



Università
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Evaluating the forecasting ability of entropy measures in predicting financial crisis

Applying Bayesian logistic regression and MCMC

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Abstract:

The objective of this study is to come up with an early warning indicator for systemic risk. The research relies on panel data of two systemic risk measures: Marginal Expected Shortfall and Delta CoVaR. We apply entropy on these measures as defined by Shannon. Bayesian logistic regression is used to predict the likelihood of occurrence of a bank crisis and Markov Chain Monte Carlo is employed for sampling from a proposed probability distribution. We then analyse the crisis indicator and assess the forecasting power of the entropy measures in predicting banking crises. The crisis indicators proved to be relevant in predicting financial disturbances.

1. Introduction

In the aftermath of the 2008 financial crisis, numerous studies were carried out to include risk measurement- related to systemic risk- within the macroprudential regulatory framework. The objective of such research is to assess a specific bank's contribution to the whole financial systemic risk, considering phenomena such as extreme risk and the risk of contagion, the causality between a specific bank risk and that of the system , and the joint risk of the system and the banks.

One of the objectives of this article is to establish an “**early warning indicator**” for systemic risk using entropy measures, in continuum with the notion of the same name introduced by Billio et al. (2015). In the context of financial systems, the concept of Early warning system has been extensively debated in the literature, including by Alessi and Detken (2011) who made use of macroeconomic variables. Demirgüç-Kunt and Detragiache (1999) determined an early warning system that emits a signal when likelihood of a crisis overpass a defined cutoff.

The remainder of this paper is organized as follows. Section 2 introduce systemic risk measures employed in this research as well as the notion of entropy. The following section presents the model, then section 4 exhibits results of the empirical results. Finally section 5 concludes



2. Theory

2.1 Literature review

Systemic risk can be defined as “the propensity of a financial institution to be under-capitalized when the financial system is under-capitalized.” (Acharya et al, 2010). The most recent literature on this concept can be divided into parts (Tarashev et al. ,2010). One category of the literature sees the financial system as a portfolio of banks. We can cite Segoviano and Goodhart (2009), who suggest a range of measures of the financial stability of a portfolio of banks by evaluating the dependence among the banks. Zhou (2010) suggests two measures in continuation with their study: a systemic impact index and a vulnerability index. The first one, measures disturbances in the whole system, given that a specified bank fails, while the second measure is the reverse measure, that is the probability of a bank default, given that there is at least another default in the system.

The other category in the literature emphasises the participation of banks taken separately to the systemic risk. Many techniques have been proposed, such as the conditional value at risk (CoVaR) method (Adrian and Brunnermeier, 2010), which measure the systemic importance of single banks as well as the distress in a particular bank, given that another is in financial distress. Other techniques consider the opposite problem, that is a measure of bank’s risk, given the system risk level. As an illustration, one could see the article about Marginal Expected Shortfall (MES) by Acharya et al. (2010), which proposes measures of banks contribution to systemic risk.



2.2 Introduction to MES and CoVaR:

These methods, as well as other analogous approaches, are essentially based on daily prices (or returns) to forecast the value at risk (VaR) as a first stage in the calculation of systemic risk measures. It is the foundation upon which all subsequent calculations are based.

MES and CoVaR originate from the joint distribution of the system and the risk of the financial institution. CoVaR is the conditional probability of the risk of the system given the institution is under distress, whilst MES is the expected conditional risk of the institution given the system being under distress. CoVaR directly uses VaR whilst MES is based on ES, which then itself is based on VaR. Similarly, it is possible to define CoES and MVaR.

Marginal risk measure	Conditional on system	Conditional on system
VaR	MVaR= $\text{pr}[R_i \leq Q_i R_S \leq Q_S]$	CoVaR= $\text{pr}[R_S \leq Q_S R_i \leq Q_i]$
ES	MES= $E[R_i R_S \leq Q_S]$	CoES= $E[R_S R_i \leq Q_i]$

Table 1: Source: Danielson et al. (2011)

where,

R_S : risky outcomes of the whole financial system (daily returns)

R_i : risky outcomes of financial institution i (daily returns)

We then introduce the joint density of an institution and the system outcomes:

$$f(R_i, R_S)$$

Thus the marginal density of the institution is $f(R_i)$, and the conditional densities are $f(R_S | R_i)$ and $f(R_i | R_S)$. Moreover, if we then take into account the marginal density of the system as being a normalizing constant, we obtain via the Bayes theorem:



$$f(R_i|R_s) \propto f(R_s|R_i) f(R_i)$$

We assume VaR as a risk measure, we then set Q as the event such that:

$$\text{pr}[R \leq Q] = p$$

where Q is some extreme negative quantile and pr is the probability. It follows that, CoVaR_i is the value at risk of the financial system given that the institution i is under financial distress. i.e.,

$$\text{CoVaR}_i = \text{pr}[R_s \leq Q_s | R_i \leq Q_i]$$

Now, we consider using the Expected Shortfall (ES):

$$E[R | R \leq Q]$$

It can be defined as a risk measure employed to assess the credit or market risk of a portfolio, the ES at the $q\%$ level being the expected return of the portfolio in the most severe $q\%$ of the cases.

we obtain MES : institutions expected equity loss in tail market outcomes:

$$\text{MES}_i = E[R_i | R_s \leq Q_s]$$

In the same manner, MVaR can be defined as :

$$\text{MVaR}_i = \text{pr}[R_i \leq Q_i | R_s \leq Q_s]$$

and CoES as:

$$\text{CoES}_i = E[RS | R_i \leq Q_i]$$

2.3 MES

The first Systemic Risk Measure we study is MES with 5% probability as proposed by Acharya et al. (2010). It is defined as the institution's expected equity loss given that the system is in a tail event. In other words, it is the average return of each institution during the 5% worst days for the market. Its formal definition is given by (1), i.e.

$$\text{MES}_i = E[R_i | R_s \leq Q_s]$$

where Q is some extreme negative quantile and R_i is the institution's risky outcome on which the VaR is calculated



2.3 Δ CoVaR

The notion of CoVaR relies on quantile regression¹.

The marginal contribution of an institution, Δ CoVaR, is the difference between the CoVaR conditional on the institution is under distress and CoVaR calculated in the median state of the same institution. That is:

$$\Delta CoVaR_{t,i} = CoVaR_t^{R_i = VaR_i^p} - CoVaR_t^{R_i = VaR_i^{50\%}}$$

where $CoVaR_i = \text{pr}[R_s \leq Q_s | R_i \leq Q_i]$

which implies that Δ CoVaR is equal to :

$$\Delta CoVaR_{t,i}(p) = \hat{\beta}_s^i [VaR_{t,i}(p) - VaR_{t,i}(50\%)]$$

Given that the financial returns are (almost) symmetrically distributed, one expects VaR calculated at 50% to be equal to zero. Hence $\hat{X}_q^{system,i} = \hat{\alpha}_q^i + \hat{\beta}_q^i X^i$ suggests that Δ CoVaR is simply a linear function of VaR.

2.4 Entropy measures

Entropy can be defined as the expected value of the information contained in every message that we receive. In our case the message is a sample drawn from a distribution. Hence, entropy is linked to the uncertainty of our source of information, and its level raises for more sources of greater randomness. The source is also characterized by the probability distribution of the samples drawn from it (for example the MES). The lower the probability of occurrence of an event is, the more information we get when it happens.

¹ For more details about the quantile regression leading to the formulas of CoVaR and Δ CoVaR, refer to Appendix 2



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It is common to use a function of the probability distribution that sums up the content of the distribution. In this regard, the entropy is one of the most widely used probability function.

We consider, the cross section histogram for the MES (or ΔCoVaR)

$$\hat{\rho}_t = (\hat{\rho}_{1t}, \dots, \hat{\rho}_{mt})$$

Where:

$t=1, 2, \dots, T$, with $T=7044$

$m=3117$, is the number of assets traded in the market at time T , and

$\hat{\rho}_{kt}$ is the estimated possibility to have a MES (or ΔCoVaR) in a given interval $[X_{k-1}, X_k]$

$\hat{\rho}_1$ and $\hat{\rho}_2$ are two vectors of probabilities associated to the cross-section distribution of a given feature of the financial assets measured over time t on the market. In this research, we apply entropy to $\hat{\rho}_t$. Entropy have a different acceptation according to the authors

The Shannon entropy (Shannon, 1948), also known as Gibbs-Boltzman- Shannon, is defined as:

$$\text{ENT}_t = - \sum_{k=1}^K \hat{\rho}_k \log \hat{\rho}_k$$

The probability distribution of the events, together with the information amount of every event, forms a random variable whose expected value is the average amount of information, that is entropy, generated by this distribution.



We should note that in the data used in this article, entropy has already been applied to MES and ΔCoVaR .

The aggregate data relies on employing entropy applied to a feature distribution estimated on the market such as the cross-sectional systemic risk measures at a given point in time (Billio et al, 2015). As a matter of fact, movements of entropy applied to these risk measures, may show first indications of a change in systemic risk. Hence, the pattern of the movement of the entropy could be employed as a “quasi-real time early warning indicator” of financial turmoil (Billio et al, 2015)

When we get closer to a systemic shock, the financial institutions (systemic relevant) that are at the origin of the event, could be among the first to respond, hence leading to a structural change in the cross-sectional distribution. In this respect, entropy can recognize the MES and ΔCoVaR in the cross-sectional distribution.

3. The model

3.1 Methodology:

Data on crisis dates are retrieved from the website of the ECB (see Billio et al , 2015)

The dates considered in the dataset start from January 1986 until January 2013. The crisis dummy are constructed on a quarterly basis, therefore we assume that the crisis indicator will equal 1 for all days in a given quarter, provided the indicator is equal to 1 for that quarter.

