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Ca' Foscari
Dorsoduro 3246
30123 Venezia

Risk analysis and application in Stable financial market

Relatore

Ch. Prof. Marco Corazza

Laureando

Francesco Groppo

Matricola 815838

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For those who constantly have supported me,
or at least, never doubted on my perseverance.
To Professor Corazza and Mr. Donati, firm guides
and precious companions in this financial travel.
To my family, to my friends, to Siusi.
To Kenji. May all his dreams come true!

"If goldfish are kept in a small bowl, they will remain small.

With more space, the fish can double, triple, or quadruple its size."

Edward Bloom –Big fish

Abstract

Stable distributions have been used by a growing number of authors in past four decades since they provide the attractive possibility to extend the standard assumption of normal behavior for financial random variables. In this work it is explained why and how it is possible and it is convenient to adopt such generalization and the statistical identifications of the Stable univariate and multivariate distributions are given. The study will then be specifically focused on the financial returns loss distribution and on two risk assessment procedures. Tools like the VaR measures and the expected shortfall one are analized with respect to the different evaluation they produce about tail risky event. The Stable estimation is carried with different methodologies, so that great part of the analytical work is eventually devoted to the comparison between those different procedures of fit.

Key words: Stable distribution, characteristic function, VaR, expected shortfall, maximum likelihood, quantile procedures.

Summary

Introduction

In April and May 2012 the students of the second cycle degree on Economics had the chance to follow as optional course an experimental one on econophysics. The aim of the course was to introduce this recent interdisciplinary field, and to suggest a new perspective for pursuing economic analysis: the adoption of scientific methods and knowledge developed in the physics disciplines.

The intuition followed is that economic systems can be considered complex ones, as the soon as the physic terminology is considered. A complex system is composed by interconnected parts that as a whole exhibit one or more properties that arise in a non–trivial way from the different ones characterizing the individual parts. If we adopt this assumption, ensues that all the tools of analysis that the physical literature has developed to study complex systems, can be applied also to the economics ones.

The lectures were organized by an associate professor of physics, Prof. Francesco Gonella, and that was the added value of the course: it was possible to depart for a while from our standardized approach as economic students and to question our generalized assumption with a scientific approach, aided by an extremely competent academic figure.

Unluckily this chance was limited to the thirty hours length of the course, so there was time enough just for a quick survey of the literature. Anyhow we got familiar with some notions like chaos laws and catastrophe theory, fractals behavior, similarity between thermodynamic laws and economic dynamics, non normal behaving processes.

That course really inspired me, as I often realized that for the sake of simplification, most of the models I had the chance to study lacked of a realism or were connoted by an excessively weak explanatory power. My disposition towards more complex and more descriptive models was though stimulated.

The chance to analyze one of such more complex frameworks comes after the course of Financial Economics, when I defined with Prof. Corazza one the main concepts that characterize this thesis, the rejection of the assumption of normally distributed financial variables.

The combination between his long experience and preparation, and my enthusiastic interest on this issue, helps me through the development of this work, that eventually occurs to involve also a third contribution, by a person that I would like to thank particularly.

I was indeed followed and assisted by the high skilled eyes of Doc. Riccardo Donati, a financial analyst specialized on risk evaluation and portfolio optimization with techniques coming from physics sciences. Once again destiny wanted my final dissertation to be connected a physicist.

I am grateful to Doc. Donati for the time he has generously devoted to follow me in the data analysis. He also gave me the possibility to exploit the professional output of a software like Mathematica, for the implementation of different estimators and measures of risk. Such a keen and qualified attention to my case-study gives to this work an additional validity and scope.

This dissertation aim then to revise one of the standard assumption in financial models, determining and describing an alternative path to follow: the use of Stable distribution to explain returns volatility.

We will define how and with which instruments it is possible to derive a Stable characterization for a financial variable and we will compare in terms of accuracy and efficient several estimators.

We will eventually consider the empirical consequences of the adoption of this different approach for what regards techniques of risk analysis and risk hedging. It will be proposed an analysis of the risk profile associated to the dynamics of 188 assets from the New York Stock Exchange market

The work is organized as follows:

- Chapter 1 is devoted the description of the distribution function we choose to use as reliable alternative of the standard normal: the Stable distribution. A good statistical connotation is given along with the main properties related to this probability law.
- Chapter 2 provides an overview on the wide field of risk management. The general definitions of risk are provided and the historical evolution of the regulatory system for financial operator is summarized. The end of the chapter describes two risk measures, their usage and their properties.
- Chapter 3 is dedicated to the detailed connotation of the analytical tools we adopted to fit the data. The different estimation techniques available for the Stable distribution are explained, with a special focus to the one that were implemented in our analysis.
- Chapter 4 presents the main results of our comparative studies. The data will express the difference in terms of risk assessment of the adoption of the Stable distribution.

We report in the Appendix summarizing tables containing all the output passages of our study.

Chapter 1: Quick introduction to α-Stable distributions

In this chapter a particular kind of probability distribution is introduced. The literature commonly refers to it as Stable, α-Stable, Lévi or Pareto Stable distribution, emphasizing now the main property of the distribution law, stability, and then two authors that are mostly related to it. The first one, Pareto is accounted for the proper definition of characteristic behavior of the tail distribution, Lèvi instead is quoted for his seminal work in the 1990s and in general for his contributions in the characterization on non-Gaussian Stable laws.

Stable distributions have been the main subject of a new wave of development regarding basics models and applications, since they identify distribution laws that, up to some extent, are versatile, give more realistic results and are supported by empirical evidences. They are increasingly adopted and used in different areas: engineering, computer science, physics, astronomy, geology, medicine.[1]

Regarding economics, the most interesting results come from finance, in field like asset returns modelling, portfolio analysis, risk assessment and management, derivative and option pricing, econometric modelling.

In order to understand the power and the amplitude gained in economic applications by those particular distribution laws, it is useful to trace back the main moments and historical passages of bibliographic evolution, taking as reference the works by Mandelbrot in early 1960s.

Up to the analysis of Mandelbrot, the common fashion and the standard usage in economic theories was to rely on the assumption that asset returns, as many other financial variable, follow in distribution the Normal, Gaussian law. Also nowadays, as my undergraduate experience confirms, basics economic models rely so much on the "bell shaped" distribution, that it happens rarely to depart from it.

As we will also have the chance to demonstrate, empirically this assumption is most of the times violated. But even in front of detailed analysis and studies, even if skepticism and concern regard discrepancies between models and reality was growing, still Gaussian distribution was regularly implemented. We can justify this persistence considering two aspects .

First, the computational burden required to analyze and use distributions of different nature was heavy. As we will see, if we want to use the most efficient statistical tool to fit distributions more complex rather than the Normal one, we have to generate and deal with huge amount of data. Computers and processors were not enough developed to provide appreciable aids expecially when it comes the need to work with a wide datasets and long time series.

Considering a second and more influent aspect, a Normal distribution is characterized by a series of properties that are highly desirable for the well behavior of a model:

[1] Nolan(2013) for an integrated bibliographic review of Stable

- Just two parameters are sufficient to identify such a law. The mean, that describes central location or the shift of the distribution with respect to the origin and the variance, or the standard deviation statistics, that explains the amount of volatility around the mean, as an index of dispersion. Other higher order moments can be calculated to enrich the data description, but as assumption fixed values for skewness and kurtosis are set.
- It is a continuous distribution, twice differentiable and defined in the whole support of real numbers, generally behaving well in optimization problem.
- It is possible, moreover, to reduce any Normal variable to a standardized one for which densities of each value and quantiles are (and were) well-known. For this reason it is straightforward to use in the calculus of probability, and in hypothesis testing, expecially if we consider also the theoretical support that comes from the Central Limit theorem.

  The latter describes the conditions that have to be met in order to make the a sum of random variables converges to a Normal distribution. It is easy to understand how important is the possibility to exploit a theorem like that in economics or finance. As an example, consider a portfolio of assets, were singular repeated records in time are aggregated to compose one unique patter of investment. We can derive the main characteristics and the risk profile of the portfolio just considering the statistical behavior of its component.

Mandelbrot demonstrated that despite of all those useful properties and the desirable behavior, shortcomings deriving from a Normal model, too far from reality were not to be underestimated. He examined different times series in the financial world, spanning from commodity prices, to assets returns, to interest rates, and he derived that a more suitable distribution for that kind of variables was the Stable one.

This study was developed further by Fama and Roll in their publication in 1968, and fueled a stream of publication investigating the real nature of assets returns. The analysis was so deep that during the 70s not only Normal distribution was questioned, but also some evidence against the Stable started to be collected. The debate focused in particular on one key aspect that need to be precisely modeled: the tails of the distribution.

In finance, part of the most dangerous component of risk is the one related to big, extreme, sudden shocks that may occur in the markets. Those events are represented in a generic distribution by the tail part, and are commonly associated with small probabilities, since their occurrence is to be considered rare. Their impact however might be catastrophic: talking about money and investment positions, despite the fact that there is no upper bound for profits, losses are clearly bounded and the capital that vanishes after a crisis need significant time and efforts before being recovered.

This is why it is so important to find the right distribution, the one that nor underestimate nor overestimate the weight of the tails.

Normal distribution was questioned for having thin tails compared to the empirical density commonly measured for financial data. In general then, it is considered inadequate to describe risky events because leads the analysts to underweight extrema values. The solution identified by the Stable distributions seemed at the beginning a better one, depicting a power law decay[2], but subsequent studies pointed out that was not the most accurate one, this time because of a too fat tail component.

Tempered-Stable, Student t-distribution, or the hyperbolic one were suggested time by time as better compromise, but all these alternatives lacked of a properties that is essential in portfolio theory and risk management: stability. Only for Stable distributed returns, a linear combination of different series is again Stable distributed.

For this reason in the last three decades, aided by new computational techniques, models adopting the Stable assumption have been rediscovered and revisited, in the attempt of relaxing the Normal assumption and develop generalized framework to study economics and finance issues.

Rachev and Mittnick are without any doubt two authors that have been working extensively on the topic; in particular the two books published in 2000 and 2011 have been reference point for most of the contents included in this study.

Another eminent author is the American Professor Nolan. This author in particular is one of the most integrated reference regarding the analytical aspects, since it has been working on the computational characteristics of both univariate and multivariate Stable distribution for years, expecially oriented towards economic and finance issues.

We will use (as many other authors have done) his program STABLE and MSTABLE for the calculation of the probability density function of Stable distribution. We will as well use his notation, harmonizing all the different ways of representing Stable parameters and connected functions that we have encountered in the jungle of publication analized.

Nolan accounts also for a bibliographic review revised in February 2013. This work reports in details a complete list of papers and books dealing with Stable distributions in different field. We have to mention for the part that regards our study McCulloch, DuMouchev, Hills, Koutrouvelis, Paulauskas, Frain, Kim, Zolotarev and Ortobelli, among the others.

Kuzobowski, Parnoska and Rachev in an article in 2003 report how other publications references. We mention for what regards derivative pricing and expansion of models in option pricing, authors like Dostoglou, Janicki, Cartea, Hurst, Platen, Karandikar, Rüschendorf and Samorodnitsky. Regarding risk management, estimation of risk and relationship between the Stable distribution and coherent measure of risk Bassi, Embrechts, Paolella, Gamrowski and Mittnik. Extension of the optimal portfolio theory, and on the

[2] Definitions and technicalities will follow in the chapter

CAPM or APT were designed by Fama, Bawa, Huberman, Elton, Gruber, Uchaikin, Belkachem, Walter, Chamberlain, Cheung, Kwan and Gamba. Other studies were carried towards exchange rate dynamics, real estate prices, insurance issues, commodity price behavior, covering as we can see a wide variety of aspects in financial works.

A role is played also by my supervisor, with papers dealing with the fractal structure of commodity futures markets or foreign currency ones, along with a specific work on the implementation of the Koutrouvelis algorithm. Once again we thank him for being reference and guide in this jungle of formulas and brand new concepts that compose the Stable world.

Throughout our analysis we will specifically refer to some of those publications, and we recommend for additional investigation over details, demonstrations, corollaries and specific examples to rely on the bibliography at the end of the work.

Chapter 1 continues with the description of Stable distributions. In a first part a proper definition is reported. We will then define better the role played by the four related parameters, and focus on the characteristic function, a statistical tool that help to identify a distribution when the probability density function is not defined. Some properties of the Stable distribution are then described while the end of the chapter is dedicated to the description of the multivariate Stable laws.

Why not normal?

This class is defined "Stable" because it collects different probability laws that commonly show, among others, the property of summation stability. It is a simple definition, but it has strong statistical implication, as we will have the chance to see. We will analyze the main consequences of this behavior and how generally it comes in handy, considering the approximation of a random variable in finance.

It is useful, nevertheless to start from the beginning, to understand the specific arguments raised against the Normal distribution. We will use one of our time series to match all those theoretical observations with an empirical example.

We have already mentioned that different authors (Rachev and Mittnik(2000), Nolan(2003), Kozubowski, Panorska and Rachev(2003), Fama(1963, 1965), McCulloch(1996)) had the chance to observe that the use of the Normal distribution as approximation of the behavior for financial variables raises some issues, now we report are those criticalities.

Suppose we follow the usual assumption and we compute the values that we need to define IBM daily returns as a normally distributed random variable. The result of the estimation are reported in Table 1[3]:

Table 1: Normal parameter estimation for IBM and Jarque-Bera test

	IBM US Equity		
Observations	11113		
Mean	0.023219	Std. Dev.	1.67051
Maximum	12.36647	Skewness	-0.25060
Minimum	-26.0884	Kurtosis	14.55767
Jarque-Bera	61969.34		
Probability	0.000000		

The number of observation is statistically high, it represents more than forty-five years of records about returns obtained from the daily closing price of the asset IBM. The maximum and minimum value are highlighted and the two defining statistics for the Normal distribution are identified as the mean and standard deviation.

As we expected the mean is positive, even if low, otherwise we should wonder how and why the assets still survives and persists in the market and still attracts investors. The value for standard deviation in pretty high compared to the mean and somehow justify the huge spread between maximum and minimum.

The term "Skewness" report a measure of asymmetry of the data with respect to the mean and exploits the third moment of the distribution. In particular it is defined as:

[3] For this first, simple analysis, we used the Eviews® Software.

$$\zeta(X) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{X_i - \bar{X}}{\hat{\sigma}}\right)^3,$$

where $\bar{X}$ represents the mean of all the observations and $\hat{\sigma}$ is an estimator for the standard deviation:

$$\hat{\sigma} = s\sqrt{(N-1)/N}\,,$$

based on the biased estimator for index of sample variance:

$$s = \sqrt{\left(\sum_{i=1}^{N}(X_i - \bar{X})^2\right)/(N-1)}\ .$$

If the distribution is symmetric, as in the case of a variable behaving like a Normal, the skewness is zero. For all the other asymmetric distributions, positive skewness implies that the right tail is more elongated, negative skewness the converse.

The Kurtosis value instead refers to the peakedness or to the flatness of the distribution, giving an hint about what should be the shape of the distribution in the central part, and it is computed as:

$$\kappa(X) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{X_i - \bar{X}}{\hat{\sigma}}\right)^4.$$

The value of kurtosis for a Normal distribution is 3. If the kurtosis exceeds this reference, the distribution is leptokurtic, excessively peaked. If the kurtosis is lower, the distribution is platykurtic, or excessively flat. IBM US Equity is though connoted by a leptokurtic shape

Jarque-Bera value is instead a test statistic that assesses whether or not the distribution should be considered Normal. Exploiting both the skewness and kurtosis parameter, though all the first four sample moments, it is defined by:

$$Jarque - Bera = \frac{N}{6}\left(S^2 + \frac{(K-3)^2}{4}\right).$$

The null hypothesis checked is that the distribution analized is normal. Under this hypothesis, the Jarque-Bera statistic is distributed like a χ^2, a “Chi-squared” distribution, with two degrees of freedom. The probability reported below the test is the probability that the Jarque-Bera statistic exceeds (in absolute value) the value observed under the null hypothesis.

Such a low value (that probability is equal to zero at least in the first six value after the point, decimal separator) would lead us to reject the hypothesis of normal distribution at a level lower than 0.0001%, that means for all the most used confidence values.

Figure 1 shows a histogram created ordering the daily returns of the asset IBM. Each time we refer to returns, we consider the logarithmic ratio of two consecutive closing prices P_t of a time series:

$$X_t = \log\left(\frac{P_t}{P_{t-1}}\right) * 100 .$$

We identified classes from a negative lower bound of -4 to a positive upper bound of 4, using a class length of 0.0625. In this way we reported most of the records from the dataset (97.2% of the total) in 129 classes.

In the figure below, we can think at the histogram like an approximation of the probability density function, since the values have been harmonized to sum to one. In such a way it is possible to make a visual comparison with the Normal distribution that we can derive considering the empirical mean and the standard deviation reported in Table 1.

As we can notice, there are several discrepancies. The less evident one is that while the Gaussian distribution does not allow for any kind of asymmetry by definition, while we can see that the left part of the observations for the IBM asset appears to be more dense. To better catch this behavior it is useful to compare the classes on the left of the modal value that are characterized by frequency between the values 0.01 and 0.03 to the correspondent observation in the right. The left part is closer to the dotted line of the Normal distribution, while the right part appears to fall more steeper.

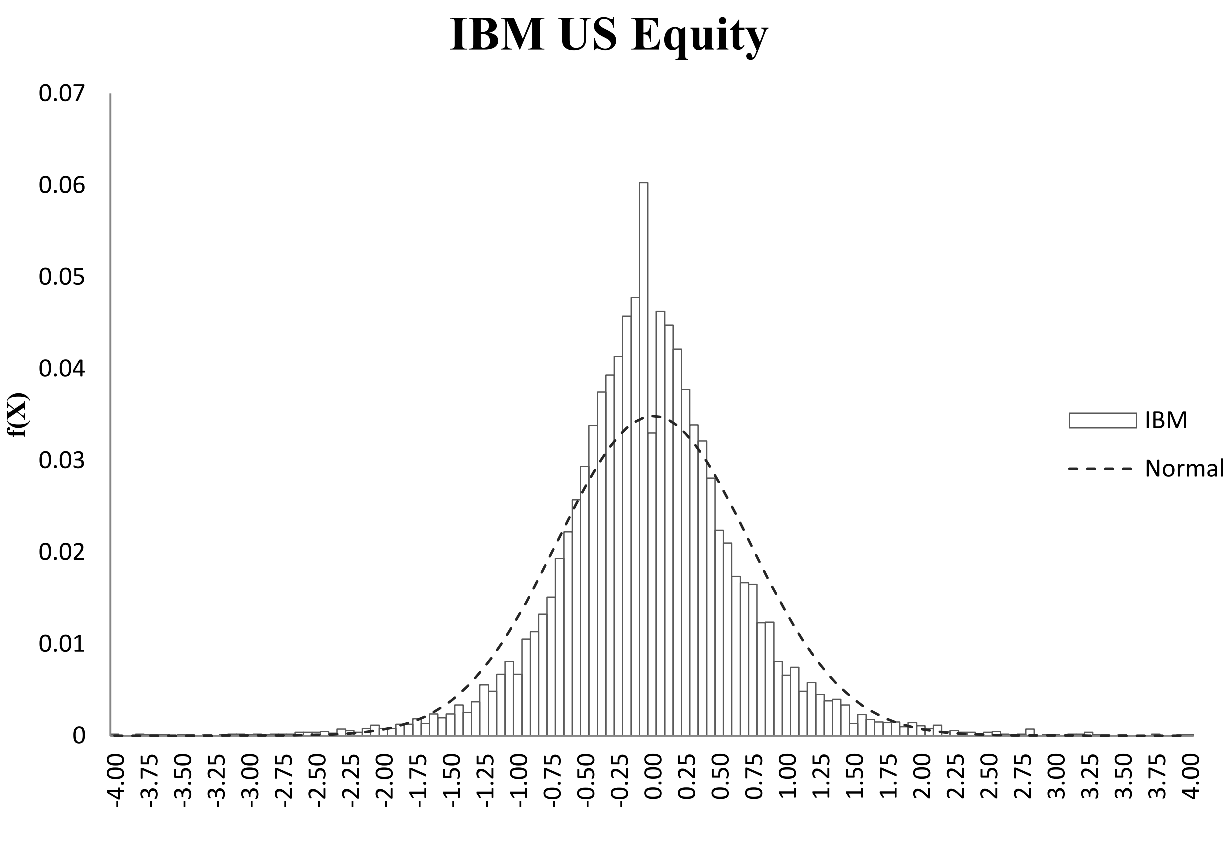

Figure 1: IBM histogram returns and comparison with the Normal distribution associated.

This is in support of the claim that usually the distribution of the returns of an asset is skewned to the right so that the tail in the left appears thinner and longer that the one in the right.

More clear and noticeable is the difference between the two shapes: the Normal density around the mean appears to seriously under match the empirical distribution. As we can see, the histogram is much more leptokurtic, peaked, where axis values between -0.5 and 0.5 are assigned with a number of observation much more higher than the relative frequencies that characterize the Normal shape.

It seems like small variations in the asset value have higher frequencies compared to the one that is normally predicted. This account for all the trading days characterized by quiet market activities that cause just little changes in the price evaluation.

Figure 2 highlights a different aspect: the behavior in the left tail zone. We can compare again a Normal tail (dotted line) and the histogram for the IBM asset, with the help of a tendency line (solid line) for the last variable, that has been plotted as polynomial regression of sixth order.

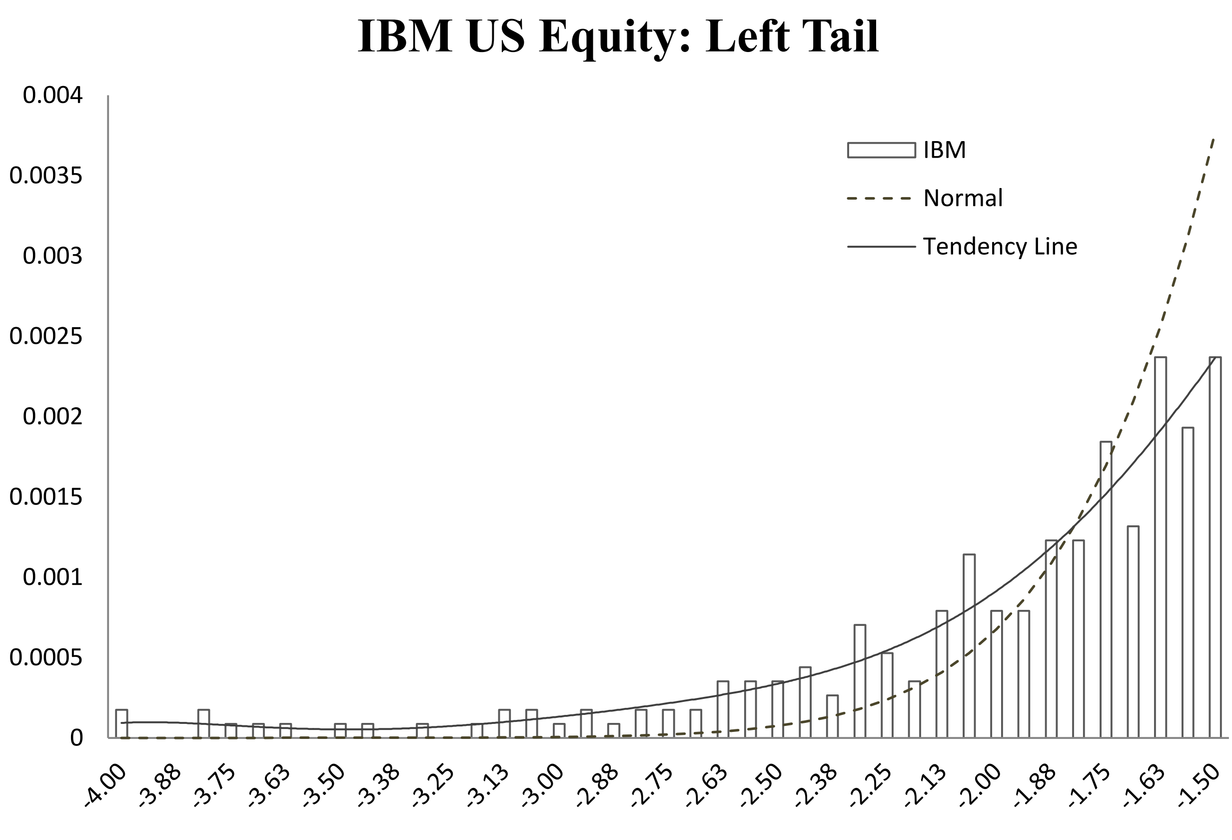

Figure 2: IBM histogram returns and comparison in the left tail with the Normal distribution associated.

As we can observe, the empirical tail is much more populated compared to the Normal one. Those observation accounts for really bad days in the markets, and cannot be properly considered if the Normal assumption holds.

We can further load the argument, observing that, the IBM time series reports seven records with losses greater than the value of 4 (in absolute value). Such values in a Normal distribution are so rare that in order to have 7 such records we should dispose of a dataset of more than 1000000000 observation, that is to say thousands of times the one we are using.

For all the reasons exposed above, the explanatory power in a model characterized by the assumption of the Normal behavior of the variables is definitely weak, with just the benefit of a framework that is easy to handle from the computational point of view.

Those models are indeed quick and simple to understand, replicate and implement in computers and routines, and are related with other benefits. For normally distributed random multivariate we can use the correlation as reliable measure of dependence and we can assume as well that the sum of so-distributed variables is Normal as well, deriving mean and standard deviation with few passages.

The technological development of the last few decades have brought the possibility to easily extend those general models to the use of more complex classes of distributions. It is now possible to run estimators and algorithms that allow us to shape and describe laws defined by more than just two parameters, without wasting too much time, and that can better fit the empirical data.

Among those kind of laws, Stables (a set in which the Normal one represents just a particular case) are the ones that permit to extend some general results of probability theory, like the law of large numbers or the propriety of domain of attraction. In this way, maintaining a parallelism with the Normal assumption, we are able to approximate better the volatility of the financial variables, allowing moreover for skewness and kurtosis and representing heavy tails behaviors.

Four defining parameters

There have been identified in statistical literature two alternative defining formulas, equivalent one to the other (Samorodnitsky and Taqqu, 1994), that depict the main property of Stable distribution,. The first one is the following:

$$X_1 + X_2 + X_3 + \cdots + X_n \triangleq c_n X + d_n \,,$$

where with $X_1 + X_2 + X_3 + \ \ldots + \ X_n$ we consider the sum of n random variables X_i, that are independent one to the others (the values observed for one random variable do not affect the other's probability distribution) and identically distributed as the random variable X (each X_i behaves according to the same probability distribution of X). Whit $\triangleq$ we indicate that the terms in the left side have the same distribution of the terms in the right. c_n represents a constant belonging to the positive R_+ set and d_n is another general constant.

The formula introduced above equates then in distribution a sum of variables X_i with a linear transformation of another random variable X, that is in fact multiplied by a first constant element c_n and shifted by a second one d_n. This tells us that for Stable laws, the sum operator, generates a result that it is once again Stable in distribution.

We can redefine c_n as a norming operator, equal to $n^{\frac{1}{\alpha}}$, where n is the number of addends[4]. This norming constant is crucial and describes the stability property of the distribution thanks to the parameter α, the first of the four that we will need to identify our distribution.

The presence of d_n allows instead to a shift and to a first classification among Stable distribution: for $d_n = 0$, the sum of random variables $X_1 + X_2 + X_3 + \cdots + X_n$ is connoted as strictly Stable.

As second main definition for stability, we consider a linear combination of two objects such that:

$$AX_1 + BX_2 \triangleq CX + D\,,$$

where again X_1 and X_2 are two random variables independent one to the other and identically distributed as the variable X, and A, B and C three positive real numbers. It is possible to derive the relationship between the constant terms and identify the parameter α also for this definition and, in the particular case in which the D term is equal to zero, as:

$$C = (A^\alpha + B^\alpha)^{\frac{1}{\alpha}}\,.$$

The distributions that are defined in such two ways can be grouped and described by four parameters, identified by Nolan(2009, 2013) with the four Greek letters $\alpha, \beta, \gamma, \delta$.[5]

$$X \sim S(\alpha, \beta, \gamma, \delta)\,.$$

This set collects different already known distributions, that we can obtain with particular combinations of values for the four parameters. For instance, with this notation we can describe also Cauchy laws, Lévi laws and the Normal distribution. The last one in particular have been widely used in various models, exactly because it does show summation stability and so allows to exploit a normalization procedure and the general result of the Central Limit Theorem.

The parameter α is identified as index of stability and have range $\alpha \in (0,2]$ in the real set. The upper bound represents the case of a Normal distribution, so that for Gaussian distribution $c_n = \sqrt{n}$ and $C^2 = A^2 + B^2$. As the value of α decreases, the shape of the distribution gets more and more peaked and the tails became fatter.

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[4] Feller(1966)

5Several way to define Stable laws and their parameters have been used by different authors in literature. One example is the notation that appears in Kozubowski, Panorska & Rachev(2003): $X \sim S_\alpha(\sigma, \beta, \mu)$. We will adopt the formalization of Nolan, since it rules out confusing issues, regarding the use of σ and μ.

The parameter generates an exceptional case for the value $\alpha = 1$, because of computational issues. We will analyze this peculiarity (related also to the Cauchy distribution) once introduced the characteristic function that generally defines a Stable law.

The value of α is connected also with the definition of the absolute moments of the distribution that are generally defined as:

$$E|X|^p = \int |x|^p f_{(X)} dx \, .$$

In particular will have finite values for those moments only for $p < \alpha$. This implies that for values of α less than 2, the variance will not be determined as a discrete value, and for values less than one also the mean will be indefinite. It is clear that sample moments of all order will always exist and be computable, but there will not be any convergence pattern to the population moments, depending on the value of α.

This fact influences the way in which parameters are estimated since the various estimation procedure have to exploit more than just asymptotical properties.

The second parameter β is a symmetry indicator. It represents the skewness of the distribution and has range $\beta \in [-1,1]$. Negative values assess that the distribution shows negative skewness, therefore the left tail, compared with the right one, appears elongated. Positive values indicates the opposite attitude. For those reason, symmetric Stable distributions are characterized by β parameter equal to 0. The effect of this second parameter is somehow related to the value of α: the more close to 2 is the latter, the harder will be for the former to depict patterns of asymmetry.

Figure 1, 2 and 3 show the probability density function for some differently combined values of α and β. The software used for the simulation is STABLE[6].

Figure 3 explains the role of α. Fixed the symmetric parameter to zero and the other one to their standard value, α define for small values distribution with peaked shape and long and fat tails while when it increase it makes the distribution degenerate in the classical "bell" shape of the Gaussian one, and leptokurtic behavior disappear.

Figure 4 shows instead how it is possible to describe asymmetry using this class of distribution. The extreme value of $\beta = 1$ represents the maximum positive skewness obtainable for each value of α, with extremely fat right tail. As we decrease the β parameter, the density function reshapes to a centered one. For negative values of β we can derive the shape simply considering the reflection of the graph using as axis the vertical one through the value 0.

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[6] The program STABLE is available at www.mathstat.american.edu following the link to the personal page of John P. Nolan.

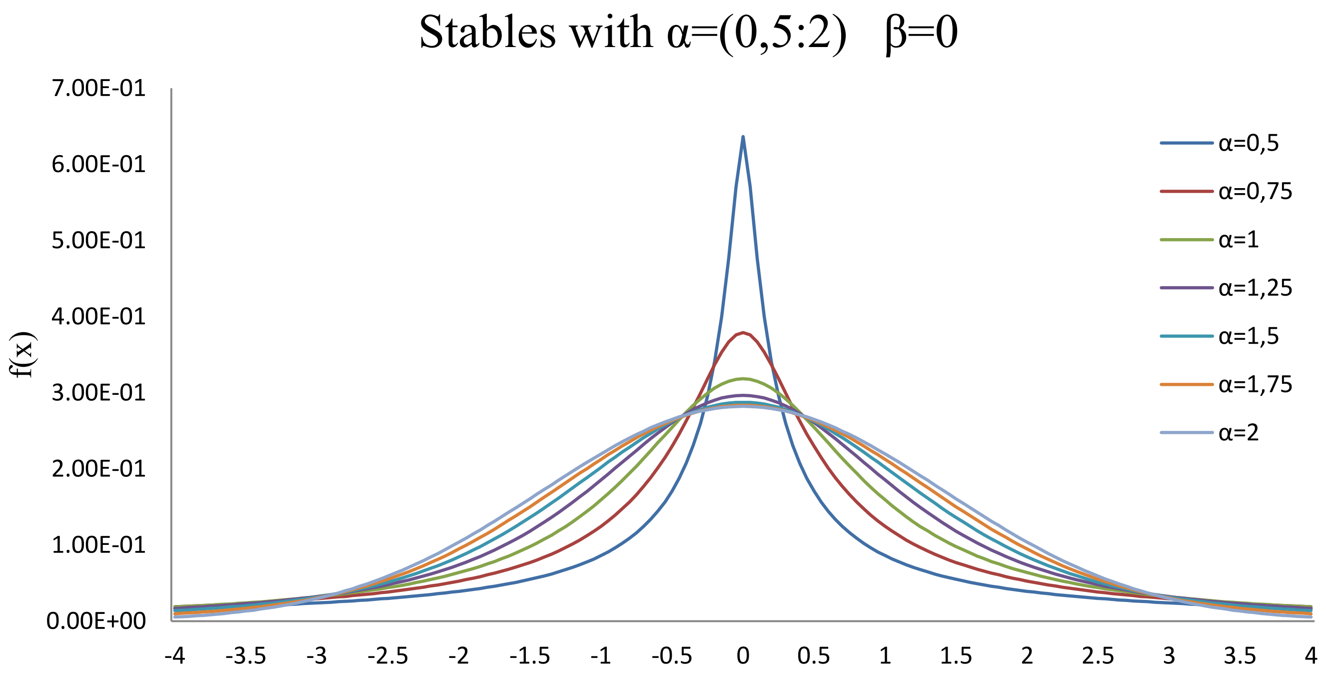

Figure 3: Shape of the probability density function of a symmetric Stable distribution, with α parameter that takes values from 0.5 (most peaked line) to 2 (most rounded shape). [$\gamma = 1, \delta = 0$]

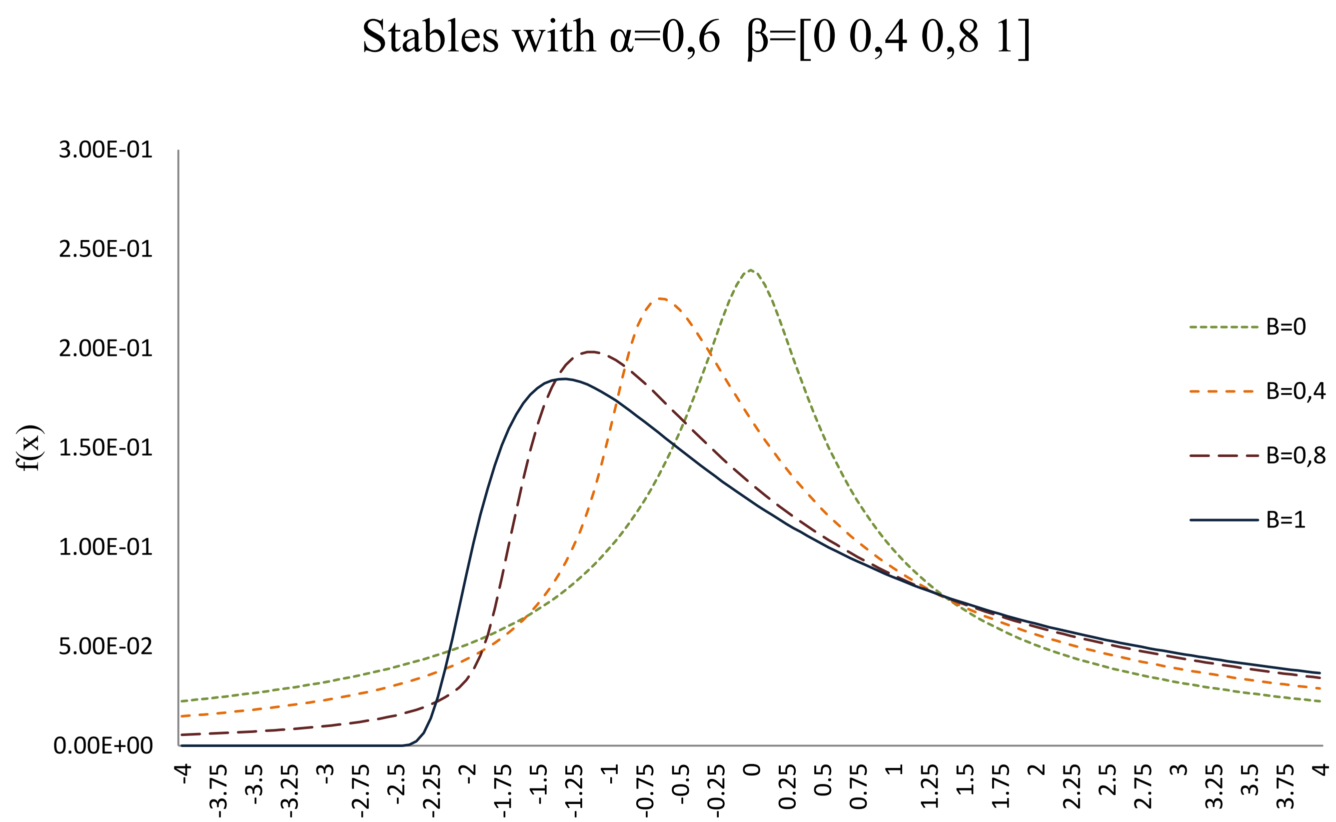

Figure 4: Shape of the probability density function of a Stable distribution with α parameter equal to 0.6 and β parameter that takes values 1 (most elongated right tail), 0.8, 0.4, 0 (symmetric shape). [$\gamma = 1, \delta = 0$]

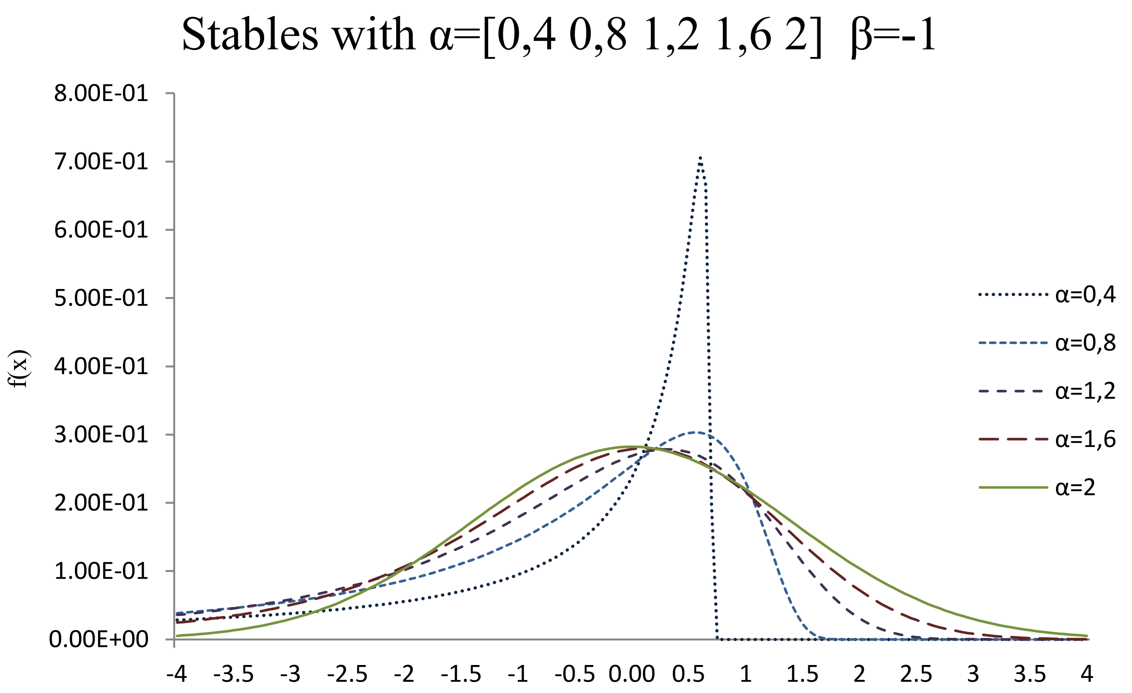

Figure 5: Shape of the probability density function of a distribution with α parameter that takes value 0.4(most peaked and skewned), 0.8, 1.2, 1.6, 2(most flatter, symmetric shape) and β parameter equal to -1. [$\gamma = 1, \delta = 0$]

Figure 5 helps us to understand how the first two parameters are related. We can see the changes in the shape of the distribution as α increase, maintaining the β parameter to his lower bond $\beta = -1$. The distribution that it is strongly asymmetric for values of α lower or around one, degenerates again into the flat symmetric Gaussian curve.

The third parameter is γ, scale parameter, that accepts only positive real values $\gamma \in (0, \infty)$, the fourth is a shift parameter $\delta \in (-\infty, \infty)$, spanning on all the set of real numbers, that defines the location. In the previous representation of Figures 3, 4 and 5, those two values have been kept to which are commonly identified as the "standardized" values: $\gamma = 1$ and $\delta = 0$. We will see how it is easy to reconnect every stable to a standardized one, studying the property of the distribution.

There are two possible equivalent way to consider by the meaning of the parameters, as according to Nolan(2009), differentiated by the definition of the fourth parameter: $S(\alpha, \beta, \gamma, \delta_0; k = 0)$ and $S(\alpha, \beta, \gamma, \delta_1; k = 1)$. Those two different notation generates different formulas in computations but define eventually the same kind of characteristic function.

We will use the second one, that is the most common in literature and the most used by software despite of Nolan(2003), reporting slightly better numerical behavior and intuitive meaning of the $k = 0$ parametrization. Nevertheless for completion we will explain the use and the peculiarity of both of them, introducing the characteristic function.

The support for the probability density function is generally identified with the whole set or real numbers $(-\infty, \infty)$, but just for the two extrema case of the β parameter, it is constrained above or below the location parameter, with a little distinction connected with the value of k and the specification for α. We can notice this behavior in both Figure 4 and Figure 5, where for values $\beta = 1$ and $\beta = -1$ respectively, the support is broken to respectively below and up of a certain value.

Characteristic function

One problematic issue regarding Stable laws is the fact that it is not possible to define in closed form the density function of the distribution, apart from three exceptions that we will report. In order to identify all the features of such distribution, we need then to use the statistical tool represented by the characteristic function.

The latter is a function that performs the mapping of the realizations to the complex plane: it is defined from the probability density function, applying what in non probabilistic context is called an inverse Fourier transformation[7].

Technically, starting from a finite point u, the characteristic represents the expectation of a transformed variable e^{iuX}:

$$\phi(u) = E\left[e^{iuX}\right] = \int_{-\infty}^{\infty} e^{iuX}\, dF(x) = \int_{-\infty}^{\infty} \cos iuX\, dF(x) + i\int_{-\infty}^{\infty} \sin iuX\, dF(x)\,.$$

The characteristic function $\phi(u)$ is important because it allows to derive another generalized definition for the population moments. We need first to define the logarithm of the characteristic function as characteristic exponent $\varphi(u)$:

$$\varphi(u) = \log \phi(u)\,,$$

and with $\varphi(u)^{(n)}$ we consider the n-th derivate of the characteristic exponent. If we compute this derivate around the value of 0, it is possible to describe a the central moments, using the cumulant $c_n(X)$[8]. Here we report the notation that is possible to use to identify the first four moments of a generic distribution, knowing the characteristic function, under the assumption that it is differentiable:

$$c_n(X) = \frac{\varphi^{(n)}(0)}{i^n}\,,$$

$E[X] = c_1(X)$	$E[X^2] - (E[X])^2 = c_2(X)$
$\zeta(X) = \dfrac{c_3(X)}{(c_2(X))^{\frac{3}{2}}}$	$\kappa(X) = \dfrac{c_4(X)}{(c_2(X))^2} + 3$

[7] Billingsley(1979) as reference. Further information about the fast Fourier information also on Chapter 3

[8] Rachev, Kim, Bianchi and Fabozzi (2011), the cumulants are a set of quantiles to be identified in a distribution.

As we can see, the first and second moments can be equalized respectively to the first and second cumulant, while to obtain the third and the fourth there is the need to consider respectively adjusted ratios of the third and the second cumulants, and of the fourth and second ones.

The characteristic function has other useful properties it is worth to report. If we define the characteristic function of a random variable X_1 as $\varphi_{X1}(u)$ and we consider another independent random variable X_2 described by $\varphi_{X2}(u)$, then we will have the following relationship to describe $X_1 + X_2$:

$$\varphi_{X1+X2}(u) = \varphi_{X1}(u)\varphi_{X2}(u) .$$

Or in the more general case of a sum of $X = \sum_{i=1}^{n} X_i$:

$$\varphi_X(u) = \prod_{i=1}^{n} \varphi_{Xi}(u).$$

The formula above states that it is possible to derive the characteristic function of a random variable X just multiplying the characteristic function of the variables that compose in summation X.

Those properties makes the characteristic function extremely interesting as a tool to use in the computation process. It is immediate the financial parallelism with the definition of a portfolio of assets: knowing the properties of its single components we can derive the overall characteristics.

The characteristic function is moreover a relation in a one-to-one correspondence with the distribution one of a random variable. That's why, as already mentioned, it is the only way to identify unambiguously a distribution with density function in no closed form.

The following equation identify the characteristic function of a Stable distribution defined for $k = 0$, first of the two notations individualized by Nolan, $S(\alpha, \beta, \gamma, \delta_0; 0)$:

$$\phi(u; \alpha, \beta, \gamma, \delta_0) = E\left[e^{iuX}\right] = \begin{cases} exp\left(i\delta_0 u - |\gamma u|^{\alpha}\left[1 + i\beta(sign\, u)\left(tan\frac{\pi\alpha}{2}\right)(|\gamma u|^{1-\alpha} - 1)\right]\right), & \alpha \neq 1 \\ exp\left(i\delta_0 u - \gamma|u|\left[1 + i\beta\frac{2}{\pi}(sign\, u)\log\gamma|u|\right]\right), & \alpha = 1 \end{cases}.$$

We can notice the already mentioned occurrence of a particular specification for $\alpha = 1$, since the first line in the graph parenthesis would have no definite form for values of the tangent function equal to $\frac{\pi}{2}$. Therefore the introduction of the $\frac{2}{\pi}\log|u|$ term in the second line.

The alternative stable parameterization, defined as $S(\alpha, \beta, \gamma, \delta_1; 1)$, for $k = 1$ leads to the following characteristic function:

$$\phi(u; \alpha, \beta, \gamma, \delta_1) = E\left[e^{iuX}\right] = \begin{cases} exp\left(i\delta_1 u - |\gamma u|^{\alpha}\left[1 - i\beta(sign\, u)\left(tan\frac{\pi\alpha}{2}\right)\right]\right), & \alpha \neq 1 \\ exp\left(i\delta_1 u - \gamma|u|\left[1 + i\beta\frac{2}{\pi}(sign\, u)\log|u|\right]\right), & \alpha = 1 \end{cases}.$$

For completeness we report also the relation between the two location parameter:

$$\delta_0 = \begin{cases} \delta_1 + \beta\gamma tan\frac{\pi\alpha}{2}, & \alpha \neq 1 \\ \delta_1 + \beta\frac{2}{\pi}\gamma\log\gamma, & \alpha = 1 \end{cases}, \qquad \delta_1 = \begin{cases} \delta_0 - \beta\gamma tan\frac{\pi\alpha}{2}, & \alpha \neq 1 \\ \delta_0 - \beta\frac{2}{\pi}\gamma\log\gamma, & \alpha = 1 \end{cases}.$$

If the stable distribution is symmetric, though ($\beta = 0$), the two definition of the location parameter δ_0 and δ_1 coincide. If the distribution is instead skewned, the shift between δ_0 and δ_1 is identified by $\beta\gamma tan\frac{\pi\alpha}{2}$, a value that tends to infinity for values of α closed to 1 (Nolan(2003)).

Analizing this characteristic function, we can see how for some values, the Stable distribution gains the property of other known laws and enables us to define a closed form.

For $\alpha = 2$ and $\beta = 0$ we can derive the closed form density function of a Normal $X{\sim}Normal(\mu\,,\sigma^2)$ as the usual Gaussian:

$$f_{(X)} = \frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

The relationship between the scale γ and shift δ measures and the traditional values of the mean π and the standard deviation σ are the following:

$$\gamma = \frac{\sigma}{\sqrt{2}}, \qquad \delta = \mu\,.$$

Though $X{\sim}Normal(\mu\,,\sigma^2) \;\triangleq X{\sim}S\left(2,0,\frac{\sigma}{\sqrt{2}},\mu\right)$

For values of the parameter $\alpha = 1$ and $\beta = 0$ we can derive the closed form density function of a Cauchy distribution, $X{\sim}Cauchy(\gamma_c,\delta_c)$:

$$f_{(X)} = \frac{1}{\pi}\frac{\gamma_c}{\gamma_c^2 + (x-\delta_c)^2},$$

where the remaining parameter γ and δ for the correspondent Stable variable are $\gamma = \gamma_c$ and $\delta = \delta_c$:

$$X{\sim}Cauchy(\gamma_c,\delta_c) \;\triangleq X{\sim}S(1,0,\gamma_c,\delta_c)$$

Eventually we obtain a third defined density function, the one for a Lévi distribution with values $\alpha = \frac{1}{2}$ and $\beta = \pm 1, X{\sim}L\grave{e}vi(\gamma_l,\delta_l)$:

$$f_{(X)} = \sqrt{\frac{\gamma_l}{2\pi}}\frac{1}{(x-\delta_l)^{\frac{3}{2}}}e^{\frac{-\gamma_l}{2(x-\delta_l)}},$$

where the remaining parameter for the correspondent Stable variable are $\gamma = \gamma_l$ and $\delta = \gamma_l + \delta_l$:

$$X \sim Lèvi(\gamma_l, \delta_l) \triangleq X \sim S\left(\frac{1}{2}, \pm 1, \gamma_l, \delta_l\right).$$

Properties

We have already introduced the defining property of Stable distributions. We consider here further implications of the stability property and some related probabilistic aspects that derives: power function for the decay of the tails and infinite divisibility property.

Stability effects

Given n number of $X_1, X_2, \dots, X_n$ independent random variables X_i, distributed with the same stability index α, as Stable $X_i \sim S(\alpha, \beta_i, \gamma_i, \delta_i)$, $i = 1, 2, \dots, n$, then:

$$X = \sum_{i=1}^{N} w_i X_i \,,$$

is Stable as well, where w_i are the constant terms that represent a certain weight for the linear combination.

While the parameter α of X remains constant by hypothesis, the others are instead identified with the following values:

- For $\alpha \neq 1$,	- For $\alpha = 1$,
$\beta = \frac{\sum_{i=1}^{n} sign(w_i)\, \beta_i (\lvert w_i \rvert \gamma_i)^\alpha}{\sum_{i=1}^{n} (\lvert w_i \rvert \gamma_i)^\alpha}$ $\gamma = \left(\sum_{i=1}^{n} (\lvert w_i \rvert \gamma_i)^\alpha\right)^{\frac{1}{\alpha}}$ $\delta = \sum_{i=1}^{n} w_i \delta_i + tan\frac{\pi\alpha}{2}\left(\beta\gamma - \sum_{i=1}^{n} \beta_i w_i \gamma_i\right)$	$\beta = \frac{\sum_{i=1}^{n} sign(w_i)\, \beta_i \lvert w_i \rvert \gamma_i}{\sum_{i=1}^{n} \lvert w_i \rvert \gamma_i}$ $\gamma = \sum_{i=1}^{n} \lvert w_i \rvert \gamma_i$ $\delta = \left(\sum_{i=1}^{n} w_i \delta_i\right) +$ $+\frac{2}{\pi}\left(\gamma\beta \log\gamma - \sum_{i=1}^{n} w_i \gamma_i \beta_i \log\lvert w_i \gamma_i \rvert\right)$

Starting from this broad definition, there are some particular cases that are useful to highlight. For instance, given a Stable $X_1 \sim S(\alpha, \beta_1, \gamma_1, \delta_1)$ and the relation $X = aX_1$, X will be a Stable $X \sim S(\alpha, \beta, \gamma, \delta)$ with the same α and characterized by:

$$\beta = sign(a)\beta_1 \,,$$
$$\gamma = |a|\gamma_1 \,,$$
$$\delta = \begin{cases} a\delta_1 & \alpha \neq 1 \\ a\delta_1 - \frac{2}{\pi} a\,(\log a)\gamma_1\beta_1 & \alpha = 1 \end{cases}.$$

If we substitute the special value $a = -1$,we derive $X = -X_1$, and coefficients will be:

$$\beta = -\beta_1, \quad \gamma = \gamma_1 \,, \quad \delta = \delta_1 \,:$$

inverting a distribution cause nothing but the reflection of all the points. Apart from the symmetry parameters there are no changes in the other indexes of the distribution.

Combining different effects coming from the properties above shown, we can define also for a Stable distribution a procedure of standardization that, in analogy with the Normal one, shift and reshape the values of the distribution to a reference one. In particular, at a Stable distributed like $X \sim S(\alpha, \beta, \gamma, \delta)$ we can assign the correspondent Standardized one $X' \sim S(\alpha, \beta, 1, 0)$, just computing for each value X_i the transformation:

$$X'_i = \frac{X_i - \delta}{\gamma}.$$

Tail behavior

The second property that we want to highlight regards that fact that the tails of a Stable distribution (apart from the boundary case for $\alpha = 2$), follow a power function in their decay. This property allows us to capture and analyze extreme events, and it is represented as:

$$P(|X| > x) \propto cx^{-\alpha}, x \to \infty,$$

where c is again a constant real term.

This characteristic aspect of the distribution have been exploited expecially for the identification of the α exponent. The Hill estimator[9] is one of the simplest estimator for the stability parameter, and even if it is a measure that is sensible to the number of observations, it provides a first indication about the stability index in a relative quick way[10].

There are also drawbacks of this tail behavior, related to the fact that variance is not finite. For this reason is not possible to adopt efficiently Stable distributions in some models. For example, developments of the model in option pricing, by Black and Scholes, using Stable distributions have not been satisfactory[11] .

