profit and are not suitable for long-term buy and hold strategies as portfolio hedging.

Furthermore, it can be observed that the tail risk pricing presents the highest cost over the last two decades and that the volatility of VIX continued to be high as the reflection of investors' expectation of a possible future deflationary collapse. Although monetary policies have contributed to lower down spot volatility, its expectation remained high by producing so called "bull market in fear" scenario. In which as soon as the central bank stimulus will finish, the market will move on its own fate reflecting the real volatilities. And finally, in the last paragraph a future perspective of hyperinflation is analysed to suggest an idea of a possible new future risk to take into account.

In the conclusion, the volatility represents an emerging asset class with a wide range of opportunities to bet on future volatility movements, but also a huge danger for inexpert investors. In order to evaluate possible mispricing in volatility products, an investor have to deeply understand the exposures and pitch perfect market timing when the volatility spikes. Both of them are not an easy issue, as the first requires a high level of technical skills and the second a good instinct to timely intercept the imminent economic drawdowns.

Appendix

Covariance Analysis: Ordinary
Sample: 1/03/1990 8/12/2015
Included observations: 6454

Correlation t-Statistic Probability	VIX	S_P_500
VIX	1.000000	
S_P_500	-0.704959	1.000000
	-79.83855	-----
	0.0000	-----

Covariance Analysis: Ordinary
Sample: 1/02/2008 8/12/2015
Included observations: 1917

Correlation t-Statistic Probability	S_P_500	VIX
S_P_500	1.000000	
VIX	-0.743835	1.000000
	-48.70215	-----
	0.0000	-----

Appendix 1. EViews output: Correlation between SPX and VIX from 1990 to 2015, 2008 to 2015

Dependent Variable: VIX
Method: Least Squares
Date: 09/11/15 Time: 12:30
Sample: 2/01/1990 8/12/2015
Included observations: 6433

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003238	0.000563	5.749046	0.0000
S_P_500	-3.953070	0.049670	-79.58622	0.0000

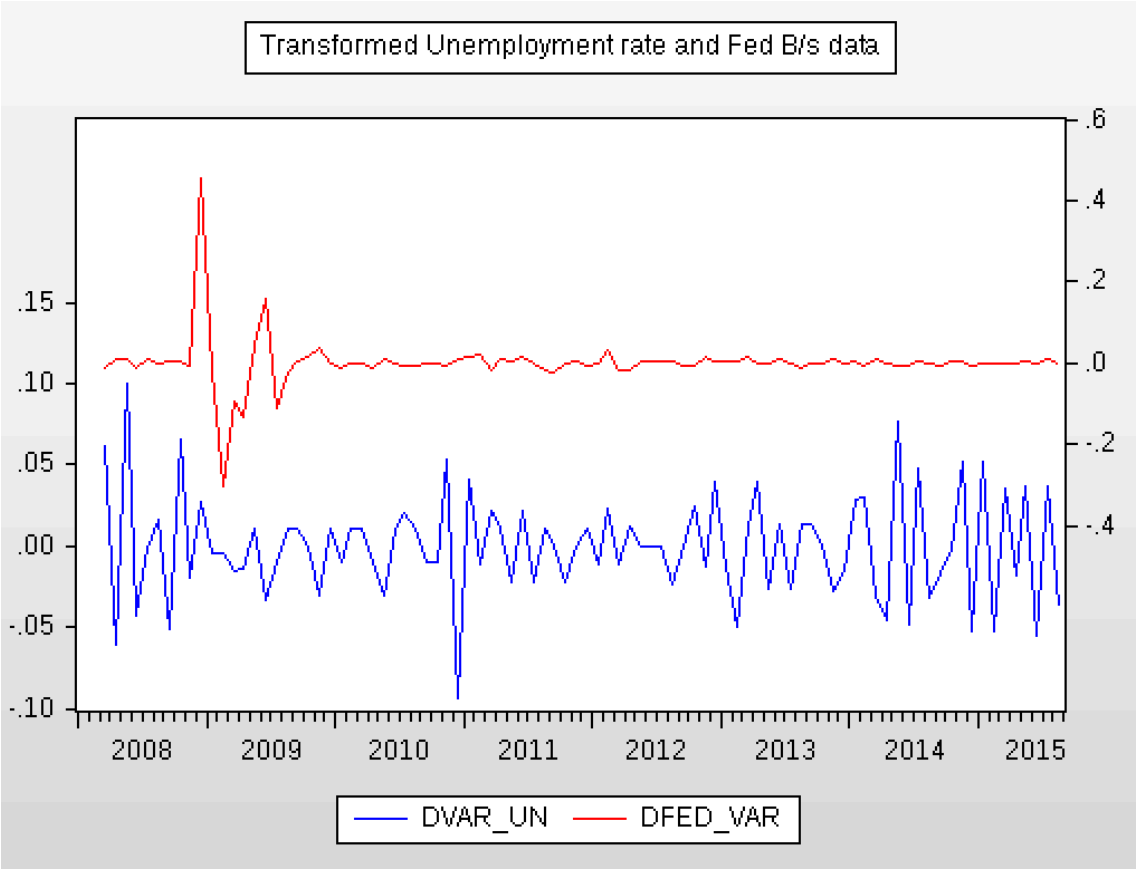
R-squared	0.496199	Mean dependent var	0.001849
Adjusted R-squared	0.496121	S.D. dependent var	0.063609
S.E. of regression	0.045152	Akaike info criterion	-3.357245
Sum squared resid	13.11101	Schwarz criterion	-3.355141
Log likelihood	10800.58	Hannan-Quinn criter.	-3.356517
F-statistic	6333.966	Durbin-Watson stat	2.227656
Prob(F-statistic)	0.000000		

