Master’s Degree *programme* in Economics - Models and methods in economics and management

**Final Thesis**

**Different Models in International Trade**
From the Krugman-Melitz framework to the new Addilog Theory of trade

**Supervisor**
Ch. Prof. Federico Etro

**Graduand**
Silvia Griselda
Matriculation Number 832591

**Academic Year**
2014 / 2015
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Introduction

The study of international trade has always dealt with comparative advantage, increasing return to scale, and the so called love for variety.

The born of the international economics can be attributed to David Hume, a Scottish Philosopher lived in the 18th century. Although he was better known for his analysis of philosophical, historical and political issues, in his *Essays, moral, political and literary* (1758) he analyzed, in a philosophical framework, his ideas on political and international relations. His main contribution for the international economics regards the attack against the British Mercantilist, a political view common in his period. According to this vision, the economic prosperity could be achieved only through the limitation of imports and the encouragements of exports to maximize the amount of gold, and therefore of wealth, in the country. Nevertheless, Hume stated two considerations against this argument. First and foremost, he claimed that the prices in a country change accordingly to the money supply. Therefore, while net exports and the amount of gold increases, the price of goods would rise consequently, and thus, the real wealth would not increase. Secondly, because of the increase of the domestic prices due to the gold inflow, the domestic goods become relatively more expensive, therefore, the exports decline, and the foreign goods become cheaper and, therefore, the imports increase.

The mercantilism was later attacked also by Adam Smith, who, however, did not consider the Hume’s arguments. Adam Smith, in his most famous book *The wealth of the Nations*, first developed the concept of absolute advantage argument. Namely, if a country can produce a good with lower costs, while another country can produce the same good at higher costs but a different good at lower costs, it is beneficial to both countries to export the good with lower production costs and to import the which one with higher production costs. In particular, the philosopher took as example England, where the production of wool was relatively cheap, and France, which produced wine at lower cost. However, only successively these concepts were analytically developed.
The theory of international trade was analytically studied first by David Ricardo, an English economist lived between the 18th and 19th centuries. He developed the so called “theory of comparative advantage”, where the advantages from the free trade can be realized both by advanced country and capitalistically underdeveloped nations.

Although the theory of comparative advantage is considered by someone to be the most important concept in the international trade, this concept is quite counterintuitive, with respect to absolute advantage, stated by Adam Smith. According to the comparative advantage, when two countries differ in their productive capacities, even if one possess absolute advantages in all goods, it will obtain benefits from the international trade, if it export the good that it can produce most-best with respect to the other country, and import the less-best produced products (even in the case these goods are produced less efficiently abroad).

However, the Ricardian theory was not completely able to explain the boom in international trade occurred after the second post world period and the fact that, in the real world, the bulk of the commerce occurred between similar countries. Although in that period some empirical papers started to use the concept of economies of scale, there was not a theoretical and supported theory for the empirical evidences in international trade.

It was necessary to wait for the year 1979, when Paul R. Krugman wrote a paper, known as “Increasing Return, Monopolistic competition and International Trade”, recognized as one of the most important paper for the international trade, for the introduction of a new trade theory. This paper, which helped the famous economist to win the Nobel Memorial Prize in 2008, starting by the unquestionable masterpiece of Dixit-Stiglitz (1977), showed an alternative explanation to the international trade, based on the fundamental concepts of economies of scale and the so-called “love for variety” preferences. In the first chapter I am going analyze deeply the Krugman model. In particular, following the paper written in 1979, I am going to analyze the model in autarky, and successively I am going to investigate the consequences of openness to international trade, considered as the same the changes in population growth and factor mobility. The model deduces that, in presence of internal economies of scale, and therefore in economic frameworks characterized by monopolistic competition, the openness to trade causes lower prices and higher varieties. The innovative aspect of the model relies on the fact that the international trade leads to benefits also in the case of similar destinations.
Although the Krugman model (1979) helped to solve some puzzles in trade theory, there were still some unsolved questions. Many empirical studies have found that in reality, not all domestic producers export to foreign markets and, more important, exporters represent the minority of producers. In the article “Firms in International Trade” these and other empirical evidences are described and interpreted. Principally, the article illustrates that exporters are usually bigger than non-exporters, and that heterogeneity in the production function is present, especially when exporters are compared to non-exporters. This last evidence seems to suggest that the most productive firms self-select themselves into export markets. However, this fact can also reflect the so-called “learning by exporting”. The authors of the article assess that, although the “learning by exporting” mechanism cannot completely be excluded, the main reason of the higher productivity of firms is due to self-selection, as in Melitz (2003).

The empirical evidences have been the start point for the development of theoretical model for international trade which could take into consideration two main assumptions: the fact that, within sector, there is heterogeneity in size and productivity, and the fact that only most productive firms are engaged in foreign trade. It was in 2003 that these assumptions started to be considered essential not only empirically, but also in theoretical model.

Therefore, in 2003, the New Theory of International Trade experiences a further boost thanks to the publication of a pioneering article “The impact of Trade on Intra-industry Reallocations and Aggregate Industry productivity” written by M. Melitz. This article gives a theoretical explanation for the numerous evidences regarding the differences in productivity of exporters. According to the model, exposure to trade induces a substantial reallocation of resources towards more-productive firms, and forces the least productive companies to exit the market or shut-down. In the Chapter 3 I am going to illustrate the model in closed economy, where the heterogeneity between firms is elegantly introduced through the use of a Pareto distribution of production. The chapter will successively move on to the case of open economy.

The main result of the Melitz model regards the reallocations of resources across firms within the same industries. In particular, the low productivity firms exit the market, whereas the intermediate productivity firms that survive face a contract of both revenues and sales. At the same time, the high productivity firms enter in the export markets and expand their market shares. In this framework, the single firm does not change its productivity level, but it is the overall
distribution that changes due to the self-selection of the most productive firms and the reallocation effects within sectors, due to trade liberalization.

Although, Melitz approach can be considered innovative for several reasons, it leaves aside certain assumptions. In the article “Market Size, Trade, and Productivity” M. Melitz and G. Ottaviano relax some hypothesis included in the Melitz Model. In more details, the article starts from the assumption of heterogeneous firms, but considers endogenous mark-ups, elastic labor supply and dismisses the hypothesis of CES utility function. The result involves in welfare gains due to the reallocation of resources towards more productive firms; although, differently from previous literature, the exit from the market of less efficient firms is uniquely due to the higher competition which affects the demand elasticity. The chapter is organized with a section which investigates the model in closed economy, and another which describes the economic framework after trade liberalization. From the first section, the differences between the Melitz and the Melitz-Ottaviano models are evident. In details, the latter leads market size to influence the equilibrium distribution of firms and their performance. In particular, bigger markets face more variety produced by more productive firms that set lower mark-ups. Moreover, the characteristics of firms operating in bigger markets are slightly different from that in smaller markets: they obtain greater sales and profit, even if they are less likely to survive in the market. As far as the case of open economy is concerned, the Melitz-Ottaviano model infers that when two countries decide to reduce the trade barriers between them, the firms respond by reducing their mark-ups, because of the stronger competition they face. Melitz-Ottaviano (2008) was not the only extension of Melitz Model (2003).

In the section “The impact of Trade on Intra-industry Reallocations and Aggregate Industry productivity”, I am going to analyze the namesake article written by Thomas Chaney, in 2008. The economist developed an extension of the Melitz model (2003), where he focuses on the elasticities of intensive and extensive margin, and the impact of trade costs on them. The innovative result of the model relies on the fact that when goods are more substitutable, trade flows are less sensitive to changes in trade barriers. This conclusion is opposite with respect to which one was in revealed in Krugman model (1979): according to Krugman, higher elasticity of substitution between goods, higher is the impact of trade barriers on goods international traded; whereas, according to Chaney, the elasticity of substitution between goods decreases the effects that trade barriers have on international trade.
In the chapter entitled “Gravity equations: Workhorse, Toolkit, and Cookbook”, the more important and used definitions of gravity equation are described (the General Gravity Equation, the Structural Gravity Equation and the Naïve Gravity Equation). Moreover, the section aims to analyze the methods to estimate the welfare equation, as well as to demonstrate as the gravity equation can be a powerful instrument to calculate the welfare changes after trade liberalization policies. The importance of gravity equation is further analyzed empirically. In the following section the article “The empirics of firm heterogeneity and international trade”, wrote, in 2011, by A.B.Bernard, J.B. Jensen, S.J.Redding, and P.K. Schott has been investigated in order to explain how the empirical evidences have helped to the creation and development of new theoretical models and have rises several new questions, previously underestimated in the international trade literature. In this chapter, one key role is playing by the gravity equation, for which the mere analysis of number of firms or products exported have been overcame by the studies of the intensive margin, namely the value that a firm trades per product per country. In addition, the chapter analyzes the firms that produce and export more than one product, the feature of importers, the product quality of imported and exported goods, the role of the intermediaries in the international trade and of the multinational firms.

The last chapter regards a further extension of the Melitz model that tries to include all models of international trade. In the paper “The addilog Theory of Trade” P. Bertoletti, F. Etro, and I. Simonovska developed a theoretical framework different from one used in the previous literature, by assuming “Indirect Additivity”. In mode details, in their model, preferences can be represented by an additively separable indirect utility function and they lead to relative demand of two goods independent on the prices of the other goods available in the market. The case of CES preferences, is included both in case of “indirect additive” preferences, and in “direct additive”, that were used in previous models analyzed in this report. The main result of this model regards the mark-up, which depends on destination per capita income, but not on the size of the destination country. Another important conclusion regards the welfare gains from trade, that can be bestowed uniquely to the increase of number of varieties available; they are found to be lower than what estimated by models that assume direct-additive and homothetic preferences.
Krugman Model:

Increasing Return, Monopolistic competition and International Trade

In 1979 Paul R. Krugman wrote a paper, known as “Increasing Return, Monopolistic competition and International Trade”, which helped him to win the Nobel Memorial Prize in 2008. With his paper, Krugman introduced a new trade theory, which showed an alternative explanation to the international trade with respect to the theory of comparative advantage, as explained by Ricardo.

The main causes of the developed of this new theory were many empirical evidences, which could not be explained by the Ricardian theory. In particular, the postwar period experienced a boom in the international trade, explained by the literature, and principally by Balassa (1967)¹ and Kravis (1971)², through the economies of scale. However, both articles lack to develop a simple formal model to explain such empirical evidences. In addition, while according to the classic theory the bulk of the commerce is between different countries, namely countries with different technology, endowments and preferences, in the real world international trade occurs mainly between similar countries.

Krugman, by using the well known Dixit and Stiglitz (1977)³ model, developed a theory able to explain the international trade as result of economies of scale and not as the difference in the endowments or in technology of countries. Moreover, differently from the previous literature, he considered internal economies of scale, that cause monopolistic competition, neglecting external economies of scale typical of perfectly competitive market.

Internal economies of scale occur in presence of fixed cost: in this framework, when firms produce more output, their average costs per unit decline. This type of economies of scale is independent from the change in the size of the industry. In addition, as mentioned above, internal economies

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of scale characterizes non-perfectly competitive markets, since larger firms face a cost advantage with respect to smaller firms. On the other side, external economies of scale occur when the increase in the size of the industry decrease the cost function of any firm in the industry. Typical examples of this are the improvement of the regulating environment or of technologies. Differently from the previous ones, external economies of scale characterized perfect competitive markets with many small firms that compete between them. In an economic framework with monopolistic competition, the gain from the trade is due to lower prices, due to internal economies of scale, and greater productivity diversity.

The Krugman started to develop his model in closed economy; he successively analyzed the consequences of openness to international trade and the impacts on changes in population growth and factor mobility.

Closed Economy

As mentioned above, Krugman model reclaims one of the most used market structures in recent literature, the monopolistic competition model à la Dixit-Stiglitz.

All individuals in the economy possess one unit of labor, and they face the same utility function over a set of goods, indexed by \( i \), where \( n \) is the number of varieties consumed. The utility function can be expressed as

\[
U = \sum_{i=1}^{n} v(c_i)
\]  

(1)

Where \( c_i \) represents the consumption of \( i \). Consequently, the utility is equal to the sum of the different utility for \( i \). It possess the following characteristics: \( u(0) = 0, u'>0 \) and \( u'' > 0 \); therefore, when individual consumes more amount the utility increases, but it increases at descending rate according to the “love for variety” assumption. The individuals prefer to consume different goods, with respect to great quantity of the same good. The elasticity is defined as \( \varepsilon_i = v'/(v''c_i) \) and assumed \( (\partial \varepsilon_i)/(\partial c_i) < 0 \).

As far as the production function is concerned, the labor is used in the production of each variety \( i \) according to a linear function

\[
l_i = \alpha + \beta x_i
\]  

(2)
Where $\alpha$ and $\beta$ are respectively the fixed and variable costs. Furthermore, $x_i$ is the output of good $i$. The model assumes that the outcome is equal to the consumption of all goods, so $x_i = LC_i$

The consumers maximize the utility subject to the budget constraint, and thus the first order condition:

$$v'(c_i) = \lambda p_i \Rightarrow p_i = \frac{v'(x_i)}{\lambda}$$

(3)

At the same time the firms maximize profits with respect to the price

$$\Pi_i = p_i x_i - (\alpha + \beta x_i)w$$

(4)

Where the price is equal to

$$p_i = \frac{\varepsilon}{\varepsilon - 1} \beta w$$

(5)

At this point the author omits the shorthand notation, because of the symmetry between goods, that will be produced in the same amount and at the same price.

The zero profit condition implies that firms enter in the market as long as they earn positive profits, therefore:

$$\Pi = px - (\alpha + \beta x)w = 0 \Rightarrow \frac{p}{w} = \beta + \frac{\alpha}{x} \Rightarrow \frac{p}{w} = \frac{\alpha}{LC}$$

This last equation can be represented in the $(c, \frac{p}{w})$ plan as a decreasing rectangular hyperbola, represented in the figure 1 as ZZ. In the figure 1 it is also showed the PP, a function between price and consumption as in the equation (5). The intersection between the two curves is unique and represents the individual consumption in equilibrium.
Finally, the amount of goods produced can be determined by the assumption that $x_i = Lc_i$, and they are equal to

$$n = \frac{L}{\alpha + \beta x}$$ \hspace{1cm} (5b)

**Effects of trade**

In this section I plan to investigate the changes in the market when labor force increases, trade growths, and migration, or mobility factor, rises.

As far as the increase in the labor force is concerned, an increment in $L$ affects the equation $ZZ$, as shows in figure 2; however, it does not influence the PP function.
The equilibrium implies that the consumption falls, as well as $p/w$, while the output and the number of goods produced increased. The individual welfare rises because of the increase in real wage, considered as the inverse of $p/w$ and the increases in the choice of goods.

With regards to the model in open economy, the author considers two identical countries, namely two countries with the same taste and technology. Moreover, the model assumes zero transportation costs. While in the previous literature, these two countries would not trade, Krugman demonstrates that they trade and obtain gains.

First of all, the symmetry between countries leads to equal wages rates, prices, and effects experienced with international trade.

The first result is comparable to a growth in the labor force of each country, which in turn increases the output and the number of goods available in the economy. Also the welfare rises for the same reason mentioned above: increase in real wage and in varieties. Therefore, the economies of scale allow gains from the trade even in the case of similarity between tastes, technology and factor endowment.

Although the model is not able to determinate the direction of flow, because of the assumption of economies of scale, it infer that each good will be produced only in one country. By funneling all production in one single plant, the producers can exploit internal economies of scale. The volume
of trade can be solve by maximize the utility function. In this case the utility is slightly different from (1), since it can be split in domestic consumption and consumption of exported goods (which varies between $n + 1$ and $n + n^*$. Therefore:

$$U = \sum_{i=1}^{n} v(c_i) + \sum_{i=n+1}^{n+n^*} v(c_i)$$

And

$$n = \frac{L}{\alpha + \beta x}$$

$$n^* = \frac{L^*}{\alpha + \beta x}$$

Where $L^*$ is the population size in open economy.

Finally, as far as the factor mobility is concerned, this could be considered as comparable to international trade. In fact, in the case of perfect mobility of labor and no trade in goods, the migration of population from one country to another, characterized by greater labor force, will result in the same gains as openness to trade. Therefore, the country with higher labor force faces greater variety of goods and greater scale of production.

To conclude, the virtue of the Krugman model can be found in its simplicity to describe an economic environment characterized by increasing returns to scale, and its ability to explore the implication of them for the international trade, which were previously associated only to the Ricardian comparative advantage. The model is able to predict that in presence of internal economies of scale, and therefore in a market characterized by monopolistic competition, the openness to trade causes lower prices and higher varieties, and therefore leads to benefits also in the case of similar destinations.
The impact of Trade on Intra-industry Reallocations and Aggregate Industry productivity

In this chapter I am going to analyze the Melitz model, as introduced by Mark Melitz, in 2003, with the article “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity”. The Melitz model aims to explain several empirical evidences that the Krugman model, and other previous theories, were not able to explain. For instance, the empirical data demonstrate that export firms are relatively few, but they are present in any industry.

The revelation of the model is the presence of sunk costs, what firms have to pay to enter in the market. In this framework, potential entrants face uncertainty about their production functions, that become known after paying the sunk costs and remain stable; moreover, firms deal with a constant and exogenous probability to exit the market. The presence of both fixed and variable costs in the export market allows only more productive firms to operate and export abroad. In equilibrium, the number of firms that exit the market, because they draw a productivity function below the zero cutoff profit condition, is equal to the number of new entrants.