As possible solution, smoothly truncated Stable distributions have been developed in the literature. The idea is to replace the heavy tails of the Stable distribution, above and below a defined value, with some thinner tail of a Normal distribution properly choosen. In this way we benefit from a better explanation of the central data behavior and at the same time we are able to describe a tail behavior that it is easier to replicate in the model.

For a Stable that have density function $g(x)$, the left tail is substituted, below the value a, with the density function of a Normal $h_1(x)$, while the right tail is approximated, above the value b by the density function $h_2(x)$ of a second Normal law.

[9] Hill(1975)
[10] Racz, Kertesz and Eisler(2009) for a straightforward introduction to the Hill estimator and the shifted alternative procedure for heavy tail distributions.
[11] See as brief review Bianchi, Rachev, Kim and Fabozzi(2010)

The defining parameters for the two Normal distributions are choosen to grant that the resulting density function for the truncated Stable is a continuous function. The point a and b are defined under the attempt to produce a resulting distribution with zero mean and variance equal to one.

A density function defined in this way grant the possibility to study extreme events, in a framework where moments are defined. Those mixed type of distribution are the result of the experimentation phase we were talking in the introduction, when also the Stable distribution started to be questioned. Nevertheless a mixed law so defined lose all the overall properties that makes Stable distribution attractive despite of the possibility to have undefined second and first moments.

Infinitely divisible distribution

A third property of α-Stable distributions is that they are infinitely divisible.

A random variable X can be defined as infinitely divisible if for any integer n, there is a series $X_1, X_2, \dots$, X_n of independent and identically distributed random variables X_i such that:

$$X \triangleq \sum_{i=1}^{n} X_i ,$$

where again we equate in distribution left and right side of the equation. As we know a Stable distributions $X \sim S(\alpha, \beta, \gamma, \delta)$ respect this condition considering a sum of n Stable $X_i \sim S(\alpha_i, \beta_i, \gamma_i, \delta_i)$.

If they share the same kind of distribution, it is true as well that the two characteristic functions $\phi(u)$ are the same and that there is a defined relationship between the parameter of X and X_i. In particular the values for α and β will be the same, while for the scale and shift parameter we will have:

$$\gamma = \frac{\gamma_i}{n} , \qquad \delta = \frac{\delta_i}{n} .$$

This property is generally used considering the application of stable distribution in jump processes like Poisson or Brownian motion[12].

Multivariate definition

A random vector $\boldsymbol{X}$ can be defined Stable in $\mathbb{R}^d$, a d-dimensional real space, if it follows one of the two alternative definitions of stability that we have introduced for univariate Stable distributions:

$$\boldsymbol{X}_1 + \boldsymbol{X}_2 + \boldsymbol{X}_3 + \cdots + \boldsymbol{X}_n \triangleq c_n \boldsymbol{X} + \boldsymbol{b}_n ,$$

$$A\boldsymbol{X}_1 + B\boldsymbol{X}_2 \triangleq C\boldsymbol{X} + \boldsymbol{D} ,$$

where we use bold characters to identify the matrix notation of vector, for example: $\boldsymbol{X} = (X_1, X_2, X_3, \dots, X_d)$. For $i = 1, 2, 3, \dots, n$, each $\boldsymbol{X}_i$ is an independent and identically distributed copy of $\boldsymbol{X}$ and again, the two definitions equate their left and right term in distribution with the operator $\triangleq$.

[12] See Rachev and Mittnik(2000) for details.

As the definitions are analogous, we can derive some other common features between univariate and multivariate. Regarding the first formula, the constant term c_n:

$$c_n = n^{\frac{1}{\alpha}} ,$$

again introduces the stability parameter α. Considering the second one, we have the correspondent definition of stability in strict sense for $\boldsymbol{D} = \mathbf{0}$, and again for such value of the vector of constants $\boldsymbol{D}$, we obtain the following relationship:

$$C = (A^{\alpha} + B^{\alpha})^{\frac{1}{\alpha}} ,$$

among the constant C , A and B, terms composing the linear combination of the $\boldsymbol{X}_i$ vectors.

We have seen that one of the major problems related to this class of Stable distributions is the lack of a closed form that can express the probability density function. Therefore, also in this multivariate case the characteristic function is the only way to identify and describe Stable random vectors.

Considering multivariates, the characteristic function will depend on α, on a shift parameter $\boldsymbol{\delta}^0$ and on a finite measure $\boldsymbol{\Gamma}$ that incorporates the effect of both β and γ parameters. To define in distribution a vector as multivariate Stable we will then use the following notation, given the stability index α and d, the dimensions of the real space:

$$\boldsymbol{X} \sim S\,(\alpha, \boldsymbol{\Gamma}, \boldsymbol{\delta}^0; k).$$

The new parameter introduced is named spectral measure $\boldsymbol{\Gamma}$, and represent the measure mentioned above, while k distinguishes again between the two alternative parameterization by Nolan(2003).

We proceed with the definition of the characteristic function; the probability measure defined for any real vector $\boldsymbol{u}$ as for multivariate Stable is identified as:

$$\phi(\boldsymbol{u}) = E\left[e^{i\langle \boldsymbol{u}, \boldsymbol{X}\rangle}\right] = e^{-\boldsymbol{I}_{\alpha} + i\langle \boldsymbol{u}, \boldsymbol{\delta}^0\rangle}$$

With the $\langle \boldsymbol{a}, \boldsymbol{b}\rangle$ notation we represent the linear combination of any 2 vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ of length n:

$$\langle \boldsymbol{a}, \boldsymbol{b}\rangle = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n .$$

$\boldsymbol{I}_{\alpha}$ is instead a function for every real $\boldsymbol{u}$ such that:

$$\boldsymbol{I}_{\alpha}(\boldsymbol{u}) = \int_{S^d} \psi_{\alpha}(\langle \boldsymbol{u}, \boldsymbol{s}\rangle)\, \boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s}) ,$$

where ψ_{α} is defined as:

$$\psi_\alpha(u) = \begin{cases} |u|^\alpha \left(1 - i\beta sign(u) tan\frac{\pi\alpha}{2}\right) & \text{for } \alpha \neq 1 \\ |u| \left(1 + i\beta \frac{\pi}{2} sign(u) \log|\mathrm{u}|\right) & \text{for } \alpha = 1 \end{cases}.$$

Particular attention deserves the spectral measure $\mathbf{\Gamma}$**:** a finite Borel measure on the unit sphere defined in $\mathbb{R}^d$. It includes all the information we need to identify the symmetry, shape and shift parameter of the multivariate distribution.

One way to determine the relationship between $\mathbf{\Gamma}$ and the parameters is to consider every one-dimensional projection multivariate random variable:

$$\boldsymbol{v} \cdot \boldsymbol{X} = \sum_i u_i X_i \,.$$

Each projection $\boldsymbol{v} \cdot \boldsymbol{X}$ will be Stable distributed $\boldsymbol{v} \cdot \boldsymbol{X} \sim S\,(\alpha, \beta(\boldsymbol{v}), \gamma(\boldsymbol{v}), \delta(\boldsymbol{v})\,; k)$ with a common α index and with values of $\gamma(\boldsymbol{u})$, $\beta(\boldsymbol{u})$ and $\delta(\boldsymbol{u})$ determined in such ways:

$$\gamma(\boldsymbol{v}) = \left(\int_{S^d} |\langle \boldsymbol{v}, \boldsymbol{s}\rangle|^\alpha \mathbf{\Gamma}(\mathrm{d}\mathbf{s})\right)^{\frac{1}{\alpha}},$$

$$\beta(\boldsymbol{v}) = \frac{\int_{S^d} |\langle \boldsymbol{v}, \boldsymbol{s}\rangle|^\alpha \mathrm{sign}\langle \boldsymbol{v}, \boldsymbol{s}\rangle \mathbf{\Gamma}(\mathrm{d}\mathbf{s})}{\int_{S^d} |\langle \boldsymbol{v}, \boldsymbol{s}\rangle|^\alpha \mathbf{\Gamma}(\mathrm{d}\mathbf{s})},$$

- For $\alpha \neq 1$,	- For $\alpha = 1$,
$\delta(\boldsymbol{v}) = \\ = \begin{cases} \langle \boldsymbol{\delta}^0 \boldsymbol{v}\rangle & \text{for } k = 1 \\ \langle \boldsymbol{\delta}^0 \boldsymbol{v}\rangle + \left(tan\frac{\pi\alpha}{2}\right) \beta(\boldsymbol{v})\gamma(\boldsymbol{v}) & \text{for } k = 0 \end{cases}$	$\delta(\boldsymbol{v}) = \\ = \begin{cases} \langle \boldsymbol{\delta}^0 \boldsymbol{v}\rangle - \frac{2}{\pi}\int_{S^d} \langle \boldsymbol{v}, \boldsymbol{s}\rangle \log|\langle \boldsymbol{v}, \boldsymbol{s}\rangle| \mathbf{\Gamma}(\mathrm{d}\mathbf{s}) & \text{for } k = 1 \\ \langle \boldsymbol{\delta}^0 \boldsymbol{v}\rangle - \frac{2}{\pi}\int_{S^d} \langle \boldsymbol{v}, \boldsymbol{s}\rangle \log\langle \boldsymbol{v}, \boldsymbol{s}\rangle \mathbf{\Gamma}(\mathrm{d}\mathbf{s}) + & \\ \quad + \frac{2}{\pi}\beta(\boldsymbol{v})\gamma(\boldsymbol{v}) \log\gamma(\boldsymbol{v}) & \text{for } k = 0 \end{cases}$

for $S^d = \{\boldsymbol{v} \in \mathbb{R}^d \colon |\boldsymbol{v}| = 1\}$ unit sphere in $\mathbb{R}^d$.

The spectral measure express also the dependence structure between the individual random variables that composes the vector $\boldsymbol{X}$ and the others. This is true apart from the cases in which $\mathbf{\Gamma}$ is discrete and is concentrated on the interception of the unit sphere with the coordinate axes. In that particular cases the components of $\boldsymbol{X}$ are independent[13].

[13] For details, Kuzubowsy, Panorska and Rachev(2003)

Generalized CLT

Considering Stable distribution and multivariate Stable ones, there is one last crucial property that we haven't yet mentioned and that is fundamental for understanding their wide adoption. We introduce it starting from a well known theorem and a basic definition.

The Central Limit Theorem (CLT) assess that a normalized sum of independent and identically distributed random variables, with finite variance converges to a Normal distribution $X \sim Normal(\mu\,, \sigma^2)$ as the number of variables summed increases.

We can notice that one of the assumption is systematically broken for any Stable distribution that has the α parameter different from the value 2, though an infinite variance. For all this values the CLT cannot be applied.

If we want to relax that assumption, The Generalized Central Limit Theorem proves that the only possible resulting limit for a normalized sum of random variables is to a Stable distribution.

We then define the sum of n random variables X_i in the domain of attraction of a certain distribution Y if for a certain positive a_n and $\boldsymbol{b}_n$ vector on the real d-dimensional plane the following convergence holds:

$$a_n(X_1 + X_2 + X_3 + \cdots + X_n) + \boldsymbol{b}_n = Y$$

Therefore the only possible non degenerate distributions with a domain of attraction, for random variables with infinite variance, are Stable.

This result is widely used in the building procedures of financial portfolios, since it gives a strong theoretical justification to the adoption of Stable distribution.

Chapter 2: Risk measures

We have defined so far how should be distributed our set of random variables collected from the financial market. It is time now to consider a different aspect related to financial data, depict a more general framework and to find a way to compare and assess risk profiles.

This is a wide field to consider, as in economic theory the problem of risk management can be proposed in many different perspective and analized with many different models. We would like to start with a general definition of the risk defined as financial, and then develop our arguments as an insight on one of its component: the market risk.

We will propose as well the measures that better represent it and that help to define models that aim to hedge it. Concerning our study, those measures (VaR and expected shortfall) will be used to compare the differences in risk evaluation, derived from the implementation of various estimation procedures for a Stable distributions.

Risk assessment: a classification

Corporation, investors and financial institution have the chance to use a strategic tool to gather, borrow, lend capital or assets: the financial market. This opportunity comes at a cost: operators face day by day risks deriving from their exposure and from their investment decisions.

This idea is so clear and general that it makes challenging to define those risks in a proper way. As the financial markets developed, with the introduction of new securities and new financial objects, dangers and hazards have become incredibly unpredictable. What is worse is that most of the time there exist connections that are hard to spot, between events all around the world and trading decisions. This effect is due both to the enlargement of the number of operators that is now able to access to the financial market and to the evolution of technological devices that makes it simple to trade, invest and move capital with just some clicks.

One common classification that have been developed in attempt to aggregate some source of risk is the distinction between credit risks, liquidity risks, operational risks and market risks[14].

Credit risks

Credit risks are related to all the events in which the counterpart is not able to respect and fulfill the contractual obligation or it is not intended to do so. In those situations, most of the time is difficult to have the transaction completed, expecially if the counterpart operates in another country or is a fictitious entity, so that it may happened, even after years of litigation, to have the capital invested repaid just by crumbs.

Under this classification fall also all the losses connected to a downgrade of the obligation reliability coming by credit agencies, an event that usually determine a drop in the market value of the asset. The Italian dynamics of government bonds in last two years helps to explain this source of risk: as situation gets instable

[14] Details in Jorion(2007)

or unpredictable, also assets that are historically considered as prime, or high grade can be labeled with ratings that express the enhanced likelihood of a default possibility.

Credit risk is though usually specific, depending on a singular actor, but may for some extent be considered also generic if we include in it also the so-called sovereign risk. The latter describes situations in which the impossibility for the counterpart to respect the obligation is due to the intromission in the contractual activities of a superior institution, like a country or a financial regulator. For example, in an autarchic regime it may happened that some capital restrictions are imposed on money flows to foreign country. So even if the counterpart is willing and ready to complete the obligation terms, it has not the possibility to proceed in the transaction.

To cope with this type of risk, can be implemented different measures of control for credit exposure, like for instance it is possible to use insurance strategies or to hedge the risk with derivates. Moreover, since the trading system pricing is now risk-based, it is easier to recombine the portfolio components to enhance credit quality or to create some diversification among them to balance the risk composition.

Recently, according with the growing regulation that is emerging to guarantee a better functioning market, institutions have also started to implement risk evaluation techniques related to the ones applied for facing market risks that as we will see have probabilistic and mathematical foundation.

Liquidity risks

The second class, liquidity risks, may take as well two forms: assets liquidity risk or funding liquidity risk. The first one occurs when it is not possible to complete a transaction at the current market price: the asset cannot be traded in time to avoid losses or to effectively realize the profit desired. This may be related to low market activity, so that there are few chances to meet a counterpart interested also in a long trading period, or to those cases in which the asset is not interesting enough and is not able to generate a proper demand and so it is ignored by the operators.

Sometimes it may also happen that a small market is indeed structured in such a way that a significant transaction can quickly affect the price of an asset. This imply that even if the transaction takes place the structure of benefits deriving from the trade is altered.

Funding liquidity risk instead refers to situations where the cash-flow needs are not correctly planned, and the investor is forced to early liquidate a position, without the chance to maintain the loss as a “paper” one and wait for the price to recover.

This is common expecially in platform trading complex financial assets as derivates where, for the stability of the system, it is required to settle regularly the personal position and balance losses as they are realized. If there is not capital enough to cover sub sequential bad realizations, it may happened to be push out from the market, without the chance to wait the situation to get better. In this way the investor is forced to close the position at an uneconomic price loosing relevant parts of the capital invested.

To deal with those possible liquidity troubles, some analytical tool adapted again from the market risk measures are available, as well as some scenario-analysis planning or, again, some technique of diversification regarding this time liquidity providers. It is necessary to determine in advance what could be the dynamic of cash flows and to check periodically the adequacy of the capital allocated.

Operational risks

Operational risks, the third class of financial risk listed, are related to losses from internal sources like errors (made by a person, coming from a machine default or as consequences of a computational error) or from external happenings such as fraud events or failure of controls.

In the first case, we are not talking just about mistakes or about the generation incorrect solution or values as output of an analysis procedure; those are problem indeed questionable and that can be avoided with expertise and practice, but are not the only source of operational internal risk.

We consider internal errors also those cases in which the result is correct and properly derived from a structured model apart from the fact that the latter has been wrongly specified. It might be the case, for instance, that in modeling risk structure or patterns for market variables, the analysts make some theoretical errors or are lead to assume propositions that are not matched by the real world or empirically confirmed.

A simple assumption like the Normal behavior of returns has been accepted and implemented correctly in models for years. The results of those models were however not generally reliable and generated though operational risks, as in all the situations where losses probability have been under-estimated.

Considering external sources for operational risk, the problem is related to the reliability of information. Prospects or news may be falsified on purpose to generate specific reactions from the market. Illegally, there are different way in which it is possible to take advantage of the weakness of operators that may lack of experience or that may be too small or not enough organized to respond.

Questionable behaviors are monitored and penalized, but most of the times the punishment effect comes with a timing that do not help the damaged operator. Anyhow we can include also this type of legal risks in this class considering then, penalties, fines or other measures of punishment coming from a supervisor as consequence of illicit actions.

Market risks

The last class composing financial risk is the market one. We will focus in particular on this last category and for it we will we expose the most adopted measures of risk. Market risk considers one of the simplest indicator in the market, price, and its dynamics.

This class include all the losses or undesired effects coming from the volatile behavior and instability in the returns composition. This volatile behavior may came both from changes due to market activity on the assets as well as more general macroeconomic factors that may be in action and affect the overall performance of the market.

We can distinguish thought between specific and general risk. The former refers to the risks associated to changes in an asset price derived just by information from or about the assets' issuer. In this way it is related only to the obligation considered and cannot affect but relatively other financial instruments that are not connected to it.

General market risk derives instead from changes in overall levels of different equities or commodity prices or interest and exchange rates. It accounts also for all the correlated pattern or dependences that can be identified with economics macro variables as indexes of production, employment, consumption.

This component of risk recently suffers from wider and wider influences, as we consider also political or social macroevents that have the power through moral or practical and behavioral arguments to modify balanced patterns in the financial market.

Market risk is though the term more influent in the composition of the aggregated risk related to a certain position and then it was the first that started to be investigated and regulated along with the credit one. Before talking about the institutional intervention and before describing the international agreements that followed to harmonize different perspective and habits, we prefer to report how the issue of developing a coherent risk measure was faced by economic literature and models.

Market risk from portfolio management to Basel(s)

For an optimal portfolio management it is crucial to identify a combination of assets that grants the lower exposition to risk and maximize the profits. Starting point is a proper investigation about the characteristics of each component of the portfolio in terms of profitability, in terms of distribution of returns and in terms of structure of dependence among the assets.

Given those information, just if we define a way to measure the risk, different portfolios can be ordered and it can be identified a set of them that is most preferred. While is obvious that holding a more profitable portfolio will mean to face an higher risk compared to a position that grants lower returns, it has to be defined also a way to compare portfolios with same returns in order to be able to select the convenient ones. Each portfolio has then so be analyzed in terms of both expected value and a certain measure of risk.

The first entity is usually well defined and easily comparable. Some problems might arise in distribution like the Stable where for the α parameter assuming values below one the first moment is not defined and so the information that comes from the computation of the sample mean is not reliable.

Nevertheless there are different studies that provide evidences reinforcing the ideas that financial variable modeled with stable parameter less than one a case rarely encountered in empirical situations. Some cases can be found considering particular kind of securities that follow a general tendency of micro movements, interrupted occasionally by some high jump, that changes dramatically the reference level. Processes like those will have indeed extreme fat tails, and leptokurtic characteristic associated to a low stability parameter.

A part from those cases, Kim 2002 suggest an interval for financial returns like $1.5 < \alpha < 2$. Rachev and Mittnik 2000 explaining estimation methodologies assess again that generally financial returns are identified by stability parameter: $1.6 < \alpha < 1.9$.

We also can contribute and support those empirical results considering the dataset we used, and the most efficient estimator adopted, the Maximum Likelihood implemented by the software Mathematica®. The 188 assets of the NYSE market that we studied are described by values for α embedded in a set that span from a minimum of 1.451041412 to the maximum 1.890417318, confirming again that in all such cases the first moment is finite. Even adopting less accurate estimators, the minimum values for the stability never assumed values below 1.3.

If we can agree to use expected value to assess profitability, the second measure, a quantization of the risk connected to the portfolio, has proved to be the harder one to identify, since there is no analytical and general way to define dangers connected to a position. Three approaches are usually pursued for risk measures assessment: identify indicators as sensitivity measures, as volatility measures or as downside measures.

By sensitivity we mean the ratio of variation of a market variable of interest to a forfeit sudden change of the underlying random parameters that causes the change. Regarding the price of an asset it represents the change in value derived from a modification of an exogenous factor. Sensitivity presents though a measure of local (not necessarily specific, on the contrary mostly general) risk, since it identifies a change related to a just a small modification of the environment and that is relative to the current price position.

The technique of risk managing related to this type of measure is to limit the exposure to external random factors, by selecting position with low sensitivity. As we can infer sensibility is not so easy to use as aggregate index, for instance considering a portfolio, because it is relative to just singular observation in time.

In order to have a risk indicator that is not just local, a volatility measure is used to characterize both sensitivity and instability of an asset. It represent the dispersion around the average mean of the variable observations, and it is statistically identified as standard deviation. It is calculated as the root of a probability weighted sum of the squared deviations from the mean, as the formula presented in Chapter 1 expose. Volatility measures depends on the sample size or as a consequence on the time span in which the variation are considered; daily, weekly or monthly.

Markowitz with his works during the 1950s is considered the father of portfolio theory and set a framework that it is still the starting point for many considerations about the behavior of assets and the way to correctly manage them.

The first statistic he used as risk measure was indeed the variance of the portfolio, identified by the weighted sum of the Variance-Covariance matrix of the assets composing the portfolio. The singular components of profitability and risk of each assets were linearly aggregated thanks to the assumption of normal distribution of asset returns.

Shortcomings of this approach were quickly identified. First of all, concerning most of the actors in financial markets, risk materializes just when earnings deviate in an adverse way, while volatility measures captures both downside and upside changes. There is no point to account for variation that led to positive results, in a measure or risk.

The author himself (Markowitz (1959)) tried to develop a possible alternative to the variance index, analizing the semi-variance measure. Semi-variance statistics is a modification of the variance that simply considers the dispersion of just one side, above or below the mean.

Still there were some side problems. In the original model, indeed, the risk measure was strictly connected to the distribution law of the return of the assets, and correct result were obtained just in the case in which a particular class of distribution, Elliptic, were analized.

Elliptical distribution can be briefly defined as probability laws that are invariant under rotation. The properties related to such behavior grants that most of the risk measures we will see in this section are coherently applicable, also the simpler one as variance and semi-variance. Moreover, for elliptical distributions the way in which dependence is measured can be simply related to the correlation statistic[15].

Since Elliptical is not always the case encountered, for all the other, generic, distributions there was the need to identify new way to account for risk. It was necessary to establish a measure that could represent a downside risk exposure, and that could account both for potential losses and probability of occurrence in a single value.

Not just to find a more specific solution in portfolio theory, but also in response to a widespread need of regulation from the market, the VaR (Value at Risk) measure was defined in early 1990s. The intuition was to exploit the attempt to define the economic capital, the amount that is determined and set aside by each actor, in order to lower at minimum the probability of default, or bankruptcy. The term “economic” strengths the idea that this capital, this quantity has to be a real value, comparable in monetary terms and that can account in this way for different sources of risk.

In principle every investor or institution should be free to set its own economic risk capital, and the frictionless ideal capital market would select and maintain operative good actors while bad ones are condemned to disappear. In practice, financial markets are far from perfect and in particular there is a strong series of interrelated externalities that might make a specific risk systemic, or general for everyone in the market.

So most of the times, if the market dooms a certain institutions to fail, without any intervention by regulator or supervisor, this is likely to trigger some complex chain reaction and compromise also different other positions. If then the institution or the actor are relatively big in the market considered, a failure may cause a

[15] Results obtained by Embrechts, McNeil and Straumann(2001), for more details on Elliptical and Spherical distribution, see Fang, Kotz and Ng(1990).

shocks that vanishes much more than just their economic capital, contaminating activities related and damaging preexisting general environment dynamics.

For this reason the choice of the amount of money to maintain as economic capital need to be assisted and commonly shared.

Milestone was the Basel Capital Accord in 1988 of G10 countries, centered on credit risks, that for the first time tried to define the capital requirements that needed to be met by commercial banks. The Basle Committee of Central-Bank Gubernators existed since 1974 and, whit no binding or legal force, acted just as consultant body for monetary and financial matters.

Before this agreement there was just a simple system of deposit insurances, granted by central banks or by governments, to prevent the risk of dramatic defaults. However that a solution was opened to moral hazard behaviors, since in that way risk hedging was basically sponsored by an external institution. The system needed though intensive control on risk taking activities and was excessively cumbersome[16].

The Accord aim was to strengthen the stability of the international banking system and to harmonize the procedures. For the economic capital to be used to cover credit risks it was initially fixed a minimum reference, corresponding to 8% of the total risk weighted assets with an additional discretional value spanning from 1 to 4%. Those new dispositions were immediately applied by the ten participating countries and sooner were shared and accepted by many others: it was the beginning of a new wave of international guidelines and regulations.

At that moment was defined also the composition of such capital at risk. It was distinguished between a core part, representing mainly the difference in the liquidation value of assets and liabilities, and a supplementary capital, adjusted to investments of low quality to account for riskier activities in specific. This second capital definition were bounded to a maximum of half of the total exposure of the bank.

In addition to those recommendations, a strict control was imposed over large risk investment, defined as positions that exceeded 10% of the institution capital to foreseen dangerous positions.

A step forward seemed to be the definition of the Value at Risk, or VaR, in 1993, a measure that quickly became widespread in use, thanks to its immediateness and quick application. In order it was considered by the G30 in July 1993, that reported about derivatives and the VaR approach to risk, by the EU in the same year in the Capital Adequacy directive "EEC 6-93" and by the Bank of International Settlements in the Fisher report.

In 1996 the Basel Committee amended the Accord of 1988 suggesting new guidelines to assess capital needs from credit and market activities. The new qualification suggested a standardized model shared among

[16] Jorion(2007) for detail in the dynamics before Basel

all the institutions and an internal one specifically customized to assess the amount of capital reserves to hold in order to face market risks.

The standardized model was based on general principles and was defined for the integration of the overall market risks, computed then with respect to exchange rate indexes, equity global volatilities, commodity fluctuation and interest rate related risks.

The internal one was based on the possessed investment positions and computed as the VaR measure at 99% confidence interval, considering ten trading days and adjusted by a multiplicative factor between three and four, severe in proportion to the quantity of risky assets owned.

In the years that follows, most of the financial institution adopted VaR measure as first evaluation of risk and numerous software were developed to speed up further the diffusion of the risk estimator.

In 2004 the Basel Accord was completely revised and extended to better cover market, credit and operational risks, structuring three pillars of regulatory action: minimal capital requirements, supervisory review, market discipline.

The first pillar regards the definition of the economic capital or minimum capital in line with the operator loss potential. This capital is computed referring to both the systematic or general market risk and the specific one. It considers then both VaR as the current measure in time t, VaR_t, and the average VaR over the previous 60 trading days, $\left(\frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}\right)$ accounting for a measure that reflects past risky behaviors. The market risk capital requirement in time t, C_t is then equal to:

$$C_t = A_t \max\left\{VaR_t\,, \frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}\right\} + S_t\,,$$

where A_t represents an adjustment factor that takes again value between three and four depending on how many times the VaR threshold has been actually exceeded in the last trading year. S_t is instead an additional capital charge accounting for specific risk.

The second pillar describes the set of constrains and controls over managerial position, directors, agency delegations or internal and external audit in general. Institutions cannot lose control over the activities of the most influent financial actors and then have to assess continuously that there is an adequate capital set aside to cover the overall risk position.

Last pillar was defined in the attempt to diminish systemic risk sources, setting new guidelines to make the market discipline more clear and transparent. Information processes were particularly revisited and so the standards for reporting risk profiles and profit chances.

After this first phase of enthusiasm and overconfidence upon this new settings, some criticisms started to arise. It was pointed out that if all the operators adopt the same risk evaluation technique there might come as consequence that the same behavior and standardized reaction are considered without differentiation. For instance, if in a critical situation all banks takes similar exit strategies it may happened that the situation became actually worse, slackening the recovery process. Those arguments were supported by analysis on pro-cyclical effects of those type of regulation, as capital requirements influence the availability of liquidity in the markets.

What is more, in a similar framework where risk is apparently well edged, operators may be pushed to behave and abuse of confidence, assuming position riskier than the one that they would normally take. This may be fatal expecially if the model specification in incorrect, like in a parametric approach where the VaR is calculated as percentile in a standardized normal distribution.[17]

We have then to consider also that the real implementation of all the Basle II procedures took longer that what was expected, expecially in the United States, where some of the amendments at the first Accord were planned to start being only effective in 2009. This time gap permitted to some institution to exploit the period of regulatory transition adopting as long as possible just the most favorable capital constraint.

Eventually in 2008 came the disaster. From a limited sector of the American economy, the sub-prime market, that accounted at that time for 13% of the real estate American market, the financial crisis sooner affected other structured tools in the financial market ending to contaminate also real economy dynamics and stability of the markets all over the world.

We do not aim to specifically describe the causes and factor behind the crisis nor we are interested in discussing here macroeconomics sources of imbalances such as government deficits, FDI movements, low interest rate patterns and leveraging and deleveraging procedures. We suggest for an overlook over the topic and expecially for what concern federal reserve in the United States policies other readings.[18]

We want to report just some of the microeconomics causes that are indeed connected to risk evaluation and to the second Basle agreement. The lack of a clear procedure to deal with events on the tail of returns distribution, made risk edging substantially centered on just smaller risky events and economic capital insufficient to cover extreme losses. Moreover, such a standardized risk evaluation methodology makes the dynamic of different markets much more correlated, creating parallel path of evolution. For this reason every theoretical advantage in differentiation was vanished when, in the worst moment of the crisis, globalization made all the prices fell, against any prediction from economic models.

The reaction for what regards regulations and regulators was a third Basle Agreement in 2010, basically focused on recovering and maintaining the overall stability of the system. The main approach was to revise

[17] Details further in the chapter.
[18] Among the others, we recommend Bernanke(2013)

and enhance most of the capitals requirements. The aim was to enable expecially banks and central institution to absorb shocks deriving from the financial market, independently from their origins, to prevent a subsequent collapse of other sectors, or other diseases in aspects of the real economy.

A robust bank system is indeed fundamental to grant a sustainable economic growth, because of the mediator role that it play connecting investors and capital lenders. Moreover in order to avoid pro-cyclical mechanisms, the financial institution have to maintain a certain buffer level that can grant that even in period of crisis, services and assistance.

Basle III confirmed the three main pillars described in the second agreements, harshening quality and quantity of capital requirements, with the introduction of some new index, as the leverage ratio.

The new classification for the economic capital distinguishes now between Tier 1 and Tier 2, entities comparable to the core and supplementary capitals, with the difference that it is now determinate a buffer level that have to be kept and maintained constantly. The buffer management needs moreover to be done with internal availabilities like, for example, diminishing the dividends distribution in proportion with the seriousness of the weight of capital to rebuilt.

Also the risk assessment procedure has slightly changed, with the introduction of the new stressed VaR index, a value that is obtained considering a year of prolonged and acute financial bad occurrences, and with the introduction of backtesting procedure.

Would this new regulation actually prevent other major shocks in the market? Is the new risk assessment procedure adequate and effective? Next few years will tell us in what way Basle III had changed and strengthen our financial world. Conscious that those such big and complex issues cannot find more space in a simple analysis such this one, we turn back to more simple question. Chapter 2 continues with the analytical description of the VaR index and its characteristics. The attention will then shift to the introduction of alternatives, and the expected shortfall index will be introduced.

VaR: the Value at Risk

It is time to describe better this renowned VaR measure, and to understand how it is implemented and what are its limits. We start with some technical remarks.

From now on we will refer as X like a random variable representing negative profits (also called loss distribution). We define $F_X(x)$ the corresponding cumulative distribution function determined as $F_X(x) = P(X \leq x)$. The aim is to identify the risk of holding a certain assets or a specific portfolio over a defined period τ considering a predetermined confidence interval k.

One type of indicator that might be considered for this evaluation is the maximum potential loss, the one that account for $F_X(x)$ being equal to one or alternatively $\inf\{x \in \mathbb{R}: F_X(x) = 1\}$. This value accounts for the worst situation ever that might happen, the loss with higher amptitude. The support of the distribution function of a variable like that is nevertheless unlikely to be bounded so the maximum value of loss is usually infinite, for theoretic distributions, making this indicator useless.

The idea is then to widen the extreme risk evaluation to a confidence interval and to evaluate the maximum loss that is not exceeded for a determined high level of probability. Given a $k\%$ confidence level, for $k \in (0,1)$:

$$VaR_k = \inf\{x \in \mathbb{R}: P(X > x) \leq 1 - k\} = \inf\{x \in \mathbb{R}: F_X(x) \geq k\}.$$

The VaR, at a k level is then given by the smallest loss x for which the probability that the loss distribution X exceeds x is smaller than $(1 - k)$. As alternative interpretation, if we consider portfolio theory, a $k\%$ VaR, or VaR_k, is the lower limit on the proportion of a portfolio that can be lost $k\%$ of the time.[19]

Identification techniques

The literature generally identifies three approaches for the determination of the VaR index: a parametric method, the non-parametric historical method and a stochastic simulation like the Monte Carlo simulation technique.

The parametric approach is the most direct one: assuming that we have fitted the sample data with some available technique, we can exploit the knowledge obtained about the distribution function. We can in fact derive from the identified parameters the probability density function, plot its values and make statistical inference. We have then just to check for the quantile interested, $k\%$, and record the value that it is associated.

For example we can directly obtain the VaR from random variable from its normal loss distribution $X \sim N(\mu, \sigma)$ as:

$$VaR_k = \mu + \sigma\Phi^{-1}(k),$$

where $\Phi^{-1}(k)$ is the inverse of the standard normal distribution function Φ, plotted in k.[20]

With the same procedure it is possible to determine the VaR of any location-scale distribution, like for a t-Student. For different, more elaborated distribution, as the Stable ones, those passages are relatively more complex since there is not a probability density function to exploit and the computations have be implemented using cumulants, as we will see in Chapter 4.

Without imposing a priori an assumption about the sample distribution, the first of the two non parametric approaches exploits historical records of the variable. Assuming that past dynamics are likely to be reproduced also in the future, the data are ordered and considered in their relative frequencies terms. In this

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[19] Frain(2008)

[20] Other detail regarding inverse distribution function of quantile function for the definition of VaR in McNeil, Frey and Embrechts(2005)

way it is possible to derive the empirical correspondent to the density function, an object in which we can identify percentiles and evaluate confidence levels.

More time consuming and structured is the generation with an appropriate algorithm of several simulations, in order to complete the description of a framework where all the possible outcomes that can derive from a supposed distribution law are represented. This is the technique that the Monte Carlo simulation method exploits, adopting a step procedure. First there is the need to define the domain of the possible input parameters and data. Then randomly inputs are derived from this fixed domain, according with a certain probability distribution. Evolutionary trajectory are simulated for different prices and the repeated generation grant the composition of a distribution. In the final part the results are aggregated and the required analysis (quantile computation) is computed.

As we can notice those methodologies are not excessively complex, making the VaR calculation easier to derive even with simple application and packages. There is no need of particular programming of deep mathematical knowledge. Several programs can be downloaded freely that contains more or less structured and precise application for Excel, R, Stata, Matlab.

Critiques

Although widespread and popular, the VaR measure became soon object to some critiques. The major one is that, because of the way in which it is defined, VaR at a k level does not provide any information about the amplitude of losses occurring with probability less than $(1-k)$. This means that two different distributions may have the same value VaR_k, then may be treated in exactly in the same way even if they show a completely different behavior in the tail, and different values characterizing the bigger losses.

Moreover, as it is defined, it relies on the choice of two parameters, a horizon of time τ and a confidence level k. While we might agree for a standard value for the confidence level, the choice of the horizon of time is more delicate since it should reflect the period in which a financial institution or an investor maintain a certain position in the market. This amount of time is related to fiscal, legal and liquidity constraints and so will widely vary across the market, making the comparison between value of risk biased.

This problem gets more complicated if we consider instead of just a single security a portfolio of assets. Trading position necessarily changes and are constantly monitored and optimized, though a VaR measure extrapolated for a certain time horizon or a different range of time might not represent the real risk exposition.

But most recurrent is a theoretically funded critique that denounces the lack of the property of subadditivity for VaR, as a risk measure.

This argumentation followed the intense literature disquisition upon the definition of coherent risk measure. The discussion aimed at the identification of the property that such a measure should respect in order to be reliable, comparable and unbiased. We report the commonly accepted framework and describe the properties that describe such coherent measures.

Coherent risk measure

First of all, given a set of random variable X_i, we define $\rho(\,)$ as a function that assigns a real number to each X_i, depending just on its probability distribution. We want the function $\rho(\,)$ to be also a scalar measure. For this reason it has to respects the basic assumptions about norms, the mathematical function: separated points, symmetry and triangle inequality.

The first one simply states that the measure is null, $\rho(X) = 0$, for each case in which we consider an X object that is equal to zero. For all the other values the measure has to be strictly positive: $\rho(X) > 0$. In our case it is straightforward to derive that if our portfolio is completely empty, it is impossible to evaluate some kind of risk connected to it. This assumption about norm is considered by Artzner, Delbaen, Eber and Heat(1999) as "Relevance" axiom.

The second one, symmetry, states that $\rho(X)$ is invariant under all the permutation of the element of X.

$$\rho(X_1, X_2) = \rho(X_2, X_1)\,.$$

It means that the risk associated to our portfolio does not change if we simply switch the order in which the assets are considered, what matters are the measurable characteristics of each single element.

The last, triangle inequality, is also one of the property that have been identified as desirable for a coherent risk measure, and that it is also referred as subadditivity. The norm of a sum of two elements must be at most large as the sum of the two separate norms of the objects. In our case, an example could be that opening a position on more than one asset have to lead to an amount of risk lower or at least equal to the sum of the risk associated to the separate assets:

$$\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)\,.$$

If this is the case, we have a proper incentive to differentiate our capital among different investments, with the benefit that our exposure to risk is going to diminish.

In addition, Szegö(2005), recalling the work by Artzner, Delbaen, Eber and Heat(1999), recommends as desirable properties that a coherent risk measure should express:

- Positive homogeneity: a risk measure $\rho\colon X \to \mathbb{R}$ is such that the risk scales consequently to the size of the position opened:

  $$\rho(\theta X) = \theta\rho(X)\,.$$

  If we double the amount of money invested in an asset, $\theta = 2$, the risk associated to it will double consequently. We can further exploit this definition, considering a set of n assets and the implication of subadditivity axiom, so in the relation:

$$\rho(nX_i) = \rho(X_i + X_i + \cdots + X_i) \leq n\rho(X_i) ,$$

it holds just the equality relationship, since there is no possible diversification.

- Transitional invariance: if we add in the portfolio an asset with sure return R, we lower the risk of losses in the portfolio by a constant amount:

$$\rho(X_1 + R) = \rho(X_1) - a .$$

If the return R equates the constant amount a, this property ensures also that the risk measure adopts as units the same of the one used for describing net worth.

- Monotonicity: if the future net loss $X_1$is greater than X_2, the first asset is related to a higher risk:

$$\text{if} \qquad X_1 \geq X_2 \qquad \text{then} \qquad \rho(X_1) \geq \rho(X_2) .$$

If a measure matches both monotonicity and transitional invariance, it can be defined as a monetary risk measure.

Even if satisfying transition invariance, positive homogeneity and monotonicity, VaR fails in general to respect subadditivity (considering also non-Elliptical distribution). Therefore seems not to be able to aggregate risk in a logical manner, preventing diversification in some circumstances. For this reason, authors refers often to VaR as weakly coherent.

Lacking of subadditivity, VaR measure is also not always convex, so it could happen:

$$\rho(\lambda X_1 + (1-\lambda)X_2) > \lambda\rho(X_1) + (1-\lambda)\rho(X_2).$$

This behavior cause ambiguities each time we consider optimization problems, since the existence of a unique solution cannot be granted[21].

This theoretical argument is necessarily to take in consideration, but if we considers which are the cases when VaR is effectively non coherent, we can admit that most of the time this drawback is not so severe as it seems.

Considering Stable distribution, for example, it has been proved that VaR is not subadditive just for a specific set of values of the α parameter: $0 < \alpha < 1$. We know that for those values the Stable distribution lack of a determined first moment and for this reason the VaR measure is no more convex and though coherent. But as we have already mentioned considering financial returns minimum values for the stability parameter are generally not so low.

Those recent arguments gave back some justification to the spread application of VaR, but still the improper description of extrema events calls for the adoption of some other risk measures, such as expected shortfall.

[21] Szegö(2005), depicts the problem plotting an efficient frontier in a Mean-VaR model with multiple maxima

Expected shortfall (ES)

Evaluating alternatives, different authors focused on how to integrate or develop the VaR measure. Under the broad definition of Conditional VaR or CVaR, we can collect measures with minor technical modifications, but that are inspired by the same idea and that belongs to the set of coherent risk measures.

For a continuous random variable representing negative profits, Rockafellar and Uryasev(2002), define the CVaR at a certain confidence level k as the expected value of the losses greater or equal to the VaR computed at the same confidence level:

$$CVaR_k = \mathbb{E}(X|X > VaR_k(X)) ,$$

we remind the use of X like loss distribution.

In other words, CVaR just exploits the VaR measure as a reference and then evaluates the composition of the tail after the k-th quantile, using the expected value statistic.

If the loss function shows discontinuities, the definition slightly changes, allowing for different notation, such as CVaR$^+$ CVaR$^-$, upper or lower CVaR. The work by Rockafellar and Uryasev(2002) provides a deep explanation, with also a comparison to other equivalent definition. It is also interesting to notice how CVaR remains coherent even if it can be defines as linear combination between two different non-coherent measures: VaR and CVaR$^-$.

Regarding our analysis, it is sufficient to understand the definition of Expected Shortfall, the CVaR risk measure we will use in our computations. We report and comment the definition by Acerbi and Taske(2002):

$$ES^k(X) = -\frac{1}{1-k}\left(\mathbb{E}\left[X\mathbf{1}_{\{X\geq x^{(k)}\}}\right] - x^{(k)}\left(P\left[X \geq x^{(k)}\right] - (1-k)\right)\right),$$

Where $x^{(k)}$ represent the quantile at k value and $\mathbf{1}$ is a dichotomous variable that returns one if the relationship to be checked (the one in graph parenthesis) is true and zero if the relationship is false, vanishing the first term:

$$\mathbf{1} = \begin{cases} 1 & for\ X \geq x^{(k)} \\ 0 & for\ X < x^{(k)} \end{cases}.$$

The $ES^k(X)$ formula integrates both the case of a continuous and discontinuous loss function given a single and complete definition.

The term $x^{(k)}\left(P\left[X \geq x^{(k)}\right] - (1-k)\right)$ represents the exceeding part that have to be subtracted from the first component $\mathbb{E}\left[X\mathbf{1}_{\{X\geq x^{(k)}\}}\right]$ if the $\left[X \geq x^{(k)}\right]$ element has probability greater than $1-k$, hence if we are in a situation where we can observe discontinuity. This means that if the function is otherwise continuous, $P\left[X \geq x^{(k)}\right] = 1-k$, so the second term simply vanishes, and the risk measure is computed as the simple expected value of the for each observation of X greater that the $1-k$ quantile

This formula equates, for a continuous loss distribution, all the variants and alternative definition to expected shortfall, as tail conditional expectation (TCE) or worst conditional expectation (WCE), setting a reference for further comparison.

As it respects all the properties of a coherent risk measure expected shortfall is also convex. For this reason it is the risk measure more used in recent publication expecially in comparison with the VaR measure .

We consider as imediate example again the simplest location-scale distribution, the normal $X \sim N(\mu, \sigma)$ for a loss function X. The expected shortfall can be plotted as :

$$ES_k = \mu + \frac{\sigma}{k} \int_{1-\mathrm{k}}^{1} \Phi^{-1}(u) du \,,$$

where $\Phi^{-1}(u)$ is the inverse of the standard Normal distribution function Φ, plotted for values of u spanning from the VaR one to the least at the bottom right.

Obviously the value of expected shortfall exceeds the one given by the VaR or it is for the limit case equal. For this reason every model that compute as risk measure this measure is giving much more weight to the risk from extrema events. A simple comparison between the values for different confidence levels k is available in McNeil, Frey and Embrechts 2005 and in various different publications.

Figure 6 presents a graphical illustration of the loss distribution function $X \sim S(\mathbf{1.55}, -\mathbf{0.95}, \mathbf{1}, \mathbf{0})$. Note the two different pieces of information that we can derive from the 2 risk measures:

VaR value (Dark red vertical line), defines the lower bound from which we can accumulate the density of observation that account for 5% of the total distribution (Dark red area). This area contains all the extreme bad events that may occur and that are assigned to such confidence level.

The vertical blue line has been positioned at the expected shortfall value. This indicator explain which is the average loss that we can expect considering just the value on the dark red area, or in other words, all the values that exceed the VaR reference.

In Chapter 4 VaR and expected shortfall will be calculated with a parametric approach for our dataset. As it has already described, to proceed with a that kind of calculation there is the need to define an assumption for the distribution of the data and to derive the descriptive parameters.

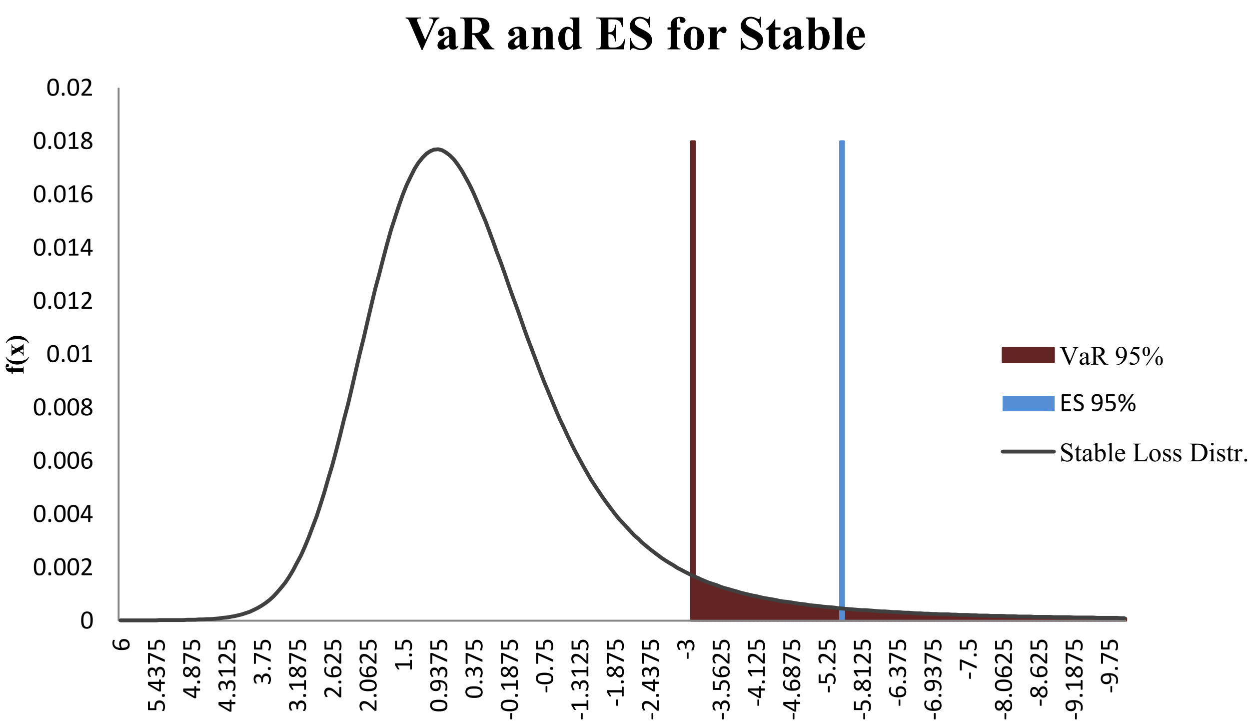

Figure 6: VaR and ES for a Stable Loss distribution function $X \sim S(1.55, -0.95, 1, 0)$

Our starting assumption is that the Loss distribution of financial returns follow a Stable distribution, though we need to estimate the four parameters α, β, γ and δ.

The next Chapter is then devoted to a quick description of the methodologies commonly used in literature, while a special focus is given to the ones that we decided to implement, in the analysis of our dataset.

Chapter 3: Stable estimation

In this chapter we discuss the parameter estimation methodologies that are available for Stable distributions. There are indeed multiple ways to derive from the raw data, aggregated or in time series, the four parameters that defines univocally the Stable law. Some of them are constructed in attempt to exploit the properties and the characteristics of the distribution, others rely just on general statistical procedures.

Also the aim may be different. Sometimes the purpose is to provide the best approximation available in terms of accuracy, while for other methodologies algorithms are simplified looking for a quick and more efficient procedure to fit the data. As usual trade-off and mixed procedures have been studied so that it is possible to customize the estimation procedure with respect to the need of the analyst.

We provide here a brief description of most of the estimator that have been defined in the past forty years, highlighting the real contribution that each of them have brought in the literature. Among those, we will describe more specifically the ones that we have implemented in our data analysis.

We can group them in different classes. There are estimators that exploit the observation of sample moments or other ordered statistics, adopting a quantile approach. There are regression-type estimators, that adopt some transformations and relate in linear way, where it is possible, the parameters, exploiting Stable properties. There are estimators that focus on matching the characteristic function of the distribution with the sample, empirical one. There are eventually estimators that are specific for testing and analyzing the tails behavior of the distribution.

Some of the more complex procedure usually narrow they parameters space adopting as a reference the output of some simpler and fast estimator obtained in an earlier step. In this way the iterative procedures converges more rapidly, avoiding also the possibility that the iteration stops in a sub optimal solutions.

Regarding multivariate Stable estimation, there are two main methodologies. The fist one is related again to the fit of the empirical characteristic function, this time focusing on the tail behavior. The second approach is based on the analysis of different projection of the data on the multivariate dimensions to provide several estimates that are then aggregated and evaluated.

Most of the time those procedure were implemented in an earlier moment, analizing symmetric distribution, so imposing as assumption $\beta = 0$. It is an hypothesis that is reasonable to adopt in a fist stage approach, since it is more important and sensible the estimation of the stability parameter α. In a second stage the estimator is generally developed for the determination of all the four parameters in detail.

There are just few publications that compare the different fits, the accuracy and the quickness of the different estimators, the paper by Höpfner and Rüschendorf in 1999 were are compared some estimators for the α and γ parameters in a symmetric Stable distribution. The work by Akgiray and Lamoureux in 1989 compares the iterative regression technique with the fractile one. Rachev and Mittnik(2000) compare both tails estimators and the fast Fourier transformation along with the maximum likelihood procedure. One latest large simulation study, by Ojeda 2001 makes a classification of the most accurate estimator, assessing at the first place the maximum likelihood procedure.

We will summarize the properties and the characteristics of all the different methodologies.

Univariate estimation

A- Tail estimators.

The tail behavior of a Stable has been described as a power law decay in Chapter 1:

$$P(|X| > x) \propto cx^{-\alpha}, \qquad x \to \infty\,.$$

As the reference x tends to infinity, the decay is proportional to the stability parameter, and can then be assessed easily. This is a behavior that interest a large number of phenomena and that gives us the opportunity to derive an estimation of the most important parameter of the distribution using just a particular subset of the data: the extreme part.

We exploit this property and focus then on the α parameter estimation: we report here the different methodologies that goes under the classification of tail estimator.

One of the first methods developed was the procedure described by Hill(1975). Using a sample of N observations $X_1, X_2, \dots, X_N$ and identified in it a subset of k, representing the values in the right tail of a loss function X, the Hill estimator is defined as:

$$\hat{\alpha}_{Hill} = k\left[\sum_{i=1}^{k} ln\,\frac{X_{N+1-i:N}}{X_{N-k:N}}\right]^{-1},$$

where $X_{i:N}$, represents an ordered statistics in the position i that respects the following relationship: $X_{1:N} \leq X_{2:N} \leq \cdots \leq X_{N:N}$.

This estimation is consistent for Stable distribution and for large values of n and k,it has been proved that it is asymptotically normal so that:

$$\left(\hat{\alpha}_{Hill}{}^{-1} - \alpha^{-1}\right)k^{\frac{1}{2}} \sim N\left(0, \hat{\alpha}^{-2}\right).$$

for values of k and N sufficiently large. This means that the Hill estimator can be tested and standard inference can be used.

One of the problems related to this procedure derives from the arbitrary choice of the parameter k. It determines the value for which the tail area is starting, so it should account for a reasonably small part of the data set. At the same time it selects the number of observations to use in the statistic and, as we know, much more observations we consider, much more easier will be to exploit asymptotical properties and limits.

It is a trade-off between precision and realistic representation. If the value is small, it does represent the tail part but it may provide an estimation that is not close enough to the real value of the stability parameter. On the other hand, if a really large k value is choosen we certainly obtain estimates characterized by low variance and though more reliable, but using also data that belongs to central part of the distribution and that should not be considered.

This dilemma has been analized and discussed in different works, Mittnik, Paolella and Rachev(1998) in particular, observed another aspect coming up, regarding Hill estimators computed for too small ks. The value for $\hat{\alpha}_{Hill}$ may break the upper bound, $\alpha = 2$, theoretically limiting the parameter space of the Stable distribution. A result from the estimation greater than two not only leads to reject the hypothesis of a Stable distributed random variable but also allow to consider finite the value that we obtain from the sample regarding the second moment, the variance statistics.

Lux(1996), compared the result of the estimation for random variables generated as Stable distributions whit index $1 \leq \alpha \leq 2$. Computing Hill for k representing the 15%, 10%, 5%, 2.5% of the dataset extreme right end, he derived that the most appropriate percentage to take as reference is the first one: 15%. For the other subset, expecially if they account for less than one tenth of the observation, $\hat{\alpha}_{Hill}$ tends to overestimate the value of the real index of stability. As support for those results, DuMouchel, Loretan and Philips(1983) studied foreign exchange rates and identified the overestimation between values 2.5 and 4 for the α parameter, for k representing the 10% of the observation.

