Income Inequality and Economic Growth

Does the Distribution of Income Affect Economic Growth?
The Case of U.S.

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Anno Accademico
2013 / 2014
Abstract

This work tries to investigate the relationship between the distribution of wealth and economic growth. The analysis starts from a review of the most important contributions of the wide literature on the topic, with the aim of presenting them under a common theoretical framework. The results, in line with some recent literature, show that, both on the theoretical and empirical point of view, under some initial conditions, an increase in inequality has a positive effect on subsequent growth; this findings are also confirmed by an empirical analysis of the case of United States.
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Introduction

The problem of finding the best allocation of resources in terms of equity and efficiency was, and is still now, one of the biggest issues of the economic science. At the core of this problem, lies the relationship between the distribution of wealth and economic growth; suppose, for example, that there is a strong negative relation between inequality and economic growth, if this was true, then there would be no more the problem of a trade-off between equity and efficiency, for the fact that the most egalitarian allocation is, at the same time, the most productive.

The scope of this thesis is to investigate the relationship between inequality and growth, searching through the mole of contributions the literature on the topic has to offer, in order to find at least some of the linkages between the two subjects.

Since the XVIII century, economists like Adam Smith, David Ricardo, Carl Marx and Vilfredo Pareto, just to cite a few, struggled to find out whether it was possible to find a way for the allocation of resources that was the most desirable in terms of efficiency and equity, since then the contributions on the topics are countless. For the many implication of the subject, economic science got soon mixed with other disciplines like sociology, psychology, philosophy and mathematics and the analysis took different directions, sometimes also according to the major ideological ideas that were dominant in the current period and in the specific nation. However, in spite of the mole of contributions on the topic some questions remain still open: in particular, does the distribution of resources affects economic growth?
And is there a best allocation of resources that is able to improve economic growth and make it a durable phenomenon, not subject to crisis and recessions? It is also interesting the opposite side of this relation: does economic growth help, once takes place, in reducing inequalities or does it tend to exacerbate them?

A univocal answer to these questions, which has not been found yet, it’s in my opinion almost impossible to find: each country followed it’s peculiar development path, and the economic performance of every nation is such strictly connected to so many variables like time, culture, government, etc. that looks impossible to find general rules governing these phenomena, however it may still be possible to identify some sort of trends in each country and similarities in the patterns of development of different nations. As said above, an answer to these questions is beyond the scope of this work, not to say beyond the limits of my knowledge, nevertheless I will try in this essay to review some of the contributions to the subject and to group them under a common framework, at least to give me a better understanding on how these things work.

The work is organized as follows: in the following of this introduction I will try to put the analysis into the historical contest, because I believe that all the theories shown in this work have to be seen in the light of the period in which they were born; for the reason that economic thought, being so deeply interrelated with other disciplines, inevitably reflects the social contest in which was developed.

In Chapter 1, I will briefly review what in my view are some of the major contributions to the topic, starting with the pioneering work of Simon Kuznets, these works will be exposed according again to the major questions cited above: the effects of income inequalities on growth and the reverse relation: the consequences of growth for the distribution of income.

In Chapter 2, I intend to analyze a little deeper the concepts of growth and wealth, focusing the attention to their composition: in fact both phenomena are
the sum of different factors that evolved in different manners through time: for example, economic growth at a country level is the sum of population growth and economic growth and the same is true for the composition of wealth for which there is not a singular source\(^1\). Following the analysis of [Picketty, 2014b], I will decompose the two objects in their major determinants, in order to have a better understanding of the two phenomena. I believe that this decomposition is important in analyzing the relationship between growth and the distribution of income because the composition of one strongly affects the behavior of the second and vice-versa; actually, sometimes the composition can be even more important than the level itself.

In Chapter 3, following the exposition of [Fiaschi, 2007], I’ll display the relation between the composition of income at a country level and economic growth in a more analytical fashion, in order to try to put the theories presented in the previous chapters in a common analytical framework; for this objective, I found the exposition of [Fiaschi, 2007] clear and complete, for the reason that can be interpreted in different ways in the light of the theories shown above in the work.

In Chapter 4, I will take a closer look at the case of the U.S., the reason for limiting the choice to the case of U.S. is mainly the availability of data that are disposable for a wider time span than in the case of other nations; however, limiting the analysis to a single country can tell only a partial story of the problem, because every nation followed its characteristic path of development; furthermore the economic history of U.S. strongly differs from the ones of European countries, not to mention the ones of emerging countries. Anyways, in the case of U.S., the availability of data for a relatively wider time period makes the analysis more interesting; moreover, I believe that a comparison between the evolution of different countries, as happens with panel regressions, while highlights many in-

\(^1\)For example wealth can derive from labor, from inheritance, etc.
teresting features about cross-country differences, on the other hand it can hide some interesting country specific effects, mainly because the multidimensionality of inequality and growth (how inequality and growth are composed in detail) can, in some cases, cancel out at a cross-country level.

**Growth in an Historical Perspective**

When we look back at economic history, it is interesting to note that average economic growth rate has been close to 0% for the most of human history: of course data are incomplete, but it is possible to infer that, until the XVIII century, the average growth rate of the world total production was close to 0% [Picketty, 2014b]. In detail, as it will be shown in chapter 2, if we decompose the growth rate as the sum of demographic growth and the changes in per capita production, it is possible to note that demographic growth was the major determinant of the dynamics of the world production almost until the middle of the XVIII century, when the Industrial Revolution took place.

For a very long period of time, growth of output has been determined almost entirely by the dynamics of the population, which in turn, was strongly affected from food availability, from plagues, wars and from the environment. Of course, although it can be inferred that the average rate of output growth was close to 0%, it had some ups and downs: take the city of Rome for example, its population reached a peak during the Roman Empire and then decreased consistently during the middle ages and it took more than a millennium for the city to regain the same population it had during the Roman Empire. Detailed data about the population are not precise, but they can be useful just to have an idea of the movements, some authors speak about something like a million of inhabitants in the II century, 30000
around year 1000, and we have to wait until the first half of the XX century to see again a million of people. [Massa; Bracco; Guenzi; Davis; Fontana; Carreras, 2005]. It is reasonable to think that wealth followed a quite similar dynamic.

Moreover, there was always a strict connection between population and food availability to the point that the economist Thomas Malthus, in writing his "Essays on the Principle of Population" in 1798, was extremely worried about the steady increase in the population that was taking place at the time. Looking backward at what happened in the XVIII century during the Industrial Revolution, it is possible to imagine how the increase in population growth was source of concerns, the radical change that took place that time was in the productivity: in fact, new technologies in the agricultural sector made possible the production of more food, this led to higher availability of capital, which had a major role in the start of the industrialization process.

In short, more disposable capitals allowed many small entrepreneurs to start their own small firm, a new social class made of entrepreneurs emerged: the middle class. Many small industries began to resemble the idea of perfect competition, however, during the XIX century, some firms became bigger than others and they started to accumulate more and more power, putting them in an increasingly dominant position compared to smaller realities; the situation of perfect competition started to evolve toward oligopoly and in some cases to monopoly; wealth increased steadily until the first decade of the XX century and capital became more concentrated in the hands of few. During the XX century, two World Wars radically changed this situation, lots of capitals were destroyed as a consequence of these wars, and only in the second half of the last century economic growth took place again.

From this brief and of course incomplete summary of what happened in the
last three centuries, it’s possible to note how history, ideologic, economic and political ideas, were all strictly interrelated. All these changes generated fears and hopes in people’s minds and this is reflected in economic and social analysis of the period. It should be no surprising that the idea of perfect competition and Adam Smith’s invisible hand were seen as sources of hopes at the beginning: wealth was increasing steadily in the XIX century, at least for a part of the population, raising positive expectation about the future inside people’s minds.

Soon the concentration of power in hands of few started to rise concerns among some authors: Marx published The Communist Manifesto in 1848 predicting the end of the capitalism caused either by an steady decrease in the return from capital, either from an indefinite accumulation of capital [Picketty, 2014b]. This expansion of capitalism however, didn’t last too long and it came to a halt at the beginning of the XX century, it somehow collapsed during the wars and then it started again. Once more, in the middle of the XX century, the economic boom, generated optimism among people and economic scientists, it seemed that inequalities were reducing and wealth was increasing for a major part of the population. It is in this period, in the fifties, that Simon Kuznets in his: Economic Growth and Income Inequality [Kuznets, 1955], developed a theory about the role of growth that, at first, it would lead to an increase in inequalities and after in a decrease in them; from his point of view, it seems that it’s enough to wait some time and economic growth is going to benefit almost everybody.

In analyzing the dynamics of economic growth and income inequalities, all of this has to be taken into account: on one side it looks impossible to develop a theory containing all these variables in order to explain which effect growth had on the distribution of wealth or vice-versa, on the other side it is still possible to find some common patterns: economic growth has never been unlimited, it seems that it always came to a slowdown, a crisis of some sort followed and then it started
again somehow. The dynamic of the concentration of capital followed a similar pattern, during steady growth there was an increase in the concentration of capital in the upper part of the income distribution, this concentration decreased during the wars and then it started again to regain it’s previous values.

Many of the theories proposed to explain the dynamics of growth failed in predicting what really happened: fortunately the catastrophic predictions of Malthus, Ricardo and Marx didn’t realize, but at the same time the optimistic predictions of Adam Smith and Kuznets didn’t materialize as well, with economic growth generally benefitting more the upper side of the income distribution.
Chapter 1

Previous Literature

In this chapter there is a little overview on the literature concerning economic growth and inequalities; the topic is really wide and the contributions are too many to enumerate them all, hence here are only some of the most important.


In particular, I will focus on the effects of inequality on economic growth; I will also take in consideration the reverse causality of the story: how economic growth affects the distribution of income. In doing this I will expose different approaches and different theories on this relationships and some related issues.
1.1 The Kuznets’s Hypothesis

Talking about growth and inequalities, a good starting point it’s the pioneering article of the economist Simon Kuznets: Economic Growth and Income Inequality, which is one of the first attempts to explain the relationship between growth and inequalities. In his article [Kuznets, 1955] Kuznets tried to discover whether, during economic growth, the distribution of income tents towards more equality or not and which factors determine the levels of inequalities in the long run.

Kuznets starts describing certain desirable characteristics an ideal analysis on the evolution of inequalities should have, particularly important is the problem of tracing the evolution of each unit of income through time and the intergenerational changes between income groups in the very long run; however, the scarceness of data was a substantial problem at the time and still remains one of the major issues.

Kuznets analysis starts observing that, in countries like UK, United States and Germany, first inequalities rose up to some point in the XIX and XX century, and then gradually declined. Kuznets finds two main groups of forces increasing inequalities in developed countries: the concentration of savings in the upper brackets of the wealth distribution and the shift from the agricultural sector to the industrial sector that takes place with industrialization and urbanization.

The concentration of savings operates because only the upper-income groups in the population are able to save, hence, in the first stages of economic development, the concentration of assets is even more unequally distributed, and since assets generate income, the cumulative effect of the concentration of savings leads to a more unequal distribution.

The second source of increasing inequality is the migration from the agricultural sector, which is indeed a low income sector, to the industrial sector; the increasing inequality in this case spurs because income inequalities in the rural
sector are usually lower than in the industrial sector.

The combination of these two factors would suggest that inequality is increasing with economic growth, but the fact that from some point on, for some countries it is observed a decline in inequalities suggests that there must be some other forces reducing inequalities. Kutnetz finds at least four different forces through which, in a second phase of development, growth leads to more equality.

One factor it is political and refers to the institutional interventions aimed to avoid the concentration of wealth in the hands of few; this channel operates in different ways, from the redistributive effects of taxes, to the phenomenon of inflation associated with economic growth, that reduces the value of accumulated wealth, and to the institutional interventions to put restrictions on accumulated wealth.

A second factor is demographic and derives from the differential rates of increase in population between rich and poor, being smaller the first and larger the latter; moreover the effect of immigration amplifies this effect, because usually immigrants enter in the lower part of income distribution of the population: even if the effect of savings is to increase more the income of those in the upper part of the distribution, the overall effect is much diluted: the increase in savings in the lower part of the distribution is then relatively smaller compared to the increase in the upper part, but concerns a larger number of individuals.

A third force in reducing inequalities is very closely related to a dynamic economy as happens when fast growth takes place and there is a kind of turnover with older assets replaced by new ones, and older industries replaced by new ones: this is something that has to do with the "disruptive" nature of capitalism as advanced by the economist Joseph Shumpeter, where new firms tend to overcome the older generating some kind of turnover in the property assets.
A last force is related to the role of service income, in fact only a small part of the revenues of the upper income groups derives from properties and a major part derives from capital income, but the long run rise in this source of income is much smaller than the growth of service income for the lower income groups of the population, furthermore a large part of the rising incomes is due to interindustry shifts.

However it should be interesting to note that these last three factors are typical of a growing economy, just like it was for United States and for much of Europe during the postwar period during the fifties when Kuznets developed this theory; major attention at the time was devoted to steadily growing economies, but growth came to a slowdown in latest decades of the XX century, and things could be much different in a stable economy or during a recession.

In his paper Kuznets adds another and maybe more important political factor, related to the increasing political power of the lower income groups of the population in democratic societies which asks for more redistributive policies to balance out the effects of industrialization and urbanization.

### 1.2 The Beginning of the Kuznets’s Curve

To examine when all this process begun, Van Zanden, in "Tracing the Beginning of the Kuznets’s Curve" [Van Zanden, 1995] tries to trace the beginning and the upswing part of the curve. Although the data are not always comparable,\(^1\) it is still possible to see some common trends in the Gini coefficients in some countries and cities of Europe before 1800.

\(^1\)In fact some data refer to income inequalities, where income is derived from data on tax collection, some others data refer to data on the rental value of houses which seems to be a good proxy for the income and wellbeing of households.
In particular it is possible to observe two facts: first inequalities are larger in cities than in the countryside; second, inequalities tend to grow during the period under examination both for cities and for the countryside; these two facts seem to confirm the Kuznets's hypothesis concerning the upswing in the curve: according to [Van Zanden, 1995], premodern economic growth was coupled with increasing inequality and economic benefits of development were highly unevenly distributed. Before the XX century, it is shown that inequality generally increased overall throughout Europe, and only in the last hundred years the inequalities in the distribution of income decreased until around 1975; [Van Zanden, 1995] argue that there was a "Super Kuznets's Curve" from the end of the premodern age where economic growth was first accompanied with rising inequalities until around 1900 and then started diminishing them. The hypothesis of [Van Zanden, 1995] is that the change of the trend was due to the transition from "premodern growth" to "modern economic growth" consisting in a general shift of labor from agriculture to industry and services: hence from a sector with low income and productivity to industry and services, sectors with higher incomes and productivity.

1.3 Inequality and Growth in the long run

As said above, the literature on inequality and growth is very wide, and the relation is complex; for instance, on one side growth affects the distribution of incomes, on the other side the distribution of incomes affects subsequent growth generating in this way a cyclical relation; following [Barro, 2000], it is possible to summarize this second causality with four main channels trough which this relationship has been analyzed in the literature : credit-market imperfection, political economy, social unrest and saving rates.
CHAPTER 1. PREVIOUS LITERATURE

1.3.1 Credit Market Imperfections

In the case of credit market imperfection, growth is dampened by inequality because the poor or who have not enough assets to make an investment, either cannot borrow at all or cannot borrow at a fair interest rate; hence the poor part of the population does not have enough capital to make profitable investments and this is detrimental to growth. The same mechanism however can work also in the opposite direction: for example, if an investment requires some fixed cost above some minimum level to be efficient\(^2\): in this case, some concentration of capital in a developing economy makes it possible for some entrepreneurs to have the minimum capital necessary to make some investment or to start their own enterprise. This is basically what happened in the early stages of the Industrial Revolution; furthermore, in the case of a growing economy it is possible to imagine that, as the economy develops, legal and economic institution will develop consequently, creating more favorable conditions for subsequent investments: hence in poor economies the effect of inequality will be to promote growth in a first stage rather than dampen it; only subsequently, with the introduction of legal and economic institutions, inequalities will start to reduce.

1.3.2 Political Economy

This section analyzes what generally happens in democratic societies, where population, through the voting process and subsequent taxation can implement redistribution. In fact, according to the median voter theorem, if population votes on a

\(^2\)Barro, 2000] cites as an example education, which requires to be carried up to some minimum level, (e.g. secondary school) to be efficient
tax for redistribution and the median income is below the mean, then all voters on the tails of the distribution would balance each other and the median preferred tax rate would be chosen, which in this case would be in favor of a more equal distribution of wealth: hence there will be redistribution. The higher the inequality, the larger the redistribution, but this requires higher taxes that will have distortionary effects on the decisions of the agents, hence reducing the incentive on investments. In the end, through this channel, higher inequality will reduce economic growth because of the distortionary effects of a higher taxation.