Through a highly tractable framework characterized by heterogeneous firms, the Melitz Model analyzes what are the impacts of international trade within industries. The model uses, to delineate the consumers’ preferences, a CES (Constant Elasticity of Substitution) utility function, as introduced by Avinash Dixit and Joseph Stiglitz in 1977. This utility function expresses “Love of Variety” preferences, and displays elasticity of substitution between differentiated goods as constant.

The main assumptions of the model can be summarized as the following:

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The production requires only one factor, labor, which is considered as *numeraire* \( w = 1 \) and inelastic; hence when salary increases the amount of labor does not change. Moreover, the supply of labor is indexed as \( L \), an aggregate level of economy’s size.

To enter in the market firms have to pay a sunk cost of entry, \( f_e \) that is equal to all firms. After paying the fixed cost, the firms draw their productivity levels \( \varphi \).

The productivity level, indexed as \( \varphi > 0 \) is different for every producer, and it follows a distribution labeled as \( g(\varphi) \). The distribution of productivity is Pareto distribution. Therefore the marginal costs vary across firms and can be denoted as \( 1/\varphi \). The higher productivity level means that firms need to hire fewer workers to produce a symmetric variety or they produce a higher variety with the same number of employees. Once the firms pay the sunk cost they observe their productivities which remains fixed thereafter. After firms have observed \( \varphi \), they decide whether to exit the market or produce according to their production functions.

In every period, firms face a probability \( \delta \) of exit, common to all firms.

### Closed Economy

First of all, the model analyzes an economy in autarky, and later it examines the impacts of the world trade of the model in open economy. Therefore, over a continuum of goods labeled by \( \omega \), where the set \( \Omega \) represents all the available goods in the economy, the utility function in the case of CES preferences is given as

\[
U = \left[ \int_{\omega \in \Omega} q(\omega)^{\rho} \, d\omega \right]^{\frac{1}{\rho}} \tag{6}
\]

where \( \rho \) is a parameter known as “love of variety parameter”, which varies between 0 and 1 to ensure the concavity of the utility function.

According to the formula, the goods are not perfect substitutes and their elasticity of substitution can be represented by the parameter \( \sigma = 1(1 - \rho) > 1 \).

The budget constraint is given by

\[
\int_{\omega \in \Omega} p(\omega)q(\omega) \, d\omega = E \tag{7}
\]
The utility maximization problem regards the optimization of the utility given the budget constraint, and therefore the first order conditions (FOCs) for any two varieties $\omega$ and $\mu$, belonging to $\Omega$, are

\[
\left( \frac{q(\omega)}{q(\mu)} \right)^{\rho-1} = \frac{p(\omega)}{p(\mu)} \rightarrow q(\omega) = q(\mu) \left( \frac{p(\omega)}{p(\mu)} \right)^{\frac{1}{\rho-1}}
\]

By substituting (last expression) into the budget constraint, recalling that $\sigma = 1(1 - \rho)$, it can be found that:

\[
q(\mu) = \frac{p(\mu)^{\frac{1}{\rho-1}} E}{\int_{\omega \in \Omega} p(\omega)^{\rho-1} d\omega} = p(\mu)^{-\sigma} \left( \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right)^{-1} E
\]

In order to find the aggregate price, the indirect utility level has been set equal to one and we solve for $E$, such that

\[
P = \frac{\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega}{(\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega)^{1-\sigma}} = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}
\]

With these two aggregates we can easily find the demand for a certain variety and the equilibrium firm revenue, respectively

\[
q(\omega) = Q \left[ \frac{p(\omega)}{p} \right]^{-\sigma}
\]

\[
r(\omega) = R \left[ \frac{p(\omega)}{p} \right]^{1-\sigma}
\]

As far at the production side, the model assumes that there is a continuum of firms, and each of them choose to produce a different horizontally-differentiated variety. The production function is characterized by a constant variable cost and a fixed production cost $f > 0$, equal to all firms and indexed as $\ell(\varphi) = q/\varphi - f$. It is important to underline that it differs from $f_e$, which indicates the investment costs needed to export.

The profit maximization problem can be considered as
where the first ordered condition implies the following pricing rule:

\[
p(\varphi) = \left( \frac{\sigma}{\sigma-1} \right) \frac{1}{\varphi} = \frac{1}{\rho \varphi}
\]

By substituting the pricing rule (11) into the equilibrium revenue we have that

\[
r(\varphi) = RP^{\sigma-1}(\rho \varphi)^{\sigma-1}
\]

The profit function, computed as different between the revenues and costs, is

\[
\pi(\varphi) = r(\varphi) - \frac{r(\varphi)}{\sigma} - f = \frac{r(\varphi)}{\sigma} - f
\]

Figure 3: Closed Economy, Profits and Productivity

The figure 3 explains the relationship between profits and productivity level. Under a certain level of \(\varphi\) the profits are negative since the revenues do not cover the fixed cost \(f\). However, the profits increase with the productivity level, since the marginal costs decrease according to a convex function.

In addition, it is important to highlight that the ratios of output and revenues of any two firms depend exclusively on the ratio of their productive levels, and therefore, firms that have higher
productivity level $\varphi$ will produce larger output and obtain higher revenues, although they will charge lower price with respect to less productive firms. Moreover, the profits of the former will be higher than the latter’s. Analytically

$$\frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$$  \hspace{1cm} (11a)

At the aggregate level, the equilibrium is characterized by $M$ firms, each one with a productivity level $\mu(\varphi)$. The price index in (8) can be therefore written as

$$P = \left[ \int_0^\infty p(\varphi)^{1-\sigma} M\mu(\varphi)d\varphi \right]^{\frac{1}{1-\sigma}} = \left[ \int_0^\infty \frac{1}{\rho\varphi}^{1-\sigma} M\mu(\varphi)d\varphi \right]^{\frac{1}{1-\sigma}}$$

$$= M^{\frac{1}{1-\sigma}} \frac{1}{\rho \left[ \int_0^\infty \varphi^{1-\sigma} \mu(\varphi)d\varphi \right]^{\frac{1}{\sigma-1}}}$$

In addition the author defines a measure of the weighted average of the firm productivities, indexed as $\bar{\varphi}$ and defined as

$$\bar{\varphi} = \left[ \int_0^\infty \varphi^{\sigma-1} \mu(\varphi)d\varphi \right]^{\frac{1}{1-\sigma}}$$  \hspace{1cm} (12)

The aggregation level allows a comparison between the Melitz model and the Krugman model: in the former, an industry with $M$ firms with any distribution of productivity $\mu(\varphi)$ generate the same aggregate outcome as an industry, described in the latter model, with $M$ representative firms, all of them sharing the same productivity level $\varphi = \bar{\varphi}$.

**Firms’ entry and exit**

To analyze the firms in the market, the free entry condition has to be considered. In the model there is an unbounded number of potential entrants. As stated above, while before entering all firms are identical, once they entered in the market and they paid the fixed sunk cost $f_e$.

The firm’s decision to enter in the market depends on its profits, as well as the probability of exogenous exit $\delta$. Therefore, the value of profits will be positive and equal to the expected value of them, in the case this value results positive, and otherwise it will be equal to zero. Analytically:
As consequences, any entering firm with productivity level \( \varphi \), lower than the cutoff level \( \varphi^* \), will immediately exits and it will never produce.

The \textit{ex post} productivity distribution \( \mu(\varphi) \) is conditional to the productivity level distribution \( g(\varphi) \) and the probability of successful entry \( 1 - G(\varphi^*) \):

\[
\mu(\varphi) = \begin{cases} 
\frac{g(\varphi)}{1-G(\varphi^*)} & \text{if } \varphi > \varphi^* \\
0 & \text{otherwise}
\end{cases}
\]  

(13)

To summarize, the closed economy general equilibrium is uniquely characterized by three parameters: \( \varphi^*, P, R \), and all other variables are function only of this triple.

Moreover, it will be useful to indicate the weighted average productivity measure according to the free entry condition and depending on the cutoff level.

\[
\bar{\varphi}(\varphi^*) = \left[ \frac{1}{1-G(\varphi^*)} \int_0^\infty \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{1-\sigma}
\]  

(14)

**Equilibrium in closed economy**

The equilibrium is determined by the free entry condition (FE) and the Zero Cutoff Profit condition (ZCP). Both of these two conditions can be written with respect to the cutoff level.

First of all, according to the ZCP firms remain in the market if their profits are positive. To find the conditions analytically, we start computing the average revenues as

\[
\bar{r} = \frac{R}{M} = r(\bar{\varphi})
\]

From the equation (11a)

\[
\frac{r(\varphi_1)}{r(\varphi_2)} = \left( \frac{\varphi_1}{\varphi_2} \right)^{\sigma-1} \Rightarrow \bar{r} = \left( \frac{\bar{\varphi}(\varphi^*)}{\varphi^*} \right)^{\sigma-1} r(\varphi^*)
\]

At same time, the average profits are
From these two definitions we can derive the zero profit conditions, according to what firms enter in the market until the profits are equal to zero.

\[
\pi(\varphi^*) = \frac{r(\tilde{\varphi})}{\sigma} - f = 0 \quad \Rightarrow \quad r(\varphi^*) = \sigma f
\]

\[
\bar{\pi} = \left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} \frac{r(\varphi^*)}{\sigma} - f = \left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} \frac{\sigma f}{\sigma} - f
\]

\[
\Rightarrow \bar{\pi} = \left( f \left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1 \right)
\]

(15)

The FE condition is different from ZCP and implies that, the firms that enter in the market expect non negative profit. In other words, firms consider the entry profitable only if the expected profits are positive. Therefore, the model defined the present values of the average profits as

\[
v_e = p_{in} \bar{v} - f_e = \frac{1-G(\varphi)}{\delta} \bar{\pi} - f_e
\]

(16)

The equation (10) represents the ZCP condition, while the equation (11) the FE conditions.

The equilibrium can be showed by drawing the two functions in \((\varphi, \pi)\) space, as in Figure 4. The FE function is increasing in \(\varphi^*\): when the cutoff productivity level increases fewer firms will be able to enter and operate in the market, and those who enter will earn higher profits. In other words, the average profits conditional on the entry is higher. On the other hand, ZCP is decreasing in \(\varphi^*\) because the probability of successful entry decreased when the cutoff productivity level increases. Moreover, it cuts the FE condition from above only once, and hence there is a unique equilibrium \((\varphi^*, \bar{\pi})\).
According to the author, in the steady state, all the aggregate variables have to remain constant. In particular, the number of new entrants that successful enter $p_{in}M_v$ has to be equal to the number of firms that exit because of bad shock $\delta M$. In this framework the distribution of productivity is not affected, because successful entrants and incumbents that fail have the same distribution productivity level.

As far as the labor clearing condition is concerned, the labor used for production $L_p$ and the labor used for investment purpose to enter in the industry $L_e$ must add to the aggregate labor available $L$. Analytically $L = L_p + L_e$. At the same time the salary of the workers, or in other words the aggregate payments to the production workers, reflects the difference between revenues and profits $L_p = R - \Pi$.

The payments to labor used in production used in entry are equal to the total firm profits:

$$L_e = M_e f_e = M \bar{\pi} = \Pi$$
With these conditions a complete characterization of the equilibrium can be found where the mass of production firms in any period is equal to

$$M = \frac{R}{\bar{v}} = \frac{L}{\sigma (\bar{v} + f)}$$  \hspace{1cm} (17)$$

From this equation we can notice that the mass of firms is increasing in the country size $L$, even if all the other firm level variables ($\varphi^*, \bar{\varphi}, \bar{\pi}, \bar{f}$) are independent from the country size.

The equilibrium price index is equal to

$$P = M^{1-\sigma} p(\bar{\varphi}) = M^{1-\sigma} \rho \bar{\varphi}$$

As far as the welfare is concerned, the benefit per worker is given by

$$W = P^{-1} = M^{\frac{1}{\sigma-1}} \rho \bar{\varphi}$$  \hspace{1cm} (18)$$

The expression (18) tells us that welfare is increasing in $M$, therefore in the number of firm. Namely, larger country experience higher welfare not because of their size, but because of the higher number of firms and the higher productive variety. In fact, the number of firms, form (17), depends on the country size. This result was the same as in Krugman (1980), where a similar expression for welfare was derived. However, in the Krugman model the aggregate productivity level of the firm $\bar{\varphi}$ was considered as exogenous, and hence not affected by the trade. In the Melitz model, on the other hand, the aggregate productivity level and the average firm level are considered endogenously and affected by trade.

**Open economy model**

When Melitz analyses the equilibrium in open economy with no trade costs, he treats the impact of openness as Krugman. In other words, the international trade is considered as well as an increase in the country size in open economy. Change in country size does not affect the firm level outcomes: the market will be characterized by the same number of firms in each country which produce the same amount of good and earn the same profits. The welfare gains due to international trade are only caused by the increasing amount of varieties available to consumers. However, different from Krugman, Melitz consider both variable ad fixed cost of export.
He based this assumption of an analysis from Roberts and Tybout (1977b). These authors interviewed several managers who were asking about export decision. The survey discovered that there are many fixed costs that firms have to face to enter in foreign market. These costs regards information about foreign consumers and their preferences, research about the regulatory environment in foreign countries and the costs necessary to adapt the production to those standards, and the construction of new distribution channels abroad. Furthermore, the authors discovered that it is important to analyze different firms from different backgrounds with different behaviors, in order to understand the implications of exporting final products to open markets. The collection of microeconomic data sets from each region provides a high detailed view of how production and sales were conducted. Decisions regards exporting are supported by econometric models that take in consideration a wide range of factors. Export supply has high level of variance; devaluation may increase the number of employers, and, at the same time, creates irreversible trends that can make the firms suffering earnings shocks and eventually driving them out of market.

The trade costs, described by Roberts and Tybout (1977b), are fixed and independently of the amount of goods exported. In addition, these fixed costs are paid after the firm gains knowledge about its productivity; in other words, a firm knows $\varphi$ and successively it decides whether to export or not. As far as the variable costs are concerned, they are treated as iceberg variable costs, namely for each unit of good exported to a foreign country $\tau > 1$ unit must be exported.

Another assumption used in the model regards the symmetry between countries. This assumption it is not necessary for the determination of the equilibrium, and can be relaxed for more realistic result. However, it becomes useful to analyze the effect of trade independently on the price equalization effects, and therefore analyzing the model with symmetric countries. This later assumption implies that the wage in one country is considered as numeraire and it is equal to all countries ($w_1 = w_2 = \cdots = w = 1$). Furthermore, in the model each country can trade with $n \geq 1$ countries, and to export firms must pay a fixed export cost $f_x$.

Considering the equilibrium in the open economy, it can be noticed that, since firms face the same elasticity demand, export prices are constant and equal to the domestic price multiplied by the variable cost trade $\tau$:

---

\[ p_x(\varphi) = \tau p_d(\varphi) = \frac{\tau}{\rho \varphi} \]

Where according to equation (11) \( p_d(\varphi) = 1/\rho \varphi \).

Consumers optimization implies that market revenues can be considered as a constant fraction of the domestic market revenue and therefore,

\[ r_x(\varphi) = \tau^{1-\sigma} r_d(\varphi) = \tau^{1-\sigma} R(\rho \varphi)^{\sigma-1} \]

Finally the total firms’ revenues depend on their condition of exporters or not:

\[ r(\varphi) = \begin{cases} r_d(\varphi) & \text{if they operate only in the domestic market} \\ r_d(\varphi) + nr_x(\varphi) = (1 + n \tau^{1-\sigma}) r_d(\varphi) & \text{if they export} \end{cases} \]

In addition, since export costs are equal across all countries there is no reason to export only in a particular country: a firm either export to all destinations or never export. Furthermore the profit from domestic sales and international sales can be expressed as

\[ \pi_d(\varphi) = \frac{r_d(\varphi)}{\sigma} - f \]  \hspace{1cm} (19a) \]

\[ \pi_x(\varphi) = \frac{r_x(\varphi)}{\sigma} - f_x \]  \hspace{1cm} (19b) \]

While total profits are

\[ \pi(\varphi) = \pi_d(\varphi) + \max(o, n \pi_x(\varphi)) \]

As in the closed economy the firm value and the domestic cutoff is given by, respectively,

\[ v(\varphi) = \max \left( 0, \frac{\pi(\varphi)}{\delta} \right) \]

\[ \varphi^* = \inf(\varphi: v(\varphi) > 0) \]

The exporting cutoff is the lower productivity level such that the profits of exporting are greater than zero. Analytically

\[ \varphi^*_x = \inf(\varphi: \varphi \geq \varphi^* \text{ and } \pi_x(\varphi) > 0) \]
According to this expression the cutoff firm, namely the which with \( \varphi^* = \varphi^*_x \), obtains null total profit but positive export profit.

On the other hand, when \( \varphi^*_x > \varphi^* \) the firm produces only in the domestic market: it does not enter in the export market, because, otherwise it would obtain negative profits. Finally the firms with productivity level above both thresholds obtain positive profits in both domestic and foreign market. Analytically this can be written as

\[
\begin{align*}
\varphi < \varphi^* & \quad \text{firm does not enter in the domestic market} \\
\varphi^* \leq \varphi < \varphi^*_x & \quad \text{firm operates in domestic market but does export} \\
\varphi > \varphi^*_x & \quad \text{firm operate both in domestic and foreign market}
\end{align*}
\]

The free entry condition in the open economy depends on the probability of successful entry \( 1 - G(\varphi^*) \), the expected domestic profit \( \bar{\pi}_d \), the probability of exporting \( p_x = \frac{1 - G(\varphi^*_x)}{1 - G(\varphi^*)} \) and the expected profit of exporting.

\[
\nu_e = \frac{[1 - G(\varphi^*)] \, \bar{\pi}_d + p_x \bar{\pi}_x}{\delta} = f_x
\]

In addition the equilibrium mass of exporting and the total mass of varieties in any country are

\[
M_x = p_x M \\
M_t = M + nM_x
\]

In other words the total number of varieties \( M_t \) is composed by the number of varieties produced domestically and the number of foreign varieties from any country \( n \).