As regards the construction of the crisis indicator, different types of signals have been proposed in the literature to identify financial crises, such as banking crisis,



or sovereign debt crisis. The focal point will be the Euro zone which has seen recently a sovereign debt crisis unfold, and which is an area characterised by a fragile banking system (Billio et al. ,2015)

The proxy variable used as a crisis indicator is defined as:

$$Y_t = \begin{cases} 1 & \text{if more than one country is in crisis at time } t \\ 0 & \text{otherwise} \end{cases}$$

Aggregate data of banks in the Euro zone, including 437 financial institutions is retrieved from the Industrial Classification Benchmark (ICB) class. We opt for the MSCI Europe index a proxy for the European market, which provides a complete summary of fifteen European countries, where the considered institutions are based.

Following the methodology in Billio et al. (2012) a rolling window approach is used to estimate systemic risk measures, with a window size of 252 daily observations, which is approximately equivalent to a year of daily observations. Our research relies on daily data and it uses systemic risk measures as a proxy for daily losses in stock prices (Bilio et al, 2015). We analyse multiple series considering a panel data framework that is focusing on the cross-section dimension at each point in time. We report in this paper the estimation for Shannon entropy

3.2 Logistic regression

The logistic model is used to forecast a binary response depending on one or more explanatory variables.

The probabilities describing the possible outcomes of a single trial are modelled, as a function of the explanatory (predictor) variables, using a logistic function:



$$P(Y_t = 1) = \Phi(\hat{\alpha} + \hat{\beta}E_t)$$

We denote E_t the entropy index at time t and with Y_t being the crisis indicator at time t , from the database referred to in Billio et al (2015).

In the context of a logistic regression, $Y \sim$ a Bernoulli distribution. Moreover, the estimated probabilities are included in the interval $[0,1]$ through the logistic function.

We have two outcomes:

“0”: failure

“1”: success

Logistic regression predicts the odds of success based on the values of the predictors. The odds can be defined as the probability that a particular outcome is “success” divided by the probability that it is a failure.

$$\varphi(E_t) = \frac{e^{\alpha + E_t\beta}}{1 + e^{\alpha + E_t\beta}}$$

As many other forms of the regression, logistic regression relies on the use of one or more predictors variables that can be either continuous or nominal. However, unlike ordinary linear regression, logistic regression is employed to predict binary outcomes, considering a Bernoulli distribution for the dependant variable, rather than a continuous framework.

Considering this element, it is required that the logistic regression applies the natural logarithm to the odds of the dependent variable being “success” (also called logit) to come up with a continuous measure, bringing about a transformation of the dependant variable

The logit of success is then fitted to the predictors using linear regression analysis. The predicted value of the logit is converted back into predicted odds via the exponential function. Hence, even though the dependent variable takes values zero or one, the logistic regression estimates the odds as continuous variable. Moreover, in our research, a prediction stating whether the dependant variable is a “success” or a “failure” is necessary. This categorical prediction can be computed



from the odds of success, with the odds above a chosen threshold value can be interpreted as a prediction of success.

This function is convenient because, the explanatory variable can take any value from negative to positive infinity, while output is bounded between 0 and 1, and thus can be seen as a probability. The logistic function $\varphi(x)$ is defined as follows:

$$\varphi(x) = \frac{e^{\alpha+\beta x}}{e^{\alpha+\beta x} + 1} = \frac{1}{1 + e^{-\alpha-\beta x}}$$

→ We introduce the logistic transform:

Using the inverse of the logistic function, g , the logit (log odds):

We have: $1 - \varphi(x) = \frac{e^{-\alpha-\beta x}}{1 + e^{-\alpha-\beta x}}$

$$g(\varphi) = \log \frac{\varphi}{1-\varphi} = \log \frac{\frac{1}{1+e^{-\alpha-\beta x}}}{\frac{e^{-\alpha-\beta x}}{1+e^{-\alpha-\beta x}}} = \alpha + \beta x_i$$

with g referring to the logit function.

and equivalently :

$$\frac{\varphi}{1-\varphi} = e^{\alpha+\beta x_i}$$

The odds of the dependent variable being a success (given some linear combination \mathbf{x} of the predictors) is equivalent to the exponential function of the linear regression expression. This illustrates how the logit serves as a link function between the probability and the linear regression expression. Given that the logit ranges between negative and positive infinity, it provides an adequate criterion upon which to conduct linear regression and the logit is easily converted back into the odds.

In order to study the effectiveness of the entropy-based indicators in detecting conditions of financial distress we set a logistic model with entropy indicators for MES and ΔCoVaR



3.2 Bayesian logistic regression :

We consider an exponential prior on $\log \alpha$ and a flat prior on β :

$$\pi_{\alpha}(\alpha|\beta)\pi_{\beta}(\beta) \propto \frac{1}{b} \exp\{\alpha \exp\{-\exp\{\alpha\}/b\}\} d\alpha d\beta$$

We choose $b = \exp(16)$ (see Robert and Casella, 2004)

The likelihood:

$$L = \prod_{t=1}^T \varphi_t^{y_t} (1 - \varphi_t)^{1-y_t}$$

$$L = L(\alpha, \beta | y_1, \dots, y_T) \propto \prod_{t=1}^T \left(\frac{1e^{\alpha+\beta E_t}}{1+e^{\alpha+\beta E_t}} \right)^{y_t} \left(\frac{1}{1+e^{\alpha+\beta E_t}} \right)^{1-y_t}$$

$$\text{Log } L = \sum_{t=1}^T [y_t \log(\varphi_t) + (1 - y_t) \log(1 - \varphi_t)]$$

$$\text{Log } L(\alpha, \beta | y_1, \dots, y_T) \propto \sum_{t=1}^T [y_t \log\left(\frac{1}{1+e^{-\alpha-\beta E_t}}\right) + (1 - y_t) \log\left(\frac{e^{-\alpha-\beta E_t}}{1+e^{-\alpha-\beta E_t}}\right)]$$

The posterior distribution is:

$$\pi(\alpha, \beta | y_1, \dots, y_T) \propto L(\alpha, \beta | y_1, \dots, y_T) \pi(\alpha, \beta)$$

3.3 Sampling via the Metropolis-Hastings algorithm:

The Hammersley-Clifford Theorem (Hammersley and Clifford (1970) and Besag (1974)) states that samples can be obtained from the target distribution by sampling from a number of conditional distributions.

In practice, MCMC gets around the issue of simulating from a complex target distribution by simulating from simpler conditional distributions. If it is possible to sample directly from the full conditional distribution, then the Gibbs sampler can be used (Geman and Geman (1984)).



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Alternatively, we use the Metropolis-Hastings algorithm (Metropolis et al. (1953)) to generate samples from the posterior distribution presented above and to approximate the posterior mean.

Metropolis-Hastings algorithm:

Let π be the target distribution and $\{X^t\}_{t=1}^T$ a sequence of random variables generated with a dependent Metropolis-Hastings with proposal $q(\cdot | X^{(t)})$

At the t -th iteration, given $X^{(t)}$

1. Generate $Y^{(t)} \sim q(y|X^t)$

2. Take

$$X^{t+1} = \begin{cases} Y^t & \text{with probability } \alpha(X^t, Y^t) \\ X^t & \text{with probability } 1 - \alpha(X^t, Y^t) \end{cases}$$

where

$$\alpha(X^t, Y^t) = \min \left\{ \frac{\pi(y)q(x|y)}{\pi(x)q(y|x)}, 1 \right\}$$



4. Empirical results:

The objective is to examine the characteristics of the MCMC output (raw output, ergodic averages, autocorrelation function), the statistical properties (convergence diagnostics), and finally the crisis prediction.

In this section a detailed analysis of results based on MES as an explanatory variable will be undertaken. The same output when considering the delta-CoVar as an explanatory variable can be found in the appendix², with comments when necessary.

² See Appendix B



Marginal expected shortfall (MES):

4.1.1 Raw output

Alpha:

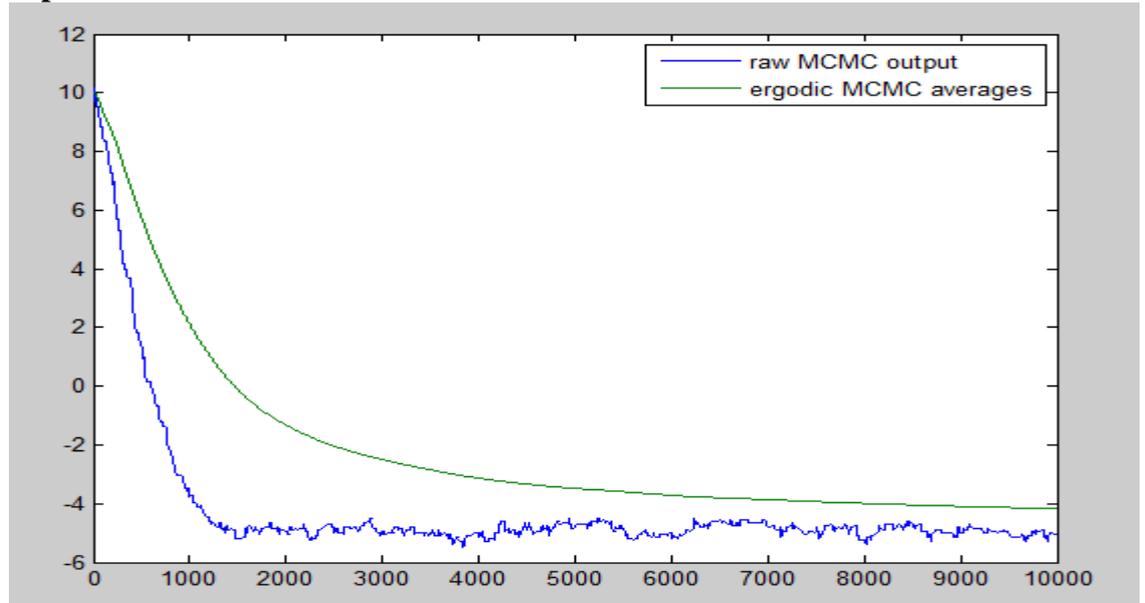


Figure 1:

After 1500 iterations, the raw MCMC output of the Y-Intercept becomes more stable and fluctuate around -5.

One of the characteristics of the Markov chains is that the posterior distribution (stationary distribution) is the asymptotic distribution. Therefore the ergodic average is employed to approximate the desired posterior expectations.