A negative effect of inequality on growth could also happen without transfers, in this case the rich elite needs to devolve some resources to lobbying and to buy votes in order to prevent the redistribution that would otherwise be implemented through the democratic process, but this also requires resources that could otherwise be invested and generate a higher rate of growth.

1.3.3 Sociopolitical Unrest

Great disparities of opportunities and wealth tend in many cases to generate crime and riots and in some cases revolutions, this devotes resources in unproductive and detrimental actions, and at the same time discourages investment because threatens property rights making investments more risky. This process is characteristic of Africa, where continuous riots and civil wars dampened productive activities and discouraged foreign investments. Through this channel, economic inequality tends to have a negative effect on growth.

1.3.4 Saving Rates

According to the Keynesian theory, it is believed that saving rates rise with the level of income: the more you earn, the more you save; in this case a redistribution
of income would reduce the relative proportion of savings in the economy; for this reason higher inequality, at least up to some point, would generate higher savings and, in this way, would foster economic growth.

The major problem with all these theories is that their effects have different signs and in the end, in most cases, they balance each other; this might be one of the reasons for which the empirical evidence on "The Curve" still remains unclear 50 years after Kuznets’s first article.

[Barro, 2000] shows that inequality has a negative effect on growth for poor countries while it has a positive effect for rich countries, although the overall relationship tends to be weak. Figure 1.1 depicts on the vertical axis the normalized growth rate as a function of the Gini coefficient holding others variables constant; it is easy to see that the effect of inequality is almost null. If instead the sample of countries is divided into two groups of rich and poor countries, where poor countries have an average income below $2070 1985 U.S. Dollars, and rich countries have average income above that level, it is possible to see, as depicted in figure 1.2, that the relation is negative for low income countries and positive for high income countries; this suggests that the average level of income, which indicates whether a country is developed or not, may have an important role in analyzing the relationship between growth and inequality.

1.4 Inequality and Growth in the Short Run

Some of the channels listed above, specially the Political Economy approach, suggest that inequalities might have a negative effect on growth, at least in the long run, because some resources are devoted to redistribution instead of production,
Figure 1.1: Growth rates and Gini coefficient. Source: [Barro, 2000].

Figure 1.2: Growth rates and Gini coefficients for countries with average income below and above $2070 1985$ U.S. Dollars. Source: [Barro, 2000].
in this way growth might be affected negatively in the short run.

In exposing the contributions to the analysis of the relation between income inequality and growth in a short term perspective, a good starting point is the work of [Forbes, 2000], where empirical analysis shows that, in the short run, an increase in inequality can have positive effects on a country economic performance. Again, this seems to be consistent with what happened just before the Industrial Revolution: in fact some authors [Massa; Bracco; Guenzi; Davis; Fontana; Carreras, 2005], agree on the big role that the increase in the private properties, derived from the phenomenon of the enclosures in England, had in paving the way to the Industrial Revolution. In fact, during the XVIII century, first in England and then across the rest of Europe, most of the "common lands" formerly used for grazing, became private properties and were used for private farming; this fact determined an increase in inequalities on one side, but on the other side increased the agricultural production, making more food available for the population, favoring the demographic revolution and, more importantly for economic growth, freeing the necessary capitals for the Industrial Revolution that took place shortly after.

Forbes uses a new dataset of higher quality, in this model growth is the dependent variable and the explanatory variables are: initial income inequality, income, gender, male and female education, market distortions, with the adjoint of dummy variables for specific time and within countries effects:

\[
Growth = \beta_1 \text{Inequality}_{i,t-1} + \beta_2 \text{Income}_{i,t-1} + \beta_3 \text{MaleEducation}_{i,t-1} + \\
\quad + \beta_4 \text{FemaleEducation}_{i,t-1} + \beta_5 \text{PPPI}_{i,t-1} + \alpha_i + \eta_t + u_{i,t} \quad (1.1)
\]

3This model differs from [Perrotti, 1996] only in the addition of dummy variables.
In equation 1.1, inequality is measured with the Gini coefficient, data comes from [Denninger and Squire, 1998], the rest of data come from World Bank STARS data set; education is measured as average years of secondary schooling both for male and female, data come from [Barro and Lee, 1996], \textit{PPPI} accounts for market distortions: it is used as a proxy the price level of investment measured as the Purchase Power Parity of investment/exchange rate referred to U.S..

In this analysis, where the results are shown in table 1.4, four different estimation technics are used, but there is always a positive and statistically significant coefficient on inequality that indicates a positive effect of inequality on growth.

<table>
<thead>
<tr>
<th>Estimation method</th>
<th>Five-year periods</th>
<th>Ten-year periods: fixed effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed effects (1)</td>
<td>Random effects (2)</td>
</tr>
<tr>
<td>Inequality</td>
<td>0.0036</td>
<td>0.0013</td>
</tr>
<tr>
<td>Income</td>
<td>(0.0015)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>Male Education</td>
<td>(0.020)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Female Education</td>
<td>(0.031)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>\textit{PPPI}</td>
<td>(0.032)</td>
<td>(0.016)</td>
</tr>
</tbody>
</table>

|                  | 0.00038           | 0.0009                    | 0.0013                     | 0.0013               | 0.0003            |
|                  | (0.0003)          | (0.0002)                  | (0.0000)                   | (0.0001)             | (0.0003)          |

\begin{tabular}{l|ccc|c}
\hline
Estimation method & Fixed effects & Random effects & Chamberlain’s π-matrix & Arellano and Bond & Ten-year periods: fixed effects \\
\hline
Inequality       & 0.0036         & 0.0013         & 0.0016                   & 0.0013              & 0.0013            \\
Income           & (0.0015)       & (0.0006)       & (0.0002)                 & (0.0006)            & (0.0011)          \\
Male Education   & (0.020)        & (0.006)        & (0.004)                  & (0.008)             & (0.016)           \\
Female Education & (0.031)        & (0.015)        & (0.010)                  & (0.022)             & (0.028)           \\
\textit{PPPI}    & (0.032)        & (0.016)        & (0.006)                  & (0.018)             & (0.030)           \\
                  & 0.00038        & 0.0009         & 0.0013                   & 0.0013              & 0.0003            \\
                  & (0.0003)       & (0.0002)       & (0.0000)                 & (0.0001)            & (0.0003)          \\
\hline
\end{tabular}

Table 1.1: Estimation results for equation 1.1 with different techniques. Source: [Forbes, 2000].

In particular, the Arellano and Bond fixed effects estimation technique controls for unobservable fixed effect and focuses on variations in each country across time, hence the coefficients describe how changes in inequality are related to growth in each country.

The other main difference between this work and other empirical analysis on the
relation income distribution-growth is the time span: in fact this work utilizes 5-year periods, estimating a short and medium run relationship, while other analyses focus over a range of 25-30 years. It is possible to note that, estimating the relation over a 10-year time span, the coefficient of inequality becomes statistically insignificant. Various tests are performed by Forbes to test why the coefficient on inequality is positive in this work and negative in others as in [Perrotti, 1996], it turns out that the length of period under consideration does affect the relation between inequality and growth.

Another interesting result comes from table 1.4 where it is possible to see that, including country effects in the pooled model, the relation becomes positive and significant, while in the pooled OLS was insignificant; this suggest that there might be some omitted variable bias from non accounting for country specific effects. Interestingly, among some hypothesis on the causes of the negative bias induced from omitted variables generating a negative coefficient there is a higher share of government spending on basic health and education, which is a form of redistribution that is negatively correlated with inequality and positively correlated with growth.

The results of the paper show that the relation between inequality and growth is complex and that there might be a trade off between growth and the reduction of inequality specially in the short run: intuitively this might be due to the distortionary effects of taxation as advocated by neoclassical economic theory.

1.5 Social Conflict and Growth

According to neoclassical growth theory, because of the law of diminishing returns and with the help from the diffusion of technologies, poor countries should catch up
with rich countries: this phenomenon is also referred as convergence; anyway empirical evidence shows that this doesn’t always happens: for instance convergence has showed up for some countries in the East Asia while not in South America or Africa. Theories of conditional convergence explain the lower growth rates in poorer countries with the lower rates of accumulation in physical and human capital. In addition investment rates have a negative correlation with political instability, mainly because investments in such countries are more risky due to the possibility of expropriation of private properties. [Benhabib and Rusticini, 1996] develop a game theoretic model that describes the relation between levels of wealth, sociopolitical unrest and incentives in savings. In this way the four channels through which inequality affects growth: credit market imperfections, political economy, sociopolitical unrest and saving rates are included in a single framework.

In the model, organized groups can try to capture a larger share of output by appropriation or manipulation of the political system, this in turn will cause a disincentive in accumulation of assets because of the fear of expropriation by some other groups. The stability of political institutions has a key role in this context, because it prevents the risks of such expropriations; the point is that generally such stable political institutions are mainly prerogatives of rich countries, while in poor countries the risk of expropriation reduces investment and potentially leads to economic stagnation.

In this framework the relationship between growth and wealth distribution is not linear and there are multiple steady states according to different preferences and technologies, influencing the strategic behavior of groups of agents which in turn dampens growth both at high or low levels of wealth. There are essentially two opposite cases: when low wealth can lead to low growth and potentially to a growth trap, and the case when high levels of wealth, associated with binding incentive constraints, leads to lower growth.
[Benhabib and Rusticini, 1996] show that, without concerning on the wealth level, when incentive constraints are binding\(^4\), an egalitarian second-best equilibrium is also the fastest growth sub-game perfect equilibrium. In the case of low levels of wealth, where with a concave utility function the marginal utility of consumption is high, it is shown that, if the marginal utility of capital is not high enough, then the incentives to accumulation are reduced, especially if the level of consumption is low and the marginal utility from consumption is high, so that agents prefer to consume in the present instead of saving for the future, even if expected returns in the future are higher.

\[1.6\] The Possibility of Different Paths

In the previous sections were exposed some of the theories about the relationship between growth and income inequalities and vice-versa. If the evidence on the Kuznets’s curve is weak, it is possible, following [Accemoglu and Robinson, 2002], to find some different patterns in the behavior of different economies in the last centuries: in fact in some countries, particularly in Europe, inequality first increased with industrialization and then generally decreased at least up to some point in time around the 70’s or 80’s; in other countries like in East Asia in recent decades, there has been growth without generating too big inequalities; at the same time however, for some other countries, growth brought increasing inequalities as in the case of some South American countries. Things went even worse for a last group of countries, for whom growth didn’t take place at all like happened in most sub-Saharan African countries.

Two things come to mind when thinking about these different patterns: all the

\(^4\)For example because there is a real risk of expropriation
countries in cited above had different political institutions derived from different historical backgrounds, moreover development took place in different historical situations: for example, when the first and second Industrial Revolutions took place in Europe and U.S., generally it was a process mostly "beginning from the inside" meaning that it started right there, while on the contrary, when industrialization took place, say in East Asia or South America, the process had already been experienced in Europe almost half century earlier, hence there might be substantial differences in between.

An interesting theory explaining these different patterns is proposed by Acemoglu and Robinson [Accemoglu and Robinson, 2002], this theory also takes into account the four main channels discussed above through which inequality and growth are correlated. The main idea is that, when industrialization takes place, political and economic institutions have a strong role in shaping the future path of growth: in fact, basically two different situations can arise: in one case strong political institutions may foster more redistributive policies investing in redistributive policies, for example in higher level education, to promote further growth. In the opposite case, with no political concern about inequalities, the process of industrialization will increase the differences in wealth of the population in the first stage, at least until a point where the poor segments of the population will ask for more political power; subsequently, the increased equality in the political spectrum obtained by the poor will enforce more redistribution making inequality decline. In other situations, the poor will not be able to organize to obtain political powers and won’t be able to obtain redistribution. [Accemoglu and Robinson, 2002] define these two alternative paths other than the Kuznets’s curve as the East Asian Miracle and Autocratic Disaster.

In short, the model developed by Accemoglu and Robinson consists in a popu-
loration of agents that can either be rich or poor, since all agents in each group have the same preferences, each group can be treated as a single agent with a different weight according to the proportion of the population that is part of the group. To produce income, agents can use a market technology, which is more productive or an informal technology, less productive, but not subject to taxation. Agents decide how to allocate income in consumption or bequests; bequests are the next generation’s endowment.

Suppose that in the first period the political power is in the hands of the rich elite, the rich group will prefer a zero tax rate, so no redistribution will be implemented, while, if there is full democracy, the preferred tax rate will be chosen according to the median voter theorem. In the first case the poor, to obtain redistribution, can organize among themselves to start a revolution; such revolution, once started always succeed, but it is costly in the sense that a part of the capital stock gets destroyed in the revolution. The rich elite, in turn, in order to prevent the revolution, can extend the franchise to the poor or decide for higher redistribution; furthermore it is assumed that the effects of revolution and democracy are irreversible\(^5\).

The economy works as follows: at time \(t = 1\) education bequests are received by the next generation, the rich part of the population can decide weather or not extend the right to vote, in response the poor can decide whether or not to start a revolution; if revolution does not take place, then a tax rate is voted; subsequently capital is allocated in the formal and informal sector for producing an output; at the end, consumption and bequests are chosen. Some initial conditions, such as a minimum level of wealth, will ensure that rich are rich enough to leave bequest and that growth can happen.

Without the treat of a revolution there are essentially three possible paths: au-

\(^5\)This means that as democracy sets in, the country does not revert to autocracy.
tocracy and only the rich accumulate, autocracy and everyone accumulates, and democracy. In the first case only the rich are able to accumulate while the poor cannot, hence there will be increasing inequality. In the second case there is autocracy, but both rich and poor can accumulate, with diminishing marginal returns both groups will converge to the same stock of human capital, hence inequality will be decreasing and the level of output will be higher than in the first case where a fraction of agents were unable to save. The third case is democracy, the median voter theorem applies and the median income voter’s preferred tax rate is chosen. In this case, even with taxation, the rich will still be able to save, nevertheless in this case, whether the poor will be able to accumulate or not, depends on the presence of a sufficiently redistributive taxation: if transfers are sufficient, then the poor will be able to save and inequality will reduce, instead, if the poor are unable to accumulate, then inequality will increase despite the transfers. Anyway it is still possible that, with the increasing level of income of the rich, from some point on, transfers will be sufficient to ensure accumulation by the poor as well, and this will induce a decline in inequality. This last case is the more interesting since tells that, in the case of a democracy, inequality will rise up to some point then decline, as stated by [Kuznets, 1955].

It is interesting to note that, in absence of democracy, there is no Kuznets’s curve, inequality increases or decreases depending only on whether the poor are able to accumulate; in the case of democracy and redistributive taxation, inequality can still monotonically increase or decrease if the poor are so poor that are unable to save, but if the income of rich gets high enough, so that transfers allow the poor to save, then, from some point on there will be a decline in inequality.

Another interesting implication is that, in this model, inequality is harmful to development since a part of the population is not able to save. Democracy in turn has a positive effect only when redistribution allows the poor to save; the effect
tend to be negative otherwise, especially if costs of redistribution are high as also explained in [Persson and Tabellini, 1994] and in [Alesina and Rodrik, 1994].

The same framework can be analyzed according for the threat of revolution: the rich elite is still in power, but it may have to implement some redistribution in order to avoid a revolution. [Accemoglu and Robinson, 2001] cite different historical examples in which the extension of franchise was more due to the intention of avoid riots than to an egalitarian concern. In the model, a promise for redistribution is not credible since after the rich propose redistribution and the poor do not undertake the revolution, the elite can still cheat and adopt another level of transfers, so that, only under the constant threat of revolution, democracy will be adopted. Moreover, the revolution will take place only if the payoffs of the poor agents after the revolution will be higher than what they’ll get under autocracy: this case will happen more likely if a society is more inegalitarian and the rich elite is large enough to ensure a sufficient return from the revolution.

The threat of revolution can happen in two cases: when only the rich accumulate and when both groups do so. When only the rich accumulate, again there will be increasing inequalities up to some point when the revolution can take place if the payoffs of the poor are high enough, otherwise revolution will not take place and there will be increasing inequality. In the case when all agents accumulate, as seen before, inequality will tend to decrease and there won’t be any possible revolution in the model except in the first period or in the case the elite in a second moment excludes the poor from the accumulation process.

1.6.1 The Path of the Kuznets’s Curve

Accemoglu and Robinson, [Accemoglu and Robinson, 2001], argue that, if the economy starts ruled as autocracy, the rich accumulate and the poor do not, then
inequalities will increase up to some point; subsequently, if the threat of revolution is credible, the elite will extend political powers to the poor in order to avoid the revolution and from that point on inequality will decrease: this sequence of events seems to describe what happened in Britain, France, Sweden and Germany in the XIX century.