At the aggregate level the weighted productivity average \( \tilde{\varphi}_t \) become, following the (9)

\[
\tilde{\varphi}_t = \left[ \frac{1}{M_t} \left[ M \tilde{\varphi}^{\sigma-1} + nM_x (\tau^{-1} \tilde{\varphi}_x)^{\sigma-1} \right] \right]^{\frac{1}{\sigma-1}}
\]  

(19c)

Similarly to the case of autarky, the weighted productivity average summarizes completely the effects of the productivity level distribution \( \mu(\varphi) \) on the aggregate outcome \( P, R, \) and \( W \), which can be written as function of it.

From (3):
\[
P = \left[ \int_{\omega \in \Omega} p (\omega)^{1-\sigma} \ d\omega \right]^{\frac{1}{1-\sigma}} = \left[ M p_d (\bar{\phi} (\varphi^*))^{\sigma-1} + M_x (\tau p_d (\bar{\phi} x (\varphi^*))^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]
\[
= \frac{\sigma}{\sigma - 1} \left[ M (\bar{\phi} (\varphi^*))^{\sigma-1} + M_x \left( \frac{\bar{\phi} x (\varphi^*)}{\tau} \right)^{\sigma-1} \right]^{\frac{1}{1-\sigma}}
\]
\[
= M_t^{\frac{1-\sigma}{\sigma - 1}} \frac{\sigma}{\phi_t} = M_t^{\frac{1-\sigma}{\rho \phi_t}} (20a)
\]

At the same manner the aggregate revenues and welfare per worker are

\[
R = M_t r_d (\tilde{\phi}_t) (20b)
\]
\[
W = \frac{R}{L} M_t^{\frac{1}{1-\sigma}} \phi \tilde{\phi}_t (20c)
\]

**Equilibrium in open economy**

First of all the model compute the ZPC, namely

\[
\pi_d (\varphi^*) = 0 \iff \pi_d (\bar{\phi} (\varphi^*)) = f \left( \left( \frac{\bar{\phi} (\varphi^*)}{\varphi^*} \right)^{\sigma-1} - 1 \right)
\]
\[
\pi_x (\varphi^*_x) = 0 \iff \pi_x (\bar{\phi} (\varphi^*_x)) = f_x \left( \left( \frac{\bar{\phi} (\varphi^*_x)}{\varphi^*_x} \right)^{\sigma-1} - 1 \right)
\]
\[
\bar{\pi} = f \left( \left( \frac{\bar{\phi} (\varphi^*)}{\varphi^*} \right)^{\sigma-1} - 1 \right) + n p_x f_x \left( \left( \frac{\bar{\phi} (\varphi^*_x)}{\varphi^*_x} \right)^{\sigma-1} - 1 \right) (21)
\]

The export cutoff level can be written as function of the cutoff level

\[
\frac{r_x (\varphi^*_x)}{r_d (\varphi^*)} = \frac{f_x}{f} = \tau^{1-\sigma} \left( \frac{\varphi^*_x}{\varphi^*} \right)^{\sigma-1} \Rightarrow \phi^*_x = \tau \phi^* \left( \frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} (22)
\]

The free entry condition remains unchanged and equal to

\[
\bar{\pi} = \frac{\delta f_e}{p_{in}}
\]
As shown in Figure 5, the openness to international trade raises the zero profit cutoff productivity level. Moreover, the low productivity firms (between \( \phi^* \) and \( \phi^* \)) that operated in autarky, now have to exit the market. At the same time intermediate productive firms (between \( \phi^* \) and \( \phi^* \)) decrease their revenues in the domestic market. Finally, firms with productivity level greater than \( \phi^* \) enter in the export market and increase their revenues.

The mass of firms present in the economy is

\[
M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{p} + f + p_x n f_x)}
\]

The impact of trade: from autarky to open economy

This section analyzes the impact of incremental trade liberalization from autarky to an open economy condition. Although, it is important to note that this analysis regards the long run consequences of trade.
We call $\varphi^*_{a}$ and $\bar{\varphi}_{a}$ the cutoff and average productivity level in autarky.

By comparing the closed and open economy conditions, it is clear that the FE condition is the same in both cases. However, as far as the ZCP condition is concerned, after the internationalization the ZCP shifts above, as displayed in the figure 6.

The exposure to the international trade changes the productivity level distribution: less productive firms exit the market, intermediate productive firms trade only domestically and high productive firms export and increase their revenues. Therefore, the cutoff productivity level increases ($\varphi^* > \varphi^*_a$), as well as the average profit per firm ($\pi^* > \pi^*_a$).

Moreover, from equation (17) and (23) it can be seen that the number of firms in autarky is more than the number of firms in open economy. Although, the number of domestic firms decreases, consumers enjoy greater productivity variety, since they can buy products from all countries. In other words, the number of domestic firms declines because of the increase in the cutoff productivity level; however, this decline is lower than the number of foreign total exporters.
Finally, the impact of trade on a share of a firm with productivity level $\varphi^* \geq \varphi_a^*$ can be analyzed by considering the following inequality:

$$r_a(\varphi) < r_d(\varphi) < r_d(\varphi) + nr_x(\varphi)$$

where $r_a$ represents the revenues in autarky. The inequality shows that all firms experience a decline in the domestic sales in open economy; however, as far as the exporting firms are concerned, the decrease in the domestic sales is more than compensated by a rise in exports sales, and therefore exporting firms face higher revenues. In this framework, all firms can be divided in two groups, one representing the companies that gain profits due to internationalization, and another group containing companies that lose profits. This concept is can be clearly seen in the figure 5, which integrates Figure 3 with the graph representing revenues and productivity level. The green area represents the increase in the profits of the more productive firms that export; the red area, on the other hand, represents the decline of profits of lower productive firms, that still export.

According to Melitz model, the exposure to trade acts as a “Darwinian Evolution”, where the most efficient firms thrive and grow by increasing both their market share and profits. At the same time, less efficient firms export but face profit losses without losing market share, while even less efficient firms do not export and therefore incur in both market share decline and profit losses. Finally, the less efficient firms are forced to exit the market.
The facts that less efficient firms are driven out from the industry may have two reasons. First of all, the most intuitive one regards the fact that international trade, by increasing the number of competitors, enhances competition with importers that are more productive than domestic firms. However, the Melitz model is not able to capture this effect because of the CES assumption, which implies that the price elasticity of demand does not get affected by the number of prices of competing varieties. Therefore, in the current model, the exit of less productive firms is explained only by the increase in the demand of labor, consider as unique input. Precisely, the international trade provides new opportunities for profits to the most efficient firms, that are hence, willing to pay higher wages for entering in the market. The increase of the average wage, due to the higher labor demand, forces the least productive firms that cannot afford these high salaries, to exit the market.
The impact of trade: the impact of trade liberalization

The previous section analyzed the impact of trade from close to open economy. However, in the real world, a complete autarky condition is really rare. For this reason this section tries to explain the effect of greater openness to international trade. In particular, the increase in the exposure of international trade may be due to an increase in the number of available partner, but also to a decline in variable or fixed costs. In all these case the effects on economy are similar to that analyzed in the previous section: the higher exposure to trade will increase welfare because of the reallocation of the market share towards more productive firms.

As far as effect of increase of \( n \) is concerned, from equations (19) and (20) it can be see that by increasing the number of commercial partners the ZCP curves shifts up. The equilibrium results in higher both cutoff and export cutoff productivity level. As in previous section, higher \( n \) obliges least productive firms to exit the market, creating new reallocation of share and profits in favor of more productive firms.

With regards to the changes in the exporting costs, two different cases have to be analyzed separately. On one side, the reduction of variable cost \( \tau \) causes a shift upward of the ZCP; however, differently from before, the new export productivity level will be greater than before. This means that a greater exposure to international trade imposes the exit of less productive firms, but also generates the entrance of new firms in the export market. By decreasing the costs, more firms find profitable to export. On the other side, the decline in fixed costs increase the cutoff level, therefore a greater number of firms is forced to exit the market, but, at the same time, decreases the export cutoff, hence more firms find profitable operate in the export market. The most important difference regards the benefits: only new exporter will benefit from these declines by increasing their combined sales, because incumbents have already paid higher fixed costs.
Melitz Model: conclusion

The Melitz model tries to address several questions about the international commerce: first of all, the effect of liberalization on the firm productivity distribution; secondly, all firms are affected by the international trade at the same manner; and finally, what are the impact on the welfare and the aggregate productivity. The model tries to answer these questions in a new framework, where firms’ heterogeneity level plays a crucial role in the intra-industry reallocation.

The main conclusions regard the positive effects of international trade on welfare. However, these welfare gains are due to the change in the reallocation of firms towards the more productive ones.

The opening of trade leads to reallocations of resources across firms within the same industries. In this framework, the low productivity firms exit the market, whereas the intermediate productivity firms that survive face a contract of both revenues and sales. At the same time, the high productivity firms enter in the export markets and expand their market shares. The main result regards the change in the industry composition, due to the improvements in aggregate industry productivity and the reallocation of resource towards more efficient firms. It is important to underline that the firms do not change their productivity level, but it is the overall distribution that changes.

Finally, different from Krugman, Melitz model makes the crucial assumption of the existence of sunk costs of exporting, in order to predict the self-selection of the most productive and the reallocation effects within due to trade liberalization. Although, Melitz approach can be considered innovative for several reasons, it leaves aside certain assumptions. For example, the fact that labor supply is considered as inelastic seems quite unrealistic in the reality. In the following sections I am going to analyses some extension of the Melitz model.
In the Melitz Model, the welfare gains from trade were due to the increase in the competition for scarce labor resources. In this framework, the labor was considered the only production factor and considered as completely inelastic, the higher competition boosts the real wage and induces the least productive firms to exit the market.

In the article “Market Size, Trade and Productivity” the author, Marck J. Melitz and Giancarlo I. P. Ottaviano, introduce a new model based on the Melitz model (2003). In Melitz-Ottaviano model (MO), however, the higher factor market competition plays no role in the determination of the effects of international trade. This is due relax of the assumption of CES utility function. In fact, according to this model, the labor supply is perfectly elastic. Thus the exit from the market of less efficient firms is uniquely due to the higher competition which affects the demand elasticity.

As in Melitz (2003), MO model analyzes firstly the model with close-economy, where different results with respect to Melitz lead market size to influence the equilibrium distribution of firms and their performance. In particular, bigger markets face more variety produced by more productive firms that set lower mark-ups. The characteristics of firms operating in bigger markets are slightly different from that in smaller markets: they obtain greater sales and profit, even if they are less likely to survive in the market. Successively, MO model analyzes the framework in open economy: also in this case the differences with the Melitz model are evident. When two countries decide to reduce the trade barriers, the firms reduce their mark-ups due to the stronger competition they face.

Melitz-Ottaviano Model in closed economy

The model considered is similar to Melitz (2003), but it considers a demand system where the price elasticity of residual demand is no longer fixed, but indigenized and variable according with the degree of competition in the market. The preferences of the consumers are described by the following utility function:
Where $q_0^c$ represent a homogeneous good, considered as numeraire, while $q_i^c$ represent a the amount a differentiate good. The love for variety is considered by the positive parameter $\gamma$; in particular, when $\gamma$ is equal to 0 the goods are perfect substitutes, while when $\gamma$ increases the consumers start to care more and more about the differentiation of goods. The other two parameters $\alpha$ and $\eta$, both positive as well, indicate the substitutability of variety with the numeraire good, and precisely, larger $\alpha$ and lower $\eta$ shift out the demand for differentiated variety with respect to the numeraire.

In this framework the prices have to be lower than an upper bound for which the demand for variety is zero; analytically:

$$p_i \leq \frac{1}{\eta N + \gamma} (\gamma \alpha + \eta N \bar{p}) \equiv p_{max}$$ (25)

where $\bar{p}$ is the average price. From the equation (25) it is evident how the threshold decreases when the competition becomes stronger: when the number of firms $N$ increases and the average price decreases $p_{max}$ declines.

The price elasticity of the demand is equal to

$$e_i = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} = \left[ \frac{p_{max}}{p_i} - 1 \right]^{-1}$$

It can be noticed that this elasticity depends on $\bar{p}, N$ and $p_{max}$. Therefore, when the average price decreases (stronger competition), the upper threshold declines as well. An increase in the number of firms (stronger competition) displays the same effects. This result is the first difference with respect to the Melitz model, where the preference were characterized by CES, namely constant elasticity of demand.

As far as the welfare is concerned, it can be evaluated through the indirect utility function associated with (24):

$$U = I^c + \frac{1}{2} \left( \eta + \frac{\gamma}{N} \right)^{-1} (\alpha - \bar{p})^2 + \frac{1}{2} \frac{N}{\gamma} \sigma_p^2$$ (26)
Where $I^c$ represents the consumers’ income and $\sigma^2$ represents the variance between prices. The welfare, as described by (26) depends on the degree of competition in the market: when the average price decreases, the consumers experience a welfare gain. Moreover, it is proportional to the variance between prices; namely when the prices differ more, the consumers re-optimize their purchase towards both lower prices varieties and numeraire good. Finally, the love of variety is recalled in this formula by the link between N and welfare: when the number of firms increases, the consumers can choose between more varieties and their welfare rises.

As far as the production side is concerned, it is very similar to which developed in Melitz (2003). The assumptions behind the model are:

- The homogenous good is produced under constant return to scale at unit cost;
- Before starting to produce, firms do not know the production function and to enter in the market they have to pay an initial investment sunk cost equal to $f_c$;
- When the initial investment is made, the differentiated goods are produced according to a common cost distribution $G(c) \in [0, c_M]$;
- With respect to Melitz (2003), firms do not have to pay further overhead costs.

In this framework the firms maximize their profits based on their linear residual demand functions, taking both N and $\bar{p}$ as given. In addition, the model is characterized by monopolistic competition with free entry. Therefore, if $c_D$ is the endogenous cutoff for which firms are indifferent about remain or not in the industry, all statistics can be summarized as

$$\text{price: } p(c) = \frac{1}{2}(c_D + c)$$

$$\text{mark-up: } \mu(c) = p(c) - c = \frac{1}{2}(c_D - c)$$

$$\text{revenues: } r(c) = \frac{L}{4\bar{y}}[(c_D)^2 - c^2]$$

$$\text{profits: } \pi(c) = \frac{L}{4\bar{y}}(c_D - c)^2$$

The firms with lower $c$, namely the more productive firms, set lower prices, but higher mark-ups. Therefore the productivity gains are not entirely passed on the consumer. Moreover, more efficient firms obtain higher revenues and earn higher profit.
The free entry condition implies that no firms will enter in the market if they earn negative profits. Hence,

\[ \int_0^{c_D} \pi(c) dG(c) = f_e \Rightarrow \frac{L}{4Y} \int_0^{c_D} (c_D - c)^2 dG(c) = f_e \] (27)

The model assumes that the distribution of cost \( G(c) \) follows a Pareto distribution and analytically

\[ G(c) = \left( \frac{c}{c_M} \right)^k \text{ with } k \geq 1 \]

The free entry condition therefore becomes

\[ \frac{L}{4Y} \int_0^{c_D} (c_D - c)^2 \left( \frac{c}{c_M} \right)^{k-1} dc = f_e \] (28)

At the same time, the cutoff level is equal to

\[ c_D = \left( \frac{\gamma(2(k+1)(k+2)c_M^k f_e)}{L} \right)^{1/(k+2)} \] (28b)

First of all, since \( k \geq 1 \), the exponential of the equation has to be positive. Secondly the average productivity is higher, and the \( c_D \) is lower when the market size \( L \) is greater; at the same manner, the varieties are closer substitutes, namely \( \gamma \) is lower when there are lower sunk cost \( f_e \). All these factors increase the competition, causing an increase in the elasticity of demand and a decline in the mark-up.

As far as the analysis of the welfare is concerned, we can compute the welfare as

\[ U = 1 + \frac{1}{2\eta} (\alpha - c_D) \left( \alpha - \frac{k + 1}{k + 2} c_D \right) \]

We can notice that the welfare is monotonically decreasing in \( c_D \): higher \( c_D \) causes an increase in the product variety \( N \) and a decrease in the average price \( \hat{p} \).

From the MO model in autarky, it can be noticed that trade integration lead to welfare gains that differ from the standard variety effects as predicted by Krugman, or the reallocation effect described by Melitz; the welfare gains regard pro-competitive effects. Furthermore, in this framework, larger markets, that can be thought as integrated world economy with no trade costs, are characterized by lower average mark-ups and prices due to the endogenous change in the cut-
off $c_D$. Moreover, they face bigger and more efficient firms, with higher revenues and profits and they lead to greater welfare due to products differentiation, lower price and higher variance between them.

**Melitz-Ottaviano Model in open economy**

This section aims to analyze the model in open economy, by considering two countries $H$ and $F$. Differently from Melitz, these countries are not considered symmetric, but they can differ both in term of size $L^l$ (with $l = H, F$) and deliver cost $\tau^l > 1$.