For this reason, another estimator, the one proposed by Pickands(1975) was taken in consideration, to be modified and improved. The formula is the following:

$$\hat{\alpha}_{Pick} = \ln 2 \left[\ln \frac{X_{N-k+1:N} - X_{N-2k+1:N}}{X_{N-2k+1:N} - X_{N-4k+1:N}} \right]^{-1},$$

where again, $X_{i:N}$, represents an ordered statistics in the position i. This time a restriction is imposed to the value of k, again identifying the tail population: $4k < N$. Pickands initially was demonstrated to be just weakly consistent, then Dekkers and De Hann(1989), defined some general condition under which the estimate gains strong consistency and asymptotically normal behavior.[22]

Mittnik and Rachev(1996) saw the possibility to modify the estimator, since for Stable distribution, the performance of standard Pickands were poor. They exploited the Bergström expansion and different level of its truncation.

Bergström considered a standard symmetric Stable distribution as function of each value x and derived this asymptotic expansion:

[22] Details about the conditions and the assumption on the paper.

$$S(x) = 1 + \frac{1}{\pi}\sum_{m=1}^{\infty}(-1)^m \frac{\Gamma(\alpha m)}{m!} x^{-\alpha m} sin \frac{\alpha m \pi}{2}.$$

Truncating this infinite sum for value of $m = 1$ allows to derive the so called unconditional Pickands estimator, as considered in this way:

$$\widehat{\alpha}_{UP} = \ln 2 \left[\ln \frac{X_{N-k+1:N}}{X_{N-2k+1:N}}\right]^{-1}.$$

If the truncation occurs at different levels for m, the tail estimator is the result of a system composed by a number of equations that grown with the truncation. The higher the latter, the smoother is the estimation, but again a tradeoff arises since adding bias may result as we consider values distant from the tails.

For this reason just the value of or $m = 2$ is considered, obtaining a system that with non linear lest square approximation, produces the so called modified unconditional Pickands estimator.

Comparing those three procedure, a shared characteristic is the dependence on the k parameter, that strongly affects the estimation result. Between Hills and standard Pickands the gain of the second one in terms of smaller optimal k value is balanced with a noticeably higher variance. The modified unconditional Pickands offers instead some numerical advantages, since the optimal value of tail sample to consider is obtained for two different levels, connected with confidence intervals of different amplitude.[23]

B- Quantile technique.

Those methods focus on the relation between the Stable parameters and the characteristic function. In particular there have been identified some linear estimators based on different quantiles values. The statistics of the variable are again ordered to allow for the identification of a corresponding quantiles value of the Stable distribution.

We will refer to the ordered observations with a slightly different terminologies with respect to the one used for tail estimators. Instead of refer at them as $X_{i:N}$ and then specifying their position i in the whole N ordered set of data, we report the probability p that is cumulated up to the i-th observation: $X_{p:N}$.

Since the probability cumulated up to the maximum value N is one, we drop the identification of the whole population leaving just the notation X_p to refer at each quantile.

The first quantile approach was developed by Fama and Roll in two works in 1968 and 1971 in their fractile method. Considering a symmetric Stable distribution, so $\beta = 0$, they obtained fair estimates for the scale parameter γ considering the quantity:

$$\hat{\gamma} = \frac{0.827}{2}\left(\hat{X}_{0.72} - \hat{X}_{0.28}\right),$$

[23] For a deeper comparison consider Mittnik and Rachev(2000)

where 0.827 refers to the value of the 0.72 fractile of the standard symmetric Stable with an approximation of ± 0.03. Since the relationship above is substantially a linear combination of ordered statistics, it is also asymptotically normal distributed. The authors assessed a bias limited to a value of 0.4% for the estimator, considering just Stable distribution with finite mean ($1 \leq \alpha \leq 2$).

To determine the α parameter is necessary instead to define first the quantity $\hat{z}$:

$$\hat{z} = \frac{\hat{X}_f - \hat{X}_{1-f}}{2\hat{\gamma}} = 0.827\frac{\hat{X}_f - \hat{X}_{1-f}}{\hat{X}_{0.72} - \hat{X}_{0.28}},$$

that depends on the f large value, and provides an estimator of the value for the f fractile of a symmetric standard Stable. The third term in the equation is derived substituting the estimator for γ.

Checking the tables that the authors provide, it is possible to derive the stability parameter value, even if a trade-off related to the amplitude of f again arise, making the selection of the optimal choice a procedure that cannot be followed analytically.[24]

McCulloch(1986) went deeper in the analysis, using quantiles 5th, 25th, 50th, 75th and 95th he was able to recomputed tabulated functions and to match them with different the Stable distribution. His estimator is considered as a generalization of the Fama and Roll procedure, since it leads to the identification of all the four parameter, relaxing though the symmetry assumption. This procedure will be implemented as early phase for the Koutrouvelis algorithm, so we postpone the description further in the chapter.

C- Sample characteristic function techniques.

In his publications in 1972, Press revised and integrated some of the existing methods based on the empirical characteristic function and on the determination of the moments from its logarithmic and arctan transformation. It has been shown in Chapter 1 how the characteristic function $\phi(u)$ may be used in the definition of the cumulants $c_n(X)$, values related to the moments of the distribution. Press started then from the empirically derived characteristic function:

$$\hat{\phi}(u) = \frac{1}{N}\sum_{j=1}^{N} e^{iuX_j}.$$

He noted that in this way, the $\hat{\phi}(u)$ is computable for all the values of u, and that since it is defined as a stochastic process $\{\hat{\phi}(u), -\infty < u < \infty\}$, for each of those value u, $|\hat{\phi}(u)| < 1$. From this last observation he derived that all the moments generated by $\hat{\phi}(u)$ are necessarily finite, and that $\hat{\phi}(u)$, for any fixed value of u is the sample average of X_j, independent and identically distributed random variables.

[24] Detail of a Monte Carlo simulation and its results explain the trade off in the paper.

The law of large number allows us to exploit this result, and to assess the consistency of the estimator. Different estimators can be defined then considering then the norm distance $|\phi(u) - \hat{\phi}(u)|$. Press proposed two solutions among the others, the Minimum Distance estimator and the Minimum rth-Mean Distance.

The first adopts the sup mathematical function to identify the value for $|\phi(u) - \hat{\phi}(u)|$ considering the vector of coefficients $\theta_1(\alpha, \beta, \gamma, \delta)$.

$$\theta_1(\alpha, \beta, \gamma, \delta) = \mathrm{sup_u}|\phi(u) - \hat{\phi}(u)|.$$

The second one considers an integer calculation of the norm, multiplied by a convergence factor $W_{(u)}$:

$$\theta_1(\alpha, \beta, \gamma, \delta) = \int_{-\infty}^{\infty} |\phi(u) - \hat{\phi}(u)|^r W_{(u)} du \,.$$

The estimator depends on the choice of the r value, and on the definition of $W_{(u)}$, but as the author states, there's no clear way to assess the efficiency of them without simulations. Some alternatives are suggested, like $W_{(u)} = (2\pi)^{-\frac{1}{2}} \exp\left\{-\frac{u^2}{2}\right\}$ or $W_{(u)} = e^{-|u|}$.

Press proposed also a revised version of the method of moments. Thanks to the properties of the characteristic function, starting from a different notation:

$$|\phi(u)| = \exp\{-\gamma|u|^{\alpha}\},$$

and using the log transformation that we have already denoted as $\varphi(u)$:

$$\varphi(u) = \log \phi(u) \,,$$

he derived the following quantities, computable for different value of u: $\varphi(u) = -\gamma|u|^{\alpha}$. Substituting two value u_1 and u_2 it is possible to derive two equation that solves as estimator for α and for γ. The remaining parameters β and δ are derived considering the imaginary part of $\varphi(u)$, and exploiting the transformation in $arctan$ of the characteristic function in polar coordinates.

Also this procedure is part of the Koutrouvelis algorithm so the complete definition of the method of moments is postponed to the next section.

D- Regression-type technique and Koutrouvelis algorithm.

Those methods develop the idea is that is possible to exploit the sample distribution value Y_i corresponding to the observation X_i, and to derive a transformation of it, Y_i^*, that is linearly related to the real value of the distribution X_i^*. The regression is described in the classical form:

$$Y_i^* = \beta_0 + \beta_1 X_i^* \,.$$

The standard procedures in econometrics suggest to adopt the tool of the ordinary-least-square fit or use the weighted-least-square version, to define the values for the $\beta_i$coefficients, allowing in this way to calculate the fitted distribution $\widehat{G}$ of the stable law.

$$\widehat{Y_i^*} = \widehat{\beta_0} + \widehat{\beta_1} X_i^*, \qquad \hat{G} = \exp\left\{-e^{\widehat{Y_i^*}}\right\}.$$

The step that follows is the assessment of the effective validity of the linearity relationship. A simple technique is the visualization of a (Y^*, X^*) plane, where the transformed sample distribution Y^* should approximate a straight line once plotted against X^*.

If there are significant deviations from this pattern, it has to be assessed the statistical relevance. It is possible to calculate the residuals ε_i as difference between Y^* and its fitted value $\widehat{Y^*}$, and to investigate over their behavior.

Computing sample standard deviation and sample mean, it can be fixed a confidence band that should bound the appropriate values for a certain probability level. Outbound values will gave a qualitative assessment of the goodness of the coefficient of the regression besides giving the change of an analysis of the autocorrelation behavior.

Authors that have implemented the regression technique in their algorithms were Paulson, Holcomb and Leitch(1975), Feuerverger and McDunnough(1981) and Press(1972), that as we have seen introduced the method of moments. Koutrouvelis(1980, 1981) and Kogon and Williams(1998) then further developed those contributions.

In our study, the algorithm of Koutrouvelis have been implemented. We adopted a code version in the Gauss programming language.

The procedure by Koutrouvelis is also called iterative regression method, and is composed by three steps. In the first phase five sample quantiles are taken from the empirical distribution to produce a preliminary estimation of the four parameters of the Stable distribution.

In the second step the third and the fourth estimated parameters are used to standardize each observation, in order to obtain a time series that is distributed like a Stable $X \sim S(\alpha, \beta, 1, 0)$. Using an approximation of the sample characteristic function $\varphi(u)$ and following the regression principles introduced previously, it is possible to obtain two regression relationships: the first between α and δ and a second one between β and δ.

The last passage is to compute the final estimation $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ and $\hat{\delta}$, iterating a regression procedure and eventually de-standardizing the Stable distribution, to obtain the specific final values.

First step

In the first phase adopts a quantile approach, obtaining initial and consistent value for the four parameters. The method adopted is the one by McCulloch(1986). We will refer to the ordered observations considering their determination as quantiles X_p.

Fundamental are the one related to a probability value of $0.05,\ 0.25,\ 0.50,\ 0.75,\ 0.95$ thus identified as $\mathrm{X}_{0.05}$, $\mathrm{X}_{0.25}$, $\mathrm{X}_{0.50}$, $\mathrm{X}_{0.75}$ and $\mathrm{X}_{0.95}$.

Those quantiles are used to compute two theoretical quantities υ_α and υ_β :

$$\upsilon_\alpha = \frac{\mathrm{X}_{0.95} - \mathrm{X}_{0.05}}{\mathrm{X}_{0.75} - \mathrm{X}_{0.25}},$$

$$\upsilon_\beta = \frac{\mathrm{X}_{0.95} + \mathrm{X}_{0.05} - 2\,\mathrm{X}_{0.50}}{\mathrm{X}_{0.95} - \mathrm{X}_{0.05}}.$$

As the subscript suggest, those two quantities, can be expressed as two functions of the first two parameters of a Stable distribution. We have in this case two equation in two unknowns that, if verified simultaneously, derive their theoretical values. For this reason we can build a system such:

$$\begin{cases} \upsilon_\alpha = \omega_1(\alpha,\beta) \\ \upsilon_\beta = \omega_2(\alpha,\beta) \end{cases}.$$

Considering the empirical quantiles, it is clear that with the same procedure we can derive the estimation of the parameters $\hat{\alpha}$ and $\hat{\beta}$ consistently from the two quantities:

$$\hat{\upsilon}_\alpha = \frac{\hat{\mathrm{X}}_{0.95} - \hat{\mathrm{X}}_{0.05}}{\hat{X}_{0.75} - \hat{\mathrm{X}}_{0.25}},$$

$$\hat{\upsilon}_\beta = \frac{\hat{\mathrm{X}}_{0.95} + \hat{\mathrm{X}}_{0.05} - 2\,\hat{\mathrm{X}}_{0.50}}{\hat{\mathrm{X}}_{0.95} - \hat{\mathrm{X}}_{0.05}},$$

adopting the proper linear regression from the tables by McCulloch.

Some proposition simplify further this estimation: if we consider a symmetric Stable random variable, it can be demonstrated that alternative ways of computing υ_α are possible:

$$\upsilon_\alpha = \frac{\delta - X_{0.05}}{\delta - X_{0.25}} = \frac{\delta - X_{0.05}}{X_{0.75} - \delta} = \frac{X_{0.95} - \delta}{\delta - X_{0.25}} = \frac{X_{0.95} - \delta}{X_{0.75} - \delta}.$$

For this special case of Stable where we have the standard value for the shift parameter $\delta = 0$, we obtain:

$$\upsilon_\alpha = \frac{\mathrm{X}_{0.05}}{\mathrm{X}_{0.25}} = -\frac{\mathrm{X}_{0.05}}{\mathrm{X}_{0.75}} = -\frac{\mathrm{X}_{0.95}}{\mathrm{X}_{0.25}} = \frac{\mathrm{X}_{0.95}}{\mathrm{X}_{0.75}}.$$

And, of course: $\upsilon_\beta = 0$.

For the identification of the third parameter, γ, is considered a third theoretical quantity:

$$\upsilon_\gamma = \frac{X_{0.75} - X_{0.25}}{\gamma},$$

computing again the interpolation passages suggested by McCulloch for the values of α and β. It is straightforward then to invert it and define:

$$\gamma = \frac{X_{0.75} - X_{0.25}}{\upsilon_\gamma}.$$

Again, with a procedure similar to the one adopted for the first two parameters, we derive for a symmetric distribution, standardized with the shift parameter equal to zero, that the following relationships hold:

$$\upsilon_\gamma = \frac{2X_{0.25}}{\gamma} = \frac{2X_{0.75}}{\gamma}.$$

If we want to relax the assumption of a zero centered distribution, we need to derive δ from one last theoretical quantity:

$$\upsilon_\delta = \frac{\xi - X_{0.50}}{\gamma}, \qquad \xi = \begin{cases} \delta + \beta\gamma \tan\left(\frac{\alpha\pi}{2}\right) & \alpha \neq 1 \\ \delta & \alpha = 1 \end{cases}.$$

so that the value of δ is related to the identification of the first three parameters and their correspondent theoretical quantities.

Second step

As we know, a Stable random variable can be standardized adopting the transformation

$$X'_i = \frac{X_i - \delta}{\gamma}.$$

In the second step of the Koutrouvelis algorithm, this transformation is computed adopting the values of $\hat{\gamma}$ and $\hat{\delta}$ previously derived.

With a standard Stable, $S(\alpha, \beta, 1, 0)$ it is possible to exploit the results coming from the sample characteristic function methodologies, in particular considering the method of moments, and to obtain two relationship. The first one is function of both the characteristic exponent α and the shape parameter γ, the second one is function of the remaining two parameters, β and δ.

To obtain such relationships, there is the need to rearrange the characteristic function and, considering the notation identified by $k = 1$ by Nolan, we describe it in polar coordinates:

$$\phi(u; \alpha, \beta, \gamma, \delta) = E\left[e^{iuX}\right] = \exp(-\gamma|u|^\alpha)\{cos[f(\alpha, \beta, \gamma, \delta)] + isin[f(\alpha, \beta, \gamma, \delta)]\},$$

where we have summarized as function $f(\alpha, \beta, \gamma, \delta)$ the term:

$$f(\alpha, \beta, \gamma, \delta) = \delta u - \gamma |u|^{\alpha} \beta(sign\, u)\omega(u, \alpha),$$

and where $\omega(u, \alpha)$ reports the alternative behavior of the characteristic function once the value for $\alpha = 1$ is reached:

$$\omega(u, \alpha) = \begin{cases} tan\left(\frac{\pi\alpha}{2}\right), & \alpha \neq 1 \\ \frac{2}{\pi}\log|u|, & \alpha = 1 \end{cases}.$$

If now we take the absolute value of the characteristic function so rewritten, we raise it to the second power and we impose a double logarithmic transformation, we obtain a relation in α, that can be matched with a linear regression from the sample characteristic function:

$$\log\left(-\log(|\phi(u)|^2)\right) = \log(2\gamma^{\alpha}) + \alpha\log(|\mathrm{u}|)$$

This is the first relationship that we need to compute and as we have anticipated depends on just α and γ. To obtain the second one, the procedure suggested by Koutrouvelis is to transform and separate the real component of the characteristic function from the imaginary one:

$$\arctan\left(\frac{\Im(\phi(u))}{\Re(\phi(u))}\right) = -|\gamma u|^{\alpha}\beta(sign\, u)tan\left(\frac{\pi\alpha}{2}\right) + \delta u\,,$$

where the two functions $\Im(\phi(u))$ and $\Re(\phi(u))$ represent respectively the imaginary and real component of the characteristic function $\phi(u)$. To this transformation, adopting the standardization assumption, we can associate a regression that exploit the sample characteristic function and isolates the remaining parameter β and δ.

In this way, from the first regression, we obtain the estimated parameters $\hat{\alpha}_{0,1}$ and $\hat{\gamma}_{0,1}$, from the second one $\hat{\beta}_{0,2}$ and $\hat{\delta}_{0,2}$. It is important to notice that all the estimated values derive somehow from the first definition of the parameter α: potential distortions in the each passages may be related to initial ones, amplifying the effects.

Third step

In this last moment, the procedure described in the second step is iteratively repeated, to obtain the values for which they tend asymptotically. Eventually, the distribution is de-standardized, to obtain values coherent to the original Stable $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\delta}$.

E- Maximum Likelihood techniques.

This extensively discussed statistical technique, started to attract the attention of authors in Stable literature since 1971, when DuMouchel developed an algorithm to define in maximum likelihood the parameter of a generic Stable distribution.

Yang in 1990 revisited the procedure under the assumption of symmetric distribution, while Chen, one year after obtained a more straightforward version for the generic case. Again, in the asymmetric case Nolan 1999 and 2001 specified an algorithm that was supported by a fast pre-computed approximation to the Stable density.

We summarize the main purpose and structure of the maximum likelihood technique and then we propose some of the solution that has been identified in the specific case of the Stable distribution.

Given N random variable X_i independent and identically distributed according to a probability density function that it is unknown, we make the assumption that the distribution belongs to the Stable family. We identify though the vector θ of unknown parameters to be estimated, as:

$$\theta(\alpha, \beta, \gamma, \delta),$$

knowing that it accepts values in a multi-dimensional parameter space $\Theta = (0, 2] * [-1, 1] * (0, \infty) * (-\infty, \infty)$. The aim is to find the probability density function that among the Stable that is most likely the one that have produced the sample data.

So a likelihood function is defined, as:

$$L(\theta|X_i),$$

and have to be maximized. In practical procedures it is usual to consider a logarithmic transformation of such a function (log likelihood), since it maintains a monotonic relation and leads to the same optimization results. The main advantage is the simplification of the calculation procedures, as for instance the definition of first and second stages of differentiation.

The maximization procedure consists in an iterative process that progressively reduce the parameter space, until it is identified a subset in which small changes in the values of the single parameters cannot improve further the likelihood performance.

One problem related to this procedure is the so called local maxima problem: The ending point of the maximization procedure may prematurely comes if the function reaches a local maxima. This depends on the values with whom the iterative algorithm is initiated, and there are no general solutions to adopt. Usually then, a global maximum is assessed as such if it is reached after imposing different starting values for the parameter vector.

A maximum likelihood procedure for Stable may be affected by those problems in proximity to the bounds of the parameters. As Nolan points out in the description of his algorithm, extreme value for the symmetry parameter β may cause the premature end of the iterative research, biasing the results. For this reason it is preferred to begin the likelihood algorithm with a preliminary estimation using a simpler method and narrowing the parameter space Θ.

This estimator possesses different attractive limiting properties, that is to say that for N high values of the population, the estimator is consistent, exploits asymptotic normality and it is efficient. The disadvantages are mainly related to the fact that depending on the internal settings of the algorithm, it can represent the slowest estimation technique, requiring also calculation analytically burdensome.

For the estimation of the Stable parameters, the models differs mainly on the way to compute the Stable density. The first version of DuMouchel(1971) adopted a technique that grouped the dataset into bins, exploiting a combination of means to compute an approximation of the log likelihood function. This procedure is called fast Fourier transformation, and represents an algorithm, a computational tool, that enable a quicker implementation of the discrete Fourier transform.

The latter converts the input values, complex numbers, to a list of coefficient determining in order by frequencies, a finite complex sinusoids. It is then a method that allows a transformation of the characteristic function into something comparable to a probability density function.

This procedure was developed by the author is other publications in the 1970s. Some contributions came also from Brosen and Yang(1990), Zolotarev(1986), McCulloch(1998) and Brant (1984), that proposed a version of the algorithm that was based just on the characteristic function and not on one of its transformation.

The two maximum likelihood estimator we implemented in our study comes both from the American author Nolan. The first, more accurate comes from the package StableDistribution for the software Mathematica. It exploits by wrapper function a executable code Fortran improved towards the years, since 1997, and that now allow for quicker and more accurate estimation compared to the others likelihood algorithms.

In particular the tail density approximation is highly accurate, and for values greater that $\alpha = 0.4$, the density calculation is aided by a pre computed spine approximation. The starting values for the parameter are identified by the quantile estimator of McCulloch(1986), but then the Fisher information matrix[25] is computed for a grid of parameter values to obtain large sample confidence intervals estimates for the parameters.

[25] The Fisher information in statistic is a dispersion measure that assess the variance of the score. Score in turn is defined as the partial derivative of the log likelihood function with respect to the parameter θ. Details in Nolan 1997.

The second maximum likelihood estimation is obtained directly from the Fortran code, using the fastest estimation, provide a slightly less accurate approximation.

Multivariate estimation

Compared to the quality and quantity of works published around univariate estimation procedures, publications related to the estimation of multivariate Stable variable are relatively few.

This comes despite the fact that multivariate distributions are key elements in much of the application of portfolio theory or in asset-pricing frameworks. The generalization of the assumption of Stable joint behavior is indeed interesting for the all the properties exposed in Chapter 1, but mainly regarding the Stable domain of attraction.

Thanks to this property, we can consider random variables that behave according to different distribution law and consider in the long term, that their normalized sum is approximated by a Stable multivariate, of which we report again the characteristic function:

$$\phi(\boldsymbol{u}) = E\left[e^{i\langle \boldsymbol{u}, \boldsymbol{X}\rangle}\right] = \exp\left[i\langle \boldsymbol{u},\ \boldsymbol{\delta}^0\rangle - \int_{S^d} \psi_\alpha(\langle \boldsymbol{u}, \boldsymbol{s}\rangle)\, \boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s})\right]$$

where ψ_α is defined as:

$$\psi_\alpha(\boldsymbol{u}) = \begin{cases} |\boldsymbol{u}|^\alpha \left(1 - i\beta sign(\boldsymbol{u}) tan\frac{\pi\alpha}{2}\right) & \text{for } \alpha \neq 1 \\ |\boldsymbol{u}| \left(1 + i\beta\frac{\pi}{2} sign(\boldsymbol{u}) \log|\mathbf{u}|\right) & \text{for } \alpha = 1 \end{cases}.$$

We recall that the use of bold characters identifies the matrix notation and though vector and matrix elements, and that the $\langle\ ,\ \rangle$ notation stand for the linear combination operation.

Such distribution is identifiable thanks of just its stability parameter α, the spectral measure $\boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s})$ and the shift operator $\boldsymbol{\delta}^0$**.** We can derive that the focus regarding multivariate estimation is just around those three parameters.

A- Empirical characteristic function technique.

A first approach for the estimation of the spectral measure was proposed by Press(1972) along with his univariate sample characteristic function method definition. Not surprisingly, the approach is similar: given a set of N Stable random vectors $\boldsymbol{X}_i$, that are independent and identically distributed, with spectral measure $\boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s})$ defining their multivariate behavior, we can identify the sample counterpart of the characteristic function $\phi(\boldsymbol{u})$ as:

$$\hat{\phi}(\boldsymbol{u}) = \frac{1}{N}\sum_{i=1}^{N} \exp\left\{i\langle \boldsymbol{u}, \widehat{\boldsymbol{X}}\rangle\right\}.$$

As in the univariate case, also for multivariate it is possible to adopt the method of minimum distance or minimum rth-mean distance. But more interesting is the extension of the method of moments.

Again, we can consider the log transformation of the characteristic function, the characteristic exponent:

$$\hat{\varphi}(\boldsymbol{u}) = \log \hat{\phi}(\boldsymbol{u}) \, .$$

This function can be related directly to the two parameters α and $\boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s})$:

$$\hat{\varphi}(\boldsymbol{u}) = -\frac{(\boldsymbol{u}'\boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s})\boldsymbol{u})^{\frac{\alpha}{2}}}{2} \, .$$

And as well it is possible to compute $\hat{\varphi}(\boldsymbol{u})$ for two different values of $\boldsymbol{u_1}$ and $\boldsymbol{u_2}$ as $\boldsymbol{u} = \boldsymbol{u_i} = s_i\boldsymbol{e}$, where $\boldsymbol{e}$ represents the unit vector and s_i are scalar values. With this substitution we have defined two equation simultaneously solvable. Modifying those equation it can be derived an estimator for $\hat{\alpha}$:

$$\hat{\alpha} = \frac{\log \hat{\varphi}(\boldsymbol{u_1}) - \log \hat{\varphi}(\boldsymbol{u_2})}{\log \hat{\varphi}\left(\frac{s_1}{s_2}\right)}$$

that is consistent since the $\hat{\phi}(\boldsymbol{u})$, as in the univariate case is consistent as well.

Selecting a new value $\boldsymbol{u_3}$ and inverting the formula for the characteristic exponent we obtain:

$$\boldsymbol{u}'\hat{\boldsymbol{\Gamma}}(\mathrm{d}\boldsymbol{s})\boldsymbol{u} = \left(-2\hat{\varphi}(\boldsymbol{u})\right)^{\frac{2}{\hat{\alpha}}} \, .$$

Developing the spectral measure, that we remind is a quadratic function, a system of several equation can be derived. Solving by determinants, the consistent estimation for the spectral measure is computed.

The last step is to identify the shift operator $\boldsymbol{\delta}^0$, and the intuition is to exploit again the $arctan$ transformation and the already derived estimates for the first two parameters.

The great difference between the univariate and this multivariate application of the method of moments is related to the huge amount of calculations to complete for the solution of the system of equations from the spectral measure. This makes this method weak, since extremely time consuming although consistent, efficient and asymptotically normal.

B- Projection method.

This second methods derives from a first definition by McCulloch(1994) and is based on one-dimensional projections of the dataset. For different $\boldsymbol{u}$ vectors, belonging to the $\mathbb{R}^d$ dimensional space, we consider the projection of the random vector $\langle \boldsymbol{u}, \boldsymbol{X} \rangle$ in one dimension. The result is a Stable univariate, so it can be characterized by the four parameters.

While α maintains its value, the other three parameters can be obtained as:

$$\gamma(\boldsymbol{u}) = \left(\int_{S^d} |\langle \boldsymbol{v}, \boldsymbol{s}\rangle|^{\alpha} \boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s})\right)^{\frac{1}{\alpha}},$$

$$\beta(\boldsymbol{v}) = \gamma(\boldsymbol{u})^{-\alpha} \int_{S^d} |\langle \boldsymbol{v}, \boldsymbol{s}\rangle|^{\alpha} \mathrm{sign}\langle \boldsymbol{v}, \boldsymbol{s}\rangle \boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s}),$$

$$\delta(\boldsymbol{v}) = \begin{cases} \langle \boldsymbol{\delta}^0 \boldsymbol{v}\rangle & \text{for } \alpha \neq 1 \\ \langle \boldsymbol{\delta}^0 \boldsymbol{v}\rangle - \dfrac{2}{\pi}\displaystyle\int_{S^d} \langle \boldsymbol{v}, \boldsymbol{s}\rangle \log|\langle \boldsymbol{v}, \boldsymbol{s}\rangle| \boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s}) & \text{for } \alpha = 1 \end{cases}.$$ [26]

This is how the method works. It is considered a sample of $\boldsymbol{X_i}$ vectors and fixed a grid of $\boldsymbol{u_i}$ vector values in S^d. For each grid vector, it is defined the dimensional projection $\langle \boldsymbol{u}, \boldsymbol{X}\rangle$, with the characteristics exposed above. Some univariate estimation technique is then applied for the definition of $\hat{\alpha}(\boldsymbol{u_i})$, $\hat{\beta}(\boldsymbol{u_i})$, $\hat{\gamma}(\boldsymbol{u_i})$ and $\hat{\delta}(\boldsymbol{u_i})$.

All those one dimensional parameters are aggregated to recover the multivariate α value and the spectral measure $\boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s})$. Regarding the identification of the stability parameter, the simple average of all the one dimensional estimated values is sufficient:

$$\hat{\alpha} = \frac{1}{N}\sum_{i=1}^{N} \hat{\alpha}(\boldsymbol{u_i}).$$

The definition of the spectral measure estimation is more complicated. We recall the definition from Chapter 1 of $\boldsymbol{I}_{\alpha}(\boldsymbol{u})$, the function related to the definition of the spectral measure:

$$\boldsymbol{I}_{\alpha}(\boldsymbol{u}) = \int_{S^d} \psi_{\alpha}(\langle \boldsymbol{u}, \boldsymbol{s}\rangle)\, \boldsymbol{\Gamma}(\mathrm{d}\boldsymbol{s}).$$

This function became important in this stage since it can be derived from the univariate estimation of the projection as:

$$\hat{\boldsymbol{I}}_{\alpha}(\boldsymbol{u_i}) = \begin{cases} \hat{\gamma}(\boldsymbol{u_i})\left(1 - i\hat{\beta}(\boldsymbol{u_i}) \tan\dfrac{\pi\hat{\alpha}}{2}\right) & \alpha \neq 1 \\ \hat{\gamma}(\boldsymbol{u_i})\left(1 - i\hat{\delta}(\boldsymbol{u_i})\right) & \alpha = 1 \end{cases}.$$

To define eventually the spectral measure estimated value $\hat{\boldsymbol{\Gamma}}(\mathrm{d}\boldsymbol{s})$ we need to proceed with a discrete approximation, and then to invert the approximation to the characteristic function $\hat{\phi}(\boldsymbol{u})$. We know that the discrete characteristic function for multivariate can be defined also as:[27]

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[26] Just the $k = 1$ is reported for the sake of brevity

[27] See Nolan 2008

$$\phi(\boldsymbol{u})^* = \exp\left\{-\sum_{i=1}^{N} \psi_\alpha(\langle \boldsymbol{u}, \boldsymbol{s_i}\rangle)\boldsymbol{\vartheta_i}\right\},$$

where star * is used to differentiate the discrete definition from the continuous one and where $\boldsymbol{\vartheta_i}$ represents the weight at each points $\boldsymbol{s_i}$.

We can though derive from the estimation of $\widehat{\boldsymbol{I}}_\alpha(\boldsymbol{u_i})$ the matrix $\widehat{\psi}_\alpha(\langle \boldsymbol{u}, \boldsymbol{s_i}\rangle)$ and eventually compute the discrete value $\widehat{\boldsymbol{\Gamma}}(\mathrm{d}\boldsymbol{s})$.

Remarks on Stable estimation

The work upon the Stable estimation is not completed. As inference techniques develop it is possible to reconsider the models proposed above and to make modification, and provide mixed models with multiple approaches.

For example, a competitor method to the maximum likelihood procedure by Nolan is the Modified Method of Scoring, resulted from the early intuition of Klebanov, Manija and Melamed(1984), and developed further by Melamed and Rachev 1994. This approach provides a mixed implementation of regression techniques, and the maximum likelihood implementation. The result is a consistent, asymptotically Normal and asymptotically efficient estimator.

It is missing a clear evaluation of the accuracy and behavior of the different estimators. A Monte Carlo simulation and an extensive test procedure for all the techniques above exposed could be an interesting research to pursue, analyzing different random variables in the financial market.

Chapter 4: Data analysis

This chapter is dedicated to the description of the data analysis we have carried out, in order to highlight: properties of Stable distributions, diversion from the Normal behavior assumption, differences among some univariate estimation methodologies and results from the VaR and expected shortfall computations.

A back test of the estimators, is eventually exposed as closing analisys. Some simulated series are estimated and the discrepancies are assessed as approximation for the uncertainty measure of the estimator.

The dataset is composed by 188 assets from the New York Stock Exchange market, for which we have downloaded the time series related to the closing price Bloomberg index.

The variable of interest is the daily return X_t, computed as the logarithmic ratio of two consecutive closing prices P_t:

$$X_t = \log\left(\frac{P_t}{P_{t-1}}\right) * 100 .$$

We are responsible of just one intervention in the dataset: all the values equal to zero have been removed. The latter action was considered after the observation of the excessive and unjustified frequency characterizing the observation 0, modal value for all the initially derived time series. We justify this modification considering two point of view.

From a statistical point of view, all the observation should be comparable to the continuous stream of values of a distribution function. Therefore they are supposed to cumulate density or probability just when considered in class, while taken as singular value they should be characterized by a single associated null (infinitesimal) value of probability. The original series accounted instead for a specific peak of density concentrated in the value zero, distorting the estimation of the fitted distribution.

Considering a different aspect, the generation of the 0 values comes from two subsequent equal records in the time series. This situation is perfectly feasible but, expecially when the contractual obligations are registered with four digits, it is unlikely to happen with a frequency so high. In the original dataset, the component of zero valued returns attested for about 5-10% of the whole population. We can explain this redundancy considering that it is a routine procedure of most of the data provider to replace a missing observation with the spot price of the day before. This can happen automatically, leaving to the analyst the task to correctly interpret the data.

For both reason, we proceeded in all the estimation leaving out this subset.

Basic descriptive statistics

Table A in Appendix summarize the preliminary descriptive statistics of the singularly considered asset. For each equity we computed mean, median, maximum and minimum value, standard deviation, skewness, kurtosis, number of observation and Jarque-Bera test.

The 188 time series vary in all their aspects.

On average, they contain 7409 observations, where the shorter time series is composed by 294 records and the longest one by 11113 records. The dataset nevertheless is not so heterogeneous: 170 assets out of the total have between the 6000 and 7000 records.

The mean return is generally positive (aggregate average value of 0.0352646). It assumes strictly negative values just for just 7 series, reaching the minimum of -0.0325692. As expected, the standard deviation of all the series is pretty high, on average 2.265447.

The median value is negative for one third of the total of series (62 out of 188). This last value, considering that the mean returns are instead mostly positive accounts for some pattern of asymmetry. A confirm comes from the analysis of the skewness parameters: we can notice that for two third of the series (121 out of 188) it takes negative values.

Nevertheless the amplitude of the asymmetric behavior is not extreme, as 158 value out of 188 have a value for this statistic $-1 < \zeta(X) < 1$. We recall that the Normal value of skewness equal to zero.

The study of the kurtosis value $\kappa(X)$, instead, is clearly against the assumption of normality, that we recall is characterized by $\kappa(X) = 3$. The equity with minimum kurtosis reports an index of 5.142365, but on average, the value is much more higher clearly assess the leptokurtic tendency at 24.79866. The financial returns that we treated are then all more peaked that a Normal distribution, and characterized by fat tails.

All the series have been tested with the Jarque-Bera statistic introduced in Chapter 1. We remind that it is a χ^2 distributed statistics testing the Normal behavior of the random variable and that to accept the null hypothesis, it should take values around zero. On average, the test for all the time series takes value 901597.5481, though the null hypothesis is rejected for all the series with the highest level of probability.

It is curious to notice that the minimum level for the Jarque-Bera, 78.671232 is reached by the shorter time series the NSM US Equity. It is nevertheless a value that still strongly reject the null hypothesis.

This is all what concerns a fist immediate evaluation of simple descriptive statistics. The second step was the implementation of five different Stable estimation to derive the four characterizing parameters for all the assets.

Stable parameter estimation: results

We tested the four different approaches using three different tools: in order the appendix, in Table B, reports the parameters estimation from the software Mathematica with a Maximum Likelihood procedure, the fit provided by a Gauss code implementing the Koutrouvelis algorithm, and eventually a Fortran application by Nolan, reporting a quantile estimation procedure (McCulloch), the sample characteristic method (Kogon and Williams) and again a maximum likelihood fit.

The first estimation will be taken as reference for the others. Table 2 below reports some aggregate statistics about the parameters of the 188 NYSE assets derived from the Mathematica maximum likelihood procedure. The subscript M helps to avoid confusion.

The stability parameter α, as already reported assume values in between the interval $1.45704 \leq \hat{\alpha}_M \leq 1.890414$; for all the assets, the first moment is then finite. It is then confirmed the hypothesis of non-normality: we remind that a normal variable is characterized by $\alpha = 2$, while the maximum value obtained from our data has been just 1.89042.

The index of symmetry, β is almost near to zero, and quite centered. Reversing the asymmetry balance derived from the skewness statistic, now just 43 out of 188 estimates have a negative value. This gives some justification to the assumption that some authors follow about the symmetric behavior of financial returns.[28]

The scale and location parameter have relative interest for now. We notice just that in 6 cases out of the total, the δ index is negative, a value that occurs almost for all those assets that have mean return below zero.

Table 2: Main statistics for the Mathematica maximum likelihood estimations.

	$\hat{\alpha}_M$	$\hat{\beta}_M$	$\hat{\gamma}_M$	$\hat{\delta}_M$
Mean	1.71163946	0.03738965	0.005345311	0.000181554
St.Dev	0.067163262	0.0856623	0.001328182	0.000149791
Min	1.457041412	-0.238142629	0.003247179	-0.000140935
Max	1.890417318	0.253158113	0.011526634	0.001714861

Figure 7 helps to visualize how much the results vary. The graph plots with the long dots line a Stable characterized by the average value of all the Mathematica parameters estimates, with the smaller dots it reports the shape of the normal distribution computed for the average of all the means returns, considering the average standard deviation of all the series.

Those two shapes can be considered as representative to evaluate how different are the estimates considering the two different approaches: normality or Stable behavior of returns. The main difference that comes up is the leptokurtic behavior, represented by the peaked central area of the Stable.

[28] Fama and Roll 1971 among the others.

The detail on the right of the graph proposes a close up on the left tail behavior, where we can see the normal curve that starting from the higher right values dramatically decreases ending up as the lowest one on the left.

The two solid lines are the estimated curves of the assets that account for the higher and lower value of $\hat{\beta}_M$; we can notice that one with the maximum value, 0.253, has the fattest left tail

The two curves identified by the points lines are the two Stable that represent the higher and the lower value of $\hat{\alpha}_M$, so determines the two limit cases of leptokurtosis observed.

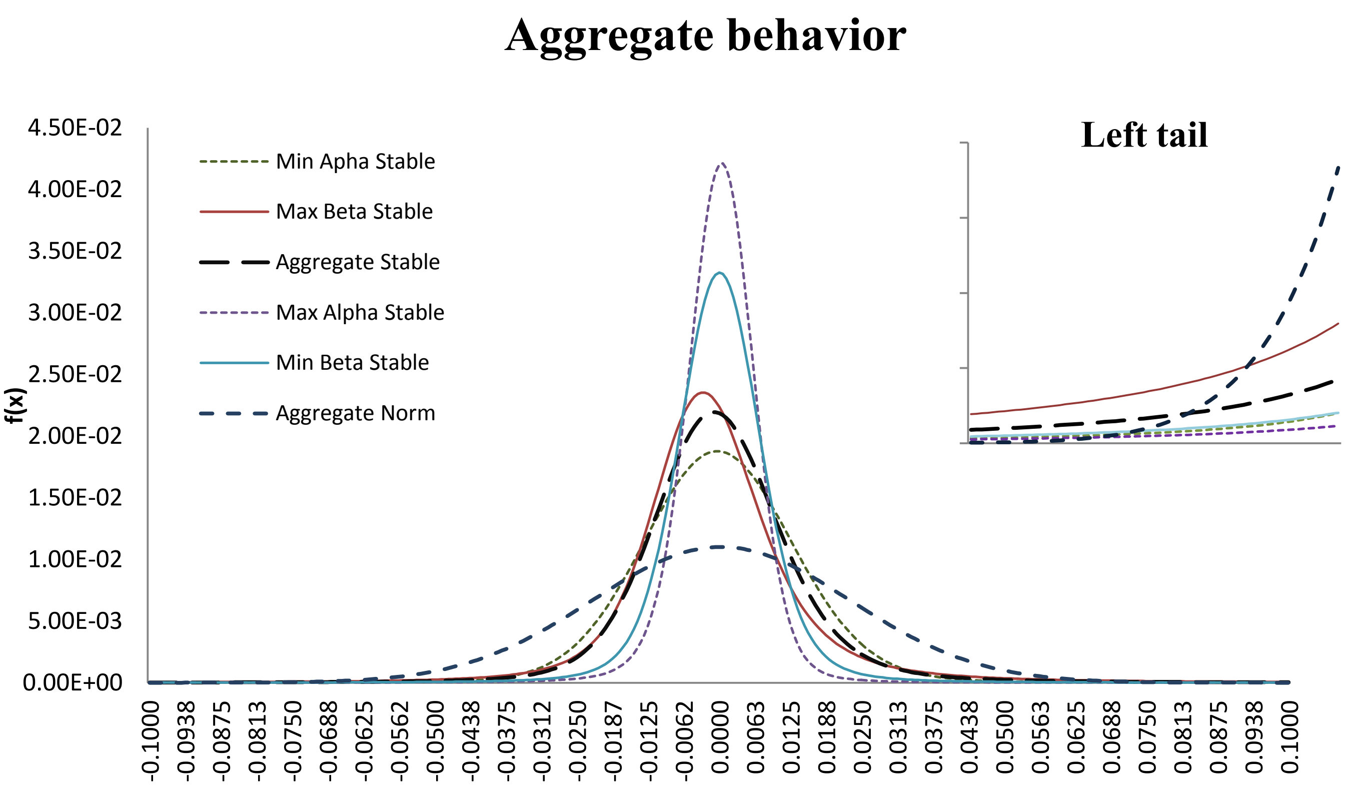

Figure 7: Comparison between Aggregate Stable $X \sim S(1.711, 0.0373, 0.00534, 0.000182)$, aggregate normal $X \sim N(0.0352, 0.0226)$, maximum beta Stable $X \sim S(1.676, 0.253, 0.00872, 0.000182)$ minimum beta Stable $X \sim S(1.778, -0.238, 0.00420, 0.0000214)$ maximum alpha Stable $X \sim S(1.89, 0.183, 0.00942, 0.0000475)$ minimum alpha Stable $X \sim S(1.457, 0.0263, 0.00772, -0.0000119)$.

We report and discuss the following tables 3 and 4, where in the same approach (aggregated statistics) are summarized the characteristic for the other estimators output.

Hereafter we can see the statistics from the Koutrouvelis estimation in Table 2. The code in Gauss confirms general similarities with the results showed in Table 1. α values belongs to the upper part of the subset of the parameter space where the first moment is defined, in this case $1.479312 \leq \hat{\alpha}_K \leq 1.920478$. $\hat{\beta}_K$, slightly positive index of asymmetry and a pair of assets with negative parameter location.

Table 3: Main statistics for the Koutrouvelis regression method estimations.

	$\hat{\alpha}_K$	$\hat{\beta}_K$	$\hat{\gamma}_K$	$\hat{\delta}_K$
Mean	1.741377301	0.112371855	0.012266311	0.000654623
St.Dev	0.07056587	0.137188678	0.003060735	0.000394867
Min	1.479312084	-0.309630911	0.007491175	-6.27936E-05
Max	1.920478341	0.606541392	0.026674862	0.003988585

We complete the exposition of the general performances, reporting also the results for the three procedure in STABLE, and so in order the quantile approximation estimator (subscript Sq), an alternative maximum likelihood (subscript Sm) and the empirical characteristic function estimator (subscript Sc).

Table 4: Main statistics for the STABLE program estimations.

STABLE: quantile method

	$\hat{\alpha}_{Sq}$	$\hat{\beta}_{Sq}$	$\hat{\gamma}_{Sq}$	$\hat{\delta}_{Sq}$
Mean	1.624990	0.047914	0.005167	0.000234
St.Dev	0.087106	0.157721	0.001314	0.000249
Min	1.374054	-0.818971	0.003237	-0.001675
Max	2	1	0.011933	0.001492

STABLE: maximum likelihood

	$\hat{\alpha}_{Sm}$	$\hat{\beta}_{Sm}$	$\hat{\gamma}_{Sm}$	$\hat{\delta}_{Sm}$
Mean	1.711550	0.066638	0.005347	0.0002378
St.Dev	0.071606	0.090267	0.001326	0.0001515
Min	1.4579	-0.2179	0.003244	-0.0000139
Max	1.8731	0.35	0.011507	0.0017848

STABLE: sample characteristic function

	$\hat{\alpha}_{Sc}$	$\hat{\beta}_{Sc}$	$\hat{\gamma}_{Sc}$	$\hat{\delta}_{Sc}$
Mean	1.775399	0.141494	0.005431	0.000277
St.Dev	0.064574	0.168896	0.001353	0.000191
Min	1.5348	-0.3542	0.003289	-0.000093
Max	1.9274	0.9907	0.011678	0.002102

We can notice that the quantile method is the one that allow for the most wider range of results. It is worth to notice that it attest an $\hat{\alpha}_{Sq}$ value equal to two to two of the equity, accepting though the hypothesis of Normal distribution. Also with respect to the symmetry parameter, we can note that the minimum and maximum value observed for β, almost coincide with its parameter space.

The STABLE maximum likelihood proposes results quite similar to the Mathematica estimates, while the sample characteristic function, despite of having the narrower range for the parameter of stability (${\hat{\alpha}_{Sc}}^{Max} - {\hat{\alpha}_{Sc}}^{min} = 0.3926$), it leaves the $\hat{\beta}_{Sc}$ estimator vary a lot, expecially for positive values, where the upper bound is almost reached.

One last remark from this initial overlook regards the estimated scale parameter $\hat{\gamma}_{Sc}$. The aggregated statistics exposed above seems to agree and shares average value, minimum, maximum and standard deviation almost up to the second significant digit, with the exception of the Koutrouvelis results. We continue a more detailed comparison in the next part.

Stable parameter estimation: comparison

In this section we focus on the difference among the different methods, considering their singular estimation for the 188 assets.

As first, crude comparison we decided to do the evaluation of a simple percentage difference. As reference was taken the values of $\hat{\alpha}_M$, $\hat{\beta}_M$, $\hat{\gamma}_M$ and $\hat{\delta}_M$ the estimates of the Mathematica maximum likelihood. The complete comparison can be found in the appendix, Table C. We report here in Table 4 a selection 25 assets of the dataset for some remarks.

We will adopt this kind of table also for all the subsequent comparison, so we describe some of its characteristics. The assets are listed in the first column, and the value that pertains to them follow in their respective row. Four main columns then groups the values relative to the α, β, γ and δ Stable parameters, specified also by the type of estimation procedure.

A visual connotation is given to the cells thanks to a the conditional formatting procedure of Excel. In particular, given an interval, such as the four columns of values for one parameter, maximum and minimum records for diversion are taken as reference to create class of values associated to different brightness of the two colors red and blue. Consequently, percentages close to zero have no color associated, while as the absolute value of the record grows, we will have a deeper red color for cells related negative deviations and a deeper blue color for cells connoted by positive ones.

We can immediately deduce that a comparison in percentage terms is not the one that better help us to evaluate the accuracy of the fits, or at least, not for all the parameters.

Concerning the α parameter, the comparison seems to work, and we can already notice some recurrent pattern. The quantile procedure by McCulloch seems to systematically overestimate the first stable parameter, while the converse remark can be addressed to the Characteristic function method. Also the Koutrouvelis estimation appears to always surpass the estimated value by Mathematica. The STABLE maximum likelihood seems instead the method more coherent, and there is no surprise, since the estimation comes from a similar procedure.

The percentages that regards the deviations in the β estimates along the procedures are instead severely affected by the fact that the parameter is for most of the Mathematica estimations really close to zero.

Table 5 - Selection from appendix, Table C: percentage comparison $[100 * (\hat{\theta}_i - \hat{\theta}_M)/\hat{\theta}_M]$ between the four estimators $\hat{\theta}_i$ K=Koutrouvelis (Gauss code), Q=Quantile method (STABLE), ML=Maximum likelihood (STABLE), ChF=Characteristic function method (STABLE) with reference to the maximum likelihood parameters computed by Mathematica $\hat{\theta}_M$.

	α				β				γ				δ			
	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF
AA	1.471%	-5.927%	-0.542%	3.108%	188%	115%	47%	244%	129.57%	-3.97%	-0.18%	1.28%	488.54%	-34.20%	57.27%	153.08%
AFL	0.993%	-2.718%	-0.184%	3.619%	65%	-14%	21%	68%	128.64%	-1.40%	-0.03%	1.90%	206.72%	17.63%	15.65%	16.98%
APD	2.255%	-4.958%	-0.370%	4.429%	164%	36%	52%	248%	129.32%	-3.23%	-0.08%	1.76%	242.13%	12.87%	25.13%	48.96%
ARW	1.355%	-2.885%	-0.508%	3.355%	66%	68%	27%	116%	129.53%	-1.08%	-0.19%	1.49%	383.72%	-19.40%	75.94%	130.49%
ASH	1.240%	-2.151%	-0.108%	3.076%	494%	-270%	677%	911%	129.92%	-1.33%	-0.03%	1.50%	178.89%	7.05%	33.49%	32.73%
BC	1.231%	-4.977%	0.217%	3.633%	144%	70%	42%	146%	128.96%	-3.00%	0.12%	1.77%	335.51%	-26.89%	30.23%	62.75%
BCR	2.653%	-8.964%	-0.498%	4.685%	143%	34%	90%	412%	129.32%	-6.71%	-0.17%	1.53%	176.10%	27.11%	19.10%	44.19%
BMY	1.616%	-4.388%	-0.354%	3.457%	96%	-6%	49%	372%	129.36%	-2.61%	-0.06%	1.45%	188.06%	89.91%	18.13%	74.45%
C	0.333%	-6.107%	0.030%	2.275%	127505280%	59810527%	56482746%	112730246%	128.60%	-5.22%	0.03%	1.20%	384.49%	-93.30%	50.45%	84.81%
CA	1.287%	-7.425%	-0.132%	4.311%	174%	79%	129%	401%	127.49%	-5.27%	-0.02%	1.82%	207.19%	54.78%	27.06%	53.40%
F	1.054%	-2.372%	-0.364%	2.462%	84%	111%	24%	97%	129.47%	-1.76%	-0.08%	1.11%	306.41%	-81.73%	37.38%	62.46%
FDX	2.817%	-6.756%	-0.589%	5.194%	94%	14%	14%	131%	129.22%	-4.50%	-0.23%	2.00%	259.29%	-30.26%	21.47%	39.59%
FL	2.444%	-5.205%	-0.294%	4.316%	55%	44%	17%	29%	130.63%	-4.33%	-0.11%	1.90%	251.51%	-154.11%	44.19%	-35.75%
FRM	1.637%	7.470%	0.022%	2.494%	149%	-100%	44%	248%	131.42%	3.53%	-0.17%	1.31%	-8788.52%	20142.04%	-2231.01%	-5749.61%
FWLT	1.051%	-2.863%	-0.065%	3.676%	54%	127%	37%	80%	129.32%	0.05%	-0.02%	2.11%	2443.68%	-3771.91%	840.63%	1054.65%
GAS	1.705%	-4.266%	1.656%	2.698%	6%	93%	-24%	-32%	130.77%	-4.02%	0.74%	1.11%	160.79%	120.12%	24.41%	40.16%
GCI	1.131%	-3.495%	0.198%	3.501%	153%	104%	41%	194%	129.03%	-2.47%	0.13%	1.77%	340.83%	-71.87%	28.69%	88.84%
GR	2.552%	-5.704%	-0.231%	5.000%	100%	-3%	20%	90%	129.68%	-3.82%	-0.06%	2.15%	262.31%	0.75%	21.52%	20.57%
GRA	2.215%	-7.346%	-0.007%	6.618%	-1137%	-1458%	-628%	-292%	129.14%	-4.85%	0.04%	3.71%	-725.37%	-1581.29%	-306.91%	38.25%
GT	1.677%	-3.987%	0.298%	4.664%	147%	132%	37%	216%	128.96%	-2.62%	0.15%	2.29%	425.40%	-187.46%	39.05%	135.30%
GWW	2.261%	-6.737%	-0.493%	4.328%	91%	-53%	35%	153%	129.66%	-3.78%	-0.13%	1.75%	178.16%	46.30%	15.29%	23.82%
HAL	1.771%	-2.551%	-0.036%	3.350%	378%	336%	158%	587%	129.73%	-2.22%	0.07%	1.36%	490.39%	15.73%	88.73%	192.87%
HAS	2.175%	-8.009%	-0.476%	4.012%	106%	-26%	20%	37%	129.48%	-6.08%	-0.16%	1.42%	231.29%	71.26%	16.28%	-8.60%
HD	2.335%	-6.737%	-0.499%	4.644%	138%	-18%	40%	133%	129.24%	-3.29%	-0.15%	1.89%	189.04%	41.19%	13.49%	13.07%
HES	1.390%	-3.479%	-0.343%	3.021%	-4072%	-3780%	-1690%	-7877%	129.67%	-1.86%	-0.05%	1.33%	301.65%	-57.51%	39.19%	121.41%

For this reason even little difference in the estimation procedure can generate variation that are then exploded to incredibly high values. Also the conditional formatting became useless, as the minimum and maximum level are extremely large. As an example, we can notice in the Table 5 the incredible percentage deviation of all the symmetry parameter estimations for the C US Equity. This result derives from the value of the Mathematica β estimation: 0.00000004249078.

γ columns are substantially affected by the same problem, but most important is that a particular pattern arise: the Koutrouvelis estimate appears biased. The percentage is indeed always assessed around 129% of positive deviation (dark blue column of cells). It is clear that there is a specification problem in the algorithm definition, in particular in some passages of standardization/de-standardization of the Stable distribution. We will turn back on this issue further in the chapter.