1.6.2 The Autocratic Disaster

The case of the autocratic disaster happens if the economy starts governed by the elite and the poor do not accumulate, but the revolution constraint does not bind\(^6\): in this situation democracy never sets in and inequality will rise. This path seems to describe what happened in some countries in Africa or in the Middle East, and now, because of the worsening of the condition of the poor, riots are becoming more and more frequent forming some sort of vicious circle.

1.6.3 The East Asian Miracle

In this case the economy start with autocracy, but in this case all agents are able to accumulate even without democracy, because the elite shares the profits of growth with the poor, this time inequality is destined to decline and growth is higher than in the previous case since there is more accumulation of capital from both groups. This seems to be the case of some East Asian countries like Thailand or China.

\(^6\)This means that the threat of revolution is not credible, for instance this can happen if the poor are not able to organize to start the revolution.
1.6.4 Revolution

In this case the economy starts ruled by autocracy, the rich accumulate while the poor do not, but at some point in time revolution becomes effective and subsequently the poor enforce redistribution. This case is different from the one of the Kuznets’s curve because now the role of the informal technology, upon which is not possible to levy taxes, is big relative to the market sector, hence redistribution is not sufficient and for the poor is better to engage in revolution\(^7\). It is interesting to note that this path can happen also as a consequence of the autocratic disaster.

The model proposed by [Accemoglu and Robinson, 2001] turns to be interesting because it’s able to explain the Kuznets’s curve where and when it happened, and at the same time explains other different possible paths that are consistent with what happened in other countries, indicating that the Kuznets hypothesis may be only one among other possibilities such as the autocratic disaster or the East Asian miracle; in this model, growth and development have different characteristics that are different from time to time and between countries. Kuznets itself [Kuznets, 1955] warned in the end of his article about the fact that it would have been dangerous to take the curve as a consolidate path which all countries will have to go through sooner or later.

1.7 Distributive Politics and Growth

In the previous sections were analyzed various contributions explaining the distributional consequences of growth and development and the effects that an uneven

\(^7\)It is possible to think about informal technology as education, in this case the big difference between the two groups of the population lies in the education, on which it is impossible to levy taxes, in this case it is this difference that move the poor to engage in the revolution.
income distribution has directly on growth through the four channels listed above, but in the same time it is possible to note that inequality influences growth even through another mechanism. This time it is redistributive politics itself that retard growth; [Alesina and Rodrik, 1994] find in this case a strong negative correlation between growth and income inequalities.

In the analysis of [Alesina and Rodrik, 1994], a high level of inequalities, through the political process generates a demand for redistribution, larger redistribution imply higher taxation, which by it’s nature is distortionary and has a negative effect on the economy because affects negatively the incentives to produce and save. Although in this way inequalities are shown to retard growth, on the other side there is the fact that inequalities, without the implementation of redistributive policies may retard growth through other mechanisms.

Furthermore, what is important in the outcome of the voting process is the composition of the electorate: [Alesina and Rodrik, 1994] argue that a large middle class may have a positive effect on subsequent growth, by intuition this makes sense: since the tax rate is chosen by the median income voter, the larger the middle class the more the median voter will represent such population, hence the policy chosen will benefit a larger amount of agents. To the contrary, in the case of a small middle class with large groups at the extremes of the wealth distribution, the median voter will represent fewer individuals, hence the policy chosen will benefit only a smaller amount of population. This fact highlights the importance of considering also how inequalities are spread among the population.

The insight from this model is that not only there are effects from growth to inequalities, but there is also a reverse effect from inequality to growth, this because higher inequalities will induce a higher demand for redistribution that, in turn, creates distortions in the economy. In short, the higher the inequality, the higher the distortions in the economy generated by taxation, the more there will
be a negative effect on the economy.

This evidence reminds of the substantial impossibility of the second welfare theorem which basically states that it is impossible to choose and implement different Pareto-Efficient equilibria, even if we admit they were theoretically attainable, simply with appropriate lump-sum wealth transfers, because of the impossibility of application of lump-sum taxes.

1.8 Does Democracy Imply Full Redistribution?

In the previous analysis, in discussing the political economic mechanism linking income distribution and growth, the median voter theorem had a central role in the analysis: if there is full democracy and the median voter has an income below the mean income, when voting on a unidimensional space over a tax rate, under the assumption of single peaked preferences, all voters on the left of the median voter will balance with the ones on the right of the median, hence the median voter’s preferred tax rate is chosen; this imply that, when the median voter income is lower than the mean, it will be chosen a tax rate that implement full redistribution.

This theorem has been widely used as a benchmark case in theoretical analysis, but in reality it is common to note that radical redistribution is quite an exception: even if redistribution is chosen, this it is only to a limited degree. On this regard [Perrotti, 1996], [Alesina and Rodrik, 1994] find that, in analyzing how income distribution affects growth, the fact that a country is democratic or not has an insignificant effect on subsequent growth. This fact, although seems contrary to common intuition, can be explained by the fact that in a democratic society redistribution is implemented only to a limited extent, while in an autocratic society, to the contrary of theoretical prediction, some redistribution takes place.
anyway in order to prevent social riots as seen before in the theory proposed by [Accemoglu and Robinson, 2002].

There are three main arguments trying to explain why democratic preferences lead to redistribution only to a limited extent, one is concerned with the political process, one focuses on the effect of redistribution to the tax base\(^8\), the other with the self-interest of the poor.

### 1.8.1 The Political Process

Concerning the political process retaining a poor majority to implement full redistribution, it is also necessary to distinguish between a direct democracy and a representative democracy framework. In a direct democracy framework, a first explanation of the limited degree of redistribution is the relative limited participation to the vote of the poor majority compared to the rich elite; in fact what generally happens is that, due to some cost of vote or indifference, not 100% of population do vote, and since the incentive to vote is positively correlated with an agent’s income, as shown by [Conway, 1985] and [Esdall, 1985], if relatively few poor voters participate to the elections, then the pivotal voter will be more to the right side of the income distribution.

Another explanation relates to the fact that in reality the policy space is not unidimensional, then a single Condorcet-winner simply does not exist, in this case it is possible that voter form coalitions and the rich elite can "bribe" the middle class by according them some transfers in order for them not to sustain full redistribution.

[Harms and Zink, 2003] propose the following interesting example: suppose there are three groups with given income: \(y_1 < y_2 < y_3\), and that the median income is below the mean: \(y_2 < \frac{1}{3} \sum_{i=1,2,3} y_i = \bar{y}\). The groups can choose between

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\(^8\)This effect is also referred as the distortionary effect of taxation, which was explained above.
a 100% tax rate with revenues equally divided among them (full redistribution),
and a scheme where only $y_3$ is taxed at $t = (\bar{y} - y_2)/y_3$ and person 2 gets the
proceeds as an individual transfer such that his post tax income is equal to the
average and nothing is given to the poor. While the middle class in this case is
indifferent between the two schemes, the higher income group will prefer the sec-
ond scheme for it implies less taxation since nothing is given to the poorest group,
hence it would be reasonable for the rich to bribe the middle group to prefer the
second option, mining in this way the outcome of democratic process.

In a representative democracy framework, in adjoint to the previous factors,
politicians are not fully committed to their electors and they are subject to pres-
sures from some others interest groups that are more likely to be expression of
the richer part of the electorate: the incentive for lobbing and other activities are
again positively correlated with the level of income; the result is that the outcome
of the political process will be a chosen redistribution lower than the one predicted
by the median voter theorem.[Harms and Zink, 2003]

1.9 The Effect of Redistribution on the Tax Base

One of the main issues with the median voter theorem lies in the distortions
caused by taxes in the production decisions of an agent: if an agent can modify
his behavior in response to the level of taxation, a tax on income will reduce his
work supply reducing overall income and then tax revenues.

Denoting the median income as $\tilde{y}$, the median voter indirect utility function
is:

$$\tilde{V}(t) = U((1-t)\tilde{y} + ty);$$  \hspace{1cm} (1.2)
which is maximized at $t = 1$ if $\tilde{y} < \bar{y}$; but if $\bar{y}$ is decreasing in $t$ the optimal tax rate will be $t < 1$. [Meltzer and Richard, 1981] demonstrate that, with different exogenous productivity and endogenous labor supply, no agent will choose complete redistribution because this will drive the tax base to zero, furthermore the higher the productivity of the median voter, the lower will be his preferred tax rate since his personal loss is increasing with innate productivity.

1.9.1 "The Self-Interest of the Poor"

These were some of the major mechanisms operating at the political level altering the prediction of full redistribution given by the median voter theorem; however there might be other reasons for the the poor not to implement full redistribution even if if they could. One of these motivations concerns the agents expectations about the future and is related to the prospect of upward mobility hypothesis (POUM), proposed by [Hirschman, 1973]: an agent expecting his future income to grow will make him more likely to tolerate present inequalities, hence when voting over a tax rate he is not only concerned with his present income, but also with his future income$^9$; hence in the case of positive expectation about the future, he will choose a tax rate lower than the one he would prefer looking only at his current income. However, there is not full consensus on this hypothesis, in particular [Bénabou and Ok, 2001] argue that this effect might be dominated by the demand of social insurance; in this case the decisive voter chooses a higher level of redistribution because of risk aversion.

$^9$This expectations are typical of a growing economy.
1.10 The Determinants of Preferences for Redistribution

In investigating the linkages between growth and redistribution it is useful to ask also what determines the demand for redistribution. Is it only the poor that asks for a more equal society or also the rich part of society seeks for a reduction in inequalities, even if this contrasts with wealth maximization? Do other regarding preferences have a role in deciding the appropriate level of taxation? A deeper understanding of this forces may allow to better explain why different countries display different levels of inequalities and how this interacts with economic growth.

On this topic, [Corneo and Grüner, 2000] found three competing forces: "the homo economics effect", the "public values effect" and "the social rivalry effect". What have been discussed so far, is in large part related to the "homo economics effect", and the median voter theorem is a clear example of this: all agents select the level of taxation in order to maximize their utility which so far was a function of income: the lower the income, the higher the desired level of redistribution; even when the rich elite extends the right to vote or allow for larger redistribution, it does so because it is maximizing his utility: in fact it faces a trade off between reducing inequality or facing the risk of social riots.

It is possible however that individuals are concerned not only with their personal income, but also with the wealth of a society as a whole, and preferences for a more equal society may reflect this concern for social welfare. According to the "public values effect", other motivations affect an individual’s preferences for redistribution: for example, someone who thinks that the family background is important in determining an individual’s future income, will favor more redistribution; the same is true for someone who believes that luck has a relevant role in deciding his wealth; to the contrary, if an individual thinks that his wealth derives
mainly from his effort, he would more likely oppose redistribution.

Another mechanism is the "social rivalry effect": according to this, an individual’s preferences for redistribution depend on the relative living standards of the individual: one individual may in fact derive some utility from his "social ranking" or "social position" which is certainly influenced by his or her relative income and the government intervention for reducing inequality may affect negatively this status. For example, assuming that redistribution in the form of scholarships allows poor students to enroll in university among the sons of wealthy individuals, the parents from the elite may dislike this if it does diminish their feeling of being part of an elite.

[Corneo and Grüner, 2000] tried to test these three effects on twelve countries using data from the Social Survey Programme (ISSP)\textsuperscript{10}, they found that there is a significant negative correlation of an individual’s relative income and his preferences on political redistribution according to the "homo economics effect".

Considering the "public values effect" they find again that this effect is strongly significant: this means that an individual who believes that hard work is the key in obtaining a good wealth status will more likely oppose redistribution; to the contrary agents believing that family background matters, will opt for a more egalitarian society.

Finally the "social rivalry effect" is significant as well, with high income classes displaying less favor for redistribution in comparison to adjacent income classes. These findings suggest that, when investigating the demand for redistribution in different countries, the "homo oeconomicus effect" is only one of the mechanisms shaping individual preferences for a more or less equal society, and other factors may be at work.

\textsuperscript{10}1992 Social Inequality II Module
1.11 The Director’s Law

When someone thinks about redistribution, generally the idea is that of a process through which money is collected mostly from the higher income groups of the population and transferred in different ways, such as the provision of services and direct transfer and other means, to the poorest part of population with the objective to reduce wealth inequalities. However, this is not what happens in most cases in reality. Aaron Director noted almost 50 years ago that public expenditures are mainly benefiting the middle classes.

Director’s intuition is the following: since the government is the only entity that has coercive power to collect taxes and redistribute them, the coalition in power will choose the redistributive program that will benefit the most his part, it turns out that, under some conditions as in the median voter case, the coalition in power will be the middle class. In this case, redistribution will be a process that mainly benefit the middle class to the detriment of both the tails in the income distribution: relatively higher taxes for the rich and relatively fewer services for the poor.

[Stigler, 1970], in order to provide some confirms of Director’s law, gives some examples of policies that benefit the most the middle class: here are some: sometimes redistribution takes the form of tax exemption, institutions that largely benefit tax exemption as churches, medical and educational institution are mainly directed to the middle classes; another example is farm assistance through raising prices, the ones who mainly benefit from these policies are farm owners, belonging to a middle class, rather than workers.

The Director’s law is based on two things: the formation of a majority coalition and the possibility for the government to levy taxes and to redistribute them. A coalition is usually formed on different basis: it can be environment, it can be religion as happens for example in muslim countries, it can be also wealth. It is
reasonable to believe that a political party, trying to capture the higher number of votes at the election, will pose itself as much as possible in the middle of the political spectrum, as in Hotelling’s minimum differentiation principle: hence it is more likely that the middle class will be better represented in the government. It is clear that, once in office, the representers of the middle class will implement policies that relatively benefit more its voters than other groups.

Another important issue is the possibility to enforce taxes and to redistribute them, and this aspect has changed considerably through time: for example, in the XIX century in the U.S., a tax on income was applied for the first time in 1861, before that time revenues were mainly from customs duties, excises and commodity taxation; moreover the government was offering fewer services than nowadays and these were directed in larger part to higher income groups. [Stigler, 1970] argues that in the XIX century, being both taxes and services unrelated to income, it is not surprising that the state was implementing so little redistribution. In the XX century instead, on one side income has been a more and more frequent a preferred base for taxes, on the other side the provision of services expanded and the ability of the state to provide those services to a larger part of the population also improved: hence the government needed more resources.

In short, redistribution is a phenomenon closely related to the use of income as a tax base and to a large provision of services to the whole population; however this phenomenon is something that took place only from the XX century on; this fact might be helpful to explain why the Kuznets’s curve appeared in some countries and not in others depending on when a tax on income was introduced. The idea expressed in [Stigler, 1970] is that, in a first phase of industrialization, when the state provides fewer services and taxes are more likely to be collected from excises and commodity taxation\textsuperscript{11}, inequality may rise because there is not

\textsuperscript{11}This happens also because are easier to calculate and collect.
substantial redistribution of the benefits from growth; afterwards, as development
taxes place and more resources are required\textsuperscript{12}, the tax base shifts to income and
more services are provided by the government, and these services are the channels
through which redistribution takes place in this case. This reduction of inequality
generally happens benefitting in major part the middle class at the expenses of
the tails of the income distribution.

1.12 Inequality at which Level?

Following the analysis of [Picketty, 2014b], when talking about inequalities it’s
important to distinguish between inequalities inside each nation and inequalities
between nations. Of course at the global level overall inequality is composed by
the two, but sometimes inequalities at the national level tend to be cancelled out
at the global level and, to the contrary, sometimes they tend to be exacerbated.

If we look in figure 1.3 at the distribution of output at the global level from
1700 to 2012, where GDP is expressed in purchasing power parity in order to make
possible some sort of comparison between continental blocks.\textsuperscript{13}

At first glance it appears that, at the global level, some convergence is in fact
in act for all continents excluding Africa, which has almost the same share of
global output since 1820. For the rest of the continents it is possible to see the
effects of the industrialization that took place in Europe before anywhere else: in
fact European output increased from a 30\% in the XVII century to about half
of global output on the eve of the first world war, then declined almost steadily
toward 2012 accounting for something around 25\% of global output. America

\textsuperscript{12}It is the case of the increasing need for infrastructures during a development phase.
\textsuperscript{13}However differences in inequality between countries take different dimensions, hence it is
impossible to fully compare inequalities only looking at GDP expressed in purchasing powers.
came a bit later, although the pattern is in some sort masked by the differences between North America and Latin America; however from 1820 there has been a significant increase in the share of global output that reached its peak around 1950, with a share of about 40% of the total, then declined to a 30%. The large increase in the share of European and American output came generally at the expenses of Asia, which sees its share decreasing from 60% in 1700 to only a 20% in 1950 for then regaining a share of a bit more than 40%. In the graph it is clear the role of industrialization that first took place in Europe, then in America and in the end in Asia; at the global level, if we exclude Africa, the convergence between continental blocks it’s evident: at the global level it appears that growth followed the predictions of the Kuznets’s curve, first inequalities were increasing with industrialization, then were gradually decreasing in the following stages of development.
If we look at the evolution of per capita GDP in figure 1.4, the dynamics are much more attenuated by the demographic evolution: pro capital inequalities grew for Asia until 1950 then started declining, in turn the share of Europe and America reached it’s peak in 1990 and it’s now declining. The case of Africa, that in the graph it’s grouped with Asia, it’s a lot different, with a stable share of global output since 1700 and a little increase in its percentage of world population it is possible to deduct that no convergence of per capita GDP is in act for Africa at all, moreover the negative trend of Africa might cancel out part of the good performance of Asia.