As in the autarky, the expression (25) reveals that the positive threshold for the two markets is equal to

$$p^l = \frac{1}{\eta N^l + \gamma} (\eta\alpha + \eta N^l \bar{p}^l) \quad (6)$$

In this framework, the upper bound cost for firms selling in the domestic market $c^l_D$ and the upper bound for exporter from country $l$ to $h$, indexed as $c^l_h$ are:

$$c^l_D = \sup\{c: \pi^l_D(c) > 0\} = p^l_{\text{max}} = \left[ \frac{y^2(k+1)(k+2)(c_M)^k f_e}{L^l} \frac{1 - \rho^h}{1 - \rho^l \rho^h} \right]^{\frac{1}{k+2}} \quad (29b)$$

$$c^l_h = \sup\{c: \pi^l_h(c) > 0\} = \frac{p^h_{\text{max}}}{\tau^h}$$

Where $\rho^l = (\tau^l)^{-k}$ indicates the openness to trade

The most productive firms, from any country, with costs less or equal than $c^l_h$, will export, while firms with costs between $c^l_D$ and $c^l_h$ will only produce for the domestic market. As far as the comparison between autarky and open economy is concerned, we can notice that the cut-off condition in closed economy (28b) is greater than which in open economy (29b): as in Melitz 2003 trade forced least productive firms to exit the market, and this in turn increases the average productivity. However, while in the Melitz the exit by the less efficient firms was due to their difficulty to acquire labor, and the more expensive, labor resources, in this model the competition on scarce resources does not play any role, and the exit of less productive firms is caused by the rise in market competition.
Successively, Melitz and Ottaviano focused on the consequence of the trade liberalization when market size differs across countries. They discovered that bigger markets attract more firms, that are forced to operate in a more competitive market. Therefore, the cutoff cost in domestic market decreases as the market size increases, as consequence, bigger market experience higher average productivity.

As far as the effects on change in transport costs are concerned, the two authors analyzed a fall in transport cost, when the costs of trade are symmetric between countries, and they called this mechanism “bilateral liberalization”. This decline boosts the competition in both markets, with higher average productivity and higher products variety, that in turn create a pro-competitive effect by lowering the mark-up. With regards to the market size, it plays a key role in this framework. In particular, when trade costs decline, the firms are attracted by bigger markets, which deal with a bigger number of firms, but with lower welfare gains with respect to smaller country.

As far as the unilateral liberalization is concerned, this regards a decrease of trade costs in one country, while the costs in the other country remain constant. The impact of this on welfare is different across countries: in the protected country, namely which did not decrease the trade costs, the productivity and welfare raise, while the liberalizing country faces a decrease of both of them. This result is due to the fact that the protected countries are considered by firms as more attractive locations with respect to liberalizing countries, because in the former they have access to the domestic market and cheaper access to the export market. In addition, in the liberalizing country the number of domestic firms decreases more than the increase in the number of new exporters: the competition in these countries is weaker and the welfare gains are lower.

Although it is important to notice that the effects of liberalization is not constant over the time: even if in the long run the welfare effects are different in protected and liberalizing countries, in the short run the liberalization causes a welfare gains in any country.
Conclusions

The model developed by Melitz and Ottaviano and inspect in this section, refers to heterogeneous firms model. Although, he can be considered slightly different from the Melitz model developed in 2003. First of all, he relaxes the assumption of CES preferences, as developed by Dixit-Stiglitz. Secondly, he considers endogenous mark-ups, that change with the degree of competition in the market. In this framework, the trade liberalization produces some important adjustments: larger markets experience a tougher competition, caused by the lower average mark-up and by the higher aggregate productivity, with respect to smaller markets.

Moreover, the article aims to demonstrate that different trade liberalization mechanisms have different impacts on the economies involved. In case of bilateral competition, both countries involved experience a higher welfare, due to the increase in competition level; at the same time, in case of unilateral liberalization, the protected country experiences increment of welfare, while the country which adopted liberalization policies, does not face welfare improvements.
The impact of Trade on Intra-industry Reallocations and Aggregate Industry productivity

Thomas Chaney, in 2008, wrote a paper in which he developed an extension of the Melitz model. In particular, he started from the assumption of firm heterogeneity, introduced by Melitz, to demonstrate that the elasticity of substitution between goods decreases the effects that trade barriers have on international trade. This conclusion does not match with the results of the Krugman model (1979), where the elasticity of substitution has a positive impact of the trade barriers and on trade flow. In other words, the Krugman model (1979), by considering identical firms, predicts that the higher the elasticity of substitution between goods, the higher is the impact of trade barriers on goods international traded. The Chaney model, however, assumes different firms and Pareto productivity distribution, and arrives to opposite results.

The Pareto distribution, introduced by Vilfredo Pareto, is a heavy tailed distribution which depends on two main parameters $\alpha$ and $\beta$, that are constant and greater than zero. The distribution function can be written as

$$F_X(x) = 1 - \left(\frac{\alpha}{x + \alpha}\right)^\beta$$

Thanks to his shape, where a tail is heavier than the other, the Pareto distribution is known for its great ability to describe the allocation of wealth across individuals, where few people own the majority of wealth, and to model extreme losses in the insurance sector. It has been widely used in the international trade literature to describe the productivity distribution between firms. The first economists who use it to describe the productivity distribution across firms were Herbert A. Simon and Charles P. Bonini\(^6\), who found that Pareto distribution can be considered as a good approximation for the productivity between firms. In fact, the shape of Pareto distribution well

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describes the condition according to which only the greater firms export. Successively, Jonathan Eaton, Sam Kortum and Francis Kramarz\(^7\) in 2007, found empirical evidence for the Paretian distribution, by analyzing the distribution of firms size for French data.

The importance of the Chaney model relies on the importance of the extensive margin analysis with respect to elasticity of substitution between goods. In particular, in a world where there are different countries that trade facing different trade barriers, the impact of reduction of cost on trade flows is negatively related to the elasticity of substitution.

Finally, there is a rich empirical literature which analyzes the relationship between trade barriers and margins of trade. The Chanel model has been empirically estimated by Crozet and Koening (2007)\(^8\) by using French data. Furthermore, Koening (2005)\(^9\) found empirical evidences supporting the fact that when good are homogenous, the distance elasticity on intensive margin is greater, while the distance elasticity on extensive margin is lower.

**The Chaney Model**

The Chanel Model reclaims the Krugman model, according to which there are \(N\) identical firms and there are no fixed costs of transportation. As explained above, this assumption was dismissed by Melitz in 2003, which introduced both fixed and sunk costs to export. The innovative aspect of the Krugman model (1979)\(^5\), regards the fact that, even if the countries are identical, they trade and exchange goods between themselves because of the preference of consumers for variety. In addition, the model predicts that export flow depends on the size of the countries (using GDP as proxy), the trade barriers, and the elasticity of substitution between goods indexed as \(\sigma\). Analytically,

\[
Export_{AB} = \text{Constant} \frac{GDP_A GDP_B}{(\text{Trade barriers}_{AB})^\sigma}
\]

Therefore, as in Krugman, when goods can be easily substituted, and thus elasticity of substitution between goods is high, the competition between firms is stronger and changes in trade barriers


effects the international flow deeper. Although in the Krugman model, there are no indications of the differences between intensive and extensive margin.

To obviate to this limitation, Chaney relaxed some assumptions used in Krugman model. First of all, the firms are not identical, but they differ for their productivity. Secondly, the costs of transport are composed both by fixed and variable costs. Thirdly, this model allows the analysis of the effects of change in costs of transport in the two margins. In particular, changes in barriers trade, and therefore in transportation costs, have different impacts on intensive and extensive margin, according to the elasticity of substitution. After changes in trade costs, the size of the exports change, and these are more significative when the elasticity between good is high. On the other side, when the parameter $\sigma$ is greater, the effects of change in barriers on the number of exporters and the number of product export are lower. This result depends on the heterogeneity between firms: when elasticity of substitution is high, the competition between firms is stronger. Therefore, after a decrease in trade barriers, less productive firms enter in the market, but their impact on the total trade flow is low, because they obtain small market shares; at the same time, more productive ones hold greater market share. When goods are greater substitutes, a decline in trade costs induces the entrance of new firms, but their impact is not significative. As consequence, the impact on intensive margin is higher, while the impact on the number of product and firms is lower. On the other hand, when goods are more differentiate, the competition is weak and the less productive firms that enter in the market are able to acquire higher market shares. This result is summarized in the formula by the introduction of $\varepsilon$ function, which can be identified as the elasticity of aggregate trade with respect to trade barriers. This function depends and is decreasing in the elasticity of substitution between good.

Analytically,

$$Export_{AB} = \text{Constant} \frac{GDP_A GDP_B}{(Trade \ barriers_{AB})^{\varepsilon(\sigma)}}$$

The model considered $N$ different countries that produce goods using labor as unique production factor. The number of consumers is different in each country and it can be written as $L_n$. As far as the goods available are concerned, there is one single homogenous good, and other $H$ different goods, over a continuum set $\Omega$. Therefore, every individual maximized his utility in which the number of goods is $H + 1$. 
The utility function is equal to

\[ U \equiv q_0^{\mu_0} \prod_{h=1}^{H} \left( \int_{\Omega_h} q_h(\omega) \frac{\sigma_h^{-1}}{\sigma_h} d\omega \right)^{\frac{\sigma_h}{\sigma_h-1}} \mu_h \]  

(30)

where \( \mu_0 + \sum_{h=1}^{H} \mu_h = 1 \). In addition, as in Stiglitz-Dixit preferences, the elasticity of substitution between two goods must be greater than 1.

The other assumptions of the models can be summarized as follow:

- The homogenous good is considered as numeraire and its production function faces constant return to scale with labor.
- The other differentiate goods are produced according to increasing return-to-scale production function, and as in Melitz (2003), firms draw a productivity \( \varphi \) from a Pareto distribution between \([1, +\infty)\). Furthermore, the Pareto distribution depends on a parameter \( \gamma_h \) greater than \( \sigma_h - 1 \), and can be written as

\[ G_h(\varphi) = 1 - \varphi^{-\gamma_h} \]

- The shape parameter \( \gamma_h \) can measure the homogeneity between sector: in particular, when \( \gamma \) is greater the sector characterized by the majority of output concentrated in smallest and least productive firms.
- The countries differ from each other both in term of productivity, where the marginal costs is equal to the wage \( w_n \), and of size.
- The transportation costs, or trade barriers, can be divided in two parts. On one side, there are iceberg variable transportation costs equal to \( \tau \). As consequence, the variable cost of trading the good \( h \) from country \( i \) to \( h \) is equal to \( \tau^h_{ij} \). On the other side, there is a fixed costs that every firm \( i \) in the sector \( h \) which want to export in country \( j \) has to pay, and it can be indexed as \( f^h_{ij} \).
- The number of potential entrants that aim to export in a potential country depends on the size and the wealth of its. Therefore, the number of potential entrants is proportional to \( w_n L_n \)
- The model does not assume free entry; the number of firms is fixed and the extra profits they obtain are redistributed between workers as units of numeraire goods: the parameter \( \pi \) identifies the dividend per share.
The total income of workers $Y_j$ is equal to the income from labor and the dividends they get; $Y_j = (1 + \pi)w_n l_n$

In this framework the costs function to export from $i$ to country $j$, for a firm in the sector $h$ is equal to

$$c_{i,j}^h(q) = \frac{w_i \tau_{i,j}^h}{\varphi} q + f_{i,j}^h$$

(31)

Moreover, the demand function is isoelastic, therefore the price elasticity is constant, and the optimal price depends on the mark-up and is equal to

$$p_{i,j}^h(q) = \frac{\sigma_h}{\sigma_h - 1} \frac{w_i \tau_{i,j}^h}{\varphi}$$

If we consider $P$ the price index for the amount of goods export from country from $i$ to country $j$, for a firm in the sector $h$ is

$$x_{i,j}^h(\varphi) = p_{i,j}^h(\varphi)q_{i,j}^h(\varphi) = \mu_h Y_j \left( \frac{p_{i,j}^h(\varphi)}{P_j^h} \right)^{1-\sigma_h}$$

(32)

The price index and the dividend per share depends on the number of firms that enter in the export market, which depends on the threshold $\varphi_{k,j}^h$; they are equal respectively to

$$P_j^h = \left( \sum_{k=1}^{N} w_k l_k \int_{\varphi_{k,j}^h}^{\infty} \left( \frac{\sigma_h}{\sigma_h - 1} \frac{w_i \tau_{i,j}^h}{\varphi} \right)^{1-\sigma_h} dG_h(\varphi) \right)^{\frac{1}{1-\sigma_h}}$$

(33)

$$\pi = \frac{\sum_{h=1}^{H} \sum_{k=1}^{N} w_k l_k \int_{\varphi_{k,j}^h}^{\infty} \pi_{k,j}^h(\varphi)dG_h(\varphi)}{\sum_{n=1}^{N} w_n l_n}$$

(34)

where $\pi_{k,j}^h(\varphi)$ represents the net profit of a firms that export from $k$ to $j$, operating in sector $h$, and with a productivity $\varphi$, and it is equal to

$$\pi_{k,j}^h(\varphi) = \left[ p_{k,j}^h(\varphi) - c_{k,j}^h(\varphi) \right] q_{k,j}^h(\varphi) - f_{k,j}^h(\varphi)$$

(35)

Since now, the model analyzes only one sector, but the analysis for the other is similar; the coefficient, therefore, will be implied.
The model in global equilibrium

The global equilibrium considers a static point, where firms decide which market to enter and the price for selling the goods. This decision depends on the competition degree in the market, which in turn depends on the number of firms that decides to enter in the market. However, every firm considers the strategy of competitors, as well as the decision of consumers, as given.

More in details, the firms that decide to export are a subset of the total firms, because not every firm can generate enough profit to cover the fixed costs. Thus, only the more productive firms enter in the export market, while the less productive firms will operate domestically.

To obtain the threshold, we use the expression (35) and considering that firms enter in the market if they obtain non negative profits. As consequence,

\[ \pi_{kj}(\varphi) = \frac{\mu Y_j}{\sigma} \left[ \frac{\sigma}{\sigma - 1} \frac{w_{tij}}{\varphi p_j} \right]^{1-\sigma} - f_{ij} = 0 \Rightarrow \]

\[ \bar{\varphi}_{ij} = \left( \frac{\sigma f_{ij}}{\mu Y_j} \right)^{\frac{1}{\sigma-1}} \left( \frac{\sigma}{\sigma-1} \right)^{\frac{w_{tij}}{p_j}} \]

(35b)

The price index is not as given, but it depends on the characteristics of the country of destination. By using (33) and (35b), we can rewrite the price index depending on a remoteness index \( \theta_j \), which describes the impact of fixed costs and firms heterogeneity on aggregate prices. Therefore,

\[ P_j = \lambda_2 Y_j^{\frac{1}{\sigma-1}} \theta_j \]

(36)

Where \( \theta_j^{-Y} \equiv \sum_{k=1}^{N} (Y_k / Y) (w_{t_k})^{-Y} f_{k_j}^{-\left[\frac{Y}{\sigma-1} - 1\right]} \), \( Y \) is the world output, and

\[ \lambda_2 = \left( \frac{\gamma - (\sigma - 1)}{\gamma} \right) \left( \frac{\sigma}{\mu} \right)^{-\frac{Y}{\sigma-1}} \left( \frac{\sigma}{\sigma-1} \right)^{\frac{Y}{\gamma}} \left( \frac{1+\gamma}{\gamma} \right) \].

By using the equation (35b) and (36), the exports level, the productivity threshold, the aggregate output and the dividends per share can be found.

\[ \chi_{ij}(\varphi) = \begin{cases} \lambda_3 \left( \frac{Y_j}{Y} \right)^{\frac{Y}{\sigma-1}} \left( \frac{\theta_j}{w_{tij}} \right)^{\frac{\theta_j}{\sigma-1}} \varphi^{\sigma-1} & \text{when } \varphi \geq \bar{\varphi}_{ij} \\ 0 & \text{o otherwise} \end{cases} \]

(37)
Where $\lambda_3, \lambda_4,$ and $\lambda_5$ are constants

$$
\lambda_5 = \frac{\sum_{h=1}^{H} \left( \frac{\sigma_h - 1}{\gamma_h} \right) \mu_h}{1 - \sum_{h=1}^{H} \left( \frac{\sigma_h - 1}{\gamma_h} \right) \mu_h}
$$

$$
\lambda_4 = \left[ \frac{\sigma \gamma}{\mu \gamma - (\sigma - 1) \gamma_h - 1 + \lambda_5} \right]^\frac{1}{\gamma}
$$

$$
\lambda_3 = \sigma \lambda_4^{1-\sigma}
$$

It is important to underline that the main results of the global equilibrium depend on the size of the country of destination, $L_i$, the marginal cost or wage $w_i$, both the variable and trade costs, $\tau_{ij}$ and $f_{ij}$, and the index of remoteness, $\theta_j$.