Appendix 2. Eviews Regression Output VIX explained by SPX

Covariance Analysis: Ordinary
Sample: 2008M01 2015M06
Included observations: 90

Correlation t-Statistic Probability	FED B S	Unemp. Rate	GDP Var
FED B/S	1.000000		
Unemployment rate	-0.256339	1.000000	
	-2.487797	-----	
	0.0147	-----	
GDP Var	0.466610	0.157299	1.000000
	4.948980	1.494197	-----
	0.0000	0.1387	-----

Appendix 3. Fed B/S expansion correlation to unemployment rate and GDP



Appendix 4. Adjusted Unemployment rate and Fed variation data[163].

[163] Unemployment rate and Fed B/S have been transformed into differentiated simple variation.

Dependent Variable: DVAR_UN
Method: ARMA Maximum Likelihood (OPG - BHHH)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DFED_VAR(-1)	0.067138	0.054612	1.229353	0.2223
AR(1)	-0.278818	0.127960	-2.178945	0.0321
MA(1)	-0.759590	0.102260	-7.428054	0.0000
SIGMASQ	0.000487	8.23E-05	5.918764	0.0000

R-squared	0.530577	Mean dependent var	-0.000883
Adjusted R-squared	0.514009	S.D. dependent var	0.032396
S.E. of regression	0.022585	Akaike info criterion	-4.684306
Sum squared resid	0.043355	Schwarz criterion	-4.572457
Log likelihood	212.4516	Hannan-Quinn criter.	-4.639223
Durbin-Watson stat	1.939462		

Inverted AR Roots	-.28
Inverted MA Roots	.76

Appendix 5. Eviews output of US differentiated unemployment rate variation ARMA (1,1) model.

