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Titolo

Competition, Mergers and Exclusive Dealing in Two-Sided Markets with Zero-Price Constraints: The Case of Search Engines

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1 Introduction

At the beginning of the last decade scholars have started to review traditional economic problems from the point of view of two-sided markets. Today, economics of two-sided markets is considered as one of the most actively developing research areas in the industrial organization field. Up to date, a multitude of classical issues (such as price discrimination, market dominance, innovation, etc.) have been analyzed within the above-mentioned approach.

In this dissertation, a two-sided market framework has been employed in order to investigate a competition between search engines and subsequent issues it could bring. Among such issues, the main stress is made on excessive pricing, exclusive dealing, and mergers among search engines. The motivation for the latter point is explained by the recent case of a merger between Yahoo! and Microsoft's search engines.

Although the theory on competition among two-sided platforms has been significantly developed, some aspects are still remained partially explored. In particular, a little research has been performed on two-sided markets with zero-price constraints imposed in equilibrium on one side. As a result, very few attempts have been made to reflect the realities of the search engine market through the prism of two-sidedness and network externalities (Jeon, Jullien, and Klimenko, 2012; Lianos and Motchenkova, 2012). Thus, Jeon, Jullien, and Klimenko (2012), and Lianos and Motchenkova (2012) look at the search engine competition when searchers single-home and, therefore, experience transportation costs. In reality, only some part of users incurs such costs, whereas other part (tech-savvy users) does not. Moreover, most of users switch search engines, and there exist empirical studies confirming this fact (White and Dumais, 2009).

Similarly to the search engine competition, a merger issue has been analyzed scarcely within the two-sided market framework. For example, Malam (2011) extends the classic two-sided single-homing model developed by Armstrong (2006) to the competition between three ad-sponsored media platforms and analyses mergers between two of them. However, such competition generally is not characterized by a single-homing assumption imposed on both sides – at least advertisers multi-home, since this group of agents does not possess any preferences towards one of the platforms.

In addition, it is important to mention that the search engine competition has been mainly studied in the literature from the advertiser point of view, particularly, in terms of position auctions (Varian, 2007; Athey and Ellison, 2011; Chen and He, 2011). Moreover, this literature stream provides the analysis assuming the existence of a monopolistic search engine. However, if one focuses on competitive outcomes resulting from the search engine competition, it would be appropriate to employ broader two-sided market framework.

Thus, the main aim of the dissertation is to investigate possible issues of the search engine competition by employing a two-sided market framework.

To reach the assigned aim, following objectives are set: (i) to explain the concept of the two-sided market, (ii) to present an oligopoly model of search engine competition, which is capable to examine the welfare impacts of possible search engine practices, such as exclusive dealing and mergers. By executing above-mentioned objectives, the author expects to cover a gap in the existing literature on search engine competition.

The master dissertation is organized as follows. In Section 2, firstly, we present a relevant literature analysis on classical definitions of two-sided markets, and, secondly, we examine how exclusive dealing could be applied within a two-sided market framework. In Section 3, we set a duopoly model of the search engine competition where platforms are constrained to charge zero prices to one side of the market (users), and examine the impacts of exclusive dealing in this particular set up. Section 4 presents an extended triopoly model of competition, and investigates probable welfare implications when two out of three platforms merge. Finally, we conclude in Section 5.

2 Literature review

2.1 Platform competition in two-sided markets

Before providing a competitive analysis in the two-sided market framework, firstly, it is important to identify a two-sided market itself. However, even if the theoretical concept of a two-sided market generally can be considered as clear, existing definitions of this phenomenon suffer from the excessive specificity, over-inclusiveness, or too much vagueness (Hagiu and Wright, 2011). This leads to the disagreement among main literature contributors about the unique definition.

Majority of definitions of a two-sided market, suggested by founders of the field, aim at considering indirect network effects and an internalization of end-user externalities. Caillaud and Jullien (2003) maintain that the presence of indirect network effects must be considered as a relevant feature for informational intermediation together with the possibility of using nonexclusive services and practice of price discrimination based on user's identity. Rochet and Tirole (2003) define noninternalized externalities among end-users as a starting point in the theory of two-sided markets; authors state that most markets with network externalities can be recognized as two-sided. Armstrong (2006) defines a two-sided market as the place where two or more groups of agents interact via platforms (intermediaries serving this market); such interaction results in the creation of the positive (negative) externalities.

Relatively structured definition of a two-sided market was suggested by Evans (2003). For a two-sided market to exist, three necessary conditions must hold: (i) there are two or more distinct groups of consumers; (ii) there are externalities associated with two groups of customers, who are connected or coordinated in some fashion; (iii) an intermediary has to internalize the externalities created by the agents of one group from the interaction with the agents from other group.

In order to structure the theory of two-sided markets, the pioneers of the field, Rochet and Tirole (2006), in their subsequent work have developed a formal and precise definition aimed at separating the notions of two-sided and traditional one-sided markets. According to authors

> *"The market is two-sided if the platform can affect the volume of transactions by charging more to one side of the market and reducing the price paid by the other side by an equal amount; in other words, the price structure matters, and platforms must design it so as to bring both sides on board."*[1]

Whilst, the definition of a traditional one-sided market is formulated as follows

> *"The market is one-sided if the end-users negotiate away the actual allocation of the burden (i.e., the Coase theorem applies); it is also one-sided in the presence of asymmetric information between buyer and seller, if the*

[1] Rochet and Tirole (2006).

transaction between buyer and seller involves a price determined through bargaining or monopoly price-setting, provided that there are no membership externalities."[2]

In addition, Rochet and Tirole (2006) define a set of characteristics making a market two-sided: (i) the existence of transaction costs among end-users, (ii) platform-imposed constraints on pricing between end-users, and (iii) fixed fees to access.

However, the definition of Rochet and Tirole (2006) is not exhaustive; it aims to explain the general nature of a two-sided market and, thus, suffers from the over-inclusiveness. Rysman (2009) together with Hagiu and Wright (2011) point out this shortcoming. However, Rysman (2009) states that broadness of a two-sided market definition is not a problem. Generally, the question is not how one defines whether a market is two-sided or one-sided; it is important to clarify how significant the two-sided issues are in determining an outcome of interest (Rysman, 2009). For example, the structure of fees set by a typical retailer to its consumers and prices paid to its suppliers will matter in realistic settings (Hagiu and Wright, 2011). This leads to an important generalization of the existing literature on two-sided markets: one of two sides must be charged with a lower price in order to create indirect benefits resulting in the appearance of additional participants on the other side. One of classic examples of such pricing technic is game consoles (Sony Playstation, Xbox). Gamers (holders of consoles) are loss leaders (subsidized side) and game developers are the category of agents from which platforms extract profits (for example, by charging royalties). Moreover, if we are in a situation when two or more platforms compete with each other (as it usually happens), there exists an additional motivation for lowering prices to consumers: lower prices help to take away consumers from the rival platforms, driving the latter out of the market (Hagiu, 2009).

Nevertheless, one may find a common trend in the burgeoning literature in the way to determine a notion of the two-sided market. Existing definitions aim at underlining three features (Weyl, 2010): (i) provision of distinct services to both sides of the market that allows to charge different prices; (ii) participant's benefit from membership depends on the number the of users on the one side; (iii) intermediaries (platforms) typically set uniform prices to both sides of the market.

Two pioneering works that have determined three above-mentioned characteristics, particularly pricing choices, in two different ways are the ones introduced by Rochet and Tirole (2003), and Armstrong (2006). The main differences between two articles consist in defining a tariff charged by the platforms and costs they incur while serving agents. According to Rochet and Tirole (2003), a platform charges usage fees and incurs per-transaction costs, whereas Armstrong's (2006) model is characterized by membership fees and per-agent costs. Other significant difference between two models lies in the definition of users' heterogeneity. Rochet and Tirole (2003) assume users to be heterogeneous due to the homogeneity of membership values (benefit or cost from participating in the service if no users participate on the other side) and the

[2] Rochet and Tirole (2006).

heterogeneity of interaction values (benefit or cost of participation for every user that participates on the other side). Contrary to Rochet and Tirole (2003), Armstrong (2006) maintains that users are heterogeneous because of the heterogeneity of membership values and the homogeneity of interaction values. Weyl (2010) provides graphical representation of what is stated above (Figure 2.1).

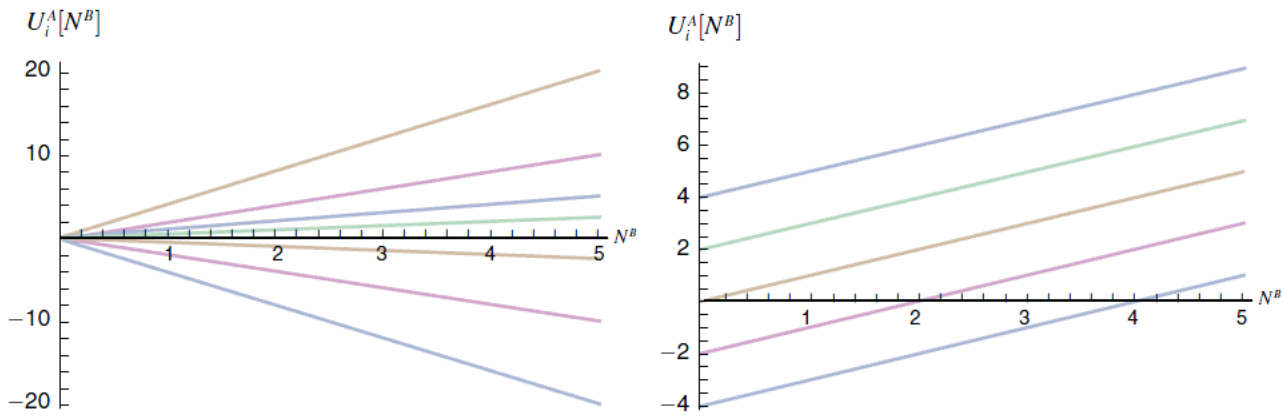

Figure 2.1. Definition of users' heterogeneity by Rochet and Tirole (2003) [left] and Armstrong (2006) [right]

On the Figure 2.1, $U_i^A[N^B]$ is the utility that user i located on the side A of a two-sided market gains depending on N users situated on the side B.

Afterwards, in a consequent paper of Rochet and Tirole (2006) there has been developed a model pointed at generalization of both Rochet and Tirole (2003) and Armstrong (2006) models. Nevertheless, Armstrong's (2006) article has become a basis for a considerable amount of literature on two-sided markets and tends to be the most attractive framework within numerous applications.

Moreover, the results provided in this dissertation rely on the model of Armstrong (2006) and other models derived from it. Therefore, it makes sense to provide a brief analysis of the paper of interest.

2.1.1 Armstrong's (2006) classic model on competition in the two-sided market

Armstrong (2006) identifies possible configurations, which could be present in the two-sided market. Suppose there are two firms (platforms) linking both sides of a market and competing with each other for agents. In turn, depending on possible network benefits, agents on both sides might wish to join one platform or both. According to the terminology used in the two-sided market, it is acceptable to say that an agent "single-homes" when she joins only one platform, and "multi-homes" when she chooses to subscribe to multiple (in our case two) platforms simultaneously. Thus, the configurations of two-sided markets can be identified as follows: (i) both groups of agents single-home; (ii) one group single-homes while other multi-homes; (iii) both groups multi-home. Armstrong (2006) concentrates his attention on the issues (i) and (ii), denominating the configuration (ii) as "competitive bottlenecks". He founds no obvious arguments for configuration (iii) to exist since there is no incentive for one side to multi-home if other side has already done this.

However, let us firstly refer to the differences in models presented in articles of Armstrong (2006), and Rochet and Tirole (2003), and determine how an agent j's utility is specified according to different frameworks.

Both articles define the utility of an agent j from single-homing on platform i as follows

$$u_i^j = \alpha_i^j n^i + \zeta_j^i$$

where n^i is the number of agents on the other side of platform i, α_i^j is the benefit that agent j enjoys from each additional agent on the other side of the market, and ζ_j^i is a fixed benefit that agent j obtains from using platform i. Rochet and Tirole's (2003) model specification assumes ζ_j^i not to be dependent on either a platform or an agent and is set at zero, instead α_i^j varies across agents and platforms. On the contrary, Armstrong (2006) puts an assumption on α_i^j to be homogeneous across agents, but heterogeneous with each side of the market. ζ_j^i in Armstrong's (2006) configuration depends both on the agent and on the platform. Different ways of defining the agent j's heterogeneity lead to different price structures for both sides of the markets in two above-mentioned articles, especially in the case of a monopoly platform.

Let us now look more precisely on how two articles define the structure of platforms' fees and costs. According to Rochet and Tirole (2003) the net utility that agent j obtains from using platform i is given by $u_i^j = (\alpha_i^j - \gamma^i)n^i$, where γ^i is a per-transaction fee and $\zeta = 0$. On the other side, if a platform has n_1 and n_2 agents on sides 1 and 2, respectively, and incurs constant cost of c for every transaction executed by an agent, then platform i's total cost is given by multiplicative formula cn_1n_2, used by Schmalensee (2002) previously. In contrast, Armstrong (2006) assumes platform i's charges to be defined on a lump-sum basis. Thus, agent j's net utility is $u_i^j = \alpha n^i + \zeta_j^i - p_i$, where p_i is a lump-sum fee charged by platform i. Given certain per-agent costs f_1 and f_2, and the total number of agents on each side equal to n_1 and n_2, then the total cost of the platform can be defined as $f_1n_1 + f_2n_2$. As one can notice, the approaches of Armstrong (2006) and Rochet and Tirole (2003) differ not only in the assessment of an agent's heterogeneity, but also in the identification of a cost function. However, both models are designed in order to outline realities of a particular market. Thus, Rochet and Tirole's (2003) model is well matched in the credit card context, whereas Armstrong's (2006) approach can be reasonably used in order to model a competition in the newspaper or retail industry.

In his paper, Armstrong (2006) derives equilibrium prices and platform's profits under three market configurations: (i) two-sided market is served by a monopoly platform; (ii) two-sided market is served by two competing platforms with both sides single-home; (iii) two-sided market is served by two competing platforms with one single-homing side and other multi-homing. Point (iii) is presented within a general framework.

Monopoly platform and welfare analysis

Although it is difficult to find two-sided markets with the pure monopoly, one might examine the case when a monopoly appears on the local level. Armstrong (2006) refers to the situation when night clubs and shopping malls are generally attended by the group of people living relatively closely to these establishments and not considering any other options.

As on all two-sided markets, two groups of agents, denoted by 1 and 2, with n_1 and n_2 members, respectively, are presented on the market. The utilities of group-1 and group-2 agents are determined in the following way

(2.1) $$u_1 = \alpha_1 n_2 - p_1; \qquad u_2 = \alpha_2 n_1 - p_2,$$

where α_1 (α_2) measures the benefit that a group-1 (group-2) agent enjoys from interaction with each group-2 (group-1) agent, and p_1 (p_2) is the price charged by the platform 1 (2) for the group-1 (group-2) agent. Since group-1 and 2 agents consist of n_1 and n_2 participants, and the utilities of these agents are u_1 and u_2, respectively, one could specify the number of members on each side as a function of the utilities

$$n_1 = \phi_1(u_1); \qquad n_2 = \phi_2(u_2),$$

where $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are continuously increasing, differentiable functions on a concave support. Expressions above are simply telling us that agents on one side care about the number of agents on the other side. Such interpretation gives rise to the "chicken & egg" problem formulated by Caillaud and Jullien (2003) in their article: in order to attract buyers, a platform should have a large base of registered sellers, but these sellers will be willing to register only if they expect a sufficient number of buyers to join.

As it has been stated above, the cost side of the platform is represented by per-agent costs f_1 and f_2 for serving each group-1 and group-2 agent, respectively. Hence, the monopoly's profit can be expressed as

$$\pi = n_1(p_1 - f_1) + n_2(p_2 - f_2).$$

Armstrong (2006) proceeds with the assumption that a platform maximizes utilities $\{u_i\}_{i=1,2}$ rather than prices $\{p_i\}_{i=1,2}$. From equations in (2.1) we express prices as a functions of utilities, i.e. $p_1 = \alpha_1\phi_2(u_2) - u_1$ and $p_2 = \alpha_2\phi_1(u_1) - u_2$. Rewritten in terms of utilities monopolistic profit becomes

(2.2) $$\pi = \phi_1(u_1)[\alpha_1\phi_2(u_2) - u_1 - f_1] + \phi_2(u_2)[\alpha_2\phi_1(u_1) - u_2 - f_2].$$

Welfare analysis

Let aggregate consumer surplus (the sum of all utilities) be $v_i(u_i)$ for $i = 1,2$. $v_i(\cdot)$ satisfies the envelope condition $v_i'(u_i) \equiv \phi_i(u_i)$. The social welfare can be expressed as the composition of aggregate surpluses of group-i ($i = 1{,}2$) agents and the monopoly platform's profit

$$\omega = \pi^M(u_1, u_2) + v_1(u_1) + v_2(u_2).$$

To find a welfare-maximizing outcome we proceed with the first order conditions of ω with respect to u_1 and u_2.

$$\frac{\partial\omega}{\partial u_1} = \phi_1{}'(u_1)[\alpha_1\phi_2(u_2) - u_1 - f_1] + \alpha_2\phi_2(u_2)\phi_1{}'(u_1) - \phi_1(u_1) + \phi_1(u_1) = 0;$$

$$\frac{\partial\omega}{\partial u_2} = \phi_2{}'(u_2)[\alpha_2\phi_1(u_1) - u_2 - f_2] + \alpha_1\phi_1(u_1)\phi_2{}'(u_2) - \phi_2(u_2) + \phi_2(u_2) = 0.$$

Thus, by solving $\frac{\partial\omega}{\partial u_1}$ and $\frac{\partial\omega}{\partial u_2}$ for u_1 and u_2, respectively, a welfare-maximizing outcome in terms of utilities is given by

(2.3) $$u_1^{SO} = (\alpha_1 + \alpha_2)n_2 - f_1; \qquad u_2^{SO} = (\alpha_1 + \alpha_2)n_1 - f_2.$$

Knowing that $p_1 = \alpha_1\phi_2(u_2) - u_1$ and $p_2 = \alpha_2\phi_1(u_1) - u_2$, we can solve $\frac{\partial\omega}{\partial u_1} = 0$ and $\frac{\partial\omega}{\partial u_2} = 0$ for the socially optimal prices, which are

(2.4) $$p_1^{SO} = f_1 - \alpha_2 n_2; \qquad p_2^{SO} = f_2 - \alpha_1 n_1.$$

Contrary to the logic of the one-sided market, a socially optimal price for each side of a two-sided market is not equal to the marginal cost; in particular, it is even below it (given α_1, $\alpha_2 > 0$). The price decrease is caused by existing positive network externalities, that is to say, a socially optimal price is adjusted downward by the external benefit that an extra group-1 agent brings to the group-2 agents on the platform.

Let us now find the optimal prices for both sides of the market when monopoly is aiming at maximizing its profit despite increasing a social welfare, i.e. we differentiate the expression (2.2) with the respect to u_1 and u_2.

$$\frac{\partial\pi^M}{\partial u_1} = \phi_1{}'(u_1)[\alpha_1\phi_2(u_2) - u_1 - f_1] + \alpha_2\phi_2(u_2)\phi_1{}'(u_1) - \phi_1(u_1) = 0;$$

$$\frac{\partial\pi^M}{\partial u_2} = \phi_2{}'(u_2)[\alpha_2\phi_1(u_1) - u_2 - f_2] + \alpha_1\phi_1(u_1)\phi_2{}'(u_2) - \phi_2(u_2) = 0.$$

By analogy with socially optimal prices we can solve two first order conditions, $\frac{\partial\pi^M}{\partial u_1}$ and $\frac{\partial\pi^M}{\partial u_2}$, for monopolistic profit-maximizing prices, which are given as follows

(2.5) $$p_1^M = f_1 - \alpha_2 n_2 + \frac{\phi_1(u_1)}{\phi_1{}'(u_1)}; \qquad p_2^M = f_2 - \alpha_1 n_1 + \frac{\phi_2(u_2)}{\phi_2{}'(u_2)}.$$

Contrary to welfare-maximizing prices, monopolistic profit-maximizing prices are larger by the value related to the elasticity of the group i's participation. In his paper Armstrong

(2006) characterizes monopoly's prices in the context of a Lerner's elasticity pricing rule, which is $\frac{p-MC}{p}=\frac{1}{\eta}$, where $\eta=\frac{\phi\prime(u)/\phi(u)}{\Delta p/p}$ is the price elasticity of demand.

