Markov-switching correlation models for contagion analysis in commodity and stock markets

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1) Introduction:

Commodity securities such as oil and gold have been subject to an increasing interest within this period of Great Depression followed by the EU Sovereign Debt crisis, being for instance an extremely popular investment class as part of a long-term diversification strategy (LTDS), offered by fund managers. In the meanwhile there is little evidence of a lasting impact of financial investment on commodity prices (see Fattouh et al. 2012 for a recent survey). This leads to considering trading motive of non-commercial financial investors as "noise-trading"- according to the microstructural framework- based on systemic risk perception rather than idiosyncratic and market-specific considerations. Hence, a large literature investigated correlation dynamics between equity and commodities market variations during the last decade, extending the multivariate GARCH model frameworks or multivariate stochastic volatility models to better capture the structure of time-dependence between the investment classes. Notably we underline here: Killian and Park (2009) show that equity-price reaction to oil price shocks were depending on their nature; Cassassus and Higuera (2011) showing that oil-prices are good predictors of equity prices; Chang et al. (2011) showing evidence of volatility spillovers between the two markets, i.e. only volatility changes and spreads out across markets.

With this respect, Lombardi and Ravazzolo (2012) offer an original contribution to this literature by investigating oil-equity prices co-movements to improve forecasts either direction using a parsimonious constant parameter Bayesian framework and deriving a Bayesian dynamic conditional correlation model. The model can capture the shift observed after 2008 (see Della Corte and al.,2010) and over-performs a no-change random walk model when conducting density forecast (see Lombardi and Ravazzolo, 2012). This gain permits a better portfolio management framework incompounding commodity prices as a new class of assets. Joint-modeling is shown to generate better profitability "relatively to constant parameter models and passive strategies, especially in times of large swings". Improving the gains of standard DCC GARCH, with a parsimonious, stable and efficient model and suitable Bayesian inference, which allows for an easy-to-handle active Portfolio Management tool.

This paper builds on a simplified version of the Markov-Switching correlation model offered by Casarin, Tronzano, and Sartore (2013). Hence we extend here the oil-equity market linkage paper based on dynamical correlation by applying a two-redeem state Markov-Switching model to account for shifts in correlations and allow a more stable portfolio management.
In fact contagion would imply serious issues on diversification, leading to underestimate global risk, and thus the related possible loss, especially while computing Value at Risk and Expected Shortfall.

We apply on this purpose, following Casarin and al. (2013), a multivariate stochastic correlation model where the parameters of the correlation dynamics and those of the log-volatility process are driven by a single latent Markov chain (MS-DC-MSV). It has this advantage to assume independent innovation process for the mean returns and Variance-Covariance matrix, which is in a way closer to empirical findings by allowing independent exogenous shocks.

We follow then a suitable Bayesian inference procedure, based on MCMC estimation algorithm, to both parameters, covariances and correlation dynamics. We can point out that Bayesian inference allows to account of higher moments than the only mean-variance.

We report some of the outputs, especially estimated time-varying stochastic correlations and volatilities. We will end by commenting on possible empirical evidence of contagion effect (contagion as defined above).
2. Theoretical part :

2.1. Introduction :

We introduce Markov Switching stochastic correlation with two redeem states (low volatility, high volatility) to model the time-varying volatility and correlation. This modeling is built on a simplified version of the Bayesian Markov-Switching DC-MSV model applied in Casarin and Al. (2013).

Both the Dynamical Correlation Model offered by Engle (2002) as extension of the Constant conditional correlation model of Bollerslev, and MSV type modeling account for time-varying correlation structure, consistently with the empirical evidence related to financial time-series stylized facts.

First we will briefly introduce the DCC-GARCH to get an intuition on the model used for the empirical part of the paper, also because it has been subject to a very large literature and extensions. This type of modeling is very popular for its parsimonious modeling and competitive results with respect to its previous ARCH and VEC type models. For a survey on Multivariate GARCH models one can refer to Bauwens and al. (2006).

Then we specify the model MS-DC-MSV, which is in direct line with a recent approach, modeling time-varying and stochastic correlation structure using multivariate SV models. Casarin et al. refers for this approach to Gourieroux et al. (2004) on the one hand, on the other on Philipov and Glickman (2006), and Asai and McAleer (2009) on the other. To this purpose, they assume that the covariance matrix is a function of a Wishart process, which introduces stochastic behavior in the DC modeling. Wishart distribution has convenient properties, and is often used in Bayesian approach.