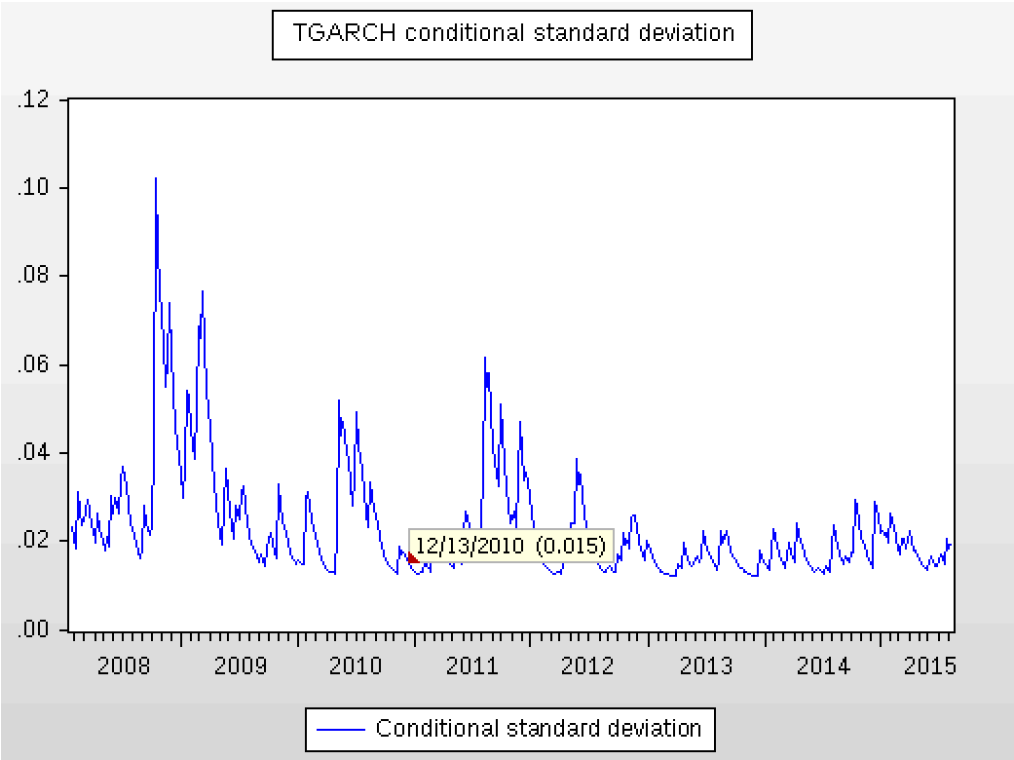
Dependent Variable: DGDP_VAR
Method: ARMA Maximum Likelihood (OPG - BHHH)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DFED_VAR(-1)	-0.438022	2.422953	-0.180780	0.8570
AR(1)	0.546459	0.085452	6.394938	0.0000
SIGMASQ	0.968257	0.147998	6.542375	0.0000

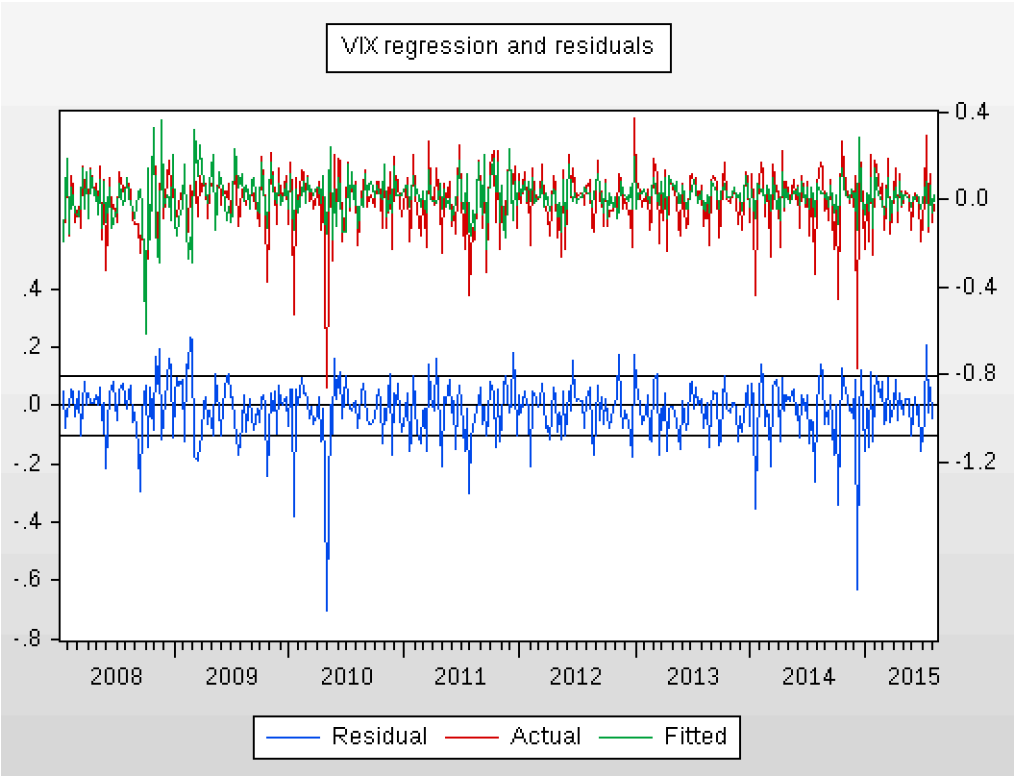
R-squared	0.300035	Mean dependent var	-0.028090
Adjusted R-squared	0.283757	S.D. dependent var	1.182799
S.E. of regression	1.001016	Akaike info criterion	2.877021
Sum squared resid	86.17489	Schwarz criterion	2.960907
Log likelihood	-125.0274	Hannan-Quinn criter.	2.910833
Durbin-Watson stat	1.688595		