Regarding δ, it is once more problematic to assess any interesting pattern, because it is as well a value too close to zero for most of the estimated $\hat{\delta}_M$ of the assets.

For all those reasons the comparison measures have been diversified, and the final solution is the one reported in Table 6 (again considering just a subsample of the total assets).

For the stability parameter and the symmetry one we measure simply the linear impact in difference. The two parameter have both small parameter spaces, and the amplitude is identical. A good measure of deviation then, is simply the difference between the reference value of $\hat{\alpha}_M$ or $\hat{\beta}_M$, as results of the Mathematica estimation, and the correspondent parameter $\hat{\alpha}_i$ or $\hat{\beta}_i$ of the other methods.

The scale parameter γ can be compared considering instead the logarithmic difference, multiplied by 100, again to avoid results too close to zero. It is a measure that for small values and being the two sufficiently near, gets close to the percentage variation and helps to evaluate both increases and decreases without being constrained to relate to the reference value.

The shift parameter needs instead a measure that accounts not only for the difference between the estimates, $\hat{\delta}_M - \hat{\delta}_i$ but considers also an adjustment by the scale parameter. This adjustment comes from the division by an average of the scale parameter: $(\hat{\gamma}_M - \hat{\gamma}_i)/2$. This ratio makes the simple difference harmonized though comparable.

Considering those new comparison tools, we can look at Table 6 and comment the results, with a particular attention to the last four row, reporting the upper bound reached for the measured diversion, the lower one, the correspondent range and the average deviation.

Stability parameter

Regarding the stability parameter, the method that more closely replicate the values of Mathematica is the similar method of maximum likelihood of the STABLE program. We consider this one despite the

Koutrouvelis shows a lower range amplitude, because $\hat{\alpha}_{Sm}$ is almost on average centered in the same values estimated by the $\hat{\alpha}_M$.

Nevertheless it is impressive the performance of the iterative regression method by Koutrouvelis, since it obtain just a slightly abundant estimation, compared to $\hat{\alpha}_M$. This pattern can then be adjusted if empirical estimation are evaluated, for financial dataset of similar origin.

The sample characteristic function procedure share some similarity with the Koutrouvelis algorithm, ending as well in a light over estimation of α. The fact that on average the distance from the reference Mathematica value double the one of Koutrouvelis, also in terms of range width, allow us to classify this estimator as the second best, with respect to the two maximum likelihood methods

As expected, the quantile procedure is the one performing poorer, where almost all the $\hat{\alpha}_{Sq}$ values under estimate the stability parameter identified by our reference. There are just few cases (7 out of the total) for which the opposite behavior is assessed, where the worst deviation account for the hypothesis of the normal distribution of the FRM US Equity returns.

Symmetry parameter

Concerning β, the comparison of the estimators confirms the best replica fit of the STABLE maximum likelihood, followed by the Koutrouvelis regression technique, the empirical characteristic function one and the quantile approximation.

It is worth to notice that all those alternative parameters estimation tend to identify a higher interval of variation for the symmetry parameter compared to the Mathematica estimations, and that is as to say that the most accurate is the estimation method, the more centered it appears the distribution.

We can suggest a possible explanation of this shared behavior, taking the case of $\hat{\beta}_K$, the Koutrouvelis value. We know that the estimation of this second parameter comes from the determination of the first one, inheriting possible rumors and errors.

We have noticed that the stability estimation is slightly abundant, and as we have exposed in Chapter 1, a higher α, makes harder for the β parameter to account for asymmetric behavior. Therefore there is the need of higher values of β to express the same patterns of symmetry.

For this reason with respect to $\hat{\alpha}_M$ and $\hat{\beta}_M$, larger approximations like the ones from the Koutrouvelis and the characteristic function method, can be justified, and we can notice also that for those two techniques, the deviation of $\hat{\beta}_i$ is in fact proportional to the average distance $\hat{\alpha}_M - \hat{\alpha}_i$.

A stranger pattern characterize instead the quantile method values of $\hat{\beta}_{Sq}$, since it reports the wider parameter space (1.468 out of 2) even if the stability estimates are lower than the reference $\hat{\alpha}_M$.

Table 6 - Selection from appendix, Table D: comparison between the four estimators with reference to the maximum likelihood computed by Mathematica.
For α: $\hat{\alpha}_M - \hat{\alpha}_i$. For β: $\hat{\beta}_M - \hat{\beta}_i$. For γ: $ln(\hat{\gamma}_M/\hat{\gamma}_i) * 100$. For δ: $(\hat{\delta}_M - \hat{\delta}_i) * [(\hat{\gamma}_M - \hat{\gamma}_i)/2]^{-1} * 100$

	α				β				γ				δ			
	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF
AA	-0.026	0.103	0.009	-0.054	-0.135	-0.083	-0.033	-0.175	-83.104	4.050	0.179	-1.267	-5.8%	0.7%	-1.1%	-1.1%
AFL	-0.017	0.046	0.003	-0.061	-0.084	0.018	-0.027	-0.088	-82.696	1.412	0.031	-1.885	-9.1%	-1.3%	-1.1%	-1.1%
APD	-0.039	0.086	0.006	-0.077	-0.088	-0.019	-0.028	-0.134	-82.993	3.279	0.079	-1.746	-5.4%	-0.5%	-0.9%	-0.9%
ARW	-0.024	0.050	0.009	-0.059	-0.101	-0.104	-0.041	-0.177	-83.088	1.084	0.188	-1.484	-4.3%	0.4%	-1.4%	-1.4%
ASH	-0.021	0.036	0.002	-0.051	-0.017	0.009	-0.023	-0.031	-83.254	1.335	0.033	-1.489	-3.0%	-0.2%	-0.9%	-0.9%
BC	-0.020	0.082	-0.004	-0.060	-0.100	-0.049	-0.029	-0.101	-82.840	3.050	-0.122	-1.759	-8.1%	1.1%	-1.2%	-1.2%
BCR	-0.045	0.153	0.009	-0.080	-0.045	-0.011	-0.028	-0.130	-82.997	6.944	0.166	-1.521	-5.7%	-1.5%	-1.0%	-1.0%
BMY	-0.028	0.077	0.006	-0.061	-0.056	0.003	-0.028	-0.215	-83.013	2.642	0.056	-1.435	-5.5%	-4.4%	-0.9%	-0.9%
C	-0.005	0.098	0.000	-0.036	-0.054	-0.025	-0.024	-0.048	-82.681	5.356	-0.031	-1.198	-5.5%	2.3%	-1.2%	-1.2%
CA	-0.022	0.124	0.002	-0.072	-0.046	-0.021	-0.034	-0.107	-82.193	5.412	0.016	-1.802	-6.3%	-2.8%	-1.3%	-1.3%
F	-0.019	0.042	0.006	-0.043	-0.124	-0.163	-0.036	-0.143	-83.059	1.780	0.084	-1.103	-5.7%	2.5%	-1.2%	-1.1%
FRM	-0.030	-0.139	0.000	-0.046	-0.363	0.243	-0.107	-0.603	-83.906	-3.466	0.167	-1.305	-3.8%	14.2%	-1.6%	-1.6%
FWLT	-0.017	0.046	0.001	-0.059	-0.042	-0.100	-0.029	-0.063	-82.997	-0.048	0.023	-2.091	-2.7%	6.8%	-1.5%	-1.5%
GAS	-0.031	0.077	-0.030	-0.048	0.005	0.080	-0.021	-0.028	-83.627	4.106	-0.736	-1.108	-2.7%	-3.4%	-0.7%	-0.7%
GCI	-0.019	0.058	-0.003	-0.058	-0.081	-0.055	-0.022	-0.103	-82.868	2.497	-0.131	-1.759	-6.6%	2.3%	-0.9%	-0.9%
GR	-0.043	0.096	0.004	-0.084	-0.123	0.003	-0.025	-0.111	-83.150	3.895	0.063	-2.125	-7.7%	0.0%	-1.0%	-1.0%
GRA	-0.033	0.110	0.000	-0.099	-0.049	-0.063	-0.027	-0.013	-82.918	4.973	-0.035	-3.643	-2.8%	-10.3%	-2.0%	-1.9%
GT	-0.028	0.066	-0.005	-0.077	-0.104	-0.093	-0.026	-0.153	-82.837	2.652	-0.153	-2.263	-7.3%	5.4%	-1.1%	-1.1%
GWW	-0.039	0.116	0.009	-0.075	-0.065	0.038	-0.025	-0.109	-83.142	3.857	0.129	-1.739	-6.3%	-2.8%	-0.9%	-0.9%
HAL	-0.031	0.045	0.001	-0.059	-0.101	-0.090	-0.042	-0.157	-83.175	2.241	-0.069	-1.353	-3.9%	-0.2%	-1.2%	-1.1%
HAS	-0.037	0.136	0.008	-0.068	-0.143	0.035	-0.027	-0.050	-83.065	6.269	0.162	-1.415	-9.8%	-5.2%	-1.1%	-1.1%
HD	-0.040	0.116	0.009	-0.080	-0.109	0.014	-0.031	-0.105	-82.959	3.350	0.150	-1.875	-9.7%	-3.5%	-1.1%	-1.1%
HES	-0.024	0.061	0.006	-0.053	-0.088	-0.082	-0.036	-0.170	-83.147	1.875	0.055	-1.322	-4.9%	1.5%	-1.0%	-1.0%
	*** ***				*** ***				*** ***				*** ***			
Up bound	-0.001	0.251	0.017	-0.019	0.077	0.651	-0.012	0.186	-79.156	8.863	0.588	-0.694	0.7%	14.2%	-0.5%	-0.5%
Dw buond	-0.052	-0.139	-0.043	-0.099	-0.397	-0.817	-0.107	-0.808	-87.872	-3.466	-1.007	-3.643	-16.6%	-10.3%	-2.0%	-1.9%
Range	0.051	0.390	0.060	0.080	0.474	1.468	0.094	0.994	8.716	12.328	1.595	2.948	17.3%	24.5%	1.5%	1.5%
Average	-0.030	0.087	0.000	-0.064	-0.075	-0.011	-0.029	-0.104	-83.054	3.465	-0.042	-1.589	-5.3%	-1.3%	-1.0%	-1.0%

This is explained just by the composition of the theoretical quantity $\hat{\upsilon}_{\beta} = \left(\widehat{X}_{0.95} + \widehat{X}_{0.05} - 2\,\widehat{X}_{0.50}\right)/\left(\widehat{X}_{0.95} - \widehat{X}_{0.05}\right)$ from which it is derived, where a huge role is played by the median value, $\widehat{X}_{0.50}$.

Scale parameter

The evaluation of the performances of the different estimators with respect to the third parameter is somehow affected by the unclear results from the Koutrouvelis column, that cuts horizontally with a dark red the table.

This pattern may arise from different scenarios, but since the remaining estimators tend to confirm the both range and value of $\hat{\gamma}_M$, we deduce that the problem is specific of the Koutrouvelis algorithm. Again though, the origin of the problem may be twofold: it might derive from the theoretical nature of the algorithm itself or if can be a specific bug in the code we implemented.

We have not the confidence to consider the first hypothesis, also because as we have seen in Chapter 3, the algorithm is based on preliminary phases that considers quantile estimation and characteristic function approximation. Both of the procedures, considered separately as in Table 6, do not give us any bad feelings about how distortive could result an iterative procedure based on them. The algorithm should though properly work and asymptotically tend to a value closer to the $\hat{\gamma}_M$ specification.

We accept then the idea that there may be some misspecification in the version of the algorithm we adopted, since is highly improbable a log deviation such that: high, unidirectional and concentrated. Moreover the fact that this issue has come out just for the third (and fourth, as we will see) parameter, while the first two estimates $\hat{\alpha}_K$ and $\hat{\beta}_K$ are apparently highly reliable, provides a path to follow in the research of the bug.

We know that in the Koutrouvelis algorithm a key part is played, in the second step, by the subsequent standardization and de-standardization of the Stable distribution. Thus we are focusing our attention on the part of the code that deals with those procedure. Unluckily by the time of this publication we have not yet identified the specific problem.

For what concerns the other estimators, once again, the one providing results most similar to the Mathematica ones is the STABLE maximum likelihood. Also the characteristic function method works properly, with just the tendency of over evaluating the scale parameter.

The results from the quantile approximation are more hazy. All the values but four are associated to a quite consistent positive log deviation.

Location parameter

The estimates regarding the final parameter are evaluated in deviation and harmonized by the scale parameter value. This is why, again, the column related to the Koutrouvelis estimator is not performing so well.

The STABLE maximum likelihood and the empirical characteristic function method are the best performing and, surprisingly, almost identical. We can derive the better estimation among the two just if we consider two digits after the decimal separator, discovering that this time, the most similar fit to the Mathematica procedure is the sample characteristic method.

The quantile procedure is not so reliable, because despite having an average percentage deviation that is not extremely big, its values are highly volatile.

Stable parameter estimation: remarks about the implementation

Another aspect that this study proposed to analyze was a comparison between the estimators not only in terms of precision, but also considering their quickness. How much times does it take for an estimator to compute its evaluations is a key variable, if we consider the modern financial market.

An efficient procedure that correctly guess with high approximation the characteristic of singular assets and then evaluate the risk exposition may take an enormous amount of time if the number of assets is high and the observations to consider for each of them are in the order of thousands. As the time pass, the solution that eventually is obtained became less and less efficient, since price patterns may have changed, or other different variables may have influenced and changed the modeled framework.

Effective benefits from a highly structured estimation system may though be not so interesting compared to the costs that can be derived to maintain it, in terms of functionality and also in operational terms, as we consider the work of an analyst beyond the estimator procedure.

Unluckily we had not the opportunity to process all our computations with the same processors. The estimates from Mathematica have been implemented in a eight core processor at 3.4 Ghz. The Gauss code implementation needed instead a 32 bit operative system and though it has been used a 3.00 Ghz processor. Eventually all the calculations exploiting the Fortran executable software STABLE were carried on a dual core, 2.00 Ghz processor.

The Mathematica maximum likelihood estimation took on average one second per time series analized, the Koutrouvelis algorithm took 5.8 seconds and for what concerns the three STABLE estimates, (quantile method, sample characteristic function method and quick maximum likelihood procedure), the total amount of time to fit an asset can be assessed around 2 seconds.

We highlight again that the timing is not directly comparable, and it is reported just to make a rough evaluation. We need to specify also that the Gauss code we implemented integrates the Stable parameters estimation with descriptive statistics and creates a double histogram of frequencies that provide an immediate visual representation of the results. For this reason the gross measure of 5.8 second has to be reduced by an undefined amount of time between 0.5 and 1 second.

The remark that concerns the STABLE program is centered instead on the fact that it has to be commanded through an interface, and that a more accurate estimation of the time consumed for the data fit is non-trivial.

An ideal solution could be the comparison of those different techniques adopting the same processor and the same code language, to assess eventually the real characteristics of each estimator considering the trade-off between speed and approximation.

This will likely be a topic to develop in further studies.

Value at Risk and expected shortfall

The last issue we cover is the evaluation of our estimated Stable dataset in terms of risk assessment. Following the guidelines reported in Chapter 3, we will proceed computing the Value at Risk measure and the expected shortfall, and make some evaluations.

Stable and normal distribution

The reference value is again from the estimates of Mathematica software. In the appendix, Table E, are reported all the value of ${VaR^M}_{0.99}$, that is to say the value at risk computed at the 99% confidence level. The time gap considered is of one day, and the estimation procedure is the parametric one. In the table are reported also the ${ES^M}_{0.99}$ measures, related again to the confidence value $k = 0.99$.

Those two measures are compared first of all to the values computed following the assumption of normal distribution of the assets, and then related also to a different parametric procedure implemented with sample generations.

The second mentioned procedure was based on the generation of Stable random variable, characterized by the use of the parameters $\hat{\alpha}_M$, $\hat{\beta}_M$, $\hat{\gamma}_M$ and $\hat{\delta}_M$ from the Mathematica maximum likelihood estimator. To each assets was then associated a vector of 100000 observations distributed with the same stability, symmetry, shape and location parameter. To assess the value of VaR and expected shortfall, the quantile procedure was followed, so for each vector we create the correspondent loss function and we identified the 0.99 quantile as VaR. The expected shortfall was then derived as:

$$ES^k(X) = -\frac{1}{1-k}\mathbb{E}\left[X\mathbf{1}_{\{X \geq x^{(k)}\}}\right],$$

where we remind $\mathbf{1}$ being a dichotomous parameter.

Table 7 help us to understand the differences between the risk assessment methodologies. A selection of 26 assets is taken to depict the general scenario. Again the statistics at the bottom, refers not just to this subset of value, but to the whole dataset.

The first two columns of values report the VaR and the expected shortfall as computed from Mathematica. The mean VaR is -0.0266687, and the average value for the expected shortfall is -0.0593724. Those measure account for the risk profile in a Stable financial world.

The remaining four columns are instead the simple difference between those values with the quantile parametric measure and the value derived following the normality assumption. We have chosen this comparison method considering that both the measure of risk are considered monetary ones. For this reason the value listed on the last four columns can be interpreted as additional of lacking economic capital compared to the one taken following the Stable assumption.

Table 7- Selection from appendix, Table E: comparison between VaR and expected shortfall deriver by the Mathematica software, a quantile parametric calculation (Q) and the value for the correspondent normal measures of risk.

			$VaR^M{}_{0.99} - VaR^i{}_{0.99}$		$ES^M{}_{0.99} - ES^i{}_{0.99}$	
$VaR^M{}_{0.99}$	$ES^M{}_{0.99}$		**Q**	**N**	**Q**	**N**
-0.0266981	-0.0564898	**AA**	-0.0004460	-0.0028033	0.0000463	-0.0290335
-0.0185550	-0.0349041	**ABT**	-0.0001223	-0.0053632	0.0007552	-0.0074241
-0.0355615	-0.0746373	**ADI**	-0.0003479	-0.0044376	-0.0015708	-0.0387471
-0.0241875	-0.0555644	**AFG**	-0.0000611	-0.0032399	0.0064857	-0.0312112
-0.0283299	-0.0646060	**AFL**	-0.0005698	-0.0031323	0.0098669	-0.0357232
-0.0285853	-0.0602382	**AVT**	0.0001532	-0.0038880	0.0051500	-0.0316949
-0.0239151	-0.0511853	**AVY**	0.0000615	-0.0034949	-0.0013490	-0.0277372
-0.0313190	-0.0695846	**AXE**	-0.0003115	-0.0050267	0.0095875	-0.0394549
-0.0279281	-0.0610198	**AXP**	-0.0003377	-0.0041950	0.0027550	-0.0337014
-0.0226532	-0.0458660	**BA**	0.0018706	-0.0028934	0.0106914	-0.0230103
-0.0303761	-0.0828766	**AMN**	-0.0001499	-0.0080154	0.0050680	-0.0570321
-0.0303208	-0.0691964	**TXI**	-0.0010526	-0.0048314	-0.0071716	-0.0398697
-0.0193294	-0.0406834	**BDX**	-0.0007263	-0.0023853	-0.0017084	-0.0213370
-0.0314412	-0.0731767	**BEN**	-0.0003186	-0.0066419	-0.0030195	-0.0448890
-0.0180075	-0.0388992	**BF-B**	0.0000486	-0.0030535	0.0201802	-0.0217473
-0.0245979	-0.0533779	**BGG**	-0.0006796	-0.0044543	-0.0026416	-0.0302497
-0.0274939	-0.0640570	**BT**	0.0003471	-0.0058447	-0.0041829	-0.0392859
-0.0370617	-0.0922253	**BWS**	0.0001578	-0.0090391	-0.0042823	-0.0600620
-0.0251476	-0.0582168	**CI**	0.0003545	-0.0035615	-0.0003662	-0.0335075
-0.0178255	-0.0361597	**CL**	-0.0003385	-0.0013177	0.0051618	-0.0172466
-0.0218857	-0.0512194	**CLX**	0.0001634	-0.0045510	0.0254419	-0.0311771
-0.0300880	-0.0707014	**CMI**	0.0002956	-0.0061714	0.0110756	-0.0430597
-0.0261306	-0.0651471	**CNA**	-0.0001949	-0.0042085	-0.0007065	-0.0399071
-0.0301810	-0.0676727	**CNW**	-0.0005349	-0.0045783	-0.0013205	-0.0381336
-0.0646015	-0.1990660	**COO**	-0.0000261	-0.0227715	-0.0229052	-0.1511294
-0.0235507	-0.0460058	**COP**	0.0002632	-0.0025767	0.0018083	-0.0219964
-0.0272263	-0.0641294	**CP**	0.0001446	-0.0064386	-0.0007429	-0.0401845
-0.0260459	-0.0561295	**CR**	0.0000031	-0.0040545	0.0018801	-0.0311027
*** ***			*** ***		*** ***	
-0.0147600	-0.0291614	**Up Bound**	0.0018706	0.0089572	0.0254419	-0.0074241
-0.0708305	-0.2055850	**Dw bound**	-0.0082689	-0.0241356	-0.0269712	-0.1522956
0.0560706	0.1764237	**Range**	0.0101394	0.0330928	0.0524131	0.1448715
-0.0266687	-0.0593724	**Average**	-0.0001134	-0.0038840	-0.0002570	-0.0332175

The table 7 allows for multiple considerations. The first one is about the role as risk measure played by the VaR. We have specified in Chapter 2 that for the range of stability parameter that characterize our dataset, the measure can be considered coherent, but still its adoption can lead to evaluation errors. Consider as example the two assets on the green colored cells.

They are characterized by a similar value of VaR, -0.0303, so an operator that is assessing risk profile just on the basis of this measure would likely consider the two assets equally. This is not the case, as the cell just on the right explain: the measure of the expected shortfall is substantially different, accounting for a loss tail riskier for the AMN US Equity, rather than the TXI one.

We choose to make this remark for the measure under the Stable hypothesis to stress the fact that it is important not just to correctly specify the model, but also to consider the proper tools of analysis.

For what concerns the different procedures, we can observe that the distances between the Mathematica measures of VaR and the ones from the quantile methods are generally identified at the fourth digit after the decimal separator.

The difference with the normal VaR is more evident, and as expected, the thinner tail of the distribution overestimate the value at risk for most of the assets: just three of them are characterized by a higher VaR compared to the reference. This deviation is on average -003884.

More interesting are the dynamics of the expected shortfall. The difference between the reference computation ${ES^M}_{0.99}$ is not problematic, if we consider the alternative derived value using Stable parameters. The mean deviation for the quantile approximation is -0.0002570 and the range of variation is symmetric with respect to this value.

If we consider the normal derived value for the expected shortfall, all the values reported a systemic under estimation of the extreme risk. On average the value are -0.0332175 smaller than the Mathematica value. This was expected, from all the consideration we took in Chapter 2, about how this value is derived. If the reference for the computations is a lower VaR value and the tail behavior is Gaussian, the result will be a lower conditional economic capital.

Comparison among the Stable estimation methods

We report now an evaluation of the differences between the VaR and expected shortfall measure, as derived from the parameters obtained with four different estimators.

From Table 8 we can derive some general conclusion. We remind that to obtain the VaR and expected shortfall measures the main focus is on the tail behavior. As we have seen, in that part a Stable distribution can be described as power low, meaning that among the four, the most influent parameter for the amplitude of those risk measures is necessarily the stability parameter $\hat{\alpha}_i$.

This can be inferred with a glance to the first three columns of values, the ones relative to the VaR comparison. For the estimates derived from the Koutrouvelis algorithm, the VaR measure appears abundant. The $\hat{\alpha}_K$ showed in Table 6 to be quite always surpassing the reference value, determining a thinner tail behavior. The other VaR computations, from STABLE maximum likelihood and quantile approach are instead quite centered on the reference value, with a range almost symmetric with respect to zero.

For the expected shortfall measure, the evaluation we can make are similar to the previous table. Since it is a VaR dependant measure, it inherits and amplifies the distortion. This makes the Koutrouvelis estimates again less coherent with the reference, generating an impressive upper bound of deviation.

	$VaR^M{}_{0.99} - VaR^i{}_{0.99}$			$ES^M{}_{0.99} - ES^i{}_{0.99}$		
	K	**ML**	**Q**	**K**	**ML**	**Q**
AA	0.0285053	-0.0000322	0.0031594	0.0537591	0.0003693	0.0147914
AIG	0.0417426	-0.0003208	0.0025789	0.1179531	-0.0006860	0.0141290
AIR	0.0367628	-0.0002653	0.0026173	0.0708769	-0.0002110	0.0136485
ALK	0.0340611	-0.0006789	-0.0035759	0.0565085	-0.0020332	-0.0113975
DOV	0.0234733	0.0000160	0.0036066	0.0430955	0.0004788	0.0146210
DOW	0.0269692	-0.0000449	0.0035198	0.0537643	0.0001816	0.0155876
F	0.0295707	-0.0001930	-0.0004591	0.0540927	-0.0000901	0.0002050
FDX	0.0232122	-0.0000415	0.0038222	0.0372734	0.0003357	0.0175981
GCI	0.0289585	-0.0004479	0.0017044	0.0644783	-0.0014403	0.0084115
GPC	0.0108076	-0.0084063	-0.0058235	0.0097744	-0.0246218	-0.0134405
GR	0.0249214	-0.0001973	0.0039497	0.0482386	-0.0001794	0.0175733
GRA	0.0758830	-0.0012210	0.0110360	0.1990635	-0.0036400	0.0815425
GT	0.0355672	-0.0007821	0.0020581	0.0770789	-0.0024364	0.0110466
GWW	0.0216910	0.0000399	0.0044393	0.0409800	0.0005617	0.0185244
HAL	0.0333914	-0.0004927	0.0006880	0.0597976	-0.0014648	0.0044210
HAS	0.0270378	-0.0000650	0.0062836	0.0509066	0.0005816	0.0288057
PNW	0.0252606	0.0000976	0.0015873	0.0527792	0.0007521	0.0067917
PPG	0.0237387	-0.0000636	0.0035395	0.0440825	-0.0000873	0.0134959
R	0.0276607	-0.0003100	0.0039217	0.0500005	-0.0007125	0.0148974
RDC	0.0391311	-0.0003313	-0.0048463	0.0428326	0.0000986	-0.0192227
WMB	0.0423816	-0.0006104	0.0031730	0.1008993	-0.0021176	0.0154457
WMS	0.0391349	-0.0006862	0.0018409	0.0705593	-0.0012924	0.0099287
WMT	0.0239296	-0.0000326	0.0046060	0.0415526	0.0002244	0.0200480
WRB	0.0235245	-0.0001266	0.0049712	0.0459160	-0.0003674	0.0226763
WY	0.0255405	-0.0012356	0.0019316	0.0410743	-0.0052953	0.0105971
XOM	0.0216129	-0.0011241	0.0017792	0.0406632	-0.0043988	0.0063305
	*** ***			*** ***		
Up Bound	0.0758830	0.0129599	0.0179051	0.2129834	0.0351714	0.0815425
Dw bound	0.0066337	-0.0136590	-0.0104323	0.0002087	-0.0372394	-0.0235483
Range	0.0692493	0.0266189	0.0283374	0.2127747	0.0724108	0.1050908
Average	0.0293095	-0.0003858	0.0034702	0.0589711	-0.0010357	0.0160319

Also the effects on the quantile approach derived values generates an unbalance since the range of variation is no more centered on zero. The STABLE maximum likelihood keep performing well, and again we justify those results considering the high similarity of the two software. Table 8- Selection from appendix, Table F: comparison between VaR and expected shortfall as derived by the Mathematica software and the same risk measure, computed with Stable parameters from K=Koutrouvelis (Gauss code), Q=Quantile method (STABLE), ML=Maximum likelihood (STABLE).

Stable estimation: back test

As last analysis we tried a back test procedure, to control for eventual bias in the estimator procedure.

We generated 50 series of 7000 observations, as Stable distributed random variable. The number of records has been choosen to maintain coherence with the original dataset. For the same reason the values for the stability parameter were choosen inside the range defined by the estimation of Mathematica, so: $1.4570 < \alpha < 1.8904$. Concerning the other three parameters we have randomly choosen between their mean, maximum and minimum values.

Then we run our five estimators, to fit those 50 artificial sample. Table 9 summarize the results.

Our setting is generally confirmed. The better fit for the Stability parameter is the one of the Mathematica maximum likelihood, that has the closest value and the minimum standard deviation among the five estimators. The second best fit is somehow shared between the Koutrouvelis algorithm and the sample characteristic function method, since the first has lower standard deviation and the second is more centered. It follows quantile procedure and STABLE maximum likelihood.

The overall tendency is to overestimate the original value, but is an error that occurs most of the time at the second digit after the decimal separator. This should be consider as uncertainty measure for all the table shown above.

We can notice for all the five estimator some shared patterns: lines that appear all red (sample 2, 4) or blue (sample 7, 16) reporting similar deviations. It is a phenomenon that recurs also for the β parameter (line 5, 11) and for the scale one as well (line 3, 8, 17, 18). Those systemic deviations seem nevertheless to be parameter specific rather than generic for the whole sample estimation. It should be interesting to study what are the analytical justification beyond those little bias.

The symmetry parameter appears instead to be best estimated by the Koutrouvelis method, even if it is not the one reporting also the smaller standard deviation. Then follows the two maximum likelihood methods, and the other two estimators. Coherently with the α general abundant estimation, for this parameter the value is generally underestimated. It is confirmed the dynamics already explained about the balance between the first two Stable parameter

The γ parameter is the most problematic to discuss. All the estimators have similar standard deviation and comparable average value (apart from the Koutrouvelis one that seems to be less accurate). The tendency of all the estimator is to report abundant outputs.

Table 9- Selection from appendix, Table G: comparison between the original value and the five estimators. Measures: for α: $\alpha - \hat{\alpha}_i$, for β: $\beta - \hat{\beta}_i$, for γ: $ln(\gamma/\hat{\gamma}_i) * 100$, for δ: $(\delta - \hat{\delta}_i) * [(\gamma - \hat{\gamma}_i)/2]^{-1} * 100$. M ML= Mathematica maximum likelihood.

	α					β					γ					δ				
	M ML	**K**	**Q**	**ML**	**ChF**	**M ML**	**K**	**Q**	**ML**	**ChF**	**M ML**	**K**	**Q**	**ML**	**ChF**	**M ML**	**K**	**Q**	**ML**	**ChF**
1	-0.0291	-0.0357	-0.0189	-0.0267	-0.0315	0.0406	0.0802	0.0533	0.0409	0.0672	1.7622	1.6412	3.1776	1.9032	1.6224	2.241%	3.577%	0.566%	2.183%	4.219%
2	-0.0370	-0.0392	-0.0489	-0.0369	-0.0366	-0.0528	-0.0819	-0.0369	-0.0532	-0.0783	-0.0785	-0.3719	0.0148	-0.2716	-0.2356	-3.747%	-17.989%	5.152%	-3.803%	-5.511%
3	-0.0088	-0.0157	0.0012	-0.0087	-0.0151	-0.0082	-0.0534	-0.0106	-0.0077	-0.0294	3.1095	3.0031	3.6353	3.0139	2.7788	-1.280%	-14.214%	2.787%	-1.304%	-2.142%
4	-0.0340	-0.0319	-0.0458	-0.0343	-0.0357	-0.0492	-0.0172	-0.0381	-0.0491	-0.0350	-1.4716	-1.4003	-2.4184	-1.5082	-1.5900	-1.361%	-6.031%	4.213%	-1.353%	-0.675%
5	-0.0037	-0.0095	-0.0305	-0.0047	-0.0062	0.1108	0.0872	0.1170	0.1101	0.0746	-0.3713	-0.6173	-1.6747	-0.4429	-0.4714	6.538%	3.626%	2.839%	6.470%	4.588%
6	-0.0011	-0.0011	-0.0430	-0.0062	0.0023	-0.0092	0.0092	-0.0196	-0.0088	-0.0051	0.7931	0.4515	-1.0303	0.5786	0.8547	-0.790%	-6.566%	3.068%	-0.777%	-0.624%
7	0.0098	0.0103	0.0294	0.0068	0.0118	0.0930	0.0957	0.1184	0.0924	0.1190	-0.3006	-0.0681	0.4128	-0.3815	-0.3268	3.732%	0.858%	-0.511%	3.619%	4.751%
8	-0.0127	-0.0208	0.0030	-0.0043	-0.0324	0.0741	0.0827	0.0264	0.0729	0.0630	2.6667	2.7220	3.4466	2.8819	2.1994	4.329%	2.164%	3.418%	4.343%	3.863%
9	-0.0151	-0.0246	0.0250	-0.0106	-0.0417	0.0622	0.0331	0.0598	0.0666	0.0384	-1.7980	-1.3833	-0.4847	-1.6270	-2.2968	0.032%	-7.726%	-2.123%	0.154%	-0.500%
10	-0.0069	-0.0089	-0.0486	-0.0222	-0.0066	0.0685	0.0634	-0.0553	0.0634	0.1174	0.9697	0.3482	-0.2739	0.3848	0.6170	4.302%	1.725%	12.042%	4.379%	5.300%
11	-0.0209	-0.0233	-0.0093	-0.0509	-0.0146	-0.1560	-0.1788	-0.2121	-0.1898	-0.1265	-0.1506	-0.3771	0.4408	-0.7589	-0.0937	-0.092%	-4.894%	12.889%	0.606%	0.213%
12	0.0108	0.0073	0.0145	-0.0170	0.0008	0.0355	0.0148	0.0599	0.0135	-0.0174	2.1753	2.0987	2.3638	1.6319	1.8870	-1.598%	-7.662%	4.592%	-1.157%	-2.051%
13	0.0162	0.0289	-0.0214	0.0119	0.0381	0.0530	0.1692	-0.0778	0.0540	0.1077	-0.8538	-1.0222	-1.2967	-0.9252	-0.5322	0.182%	-1.181%	2.948%	0.174%	0.568%
14	-0.0185	-0.0161	-0.0260	-0.0172	0.0046	-0.0108	-0.0180	-0.0616	-0.0107	-0.0103	-0.1743	-0.5165	-0.5214	-0.3176	0.5791	1.762%	-1.991%	20.048%	1.703%	0.902%
15	-0.0459	-0.0485	-0.0610	-0.0465	-0.0404	0.0127	0.0230	0.0026	0.0119	0.0259	0.0800	-0.2777	-0.9611	-0.0938	0.1931	0.922%	-0.931%	11.316%	0.831%	1.643%
16	0.0269	0.0300	0.0107	0.0269	0.0518	-0.0419	0.0219	-0.0585	-0.0412	0.0268	0.6519	0.5118	0.0298	0.6343	1.6969	-3.951%	0.482%	14.789%	-4.001%	0.209%
17	-0.0201	-0.0051	-0.0337	-0.0213	0.0052	-0.0587	0.0416	-0.0776	-0.0556	0.0012	2.3951	2.4194	2.5074	2.4351	3.2357	-0.287%	3.778%	19.572%	-0.098%	1.273%
18	-0.0073	-0.0141	-0.0134	-0.0117	-0.0271	-0.0519	-0.0122	-0.0241	-0.0533	-0.0129	-3.0704	-2.9169	-3.2749	-3.2504	-3.7744	-6.424%	-4.153%	13.284%	-6.356%	-3.641%
19	-0.0036	0.0042	-0.0307	-0.0054	0.0106	0.0052	0.0138	-0.0246	0.0038	-0.0146	-1.2471	-1.5708	-2.3664	-1.4398	-1.0494	-1.658%	-1.716%	7.151%	-1.732%	-3.086%
20	0.0046	0.0045	0.0225	0.0097	0.0134	0.0285	0.0391	0.0215	0.0281	0.0260	0.1234	-0.0162	1.0719	0.1815	0.3045	1.330%	1.667%	9.621%	1.207%	1.082%
	*** ***					*** ***					*** ***					*** ***				
Dw bound	-0.0469	-0.0549	-0.0640	-0.0565	-0.0509	-0.1560	-0.1788	-0.2798	-0.1898	-0.1265	-3.0704	-2.9169	-3.2749	-3.2504	-3.7744	-6.42%	-17.99%	-2.12%	-6.36%	-5.51%
Up bound	0.0305	0.0378	0.0724	0.0320	0.0518	0.2694	0.2738	0.2391	0.2708	0.3712	3.1095	3.0031	3.6353	3.0139	3.2357	6.54%	3.78%	25.44%	6.47%	5.30%
Range	0.0774	0.0927	0.1364	0.0886	0.1027	0.4254	0.4526	0.5189	0.4606	0.4977	6.1798	5.9200	6.9102	6.2644	7.0101	12.96%	21.77%	27.56%	12.83%	10.81%
Average	-0.0056	-0.0073	-0.0089	-0.0097	-0.0057	0.0159	0.0219	0.0050	0.0117	0.0261	0.3722	0.2948	0.2438	0.2666	0.3526	0.17%	-2.49%	6.45%	0.18%	0.25%
St. Dev.	0.0189	0.0207	0.0287	0.0224	0.0236	0.0690	0.0771	0.0928	0.0704	0.0853	1.3415	1.3361	1.7075	1.3874	1.3850	2.38%	5.12%	6.36%	2.37%	2.51%

The shift parameter attest again the best approximation to the maximum likelihood methods, followed by the characteristic function procedure and by the remaining two. For this parameter there are no general tendency also if we can note some estimator specific bias, like for Koutrouvelis.

The estimators are then differently characterized, and their procedures of fit are, even if related, quite different. Considering expecially the estimation of the stability parameter, we have seen that generally the results are reliable, but it is important to remind how this last test have been implemented.

We have considered just a portion of the parameter space of the Stable distribution. This choice is related to the fact that we focused our study on just a particular kind of financial assets. Enlarging the dataset to different other securities may change substantially the results and confute our results

This summarize and end our analytical work. Many other question arose in the data analysis phase, and hopefully we will be able to dedicate further studies on them.

Conclusions

Chapter 4 has questioned Stable behavior, Stable estimators, adequacy of measures of market risk. The derived remarks and comments are so many and different that there is the risk to lose the focus on the overall new perspective assumed.

We summarize then the main results obtained:

- The analysis of 188 log return series from common assets of the NYSE allowed the comparison between the Stable distribution fit and the normal one. Tested by Jarque-Bera, the normal assumption is always rejected.
- The estimation of the Stable parameters has been differentiated, in order to permit a comparison between different easily applicable procedures and the professional one of maximum likelihood. Difference in terms of accuracy and systematic deviations have been extensively discussed.
- The estimator procedures have been compared in terms of time required for the calculation, deriving simple remarks to consider for a further complete analysis.
- VaR and expected shortfall have been computed. The difference between the use of Stable approximation and the usually adopted normal distribution have been identified. If normal assumption is followed, the risk measures are under estimating extreme events.
- The last implemented test on the estimators allow to understand up to which approximation is it possible to rely on the results obtained with the different implemented estimators.

All those considerations derive just from the creation of a framework slightly more complex than the usual one, where it is possible to connote with more realism variables and processes. This happens indeed just because the assumption of normality is relaxed.

As we realized during the development of this work, the costs of this more complex framework in terms of computational procedure or technical representation is not so heavy. We shown that with tools of analysis freely available it is possible to obtain reliable output and base on them researches and studies.

For this reason at the end of this paper, we would like to suggest some ideas to steer new applications.

The comparison between the estimators of Stable random variables is not complete. A Monte Carlo simulation, testing in depth different fits should be carried, possibly deriving standardized formalizations of the codes to use and implementing them in the same processor to evaluate also efficiency.

Regarding risk assessment, the possible applications are multiple. There are already examples in the literature of portfolio optimization that adopt Stable distribution combined with the expected shortfall measure, but again, such redefinitions of models like the CAPM models[29] or multifactor depending pricing

[29] Sharpe(1964)

models are not exhaustive. Multivariate properties and new forms of interdependence can be much more exploited, expecially now that the calculus capability of our computers is exponentially growing.

Eventually we had the chance to notice that the use of Stable distributions in economics is mostly related to the financial field. It should be interesting to exploit those developed tools also in other economics sectors, looking for Stable random variables inside a the production structure of a company, or among some of the macroeconomics entities that governs our market and their dynamics.

It is impossible to define a complex system in all its components and to catch and understand the dynamics that it produces. Nevertheless, as analysts we have to be flexible enough and to derive the compromise more adequate to the research fulfilled. We believe that Stable distribution can represent a convenient tool to adopt to unburden the consequences of such a compromise.

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Appendix

Table A: Descriptive statistics for the 188 NYSE assets

	AA_US_EQUITY	ABT_US_EQUITY	ADI_US_EQUITY	ADM_US_EQUITY	ADP_US_EQUITY	AEP_US_EQUITY	AET_US_EQUITY	AFG_US_EQUITY	AFL_US_EQUITY	AIG_US_EQUITY	AIR_US_EQUITY	ALK_US_EQUITY	AM_US_EQUITY
Mean	0.008906245	0.051345795	0.054055518	0.036640221	0.051786432	0.013026227	0.062416233	0.029568753	0.074127462	-0.000536666	0.024275238	0.047323808	0.008431318
Median	-0.070696362	0.06920487	-0.036663612	0.031093143	0.047283595	0.076016727	0.044434571	-0.059844406	0.047653086	0.015779562	-0.182080318	-0.139811278	0.071454094
Maximum	20.87336563	11.74816508	18.99080907	15.98868332	16.40904639	18.10016735	16.34187766	37.41066064	26.45053382	50.68186091	21.6340804	27.21997083	41.38158873
Minimum	-27.58476148	-17.59939806	-21.47887469	-22.18031969	-27.89447587	-25.85739883	-22.70278894	-25.76110234	-45.98540427	-93.62578055	-31.64245413	-33.64063409	-55.17355265
Std. Dev.	2.351952755	1.698138032	3.109323064	2.148161761	1.773018725	1.490235887	2.395324256	2.113667436	2.519756664	3.259347016	2.983387004	2.920963736	2.636529968
Skewness	-0.247965314	-0.244049166	0.102195969	-0.217972601	-0.778358217	-0.478517325	-0.637658351	1.19036846	-0.72601397	-3.54958163	0.012517274	-0.120044895	-2.018086468
Kurtosis	12.12731918	7.607994781	6.488737659	9.672584033	20.93879951	25.10685817	13.32343591	35.63517079	26.35146025	146.5400326	9.578162912	11.26113403	67.215035
Jarque-Bera	27520.56665	7043.663391	3838.465031	13913.32769	105129.5976	147317.1535	13998.34415	338482.081	171860.5031	6395037.809	13104.46263	21211.23135	1217636.738
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7905	7873	7543	7468	7782	7221	3105	7587	7535	7431	7268	7453	7059

	AMD_US_EQUITY	AME_US_EQUITY	AMGN_US_EQUITY	AMN_US_EQUITY	AON_US_EQUITY	APA_US_EQUITY	APD_US_EQUITY	ARW_US_EQUITY	ASH_US_EQUITY	AVP_US_EQUITY	AVT_US_EQUITY	AVY_US_EQUITY	AXE_US_EQUITY
Mean	0.002188385	0.065972832	0.093280394	0.038450592	0.040596839	0.033881676	0.036691143	0.023138473	0.023007026	0.012019595	0.018636475	0.037269096	0.056241544
Median	-0.161095484	0.092703546	0.016354567	0.027569221	0.094824673	0.024444349	0.016301247	-0.127186027	0.035249009	-0.090428024	-0.0365297	0.07191658	0.091911771
Maximum	23.84110234	14.54985124	22.44686398	24.98546842	19.95830855	19.32397794	13.67534689	33.02416869	15.2539725	18.49223385	25.48922496	17.75138552	23.36148512
Minimum	-47.69240721	-16.73210812	-22.31521999	-40.64828257	-36.14030851	-20.13389213	-24.85372421	-37.03737883	-28.95460059	-38.54308349	-23.51918898	-16.59851375	-28.58713609
Std. Dev.	3.933784396	1.98665281	2.66243414	2.234930443	1.93769717	2.5658145	1.890572766	3.094110222	2.092916409	2.176707045	2.45871297	1.998162783	2.631613039
Skewness	-0.613272244	-0.095317012	0.080229609	-0.901859318	-1.666450028	0.122143883	-0.232916582	0.017368514	-0.70808919	-0.886587563	-0.125568413	-0.204786469	-0.201398538
Kurtosis	13.00708225	8.742980804	9.005395415	32.58077155	40.95515401	5.947400863	10.29827231	13.08046543	16.42305823	26.09253129	10.55120395	9.527215051	11.56749532
Jarque-Bera	32751.22257	5332.434431	10579.03404	246502.6419	464181.695	2697.673815	17423.91802	31251.45843	58192.62305	207277.4363	18461.83639	13649.70587	20276.13856
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7733	3876	7035	6736	7674	7402	7819	7381	7666	9274	7762	7659	6615

	AXP_US_EQUITY	BA_US_EQUITY	BAX_US_EQUITY	BC_US_EQUITY	BCR_US_EQUITY	BDX_US_EQUITY	BEN_US_EQUITY	BF_B_US_EQUITY	BGG_US_EQUITY	BK_US_EQUITY	BLL_US_EQUITY	BMS_US_EQUITY	BMY_US_EQUITY
Mean	0.043322325	0.035599163	0.034054107	0.040817166	0.056970328	0.047953928	0.080488008	0.049766523	0.017258791	0.037367006	0.053113978	0.053140375	0.034763739
Median	0.017870669	0.015150367	0.067773639	-0.053092648	0.05075114	0.082101811	0.090912002	0.069055039	-0.035937201	0.024411195	0.090244569	0.088948198	0.071958579
Maximum	18.77115508	14.37773538	12.88327483	41.70068685	19.8316147	16.44723276	26.38470767	14.90536187	13.12806602	22.15943643	14.99993087	11.77830357	14.73791723
Minimum	-30.42122296	-19.38930814	-30.49539265	-51.59407244	-20.64919177	-25.36027588	-24.92367684	-10.41139299	-15.35721435	-31.68744401	-27.33899325	-19.90542985	-24.57103225
Std. Dev.	2.34641355	1.986434978	1.923890499	3.028015168	2.023665989	1.708023652	2.499338598	1.510178245	2.011988715	2.302042699	1.889584324	1.825047554	1.749184301
Skewness	-0.256802469	-0.199254006	-1.300878975	-0.48779394	-0.08374537	-0.506291513	-0.071331353	0.382003702	-0.096325169	-0.055537457	-0.223399948	-0.20691871	-0.490398505
Kurtosis	12.59313871	8.329724253	22.12167979	27.68261505	8.731544104	16.5334142	11.12713261	7.979229005	7.460010228	17.06590879	12.49900839	8.095416533	14.04918042
Jarque-Bera	30352.56448	9384.716896	119279.1093	192233.1504	10595.14527	60172.69138	18554.83727	6455.129074	6320.752777	64000.15405	28640.18495	8092.993117	40471.99551
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7893	7885	7687	7561	7734	7841	6740	6105	7612	7763	7601	7432	7894

	BOH_US_EQUITY	BP_US_EQUITY	BT_US_EQUITY	BWS_US_EQUITY	C_US_EQUITY	CA_US_EQUITY	CAG_US_EQUITY	CAT_US_EQUITY	CB_US_EQUITY	CBE_US_EQUITY	CBRL_US_EQUITY	CBT_US_EQUITY	CCK_US_EQUITY
Mean	0.041458321	0.021045657	0.014720519	0.019212717	0.017340409	0.065432141	0.048152427	0.031724984	0.046286994	0.025339616	0.057003975	0.030162332	0.03331554
Median	0.081251277	0.093309702	0.01572698	-0.074103375	-0.021597721	0.080645493	0.080808085	0.011039947	0.023821961	0.043651419	0.119189525	-0.001984319	0.080364566
Maximum	12.94580672	14.72598873	29.34157228	29.69249395	45.63162181	26.71555923	14.17940506	13.73497369	15.53657468	24.24689768	22.38306005	19.76536411	37.33767936
Minimum	-25.50765984	-17.19223524	-22.16609714	-22.57303192	-49.46962418	-55.20042426	-23.93512198	-24.36203785	-13.41960653	-28.83553162	-19.8933697	-19.92760699	-41.52212831
Std. Dev.	1.833021633	1.772281717	2.154063223	2.77869135	3.06681548	3.151919575	1.783790069	2.079964049	1.726907004	2.061363193	2.575708552	2.206000479	2.774823992
Skewness	-0.51484482	-0.341600083	0.026063186	-0.134587126	-0.544276913	-1.411095985	-0.648578762	-0.323604158	0.255463524	-0.391127524	-0.01587726	-0.274498413	0.292826635
Kurtosis	15.89528734	9.680645037	15.21753581	13.47692536	37.11622533	31.19139346	15.41911159	9.488399138	9.915798657	18.75368386	8.619181269	11.47266789	33.6075051
Jarque-Bera	46285.86026	14778.91851	41752.31423	34406.88497	309140.339	242220.5888	49523.12133	14041.6362	14811.50979	79094.63949	8613.725993	22791.66418	302781.5386
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	6638	7865	6713	7518	6368	7242	7623	7926	7392	7630	6547	7588	7754

	CEG_US_EQUITY	CI_US_EQUITY	CL_US_EQUITY	CLX_US_EQUITY	CMI_US_EQUITY	CNA_US_EQUITY	CNW_US_EQUITY	COO_US_EQUITY	COP_US_EQUITY	CP_US_EQUITY	CPB_US_EQUITY	CR_US_EQUITY	CSC_US_EQUITY
Mean	0.02205625	0.033009576	0.053838067	0.055184549	0.042247752	0.024725031	0.020604062	0.018894229	0.030497298	0.065479185	0.041895685	0.034953053	0.032144276
Median	0.162206037	0.101303617	0.025505324	0.080638611	-0.050869141	-0.066755677	-0.123970617	-0.0809127	0.065707647	0.051384267	0.032633005	0.13330373	-0.030316811
Maximum	23.18020638	21.13946588	18.20803314	12.43429666	19.94195223	24.10587564	21.79139755	53.89203132	15.73755015	13.87213076	13.63450375	26.45996622	17.00996398
Minimum	-44.47775112	-47.91014934	-21.46569374	-17.77304655	-20.74923231	-39.47637951	-22.87484235	-40.54651081	-22.7394762	-14.27707562	-14.10785983	-26.58748191	-50.34591079
Std. Dev.	1.788586228	2.160575028	1.668155347	1.73905223	2.393229757	2.173485011	2.551015573	4.151824662	2.082554696	2.090510364	1.660263571	2.166036529	2.406047223
Skewness	-3.627795398	-1.857060443	-0.090403656	-0.107421442	-0.083522214	-0.701895515	-0.082580478	0.618210575	-0.252247543	-0.216338667	0.206734629	-0.160501679	-1.473101887
Kurtosis	85.14675456	47.01018938	13.35515473	9.65617371	10.03199229	30.33500342	8.701004367	19.75160865	9.411474095	8.085727236	9.02288559	13.64953591	34.87621536
Jarque-Bera	1995731.737	609591.0822	34726.04683	14177.47429	16244.88902	240266.0374	10372.59844	76473.08621	13239.13171	3178.323406	11835.87346	35989.19346	330873.6542
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7043	7500	7770	7672	7880	7697	7653	6505	7682	2928	7794	7609	7749

	CSX_US_EQUITY	CVS_US_EQUITY	CVX_US_EQUITY	DBD_US_EQUITY	DD_US_EQUITY	DE_US_EQUITY	DIS_US_EQUITY	DNB_US_EQUITY	DOV_US_EQUITY	DOW_US_EQUITY	DUK_US_EQUITY	ED_US_EQUITY	EIX_US_EQUITY
Mean	0.042064302	0.042912883	0.031899065	0.029018309	0.025402681	0.035009317	0.045534524	0.053915203	0.042944541	0.018804007	0.030124594	0.030544352	0.027916304
Median	0.055030727	0.032600336	0.105343253	-0.027874827	-0.00109607	0.024106082	0.078895468	0.058130378	0.053213624	0.031740994	0.122215746	0.120862238	0.148360628
Maximum	14.10317502	16.73130723	18.94149053	47.64092716	10.85589879	14.88456276	17.47679002	12.59532459	14.5979579	16.91585703	14.97803886	9.047964584	30.25706849
Minimum	-20.34625715	-26.12579823	-18.23215568	-25.26630462	-20.17989097	-18.14979231	-34.38176848	-15.17756952	-17.76811772	-23.30163257	-18.02601916	-17.95399138	-42.66531884
Std. Dev.	2.045057831	1.94771222	1.713792896	2.105275579	1.807767539	2.157687567	2.082423987	1.636234588	1.88046656	2.075179586	1.479135656	1.267715261	1.942892319
Skewness	-0.188715292	-0.686700572	-0.108476516	1.350192287	-0.270360248	-0.159474271	-0.661713831	-0.58255903	-0.037450392	-0.425606401	-0.430052084	-0.396875132	-1.497704743
Kurtosis	8.104471787	15.89623712	10.59398809	45.84940602	8.272568944	7.534581185	17.62063928	13.73831979	7.446829342	11.26071988	15.35019218	11.52681001	73.17813722
Jarque-Bera	8552.583515	54762.84402	18981.25866	594945.6256	9277.502526	6772.676701	85065.39035	15254.47332	6362.534926	22697.79499	47033.81863	22550.98251	1518009.926
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7835	7814	7893	7746	7926	7866	9473	3138	7720	7899	7365	7380	7384

	EMR_US_EQUITY	ETN_US_EQUITY	ETR_US_EQUITY	EXC_US_EQUITY	F_US_EQUITY	FDX_US_EQUITY	FL_US_EQUITY	FRM_US_EQUITY	FWLT_US_EQUITY	GAS_US_EQUITY	GCI_US_EQUITY	GPC_US_EQUITY	GR_US_EQUITY
Mean	0.036772309	0.043689891	0.023518738	0.022114551	0.026215218	0.034582337	0.021643686	-0.003388127	-0.028354314	0.027804419	0.016981183	0.035910092	0.033951271
Median	-0.019667617	0.066520324	0.064998377	0.119904263	-0.126859739	-0.092948886	-0.194567504	-0.385728899	-0.310077768	0.089605741	-0.04974712	0.023250988	-0.015886885
Maximum	14.32206687	17.55982594	17.48027244	15.87232165	25.86502398	13.85861633	18.33373009	22.31435513	37.15635564	8.217799185	33.22146464	9.649873116	14.70046503
Minimum	-17.34648931	-29.77083733	-19.9648723	-12.54935576	-46.49159225	-64.4351937	-32.91595898	-36.73797741	-65.69441319	-10.37217021	-27.40764204	-11.60738783	-28.76820725
Std. Dev.	1.71042445	1.837115507	1.744145241	1.66138737	2.54822227	2.272751546	2.607644701	4.166995879	3.819013734	1.359405551	2.188757425	1.544331457	2.106929268
Skewness	-0.159748164	-0.417670394	-0.642420788	-0.113759694	-0.784629088	-2.840564462	-0.021147235	0.056989217	-1.020539767	-0.129634037	-0.146320199	0.111185844	-0.46023184
Kurtosis	9.915226975	17.46574959	16.65913327	9.460813936	28.26948955	86.68917145	11.63890761	5.995895809	31.2718867	7.142707504	24.96250128	7.101344383	13.77962377
Jarque-Bera	15772.47373	68261.84873	55470.80991	12381.43748	209129.4059	2336002.678	23776.67494	2368.438863	249442.5859	4521.959033	157093.581	5370.434514	36660.26553
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7899	7803	7073	7110	7830	7968	7646	6324	7451	6299	7815	7640	7517