Based on the Pareto distribution, the aggregate export goods are

$$
X_{ij}^h = w_i L_i \int_{\bar{\phi}_{kj}}^{\varphi} x_{ij}^h(\varphi)dG_h(\varphi) = w_i L_i \int_{\bar{\phi}_{kj}}^{\varphi} \lambda_3 \left( \frac{Y_j}{Y} \right)^{\gamma} \left( \frac{\theta_j}{w_i \tau_{ij}} \right)^{\lambda - 1} \frac{\varphi^{\sigma - 1}}{\gamma_h} d\varphi
$$

And the productivity threshold

$$
\bar{\varphi}_{kj}^h = \lambda_4 \left( \frac{Y_j}{Y} \right)^{\gamma} \left( \frac{w_i \tau_{ij}}{\theta_j} \right)^{\lambda - 1}
$$

Putting together these last two equations we obtain that the aggregate exports are equal to

$$
X_{ij}^h(\varphi) = \mu_h \left( \frac{Y_j}{Y} \right)^{\gamma} \left( \frac{w_i \tau_{ij}}{\theta_j} \right)^{-\gamma_h} \left( f_{ij}^h \right)^{-\gamma_h - 1}
$$

This last equation is one of the main results of the model; according to it the elasticity with respect to variable trade costs, $\gamma$, is different with respect to Krugman Model (1979), where the economy was characterized by identical firms. In that model, the elasticity of substitution with respect to
variable trade barriers was $\sigma - 1$. However, in this model firms are not identical, but differ from the productivity function. Therefore, when the variable trade costs decreases, each firms increase the size of experts, and new firms enter in the market, since more of them become able to obtain the profits necessary to repay the fixed costs.

In addition, in sector where the good are more homogenous, the effects of changes in transportation costs are magnified. In fact, in these sectors the competition is higher, and when trade costs changes, a greater number of firms enter in and exit from the market.

**The intensive and Extensive margins**

In this section, I am going to try to explain the relationship between trade costs changes on the extensive and intensive margins, and the impact of elasticity of substitution in this relationship. The model concludes that elasticity of substitution affects the two margins in opposite directions. For higher elasticity of substitution, the impact of trade barriers on intensive margin is high, while is low on the extensive margin. Furthermore, the effects on the extensive margin are greater than the opposite effects on intensive margin.

The model considers three different types of elasticity: the well known elasticity of substitution between goods, $\sigma$, the elasticity of trade with respect t variable costs $\zeta$, and the elasticity of trade with respect to fixed costs, $\xi$. Analytically,

$$\zeta = - \frac{d \ln X_{ij}}{d \ln \tau_{ij}}$$

$$\xi = - \frac{d \ln X_{ij}}{d \ln f_{ij}}$$

The model predicts that the elasticity of substitution has no effects on the elasticity of trade with respect to variable costs, while has a negative impact on the elasticity of trade with respect to fixed costs.

$$\frac{\partial \zeta}{\partial \sigma} = 0 \text{ and } \frac{\partial \xi}{\partial \sigma} < 0$$

To arrive at this result, the impact of trade barriers on trade flows has been decomposed in the intensive and extensive margin. The former measures how much an existing export changes the
size of its exports when trade costs change; whereas the latter assess how much the new entrants, entered because of change in trade costs, exports. The two effects are identified by differentiating the aggregate export. Thus

\[ dx_{ij} = \left( w_i L_i \int_{\bar{\varphi}_{ij}}^{\varphi} \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} dG(\varphi) \right) d\tau_{ij} - \left( w_i L_i x(\bar{\varphi}_{ij}) G'(\bar{\varphi}_{ij}) \frac{\partial \bar{\varphi}_{ij}}{\partial \tau_{ij}} \right) d\tau_{ij} \]

\[ + \left( w_i L_i \int_{\bar{\varphi}_{ij}}^{\varphi} \frac{\partial x_{ij}(\varphi)}{\partial f_{ij}} dG(\varphi) \right) df_{ij} - \left( w_i L_i x(\bar{\varphi}_{ij}) G'(\bar{\varphi}_{ij}) \frac{\partial \bar{\varphi}_{ij}}{\partial f_{ij}} \right) df_{ij} \]

Where the first and the third brackets represent the intensive margin, while the others the extensive margin.

The effects of both margins on the elasticity of trade with respect to variable costs can be studied with the following expression:

\[ \zeta \equiv - \frac{d \ln x_{ij}}{d \ln \tau_{ij}} = (\sigma - 1) + (\gamma - (\sigma - 1)) = \gamma \]

\[ \frac{\partial \zeta}{\partial \sigma} = 0 \]

The equation has two main terms: in the first bracket, it is represented by the intensive margin elasticity, and is increasing in elasticity of substitution between goods, while the second term represents the extensive margin elasticity, and it is decreasing in the elasticity of substitution between goods.

As far as the impact on two margins on the elasticity of trade with respect to fixed costs, it can been written that

\[ \zeta \equiv - \frac{d \ln x_{ij}}{d \ln f_{ij}} = 0 + \frac{\gamma}{\sigma - 1} - 1 \]

\[ \frac{\partial \zeta}{\partial \sigma} < 0 \]
This expression discloses a different result in respect to variable costs. The elasticity of substitution between goods has not effects on intensive margin elasticity, while negatively affects the extensive margin elasticity.

Conclusion

The Chaney model employs the firm heterogeneity to understand the effects of elasticity of substitution between goods on the intensive and extensive margins. In particular, when goods are highly differentiated, changes in trade costs do not affect deeply the demand of goods. At the same time, when goods are differentiated, or in other words when $\sigma$ is low, even the less productive firms are able to acquire a relative high market share, since the consumers are willing to buy a goods at higher prices because there are no perfect substitutes. In this framework, when the trade costs decline, new entrants, with lower productivity function enter in the export market, and they own large market share, therefore strongly affecting the extensive margin.
Firms in International Trade

Andrew B. Bernard, J. Bradford Jensen, Stephen J. Redding, and Peter K. Schott, in 2007, wrote an empirical article aiming to explain many empirical evidences which the previous theories of international trade were not completely able to explain. In particular, since 1990s, the role of the firms in the international trade became more and more important. In fact, firms which export possess different features with respect to firms which only serve the domestic market, and these features are present also before exporting. More in details, exports are larger, more productive, more skill and capital intensive, and they pay higher salary with respect to domestic firms. This article is going to explain in details these differences and how these differences match the economic theory of heterogeneous-firm models.

As showed by Melitz and Melitz-Ottaviano models, the productive advantage of exporters has an impact on the macroeconomics outcome in term of self-selection. When the barriers to trade decline the reallocation of more productive firms, due to the surviving of high-productive and the exit of low-productive firms, raises the aggregate productivity. Therefore, this paper aims to explain how the focus of international trade has shifted from countries and industries to firms.

From old to new trade theory

The traditional theory of international trade has always dealt with comparative advantage both in terms of productivity differences between countries (as conceived by Ricardo) and in term of differences between industries in factor intensity or difference in endowments between countries. All these sets of theories share the conviction that countries export goods from one industry and import goods for a different one.

The crisis of the old theories took place where the abundance of data showed how the international trade occurs between similar countries and within industry. At the beginning of 1980s, several economists, from Krugman in particular, started to analyze models in which the welfare gains were due to a production function characterized by economies of scale and consumer preferences characterized by “love for variety”. In this framework, international trade
rise the welfare not because of difference in the opportunity costs of production across countries, but by forming the greater varieties, that consumers can enjoy.

Although the new theories were able to explain the trade between similar countries, they still lack to consider the heterogeneity characterizing the exporting firms. In fact, all models, to facilitate the general equilibrium analysis, consider a representative firm, underestimate the impact of firms’ differences in the aggregate productivity.

The first evidence that the article wants to analyze is the fact that firms which export are a relatively small share with respect to the total number of firms. The data state that in U.S., in 2000, only 4% of firms served international markets. This trend is demonstrated by the table 1, which underlines the export data from a manufactory industry, one of the most predisposed to exporting.

Table 1: Exporting by US Manufacturing Firms in 2002

<table>
<thead>
<tr>
<th>NAICS industry</th>
<th>Percent of firms</th>
<th>Percent of firms that export</th>
<th>Mean exports as a percent of total shipments</th>
</tr>
</thead>
<tbody>
<tr>
<td>311 Food Manufacturing</td>
<td>6.8</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>312 Beverage and Tobacco Product</td>
<td>0.7</td>
<td>23</td>
<td>7</td>
</tr>
<tr>
<td>313 Textile Mills</td>
<td>1.0</td>
<td>25</td>
<td>13</td>
</tr>
<tr>
<td>314 Textile Product Mills</td>
<td>1.9</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>315 Apparel Manufacturing</td>
<td>3.2</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>316 Leather and Allied Product</td>
<td>0.4</td>
<td>24</td>
<td>13</td>
</tr>
<tr>
<td>321 Wood Product Manufacturing</td>
<td>5.5</td>
<td>8</td>
<td>19</td>
</tr>
<tr>
<td>322 Paper Manufacturing</td>
<td>1.4</td>
<td>24</td>
<td>9</td>
</tr>
<tr>
<td>323 Printing and Related Support</td>
<td>11.9</td>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>324 Petroleum and Coal Products</td>
<td>0.4</td>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>325 Chemical Manufacturing</td>
<td>3.1</td>
<td>36</td>
<td>14</td>
</tr>
<tr>
<td>326 Plastics and Rubber Products</td>
<td>4.4</td>
<td>28</td>
<td>10</td>
</tr>
<tr>
<td>327 Nonmetallic Mineral Product</td>
<td>4.0</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>331 Primary Metal Manufacturing</td>
<td>1.5</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>332 Fabricated Metal Product</td>
<td>19.9</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>333 Machinery Manufacturing</td>
<td>9.0</td>
<td>33</td>
<td>16</td>
</tr>
<tr>
<td>334 Computer and Electronic Product</td>
<td>4.5</td>
<td>38</td>
<td>21</td>
</tr>
<tr>
<td>335 Electrical Equipment, Appliance</td>
<td>1.7</td>
<td>38</td>
<td>13</td>
</tr>
<tr>
<td>336 Transportation Equipment</td>
<td>3.4</td>
<td>28</td>
<td>13</td>
</tr>
<tr>
<td>337 Furniture and Related Product</td>
<td>6.4</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>339 Miscellaneous Manufacturing</td>
<td>9.1</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td><strong>Aggregate manufacturing</strong></td>
<td><strong>100</strong></td>
<td><strong>18</strong></td>
<td><strong>14</strong></td>
</tr>
</tbody>
</table>
The table exhibits in the first column the sector, in the second the share of the industry in the whole market, the third the percentage of firms which export in the sector and in the fourth the average amount of good traded. First of all, the aggregate share of exporting firms is relatively small, around 18% and between industries the range widely varies. For example, electronic sector and computer faces a percentage around 40%, while the apparel manufacturing sector is characterized by a share of just 8% of exporters. These data seems to be consistent with the old theories of international trade: since United States are a skill-abundant states, they export in skill-intensive sectors, as computer and electronics, because they possess comparative advantage. However, the old theory is not able to explain while export occurs in all manufacturing industries. Furthermore, the amount of goods that firms export is relatively low, around 14%, even if, also in this case, there are differences across industries.

The second evidence that the data reveal is that exporters are different for many reason with respect to firms which operate exclusively in the domestic market.

To show that, the authors run the following regression where the explanatory variable consists in a dummy variable indicating whether firm is exporting or not, and the dependent variables are indicated in the first columns of table 2. The data are based on the same database as in the Table 1.

The table displays three regression, a normal OLS, another taking into consideration industry fixed effects such as firms characteristics across industries, and the final taking into consideration both fixed effects and firms’ size, computed using log of employment as proxy.
First of all, the data in the second column are lower than which in the first column; in any cases, the exporters are greater than domestic firms, both in terms of employees, 97% more, and in terms of shipment, 108% greater (second column). In addition, many studies demonstrated that exporters increase their employees and outcome faster than domestic firms, because of both initial high productivity and faster growth after commencing abroad. This mechanism underlines the key role of trade liberalization also for boost aggregate productivity and reallocation across firms.

Secondly, exporters are more productive; because both the workers are more efficient by 26%, and the total factor productivity is 3% higher. This higher productivity of exporters is also present before the entrance into international markets. Moreover, the presence of sunk costs leads many economists to think that only more productive firms can afford to enter in the international market, whereas there are little evidences that firms become more productive after compete in the international market. However, this may be not the case in low-income countries, such as Sub-Saharan African, where many studies reports an increasing productivity after openness to trade.  

Finally, exporters pay a salary that is 6% higher than domestic firms, and they are more capital intensive by 12%, and skill-intensive by 11%. These results are not completely consistent with old theories of comparative advantage, but are rational to the theory expressed by Alvarez and Lopez  

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**Table 2: Export Premia in U.S. Manufacturing**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log employment</td>
<td>1.19</td>
<td>0.97</td>
<td>0.08</td>
</tr>
<tr>
<td>Log shipments</td>
<td>1.48</td>
<td>1.08</td>
<td>0.06</td>
</tr>
<tr>
<td>Log value-added per worker</td>
<td>0.26</td>
<td>0.11</td>
<td>0.10</td>
</tr>
<tr>
<td>Log TFP</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Log wage</td>
<td>0.17</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Log capital per worker</td>
<td>0.32</td>
<td>0.12</td>
<td>0.04</td>
</tr>
<tr>
<td>Log skill per worker</td>
<td>0.19</td>
<td>0.11</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Additional covariates: None, Industry fixed effects, Industry fixed effects, log employment.

---

10 Van Biesbroeck (2005) showed a raise in productivity for manufactoring firms in Africa.
According to which exporters are usually more capital and skill intensive in both developed and developing countries.

As far as the source of trade is concerned, the empirical literature evidence that the welfare gains from the openness to international trade is due to the exit from the market of low-productive firms and the entry of high-production firms, a mechanism which increases the aggregate productivity. The main contribution to this theory, which adds a new source of gains different from the economies of scale and the increase in product variety available, predicted by the New Trade Theory, are Pavcnik (2002)\textsuperscript{12}, Trefler (2004)\textsuperscript{13}, and Bernard, Jensen, and Schott (2006a)\textsuperscript{14}. According to Pavcnik, after the trade liberalization in Chile in 1980s the aggregate productivity grew by 19%. Of these share, two-third can be attributed to reallocation of resources from low to high productive firms. In this framework, Trefler addresses the issues concerning the impact of trade liberalization, by stating that openness policies are in many case accompanied with other economic reforms which can themselves increase the aggregate productivity. He demonstrates that, in Canada, after the reduction of trade barriers, the industry productivity increased twice more than plant productivity. Finally, Bernard, Jensen, and Schott showed that when trade costs decreases, the lowest productive plants are more likely to exit from the market.

Ultimately, the abundance of empirical data has led to the development of a richer theoretical literature, in which the Melitz model (2003) plays a key role in underlining the importance of firm heterogeneity for the creation and enhancement of international trade, and for boosting the aggregate productivity. As explained above, the welfare gains after the reduction of barriers to trade is mainly due to the reduction of the productivity cutoff, which increases the number of firms which export and boosts the labor demand. This mechanism bids up the wages and reduces the profits of firms which uniquely operate domestically inducing them to exit from the market. As a result, a new source of gain, due to the reallocation of resources from low-productive to high-productive firms, prevails. Forasmuch as, the Melitz model is able to explain the empirical evidences according to which, after a reduction in trade costs, exporters increase their size and

employees more than non-exporters. In addition, this effect occurs in any industry (from the table 1 it can be noticed that in any industry some firms export), but is more evident in industries which possess a comparative advantage (as the case of U.S. computer and electronic sectors). Beyond shadow of doubt, trade liberalization affects also the distribution of income across factor: the increase in average productivity reduces the price of goods and increases the real income of all factors.

**New empirical challenges**

In this section I am going to analyze the Linked-Longitudinal Firms Trade Transaction Database (LFTTD) in order to understand the so called gravity model, in which distance between countries play a key role in the determination of trade flow. The LFTTD is a U.S. database; the authors used data from 1992 and 2000, and they investigate data on the products that firms export, both in term of value and quantity, the date of the shipment, the domestic and destination country, and the transportation mode use to export. Before briefly discussing the empirical evidence and the theory implications, it is compulsory to spend a few words about the fundamental difference between extensive and intensive margin. As far as the former is concerned, it is equal to the amount of goods which a firm exports and the number of the destinations; on the other side, internal margins regard the value that a firm trades per product per country.

First of all, the data reveal that international trade is concentrated between firms, both in terms of outcome and employment. In particular, data show that in 2000 the top 1% firms produced the 80% of the total trade value, whereas the 10% produced the 95% of the total trade value. Likewise, the top 1% had an employment share equal to 14%, while the top 10% had one equal to 24%. There are several theories with which these phenomena can be explained. In first place, this unequal distribution of trade can be due to the corresponding unequal distribution of productivity across firms. On second place, the firms may face a high elasticity of substitution between varieties, and thus, consumers easily shift from low-priced varieties to high-values ones after small changes in prices and quality. On third place, this phenomenon could be explained by the presence of sunk costs. In particular, sunk costs create economies of scale that foster the concentration of trade between few firms. Besides these, sunk costs may vary across countries and products market, allowing only more efficient producers to export to more destinations a wide range of products.
Secondly, data do not fit perfectly with the old and new theory. Both of them, in fact, predict trade flows higher than which revealed from the data. In fact, in presence of no costs of trade and identical and homogenous preference, the theories states that the trade flows should be equal to the share of the rest of the world in term of GDP. Although, the data affirm a significant lower level for international commerce. Moreover, if old theory can explain the absence of trade in term of high trade barriers or complete specialization, it is not able to develop a complete explanation regards the facts that some firms export and some only serve the domestic market. The table 3 helps us to understand the scarcity of international trade.