Inverted AR Roots	.55

Appendix 6. Eviews output of US differentiated GDP variation AR (1) model.



Appendix 7. SPX conditional standard deviation modelled by TGARCH.



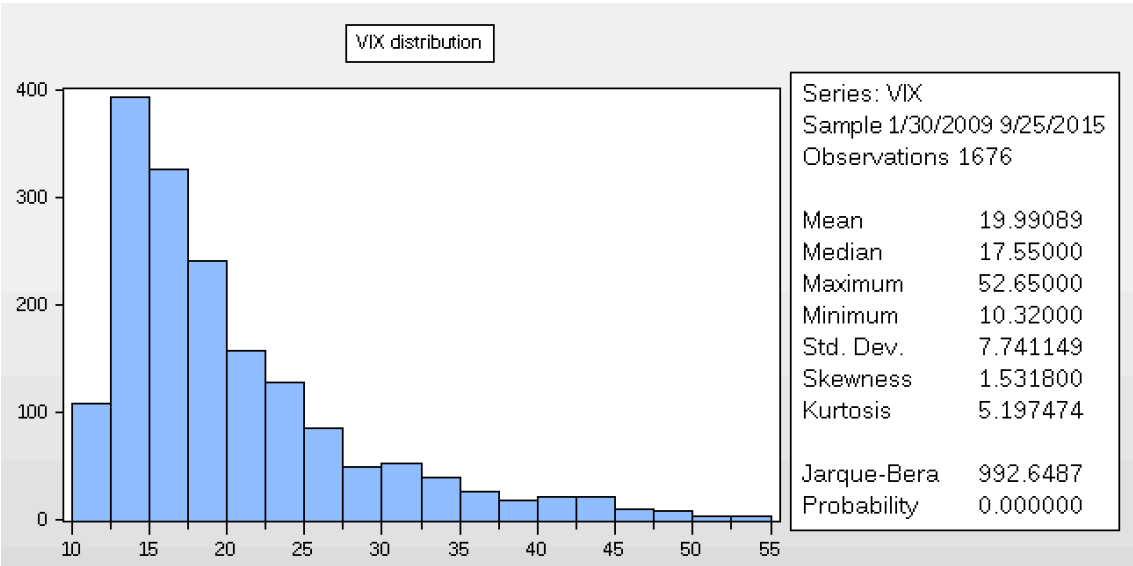
Appendix 8. VIX regression actual, fitted and residuals.[164]

[164] The results should be observed as swapped from negative to positive. Thus, the highest spikes are upward.