From the figure one, we can see that the chain is fast in converging, and that the number of simulations (10000) is suitable as we can see that the chain is not far from converging.

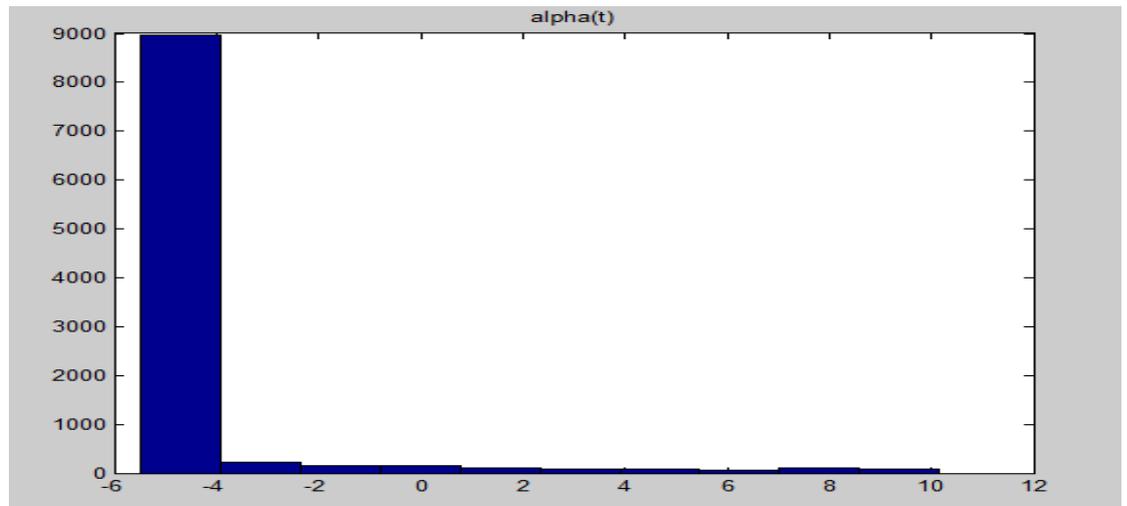


Figure 2: histogram of the M.-H. sample for α

The different values given by the simulations are concentrated around -5, which is the value the chain is converging to.

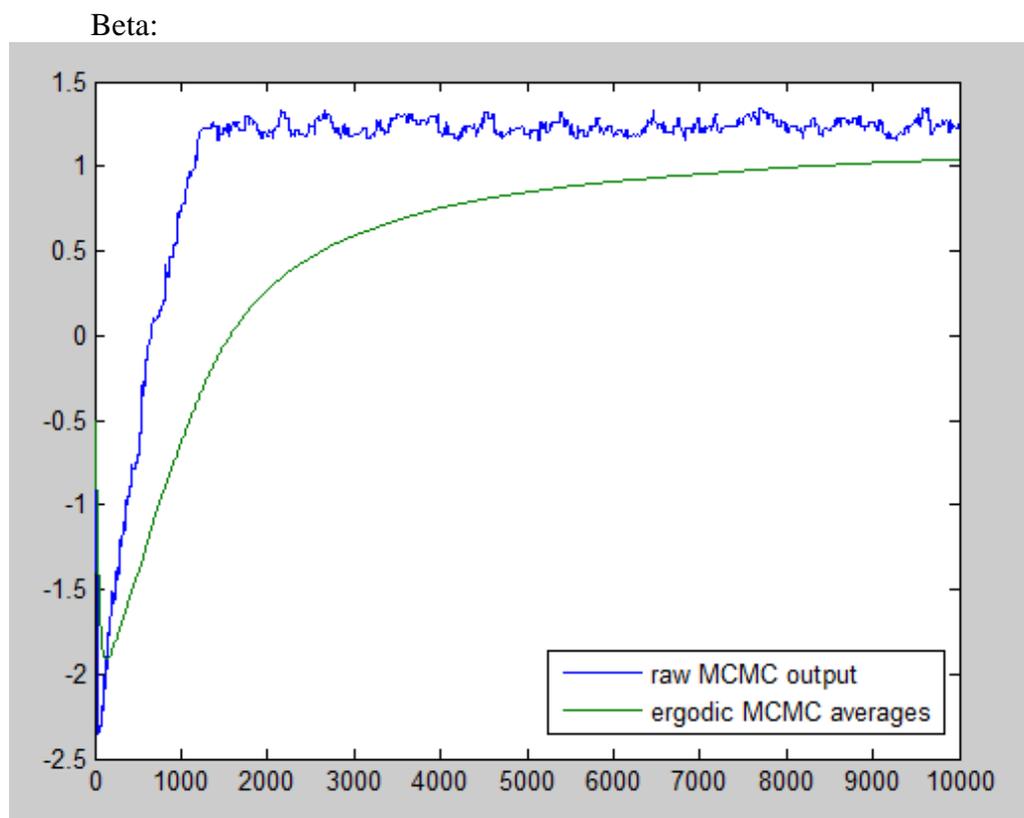


Figure 3:



The slope ($\hat{\beta}$) in the equation:

$$\Phi(\hat{\alpha} + \hat{\beta}E_t) = P(Y_i = 1)$$

As for α , we can note that chain is rapidly converging.

E_t here corresponds to entropy of Shannon applied to the MES, and we need to interpret the coefficient of the slope in the light of the following equation :

$$g(\varphi) = \log \frac{\varphi}{1 - \varphi} = \alpha + \beta x_i$$

Recalling that $MES = E[R_i/R_S \leq Q_S]$, then $\hat{\beta}$ being equal to 1.25 (and converging to 1), means that an increase of the MES by one unit will lead to a slightly higher increase in the probability of failure.

Thus, MES does reasonably well in explaining the probability of success (failure), and as we could expect a higher expectation of low or negative returns from individual banks, given low or negative returns in the system MES is associated with a higher probability of failure.

Histogram Beta:

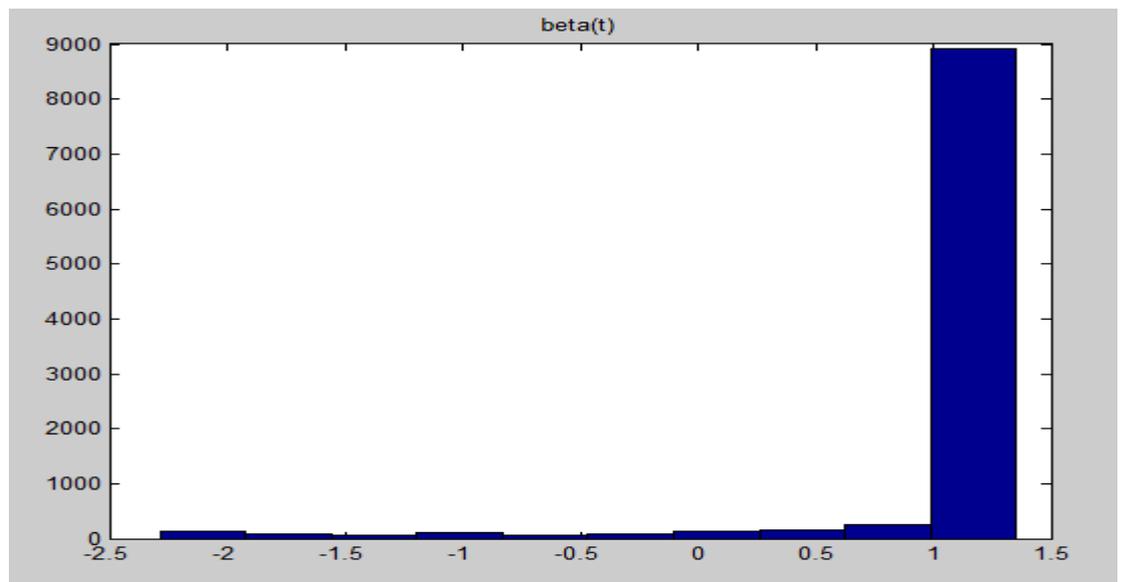


Figure 4: histogram of the M.-H. samples for β



4.1.2 Ergodic averages:

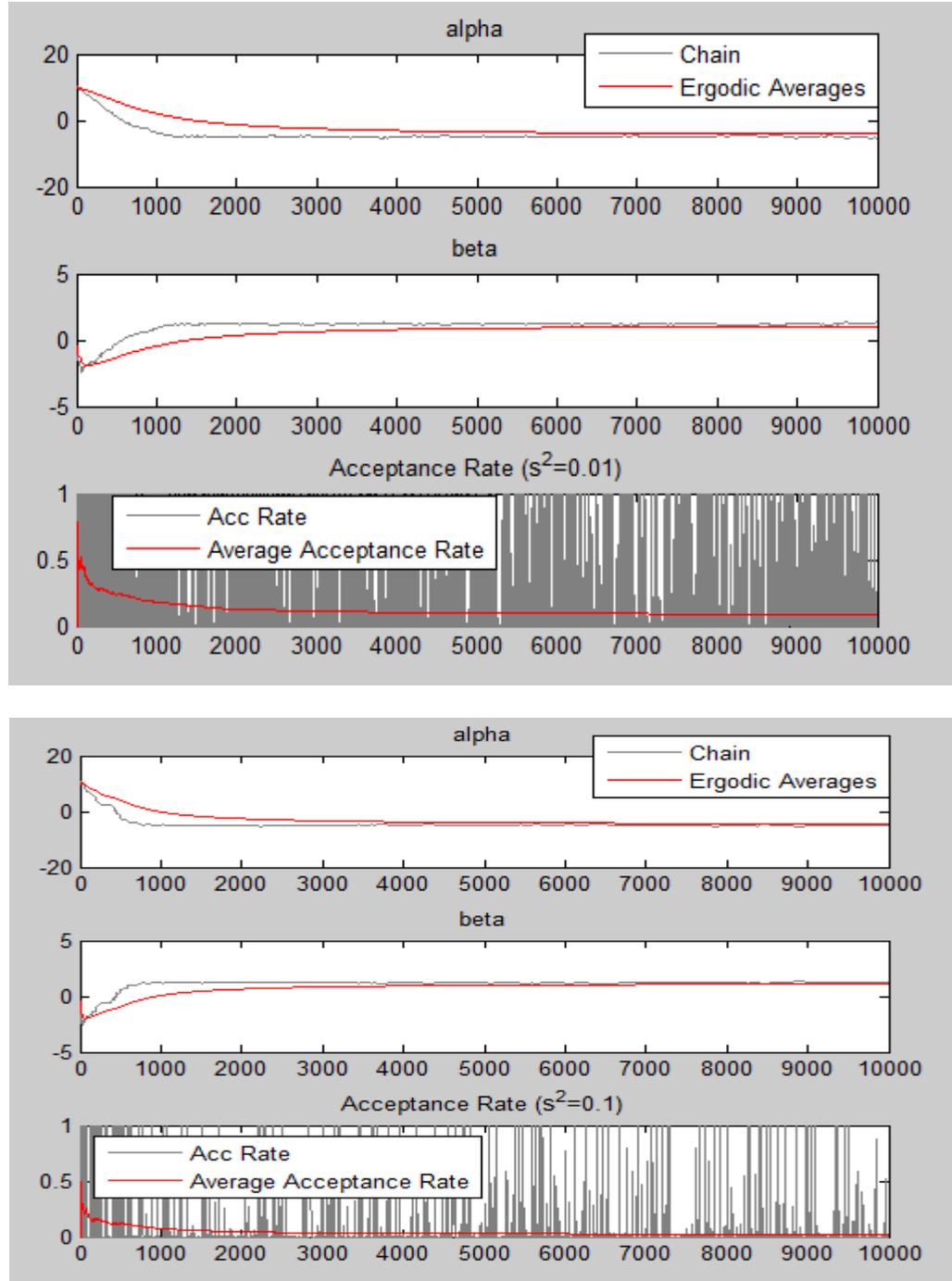


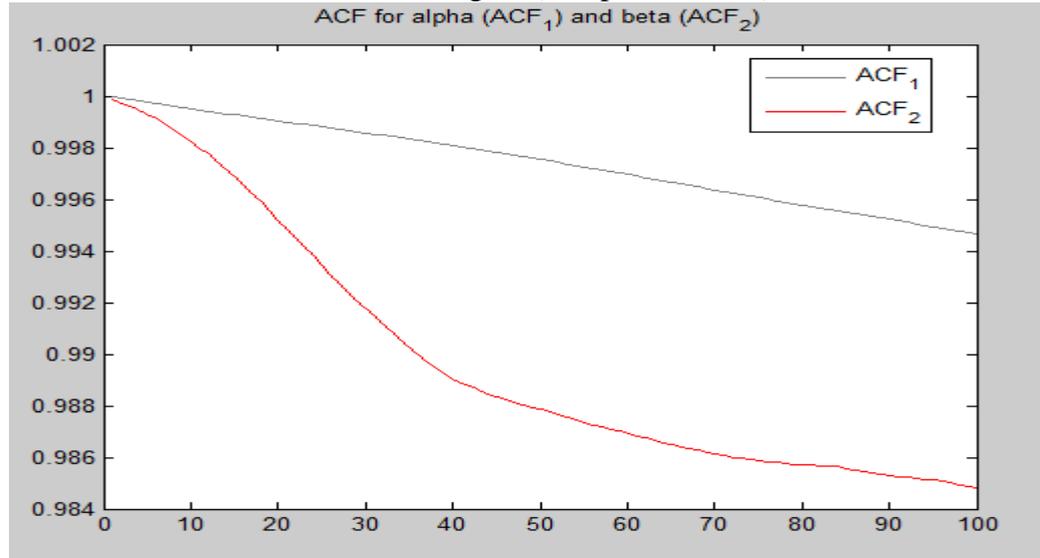
Figure 5: Ergodic averages and acceptance rate for different values of the scale σ^2 of the random-walk proposal



The grey lines and red line represent respectively the raw and average acceptance rates for the M.-H.

4.1.3 Autocorrelation function

Autocorrelation function for both sigma (of alpha and beta)= 0.01



Autocorrelation function for both sigma (of alpha and beta)= 0.1

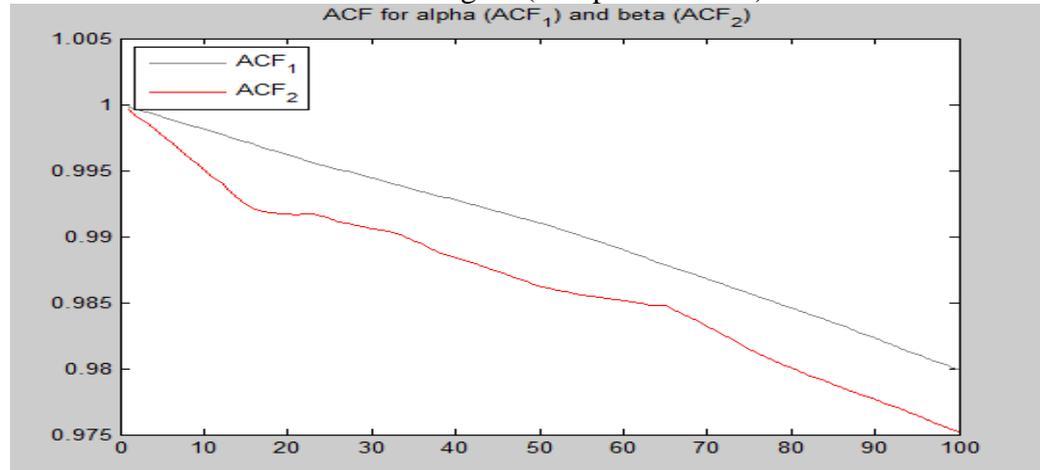


Figure 6: ACF of the M.-H. chain for different values of the scale σ^2 of the random-walk proposal.



According to figure 5 and 6, we observe a high Acceptance rate and slow convergence for $\sigma^2 = 0.01$. However for $\sigma^2 = 0.1$, we note a low acceptance rate and fast convergence

4.3 Crisis prediction:

In order to assess how well our statistical models fit our observations, we can rely for instance on a list of generally prevalent goodness-of-fit-measures (Green, 2008). In our context, we apply the *percent of accurately predicted indicators* (Billio et al. 2015). With regard to the dependant variable, that is the crisis indicator, we determine a cutoff as the times, in terms of percentage in which it is equal to one.

More specifically, $\text{cutoff} = \frac{\sum_{t=1}^T C_t}{T}$

where C_t is the crisis indicator defined previously as :

$$C_t = \begin{cases} 1 & \text{if more than one country is in crisis at time } t \\ 0 & \text{otherwise} \end{cases}$$

In our dataset, for the period January 1986 - January 2013, the cutoff is equal to: 0.4725, but we decide to keep 0.5, as it gives very similar results, and has the advantage of corresponding to the mean of possible values the crisis indicator could take.

If \hat{C}_t is the predicted probability of crisis returned by the logit model, we can define a binary variable \tilde{C}_t such that:

$$\tilde{C}_{ot} = \begin{cases} 1 & \text{if } \hat{C}_t \geq 50\% \\ 0 & \text{otherwise} \end{cases}$$



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We also consider:

***Conservative approach :**

We define the conservative approach, as one in which the banking system resists to change and is not really willing to implement changes towards a more cautious management. In mathematical terms, it could be defined as follows:

$$\tilde{C}_{1t} = \begin{cases} 1 & \text{if } q_{0.05}(C_t) \geq 50\% \\ 0 & \text{otherwise} \end{cases}$$

***Less conservative approach:**

On the other hand, under this scenario, the banks are more flexible and intend to undertake changes, and hence this approach acts more like a prevention, although it could lead to requiring changes from financial institutions, whereas these are not really adequate, since the underlying methodology is based on the fact that it would only take 5% of the financial institutions to fail, in order to consider a given period as a crisis. It could be presented as follows:

$$\tilde{C}_{2t} = \begin{cases} 1 & \text{if } q_{0.05}(C_t) \geq 50\% \\ 0 & \text{otherwise} \end{cases}$$



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Explanatory variable: MES

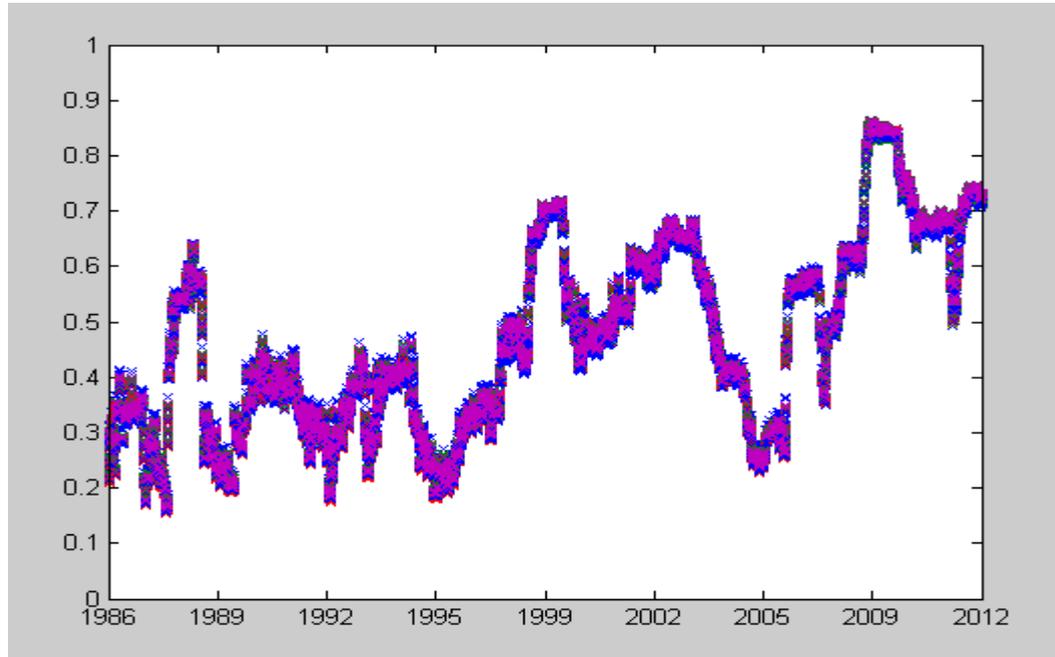


Figure 7: Simulations of the predicted probabilities of financial crisis

We use the figure 7 as it enables us to compare the real dates in the X-axis with the matlab serie dates contained in graphs below