If we introduce group-1 and group-2 price elasticities of demand for each side of the market as $\eta_1(p_1|n_2)=\frac{p_1\phi_1'(\alpha_1 n_2-p_1)}{\phi_1(\alpha_1 n_2-p_1)}$ and $\eta_2(p_2|n_1)=\frac{p_2\phi_2'(\alpha_2 n_1-p_2)}{\phi_2(\alpha_2 n_1-p_2)}$ with $\Delta p_i=1$, respectively, then we can state that the profit-maximizing pair of prices satisfies the classic Lerner formulae

(2.6) $$\frac{p_1^M-(f_1-\alpha_2 n_2)}{p_1^M}=\frac{1}{\eta_1(p_1^M|n_2)}; \qquad \frac{p_2^M-(f_2-\alpha_1 n_1)}{p_2^M}=\frac{1}{\eta_2(p_2^M|n_1)}.$$

If we substitute in equation (2.6) p_1^M and p_2^M, we will investigate the equality in Lerner indices. Equation (2.6) gives us an important intuition of the practical functioning of the two-sided markets. Platform solves "chicken-egg" problem by setting a subsidized price ($p_1<f_1$) to the group with high elasticity of demand and/or with low external benefit from the interaction. Sometimes subsidy can be so large, that price will be negative or set to zero (in the case if negative prices are not feasible). However, not all literature follows the same intuition. Rochet and Tirole (2003) have derived a rather unusual result according to which a more elastic group of customers must be charged more. The same result was reached also by Roson (2005). Anyhow, the reality confirms the logic of Armstrong (2006): a low price on one side not only attracts elastic consumers on that side but also, as a result, leads to higher prices and more participation on the other less elastic side (Rysman, 2009). As a typical example of what is stated before may serve Abobe PDF Reader. Since there is a lot of free software reading PDF format, readers themselves are very price elastic. In turn, publishers need software to create PDF files (Adobe Acrobat) and do not really care of its price since once they buy it, they are able to create numerous amounts of PDF documents and spread them between readers for a particular charge.

Two-sided single-homing

The model where both sides are allowed only to single-home can be viewed as an extension to the above-mentioned monopoly platform, with the difference that there are two platforms serving the market and competing with each other. Thus, group-1 and group-2 agents get utilities of u_1^i and u_2^i if they join a platform $i=A,B$. $\{u_1^i,u_2^i\}$ are determined in the similar manner to those expressed in (2.1):

(2.7) $$u_1^i=\alpha_1 n_2^i-p_1^i; \qquad u_2^i=\alpha_2 n_1^i-p_2^i,$$

where $\{n_1^i,n_2^i\}$ are the extents of members of two agent groups attracted by platform i, and $\{p_1^i,p_2^i\}$ are prices charged by platform i to these two groups, respectively. Let us now derive the location of the indifferent single-homing agent on both sides by using a Hotelling location model. This model is also known as a Linear city model, where a city has only one street with length equal to one, and inhabitants uniformly distributed along this street.

Given that agents are uniformly distributed along the unit interval, it is possible to express utilities suggested by platforms A and B to the group-1 agents as follows:

(2.8) $$u_1^A = \alpha_1 n_2^A - p_1^A - t_1 n_1^A; \qquad u_1^B = \alpha_1 n_2^B - p_1^B - t_1(1 - n_1^A),$$

where $t_1 > 0$ is a transportation cost incurred by group-1 agent in order to move from platform A to platform B. Since the mass of group-1 agents equals one, we have $n_1^B = 1 - n_1^A$. Under the condition when $u_1^A = u_1^B$, one would expect a group-1 agent to be indifferent between two platforms. By solving the simultaneous equations in (2.8) for n_1^A one can get $n_1^A = \frac{1}{2} + \frac{u_1^A - u_1^B}{2t_1}$. Following the same logic one can solve also for n_2^A and obtain $n_2^A = \frac{1}{2} + \frac{u_2^A - u_2^B}{2t_2}$, where $t_2 > 0$ is a transportation cost parameter incurred by a group-2 agent. The generalization of these results leads to the following demand configurations

(2.9) $$n_1^i = \frac{1}{2} + \frac{u_1^i - u_1^j}{2t_1}; \qquad n_2^i = \frac{1}{2} + \frac{u_2^i - u_2^j}{2t_2}.$$

Substituting (2.7) into (2.9) and setting $n_1^i = 1 - n_1^j$ and $n_2^i = 1 - n_2^j$, leads to the following market shares, which depend on number of agents located on the opposite side of platform i:

(2.10) $$n_1^i = \frac{1}{2} + \frac{\alpha_1(2n_2^i - 1) - (p_1^i - p_1^j)}{2t_1}; \qquad n_2^i = \frac{1}{2} + \frac{\alpha_2(2n_1^i - 1) - (p_2^i - p_2^j)}{2t_2}.$$

In order to obtain market shares as a function of platforms' prices one can solve simultaneous equations in (2.10) and get the following results:

(2.11) $$n_1^i = \frac{1}{2} + \frac{1}{2}\frac{\alpha_1(p_2^j - p_2^i) + t_2(p_1^j - p_1^i)}{t_1 t_2 - \alpha_1 \alpha_2}; \quad n_2^i = \frac{1}{2} + \frac{1}{2}\frac{\alpha_2(p_1^j - p_1^i) + t_1(p_2^j - p_2^i)}{t_1 t_2 - \alpha_1 \alpha_2}.$$

In order to support market-sharing equilibrium it is crucial to make the next assumption: $t_1 t_2 - \alpha_1 \alpha_2 > 0$, that is to say network externality parameters $\{\alpha_1, \alpha_2\}$ must be smaller comparing to the preference parameters $\{t_1, t_2\}$, otherwise equilibrium will converge to the case where one platform corners both sides of the market. One may notice from the equation (2.11) that platform i's market shares for both groups of agents have a direct relation with prices set to these agents by a competitor, provided that $\alpha_1, \alpha_2, t_1, t_2 > 0$.

Suppose that each platform incurs the same per-agent costs of $\{f_1, f_2\}$ for serving group-1 and group-2 agents, respectively. Then, one can express platform i's profit under single-homing condition as follows:

$$\pi_i^{SH} = (p_1^i - f_1)\left[\frac{1}{2} + \frac{1}{2}\frac{\alpha_1(p_2^j - p_2^i) + t_2(p_1^j - p_1^i)}{t_1 t_2 - \alpha_1 \alpha_2}\right] + (p_2^i - f_2)\left[\frac{1}{2} + \frac{1}{2}\frac{\alpha_2(p_1^j - p_1^i) + t_1(p_2^j - p_2^i)}{t_1 t_2 - \alpha_1 \alpha_2}\right]$$

π_i^{SH} is concave in platform i's prices if and only if $t_1 t_2 - \alpha_1 \alpha_2 > 0$. Let us find the best response of platform i to the platform j's prices, that is to say we have to maximize π_i^{SH} with the respect to p_1^i and p_2^i. Thus, first order conditions are as follows:

$$\frac{\partial \pi_i^{SH}}{\partial p_1^i} = \frac{1}{2} - \frac{(p_1^i - f_1)t_2}{2(t_1 t_2 - \alpha_1 \alpha_2)} + \frac{(p_1^j - p_1^i)t_2 + (p_2^j - p_2^i)\alpha_1}{2(t_1 t_2 - \alpha_1 \alpha_2)} - \frac{(p_2^i - f_2)\alpha_2}{2(t_1 t_2 - \alpha_1 \alpha_2)} = 0;$$

$$\frac{\partial \pi_i^{SH}}{\partial p_2^i} = \frac{1}{2} - \frac{(p_2^i - f_2)t_1}{2(t_1 t_2 - \alpha_1 \alpha_2)} + \frac{(p_2^j - p_2^i)t_1 + (p_1^j - p_1^i)\alpha_2}{2(t_1 t_2 - \alpha_1 \alpha_2)} - \frac{(p_1^i - f_1)\alpha_1}{2(t_1 t_2 - \alpha_1 \alpha_2)} = 0.$$

In the case of symmetric equilibrium where each platform offers the same pair of prices (i.e. $p_1^j = p_1^i = p_1$ and $p_2^j = p_2^i = p_2$) one can find the expressions for p_1 and p_2 from the first order conditions:

(2.12) $$p_1^{SH} = f_1 + t_1 - \frac{\alpha_2}{t_2}(\alpha_1 + p_2 - f_2); \qquad p_2^{SH} = f_2 + t_2 - \frac{\alpha_1}{t_1}(\alpha_2 + p_1 - f_1).$$

The price for group-1 agents is adjusted downward by the factors connected to cross-group externalities. The term $(\alpha_1 + p_2 - f_2)$ is composed of two parts: $(p_2 - f_2)$ represents platform's profit from an extra group-2 agent, and α_1 shows how much a platform is able to extract from its group-1 agents without losing a market share when it has an extra group-2 agent on board. The number of such extra group-2 agents is represented by α_2/t_2 term. Integrally, the term $\frac{\alpha_2}{t_2}(\alpha_1 + p_2 - f_2)$ measures the external benefit, which a platform enjoys from attracting an extra group-1 agent. In contrast, the equilibrium price without cross-group externalities is simply equal to $f_1 + t_1$.

We can proceed with the simplification of (2.12) by solving its two simultaneous equations. The result of such simplification is given as follows:

(2.13) $$p_1^{SH} = f_1 + t_1 - \alpha_2; \qquad p_2^{SH} = f_2 + t_2 - \alpha_1.$$

Equations in (2.13) represent the unique symmetric equilibrium when both sides of the market single-home, and there are two platforms serving these sides. One can notice that symmetric prices in (2.13) do not depend on their own side-specific externality parameters, as well as on the price set for the agents on the opposite side. Armstrong (2006) claims it to be a shortcoming caused by Hotelling specification of a consumer's demand. Once the assumption on the fixed market shares is imposed, there will not be a possibility to perform the welfare analysis since any symmetric pair of prices will yield the same total surplus. However, one may notice from (2.13) that in the case when group-specific transportation and per-agent costs are relatively equal, a platform will set a lower price to that group of agents, which is more valuable for the opposite group.

Let us now derive the Lerner formula within competing platforms and single-homing agents framework. To derive platform's own elasticity of demand we can use expression in (2.10). Knowing that $\eta = \frac{dn/n}{dp/p}$, $dp_1^i = 1$, and by fixing number of group-1 and group-2 agents on the equal level, i.e. $n_1^i = n_2^i = 1/2$, then expression for η can be derived in the following steps:

$$\frac{dn_1^i}{dp_1^i} = \frac{\partial}{\partial p_1^i}\left(\frac{1}{2} - \frac{(p_1^i - p_1^j)}{2t_1}\right) = -\frac{1}{2t_1}.$$

Knowing that in the symmetric equilibrium prices are equal ($p_1^j = p_1^i = p_1^{SH}$), this leads us to:

$$\eta_1^{SH} = \frac{dn_1/n_1}{dp_1/p_1} = -\frac{p_1^{SH}}{t_1}$$

We are interested in the positive value of elasticity, thus $L = 1/\eta_1^{SH} = t_1/p_1^{SH}$. The same holds for the group-2 agents: $1/\eta_2^{SH} = t_2/p_2^{SH}$. Thus, bearing in mind above-mentioned conditions one can rewrite Lerner formula as follows:

(2.14) $$\frac{p_1^{SH} - (f_1 - 2\alpha_2 n_2)}{p_1^{SH}} = \frac{1}{\eta_1^{SH}}; \qquad \frac{p_2^{SH} - (f_2 - 2\alpha_1 n_1)}{p_2^{SH}} = \frac{1}{\eta_2^{SH}}.$$

If one substitutes p_1^{SH} and p_2^{SH} from (2.13) into (2.14), and assume number of agents to be equal in equilibrium, it is possible to notice that equalities in (2.14) hold. Comparing expressions in (2.14) with the monopolistic ones in (2.6), it can be obviously seen that a duopolist puts a double emphasis on the group-2 (group-1) external benefit parameter when it makes a decision to price group-1 (group-2) agents. This can be explained by the possible outcomes for platform i: if it sets a higher price to the side-k agents comparing to its rival, it pushes this group of agents to leave a platform and join the competitor. In the case of monopoly, dissatisfied with a high price agents simply disappear from the market.

In a symmetric equilibrium platform i's profit can be represented as follows

(2.15) $$\pi_i^{SH} = \frac{t_1 + t_2 - \alpha_1 - \alpha_2}{2}.$$

The equation (2.15) can be derived if one substitutes $p_1^{SH} = f_1 + t_1 - \alpha_2$, $p_2^{SH} = f_2 + t_2 - \alpha_1$ and $n_1^i = n_2^i = 1/2$ into $\pi_i^{SH} = (p_1^{SH} - f_1)n_1 + (p_2^{SH} - f_2)n_2$. Under the assumption of $t_1 t_2 - \alpha_1 \alpha_2 > 0$, the profit is positive. Moreover, positive cross-group externalities ($\alpha_1, \alpha_2 > 0$) push platforms to compete more severely for the market shares by reducing prices and, therefore, profits.

Competitive bottlenecks: a general framework

In this section Armstrong (2006) represents the general framework of a two-sided market model where one group (say, group 1) remains to operate only with one platform (single-homes) while the other group (say, group 2) deals with both platforms simultaneously (multi-homes); that is to say group-2 agents put more weight on network benefits and, therefore, are willing to cover all the population of group-1 agents presented in the two-sided market. The crucial assumption of this model is that group-2 agents decide to join one platform independently from the decision to join the other (put simply, platforms do not compete for group-2 agents).

According to Armstrong (2006), such two-sided market model set-up is called "Competitive Bottlenecks". This particular set-up has been successfully used in various extensions by a number of authors, including Armstrong himself. Particularly, Armstrong and Wright (2007) have employed a "competitive bottlenecks" framework together with the conception of product differentiation in order to analyze exclusive dealing issues in two-sided markets.

The theoretical framework of the “competitive bottlenecks” model also has been used in order to analyze such anti-competitive practices such as tying. In particular, it was successfully applied by Choi (2010) in his paper on tying in the two-sided market with multi-homing on both sides. Choi (2010) has showed that within the competitive bottlenecks set-up when users multi-home and content providers single-home, tying reduces social welfare. In the next subsection we are going to investigate the market equilibrium introduced in Choi (2010) since we will employ his framework in the further analysis of a search engine competition.

2.1.2 Competition in two-sided markets with multi-homing on both sides

Mostly, the competition in two-sided markets is analyzed via models where multi-homing is not allowed, or allowed only on one side. The reason why we do not expect to find papers on two-sided markets with both groups of agents multi-homing is common: if each member of one out of two groups joins both platforms, there is no need for any member from other group to join more than one platform (Armstrong, 2006). However, it not always the case when we are dealing with the information goods. The main features of information goods are: (i) low marginal costs (close to zero), (ii) high network externalities (usually positive) implying that the benefits to a single consumer increase with the number of consumers using the same good, (iii) possibility of the convergence to more complex products with the development of new technologies. Among such goods one can point out media players, search engines, software etc. Moreover, the users of information goods do not generally incur high transportation costs, such that they have incentives to multi-home.

One of the papers on competition in the two-sided market where multi-homing is allowed on both sides was developed by Choi (2010). In general, the paper of Choi is aimed at examining the case whether tying is welfare-enhancing if multi-homing is allowed on both sides. In particular, the author points out that tying prompts more consumers to multi-home and this, in turn, opens them an access to more exclusive content, which is beneficial to content providers. In order to investigate whether a tying is more advantageous comparing to the non-tying, firstly, the author sets up two models (with and without tying) where multi-homing is allowed on both sides and derives equilibrium market outcomes, consumer surplus, and social welfare. We are going to investigate only the first part of Choi’s (2010) paper, which is devoted to the development of the two-sided market framework where multi-homing is allowed on both sides.

Choi’s (2010) model where the “tying” component is absent can be considered as an extension to Armstrong’s (2006) “competitive bottlenecks” model with the difference that Choi (2010) assumes multi-homing on both sides of the market, whereas Armstrong (2006) provides the analysis of the situation where only one side multi-homes. However, in general, the model of Choi (2010) adopts the common logic of two-sided market models (Rochet and Tirole, 2003; Rochet and Tirole, 2006; Armstong, 2006) such that there are consumers on the one side of the market, content providers on the other side, and platforms serving as intermediaries between these two groups of agents; but it is allowed to multi-home on both sides of the market. There are two intermediaries (platforms) indexed by $i = A, B$ aiming at increasing market shares on both sides of the market. The platforms charge prices p_i and q_i to content providers and consumers, and incur per-agent costs c and d, respectively. The number of content providers and consumers participating in platform i is denoted by m_i and n_i, respectively.

Consumer side

Since Choi (2010) considers the realities of a digital media market, his two-sided market model is constructed in order to reflect such realities. Product differentiation is represented by a linear Hotelling model where platforms A and B are located at the end points of a line with length equal to 1. Consumers reside along this line with a uniform density. The mass of consumers is normalized to 1. The utility of every single consumer depends on the number of content providers on the same platform, i.e. each consumer enjoys a benefit measured by b from every additional content provider.

Consumers can either single-home or multi-home (Figure 2.2), i.e. they have three options:

- join platform *A* and get utility $u_A(q_A, x) = bm_A - q_A - tx$,
- join platform *B* and get utility $u_B(q_B, x) = bm_B - q_B - t(1 - x)$,
- multi-home and get utility $u_{AB}(q_A, q_B, x) = bm - q_A - tx - q_B - t(1 - x)$,

where t is a transportation cost parameter, and m is the total amount of content available to consumers who multi-home. If there is no content duplication then $m = m_A + m_B$. If the duplication of $\delta \leq \min\{m_A, m_B\}$ is present, we have $m = m_A + m_B - \delta$.

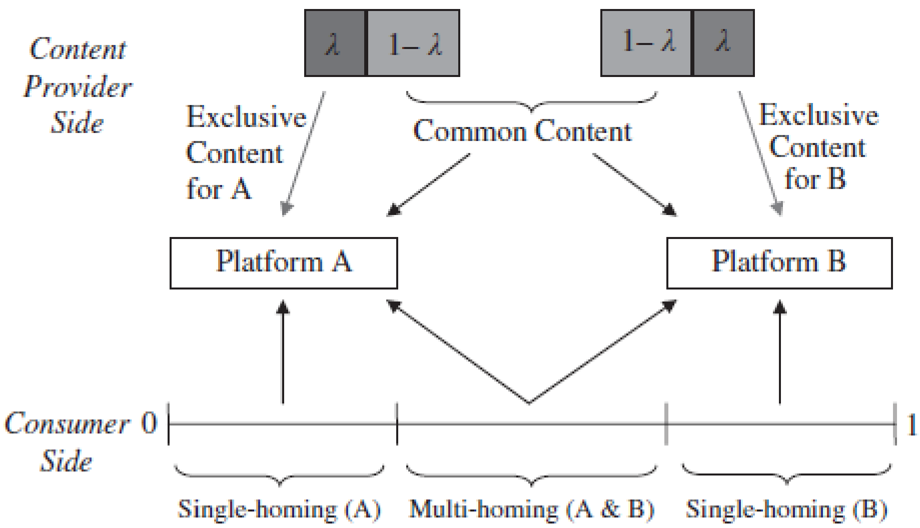

Figure 2.2. Two-sided markets with multi-homing

Content provider side

The extent of content providers is normalized to 1 for each platform. Hence, the total measure of the supplied content is also normalized to 1. Fixed costs on the content creation are not assumed to exist in this model, since the amount of content is exogenously given, and, thus, the entrance possibilities of other content providers are denied. Choi (2010) states that the presence of exogenously given amount of content is a shortcoming of the model. Each content provider enjoys a benefit measured by π from the interaction with every additional consumer. Thus, the profit of a content provider who joins platform i is given by $\pi n_i - p_i$. Naturally, the content provider will supply to platform i its content only under the condition when $\pi n_i - p_i$ is non-negative.

In order to support the multi-homing assumption on the consumer side, Choi (2010) states that there are two types of content available. These two types of content differ in feasibility of being converted into other format, that is to say, there exists the content of the first type, which is not economically feasible to transform into the other format, and the content of second type that can be transformed into other format. Thus, we have a particular proportion of first type content λ which can be encoded only for a particular format while proportion of second type content $(1-\lambda)$ can be encoded into other format. The availability of exclusive content pushes consumers to multi-home, whereas the existence of the second type content encourages content providers to multi-home.

Market equilibrium

In equilibrium both consumers and content providers multi-home. Each platform provides its consumers with the exclusive content of extent λ and the nonexclusive content of extent $(1-\lambda)$. It is also possible to mention that the measure of content duplication equals to $\delta = 1 - \lambda$. In a symmetric equilibrium consumers located closer to platform A will choose to participate in platform A, and the same for those who are located closer to platform B. However, they may wish to multi-home.