The table is split in 3 sub-tables containing data for exporting firms, exporting values and employment of exporting firms. The figures depend on the number of destination countries and the number of products delivered. The table shows that the main trade flow occurs for one country: 64% of firms export to only one destination, and most of them, roughly 42 %, deliver only one product. However, it is important to note that, as far as the export value is concerned, the one related to one country account for about 3%, while the one regarding more than 5 products account for 93%. Consequently, there are few big firms which export and ship many products in many destination. The authors state that the correlation effects between the number of product delivered and the number of destination countries is positive and equal to 0.81. Finally, It is important to note also the employment data: they reveal that firms which export in just one country have a share of 14% of the total employment, while the share increases up to 70% for firms operating in more than 5 countries. In that event, the conformation of firms, in term of employment, matters in the decision of exporting, and in particular in the number of destination markets.
Another empirical challenge regards the number of products exported. Specifically, the old theory considered the constant return of scale and perfect competition without considering firms producing more than one product. At the same time, new theory of international trade dismisses the market framework of perfect competition by assuming horizontal differentiation, but they still
assume that each firm produces a single variety of good. Although, the empirical research seems to demonstrate that when firms export, they decide to export more products. In the table 4 regards a regression using a dummy variable concerning the status of exporters as dependent variable and the number of products and the ratio between means shipments and number of products as explanatory variables, both in log form. The first column indicates the results for a bivariate ordinary least square regression, while the second includes fixed effects.

Table 4: the number of products in the international trade

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log number of products</td>
<td>0.23</td>
<td>0.27</td>
</tr>
<tr>
<td>Log mean shipments/# products</td>
<td>1.25</td>
<td>0.73</td>
</tr>
<tr>
<td>Additional covariates</td>
<td>None</td>
<td>Industry fixed effects</td>
</tr>
</tbody>
</table>

Sources: Data are for 1997 and are from the U.S. Census of Manufactures. Notes: All results are from bivariate ordinary least squares regressions of the firm characteristic in the first column on a dummy variable indicating firm export status. Regressions in column two include four-digit SIC industry fixed effects. The first dependent variable is the log of the number of five-digit SIC products produced by the firm in 1997. The second dependent variable is the log of total firm shipments divided by the number of products.

According to both regression, exporters produce more products and face a higher ratio between shipment and number of goods with respect to non-exporters: the values vary between 23% and 27% for the number of products and 125% and 73% for the ratio according to the inclusion or not of fixed effects.

**The gravity equation**

The gravity equation concerns the relationship between the trade flows and the incomes of the countries involved and their distance. This concept is one of the crucial concepts, mainly analyzed at empirical level, of international trade and has evolved over the time, starting to analyze wider variables that can influence the international trade and the relationship between more than two countries. Further analysis and explanation regarding the Gravity equation are in the following section.
This section aims to analyze the different impacts of external and internal margin, in the determination of gravity equation. Recalling that external margin regards the number of firms and number of products, whereas the intensive margin comprehends the value per product per firms, it can be demonstrated that the two margins have opposite effects on the gravity equation.

To analyze the gravity equation, a regression is run using the data from LFTTD in 2000. The explanatory variables are the GDP of the importers (since the exporters U.S. and remain constant) and the distance between the U.S. and the destination country, both values in log form. The regression is run fourth times, according to different dependent variables: the first case regards the aggregate value of exports, while for the other regression this value it has been split in number of firms, number of products and average value of exports per product per firms. All the dependent variables are considering in log form.

The results of the regression are showed in Table 5.

<table>
<thead>
<tr>
<th></th>
<th>Log of total exports value</th>
<th>Log of number of exporting firms</th>
<th>Log of number of exported products</th>
<th>Log of export value per product per firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of GDP</td>
<td>0.98</td>
<td>0.71</td>
<td>0.52</td>
<td>-0.25</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Log of distance</td>
<td>-1.36</td>
<td>-1.14</td>
<td>-1.06</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.16)</td>
<td>(0.15)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>Observations</td>
<td>175</td>
<td>175</td>
<td>175</td>
<td>175</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.82</td>
<td>0.74</td>
<td>0.64</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Source: Data are from the 2000 Linked-Longitudinal Firm Trade Transaction Database (LFTTD). Notes: Each column reports the results of a country-level ordinary least squares regression of the dependent variable noted at the top of each column on the covariates noted in the first column. Results for the constant are suppressed. Standard errors are noted below each coefficient. Products are defined as ten-digit Harmonized System categories. All results are statistically significant at the 1 percent level.

According to the table, the GDP of the destination country has a positive impact on the trade flow: when the GDP increases by 100% the export value increases approximately by 98%. At the same time, the distance has a negative effect on the international trade, approximately equal to -1.36. As far as the extensive margin is concerned, the GDP has positive impact on both the number of firm and the number of products, with coefficients respectively equal to 0.71 and 0.52. On the other size the distance impacts negatively the two variables of the extensive margin, with coefficients equal to -1.14 in the case of firms, and -1.06 in case of product. However, different
effects can be seen for the intensive margin. In particular, the GDP negatively affects the export value per firms per products (the coefficient is equal to -0.25), while the distance positively affects it (the coefficient is equal to 0.84). This data may be explained by the fact that the costs of exporting do not depend on the export value, but on the weight of shipment. Therefore, increase in distance and decline of GDP of destination countries may shift the export toward higher-value commodities, namely goods which have higher prices and weight less.

Import: empirical data

Over the past, data on the imports were not available. As consequences, model concerning the heterogeneity between firms did not consider firm import behavior and do not take into consideration imports of intermediate goods.

Today data on import are abundant; the table 6 considers import and export data of U.S. from LFTTD database. From the table 6, we can see some similarities between import and export data. First of all, imports are relatively rare, with a share around 14%, even if the value vary across industry, from a nadir of 3% for printing and related products to a peak of 43% of leather and allied products. According to the author or the article, the correlation between firms which export and imports is 0.87.

The same database is used to run a regression to study the difference between imports and exports. The regression is run using as explanatory variables data as employment, shipment, value-added per worker, total factor productivity, wage, capital per worker and skill per workers. All these variables are considered in log form. Dummy variables explaining the export, import and both export and import status are used as dependent variables. The results are showed in Table 7.
The first consideration regards the fact that importers, as well as exporters, are bigger, more productive, more skill and capital intensive and they pay higher salary with respect to firms which only operate in domestic market. As far as the firms which interact in the international market both exporting and importing they are bigger in term of employment and they pay higher salary.
As in the previous section, the impact of the gravity equation is analyzed according to the import data, and specially, the aggregate value is decomposed to study separately the extensive and intensive margin. The results are showed in table 8. The regressions used are the same as in the table 5, but the GDP of source country is used instead of the one of destination country.

Table 7: Difference between Exporters and Importers in Manufacturing sector in 2007

<table>
<thead>
<tr>
<th></th>
<th>(1) Exporter premia</th>
<th>(2) Importer premia</th>
<th>(3) Exporter &amp; importer premia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log employment</td>
<td>1.50</td>
<td>1.40</td>
<td>1.75</td>
</tr>
<tr>
<td>Log shipments</td>
<td>0.29</td>
<td>0.26</td>
<td>0.31</td>
</tr>
<tr>
<td>Log value-added per worker</td>
<td>0.25</td>
<td>0.23</td>
<td>0.25</td>
</tr>
<tr>
<td>Log TFP</td>
<td>0.07</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>Log wage</td>
<td>0.29</td>
<td>0.23</td>
<td>0.33</td>
</tr>
<tr>
<td>Log capital per worker</td>
<td>0.17</td>
<td>0.13</td>
<td>0.20</td>
</tr>
<tr>
<td>Log skill per worker</td>
<td>0.04</td>
<td>0.06</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Sources: Data are for 1997 and are for firms that appear in both the U.S. Census of Manufacturers and the Linked-Longitudinal Firm Trade Transaction Database (LFTTD).
Notes: All results are from bivariate ordinary least squares regressions of the firm characteristic listed on the left on a dummy variable noted at the top of each column as well as industry fixed effects and firm employment as additional controls. Employment regressions omit firm employment as a covariate. Total factor productivity (TFP) is computed as in Caves, Christensen, and Diewert (1982).

Table 8: The gravity equation and the extensive and intensive margin for importers

<table>
<thead>
<tr>
<th></th>
<th>Log of total import value</th>
<th>Log of number of importing firms</th>
<th>Log of number of imported products</th>
<th>Log of import value per product per firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of GDP</td>
<td>1.14*** (0.06)</td>
<td>0.82*** (0.03)</td>
<td>0.71*** (0.03)</td>
<td>−0.39*** (0.05)</td>
</tr>
<tr>
<td>Log of Distance</td>
<td>−0.73*** (0.27)</td>
<td>−0.43*** (0.15)</td>
<td>−0.61*** (0.15)</td>
<td>0.31 (0.24)</td>
</tr>
<tr>
<td>Observations</td>
<td>175</td>
<td>175</td>
<td>175</td>
<td>175</td>
</tr>
<tr>
<td>R²</td>
<td>0.59</td>
<td>0.78</td>
<td>0.74</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Sources: Data are from the 2000 Linked-Longitudinal Firm Trade Transaction Database (LFTTD).
Notes: Each column reports the results of a country-level ordinary least squares regression of the dependent variable noted at the top of each column on the covariates listed on the left. Results for constants are suppressed. Standard errors are noted below each coefficient. Products are defined as ten-digit Harmonized System categories.
*, **, and *** represent statistical significance at the 10, 5, and 1 percent levels, respectively.

First of all, the GDP of the domestic country has a positive impact on the trade flow: when the GDP increases by 100% the import value increase approximately by 114%. At the same time, the distance has a negative effect on the international trade, approximately equal to -0.73. Therefore, the GDP changes have greater effects on import than exports (the same estimation for export was
equal to 98%), while the distance have greater effect on export (the corresponding coefficient was -136%). As far as the extensive margin is concerned, the GDP has positive impact on both the number of firm and the number of products, with coefficients respectively equal to 0.82 and 0.43. On the other hand, the distance negatively affects the two variables of the extensive margin, with coefficients equal to -0.43 in the case of firms, and -0.61 in case of products. As for the exports, different effects can be noticed for the intensive margin; in particular, the GDP negatively affects the export value per firms per products (the coefficient is equal to -0.39), while the effects of distance is positive, but it is not significant.

To conclude, the whole analysis underlines the fundamental role of firms and products in the international trade, that are always been underestimated by the old and new theories. Even if the recent abundance of the data have allowed the development of new and more developed explanations, many evidences have to been explained further. For instance, there are no models explaining the decision of firms about what products export or the destination countries.
Gravity equations: Workhorse, Toolkit, and Cookbook

The gravity equation was firstly introduced in 1962 by Tinbergen in an article titled “Shaping the World Economy: Suggestion for an International Economic Policy” and it has been always considered by the economist in the same manner as a Newtonian physic law able to solve some empirical evidence. However, Keith Haed and Thierry Mayer, in their article Gravity equations: Workhorse, Toolkit, and Cookbook try to demonstrate as gravity equation can be estimated, and how it can be consider a powerful instruments for calculate welfare changes occurring after changing in trade policy.

Only after 1995, the gravity equation was considered as fundamental part in the international trade. In first place, economists realize that the gravity equation may be used to measure and analyze the huge amount of trades that the previous theories were not able to explain. In second place, it has been discovered, thanks to the contributions of Eaton and Kortum (2002)\(^\text{15}\) and Anderson and Van Wincoop (2003)\(^\text{16}\) how the gravity equation also had many micro-foundations. Moreover, it became clear that the multilateral resistance terms, used in different economic models, could be explained using the importers and exporters fixed effects.

In third place, gravity equation was considered in the light of the new paradigm of heterogeneous firm literature (Melitz (2003) and Melitz-Ottaviano (2008)). In particular, it has been considered as appropriate instruments to measure the intensive and extensive margin in the adjustment occurring after trade shocks. In this period, the gravity equation was assimilated to the theory to measure the welfare gains from trade.

Gravity equation: the empirical framework


As mentioned before, the gravity equation finds its basis in the empirical framework. By analyzing the data, the first regularity rises in the fact that the economic size of the destination country positively affects the exports. At the same time, the size of the origin country positively affects the imports amount.

In the figure 8, the authors show the correlation between countries’ size and trade flow between Europe and Japan. The two destinations have been chosen because the distance between the places is large enough to neglect the difference in distance between Japan and any other country in Europe. Moreover, the two locations have different language, religion, currency and historical background. The GDP of the country is used as proxy for economic size and the Greece GDP is normalized to one (all other GDP are divided by Greek’s one). The figure (8a) represents the export toward Japan, while the figure (8b) the imports from Japan. In the vertical axis the amount of export is represented (divided by the Greece exports), while in horizontal axis the size of the different European countries is displayed.

The figures show the presence of a relationship between market size and international trade: the line represents a simple regression using log trade flow as function of log GDP.

The second regularity which appears from the data represent the negative relationship between physical distance and trade. In this case the authors consider the trade between France and the other countries in the Europe. Figure 9 shows in the vertical axis the ratio between exports or
imports and country GDP, and in the horizontal axis the distance measured in km. The negative relationship is quite evident, even if there are many deviations from the benchmark.

**Figure 9: negative relationship between trade and distance**

Gravity equation: Micro foundations

When the economic literature started to consider the gravity equation both in a theoretical and empirical framework, many definitions of its have been used. Three main definitions will be explained below.

The first definition, known as “General Gravity Equation” regards a set of models summarized the bilateral trade equations that can be expressed as multiplications of factors and

\[ X_{ni} = G S_i M_n \phi_{ni} \]  

(Def. 1)

Where \( G \) is a constant, known as “gravitational constant”, \( S_i \) summarizes the capabilities of exporter \( i \), \( M_n \) include all characteristics of the destination market \( n \), while \( \phi_{ni} \) comprehends trade costs between the two countries and their elasticity and is between 0 and 1. Moreover this term may comprehend the third-country effects.

The second definition, known as “Structural Gravity Equation” can be expressed
\[ X_{ni} = \frac{y_i x_n}{\Omega_i \Phi_n} \phi_{ni} \] (def. 2)

Where \( \frac{y_i}{\Omega_i} \) can be considered \( S_i \), while \( \frac{x_n}{\Phi_n} \) can be included in \( M_n \). Moreover, \( Y_i \) represents the value of production, whereas \( X_n \) is the value of the expenditure of the importers in all sources countries.

With respect to definition 1, definition 2 lacks of the constant gravity but contains the multilateral resistance coefficients, \( \Omega_i \) and \( \Phi_n \), that can be solved for a given set of trade costs, and therefore they allow more complicate analysis of the impacts of the change in trade costs.

The structural gravity equation depends on two main assumptions. The first assumption regards the division of expenditure between exporters. In particular, if we think about the total expenditure of the importer \( n \), \( X_n \) should be divided by all destination countries. The share allocated to each country \( i \) is called \( \pi_{ni} \), such that \( X_{ni} = \pi_{ni} X_n \).

The structural gravity equation requires that the budget share is independent of income and

\[ \pi_{ni} = \frac{S_i \phi_{ni}}{\Phi_n} \]

Where \( \Phi_n \) represents the opportunities of consumers in \( n \), or, in other words, the degree of competition in the market. This assumption excludes many economic models, such as the one based on quasi-linear demand system. As far as the Melitz-Ottaviano model (2008), it does not fit in the structural gravity equation, since it does not rely on the above mentioned assumption, but can be included in the models following the first definition.

The second assumption states that the production of \( i \) should be equal to the sum of shipments to all countries; therefore

\[ Y_i = \sum_n X_{ni} = S_i \sum_n \frac{\phi_{ni} X_{ni}}{\Phi_n} \]

Many of established theories are included in this definition of gravity equation. In particular all these models consider trade costs, that in turn comprehend transportation costs, costs for travelling for services linked or not to the goods, costs of research and other transaction costs, in the iceberg form, namely they consider \( \tau_{ni} \) the share of GDP of \( n \) allocated to export from \( i \).

Between these theories lies the Krugman model (1979), based on CES preferences.
Finally, the third definition, known as “Naïve gravity equation”, expresses the bilateral trade as proportion of the product of the country size. Analytically,

\[ X_{ni} = G Y_i^a Y_n^b \phi_{ni} \]  

(def. 3)

While \( Y_i^a \) and \( Y_n^b \) are respectively the value of the production in domestic and destination countries and \( a \) and \( b \) are defined as \( a \neq b \neq 1 \). In addition, the equation imposes that \( \phi_{ni} \) is constant, which seems as an implausible restriction.

### Gravity equation: Theory-consistent estimation

After considering the different definitions used to describe gravity equation and their application in the economic models, this section considers the estimation methods used to estimate the gravity equation and, in particular, the multilateral resistance term.

At the beginning, the econometricians were used to consider the remoteness between countries a good proxy for the multilateral resistance term. This approach was easily dismissed in favor of more appropriate and structural approach. Anderson and Van Wincoop (2003)\(^{17}\) used a non-linear least square methods and fixed effects estimation, that can be considered more appropriate, but difficult to assess in the case, high frequent in practice, of large datasets.

The estimation process involves the following regression:

\[ \ln X_{ni} = \ln G + \ln S_i + \ln M_n + \ln \phi_{ni} \]

Where GDPs are used as proxy for \( S_i \) and \( M_i \). However, these two variables are in practice different across countries, due to differences on comparative advantages and consumers’ preferences. Therefore, fixed effects estimations seem more plausible.

Another estimation method regards the Monte Carlo simulation, run, in the paper “Gravity Equation: Workhorse, Toolkit, and Cookbook” considering data from 170 countries (regards GDP, distance and existence of Regional Trade Agreement), to calculate the multilateral resistance terms.