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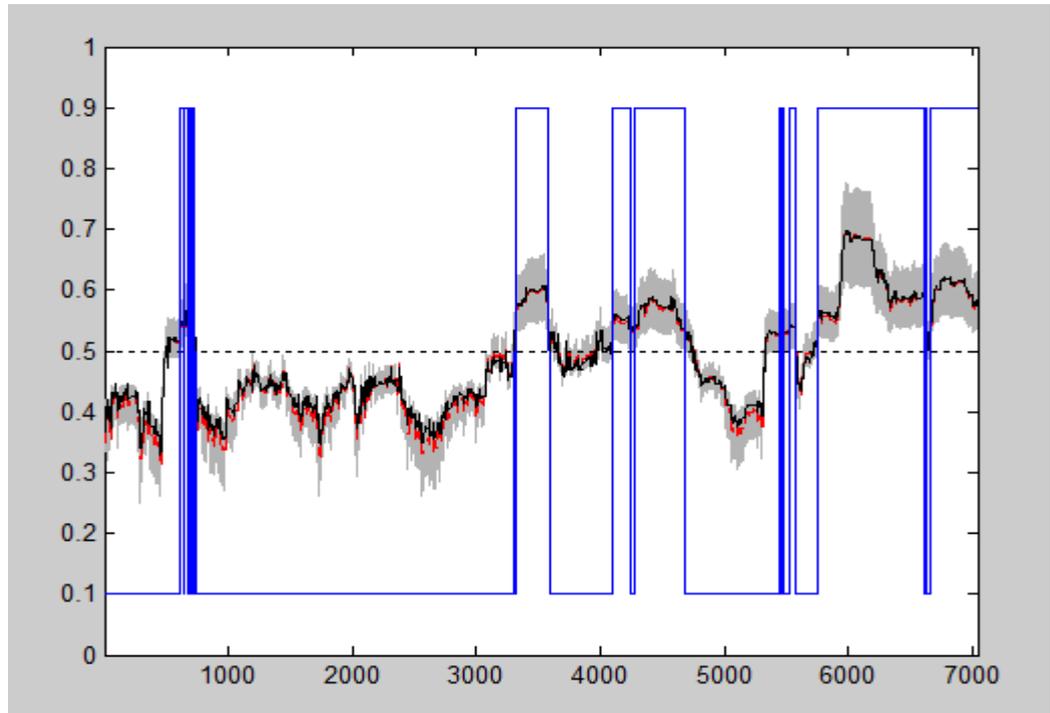


Figure 8: Crisis indicator \tilde{C}_{1t} (quantile 5%)

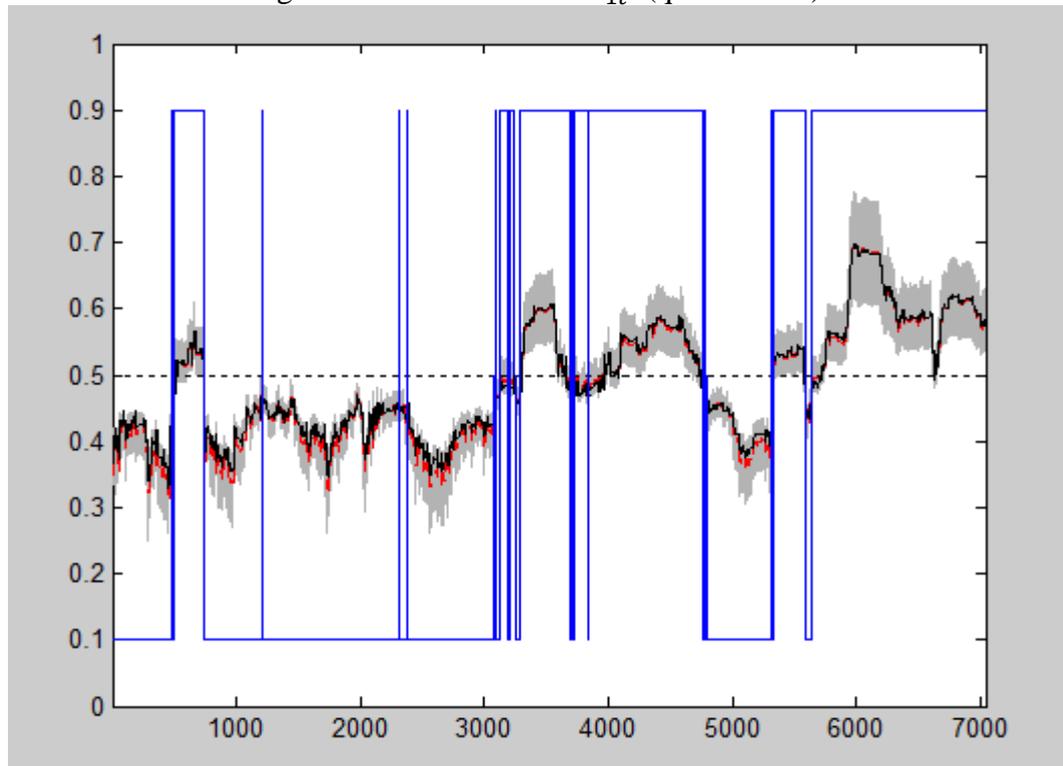


Figure 9: Crisis indicator \tilde{C}_{2t} (quantile 95%)



The grey area corresponds to the interval of simulations of predicted probabilities of failure, between the 5% quantile and the 95% quantile, the red line to the mean, and the black line to the median (50% quantile).

From figure 7 (real dates), 8 and 9, the following crisis periods have been predicted:

-Period 1: 1987-1988 corresponding to: (600-700) : Black Monday 1987, all over the world stock markets crashed, losing considerable value in a very short time. The crash started in Hong Kong and expanded towards Europe, and then impacting the US after other markets had already dropped consequently in value.

-Period 2: 1999-2000 corresponding to: (3300-3500) : Dot com bubble : The early 2000s recession saw a deterioration in the economy, which took place mainly in developed countries. The economic distress affected the European Union during 2000 and 2001. Economists were able to predict this recession, because the economic boom of the 1990s, had already come to an end in East Asia during the 1997 financial crisis.

- Period 3 : 2001-2003 4100-4700 : The early 2000s recession affected the USA in 2002-2003

-Period 4 : 2008-2013 5800 -7044 : The financial crisis of 2007-2008, followed by the 2008-2012 global recession, that led to European sovereign debt-crisis.

Now from another perspective, we have the 1990 decade that saw high-income countries such as the United States, and western European countries going through continuous economic growth for the major part of the decade. During this period, the conservative approach doesn't show any sign of potential crisis, however the less conservative does, although these are marginal (very short period) and spurious results.

The analysis of the differences between the graphs 8 and 9, respectively corresponding to a conservative and less conservative approach will be based on the following four focal points:



1) The detection of the beginning of a crisis is made in anticipation in the less conservative method in comparison to the other approach.

2) The duration is longer for the less conservative approach: which is a more cautious mentality.

*The dotcom bubble 1997-2000 , which was followed by a crash in 2000-2001 is shown to have started via early warnings in the less conservative approach –graph 9-(many very short crisis periods in the years 1997-1998), then followed by a similar signal to the one in the conservative approach- graph8-

* The 2008: In figure 9, we can see a highlight of the period starting from 2006-early 2007, which corresponds to the beginning of the slope going upward, while it was way below the threshold. In this respect, the conservative approach could be much more appropriate in the prediction of systemic risk, as in the conservative approach the crisis indicator (blue vertical line didn't really signal the real commencement of the crisis until around 2008 (corresponding to 5800), despite some very short signal for a crisis that started around 2007, which is still later than with the less conservative approach (2006-2007)

3) The detection of crisis depend on cycles. Theoretically, this corresponds to periods of growth (boom) followed by the burst of the bubble. This phenomenon is accurately described by Adrian et al. (2011, p. 2) who states: “risk typically builds up in the background in the form of imbalances and bubbles and materializes only during a crisis”. Therefore, measures based on contemporaneous price volatility could be misleading, especially that policies relying on these measures are procyclical, which can make the burden even harder to cope with during financial distress. Rather, according to the same authors what could be more appropriate is to build a countercyclical future oriented measure based on many institutional attributes, such as leverage and maturity mismatch. Graphically, we need to detect when curve of the simulations of the probability of failure change in tendency, that is an inflection point in the case where the curve change switch from a downward slope to an upward slope. For example, prior to the 2008 financial crisis (subprime), we can go back as far as the year 2004-2005



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to notice this change in trend, and which corresponds in reality to the year in which the housing bubble reached its peak. This is better captured by the less conservative approach, although a little bit later (2006), and also by the conservative approach (graph 8) even more later (2007), via early punctual warnings followed by a larger signal. This last approach does not however cover the full duration of the crisis. The late signal in both approaches could be explained by the fact that, although the curve of the predicted probability of default had already started to be upward-sloping, it needs to cross the 0.5 cutoff.

4) However, one of the downsides of the less conservative approach is that we can observe **spurious signals** including one in 1990 (1200 in matlab date series), which follows an upper trend in the slope of the predicted probability of crisis but without reaching the 0.5 cutoff.