As it has been already defined before, the consumer's utility from membership in only platform A equals to $u_A(q_A, x) = bm_A - q_A - tx$. Under the assumption that the amount of content providers is normalized to 1 ($m_A = 1$) for a platform A, we have $u_A(q_A, x) = b - q_A - tx$. Subsequently, when a consumer decides to multi-home, her utility becomes $u_{AB}(q_A, q_B, x) = bm - q_A - tx - q_B - t(1-x)$. Since each platform has the number of the unique content equal to λ and the duplicative content equal to $(1-\lambda)$, we can draw a result for the total amount of content available for multi-homing consumers. $m = m_A + m_B - \delta = [\lambda + (1-\lambda)] + [\lambda + (1-\lambda)] - (1-\lambda) = 1 + \lambda$. Thus, the utility of consumers who multi-home can be rewritten as $u_{AB}(q_A, q_B, x) = b(1+\lambda) - (q_A + q_B) - t$.

Suppose now that the consumer is indifferent between single-homing on platform A and multi-homing, that is to say $u_A(q_A, x) = u_{AB}(q_A, q_B, x)$. Consumer's location in this case is given by

$$x = 1 - \frac{\lambda b - q_B}{t}.$$

Identically, the location of consumer who is indifferent between single-homing on platform *B* and multi-homing, $u_B(q_B, y) = u_{AB}(q_A, q_B, y)$, is given by

$$y = \frac{\lambda b - q_A}{t}.$$

In order not to create the confusion between two different locations, we make the following notations: $x = n_A$ and $y = 1 - n_B$.

Choi (2010) demonstrates consumer's location graphically (Figure 2.3).

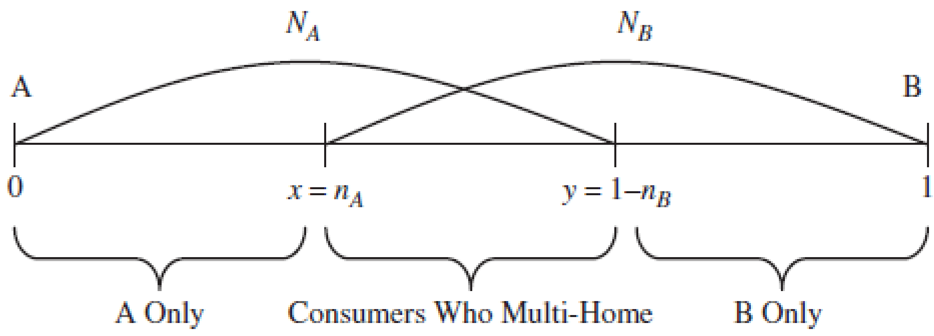

Figure 2.3. The choice of consumers

Thus, the number of consumers who single-home on one platform is given by:

$$n_i = 1 - \frac{\lambda b - q_j}{t}, \text{ where } i = A, B \text{ and } j \neq i.$$

Accordingly, the number of multi-homing consumers is measured by the distance between y and x (Figure 2.3):

$$n_M = y - x = \frac{2\lambda b - (q_A + q_B)}{t} - 1.$$

The total number of consumers participating in platforms A and B are denoted as N_A and N_B, respectively (Figure 2.3). Then, the equations for N_A and N_B can be formulated as follows:

$$N_A = y = n_A + n_M = \frac{\lambda b - q_A}{t};$$

$$N_B = 1 - x = n_B + n_M = \frac{\lambda b - q_B}{t}.$$

Subsequently, $n_M = N_A + N_B - 1$. The situation when some consumers multi-home requires $N_A + N_B > 1$. Choi (2010) maintains that the content provider will supply platform A with exclusive content when $\pi N_A - p_A \geq 0$. If a content provider has already supplied his content to platform B, it will wish to multi-home (i.e. to duplicate existed content in format A) under the condition $\pi n_A - p_A \geq 0$. Thus, if consumers multi-home, then $N_A > n_A$. This means that the platform A can charge content providers in two ways and with two different outcomes:

- charge πN_A and attract only λ exclusive content providers,
- charge πn_A and attract both exclusive and nonexclusive content providers.

Choi (2010) analyses the maximization problem with the assumption that platform A serves both exclusive and nonexclusive content providers, where the latter group multi-homes. Under the afore-mentioned condition the optimal price charged by platform A in this case is given by:

$$p_A^*(q_B) = \pi n_A = \pi\left(1 - \frac{\lambda b - q_B}{t}\right).$$

However, if platform A wishes to serve only exclusive content providers, the optimal price established for them will be independent of platform *B's* price to consumers.

On the consumer side, platform A has to solve the following optimization problem:

(2.16) $$\max_{q_A}(q_A - d)N_A = (q_A - d)\frac{\lambda b - q_A}{t}.$$

It can be seen that profit from content provider side is not considered in the maximization problem because the optimal price p_A^* is a function of q_B.

F.O.C. of equation (2.16) is

$$\frac{\partial}{\partial q_A}\left((q_A - d)\frac{\lambda b - q_A}{t}\right) = \frac{b\lambda + d - 2q_A}{t} = 0.$$

Solving F.O.C. of (2.16) for q_A, we find the equilibrium price charged to the consumers by the platform $i = A, B$:

(2.17) $$q_i^* = \frac{\lambda b + d}{2}.$$

Since platforms A and B set equivalent prices to consumers, then

(2.18) $$N_A = N_B = \frac{\lambda b - \frac{\lambda b + d}{2}}{t} = \frac{\lambda b - d}{2t}.$$

For (2.18) to be consistent with consumer side multi-homing, we need $N_A + N_B = \frac{\lambda b - d}{2t} + \frac{\lambda b - d}{2t} > 1$, or $\lambda b - d > t$. Choi (2010) names this assumption as $A1$.

$A1$ $$\lambda b - d > t.$$

The condition $A1$ means that consumers multi-home when benefits from interaction and amount of exclusive content are relatively high comparing to transportation and per-agent costs. Choi (2010) states that under two-sided multi-homing (even besides assumption $A1$) neither platform is going to deviate since the attraction of both types of content providers yields higher payoff comparing to the attraction of only exclusive content providers. For analytical simplicity on this step Choi (2010) assumes $c = d = 0$ to hold for the rest of the paper.

Bearing in mind the assumption on zero per-agent costs, Choi (2010) derives the second (no deviation) condition and denotes it as $A2$:

$$A2 \qquad \frac{\lambda[\lambda\pi + 2b(1+\lambda)]}{4} \leq t.$$

Proof of $A2$*.* See Appendix A.

Proposition 2.1. Under the conditions $A1$ *and* $A2$ *to hold, there exists an equilibrium with consumers and content providers multi-homing when* $\lambda b > t \geq \frac{\lambda[\lambda\pi+2b(1+\lambda)]}{4}$*.*

By assuming a zero transportation cost, i.e. by solving inequality $0 > \frac{\lambda[\lambda\pi+2b(1+\lambda)]}{4} - \lambda b$, it is easy to find out that parameter space is non-empty when $\frac{2(1-\lambda)}{\lambda} > \frac{\pi}{b}$, which implies that a large amount of the exclusive content must be accompanied by the lower ratio of the content provider's benefit from addition consumer (π) to consumer's benefit from additional provider (b).

The next section of Choi's (2010) paper is devoted to the analysis of tying in two-sided markets with multi-homing. Choi (2010) analyses tying from a particular point of view. A firm, which owns platform A, is a monopolistic supplier of product M, which is essential to have in order to access either platform A or platform B. Depending on the policy of this firm, it can decide to bundle product M with platform A, or supply them independently on the market. If this monopolistic firm ties two goods together, the welfare implications are ambiguous and depend on consumers' status, i.e. we must know whether consumers multi-home or not. If consumers multi-home then social welfare increases, but aggregate consumer surplus decreases and vice versa. However, this analysis is not of our interest, such that we proceed with the analysis of the exclusive dealing in two-sided markets.

2.2 Competitive bottlenecks and exclusive dealing in the two-sided market

Unlike a one-sided market framework, which is mostly characterized by the buyer's involvement in signing the exclusive contracts, in two-sided markets with network externalities one might expect the exclusive dealing to occur with both sides simultaneously. In fact, in traditional one-sided markets incumbent firms are only anxious about signing exclusive contracts with all the consumers in order to deny the possibility of having potential entrants (Rasmusen al. 1991; Segal and Whinston, 2000). However, sometimes it might happen that an incumbent involves in the exclusive dealing only some part of consumers, thereby, giving its potential rivals an opportunity to enter (Wright, 2008). On the contrary, depending on different parameters (network benefits or transportations costs), a platform serving the two-sided market can bind one side with attractive exclusive contracts resulting in lower prices, and exploit positive network effects on the other side by charging more to its consumers; or it can offer exclusive contracts to both sides of the market.

An extensive analysis of the exclusive dealing in two-sided markets is performed by Armstrong and Wright (2007). Based on the classic two-sided market framework developed by Armstrong (2006), authors extend the analysis with two additional cases: when there exists a product differentiation on only one side of the market (competitive bottlenecks), and when there is no product differentiation on neither side. Let us introduce the general framework developed by Armstrong and Wright (2007) in their article.

2.2.1 Armstrong and Wright's (2007) generalized framework of the two-sided market

There are two groups of agents, sellers (S) and buyers (B), with the number of agents in each group normalized to one. Each agent from group $k = S, B$ enjoys a benefit of $b_k n$ from interaction with n agents from the other group. The two-sided market is represented by two symmetric platforms denoted by $i = 1, 2$. Agents are allowed to single-home or multi-home on one or both platforms, respectively. Independently of subscribing on either platform, an agent obtains an intrinsic benefit of v_k^0 from such subscription. As in Armstrong (2006), agents from group $k = S, B$ are uniformly distributed along a unit interval, and the two platforms are located at two endpoints. Therefore, a group-k agent incurs a transportation cost of $t_k x$ from travelling to the platform 1, and $t_k(1 - x)$ from travelling to the platform 2. These transportation costs may imply costs needed to reach the platform, signing up a contract, learning how a platform works, or forming a habit to use a particular platform.

The total number of agents on platform i is equal to $n_k^i + N_k$, where n_k^i is the number of agents who subscribed exclusively to platform i, and N_k is the number of multi-homing agents. Platforms incur the same per-agent costs (the costs of serving an agent from a group k), which are $f_k \geq 0$. The subscription price that platform i sets to the agents from group k is denoted by p_k^i, and it is assumed to be non-negative. Such assumption differs from that one assumed by Armstrong (2006), where a platform could charge negative prices to one side in order to exploit the network benefits on the other side in a greater way. The analysis of two-sided markets with negative prices charged to one of two sides boils down to the analysis of tying, where "tying" component is deployed by platforms as a tool to introduce implicit subsidies (Amelio and Jullien, 2012). However, in media markets the minimum price is generally bounded at zero level. Imagine yellow pages directories or search engines, which do not charge a consumer for using their services.

Under the afore-mentioned assumptions, the utility of a group-k agent, located at $x \in [0, 1]$ and joined only platform 1 is given by

(2.19) $$v_k^1(x) = v_k^0 - p_k^1 - t_k x + b_k(n_l^1 + N_l),$$

where l denotes agents from the opposite group, such that $l \neq k$. When a group-k agent joins only platform 2, her utility is expressed as

(2.20) $$v_k^2(x) = v_k^0 - p_k^2 - t_k(1 - x) + b_k(n_l^2 + N_l).$$

The utility of a multi-homing agent is given by

(2.21) $$v_k^{12}(x) = v_k^0 - p_k^1 - p_k^2 - t_k + b_k(n_l^1 + n_l^2 + N_l).$$

It could be seen from (2.21) that when the agent multi-homes, she incurs a total transportation cost consisting of transportation costs of joining two platforms separately. The profit of platform i equals

(2.22) $$\pi^i = \left(p_S^i - f_S\right)\left(n_S^i + N_S\right) + \left(p_B^i - f_B\right)\left(n_B^i + N_B\right).$$

The prices are chosen simultaneously, and after observing these prices group-k agents simultaneously make a decision on which platform to join.

Armstrong and Wright (2007) apply this model to three possible situations: (i) strong product differentiation on both sides, (ii) product differentiation on only one side, and (iii) pure network effects. We are going to investigate in this subsection only (i) and (ii) cases, since we will refer to them in the further analysis.

Strong product differentiation on both sides

The situation with a strong product differentiation on both sides almost fully coincides with the two-sided single-homing case described in Armstrong (2006). The only difference between the two models is that Armstrong and Wright (2007) impose non-negativity constraints on prices. The assumptions imposed on transportation costs remain the same, i.e. $t_S > b_S$, $t_B > b_B$, and $4t_S t_B > (b_S + b_B)^2$. In addition, to support a single-homing equilibrium, Armstrong and Wright (2007) assume intrinsic benefits of v_S^0 and v_B^0 to be sufficiently high in order to be sure that all agents wish to subscribe to at least one platform in equilibrium.

Since both sides multi-home, we have $N_S = N_B = 0$ and $n_k^1 = 1 - n_k^2$. In order to find the location of the indifferent group-k agent, one has to equate $v_k^1(x) = v_k^2(x)$.

$$v_S^0 - p_S^1 - t_S x_S + b_S n_B^1 = v_S^0 - p_S^2 - t_S(1 - x_S) + b_S n_B^2;$$

$$v_B^0 - p_B^1 - t_B x_B + b_B n_S^1 = v_B^0 - p_B^2 - t_B(1 - x_B) + b_B n_S^2.$$

If we proceed with solving these equations for $x_S = n_S^1$ and $x_B = n_B^1$ and expressing n_S^1 and n_B^1 as a function of prices, we will end up with the same result as in Section 4 of Armstrong (2006). Specifically, the demand configurations for the platform 1 can be expressed as follows

(2.23) $$n_S^1 = \frac{1}{2} + \frac{1}{2}\frac{b_S(p_B^2 - p_B^1) + t_B(p_S^2 - p_S^1)}{t_S t_B - b_S b_B}; \quad n_B^1 = \frac{1}{2} + \frac{1}{2}\frac{b_B(p_S^2 - p_S^1) + t_S(p_B^2 - p_B^1)}{t_S t_B - b_S b_B}.$$

Consequently, the demand configurations for platform 2 are

(2.24) $$n_S^2 = \frac{1}{2} + \frac{1}{2}\frac{b_S(p_B^1 - p_B^2) + t_B(p_S^1 - p_S^2)}{t_S t_B - b_S b_B}; \quad n_B^2 = \frac{1}{2} + \frac{1}{2}\frac{b_B(p_S^1 - p_S^2) + t_S(p_B^1 - p_B^2)}{t_S t_B - b_S b_B}.$$

Substituting the values from equations (2.23) and (2.24) into the profit functions in (2.22), and solving four first order conditions $\partial \pi^i / \partial p_k^i = 0$ for $i = 1, 2$ and $k = S, B$ we obtain the equilibrium prices which are

(2.25) $$p_S = f_S + t_S - b_B; \qquad p_B = f_B + t_B - b_S.$$

Per-platform profit is

(2.26) $$\pi^1 = \pi^2 = \frac{t_S + t_B - b_S - b_B}{2} > 0.$$

The non-negativity of equilibrium prices in (2.25) hold under the two conditions: $f_S + t_S \geq b_B$ and $f_B + t_B \geq b_S$. These conditions guarantee the situation when both platforms have equal shares in both markets and the profit of each platform is equal exactly to (2.26).

One can notice from (2.25) and (2.26) that either equilibrium prices or symmetric profits increase in transportation costs, t_k, and decrease in the network benefit parameters, b_k. Particularly, under $f_S + t_S \geq b_B$ and $f_B + t_B \geq b_S$, the price-cost margin $(p_k - f_k)$ for group k is equal to the product differentiation parameter (t_k) reduced by the externality parameter of the other group (b_l). When $b_S > b_B$, sellers evaluate each additional buyer higher comparing to the buyers' evaluation of each additional seller, and, therefore, platforms have to subsidize the buyer side of the market in order to create positive flow of buyers. In turn, increased number of buyers will increase the number of sellers on the platform.

If sellers are characterized by a considerably high degree of a network benefit parameter, then the platform may wish to pay buyers for joining the platform, or, in other terms, charge negative prices to buyers. Given the non-negativity constraint on the prices, platform remains with offering buyers a free entry. This is a quite common platform strategy in media markets. Moreover, if a platform cannot charge negative price to one side, it starts to compete more intensively for the other side users in order to exploit more the positive externalities that run in both directions.

When a two-sided market is characterized by a strong product differentiation, it basically means that agents do not have any incentives to multi-home. However, if some of agents from one group multi-home, this provides benefits to single-homing agents located on the other side of the market. Imagine a situation when one group (say, sellers) has a high value of network parameter b_k, and other group (say, buyers) is characterized by relatively small t_k, i.e. it is easy for them to multi-home. Then, the total welfare may increase due to this multi-homing action. That is to say, that above-mentioned single-homing configuration is not necessarily could be considered as welfare maximizing.

Product differentiation on only one side and exclusive contracts

The situation with a product differentiation on only one side allows us to relax assumptions concerning transportation cost and network benefit parameters, i.e. that the former must be necessarily larger than the latter. If a product differentiation exists on the buyer's side, it means that buyers see both platforms as heterogeneous, whereas sellers consider them as homogenous. Think of newspapers. Buyers have preferences on a particular newspaper contrary to the advertisers, who view a newspaper as a place where they could put their ads. Buyers are steeped, because newspapers generally supply a bundled service to buyers (newspapers with news and entertainments) where the interaction with sellers (advertisers) is only the one aspect of platform's services for buyers. Another reason explaining why buyers could be loyal towards a particular platform is that buyers can incur relatively high transportation costs to reach this platform. For example, buyers have to travel to neighboring shopping malls or supermarkets in order to decrease their transportation costs, whereas sellers can be indifferent when they make a distribution decisions.

Hence, if we observe a situation when a product differentiation parameter on the buyer side is high compared to a network benefit parameter, the buyers approach single-homing. In turn, when sellers view platforms as homogenous, they (sellers) will want to multi-home and, therefore, maximize their interaction with buyers.

To employ the initial model to the case when sellers multi-home and buyers single-home, Armstrong and Wright (2007) have modified initial assumptions imposed on transportation costs, network benefits, and intrinsic benefits in the next way: $\upsilon_S^0 = 0$ and υ_B^0 is sufficiently high to ensure the situation when buyers subscribe to at least one out of two platforms in equilibrium; $t_S = 0$ and $t_B > b_B$. In addition to these two assumptions, Armstrong and Wright (2007) add the third one, which guarantee the platforms' desire to serve sellers in equilibrium: $f_S \leq \min\left\{\frac{1}{2}t_B, \frac{1}{4}(b_S + b_B)\right\}$.

Given these assumptions and single-homing buyers, there are four consistent demand configurations: (i) all sellers multi-home, (ii) all sellers single-home on platform 1, (iii) all sellers single-home on platform 2, (iv) neither seller joins either platform. Let us characterize every configuration.