Comes natural at this point to ask which sources determined convergence, in this regard marginal economic theory says that, because of decreasing marginal productivity of capital, if capitals are free to flow from one country to the other and in absence of restrictions or at least without too strong restrictions; rich countries,
where capital stock is high and marginal productivity low, will seek to invest in poorer countries where marginal productivity of capital is high, searching for better returns: this in turn will increase productivity in poor countries favoring convergence. However, as [Picketty, 2014b] points out, this mechanism might imply convergence in output per head, but not necessarily the convergence of income per head: rich countries may own poor countries\textsuperscript{14}, hence increasing their own income per head in a major proportion than the increase in the income per head of the poor countries; something similar to what happened during the colonialism in some ways. In this perspective, capital mobility was not the major force promoting convergence; what favored it in Asian countries like Taiwan, Thailand or South Korea, was in major part the fact that these countries invested their own resources in the creation of the necessary physical and human capital, and this appears to be the true key of their rapid growth. This is almost evident if we compare Africa, where little investment in human capital has been done, and major resources like mines are owned by rich countries and Asia, where massive investments in education were maid. Asian countries in the end benefitted more from free trade and the diffusion of knowledge and technology than from free capital flows [Picketty, 2014b]. This is somehow similar with what happened in the early stages of the Industrial Revolution, where entrepreneurs at the beginning started to finance themselves for the necessary capitals and a big role in the economic progress was due in larger part to the diffusion of knowledge, derived from the recent scientific innovations, that was diffused by important institutions like schools and universities that were created in the period.

The issue of inequality inside nations, on which is the focus of this work, it may not explain completely the global dynamics of wealth inequalities, but relies on a much broader set of data, hence there are more information, and can give

\textsuperscript{14}This phenomenon is referred as Neocolonialism
many insights on the dynamics of growth and wealth inequalities, moreover the
differences across nations can give us information about which economic policies
proved efficient in the past and whose were not, and furthermore on the different
paths taken by different nations due to the peculiarities in the developing process
of each country.
Chapter 2

Wealth and Growth

2.1 Growth and Wealth Inequalities

In this chapter, in analyzing the dynamics of growth and wealth inequalities, I will follow the approach adopted by [Picketty, 2014b] and [Romer, 2006]; specifically, growth will be decomposed in population growth and the growth of GDP per capita, for the reason that, although both contribute to the general dynamics of growth, they come from different sources. In fact, ceteris paribus, national GDP\(^1\), can grow either because of a demographic increment: more people earn the same income, either because of a growth in per capita production: the same number of people product and earn more.

A similar distinction should be made about inequalities: in particular, instead of focusing on Gini coefficient for a given nation\(^2\), it is possible to use and compare percentiles of the income distribution, that can tell how much wealth is in the hand of the top 1% wealthiest of the population, how much in the top 10%,

\(^1\)At the moment, national GDP, although other measures of economic welfare are under development, is the best indicator of economic development available and for which there are sufficient data from the past.

\(^2\)The Gini coefficient gives us a really synthetic information, but tells nothing about how wealth is in fact distributed.
how much on the bottom 90% and so on; in this way it is possible to have a more complete idea about the distribution of income. Furthermore wealth is decomposed in income from capital an income from labour; this distinction is of some importance in this contest, specially once we consider that capital generally offers a better return than labour: hence the different sources of wealth can also be important in accounting for the increasing of inequalities. In particular, in this chapter it is considered the ratio between the stock of capital and the flow of income from labour, this has also an interesting interpretation since it also tells how many years of income from labour are necessary to equate the stock of capital.

2.2 The Composition of Wealth

In analyzing the relationship between the distribution of wealth and growth it is important to understand how wealth is composed: wealth can have many sources, but, for simplicity, we can group them into three broad categories: inheritance, income from labor and income from capital [Picketty, 2014b].

The role of these three components is fundamental for the dynamics of inequalities: let’s think of the fact that inheritance is leaved as a bequest from one generation to another, hence inequalities are only translated ahead in time: of course inequalities in endowments can increase in time, due to more accumulation of wealth, or decrease: it can be the case of capital destruction during a war, or again the case of inflation that reduces the value of fortunes accumulated if not invested properly. The role of income from labour is much different, in this case the fact that, with education, a high number of individuals from potentially every social class can acquire more skills and competence and afterwards work as high skilled workers, can be a factor helping in reducing inequalities. Finally the role
of income from capital, in this simplification, it can be assumed to work in an opposite way from the latter: the function that remunerates capital in fact can be thought as being convex: larger amounts of capital are remunerated in a more than proportional way than smaller amounts; this seems to be reasonable and it is what is captured by the function $h$ in the model presented in chapter 3 in equation (3.26). For this reason it can be imagined that a higher share of capital in the capital labour ratio has a positive role in increasing inequalities.

### 2.2.1 Inheritance

In substance, one individual has basically three ways for accumulating wealth: income from capital, although for this he already needs some endowment; income from work, which depends mostly on skills and knowledge; and inheritance, that depends on wealth accumulated by his relatives in the previous generations. Inheritance, even if can increase or decrease trough generations according to how wealth is managed by the previous generations, ceteris paribus, it can be thought as translating inequalities in the future. Of course inheritance can grow trough time by means of investment, in this case accumulated wealth can grow, if properly invested, but can decrease as well, for example because of inflation: in fact, if not invested properly, the purchasing power of it can be eroded by a high inflation as happened mostly in the period following the first world war; or can undergo violent shocks, as again was the case of the period following the two wars, because of the massive destruction of private property.

One last reason for the large drop in the proportion of inheritance in total wealth was the large demographic growth that took place in almost all developed countries in the period that followed the wars: the population and the economy as well experienced rapid growth in the period following the fifties, in this case,
CHAPTER 2. WEALTH AND GROWTH

even if the value of inheritance remains the same, the divisor of the ratio, in this case national income, grows bigger, hence the ratio decreases consequently. Nevertheless inheritance represented in the past and now a big share of national income [Picketty, 2014b].

![Figure 2.1: Annual flow of inheritance as share of national income in France from 1820 to 2010. Source: [Picketty, 2014a](#).

As depicted in figure 2.1, the share of inheritance in French national income\(^3\), was pretty large until the beginning of the century, representing about 20% of national income, then it collapsed reaching the bottom in 1950 where accounted for only about 4%, and it’s regaining importance in the last decades. The sharp drop is due mostly on the high inflation and the destruction of properties that followed the wars, and also on the economic policies that aimed primarily to the reconstruction. Nowadays inheritance still represents a large share of national income, about 16%; this increase certainly happened at the expenses of the other two sources of income, hence it is possible to see that the role of accumulated

\(^3\)In figure 2.1 inheritance is represented both as economic flow and fiscal flow.
wealth is growing in importance again in the latest decades.

This change in the structure of wealth represents a major change in the structure of inequalities; in fact, other thing being equal, when inheritance forms a larger share of national income, it is reasonable to suppose that the task of reducing inequalities becomes harder [Picketty, 2014b].

### 2.2.2 Income from Capital

A second source of an individual’s fortune is income from capital, to be sure to make an investment, one already needs some capital to invest, but it is reasonable to assume, as explained in the model in chapter 3, that there are some convexities in the function of returns on capital. Intuitively, an individual which owns more capital can access investments offering better returns than an individual whom owns less: this is due in large part to the fact that often there are some minimum capital requirements for some investments and to the presence of asymmetric information in the market of capital. Generally, an individual with more economic means has some advantages in spotting better investment opportunities than another; furthermore, investing more wealth is possible to implement a better diversification of investments, hence reducing the risk of bearing big losses [Picketty, 2014b].

To have an idea of the evolution of the structure of capital ownership in the past, we can look at figure 2.2 where are shown the shares in capital ownership of the richest 1% and 10% in Europe and United States. Again the pattern is something similar to the pattern of inheritance: there was a peak at the beginning of the XX century, then a drop until 1970, then the shares are increasing again for both groups in both nations.

---

4It is reasonable to assume that capital is remunerated in a more than proportional way.
2.2.3 Income from Labor

The last main source of wealth is income from labor: the total of annual national income is composed by income from labor and income from capital\(^5\), hence the two will follow symmetrically opposite patterns: when the share of capital income increases, the share of labor income decreases and vice-versa.

In figure 2.3 it is graphed the evolution of the shares of capital and labor income as percentage of national income, it is possible to see that income from capital, as seen before in the case of Europe and United States, suffered a sharp decrease right after the first world war, when accounted for almost 40\% of French income and fell by almost half of his value to 20\%; afterwards it recovered a bit, then fell

\(^5\)Income from capital obviously account also for the role of inheritance.
again until 1970, after that regained a level of almost 30%; the share of income from work followed a symmetrically opposite pattern, accounting for as much as 80% in 1915 and 1970 and now it is a little above 70%.

In conclusion, wealth changed radically its composition during the last century for different causes, among the main causes there are the economic shocks of two world wars and the economic policies that followed these events pursuing the reconstruction; nevertheless all these changes, it is possible to see that wealth is somehow regaining a composition quite similar to the one it had at the beginning of the XX century [Picketty, 2014b].

Following the analysis of [Picketty, 2014b], one variable on which is interesting to focus is the capital income ratio, defined here as $\chi$, that expresses the importance of the stock of capital for a given nation at a point in time; this ratio can be also interpreted as the number of year of national income of a given nation necessary
to produce the stock of capital. To have an idea of some orders of magnitude, suppose $\chi = 6$, which is approximately the value of the ratio in the case of Italy nowadays, this means that the total stock of capital is equal to 6 year of national income. This ratio is linked to the rate of return from capital $r^k$ from the identity:

$$\alpha = r^k \times \chi; \quad (2.1)$$

which, in the long run, is determined as:

$$\chi = \frac{s}{g}; \quad (2.2)$$

from the last equation it is easy to note that the $\chi$ ratio depends negatively on the rate of growth: $g$, and positively on savings: $s$: if a country saves more, it will increase its stock of capital as it’s clear; on the other side, if a nation experience low growth, the relative importance of capital, hence accumulated wealth, will be greater.

In figure 2.4 from [Picketty, 2014b] is depicted the evolution of the capital/income ratio for Germany, Britain and France from 1870 to 2010: the interesting fact is that the ratio reached its top before the first world war, then reached its lower point in 1950: right after the second world war, for then regaining importance in last decades reaching values similar to those prior 1910. This pattern suggest that the wars and the subsequent policies may had a strong role in shaping the dynamics of the ratio.

Another insight that this graph might suggest is that the large values attained by the ratio in the years prior to 1910 might had an important role in the causes of the great sociopolitical instability of the time, that then culminated in the war, something similar to what was previously discussed in section 1.3.3 about the sociopolitical unrest.
The importance of capital in a given country can be reassumed by the capital/income ratio, furthermore this ratio has interesting implications in the dynamics of inequalities: at least, the higher the ratio, the more inequalities in distribution of wealth will be persistent, not to mention the fact that the higher the importance of capital in a given society, the more inequalities are likely to be larger.

This ratio, as expressed in equation (2.2), depends negatively on the rate of growth of the economy and positively on the rate of savings: the higher is the rate of growth, the higher will be the flow of national income in proportion to the capital stock.
2.3 Growth

After having investigated the composition of wealth, it’s now time to turn the attention to the composition of economic growth. In some respects, growth is one of the topics on which economists focused more in the past, to quote Robert Lucas (1988): "Once one starts to think about [economic growth], it is hard to think about anything else" [Romer, 2006].

Inevitably there is a lot of talk about growth: it’s one of the major issues in economic news, but not often it is analyzed in detail; for sure, steady economic growth it’s a relatively recent phenomenon: from the beginning of human history, the average economic growth rate must have been reasonably small [Picketty, 2014b] until the XVIII century, afterwards the rate of growth started to be positive and constant although with some slowdowns. During the fifties, the rate of growth for developed countries was so high and constant to the point that economists started thinking that it was a standard fact for developed countries, endogenous technological progress being the key for constant growth; nevertheless growth came to a slowdown in the last decades in most of developed countries.

Here, following [Picketty, 2014b], I briefly discuss two things about growth not always taken into account: it’s composition and the effects on the long run of even a small rate of growth due to the compound effect. Basically, growth of national income can be composed in two components: percentage economic growth of per capita output: $g^y$, and percentage demographic growth: $g^d$, such that the overall percentage of growth of national income $g^Y$ is:

$$g^Y = g^d + g^y.$$  \hspace{1cm} (2.3)

\footnote{Endogenous growth theory argues that growth is not exogenous, but it’s determined by human capital and innovations.}
2.3.1 The Composition of Economic Growth

Let’s see how this two components evolved trough history, first I take in consideration the population growth: of course historical data for ancient times are only hypothesis, nevertheless the values indicated by [Picketty, 2014b], both for population and pro-capita output, seem to be reasonable at least for the fact that a higher rate of growth, because of its compound effect during time, on which I’ll say something more later, is incompatible with the values we observe now and some minimum values of subsistence in ancient times: for example, a higher average rate of growth from antiquity to now implies or that population was really small in the beginning, or that the current population would be much higher.

![World population growth rate estimates](image)

**Figure 2.5: World population growth rate estimates.** Source:[Picketty, 2014a].

In figure 2.5 are shown the estimates of population growth from year 0 to 2012 and the U.N. projections until 2100. Population growth was below 0.5% until the XIX century, then it rose sharply, with a sharp increase as big as 1.9% in
the middle of the last century: something that would have looked catastrophic to Malthus at the end of the XVIII century, when population growth rates were in the order of 0.6%; however the demographic explosion ended in the nineties, and according to projections of the United Nations, the rate of demographic growth is expected to decline and reach levels similar to those prior the Industrial Revolution. It happened that, what looked as an unsustainable increase in population, with the means available on the eve of the industrialization, was made possible by the increase in the productivity, specially in the agricultural sector, that made available a larger amount of food.

The increase in productivity that happened from 1700 on, introduces us to the second component of growth: the growth of per capita output. Again the data for ancient times are only approximation, but could be enough to give an idea of the pattern followed by this variable.

Figure 2.6: Estimates for the rate of growth of world per-capita output. Source:[Picketty, 2014a].
In figure 2.6, the effect of the Industrial Revolution is even more striking, the rate of growth in per capita output has been close to 0 for almost all human history since 1700, it rose sharply thereafter, and it’s expected to reach a peak in the near future; nevertheless the projections shown in [Picketty, 2014b] estimate that it will start to decrease in the second half of the century; anyway, leaving projections aside, it is interesting to focus on the dynamics of the last 5 centuries.

Comparing figures 2.5 and 2.6 it is possible to note that growth in per capita output always followed the demographic dynamics; this is consistent with the endogenous growth models as in Aghion-Howitt [Romer, 2006], where technological innovation is positively correlated with population growth in a probabilistic fashion: more individuals basically translates, ceteris paribus, in more chances for new technologies to be discovered.

So far we have seen that population and economic growth followed a similar pattern in the past, in table 2.1 it is clear how the two variables affected global economic growth, in particular it is interesting to note that, in the last century, demographic dynamics accounted for almost half of economic growth; at this point it is natural to question what will be the growth rate in the future, in particular in light of the fact that demographic growth is expected to decrease [Picketty, 2014b].

2.3.2 The Compound Effect of Economic Growth

Let’s now turn to the second issue about growth cited above: the compound effect of growth. As stated before, growth is a phenomenon relatively new in human history, regarding at most only the last five centuries, nowadays a rate of growth of 3% is considered as normal for developed countries\footnote{However in the last decade, after the economic crisis, a rate of growth in the order of 3% is highly above the expectations.}, it is also assumed as a
standard parameter in E.U. economic policies for a member state, but on one hand, such a high rate of growth was a character only in the last century; on the other hand, in a larger time perspective, this is an enormous rate of growth if we consider the compound effect it might have. The fact is that for the rate of growth is valid the same behavior of compounded interest rates, hence an annual rate of growth in the order of 3%, although considered the norm, can induce radical transformation in societies [Picketty, 2014b].

To give some examples, an annual growth rate in population of about 2.5% a year means an increase in population after 30 years of about 110%, which means that, in the time span of one generation, population more than doubles; even a smaller rate of growth in the order of 1% a year means an increase of 35% after 30 years, and that population almost triples in 100 years; the same reasoning obviously apply to economic growth, even if it’s easier to understand the magnitude in terms of population.