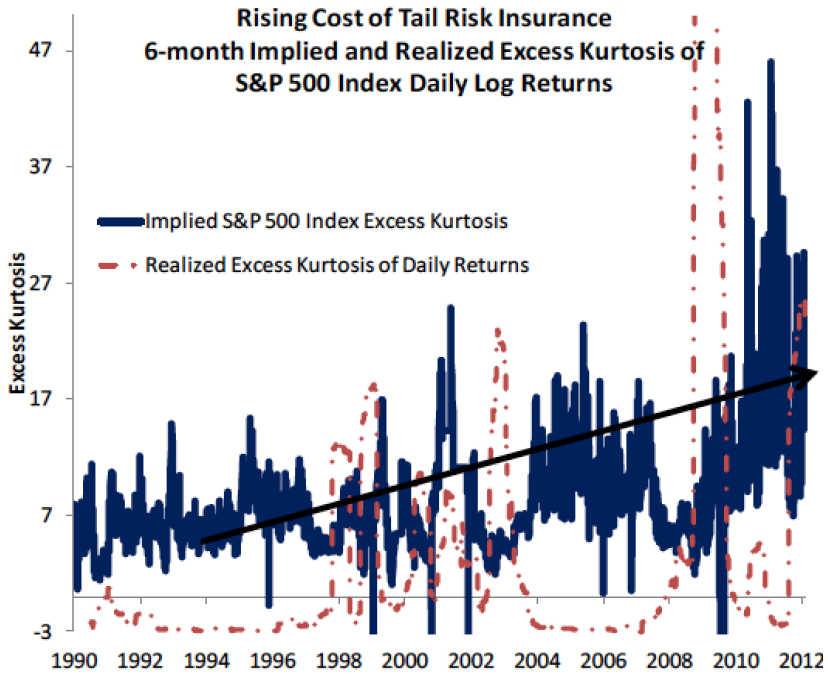
Covariance Analysis: Ordinary
Sample: 1/21/2008 9/21/2015
Included observations: 1965

Correlation t-Statistic Probability	VSTOXX_50	STOXX_50
VSTOXX_50	1.000000	
STOXX_50	-0.482765 -24.42395 0.0000	1.000000 ----- -----

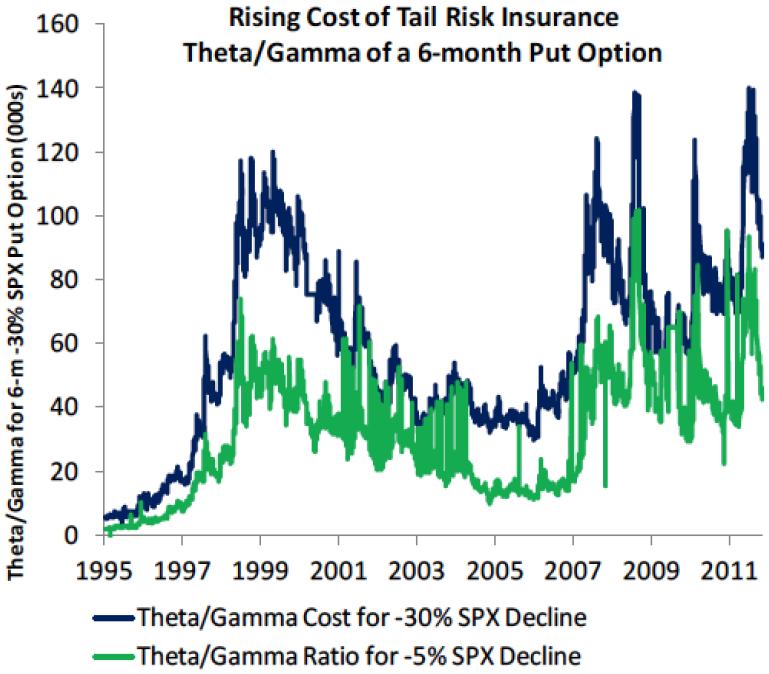
Appendix 9. Correlation between VSTOXX_50 and STOXX_50.



Appendix 10. VIX distribution over last 6 years.



Appendix 11. SPX 6-month realised versus implied kurtosis[165].



Appendix 12. Theta/Gamma cost of out-the-money SPX put options[166].

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[165] Graph source: Artemis Capital Management LLC (March 30, 2012), *Volatility at World's End: Deflation, Hyperinflation and the Alchemy of Risk*, Letter to Investors from Artemis Capital Management, LLC. p. 7.

[166] Graph source: Ibid.

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