(i) All sellers multi-home

Under this configuration all the buyers joining platform $i = 1, 2$ will enjoy the utility expressed in (2.19) and (2.20). On the contrary, sellers' utility will not include transportation costs and, therefore, is

(2.27) $$\upsilon_S^1 = b_S n_B^1 - p_S^1$$

when they join only platform 1, and

(2.28) $$v_S^2 = b_S n_B^2 - p_S^2$$

when they join only platform 2. When sellers multi-home, they get the utility of

(2.29) $$v_S^{12} = b_S - p_S^1 - p_S^2.$$

Since buyers single-home, the consistent demand configurations for the them do not change and could be drawn from equation (2.10)

(2.30) $$n_B^1 = \frac{1}{2} + \frac{b_B(n_S^1 - n_S^2) - (p_B^1 - p_B^2)}{2t_B}; \quad n_B^2 = \frac{1}{2} + \frac{b_B(n_S^2 - n_S^1) - (p_B^2 - p_B^1)}{2t_B}.$$

Given the equal distribution of sellers throughout both platforms, i.e. $n_S^1 = n_S^2 = 1/2$, the number of buyers joining the platform $i = 1, 2$ can be expressed as

(2.31) $$n_B^1 = \frac{1}{2} + \frac{p_B^2 - p_B^1}{2t_B}; \qquad n_B^2 = \frac{1}{2} + \frac{p_B^1 - p_B^2}{2t_B}.$$

The sellers will have an incentive to multi-home when their utility of multi-homing is (a) higher than the utility of joining only platform 1, i.e. $b_S - p_S^1 - p_S^2 \geq b_S n_B^1 - p_S^1$, (b) higher than the utility of joining only platform 2, i.e. $b_S - p_S^1 - p_S^2 \geq b_S n_B^2 - p_S^2$, and (c) higher than the utility of joining neither platform, i.e. $b_S - p_S^1 - p_S^2 \geq 0$. Given demand configurations presented in (2.31), we can rewrite the inequalities (a) and (b) in terms of prices as

(2.32) $$p_S^1 \leq \left(\frac{1}{2} + \frac{p_B^2 - p_B^1}{2t_B}\right) b_S,$$

and

(2.33) $$p_S^2 \leq \left(\frac{1}{2} + \frac{p_B^1 - p_B^2}{2t_B}\right) b_S.$$

Hence, the profit of platform 1 is given by

(2.34) $$\pi^1 = (p_S^1 - f_S) + (p_B^1 - f_B)\left(\frac{1}{2} + \frac{p_B^2 - p_B^1}{2t_B}\right),$$

and the profit of platform 2 is

(2.35) $$\pi^2 = (p_S^2 - f_S) + (p_B^2 - f_B)\left(\frac{1}{2} + \frac{p_B^1 - p_B^2}{2t_B}\right).$$

(ii) Sellers single-home on platform 1

If all sellers join only platform 1, i.e. $n_S^1 = 1$, the buyers' demand configuration can be expressed as follows

(2.36) $$n_B^1 = \frac{1}{2} + \frac{p_B^2 - p_B^1 + b_B}{2t_B}; \qquad n_B^2 = \frac{1}{2} + \frac{p_B^1 - p_B^2 - b_B}{2t_B}.$$

Similarly to the previous case, it is possible to express the sellers' desire to single-home on platform 1 through utilities. Thus, the inequalities could be written as follows: $b_S n_B^1 - p_S^1 \geq 0$, $b_S n_B^1 - p_S^1 \geq b_S n_B^2 - p_S^2$, and $b_S n_B^1 - p_S^1 \geq b_S - p_S^1 - p_S^2$. By transforming the utility inequalities in price inequalities, one can obtain

(2.37) $$p_S^1 \leq \left(\frac{1}{2} + \frac{p_B^2 - p_B^1 + b_B}{2t_B}\right) b_S,$$

(2.38) $$p_S^2 \geq \left(\frac{1}{2} + \frac{p_B^1 - p_B^2 - b_B}{2t_B}\right) b_S.$$

In turn, the profits of platforms 1 and 2 are

(2.39) $$\pi^1 = (p_S^1 - f_S) + (p_B^1 - f_B)\left(\frac{1}{2} + \frac{p_B^2 - p_B^1 + b_B}{2t_B}\right),$$

(2.40) $$\pi^2 = (p_B^2 - f_B)\left(\frac{1}{2} + \frac{p_B^1 - p_B^2 - b_B}{2t_B}\right).$$

(iii) Sellers single-home on platform 2

Analogously to the case when sellers single-home on platform 1, it is possible to find the optimal demand configurations, prices, and profits of the platforms.

When $n_S^2 = 1$, the optimal demand configuration are as follows

(2.41) $$n_B^1 = \frac{1}{2} + \frac{p_B^2 - p_B^1 - b_B}{2t_B}; \qquad n_B^2 = \frac{1}{2} + \frac{p_B^1 - p_B^2 + b_B}{2t_B}.$$

In terms of utilities, sellers single-home on platform 2 if $b_S n_B^2 - p_S^2 \geq 0$, $b_S n_B^2 - p_S^2 \geq b_S n_B^1 - p_S^1$, and $b_S n_B^2 - p_S^2 \geq b_S - p_S^1 - p_S^2$. Again, rewriting these inequalities in terms of prices we get

(2.41) $$p_S^1 \geq \left(\frac{1}{2} + \frac{p_B^2 - p_B^1 - b_B}{2t_B}\right) b_S,$$

(2.42) $$p_S^2 \leq \left(\frac{1}{2} + \frac{p_B^1 - p_B^2 + b_B}{2t_B}\right) b_S.$$

The subsequent profits are

(2.43) $$\pi^1 = (p_B^1 - f_B)\left(\frac{1}{2} + \frac{p_B^2 - p_B^1 - b_B}{2t_B}\right),$$

(2.44) $$\pi^2 = (p_S^2 - f_S) + (p_B^2 - f_B)\left(\frac{1}{2} + \frac{p_B^1 - p_B^2 + b_B}{2t_B}\right).$$

The last configuration described by Armstrong and Wright (2007) refers to the situation when neither platform attracts sellers.

(iv) Sellers join neither platform

The proportion of buyers on both platforms is identical to the first configuration, i.e. to (2.31). If sellers are not willing to join neither platform, it means they obtain negative utilities either from single-homing or multi-homing, i.e. $b_S n_B^1 - p_S^1 < 0$, $b_S n_B^2 - p_S^2 < 0$, and $b_S - p_S^1 - p_S^2 < 0$. Therefore, the price inequalities in (2.32) and (2.33) have the reversed signs when sellers join neither platform. Hence, the platforms' profits consist only from sales to buyers. Think of newspapers, which could be sold without any advertisements inside.

Notice that when both platforms offer equal prices on each side, $p_S^1 = p_S^2 = p_S$ and $p_B^1 = p_B^2 = p_B$, one can derive the following conditions for configurations to be consistent. For example, under above-mentioned conditions, one can find from inequalities (2.32) and (2.33) the consistency condition for the configuration (i) that is $p_S \leq b_S/2$. Similarly, the configurations (ii) and (iii) are consistent when $p_S > b_S(1 - b_B/t_B)/2$. Instead, the configurations (ii), (iii), and (iv) are consistent under $b_S/2 < p_S < b_S(1 + b_B/t_B)/2$. Hence, one can notice that there is no possibility for configurations (i) and (iv) to be simultaneously consistent. As Armstrong and Wright (2007) have noticed, this happens because these two configurations are represented by the different assumptions on how agents manage their choices with specified prices.

Before proceeding with the exclusive dealing analysis, it is important to mention that if we impose three more assumptions on network benefits, transportation costs, and per-agent costs, particularly $b_S, b_B > 0$, $b_S \geq t_B$, and $b_B \geq f_B$, in addition to those ones, which have been introduced at the beginning (i.e. $v_S^0 = 0$, v_B^0 is sufficiently high, $t_S = 0$, $t_B > b_B$, and $f_S \leq \min\left\{\frac{1}{2}t_B, \frac{1}{4}(b_S + b_B)\right\}$), there will exist a range of symmetric equilibria with buyers single-homing and sellers multi-homing, characterized by

(2.45) $$p_S = b_S/2,$$

and p_B lying in the interval

(2.46) $$\max\{0, f_B + t_B - b_S\} \leq p_B \leq f_B.$$

Armstrong and Wright (2007) have represented four afore-mentioned configurations, containing also new assumptions on parameters, graphically (Figure 2.4), where axes are platform 1s prices expressed in terms of the platform 2s prices. We will refer to this equilibrium pair of prices in the further analysis of tying of exclusive contracts.

Exclusive contracts

Generally, when a platform observes that agents multi-home, it would rather prefer to have them single-homing on itself. In order to prevent agents from multi-homing, a platform might suggest them to sign exclusive contracts. It is rather cheap and profitable to offer such contracts, since the more one side agents (say, sellers) are present on one platform, the more other side agents (say, buyers) it is possible to attract due to the bigger amount of the exclusive content. Consequently, then the platform will have a possibility to sing up exclusive contracts with more sellers. However, as it has been noticed by Armstrong and Wright (2007), given the assumption on the firms' symmetry in the actual model, it is not obvious to define whether tying exclusive contracts can be viewed as an advantage to platforms in equilibrium since by forcing sellers to single-home, a platform can instead prompt them to switch on its rival. Nevertheless this fact, in the real world exclusive dealing practices are generally seen as the anti-competitive action.

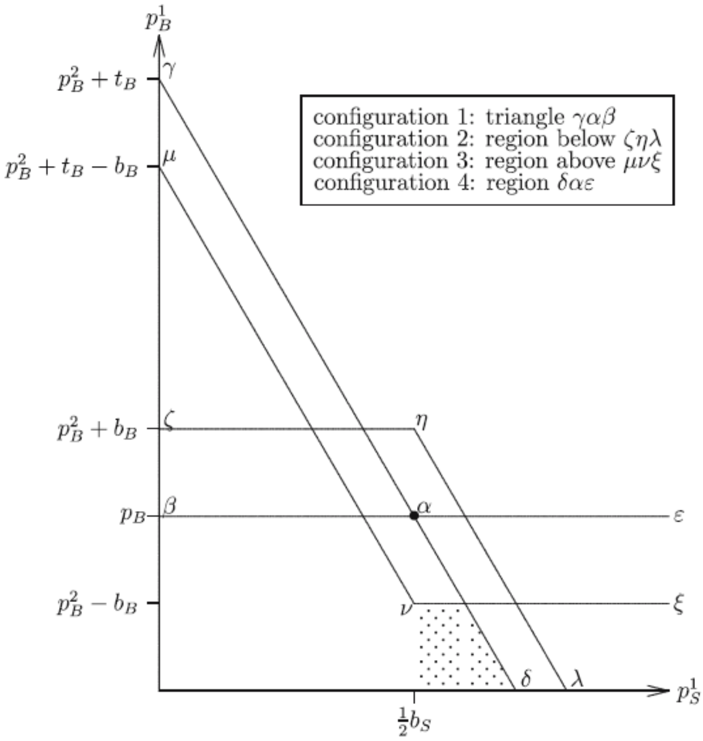

Figure 2.4. Armstrong and Wright's (2007) illustration of demand configurations under the product differentiation on only one side when platform 2 sets prices $p_2^S = b_S/2$, and p_2^B

The model of Armstrong and Wright (2007), which has been described so far, assumes that platforms simultaneously set up their prices in the first stage, and agents simultaneously choose the platform to join in the second stage. Starting from the competitive bottlenecks equilibrium characterized by prices in (2.45) and (2.46), a platform could have a motive to offer the exclusive contracts in order to discard the sellers' incentives to multi-home.

The platform might deviate from the multi-homing equilibrium, characterized by the prices $p_S = b_S/2$ and p_B, by offering to sellers new exclusive price of $b_S/2 - \varepsilon$ for some small ε. Simultaneously, this platform can increase its price for buyers to $p_B + b_B$, and set its price for sellers, who still prefer to multi-home, on the excessively high level, such that these sellers do not consider any multi-homing opportunity. When sellers are suggested to sign up exclusive contracts with platform 1, they will do so since they do not obtain any surplus from platform 2 anyway. On the contrary, the structure of buyers does not change since their surplus does not decrease due to the increased network benefits resulting from a higher amount of exclusive sellers on platform 1. By the iterating best responses, the equilibrium will converge to the configuration (ii), when all sellers single-home on platform 1. Therefore, the platform 1's profit will increase by $b_B - \varepsilon > 0$.

The same result cannot be achieved without using exclusive contracts, i.e. when the platform simply offers better terms to all of its sellers and charge more to all of its buyers without signing any exclusive contracts. The only way for platform 1 to prompt sellers to single-homing is to undercut platform 2 on the buyers' side. To perform such undercutting, platform 1 has to charge buyers with a lower price. Since buyers have already been subsidized, such a strategy will not be profitable. In the case when there exists a zero-price constraint on the buyer side and there is no possibility to charge negative prices by using tying (Amelio and Jullien, 2012) such strategy is not even applicable. On the contrary, with the use of exclusive contracts it is possible to undercut the rival on the sellers' side and squeeze more profits from the buyers who are not going to change the platform, since all the sellers will sign up exclusively to their platform.

However, even if all sellers join platform 1, platform 2 does not leave the market (as it is generally happens in traditional one-sided markets) and will still continue providing particular services to its buyers. Armstrong and Wright (2007) state, therefore, that tying exclusive contracts in the two-sided markets involves partial foreclosure. Such a situation reflects the assumption imposed on transportation costs on the buyer side, i.e. when $t_B > b_B$ the buyers perceive a strong product differentiation, such that they are not willing to change a platform for any reason. For example, a platform can offer to its buyers some content produced exclusively by itself (think of newspapers). Surely, if buyers experience low transportation costs and, thus, multi-home, one platform could fully foreclose other from the both sides of the market by using exclusive contracts.

However, Armstrong and Wright (2007) claim that *"the equilibrium analysis of exclusive dealing with partial exclusion turns out to be messy, with equilibria in which both platforms offer exclusive dealing arising under some parameter constellations and not under others"*. Authors also state that it is much easier to analyze exclusive dealing under pure network effects set up, i.e. when both sides multi-home and do not incur any transportation costs.

3 Competition and exclusive dealing in two-sided markets with zero price constraints on one side

In this section we set up a model within the two-sided market framework where a zero price constraint is imposed on the one side in equilibrium; in this study the issue of the exclusive dealing in this particular set-up is analyzed either.

3.1 Single-homing on both sides of the market: a general framework with zero-price constraint

Before modeling the search engine competition, let us introduce the basic framework of the platform competition in the two-sided market with both sides single-homing, which has been developed by Armstrong (2006), and investigate the possible strategic impact of a zero-price constraint imposed on one side. We make zero-price constraint assumption on the buyer side. Subsequently, since a platform offers free content to consumers, there is a zero price charged to them in the equilibrium. In the case of ad-sponsored media platforms, by offering zero prices to the one group, an intermediary creates advertising opportunities for the other group that is a key element for the further successful activities.

The majority of existing literature on two-sided markets considers platform competition from the point of view where both sides of the market are charged with a particular price (could be also negative). Therefore, before modeling a particular framework and looking at the issue of exclusive dealing in two-sided markets, it is necessary to model the platform competition with zero price constraint on one side, and find equilibrium pair of prices, set by two platforms and charged to the one of two groups under the market configuration with both sides single-homing.

Let us assume two competing platforms to be ad-sponsored, and, therefore, serve users (group-1 agents) and advertisers (group-2 agents). The model notations are:

u_1^i, u_2^i –utilities of group-1 (users) and group-2 (advertisers) agents who have joined the platform $i = A, B$, respectively;

n_1^i, n_2^i – the number of consumers and content providers who have joined the platform $i = A, B$ exclusively. There is a unit measure of the group-1 agents and a unit measure of the group-2 agents;

t_1, t_2 – transportation costs incurred by group-1 and group-2 agents, respectively;

α_1, α_2 – the benefit that a group-1 (group-2) agent enjoys from an interaction with the additional group-2 (group-1) agent;

f_1, f_2 – per-agent costs that platform $i = A, B$ incurs while serving each group-1 and group-2 agent, respectively;

p_2^i – the price charged by platform $i = A, B$ to a group-2 agent. Recall that $p_1^i = 0$ by the initial condition.

When both sides single-home, we obtain the same framework as in Armstrong (2006) with one difference that users have a free access to platform's services. Therefore, the utility of group-1 agent, located at $x_1 \in [0, 1]$ when she joins platform $i = A, B$ can be determined as follows

(3.1) $$u_1^A = \alpha_1 n_2^A - t_1 x_1; \qquad u_1^B = \alpha_1 n_2^B - t_1(1 - x_1).$$

In turn, the utility of group-2 agent located at $x_2 \in [0, 1]$ as well when she joins platform $i = A, B$ can be expressed as

(3.2) $$u_2^A = \alpha_2 n_1^A - t_2 x_2 - p_2^A; \qquad u_2^B = \alpha_2 n_1^B - t_2(1 - x_2) - p_2^B.$$

To support the unique equilibrium where both sides single-home, one has to apply the following assumptions (Armstrong and Wright, 2007): $t_1 > \alpha_1$, $t_2 > \alpha_2$, and $4t_1t_2 > (\alpha_1 + \alpha_2)^2$. Higher values of the parameters t_1 and t_2 guarantee that agents incur higher transportation costs comparing to possible benefits, such that they prefer not to multi-home. The last assumption stands for concavity of platforms' profit functions.

The demand configurations of group-1 agents who are indifferent between single-homing on A or B (i.e. $u_1^A = u_1^B$) are given as follows

(3.3) $$n_1^A = \frac{1}{2} + \frac{\alpha_1(n_2^A - n_2^B)}{2t_1}; \qquad n_1^B = \frac{1}{2} + \frac{\alpha_1(n_2^B - n_2^A)}{2t_1}.$$

In the similar manner one could derive the demand configurations of group-2 agents who are indifferent between single-homing on either platform.

(3.4) $$n_2^A = \frac{1}{2} + \frac{\alpha_2(n_1^A - n_1^B) - (p_2^A - p_2^B)}{2t_2}; \qquad n_2^B = \frac{1}{2} + \frac{\alpha_2(n_1^B - n_1^A) - (p_2^B - p_2^A)}{2t_2}.$$

Since there are no multi-homing users and advertisers, $n_1^A = 1 - n_1^B$, and $n_2^A = 1 - n_2^B$. Solving simultaneous systems of equations (3.3) and (3.4) for n_1^i and n_2^i as functions of p_2^A and p_2^B, where $i = A, B$, we get

(3.5) $$n_1^A = \frac{1}{2} + \frac{\alpha_1(p_2^B - p_2^A)}{2(t_1t_2 - \alpha_1\alpha_2)}; \qquad n_1^B = \frac{1}{2} + \frac{\alpha_1(p_2^A - p_2^B)}{2(t_1t_2 - \alpha_1\alpha_2)}.$$

(3.6) $$n_2^A = \frac{1}{2} + \frac{t_1(p_2^B - p_2^A)}{2(t_1t_2 - \alpha_1\alpha_2)}; \qquad n_2^B = \frac{1}{2} + \frac{t_1(p_2^A - p_2^B)}{2(t_1t_2 - \alpha_1\alpha_2)}.$$

Suppose per-agent cost that platform $i = A, B$ incurs while serving group-1 agents is assumed to be zero, i.e. $f_1 = 0$. Then, the profit of platform i can be expressed as follows

(3.7) $$\pi^i = \left(p_2^i - f_2\right) n_2^i.$$

Profit is a concave function of platform prices to group-2 agents. Substituting equations from (3.6) into (3.7) and solving two first-order conditions $\frac{\partial \pi^A}{\partial p_2^A}$ and $\frac{\partial \pi^B}{\partial p_2^B}$, it is possible to obtain the unconstrained solution, which gives the Nash equilibrium prices. Therefore, the optimal price charged to group-2 agents is

(3.8) $$p_2^A = p_2^B = f_2 + t_2 - \frac{\alpha_1}{t_1}\alpha_2.$$

Equation (3.8) holds if $f_2 + t_2 \geq \frac{\alpha_1}{t_1}\alpha_2$. It could be seen that the adjustment factor $\frac{\alpha_1}{t_1}\alpha_2$ consists of two parts: $\frac{\alpha_1}{t_1}$ that is the number of additional group-1 agents which could be attracted when one additional group-2 agent joins the platform, and α_2 is basically equal to the benefit that group-2 agent enjoys from an interaction with the additional group-1 agent.

Comparing the equilibrium pair of prices from (3.8) with those ones obtained by Armstrong (2006) in a single-homing equilibrium when p_1^i is non-negative, i.e. $p_2^{SH} = f_2 + t_2 - \alpha_1$, one may notice that the latter price does not depend on its own externality parameter α_2, whereas the former price does. Thus, when $\frac{\alpha_2}{t_1} < 1$, platform i will charge group-2 agents less when it faces zero-price constraint on a consumer side comparing to the unconstrained case.

The optimal demand configurations given symmetric prices obtained in (3.8) are, therefore, $n_1^A = n_1^B = \frac{1}{2}$ and $n_2^A = n_2^B = \frac{1}{2}$. In equilibrium, platforms make the following symmetric profits

(3.9) $$\pi^A = \pi^B = \frac{t_1 t_2 - \alpha_1 \alpha_2}{2t_1} > 0$$

Profit obtained in (3.9) is higher than the profit obtained when $p_1^i > 0$, i.e. $\frac{t_1 t_2 - \alpha_1 \alpha_2}{2t_1} > \frac{t_1 + t_2 - \alpha_1 - \alpha_2}{2}$, under the condition $\alpha_2 > t_1$, i.e. advertisers enjoy a higher network benefit comparing to the transportation costs incurred by users.

Returning to the equation (3.8), one might notice that if t_2 decreases, the competition for content providers becomes fiercer, and it pushes prices down. The same logic applies when t_1 decreases, i.e. if users' preferences are weak, platforms start to compete intensively for content providers by lowering their prices. Platforms also resort to price decreasing practices to group-2 agents when network benefit parameters α_1 and α_2 increase.

Strategic impact of zero-price constraint on the one side

It could be proved that if a zero-price constraint is bounded on group-1 agents, the competition for the group-2 agents will be more intense, and it will end up, therefore, in lower equilibrium prices for group-2 agents.