The approach here is:

- to assume that the inverse covariance matrix follows a Wishart distribution conditionally on the past information (following Asai and McAleer).
- to offer an extension to that, using the Markov-switching approach of So and al. (1998)
2.1.1. DCC-GARCH model:

The following approach consists in estimating variances and portfolio conditional correlations, using the 2-step parameters estimation method offered by Engle(2002).

Thus we first estimate the variables respective conditional variances, with an univariate GARCH model. Then we embed the estimated standardized error, used as inputs to model autoregressive dynamic correlations within the portfolio.

❖ Step 1: univariate estimations: conditional variance estimation

The conditional matrix of Variance-Covariance is given by the following equation:

\[ H_t = D_t \cdot R_t \cdot D_t \]

With \( D_t = \text{diag} \{ \sqrt{h_{i,t}} \} \), Diagonal matrix of Conditional volatility.

\[
D_t = \begin{pmatrix}
\sqrt{h_{1,t}} & 0 & 0 \\
0 & \sqrt{h_{2,t}} & 0 \\
0 & 0 & \sqrt{h_{3,t}}
\end{pmatrix}
\]

\( R_t \), the matrix of conditional correlations.

Hence we reduced drastically the number of parameters to estimate. We also could have preferred the two-steps approach, which leads us to the estimation of the diagonal matrix through a N univariate GARCH on our N assets. We could then define a standardized error vector for each univariate process, computed from the following:

\[ \varepsilon_t = D_t^{-1} r_t \]

With \( r_t \) returns.
One should underline, that the estimation of the conditional correlation matrix will give us a definite positive matrix: diagonal elements will be equal to 1, the non-diagonal elements -1.

\[ R_t \text{ defined as follows:} \]

\[ R_t = diag(Q_t)^{-1} Q_t diag(Q_t)^{-1} \]

Where \( Q_t = S \left( tt' - A - B \right) + A \epsilon_{t-1} \epsilon_{t-1}' + B Q_{t-1} \)

\( t \) unit vector.

\( S \) unconditional correlation matrix of the error function:

\[ S = \frac{1}{T} \sum_{t=1}^{T} \epsilon_t \epsilon_t' \]
2) **Step 2 : Modeling autoregressive dynamical correlations :**

Engle et Sheppard (2001) have shown that $R_t$ and $Q_t$ could be estimated in the second step by

Maximizing partially the Log-Likelihood, expressed as a volatility driven part plus a correlation term.

Log-likelihood expressed then:

$$L(\theta, \phi) = L_{\nu}(\theta, \phi) + L_{C}(\theta, \phi)$$

$L_{\nu}$ volatility term following:

$$L_{\nu}(\theta) = -\frac{1}{2} \sum_t (n \log(2\pi) + \log|D_t|^2 + r_t' \ D_t^{-2} r_t)$$

$L_C$ the correlation term expressed as following:

$$L_{C}(\theta, \phi) = -\frac{1}{2} \sum_t (\log|R_t| + \epsilon_t' R_t^{-1} \epsilon_t - \epsilon_t' \epsilon_t)$$

Thus the log-likelihood will be:

$$\log L(\theta) = -\frac{1}{2} \sum_{t=1}^{T} (\log|D_t R_t D_t| + \tilde{\epsilon}_t' \tilde{\epsilon}_t)$$
2.2. A Markov-Switching Stochastic Correlation Model:

Let \( y_t = (y_{1t}, \ldots, y_{mt})' \in \mathbb{R}_m \) be a vector-valued time series, representing the log-differences in the spot exchange rates;

\[ h_t = (h_{1t}, \ldots, h_{mt})' \in \mathbb{R}_m \quad \text{the log-volatility process,} \]

\( \Sigma_t \in \mathbb{R}_m \times \mathbb{R}_m \) the time-varying covariance matrix, and \( s_t \) a two-state Markov-Chain. Consider here a special case of the stochastic correlation model (MSSC) given in [3].

\[
y_t = A y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma_t) \quad (1)
\]

\[
h_t = b_{00} + b_{01} s_t + (B_{10} + B_{11} s_t) h_{t-1} + \eta_t \quad \eta_t \sim \mathcal{N}(0, \Sigma \eta) \quad (2)
\]

with \( \varepsilon_t \) and \( \eta_t \) independent for every \( s_t \), and \( \mathbb{N}_m(\mu, \Sigma) \) the \( m \)-variate normal distribution, with mean \( \mu \) and covariance matrix \( \Sigma \), and \( A, A, b_{00}, b_{01}, B_{10}, B_{11} \) parameters to be estimated. The probability law governing \( s_t \) is \( s_t \sim P(s_t = j \mid s_{t-1} = i) = p_{ij} \), with \( p_{ij} \) for \( j = 0, 1 \).