Cross comparison with the crisis indicator based on the entropy applied to the delta-CoVaR:

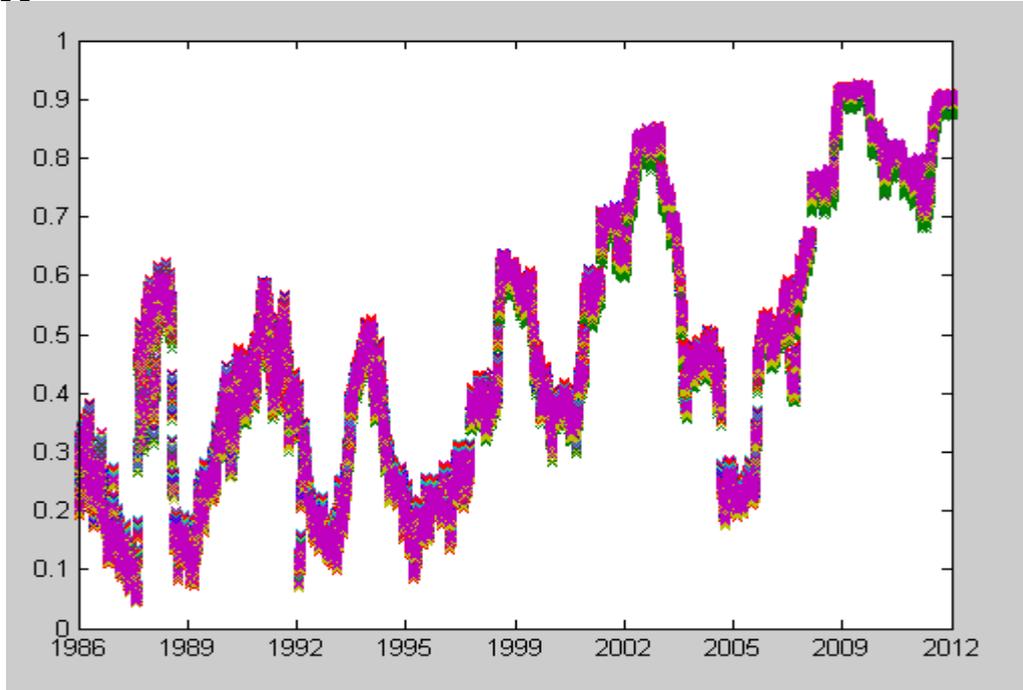


Figure 10: Simulations of the predicted probabilities of financial crisis

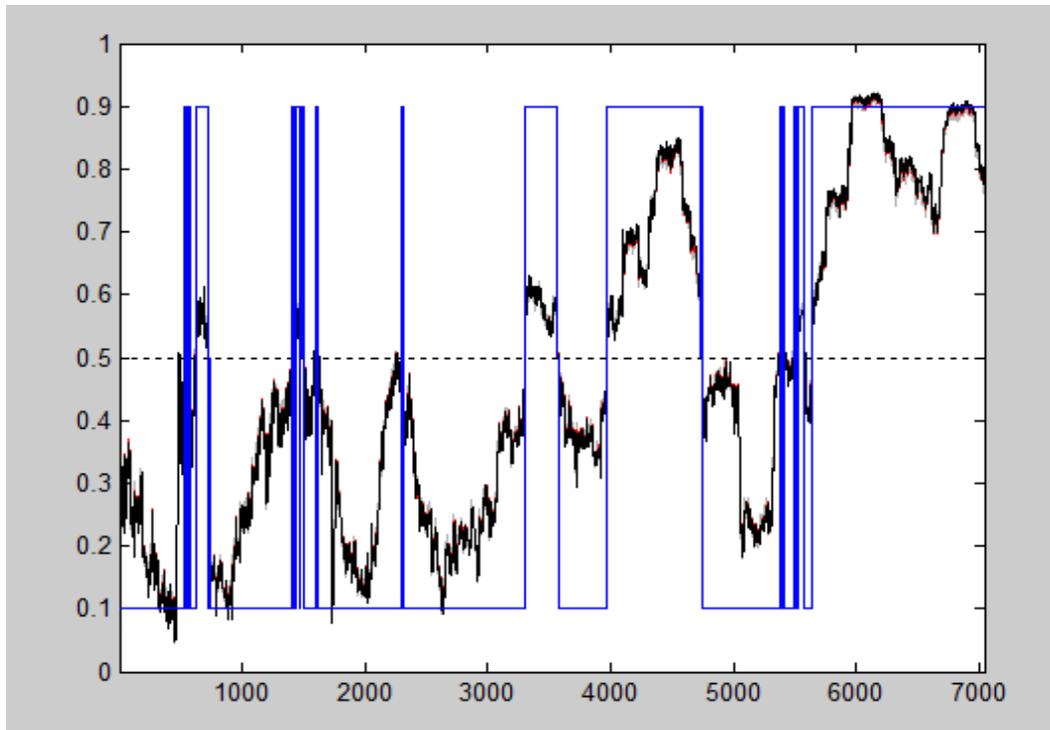


Figure 11: Crisis indicator \tilde{C}_{1t} (quantile 5%)

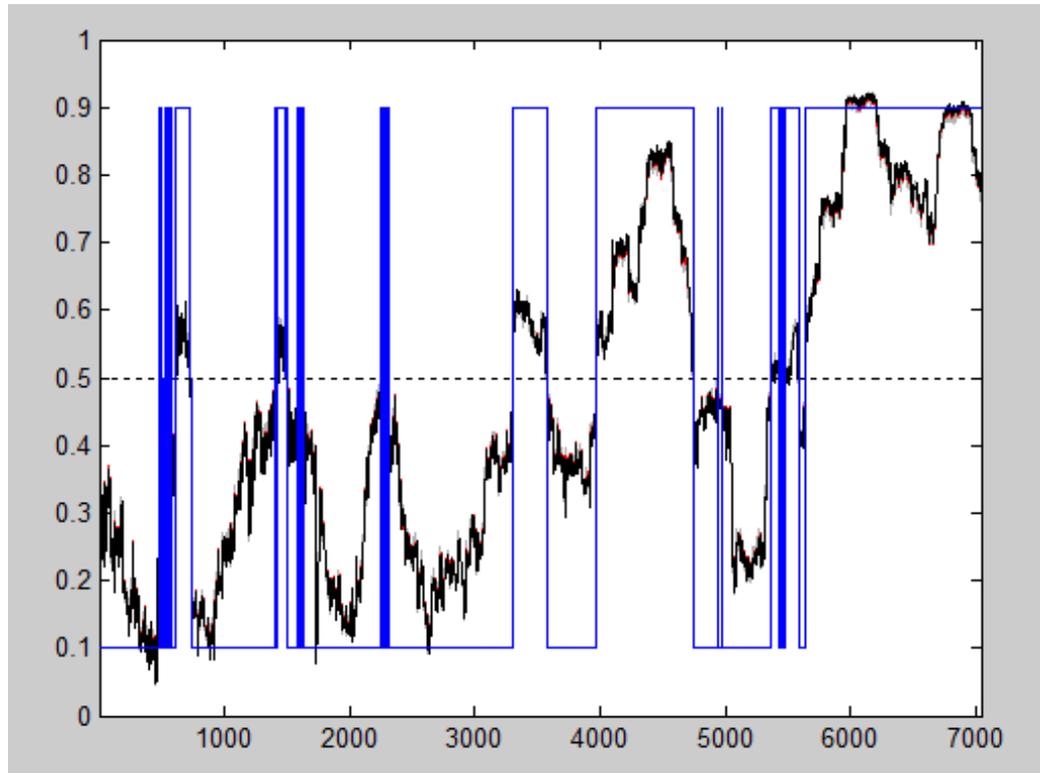


Figure 12: Crisis indicator \tilde{C}_{2t} (quantile 95%)

When we consider the entropy applied to the delta-CoVaR as explanatory variable, we first observe that the interval between the 5% and the 95% quantile of the simulations of the predicted probability of failure, is much thinner, than in the previous case with the MES. Hence we have nearly and overlap of the 5% and 95% quantiles with the mean and the median. This will make the distinction between the figure 11 (conservative approach) and the figure 12 (less conservative approach) less emphasised than in the analysis with MES.

Also, the frequency of detection of crisis is higher under the delta-CovaR, although many of these take place in the 1990's, which is not an accurate statement given the actual state of the economy during this period. This is possibly due to some spurious signals.



5. Conclusion:

In our sample of European financial institutions, using the Bayesian logistic regression and the MCMC sampling, the output obtained via Matlab³ shows expected results given by the univariate regression using entropy measures applied to the MES and the delta-CoVaR. Indeed, the first entropy (applied to MES) does well in explaining the probability of failure, and as we could expect a higher expectation of low or negative returns from individual banks, given low or negative returns in the system is associated with a (slightly) higher probability of a crisis in the system- having at least one country in crisis at a time t . Moreover, the second entropy applied to delta-CovaR performs similarly in explaining the probability of success (failure), and as we could expect a higher dependency of the system to banks' individual returns in the tail of the distribution is associated with a (slightly) higher probability of a crisis in the system. Statistical tools show that the convergence properties of the two considered MCMC chains are valid to use.

Using a crisis indicator based on 10000 simulations, for both cases the MES and CoVaR, and considering the 5% quantile and 95% quantile of the simulations of the predicted probability of failure, as determinants in considering whether there is a crisis or there is no crisis for a given interval of time, show that the less conservative approach (quantile 5%) enable us to detect the beginning of a crisis in advance. Also, the duration of crisis with the less conservative approach is longer and more representative of the reality (length of financial crisis between 1986 and 2013) as we look at it through backtesting. Finally, the results given by the entropy applied to MES are more accurate than with the delta-CoVaR.