This should lead to the conclusion that such rapid economic growth might be something extraordinary that took place only in the last centuries and nothing

<table>
<thead>
<tr>
<th>Average annual growth rate</th>
<th>World output</th>
<th>World population</th>
<th>Per capita output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1700</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>1700-2012</td>
<td>1.6%</td>
<td>0.8%</td>
<td>0.8%</td>
</tr>
<tr>
<td>incl.: 1700-1820</td>
<td>0.5%</td>
<td>0.4%</td>
<td>0.1%</td>
</tr>
<tr>
<td>1820-1913</td>
<td>1.5%</td>
<td>0.6%</td>
<td>0.9%</td>
</tr>
<tr>
<td>1913-2012</td>
<td>3.0%</td>
<td>1.4%</td>
<td>1.6%</td>
</tr>
</tbody>
</table>

Table 2.1: Estimated composition of world growth rate from year 0 to 2012. Source:[Picketty, 2014a].
indicates that it will be the norm in the future, leaving aside concerns about the sustainability of a such large increase. In particular, in investigating the dynamics of growth and inequalities, when looking at the future, it makes little sense to assume that growth will continue at such a high rate; this fact might also cast some doubt on the dynamics of the Kutnetz curve. In fact, even if we assume that growth has some positive effects in reducing inequalities, the fact that growth might be only transitory leaves open the question of how inequalities will evolve in a future of slow growth.

2.4 Rates of Growth and Rates of Return

So far we took a brief review of the dynamics of wealth and growth in their forms, the most interesting fact was that a high grow regime was a characteristic only of a large part of last century, while the average rate of growth was far below 1% until the middle of the XIX century as indicated in figure 2.7, and it’s expected to fall again in the years to come [Picketty, 2014b].

At the same time, the return on capital, even after deducing taxes and depreciation, was significantly above that level for a long time and was smaller than the rate of growth only for several decades in last century.

In figure 2.8 are plotted the rates of world economic growth and the average rate of return on capital after deducing taxes and accounting for capital losses; it is evident that estimated returns on capital were significantly higher than the rate of growth of the global economy for most of the period under exam, except in the XX century. Intuitively the reason for the return on capital to be higher than the rate of growth of the economy is that, in the opposite case, if agents have time preferences, for which certain immediate consumption should be more preferred to uncertain consumption of the same amount in the future, then there will be too
Figure 2.7: Average rate of growth of world output. Source: [Picketty, 2014a].

Figure 2.8: Returns of capital (after taxes and capital losses) and rate of growth of world output. Source: [Picketty, 2014a].
little incentive to save.

An interesting point of view concerning the fact that the rate of growth is smaller than the rate of return is expressed by [Picketty, 2014b], that considers the fact that \( r^k < g \) as one of the major forces in amplifying inequalities in the distribution of wealth. The intuition of the author is the following: if the return of capital is to be higher than the rate of growth of the economy, it only takes to reinvest in capital a small fraction of wealth for the latter to increase at the same pace of the economy; this fact, combined with the likelihood that returns from capital are more than proportional to it’s size, turns out to be a major force in increasing inequalities. Furthermore it might be interesting to note that during the years in which [Kuznets, 1955] developed his theoretical model\(^9\), are the same years in which the rate of return from capital fell below the rate of growth of the economy, a fact that according to Picketty contributed to a very large extent to the reduction of inequalities in the period considered by Kuznets\(^{10}\).

\(^8\)For a proof of this inequality, see Appendix A.
\(^9\)In fact, as mentioned before according to Kuznets’s theory, as economic growth takes place, inequalities first increase for some time and then decrease in the advanced stages of economic development.
\(^{10}\)This fact again highlights the necessity of considering each economic theory inside the historical contest in which it was developed.
Chapter 3

Theoretical Framework

In this chapter I will use the model from [Fiaschi, 2007], other contributions are from [Romer, 2006] and [Solow, 1956]; the model is an exemplification of preexisting models and it is a good tool to explain some of the dynamics listed above. In particular, it will be left aside the role of voting and redistribution, and the focus will be on the role of an imperfect capital market for the evolution of inequalities and growth.

3.1 The Basic Model

3.1.1 The Allocation Between Savings and Consumption

The basic model consist in an economy consisting in $N$ dynasties formed by one representative individual $i$, born at time $t$, that gets a bequest $b_i^t$ from his parents. Each bequest represents the individual’s endowment, which is divided in consumption $c_i^t$ and investment $s_i^t$ to get:

$$b_i^t = s_i^t + c_i^t;$$  \hspace{1cm} (3.1)
Each agent then, has some income from work \( w_{t+1} \) and from capital \( r_{t+1} \). Assume for simplicity that the endowment of labor for dynasty \( i \) is 1 so that the aggregate stock of labor is \( N \); furthermore let’s assume that labor supply is anelastic. At time \( t + 1 \) individual \( i \) allocates his income according to:

\[
y^i_{t+1} = r_{t+1}s^i_t + w_{t+1} = c^i_{t+1} + b^i_{t+1}; \tag{3.2}
\]

Each agent gets an income from work plus interests from past savings and subsequently allocates his wealth in consumption \( c^i_{t+1} \), and bequests \( b^i_{t+1} \).

Suppose now individuals have log-linear utility function:

\[
U^i = \log(c^i_t) + \beta[\log(c^i_{t+1}) + \rho \log(b^i_{t+1})]; \tag{3.3}
\]

Each agent then chooses a level of savings and future bequests in order to maximize his utility according to the first order conditions to get:

\[
b^i_{t+1} = \frac{\phi r_{t+1} b^i_t + w_{t+1}}{(1 + \phi)(1 + \rho)}; \tag{3.4}
\]

\[
s^i_t = \frac{\phi b^i_t - w_{t+1}/r_{t+1}}{(1 + \phi)}; \tag{3.5}
\]

\[
y^i t + 1 = \frac{\phi}{1 + \phi}[r_{t+1} b^i_t + w_{t+1}]; \tag{3.6}
\]

where \( \phi = \beta(1 + \rho) \).

Let’s note in equation 3.5 how savings depend negatively on the ratio between work remuneration and capital returns: \( w_{t+1}/r_{t+1} \); the higher the first compared to the returns on capital, the lower will be the incentive to save.

Substituting in the utility function we get the indirect utility function:
\[ U^t = (1 + \phi) \log(r_{t+1}b_t^i + w_{t+1} - \log(1 - r_{t+1}) + D, \quad (3.7) \]

here, \( D \) is a constant in function of the parameters chosen in the model.

### 3.1.2 The Production Side

Following [Romer, 2006], it is determined the remuneration of capital and labor. In an economy composed by many firms and a constant-return technology, where the average level in the stock of capital has a positive effect on the profits of a single firm in order to take into account the likelihood of positive externalities from a higher average stock of capital, the profits of firm \( j \) are:

\[ y^j_t = A(\bar{k}_t)^{\beta}(l^j_t)^{\alpha}(k^j_t)^{1-\alpha}; \quad (3.8) \]

In equation 3.8, \( A \) is a parameter to take account of the effectiveness of the factors, \( k^j_t \) is the stock of capital of firm \( j \) and \( \bar{k}_t \) the average stock of capital. With such a production function, \( \beta \geq \alpha \) is a necessary condition for long-term growth: as seen before, savings depend negatively on the ratio between work remuneration and capital returns.

To simplify, if we suppose \( \alpha = \beta \), the model reduces to an \( AK \) model with constant interest rate\(^1\) so that the overall production becomes:

\[ Y_t = AK_t \quad (3.9) \]

and the factors remuneration:

\[ r_t = (1 - \alpha)A, \quad (3.10) \]

\(^1\)This is a special case of a Cobb-Douglas production function.
\[ w_t = \alpha AK_t \tag{3.11} \]

### 3.1.3 The Growth Rate of the Economy

In this model, following the intuition of Solow [Solow, 1956], in a closed economy, the level of savings in the previous period will determine the aggregate stock of capital in the next period, so we have:

\[ S_t = K_{t+1} = \frac{\phi(1 - \alpha)B_t}{1 + \phi(1 - \alpha)}. \tag{3.12} \]

Plugging (3.12) in the equations for the remuneration of factors, in period \( t+1 \) we get:

\[ r_{t+1} = (1 - \alpha)A, \tag{3.13} \]

\[ w_{t+1} = \frac{\phi\alpha(1 - \alpha)AB_t}{1 + \phi(1 - \alpha)N}; \tag{3.14} \]

indicating with \( B_t \) the sum total of bequests and with \( s_t \) the total of savings at time \( t \).

It follows that the bequests to the next generation will be:

\[ b_{t+1}^i = g \left[ b_t^i + \frac{\alpha\phi(\bar{b}_t - b_t^i)}{1 + \phi} \right]. \tag{3.15} \]

where \( B_t/N \) is the average bequest in the economy, hence the rate of growth of the overall economy will be:

\[ \frac{K_{t+1} - K_t}{K_t} = g = \frac{\rho\phi(1 - \alpha)A}{(1 + \rho)[1 + \phi(1 - \alpha)]}. \tag{3.16} \]

Still we have to keep in mind that, in this model, because of the assumption
\( \alpha = \beta \), the rate of growth of resources will be equal to the rate of growth of income from labor. Bearing this in mind, it is possible to find a lower bound for the interest rate necessary to have a positive incentive in savings: imposing the condition that the rate of growth must be bigger than one: \( g > 1 \) we have:

\[
(1 - \alpha)A > \frac{(1 + \rho)[1 + \phi(1 - \alpha)]}{\rho \phi}.
\]

(3.17)

In the end the level of bequests at time \( t + 1 \) it will be:

\[
B_{t+1} = gB_t.
\]

(3.18)

From this model it follows that how resources are distributed across individuals has no effect on the level of savings, this is due to the fact that all agents get rewarded from productive factors at the same rate\(^2\), hence individuals leave bequests directly proportional to their income, and savings are a linear function of what they get as bequest.

### 3.1.4 The Distribution of Resources

From the previous analysis we can derive that:

\[
\bar{b}_{t+1} - b_{t+1}^i = g\psi(\bar{b}_t - b_t^i);
\]

(3.19)

here \( \psi = [1 + \phi(1 - \alpha)]/(1 + \phi) < 1 \).

From the previous equation it follows that the difference in endowments in the subsequent period will tend to 0 for \( g\psi < 1 \) and will tend to get larger in the other case. Furthermore, if we then express inequalities as the ratio between the endowment of an individual and the average endowment, we get:

\(^2\)In fact in this case what matters for growth is the total level of bequests, not its distribution.
An important conclusion is that, since $\psi < 1$, inequalities will tend to decrease until complete equality is reached. Unfortunately this conclusion is not supported by empirical evidence, in the following sections it will be relaxed the assumption of the same rate of reward for productivity factors and see how this affects the model.

### 3.2 Incomplete Market of Capital

In this section, again following the exposition of [Fiaschi, 2007], we will exploit the fact that, in an imperfect capital market, it is possible that different productivity factors will have different rates of return: this fact was at the basis of the previous conclusions in a perfect capital market: hence, in this case, the growth rate it will be allowed to depend also on the distribution of resources. Notwithstanding, it is shown in [Fiaschi, 2007] that, if savings are a concave function on income, then inequalities are a non-increasing function of time; to take account of the effects of initial inequality on income, we have to assume furthermore some convexities in the production function. The previous model is now modified to take account of this second case to investigate the dynamics of inequalities: in particular there will be considered two kind of investments: one with higher returns that requires an investment above some minimum threshold, and a second investment with lower returns that is available to everybody.

Let’s take the utility function given from 3.3, but now there are two kinds of investment, one gives a constant return on savings $s^i_t$, the other investment: $e^i_t$ rewards according to the function $h$. It can be though that the constant return

\[
\frac{b^{i}_{t+1}}{b_{t+1}} = \psi \frac{b^i_t}{b_t} + 1 - \psi.
\]  

(3.20)
derives from capital invested in typical activities of a poor economy (agrarian); furthermore each agent chooses how to allocate his labor $l_i^t = 1$ in poor-skill job rewarded at rate $w$, and a high-quality job again rewarded according to the function $h$, where the return depends also on the resources invested in this activity: this accounts for the role of human capital: in fact a person who invested in knowledge it is supposed to make a higher quality job and earn generally more.

Another crucial assumption is that in this case there are credit constraint, hence it is not possible to borrow, such that investment $s_t^i$ has to be nonnegative:

$$s_t^i \geq 0 \quad \forall i, \forall t. \quad (3.21)$$

In this case the consumption will be:

$$c_t^i = b_t^i - s_t^i - e_t^i, \quad (3.22)$$

and in the second period:

$$c_{t+1}^i = rs_t^i + wl_t^i + (1 - l_t^i)h(e_t^i) - b_{t+1}^i, \quad (3.23)$$

so that the bequests will be:

$$b_{t+1}^i = \frac{\rho [rs_t^i + wl_t^i + (1 - l_t^i)h(e_t^i)]}{1 + \rho}. \quad (3.24)$$

If we plug equation 3.24 into the utility function we get the new indirect utility function:

$$U^i = \log(b_t^i - e_t^i - s_t^i) + \phi \log(rs_t^i + wl_t^i + (1 - l_t^i)h(e_t^i)) + D; \quad (3.25)$$

where $D$ is again a constant as function of the parameters of the model.
CHAPTER 3. THEORETICAL FRAMEWORK

Now we will focus on the case in which the rewarding function $h$ has some convexities: in this case investment $e^i_t$ will give a higher reward than $s^i_t$ only above a minimum level $\varepsilon$, while for lower levels it will give no reward. It is reasonable to assume that that investing time in $e^i_t$, which can be seen as education, gives a better return than the poor-skill job $w^i_t$. We model the $h$ function following [Fiaschi, 2007] as:

$$h(e^i_t) = \begin{cases} 
0 & \forall t < \varepsilon \\
Ae^i_t & \forall t \geq \varepsilon 
\end{cases} \quad (3.26)$$

The differential in returns is obtained imposing the condition:

$$A > r, \quad (3.27)$$

and the better reward of investment $e^i_t$ as:

$$Ae^i_t > w. \quad (3.28)$$

Under these assumption each agent can earn income working in a low-skill job, or investing in low returns activity $s^i_t$, or investing in some entrepreneurship or in some kind of education requiring both time and some investment above a minimum ledge $\varepsilon$. The individual’s maximization problem becomes now:

$$\max_{e^i_t, s^i_t, l^i_t} U^i = \log(b^i_t - e^i_t - s^i_t) + \phi \log(rs^i_t + wl^i_t + (1 - l^i_t)h(e^i_t)) \quad (3.29)$$
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\[
\begin{align*}
  h(e_i^t) &= \begin{cases} 
    0 & \forall e_i^t < \bar{e} \\
    Ae_i^t & \forall e_i^t \geq \bar{e}
  \end{cases} \\
  s.t. & \\
  s_i^t \geq 0; \\
  s_i^t \leq 1.
\end{align*}
\]

(3.30)

For the complete solution of the problem we refer again to the appendix in [Fiaschi, 2007]; it is shown that, if an individual invests in education, it does not allocate time in unqualified job: hence if he chooses not to invest in \( e \) the optimal choice will be:

\[
((l_i^t)^*, (s_i^t)^*, (e_i^t)^*) = \left(1, \max\left[0, \frac{\phi r b_i^t - w}{r(1 + \phi)}\right], 0\right),
\]

whether if he chooses to invest in \( e \):

\[
((l_i^t)^*, (s_i^t)^*, (e_i^t)^*) = \left(0, 0, \max\left(e, \frac{\phi b_i^t}{1 + \phi}\right)\right) \text{ for } b_i^t > e.
\]

(3.31)

(3.32)

Since the discontinuity in the \( h \) function, it is not possible to take the derivatives to determine the optimal values of investment; defining the difference in utilities between investing in education or not as: \( U^e(b_i^t) - U^s(b_i^t) \), it follows that \( i \) will invest in \( e \) if \( b_i^t \geq b^* \) which is defined from the equality: \( U^{**}(b^*) = U^{**e}(b^*) \): in other words, each agent will invest in education if it has the necessary resources to make it more profitable than in the other case. If we take a closer look at the utilities of both strategies, that, in the case without investment in education becomes:

\[
U^s(b_i^t) = \log\left(b_i - \max\left[0, \frac{\phi r b_i^t - w}{r(1 + \phi)}\right]\right) + \phi \log\left(r \max\left[0, \frac{\phi r b_i^t - w}{r(1 + \phi)}\right] + w\right);
\]

(3.33)
CHAPTER 3. THEORETICAL FRAMEWORK

while with investment in education:

\[ U^e(b_t^i) = \log \left( b_t - \max \left[ \epsilon, \frac{\phi b_t}{1 + \phi} \right] \right) + \phi \log \left( A \max \left[ \epsilon, \frac{\phi b_t}{1 + \phi} \right] \right). \quad (3.34) \]

From this it’s possible to observe some facts: first that the difference between the two utilities is increasing in the level \( b_t \): this is basically due to the convexity of the \( h \) function, that is accessible only above a minimum level of wealth \( b^* \). Moreover, the necessary minimum resource to invest in the more remunerative factor depends negatively on the level of \( \epsilon \); in fact the negative component of \( \epsilon \) is larger than the positive one: the higher the level of the minimum threshold of the investment in \( e \), the smaller will be, ceteris paribus, the necessary endowment to make the investment in education profitable.