\(^{17}\) See footnote 16
In the following table different estimation methods have been considered. In particular, it shows the mean of 1000 repetitions, the average standard errors in parenthesis and the standard deviation in square brackets.

<table>
<thead>
<tr>
<th>Table 9: Different Estimator for the Gravity Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Censoring Estimates</strong></td>
</tr>
<tr>
<td></td>
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<td></td>
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<tr>
<td>OLS</td>
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<td></td>
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<tr>
<td>SILS</td>
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<tr>
<td>LSDV</td>
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<tr>
<td>DDM</td>
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<td>BVU</td>
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<tr>
<td>TETRA</td>
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</tbody>
</table>

Notes: Top value in each cell is the mean estimate (based on 1000 repetitions). The true parameters are -1 for distance and 5 for RTA. Average standard error is "( )" and standard deviation of estimate in "[ ]". Table 2 defines the estimators.

The table shows that the simple OLS regression is a poor estimation, since it lacks to consider export and import dummies; in addition, the data are biased towards zero. Secondly, the Anderson and Van Wincoop methods, named SILS in the table, is a good and unbiased estimator only in case of no missing data. However, by including fixed effects, LSDV methods, the process gives more precisely values also in presence of missing data. However, all these estimations do not take into consideration heteroskedastic errors and structural zeros, occurring in the case of no trade between countries.

**Gravity equation: Policy Impacts**
One of the main goals behind the estimation of the gravity equation was the analysis of the impact of various policies and variables different from distance on trade. The authors of the article “Gravity equations: Workhorse, Toolkit, and Cookbook” collected sample of estimation after 2005, published in *Journal of International Economics* and *Reviews of Economics and Statistics*, summarized in the Table 10.

**Table 10: Estimates of typical gravity variables**

<table>
<thead>
<tr>
<th></th>
<th>All Gravity</th>
<th></th>
<th></th>
<th>Structural Gravity</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>median</td>
<td>mean</td>
<td>s.d.</td>
<td>#</td>
<td>median</td>
<td>mean</td>
</tr>
<tr>
<td>Origin GDP</td>
<td>.97</td>
<td>.98</td>
<td>.42</td>
<td>700</td>
<td>.86</td>
<td>.74</td>
</tr>
<tr>
<td>Destination GDP</td>
<td>.85</td>
<td>.84</td>
<td>.28</td>
<td>671</td>
<td>.67</td>
<td>.58</td>
</tr>
<tr>
<td>Distance</td>
<td>-.89</td>
<td>-.93</td>
<td>.4</td>
<td>1835</td>
<td>-1.14</td>
<td>-1.1</td>
</tr>
<tr>
<td>Contiguity</td>
<td>.49</td>
<td>.53</td>
<td>.57</td>
<td>1066</td>
<td>.52</td>
<td>.66</td>
</tr>
<tr>
<td>Common language</td>
<td>.49</td>
<td>.54</td>
<td>.44</td>
<td>680</td>
<td>.33</td>
<td>.39</td>
</tr>
<tr>
<td>Colonial link</td>
<td>.91</td>
<td>.92</td>
<td>.61</td>
<td>147</td>
<td>.84</td>
<td>.75</td>
</tr>
<tr>
<td>RTA/FTA</td>
<td>.47</td>
<td>.59</td>
<td>.5</td>
<td>257</td>
<td>.28</td>
<td>.36</td>
</tr>
<tr>
<td>EU</td>
<td>.23</td>
<td>.14</td>
<td>.56</td>
<td>329</td>
<td>.19</td>
<td>.16</td>
</tr>
<tr>
<td>CUSA/NAFTA</td>
<td>.39</td>
<td>.43</td>
<td>.67</td>
<td>94</td>
<td>.53</td>
<td>.76</td>
</tr>
<tr>
<td>Common currency</td>
<td>.87</td>
<td>.79</td>
<td>.48</td>
<td>104</td>
<td>.98</td>
<td>.86</td>
</tr>
<tr>
<td>Home</td>
<td>1.93</td>
<td>1.96</td>
<td>1.28</td>
<td>279</td>
<td>1.55</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Notes: The number of estimates is 2508, obtained from 159 papers. Structural gravity refers here to some use of country fixed effects or ratio-type method.

The table displays the summary statistic considering all paper in the first column and only the structural gravity paper, namely the paper which model could be considered included in the second definition, in the second column.

As far as the origin GDP elasticity, this is equal to 0.98, and therefore similar to the prediction showed in the Figure 8. The destination GDP elasticity is equal to 0.84, meaning that when the destination GDP grows by 100%, the trade flow increases by 84%.

The average distance elasticity is equal to -0.93, therefore, the distance between countries negatively affects the trade. The congruity and the common language elasticities are both around 0.5, while the effect of colonial link is estimated equal to 0.92. As far as the effects on common currency, the table shows elasticity equal to 0.79; although, this value highly varies in the
literature. In fact, Rose (2000)\textsuperscript{18} estimates a value of 1.21, which means that in presence of common currency the trade flow should increase by 121%. However, at the same time Santos Silva and Tenreyro (2006)\textsuperscript{19} estimated the elasticity of trade with respect to common currency in Euro Zone; considering the high level of integration between Euro countries, they found no effects of common currency on trade. It is important to underline that table 10 does not consider the endogeneity issues regards the fact that when countries sign a trade agreement, as RTAs, they are already highly integrated.

The second goal of the estimation of gravity equation lies on its ability to estimate the elasticity of trade with respect to trade costs. In particular, Alkolakis and al (2012b)\textsuperscript{20} shows that the welfare gains from trade can be calculate using the import ratio, an observable variable and the trade cost elasticity. The authors of the article “Gravity equations: Workhorse, Toolkit, and Cookbook” collect sample from 32 papers, to estimate trade elasticity according to the regression:

$$\ln X_{ni} = \ln S_i + \ln M_n + \epsilon \ln \tau_{ni}$$

The results are showed in table 11.

The results clearly indicate a high variance in the estimation of elasticity; the standard deviation is more than twice as large as the mean. Although, this high variance may due to the fact that the elasticity has been estimated at industry level; furthermore, in many cases the variance depends on the estimation methods: structural gravity equation yield higher standard deviation with respect to naïve gravity equation.

To conclude the paper “Gravity equations: Workhorse, Toolkit, and Cookbook” summarizes the different definition of the gravity equation in the literature and the several methods used to estimate it. Moreover, the authors show many uses of gravity equations in the assessment of the impact of the international trade.

Table 11: Price elasticity in gravity equation

<table>
<thead>
<tr>
<th>Estimates:</th>
<th>median</th>
<th>mean</th>
<th>s.d.</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>-3.19</td>
<td>-4.51</td>
<td>8.93</td>
<td>744</td>
</tr>
<tr>
<td>Naive gravity</td>
<td>-1.31</td>
<td>-1.35</td>
<td>5.17</td>
<td>122</td>
</tr>
<tr>
<td>Structural gravity</td>
<td>-3.78</td>
<td>-5.13</td>
<td>9.37</td>
<td>622</td>
</tr>
</tbody>
</table>

Split structural estimates by:

<table>
<thead>
<tr>
<th>Estimation method:</th>
<th>median</th>
<th>mean</th>
<th>s.d.</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country FE s</td>
<td>-3.5</td>
<td>-4.12</td>
<td>8.2</td>
<td>447</td>
</tr>
<tr>
<td>Ratios</td>
<td>-4.82</td>
<td>-7.7</td>
<td>11.49</td>
<td>175</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Identifying variable:</th>
<th>median</th>
<th>mean</th>
<th>s.d.</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tariffs/Freight rates</td>
<td>-5.03</td>
<td>-6.74</td>
<td>9.3</td>
<td>435</td>
</tr>
<tr>
<td>Price/Wage/Exchange rate</td>
<td>-1.12</td>
<td>-1.38</td>
<td>8.46</td>
<td>187</td>
</tr>
</tbody>
</table>

Notes: The number of statistically significant estimates is 744, obtained from 32 papers.
The empirics of firm heterogeneity and international trade

The authors A.B.Bernard, J.B. Jensen, S.J.Redding, and P.K. Schott wrote, in 2011, an article entitled “The empirics of firm heterogeneity and international trade”, in order to explain how the empirical evidences have helped to the creation and development of new theoretical models and how these empirical results have rose several questions that have been previously underestimated in the international trade literature. More in details, one key role is playing by the gravity equation, for which the mere analysis of number of firms or products exported have been overcame by the studies of the intensive margin, namely the value that a firm trade per product per country.

Gravity equation and the Extensive and Intensive Margins

In the last year, the gravity equation has started to play a fundamental role in both the empirical and theoretical analysis. In the section “Gravity equation: workhorse, toolkit, and cookbook” we analyzed the different definitions of it in the theoretical models, and the methods to estimate it. In this section, however, I am going to further explain the important role of gravity equation with respect to the extensive and intensive margin.

The gravity equation relates the value of the total flow traded international with respect to the economics size of the countries, often approximated with GDP, and the distance, that can be though as physical distance, but in wider vision, as the trade costs between countries. In the Melitz model, the gravity equation states that the aggregate bilateral trade can be split into extensive and intensive margin. Analytically, considering countries $i$ and $j$ the trade flow between countries is

$$X_{ij} = M_{ij} \frac{X_{ij}}{M_{ij}}$$
where $M_{ij}$ is the number of firms that export from $i$ to $j$, and thus the extensive margin, whereas the $X_{ij}/M_{ij}$ represents the average firm exports, and therefore the intensive margin.

According to the Melitz model (2003), changes in the trade costs affect the extensive margin, while the intensive margin is independent from the trade costs. In fact, rising of trade barriers has two opposite effects on intensive margin: on one side, higher variable trade costs decrease the average firms’ exports and, on the other side, higher costs lead less productive firms to exit the market, and therefore they lower $M_{ij}$. If the productivity distribution is assumed to be a Pareto distribution, as in Melitz (2003) these two effects offset each other.

In addition, the Melitz model implies that extensive margin is proportional to market size: the number of firms in larger market is greater, because even less productive firms may obtain sufficient variable profits to cover the initial fixed costs, and therefore, find profitable stay in the market. The empirical data seem to confirm this thesis. Specially, analyzing the French market, it has been found that there is a log linear relationship between the number of firms and the market size$^{21}$.

**Multi-products firms analysis**

As mentioned in the section “Firms and International trade”, international trade activity is concentrated around a small number of firms. Moreover, larger exporters export more and more products, with respect to small exporters, that usually trade only one product.

To give a theoretical explanation to these empirical evidences, some models dealing with more than one product and more than one destination country have been developed. A great contribution in this sense can be found in Bernard et al. (2011)$^{22}$, where Melitz model (2003) has been used to develop a more complicate framework for multi-products and multi-destinations models.

The model assumes that

- To enter in the market the firms have to pay a sunk cost.

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The firms face ex-ante uncertainty about their productivity function that will be revealed after they pay the sunk cost. The productivity is the same across products.

Firms can choose to produce one or more goods over a continuum set.

Firms can decide in what destinations they want to export.

Firms face fixed costs to serve each market and supply each product.

In this framework, only the more productive firms obtain profits that allow them to cover the fixed costs to export many products in different destination. At the same time, the intermediate-ability firms export in one or few markets, one or few products; the low productive firms exit.

This model identifies a further source for gains from trade. At the same way trade liberalization shifts the reallocation of resource from low-productivity firms to high productivity ones, (Melitz, 2003), in this model, trade liberalization induces firms to drop products in what they have lower productivity. Therefore the reallocation of resource is towards the more productive firms, but also the more productive products.

The relationship between extensive margin and the gravity equation is slightly different with respect to the standard one. In particular, the extensive margin is split into extensive margin or firms and products where firms export, indexed by $O_{ij}$, and the intensive margin conditional on positive trade, equal to

$$\bar{x}_{ij} = X_{ij}/O_{ij}$$

At the same time, we can define the density factor as

$$D_{ij} = \frac{O_{ij}}{M_{ij}N_{ij}}$$

where $N_{ij}$ represents the number of exported products. Therefore, the number of firm-products observation with positive trade is equal to the extensive margin multiply by the number of exporting products and the density factor.

$$O_{ij} = M_{ij}N_{ij}D_{ij}$$

All these variables are used in the regression displayed in the following table.
Table 12: Gravity equation and Margins of Trade

<table>
<thead>
<tr>
<th></th>
<th>ln(Value)</th>
<th>ln(Avg Exports)</th>
<th>ln(Obs)</th>
<th>ln(Firms)</th>
<th>ln(Products)</th>
<th>ln(Density)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(Distance)</td>
<td>-1.37</td>
<td>0.05</td>
<td>-1.43</td>
<td>-1.17</td>
<td>-1.1</td>
<td>0.84</td>
</tr>
<tr>
<td>ln(GDP)</td>
<td>1.01</td>
<td>0.23</td>
<td>0.78</td>
<td>0.71</td>
<td>0.55</td>
<td>-0.48</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Constant</td>
<td>7.82</td>
<td>6.03</td>
<td>1.8</td>
<td>0.52</td>
<td>3.48</td>
<td>-2.2</td>
</tr>
<tr>
<td></td>
<td>1.83</td>
<td>1.07</td>
<td>1.81</td>
<td>1.59</td>
<td>1.55</td>
<td>1.37</td>
</tr>
<tr>
<td>Observations</td>
<td>175</td>
<td>175</td>
<td>175</td>
<td>175</td>
<td>175</td>
<td>175</td>
</tr>
<tr>
<td>Fixed Effects</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.82</td>
<td>0.37</td>
<td>0.75</td>
<td>0.76</td>
<td>0.68</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Source: Bernard et al. (2011).

Notes: Table reports results of OLS regressions of U.S. export value or its components on trading-partners' GDP and great-circle distance (in kilometers) from the United States. All five columns are country-level regressions. Heteroscedasticity robust standard errors are noted below each coefficient. Data are for 2002.

The table 12 represents a regression run using U.S. data in 2002. The independent variables, as usual in equation gravity, are distance and GDP, both taken in log form. As far as the dependent variables are concerned, many regressions are run, using each one different of the following variables: the aggregate bilateral trade in the first column, the internal margin in the second, the variable $O_{ij}$ representing the intensive margin conditional to export in the third column, the number of firms in fourth column, and the number of products in the fifth and finally the density factor. All the dependent variables are taken in their log form.

The distance affects negatively the aggregate bilateral trade and the variable $O_{ij}$, with coefficient equal to -1.37 and -1.43 respectively. The effects on the internal margin seem to be positive, but it does not demonstrate statistical significance. Moreover, both values characterizing the extensive margin have negative coefficients: for the number of firms the value is -1.17, while for the number of products the value is -1.1. However, the more interesting datum regards the coefficient of the density, which is positive and equal to 0.84. This means that when the distance increases by 100% the density increases by 84% because the number of firms-products observations $O_{ij}$ increasing less than proportionally than the ration between number of firms and products. In other words, when trade costs decrease, a greater number of firms find profitable to export in more countries and more products: therefore, the decline in trade’s barriers increases export for a given firms and
products, but at the same time increases the number of firms that export less products in less destination.

Firms importing

The recent abundance of import data has allowed a deeper focus on the behavior of importers. Empirical evidences reveal that importers are bigger; they have a greater productivity, are more skill and capital intensive and pay higher salary with respect to firms that not export. However, there are few studies about the impact of trade liberalization in the import market for intermediate goods.

The contribution of Amiti and Konings (2007)\textsuperscript{23} helps to understand this relationship, by studying the manufacturing firms in Indonesia. In particular, they proved that the reduction of costs related to importation, increases the productivity of firms that use imported inputs by a factor twice as grater as the one occurring in case of reduction of exporting costs.

There are many factors that may affect the production of firms through the importation of intermediate goods. Specially, boosting importation of intermediate goods may lead to improvement of technologies by learning from foreign, as well as by increasing the varieties available. Moreover, the intermediate goods imported might be higher in quality, with respect to the domestic one.

According to Goldberg at al. (2010)\textsuperscript{24} after a reduction of import tariffs, the firms experience a greater increase in the outputs produced and in the amount of variety, and a decline of prices.

Product Quality

Empirical evidences show that exporters that earn more profits, use in their production function higher quality inputs to produce higher quality goods that they sold at higher prices. In other words, countries with more capital and skill endowment trade higher quality varieties. The higher quality is reflected in the market by higher prices. However, the presence of high prices raises the


question whether they are due to higher quality or lower productivity, and thus higher marginal costs.

Many empirical contributions seem to establish that higher prices reflect higher qualities. Kugler and Verhoogen (2011)\textsuperscript{25} exposed, by analyzing on Columbia manufacturing industry, that larger firms that export, employ more expensive inputs to produce products that they sells at higher prices, with respect to smaller firm that do not export. In addition, Baldwin and Harrigan (2011)\textsuperscript{26} and Johnson (2010)\textsuperscript{27} contribute to identify when the firms charge higher prices due to lower productivity (and therefore higher marginal cost), or they charge higher prices because they trade higher products goods. Firms charge higher prices in both cases, but the higher quality is the only channel that affects the firms’ selection in the export market.