³ The Matlab code is available upon request



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Appendix A:

Delta Covar results :

Alpha:

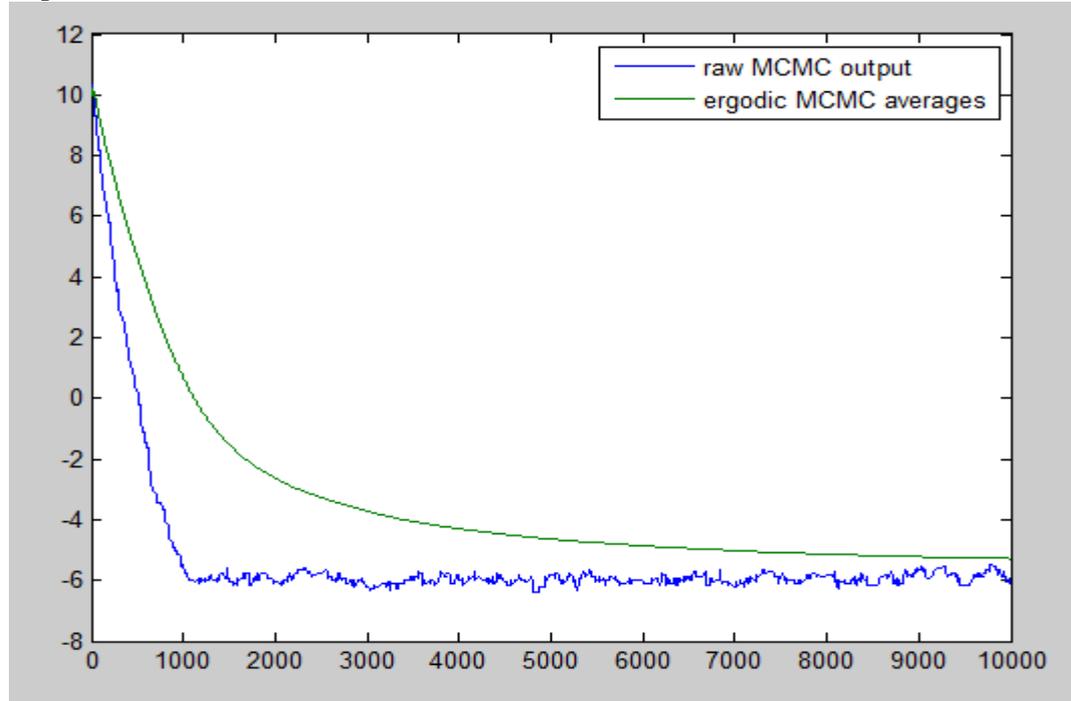


Figure 13

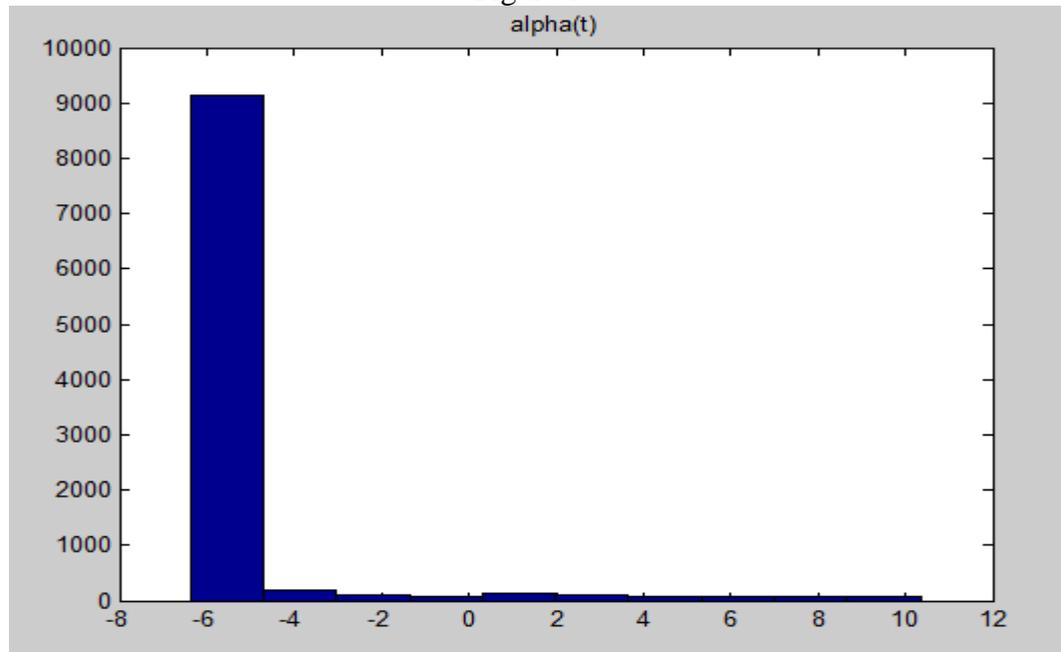


Figure 14: Histogram alpha



Beta:

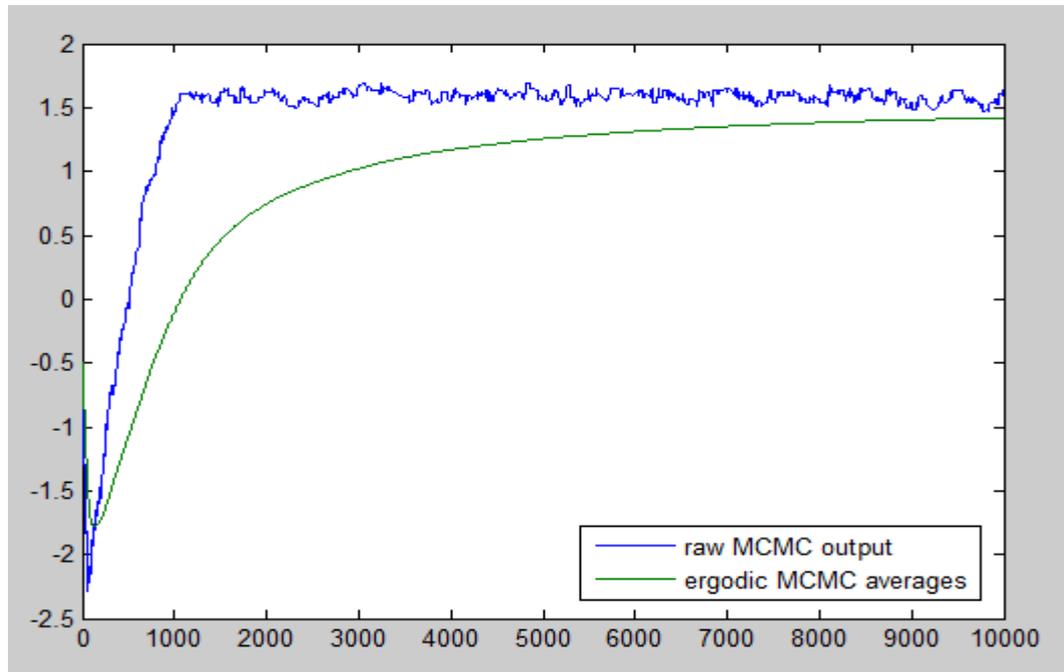


Figure 15

The value of the slope is about 1.5 after 10000 iterations. If we keep in mind that the definition of delta covar is $\text{CoVaR} = \text{pr}[RS \leq QS | Ri \leq Qi]$ then $\hat{\beta}$ being equal to 1.5, means that an increase of the delta covar by one unit will lead to a slightly higher increase in the probability of failure.

Thus, delatCovaR does reasonably well in explaining the probability of success (failure), and as we could expect a higher dependency of the system to banks' individual returns in the tail of the distribution is associated with a higher probability of failure.