	GRA_US_EQUITY	GT_US_EQUITY	GWW_US_EQUITY	HAL_US_EQUITY	HAS_US_EQUITY	HD_US_EQUITY	HES_US_EQUITY	HNZ_US_EQUITY	HON_US_EQUITY	HPQ_US_EQUITY	HRS_US_EQUITY	HST_US_EQUITY	HSY_US_EQUITY
Mean	0.038266174	0.006879791	0.053389545	0.014111835	0.078519841	0.102957003	0.023933632	0.045179414	0.029825451	0.030979637	0.022859432	0.042435828	0.057188652
Median	0.064578627	-0.159109023	0.068685798	-0.031540767	0.084829639	0.119367367	-0.020962163	0.069141952	0.05272638	0.087482842	0.073348847	0.085180373	0.059049308
Maximum	30.4670898	18.11246686	15.89570839	21.79448764	27.94861216	20.38579492	21.20226911	18.127383	27.17109817	15.94651299	18.3061495	23.8870577	22.53810463
Minimum	-41.90707602	-33.64722366	-14.73247148	-55.24472985	-27.85733563	-33.87737336	-27.95848622	-10.0087214	-34.8038361	-22.68132487	-23.56762749	-43.04069669	-27.16562229
Std. Dev.	4.623908647	2.717652631	1.712505887	2.721054158	2.354624149	2.415629284	2.348690401	1.54083984	2.122184875	2.45437955	2.181694114	2.719293458	1.6505423
Skewness	-0.359921891	-0.495311846	0.087725612	-1.098298517	-0.189892926	-0.726353864	-0.557441362	0.254660993	-0.541623368	-0.308760273	-0.171747053	-0.564463796	-0.152490247
Kurtosis	13.72849199	11.95899498	8.36420676	29.26132075	14.56485924	17.50409234	12.50566808	9.095687124	26.3925448	9.364722043	10.42927697	23.1538717	21.09968816
Jarque-Bera	17800.46473	26211.64436	9371.388454	228687.1373	42158.67877	66142.95896	29987.77559	12063.80328	152359.5146	13643.96978	17888.86939	127159.2973	105967.3114
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	3695	7743	7808	7903	7557	7471	7857	7738	6668	8008	7762	7490	7761

	HUM_US_EQUITY	IBM_US_EQUITY	IFF_US_EQUITY	IP_US_EQUITY	IR_US_EQUITY	JCP_US_EQUITY	JPM_US_EQUITY	K_US_EQUITY	KMB_US_EQUITY	KO_US_EQUITY	KR_US_EQUITY	LLY_US_EQUITY	LMT_US_EQUITY
Mean	0.04480817	0.023218671	0.031058516	0.019553469	0.033853764	0.013230713	0.026645301	0.041794873	0.045478038	0.038269977	0.032959695	0.0355608	0.03049156
Median	0.025141421	-0.027288853	0.034299434	0.030602886	0.118495155	-0.124610608	0.023504525	0.043704508	0.029229602	0.080482074	0.09601537	0.070696362	-0.012400025
Maximum	14.66034742	12.36646946	14.94220526	19.78967686	16.20243309	17.21572785	22.39171602	21.16123537	10.20455742	17.95812654	26.04149461	15.30932645	13.7080671
Minimum	-29.74382474	-26.08838793	-17.39231871	-31.40832417	-36.08739975	-21.9623344	-32.4616895	-30.50463604	-29.31417793	-28.35688142	-115.6277931	-34.50016115	-14.79873321
Std. Dev.	2.69718594	1.670514662	1.738051168	2.158022831	2.197122393	2.413379754	2.459014828	1.629973128	1.52913433	1.598263202	2.586509303	1.797846006	1.793556508
Skewness	-0.885389994	-0.250600758	-0.077912223	-0.351129147	-0.65010013	-0.014718168	-0.115406309	-0.400744474	-0.876752033	-0.384848373	-14.05517789	-1.265177963	-0.216889852
Kurtosis	13.67115909	14.55767404	9.39637791	15.52215793	16.80915733	9.21877694	16.14851087	25.49880602	24.75054681	17.70684776	587.7065892	30.45967713	9.742200327
Jarque-Bera	37496.60466	61969.34417	13159.83927	52039.29131	62749.03498	12625.46693	56499.83245	164173.4883	156339.549	97371.7912	108384294.1	252179.7773	8564.223818
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7691	11113	7715	7940	7828	7835	7841	7774	7880	10775	7591	7959	4503

	LNC_US_EQUITY	LPX_US_EQUITY	LUV_US_EQUITY	MAS_US_EQUITY	MAT_US_EQUITY	MCD_US_EQUITY	MDT_US_EQUITY	MMM_US_EQUITY	MRK_US_EQUITY	MRO_US_EQUITY	MSI_US_EQUITY	MTRN_US_EQUITY	MUR_US_EQUITY
Mean	0.023715342	0.012281882	0.048061708	0.022621108	0.038459666	0.056924913	0.05649817	0.035285391	0.039285058	0.023657025	0.029209106	0.013401193	0.026046708
Median	0.063938686	-0.17559267	-0.059898175	-0.06832935	-0.073975518	0.097961806	0.104432911	0.050193245	0.076277654	0.035041148	0.04831319	-0.073238004	-0.02475902
Maximum	36.2348718	21.59381676	15.75543591	16.7682686	18.92419996	10.31014498	11.72721032	10.92028753	12.25089968	20.99077596	17.50947681	28.76023499	15.88853385
Minimum	-50.89086028	-33.64722366	-27.52980133	-18.14003223	-35.96482843	-18.17385722	-17.47363507	-30.08169594	-31.17092752	-45.82994363	-26.23633521	-23.55255929	-18.80522315
Std. Dev.	2.800753509	2.983132118	2.48702602	2.37270806	2.752404577	1.648389634	1.977820339	1.545677742	1.737712825	2.279897231	2.640672136	2.845426742	2.13034708
Skewness	-1.276494772	-0.425915866	-0.325727687	-0.163667792	-0.761762462	-0.194153072	-0.258038333	-0.948659246	-0.949687082	-1.033080346	-0.496540513	-0.098693838	-0.105938657
Kurtosis	59.03217785	13.55952129	8.7283254	8.056607765	17.84763499	8.474209493	7.889294228	24.7421831	20.98634067	30.45458963	11.09952176	11.4248468	7.937187435
Jarque-Bera	1023407.5	35684.17986	10629.26931	8223.946775	69486.73414	9903.720531	7951.427267	157603.1554	108888.7628	239724.9356	22185.05612	21962.00896	7839.048477
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7807	7631	7675	7687	7486	7892	7895	7941	7989	7590	7996	7422	7704

	NAV_US_EQUITY	NBL_US_EQUITY	NC_US_EQUITY	NEE_US_EQUITY	NEM_US_EQUITY	NOC_US_EQUITY	NSC_US_EQUITY	NSM_US_EQUITY	NUE_US_EQUITY	NVO_US_EQUITY	NWL_US_EQUITY	OMX_US_EQUITY	OXY_US_EQUITY
Mean	-0.031406709	0.032383584	0.016519513	0.032990724	0.006889127	0.030797945	0.037337069	0.388028834	0.046071383	0.062388946	0.049294414	-0.008652999	0.025215206
Median	-0.217260194	-0.011512763	-0.04804193	0.113417274	-0.146133629	0.040654824	0.046551324	0.216283542	0.041494641	0.087044834	0.114155264	-0.187617481	-0.012067821
Maximum	24.96326957	21.37481193	19.87609137	13.05266131	22.45338887	21.32502331	14.33494144	15.57400614	40.73613963	16.42302581	18.67331876	37.89457592	16.64323647
Minimum	-38.97313268	-18.27094795	-73.75996049	-18.54761335	-42.81480284	-15.80946244	-18.92454479	-10.79392609	-40.16079412	-30.3038706	-31.90229371	-25.32732928	-24.48947972
Std. Dev.	3.677005948	2.575755046	2.831232881	1.37228324	2.652357317	1.874725094	1.950889557	3.092153816	2.465696232	1.974117151	2.225480609	2.863453428	2.093448207
Skewness	-0.020585048	0.157520775	-2.667227507	-0.368055429	-0.462164214	-0.081878643	-0.177746387	0.246175947	-0.279235113	-1.321049007	-0.593805013	0.230417237	-0.274196722
Kurtosis	10.62704041	7.426197497	72.9690877	17.3227466	18.50275751	10.31123294	8.154856516	5.485906478	27.34746853	26.56505898	15.53270051	16.94691571	11.37837628
Jarque-Bera	16867.87384	6218.908498	1593161.112	63393.19281	79391.66668	17390.22015	8684.936451	78.67123217	194491.019	174755.8812	49253.95244	62702.68204	22168.65981
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	6959	7580	7765	7397	7900	7804	7807	294	7870	7459	7459	7728	7547

	PBI_US_EQUITY	PCG_US_EQUITY	PEP_US_EQUITY	PFE_US_EQUITY	PG_US_EQUITY	PGN_US_EQUITY	PH_US_EQUITY	PKI_US_EQUITY	PNW_US_EQUITY	PPG_US_EQUITY	R_US_EQUITY	RDC_US_EQUITY	ROK_US_EQUITY
Mean	0.025105449	0.018916975	0.053077937	0.044781696	0.043977025	0.025243815	0.041106627	0.02818107	0.015823345	0.045597156	0.030511111	0.010327386	0.040718978
Median	0.108283714	0.123212435	0.047532283	0.02845355	0.077679962	0.114587023	0.034838009	0.072176113	0.0856582	0.101394431	0.075187973	-0.142597467	0.061620624
Maximum	18.46838173	26.86338775	14.96562902	9.754698423	20.04953034	10.14001144	12.80722678	18.42281351	64.55191316	13.83243385	12.48212115	22.42253472	15.26493033
Minimum	-33.43724012	-45.77764027	-15.41587458	-18.99494085	-36.00515319	-11.50693298	-28.00816369	-37.86538507	-26.46925542	-23.09867288	-20.76419017	-25.13144283	-22.47840294
Std. Dev.	1.90502172	1.972940507	1.665963513	1.824668309	1.557096428	1.305560597	2.104406845	2.405558247	1.859777105	1.847211114	2.194105809	3.323104978	2.36616817
Skewness	-1.115247126	-2.462702394	-0.03596036	-0.228326491	-2.516191657	-0.189326678	-0.330586688	-0.993514729	5.432187891	-0.116121522	-0.327808074	0.041719922	-0.379648563
Kurtosis	23.59985197	86.49151912	9.18743688	7.344461109	69.02522444	8.977041694	10.40335601	21.69326849	222.7250958	9.994855988	8.153510795	5.142365242	9.364020704
Jarque-Bera	139227.8188	2111935.275	12429.74443	6314.046025	1448945.831	10578.19067	17748.04427	111625.9837	14442516.89	16013.01488	8563.228065	1396.244903	6981.428026
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7783	7246	7791	7941	7931	7078	7710	7581	7162	7846	7615	7290	4079

	RSH_US_EQUITY	RTN_US_EQUITY	S_US_EQUITY	SLB_US_EQUITY	SNE_US_EQUITY	SO_US_EQUITY	SUN_US_EQUITY	T_US_EQUITY	TAP_US_EQUITY	TEN_US_EQUITY	TGT_US_EQUITY	THC_US_EQUITY	TNB_US_EQUITY
Mean	0.001588562	0.02349587	0.01215073	0.022218043	0.018869017	0.023986529	0.021422055	0.028034642	0.027432366	0.013534762	0.053420729	0.01608866	0.02449123
Median	-0.050748542	0.051242635	0.090618036	-0.015817412	-0.150263519	0.096897893	0.052949404	0.080064947	0.04625347	-0.131061618	0.037085111	-0.092555461	0.019918335
Maximum	20.90917979	23.71297933	25.19639898	13.90175387	16.89917509	10.49574586	19.62444704	18.43910393	15.24280265	38.62337462	16.38487271	43.83301217	20.74560003
Minimum	-35.40251969	-57.27834983	-31.6461037	-25.48965751	-18.14893731	-15.63423478	-30.3185699	-24.76345713	-31.21884826	-41.19797891	-39.99030016	-62.90559866	-33.61533082
Std. Dev.	2.816182585	1.89277382	2.76225026	2.279593499	2.275771743	1.41718383	2.217820709	1.712540459	2.261187001	4.407035797	2.107578123	3.01175947	2.057155761
Skewness	-0.81293783	-3.783102396	-0.669287143	-0.327936991	0.166400765	-0.175661795	-0.339683367	-0.250190933	-0.804361496	0.010686106	-0.813316093	-1.609435778	-1.181867758
Kurtosis	16.68948015	123.2328995	20.10912127	8.876641282	7.539281259	8.115930542	14.44127496	15.30336812	15.06503314	14.80097744	23.47404581	53.85648938	27.95681379
Jarque-Bera	62437.81219	4742792.025	92293.26581	11617.14897	6691.21348	9107.241449	41335.97015	44501.42364	43390.31406	23193.17959	139433.2123	819272.6202	195520.2215
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7885	7843	7521	7974	7752	8312	7552	7044	7029	3997	7933	7572	7467

	TSO_US_EQUITY	TXI_US_EQUITY	TXN_US_EQUITY	TXT_US_EQUITY	UIS_US_EQUITY	UN_US_EQUITY	UNP_US_EQUITY	UTX_US_EQUITY	VFC_US_EQUITY	VLO_US_EQUITY	VZ_US_EQUITY	WHR_US_EQUITY	WMB_US_EQUITY
Mean	0.022807118	0.029721662	0.035000687	0.026981051	-0.032569172	0.045460164	0.037372476	0.04373086	0.060368862	0.045044091	0.026759408	0.032720938	0.02698764
Median	-0.269905697	-0.123454552	-0.046500815	-0.019227211	-0.190585597	0.0981502	0.031245118	0.055040085	0.069842817	0.089997143	0.066923521	-0.012705439	0.094413854
Maximum	28.86214807	24.78361639	21.56579771	39.77515712	52.48118657	31.61133322	9.223364953	12.7925746	13.59815068	16.53334378	13.65623394	17.5507955	62.98401867
Minimum	-35.6950768	-27.80733004	-31.65907724	-38.44185074	-46.41039071	-43.90876221	-24.08051861	-33.19507519	-30.24971582	-22.31435513	-19.2939511	-23.50721222	-94.28050455
Std. Dev.	3.516177548	2.544299728	2.686682149	2.432998133	3.789144382	1.664804835	1.834710706	1.77856923	1.993823249	2.752658814	1.636533262	2.244698678	3.171599555
Skewness	-0.078363704	-0.036494204	-0.224537933	-1.055681819	-0.316240335	-1.622146966	-0.357040136	-0.881455235	-0.37699342	-0.346785597	-0.051927212	-0.068621023	-3.265819713
Kurtosis	10.44726513	11.89620901	10.48078727	39.42006017	28.43957983	85.05528819	9.644352048	21.89513194	14.03932273	7.663173136	10.54690186	8.75023314	144.7981819
Jarque-Bera	16366.15286	24839.23735	18793.82743	430428.3333	201394.9709	2199536.151	14720.17068	118994.231	39607.76863	3611.761514	16840.72551	10716.46436	6468792.685
Probability	0	0	0	0	0	0	0	0	0	0	0	0	0
Observations	7079	7532	8031	7762	7464	7828	7911	7930	7764	3900	7095	7774	7705

	WMS_US_EQUITY	WMT_US_EQUITY	WRB_US_EQUITY	WY_US_EQUITY	XOM_US_EQUITY	XRX_US_EQUITY
Mean	0.053593003	0.075408011	0.060097646	0.014453252	0.038382319	-0.001032071
Median	-0.18115947	0.103252461	0.04039588	-0.08246423	0.115353101	-0.093751706
Maximum	41.33750976	14.66034742	40.54651081	13.13668896	16.47552331	32.97532864
Minimum	-32.74936774	-18.54761335	-36.46287698	-31.13803833	-26.6946174	-29.77122468
Std. Dev.	3.723233681	2.062322115	2.050215191	2.136721448	1.566720919	2.512821958
Skewness	0.373006581	0.025182587	0.686700793	-0.456409834	-0.490199385	-0.504631396
Kurtosis	11.10885881	7.586458352	46.19297536	11.93371405	21.34802568	22.41725066
Jarque-Bera	19171.9154	8258.345308	501973.6392	26165.7757	110089.3946	124535.3892
Probability	0	0	0	0	0	0
Observations	6939	9421	6451	7787	7826	7906

Table B: Estimated values for Mathematica M ML=Maximum Likelihood, K=Koutrouvelis (Gauss code), S Q=Quantile method (STABLE), S ML=Maximum likelihood (STABLE), S ChF=Characteristic function method (STABLE)

	α					β					γ					δ				
	S ML	S K	S Q	S ML	S ChF	S ML	S K	S Q	S ML	S ChF	S ML	S K	S Q	S ML	S ChF	S ML	S K	S Q	S ML	S ChF
AA	1.73449281	1.76000232	1.631696	1.7251	1.7884	0.07185531	0.20696492	0.154613	0.105300	0.2471	0.00572262	0.01313744	0.00549549	0.00571236	0.00579561	0.0001129	0.0006642	0.0000743	0.0001775	0.0002856
ABT	1.81530414	1.83884724	1.678778	1.8508	1.8639	0.02174009	0.12165468	0.031519	0.050200	0.2634	0.00454005	0.01038257	0.00430885	0.00458055	0.00457169	0.0002449	0.0007181	0.0003478	0.0002686	0.0003689
ADI	1.73297807	1.77490328	1.601393	1.7227	1.8108	0.15099089	0.30373646	0.155519	0.192900	0.3317	0.00788926	0.01809956	0.00750980	0.00787058	0.00802051	0.0003011	0.0011747	0.0004287	0.0004192	0.0004267
ADM	1.78890874	1.81574555	1.783541	1.8099	1.8334	0.03021816	0.11542179	0.115030	0.063400	0.1800	0.00548693	0.01268362	0.00545321	0.00551781	0.00555583	0.0001737	0.0005860	0.0002729	0.0002163	0.0002962
ADP	1.72986985	1.77048525	1.628041	1.7218	1.8105	0.11215368	0.21778955	0.128157	0.136800	0.2920	0.00437481	0.01002652	0.00422157	0.00436924	0.00445756	0.0002848	0.0008567	0.0004501	0.0003252	0.0003792
AEP	1.74521489	1.76867326	1.679438	1.7394	1.7870	-0.17036532	-0.20835595	-0.163893	-0.149400	-0.1516	0.00354192	0.00817003	0.00345092	0.00353980	0.00358265	0.0000017	-0.0000154	0.0001223	0.0000182	0.0000568
AET	1.64847946	1.66256511	1.596119	1.6534	1.7075	-0.01308368	0.02726409	0.081038	0.010400	0.1327	0.00529983	0.01211472	0.00517908	0.00531041	0.00539942	0.0003681	0.0011000	0.0004087	0.0004232	0.0006493
AFG	1.66903843	1.68554972	1.580666	1.6683	1.7299	0.08728497	0.12676185	0.140624	0.109300	0.0893	0.00462297	0.01055448	0.00446360	0.00462148	0.00469749	0.0001045	0.0003379	0.0000756	0.0001454	0.0000574
AFL	1.67507542	1.69170428	1.629550	1.6720	1.7357	0.12901540	0.21300255	0.111327	0.156300	0.2166	0.00564507	0.01290662	0.00556590	0.00564332	0.00575249	0.0004101	0.0012577	0.0004823	0.0004742	0.0004797
AIG	1.51288598	1.51346988	1.472224	1.5121	1.5348	0.02284138	0.08670101	0.016217	0.039100	0.0996	0.00473778	0.01086298	0.00463952	0.00473765	0.00479173	0.0001612	0.0008922	0.0001294	0.0002134	0.0004067
AIR	1.69298071	1.73120259	1.606850	1.6853	1.7654	0.12932666	0.22705348	0.244927	0.164100	0.2772	0.00713061	0.01643130	0.00694661	0.00711745	0.00727234	0.0001858	0.0007836	0.0000473	0.0002860	0.0003500
ALK	1.77312365	1.80550535	1.759569	1.7755	1.8377	0.14707903	0.28923236	0.488591	0.193200	0.4817	0.00745060	0.01713240	0.00740425	0.00746137	0.00757768	0.0002902	0.0010229	0.0003108	0.0003781	0.0005933
AM	1.62894745	1.64358058	1.480428	1.6327	1.6788	0.01567225	0.09045978	-0.015402	0.037700	0.0466	0.00515750	0.01176914	0.00480073	0.00516454	0.00523123	0.0000979	0.0005683	0.0002516	0.0001483	0.0001308
AMD	1.75738209	1.78546095	1.699910	1.7495	1.8096	0.06545276	0.18964173	0.241936	0.124100	0.3073	0.00976505	0.02250566	0.00956827	0.00975067	0.00989415	0.0000950	0.0008059	0.0000608	0.0002611	0.0005016
AME	1.66138132	1.69407180	1.570926	1.6632	1.7507	-0.01606067	0.04096312	-0.049946	0.007000	-0.0211	0.00470442	0.01071702	0.00451379	0.00471013	0.00481116	0.0002439	0.0007668	0.0002749	0.0002891	0.0002398
AMGN	1.67644282	1.72377781	1.563757	1.6728	1.7694	0.10670365	0.17161790	0.130717	0.137400	0.2476	0.00638386	0.01467503	0.00618210	0.00638012	0.00654399	0.0004656	0.0012611	0.0005412	0.0005447	0.0005951
AMN	1.53893743	1.57201899	1.440554	1.5394	1.6243	0.06611813	0.17032529	0.073688	0.081700	0.1305	0.00446287	0.01018291	0.00422046	0.00446694	0.00458439	0.0002740	0.0011080	0.0004182	0.0003199	0.0003397
AON	1.70067986	1.72652300	1.624988	1.6935	1.7545	0.04016297	0.08061712	0.013507	0.061300	0.1495	0.00435566	0.01001244	0.00425060	0.00434911	0.00442140	0.0002382	0.0006758	0.0004382	0.0002744	0.0003566
APA	1.84855596	1.87723453	1.851514	1.8591	1.8947	0.07613251	0.22213010	0.117970	0.150000	0.1522	0.00704774	0.01624542	0.00697388	0.00705570	0.00711740	0.0000892	0.0004594	0.0002238	0.0001584	0.0001204
APD	1.74195312	1.78123703	1.655591	1.7355	1.8191	0.05394259	0.14215071	0.073144	0.082000	0.1875	0.00478115	0.01096392	0.00462690	0.00477738	0.00486534	0.0001742	0.0005959	0.0001966	0.0002180	0.0002595
ARW	1.74436873	1.76801348	1.694052	1.7355	1.8029	0.15292048	0.25364563	0.256565	0.194400	0.3299	0.00758161	0.01740238	0.00749990	0.00756736	0.00769493	0.0001414	0.0006841	0.0001140	0.0002488	0.0003260
ASH	1.67390634	1.69466496	1.637907	1.6721	1.7254	0.00341089	0.02025179	-0.005786	0.026500	0.0345	0.00470740	0.01082303	0.00464496	0.00470584	0.00477802	0.0001312	0.0003660	0.0001405	0.0001752	0.0001742
AVP	1.66717572	1.69121013	1.578239	1.6662	1.7393	0.09183095	0.15037632	0.145630	0.115200	0.2096	0.00491279	0.01121496	0.00475760	0.00491342	0.00501293	0.0001395	0.0005102	-0.0000079	0.0001875	0.0002555
AVT	1.74324943	1.79511866	1.607462	1.7363	1.8025	0.02217801	0.14914662	0.061504	0.058900	0.1238	0.00608973	0.01431637	0.00577192	0.00608265	0.00616002	0.0000486	0.0004780	0.0000453	0.0001169	0.0001571
AVY	1.72837089	1.76564366	1.614623	1.7201	1.7926	0.02948899	0.10567791	0.029278	0.056200	0.1083	0.00499164	0.01132483	0.00479645	0.00498305	0.00506519	0.0001468	0.0006214	0.0003051	0.0001931	0.0002156
AXE	1.69891687	1.73596184	1.619720	1.6909	1.7747	0.07085257	0.16910731	0.040161	0.102400	0.2298	0.00631834	0.01450963	0.00615196	0.00630766	0.00644194	0.0003046	0.0010738	0.0005151	0.0003811	0.0004983
AXP	1.71537519	1.74764952	1.568890	1.7067	1.7791	0.02139821	0.10526203	0.050540	0.053100	0.1260	0.00567411	0.01301360	0.00535400	0.00566416	0.00575082	0.0001971	0.0007436	0.0002321	0.0002593	0.0003168
BA	1.76926668	1.80260914	1.682104	1.7736	1.8321	0.03273107	0.10973383	0.067572	0.065100	0.2099	0.00511027	0.01174000	0.00494832	0.00512197	0.00518074	0.0001731	0.0005683	0.0001864	0.0002175	0.0003038
BAX	1.80143675	1.82199301	1.783512	1.8354	1.8406	-0.03415370	0.08123793	0.066555	-0.014400	0.1703	0.00486821	0.01120027	0.00480046	0.00491144	0.00491401	0.0001968	0.0006684	0.0003727	0.0002307	0.0003493
BC	1.65101473	1.67133838	1.568846	1.6546	1.7110	0.06954888	0.16972219	0.118363	0.098800	0.1710	0.00658971	0.01508812	0.00639176	0.00659775	0.00670667	0.0002628	0.0011446	0.0001921	0.0003423	0.0004278
BCR	1.71132853	1.75673578	1.557919	1.7028	1.7915	0.03168476	0.07699433	0.042423	0.060100	0.1621	0.00503188	0.01153935	0.00469432	0.00502353	0.00510898	0.0002666	0.0007361	0.0003389	0.0003175	0.0003844
BDX	1.74637698	1.77176048	1.625185	1.7398	1.8031	0.02526573	0.11467127	-0.001841	0.052200	0.1564	0.00419065	0.00959843	0.00401921	0.00418790	0.00424202	0.0002379	0.0007441	0.0003522	0.0002732	0.0003365
BEN	1.66316278	1.69337904	1.574722	1.6640	1.7540	0.05475758	0.16139919	0.013907	0.082400	0.1237	0.00589000	0.01339671	0.00568635	0.00589459	0.00603087	0.0003509	0.0012288	0.0004390	0.0004166	0.0003961
BF-B	1.72008840	1.76280565	1.561329	1.7116	1.8036	0.07957176	0.12377183	0.027681	0.097400	0.1286	0.00379864	0.00868073	0.00357913	0.00379010	0.00385948	0.0002041	0.0005211	0.0003581	0.0002293	0.0002030
BGG	1.71993462	1.76092349	1.607932	1.7115	1.7973	0.01349893	0.07811171	0.075349	0.040500	0.0732	0.00500389	0.01149849	0.00480988	0.00499463	0.00509624	0.0000713	0.0003366	0.0000473	0.0001188	0.0001234
BK	1.63609544	1.65626743	1.555367	1.6416	1.7027	0.05773500	0.12382947	0.038738	0.080700	0.0814	0.00504899	0.01153171	0.00485971	0.00505932	0.00514726	0.0001465	0.0005907	0.0002191	0.0001963	0.0001369
BLL	1.72852729	1.76719852	1.617954	1.7198	1.7971	0.04068287	0.11192629	-0.004339	0.066900	0.0322	0.00467435	0.01075023	0.00445984	0.00466647	0.00474543	0.0002045	0.0006211	0.0003707	0.0002458	0.0001702
BMS	1.74494670	1.78520383	1.616511	1.7388	1.8224	0.05744369	0.16052725	0.011547	0.084900	0.1792	0.00466707	0.01067878	0.00445574	0.00466406	0.00474349	0.0002415	0.0007948	0.0003633	0.0002830	0.0003095
BMY	1.75270610	1.78102517	1.675803	1.7465	1.8133	0.05789000	0.11362526	0.054629	0.086100	0.2730	0.00435277	0.00998362	0.00423929	0.00435034	0.00441567	0.0002090	0.0006021	0.0003969	0.0002469	0.0003646
BOH	1.61992127	1.65373489	1.529750	1.6226	1.7092	-0.00332007	0.00461065	0.004494	0.014300	0.0011	0.00408856	0.00935004	0.00394342	0.00409502	0.00419661	0.0001626	0.0004050	0.0003647	0.0001985	0.0001717
BP	1.78616271	1.80823378	1.717986	1.8060	1.8351	-0.10078451	-0.03766622	-0.085620	-0.076400	0.0384	0.00454566	0.01042840	0.00446020	0.00457328	0.00459528	0.0000842	0.0003621	0.0002849	0.0001222	0.0002126
BT	1.66635694	1.69279471	1.579317	1.6665	1.7502	-0.00075938	0.05054284	0.000918	0.022800	0.0460	0.00500481	0.01140085	0.00483709	0.00500728	0.00511983	0.0000566	0.0003230	0.0001039	0.0001050	0.0001164
BWS	1.61001340	1.63350881	1.501225	1.6113	1.6927	0.00827626	0.04677627	0.071049	0.033900	0.0314	0.00605563	0.01379898	0.00581584	0.00606099	0.00621239	0.0000754	0.0003720	-0.0000150	0.0001517	0.0001020
C	1.60351768	1.60886534	1.505596	1.6040	1.6400	0.00000004	0.05417803	0.025414	0.024000	0.0479	0.00592381	0.01354185	0.00561486	0.00592562	0.00599518	0.0001400	0.0006783	0.0000094	0.0002106	0.0002587
CA	1.67250246	1.69402158	1.548323	1.6703	1.7446	0.02662007	0.07289593	0.047600	0.060900	0.1333	0.00711838	0.01619350	0.00674335	0.00711721	0.00724781	0.0003531	0.0010847	0.0005465	0.0004486	0.0005416
CAG	1.71855321	1.75430154	1.610003	1.7103	1.7885	0.00712987	0.06643932	0.007374	0.030700	0.0470	0.00432427	0.00993288	0.00411832	0.00431655	0.00439595	0.0002162	0.0006498	0.0003652	0.0002509	0.0002483
CAT	1.73800429	1.77368410	1.668247	1.7301	1.8118	0.05697564	0.17602069	0.092106	0.087300	0.2276	0.00521003	0.01195331	0.00510067	0.00520354	0.00530533	0.0001876	0.0007332	0.0002291	0.0002399	0.0003226

	α					β					γ					δ				
	S ML	S K	S Q	S ML	S ChF	S ML	S K	S Q	S ML	S ChF	S ML	S K	S Q	S ML	S ChF	S ML	S K	S Q	S ML	S ChF
CB	1.65368027	1.68199622	1.573273	1.6574	1.7313	0.07849001	0.11313456	0.070351	0.095500	0.1164	0.00398156	0.00910116	0.00386347	0.00398605	0.00406622	0.0002061	0.0005535	0.0002500	0.0002348	0.0002107
CBE	1.71789209	1.74689593	1.605602	1.7091	1.7767	-0.01949760	-0.01055686	0.000993	0.008500	0.0526	0.00490843	0.01125408	0.00466230	0.00489781	0.00496923	0.0001151	0.0003071	0.0001919	0.0001574	0.0002080
CBRL	1.72218896	1.76149001	1.627690	1.7142	1.8013	0.00507690	0.07488362	-0.024704	0.041700	0.0845	0.00639766	0.01467507	0.00618364	0.00638804	0.00652012	0.0002241	0.0007734	0.0004482	0.0003008	0.0003236
CBT	1.68624105	1.71858916	1.579785	1.6810	1.7485	0.08385959	0.19275564	0.091256	0.109200	0.1713	0.00516488	0.01188423	0.00490790	0.00516011	0.00524665	0.0001985	0.0008021	0.0002366	0.0002521	0.0002668
CCK	1.55484205	1.55747782	1.506166	1.5556	1.5755	0.00483865	0.01220308	-0.048802	0.025100	0.0597	0.00502002	0.01155797	0.00490639	0.00502406	0.00506368	0.0001217	0.0003967	0.0001762	0.0001839	0.0003032
CEG	1.76041165	1.77993867	1.733798	1.7600	1.7942	-0.14018537	-0.13659356	-0.273276	-0.116600	-0.0521	0.00399750	0.00919373	0.00395074	0.00400120	0.00403144	0.0001426	0.0003797	0.0003936	0.0001693	0.0002441
CI	1.67100210	1.68330706	1.583885	1.6696	1.7270	0.01412867	0.09587077	-0.029341	0.035900	0.0918	0.00468302	0.01067464	0.00455558	0.00468248	0.00476157	0.0002241	0.0008254	0.0003679	0.0002657	0.0003258
CL	1.75311266	1.78373278	1.638816	1.7466	1.8143	0.18595387	0.32207402	0.144425	0.210300	0.3821	0.00418162	0.00957798	0.00398775	0.00417889	0.00423208	0.0002986	0.0009073	0.0003613	0.0003358	0.0003774
CLX	1.66023214	1.68919003	1.581303	1.6620	1.7449	0.04508781	0.12233151	0.002730	0.065000	0.0948	0.00405862	0.00926759	0.00397233	0.00406353	0.00415756	0.0002477	0.0007898	0.0003561	0.0002820	0.0002745
CMI	1.65685147	1.68568689	1.546963	1.6589	1.7398	0.05997504	0.12223355	0.084681	0.085100	0.1672	0.00556487	0.01269169	0.00533061	0.00557075	0.00568929	0.0002414	0.0008020	0.0000590	0.0002994	0.0003627
CNA	1.60262496	1.61908165	1.492250	1.6030	1.6615	0.11784090	0.19757708	0.159478	0.137000	0.1210	0.00447536	0.01021697	0.00424383	0.00447845	0.00455224	0.0001609	0.0006626	0.0002241	0.0002057	0.0000807
CNW	1.68235432	1.72614107	1.614406	1.6769	1.7641	0.14593457	0.28613110	0.219977	0.175000	0.3280	0.00610292	0.01406551	0.00595181	0.00609514	0.00623867	0.0002257	0.0009801	0.0000881	0.0003002	0.0003950
COO	1.45704141	1.47931208	1.374054	1.4579	1.5503	0.02634290	0.04356124	0.069256	0.048300	0.0284	0.00772519	0.01758741	0.00734818	0.00772824	0.00799781	-0.0000119	0.0000920	0.0002569	0.0001303	-0.0000926
COP	1.80433812	1.83258420	1.777110	1.8429	1.8504	-0.08508493	-0.05061800	-0.118609	-0.042900	-0.0810	0.00542939	0.01252644	0.00532883	0.00547820	0.00548881	0.0000791	0.0002997	0.0001392	0.0001331	0.0001123
CP	1.66003797	1.69005547	1.549867	1.6615	1.7533	0.00664185	0.08428220	0.053117	0.031000	0.1147	0.00495940	0.01127102	0.00472315	0.00496482	0.00508222	0.0003230	0.0010567	0.0003770	0.0003733	0.0004598
CPB	1.67248906	1.71795121	1.583147	1.6705	1.7589	0.10785961	0.18044068	0.092678	0.127500	0.1993	0.00394864	0.00908773	0.00381408	0.00394902	0.00403966	0.0002155	0.0006336	0.0003328	0.0002491	0.0002508
CR	1.73058877	1.76199892	1.656247	1.7233	1.7877	-0.06193210	-0.02753320	-0.164089	-0.032700	-0.0517	0.00525351	0.01210182	0.00508596	0.00524578	0.00532866	0.0000956	0.0003587	0.0002408	0.0001407	0.0001355
CSC	1.68229619	1.71949028	1.540262	1.6769	1.7523	0.15515160	0.31537379	0.159883	0.181800	0.3023	0.00552565	0.01268845	0.00518141	0.00551913	0.00561775	0.0003145	0.0012279	0.0003947	0.0003773	0.0004149
CSX	1.77332183	1.80631645	1.669642	1.7804	1.8390	0.04233157	0.13030841	0.037868	0.077400	0.1572	0.00530208	0.01215316	0.00512699	0.00531801	0.00537544	0.0001895	0.0006203	0.0003134	0.0002363	0.0002620
CVS	1.73456770	1.76358370	1.640890	1.7256	1.7952	0.08747788	0.16811236	0.118814	0.115500	0.2677	0.00473943	0.01087048	0.00457975	0.00473253	0.00480651	0.0002676	0.0008011	0.0003763	0.0003142	0.0004069
CVX	1.80474707	1.82800274	1.757477	1.8395	1.8538	-0.09134061	-0.06366493	-0.250680	-0.047300	-0.1519	0.00447105	0.01025400	0.00439391	0.00451244	0.00451682	0.0000867	0.0002878	0.0001751	0.0001291	0.0000790
DBD	1.65502283	1.68348282	1.540027	1.6580	1.7414	0.05186111	0.11067946	0.047323	0.073800	0.1010	0.00471941	0.01073411	0.00449817	0.00472583	0.00482833	0.0001307	0.0004837	0.0000144	0.0001739	0.0001536
DD	1.75286983	1.78431079	1.705656	1.7474	1.8272	0.06805010	0.18961150	0.121369	0.096100	0.2715	0.00461314	0.01110762	0.00452909	0.00461123	0.00469691	0.0001589	0.0006089	0.0001753	0.0001995	0.0002870
DE	1.76010573	1.79386447	1.643700	1.7565	1.8271	0.08518846	0.19069140	0.120328	0.120200	0.3082	0.00553849	0.01269281	0.00530801	0.00553826	0.00561562	0.0002147	0.0007290	0.0003771	0.0002706	0.0003770
DIS	1.73057109	1.75494214	1.627168	1.7219	1.7963	0.07323534	0.13865044	0.054432	0.101500	0.2421	0.00508412	0.01121983	0.00489547	0.00507665	0.00516167	0.0002562	0.0008387	0.0004641	0.0003069	0.0004011
DNB	1.60110716	1.62483629	1.492123	1.6026	1.6881	-0.09933249	-0.11955770	-0.090449	-0.084700	-0.0978	0.00354858	0.00807161	0.00337397	0.00355257	0.00364378	0.0001873	0.0003962	0.0000204	0.0002116	0.0002443
DOV	1.73118913	1.77321113	1.644031	1.7227	1.8112	0.07211683	0.15446161	0.034827	0.099000	0.2093	0.00474273	0.01089346	0.00460369	0.00473619	0.00483275	0.0002159	0.0006730	0.0002994	0.0002610	0.0003009
DOW	1.69791097	1.73506227	1.609007	1.6911	1.7683	0.00888501	0.09549904	0.026217	0.035700	0.0847	0.00496264	0.01142301	0.00479358	0.00495676	0.00505270	0.0001009	0.0005079	0.0001993	0.0001506	0.0001841
DUK	1.73021378	1.75617426	1.618551	1.7223	1.7785	-0.17191008	-0.22892196	-0.175689	-0.151600	-0.1162	0.00355045	0.00816252	0.00338194	0.00354660	0.00358714	0.0000903	0.0001610	0.0002509	0.0001069	0.0001894
ED	1.80742765	1.83927541	1.740262	1.8478	1.8606	-0.18928621	-0.25171180	-0.331103	-0.166700	-0.3036	0.00332970	0.00767892	0.00328236	0.00336202	0.00337279	0.0000792	0.0001752	0.0002217	0.0001122	0.0000734
EIX	1.67752976	1.69684871	1.599603	1.6758	1.7138	-0.15617874	-0.18824178	-0.209494	-0.137300	-0.1677	0.00396615	0.00913024	0.00381544	0.00396605	0.00400781	0.0000277	0.0000125	0.0002393	0.0000530	0.0000554
EMR	1.73338304	1.76996877	1.620427	1.7245	1.8065	0.11908778	0.19509406	0.173512	0.144800	0.3550	0.00426767	0.00979583	0.00409295	0.00426147	0.00434021	0.0002321	0.0006720	0.0002467	0.0002727	0.0003762
ETN	1.73870782	1.76937696	1.617554	1.7310	1.8042	0.06964543	0.16525231	0.015928	0.095600	0.1799	0.00453602	0.01039816	0.00433962	0.00453111	0.00459927	0.0002135	0.0007004	0.0003216	0.0002543	0.0002787
ETR	1.76571078	1.78196992	1.804746	1.7685	1.7909	-0.09003508	-0.02568138	-0.194363	-0.064000	-0.0691	0.00417563	0.00968358	0.00427541	0.00418087	0.00421646	0.0000988	0.0003828	0.0001182	0.0001291	0.0001265
EXC	1.77834172	1.80998507	1.793606	1.7935	1.8257	-0.23814263	-0.30963091	-0.393353	-0.217900	-0.2508	0.00420311	0.00975071	0.00419251	0.00422096	0.00426476	0.0000215	0.0000215	0.0001735	0.0000560	0.0000787
F	1.75498854	1.77349240	1.713358	1.7486	1.7982	0.14672796	0.27044876	0.309600	0.182500	0.2895	0.00608457	0.01396211	0.00597721	0.00607944	0.00615204	0.0001873	0.0007610	0.0000342	0.0002573	0.0003042
FDX	1.72395504	1.77251190	1.607487	1.7138	1.8135	0.20990561	0.40826586	0.240329	0.238800	0.4858	0.00547748	0.01255537	0.00523123	0.00546504	0.00558710	0.0003069	0.0011028	0.0002141	0.0003729	0.0004285
FL	1.68114604	1.72222684	1.593648	1.6762	1.7537	0.17108269	0.26497386	0.246739	0.200200	0.2207	0.00614380	0.01416940	0.00587789	0.00613679	0.00626066	0.0001725	0.0006062	-0.0000933	0.0002487	0.0001108
FRM	1.86098333	1.89144121	2.000000	1.8614	1.9074	0.24315721	0.60654139	0.000000	0.350000	0.8464	0.01152663	0.02667486	0.01193310	0.01150740	0.01167800	-0.0000083	0.0007190	-0.0016752	0.0001764	0.0004676
FWLT	1.59843487	1.61522847	1.552669	1.5974	1.6572	0.07877269	0.12107960	0.178670	0.108100	0.1416	0.00769645	0.01764986	0.00770014	0.00769467	0.00785904	0.0000140	0.0003549	-0.0005122	0.0001312	0.0001611
GAS	1.79526517	1.82588043	1.718686	1.8250	1.8437	-0.08653980	-0.09178439	-0.166626	-0.066000	-0.0586	0.00354458	0.00817997	0.00340198	0.00357076	0.00358406	0.0000972	0.0002535	0.0002140	0.0001210	0.0001363
GCI	1.65321940	1.67191848	1.595439	1.6565	1.7111	0.05314121	0.13443839	0.108188	0.074700	0.1565	0.00476223	0.01090687	0.00464479	0.00476849	0.00484675	0.0001527	0.0006730	0.0000429	0.0001965	0.0002883
GPC	1.74860938	1.78571117	1.647060	1.7431	1.8213	0.09233317	0.17822941	0.115191	0.115300	0.1113	0.00394094	0.00903447	0.00380085	0.00393695	0.00400273	0.0001547	0.0004628	0.0002852	0.0001829	0.0001255
GR	1.67838294	1.72122286	1.582649	1.6745	1.7623	0.12354170	0.24664525	0.120222	0.148500	0.2349	0.00498118	0.01144062	0.00479087	0.00497806	0.00508815	0.0002411	0.0008734	0.0002429	0.0002930	0.0002907
GRA	1.49730875	1.53047560	1.387313	1.4972	1.5964	-0.00432189	0.04480286	0.058671	0.022800	0.0083	0.00907328	0.02079076	0.00863314	0.00907647	0.00940987	-0.0000577	0.0003608	0.0008546	0.0001194	-0.0000798
GT	1.64669739	1.67430910	1.581040	1.6516	1.7235	0.07080176	0.17508622	0.164234	0.097300	0.2239	0.00617660	0.01414184	0.00601497	0.00618603	0.00631796	0.0001749	0.0009189	-0.0001530	0.0002432	0.0004115
GWW	1.72781801	1.76688829	1.611408	1.7193	1.8026	0.07106852	0.13571386	0.033427	0.095600	0.1797	0.00427285	0.00981292	0.00411117	0.00426735	0.00434779	0.0002494	0.0006938	0.0003649	0.0002876	0.0003088
HAL	1.76312777	1.79435548	1.718146	1.7625	1.8222	0.02677133	0.12806043	0.116716	0.069200	0.1838	0.00676161	0.01553375	0.00661177	0.00676629	0.00685371	0.0000882	0.0005208	0.0001021	0.0001665	0.0002583
HAS	1.70000006	1.73697033	1.563849	1.6919	1.7682	0.13524085	0.27825467	0.100013	0.162500	0.1849	0.00563531	0.01293190	0.00529287	0.00562620	0.00571561	0.0003950	0.0013085	0.0006764	0.0004593	0.0003610
HD	1.71677057	1.75686237	1.601105	1.7082	1.7965	0.07868843	0.18740878	0.064725	0.110100	0.1837	0.00588473	0.01349005	0.00569084	0.00587592	0.00599610	0.0004987	0.0014414	0.0007041	0.0005660	0.0005639

	α					β					γ					δ				
	S ML	S K	S Q	S ML	S ChF	S ML	S K	S Q	S ML	S ChF	S ML	S K	S Q	S ML	S ChF	S ML	S K	S Q	S ML	S ChF
HES	1.75381771	1.77819185	1.692798	1.7478	1.8068	-0.00215752	0.08569019	0.079391	0.034300	0.1678	0.00580613	0.01333485	0.00569826	0.00580295	0.00588342	0.0001550	0.0006226	0.0000659	0.0002158	0.0003432
HNZ	1.74663319	1.78183968	1.634891	1.7403	1.8110	0.01879607	0.05862901	-0.020000	0.042300	0.0351	0.00388866	0.00892755	0.00371241	0.00388415	0.00393936	0.0001742	0.0004693	0.0002675	0.0002007	0.0001773
HON	1.68819874	1.72210807	1.623321	1.6830	1.7557	0.01188945	0.08558366	-0.008961	0.037800	0.0976	0.00486091	0.01117550	0.00474973	0.00485724	0.00494947	0.0001591	0.0006094	0.0002093	0.0002075	0.0002598
HPG	1.74382025	1.77466487	1.644158	1.7376	1.8080	-0.04757116	0.00218169	-0.063804	-0.011100	-0.0315	0.00613550	0.01406602	0.00593667	0.00612839	0.00622369	0.0000895	0.0003843	0.0002186	0.0001523	0.0001281
HRS	1.73881215	1.77072131	1.648531	1.7311	1.7971	0.02132555	0.07569543	-0.050674	0.051200	0.0531	0.00539430	0.01240365	0.00519552	0.00538527	0.00546312	0.0000836	0.0003442	0.0002084	0.0001346	0.0001097
HTS	1.63898144	1.65105477	1.568701	1.6441	1.6796	0.05755732	0.11883946	0.023976	0.082700	0.1116	0.00573794	0.01318037	0.00556790	0.00574757	0.00581443	0.0002474	0.0008838	0.0004462	0.0003097	0.0003344
HSY	1.72939696	1.76652600	1.616084	1.7209	1.8022	0.10864276	0.16429735	0.118535	0.132200	0.2632	0.00402919	0.00924218	0.00388199	0.00402386	0.00409810	0.0002855	0.0007392	0.0004754	0.0003214	0.0003667
HUM	1.73129409	1.76059549	1.686741	1.7219	1.7897	0.06869734	0.22281850	0.072613	0.105300	0.2613	0.00653402	0.01503748	0.00642695	0.00652381	0.00663575	0.0003332	0.0012957	0.0002754	0.0004144	0.0005608
IBM	1.71604174	1.75114549	1.628060	1.7077	1.7874	0.07536979	0.11683669	0.103769	0.094800	0.1607	0.00405195	0.00929806	0.00392734	0.00404432	0.00412257	0.0001185	0.0003464	0.0000665	0.0001502	0.0001630
IFF	1.72783831	1.76079393	1.608758	1.7190	1.7941	0.10838769	0.20592979	0.071286	0.131800	0.2462	0.00429343	0.00984878	0.00409549	0.00428693	0.00435773	0.0001792	0.0006182	0.0002919	0.0002182	0.0002574
IP	1.71584186	1.74227729	1.643237	1.7068	1.7684	0.02857919	0.09933653	0.030777	0.055100	0.0951	0.00511149	0.01176849	0.00498386	0.00510190	0.00518266	0.0000914	0.0004328	0.0001985	0.0001412	0.0001631
IR	1.73859629	1.77103512	1.664181	1.7310	1.8019	0.00477920	0.06279677	-0.054494	0.037500	0.1277	0.00541111	0.01242940	0.00524306	0.00540527	0.00549216	0.0001658	0.0005786	0.0004026	0.0002210	0.0003035
JCP	1.66339644	1.69317374	1.547665	1.6636	1.7561	0.08959925	0.18856081	0.145496	0.116100	0.1791	0.00571985	0.01299806	0.00546789	0.00572158	0.00586036	0.0001016	0.0005932	-0.0000494	0.0001635	0.0001465
JPM	1.65044825	1.67020685	1.544481	1.6552	1.7208	0.03532408	0.07423509	0.012836	0.060600	0.0471	0.00551840	0.01255926	0.00525746	0.00552501	0.00562141	0.0000809	0.0003300	0.0001443	0.0001363	0.0000664
K	1.70135972	1.74388232	1.568585	1.6950	1.7766	0.04063536	0.09078609	-0.010043	0.062100	-0.0037	0.00389143	0.00894200	0.00369290	0.00388672	0.00395694	0.0001489	0.0004165	0.0001686	0.0001797	0.0000798
KMB	1.71826131	1.75479518	1.604925	1.7096	1.7887	0.13310493	0.22736556	0.128744	0.153700	0.2685	0.00368647	0.00846846	0.00354405	0.00368157	0.00375010	0.0002563	0.0007377	0.0003536	0.0002886	0.0003111
KO	1.75052346	1.77690579	1.632836	1.7446	1.8126	0.07951399	0.19762957	0.018655	0.103400	0.1720	0.00398447	0.00928244	0.00382217	0.00398281	0.00403770	0.0001885	0.0006285	0.0003813	0.0002204	0.0002276
KR	1.75645610	1.78667533	1.627954	1.7515	1.8101	0.12908302	0.27504564	0.079510	0.160800	0.2662	0.00520574	0.01193336	0.00492810	0.00520439	0.00526043	0.0003014	0.0010022	0.0005950	0.0003533	0.0003730
LLY	1.76233807	1.79120345	1.634551	1.7624	1.8208	0.04339795	0.12013540	-0.021309	0.072400	0.1385	0.00445459	0.01019326	0.00426105	0.00445982	0.00450421	0.0001813	0.0005736	0.0002668	0.0002180	0.0002408
LMT	1.65726085	1.68433528	1.561053	1.6598	1.7414	0.01944240	0.09952696	0.032625	0.039900	0.0785	0.00417918	0.00951910	0.00400940	0.00418482	0.00427527	0.0001637	0.0006317	0.0000231	0.0002003	0.0002154
LNC	1.55606643	1.55801768	1.493816	1.5573	1.5812	0.00731322	0.05640973	0.000546	0.024900	0.0662	0.00478069	0.01096070	0.00462189	0.00478351	0.00483458	0.0001457	0.0006732	0.0002796	0.0001972	0.0003030
LPX	1.67278141	1.69528032	1.568505	1.6702	1.7430	0.08660080	0.14921174	0.196321	0.119100	0.1490	0.00690843	0.01574825	0.00658736	0.00690430	0.00703272	0.0001256	0.0005555	-0.0000238	0.0002158	0.0001678
LUV	1.82607593	1.84905143	1.753698	1.8522	1.8764	0.14307989	0.33780194	0.339110	0.196700	0.5559	0.00668927	0.01530663	0.00648034	0.00673132	0.00675012	0.0002843	0.0010085	0.0003138	0.0003304	0.0005288
MAS	1.69921737	1.74066253	1.602214	1.6912	1.7790	0.05424744	0.16319780	0.122134	0.084500	0.2042	0.00577324	0.01327641	0.00556129	0.00576339	0.00589177	0.0001614	0.0007298	0.0000442	0.0002273	0.0003182
MAT	1.71412337	1.74457420	1.615962	1.7039	1.7756	0.19169212	0.34938254	0.223957	0.224300	0.3340	0.00654584	0.01503008	0.00631678	0.00653061	0.00664299	0.0003219	0.0012379	0.0003516	0.0004106	0.0004092
MCD	1.79842824	1.82535340	1.727046	1.8267	1.8517	0.07410159	0.16563244	0.020151	0.108000	0.1761	0.00432124	0.00991749	0.00421502	0.00435379	0.00436936	0.0002637	0.0007446	0.0004507	0.0002870	0.0003069
MDT	1.71592522	1.75466998	1.609327	1.7074	1.7920	0.01594987	0.07672235	0.042971	0.044000	0.1572	0.00489320	0.01122960	0.00468441	0.00488594	0.00498066	0.0002798	0.0008381	0.0005519	0.0003275	0.0004230
MMM	1.71819378	1.75926950	1.598677	1.7098	1.7945	0.03639767	0.08282236	0.009511	0.058700	0.1508	0.00376617	0.00864634	0.00360773	0.00376127	0.00383200	0.0001850	0.0005191	0.0002355	0.0002157	0.0002616
MRK	1.77263439	1.79521323	1.660273	1.7806	1.8214	-0.03736716	-0.00706441	0.009242	-0.011100	0.1561	0.00435382	0.00997449	0.00416592	0.00436851	0.00439615	0.0001972	0.0005352	0.0003466	0.0002300	0.0003536
MRO	1.80750051	1.82939030	1.767088	1.8502	1.8433	-0.04005774	-0.01440054	-0.016476	0.005900	-0.0223	0.00576399	0.01329193	0.00569408	0.00581843	0.00581422	0.0000593	0.0002175	0.0001293	0.0001116	0.0000891
MSI	1.71900936	1.74960482	1.638279	1.7097	1.7801	0.05000630	0.17014015	0.024580	0.085000	0.1903	0.00638845	0.01467880	0.00617664	0.00637707	0.00648239	0.0002031	0.0008920	0.0002760	0.0002814	0.0003674
MTRN	1.65949113	1.68224410	1.560310	1.6614	1.7341	-0.01763476	-0.00178948	0.053643	0.011500	0.0229	0.00653034	0.01489710	0.00627647	0.00653349	0.00666929	0.0000297	0.0001632	-0.0001195	0.0001054	0.0001181
MUR	1.73479474	1.77344735	1.616356	1.7275	1.8082	-0.02186535	0.02452823	0.001857	0.010700	0.0643	0.00536472	0.01230628	0.00513192	0.00535907	0.00544927	0.0000904	0.0003610	-0.0001030	0.0001442	0.0001883
NAV	1.77603218	1.80036483	1.731310	1.7846	1.8190	0.12697352	0.23731275	0.301378	0.185800	0.2883	0.00924251	0.02133067	0.00918506	0.00926318	0.00934809	-0.0001409	0.0000613	-0.0001373	-0.0000139	0.0000600
NBL	1.80947214	1.83550767	1.753874	1.8502	1.8574	0.05005819	0.13749436	0.134409	0.105800	0.1397	0.00679504	0.01563174	0.00669995	0.00686145	0.00686650	0.0000975	0.0004239	0.0001852	0.0001511	0.0001625
NC	1.61685844	1.64799299	1.533209	1.6182	1.6997	0.05512241	0.15302872	0.091799	0.079600	0.1463	0.00597278	0.01366907	0.00577597	0.00597764	0.00612048	0.0001836	0.0009411	0.0001375	0.0002537	0.0003197
NEE	1.73353418	1.75595067	1.713050	1.7262	1.7772	-0.12910724	-0.14885885	-0.259718	-0.110600	-0.0938	0.00324718	0.00749117	0.00323695	0.00324381	0.00328901	0.0001074	0.0002559	0.0002263	0.0001223	0.0001638
NEM	1.73269169	1.76376404	1.659878	1.7243	1.7992	0.11089985	0.23147816	0.202962	0.144800	0.1662	0.00646898	0.01481802	0.00630462	0.00645606	0.00656849	0.0000382	0.0004240	-0.0001240	0.0001167	0.0000310
NOC	1.69829879	1.74143520	1.548705	1.6911	1.7806	0.00621218	0.05634867	0.032836	0.032800	0.1273	0.00457256	0.01047450	0.00426845	0.00456583	0.00465326	0.0001606	0.0005233	0.0002629	0.0002047	0.0002832
NSC	1.71427137	1.75749375	1.601867	1.7065	1.7963	0.02371081	0.07277893	-0.000020	0.050200	0.1567	0.00484941	0.01112510	0.00462827	0.00484127	0.00493923	0.0001892	0.0005704	0.0002021	0.0002322	0.0003071
NSM	1.80385143	1.84507866	1.552795	1.8277	1.8549	0.14290083	0.21096722	0.120661	0.185700	0.5835	0.00825912	0.01907919	0.00755865	0.00832119	0.00832096	0.0017149	0.0039886	0.0014925	0.0017848	0.0021022
NUE	1.75089563	1.77452913	1.640702	1.7441	1.8019	-0.00000019	0.06649631	0.039590	0.038300	0.2404	0.00597529	0.01368126	0.00569473	0.00597078	0.00603646	0.0002554	0.0008139	0.0002776	0.0003210	0.0005348
NVO	1.73085119	1.75881495	1.625744	1.7235	1.7908	0.03894632	0.12894813	0.015969	0.063100	0.0936	0.00468963	0.01074065	0.00452604	0.00468345	0.00475400	0.0003141	0.0009329	0.0004112	0.0003533	0.0003465
NWL	1.67414866	1.71237049	1.558694	1.6719	1.7431	0.04297430	0.14303122	0.004227	0.067900	0.0960	0.00516388	0.01189142	0.00488456	0.00516372	0.00525348	0.0002604	0.0009344	0.0005080	0.0003128	0.0003002
OMX	1.60942544	1.62515906	1.536342	1.6098	1.6658	0.08689225	0.15563172	0.186329	0.111200	0.1417	0.00601674	0.01378496	0.00585703	0.00601870	0.00612754	0.0000224	0.0004245	-0.0001113	0.0000954	0.0000867
OXY	1.79711469	1.81979325	1.713615	1.8303	1.8408	-0.08210993	-0.05360052	0.008998	-0.042500	-0.1026	0.00538456	0.01237769	0.00528396	0.00543028	0.00543526	0.0000435	0.0001797	-0.0000374	0.0000942	0.0000468
PBI	1.76333617	1.78785274	1.659910	1.7641	1.8202	-0.00898500	0.08398816	-0.070144	0.021700	0.1308	0.00471639	0.01077791	0.00455670	0.00472249	0.00477076	0.0001563	0.0005899	0.0003427	0.0001963	0.0002768
PCG	1.67781962	1.69769903	1.706144	1.6758	1.6967	-0.12489542	-0.13706583	-0.253844	-0.108100	-0.0604	0.00389002	0.00904031	0.00387732	0.00389019	0.00392048	0.0000229	0.0000770	0.0002136	0.0000477	0.0001354
PEP	1.73931481	1.77355458	1.620891	1.7318	1.8109	0.02832768	0.07587383	0.040234	0.052800	0.1284	0.00417807	0.00956912	0.00404261	0.00417164	0.00424501	0.0002405	0.0006457	0.0002824	0.0002719	0.0003064
PFE	1.77921157	1.81231144	1.642293	1.7875	1.8463	0.12077517	0.26827937	0.160540	0.153000	0.4063	0.00478200	0.01094289	0.00457405	0.00479788	0.00484439	0.0002672	0.0008585	0.0004385	0.0003033	0.0004098

