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Final Thesis

Systemic Risk Analysis

Risk measurements applied to SPY ETFs'sectors

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Introduction

This thesis delves into the intricate and multifaceted domain of financial risk management, with a particular emphasis on the utilization of copula models. The motivation behind this study stems from the inherent limitations of traditional risk measures such as Value at Risk (VaR), Conditional Value at Risk (CoVaR) and DeltaCoVaR which predominantly rely on linear assumptions. These conventional models, while instrumental, often fall short in capturing the complex, non-linear dependencies and extreme co-movements that characterize financial markets, especially during periods of market stress.

Copula models offer a sophisticated alternative, enabling a more nuanced understanding of the dependencies between different financial assets. Unlike linear models, copulas allow for the modeling of non-linear and asymmetric relationships, providing a more accurate and comprehensive picture of joint risk. This is particularly crucial for understanding tail dependencies and extreme events, which are pivotal in effective risk management.

Incorporating copula models into our analysis can significantly enhance the robustness and accuracy of risk assessment. By separating the marginal distributions of individual assets from their dependency structure, copulas provide the flexibility to model the unique behavior of each asset while simultaneously capturing their interdependencies. This results in a more holistic approach to risk analysis, leading to better-informed investment decisions and more effective portfolio diversification strategies.

Towards the end of our thesis we will implement a Gaussian copula to assess its fit to our data and explore how it can be integrated with traditional risk measures to improve our understanding of financial dependencies. Through this analysis, we aim to demonstrate the added value of copula models in enhancing the accuracy and effectiveness of financial risk management practices. By leveraging the strengths of copula models, this thesis seeks to contribute to the development of more resilient and informed risk management frameworks, capable of withstanding the complexities and volatilities of modern financial markets.

Chapter I

1.1 Risk Measures and its purposes

In the complex and dynamic landscape of financial markets, the quantification and management of risk are paramount for the stability and growth of financial institutions. Risk measures serve as essential tools for financial professionals, enabling them to assess the potential for losses and to make informed decisions to mitigate those risks. Among the myriad of risk measures utilized, Value at Risk (VaR), Conditional Value at Risk (CoVaR), and Delta Conditional Value at Risk (DeltaCoVaR) stand out for their distinct purposes and applications. VaR, a widely used measure, estimates the maximum potential loss of a portfolio over a specified time period at a given confidence level, providing a straightforward snapshot of risk exposure. However, VaR's limitations, such as its inability to capture tail risks, have led to the development of more sophisticated measures like CoVaR and DeltaCoVaR.

CoVaR extends the concept of VaR by considering the risk of a portfolio conditional on the distress of another entity, typically a systemically important financial institution, thereby offering insights into systemic risk.

DeltaCoVaR, further refining this approach, quantifies the incremental impact of an individual institution on the overall risk of the financial system. Together, these measures provide a comprehensive toolkit for financial risk assessment, each addressing specific aspects of risk and enabling more robust risk management strategies.

1.2 VaR

In the realm of financial risk management, Value at Risk (VaR) stands as a critical measure for quantifying potential losses within a portfolio over a specified time horizon and at a given confidence level. As an integral part of risk assessment, VaR provides a single, summary statistic that reflects the potential downside risk of investments. However, the calculation of VaR is not monolithic; it encompasses a variety of methodologies, each offering unique insights and benefits. This chapter delves into three prominent methods used to compute VaR: Historical Simulation, Variance-Covariance Method (Parametric VaR), and Monte Carlo Simulation.

Historical Simulation leverages actual historical returns to estimate potential future losses, providing a non-parametric approach grounded in empirical data.

The Variance-Covariance Method, on the other hand, assumes normally distributed returns and uses statistical parameters to derive risk estimates, offering simplicity and analytical clarity.

Monte Carlo Simulation, the most sophisticated of the three, involves generating a multitude of random scenarios to model potential future outcomes, capturing a wide range of possible risks. By exploring these methods, we gain a comprehensive understanding of how VaR can be computed and applied to manage financial risk effectively.

1.2.1 Historical Simulation

The Historical Simulation method for computing Value at Risk (VaR) is a non-parametric approach that relies on actual historical returns data to estimate potential future losses. This method proceeds through a series of straightforward steps.

First, one must collect a sufficiently large dataset of historical returns for the portfolio in question. This dataset should ideally encompass a broad range of market conditions to ensure robustness.