Consider a general two-sided framework developed by Armstrong (2006) and reviewed above in the Section 2.1.1, which assumes that prices on both sides are unconstrained. Recall that under $f_1 = 0$,

$$(3.10) \qquad \frac{\partial \pi_i^{SH}}{\partial p_2^i} = \frac{1}{2} - \frac{(p_2^i - f_2)t_1}{2(t_1 t_2 - \alpha_1 \alpha_2)} + \frac{(p_2^j - p_2^i)t_1 + (p_1^j - p_1^i)\alpha_2}{2(t_1 t_2 - \alpha_1 \alpha_2)} - \frac{p_1^i \alpha_1}{2(t_1 t_2 - \alpha_1 \alpha_2)} = 0.$$

From the equation (3.10) it is possible to express p_2^i as a function of competitor's side-1 and side-2 prices. Hence, the platform i's best response is given by

$$(3.11) \qquad p_2^i = \frac{p_2^j}{2} - \frac{\alpha_1 + \alpha_2}{2t_1} p_1^i + \frac{\alpha_2}{2t_1} p_1^j + \frac{1}{2}\left[f_2 + t_2 - \frac{\alpha_1 \alpha_2}{t_1}\right].$$

From (3.11) it could be seen that when platform i decreases its own side-1 price, it might increase its side-2 price. This effect is known as the "competition softening". Amelio and Jullien (2012) used this term in order to explain why platforms tend to increase the side-2 price in equilibrium when they perform tying (i.e. charge negative prices) to group-1 agents.

Suppose now the equilibrium is represented by the symmetric pair of prices to group-1 agents, i.e. $p_1^i = p_1^j = p_1$ for $i = A, B$ and $i \neq j$. Thus, equation (3.11) could be rewritten in the following way

$$(3.12) \qquad p_2^i = \frac{1}{2}\left[(p_2^j + f_2 + t_2) - \frac{\alpha_1(p_1 + \alpha_2)}{t_2}\right]$$

The best response of the platform i's side-2 price, p_2^i, to a symmetric side-1 price, p_1, can be expressed by

$$(3.13) \qquad \frac{\partial p_2^i}{\partial p_1} = -\frac{\alpha_1}{2t_1},$$

that is negative unless $\alpha_1 < 0$. The equation (3.13) represents the competition intensifying effect. In the unconstrained case, in order to attract more agents, a platform might decrease a price to group-1 agents and increase a price to group-2 agents. Therefore, increased side-2 prices can serve as the strategic substitutes for the negative side-1 prices. In the case when there is no possibility to charge a negative price to group-1 agents, the platforms are remained to compete for more group-2 agents simply by charging them less. Thus, a zero-price constraint on one side leads to the intensification of the competition for agents on the other side.

The model in this subsection describes the situation when multi-homing is not allowed on both sides of the market. Since everyone single-homes, there is no sense to deviate from the single-homing equilibrium, perform exclusive dealing and push content providers to sign exclusive contracts. In the next subsection we introduce the competition within the two-sided market framework when both sides multi-home. We also clarify how one of the platforms may deal in order to prevent multi-homing.

3.2 Search engine competition: model set-up

Search engine competition could be modeled by using a two-sided market approach. The existing literature on a search engine competition counts several works, some of which are developed within a two-sided market framework. For example, Jeon, Jullien, and Klimenko (2012) use classical "competitive bottlenecks" approach to study *"how bilingualism affects competition between a foreign search engine and a domestic one within a small country and thereby production of home language content."* Lianos and Motchenkova (2012) provide the analysis of the possible abusive behavior of the dominant search engine in the context of investments in quality improving innovations. The model developed in this subsection is close to the models of Armstrong and Write (2007) and Jeon, Jullien, and Klimenko (2012), but is extended with the two-sided multi-homing approach introduced by Choi (2010) and Doganoglu and Wright (2006). In addition, some other assumptions and modifications have been imposed on the parameters of our model, which are pointed at reflecting the realities of a search engine market.

Suppose there are two search engines, indexed by $i = A, B$, serving as intermediaries between two groups of agents, indexed by $k = 1, 2$. Group-1 agents are users (or searchers) who can either enjoy organic search and other follow up services, or make purchases by following the ad sponsored links. Group-2 agents are content providers (or advertisers) who are willing to place their content (ads) on a search engine's main web-page. There is a unit measure of group-1 agents and a continuum of group-2 agents. Advertisers are assumed to multi-home. Contrary to Jeon, Jullien, and Klimenko (2012), we do not assume full single-homing on a consumer side. Let us justify this point of view.

In general, the assumption of single-homing consumers presumes some logic. According to the model of Jeon, Jullien, and Klimenko (2012), *"consumer forms the habit to use one of the two platforms"*, and, therefore, is not subject to switching a platform in future. Jeon, Jullien, and Klimenko (2012) assume that *"for a consumer located at x, the cost of forming the habit to use platform 1 (2) is tx ($t(1-x)$)."* However, the results of various empirical studies on search engine switching behavior are not in favor of such assumption. For example, the research study performed by White and Dumais (2009) shows that

> *"Of the 14.2 million users in our log sample, 10.3 million (72.6%) used more than one engine in the six-month duration of the logs, 7.1 million (50.0%) switched engines within a search session at least once, and 9.6 million (67.6%) used different engines for different sessions (i.e., engaged in between-session switching). In addition, 0.6 million users (4.4%) "defected" from one search engine to another and never returned to the previous engine."*[3]

It is true that each search engine is actually one click away from losing its customers, and there are no exceptions. High switching rates, indicated in White and Dumais'

[3] White and Dumais (2009).

(2009) study, occur because, virtually, a searcher does not experience any switching costs. If she would like to use a rival search engine, all her actions will boil down to simple typing of that search engine's address in the browser's address line.

However, there still remains some part of consumers who are not willing to switch the preferable search engine. Such decision could be followed by users' preferences in the selection of one search engine over another, which can include familiarity, effectiveness, reputation, interface usability, or an absence of obsessive ads on the main search engine's web-page (White and Dumais, 2009). For this reason, searchers might use the same search engine for performing all queries and would not want to change it. On the contrary, some users might switch a search engine because they can pursue different purposes.

The case of multi-homing on user side

There exists other explanation why a part of searchers would not want to multi-home, which can be described by network effects. Thus, following the article of Doganoglu and Wright (2006), we assume that there exists a fraction of group-1 agents, γ, who highly value a network size and have a network benefit parameter of $\alpha_1 = \alpha_{1H}$ (high type agents), and a fraction of users $1-\gamma$, who are not characterized by a high valuation of a network side and have a network benefit parameter of $\alpha_1 = \alpha_{1L} > 0$ (low type agents). Naturally, two more assumptions arise: $\alpha_{1H} > \alpha_{1L}$ and $0 < \gamma < 1$.

Therefore, the utility of the user single-homing on platform i is given as

(3.14) $$u_1^i = \vartheta_0 + \alpha_1 n_2^i - t(x),$$

where $\vartheta_0 > 0$ is the intrinsic benefit, which in the search engine market can stand for the utility from follow up services (say, organic search). Similarly to Jeon, Jullien, and Klimenko (2012), we assume that users form a habit to use platform i, and tx is a cost of forming such habit for a user located at x. Consequently, $t(1-x)$ stands for a cost of forming the habit for a user located at $1-x$. Recall that users are not charged by neither platform, i.e. $p_1^i = 0$.

The utility of a multi-homing user is given by

(3.15) $$u_1^{AB} = \vartheta_0 + \alpha_1(n_2^A + n_2^B) - t,$$

where $t = tx + t(1-x)$ is a total transportation cost for a multi-homing user. Similarly to Armstrong and Wright (2007), we do not allow for duplicated intrinsic benefits.

In line with the research of Poolsombat and Vernasca (2006), we assume that the total number of consumers who have chosen to join platform i is given by

(3.16) $$n_1^i = \gamma h_1^i + (1-\gamma) l_1^i,$$

where l_1^i and h_1^i are the masses of low type and high type agents located on side 1 (consumer side). We assume that $l_1^A = 1 - l_1^B$ and $h_1^A = 1 - h_1^B$.

To find demand configurations of consumers, one has to solve two problems of the indifferent low type and high type agent. Thus, we must solve $u_1^A(\alpha_{1L}) = u_1^B(\alpha_{1L})$ for $x = l_1^A$, and $u_1^A(\alpha_{1H}) = u_1^B(\alpha_{1H})$ for $x = h_1^A$.

However, as we have already stated above, we do not support the assumption of single-homing users. Instead, we imply partial multi-homing assumption on group-1 agents. This means that consumers of a high type multi-home, whereas consumers of a low type single-home. Given this fact, we set $h_1^A = h_1^B = 1$. Such formulation of a problem leads us to finding demand configurations of low type users only.

Hence, when we solve $\vartheta_0 + \alpha_{1L} n_2^A - tx = \vartheta_0 + \alpha_{1L} n_2^B - t(1-x)$ for $x = l_1^A$, we get

(3.17) $$l_1^A = \frac{1}{2} + \frac{\alpha_{1L}(n_2^A - n_2^B)}{2t},$$

where n_2^i is the number of content providers (advertisers) on platform i.

According to the formula (3.16), the total number of group-1 agents on platform i is given by

(3.18) $$n_1^i = \gamma + (1-\gamma)\left(\frac{1}{2} + \frac{\alpha_{1L}\left(n_2^i - n_2^j\right)}{2t}\right), i \neq j.$$

In order to rule out the possibility of the corner solution where all low type agents choose the same platform independently of their location, the following assumption on transportation cost parameter is made

Assumption A3.1. $t > \overline{\alpha_1} = \gamma\alpha_{1H} + (1-\gamma)\alpha_{1L}.$

Assumption A3.1 also implies that low type group-1 agents will single-home as long as their product differentiation parameter, t, is bigger than their network benefit parameter, α_{1L}, i.e. $t > \alpha_{1L}$. Thus, the only group-1 agents who multi-home are high type agents.

In turn, high type agents will have an incentive to multi-home when $u_1^{AB}(\alpha_{1H}) \geq u_1^A(\alpha_{1H})$, or in other terms $\vartheta_0 + \alpha_{1H}(n_2^A + n_2^B) - t \geq \vartheta_0 + \alpha_{1H} n_2^A - t\left(\frac{1}{2} + \frac{\alpha_{1H}\left(n_2^A - n_2^B\right)}{2t}\right)$. Solving this inequality for α_{1H} we get $\alpha_{1H} \geq \frac{t}{n_2^A + n_2^B}$.

Assumption A3.2. High type group-1 agents multi-home under $\alpha_{1H} \geq \frac{t}{n_2^A + n_2^B}$.

Under the full market coverage, i.e. when $n_2^A + n_2^B = 1$, the multi-homing on high type agents is simply supported with $\alpha_{1H} \geq t$.

Content provider side (side 2) is characterized by the following assumptions: content providers (i) see both platforms as homogeneous, and, therefore, do not experience any transportation costs; (ii) enjoy a platform-specific benefit of $\alpha_2^i > 0$ from an interaction with searchers on platform i; (iii) incur a fixed cost of f when joining a platform; (iv) are charged by platform $i = A, B$ with a subscription fee $p_2^i > 0$. In the search engine market the role of the subscription fee could be captured by the average cost-per-click[4].

It is important to notice that neither Doganoglu and Wright (2006) nor Poolsombat and Vernasca (2006) do not impose assumptions (i) and (iii) on supplier side. On the contrary, authors assume that suppliers experience transportation costs (as well as buyers) and their market shares are represented by the standard Hotelling share functions.

The utility of an advertiser who has chosen platform i is given by

(3.19) $$u_2^i = \alpha_2^i n_1^i - p_2^i - f n_2^i,$$

where n_1^i stands for the total number of users on platform i.

Content provider is willing to join platform i until his utility in (3.19) is non-negative. Similarly to Jeon, Jullien, and Klimenko (2012), we assume that fixed cost, f, of a content provider joining platform i is distributed with a constant density $f = 1$. Thus, the mass of content providers on platform i is given by:

(3.20) $$n_2^i = \alpha_2^i n_1^i - p_2^i,$$

If we substitute n_2^i from (3.20) into (3.18) and solve for n_1^i as a function of prices to advertisers, we will get following demand configurations

(3.21) $$n_1^i = \frac{(1-\gamma)\alpha_{1L}(p_2^j - p_2^i) + (1+\gamma)(t - (1-\gamma)\alpha_{1L}\alpha_2^j)}{\Phi},$$

where $\Phi = 2t - (1-\gamma)\alpha_{1L}\left(\alpha_2^i + \alpha_2^j\right) > 0$.

Given that $n_2^i = \alpha_2^i n_1^i - p_2^i$ and the marginal per-content provider cost is low enough and tends to zero, the platform i's profit can be expressed as follows

(3.22) $$\pi^i = n_2^i p_2^i = \left(\alpha_2^i n_1^i - p_2^i\right) p_2^i,$$

where n_1^i is given by (3.21).

By differentiating (3.22) with the respect to p_2^i and solving $\frac{\partial \pi^i}{\partial p_2^i} = 0$, one could obtain the expression for p_2^i that is

(3.23) $$p_2^i = \frac{(1+\gamma)\alpha_2^i[t\left(4t - 3(1-\gamma)\alpha_{1L}\alpha_2^j\right) - (1-\gamma)\alpha_{1L}\alpha_2^i\left(2t - (1-\gamma)\alpha_{1L}\alpha_2^j\right)]}{8t\left(2t - (1-\gamma)\alpha_{1L}\alpha_2^j\right) - (1-\gamma)\alpha_{1L}\alpha_2^i(8t - 3(1-\lambda)\alpha_{1L}\alpha_2^j)}.$$

Proposition 3.1. Under Assumptions A3.1-A3.2, symmetric side-2 network benefit parameters $\alpha_2^A = \alpha_2^B = \alpha_2$, and $t > (1-\gamma)\alpha_{1L}\alpha_2$, there exists a symmetric equilibrium with $p_2^A = p_2^B = p_2$, where p_2 is given by

[4] Cost-per-click (CPC) is the amount of money paid by an advertiser when a searcher clicks on her paid ad. In turn, Average CPC is an average amount of money that an advertiser has been charged for a click on her ad.

$$p_2 = \frac{(1+\gamma)\alpha_2[t-(1-\gamma)\alpha_{1L}\alpha_2]}{4t-3(1-\gamma)\alpha_{1L}\alpha_2}.$$

Under the symmetric equilibrium side-2 prices, the demand configurations for group-1 and group-2 agents are as follows

(3.24) $$n_1^A = n_1^B = \frac{1+\gamma}{2},$$

(3.25) $$n_2^A = n_2^B = \frac{(1+\gamma)\alpha_2(2t-(1-\gamma)\alpha_{1L}\alpha_2)}{8t-6(1-\gamma)\alpha_{1L}\alpha_2}.$$

Hence, platform i's profit is

(3.26) $$\pi^A = \pi^B = \frac{(1+\lambda)^2\alpha_2^2(t-(1-\gamma)\alpha_{1L}\alpha_2)(2t-(1-\gamma)\alpha_{1L}\alpha_2)}{2(4t-3(1-\gamma)\alpha_{1L}\alpha_2)^2}.$$

Proposition 3.2. Given the symmetric equilibrium price in Proposition 3.1 and symmetric number of group-k agents in (3.24)-(3.25), high type group-1 agents will multi-home when $\alpha_{1H} \geq \frac{t(4t-3(1-\gamma)\alpha_{1L}\alpha_2)}{(1+\gamma)\alpha_2(2t-(1-\gamma)\alpha_{1L}\alpha_2)}$.

The result in (3.23) can be rewritten as

(3.27) $$p_2^i = \frac{n_1^i\alpha_2^i}{2+\frac{(1-\gamma)\alpha_{1L}\alpha_2^i}{\Phi}}.$$

The expression for platform i's price to advertisers depicted in (3.27) preserves the logic of the result of delivered by Jeon, Jullien, and Klimenko (2012) when consumers are only allowed to single-home. In the line with their paper, the search engine's price to advertisers directly depends on the user market share in the search engine market, i.e. the more users perform search on one search engine, the more attractive such search engine becomes to advertisers, and the more it can charge these advertisers. Moreover, as Φ increases, the competition between platforms becomes softer and they charge a higher price to advertisers.

The search engine market perfectly reflects these statements. Think of Google AdWords and Bing Ads[5]. Average cost-per-click on Bing Ads is as twice as cheap comparing to the same indicator on Google AdWords (Table 3.1). This can be basically explained by Google's search market share comparing to the position of Yahoo! Bing Network. From the Table 3.2 one could investigate that Google and its vertical sites have the largest search share and occupy 67% of the whole search market, whereas Yahoo! and Bing in sum have the search market share three times less than Google's one (around 28%).

[5] In fact, Bing Ads represent Yahoo! Bing advertising network which started to be available at the end of April 2012, when Yahoo! and Bing have finally completed their merger and launched a unified advertising platform.

Table 3.1. U.S. Average Cost-Per-Click, Q3 2012[6]

Ad Category	**Search Engine**	
	Google	**Yahoo! Bing**
Shopping and Classified	$0.72	$0.44
Financial Services	$2.88	$1.98
Travel	$0.83	$0.64
Education	$3.51	$2.07
Computer and Internet	$1.08	$0.40
Business	$1.98	$0.91

Table 3.2. U.S. Explicit Core Search[7]

Core Search Entity	**Explicit Core Search Share (%)**			
	Sep-12	**Oct-12**	**Nov-12**	**Dec-12**
Total Explicit Core Search	***100.0%***	***100.0%***	***100.0%***	***100.0%***
Google Sites	66.7%	66.9%	67.0%	66.7%
Microsoft Sites	15.9%	16.0%	16.2%	16.3%
Yahoo! Sites	12.2%	12.2%	12.1%	12.2%
Ask Network	3.5%	3.2%	3.0%	3.0%
AOL, Inc.	1.8%	1.8%	1.7%	1.8%

Table 3.3. U.S. Average Clickthrough Rates, Q3 2012[8]

Ad Category	**Search Engine**	
	Google	**Yahoo! Bing**
Shopping and Classified	3.70%	1.13%
Financial Services	3.53%	0.81%
Travel	4.14%	1.27%
Education	2.57%	0.44%
Computer and Internet	3.25%	1.35%
Business	3.12%	0.60%

One of the most important parameter determining platform i's price is a platform-specific network benefit, which advertisers get from an additional user. Within the realities of the two-sided search engine market, the role of such network benefit parameter, α_2^i, could be played by an Average Clickthrough Rate, where the Clickthrough Rate for a particular ad is measured as the number of clicks that this ad receives divided by the number of times this ad is displayed (called ad impressions). The Table 3.3 shows that the Average Clickthrough Rate among such ad categories as

[6] AdGooroo. (2012). Yahoo! Bing PPC Performance Metrics. An AdGooroo Special Report. Available at http://succeed.adgooroo.com/rs/adgooroo/images/AdGooroo_Yahoo_Bing_PPC_Performance_Metrics.pdf

[7] comScore Releases. (2012). Press Releases . U.S. Search Engine Rankings. Available at http://www.comscore.com/Insights/Press_Releases

[8] AdGooroo. (2012). Yahoo! Bing PPC Performance Metrics. An AdGooroo Special Report. Available at http://succeed.adgooroo.com/rs/adgooroo/images/AdGooroo_Yahoo_Bing_PPC_Performance_Metrics.pdf

Shopping, Financial Services, and Computer and Internet on Google is around three times higher than on Yahoo! Bing Network, whereas this network benefit parameter among Education and Business categories is more than five times higher on Google. Such situation allows Google to charge higher subscription fees, since advertisers themselves would prefer to pay more in order to ensure higher probability for their ads of being referenced by searchers.

Thus, the equation (3.27) for platform i's price to advertisers reflects the realities of price determination in the search engine market.

The case of single-homing on user side

Let us now apply the above-mentioned framework to find the symmetric equilibrium price when users single-home.

In order to push high type side-1 agents to single-home, one must induce such assumption

Assumption A3.3. High type group-1 agents single-home under $\alpha_{1H} \leq \frac{t}{n_2^A + n_2^B}$.

Following similar steps as in the case of multi-homing consumers, one can solve for the symmetric side-2 price which is given in Proposition 3.3.

Proposition 3.3. Under Assumptions A3.1 and A3.3, symmetric side-2 network benefit parameters $\alpha_2^A = \alpha_2^B = \alpha_2$, *and* $t > \alpha_2(\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L})$, *there exists a symmetric equilibrium with* $p_2^A = p_2^B = p_2$, *where* p_2 *is given by*

$$p_2 = \frac{\alpha_2[t - \alpha_2(\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L})]}{4t - 3\alpha_2[\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L}]}.$$

Proof of Proposition 3.3. See Appendix B.