As regards the conditional covariance matrix \( \Sigma_t \) we use the decomposition (see [2] ):

\[
\Sigma_t = \Lambda_t \Omega_t \Lambda_t', \quad (3)
\]

With \( \Lambda_t = \text{diag}\{\exp(h_{1t}/2), \ldots, \exp(h_{mt}/2)\} \), a diagonal matrix with the log-volatilities on the main diagonal,

and \( \Omega_t = \bar{Q}_t Q_t \bar{Q}_t' \) the stochastic correlation matrix

with \( \bar{Q}_t = (\text{diag} \{ \text{vecd} \ Q_t \})^{1/2} \) and \( Q_t^{-1} \sim \mathbb{W}_m(\nu, S_{t-1}) \)

where:

\[
S_{t-1} = \frac{1}{\nu} Q_{t-1}^{-d/2} \bar{Q}_t Q_{t-1}^{-d/2},
\]

\[
\bar{Q}_t = \bar{D}_{s,t} = \sum_{k=0,1} \{ k \} (s_t) \bar{D}_k
\]
and, $\bar{D}_k$, $k \in \{0,1\}$, is a sequence of positive definite matrices which capture the long-term dependence structure between series $i$, the different regimes, and $d$ is a scalar parameter. The correlation-switching process $s_t \sim P(s_t = j | s_{t-1} = i) = p_{ij}$


4. Directed graph:

the same hidden-switching process (latent variables), impact the covariance matrix and the stochastic (inverse) covariance matrix, which contribute contemporaneously to determine the output for every $t$. We could add progressively in the directed graph, the Matrices and parameters to be estimated corresponding aside to the directed arrows. This "spatial-time structure" impose restrictions and constraints on the estimation of the parameters iteratively, since at the end we have deterministic values of these ones.

We see here the increased flexibility given by this model, both by Markov-switching and stochastic correlation. We elude here the related suitable Bayesian inference based on MCMC and Gibbs-sampling. Nonetheless, it is the keystone of this procedure. (For the computational details see Casarin and al. (2013)).
2. Descriptive statistics:

1) Evolution of Gold (comex) in $, SP500 index, and OIL (Brent, crude oil) from 8 of June 2006 to the 4th of May 2014. Dashed lines and ellipses correspond to the start of the sub-prime crisis (15/08/2007) and Greek sovereign debt crisis (31/12/2009):

Oil prices have risen from the beginning of 2007 to 2009 from 50$ per barrel to 143.95$, to fall afterwards in few months during 2009 to a historical record of 33.73$ per barrel. We had then extreme variations on spot prices, skyrocketing by more than 280% and registering immediately after a 427% sinking. This fall were following U.S. dollar appreciation as one of the subprime crisis consequences, but raised serious issues on commodities market understanding by the importance of these variations. This has been the subject of a well-documented literature, especially rising the debate around the oil price determinants, in particular the role of speculation: do commodities, especially oil, tend to have a "financial class" behavior? For a survey on the question one can refer to Fattouh and al. (2012). Since 2011 crude oil prices fluctuate around 110€ per barrel.
The SP500 index is published by Standard and Poors (filial of Mc Graw Hill) since 1957, and is a weighted average of the 500 most exchanged NYSE and NASDAQ stock market capitalizations. In ellipse the SP500 index at the start of the subprime crisis (07/2007) and Greek Sovereign debt crisis (31/12/2009). One can see the collapse of stock markets during the Great Depression and recovery, reaching now the 2000 points (above the precedent record of 1500 points registered at the burst of internet bubble in 2000.

Gold London closing fixing in $ (unit : troy oz): We observe with clear evidence a sharp increase of gold prices after the two crisis, used as a safe haven. Fixed-income investment, in particular U.S. treasury bonds, followed by those of E.U. (cf. Greece, Spain, Portugal...) raised doubts on their validity as being a safe diversification tool, which increased investment in commodities, as a LTDS. We have reached a peak of 1795.1 $/Oz in February 2013, since then loosing 30% of its value. Is Gold correlation with SP500 still the same? Or has the commodities investments being attractive pushed Gold prices to behave differently?

2) Corresponding logreturns (DLOIL, DLGold and DLSP500) during the same period.

3) Density estimation with constant binwidth (=50), compared to a fitted normal.