Next, these historical returns are sorted in ascending order. The desired confidence level is then determined, typically set at 95% or 99%, which corresponds to the level of risk tolerance. To identify the VaR, one selects the return at the specified percentile from the sorted list. For instance, at a 95% confidence level, the VaR is the return that falls at the 5th percentile of the ordered dataset. Mathematically, this can be expressed as:

$$VaR_{Historical} = -Percentile\ of\ Historical\ Returns$$

where the percentile of historical returns is the value below which a specified percentage of observations fall. If R_i represents the i-th return in the sorted list of N returns, the VaR at a confidence level α is given by:

$$VaR_{Historical} = -R_{\lceil (1-\alpha)\cdot \mathrm{N} \rceil}$$

where $\lceil \cdot \rceil$ denotes the ceiling function, ensuring the index corresponds to the appropriate percentile. This method's strength lies in its simplicity and direct use of historical data, although it assumes that past market conditions are indicative of future risks.

1.2.2 Parametric VaR

The Variance-Covariance Method, also known as Parametric VaR, is a widely used technique for calculating Value at Risk (VaR) that assumes the returns of a portfolio are normally distributed. This method simplifies the computation by leveraging the mean (μ) and standard deviation (σ) of the portfolio's returns.

The first step involves estimating these parameters from historical return data. Once μ and σ are determined, the next step is to choose a confidence level α, such as 95% or 99%. The corresponding z-score z_α for the normal distribution is then identified. For example, a 95% confidence level corresponds to a z-score of approximately 1.645, and a 99% confidence level corresponds to a z-score of approximately 2.33.

The VaR is then computed using the formula:

$$VaR_{Parametric} = \mu + z_\alpha \sigma$$

where z_α represents the z-score for the chosen confidence level.

This formula essentially multiplies the standard deviation by the z-score to scale the potential risk according to the desired confidence level, and then adjusts for the mean return. For a more practical application, if the mean return is close to zero, the formula simplifies to:

$$VaR_{Parametric} \approx z_\alpha \sigma$$

This approach is advantageous due to its simplicity and the analytical clarity it provides, making it computationally efficient. However, its primary limitation lies in the assumption of normally distributed returns, which may not hold true in markets exhibiting skewness and kurtosis. Consequently, while the Variance-Covariance Method is a powerful tool for estimating VaR, it is essential to validate the normality assumption for accurate risk assessment.

1.2.3 Monte Carlo Simulation

The Monte Carlo Simulation method for calculating Value at Risk (VaR) is a sophisticated and flexible approach that involves generating a large number of potential future price paths for a portfolio based on its statistical properties. The first step in this process is to model the statistical characteristics of the portfolio's returns, including the mean (μ), standard deviation (σ), and, if necessary, higher moments like skewness and kurtosis.

Using these parameters, random samples of future returns are generated, typically by assuming a specific distribution, often the normal distribution, although other distributions can be used to better capture the behavior of the returns. For each simulated scenario, the portfolio's value is recalculated, resulting in a distribution of potential future portfolio values.

The next step is to sort these simulated portfolio values in ascending order. The desired confidence level α is then applied to determine the VaR. Specifically, the VaR is the value at the (1 - α) percentile of the sorted distribution of portfolio values. For instance, at a 95% confidence level, the VaR corresponds to the 5th percentile of the simulated distribution. This can be mathematically formulated as follows:

$$VaR_{Monte\ Carlo} = -Percentile_{(1-\alpha)}$$

where $Percentile_{(1-\alpha)}$ is the portfolio value at the (1 - α) percentile of the sorted simulated values. If V_i represents the i-th simulated portfolio value in a sorted list of N simulations, the VaR at confidence level α is given by:

$$VaR_{Monte\ Carlo} = -V_{\lceil (1-\alpha)\cdot N \rceil}$$

where the ceiling function, as for the Historical Simulation method, ensures the index corresponds to the appropriate percentile. The Monte Carlo Simulation method's strength lies in its ability to model complex portfolios and capture a wide range of risk factors and their interactions, making it particularly useful in scenarios where returns are not normally distributed. However, it requires significant computational power and can be timeconsuming, particularly for large portfolios or when high accuracy is needed. Despite these challenges, Monte Carlo Simulation provides a robust and detailed estimate of VaR, accommodating a variety of risk dynamics and assumptions.

1.3 CoVaR

Conditional Value at Risk (CoVaR) is an advanced risk measure that extends the concept of Value at Risk (VaR) to account for systemic risk within the financial system. Introduced by Adrian and Brunnermeier, CoVaR quantifies the potential loss of a portfolio or institution conditional on another entity being in distress. In essence, while VaR assesses the risk of a single portfolio in isolation, CoVaR evaluates how the risk of one entity affects the broader financial system. This measure is particularly relevant for understanding the interconnectedness of financial institutions and the potential for systemic crises. By capturing the spillover effects of financial distress, CoVaR provides a more comprehensive view of risk, making it a vital tool for regulators and risk managers focused on systemic stability.