Another thing we can note is that \( b^* \) depends positively on the rate of return from the constant interest rate and non-skilled job’s wage, because \( r \) and \( w \) have a positive effect on utility in the case of non investment in \( e \). Equation (3.24) provides the dynamics of accumulation in each dynasty, now according to the level of endowment, each generation will have to take different investment strategies. The dynamic of accumulation in this case becomes for the case of non investment:

\[ b_{t+1}^s = \max \left[ \frac{\rho w}{1 + \rho}, \frac{\rho \phi (rb_t^i + w)}{(1 + \rho)(1 + \phi)} \right] \quad \text{for} \quad b_t^i < b^*; \quad (3.35) \]

and for the case of investment:

\[ b_{t+1}^e = \max \left[ \frac{\rho Ae}{1 + \rho}, \frac{\rho \phi Ab_t^i}{(1 + \rho)(1 + \phi)} \right] \quad \text{for} \quad b_t^i \geq b^*. \quad (3.36) \]

At this point, as modeled by [Fiaschi, 2007], it is assumed that long-run growth it is possible only trough investing in \( e \), the idea is to model the fact that long-run growth it is possible only trough the accumulation of technology as it is also shown
in the model of growth from [Solow, 1956]. This assumption allows us to model the case of the poverty trap presented above, a case that somehow represents the situation suitable for most of the world before the industrialization; the relevant condition is therefore:

\[ r < \frac{(1 + \rho)(1 + \phi)}{\rho \phi}, \]  

(3.37)

and for the technological sector:

\[ A > \frac{(1 + \rho)(1 + \phi)}{\rho \phi}; \]  

(3.38)

This condition states that the return on capital invested in the poor sector is smaller than the inter-temporal discount rate of future consumption and endowments: individuals in this case have too little incentive to save, while in the other case the major return from technology is a sufficient incentive to continuous accumulation of capital.

So, if an individual has not enough resources \( b_i < B^* \), he is constrained to accumulate according to equation (3.35). [Fiaschi, 2007] analyzes the behavior of the function in (3.35) inside the interval \([0, \infty)\) for the possible values of interest: it is shown that there is a fixed point depending on the value of \( r \):

\[ \forall r \in \left[0, \frac{1 + \rho}{\rho \phi}\right] \quad b^{R,F} = \frac{\rho w}{1 + \rho}, \]  

(3.39)

and

\[ \forall r \in \left(\frac{1 + \rho}{\rho \phi}, \frac{(1 + \rho)(1 + \phi)}{\rho \phi}\right) \quad b^{NR,F} = \frac{\rho \phi w}{(1 + \rho)(1 + \phi)}. \]  

(3.40)

In the first case the agent has not enough endowment to invest in education

\[ \text{[However in the model of Solow the accumulation of technology is exogenous.]} \]
and will leave a bequest independent from $r$ equal to \( \frac{\rho w}{(1 + \rho)} \); in the second case instead, as $r$ appears with negative sign in the divisor, it increases the disposable wealth: the individual does not consume everything in consumption and bequests because it has enough incentive to save for the next period because of the sufficient high level of the interest rate. However it is necessary to understand that this is a result of the assumption of the imperfections of the market of capitals for which the individual cannot borrow, hence $r$ represents only the return on savings, not the interest on debt: so an increase in $r$ will have only a positive effect on wealth, whether allowing for savings to be negative this will lead to a major credit constraint for the individual. However this will not change the dynamic because there will be no growth in the long run anyway [Fiaschi, 2007].

On the other side, for a level of wealth $b_i \geq b^*$, agents will accumulate according to equation (3.36): in this case, because of the assumption that technology allows long-run growth, wealth will increase in this case at a constant rate:

$$g = \frac{\rho \phi A}{(1 + \rho)(1 + \phi)} > 1.$$ (3.41)

### 3.2.1 The Dynamics of Accumulation

The dynamics of accumulation are now illustrated using the graph 3.1 as in [Fiaschi, 2007]; since there are two possible patterns depending on the interest rate, they will be consider separately.

Let’s take the first case where:

$$r \in \left[ 0, \frac{1 + \rho}{\rho \phi} \right)$$

$$b^{R,F} = \frac{\rho w}{1 + \rho}.$$ (3.42)

This situation it is illustrated in figure 3.1, the black line illustrates the path of
the dynamic of accumulation: when individual wealth it is not sufficient to save, indeed: $b^{R,F} < b^*$, the dynamics of the agents will differ depending on the level of the initial wealth; for classes with an endowment lower than some level $b^*$, then wealth will converge to $b^{R,F}$, while other classes with wealth above that level will experiment long run growth $g$, as stated by equation (3.41).

![Equilibrium with formation of social classes. Source: [Fiaschi, 2007].](image)

It is shown in [Fiaschi, 2007] that, in this case, if it is valid that $\epsilon > \frac{w}{\phi r}$: so, if discounted income from labor is too low, every individual will see his wealth converging to $b^{R,F}$. On the other side, if it’s the case that $b_t > b^*$, then there will be growth. Finally, in the case that the minimum level of investment lies in the interval:
\[
\frac{w}{\phi r} < \bar{e} \leq \left[ \frac{w \phi}{(1 + \phi) r} \right] \left[ \left( \frac{A}{r} \right)^{\frac{\phi}{1+\phi}} - 1 \right]^{-1} ; \tag{3.43}
\]

it is possible to find an explicit solution for \( b^* \):

\[
b^* = \frac{w}{r} \left[ \left( \frac{A}{r} \right)^{\frac{\phi}{1+\phi}} - 1 \right]^{-1} . \tag{3.44}
\]

The interesting fact here is that if education requires a minimum amount of investment \( \bar{e} \), larger than the discounted wage rate \( w/(\phi r) \), then the wealth of each agent will always be smaller than the necessary endowment \( b^* \) for which the incentives for the investment in the advanced sector are profitable; hence long-run growth will not take place. This dynamic allows in the long run to the polarization of the population around two different social classes: one invest in the high return sector while the other finds occupation as non-qualified worker.

The crucial point here is that, for some interval of values of the minimum level in the high-return sector as in equation (3.43), a rise in the wage will increase the minimum level of bequest \( b^* \) beyond which it is profitable to invest in the more remunerative sector: a higher wage in the end increases the wealth of the agent, but at the same time it decreases the incentive to invest in education precluding long-term growth.

Instead, in the other case, with the interest rate sufficiently high, in particular, when \( r \) is inside the interval:

\[
r \in \left( \frac{1 + \rho}{\rho \phi} , \frac{(1 + \rho)(1 + \phi)}{\rho \phi} \right) ,
\]

which admits as fixed point:

\[
b^{NR,F} = \frac{\rho \phi w}{(1 + \rho)(1 + \phi)} ,
\]
that corresponds to the case where individuals do not face credit constraints; again the dynasties will follow different paths according once more to their initial wealth, but this time with different directions from the previous case.

As shown in figure 3.2, the black line describes the dynamic of accumulation for this case, for levels of initial wealth below some threshold $b_{NR:F}^* < b^*$, the dynasties will see their wealth converging to $b_{NR:F}^*$, while for initial wealth again above $b^*$ there will be long-run growth at rate $g$.

In detail, assuming moreover that:

$$\frac{w}{(\phi r)} < \epsilon \geq \frac{w\phi}{(1+\phi)r} \left[ \left( \frac{A}{r} \right)^{\frac{\phi}{1+\phi}} - 1 \right]^{-1},$$

(3.45)
this condition provides that it is possible to shift from one kind of investment to another without any constraint on the level of both investment, then it follows that if:

\[ A < r^{-\frac{1}{\phi}} \left[ \frac{(1 + \rho)(1 + \phi)}{\rho \phi} \right]^{\frac{1+\phi}{\phi}} \]  

(3.46)

then every class with an initial endowment below:

\[ b^* = \frac{w}{r} \left[ \left( \frac{A}{r} \right)^{\frac{\phi}{1+\phi}} - 1 \right]^{-1}, \]  

(3.47)

will have their wealth converging to \( b^{NR,F} \), otherwise as introduced before, they will experience growth \( g \).

In the last case where:

\[ A > r^{-\frac{1}{\phi}} \left[ \frac{(1 + \rho)(1 + \phi)}{\rho \phi} \right]^{\frac{1+\phi}{\phi}}, \]  

(3.48)

then every class, without any regard to their initial endowment will experience growth.

The case without credit constraint: \( b^{NR,F} \), it’s of particular interest for some reasons: first there is still a path in which wealth increases also for the individuals who do not accumulate in the long run because they cannot invest in the technological sector\(^4\). Second, if we interpret the low return activity as poor agriculture and assuming that investment in agriculture benefit \( r \), we get a situation similar to what it was before the industrialization, where the increase in the productivity in the agricultural sector was the source for creating the necessary capital for the development of the industrial sector as it was discussed in the previous chapters.

In fact, as accumulation grows in the short-run, an increase in the rate of return

\(^4\)They see their wealth converging to \( b^{NR,F} \).
$r$ can make it profitable to switch to the high technology sector, allowing in this way for long-run growth.

Another interesting point in this analysis is the reciprocal relationship between growth and the distribution of resources: with credit constraints there is likely to be a polarization between classes where some accumulate and some others do not, to the point that one class furnishes only low-skilled labor and the other invests in the high tech sector which can consist both in human capital or in technological activities with higher remuneration. In this situation are likely to rise social conflicts because growth will benefit only the rich part of the population, while the other will experience stagnant wages. However, another thing interesting to note is how the distribution of wealth determines labor choices in one sector or the other. In this case the distribution of resources has a high importance in determining the choices of the agents.

On the other side, when credit constraints are not binding, there can happen redistributive dynamics: in the sense that classes with enough endowment to invest in the more profitable sector will experience long-run growth since the beginning; while for the other classes there will be the chance to escape the poverty trap due to an increase in the return from the poor sector that will make it possible to switch to the investment in the more profitable channel, in this way the poor will put themselves on the long-run growth path as well. Moreover, in this situation the overall growth experienced will be higher than in the case of growth with credit constraints due to the fact that the same rate of growth is shared by a major part of the population.

Furthermore it is possible to interpret the part of the model describing the path of growth when there are not credit constraints in a Kuznets fashion: initially the poorer classes will experience smaller growth compared to the richer classes that can benefit from investment in a sector with higher profits, this will increase
inequalities for some time, but, in the meantime, the poorer classes will be able to save enough to invest in the sector with high technology and they will start to experience long-run growth as well, from this point on inequalities will tend to decrease.

Another important issue is the fact that, if wealth is too uniformly distributed and all classes are too poor to invest in the more remunerative sector $b_i^t < b^* \quad \forall i)$, then long-run growth doesn’t take place in the economy, this means that for a poor economy, some initial concentration in the distribution of endowments may be necessary to enhance growth\textsuperscript{5}.

### 3.2.2 Redistribution and Efficiency

The last step is to introduce in the model redistributive politics, this will allow to investigate the effects of redistribution and the issue of efficiency. Following the work of [Fiaschi, 2007] for the exposition, it will be made another assumption: let’s suppose the possibility of levy taxes in order to transfer resources from elder to younger generations. Let’s define with $\tau \in [0, 1]$ a proportional tax rate on income, and $E_t$ and $S_t$ the total amounts of resources invested in education and savings. Furthermore, let’s suppose to be in the case described above of credit constraints, where equation (3.42) is binding: this is the more interesting case for redistributive politics since for the poorest agent it is impossible to accumulate, and it is specially in this case that redistributive politics are needed. If we order agents on the basis of their endowments, starting from the poorest to the wealthiest, such that $b_i^t < b_j^t \quad \forall \quad i < j$, there will be some individual $\bar{i}$ for which it will be indifferent to invest in education or in regular savings. Now, since the introduction of a tax rate $\tau$, the total amount of endowment for individual $i$ it will be:

\textsuperscript{5}This is due by the requirement of some minimum capital necessary to the investment.
\[
\hat{b}_{i,t+1} = b_{i,t+1} + \tau \bar{Y}_t,
\]

where \( \bar{Y}_t \) is the average income in the economy:

\[
\bar{Y}_t = \frac{AE_t + rS_t + (N - \bar{n}_t)}{N}.
\]

Furthermore, let's define as: \( y(q_{it}) \) the income from investment \( q_{it} \) where:

\[
y(q_{it}) = \begin{cases} 
rq_{it} + w & \text{if } q_{it} = s_{it}, \\
Aq_{it} & \text{if } q_{it} = e_{it}.
\end{cases}
\]

(3.51)

It is possible now introduce the effect of taxation in the utility function in equation (3.3); hence the utility now becomes:

\[
U^i = \log(\hat{b}^i_t - q^i_t) + \beta \left[ \log((1 - \tau)y(q^i_t) - b_{i,t+1} + \rho \log(b^i_t + 1)) \right],
\]

(3.52)

Now, individual \( i \) will look for an optimum level of bequest in time \( t + 1 \) in order to maximize his utility:

\[
b_{i,t+1} = \left[ \frac{(1 - \tau)\rho}{1 + \rho} \right] y(q^i_t),
\]

(3.53)

and the total amount of resources at disposal of the heir will be:

\[
\hat{b}_{i,t+1} = \left( \frac{1}{1 + \rho} \right) [\rho(1 - \tau)y(q^i_t) + (1 + \rho)\tau \bar{Y}_t].
\]

(3.54)

From equation (3.54) it is possible to see the role of redistribution: in this case the endowment of individual \( i \) at time \( t + 1 \) it’s a weighted average between inheritance and the average income in the economy: the higher the taxation, the
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higher will be the role of the average income in the economy in increasing the endowment.

Substituting in the utility function, we get the indirect utility:

\[ U^i = \log(\hat{b}_i^t - q_i^t) + \phi \log((1 - \tau)y(q_i^t)) + D. \]  \hspace{1cm} (3.55)

Now, the individual, according to whether he invest in education or in the basic investment, will implement the optimal choices:

\[ e_i^t = \left( \frac{\phi}{1 + \phi} \right) \hat{b}_i^t, \] \hspace{1cm} (3.56)

or:

\[ s_i^t = \frac{(\phi r \hat{b}_i^t - w)}{(1 + \phi)r}. \] \hspace{1cm} (3.57)

With a logarithmic utility function, the introduction of taxation does not affect the investment choices: in substance what changes it’s the disposable income for the individual, but the level of endowment for which it is optimal to switch from investment in \( s \) to \( e \) remains the same as expressed in equation: (3.47).

At the level of the economy, the investment in the high return sector will be:

\[ E_t = \left( \frac{\phi}{1 + \phi} \right) \hat{B}^e_t; \] \hspace{1cm} (3.58)

while the investment in the regular sector will be:

\[ S_t = \frac{\phi r \hat{B}^s_t - (N - \bar{i}_t)w}{(1 + \phi)r}; \] \hspace{1cm} (3.59)

where \( \hat{B}^e_t \) and \( \hat{B}^s_t \) indicate the total wealth of individuals investing in \( e \) or \( s \).

It is now possible to solve for \( \hat{B}^e_{t+1} \) to get the level of bequests leaved to individual \( i \) in the next period.
\[
\hat{b}_{t+1}^i = \frac{(\rho + \tau)}{(1 + \rho)} N\bar{Y}_t = \frac{\phi(\rho + \tau)[A\hat{B}_t^e + r\hat{B}_t^s + (N - \bar{i}_t)w]}{(1 + \rho)(1 + \phi)}. \tag{3.60}
\]

In equation (3.60) it is possible to see that taxation generally has, other things being equal, a positive effect on accumulation: \(\tau\) in fact appears only in the numerator of the equation.

However if the average endowment is below the threshold \(b^*\):

\[
\frac{(\rho + \tau)\bar{Y}_t}{(1 + \rho)} < b^*, \tag{3.61}
\]

then an uniform redistribution will cause all bequests to be smaller than the minimum amount of wealth: \(\hat{b}_{t+1}^l < b^* \forall i\), for which it is convenient the investment in the high-productivity sector; in this situation, the variation in average income will be:

\[
\phi \frac{[\bar{i}w - (A - r)(\hat{B}_t^e - \hat{B}_t^s)]}{(1 + \phi)N}, \tag{3.62}
\]

that will be negative if the gains lost by not pursuing the more profitable investment are not compensated by the raise in wages: \((\hat{B}_t^e - \hat{B}_t^s) < 0\). For this reason, in this case redistribution can have a negative effect on growth; this is basically what we have seen in the previous paragraph when noted that some inequality may, under certain circumstances, foster economic growth.