Under the symmetric equilibrium side-2 prices represented in Proposition 3.3, the demand configurations for group-1 and group-2 agents are as follows

(3.24) $$n_1^A = n_1^B = \frac{1}{2},$$

(3.25) $$n_2^A = n_2^B = \frac{\alpha_2[2t - \alpha_2(\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L})]}{8t - 6\alpha_2[\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L}]}.$$

Platform i's profit is given by

(3.26) $$\pi^A = \pi^B = \frac{\alpha_2^2[2t - \alpha_2(\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L}))(t - \alpha_2(\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L})]}{2[4t - 3\alpha_2(\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L})]^2}.$$

Let us now provide the parameterization and, given the Assumptions A3.1-A3.3, investigate the possible outcomes of multi-homing among high type group-1 agents under the symmetric equilibria denoted in Propositions 3.1 and 3.3.

Suppose $\alpha_{1L} = \alpha_2 = 1$ and $\gamma = 0.5$. Then α_{1H} satisfying single-homing equilibrium must lie in the following range: $1 < \alpha_{1H} \leq -0.5 + 3.5t - \sqrt{0.25 - 0.5t + 4.25t^2}$. Assume α_{1H} equals exactly its upper bound.

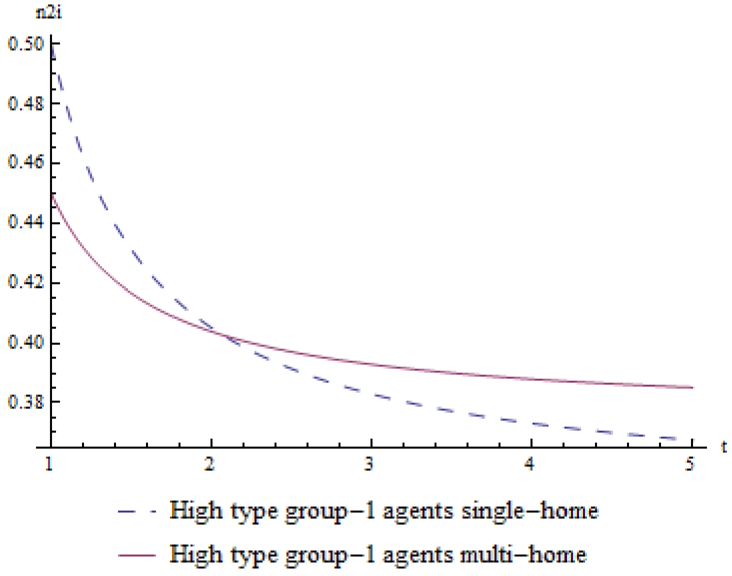

Figure 3.1. Distribution of group-2 agents under single-homing and multi-homing configurations of high type group-1 agents

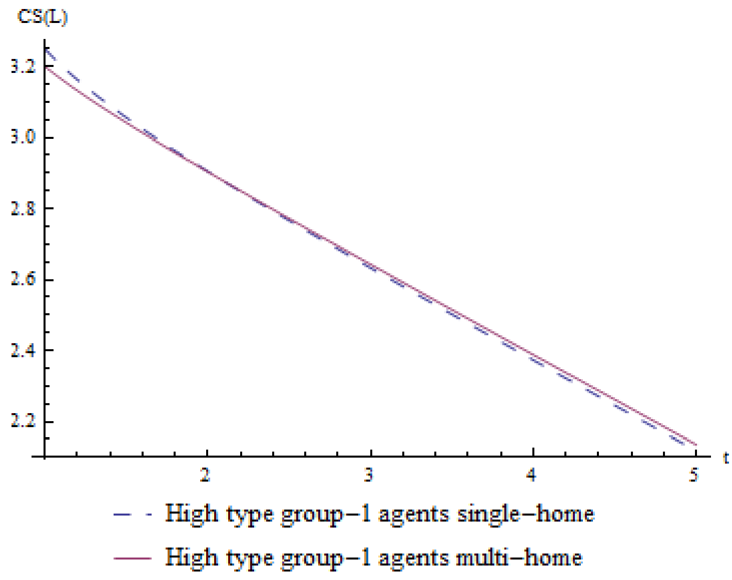

Figure 3.2. Consumer surplus of low type group-1 agents under single-homing and multi-homing configurations of high type group-1 agents

Consumer surplus of low type group-1 agents is calculated as follows

$$CS(L) = \vartheta_0 + \alpha_{1L}[l_1^A n_2^A + (1 - l_1^A)n_2^B] - \frac{t}{2}[(l_1^A)^2 + (1 - l_1^A)^2].$$

Under the applied parameterization, it can be clearly seen from Figures 3.1-3.2 that consumer surplus of low type agents decreases as t increases. Moreover, changes in consumer surplus are related to changes in the extent of group-2 agents. Since the slope of $n_2^i(t)$ under single-homing high type group-1 agents is steeper than the slope of $n_2^i(t)$ under multi-homing high type group-1 agents, after a particular point consumer surplus on the Figure 3.2 under single-homing high type group-1 agents starts to be smaller than under multi-homing high type group-1 agents.

Platforms' pricing policies preserve the logic of Poolsombat and Vernasca's (2006) paper: the symmetric equilibrium price charged to group-2 agents is higher when high type group-1 agents multi-home rather than single-home. Clearly, such an outcome is connected with the fact that advertisers enjoy higher benefits from multi-homing consumers. The more searchers view the advertisers' content, the more beneficial such situation becomes to these advertisers.

In the next subsection we investigate the exclusionary abuse that one search engine might perform in order to exclude competitors from the market.

3.3 Exclusive dealing in the search engine market

Analyzing exclusive contracts in the two-sided market, we refer to the article of Armstrong and Wright (2007), which has been already reviewed in the subsection 2.2. Recall that Armstrong and Wright (2007) have found that when there exists a competitive bottlenecks equilibrium with prices $p_S = b_S/2$ and p_B, one platform might deviate by offering sellers an exclusive price of $b_S/2 - \varepsilon$ for some small ε, and simultaneously increase its price for buyers to $p_B + b_B$. By the iterating best responses, the equilibrium converges to the configuration, when sellers single-home on the platform 1. Armstrong and Wright (2007) have discovered that even though all sellers join platform 1, the platform 2 does not leave the market (as it is generally happens in traditional one-sided markets) and still provides its buyers with the particular services that they still value, unless it is undercut from both sides. It means that tying of exclusive contracts to one side of the two-sided market involves a partial foreclosure.

In the case of search engines, a platform cannot have a possibility to charge any price to consumers, since there has been already imposed a zero-price constraint, and the existence of a user group is possible only due to the provision of a free entry. Provided that a media platform (like search engine) cannot charge one side for any reason, the partial foreclosure transforms into the full foreclosure, and, therefore it is enough to undercut a rival from only one side in order to push it out of the market.

Consider the market configuration when group-1 agents of a high type multi-home, and all sellers join only platform A, i.e. $n_2^A = 1$. Such situation occurs when seller's utility from single-homing on platform A exceeds her utility from single-homing on platform B or multi-homing, i.e. $\alpha_2 n_1^A - p_2^A - n_2^A \geq 0$, $\alpha_2 n_1^A - p_2^A - n_2^A \geq \alpha_2 n_1^B - p_2^B - n_2^B$, and $\alpha_2 n_1^A - p_2^A - n_2^A \geq \alpha_2 n_1^A + \alpha_2 n_1^B - p_2^A - p_2^B - n_2^A - n_2^B$, with n_1^i is given by (3.18). Moreover, we assume that network benefits are equal among search engines, i.e. $\alpha_2^A = \alpha_2^B = \alpha_2$.

By transforming the afore-mentioned utility inequalities into price inequalities one could get

(3.27) $$p_2^A \leq \alpha_2 \left[\gamma + (1-\gamma)\left(\frac{1}{2} + \frac{\alpha_{1L}}{2t}\right)\right] - 1,$$

(3.28) $$p_2^B \geq \alpha_2 \left[\gamma + (1-\gamma)\left(\frac{1}{2} - \frac{\alpha_{1L}}{2t}\right)\right] - 1.$$

Provided that $n_2^B = 0$, platform B is undercut from the both sides of the market, since it cannot make any profit on the market, i.e. $\pi^B = 0$. In turn, the profit of platforms 1 is given by

(3.29) $$\pi^A = p_2^A n_2^A = \alpha_2 \left[\gamma + (1-\gamma)\left(\frac{1}{2} + \frac{\alpha_{1L}}{2t}\right)\right] - 1.$$

Total platform profit under exclusive dealing is given by $\pi_E^{AB} = \alpha_2 \left[\gamma + (1-\gamma)\left(\frac{1}{2} + \frac{\alpha_{1L}}{2t}\right)\right] - 1$. Under the symmetric market equilibrium (without exclusive dealing) where the same amount of content providers join both platforms, i.e. $n_2^A = n_2^B = 1/2$, the producer surplus can be expressed as $\pi_{NE}^{AB} = p_2 = \frac{(1+\gamma)\alpha_2[t-(1-\gamma)\alpha_{1L}\alpha_2]}{4t-3(1-\gamma)\alpha_{1L}\alpha_2}$.

Given the full market coverage on the advertiser side, i.e. $n_2^A + n_2^B = 1$, we have $\pi_E^{AB} \geq \pi_{NE}^{AB}$ when α_2 lies in the following interval:

(3.30) $$\alpha_2 \geq \frac{2t}{(1-\gamma)\alpha_{1L}}.$$

Inequality (3.30) above indicates that the advertiser's network benefit parameter has to be relatively high. In a real search engine market, α_{1L} is set on the low level. This requires α_2 to be large enough. As we could be assured, in practice it does not hold that α_2 reaches high values. Thus, we can make a conclusion that if α_2 does not fall inside interval in (3.30), performing exclusive dealing and, as a result, the monopolization of a market by one search engine is clearly welfare minimizing action.

Here, it is important to provide a remark. Since we assume the full market coverage, i.e. the mass of content providers is normalized to 1, the variations in welfare occur only due to the changes in the producer surplus, since consumer surplus and advertiser surplus do not change.

As it has been already studied by Lianos and Motchenkova (2012), the monopolization of the search engine market leads not only to the welfare reduction under small α_2: due to the absence of competition, the quality of search results are lower comparing to the social optimum. Once a search engine gets a dominant market position, it starts running a number of abusive practices, which bias search results. Among the most probable abusive practices it is possible to point out the manipulation of organic search results, which happens when a search engine pushes its own links on higher positions, and, thus, violates the conception of the "search neutrality".

Nowadays, search engines have their own vertical search services (for example, Google Shopping, Bing Shopping). While performing the manipulation, a dominant search engine significantly harms its indirect competitors, the vertical search engines, by taking away a large part of searchers. Provided that searchers start clicking on the top links, with the high probability they might be redirected to the search engine's vertical service.

An important issue here is that consumers do not know that the search results they observe have been tampered. The majority of searchers are confident that their queries typed into a search engine consist of the most relevant sites. In reality, users might face with biased search results. Hence, if a dominant search engine like Google manipulates its search results, this will have a dramatic impact on both content providers who will lose the possibility of being clicked on their own links, and searchers who will basically spend much time in order to satisfy the needs.

> *"On a Google search results page, the average click-through rate (CTR) for the first, second and third positions are 36.4 percent, 12.5 percent, and 9.5 percent respectively. That means the top three listings attract more than half of all click-throughs in a given search result. Companies whose listings don't even make the first page better be destination sites, or have a strong flow of referrals from prominent sites, because they won't likely get any substantial traffic from Google. And while this hurts businesses, the larger implication is that it hurts consumers."*[9]

Thus, when one search engine gets a dominant market position, it is important that public authorities could proceed with the intervention in the market in order to avoid abusive practices of a monopolistic search engine, including the deterioration of the search results relevance (Lianos and Motchenkova, 2012).

[9] Digital Trends. (2012). Is Google's Search Manipulation Hurting Consumers? Available at http://www.digitaltrends.com/web/bias-and-google-shopping/

4 Search engine mergers and their competitive outcomes

In this section we investigate the real case of the search engine merger, and provide the implication of such merger within a two-sided market framework.

4.1 Yahoo!-Microsoft merger: a case discussion

On 29th of July, 2009, Microsoft and Yahoo! have announced an agreement aimed at improving the search experience of both searchers and content providers. In general words, Yahoo!'s search engine will be powered by Bing, i.e. search algorithms and search results of Yahoo! will be the same as of Bing. Thus, even though both companies will have their own interfaces, they will unite their consumers and advertisers into one network, and, thus, acquire larger market shares.

The merger agreement is connected only with the unification of search technologies and a creation of a unique advertiser database. In turn, the agreement does not assume merging particular products of each company, such as electronic mailboxes, instant messaging, news or other aspects of these companies' businesses. However, the main areas of competition (competition for users and advertisers) are abandoned.

The main terms of the Microsoft-Yahoo! agreement are as follows:[10]

- The agreement's period is 10 years.
- Yahoo! will make available to Microsoft its core search technologies for 10 years with their further integration into Microsoft's existing web search platforms.
- Bing's search algorithms and paid search platform will be implemented into Yahoo! sites.
- Yahoo! preserves a right to use its own technologies in other areas of its business different from search algorithms and paid search platform.
- An automated auction process for both companies, i.e. the process of forming the prices for search ads, will be performed by Microsoft AdCenter platform.
- Display advertising business, i.e. advertising on the search-engines' main web-pages, and sales force are left to be maintained separately by two companies.
- Yahoo! remains with the right to innovate the user search experience on its own, even though the company will function under Microsoft's search technology.
- The Microsoft's compensation to Yahoo! will be provided within a revenue sharing agreement on traffic, which will be generated by Yahoo!'s web-site network. Yahoo! will keep 88% of Microsoft's revenue gained from searches on Yahoo's sites.
- Microsoft entitles Yahoo! with the right to syndicate the existing search affiliate partnerships of the latter.

[10] Microsoft News Center. (2009). Microsoft, Yahoo! Change Search Landscape. Available at https://www.microsoft.com/en-us/news/press/2009/jul09/07-29release.aspx

- The agreement maintains consumer privacy on the level, which is limited by sharing data between the companies for the successful functioning and improving the combined search platform.

At the end of the first quarter of 2012, Bing and Yahoo! have finally completed their merger and launched a unified advertising platform. The united network of searchers and advertisers is known as Yahoo! Bing Network. It could be clearly seen from Figure 4.1 that each advertiser's paid ad is reflected in both Yahoo! and Bing search engines simultaneously. That was the initial idea of the merger.

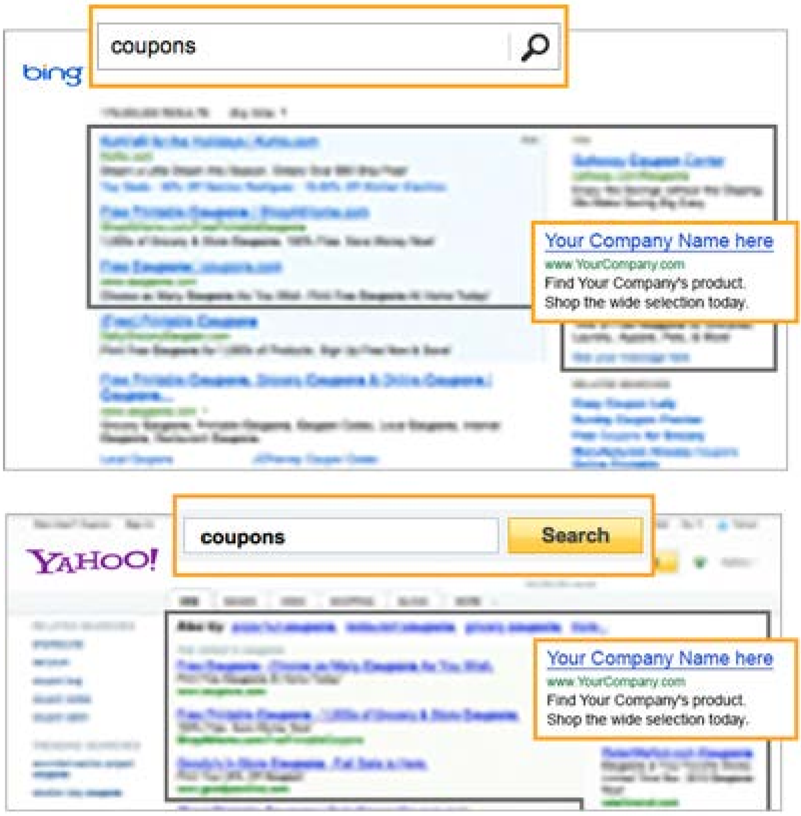

Figure 4.1. Paid ad appearance within Yahoo! Bing Network

For now Yahoo! Bing Network holds 30 per cent of a search engine market in the U.S. However, searchers who use this network have significantly higher Buying Power Index comparing to those ones using Google. Thus, according to the report of comScore Inc., an average searcher on Yahoo! Bing Network is characterized by the highest Buying Power Index among all searchers worldwide; she spends 136% more comparing to the spending of an average Internet searcher, and 77.4% more comparing to the spending of an average Google searcher (Figure 4.2).

Before the official occurrence of a merger in 2009, Yahoo! and Microsoft representatives claimed that such search alliance will allow their users to obtain more relevant results from queries; advertisers, in turn, will get better value and their ads will be more efficiently presented to the web-surfers. In the two-sided market terminology, the new Yahoo! Bing Network promised content providers to increase their network benefit parameters.

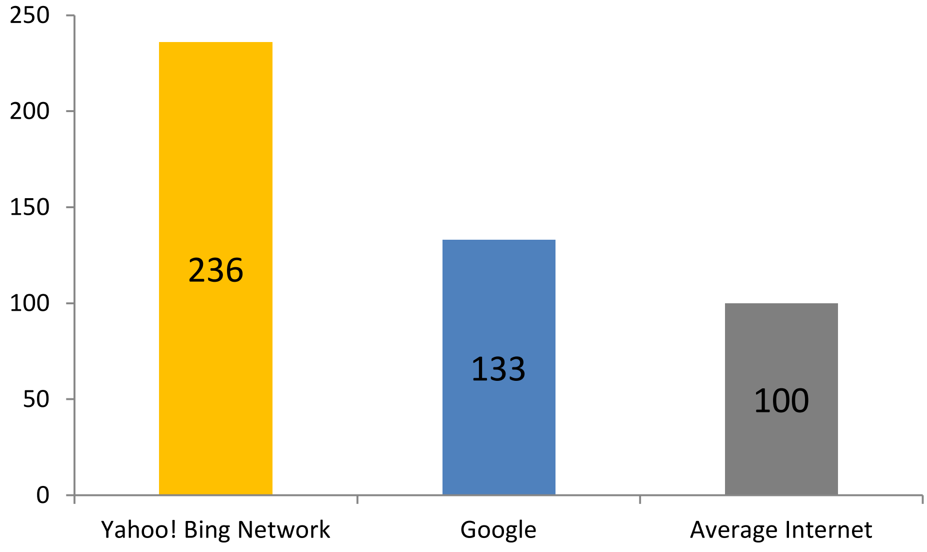

Figure 4.2. Worldwide Buying Power Index among Internet searchers[11]

However, the increase of network benefits is just one aspect of the merger, but it was frequently highlighted to companies' official representatives contrary to the other possible consequences that a merger could bring.

Alternatively, as it could be seen from equation (3.30) where the price is a function of the extent of users, as the number of searchers on one platform increases, the price for advertisers increases as well. As we have already mentioned in the previous section, such price could be represented by an average cost-per-click. An increase in cost-per-click also could be explained by the other factors different from the user market share. Think of changes in the bidding strategy. Previously, a cost-per-click was based on the price per a particular ad keyword determined separately on Yahoo! and Bing as a result of separate auctions. Subsequently, as the competition among advertisers was weaker on the separate platforms, average cost-per-clicks were lower. However, once a unified bidding mechanism was introduced, a competition inside Yahoo! Bing Network became tougher since more advertisers were willing to win a bid. This resulted in the increase of average cost-per-click on a unified auction marketplace.

The interesting fact has been observed by the leading marketing companies in the area of search engine advertising – after the official announcement of a merger and before its technical implementation, the average cost-per-click on Bing has started to increase, but, in turn, the same indicator has fallen on Yahoo!. Probably, such situation was caused with the migration of a part of advertisers from Yahoo! on Bing.

Within the previous section we have noted that network benefit parameters might be represented in terms of the average clickthrough rate, whereas a subscription fee coincides with the average cost-per-click. Provided that a unified Yahoo! Bing

[11] Yahoo! Bing Network. (2013). Yahoo! Bing Network Represents 30% Search Share in U.S. Available at http://yahoobingnetwork.com/en/community_blogpost/194/123521/yahoo-bing-network-represents-30-search-share-in-u-s

advertising platform has been launched at the end of the first quarter of 2012, it is possible to track the effects of such merger (Figure 4.3).