	α					β					γ					δ				
	S ML	**S K**	**S Q**	**S ML**	**S ChF**	**S ML**	**S K**	**S Q**	**S ML**	**S ChF**	**S ML**	**S K**	**S Q**	**S ML**	**S ChF**	**S ML**	**S K**	**S Q**	**S ML**	**S ChF**
PG	1.73934288	1.77458200	1.594900	1.7319	1.8045	0.07017353	0.20121750	-0.013742	0.091600	0.1799	0.00367273	0.00840866	0.00347668	0.00366952	0.00371985	0.0002411	0.0007796	0.0003127	0.0002696	0.0002883
PGN	1.79266425	1.82113302	1.740059	1.8206	1.8413	-0.21829881	-0.29516182	-0.313198	-0.200700	-0.3131	0.00337352	0.00777394	0.00326351	0.00339889	0.00341178	0.0000477	0.0000750	0.0002121	0.0000773	0.0000517
PH	1.75262843	1.78981289	1.657409	1.7477	1.8224	0.04407273	0.14263113	0.023892	0.075000	0.1066	0.00535635	0.01228931	0.00514986	0.00535397	0.00543510	0.0001844	0.0006477	0.0002009	0.0002357	0.0002143
PKI	1.70170629	1.73289997	1.588882	1.6946	1.7639	0.00005065	0.06270222	-0.003633	0.029300	0.0991	0.00562970	0.01292439	0.00537634	0.00562237	0.00571342	0.0001601	0.0006174	0.0003031	0.0002195	0.0002999
PNW	1.73773244	1.75643521	1.686841	1.7305	1.7716	-0.19224908	-0.23127632	-0.188338	-0.169300	-0.1545	0.00392582	0.00905844	0.00383528	0.00392195	0.00396168	-0.0000167	-0.0000628	0.0001149	0.0000040	0.0000648
PPG	1.75016815	1.78439570	1.671728	1.7448	1.8184	0.04851985	0.09936956	-0.039449	0.074400	0.1037	0.00466806	0.01070880	0.00454214	0.00466391	0.00474019	0.0001954	0.0005539	0.0003723	0.0002319	0.0002241
R	1.76064658	1.79439060	1.682856	1.7591	1.8312	0.01522067	0.10869400	-0.024401	0.051500	0.1594	0.00561945	0.01288589	0.00551460	0.00562256	0.00571403	0.0001605	0.0006186	0.0002778	0.0002169	0.0002876
RDC	1.89041732	1.92047834	1.937597	1.8731	1.9274	0.18253768	0.57964188	1.000000	0.285100	0.9907	0.00941963	0.02184997	0.00904911	0.00936439	0.00951824	0.0000475	0.0005671	0.0002706	0.0001994	0.0004196
ROK	1.69363082	1.73220690	1.589458	1.6871	1.7709	-0.02695764	0.04727290	-0.004537	0.003500	0.0956	0.00569139	0.01307213	0.00549502	0.00568171	0.00580279	0.0002061	0.0007866	0.0002545	0.0002644	0.0003905
RSH	1.72526656	1.75185801	1.593160	1.7161	1.7783	0.00837942	0.07673048	0.086353	0.047500	0.1050	0.00674513	0.01546827	0.00640168	0.00673223	0.00681699	0.0000431	0.0003834	0.0000671	0.0001288	0.0001844
RTN	1.68830671	1.72636558	1.604668	1.6832	1.7629	0.01929499	0.07311948	-0.008208	0.041800	0.0763	0.00417053	0.00959402	0.00405641	0.00416785	0.00425085	0.0001335	0.0004493	0.0002060	0.0001706	0.0001826
S	1.63583532	1.64715455	1.594886	1.6409	1.6750	0.02773293	0.11476561	-0.014752	0.054000	0.1018	0.00579554	0.01331323	0.00570766	0.00580473	0.00586824	0.0001174	0.0007435	0.0003501	0.0001844	0.0002643
SLB	1.79207808	1.82161280	1.701135	1.8071	1.8558	0.13082052	0.28303153	0.184198	0.172700	0.4490	0.00598833	0.01370569	0.00585338	0.00601594	0.00606926	0.0001766	0.0007052	0.0002915	0.0002251	0.0003743
SNE	1.74206674	1.77733286	1.645828	1.7319	1.8099	0.23981762	0.42270756	0.308858	0.270000	0.4526	0.00574765	0.01318536	0.00549108	0.00573524	0.00583030	0.0001844	0.0008122	0.0000634	0.0002549	0.0002689
SO	1.83207356	1.86669010	1.952983	1.8551	1.8819	-0.16820098	-0.22108731	-0.818971	-0.144700	-0.3542	0.00379180	0.00881467	0.00389699	0.00380847	0.00385381	0.0000473	0.0001390	0.0002816	0.0000779	0.0000072
SUN	1.75274072	1.77984167	1.692963	1.7488	1.8136	-0.01662405	0.05428195	-0.008550	0.016400	0.0584	0.00550785	0.01261504	0.00536983	0.00550489	0.00558563	0.0000745	0.0003955	0.0002140	0.0001268	0.0001614
T	1.68980628	1.73510076	1.564405	1.6855	1.7754	-0.09998326	-0.11390316	-0.106761	-0.077500	-0.1513	0.00410868	0.00943106	0.00390798	0.00410705	0.00419308	0.0000404	0.0001156	0.0001055	0.0000738	0.0000435
TAP	1.67355148	1.69930477	1.578023	1.6713	1.7483	0.04294898	0.13874709	0.047017	0.068200	0.1078	0.00525126	0.01198018	0.00508080	0.00524787	0.00535393	0.0001989	0.0007925	0.0003326	0.0002483	0.0002514
TEN	1.48610967	1.50452813	1.386825	1.4845	1.5645	0.04813372	0.14072625	0.137211	0.072900	0.1193	0.00839459	0.01915002	0.00797252	0.00839176	0.00863368	0.0001521	0.0015415	0.0006723	0.0003112	0.0003980
TGT	1.72914769	1.76670090	1.601474	1.7204	1.8017	0.09434701	0.20730184	0.076005	0.122800	0.2123	0.00516911	0.01185653	0.00496807	0.00516127	0.00525415	0.0002779	0.0009018	0.0003512	0.0003312	0.0003425
THC	1.73760128	1.75461044	1.692116	1.7290	1.7695	0.07478372	0.18937371	0.208541	0.110600	0.1714	0.00671723	0.01545098	0.00654593	0.00670592	0.00677054	0.0001590	0.0007870	0.0000727	0.0002399	0.0002829
TNB	1.69169799	1.73123678	1.573717	1.6856	1.7686	0.07400647	0.16995131	0.061316	0.098300	0.1629	0.00479373	0.01099304	0.00454551	0.00478806	0.00488230	0.0001743	0.0006487	0.0002427	0.0002210	0.0002263
TSO	1.72557501	1.75857014	1.681997	1.7143	1.7819	0.15250348	0.28490238	0.355125	0.197200	0.3630	0.00850209	0.01967285	0.00840970	0.00848142	0.00864355	0.0002264	0.0010567	-0.0000885	0.0003681	0.0005267
TXI	1.66978402	1.69389477	1.555917	1.6679	1.7408	0.11753798	0.20816432	0.180481	0.145800	0.2209	0.00590470	0.01347393	0.00560254	0.00590312	0.00601345	0.0002092	0.0008269	0.0000697	0.0002778	0.0002896
TXN	1.71478894	1.75778726	1.587174	1.7059	1.7970	0.07489769	0.16675961	0.102659	0.110300	0.2266	0.00666875	0.01527171	0.00630168	0.00665523	0.00678389	0.0001945	0.0007378	0.0001424	0.0002791	0.0003351
TXT	1.64781110	1.65999967	1.580802	1.6530	1.6926	0.01918734	0.03478660	0.064821	0.040700	0.0795	0.00503537	0.01153124	0.00489017	0.00504252	0.00510725	0.0001856	0.0004982	0.0000894	0.0002318	0.0002861
UIS	1.60839792	1.61511708	1.550612	1.6086	1.6312	0.06899103	0.13370261	0.122058	0.098600	0.1202	0.00739821	0.01703438	0.00720506	0.00739901	0.00745304	-0.0000280	0.0004247	-0.0002900	0.0000807	0.0001045
UN	1.67949110	1.72246684	1.587565	1.6769	1.7606	-0.03310901	-0.02124574	-0.028141	-0.016300	0.0165	0.00370968	0.00852716	0.00359189	0.00370875	0.00378970	0.0001968	0.0005173	0.0003725	0.0002218	0.0002642
UNP	1.73328728	1.77789935	1.632682	1.7258	1.8197	0.05265976	0.15082727	0.070772	0.079500	0.1659	0.00465873	0.01067508	0.00449232	0.00465361	0.00474887	0.0001921	0.0006396	0.0002774	0.0002348	0.0002521
UTX	1.77879499	1.80892421	1.657347	1.7902	1.8428	0.00190652	0.04883090	0.010257	0.031200	0.1166	0.00457796	0.01044508	0.00437997	0.00459707	0.00462641	0.0001941	0.0005405	0.0002572	0.0002276	0.0002700
VFC	1.68855774	1.73386072	1.562384	1.6842	1.7710	0.01118682	0.07426568	-0.023468	0.036600	0.0032	0.00478443	0.01099764	0.00454150	0.00478082	0.00487868	0.0002311	0.0006962	0.0002410	0.0002769	0.0002017
VLO	1.76065968	1.79401459	1.676099	1.7626	1.8260	-0.14347151	-0.09887628	-0.124991	-0.100600	-0.1160	0.00704143	0.01615516	0.00683728	0.00705103	0.00714084	0.0001001	0.0004979	0.0000722	0.0001827	0.0002235
VZ	1.73073281	1.76581455	1.600585	1.7224	1.8030	-0.02233089	-0.06453668	-0.019855	0.002800	0.0305	0.00407413	0.00932025	0.00388161	0.00406669	0.00413478	0.0000906	0.0001335	0.0002517	0.0001188	0.0001489
WHR	1.71272295	1.75276928	1.605976	1.7042	1.7882	0.05819970	0.13476957	0.091302	0.087600	0.1618	0.00552288	0.01268765	0.00528335	0.00551362	0.00562049	0.0001592	0.0005799	0.0001835	0.0002187	0.0002441
WMB	1.61814019	1.63151941	1.559860	1.6204	1.6641	-0.01678207	0.02714906	-0.015904	0.006700	0.0524	0.00592689	0.01359127	0.00575167	0.00593239	0.00601664	0.0001617	0.0006791	0.0003560	0.0002289	0.0003456
WMS	1.67575876	1.71309611	1.625520	1.6702	1.7513	0.25315811	0.43992134	0.302316	0.291000	0.4587	0.00872375	0.02008692	0.00859341	0.00870800	0.00891128	0.0004337	0.0018648	0.0004026	0.0005788	0.0006493
WMT	1.74199760	1.79192369	1.620915	1.7341	1.8058	0.07715391	0.17235489	0.047037	0.107300	0.1785	0.00519611	0.01183803	0.00489202	0.00519102	0.00526633	0.0003410	0.0010220	0.0005559	0.0003936	0.0004003
WRB	1.68197133	1.71888528	1.557112	1.6774	1.7504	0.16837646	0.26896116	0.133313	0.190300	0.2238	0.00462590	0.01063402	0.00440138	0.00462271	0.00470595	0.0002860	0.0008786	0.0005255	0.0003315	0.0002605
WY	1.80912221	1.83352673	1.700228	1.8474	1.8600	0.11467626	0.26119764	0.254048	0.156100	0.4078	0.00564047	0.01291198	0.00543365	0.00569753	0.00569090	0.0001191	0.0005241	0.0001046	0.0001472	0.0002847
XOM	1.81267118	1.82844459	1.770608	1.8507	1.8480	-0.15215652	-0.18455052	-0.269757	-0.139700	-0.1297	0.00399962	0.00918148	0.00393274	0.00403563	0.00403099	0.0001198	0.0002649	0.0002469	0.0001532	0.0001632
XRX	1.62047486	1.63633152	1.553642	1.6233	1.6723	-0.00997619	0.00642570	0.067480	0.010800	0.0182	0.00524623	0.01201995	0.00509489	0.00525105	0.00532859	0.0000011	0.0000963	-0.0001992	0.0000505	0.0000638

Table C: percentage comparison $[100 * (\hat{\theta}_i - \hat{\theta}_M)/\hat{\theta}_M]$ between the four estimators $\hat{\theta}_i$ K=Koutrouvelis (Gauss code), Q=Quantile method (STABLE), ML=Maximum likelihood (STABLE), ChF=Characteristic function method (STABLE) with reference to the maximum likelihood parameters computed by Mathematica $\hat{\theta}_M$.

	α				β				γ				δ			
	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF
AA	1.471%	-5.927%	-0.542%	3.108%	188%	115%	47%	244%	129.57%	-3.97%	-0.18%	1.28%	488.54%	-34.20%	57.27%	153.08%
ABT	1.297%	-7.521%	1.955%	2.677%	460%	45%	131%	1112%	128.69%	-5.09%	0.89%	0.70%	193.25%	42.03%	9.67%	50.66%
ADI	2.419%	-7.593%	-0.593%	4.491%	101%	3%	28%	120%	129.42%	-4.81%	-0.24%	1.66%	290.18%	42.39%	39.23%	41.72%
ADM	1.500%	-0.300%	1.173%	2.487%	282%	281%	110%	496%	131.16%	-0.61%	0.56%	1.26%	237.32%	57.05%	24.50%	70.46%
ADP	2.348%	-5.887%	-0.467%	4.661%	94%	14%	22%	160%	129.19%	-3.50%	-0.13%	1.89%	200.82%	58.03%	14.18%	33.16%
AEP	1.344%	-3.769%	-0.333%	2.394%	22%	-4%	-12%	-11%	130.67%	-2.57%	-0.06%	1.15%	-1009.69%	7124.85%	974.88%	3258.42%
AET	0.854%	-3.176%	0.298%	3.580%	-308%	-719%	-179%	-1114%	128.59%	-2.28%	0.20%	1.88%	198.84%	11.04%	14.98%	76.39%
AFG	0.989%	-5.295%	-0.044%	3.647%	45%	61%	25%	2%	128.31%	-3.45%	-0.03%	1.61%	223.26%	-27.71%	39.13%	-45.07%
AFL	0.993%	-2.718%	-0.184%	3.619%	65%	-14%	21%	68%	128.64%	-1.40%	-0.03%	1.90%	206.72%	17.63%	15.65%	16.98%
AIG	0.039%	-2.688%	-0.052%	1.448%	280%	-29%	71%	336%	129.28%	-2.07%	0.00%	1.14%	453.33%	-19.78%	32.36%	152.20%
AIR	2.258%	-5.088%	-0.454%	4.278%	76%	89%	27%	114%	130.43%	-2.58%	-0.18%	1.99%	321.69%	-74.55%	53.90%	88.35%
ALK	1.826%	-0.764%	0.134%	3.642%	97%	232%	31%	228%	129.95%	-0.62%	0.14%	1.71%	252.41%	7.09%	30.27%	104.42%
AM	0.898%	-9.118%	0.230%	3.060%	477%	-198%	141%	197%	128.19%	-6.92%	0.14%	1.43%	480.23%	156.85%	51.44%	33.52%
AMD	1.598%	-3.270%	-0.449%	2.971%	190%	270%	90%	369%	130.47%	-2.02%	-0.15%	1.32%	748.26%	-35.97%	174.88%	427.98%
AME	1.968%	-5.445%	0.109%	5.376%	-355%	211%	-144%	31%	127.81%	-4.05%	0.12%	2.27%	214.39%	12.71%	18.54%	-1.69%
AMGN	2.824%	-6.722%	-0.217%	5.545%	61%	23%	29%	132%	129.88%	-3.16%	-0.06%	2.51%	170.85%	16.25%	17.00%	27.80%
AMN	2.150%	-6.393%	0.030%	5.547%	158%	11%	24%	97%	128.17%	-5.43%	0.09%	2.72%	304.32%	52.59%	16.72%	23.94%
AON	1.520%	-4.451%	-0.422%	3.165%	101%	-66%	53%	272%	129.87%	-2.41%	-0.15%	1.51%	183.74%	83.98%	15.19%	49.74%
APA	1.551%	0.160%	0.570%	2.496%	192%	55%	97%	100%	130.51%	-1.05%	0.11%	0.99%	415.09%	150.91%	77.63%	34.95%
APD	2.255%	-4.958%	-0.370%	4.429%	164%	36%	52%	248%	129.32%	-3.23%	-0.08%	1.76%	242.13%	12.87%	25.13%	48.96%
ARW	1.355%	-2.885%	-0.508%	3.355%	66%	68%	27%	116%	129.53%	-1.08%	-0.19%	1.49%	383.72%	-19.40%	75.94%	130.49%
ASH	1.240%	-2.151%	-0.108%	3.076%	494%	-270%	677%	911%	129.92%	-1.33%	-0.03%	1.50%	178.89%	7.05%	33.49%	32.73%
AVP	1.442%	-5.335%	-0.059%	4.326%	64%	59%	25%	128%	128.28%	-3.16%	0.01%	2.04%	265.77%	-105.69%	34.40%	83.13%
AVT	2.975%	-7.789%	-0.399%	3.399%	572%	177%	166%	458%	135.09%	-5.22%	-0.12%	1.15%	883.18%	-6.90%	140.50%	223.02%
AVY	2.157%	-6.581%	-0.479%	3.716%	258%	-1%	91%	267%	126.88%	-3.91%	-0.17%	1.47%	323.16%	107.78%	31.49%	46.82%
AXE	2.181%	-4.662%	-0.472%	4.461%	139%	-43%	45%	224%	129.64%	-2.63%	-0.17%	1.96%	252.48%	69.08%	25.11%	63.56%
AXP	1.881%	-8.540%	-0.506%	3.715%	392%	136%	148%	489%	129.35%	-5.64%	-0.18%	1.35%	277.33%	17.77%	31.60%	60.76%
BA	1.885%	-4.926%	0.245%	3.551%	235%	106%	99%	541%	129.73%	-3.17%	0.23%	1.38%	228.21%	7.64%	25.63%	75.45%
BAX	1.141%	-0.995%	1.885%	2.174%	-338%	-295%	-58%	-599%	130.07%	-1.39%	0.89%	0.94%	239.57%	89.35%	17.19%	77.44%
BC	1.231%	-4.977%	0.217%	3.633%	144%	70%	42%	146%	128.96%	-3.00%	0.12%	1.77%	335.51%	-26.89%	30.23%	62.75%
BCR	2.653%	-8.964%	-0.498%	4.685%	143%	34%	90%	412%	129.32%	-6.71%	-0.17%	1.53%	176.10%	27.11%	19.10%	44.19%
BDX	1.453%	-6.940%	-0.377%	3.248%	354%	-107%	107%	519%	129.04%	-4.09%	-0.07%	1.23%	212.77%	48.02%	14.84%	41.45%
BEN	1.817%	-5.318%	0.050%	5.462%	195%	-75%	50%	126%	127.45%	-3.46%	0.08%	2.39%	250.19%	25.11%	18.73%	12.88%
BF-B	2.483%	-9.230%	-0.493%	4.855%	56%	-65%	22%	62%	128.52%	-5.78%	-0.22%	1.60%	155.31%	75.47%	12.38%	-0.53%
BGG	2.383%	-6.512%	-0.490%	4.498%	479%	458%	200%	442%	129.79%	-3.88%	-0.19%	1.85%	371.95%	-33.63%	66.50%	72.99%
BK	1.233%	-4.934%	0.336%	4.071%	114%	-33%	40%	41%	128.40%	-3.75%	0.20%	1.95%	303.31%	49.61%	34.05%	-6.55%
BLL	2.237%	-6.397%	-0.505%	3.967%	175%	-111%	64%	-21%	129.98%	-4.59%	-0.17%	1.52%	203.67%	81.24%	20.16%	-16.79%
BMS	2.307%	-7.360%	-0.352%	4.439%	179%	-80%	48%	212%	128.81%	-4.53%	-0.06%	1.64%	229.19%	50.47%	17.21%	28.19%
BMY	1.616%	-4.388%	-0.354%	3.457%	96%	-6%	49%	372%	129.36%	-2.61%	-0.06%	1.45%	188.06%	89.91%	18.13%	74.45%
BOH	2.087%	-5.566%	0.165%	5.511%	-239%	-235%	-531%	-133%	128.69%	-3.55%	0.16%	2.64%	149.11%	124.30%	22.08%	5.63%
BP	1.236%	-3.817%	1.111%	2.740%	-63%	-15%	-24%	-138%	129.41%	-1.88%	0.61%	1.09%	329.98%	238.34%	45.13%	152.42%

	α				β				γ				δ			
	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF
BT	1.587%	-5.223%	0.009%	5.032%	-6756%	-221%	-3102%	-6158%	127.80%	-3.35%	0.05%	2.30%	470.74%	83.60%	85.52%	105.68%
BWS	1.459%	-6.757%	0.080%	5.136%	465%	758%	310%	279%	127.87%	-3.96%	0.09%	2.59%	393.62%	-119.97%	101.34%	35.32%
C	0.333%	-6.107%	0.030%	2.275%	127505280%	59810527%	56482746%	112730246%	128.60%	-5.22%	0.03%	1.20%	384.49%	-93.30%	50.45%	84.81%
CA	1.287%	-7.425%	-0.132%	4.311%	174%	79%	129%	401%	127.49%	-5.27%	-0.02%	1.82%	207.19%	54.78%	27.06%	53.40%
CAG	2.080%	-6.316%	-0.480%	4.070%	832%	3%	331%	559%	129.70%	-4.76%	-0.18%	1.66%	200.51%	68.88%	16.04%	14.82%
CAT	2.053%	-4.014%	-0.455%	4.246%	209%	62%	53%	299%	129.43%	-2.10%	-0.12%	1.83%	290.80%	22.10%	27.85%	71.94%
CB	1.712%	-4.862%	0.225%	4.694%	44%	-10%	22%	48%	128.58%	-2.97%	0.11%	2.13%	168.51%	21.30%	13.90%	2.19%
CBE	1.688%	-6.537%	-0.512%	3.423%	-46%	-105%	-144%	-370%	129.28%	-5.01%	-0.22%	1.24%	166.90%	66.74%	36.81%	80.79%
CBRL	2.282%	-5.487%	-0.464%	4.594%	1375%	-587%	721%	1564%	129.38%	-3.35%	-0.15%	1.91%	245.09%	99.97%	34.20%	44.39%
CBT	1.918%	-6.313%	-0.311%	3.692%	130%	9%	30%	104%	130.10%	-4.98%	-0.09%	1.58%	304.05%	19.21%	26.99%	34.39%
CCK	0.170%	-3.131%	0.049%	1.329%	152%	-1109%	419%	1134%	130.24%	-2.26%	0.08%	0.87%	225.84%	44.76%	51.02%	149.08%
CEG	1.109%	-1.512%	-0.023%	1.919%	-3%	95%	-17%	-63%	129.99%	-1.17%	0.09%	0.85%	166.22%	176.00%	18.70%	71.18%
CI	0.736%	-5.213%	-0.084%	3.351%	579%	-308%	154%	550%	127.94%	-2.72%	-0.01%	1.68%	268.32%	64.17%	18.56%	45.40%
CL	1.747%	-6.520%	-0.371%	3.490%	73%	-22%	13%	105%	129.05%	-4.64%	-0.07%	1.21%	203.82%	20.96%	12.45%	26.37%
CLX	1.744%	-4.754%	0.106%	5.100%	171%	-94%	44%	110%	128.34%	-2.13%	0.12%	2.44%	218.80%	43.75%	13.82%	10.80%
CMI	1.740%	-6.632%	0.124%	5.006%	104%	41%	42%	179%	128.07%	-4.21%	0.11%	2.24%	232.20%	-75.56%	24.02%	50.22%
CNA	1.027%	-6.887%	0.023%	3.674%	68%	35%	16%	3%	128.29%	-5.17%	0.07%	1.72%	311.90%	39.28%	27.85%	-49.82%
CNW	2.603%	-4.039%	-0.324%	4.859%	96%	51%	20%	125%	130.47%	-2.48%	-0.13%	2.22%	334.19%	-60.95%	33.00%	74.99%
COO	1.528%	-5.696%	0.059%	6.401%	65%	163%	83%	8%	127.66%	-4.88%	0.04%	3.53%	-871.56%	-2253.38%	-1191.92%	676.29%
COP	1.565%	-1.509%	2.137%	2.553%	-41%	39%	-50%	-5%	130.72%	-1.85%	0.90%	1.09%	278.96%	75.98%	68.33%	41.99%
CP	1.808%	-6.637%	0.088%	5.618%	1169%	700%	367%	1627%	127.27%	-4.76%	0.11%	2.48%	227.10%	16.72%	15.56%	42.35%
CPB	2.718%	-5.342%	-0.119%	5.167%	67%	-14%	18%	85%	130.15%	-3.41%	0.01%	2.31%	193.97%	54.42%	15.60%	16.36%
CR	1.815%	-4.296%	-0.421%	3.300%	-56%	165%	-47%	-17%	130.36%	-3.19%	-0.15%	1.43%	275.38%	151.96%	47.26%	41.82%
CSC	2.211%	-8.443%	-0.321%	4.161%	103%	3%	17%	95%	129.63%	-6.23%	-0.12%	1.67%	290.39%	25.49%	19.94%	31.91%
CSX	1.861%	-5.847%	0.399%	3.704%	208%	-11%	83%	271%	129.21%	-3.30%	0.30%	1.38%	227.40%	65.42%	24.73%	38.28%
CVS	1.673%	-5.401%	-0.517%	3.496%	92%	36%	32%	206%	129.36%	-3.37%	-0.15%	1.42%	199.39%	40.64%	17.44%	52.08%
CVX	1.289%	-2.619%	1.926%	2.718%	-30%	174%	-48%	66%	129.34%	-1.73%	0.93%	1.02%	231.72%	101.84%	48.81%	-8.95%
DBD	1.720%	-6.948%	0.180%	5.219%	113%	-9%	42%	95%	127.45%	-4.69%	0.14%	2.31%	270.24%	-88.97%	33.08%	17.59%
DD	1.794%	-2.694%	-0.312%	4.240%	179%	78%	41%	299%	140.78%	-1.82%	-0.04%	1.82%	283.30%	10.35%	25.56%	80.69%
DE	1.918%	-6.614%	-0.205%	3.806%	124%	41%	41%	262%	129.17%	-4.16%	0.00%	1.39%	239.59%	75.67%	26.06%	75.62%
DIS	1.408%	-5.975%	-0.501%	3.798%	89%	-26%	39%	231%	120.68%	-3.71%	-0.15%	1.53%	227.34%	81.13%	19.78%	56.53%
DNB	1.482%	-6.807%	0.093%	5.433%	20%	-9%	-15%	-2%	127.46%	-4.92%	0.11%	2.68%	111.48%	-89.11%	12.97%	30.37%
DOV	2.427%	-5.035%	-0.490%	4.622%	114%	-52%	37%	190%	129.69%	-2.93%	-0.14%	1.90%	211.74%	38.70%	20.88%	39.39%
DOW	2.188%	-5.236%	-0.401%	4.146%	975%	195%	302%	853%	130.18%	-3.41%	-0.12%	1.81%	403.15%	97.46%	49.16%	82.34%
DUK	1.500%	-6.454%	-0.457%	2.791%	33%	2%	-12%	-32%	129.90%	-4.75%	-0.11%	1.03%	78.30%	177.75%	18.38%	109.68%
ED	1.762%	-3.716%	2.234%	2.942%	33%	75%	-12%	60%	130.62%	-1.42%	0.97%	1.29%	121.17%	179.75%	41.55%	-7.35%
EIX	1.152%	-4.645%	-0.103%	2.162%	21%	34%	-12%	7%	130.20%	-3.80%	0.00%	1.05%	-55.05%	763.41%	91.20%	99.99%
EMR	2.111%	-6.517%	-0.512%	4.218%	64%	46%	22%	198%	129.54%	-4.09%	-0.15%	1.70%	189.49%	6.28%	17.47%	62.05%
ETN	1.764%	-6.968%	-0.443%	3.767%	137%	-77%	37%	158%	129.24%	-4.33%	-0.11%	1.39%	228.11%	50.65%	19.11%	30.53%
ETR	0.921%	2.211%	0.158%	1.427%	-71%	116%	-29%	-23%	131.91%	2.39%	0.13%	0.98%	287.45%	19.65%	30.66%	28.07%
EXC	1.779%	0.858%	0.852%	2.663%	30%	65%	-9%	5%	131.99%	-0.25%	0.42%	1.47%	0.22%	707.50%	160.63%	266.23%
F	1.054%	-2.372%	-0.364%	2.462%	84%	111%	24%	97%	129.47%	-1.76%	-0.08%	1.11%	306.41%	-81.73%	37.38%	62.46%
FDX	2.817%	-6.756%	-0.589%	5.194%	94%	14%	14%	131%	129.22%	-4.50%	-0.23%	2.00%	259.29%	-30.26%	21.47%	39.59%
FL	2.444%	-5.205%	-0.294%	4.316%	55%	44%	17%	29%	130.63%	-4.33%	-0.11%	1.90%	251.51%	-154.11%	44.19%	-35.75%

	α				β				γ				δ			
	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF
FRM	1.637%	7.470%	0.022%	2.494%	149%	-100%	44%	248%	131.42%	3.53%	-0.17%	1.31%	-8788.52%	20142.04%	-2231.01%	-5749.61%
FWLT	1.051%	-2.863%	-0.065%	3.676%	54%	127%	37%	80%	129.32%	0.05%	-0.02%	2.11%	2443.68%	-3771.91%	840.63%	1054.65%
GAS	1.705%	-4.266%	1.656%	2.698%	6%	93%	-24%	-32%	130.77%	-4.02%	0.74%	1.11%	160.79%	120.12%	24.41%	40.16%
GCI	1.131%	-3.495%	0.198%	3.501%	153%	104%	41%	194%	129.03%	-2.47%	0.13%	1.77%	340.83%	-71.87%	28.69%	88.84%
GPC	2.122%	-5.807%	-0.315%	4.157%	93%	25%	25%	21%	129.25%	-3.55%	-0.10%	1.57%	199.13%	84.35%	18.21%	-18.86%
GR	2.552%	-5.704%	-0.231%	5.000%	100%	-3%	20%	90%	129.68%	-3.82%	-0.06%	2.15%	262.31%	0.75%	21.52%	20.57%
GRA	2.215%	-7.346%	-0.007%	6.618%	-1137%	-1458%	-628%	-292%	129.14%	-4.85%	0.04%	3.71%	-725.37%	-1581.29%	-306.91%	38.25%
GT	1.677%	-3.987%	0.298%	4.664%	147%	132%	37%	216%	128.96%	-2.62%	0.15%	2.29%	425.40%	-187.46%	39.05%	135.30%
GWW	2.261%	-6.737%	-0.493%	4.328%	91%	-53%	35%	153%	129.66%	-3.78%	-0.13%	1.75%	178.16%	46.30%	15.29%	23.82%
HAL	1.771%	-2.551%	-0.036%	3.350%	378%	336%	158%	587%	129.73%	-2.22%	0.07%	1.36%	490.39%	15.73%	88.73%	192.87%
HAS	2.175%	-8.009%	-0.476%	4.012%	106%	-26%	20%	37%	129.48%	-6.08%	-0.16%	1.42%	231.29%	71.26%	16.28%	-8.60%
HD	2.335%	-6.737%	-0.499%	4.644%	138%	-18%	40%	133%	129.24%	-3.29%	-0.15%	1.89%	189.04%	41.19%	13.49%	13.07%
HES	1.390%	-3.479%	-0.343%	3.021%	-4072%	-3780%	-1690%	-7877%	129.67%	-1.86%	-0.05%	1.33%	301.65%	-57.51%	39.19%	121.41%
HNZ	2.016%	-6.398%	-0.363%	3.685%	212%	-206%	125%	87%	129.58%	-4.53%	-0.12%	1.30%	169.47%	53.57%	15.22%	1.77%
HON	2.009%	-3.843%	-0.308%	3.998%	620%	-175%	218%	721%	129.91%	-2.29%	-0.08%	1.82%	283.08%	31.58%	30.46%	63.33%
HPG	1.769%	-5.715%	-0.357%	3.680%	-105%	34%	-77%	-34%	129.26%	-3.24%	-0.12%	1.44%	329.39%	144.31%	70.23%	43.16%
HRS	1.835%	-5.192%	-0.444%	3.352%	255%	-338%	140%	149%	129.94%	-3.68%	-0.17%	1.28%	311.58%	149.23%	61.00%	31.17%
HTS	0.737%	-4.288%	0.312%	2.478%	106%	-58%	44%	94%	129.71%	-2.96%	0.17%	1.33%	257.26%	80.38%	25.21%	35.19%
HSY	2.147%	-6.552%	-0.491%	4.210%	51%	9%	22%	142%	129.38%	-3.65%	-0.13%	1.71%	158.93%	66.52%	12.59%	28.46%
HUM	1.692%	-2.573%	-0.543%	3.374%	224%	6%	53%	280%	130.14%	-1.64%	-0.16%	1.56%	288.85%	-17.35%	24.36%	68.29%
IBM	2.046%	-5.127%	-0.486%	4.158%	55%	38%	26%	113%	129.47%	-3.08%	-0.19%	1.74%	192.41%	-43.83%	26.78%	37.61%
IFF	1.907%	-6.892%	-0.512%	3.835%	90%	-34%	22%	127%	129.39%	-4.61%	-0.15%	1.50%	244.98%	62.89%	21.77%	43.66%
IP	1.541%	-4.231%	-0.527%	3.063%	248%	8%	93%	233%	130.24%	-2.50%	-0.19%	1.39%	373.53%	117.16%	54.52%	78.48%
IR	1.866%	-4.280%	-0.437%	3.641%	1214%	-1240%	685%	2572%	129.70%	-3.11%	-0.11%	1.50%	249.09%	142.86%	33.34%	83.11%
JCP	1.790%	-6.958%	0.012%	5.573%	110%	62%	30%	100%	127.24%	-4.40%	0.03%	2.46%	483.86%	-148.65%	60.93%	44.19%
JPM	1.197%	-6.421%	0.288%	4.263%	110%	-64%	72%	33%	127.59%	-4.73%	0.12%	1.87%	308.00%	78.42%	68.47%	-17.86%
K	2.499%	-7.804%	-0.374%	4.422%	123%	-125%	53%	-109%	129.79%	-5.10%	-0.12%	1.68%	179.80%	13.27%	20.75%	-46.37%
KMB	2.126%	-6.596%	-0.504%	4.099%	71%	-3%	15%	102%	129.72%	-3.86%	-0.13%	1.73%	187.85%	37.97%	12.62%	21.40%
KO	1.507%	-6.723%	-0.338%	3.546%	149%	-77%	30%	116%	132.97%	-4.07%	-0.04%	1.34%	233.41%	102.29%	16.95%	20.72%
KR	1.720%	-7.316%	-0.282%	3.054%	113%	-38%	25%	106%	129.23%	-5.33%	-0.03%	1.05%	232.50%	97.40%	17.20%	23.74%
LLY	1.638%	-7.251%	0.004%	3.317%	177%	-149%	67%	219%	128.83%	-4.34%	0.12%	1.11%	216.37%	47.17%	20.21%	32.83%
LMT	1.634%	-5.805%	0.153%	5.077%	412%	68%	105%	304%	127.77%	-4.06%	0.13%	2.30%	285.96%	-85.90%	22.37%	31.58%
LNC	0.125%	-4.000%	0.079%	1.615%	671%	-93%	240%	805%	129.27%	-3.32%	0.06%	1.13%	362.20%	91.95%	35.40%	108.02%
LPX	1.345%	-6.234%	-0.154%	4.198%	72%	127%	38%	72%	127.96%	-4.65%	-0.06%	1.80%	342.26%	-118.98%	71.83%	33.58%
LUV	1.258%	-3.964%	1.431%	2.756%	136%	137%	37%	289%	128.82%	-3.12%	0.63%	0.91%	254.69%	10.37%	16.22%	85.97%
MAS	2.439%	-5.709%	-0.472%	4.695%	201%	125%	56%	276%	129.96%	-3.67%	-0.17%	2.05%	352.20%	-72.62%	40.80%	97.15%
MAT	1.776%	-5.727%	-0.596%	3.586%	82%	17%	17%	74%	129.61%	-3.50%	-0.23%	1.48%	284.59%	9.23%	27.56%	27.12%
MCD	1.497%	-3.969%	1.572%	2.962%	124%	-73%	46%	138%	129.51%	-2.46%	0.75%	1.11%	182.40%	70.94%	8.84%	16.41%
MDT	2.258%	-6.212%	-0.497%	4.433%	381%	169%	176%	886%	129.49%	-4.27%	-0.15%	1.79%	199.56%	97.26%	17.06%	51.18%
MMM	2.391%	-6.956%	-0.489%	4.441%	128%	-74%	61%	314%	129.58%	-4.21%	-0.13%	1.75%	180.66%	27.30%	16.64%	41.46%
MRK	1.274%	-6.339%	0.449%	2.751%	-81%	-125%	-70%	-518%	129.10%	-4.32%	0.34%	0.97%	171.45%	75.81%	16.68%	79.36%
MRO	1.211%	-2.236%	2.362%	1.981%	-64%	-59%	-115%	-44%	130.60%	-1.21%	0.94%	0.87%	266.74%	118.01%	88.22%	50.20%
MSI	1.780%	-4.696%	-0.542%	3.554%	240%	-51%	70%	281%	129.77%	-3.32%	-0.18%	1.47%	339.13%	35.88%	38.52%	80.89%
MTRN	1.371%	-5.977%	0.115%	4.496%	-90%	-404%	-165%	-230%	128.12%	-3.89%	0.05%	2.13%	449.43%	-502.46%	255.04%	297.57%

	α				β				γ				δ			
	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF
MUR	2.228%	-6.827%	-0.420%	4.231%	-212%	-108%	-149%	-394%	129.39%	-4.34%	-0.11%	1.58%	299.30%	-213.93%	59.53%	108.32%
NAV	1.370%	-2.518%	0.482%	2.419%	87%	137%	46%	127%	130.79%	-0.62%	0.22%	1.14%	-143.47%	-2.57%	-90.11%	-142.58%
NBL	1.439%	-3.073%	2.251%	2.649%	175%	169%	111%	179%	130.05%	-1.40%	0.98%	1.05%	334.71%	89.96%	54.97%	66.61%
NC	1.926%	-5.174%	0.083%	5.124%	178%	67%	44%	165%	128.86%	-3.30%	0.08%	2.47%	412.63%	-25.11%	38.21%	74.13%
NEE	1.293%	-1.182%	-0.423%	2.519%	15%	101%	-14%	-27%	130.70%	-0.32%	-0.10%	1.29%	138.25%	110.69%	13.89%	52.57%
NEM	1.793%	-4.202%	-0.484%	3.838%	109%	83%	31%	50%	129.06%	-2.54%	-0.20%	1.54%	1011.21%	-424.92%	205.80%	-18.87%
NOC	2.540%	-8.808%	-0.424%	4.846%	807%	429%	428%	1949%	129.07%	-6.65%	-0.15%	1.76%	225.94%	63.75%	27.47%	76.41%
NSC	2.521%	-6.557%	-0.453%	4.785%	207%	-100%	112%	561%	129.41%	-4.56%	-0.17%	1.85%	201.49%	6.84%	22.72%	62.30%
NSM	2.286%	-13.918%	1.322%	2.830%	48%	-16%	30%	308%	131.01%	-8.48%	0.75%	0.75%	132.59%	-12.97%	4.08%	22.59%
NUE	1.350%	-6.294%	-0.388%	2.913%	-34948956%	-20807650%	-20129657%	-126348548%	128.96%	-4.70%	-0.08%	1.02%	218.66%	8.67%	25.67%	109.37%
NVO	1.616%	-6.073%	-0.425%	3.464%	231%	-59%	62%	140%	129.03%	-3.49%	-0.13%	1.37%	196.97%	30.88%	12.46%	10.28%
NWL	2.283%	-6.896%	-0.134%	4.119%	233%	-90%	58%	123%	130.28%	-5.41%	0.00%	1.74%	258.85%	95.09%	20.11%	15.30%
OMX	0.978%	-4.541%	0.023%	3.503%	79%	114%	28%	63%	129.11%	-2.65%	0.03%	1.84%	1798.19%	-597.70%	326.71%	287.80%
OXY	1.262%	-4.646%	1.847%	2.431%	-35%	-111%	-48%	25%	129.87%	-1.87%	0.85%	0.94%	312.79%	-185.83%	116.45%	7.54%
PBI	1.390%	-5.865%	0.043%	3.225%	-1035%	681%	-342%	-1556%	128.52%	-3.39%	0.13%	1.15%	277.42%	119.22%	25.59%	77.08%
PCG	1.185%	1.688%	-0.120%	1.125%	10%	103%	-13%	-52%	132.40%	-0.33%	0.00%	0.78%	236.58%	833.19%	108.35%	491.72%
PEP	1.969%	-6.809%	-0.432%	4.116%	168%	42%	86%	353%	129.03%	-3.24%	-0.15%	1.60%	168.49%	17.42%	13.07%	27.41%
PFE	1.860%	-7.695%	0.466%	3.771%	122%	33%	27%	236%	128.84%	-4.35%	0.33%	1.30%	221.32%	64.12%	13.50%	53.39%
PG	2.026%	-8.304%	-0.428%	3.746%	187%	-120%	31%	156%	128.95%	-5.34%	-0.09%	1.28%	223.35%	29.69%	11.80%	19.59%
PGN	1.588%	-2.934%	1.558%	2.713%	35%	43%	-8%	43%	130.44%	-3.26%	0.75%	1.13%	57.24%	344.84%	62.07%	8.49%
PH	2.122%	-5.433%	-0.281%	3.981%	224%	-46%	70%	142%	129.43%	-3.86%	-0.04%	1.47%	251.30%	8.98%	27.83%	16.22%
PKI	1.833%	-6.630%	-0.418%	3.655%	123697%	-7273%	57749%	195560%	129.58%	-4.50%	-0.13%	1.49%	285.57%	89.31%	37.07%	87.28%
PNW	1.076%	-2.929%	-0.416%	1.949%	20%	-2%	-12%	-20%	130.74%	-2.31%	-0.10%	0.91%	276.55%	-789.20%	-123.84%	-488.53%
PPG	1.956%	-4.482%	-0.307%	3.899%	105%	-181%	53%	114%	129.41%	-2.70%	-0.09%	1.55%	183.46%	90.52%	18.70%	14.71%
R	1.917%	-4.418%	-0.088%	4.007%	614%	-260%	238%	947%	129.31%	-1.87%	0.06%	1.68%	285.53%	73.12%	35.19%	79.20%
RDC	1.590%	2.496%	-0.916%	1.956%	218%	448%	56%	443%	131.96%	-3.93%	-0.59%	1.05%	1092.76%	469.13%	319.36%	782.70%
ROK	2.278%	-6.151%	-0.386%	4.562%	-275%	-83%	-113%	-455%	129.68%	-3.45%	-0.17%	1.96%	281.67%	23.46%	28.30%	89.46%
RSH	1.541%	-7.657%	-0.531%	3.074%	816%	931%	467%	1153%	129.32%	-5.09%	-0.19%	1.07%	789.00%	55.54%	198.56%	327.60%
RTN	2.254%	-4.954%	-0.302%	4.418%	279%	-143%	117%	295%	130.04%	-2.74%	-0.06%	1.93%	236.61%	54.33%	27.81%	36.78%
S	0.692%	-2.503%	0.310%	2.394%	314%	-153%	95%	267%	129.72%	-1.52%	0.16%	1.25%	533.52%	198.28%	57.16%	125.20%
SLB	1.648%	-5.075%	0.838%	3.556%	116%	41%	32%	243%	128.87%	-2.25%	0.46%	1.35%	299.42%	65.10%	27.48%	112.00%
SNE	2.024%	-5.524%	-0.584%	3.894%	76%	29%	13%	89%	129.40%	-4.46%	-0.22%	1.44%	340.37%	-65.62%	38.20%	45.81%
SO	1.889%	6.600%	1.257%	2.720%	31%	387%	-14%	111%	132.47%	2.77%	0.44%	1.64%	193.71%	494.90%	64.49%	-84.81%
SUN	1.546%	-3.411%	-0.225%	3.472%	-427%	-49%	-199%	-451%	129.04%	-2.51%	-0.05%	1.41%	431.04%	187.38%	70.27%	116.69%
T	2.680%	-7.421%	-0.255%	5.065%	14%	7%	-22%	51%	129.54%	-4.88%	-0.04%	2.05%	185.83%	160.88%	82.56%	7.52%
TAP	1.539%	-5.708%	-0.135%	4.466%	223%	9%	59%	151%	128.14%	-3.25%	-0.06%	1.96%	298.52%	67.24%	24.86%	26.40%
TEN	1.239%	-6.681%	-0.108%	5.275%	192%	185%	51%	148%	128.12%	-5.03%	-0.03%	2.85%	913.67%	342.11%	104.61%	161.71%
TGT	2.172%	-7.384%	-0.506%	4.196%	120%	-19%	30%	125%	129.37%	-3.89%	-0.15%	1.65%	224.54%	26.38%	19.18%	23.25%
THC	0.979%	-2.618%	-0.495%	1.836%	153%	179%	48%	129%	130.02%	-2.55%	-0.17%	0.79%	395.08%	-54.28%	50.94%	77.98%
TNB	2.337%	-6.974%	-0.360%	4.546%	130%	-17%	33%	120%	129.32%	-5.18%	-0.12%	1.85%	272.26%	39.27%	26.82%	29.87%
TSO	1.912%	-2.525%	-0.653%	3.264%	87%	133%	29%	138%	131.39%	-1.09%	-0.24%	1.66%	366.80%	-139.11%	62.62%	132.67%
TXI	1.444%	-6.819%	-0.113%	4.253%	77%	54%	24%	88%	128.19%	-5.12%	-0.03%	1.84%	295.28%	-66.71%	32.82%	38.43%
TXN	2.507%	-7.442%	-0.518%	4.794%	123%	37%	47%	203%	129.00%	-5.50%	-0.20%	1.73%	279.38%	-26.80%	43.49%	72.33%
TXT	0.740%	-4.067%	0.315%	2.718%	81%	238%	112%	314%	129.00%	-2.88%	0.14%	1.43%	168.43%	-51.83%	24.90%	54.13%

	α				β				γ				δ			
	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF	K	Q	ML	ChF
UIS	0.418%	-3.593%	0.013%	1.418%	94%	77%	43%	74%	130.25%	-2.61%	0.01%	0.74%	-1617.95%	936.30%	-388.39%	-473.45%
UN	2.559%	-5.473%	-0.154%	4.829%	-36%	-15%	-51%	-150%	129.86%	-3.18%	-0.03%	2.16%	162.81%	89.28%	12.70%	34.23%
UNP	2.574%	-5.804%	-0.432%	4.985%	186%	34%	51%	215%	129.14%	-3.57%	-0.11%	1.93%	232.94%	44.43%	22.21%	31.23%
UTX	1.694%	-6.828%	0.641%	3.598%	2461%	438%	1536%	6016%	128.16%	-4.32%	0.42%	1.06%	178.45%	32.49%	17.24%	39.11%
VFC	2.683%	-7.472%	-0.258%	4.882%	564%	-310%	227%	-71%	129.86%	-5.08%	-0.08%	1.97%	201.19%	4.25%	19.79%	-12.73%
VLO	1.894%	-4.803%	0.110%	3.711%	-31%	-13%	-30%	-19%	129.43%	-2.90%	0.14%	1.41%	397.43%	-27.88%	82.50%	123.31%
VZ	2.027%	-7.520%	-0.481%	4.176%	189%	-11%	-113%	-237%	128.77%	-4.73%	-0.18%	1.49%	47.33%	177.71%	31.06%	64.23%
WHR	2.338%	-6.233%	-0.498%	4.407%	132%	57%	51%	178%	129.73%	-4.34%	-0.17%	1.77%	264.30%	15.31%	37.41%	53.37%
WMB	0.827%	-3.602%	0.140%	2.840%	-262%	-5%	-140%	-412%	129.32%	-2.96%	0.09%	1.51%	320.10%	120.22%	41.58%	113.78%
WMS	2.228%	-2.998%	-0.332%	4.508%	74%	19%	15%	81%	130.26%	-1.49%	-0.18%	2.15%	330.00%	-7.17%	33.47%	49.72%
WMT	2.866%	-6.951%	-0.453%	3.663%	123%	-39%	39%	131%	127.83%	-5.85%	-0.10%	1.35%	199.67%	63.00%	15.41%	17.37%
WRB	2.195%	-7.423%	-0.272%	4.068%	60%	-21%	13%	33%	129.88%	-4.85%	-0.07%	1.73%	207.17%	83.72%	15.88%	-8.94%
WY	1.349%	-6.019%	2.116%	2.812%	128%	122%	36%	256%	128.92%	-3.67%	1.01%	0.89%	339.90%	-12.20%	23.56%	138.92%
XOM	0.870%	-2.321%	2.098%	1.949%	21%	77%	-8%	-15%	129.56%	-1.67%	0.90%	0.78%	121.00%	106.04%	27.85%	36.17%
XRX	0.979%	-4.124%	0.174%	3.198%	-164%	-776%	-208%	-282%	129.12%	-2.88%	0.09%	1.57%	8954.88%	-18826.08%	4647.73%	5899.34%