In this model, the individual’s pattern of accumulation will depend again on the level of initial wealth, this time with the difference that the accumulation function will be less steep at a factor \((1 - \tau)\), and will shift upward of \(\tau\bar{Y}_t\).

The accumulation function for the individual in fact will be in this case:

\[
\hat{b}_{t+1}^i = \left[ \frac{(1 - \tau)\rho \phi}{(1 + \rho)(1 + \phi)} \right] y(\hat{b}_t^i) + \tau\bar{Y}_t. \tag{3.63}
\]
As shown by [Fiaschi, 2007], it is possible to note that the necessary condition to have long-run growth is:

\[
\frac{\rho \phi A (1 - \tau)}{(1 + \rho)(1 + \phi)} > 1 \Rightarrow \tau < 1 - \frac{(1 + \rho)(1 + \phi)}{\rho \phi A},
\]  

(3.64)

hence the tax rate has an upper limit, otherwise each individual will have his wealth converging to a constant value.

The condition for avoiding the formation of social classes is that each individual should have an endowment sufficient to invest in the high-tech sector and hence avoid segmentation, in this case:

\[
b^* < \hat{b}_F^t = \frac{(1 + \rho)(1 + \phi)}{(1 + \rho)(1 + \phi) - \rho \phi r (1 - \tau)} \left[ \frac{\phi \rho w (1 - \tau)}{(1 + \rho)(1 + \phi)} + \tau \bar{Y}_t \right].
\]  

(3.65)

It is possible to see that \( \hat{b}_F^t \) depends positively on the average income, according to the intuition that, in developed economies, it is sufficient a smaller level of taxation and hence less distortion are introduced in the economy.

From the model appears convenient for the government to finance redistribution with debt, and begin to tax individuals only after they are able to invest in the high-remuneration sector, this will allow to escape the so called poverty-trap.
Chapter 4

Empirical Findings: the Case of U.S.

This chapter aims to investigate the relationship between inequalities and economic growth in the case of the United States, the reason to limit the analysis to a single country is twofold: on one side the availability of larger dataset will allow to consider a wider time span, on the other side, it will allow to focus more on the country specific situation, perhaps leading interesting results; however, focusing the analysis on a single country, which moreover reached a development stage before the time period under investigation, can have also some drawbacks: in fact the variables under consideration may have little variations compared to the panel data analysis as in [Barro, 2000] and [Forbes, 2000].

This chapter is divided in two main parts, the first part is more descriptive and aims to track the behavior of economic growth and inequalities in the past century in U.S.; this will give interesting information on how wealth and its distribution evolved through time; this section also investigates for the presence of the Kuznets’s curve during the period under exam. The second part of this chapter focuses more on the econometric analysis of the effects of inequalities and education on subsequent growth.

The chapter is organized as follows: in section 4.1 there will be a description of
the data used in the rest of the chapter; in section 4.2 there will be a descriptive analysis of the evolution of inequalities and growth; in section 4.3 there will be an econometric analysis of the effect of inequality and education to economic growth, following the model presented in [Forbes, 2000], limiting the analysis to the case of United States.

### 4.1 Database Description

In this section there is an illustration of the variables used in the rest of the chapter, in table 4.1 are illustrated: the name of the variables, a brief description of the variable and the source.

Although most variables have a straightforward interpretation, some variables like the Gini coefficient and the Inverted Pareto-Lorentz coefficient need some explanations: the two variables are almost similar in measuring inequality, the use of the first rather than the other was dictated by the availability of data for a larger time period.

The Gini coefficient is a measure of inequality in a distribution and it’s values are comprised between the interval $[0, 1]$, which takes value 0 in case of perfect equality and 1 in the case of perfect inequality; in this case the coefficient it is computed for a discrete population of number $n$ with values of income $y_i, i−1,..., n$, where values $y_i$ are indexed in increasing order $y_i ≤ y_{i+1}$.

$$Gini = \frac{1}{n} \left( n + 1 - 2 \sum_{i=1}^{n} (n + 1 - i)y_i \right) \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} y_i}.$$  

(4.1)

The Inverted Pareto-Lorentz Coefficient needs a deeper introduction, being its calculation more complex\(^1\). The reason for the utility of the Inverted Pareto-Lorenz coefficient lies in the fact that historical data are not always present for every

\(^1\)For a detailed explanation I refer to [Antony Atkinson, 2011].
## CHAPTER 4. EMPIRICAL FINDINGS: THE CASE OF U.S.

### Table 4.1: Variables Description

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Income</strong></td>
<td>Natural Log of real GDP per capita at constant 2005 national prices in millions of 2005 U.S. $</td>
<td>Penn World Table 8.0</td>
</tr>
<tr>
<td><strong>Growth</strong></td>
<td>Log Difference of real GDP per capita at constant 2005 national prices in millions of 2005 U.S. $</td>
<td>Penn World Tables 8.0</td>
</tr>
<tr>
<td><strong>Inequality</strong></td>
<td>Inequality measured by Gini coefficient</td>
<td>Federal Bureau of Economic Research</td>
</tr>
<tr>
<td><strong>Male Education</strong></td>
<td>Percent of male population who have completed high school</td>
<td>U.S. Census Bureau</td>
</tr>
<tr>
<td><strong>Female Education</strong></td>
<td>Percent of female population who have completed high school</td>
<td>U.S. Census Bureau</td>
</tr>
<tr>
<td><strong>Male High Education</strong></td>
<td>Percent of male population with a college degree</td>
<td>U.S. Census Bureau</td>
</tr>
<tr>
<td><strong>Female High Education</strong></td>
<td>Percent of female population with a college degree</td>
<td>U.S. Census Bureau</td>
</tr>
<tr>
<td><strong>Total High Education</strong></td>
<td>Percent of total population with a college degree</td>
<td>U.S. Census Bureau</td>
</tr>
<tr>
<td><strong>Top 1% Average Income</strong></td>
<td>Average income of richest 1% of the income distribution (incl. capital gains)</td>
<td>World Top Income Database</td>
</tr>
<tr>
<td><strong>Top 5% Average Income</strong></td>
<td>Average income of richest 5% of the income distribution (incl. capital gains)</td>
<td>World Top Income Database</td>
</tr>
<tr>
<td><strong>Top 10% Average Income</strong></td>
<td>Average income of richest 10% of the income distribution (incl. capital gains)</td>
<td>World Top Income Database</td>
</tr>
<tr>
<td><strong>Bottom 10% Average Income</strong></td>
<td>Average income of poorest 10% of the income distribution (incl. capital gains)</td>
<td>World Top Income Database</td>
</tr>
<tr>
<td><strong>Pareto-Lorentz Coefficient</strong></td>
<td>Inequality measured with the Inverted Pareto-Lorentz coefficient</td>
<td>World Top Income Database</td>
</tr>
<tr>
<td><strong>Price Level of Investments</strong></td>
<td>Price level of capital formation U.S. GDP in 2005=1</td>
<td>Penn World Tables 8.0</td>
</tr>
</tbody>
</table>
year and for every group of income; moreover income groups changed over time, hence there is the need to interpolation of the missing values through a probability function. The function used for the interpolation is the Pareto distribution, which is determined by the coefficient $\alpha$, that is the Pareto Coefficient. Since the Inverted Pareto-Lorenz coefficient is the inverse of the Pareto coefficient, the first it is used in the work rather than the second because it is increasing in inequality and has a straightforward interpretation.

I start illustrating the Pareto-Lorenz coefficient, for then illustrate the direct relation between the Pareto coefficient and its inverse.

The Pareto law for top incomes it is given by the cumulative distribution function $F(y)$ for income $y$:

$$1 - F(y) = (k/y)^\alpha$$

(4.2)

where $(k > 0, \alpha > 1)$; $\alpha$ is the Pareto coefficient.

The corresponding density function is hence given by:

$$f(y) = \frac{\alpha k^\alpha}{y^{1-\alpha}}$$

(4.3)

The main property of this distribution is that the ratio of average income $y^*(y)$ of individuals with income above $y^*$ to $y$, does not depend on the level of income $y$:

$$y^*(y) = \alpha y/\alpha (\alpha - 1),$$

(4.4)

Hence:

$$y^*(y)/y = (\alpha/1 - \alpha) = \beta.$$ 

(4.5)
Where $\beta$ it is the Inverted Pareto-Lorentz coefficient, if for example $\beta = 3$ the average income of individuals with income above 100,000$ is 300,000$.

The relation between the two measures comes from the fact that the Gini coefficient is the measure of twice the area comprised from the perfect equality line and the Lorentz curve, hence the Gini coefficient for the Pareto distribution is:

$$Gini = 1 - 2\left(\int_0^1 F(y)dy\right) = \frac{1}{2\alpha - 1} = \frac{\beta - 1}{\beta + 1}. \quad (4.6)$$

### 4.2 The Evolution of Inequality and Growth in the U.S. Over the Last Century

In this section I will expose the evolution of the wealth distribution and economic growth in the United States over the last century.

In figure 4.1 it is plotted the Inverted Pareto-Lorenz coefficient for U.S. for the period between 1913 to 2012; it is possible to see that inequalities decreased sharply after the first world war until the 60’ and then started increasing until 2012, as mentioned by [Picketty, 2014b]; the reason I used the inverted Pareto Lorenz coefficient in this section is the possibility to track inequalities back to 1913, in any case the behavior of this coefficient is much similar to the Gini index.

In figure 4.2 it is described the behavior of real GDP in 2009 U.S. chained dollars for the United States from 1929, from the graph it stands clear that, in the case of U.S., growth was almost continuous trough the last century, making it an ideal case to confront the behavior of economic growth in the country with the pattern of inequalities.

In figure 4.3 are plotted the the values of the Inverted Pareto-Lorenz coefficients for the corresponding values of real log GDP in Billions of chained 2009 U.S. dollars, the blue line is the quadratic interpolation of the data: it is possible to see how
Figure 4.1: Inverted Pareto Lorentz coefficients for United States from 1950 to 2012. Source: [Facundo; Atkinson; Picketty; Saez, 2014].
Figure 4.2: Real GDP growth in U.S. in 2009 U.S. chained dollars. Data Source [Feenstra; Inklaar; Timmer, 2013].
Figure 4.3: Scatter plot of log real GDP for the Inverted Pareto-Lorenz coefficients. Data Source: [Facundo; Atkinson; Picketty; Saez, 2014].
inequalities first decreased then increased with the increase in U.S. real GDP.

In this case seems that, in the advanced stages of development as is the case of U.S. during the last century, the Kuznets’s curve appear to be inverted\(^2\). Again the decrease in inequality can be a consequence of the economic shocks of the two wars, the Great Depression and the increase in population, which as seen in previous chapter, is a factor that ceteris paribus contributes to decrease inequalities; what is surprising is the increase in inequalities registered in the last decades of the century, different causes have been proposed for this, for sure the economic policies pursued from Reagan played a role in this [Picketty, 2014b].

To investigate what happened in the increase in inequalities it is interesting to look at the evolution of different income groups in the society, in particular the richest 1%, 5%, 10% and the poorest 10% of the population, here incomes are before taxes, hence the difference in incomes is overestimated, in any case in figure 4.4 it is possible to note how the bottom 10% incomes stagnated during the century compared with the richest groups of the population.

From the graph it is possible to note how, almost until around the ’80 growth was benefitting all the three richest groups almost at the same pace, while, from that point on, the gap between the richest 1% and the others increased dramatically, furthermore since the top 5% and 10% groups include the richest 1%, it can be inferred that a large share of the increase in the average income of the 5% and 10% was largely due to the extremely high incomes in the top percentile.

It is interesting to note as well how the average income of the richest 1% was more volatile compared to the other groups and was more sensitive to the economic situation, in particular there were three major declines during the last century, one

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\(^2\)To be clear, this is not an evidence against the Kuznets’s curve: in fact Kuznets’s theory is concerned about early stages of development and the subsequent phases. It is still possible that the curve is valid until the fifties, what remains to explain is the increase in inequalities in the second half of the century.
Figure 4.4: Differences in the average income for the richest 1% 5% 10% and the poorest 10% from 1917 to 2012. Sources and series [Facundo; Atkinson; Picketty; Saez, 2014].
during the Great depression, one in year 2000 due to the burst of the Dot-com bubble and the last during the financial crisis in 2007.

4.3 Empirical analysis of the effect of inequality on growth

In this section I will try to investigate the effects of income inequalities on subsequent growth for the United States. For the estimation procedure, the starting point is the equation for growth in [Forbes, 2000]:

\[
Growth_t = \beta_1 Growth_{t-1} + \beta_1 \Delta Gini_{t-1} + \beta_2 \Delta Price \ level\ of \ investment_{t-1} + \\
+ \beta_3 \Delta Male\ education_{t-1} + \beta_4 \Delta Female\ education_{t-1}
\]

(4.7)

However, in this work I will focus only on the case of Unites States, this will make the estimation procedure simpler than in the case of a panel regression, and will allow to focus only on the within country specific effects. The econometric software used for the estimation is Gretl; the estimation technique for the regression is Ordinary Least Squares, which in the case of time series regression gives consistent estimates provided that Gauss-Markov assumptions are satisfied:

1. Linearity in parameters;

2. no perfect collinearity;

3. zero conditional mean;

\(^3\)As mentioned in chapter 1, the relationship between inequality and growth is really complex and multidimensional, this is one of the reason the analysis focuses on a single country.
CHAPTER 4. EMPIRICAL FINDINGS: THE CASE OF U.S.

4. homoskedasticity;

5. no serial correlation.

The first condition is satisfied by the specification of equation 4.7; the second condition is satisfied since no variables are exact combination of other variables; to control for the requirement of the last three conditions, tests on residuals were performed for every estimation to ensure the OLS estimates are consistent.

Furthermore, time series regression is fraught with other problems to deal with, and the most relevant issue to deal with is the presence of integrated variables: in fact all the variables in this case are $I \sim (1)^4$, hence the risk of running a spurious regression is real: in fact, when regressing two or more variables that are integrated or non-stationary, it is really likely to find that they are highly related even though they are independent, just because they are non stationary, however, recent econometric literature developed different tools to deal with integrated variables avoiding the risk of running a spurious regression. On this issue the notion of cointegration is of particular interest. In fact, as fund by Engle and Granger [Wooldridge, 2009], a regression involving $I \sim (1)$ variables can be potentially meaningful in the case the residuals of the regression are $I \sim (0)$; in this the case, it is possible to assume that the variables are cointegrated and there is a long-run relationship between them among time; they can diverge in the short run, but in the long run they will not diverge indefinitely and will converge to their long run levels.

In analyzing the relationship between inequality and income, the notion of cointegration has both an econometric appeal and an economic interpretation: in fact the presence of cointegration would mean in this case that inequality and income will converge to their long run values even if in the short run they can

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4Cointegration analyses was performed for all variables in the model leading to the conclusion that it is not possible to reject the hypothesis of integration of the first order for all variables. These conclusion are also supported by the economic literature [Wooldridge, 2009], hence the results of the analysis are omitted.
diverge significantly. This behavior it is often described with the imagine of a drunk man with his dog on a leash: they both can display a random walk, but in the end they are related.

In order to test for the presence of cointegration, I performed an Engle-Granger test, different specifications of the test were performed, but the all results clearly do not support the hypothesis of cointegration. As said before, this has an interesting economic interpretation, since it indicates that income and inequality, in the case in exam, are not related trough time, and they theoretically could increase or decrease substantially during time without converge to their long-run values. In this case, if we are investigating the relationship between income inequality and average GDP, the absence of cointegration suggest that there is no definite long-run relationship. However, this result may be strongly influenced by the relatively small time period for which data are available: in fact, to account for education the sample has to be limited to data from 1965 because there are some missing values prior to that date and it is not possible to calculate first differences.

The absence of cointegration forces to continue the analysis of the variables in first differences, hence the rate of growth will be explained in terms of variations of the independent variables, as in equation 4.7 defined with "Δ" to indicate the variables are first differenced; finally, all variables except Gini appear as natural logarithms.

In the next two figures it is shown the behavior of the series in level and in the first differences, it is possible to see that the series display a significant increasing trend over time, but the trend is removed almost completely if series are taken in first differences.

The output of OLS regression is shown in table: 4.3. The model it is similar to the one in [Forbes, 2000], one difference regards the fact that variables are expressed in first differences, another difference concerns the variables on education:
CHAPTER 4. EMPIRICAL FINDINGS: THE CASE OF U.S.