One can see in (2) two periods of high volatility followed by period of low volatility in accordance with stylized facts within financial time series. In (3) we will point out excess kurtosis and fat tails for the SP500

Descriptive statistics on raw data :

<table>
<thead>
<tr>
<th>Variable</th>
<th>min</th>
<th>mean</th>
<th>max</th>
<th>std.dev</th>
<th>Normality Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP500</td>
<td>676.53</td>
<td>1325.1</td>
<td>1896.7</td>
<td>251.44</td>
<td></td>
</tr>
<tr>
<td>OIL</td>
<td>33.73</td>
<td>90.711</td>
<td>143.95</td>
<td>23.308</td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td>568.1</td>
<td>1118.1</td>
<td>1795.1</td>
<td>368.42</td>
<td></td>
</tr>
</tbody>
</table>

SP500
--------
Normality Test :

- SP500 :

<table>
<thead>
<tr>
<th>Statistic</th>
<th>t-Test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skewness</td>
<td>0.0043552</td>
<td>0.079931</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>-0.16733</td>
<td>1.5362</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>2.3606</td>
<td>0.30719</td>
</tr>
</tbody>
</table>

ARCH 1-2 test: F(2,2013) =3.0333e+005 [0.0000]**
ARCH 1-5 test: F(5,2007) =1.2117e+005 [0.0000]**
ARCH 1-10 test: F(10,1997)= 60523. [0.0000]**
• OIL :

Normality Test

<table>
<thead>
<tr>
<th>Statistic</th>
<th>t-Test</th>
<th>P-Value</th>
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<tbody>
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<td>Skewness</td>
<td>-0.29856</td>
<td>5.4795 4.2644e-08</td>
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<td>Excess Kurtosis</td>
<td>-0.98082</td>
<td>9.0049 2.1580e-019</td>
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<tr>
<td>Jarque-Bera</td>
<td>110.87</td>
<td>.NaN 8.4144e-025</td>
</tr>
</tbody>
</table>

ARCH 1-2 test: $F(2,2013) = 1.6103e+005 [0.0000]**$
ARCH 1-5 test: $F(5,2007) = 64333. [0.0000]**$
ARCH 1-10 test: $F(10,1997)= 32055. [0.0000]**$

• Gold :

Normality Test

<table>
<thead>
<tr>
<th>Statistic</th>
<th>t-Test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skewness</td>
<td>0.21962</td>
<td>4.0306 5.5631e-005</td>
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<tr>
<td>Excess Kurtosis</td>
<td>-1.2902</td>
<td>11.846 2.2668e-032</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>156.19</td>
<td>.NaN 1.2100e-034</td>
</tr>
</tbody>
</table>

ARCH 1-2 test: $F(2,2013) = 5.9134e+005 [0.0000]**$
ARCH 1-5 test: $F(5,2007) = 2.3578e+005 [0.0000]**$
ARCH 1-10 test: $F(10,1997)= 1.1747e+005 [0.0000]**$

Distribution: Normal
Log likelihood: -14788.3
Domain: $\text{Inf} < y < \text{Inf}$
Mean: 1118.09
Variance: 135803

Parameter Estimate Std. Err.
mu   1118.09   8.20341
sigma 368.515   5.80284

Estimated covariance of parameter estimates:
mu     sigma
mu \quad 67.2959 \quad -1.00042 \times 10^{-13}

\sigma \quad -1.00042 \times 10^{-13} \quad 33.673

Distribution: \quad \text{Gamma}

Log likelihood: \quad -14737.6

Domain: \quad 0 < y < \infty

Mean: \quad 1118.09

Variance: \quad 139343

Parameter Estimate Std. Err.
\begin{align*}
a & \quad 8.97163 \quad 0.277347 \\
b & \quad 124.625 \quad 3.96241 \\
\end{align*}

Estimated covariance of parameter estimates:
\begin{align*}
a & \quad 0.0769213 \quad -1.06852 \\
b & \quad -1.06852 \quad 15.7007 \\
\end{align*}

Kernel: \quad \text{normal}

Bandwidth: \quad 0.0998176

Domain: \quad 0 < y < \infty
2) Model application: Empirical results

In black line the effective volatility (respectively $Q_t$), in red line the estimated latent process.

The grey area represents the 95% high probability density region.
We processed to only 500 iterations (hence with no burns) following the procedure in Casarin et al. (2013). A good part of the starting iterations should have been eliminated and the number of iterations increased (e.g.: 5000 iterations and 1000 burns) for more accurate results.
In red the posterior mean, grey area the credibility interval. These outputs are useful to detect in our case contagion effect, in the definition offered by Forbes and Rigobon (2009). One can see that we have both increased volatility and high correlation during the beginning of the subprime crisis (last quarter 2007).
Computed posterior mean, regime switching, all along the studied period.
Conclusion:
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