Table D: comparison between the four estimators with reference to the maximum likelihood computed by Mathematica. For α: $\hat{\alpha}_M - \hat{\alpha}_i$. For β: $\hat{\beta}_M - \hat{\beta}_i$. For γ: $ln(\hat{\gamma}_M/\hat{\gamma}_i) * 100$. For δ: $(\hat{\delta}_M - \hat{\delta}_i) * [(\hat{\gamma}_M - \hat{\gamma}_i)/2]^{-1} * 100$

	α				β				γ				δ			
	G	Q	ML	ChF	G	Q	ML	ChF	G	Q	ML	ChF	G	Q	ML	ChF
AA	-0.026	0.103	0.009	-0.054	-0.135	-0.083	-0.033	-0.175	-83.104	4.050	0.179	-1.267	-0.058	0.007	-0.011	-0.011
ABT	-0.024	0.137	-0.035	-0.049	-0.100	-0.010	-0.028	-0.242	-82.719	5.227	-0.888	-0.694	-0.063	-0.023	-0.005	-0.005
ADI	-0.042	0.132	0.010	-0.078	-0.153	-0.005	-0.042	-0.181	-83.038	4.929	0.237	-1.650	-0.067	-0.017	-0.015	-0.015
ADM	-0.027	0.005	-0.021	-0.044	-0.085	-0.085	-0.033	-0.150	-83.794	0.616	-0.561	-1.248	-0.045	-0.018	-0.008	-0.008
ADP	-0.041	0.102	0.008	-0.081	-0.106	-0.016	-0.025	-0.180	-82.937	3.566	0.127	-1.874	-0.079	-0.038	-0.009	-0.009
AEP	-0.023	0.066	0.006	-0.042	0.038	-0.006	-0.021	-0.019	-83.580	2.603	0.060	-1.143	0.003	-0.034	-0.005	-0.005
AET	-0.014	0.052	-0.005	-0.059	-0.040	-0.094	-0.023	-0.146	-82.675	2.305	-0.199	-1.862	-0.084	-0.008	-0.010	-0.010
AFG	-0.017	0.088	0.001	-0.061	-0.039	-0.053	-0.022	-0.002	-82.551	3.508	0.032	-1.599	-0.031	0.006	-0.009	-0.009
AFL	-0.017	0.046	0.003	-0.061	-0.084	0.018	-0.027	-0.088	-82.696	1.412	0.031	-1.885	-0.091	-0.013	-0.011	-0.011
AIG	-0.001	0.041	0.001	-0.022	-0.064	0.007	-0.016	-0.077	-82.979	2.096	0.003	-1.132	-0.094	0.007	-0.011	-0.011
AIR	-0.038	0.086	0.008	-0.072	-0.098	-0.116	-0.035	-0.148	-83.479	2.614	0.185	-1.968	-0.051	0.020	-0.014	-0.014
ALK	-0.032	0.014	-0.002	-0.065	-0.142	-0.342	-0.046	-0.335	-83.268	0.624	-0.144	-1.691	-0.060	-0.003	-0.012	-0.012
AM	-0.015	0.149	-0.004	-0.050	-0.075	0.031	-0.022	-0.031	-82.503	7.168	-0.136	-1.420	-0.056	-0.031	-0.010	-0.010
AMD	-0.028	0.057	0.008	-0.052	-0.124	-0.176	-0.059	-0.242	-83.496	2.036	0.147	-1.313	-0.044	0.004	-0.017	-0.017
AME	-0.033	0.090	-0.002	-0.089	-0.057	0.034	-0.023	0.005	-82.333	4.137	-0.121	-2.244	-0.068	-0.007	-0.010	-0.010
AMGN	-0.047	0.113	0.004	-0.093	-0.065	-0.024	-0.031	-0.141	-83.238	3.211	0.059	-2.477	-0.076	-0.012	-0.012	-0.012
AMN	-0.033	0.098	0.000	-0.085	-0.104	-0.008	-0.016	-0.064	-82.492	5.585	-0.091	-2.687	-0.114	-0.033	-0.010	-0.010
AON	-0.026	0.076	0.007	-0.054	-0.040	0.027	-0.021	-0.109	-83.235	2.442	0.151	-1.498	-0.061	-0.046	-0.008	-0.008
APA	-0.029	-0.003	-0.011	-0.046	-0.146	-0.042	-0.074	-0.076	-83.510	1.054	-0.113	-0.984	-0.032	-0.019	-0.010	-0.010
APD	-0.039	0.086	0.006	-0.077	-0.088	-0.019	-0.028	-0.134	-82.993	3.279	0.079	-1.746	-0.054	-0.005	-0.009	-0.009
ARW	-0.024	0.050	0.009	-0.059	-0.101	-0.104	-0.041	-0.177	-83.088	1.084	0.188	-1.484	-0.043	0.004	-0.014	-0.014
ASH	-0.021	0.036	0.002	-0.051	-0.017	0.009	-0.023	-0.031	-83.254	1.335	0.033	-1.489	-0.030	-0.002	-0.009	-0.009
AVP	-0.024	0.089	0.001	-0.072	-0.059	-0.054	-0.023	-0.118	-82.541	3.210	-0.013	-2.018	-0.046	0.030	-0.010	-0.010
AVT	-0.052	0.136	0.007	-0.059	-0.127	-0.039	-0.037	-0.102	-85.480	5.360	0.116	-1.148	-0.042	0.001	-0.011	-0.011
AVY	-0.037	0.114	0.008	-0.064	-0.076	0.000	-0.027	-0.079	-81.923	3.989	0.172	-1.463	-0.058	-0.032	-0.009	-0.009
AXE	-0.037	0.079	0.008	-0.076	-0.098	0.031	-0.032	-0.159	-83.136	2.669	0.169	-1.937	-0.074	-0.034	-0.012	-0.012
AXP	-0.032	0.146	0.009	-0.064	-0.084	-0.029	-0.032	-0.105	-83.008	5.807	0.175	-1.343	-0.058	-0.006	-0.011	-0.011
BA	-0.033	0.087	-0.004	-0.063	-0.077	-0.035	-0.032	-0.177	-83.175	3.220	-0.229	-1.370	-0.047	-0.003	-0.009	-0.009
BAX	-0.021	0.018	-0.034	-0.039	-0.115	-0.101	-0.020	-0.204	-83.321	1.401	-0.884	-0.936	-0.059	-0.036	-0.007	-0.007
BC	-0.020	0.082	-0.004	-0.060	-0.100	-0.049	-0.029	-0.101	-82.840	3.050	-0.122	-1.759	-0.081	0.011	-0.012	-0.012
BCR	-0.045	0.153	0.009	-0.080	-0.045	-0.011	-0.028	-0.130	-82.997	6.944	0.166	-1.521	-0.057	-0.015	-0.010	-0.010
BDX	-0.025	0.121	0.007	-0.057	-0.089	0.027	-0.027	-0.131	-82.874	4.177	0.066	-1.218	-0.073	-0.028	-0.008	-0.008
BEN	-0.030	0.088	-0.001	-0.091	-0.107	0.041	-0.028	-0.069	-82.175	3.519	-0.078	-2.364	-0.091	-0.015	-0.011	-0.011
BF-B	-0.043	0.159	0.008	-0.084	-0.044	0.052	-0.018	-0.049	-82.646	5.952	0.225	-1.589	-0.051	-0.042	-0.007	-0.007
BGG	-0.041	0.112	0.008	-0.077	-0.065	-0.062	-0.027	-0.060	-83.200	3.954	0.185	-1.829	-0.032	0.005	-0.009	-0.009
BK	-0.020	0.081	-0.006	-0.067	-0.066	0.019	-0.023	-0.024	-82.591	3.821	-0.204	-1.928	-0.054	-0.015	-0.010	-0.010
BLL	-0.039	0.111	0.009	-0.069	-0.071	0.045	-0.026	0.008	-83.284	4.698	0.169	-1.509	-0.054	-0.036	-0.009	-0.009
BMS	-0.040	0.128	0.006	-0.077	-0.103	0.046	-0.027	-0.122	-82.773	4.634	0.065	-1.624	-0.072	-0.027	-0.009	-0.009
BMY	-0.028	0.077	0.006	-0.061	-0.056	0.003	-0.028	-0.215	-83.013	2.642	0.056	-1.435	-0.055	-0.044	-0.009	-0.009
BOH	-0.034	0.090	-0.003	-0.089	-0.008	-0.008	-0.018	-0.004	-82.719	3.614	-0.158	-2.608	-0.036	-0.050	-0.009	-0.009
BP	-0.022	0.068	-0.020	-0.049	-0.063	-0.015	-0.024	-0.139	-83.036	1.898	-0.606	-1.086	-0.037	-0.045	-0.008	-0.008
BT	-0.026	0.087	0.000	-0.084	-0.051	-0.002	-0.024	-0.047	-82.329	3.409	-0.049	-2.272	-0.032	-0.010	-0.010	-0.010

	α				β				γ				δ			
	G	**Q**	**ML**	**ChF**	**G**	**Q**	**ML**	**ChF**	**G**	**Q**	**ML**	**ChF**	**G**	**Q**	**ML**	**ChF**
BWS	-0.023	0.109	-0.001	-0.083	-0.039	-0.063	-0.026	-0.023	-82.361	4.040	-0.089	-2.556	-0.030	0.015	-0.013	-0.012
C	-0.005	0.098	0.000	-0.036	-0.054	-0.025	-0.024	-0.048	-82.681	5.356	-0.031	-1.198	-0.055	0.023	-0.012	-0.012
CA	-0.022	0.124	0.002	-0.072	-0.046	-0.021	-0.034	-0.107	-82.193	5.412	0.016	-1.802	-0.063	-0.028	-0.013	-0.013
CAG	-0.036	0.109	0.008	-0.070	-0.059	0.000	-0.024	-0.040	-83.161	4.880	0.179	-1.644	-0.061	-0.035	-0.008	-0.008
CAT	-0.036	0.070	0.008	-0.074	-0.119	-0.035	-0.030	-0.171	-83.042	2.121	0.125	-1.813	-0.064	-0.008	-0.010	-0.010
CB	-0.028	0.080	-0.004	-0.078	-0.035	0.008	-0.017	-0.038	-82.673	3.011	-0.113	-2.104	-0.053	-0.011	-0.007	-0.007
CBE	-0.029	0.112	0.009	-0.059	-0.009	-0.020	-0.028	-0.072	-82.978	5.145	0.217	-1.231	-0.024	-0.016	-0.009	-0.009
CBRL	-0.039	0.094	0.008	-0.079	-0.070	0.030	-0.037	-0.079	-83.022	3.402	0.150	-1.896	-0.052	-0.036	-0.012	-0.012
CBT	-0.032	0.106	0.005	-0.062	-0.109	-0.007	-0.025	-0.087	-83.333	5.104	0.092	-1.571	-0.071	-0.008	-0.010	-0.010
CCK	-0.003	0.049	-0.001	-0.021	-0.007	0.054	-0.020	-0.055	-83.394	2.290	-0.080	-0.866	-0.033	-0.011	-0.012	-0.012
CEG	-0.020	0.027	0.000	-0.034	-0.004	0.133	-0.024	-0.088	-83.285	1.177	-0.093	-0.846	-0.036	-0.063	-0.007	-0.007
CI	-0.012	0.087	0.001	-0.056	-0.082	0.043	-0.022	-0.078	-82.393	2.759	0.011	-1.663	-0.078	-0.031	-0.009	-0.009
CL	-0.031	0.114	0.007	-0.061	-0.136	0.042	-0.024	-0.196	-82.877	4.747	0.065	-1.199	-0.088	-0.015	-0.009	-0.009
CLX	-0.029	0.079	-0.002	-0.085	-0.077	0.042	-0.020	-0.050	-82.568	2.149	-0.121	-2.409	-0.081	-0.027	-0.008	-0.008
CMI	-0.029	0.110	-0.002	-0.083	-0.062	-0.025	-0.025	-0.107	-82.447	4.301	-0.106	-2.211	-0.061	0.033	-0.010	-0.010
CNA	-0.016	0.110	0.000	-0.059	-0.080	-0.042	-0.019	-0.003	-82.546	5.312	-0.069	-1.703	-0.068	-0.014	-0.010	-0.010
CNW	-0.044	0.068	0.005	-0.082	-0.140	-0.074	-0.029	-0.182	-83.496	2.507	0.128	-2.200	-0.075	0.023	-0.012	-0.012
COO	-0.022	0.083	-0.001	-0.093	-0.017	-0.043	-0.022	-0.002	-82.270	5.003	-0.039	-3.468	-0.008	-0.036	-0.018	-0.018
COP	-0.028	0.027	-0.039	-0.046	-0.034	0.034	-0.042	-0.004	-83.601	1.870	-0.895	-1.088	-0.025	-0.011	-0.010	-0.010
CP	-0.030	0.110	-0.001	-0.093	-0.078	-0.046	-0.024	-0.108	-82.095	4.881	-0.109	-2.446	-0.090	-0.011	-0.010	-0.010
CPB	-0.045	0.089	0.002	-0.086	-0.073	0.015	-0.020	-0.091	-83.355	3.467	-0.010	-2.279	-0.064	-0.030	-0.009	-0.008
CR	-0.031	0.074	0.007	-0.057	-0.034	0.102	-0.029	-0.010	-83.446	3.241	0.147	-1.420	-0.030	-0.028	-0.009	-0.009
CSC	-0.037	0.142	0.005	-0.070	-0.160	-0.005	-0.027	-0.147	-83.129	6.432	0.118	-1.653	-0.100	-0.015	-0.011	-0.011
CSX	-0.033	0.104	-0.007	-0.066	-0.088	0.004	-0.035	-0.115	-82.949	3.358	-0.300	-1.374	-0.049	-0.024	-0.009	-0.009
CVS	-0.029	0.094	0.009	-0.061	-0.081	-0.031	-0.028	-0.180	-83.013	3.427	0.146	-1.405	-0.068	-0.023	-0.010	-0.010
CVX	-0.023	0.047	-0.035	-0.049	-0.028	0.159	-0.044	0.061	-83.005	1.740	-0.922	-1.019	-0.027	-0.020	-0.009	-0.009
DBD	-0.028	0.115	-0.003	-0.086	-0.059	0.005	-0.022	-0.049	-82.174	4.801	-0.136	-2.282	-0.046	0.025	-0.009	-0.009
DD	-0.031	0.047	0.005	-0.074	-0.122	-0.053	-0.028	-0.203	-87.872	1.839	0.041	-1.800	-0.057	-0.004	-0.009	-0.009
DE	-0.034	0.116	0.004	-0.067	-0.106	-0.035	-0.035	-0.223	-82.931	4.251	0.004	-1.383	-0.056	-0.030	-0.010	-0.010
DIS	-0.024	0.103	0.009	-0.066	-0.065	0.019	-0.028	-0.169	-79.156	3.781	0.147	-1.514	-0.071	-0.042	-0.010	-0.010
DNB	-0.024	0.109	-0.001	-0.087	0.020	-0.009	-0.015	-0.002	-82.181	5.046	-0.112	-2.647	-0.036	0.048	-0.007	-0.007
DOV	-0.042	0.087	0.008	-0.080	-0.082	0.037	-0.027	-0.137	-83.155	2.975	0.138	-1.880	-0.058	-0.018	-0.010	-0.009
DOW	-0.037	0.089	0.007	-0.070	-0.087	-0.017	-0.027	-0.076	-83.369	3.466	0.118	-1.799	-0.050	-0.020	-0.010	-0.010
DUK	-0.026	0.112	0.008	-0.048	0.057	0.004	-0.020	-0.056	-83.248	4.862	0.108	-1.028	-0.012	-0.046	-0.005	-0.005
ED	-0.032	0.067	-0.040	-0.053	0.062	0.142	-0.023	0.114	-83.560	1.432	-0.966	-1.286	-0.017	-0.043	-0.010	-0.010
EIX	-0.019	0.078	0.002	-0.036	0.032	0.053	-0.019	0.012	-83.380	3.874	0.002	-1.045	0.002	-0.054	-0.006	-0.006
EMR	-0.037	0.113	0.009	-0.073	-0.076	-0.054	-0.026	-0.236	-83.089	4.180	0.145	-1.686	-0.063	-0.003	-0.010	-0.009
ETN	-0.031	0.121	0.008	-0.065	-0.096	0.054	-0.026	-0.110	-82.958	4.426	0.108	-1.385	-0.065	-0.024	-0.009	-0.009
ETR	-0.016	-0.039	-0.003	-0.025	-0.064	0.104	-0.026	-0.021	-84.117	-2.362	-0.126	-0.973	-0.041	-0.005	-0.007	-0.007
EXC	-0.032	-0.015	-0.015	-0.047	0.071	0.155	-0.020	0.013	-84.152	0.252	-0.424	-1.456	0.000	-0.036	-0.008	-0.008
F	-0.019	0.042	0.006	-0.043	-0.124	-0.163	-0.036	-0.143	-83.059	1.780	0.084	-1.103	-0.057	0.025	-0.012	-0.011
FDX	-0.049	0.116	0.010	-0.090	-0.198	-0.030	-0.029	-0.276	-82.950	4.600	0.227	-1.982	-0.088	0.017	-0.012	-0.012
FL	-0.041	0.087	0.005	-0.073	-0.094	-0.076	-0.029	-0.050	-83.564	4.425	0.114	-1.884	-0.043	0.044	-0.012	-0.012
FRM	-0.030	-0.139	0.000	-0.046	-0.363	0.243	-0.107	-0.603	-83.906	-3.466	0.167	-1.305	-0.038	0.142	-0.016	-0.016

	α				β				γ				δ			
	G	Q	ML	ChF	G	Q	ML	ChF	G	Q	ML	ChF	G	Q	ML	ChF
FWLT	-0.017	0.046	0.001	-0.059	-0.042	-0.100	-0.029	-0.063	-82.997	-0.048	0.023	-2.091	-0.027	0.068	-0.015	-0.015
GAS	-0.031	0.077	-0.030	-0.048	0.005	0.080	-0.021	-0.028	-83.627	4.106	-0.736	-1.108	-0.027	-0.034	-0.007	-0.007
GCI	-0.019	0.058	-0.003	-0.058	-0.081	-0.055	-0.022	-0.103	-82.868	2.497	-0.131	-1.759	-0.066	0.023	-0.009	-0.009
GPC	-0.037	0.102	0.006	-0.073	-0.086	-0.023	-0.023	-0.019	-82.963	3.619	0.101	-1.556	-0.047	-0.034	-0.007	-0.007
GR	-0.043	0.096	0.004	-0.084	-0.123	0.003	-0.025	-0.111	-83.150	3.895	0.063	-2.125	-0.077	0.000	-0.010	-0.010
GRA	-0.033	0.110	0.000	-0.099	-0.049	-0.063	-0.027	-0.013	-82.918	4.973	-0.035	-3.643	-0.028	-0.103	-0.020	-0.019
GT	-0.028	0.066	-0.005	-0.077	-0.104	-0.093	-0.026	-0.153	-82.837	2.652	-0.153	-2.263	-0.073	0.054	-0.011	-0.011
GWW	-0.039	0.116	0.009	-0.075	-0.065	0.038	-0.025	-0.109	-83.142	3.857	0.129	-1.739	-0.063	-0.028	-0.009	-0.009
HAL	-0.031	0.045	0.001	-0.059	-0.101	-0.090	-0.042	-0.157	-83.175	2.241	-0.069	-1.353	-0.039	-0.002	-0.012	-0.011
HAS	-0.037	0.136	0.008	-0.068	-0.143	0.035	-0.027	-0.050	-83.065	6.269	0.162	-1.415	-0.098	-0.052	-0.011	-0.011
HD	-0.040	0.116	0.009	-0.080	-0.109	0.014	-0.031	-0.105	-82.959	3.350	0.150	-1.875	-0.097	-0.035	-0.011	-0.011
HES	-0.024	0.061	0.006	-0.053	-0.088	-0.082	-0.036	-0.170	-83.147	1.875	0.055	-1.322	-0.049	0.015	-0.010	-0.010
HNZ	-0.04	0.11	0.01	-0.06	-0.04	0.04	-0.02	-0.02	-83.11	4.64	0.12	-1.30	-4.61%	-2.45%	-0.68%	-0.68%
HON	-0.03	0.06	0.01	-0.07	-0.07	0.02	-0.03	-0.09	-83.25	2.31	0.08	-1.81	-5.62%	-1.05%	-1.00%	-0.99%
HPG	-0.03	0.10	0.01	-0.06	-0.05	0.02	-0.04	-0.02	-82.97	3.29	0.12	-1.43	-2.92%	-2.14%	-1.02%	-1.02%
HRS	-0.03	0.09	0.01	-0.06	-0.05	0.07	-0.03	-0.03	-83.26	3.75	0.17	-1.27	-2.93%	-2.36%	-0.95%	-0.94%
HTS	-0.01	0.07	-0.01	-0.04	-0.06	0.03	-0.03	-0.05	-83.16	3.01	-0.17	-1.32	-6.73%	-3.52%	-1.09%	-1.08%
HSY	-0.04	0.11	0.01	-0.07	-0.06	-0.01	-0.02	-0.15	-83.02	3.72	0.13	-1.70	-6.84%	-4.80%	-0.89%	-0.88%
HUM	-0.03	0.04	0.01	-0.06	-0.15	0.00	-0.04	-0.19	-83.35	1.65	0.16	-1.54	-8.92%	0.89%	-1.24%	-1.23%
IBM	-0.04	0.09	0.01	-0.07	-0.04	-0.03	-0.02	-0.09	-83.06	3.12	0.19	-1.73	-3.41%	1.30%	-0.78%	-0.78%
IFF	-0.03	0.12	0.01	-0.07	-0.10	0.04	-0.02	-0.14	-83.03	4.72	0.15	-1.49	-6.21%	-2.69%	-0.91%	-0.90%
IP	-0.03	0.07	0.01	-0.05	-0.07	0.00	-0.03	-0.07	-83.39	2.53	0.19	-1.38	-4.04%	-2.12%	-0.98%	-0.97%
IR	-0.03	0.07	0.01	-0.06	-0.06	0.06	-0.03	-0.12	-83.16	3.15	0.11	-1.49	-4.63%	-4.45%	-1.02%	-1.01%
JCP	-0.03	0.12	0.00	-0.09	-0.10	-0.06	-0.03	-0.09	-82.09	4.50	-0.03	-2.43	-5.25%	2.70%	-1.08%	-1.07%
JPM	-0.02	0.11	0.00	-0.07	-0.04	0.02	-0.03	-0.01	-82.24	4.84	-0.12	-1.85	-2.76%	-1.18%	-1.00%	-0.99%
K	-0.04	0.13	0.01	-0.08	-0.05	0.05	-0.02	0.04	-83.20	5.24	0.12	-1.67	-4.17%	-0.52%	-0.79%	-0.79%
KMB	-0.04	0.11	0.01	-0.07	-0.09	0.00	-0.02	-0.14	-83.17	3.94	0.13	-1.71	-7.92%	-2.69%	-0.88%	-0.87%
KO	-0.03	0.12	0.01	-0.06	-0.12	0.06	-0.02	-0.09	-84.57	4.16	0.04	-1.33	-6.63%	-4.94%	-0.80%	-0.80%
KR	-0.03	0.13	0.00	-0.05	-0.15	0.05	-0.03	-0.14	-82.96	5.48	0.03	-1.05	-8.18%	-5.79%	-1.00%	-0.99%
LLY	-0.03	0.13	0.00	-0.06	-0.08	0.06	-0.03	-0.10	-82.78	4.44	-0.12	-1.11	-5.36%	-1.96%	-0.82%	-0.82%
LMT	-0.03	0.10	0.00	-0.08	-0.08	-0.01	-0.02	-0.06	-82.32	4.15	-0.13	-2.27	-6.83%	3.43%	-0.88%	-0.87%
LNC	0.00	0.06	0.00	-0.03	-0.05	0.01	-0.02	-0.06	-82.97	3.38	-0.06	-1.12	-6.70%	-2.85%	-1.08%	-1.07%
LPX	-0.02	0.10	0.00	-0.07	-0.06	-0.11	-0.03	-0.06	-82.40	4.76	0.06	-1.78	-3.80%	2.21%	-1.31%	-1.29%
LUV	-0.02	0.07	-0.03	-0.05	-0.19	-0.20	-0.05	-0.41	-82.78	3.17	-0.63	-0.91	-6.58%	-0.45%	-0.69%	-0.69%
MAS	-0.04	0.10	0.01	-0.08	-0.11	-0.07	-0.03	-0.15	-83.28	3.74	0.17	-2.03	-5.97%	2.07%	-1.14%	-1.13%
MAT	-0.03	0.10	0.01	-0.06	-0.16	-0.03	-0.03	-0.14	-83.12	3.56	0.23	-1.47	-8.49%	-0.46%	-1.36%	-1.35%
MCD	-0.03	0.07	-0.03	-0.05	-0.09	0.05	-0.03	-0.10	-83.08	2.49	-0.75	-1.11	-6.76%	-4.38%	-0.54%	-0.54%
MDT	-0.04	0.11	0.01	-0.08	-0.06	-0.03	-0.03	-0.14	-83.07	4.36	0.15	-1.77	-6.93%	-5.68%	-0.98%	-0.97%
MMM	-0.04	0.12	0.01	-0.08	-0.05	0.03	-0.02	-0.11	-83.11	4.30	0.13	-1.73	-5.38%	-1.37%	-0.82%	-0.81%
MRK	-0.02	0.11	-0.01	-0.05	-0.03	-0.05	-0.03	-0.19	-82.90	4.41	-0.34	-0.97	-4.72%	-3.51%	-0.75%	-0.75%
MRO	-0.02	0.04	-0.04	-0.04	-0.03	-0.02	-0.05	-0.02	-83.55	1.22	-0.94	-0.87	-1.66%	-1.22%	-0.90%	-0.90%
MSI	-0.03	0.08	0.01	-0.06	-0.12	0.03	-0.03	-0.14	-83.19	3.37	0.18	-1.46	-6.54%	-1.16%	-1.23%	-1.22%
MTRN	-0.02	0.10	0.00	-0.07	-0.02	-0.07	-0.03	-0.04	-82.47	3.97	-0.05	-2.11	-1.25%	2.33%	-1.16%	-1.15%
MUR	-0.04	0.12	0.01	-0.07	-0.05	-0.02	-0.03	-0.09	-83.03	4.44	0.11	-1.56	-3.06%	3.68%	-1.00%	-1.00%

	α				β				γ				δ			
	G	Q	ML	ChF	G	Q	ML	ChF	G	Q	ML	ChF	G	Q	ML	ChF
NAV	-0.02	0.04	-0.01	-0.04	-0.11	-0.17	-0.06	-0.16	-83.63	0.62	-0.22	-1.14	-1.32%	-0.04%	-1.37%	-1.37%
NBL	-0.03	0.06	-0.04	-0.05	-0.09	-0.08	-0.06	-0.09	-83.31	1.41	-0.97	-1.05	-2.91%	-1.30%	-0.79%	-0.78%
NC	-0.03	0.08	0.00	-0.08	-0.10	-0.04	-0.02	-0.09	-82.79	3.35	-0.08	-2.44	-7.71%	0.78%	-1.17%	-1.16%
NEE	-0.02	0.02	0.01	-0.04	0.02	0.13	-0.02	-0.04	-83.59	0.32	0.10	-1.28	-2.77%	-3.67%	-0.46%	-0.46%
NEM	-0.03	0.07	0.01	-0.07	-0.12	-0.09	-0.03	-0.06	-82.88	2.57	0.20	-1.53	-3.63%	2.54%	-1.22%	-1.20%
NOC	-0.04	0.15	0.01	-0.08	-0.05	-0.03	-0.03	-0.12	-82.89	6.88	0.15	-1.75	-4.82%	-2.32%	-0.97%	-0.96%
NSC	-0.04	0.11	0.01	-0.08	-0.05	0.02	-0.03	-0.13	-83.03	4.67	0.17	-1.84	-4.77%	-0.27%	-0.89%	-0.88%
NSM	-0.04	0.25	-0.02	-0.05	-0.07	0.02	-0.04	-0.44	-83.73	8.86	-0.75	-0.75	-16.63%	2.81%	-0.84%	-0.84%
NUE	-0.02	0.11	0.01	-0.05	-0.07	-0.04	-0.04	-0.24	-82.84	4.81	0.08	-1.02	-5.68%	-0.38%	-1.10%	-1.09%
NVO	-0.03	0.11	0.01	-0.06	-0.09	0.02	-0.02	-0.05	-82.87	3.55	0.13	-1.36	-8.02%	-2.11%	-0.83%	-0.83%
NWL	-0.04	0.12	0.00	-0.07	-0.10	0.04	-0.02	-0.05	-83.41	5.56	0.00	-1.72	-7.90%	-4.93%	-1.01%	-1.01%
OMX	-0.02	0.07	0.00	-0.06	-0.07	-0.10	-0.02	-0.05	-82.90	2.69	-0.03	-1.82	-4.06%	2.25%	-1.21%	-1.20%
OXY	-0.02	0.08	-0.03	-0.04	-0.03	-0.09	-0.04	0.02	-83.24	1.89	-0.85	-0.94	-1.53%	1.52%	-0.94%	-0.94%
PBI	-0.02	0.10	0.00	-0.06	-0.09	0.06	-0.03	-0.14	-82.65	3.44	-0.13	-1.15	-5.60%	-4.02%	-0.85%	-0.84%
PCG	-0.02	-0.03	0.00	-0.02	0.01	0.13	-0.02	-0.06	-84.33	0.33	0.00	-0.78	-0.84%	-4.91%	-0.64%	-0.64%
PEP	-0.03	0.12	0.01	-0.07	-0.05	-0.01	-0.02	-0.10	-82.87	3.30	0.15	-1.59	-5.90%	-1.02%	-0.75%	-0.75%
PFE	-0.03	0.14	-0.01	-0.07	-0.15	-0.04	-0.03	-0.29	-82.78	4.45	-0.33	-1.30	-7.52%	-3.66%	-0.75%	-0.75%
PG	-0.04	0.14	0.01	-0.07	-0.13	0.08	-0.02	-0.11	-82.83	5.49	0.09	-1.27	-8.91%	-2.00%	-0.77%	-0.77%
PGN	-0.03	0.05	-0.03	-0.05	0.08	0.09	-0.02	0.09	-83.48	3.32	-0.75	-1.13	-0.49%	-4.95%	-0.87%	-0.87%
PH	-0.04	0.10	0.00	-0.07	-0.10	0.02	-0.03	-0.06	-83.04	3.93	0.04	-1.46	-5.25%	-0.32%	-0.96%	-0.95%
PKI	-0.03	0.11	0.01	-0.06	-0.06	0.00	-0.03	-0.10	-83.11	4.60	0.13	-1.48	-4.93%	-2.60%	-1.06%	-1.05%
PNW	-0.02	0.05	0.01	-0.03	0.04	0.00	-0.02	-0.04	-83.61	2.33	0.10	-0.91	0.71%	-3.39%	-0.53%	-0.52%
PPG	-0.03	0.08	0.01	-0.07	-0.05	0.09	-0.03	-0.06	-83.03	2.73	0.09	-1.53	-4.66%	-3.84%	-0.78%	-0.78%
R	-0.03	0.08	0.00	-0.07	-0.09	0.04	-0.04	-0.14	-82.99	1.88	-0.06	-1.67	-4.95%	-2.11%	-1.00%	-1.00%
RDC	-0.03	-0.05	0.02	-0.04	-0.40	-0.82	-0.10	-0.81	-84.14	4.01	0.59	-1.04	-3.32%	-2.42%	-1.62%	-1.60%
ROK	-0.04	0.10	0.01	-0.08	-0.07	-0.02	-0.03	-0.12	-83.15	3.51	0.17	-1.94	-6.19%	-0.86%	-1.03%	-1.01%
RSH	-0.03	0.13	0.01	-0.05	-0.07	-0.08	-0.04	-0.10	-83.00	5.23	0.19	-1.06	-3.06%	-0.36%	-1.27%	-1.26%
RTN	-0.04	0.08	0.01	-0.07	-0.05	0.03	-0.02	-0.06	-83.31	2.77	0.06	-1.91	-4.59%	-1.76%	-0.89%	-0.88%
S	-0.01	0.04	-0.01	-0.04	-0.09	0.04	-0.03	-0.07	-83.17	1.53	-0.16	-1.25	-6.55%	-4.05%	-1.16%	-1.15%
SLB	-0.03	0.09	-0.02	-0.06	-0.15	-0.05	-0.04	-0.32	-82.80	2.28	-0.46	-1.34	-5.37%	-1.94%	-0.81%	-0.80%
SNE	-0.04	0.10	0.01	-0.07	-0.18	-0.07	-0.03	-0.21	-83.03	4.57	0.22	-1.43	-6.63%	2.15%	-1.23%	-1.22%
SO	-0.03	-0.12	-0.02	-0.05	0.05	0.65	-0.02	0.19	-84.36	-2.74	-0.44	-1.62	-1.45%	-6.09%	-0.80%	-0.80%
SUN	-0.03	0.06	0.00	-0.06	-0.07	-0.01	-0.03	-0.08	-82.87	2.54	0.05	-1.40	-3.54%	-2.57%	-0.95%	-0.94%
T	-0.05	0.13	0.00	-0.09	0.01	0.01	-0.02	0.05	-83.09	5.01	0.04	-2.03	-1.11%	-1.62%	-0.81%	-0.80%
TAP	-0.03	0.10	0.00	-0.07	-0.10	0.00	-0.03	-0.06	-82.48	3.30	0.06	-1.94	-6.89%	-2.59%	-0.94%	-0.93%
TEN	-0.02	0.10	0.00	-0.08	-0.09	-0.09	-0.02	-0.07	-82.47	5.16	0.03	-2.81	-10.09%	-6.36%	-1.90%	-1.87%
TGT	-0.04	0.13	0.01	-0.07	-0.11	0.02	-0.03	-0.12	-83.02	3.97	0.15	-1.63	-7.33%	-1.45%	-1.03%	-1.02%
THC	-0.02	0.05	0.01	-0.03	-0.11	-0.13	-0.04	-0.10	-83.30	2.58	0.17	-0.79	-5.67%	1.30%	-1.21%	-1.20%
TNB	-0.04	0.12	0.01	-0.08	-0.10	0.01	-0.02	-0.09	-83.00	5.32	0.12	-1.83	-6.01%	-1.47%	-0.98%	-0.97%
TSO	-0.03	0.04	0.01	-0.06	-0.13	-0.20	-0.04	-0.21	-83.89	1.09	0.24	-1.65	-5.89%	3.72%	-1.67%	-1.65%
TXI	-0.02	0.11	0.00	-0.07	-0.09	-0.06	-0.03	-0.10	-82.50	5.25	0.03	-1.83	-6.38%	2.43%	-1.16%	-1.15%
TXN	-0.04	0.13	0.01	-0.08	-0.09	-0.03	-0.04	-0.15	-82.86	5.66	0.20	-1.71	-4.95%	0.80%	-1.27%	-1.26%
TXT	-0.01	0.07	-0.01	-0.04	-0.02	-0.05	-0.02	-0.06	-82.86	2.93	-0.14	-1.42	-3.77%	1.94%	-0.92%	-0.91%
UIS	-0.01	0.06	0.00	-0.02	-0.06	-0.05	-0.03	-0.05	-83.40	2.65	-0.01	-0.74	-3.71%	3.59%	-1.47%	-1.46%

	α				β				γ				δ			
	G	**Q**	**ML**	**ChF**	**G**	**Q**	**ML**	**ChF**	**G**	**Q**	**ML**	**ChF**	**G**	**Q**	**ML**	**ChF**
UN	-0.04	0.09	0.00	-0.08	-0.01	0.00	-0.02	-0.05	-83.23	3.23	0.03	-2.13	-5.24%	-4.81%	-0.67%	-0.67%
UNP	-0.04	0.10	0.01	-0.09	-0.10	-0.02	-0.03	-0.11	-82.92	3.64	0.11	-1.92	-5.84%	-1.87%	-0.92%	-0.91%
UTX	-0.03	0.12	-0.01	-0.06	-0.05	-0.01	-0.03	-0.11	-82.49	4.42	-0.42	-1.05	-4.61%	-1.41%	-0.73%	-0.73%
VFC	-0.05	0.13	0.00	-0.08	-0.06	0.03	-0.03	0.01	-83.23	5.21	0.08	-1.95	-5.89%	-0.21%	-0.96%	-0.95%
VLO	-0.03	0.08	0.00	-0.07	-0.04	-0.02	-0.04	-0.03	-83.04	2.94	-0.14	-1.40	-3.43%	0.40%	-1.17%	-1.16%
VZ	-0.04	0.13	0.01	-0.07	0.04	0.00	-0.03	-0.05	-82.75	4.84	0.18	-1.48	-0.64%	-4.05%	-0.69%	-0.69%
WHR	-0.04	0.11	0.01	-0.08	-0.08	-0.03	-0.03	-0.10	-83.17	4.43	0.17	-1.75	-4.62%	-0.45%	-1.08%	-1.07%
WMB	-0.01	0.06	0.00	-0.05	-0.04	0.00	-0.02	-0.07	-82.99	3.00	-0.09	-1.50	-5.30%	-3.33%	-1.13%	-1.13%
WMS	-0.04	0.05	0.01	-0.08	-0.19	-0.05	-0.04	-0.21	-83.40	1.51	0.18	-2.13	-9.93%	0.36%	-1.67%	-1.65%
WMT	-0.05	0.12	0.01	-0.06	-0.10	0.03	-0.03	-0.10	-82.34	6.03	0.10	-1.34	-7.99%	-4.26%	-1.01%	-1.00%
WRB	-0.04	0.12	0.00	-0.07	-0.10	0.04	-0.02	-0.06	-83.24	4.98	0.07	-1.72	-7.77%	-5.31%	-0.98%	-0.97%
WY	-0.02	0.11	-0.04	-0.05	-0.15	-0.14	-0.04	-0.29	-82.82	3.74	-1.01	-0.89	-4.37%	0.26%	-0.50%	-0.50%
XOM	-0.02	0.04	-0.04	-0.04	0.03	0.12	-0.01	-0.02	-83.10	1.69	-0.90	-0.78	-2.20%	-3.20%	-0.83%	-0.83%
XRX	-0.02	0.07	0.00	-0.05	-0.02	-0.08	-0.02	-0.03	-82.91	2.93	-0.09	-1.56	-1.10%	3.87%	-0.94%	-0.94%

Table E: Values and comparison between VaR and expected shortfall deriver by the Mathematica software (M), a quantile parametric calculation (Q) and the value for the correspondent normal measures of risk(N).

	$VaR^{i}_{0.99}$			$ES^{i}_{0.99}$			$VaR^{M}_{0.99} - VaR^{i}_{0.99}$		$ES^{M}_{0.99} - ES^{i}_{0.99}$	
	M	Q	N	M	Q	N	Q	N	Q	N
AA	-0.0266981	-0.0262521	-0.0238948	-0.0564898	-0.0565362	-0.0274564	-0.0004460	-0.0028033	0.0000463	-0.0290335
ABT	-0.0185550	-0.0184327	-0.0239182	-0.0349041	-0.0356593	-0.0274800	-0.0001223	0.0053632	0.0007552	-0.0074241
ADI	-0.0355615	-0.0352136	-0.0311239	-0.0746373	-0.0730665	-0.0358902	-0.0003479	-0.0044376	-0.0015708	-0.0387471
ADM	-0.0235233	-0.0233350	-0.0214597	-0.0462647	-0.0442805	-0.0253217	-0.0001883	-0.0020636	-0.0019842	-0.0209430
ADP	-0.0200405	-0.0196821	-0.0175584	-0.0424439	-0.0385262	-0.0201473	-0.0003584	-0.0024821	-0.0039177	-0.0222966
AEP	-0.0178034	-0.0180646	-0.0150376	-0.0380309	-0.0346050	-0.0173072	0.0002612	-0.0027658	-0.0034258	-0.0207236
AET	-0.0300334	-0.0294920	-0.0239060	-0.0719043	-0.0694978	-0.0274109	-0.0005415	-0.0061274	-0.0024065	-0.0444934
AFG	-0.0241875	-0.0241264	-0.0209476	-0.0555644	-0.0620501	-0.0243532	-0.0000611	-0.0032399	0.0064857	-0.0312112
AFL	-0.0283299	-0.0277601	-0.0251976	-0.0646060	-0.0744728	-0.0288828	-0.0005698	-0.0031323	0.0098669	-0.0357232
AIG	-0.0350452	-0.0355775	-0.0332741	-0.0994241	-0.0886258	-0.0381975	0.0005322	-0.0017711	-0.0107983	-0.0612266
AIR	-0.0349405	-0.0345232	-0.0300287	-0.0775438	-0.0783677	-0.0345447	-0.0004174	-0.0049118	0.0008239	-0.0429991
ALK	-0.0314528	-0.0318039	-0.0290859	-0.0620218	-0.0591415	-0.0333932	0.0003511	-0.0023669	-0.0028803	-0.0286285
AM	-0.0302427	-0.0292850	-0.0266749	-0.0735925	-0.0773298	-0.0305792	-0.0009576	-0.0035678	0.0037373	-0.0430133
AMD	-0.0439270	-0.0428840	-0.0399449	-0.0899892	-0.0830829	-0.0455782	-0.0010430	-0.0039822	-0.0069063	-0.0444110
AME	-0.0260928	-0.0260221	-0.0196336	-0.0613248	-0.0604641	-0.0224975	-0.0000708	-0.0064592	-0.0008607	-0.0388273
AMGN	-0.0323069	-0.0328770	-0.0264676	-0.0737448	-0.0763660	-0.0302738	0.0005701	-0.0058393	0.0026213	-0.0434709
AMN	-0.0303761	-0.0302262	-0.0223607	-0.0828766	-0.0879446	-0.0258445	-0.0001499	-0.0080154	0.0050680	-0.0570321
AON	-0.0217868	-0.0213682	-0.0193905	-0.0485514	-0.0482021	-0.0223706	-0.0004186	-0.0023963	-0.0003493	-0.0261808
APA	-0.0272680	-0.0269536	-0.0259942	-0.0479235	-0.0491602	-0.0297802	-0.0003144	-0.0012737	0.0012367	-0.0181433
APD	-0.0220833	-0.0218929	-0.0188079	-0.0464521	-0.0451699	-0.0217301	-0.0001904	-0.0032754	-0.0012822	-0.0247220
ARW	-0.0336382	-0.0330452	-0.0311135	-0.0692483	-0.0662204	-0.0359020	-0.0005930	-0.0025247	-0.0030279	-0.0333463
ASH	-0.0253514	-0.0253609	-0.0212164	-0.0585472	-0.0575060	-0.0243376	0.0000095	-0.0041350	-0.0010412	-0.0342096
AVP	-0.0257097	-0.0256474	-0.0218468	-0.0592308	-0.0553520	-0.0250877	-0.0000623	-0.0038628	-0.0038789	-0.0341431
AVT	-0.0285853	-0.0287385	-0.0246973	-0.0602382	-0.0653882	-0.0285433	0.0001532	-0.0038880	0.0051500	-0.0316949
AVY	-0.0239151	-0.0239766	-0.0204202	-0.0511853	-0.0498364	-0.0234481	0.0000615	-0.0034949	-0.0013490	-0.0277372
AXE	-0.0313190	-0.0310075	-0.0262924	-0.0695846	-0.0791721	-0.0301297	-0.0003115	-0.0050267	0.0095875	-0.0394549
AXP	-0.0279281	-0.0275904	-0.0237331	-0.0610198	-0.0637748	-0.0273183	-0.0003377	-0.0041950	0.0027550	-0.0337014
BA	-0.0226532	-0.0245237	-0.0197598	-0.0458660	-0.0565575	-0.0228557	0.0018706	-0.0028934	0.0106914	-0.0230103
BAX	-0.0207855	-0.0211391	-0.0192611	-0.0405367	-0.0382288	-0.0220252	0.0003537	-0.0015243	-0.0023079	-0.0185115
BC	-0.0358898	-0.0358504	-0.0304200	-0.0845021	-0.0768096	-0.0351219	-0.0000394	-0.0054699	-0.0076925	-0.0493802
BCR	-0.0247592	-0.0248607	-0.0201690	-0.0543945	-0.0522357	-0.0231788	0.0001014	-0.0045903	-0.0021588	-0.0312157
BDX	-0.0193294	-0.0186032	-0.0169441	-0.0406834	-0.0389749	-0.0193464	-0.0007263	-0.0023853	-0.0017084	-0.0213370
BEN	-0.0314412	-0.0311226	-0.0247992	-0.0731767	-0.0701572	-0.0282877	-0.0003186	-0.0066419	-0.0030195	-0.0448890
BF-B	-0.0180075	-0.0180561	-0.0149539	-0.0388992	-0.0590794	-0.0171519	0.0000486	-0.0030535	0.0201802	-0.0217473
BGG	-0.0245979	-0.0239183	-0.0201436	-0.0533779	-0.0507363	-0.0231282	-0.0006796	-0.0044543	-0.0026416	-0.0302497
BK	-0.0285357	-0.0280949	-0.0230549	-0.0686234	-0.0662679	-0.0265644	-0.0004408	-0.0054808	-0.0023555	-0.0420589
BLL	-0.0222192	-0.0224017	-0.0188436	-0.0475112	-0.0521161	-0.0217160	0.0001825	-0.0033756	0.0046050	-0.0257952
BMS	-0.0213377	-0.0212273	-0.0182397	-0.0445872	-0.0456174	-0.0210379	-0.0001104	-0.0030980	0.0010302	-0.0235493
BMY	-0.0196343	-0.0195624	-0.0175201	-0.0405452	-0.0412076	-0.0199639	-0.0000719	-0.0021142	0.0006624	-0.0205812
BOH	-0.0245575	-0.0245660	-0.0181891	-0.0605326	-0.0587063	-0.0208902	0.0000086	-0.0063684	-0.0018263	-0.0396423
BP	-0.0204884	-0.0202535	-0.0175942	-0.0411797	-0.0385354	-0.0203076	-0.0002349	-0.0028942	-0.0026443	-0.0208721

	$VaR^{i}_{0.99}$			$ES^{i}_{0.99}$			$VaR^{M}_{0.99} - VaR^{i}_{0.99}$		$ES^{M}_{0.99} - ES^{i}_{0.99}$	
	M	Q	N	M	Q	N	Q	N	Q	N
BT	-0.0274939	-0.0278410	-0.0216492	-0.0640570	-0.0598741	-0.0247711	0.0003471	-0.0058447	-0.0041829	-0.0392859
BWS	-0.0370617	-0.0372195	-0.0280227	-0.0922253	-0.0879430	-0.0321633	0.0001578	-0.0090391	-0.0042823	-0.0600620
C	-0.0368214	-0.0358391	-0.0310935	-0.0925286	-0.0934590	-0.0355947	-0.0009823	-0.0057279	0.0009304	-0.0569339
CA	-0.0378805	-0.0373935	-0.0316196	-0.0875898	-0.0789058	-0.0361260	-0.0004870	-0.0062609	-0.0086841	-0.0514639
CAG	-0.0212148	-0.0211814	-0.0177841	-0.0464235	-0.0429490	-0.0204023	-0.0000334	-0.0034306	-0.0034745	-0.0260211
CAT	-0.0242122	-0.0243776	-0.0208525	-0.0512025	-0.0485088	-0.0239865	0.0001654	-0.0033598	-0.0026936	-0.0272160
CB	-0.0214302	-0.0214881	-0.0172686	-0.0504659	-0.0446940	-0.0199148	0.0000579	-0.0041616	-0.0057718	-0.0305511
CBE	-0.0245081	-0.0248340	-0.0207323	-0.0533904	-0.0575024	-0.0238304	0.0003259	-0.0037758	0.0041120	-0.0295600
CBRL	-0.0312892	-0.0315582	-0.0290160	-0.0678557	-0.0669193	-0.0332045	0.0002689	-0.0022733	-0.0009363	-0.0346512
CBT	-0.0261173	-0.0260937	-0.0220306	-0.0588377	-0.0551733	-0.0251446	-0.0000236	-0.0040867	-0.0036644	-0.0336931
CCK	-0.0343938	-0.0351735	-0.0278446	-0.0919453	-0.0884188	-0.0318173	0.0007797	-0.0065492	-0.0035265	-0.0601280
CEG	-0.0191386	-0.0191298	-0.0180060	-0.0401126	-0.0397986	-0.0206915	-0.0000088	-0.0011326	-0.0003140	-0.0194212
CI	-0.0251476	-0.0255020	-0.0215861	-0.0582168	-0.0578507	-0.0247094	0.0003545	-0.0035615	-0.0003662	-0.0335075
CL	-0.0178255	-0.0174870	-0.0165078	-0.0361597	-0.0413215	-0.0189131	-0.0003385	-0.0013177	0.0051618	-0.0172466
CLX	-0.0218857	-0.0220491	-0.0173346	-0.0512194	-0.0766613	-0.0200423	0.0001634	-0.0045510	0.0254419	-0.0311771
CMI	-0.0300880	-0.0303836	-0.0239166	-0.0707014	-0.0817770	-0.0276417	0.0002956	-0.0061714	0.0110756	-0.0430597
CNA	-0.0261306	-0.0259356	-0.0219221	-0.0651471	-0.0644406	-0.0252400	-0.0001949	-0.0042085	-0.0007065	-0.0399071
CNW	-0.0301810	-0.0296461	-0.0256027	-0.0676727	-0.0663522	-0.0295391	-0.0005349	-0.0045783	-0.0013205	-0.0381336
COO	-0.0646015	-0.0645754	-0.0418300	-0.1990660	-0.1761607	-0.0479365	-0.0000261	-0.0227715	-0.0229052	-0.1511294
COP	-0.0235507	-0.0238139	-0.0209740	-0.0460058	-0.0478140	-0.0240094	0.0002632	-0.0025767	0.0018083	-0.0219964
CP	-0.0272263	-0.0273708	-0.0207876	-0.0641294	-0.0633865	-0.0239449	0.0001446	-0.0064386	-0.0007429	-0.0401845
CPB	-0.0201943	-0.0196170	-0.0166465	-0.0460489	-0.0457711	-0.0191593	-0.0005774	-0.0035478	-0.0002777	-0.0268896
CR	-0.0260459	-0.0260490	-0.0219914	-0.0561295	-0.0580096	-0.0250268	0.0000031	-0.0040545	0.0018801	-0.0311027
CSC	-0.0270949	-0.0269171	-0.0240602	-0.0606478	-0.0642040	-0.0275993	-0.0001778	-0.0030347	0.0035562	-0.0330485
CSX	-0.0232451	-0.0234671	-0.0206298	-0.0460534	-0.0476236	-0.0236091	0.0002220	-0.0026153	0.0015702	-0.0224443
CVS	-0.0217931	-0.0215317	-0.0193770	-0.0461850	-0.0476956	-0.0222273	-0.0002613	-0.0024161	0.0015106	-0.0239577
CVX	-0.0193927	-0.0192513	-0.0170657	-0.0381077	-0.0350839	-0.0196622	-0.0001413	-0.0023269	-0.0030238	-0.0184454
DBD	-0.0257836	-0.0253165	-0.0209478	-0.0607237	-0.0571737	-0.0239898	-0.0004671	-0.0048358	-0.0035501	-0.0367340
DD	-0.0207842	-0.0211840	-0.0181245	-0.0428903	-0.0581971	-0.0206962	0.0003998	-0.0026597	0.0153068	-0.0221941
DE	-0.0244522	-0.0243233	-0.0217122	-0.0499941	-0.0462379	-0.0248004	-0.0001288	-0.0027400	-0.0037562	-0.0251937
DIS	-0.0237200	-0.0234230	-0.0209894	-0.0503997	-0.0469498	-0.0241065	-0.0002970	-0.0027306	-0.0034498	-0.0262931
DNB	-0.0231607	-0.0233175	-0.0162001	-0.0584292	-0.0791927	-0.0186281	0.0001568	-0.0069606	0.0207635	-0.0398011
DOV	-0.0221355	-0.0217739	-0.0189915	-0.0470938	-0.0459242	-0.0217613	-0.0003617	-0.0031440	-0.0011696	-0.0253325
DOW	-0.0254723	-0.0252947	-0.0211302	-0.0568975	-0.0584070	-0.0242165	-0.0001776	-0.0043421	0.0015095	-0.0326809
DUK	-0.0183292	-0.0183983	-0.0147764	-0.0398930	-0.0369014	-0.0169341	0.0000691	-0.0035528	-0.0029916	-0.0229589
ED	-0.0147600	-0.0149896	-0.0127045	-0.0291614	-0.0288897	-0.0145634	0.0002297	-0.0020554	-0.0002717	-0.0145980
EIX	-0.0227885	-0.0235403	-0.0195116	-0.0527511	-0.0513193	-0.0224247	0.0007518	-0.0032769	-0.0014317	-0.0303264
EMR	-0.0194160	-0.0190834	-0.0170061	-0.0407975	-0.0428158	-0.0196318	-0.0003326	-0.0024099	0.0020183	-0.0211657
ETN	-0.0208955	-0.0206314	-0.0184322	-0.0438754	-0.0420651	-0.0212209	-0.0002642	-0.0024633	-0.0018102	-0.0226545
ETR	-0.0194942	-0.0195061	-0.0175268	-0.0403702	-0.0428985	-0.0202587	0.0000119	-0.0019674	0.0025284	-0.0201115
EXC	-0.0201457	-0.0202950	-0.0167063	-0.0416757	-0.0423888	-0.0190376	0.0001493	-0.0034394	0.0007132	-0.0226381
F	-0.0265136	-0.0259278	-0.0254279	-0.0538083	-0.0538277	-0.0292135	-0.0005858	-0.0010857	0.0000194	-0.0245949
FDX	-0.0243293	-0.0238053	-0.0226112	-0.0510364	-0.0486671	-0.0259724	-0.0005240	-0.0017181	-0.0023693	-0.0250640
FL	-0.0301265	-0.0302808	-0.0260409	-0.0672211	-0.0754251	-0.0298409	0.0001543	-0.0040856	0.0082040	-0.0373802