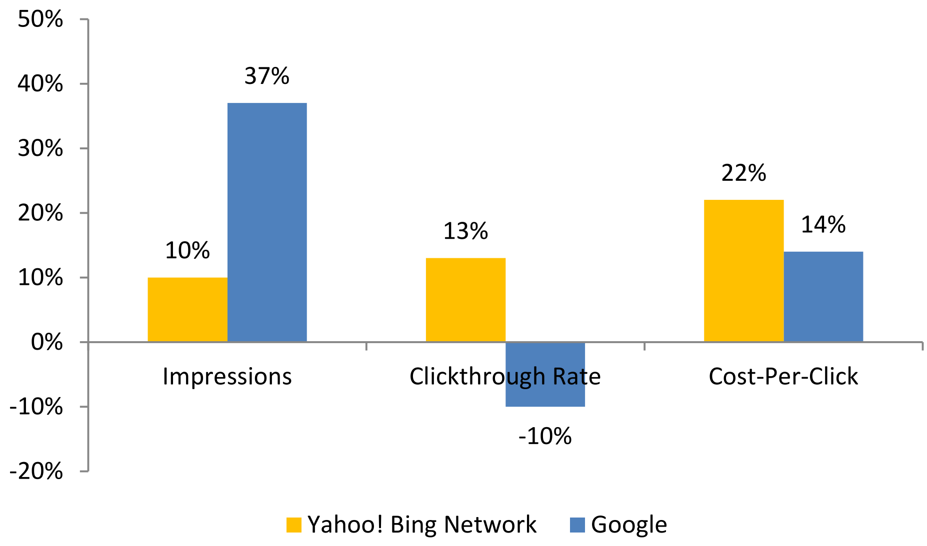

Figure 4.3. U.S. Percentage change of the main search market indicators of Yahoo! Bing Network and Google on a year-over-year basis (Q4 2011 – Q4 2012)[12]

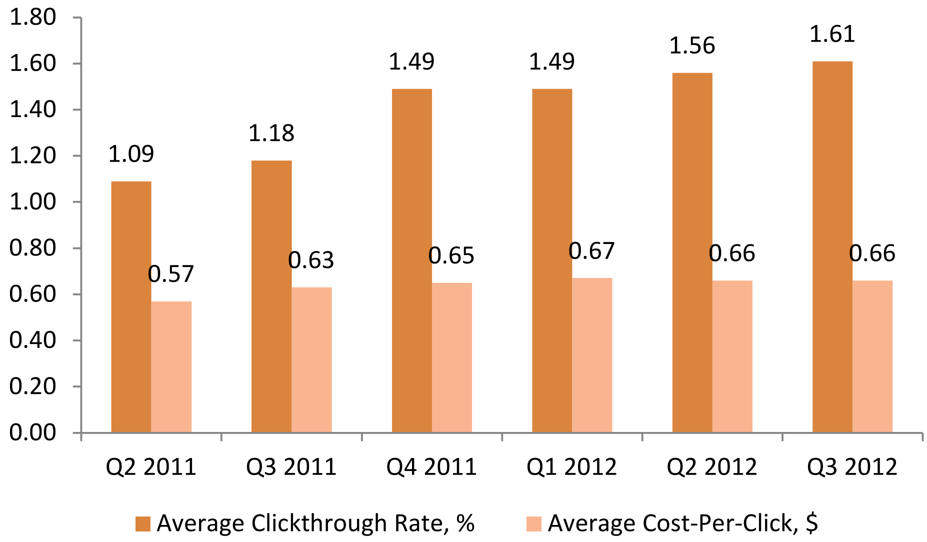

Figure 4.4. U.S. Average Clickthrough Rate and Average Cost-Per-Click of Yahoo! Bing Network[13]

From the Figures 4.3-4.4 it could be clearly seen that a merger between Yahoo! and Microsoft favored the increase in both Average Clickthrough Rate and Average Cost-Per-Click indicators. Therefore, the increase in subscription fee is compensated by the increase in network benefit parameter.

[12] Marin Software. (2012). Online Advertising Report, October-December 2012. Available at http://www.marinsoftware.com/downloads/Q4_2012_online_advertising_report_US.pdf

[13] Kenshoo. (2012). Global Search Advertising Trends, October 2012. Available at http://www.kenshoo.com/digitalmarketingtechnology/wp-content/uploads/2013/04/kenshoo-global-search-advertising-trends-q3-2012.pdf

For now, the impacts of Yahoo!-Microsoft merger could be experienced preferably by the U.S. searchers and advertisers since Yahoo! Bing Network have covered 30% of the U.S. search market; in Europe where Google owns 95% of the market (Table 4.1) the results of the merger are still imperceptible. However, provided that Yahoo! and Microsoft will continue improving their search algorithm, they might exert a competition to Google not only on the U.S. market, but also worldwide, and, therefore, prevent a dominant search engine from the employment of abusive practices.

Table 4.1. Yahoo! Bing and Google market shares in some European countries[14]

Country	Yahoo! Bing market share	Google market share
UK	6.78%	92.00%
France	4.29%	94.76%
Germany	3.00%	95.69%
Ireland	4.16%	94.67%
Italy	1.05%	97.54%
Spain	2.42%	96.96%

In the next subsection we model search engine mergers by using the two-sided market framework, and, thus, investigate its impact on platforms' prices to content providers.

4.2 Modeling the competition between three platforms in two-sided markets with zero-price constraints

Suppose there are three platforms serving a two-sided market. Our aim is to investigate the impacts of a merger between two of them. Before doing so we have to derive user demand configurations from each of these platforms. This issue is complicated given the fact that we do not know the exact location of platform $i = A, B, C$, comparing to the previous cases where two platforms were located at the end points of a of a line with length equal to 1. However, similarly to Malam (2011), we assume the platform locations to be exogenously given and symmetric. Without loss of generality, platforms A, B and C are supposed to be located at 0, $1/3$, and $2/3$, respectively (Figure 4.5).

Since we are dealing with a triopoly, in order not to face the computational difficulties, we assume hereafter that users (group-1 agents) single-home on a particular platform whereas content providers are remained to be multi-homing. Moreover, we assume group-1 agents to be of the same type, i.e. $\alpha_{1L} = \alpha_{1H} = \alpha_1$. Provided that all users single-home, there is a product differentiation on one side. Thus, we have to assume that user's preference parameter is higher comparing to the network benefits, i.e. $t > \alpha_1$.

[14] Digital Clarity. (2011). Yahoo & Bing Search Engine Merger. Available at http://www.digital-clarity.com/blog/search-engines/yahoo-bing-search-engine-merger/

Optimal group-1 demand configurations are drawn in the same manner to Malam (2011). However, contrary to the author, we do not apply Hotelling's Location Model in order to find group-2 market shares. Instead, we assume that there is a continuum of group-2 agents and their total mass is sufficiently large, such that some of them may decide not to join any of three platforms.

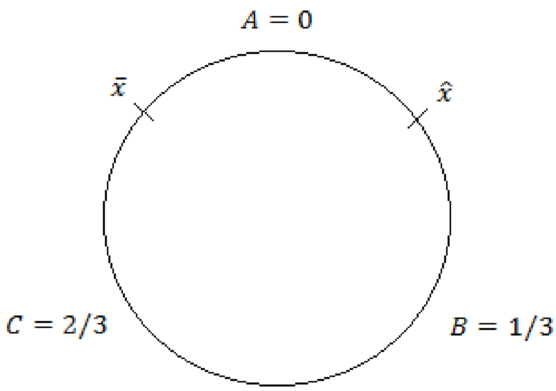

Figure 4.5. Platform i's location

Similarly to the previous section, platform i's equilibrium price to consumers is assumed to be zero, i.e. $p_1^i = 0$. Therefore, the utility of searchers is given as follows

(4.1) $$u_1^i = \vartheta_0 + \alpha_1 n_2^i - t d_i(x),$$

where $d_i(x)$ is the distance function, which gives the shortest arc to a nearly located platform.

(4.2) $$d_A(x) = \min\{x, 1-x\},$$
$$d_B(x) = \min\left\{\left|x - \frac{1}{3}\right|, \frac{4}{3} - x\right\},$$
$$d_C(x) = \min\left\{\left|x - \frac{2}{3}\right|, \frac{1}{3} + x\right\}.$$

We assume the utility from complementary services, ϑ_0, is sufficiently large such that all users join at least one platform in equilibrium. Moreover, users are uniformly distributed along the arc. Hence, the location of a searcher who is indifferent between single-homing on A or B is defined as follows

(4.3) $$\vartheta_0 + \alpha_1 n_2^A - t\hat{x} = \vartheta_0 + \alpha_1 n_2^B - t\left(\frac{1}{3} - \hat{x}\right),$$

whereas the location of an indifferent between A and C searcher is

(4.4) $$\vartheta_0 + \alpha_1 n_2^A - t(1 - \bar{x}) = \vartheta_0 + \alpha_1 n_2^C - t\left(\bar{x} - \frac{2}{3}\right).$$

Solving (4.3) and (4.4) for the indifferent agent's locations on each segment gives us

(4.5) $$\hat{x} = \frac{1}{6} + \frac{\alpha_1(n_2^A - n_2^B)}{2t},$$

(4.6) $$\bar{x} = \frac{5}{6} + \frac{\alpha_1(n_2^C - n_2^A)}{2t}.$$

The total number of users on platform A is, therefore, given by

(4.7) $$n_1^A = \hat{x} + (1 - \bar{x}) = \frac{1}{3} + \frac{\alpha_1(2n_2^A - (n_2^B + n_2^C))}{2t}.$$

Similarly we can find the location of an indifferent agent situated on the segments between B and A, and B and C. Thus, the amount of indifferent users on the segment between B and C is given by

(4.8) $$\vartheta_0 + \alpha_1 n_2^B - t\left(\hat{x} + \frac{1}{3}\right) = \vartheta_0 + \alpha_1 n_2^C - t\left(\frac{2}{3} - \hat{x}\right),$$

and on segment between B and A

(4.9) $$\vartheta_0 + \alpha_1 n_2^B - t\left(\frac{4}{3} - \bar{x}\right) = \vartheta_0 + \alpha_1 n_2^A - t\left(\bar{x} - \frac{1}{3}\right).$$

Solving (4.8) and (4.9) for $\hat{x}$ and $\bar{x}$ we get

(4.10) $$\hat{x} = \frac{1}{6} + \frac{\alpha_1(n_2^B - n_2^C)}{2t},$$

(4.11) $$\bar{x} = \frac{5}{6} + \frac{\alpha_1(n_2^A - n_2^B)}{2t}.$$

The total number of users on platform B can be expressed as

(4.12) $$n_1^B = \left(\hat{x} - \frac{1}{3}\right) + \left(\frac{4}{3} - \bar{x}\right) = \frac{1}{3} + \frac{\alpha_1(2n_2^B - (n_2^A + n_2^C))}{2t}.$$

By analogy with platforms A and B, the user demand on platform C is

(4.13) $$n_1^C = \frac{1}{3} + \frac{\alpha_1(2n_2^C - (n_2^A + n_2^B))}{2t}.$$

As in Armstrong (2006), side-1 demand configurations depend on side-2 market shares. Since we have not introduced a platform-specific utility from follow up services, the platform i's side-1 demand configuration is not in any relation with its own utility from follow up services and with utilities provided by competitors.

Analogously to the previous section, the utility of content providers who has chosen to place their ads on platform i is given by

(4.14) $$u_2^i = \alpha_2^i n_1^i - p_2^i.$$

Content providers join platform i only when their resulting net surplus, $\alpha_2^i n_1^i - p_2^i$, is non-negative. As in the duopoly case, we assume a fixed cost incurred by content providers who join platform i to be distributed with the constant density $f = 1$. Therefore, the mass of content providers on platform i is expressed as

(4.15) $$n_2^A = \alpha_2^A n_1^A - p_2^A,$$

(4.16) $$n_2^B = \alpha_2^B n_1^B - p_2^B,$$

(4.17) $$n_2^C = \alpha_2^C n_1^C - p_2^C.$$

By substituting (4.15)-(4-17) into (4.7), (4.12) and (4.13), we solve for the user location as a function of side-2 prices (p_2^A, p_2^B, p_2^C). Solving for n_1^A gives us

(4.18) $$n_1^A = \left\{3\alpha_1 \left[p_2^B\left(2t - 3\alpha_1\alpha_2^C\right) + p_2^C(2t - 3\alpha_1\alpha_2^B) - p_2^A\left(4t - 3\alpha_1\left(\alpha_2^B + \alpha_2^C\right)\right)\right] + (2t - 3\alpha_1\alpha_2^B)\left(2t - 3\alpha_1\alpha_2^C\right)\right\}/3\Gamma$$

where $\Gamma = 4t^2 - 4t\alpha_1(\alpha_2^A + \alpha_2^B + \alpha_2^C) + 3\alpha_1^2\left(\alpha_2^B\alpha_2^C + \alpha_2^A(\alpha_2^B + \alpha_2^C)\right)$ and assumed to be positive.

Provided that platforms incur relatively low per-agent costs, such that we could set them to zero, platform A's profit is given by

(4.19) $$\pi^A = p_2^A n_2^A = p_2^A(\alpha_2^A n_1^A - p_2^A),$$

where n_1^A is given by equation (4.18). Clearly, platform A's profit is concave in p_2^A.

First order condition of (4.19) with the respect to p_2^A lead us to the platform A's best response to two other platforms, which is given by

(4.20) $$p_2^A = \frac{\alpha_2^A[3p_2^B\alpha_1(2t - 3\alpha_1\alpha_2^C) + (2t - 3\alpha_1\alpha_2^B)(2t + 3p_2^C\alpha_1 - 3\alpha_1\alpha_2^C)]}{6(4t^2 - 4t\alpha_1(\alpha_2^B + \alpha_2^C) + 3\alpha_1^2\alpha_2^B\alpha_2^C)},$$

which is clearly increasing in platform A specific network benefit parameter, α_2^A, experienced by content providers.

Solving two more maximization problems for platforms B and C, we get best response price functions, which can be introduced in a generalized framework

(4.21) $$p_2^i = \frac{\alpha_2^i\left[3p_2^j\alpha_1(2t - 3\alpha_1\alpha_2^k) + (2t - 3\alpha_1\alpha_2^j)(2t + 3p_2^k\alpha_1 - 3\alpha_1\alpha_2^k)\right]}{6(4t^2 - 4t\alpha_1(\alpha_2^j + \alpha_2^k) + 3\alpha_1^2\alpha_2^j\alpha_2^k)},$$

where $i = A, B, C$; $i \neq j$, $i \neq k$, $j \neq k$.

Solving a system of three equations from (4.21) we get equilibrium prices, which depend on network benefit parameters and user preference parameters.

Proposition 4.1. Under symmetric side-2 network benefit parameters $\alpha_2^A = \alpha_2^B = \alpha_2^C = \alpha_2$ *and* $t > 3\alpha_1\alpha_2/2$*, there exists a symmetric equilibrium with* $p_2^A = p_2^B = p_2^C = p_2$*, where* p_2 *is given by*

$$p_2 = \frac{\alpha_2(2t - 3\alpha_1\alpha_2)}{12(t - \alpha_1\alpha_2)}.$$

Symmetric equilibrium is also characterized by the equal number of group-1 and group-2 agents on platforms. Thus, $n_1^i = 1/3$ and $n_2^i = \frac{\alpha_2(2t-\alpha_1\alpha_2)}{12(t-\alpha_1\alpha_2)}$.

Platform i's profit under symmetric price in Proposition 4.1 is given by

(4.22)
$$\pi^i = \frac{\alpha_2^2(2t - \alpha_1\alpha_2)(2t - 3\alpha_1\alpha_2)}{144(t - \alpha_1\alpha_2)^2}.$$

If we rewrite the equation (4.21) in terms of platforms' market shares we get

(4.23)
$$p_2^i = \frac{n_1^i\alpha_2^i}{2 + \frac{\alpha_1\alpha_2^i(4t - 3\alpha_1(\sum_{i\neq j}\alpha_2^j))}{\Upsilon}},$$

where $\Upsilon = 4t^2 - 4t\alpha_1\left(\sum_i \alpha_2^i\right) + 3\alpha_1^2\left(\prod_{i\neq j}\alpha_2^j + \alpha_2^i\left(\sum_{i\neq j}\alpha_2^j\right)\right) > 0$.

Given that platform i's price to advertisers directly depends on its market share among users, an increase in a fraction of searchers on the platform leads to an increase in its price to advertisers. This is what actually we can see on search engine market – having almost 70% of the U.S. search market, Google's average cost-per-click is more than two times higher comparing to Yahoo!-Bing Network.

In order to resist a competitor and increase their own prices, two other platforms may wish to cooperate between each other. Such cooperation can be transformed into the merger, which could bring higher profits to these platforms and higher benefits to their agents. Moreover, as it has been already mentioned, if the market is occupied with a dominant platform, an appearance of the merger between dominated platforms can be more than appropriate. The case of Microsoft-Yahoo! merger confirms these observations.

4.3 Platform mergers in two-sided markets with zero-price constraints

In this subsection we analyze the possible impacts of a merger between two platforms. Suppose that platforms B and C merge. Analogously to the Microsoft-Yahoo! case, we assume that the merger results in setting the same price to the content providers and the same network benefit parameter enjoyed by this group of agents, α_2^{BC}.

We leave user demand configurations the same as in 4.7, 4.12, and 4.13, since despite the merger both platforms remain operating as before.

As in the previous section we assume that the fixed cost, f, of a content provider joining platform i is distributed with a constant density $f = 1$. Thus, the mass of content providers on platforms A, B and C is given by:

(4.24) $$n_2^A = \alpha_2^A n_1^A - p_2^A,$$

(4.25) $$n_2^B = \alpha_2^{BC} n_1^B - p_2^{BC},$$

(4.26) $$n_2^C = \alpha_2^{BC} n_1^C - p_2^{BC}.$$

Given (4.24)-(4.26), the optimal user demand configurations are

(4.27) $$n_1^A = \frac{2t + 3\alpha_1(2(p_2^{BC} - p_2^A) - \alpha_2^{BC})}{\Lambda},$$

(4.28) $$n_1^B = n_1^C = \frac{2t + 3\alpha_1(p_2^A - p_2^{BC} - \alpha_2^A)}{\Lambda},$$

where $\Lambda = 6t - 3\alpha_1(2\alpha_2^A + \alpha_2^{BC}) > 0$.

Platform profits are defined by the following equations

(4.29) $$\pi^A = p_2^A n_2^A = p_2^A(\alpha_2^A n_1^A - p_2^A),$$

(4.30) $$\pi^{BC} = p_2^{BC}(n_2^B + n_2^C) = p_2^{BC}[(\alpha_2^{BC} n_1^B - p_2^{BC}) + (\alpha_2^{BC} n_1^C - p_2^{BC})],$$

From the first order conditions, $\partial\pi^A/p_2^A$ and $\partial\pi^{BC}/p_2^{BC}$, we get the platform i's best response function that can be expressed as

(4.31) $$p_2^A = \frac{\alpha_2^A(2t + 6p_2^{BC}\alpha_1 - 3\alpha_1\alpha_2^{BC})}{6(2t - \alpha_1\alpha_2^{BC})},$$

(4.32) $$p_2^{BC} = \frac{\alpha_2^{BC}(2t + 3p_2^A\alpha_1 - 3\alpha_1\alpha_2^A)}{12(t - \alpha_1\alpha_2^A)}.$$

Solving the system of two equations (4.31)-(4.32), we get equilibrium pair of prices

(4.33) $$p_2^A = \frac{\alpha_2^A[4t^2 + 3\alpha_1^2\alpha_2^A\alpha_2^{BC} - 4t\alpha_1(\alpha_2^A + \alpha_2^{BC})]}{3[8t^2 + 3\alpha_1^2\alpha_2^A\alpha_2^{BC} - 4t\alpha_1(2\alpha_2^A + \alpha_2^{BC})]},$$

(4.34) $$p_2^{BC} = \frac{\alpha_2^{BC}[8t^2 - 10t\alpha_1\alpha_2^A - 4t\alpha_1\alpha_2^{BC} + 3\alpha_1^2\alpha_2^A\alpha_2^{BC}]}{6[8t^2 - 8t\alpha_1\alpha_2^A - 4t\alpha_1\alpha_2^{BC} + 3\alpha_1^2\alpha_2^A\alpha_2^{BC}]}.$$

Proposition 4.2. Under symmetric side-2 network benefit parameters $\alpha_2^A = \alpha_2^{BC} = \alpha_2$ *and* $t > 3\alpha_1\alpha_2/2$, *there exists a unique Nash equilibrium with the pair of prices given by*

$$p_2^A = \frac{\alpha_2(4t^2 - 8t\alpha_1\alpha_2 + 3\alpha_1^2\alpha_2^2)}{24t^2 - 36t\alpha_1\alpha_2 + 9\alpha_1^2\alpha_2^2},$$

$$p_2^{BC} = \frac{\alpha_2(8t^2 - 14t\alpha_1\alpha_2 + 3\alpha_1^2\alpha_2^2)}{6(8t^2 - 12t\alpha_1\alpha_2 + 3\alpha_1^2\alpha_2^2)}.$$

Contrary to the symmetric equilibrium in Proposition 4.1, the equilibrium in Proposition 4.2 is not characterized by the equal number of group-1 and group-2 agents on platforms. Thus, $n_1^A = \frac{4(2t^2 - 3t\alpha_1\alpha_2 + \alpha_1^2\alpha_2^2)}{24t^2 - 36t\alpha_1\alpha_2 + 9\alpha_1^2\alpha_2^2}$ and $n_1^B = n_1^C = \frac{16t^2 - 24t\alpha_1\alpha_2 + 5\alpha_1^2\alpha_2^2}{48t^2 - 72t\alpha_1\alpha_2 + 18\alpha_1^2\alpha_2^2}$. Let us proceed with parameterization and normalize network benefit parameters as follows: $\alpha_1 = \alpha_2 = 1$. The graphical representation of $n_1^A(t)$ and $n_1^B(t) = n_1^C(t)$ is provided on Figure 4.6.