**Intermediaries**

The abundance of data recently available on import has raised the question about the role of intermediaries in the international trade. In fact, almost all models of international trade assume that foreign firms trade directly with consumers; in the reality, although, consumers purchase imported goods from retailers. Bernard et al. (2010c)\textsuperscript{28} used the LFTTD database to study the impact of retails and wholesalers in the international trade. The data show that few large firms own most of the international flows, and they are vertically integrated. More in details, the wholesales that export, account for 35% of the total number of wholesales, but only the 8% of the export value. At the same time, wholesales that import account for 42%, but only the 15% of the import flows. Moreover retails are usually smaller that wholesales: the retails that export represents 9% of total exporters and 13% of the total importers, although they account only for 1% of the total value of exports and imports.

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Multinational firms

The globalization and the more and more important role of multinational firms have raised the question whether the firms play a role in the determination of the international trade. Helpman et al. (2004)\textsuperscript{29} developed an economic framework, based on the Melitz (2003) analysis, where multinational firms can decide whether export or invest in the country by establishing an overseas affiliate. Similarly to decision to export, the author indicates that when the costs for foreign direct investments are high, only the most efficient firms decide to open affiliates, while the medium productive firms only export, and the least efficient firms trade domestically. In this framework, when the costs for direct investment decrease (due to more standardize procedure or more flexible regulations), a greater number of firms, including less productive and smaller firms, invest in overseas affiliates.

Although these models are exhaustive in explaining the theory of foreign direct investment in relation to heterogeneous firms theory, further analysis are necessary. In particular, data show that the number of firms investing overseas is not consistent with the economic theory. Larger and more productive firms invest in a number of locations lower than which predicted, while small and less efficient companies invest in a number of locations greater than which the economic theory predicts.

Conclusion

The article “The empirics of firm heterogeneity and international trade” explains empirical evidences about the differences between firms that export or not. These evidences seem to be coherent only with the new theoretical framework characterized by heterogeneous firms.

Moreover, additional findings about the mechanism through which the openness to international trade affects the aggregate economy, demonstrate how further empirical and theoretical research are necessary to clearly understand the effects of international trade.

The main result of this chapter regards the relationship between the gravity equation and the margins of trade. In particular, the gravity equation is affected by the extensive margin of firms

\textsuperscript{29} Helpman E., Melitz M.J., and Yeaple S.R., 2004, “Export Versus FDI with Heterogenous Firms”. American Economic Review. 94(1): 300-16
and product and is less affected by the intensive margin. Moreover, reduction in trade costs provokes in the internal firms organization and on its market decisions. These decisions regards the markets of destinations, the countries were allocated the production
The Addilog Theory of Trade

Since now, the models analyzed were based on CES preferences. In fact, apart from Melitz-Ottaviano, all others models were based on Dixit-Stiglitz model, where “direct additivity” was assumed. The assumption of “direct additivity” considers consumers’ preferences that can be represented by an additively separable utility function. A particular case in this set of preference regards the well known CES preferences, where, in equilibrium, the number of consumers (and therefore the market size) and the individual income do not affect the prices and the outcome.

In the paper “The addilog Theory of Trade” P. Bertoletti, F. Etro, and I. Simonovska developed a theoretical framework that can be defined as dual as on used in the previous literature. In particular, they assume preferences that can be represented by an additively separable indirect utility function. The “indirect Additivity” assumption implies one important result: the relative demand of two goods does not depend on the prices of the other goods available in the market. It is important to note that the case of CES preferences is included in both “direct additive” and “indirect additive” cases. Furthermore, the term “Addilog” is used in honor of Houthakker (1960): the model used the additive functional form introduced firstly by the economist.

The model developed in “The addilog Theory of Trade”, from now on Bertoletti-Etro-Simonovska model, integrate the “Indirect Additive Addilog” preferences to a monopolistic competition framework to establish a general model, which can include the CES preferences investigated in Krugman (1980), Melitz (2003) and Chaney (2008) models. As in Melitz model, the firms face heterogeneity productivity according to a Pareto distribution function; however, the results are slightly different from the previous model. The mark-up, that was considered as constant in Krugman, depends on destination per capita income, but not on the size of the destination country. In addition, the gains from welfare can be bestowed uniquely to the increase of number of varieties available, and these gains are lower than what estimated by models that assume direct-additive and homothetic preferences.

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The Model

The model was promoted to clarify some empirical evidences. First of all, from literature it can be inferred that welfare gains from increase in the varieties exported have been quite significant. According to Broda and Weinstein (2004)\textsuperscript{31}, the number of imported varieties in U.S. has tripled during the period between 1972 and 2001. In addition, between the new varieties, half of them have been sold by existing exporters. In line with this article, Kehoe and Ruhl (2013)\textsuperscript{32} discovered that, after trade liberalization policies in NAFTA countries, the extensive margin of trade account for almost 10% of the growth in trade. Although both articles aim to explain how consumers receive welfare gains when the number of varieties in the market increases, there is not a satisfactory measure of such welfare gains. According to the Bertoletti-Etro-Simonovska model, the rise in welfare is entirely due to the raise in number of varieties available. Secondly, the empirical evidences suggest that exporters charge higher price according to higher per capita income in destination countries, but they do not apply different prices to markets with different sizes.

As mentioned above, the model employs additively separable indirect utility as following

\[
V = \int_{\Omega} v \left( \frac{p(w)}{E} \right) dw
\]

with \( v(s) = \frac{(a - s)^{1+\gamma}}{1 + \gamma} \)

The model uses similar notation with respect to the previous models. The goods are of different varieties represented \( \omega \) over a continuum set \( \Omega \). \( s(w) \) represents the normal price as \( s(w) = p(w)/E \), where \( E \) is the income. As usual \( a \) represents the willing to pay of consumers for each variety, while \( \gamma \) is a parameter that represents the elasticity of demand. In particular, when \( \gamma \to \infty \), the demand is perfectly elastic, when \( \gamma = 1 \) the demand is linear and it is perfectly rigid when \( \gamma \to 0 \).

The article is working in progress and in this section I am going to inspect only one part of the paper. In particular, I am going to analyze a simple version of the model, where the preferences are expressed as quadratic version of indirect additive preferences, and therefore,

\[ V = \int_{\Omega} \frac{(a - s(w))^2}{2} \, dw \]

The main assumptions of the model can be summarized as the following:

- The market size is indexed as \( L \), that indicates the number of consumers.
- \( e > 0 \) represents the exogenous individual endowment. As consequences, the per capita income can be written as \( E = we \).
- To enter in the industry the firms have to pay a fixed entry cost \( F \); as consequences, the productivity function is represented by increasing returns of scale.
- The marginal costs are identically drawn from a Pareto distribution \( G(c) = \left( \frac{c}{E} \right)^k \) where \( k > 1 \).
- The number of active firms is represented by \( N \), while the number of active firms is \( n \).

The individual demand can be derived by the indirect utility function through the Roy’s identity\(^{33}\):

\[
x(w) = \frac{a - s(w)}{|\mu|}
\]

where \( |\mu| \) represents the price aggregator and can be written as

\[
\mu = -\int_{\Omega} (a - s(w))s(w) \, dw
\]

Moreover, the direct demand (namely the amount of goods as function of the price), has elasticity equal to \( \theta(s) = s/(a - s) \). In other words, the elasticity of demand depends on the price income ratio \( s \), which has a positive effect on it. Although, it is independent on all other prices.

The profit can be written as

\[
\pi = (p - MC)x(p)L = \frac{(p - cw) \left( a - \frac{p}{E} \right) L}{|\mu|}
\]

\(^{33}\) Roy’s identity allow to derive the demand of good with respect to price and income from the indirect utility function.

\[
x_i(p, E) = -\frac{\partial V/\partial p_i}{\partial V/\partial E}
\]
From now on, as in other model, the labor is considered as numeraire, therefore, the wage is imposed equal to 1.

The profit maximization\textsuperscript{34} leads to a price equal to

\[ p(c) = \frac{c + ae}{2} \]

In this framework the markup\textsuperscript{35} is equal to

\[ m(c) = \frac{ae - c}{2c} \]

The expression of the mark-up reveals some important features of the quadratic indirect additive preferences. First of all, when firms are more productive, and therefore the marginal cost \( c \) is lower, they charge a lower price. Although, they obtain a higher mark-up. Therefore, \( zx \) demonstrate from literature, and in particular by De Loecker and Warzynsky\textsuperscript{36} (2012), the most efficient firms enjoy higher mark-ups, because they do not pass-through the lower cost of production to lower prices for consumers. Moreover, the degree of pass-through is inversely related to firm productivity. In addition, this result is quite different from which found in Melitz (2003) where the mark-ups where constant, but is similar as Melitz-Ottaviano (2008). As far as the income is concerned, the mark-up is increasing in the income of consumer, represented by \( e \). This effect was not captured in both Melitz (2003) and Melitz-Ottaviano Model. The third important result, completely different from those two models, based on direct additive preferences, or quasi-linear utility, is that since the elasticity of substitution is not affected by the amount of good consumed, also the price does not depends on the quantity of other goods.

As far as the other variables in equilibrium, the consumption and profits are equal respectively to:

\[ x(c) = \frac{ae - c}{2e|\mu|} \]

\textsuperscript{34} The profit maximization problem regards the maximum of \( \pi = \frac{(p - cw)(a - \frac{p}{E})}{|\mu|} \) with respect to the price. The first order condition are equal to \( \frac{d\pi}{dp} = 0 = a - \frac{2p}{E} + \frac{c}{E} \) from which the price can be easily obtain.

\textsuperscript{35} The mark-up \( m(c) \) is obtained, as usual, from the expression \( p(c) = (1 + m(c))c \)

Moreover, the number of firms active in the market can be used knowing that firms are willing to enter in the market if they obtain non negative profits. Therefore, the cut-off \( \hat{c} \) can be found:

\[
\pi(c) = \frac{(ae - c)^2 L}{4e|\mu|} = 0 \Rightarrow \hat{c} = ae
\]

The free-entry cut-off depends on the income of consumers and from their willingness to pay, but does not depend on the market size.

With regards to price aggregator, we can derive it from the following expression of expected profits:

\[
\mathbb{E}[\pi(c)] = \int_0^{\hat{c}} \pi(c) dG(c) = \frac{a^{k+2}e^{k+1}L}{2(k+1)(k+2)|\mu|\hat{c}^k}
\]

Where the price aggregator is equal to

\[
|\mu| = \frac{a^{k+2}e^{k+1}L}{2(k+1)(k+2)\hat{c}^k F}
\]

Finally the number of active firms \( n \) and the number of firms entering in the market \( N \) can be derived from the budget constraint, and in particular:

\[
n = \frac{eG(\hat{c})}{\int_0^{\hat{c}} x(c)p(c)dG(c)} = \frac{2(k+2)|\mu|}{a^2} = \frac{2(k+2)a^{k+2}e^{k+1}L}{a^22(k+1)(k+2)\hat{c}^k F} = \frac{a^k e^{k+1} L}{(k+1)\hat{c}^k F}
\]

While the number of the firm entering is

\[
N = \frac{n}{G(\hat{c})} = \frac{n}{\left(\frac{c}{\hat{c}}\right)^k} = \frac{a^k e^{k+1} L}{(k+1)\hat{c}^k F} \frac{\hat{c}^k}{\hat{c}^k} = \frac{a^k e^{k+1} L}{(k+1)\hat{c}^k F} \frac{\hat{c}^k}{\hat{c}^k} = \frac{eL}{(k+1)F}
\]

This last expression shows an analogy with respect to Krugman: the number of firms entering is linearly proportional to the national income \( eL \). As far as the welfare is concerned, it depends on \( n \), which is linear on \( L \), but is homogenous of degree \( (k+1) \) with respect to \( e \).

\[37\text{\ Recall that the productivity distribution function is equal to } G(c) = (c/\hat{c})^k\]
From these two equations, it can be clearly inferred that, as in Krugman, where the openness to trade was compared to an increase in population size $L$. Therefore, the Bertoletti-Etro-Simonovska model confirms that international trade brings about “pure gains from variety”. This important implication can be noticed even clearer when the welfare is expressed as follow:

$$V = \frac{n}{2} \int_0^c \left( a - \frac{p(c)}{e} \right)^2 dG(c) = \frac{a^2 n}{4(k + 1)(k + 2)}$$

From this expression it can be clearly noticed that welfare increases linearly with the number of varieties. At the same time, after the trade liberalization the number of consumers $L$ increases the number of varieties of goods available and therefore the welfare. Thanks to this expression, the Krugman model can be generalized to the case of heterogeneous firms.

As far as the impact of the individual income, or labor endowment, when $e$ increases the number of varieties rise more than proportionally. However, the individual income rises also the mark-up and the free entry cut-off. As result, increase in $e$ causes the creation of new firms, by increasing $n$, and allows the less productive firms to enter in the market, by increasing $N$. On the other hand, when the labor endowment decreases, due to, for example, an economic crisis, the mark-up $f$ firms active decreases, and the less efficient firms are forced to exit the market. It is interesting to recognize that these effects are unearth by this model, but they do not emerge by any model with homothetic preferences, directly additive or quasi-linear preferences.

**Costly trade**

The model explained above can be used to analyze the impact of international trade in case of presence of fixed and variable costs of trade. This section is going to consider bilateral trade between two identical countries, where both the fixed cost $f$ and the variable costs $\tau$ are greater than zero. Both of countries face the same price aggregator $|\mu|$ and the profits are

$$\pi(\tau) = \frac{(ae - ct)^2 L}{4e|\mu|}$$

The cut-off can be easily found by imposing that firms enter in the market if they obtain non negative profits and, thus:
\[ \hat{c} = ae - \frac{4e|\mu|f}{L} \] and \[ \hat{c_x} = \frac{\hat{c}}{\tau} \]

The cut-off divides the firms between that export and not. Moreover, when international trade does not require fixed costs, changes in variable costs \( \tau \) do not affect the distribution of firms that export or not. On the other side, when fixed costs are greater than zero, the zero cut-off profit condition can be divided according to that firms trade abroad or not:

\[ \int_0^{\hat{c}} \frac{(ae - c)^2}{(ae - \hat{c})^2} dG(c) - G(\hat{c}) + \int_0^{\hat{c}} \frac{(ae - \tau c)^2}{(ae - \hat{c})^2} dG(c) - G\left(\frac{\hat{c}}{\tau}\right) = \frac{F}{f} \]

As in Melitz (2003), the market size \( L \) does not affect the domestic cut-off \( \hat{c} \), but it rises when the variable costs increase. After a trade liberalization policy, the expected profit of existing exporter increases, while that of non-exporters decrease.

**Conclusions**

The theoretical framework analyzed by P. Bertoletti, F. Etro, and I. Simonovska employs indirect additive preference to generalize most of the previous model of international trade. In particular, it contains CES preferences, but it acknowledges more realistic implication on prices. In particular, it establishes mark-up increasing in income level and incomplete pass-trough. In this framework the gain from welfare is preserved as in Krugman, and is uniquely due to increase in the varieties available.

Another important consequence of “indirect additive” preferences regards the fact that, different from other models, trade liberalization policies, namely policies aimed to reduce the costs of trade, reallocates the production across exporters and non-exporters, but do not changes the cost threshold for domestic firms. As consequence, after a trade liberalization, the number of active firms in each countries does not change. Another important implication of trade costs reduction regards the fact that the imported good become cheaper; although, the price distribution, conditional to income, of consumers remain unchanged, since at the meanwhile the consumer increases also the varieties of imported good consumed.

The third important result regards the relationship between mark-up and country sizes. In fact, while firms apply higher mark-ups in richer destination countries, they do not change mark-up in
relationship with the size of population. This result is well documented in the literature. According to Simonovska\textsuperscript{38} (2015), firms in a monopolistically-competitive environments charges higher prices, for the same products, in richer destination countries. At the same time, however, there is not significative evidence that company charges different prices in higher markets. This result in quite important, because is typical of models that employs indirect additive preferences, while it is not present in models as Melitz (2003), where direct additive preferences were used, or in Melitz-Ottaviano (2008) model, with quasi-linear preferences.

\textsuperscript{38}Simonovska, I., 2015, Income Differences and Prices of Tradables: Insights from an Online Retailer, Review of Economic Studies, 82, 4, 1612-56
Conclusions

This thesis aims to give a complete analysis of the main models concerning the international trade. I started my description from the Krugman model (1979) where the assumption of internal economies of scale and the “Love for variety” preference, leads to a welfare gain due to trade liberalization, which regards the increases in the variance that consumers can enjoy, as well as lower prices due to the exploitation of increasing marginal returns of scale. It is in 2003 that the international trade theory experienced a great boost, mainly due to the pianistic model written by Melitz (2003). The introduction of heterogeneity between firms allow economists to investigate an alternative channel of gains due to the openness to international trade. In fact, in this model the welfare’s increments are caused by the reallocation of resources towards more efficient firms. This theory seems to legitimate many empirical evidences described in the article “Firms in International Trade”, where data shows how exporters are bigger than non-exporters, and firms self-selected themselves in the export market.

Although the model developed by Melitz seems as innovative, some further extensions have been discussed. In Melitz-Ottaviano (2008), the reallocation of resources and the consequent exit of less productive firms are not due to scarce resources, but the higher competition in the market. Moreover, the Chaney model (2008) investigates the impact of elasticity of substitution between goods on the relationship between reduction of trade costs and margins of trade. The last and more recent model regards “indirect additive preferences” framework, where the welfare gains are uniquely due to the increase in the varieties available in the market.

All these models have been analyzed taking into consideration the empirical evidences. There are no doubts that international trade literature has helped to explain many of these facts. Although, there are still many puzzles that need to be resolved. The article “The Addilog Theory of Trade” seems to be the milestone to further research exploiting different preferences functions.
Bibliography