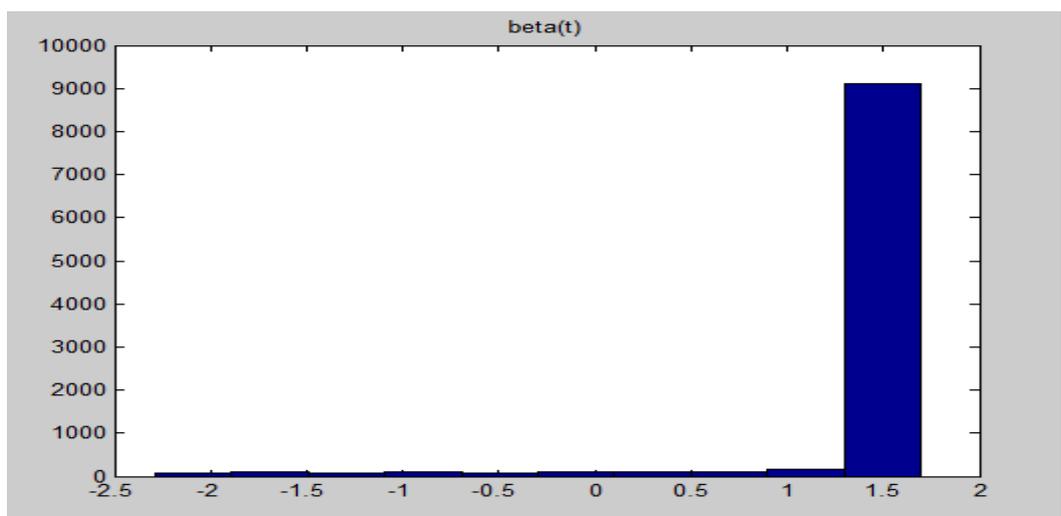


Figure 16: Histogram beta



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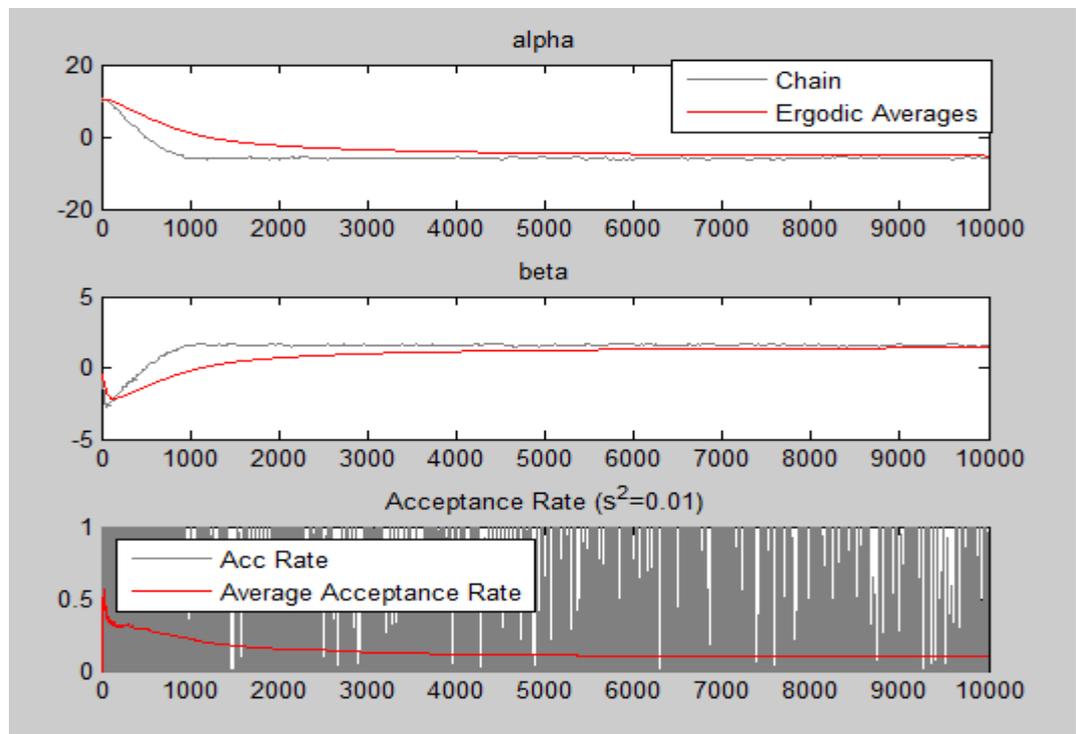
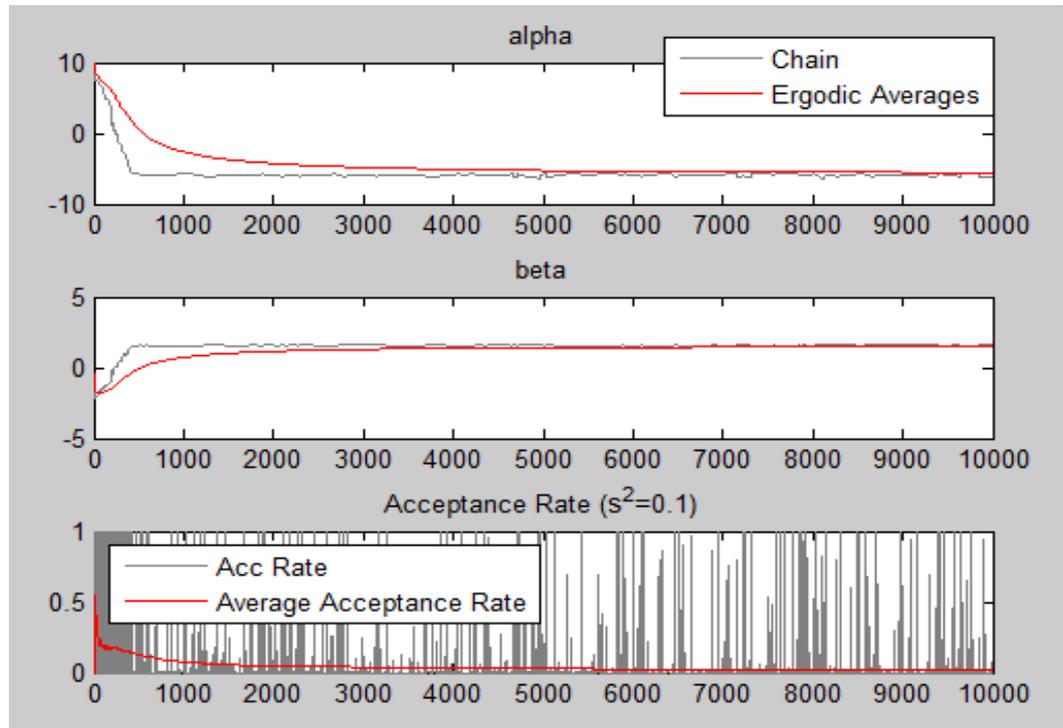


Figure 17: Ergodic averages and acceptance rate for different values of the scale σ^2 of the random-walk proposal



The grey lines and red line represent respectively the raw and average acceptance rates for the M.-H.

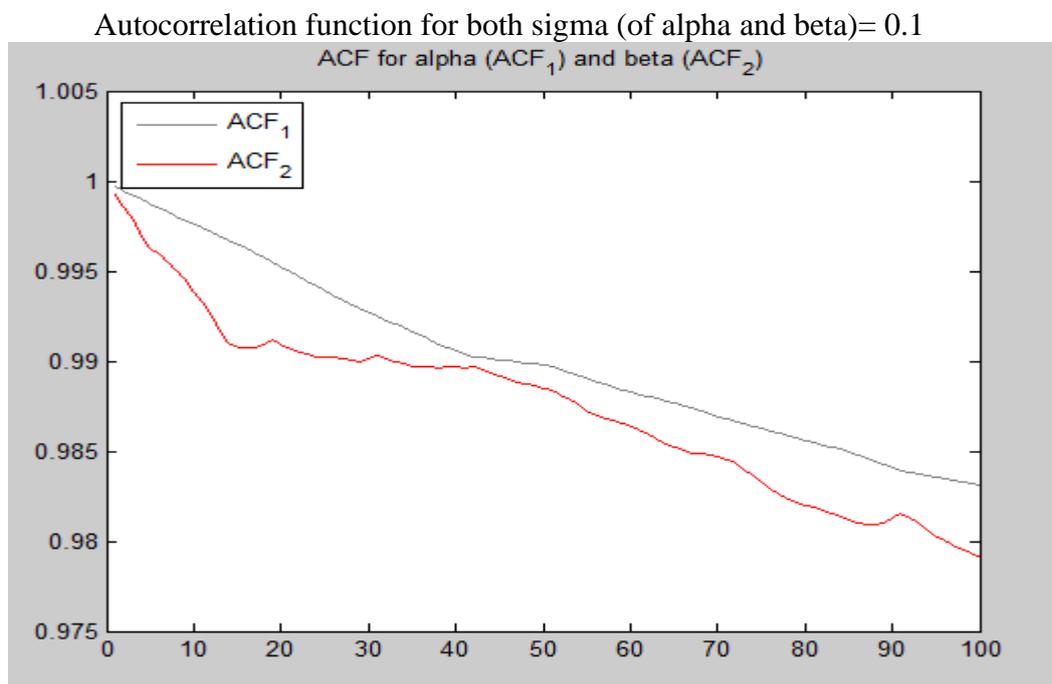
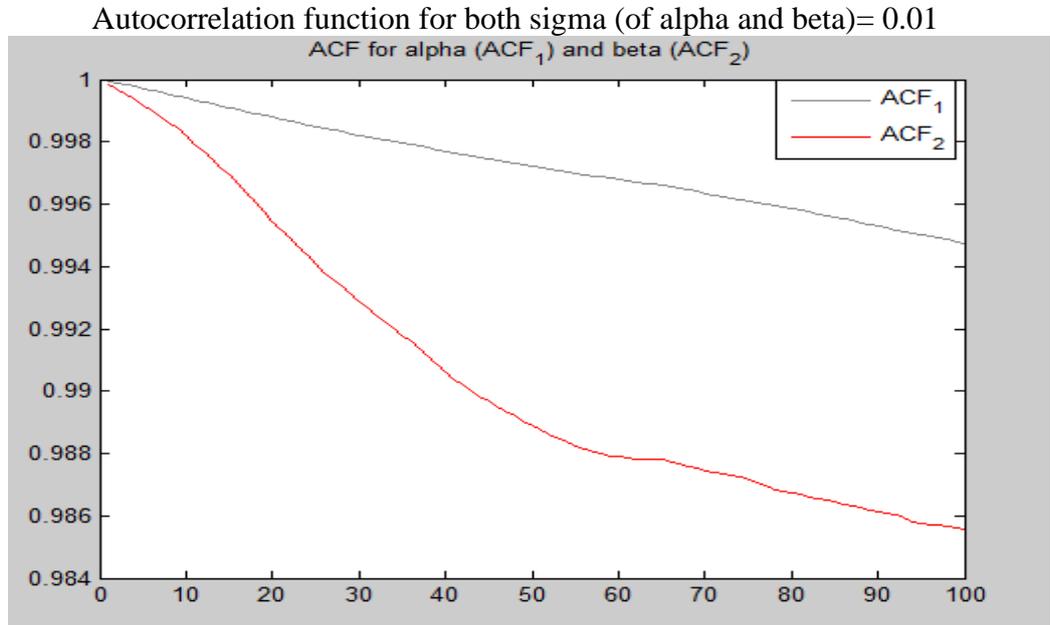


Figure 18: ACF of the M.-H. chain for different values of the scale σ^2 of the random-walk proposal.



In figure 17 and 18 , we observe a lower convergence rate and a faster convergence for a bigger $\sigma^2=0.1$. In the graph 17, this is clearer in the case of beta, than in the case of alpha.

Appendix 2:

In Adrian and Brunnermeier (2011) approach, CoVaR is determined via quantile regression. They start with the quantile regression of the financial system $\hat{X}_q^{system,i}$ on a specific financial institution or portfolio i, considering the q-th quantile, such that:

$$\hat{X}_q^{system,i} = \hat{\alpha}_q^i + \hat{\beta}_q^i X^i$$

where $\hat{X}_q^{system,i}$ indicate the projected value for a specific quantile conditional on the bank i. From the definition of the VaR, we can infer the following formula:

$$VaR_q^{system} | X^i = \hat{X}_q^{system,i}$$

Which means that the predicted value obtained from the quantile regression of the system on bank i, returns the VaR_q of the financial system given Xⁱ .

Considering a specific predicted value of Xⁱ = VaR_qⁱ results in CoVaR_qⁱ , with the conditioning event: {Xⁱ = VaR_qⁱ}.

In more conventional terms, in the context of quantile regression, the CoVaR takes the following form:

$$CoVaR_q^{system|X^i=VaR_q^i} = VaR_q^{system} | VaR_q^i = \hat{\alpha}_q^i + \hat{\beta}_q^i VaR_q^i$$

Subsequently, we get the $\Delta CoVaR_q^i$:

$$\Delta CoVaR_q^{system|i} = \hat{\beta}_q^i (VaR_q^i - VaR_{50\%}^i)$$