Figure 4.5: Time series in the levels.
Figure 4.6: Time series in first differences.
### Table 4.2: Model 1

Model 1: OLS, usando le osservazioni 1966–2011 ($T = 46$)

Variabile dipendente: *Growth*

Errori standard HAC, larghezza di banda 2 (Kernel di Bartlett)

<table>
<thead>
<tr>
<th></th>
<th>Coefficiente</th>
<th>Errore Std.</th>
<th>rapporto $t$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Gini_1$</td>
<td>1,12591</td>
<td>0,403886</td>
<td>2,7877</td>
<td>0,0080</td>
</tr>
<tr>
<td>$\Delta Price \ level \ of \ inv_1$</td>
<td>-0,0456994</td>
<td>0,110910</td>
<td>-0,4120</td>
<td>0,6825</td>
</tr>
<tr>
<td>$\Delta Male \ education_1$</td>
<td>-0,000379850</td>
<td>0,00826520</td>
<td>-0,0460</td>
<td>0,9636</td>
</tr>
<tr>
<td>$\Delta Female \ education_1$</td>
<td>0,0110964</td>
<td>0,0124004</td>
<td>0,8948</td>
<td>0,3761</td>
</tr>
<tr>
<td>$Growth_1$</td>
<td>0,364216</td>
<td>0,127894</td>
<td>2,8478</td>
<td>0,0069</td>
</tr>
</tbody>
</table>

Media var. dipendente: 0,018609

SQM var. dipendente: 0,021212

Somma quadr. residui: 0,018571

E.S. della regressione: 0,021283

$R^2$: 0,486683

$R^2$ corretto: 0,436603

$F(5,41)$: 13,10611

P-value($F$): 1,21e–07

Log-verosimiglianza: 114,4694

Criterio di Akaike: -218,9388

Criterio di Schwarz: -209,7956

Hannan–Quinn: -215,5137

$\hat{\rho}$: -0,112215

Durbin–Watson: 2,177335

Test per la normalità dei residui –

Ipotesi nulla: L’errore è distribuito normalmente

Statistica test: $\chi^2(2) = 8,06992$

con p-value = 0,0176864
in [Forbes, 2000] education is computed as average year of secondary schooling for population over 25 years old while in the this model education is computed as percentage of population over 25 years with secondary education, the price level of investment is obtained from [Feenestra;Inklaar;Timmer, 2013], and the Gini index is expressed in first differences, this was done in order to obtain a stationary series. In order to test the specification of the model, ADF test was performed on residuals indicating the absence of unit root, furthermore, as shown in figure 4.7, the correlogram of residuals indicates little serial correlation, and the test on the normality of residuals, shown in fig 4.8 suggests to be careful in accepting the hypothesis that are normally distributed since the p-value of the Jarque-Bera test is small: only 0.017.

![Figure 4.7: Correlogram of residuals from Model 1.](image)

Looking at the output of the regression it is possible to see that the the coefficient for the variations in the Gini index is positive and significant as in [Forbes, 2000], indicating that an increase in the Gini coefficient has a positive effect on economic growth in the subsequent period. However, the rest of the coef-
coefficients, excluding the lagged dependent variable are not statistically different from 0; this, combined with the small probability that errors are normally distributed, suggests it is better to try a different specification of the model. A first hint in improving the model can be drawn looking at the graph of the estimated and fitted values of the regression in figure 4.9, from the graph it is possible to see that the model fail completely to mimic the strong drop in GDP in 2009 which was due to the sub-prime crisis. For this reason in the following specification of the model it is introduced a dummy variable for 2009 in order to account for that strong economic downturn.

A second way to improve the model comes from the consideration that, in the case of an advanced country like U.S., valuing education in terms of percentage people with a secondary school degree may not be very informative, since secondary education should be considered as a standard and of little influence for growth, for this reason in the following specifications of the model I will include also the percentage of people over 25 with a college degree in order to account for the high
level education factor that was introduced in the model in chapter 3. The result of a different specification, as shown in table 4.3, confirms the intuitions. In particular it is possible to see that the result of the regression improved in terms of $R^2$, the most relevant fact it is that, in this case, the coefficients on tertiary education are more significant than those on secondary education, this evidences the importance of high level education for subsequent growth, particularly in a already highly developed country.

Following the results of 4.3, I tested a different model, accounting only for higher education, the results are shown in table 4.3.

An analysis on the correlogram of the residuals in figure 4.10 indicates only some autocorrelation in the second period, but it is still included in the confidence band. The distribution of residuals, shown in figure 4.11 indicates that it is not possible to reject the normality hypothesis, suggesting that the relationship it is well specified, this is confirmed by an analysis of the residuals in figure 4.13 where residuals appear to be white noise.
CHAPTER 4. EMPIRICAL FINDINGS: THE CASE OF U.S.

Table 4.3: Model 2

Model 2: OLS, usando le osservazioni 1966–2011 ($T = 46$)
Variabile dipendente: $Growth$
Errori standard HAC, larghezza di banda 2 (Kernel di Bartlett)

<table>
<thead>
<tr>
<th></th>
<th>Coefficiente</th>
<th>Errore Std.</th>
<th>rapporto $t$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Gini_{1}$</td>
<td>1,00926</td>
<td>0,530934</td>
<td>1,9009</td>
<td>0,0649</td>
</tr>
<tr>
<td>$\Delta Price level of inv_{1}$</td>
<td>-0,143003</td>
<td>0,104808</td>
<td>-1,3644</td>
<td>0,1805</td>
</tr>
<tr>
<td>$\Delta Male education_{1}$</td>
<td>0,00460638</td>
<td>0,0100501</td>
<td>0,4583</td>
<td>0,6493</td>
</tr>
<tr>
<td>$\Delta Female education_{1}$</td>
<td>0,00192602</td>
<td>0,0123827</td>
<td>0,1555</td>
<td>0,8772</td>
</tr>
<tr>
<td>D2009</td>
<td>-0,0662505</td>
<td>0,00583018</td>
<td>-11,3634</td>
<td>0,0000</td>
</tr>
<tr>
<td>$\Delta High male edu_{1}$</td>
<td>0,0101450</td>
<td>0,00627642</td>
<td>1,6164</td>
<td>0,1143</td>
</tr>
<tr>
<td>$\Delta High female edu_{1}$</td>
<td>0,0173957</td>
<td>0,00798708</td>
<td>2,1780</td>
<td>0,0357</td>
</tr>
<tr>
<td>$Growth_{1}$</td>
<td>0,214843</td>
<td>0,108646</td>
<td>1,9775</td>
<td>0,0553</td>
</tr>
</tbody>
</table>

Media var. dipendente 0,018609  SQM var. dipendente 0,021212  
Somma quadr. residui 0,013025  E.S. della regressione 0,018514  
$R^2$ 0,639965  $R^2$ corretto 0,573643  
Log-verosimiglianza 122,6273  Criterio di Akaike -229,2547  
Criterio di Schwarz -214,6255  Hannan–Quinn -223,7745  
$\hat{\rho}$ -0,002486  Durbin–Watson 1,911953  

Test per la normalità dei residui –
Ipotesi nulla: L’errore è distribuito normalmente
Statistica test: $\chi^2(2) = 4,17793$
con p-value = 0,123815
Table 4.4: Model 3

Model 3: OLS, usando le osservazioni 1966–2011 ($T = 46$)

Variabile dipendente: *Growth*

Errori standard HAC, larghezza di banda 2 (Kernel di Bartlett)

<table>
<thead>
<tr>
<th>Coefficiente</th>
<th>Errore Std.</th>
<th>rapporto $t$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Gini_1$</td>
<td>1,15682</td>
<td>0,455801</td>
<td>2,5380</td>
</tr>
<tr>
<td>$\Delta High male edu_1$</td>
<td>0,0162613</td>
<td>0,00606316</td>
<td>2,6820</td>
</tr>
<tr>
<td>$\Delta High female edu_1$</td>
<td>0,0154405</td>
<td>0,00726139</td>
<td>2,1264</td>
</tr>
<tr>
<td>$\Delta Price level of inv_1$</td>
<td>−0,0871769</td>
<td>0,0933031</td>
<td>−0,9343</td>
</tr>
</tbody>
</table>

| D2009               | −0,0640210  | 0,00597934   | −10,7070 | 0,0000  |

| $\Delta Growth_1$  | 0,277442    | 0,0975578    | 2,8439  | 0,0070  |

| Media var. dipendente | 0,018609   | SQM var. dipendente | 0,021212 |
| Somma quadr. residui  | 0,013351   | E.S. della regressione | 0,018270 |
| $R^2$                | 0,630951   | $R^2$ corretto      | 0,584820 |
| $F(6,40)$            | 8,50e+15   | P-value($F$)        | 1,8e–300 |
| Log-veroimalignanza  | 122,0586   | Criterio di Akaike  | −232,1172 |
| Criterio di Schwarz  | −221,1453  | Hannan–Quinn       | −228,0071 |

| $\hat{\rho}$        | 0,002639   | Durbin–Watson      | 1,896467 |

Test per la normalità dei residui –
Ipotesi nulla: L’errore è distribuito normalmente

Statistica test: $\chi^2(2) = 2,45687$
con p-value = 0,29275

Test LM per l’autocorrelazione fino all’ordine 2 –
Ipotesi nulla: Non c’è autocorrelazione

Statistica test: $LMF = 2,22314$
con p-value = $P(F(2,38) > 2,22314) = 0,122162$
**Figure 4.10:** Correlogram of residuals from Model 3

**Figure 4.11:** Distribution of residuals from Model 3.
In figure 4.12 it is plotted the performance of the model in terms of effective versus estimated values of growth, it is possible to see that the model is able to catch most of the variations in the rate of growth, although not completely: a reasonable explanation for this is the fact that the model does not account for exogenous shocks\textsuperscript{5}, furthermore the model does not account for other exogenous factors as exports and the global economic situation, however these omitted variables seem to be uncorrelated with the explicative variables, and this is confirmed by the analysis of the residuals.

From table 4.3 it is possible to note that this specification improved the performance of the regression: the adjusted $R^2 = 58\%$ and it is higher than in table 4.3, where $R^2 = 44\%$; this indicates that omitting the variables on secondary education the performance of the regression which, accounting for the number of variables included in the model, it is improved; moreover it is possible to see that Gini the coefficient on inequality is still positive and significantly different from 0, and the

\textsuperscript{5}Except in 2009 where a dummy variable was introduced to explain the recession.
coefficients on education variables are positive and significant, at a 1% level of confidence, furthermore, the coefficients on male and female education are quite of the same magnitude, as to say that higher education has almost the same effect on growth both for male and female indicating little or none gender disparity. At last, only the coefficient on the variation in the price level of investment it is not significantly different from 0.

From the previous analysis it is possible to confirm the results found in the previous literature, [Barro, 2000] [Forbes, 2000], [Perrotti, 1996], that inequality has a positive role on growth, at least in the short run; moreover in different specifications of the model, the coefficient on Gini was never negative, this indicates that in the case of U.S. rising inequalities were not detrimental to growth.

It is now possible to simulate the possible effects that a shock in inequality or education can have on growth. In order to do this, with the help of the econometric software Gretl, it is possible to include the relevant variables into a vector
autoregressive model (VAR) in which all variables in the model are regressed on their own past and on the past values of the other variables, in order to account for endogeneity in the explicative variables as well, and calculate the impulse response function of growth to a shock in the other variables.

In figure 4.14 it is plotted the impulse response function for Growth to a shock in the Gini coefficient, it is possible to see that the maximum effect is on the first lag, which means that a shock in the Gini coefficient has its maximum effect on growth in the first period and then decreases, but remains positive and declines slowly, however it is possible to see that the effect remains positive in time, which seems to exclude a negative effect of inequalities on growth; however it is to say that this it is probably due to the fact that variations in inequality in the period under exam for the U.S. could have been too small to dampen economic growth⁶.

![Figure 4.14: Impulse response function of growth to a shock in Gini.](image)

⁶In fact in the case under exam, although there are variations in the Gini index, it is still possible that these variations are too small to mine subsequent growth.
In figure 4.15, showing the effects on growth of a shock in the high education variable, it is possible to see again a positive and persistent effect of the shock, that start to decline slowly after the third lag, this indicates that higher education influences growth in a more persistent way, although the coefficient is smaller, than inequality.

Figure 4.15: Impulse response function of growth to a shock in high education.
Chapter 5

Conclusions

In this work I tried to investigate the relationship between economic growth and income inequalities, as evidenced many times throughout this work, the literature on the topic is immense and was difficult to expose all the contributions on the issue, however in chapter 1, I tried to focus on the ones I believed were the major contributions on the argument. In chapter 2, I tried to investigate a little deeper in the dynamics of economic growth and wealth, with the objective of decomposing the two main objects of this analysis in their major components, since both are multidimensional phenomena. In chapter 3, following the model proposed by [Fiaschi, 2007], I exposed a theoretical model that wish to explain the relation between economic growth and income inequality in an analytical framework, the model it is interesting since it fits many of the different channels in which inequality and growth are interrelated. In chapter 4, I analyzed the empirical relationship between wealth distribution and growth in the case of U.S., the first part of the chapter aims to describe the evolution of economic growth and incomes in United States over the last century, it was possible to see the stagnation of incomes for the lower part of the distribution and a rapid raise in higher incomes especially after the 80’s that was more likely due to the era of liberalization introduced by
Reagan.

A second fact worth of note in this section was the "U" shaped correlation between average income and inequality during the last century: in fact, in the period under exam, the relationship between average per capita GDP and inequality has a clear "U" shape, where inequality first decreased until around the middle of the century, then increased again. This seems to confirm the hypothesis of [Picketty, 2014b] that inequality in U.S. decreased in the first half of last century and increased in the second part of last century in the U.S. inequality\(^1\).

In the second part there is an econometric analysis to determine if income inequalities influenced growth and in which direction, although data availability limited the analysis to the period 1965-2012, it is still possible to find a positive correlation between inequality and growth, where increasing inequalities are followed by positive growth: at first sight, this is in sharp contrast with the analysis of [Picketty, 2014b], who asserts that increasing inequalities in the last century had a negative influence to the economic performance of a nation, also in the case of a highly developed country like the United States. To the contrary, the finding of chapter 4, are in line with the analysis of [Barro, 2000] and [Forbes, 2000], finding a positive, although small correlation between a rise in the Gini coefficient and economic growth.

However this results may be due to the fact that during the period under exam, the variations in the Gini coefficient may have been too small to dampen economic growth. Furthermore, analyzing the impulse response function of growth to a shock in the explanatory variables, it is possible to see that a shock both in inequality and education has a positive impact on subsequent growth; this effect is more remarkable in the first years and moreover does not assume negative values in the following years, ruling out a negative correlation between inequalities and growth.

\(^1\)However this results refer only to the correlation between inequality and GDP
CHAPTER 5. CONCLUSIONS

in the longer run, this in my view can be due to the country specific situation, where inequalities are not sufficient to dampen subsequent growth similar to the case presented in figure 3.2 in chapter 3. However different results may be obtained limiting the analysis to countries with higher levels of inequality, where the initial level of inequality is sufficiently high to exclude a significant part of the population from the investment in higher education and hence limiting growth.

One last remark should be done to notice that under the period under examination during the regression analysis, which focuses on the last 50 years both inequalities and GDP are rising as shown in figure: 4.5, although at different speed, hence it is possible to assume that considering a longer time period as was done in section 4.2 could have yield different results, maybe supporting the analysis of Picketty, but unfortunately the lack of reliable data forced the regression to be limited to the last 50 years.

The results of the model, if compared with the results obtained by Picketty, Forbes and Barro, suggests that, the length of timeframe under consideration is a relevant issue when analyzing the relationship between growth and inequality.
Appendix A

To prove the fact that, ceteris paribus, \( r^k > g \), even without accounting for convexities in the remuneration of high-quality investments, we refer to the model in chapter 3; as stated in equation: (3.10):

**Proof.**

\[
    r_t = (1 - \alpha)A,
\]

and in equation (3.16)

\[
    g = \frac{\rho \phi (1 - \alpha) A}{(1 + \rho)[1 + \phi(1 - \alpha)]};
\]

if we divide both equations (3.10) and (3.16) by \((1 - \alpha)A\) we obtain that, to prove the inequality, it is only necessary to show that:

\[
    \frac{\rho \phi}{(1 + \rho)[1 + \phi(1 - \alpha)]} < 1; \quad (A.1)
\]

expanding the denominator, it becomes:

\[
    \frac{\rho \phi}{1 + \rho + \phi + \rho \phi - \phi \alpha - \rho \phi \alpha} < 1; \quad (A.2)
\]
since $\alpha < 1$ the equation reduces to:

$$\frac{\rho \phi}{1 + \rho + \iota},$$

(A.3)

where $\iota = \phi - \phi \alpha + \rho \phi - \rho \phi \alpha > 0$; now, since $\phi = [\beta(1 + \rho)]$, the equation becomes:

$$\frac{\beta \rho + \beta \rho^2}{1 + \rho + \iota},$$

(A.4)

that is $< 1$, now, since $\rho < 1$ and $\beta < 1$, it follows that $(\beta \rho + \beta \rho^2) < (1 + \rho + \iota)$, hence $r^k > g$. 

$\square$
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