	$VaR^i_{0.99}$			$ES^i_{0.99}$			$VaR^M_{0.99} - VaR^i_{0.99}$		$ES^M_{0.99} - ES^i_{0.99}$	
	M	**Q**	**N**	**M**	**Q**	**N**	**Q**	**N**	**Q**	**N**
FRM	-0.0425465	-0.0417638	-0.0420861	-0.0697163	-0.0649884	-0.0485742	-0.0007826	-0.0004604	-0.0047278	-0.0211420
FWLT	-0.0465536	-0.0458433	-0.0385161	-0.1168609	-0.1126295	-0.0441577	-0.0007103	-0.0080375	-0.0042314	-0.0727031
GAS	-0.0156000	-0.0159009	-0.0135890	-0.0311480	-0.0297194	-0.0156432	0.0003009	-0.0020110	-0.0014286	-0.0155049
GCI	-0.0260722	-0.0260932	-0.0220854	-0.0612990	-0.0693391	-0.0254078	0.0000210	-0.0039869	0.0080402	-0.0358912
GPC	-0.0260722	-0.0178034	-0.0155608	-0.0612990	-0.0343277	-0.0177265	-0.0082689	-0.0105114	-0.0269712	-0.0435724
GR	-0.0250320	-0.0242352	-0.0209461	-0.0567118	-0.0514758	-0.0241059	-0.0007968	-0.0040858	-0.0052360	-0.0326059
GRA	-0.0708305	-0.0718738	-0.0466949	-0.2055850	-0.1815604	-0.0532894	0.0010432	-0.0241356	-0.0240247	-0.1522956
GT	-0.0339740	-0.0347254	-0.0272660	-0.0803824	-0.0833648	-0.0311873	0.0007514	-0.0067080	0.0029824	-0.0491950
GWW	-0.0200204	-0.0199655	-0.0169803	-0.0427050	-0.0439703	-0.0194833	-0.0000549	-0.0030401	0.0012654	-0.0232217
HAL	-0.0305142	-0.0301829	-0.0274111	-0.0625562	-0.0572081	-0.0315773	-0.0003312	-0.0031031	-0.0053481	-0.0309790
HAS	-0.0269394	-0.0263319	-0.0233768	-0.0592850	-0.0534197	-0.0268565	-0.0006075	-0.0035626	-0.0058653	-0.0324285
HD	-0.0278970	-0.0274208	-0.0241094	-0.0606633	-0.0574946	-0.0275775	-0.0004762	-0.0037876	-0.0031687	-0.0330858
HES	-0.0268601	-0.0269823	-0.0235885	-0.0560211	-0.0535699	-0.0269397	0.0001222	-0.0032716	-0.0024513	-0.0290814
HNZ	-0.0180194	-0.0180140	-0.0153585	-0.0378647	-0.0388733	-0.0177646	-0.0000054	-0.0026608	0.0010087	-0.0201000
HON	-0.0253330	-0.0252157	-0.0211768	-0.0574236	-0.0526544	-0.0244482	-0.0001174	-0.0041562	-0.0047693	-0.0329754
HPG	-0.0295004	-0.0295745	-0.0247693	-0.0624392	-0.0565591	-0.0284467	0.0000741	-0.0047311	-0.0058801	-0.0339925
HRS	-0.0254993	-0.0254393	-0.0220646	-0.0539667	-0.0543672	-0.0252640	-0.0000600	-0.0034347	0.0004006	-0.0287027
HSY	-0.0184775	-0.0184148	-0.0274347	-0.0394717	-0.0346239	-0.0315516	-0.0000626	0.0089572	-0.0048478	-0.0079200
HTS	-0.0321682	-0.0326870	-0.0165239	-0.0772052	-0.0791640	-0.0188905	0.0005187	-0.0156443	0.0019588	-0.0583147
HUM	-0.0304975	-0.0302000	-0.0270719	-0.0651676	-0.0659183	-0.0311522	-0.0002975	-0.0034255	0.0007507	-0.0340154
IBM	-0.0194877	-0.0192461	-0.0166621	-0.0422443	-0.0385821	-0.0190315	-0.0002416	-0.0028255	-0.0036622	-0.0232128
IFF	-0.0198718	-0.0197267	-0.0173177	-0.0421934	-0.0455586	-0.0199871	-0.0001452	-0.0025542	0.0033652	-0.0222063
IP	-0.0251467	-0.0249878	-0.0217279	-0.0547525	-0.0620584	-0.0248494	-0.0001589	-0.0034188	0.0073059	-0.0299031
IR	-0.0256733	-0.0258089	-0.0221959	-0.0544871	-0.0577894	-0.0254487	0.0001356	-0.0034774	0.0033023	-0.0290384
JCP	-0.0302434	-0.0302651	-0.0243153	-0.0699641	-0.0772176	-0.0278452	0.0000217	-0.0059281	0.0072535	-0.0421190
JPM	-0.0307369	-0.0301295	-0.0246951	-0.0724637	-0.0934596	-0.0285343	-0.0006074	-0.0060418	0.0209959	-0.0439294
K	-0.0194992	-0.0198007	-0.0163087	-0.0433396	-0.0524214	-0.0186054	0.0003016	-0.0031905	0.0090818	-0.0247342
KMB	-0.0170691	-0.0169871	-0.0153113	-0.0365482	-0.0364000	-0.0176672	-0.0000820	-0.0017578	-0.0001482	-0.0188810
KO	-0.0178961	-0.0181199	-0.0158970	-0.0372126	-0.0352672	-0.0182437	0.0002237	-0.0019991	-0.0019454	-0.0189689
KR	-0.0226433	-0.0228105	-0.0261060	-0.0460765	-0.0453067	-0.0300228	0.0001671	0.0034627	-0.0007698	-0.0160537
LLY	-0.0198871	-0.0197607	-0.0180242	-0.0406640	-0.0374883	-0.0206834	-0.0001263	-0.0018628	-0.0031757	-0.0199807
LMT	-0.0230398	-0.0230661	-0.0179362	-0.0540879	-0.0505162	-0.0206289	0.0000264	-0.0051036	-0.0035716	-0.0334590
LNC	-0.0325979	-0.0340196	-0.0281292	-0.0872643	-0.0916113	-0.0322792	0.0014217	-0.0044687	0.0043470	-0.0549851
LPX	-0.0359312	-0.0361224	-0.0300861	-0.0820541	-0.0926722	-0.0348946	0.0001912	-0.0058452	0.0106181	-0.0471595
LUV	-0.0260799	-0.0256796	-0.0249342	-0.0469734	-0.0465308	-0.0288601	-0.0004002	-0.0011457	-0.0004426	-0.0181133
MAS	-0.0289320	-0.0287457	-0.0240928	-0.0642828	-0.0760340	-0.0275967	-0.0001863	-0.0048392	0.0117512	-0.0366862
MAT	-0.0298556	-0.0292033	-0.0277365	-0.0639303	-0.0600217	-0.0317771	-0.0006523	-0.0021190	-0.0039086	-0.0321532
MCD	-0.0178493	-0.0181071	-0.0163343	-0.0344771	-0.0347428	-0.0186822	0.0002578	-0.0015150	0.0002657	-0.0157949
MDT	-0.0240043	-0.0247730	-0.0197682	-0.0526603	-0.0526429	-0.0226530	0.0007687	-0.0042361	-0.0000174	-0.0300073
MMM	-0.0182677	-0.0186366	-0.0154869	-0.0399004	-0.0399799	-0.0177855	0.0003689	-0.0027808	0.0000795	-0.0221149
MRK	-0.0196054	-0.0196301	-0.0174271	-0.0398074	-0.0421551	-0.0199824	0.0000247	-0.0021782	0.0023477	-0.0198250
MRO	-0.0245595	-0.0242893	-0.0230449	-0.0472205	-0.0492265	-0.0263293	-0.0002702	-0.0015146	0.0020059	-0.0208912
MSI	-0.0308721	-0.0308850	-0.0264722	-0.0670447	-0.0615868	-0.0302275	0.0000129	-0.0043998	-0.0054579	-0.0368172
MTRN	-0.0366933	-0.0368220	-0.0285108	-0.0859851	-0.0934575	-0.0327566	0.0001287	-0.0081825	0.0074724	-0.0532285

	$VaR^i_{0.99}$			$ES^i_{0.99}$			$VaR^M_{0.99} - VaR^i_{0.99}$		$ES^M_{0.99} - ES^i_{0.99}$	
	M	Q	N	M	Q	N	Q	N	Q	N
MUR	-0.0259812	-0.0268389	-0.0214798	-0.0557347	-0.0612921	-0.0247118	0.0008577	-0.0045014	0.0055574	-0.0310229
NAV	-0.0396165	-0.0397984	-0.0374875	-0.0777033	-0.0795585	-0.0428861	0.0001819	-0.0021289	0.0018551	-0.0348172
NBL	-0.0280813	-0.0281807	-0.0260171	-0.0531000	-0.0500474	-0.0298881	0.0000994	-0.0020642	-0.0030526	-0.0232120
NC	-0.0351000	-0.0337712	-0.0284908	-0.0864181	-0.0792078	-0.0327368	-0.0013288	-0.0066091	-0.0072103	-0.0536813
NEE	-0.0163655	-0.0166045	-0.0137280	-0.0354944	-0.0343108	-0.0159008	0.0002389	-0.0026376	-0.0011837	-0.0195936
NEM	-0.0298828	-0.0297848	-0.0265854	-0.0629778	-0.0658333	-0.0305362	-0.0000980	-0.0032975	0.0028556	-0.0324416
NOC	-0.0234120	-0.0236635	-0.0187094	-0.0521482	-0.0546082	-0.0214890	0.0002515	-0.0047026	0.0024600	-0.0306592
NSC	-0.0238754	-0.0243320	-0.0195062	-0.0524612	-0.0508305	-0.0223498	0.0004565	-0.0043693	-0.0016307	-0.0301113
NSM	-0.0318942	-0.0323311	-0.0296440	-0.0611155	-0.0598596	-0.0343066	0.0004369	-0.0022503	-0.0012559	-0.0268090
NUE	-0.0276767	-0.0278266	-0.0245445	-0.0580626	-0.0562110	-0.0281160	0.0001500	-0.0031321	-0.0018516	-0.0299466
NVO	-0.0221016	-0.0219173	-0.0197282	-0.0472988	-0.0446723	-0.0227536	-0.0001844	-0.0023734	-0.0026265	-0.0245452
NWL	-0.0271791	-0.0272648	-0.0223558	-0.0624885	-0.0674199	-0.0256600	0.0000857	-0.0048233	0.0049314	-0.0368285
OMX	-0.0354492	-0.0355574	-0.0290972	-0.0875099	-0.0869735	-0.0332362	0.0001082	-0.0063520	-0.0005364	-0.0542737
OXY	-0.0236877	-0.0244507	-0.0209678	-0.0467945	-0.0475828	-0.0240149	0.0007631	-0.0027198	0.0007884	-0.0227795
PBI	-0.0214563	-0.0213330	-0.0193326	-0.0443041	-0.0428439	-0.0221322	-0.0001233	-0.0021237	-0.0014602	-0.0221719
PCG	-0.0220583	-0.0219379	-0.0197666	-0.0510880	-0.0500403	-0.0228627	-0.0001204	-0.0022916	-0.0010477	-0.0282253
PEP	-0.0195016	-0.0197217	-0.0164755	-0.0414118	-0.0398100	-0.0189577	0.0002201	-0.0030261	-0.0016018	-0.0224541
PFE	-0.0200965	-0.0196220	-0.0182350	-0.0396083	-0.0372211	-0.0208783	-0.0004745	-0.0018615	-0.0023872	-0.0187300
PG	-0.0168273	-0.0171834	-0.0155492	-0.0356992	-0.0363856	-0.0179086	0.0003562	-0.0012780	0.0006864	-0.0177906
PGN	-0.0155716	-0.0155427	-0.0131143	-0.0315927	-0.0287512	-0.0151337	-0.0000289	-0.0024574	-0.0028415	-0.0164590
PH	-0.0243655	-0.0238330	-0.0210417	-0.0504376	-0.0456855	-0.0241124	-0.0005325	-0.0033238	-0.0047521	-0.0263251
PKI	-0.0287472	-0.0287270	-0.0239677	-0.0641398	-0.0630342	-0.0274117	-0.0000202	-0.0047794	-0.0011056	-0.0367281
PNW	-0.0202141	-0.0201423	-0.0188106	-0.0435958	-0.0406364	-0.0214037	-0.0000717	-0.0014035	-0.0029594	-0.0221922
PPG	-0.0212591	-0.0208660	-0.0185598	-0.0444279	-0.0462011	-0.0212234	-0.0003931	-0.0026993	0.0017732	-0.0232045
R	-0.0254945	-0.0256224	-0.0219513	-0.0524650	-0.0493882	-0.0250193	0.0001278	-0.0035432	-0.0030769	-0.0274457
RDC	-0.0340082	-0.0335563	-0.0333685	-0.0527477	-0.0517640	-0.0381597	-0.0004519	-0.0006396	-0.0009837	-0.0145880
ROK	-0.0298270	-0.0299188	-0.0236515	-0.0673871	-0.0660039	-0.0271196	0.0000917	-0.0061756	-0.0013832	-0.0402675
RSH	-0.0329432	-0.0335202	-0.0286508	-0.0708665	-0.0667081	-0.0328223	0.0005770	-0.0042923	-0.0041584	-0.0380442
RTN	-0.0216620	-0.0218843	-0.0190320	-0.0491416	-0.0483813	-0.0217768	0.0002224	-0.0026300	-0.0007603	-0.0273648
S	-0.0333156	-0.0334727	-0.0278453	-0.0800528	-0.0837621	-0.0318561	0.0001572	-0.0054702	0.0037092	-0.0481967
SLB	-0.0247257	-0.0244467	-0.0231049	-0.0473335	-0.0490340	-0.0263839	-0.0002790	-0.0016208	0.0017005	-0.0209495
SNE	-0.0246069	-0.0239065	-0.0228328	-0.0499917	-0.0485483	-0.0261879	-0.0007004	-0.0017741	-0.0014435	-0.0238038
SO	-0.0159736	-0.0159832	-0.0142894	-0.0299812	-0.0293860	-0.0163653	0.0000096	-0.0016842	-0.0005952	-0.0136159
SUN	-0.0257417	-0.0253851	-0.0222404	-0.0538038	-0.0578048	-0.0254254	-0.0003566	-0.0035013	0.0040010	-0.0283785
T	-0.0224846	-0.0230729	-0.0171962	-0.0512763	-0.0571791	-0.0197231	0.0005883	-0.0052885	0.0059028	-0.0315532
TAP	-0.0277374	-0.0279135	-0.0227095	-0.0636483	-0.0652010	-0.0261796	0.0001761	-0.0050279	0.0015526	-0.0374687
TEN	-0.0649086	-0.0651732	-0.0442617	-0.1910777	-0.1875189	-0.0504784	0.0002647	-0.0206468	-0.0035588	-0.1405993
TGT	-0.0239488	-0.0236741	-0.0210442	-0.0509481	-0.0514009	-0.0241431	-0.0002747	-0.0029046	0.0004529	-0.0268050
THC	-0.0310985	-0.0304653	-0.0302308	-0.0654487	-0.0680491	-0.0345128	-0.0006332	-0.0008677	0.0026004	-0.0309359
TNB	-0.0241113	-0.0239071	-0.0207716	-0.0538341	-0.0534993	-0.0239133	-0.0002042	-0.0033397	-0.0003349	-0.0299209
TSO	-0.0388905	-0.0382279	-0.0356115	-0.0822757	-0.0782696	-0.0407826	-0.0006626	-0.0032790	-0.0040061	-0.0414931
TXI	-0.0303208	-0.0292682	-0.0254895	-0.0691964	-0.0620248	-0.0293267	-0.0010526	-0.0048314	-0.0071716	-0.0398697
TXN	-0.0321547	-0.0315531	-0.0270245	-0.0698832	-0.0617430	-0.0310642	-0.0006016	-0.0051302	-0.0081402	-0.0388190
TXT	-0.0283003	-0.0279373	-0.0245079	-0.0672918	-0.0618423	-0.0282105	-0.0003630	-0.0037924	-0.0054495	-0.0390812

	$VaR^{i}_{0.99}$			$ES^{i}_{0.99}$			$VaR^{M}_{0.99} - VaR^{i}_{0.99}$		$ES^{M}_{0.99} - ES^{i}_{0.99}$	
	M	**Q**	**N**	**M**	**Q**	**N**	**Q**	**N**	**Q**	**N**
UIS	-0.0441484	-0.0436222	-0.0381763	-0.1093969	-0.1133981	-0.0437340	-0.0005262	-0.0059721	0.0040011	-0.0656629
UN	-0.0199887	-0.0192234	-0.0165762	-0.0462417	-0.0406261	-0.0189022	-0.0007653	-0.0034125	-0.0056156	-0.0273396
UNP	-0.0218534	-0.0215796	-0.0185546	-0.0463625	-0.0453790	-0.0214087	-0.0002738	-0.0032988	-0.0009834	-0.0249537
UTX	-0.0201224	-0.0199522	-0.0178705	-0.0405072	-0.0396251	-0.0205986	-0.0001702	-0.0022519	-0.0008821	-0.0199086
VFC	-0.0248501	-0.0253127	-0.0199064	-0.0564802	-0.0569326	-0.0227516	0.0004626	-0.0049437	0.0004523	-0.0337286
VLO	-0.0338838	-0.0349047	-0.0277890	-0.0708724	-0.0717972	-0.0318488	0.0010208	-0.0060949	0.0009248	-0.0390236
VZ	-0.0198680	-0.0199804	-0.0164670	-0.0426734	-0.0411108	-0.0188771	0.0001123	-0.0034011	-0.0015626	-0.0237964
WHR	-0.0269284	-0.0259527	-0.0225881	-0.0586093	-0.0583649	-0.0257821	-0.0009756	-0.0043403	-0.0002445	-0.0328272
WMB	-0.0360392	-0.0362415	-0.0318783	-0.0893150	-0.0824577	-0.0363613	0.0002023	-0.0041609	-0.0068572	-0.0529536
WMS	-0.0411958	-0.0394931	-0.0371900	-0.0917755	-0.0834765	-0.0428224	-0.0017027	-0.0040058	-0.0082990	-0.0489531
WMT	-0.0236252	-0.0234332	-0.0205267	-0.0498608	-0.0493046	-0.0236519	-0.0001921	-0.0030986	-0.0005562	-0.0262089
WRB	-0.0225242	-0.0218412	-0.0203211	-0.0506681	-0.0466875	-0.0234373	-0.0006829	-0.0022031	-0.0039806	-0.0272308
WY	-0.0228464	-0.0224110	-0.0215013	-0.0425988	-0.0404168	-0.0248752	-0.0004354	-0.0013451	-0.0021820	-0.0177236
XOM	-0.0173429	-0.0176287	-0.0156377	-0.0338003	-0.0342830	-0.0179527	0.0002858	-0.0017052	0.0004827	-0.0158476
XRX	-0.0317862	-0.0322707	-0.0254973	-0.0782787	-0.0747542	-0.0290533	0.0004844	-0.0062889	-0.0035246	-0.0492255

Tabella F: comparison between VaR and expected shortfall as derived by the Mathematica software and the same risk measure, computed with Stable parameters from K=Koutrouvelis (Gauss code), Q=Quantile method (STABLE), ML=Maximum likelihood (STABLE).

	$VaR^{M}_{0.99} - VaR^{i}_{0.99}$			$ES^{M}_{0.99} - ES^{i}_{0.99}$		
	K	ML	Q	K	ML	Q
AA	0.0285053	-0.0000322	0.0031594	0.0537591	0.0003693	0.0147914
ABT	0.0211493	-0.0009985	0.0039091	0.0358550	-0.0041070	0.0167008
ADI	0.0359265	-0.0002355	0.0073072	0.0613305	0.0001823	0.0322413
ADM	0.0270119	-0.0009618	-0.0006903	0.0474433	-0.0033290	-0.0016945
ADP	0.0209022	0.0000079	0.0029949	0.0381560	0.0003881	0.0133790
AEP	0.0218392	0.0000535	0.0019094	0.0445480	0.0004575	0.0078716
AET	0.0351897	-0.0006207	0.0009992	0.0812455	-0.0021033	0.0067521
AFG	0.0282416	-0.0002741	0.0027525	0.0618311	-0.0007953	0.0130809
AFL	0.0316044	-0.0002999	0.0023402	0.0677938	-0.0006047	0.0097843
AIG	0.0417426	-0.0003208	0.0025789	0.1179531	-0.0006860	0.0141290
AIR	0.0367628	-0.0002653	0.0026173	0.0708769	-0.0002110	0.0136485
ALK	0.0340611	-0.0006789	-0.0035759	0.0565085	-0.0020332	-0.0113975
AM	0.0340070	-0.0005599	0.0085868	0.0792089	-0.0018395	0.0425493
AMD	0.0479653	-0.0006186	0.0001156	0.0863810	-0.0011012	0.0047565
AME	0.0280002	-0.0003843	0.0045120	0.0604280	-0.0010429	0.0194580
AMGN	0.0335924	-0.0003676	0.0060550	0.0664507	-0.0006587	0.0273669
AMN	0.0301002	-0.0003206	0.0047371	0.0744738	-0.0007980	0.0288477
AON	0.0249584	0.0000240	0.0030056	0.0514248	0.0004690	0.0125752
APA	0.0314429	-0.0008392	-0.0007776	0.0453302	-0.0034036	-0.0022197
APD	0.0235165	-0.0000514	0.0028371	0.0426935	0.0001199	0.0119684
ARW	0.0375182	-0.0002603	0.0008984	0.0692666	-0.0002212	0.0056077
ASH	0.0301298	-0.0002298	0.0016288	0.0658925	-0.0006621	0.0065123
AVP	0.0286519	-0.0002923	0.0031455	0.0612891	-0.0008849	0.0143319
AVT	0.0298451	-0.0001489	0.0059907	0.0510792	-0.0003296	0.0254380
AVY	0.0250483	0.0000186	0.0046178	0.0475314	0.0005093	0.0197191
AXE	0.0327309	-0.0001132	0.0045796	0.0641139	0.0003301	0.0189979
AXP	0.0300985	-0.0000296	0.0067362	0.0593575	0.0003890	0.0298757
BA	0.0251290	-0.0004223	0.0027634	0.0447062	-0.0014326	0.0116472
BAX	0.0236913	-0.0011497	-0.0004822	0.0414994	-0.0046797	-0.0002154
BC	0.0387500	-0.0008013	0.0040255	0.0854569	-0.0023064	0.0194704
BCR	0.0263594	0.0000032	0.0063282	0.0507333	0.0004822	0.0286747
BDX	0.0213902	-0.0000168	0.0043460	0.0408833	0.0001772	0.0179455
BEN	0.0322785	-0.0005118	0.0054712	0.0682524	-0.0013165	0.0236156
BF-B	0.0194052	0.0000759	0.0056204	0.0367375	0.0007059	0.0243669
BGG	0.0263386	0.0000230	0.0039694	0.0503477	0.0005618	0.0175738
BK	0.0318318	-0.0006232	0.0040152	0.0719801	-0.0021710	0.0184622
BLL	0.0239987	0.0000398	0.0044956	0.0452776	0.0006087	0.0189741
BMS	0.0223063	-0.0000520	0.0052260	0.0399079	0.0000124	0.0215155
BMY	0.0222581	-0.0000461	0.0023024	0.0418825	0.0000147	0.0100390
BOH	0.0276777	-0.0003421	0.0036514	0.0631991	-0.0009412	0.0182363
BP	0.0236020	-0.0008177	0.0021930	0.0438745	-0.0029370	0.0089894

	$VaR^{M}_{0.99} - VaR^{i}_{0.99}$			$ES^{M}_{0.99} - ES^{i}_{0.99}$		
	K	ML	Q	K	ML	Q
HAL	0.0333914	-0.0004927	0.0006880	0.0597976	-0.0014648	0.0044210
HAS	0.0270378	-0.0000650	0.0062836	0.0509066	0.0005816	0.0288057
HD	0.0287507	-0.0000567	0.0059078	0.0542985	0.0005626	0.0249907
HES	0.0300925	-0.0001443	0.0018118	0.0571258	-0.0000609	0.0077897
HNZ	0.0201752	-0.0000003	0.0036683	0.0379579	0.0002329	0.0147773
HON	0.0273021	-0.0001069	0.0030578	0.0559906	0.0000969	0.0125496
HPG	0.0330008	-0.0001524	0.0054435	0.0638886	0.0001552	0.0217887
HRS	0.0285847	-0.0000324	0.0046120	0.0546677	0.0002547	0.0179020
HSY	0.0511775	0.0129599	0.0179051	0.1245137	0.0351714	0.0565728
HTS	0.0066337	-0.0136590	-0.0104323	0.0002087	-0.0372394	-0.0235483
HUM	0.0317049	-0.0000790	0.0021132	0.0588957	0.0002178	0.0084080
IBM	0.0217150	0.0000657	0.0025780	0.0423892	0.0006781	0.0109910
IFF	0.0213584	0.0000426	0.0042136	0.0401690	0.0005901	0.0176845
IP	0.0281956	0.0000479	0.0029465	0.0566531	0.0007286	0.0124215
IR	0.0285097	-0.0000500	0.0036195	0.0545568	0.0001622	0.0145508
JCP	0.0313780	-0.0004585	0.0050165	0.0656576	-0.0013684	0.0238375
JPM	0.0351948	-0.0006738	0.0058925	0.0791535	-0.0018702	0.0270959
K	0.0209680	-0.0000047	0.0052065	0.0410177	0.0002552	0.0216874
KMB	0.0182022	0.0000489	0.0031804	0.0343791	0.0005673	0.0139822
KO	0.0200881	-0.0000208	0.0040019	0.0367599	0.0001091	0.0161736
KR	0.0240176	-0.0001534	0.0048922	0.0419063	-0.0000603	0.0210869
LLY	0.0221334	-0.0002274	0.0050787	0.0403137	-0.0006599	0.0201709
LMT	0.0246473	-0.0003428	0.0037500	0.0537287	-0.0008891	0.0167387
LNC	0.0395268	-0.0004232	0.0033454	0.1050929	-0.0013104	0.0180775
LPX	0.0400474	-0.0005037	0.0035706	0.0853615	-0.0012161	0.0196201
LUV	0.0289650	-0.0010820	-0.0000179	0.0434242	-0.0045906	0.0036442
MAS	0.0296074	-0.0000741	0.0036433	0.0570361	0.0001723	0.0167611
MAT	0.0305468	-0.0001043	0.0038508	0.0555442	0.0001561	0.0175671
MCD	0.0203000	-0.0008333	0.0021462	0.0342740	-0.0032629	0.0091478
MDT	0.0258091	0.0000210	0.0037089	0.0502618	0.0005468	0.0167415
MMM	0.0198115	0.0000611	0.0041198	0.0382005	0.0001709	0.0167685
MRK	0.0229712	-0.0004276	0.0031509	0.0437624	-0.0012156	0.0139803
MRO	0.0295204	-0.0017873	0.0012914	0.0528235	-0.0065976	0.0055394
MSI	0.0326237	-0.0000646	0.0043661	0.0628214	0.0003096	0.0177382
MTRN	0.0426618	-0.0006842	0.0049826	0.0945414	-0.0017454	0.0238241
MUR	0.0283749	-0.0000529	0.0052942	0.0537206	0.0001373	0.0212136
NAV	0.0450433	-0.0013500	-0.0002893	0.0779023	-0.0046020	0.0020135
NBL	0.0323971	-0.0018326	0.0013310	0.0543523	-0.0075327	0.0067327
NC	0.0362395	-0.0005787	0.0043045	0.0813778	-0.0016120	0.0214388
NEE	0.0199462	0.0000996	0.0013785	0.0411424	0.0003416	0.0046817
NEM	0.0317624	-0.0001306	0.0020110	0.0583957	-0.0001172	0.0095773

	$VaR^{M}{}_{0.99} - VaR^{i}{}_{0.99}$			$ES^{M}{}_{0.99} - ES^{i}{}_{0.99}$				$VaR^{M}{}_{0.99} - VaR^{i}{}_{0.99}$			$ES^{M}{}_{0.99} - ES^{i}{}_{0.99}$		
	K	ML	Q	K	ML	Q		K	ML	Q	K	ML	Q
BT	0.0304360	-0.0003395	0.0040920	0.0655672	-0.0011402	0.0179788	NOC	0.0248259	-0.0000255	0.0055994	0.0494177	0.0002656	0.0264226
BWS	0.0417860	-0.0006161	0.0060176	0.0979601	-0.0015666	0.0311845	NSC	0.0255602	-0.0000011	0.0048974	0.0489481	0.0003991	0.0202889
C	0.0438122	-0.0005415	0.0054789	0.1085908	-0.0015792	0.0284547	NSM	0.0362855	-0.0013178	0.0155145	0.0575360	-0.0056665	0.0670332
CA	0.0427567	-0.0005474	0.0074802	0.0931835	-0.0015154	0.0351874	NUE	0.0313461	-0.0001414	0.0043950	0.0603118	-0.0003220	0.0187473
CAG	0.0231424	0.0000535	0.0036883	0.0452921	0.0006366	0.0160268	NVO	0.0241218	0.0000140	0.0042798	0.0467293	0.0004177	0.0174744
CAT	0.0254091	-0.0000342	0.0023385	0.0464773	0.0002310	0.0099721	NWL	0.0281078	-0.0002544	0.0056416	0.0573122	-0.0006163	0.0256289
CB	0.0241160	-0.0003380	0.0030098	0.0525555	-0.0013338	0.0133320	OMX	0.0400936	-0.0005482	0.0022638	0.0945362	-0.0014445	0.0140151
CBE	0.0283999	0.0000383	0.0042534	0.0577923	0.0006489	0.0189494	OXY	0.0280601	-0.0014745	0.0027655	0.0509086	-0.0053984	0.0110281
CBRL	0.0333888	-0.0001252	0.0054507	0.0639712	0.0002458	0.0224392	PBI	0.0237740	-0.0002777	0.0044133	0.0441203	-0.0009313	0.0170117
CBT	0.0274311	-0.0001345	0.0042488	0.0552468	0.0001161	0.0193085	PCG	0.0273649	-0.0000860	-0.0004722	0.0607296	-0.0001575	-0.0020348
CCK	0.0439349	-0.0004626	0.0039091	0.1173723	-0.0011427	0.0184235	PEP	0.0216369	0.0000200	0.0040880	0.0410307	0.0004055	0.0168308
CEG	0.0230786	-0.0001476	0.0015456	0.0459559	-0.0003612	0.0057174	PFE	0.0214596	-0.0004532	0.0038120	0.0352812	-0.0017422	0.0170199
CI	0.0284090	-0.0002292	0.0045116	0.0632622	-0.0004903	0.0188260	PG	0.0174472	0.0000365	0.0050889	0.0314604	-0.0000344	0.0204520
CL	0.0190492	-0.0000408	0.0033625	0.0328533	0.0000236	0.0144331	PGN	0.0190044	-0.0008457	0.0016242	0.0355864	-0.0030720	0.0062295
CLX	0.0234470	-0.0002926	0.0036529	0.0506201	-0.0008049	0.0153178	PH	0.0260726	-0.0001361	0.0039582	0.0466036	0.0000738	0.0161484
CMI	0.0326836	-0.0005135	0.0054867	0.0704500	-0.0016107	0.0249704	PKI	0.0315895	-0.0000665	0.0055716	0.0640385	0.0001456	0.0241363
CNA	0.0288266	-0.0003244	0.0039381	0.0680293	-0.0011298	0.0220253	PNW	0.0252606	0.0000976	0.0015873	0.0527792	0.0007521	0.0067917
CNW	0.0297354	-0.0002523	0.0019885	0.0561170	-0.0003852	0.0101872	PPG	0.0237387	-0.0000636	0.0035395	0.0440825	-0.0000873	0.0134959
COO	0.0739875	-0.0010896	0.0072419	0.2129834	-0.0035939	0.0581468	R	0.0276607	-0.0003100	0.0039217	0.0500005	-0.0007125	0.0148974
COP	0.0275131	-0.0016458	0.0009954	0.0486870	-0.0063190	0.0040516	RDC	0.0391311	-0.0003313	-0.0048463	0.0428326	0.0000986	-0.0192227
CP	0.0287243	-0.0004103	0.0043508	0.0620514	-0.0012091	0.0212086	ROK	0.0315030	-0.0001204	0.0052794	0.0638000	0.0001737	0.0227364
CPB	0.0210110	-0.0001524	0.0030785	0.0420110	-0.0002508	0.0134741	RSH	0.0367901	-0.0001110	0.0059686	0.0726444	0.0005097	0.0274698
CR	0.0295572	-0.0000084	0.0044002	0.0585336	0.0003616	0.0167932	RTN	0.0234836	-0.0000567	0.0034868	0.0476746	-0.0000103	0.0142074
CSC	0.0264031	-0.0001939	0.0060722	0.0506073	-0.0001467	0.0290200	S	0.0378679	-0.0007719	0.0028470	0.0881272	-0.0022634	0.0123453
CSX	0.0255962	-0.0005400	0.0039888	0.0458223	-0.0009962	0.0171297	SLB	0.0268986	-0.0008030	0.0026727	0.0432986	-0.0030922	0.0123598
CVS	0.0240643	0.0000155	0.0028168	0.0456205	0.0004911	0.0125437	SNE	0.0251427	-0.0000654	0.0020365	0.0403997	0.0001895	0.0108234
CVX	0.0228374	-0.0012477	0.0025511	0.0406810	-0.0051604	0.0087287	SO	0.0192157	-0.0007039	-0.0021642	0.0323327	-0.0025682	-0.0092210
DBD	0.0280176	-0.0004300	0.0053559	0.0603492	-0.0013983	0.0238893	SUN	0.0287647	-0.0001932	0.0021948	0.0543841	-0.0002640	0.0092539
DD	0.0244359	-0.0000747	0.0009253	0.0440776	0.0001904	0.0044113	T	0.0248262	-0.0000555	0.0052855	0.0508911	-0.0000309	0.0227240
DE	0.0264468	-0.0002308	0.0039537	0.0465057	-0.0004920	0.0172516	TAP	0.0295585	-0.0002766	0.0045020	0.0627380	-0.0003495	0.0200126
DIS	0.0248359	0.0000056	0.0041640	0.0486252	0.0004674	0.0177159	TEN	0.0684660	-0.0009237	0.0069177	0.1896619	-0.0022520	0.0590040
DNB	0.0274567	-0.0002302	0.0047139	0.0669297	-0.0000471	0.0235239	TGT	0.0250524	-0.0000058	0.0056239	0.0460418	0.0004215	0.0235117
DOV	0.0234733	0.0000160	0.0036066	0.0430955	0.0004788	0.0146210	THC	0.0348331	-0.0001144	-0.0000891	0.0677862	0.0003481	0.0023301
DOW	0.0269692	-0.0000449	0.0035198	0.0537643	0.0001816	0.0155876	TNB	0.0249549	-0.0000714	0.0048132	0.0492872	0.0004352	0.0215476
DUK	0.0224209	0.0001276	0.0036206	0.0468557	0.0008918	0.0153202	TSO	0.0413606	-0.0002686	-0.0012206	0.0753238	0.0000060	-0.0013060
ED	0.0177035	-0.0010802	0.0026219	0.0320912	-0.0041316	0.0091409	TXI	0.0326471	-0.0003984	0.0043522	0.0691138	-0.0008781	0.0218217
EIX	0.0282871	-0.0001191	0.0034368	0.0633030	-0.0000308	0.0146213	TXN	0.0333018	-0.0001282	0.0061815	0.0621689	0.0000069	0.0276562
EMR	0.0211253	0.0000202	0.0028693	0.0393132	0.0004701	0.0132611	TXT	0.0344231	-0.0005845	0.0024867	0.0795028	-0.0020173	0.0118670
ETN	0.0226753	0.0000053	0.0048963	0.0428225	0.0003664	0.0202669	UIS	0.0522685	-0.0008113	0.0029201	0.1265503	-0.0022622	0.0152670
ETR	0.0232431	-0.0002835	-0.0004291	0.0454075	-0.0007567	-0.0026358	UN	0.0219224	-0.0000754	0.0031210	0.0452291	-0.0002222	0.0136619
EXC	0.0244971	-0.0006945	0.0000401	0.0467857	-0.0025181	0.0000649	UNP	0.0225858	-0.0000083	0.0033912	0.0411433	0.0006263	0.0145622
F	0.0295707	-0.0001930	-0.0004591	0.0540927	-0.0000901	0.0002050	UTX	0.0227066	-0.0005465	0.0040395	0.0409409	-0.0017437	0.0164494
FDX	0.0232122	-0.0000415	0.0038222	0.0372734	0.0003357	0.0175981	VFC	0.0259871	-0.0001377	0.0061111	0.0516216	-0.0003900	0.0260462
FL	0.0315875	-0.0002903	0.0025284	0.0611555	-0.0002193	0.0133144	VLO	0.0376004	-0.0006587	0.0049942	0.0713254	-0.0016409	0.0194326

	$VaR^M{}_{0.99} - VaR^i{}_{0.99}$			$ES^M{}_{0.99} - ES^i{}_{0.99}$				$VaR^M{}_{0.99} - VaR^i{}_{0.99}$			$ES^M{}_{0.99} - ES^i{}_{0.99}$		
	K	**ML**	**Q**	**K**	**ML**	**Q**		**K**	**ML**	**Q**	**K**	**ML**	**Q**
FRM	0.0474279	-0.0011525	-0.0016119	0.0528130	-0.0043169	-0.0230630	VZ	0.0233424	0.0000552	0.0044869	0.0465462	0.0006108	0.0190647
FWLT	0.0541047	-0.0007724	0.0020666	0.1296329	-0.0020054	0.0104737	WHR	0.0286365	-0.0000248	0.0041731	0.0551493	0.0003869	0.0188192
GAS	0.0184384	-0.0008281	0.0022267	0.0330974	-0.0032945	0.0085147	WMB	0.0423816	-0.0006104	0.0031730	0.1008993	-0.0021176	0.0154457
GCI	0.0289585	-0.0004479	0.0017044	0.0644783	-0.0014403	0.0084115	WMS	0.0391349	-0.0006862	0.0018409	0.0705593	-0.0012924	0.0099287
GPC	0.0108076	-0.0084063	-0.0058235	0.0097744	-0.0246218	-0.0134405	WMT	0.0239296	-0.0000326	0.0046060	0.0415526	0.0002244	0.0200480
GR	0.0249214	-0.0001973	0.0039497	0.0482386	-0.0001794	0.0175733	WRB	0.0235245	-0.0001266	0.0049712	0.0459160	-0.0003674	0.0226763
GRA	0.0758830	-0.0012210	0.0110360	0.1990635	-0.0036400	0.0815425	WY	0.0255405	-0.0012356	0.0019316	0.0410743	-0.0052953	0.0105971
GT	0.0355672	-0.0007821	0.0020581	0.0770789	-0.0024364	0.0110466	XOM	0.0216129	-0.0011241	0.0017792	0.0406632	-0.0043988	0.0063305
GWW	0.0216910	0.0000399	0.0044393	0.0409800	0.0005617	0.0185244	XRX	0.0381095	-0.0005206	0.0023434	0.0901447	-0.0018738	0.0122524

Table G: Comparison between the original value and the five estimators. Measures: for α: $\alpha-\hat{\alpha}_i$, for β: $\beta-\hat{\beta}_i$, for γ: $ln(\gamma/\hat{\gamma}_i)*100$, for δ: $(\delta-\hat{\delta}_i)*[(\gamma-\hat{\gamma}_i)/2]^{-1}*100$. M ML= Mathematica maximum likelihood.

	α					β					γ					δ				
	M ML	K	Q	ML	ChF	M ML	K	Q	ML	ChF	M ML	K	Q	ML	ChF	M ML	K	Q	ML	ChF
1	-0.0291	-0.0357	-0.0189	-0.0267	-0.0315	0.0406	0.0802	0.0533	0.0409	0.0672	1.7622	1.6412	3.1776	1.9032	1.6224	2.241%	3.577%	0.566%	2.183%	4.219%
2	-0.0024	-0.0184	0.0111	-0.0026	-0.0159	0.0030	0.0802	-0.0018	0.0023	-0.0467	1.6233	1.6412	1.8687	1.4127	0.6325	0.955%	3.577%	4.705%	0.825%	-3.051%
3	-0.0370	-0.0392	-0.0489	-0.0369	-0.0366	-0.0528	-0.0819	-0.0369	-0.0532	-0.0783	-0.0785	-0.3719	0.0148	-0.2716	-0.2356	-3.747%	-17.989%	5.152%	-3.803%	-5.511%
4	-0.0374	-0.0354	-0.0419	-0.0384	-0.0320	0.0613	0.0273	0.0672	0.0604	0.0531	-1.0764	-1.0011	-0.6855	-1.0920	-0.8240	3.130%	-4.330%	1.403%	3.076%	2.827%
5	-0.0088	-0.0157	0.0012	-0.0087	-0.0151	-0.0082	-0.0534	-0.0106	-0.0077	-0.0294	3.1095	3.0031	3.6353	3.0139	2.7788	-1.280%	-14.214%	2.787%	-1.304%	-2.142%
6	-0.0469	-0.0549	-0.0640	-0.0473	-0.0509	0.0581	0.0562	0.0324	0.0583	0.0527	1.7556	1.6319	1.1453	1.8064	1.7603	3.939%	0.584%	5.895%	3.957%	3.503%
7	-0.0340	-0.0319	-0.0458	-0.0343	-0.0357	-0.0492	-0.0172	-0.0381	-0.0491	-0.0350	-1.4716	-1.4003	-2.4184	-1.5082	-1.5900	-1.361%	-6.031%	4.213%	-1.353%	-0.675%
8	-0.0110	-0.0128	-0.0045	-0.0112	-0.0121	-0.0096	-0.0221	-0.0294	-0.0096	-0.0054	1.1314	0.9682	1.4159	1.0885	1.0691	-2.554%	-13.199%	3.595%	-2.567%	-2.361%
9	-0.0037	-0.0095	-0.0305	-0.0047	-0.0062	0.1108	0.0872	0.1170	0.1101	0.0746	-0.3713	-0.6173	-1.6747	-0.4429	-0.4714	6.538%	3.626%	2.839%	6.470%	4.588%
10	0.0149	0.0179	-0.0024	0.0135	0.0315	0.0310	-0.0399	0.0341	0.0302	-0.0446	1.9710	1.6513	0.4850	1.8077	2.4788	1.471%	-11.133%	3.316%	1.473%	-2.685%
11	-0.0011	-0.0011	-0.0430	-0.0062	0.0023	-0.0092	0.0092	-0.0196	-0.0088	-0.0051	0.7931	0.4515	-1.0303	0.5786	0.8547	-0.790%	-6.566%	3.068%	-0.777%	-0.624%
12	0.0305	0.0378	0.0289	0.0320	0.0433	0.0374	0.0301	0.0067	0.0156	0.0494	1.4448	1.6262	1.8131	1.5619	1.8281	-0.917%	-8.956%	-0.659%	-1.869%	-0.163%
13	0.0098	0.0103	0.0294	0.0068	0.0118	0.0930	0.0957	0.1184	0.0924	0.1190	-0.3006	-0.0681	0.4128	-0.3815	-0.3268	3.732%	0.858%	-0.511%	3.619%	4.751%
14	0.0107	0.0126	-0.0167	0.0120	0.0267	-0.0051	-0.0370	0.0056	-0.0061	-0.0237	0.9055	0.4007	-0.3041	0.7502	1.1678	-3.292%	-15.546%	-0.472%	-3.381%	-4.137%
15	-0.0127	-0.0208	0.0030	-0.0043	-0.0324	0.0741	0.0827	0.0264	0.0729	0.0630	2.6667	2.7220	3.4466	2.8819	2.1994	4.329%	2.164%	3.418%	4.343%	3.863%
16	-0.0264	-0.0238	-0.0287	-0.0178	-0.0161	0.0471	0.0969	0.1111	0.0489	0.1071	-1.1653	-1.2851	-1.2615	-1.0816	-0.9318	2.101%	0.392%	1.309%	2.091%	3.876%
17	-0.0151	-0.0246	0.0250	-0.0106	-0.0417	0.0622	0.0331	0.0598	0.0666	0.0384	-1.7980	-1.3833	-0.4847	-1.6270	-2.2968	0.032%	-7.726%	-2.123%	0.154%	-0.500%
18	0.0231	0.0147	-0.0050	0.0316	0.0040	0.0612	0.0726	0.0794	0.0622	0.0661	2.0091	2.0305	1.8184	2.2677	1.5691	-0.420%	-7.137%	-1.988%	-0.335%	-0.403%
19	-0.0069	-0.0089	-0.0486	-0.0222	-0.0066	0.0685	0.0634	-0.0553	0.0634	0.1174	0.9697	0.3482	-0.2739	0.3848	0.6170	4.302%	1.725%	12.042%	4.379%	5.300%
20	0.0179	0.0117	0.0080	0.0124	0.0061	0.0237	0.0397	-0.0092	0.0230	-0.0448	2.3850	2.2601	2.0222	2.2448	2.0158	0.706%	-2.389%	8.646%	0.775%	-0.692%
21	-0.0209	-0.0233	-0.0093	-0.0509	-0.0146	-0.1560	-0.1788	-0.2121	-0.1898	-0.1265	-0.1506	-0.3771	0.4408	-0.7589	-0.0937	-0.092%	-4.894%	12.889%	0.606%	0.213%
22	0.0038	-0.0034	-0.0375	-0.0316	0.0024	-0.0757	-0.0573	-0.2798	-0.0970	-0.0844	1.0647	0.7090	-0.2765	0.4065	0.8993	-0.179%	-3.692%	10.021%	0.552%	-0.450%
23	0.0108	0.0073	0.0145	-0.0170	0.0008	0.0355	0.0148	0.0599	0.0135	-0.0174	2.1753	2.0987	2.3638	1.6319	1.8870	-1.598%	-7.662%	4.592%	-1.157%	-2.051%
24	-0.0138	-0.0171	0.0266	-0.0107	-0.0201	-0.1370	-0.1375	-0.0950	-0.1423	-0.0527	-0.0662	0.4067	1.2067	0.2305	0.0864	1.342%	-1.402%	6.158%	1.325%	2.427%
25	0.0162	0.0289	-0.0214	0.0119	0.0381	0.0530	0.1692	-0.0778	0.0540	0.1077	-0.8538	-1.0222	-1.2967	-0.9252	-0.5322	0.182%	-1.181%	2.948%	0.174%	0.568%
26	0.0136	0.0106	0.0724	0.0196	-0.0007	0.2694	0.2738	0.2391	0.2708	0.3712	0.8729	1.2045	2.3175	1.1871	0.7489	2.933%	1.621%	0.586%	2.987%	3.697%
27	-0.0185	-0.0161	-0.0260	-0.0172	0.0046	-0.0108	-0.0180	-0.0616	-0.0107	-0.0103	-0.1743	-0.5165	-0.5214	-0.3176	0.5791	1.762%	-1.991%	20.048%	1.703%	0.902%
28	-0.0013	-0.0042	0.0046	-0.0011	-0.0029	0.0337	0.0892	0.0341	0.0342	0.0678	0.3828	0.3415	1.0663	0.4109	0.3147	-1.816%	-0.451%	9.512%	-1.799%	0.335%
29	-0.0459	-0.0485	-0.0610	-0.0465	-0.0404	0.0127	0.0230	0.0026	0.0119	0.0259	0.0800	-0.2777	-0.9611	-0.0938	0.1931	0.922%	-0.931%	11.316%	0.831%	1.643%
30	-0.0006	0.0003	-0.0225	-0.0007	0.0049	-0.0075	0.0006	-0.0380	-0.0080	0.0014	0.4856	0.5310	-0.9860	0.5288	0.7478	-0.861%	-0.341%	13.852%	-0.880%	-0.442%
31	0.0269	0.0300	0.0107	0.0269	0.0518	-0.0419	0.0219	-0.0585	-0.0412	0.0268	0.6519	0.5118	0.0298	0.6343	1.6969	-3.951%	0.482%	14.789%	-4.001%	0.209%
32	-0.0079	-0.0006	-0.0286	-0.0080	0.0184	-0.0619	-0.0491	-0.1152	-0.0615	-0.0430	-1.5382	-1.7742	-2.7596	-1.6287	-0.5688	-1.035%	-0.754%	25.435%	-1.069%	-1.577%
33	-0.0201	-0.0051	-0.0337	-0.0213	0.0052	-0.0587	0.0416	-0.0776	-0.0556	0.0012	2.3951	2.4194	2.5074	2.4351	3.2357	-0.287%	3.778%	19.572%	-0.098%	1.273%
34	0.0250	0.0207	0.0052	0.0237	0.0099	0.0131	-0.0844	-0.0399	0.0085	-0.0846	0.9863	0.6636	-0.2351	0.6479	0.0737	0.934%	-4.007%	21.162%	0.810%	-3.502%
35	-0.0073	-0.0141	-0.0134	-0.0117	-0.0271	-0.0519	-0.0122	-0.0241	-0.0533	-0.0129	-3.0704	-2.9169	-3.2749	-3.2504	-3.7744	-6.424%	-4.153%	13.284%	-6.356%	-3.641%
36	0.0089	0.0067	0.0217	0.0037	0.0060	-0.0243	0.0256	0.0156	-0.0273	0.0462	0.3575	0.3498	0.9574	0.2748	0.3467	-1.914%	0.646%	14.291%	-1.769%	1.648%
37	-0.0036	0.0042	-0.0307	-0.0054	0.0106	0.0052	0.0138	-0.0246	0.0038	-0.0146	-1.2471	-1.5708	-2.3664	-1.4398	-1.0494	-1.658%	-1.716%	7.151%	-1.732%	-3.086%
38	0.0037	-0.0050	-0.0094	0.0046	0.0058	0.0800	0.0844	0.0662	0.0807	0.0531	0.5231	0.2632	-1.1504	0.5798	0.6873	3.612%	3.752%	10.292%	3.618%	2.435%
39	0.0046	0.0045	0.0225	0.0097	0.0134	0.0285	0.0391	0.0215	0.0281	0.0260	0.1234	-0.0162	1.0719	0.1815	0.3045	1.330%	1.667%	9.621%	1.207%	1.082%
40	-0.0159	-0.0172	0.0391	-0.0072	-0.0262	-0.0351	-0.0363	0.0218	-0.0343	-0.0036	0.9097	1.3583	3.1492	1.1826	0.8318	-0.516%	-0.407%	7.525%	-0.637%	0.836%
41	-0.0310	-0.0433	-0.0267	-0.0238	-0.0465	0.0105	-0.0246	-0.0395	0.0050	-0.0388	-2.1843	-2.3462	-2.6011	-2.0430	-2.5907	0.426%	-0.173%	5.353%	0.186%	-0.477%

	α					β					γ					δ				
	M ML	**K**	**Q**	**ML**	**ChF**	**M ML**	**K**	**Q**	**ML**	**ChF**	**M ML**	**K**	**Q**	**ML**	**ChF**	**M ML**	**K**	**Q**	**ML**	**ChF**
42	0.0111	0.0105	-0.0182	0.0195	0.0067	0.0014	0.0192	0.0209	0.0026	0.0570	0.5579	0.6199	-0.2280	0.7592	0.4718	0.033%	0.549%	4.725%	-0.073%	1.695%
43	-0.0086	-0.0184	-0.0259	-0.0138	-0.0144	0.0458	0.0130	0.0161	0.0460	0.0358	-0.3529	-0.4548	-0.0686	-0.5016	-0.5293	-0.744%	-1.284%	4.160%	-0.705%	-0.782%
44	-0.0159	-0.0316	0.0140	-0.0496	-0.0458	-0.0065	-0.0161	0.0610	-0.0242	-0.0235	-0.4257	-0.2710	0.7204	-1.1134	-1.0131	-1.493%	-1.491%	2.190%	-1.256%	-1.319%
45	-0.0164	-0.0056	-0.0146	-0.0565	-0.0128	0.0293	0.0624	0.0464	0.0169	0.0527	0.8698	0.9879	0.6714	0.0204	0.9404	1.183%	1.571%	4.539%	1.405%	1.442%
46	-0.0223	-0.0248	-0.0529	-0.0438	-0.0215	0.0292	-0.0009	-0.0500	0.0252	-0.0001	-1.3240	-1.4589	-1.9568	-1.6769	-1.3291	-0.804%	-1.125%	3.095%	-0.640%	-1.184%
47	0.0180	0.0152	0.0099	-0.0218	0.0165	-0.0279	-0.0516	-0.1557	-0.0429	-0.0608	-0.2062	-0.3177	-1.3038	-0.9827	-0.2782	-1.506%	-1.920%	5.818%	-1.095%	-2.166%
48	-0.0028	0.0055	0.0275	-0.0126	0.0043	0.0810	0.1133	0.2230	0.0779	0.2023	-0.9560	-0.9079	-0.4848	-1.1512	-0.8899	0.172%	0.452%	0.392%	0.175%	1.831%
49	-0.0037	0.0020	0.0001	-0.0019	0.0071	-0.0388	-0.0165	0.1311	-0.0424	0.0810	1.7690	1.5494	2.2736	1.8117	1.8824	-0.815%	-0.834%	0.388%	-0.885%	0.367%
50	0.0017	-0.0037	0.0091	0.0103	-0.0094	0.1649	0.1688	0.1305	0.1226	0.2258	0.6913	0.7035	0.7604	0.9921	0.4348	0.050%	0.121%	-0.554%	-0.292%	0.668%