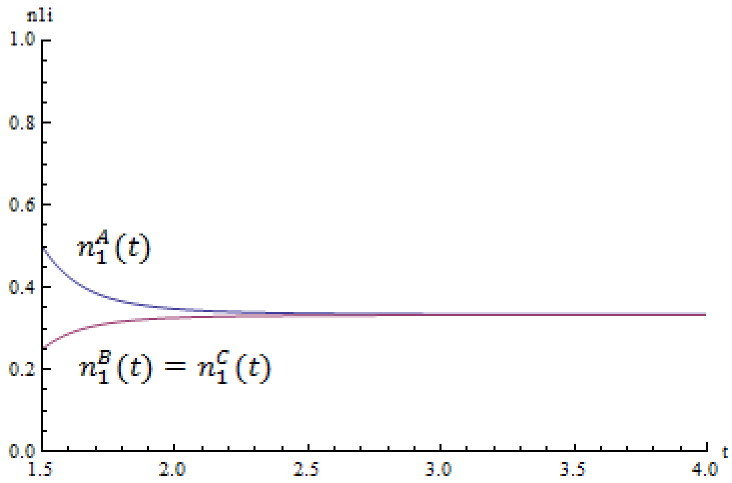

Figure 4.6. The number of group-1 agents on platform i as a function of preference parameter, $t > 3\alpha_1\alpha_2/2$, $\alpha_1 = \alpha_2 = 1$

Proposition 4.3. As $t \to \infty$, there exists an equilibrium where group-1 agents are equally distributed among three platforms, i.e. $n_1^A = n_1^B = n_1^C = 1/3$.

Comparative statics

First, let us compare prices under no merger and merger. To do so, we apply the following parameterization: $\alpha_1 = \alpha_2 = 1$. Consequently, $t > 3/2$.

On Figures 4.7-4.8 there are represented equilibrium configurations under no merger and merger.

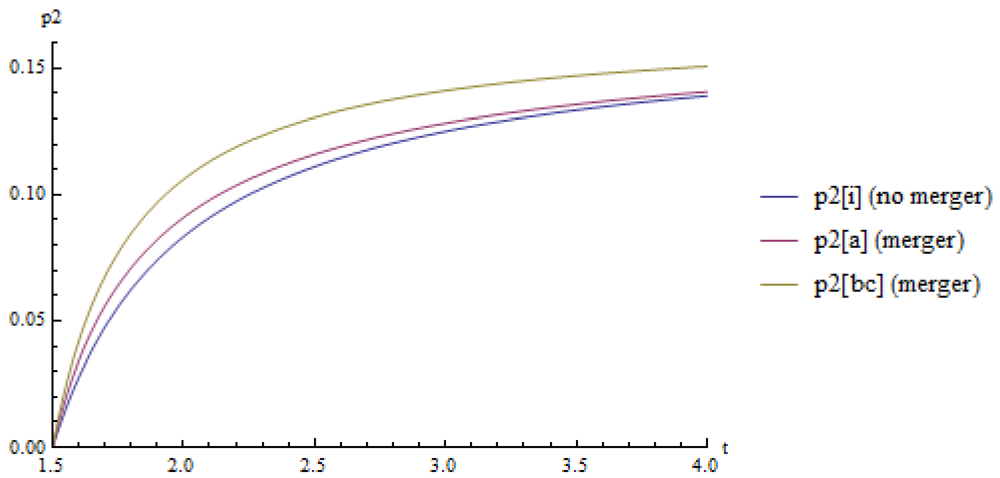

Figure 4.7. Comparative statics of equilibrium prices, $t > 3\alpha_1\alpha_2/2$, $\alpha_1 = \alpha_2 = 1$

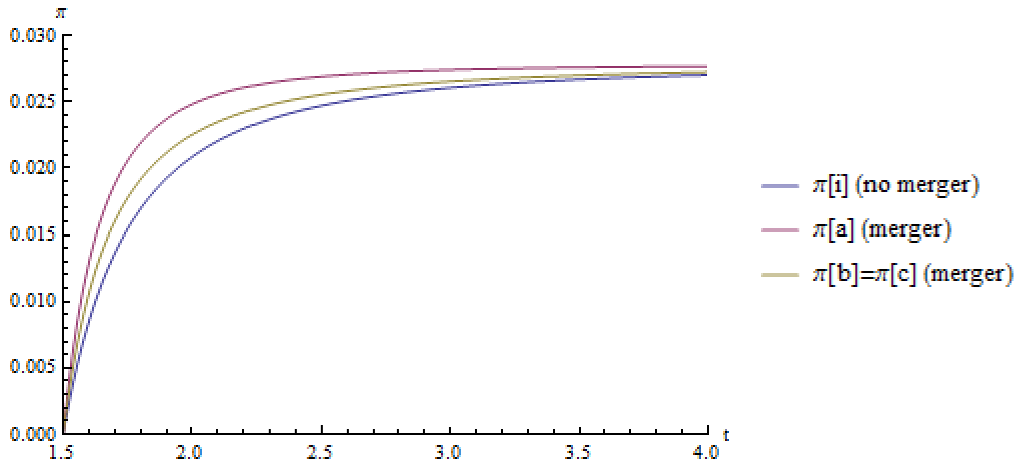

Figure 4.8. Comparative statics of equilibrium profits, $t > 3\alpha_1\alpha_2/2$, $\alpha_1 = \alpha_2 = 1$

The analysis of Figures 4.7-4.8 for platforms yields in favor of a merger. Merged platforms charge more comparing to the case, when they act on the market separately (Figure 4.7) and they earn more profits (Figure 4.8). Such price increase is counter-balanced by the additional network benefits for advertisers resulting from attraction of exclusive buyers. Advertisers prefer to pay more, but to have an access to a wider user base.

In addition, the average cost-per-click presented on a merged platform space has increased because of more aggressive bidding strategies applied by advertisers. Advertisers prefer bid on their ads more on purpose, since now they can reach more users, and, therefore, would like to have these ads shown in both search engines. Recall that, due to a merger, advertisers do not have a possibility to choose different cost-per-click bids for the different search engines.

An interesting observation resulting from a merger could be done. Our model shows that the price increase by merged platforms (say, Yahoo! Bing Network) has created the positive response from their competitor (say, Google). The reason for such an increase can be explained by the reduced platform rivalry on the market – there are remained only two major players competing. In such situation advertisers are left with take-it-or-leave-it decision when joining the platform. If their resulting net surplus is non-negative, they would prefer to stay. The real data confirms these observations – from the Figure 4.3 it could be seen that one year after merger Google's average cost-per-click among the U.S. advertisers has increased by 14%.

4.4 Merger welfare analysis

Here we analyze welfare implications of a merger under equilibrium configurations mentioned in Propositions 4.1-4.2. Within such configuration each user experiences the same intrinsic benefit among three platforms, which is equal to ϑ_0, and advertisers enjoy the same network externality parameter from interaction with each additional user, i.e. $\alpha_2^A = \alpha_2^B = \alpha_2^C = \alpha_2^{BC} = \alpha_2$.

In general terms consumer surplus can be written as follows

(4.35) $$CS = \vartheta_0 + \alpha_1(n_1^A n_2^A + n_1^B n_2^B + n_1^C n_2^C) - \frac{t}{2}[(n_1^A)^2 + (n_1^B)^2 + (n_1^C)^2].$$

Provided that $f = 1$, advertiser surplus is given by

(4.36) $$AS = \frac{1}{2}[(n_2^A)^2 + (n_2^B)^2 + (n_2^C)^2].$$

Producer surplus (platforms' profits) is equal to

(4.37) $$PS = p_2^A n_2^A + p_2^B n_2^B + p_2^C n_2^C.$$

Total welfare is, therefore, can be expressed as

(4.38) $$W = CS + AS + PS.$$

For simplicity, we provide our welfare analysis under normalized values of network benefit parameters, such that $\alpha_1 = \alpha_2 = 1$. Once we have proceeded with a normalization operation, our consumer surplus and total welfare remain to depend only on the preference parameter, t. Figures 4.9-4.10 below show this dependence.

Clearly, the normalization of network benefit parameters to one does not reflect the realities of the search engine market, but it helps to understand what happens with consumer surplus and social welfare under no merger and merger configurations as t increases.

As in the previous cases, for results in section to be sufficient, we have to impose the assumption on preference parameter, t, in order to guarantee non-negative prices, such that $t > 3\alpha_1\alpha_2/2$.

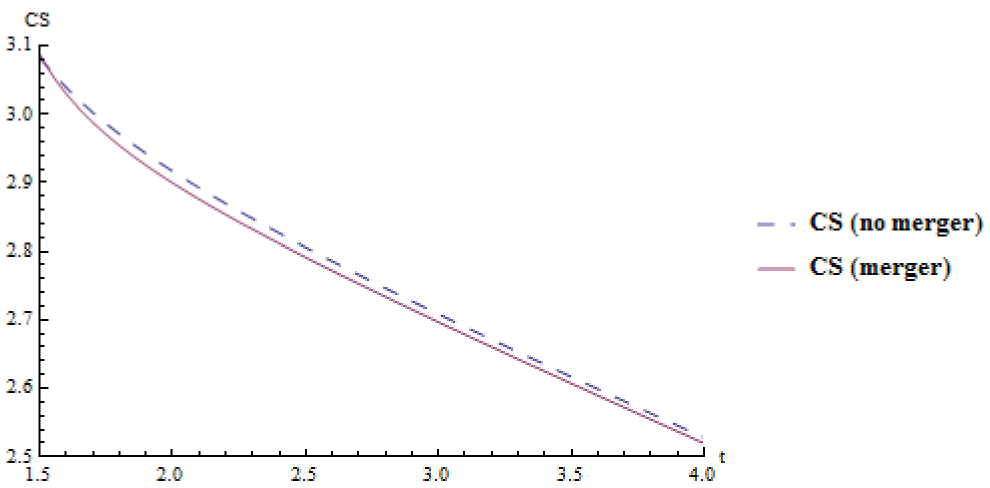

Figure 4.9. Consumer surplus under normalized network benefit parameters ($\vartheta_0 = 3$, $t > 3\alpha_1\alpha_2/2$, $\alpha_1 = \alpha_2 = 1$)

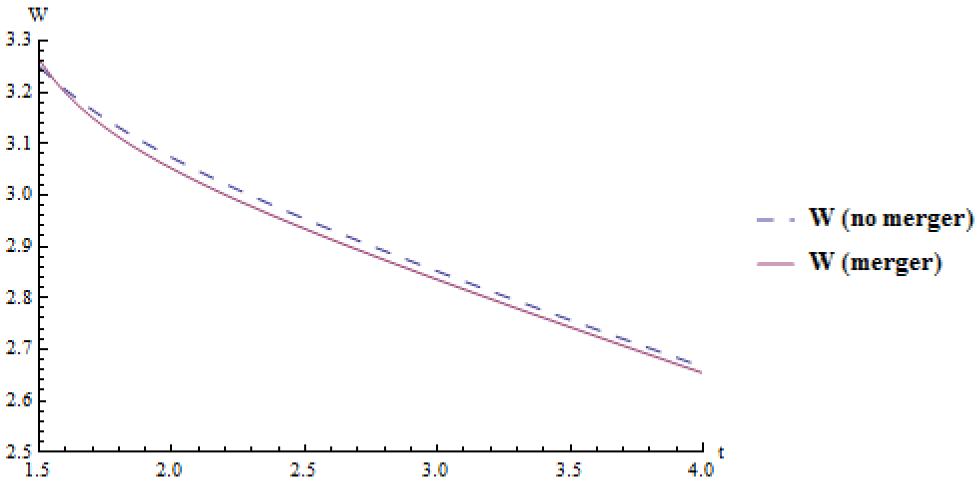

Figure 4.10. Total welfare under normalized network benefit parameters ($\vartheta_0 = 3$, $t > 3\alpha_1\alpha_2/2$, $\alpha_1 = \alpha_2 = 1$)

Proposition 4.4. Under $t > 3\alpha_1\alpha_2/2$, a merger between two platforms leads to a slight decrease in consumer surplus and total welfare, but to an increase in total platform profits.

However, as $t \to \infty$, consumer surplus and total welfare tend to be equal under no merger and merger configurations.

5 Conclusions

The main insight of this dissertation is to reflect the realities of the search engine market, especially the issue of excessive pricing. As the main analytical tool it has been decided to choose an oligopoly model of platform competition in two-sided markets, which has been successfully applied in many industries including media.

One of the fundamental features of our model is a zero-price constraint imposed in equilibrium on one side of the market. In the case of a search engine market, such constraint is represented by free services provided in the form of information search on the World Wide Web, electronic mailbox, access to the financial information, etc. On the other side, however, such free content is augmented with ads, which are the main revenue source of any search engine in particular, and media platform in general. Other feature employed in our analysis is represented by the continuum of content providers. Contrary to the majority of studies in the field of two-sided markets, we do not normalize the mass of advertisers to one, therefore assuming that the total mass of advertisers is large enough such that there always exist advertisers who have decided not to join any platform. Such assumption allows us to proceed with the welfare analysis of different equilibrium outcomes.

The solution to platforms' profit maximization problem in Section 3 yields that the symmetric equilibrium price charged to advertisers is higher when high type consumers multi-home rather than single-home. In other words, we can state that platforms' price to advertisers directly depends on a market share among agents on the subsidized side (on consumer side). Empirical observations, presented in this dissertation, confirm the validity of the theoretical model – having almost 70% of a market share among U.S. searchers, Google's average cost-per-click is more than three times higher comparing to those one of Yahoo! Bing Network.

In addition to the analysis of optimal pricing, our duopoly model of platform competition has been employed to investigate the issue of exclusive dealing. It has been found that tying exclusive contracts leads to the reduction of the producer surplus, and, consequently, the total welfare under relatively small values of advertiser network benefit parameter. It is crucial to notice that, contrary to the ordinary two-sided markets where both sides of the market are charged with a particular price, in two-sided media markets, like a search engine market, exclusive dealing ends up with the full exclusion of one platform from the market. Moreover, as it has been already noticed by a number of scholars, obtaining of a dominant market position by one search engine is fraught with some undesirable consequences, such as underinvestment in quality and the manipulation of search results.

The last section of the dissertation is motivated by the recent case of a merger between Yahoo! and Microsoft. In order to cover this case, the duopoly model of platform competition has been extended to the triopoly model. The main results of a merger analysis yield an increase in prices to the advertisers, which are set by not only two merged platforms, but also by their competitor. One of the possible reasons that can explain such increase is the reduction in platform rivalry on the market – there are remained only two major competitors. In such situation advertisers are left with take-it-or-leave-it decision when joining the platform. These theoretical observations reflect the realities of a search engine market. In addition, the conducted welfare analysis of equilibrium market configurations detects a slight decrease in the consumer surplus and total welfare, and an increase in the producer surplus as a result of the merger.

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Appendix A

Proof of inequality A2

Suppose platform A deviates and starts to serve only exclusive content providers by charging them the price of πN_A. Then, platform's profit extracted from both sides is given by:

$$\Pi_A = \lambda(\pi N_A - c) + (q_A - d)N_A = \lambda\left(\pi \frac{\lambda b - q_A}{t} - c\right) + (q_A - d)\frac{\lambda b - q_A}{t}$$

F.O.C.

$$\frac{\partial \Pi_A}{\partial q_A} = \frac{\lambda(b-\pi) + d - 2q_A}{t} = 0$$

The optimal price for consumer established by platform *A* is then:

$$\hat{q}_A = \frac{\lambda(b-\pi) + d}{2}, \text{ where } t \neq 0.$$

By substituting optimal price for consumers who participate with platform A in $N_A = \frac{\lambda b - q_A}{t}$ we can obtain the optimal number of these consumers participating in platform A:

$$\widehat{N}_A = \frac{\lambda(b+\pi) - d}{2t}.$$

By substituting optimal number of consumers and price set for them by platform *A* in initial profit equation we get:

$$\widehat{\Pi}_A = \lambda\left(\pi \frac{\lambda(b+\pi) - d}{2t} - c\right) + \left(\frac{\lambda(b-\pi) - d}{2}\right)\frac{\lambda(b+\pi) - d}{2t},$$

or by rewriting this equation given $c = d = 0$ we can obtain

$$\widehat{\Pi}_A = \frac{\pi\lambda^2(b+\pi)}{2t} + \frac{\lambda^2(b+\pi)(b-\pi)}{4t},$$

that is an *optimal deviation payoff* for platform A when it deviates.

The equation of non-deviation payoff of platform *A* thus is given by:

$$\Pi_A = [\lambda + (1-\lambda)](\pi n_A - c) + (q_A - d)N_A$$

If we substitute here values of $n_A = 1 - \frac{\lambda b - q_B}{t}$ and $N_A = \frac{\lambda b - d}{2t}$ given $q_B^* = q_A^* = \frac{\lambda b + d}{2}$ we obtain the optimal non-deviation payoff of platform *A*, that is:

$$\Pi_A^* = \left[\pi\left(1 - \frac{\lambda b - d}{2t}\right) - c\right] + \frac{(\lambda b - d)^2}{4t},$$

or under the condition when $c = d = 0$

$$\Pi_A^* = \pi\left(1 - \frac{\lambda b}{2t}\right) + \frac{(\lambda b)^2}{4t},$$

that is an *optimal collusive payoff* for platform *A.*

Accordingly, two-sided multi-homing equilibrium holds if $\Pi_A^* \geq \widehat{\Pi}_A$. Solving inequality for *t* gives us

$$t \geq \frac{\lambda[\lambda\pi + 2b(1+\lambda)]}{4}.$$

Appendix B

Proof of Proposition 3.3

Recall that total number of group-1 agents on platform i is given by $n_1^i = \gamma h_1^i + (1-\gamma) l_1^i$.

If both high and low consumer types single-home, we have to find a location of each user type.

We find low type group-1 agent's location from

$$\vartheta_0 + \alpha_{1L} n_2^A - tx = \vartheta_0 + \alpha_{1L} n_2^B - t(1-x),$$

where $x = l_1^A$.

High type group-1 agent's location can be found from

$$\vartheta_0 + \alpha_{1H} n_2^A - tx = \vartheta_0 + \alpha_{1H} n_2^B - t(1-x),$$

where $x = h_1^A$.

Consequently,

$$l_1^i = \frac{1}{2} + \frac{\alpha_{1L}\left(n_2^i - n_2^j\right)}{2t},$$

$$h_1^i = \frac{1}{2} + \frac{\alpha_{1H}\left(n_2^i - n_2^j\right)}{2t},$$

where $n_2^i = \alpha_2^i n_1^i - p_2^i$.

Solving for the total number of group-1 agents as a function of group-2 prices we get

(B.1) $$n_1^i = \frac{t - (\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L})(\alpha_2^j + \left(p_2^j - p_2^i\right))}{2t - (\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L})(\alpha_2^i + \alpha_2^j)}.$$

Let us assume that $\alpha_2^A = \alpha_2^B = \alpha_2$.

Platform i's profit is given by

(B.2) $$\pi^i = n_2^i p_2^i = \left(\alpha_2 n_1^i - p_2^i\right) p_2^i.$$

Substituting (B.1) into (B.2) and solving a system of two equations $\frac{\partial \pi^A}{\partial p_2^A} = 0$ and $\frac{\partial \pi^B}{\partial p_2^B} = 0$ for equilibrium pair of prices to side-2 agents, we get symmetric side-2 price

(B.3) $$p_2 = \frac{\alpha_2[t - \alpha_2(\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L})]}{4t - 3\alpha_2[\gamma\alpha_{1H} + (1-\gamma)\alpha_{1L}]}.$$