1.3.1 Mathematical Derivation from VaR

The mathematical derivation of CoVaR builds upon the foundation of VaR. While VaR at a confidence level α is defined as the maximum loss not exceeded with probability α, CoVaR at the same confidence level conditions this calculation on the distress of another entity. Formally, the VaR of a portfolio i at confidence level α is defined as:

$$VaR_{\alpha}^{i} = \inf\{x \in \mathbb{R}: P(L_i > x) \leq (1 - \alpha)\}$$

Where L_i represents the loss of portfolio i.

CoVaR is then defined as the VaR of portfolio j conditional on the event that portfolio i is at its VaR threshold. Mathematically, this is expressed as:

$$CoVaR_{\alpha}^{j} | \, i = inf\left\{x \in \mathbb{R}: P\left(L_j > x | \, L_i = VaR_{\alpha}^{i}\right) \leq (1 - \alpha)\right\}$$

This conditional approach necessitates calculating the joint distribution of losses for both portfolios i and j and then deriving the conditional distribution of L_j given L_i.

1.3.2 Improvements for the Analysis Given by the Implementation of CoVaR

The implementation of CoVaR significantly enhances risk analysis by addressing the limitations of traditional VaR, particularly in the context of systemic risk. By focusing on the conditional relationships between entities, CoVaR provides a deeper insight into the potential for contagion and the impact of one institution's distress on others. This is crucial for identifying systemically important financial institutions (SIFIs) and understanding their role in financial stability. CoVaR's ability to capture tail dependencies and correlated risks offers a more nuanced view of the risk landscape, enabling better-informed decisions regarding capital allocation, risk management, and regulatory oversight. Moreover, CoVaR facilitates stress testing and scenario analysis, allowing for the assessment of systemic vulnerabilities under various adverse conditions. Overall, the adoption of CoVaR enriches the analytical toolkit available to risk managers and policymakers, promoting a more resilient financial system.

1.3.3 Similarities and Differences from VaR

CoVaR shares several similarities with VaR, as both are risk measures that quantify potential losses at a given confidence level. Both metrics are integral to risk management frameworks and are used to set capital reserves and inform risk mitigation strategies. However, there are also critical differences between the two. While VaR assesses the risk of a single portfolio or institution in isolation, CoVaR evaluates risk in a conditional context, explicitly accounting for the potential impact of one entity's distress on another. This distinction makes CoVaR particularly valuable for systemic risk analysis, as it incorporates the interconnectedness and interdependencies within the financial system.
Additionally, CoVaR often requires more complex statistical techniques and joint distribution models, compared to the relatively straightforward computation of VaR. In summary, while VaR provides a snapshot of individual risk, CoVaR offers a broader, more interconnected perspective, essential for understanding and mitigating systemic risk.

1.4 Delta CoVaR

DeltaCoVaR, an extension of Conditional Value at Risk (CoVaR), provides a measure of systemic risk by quantifying the marginal contribution of a single financial institution to the overall risk of the financial system. Introduced by Adrian and Brunnermeier, DeltaCoVaR assesses how the risk profile of the financial system changes when a particular institution transitions from a normal state to distress. This metric captures the incremental risk that a single institution adds to the system, making it an essential tool for identifying systemically important financial institutions (SIFIs) and understanding their potential impact on financial stability.

1.4.1 Mathematical Derivation

The mathematical derivation of DeltaCoVaR starts with the concept of CoVaR. Recall that CoVaR is defined as the VaR of a portfolio or system conditional on another institution being in distress. Formally, CoVaR for institution j given institution i at confidence level α is:

$$CoVaR_{\alpha}^{j}|\, i = inf\{x \in \mathbb{R}: P\left(L_j > x|\, L_i = VaR_{\alpha}^{i}\right) \leq (1 - \alpha)\}$$

DeltaCoVaR measures the difference between the CoVaR of the financial system when an institution i is in a normal state versus when it is in distress. Mathematically, it is defined as:

$$\Delta CoVaR_{\alpha}^{i} = CoVaR_{\alpha}^{j}\big|\, i - CoVaR_{\alpha}^{j}\big|\, median$$

where $CoVaR_{\alpha}^{j}|\, i$ represents the CoVaR when institution i is in distress, and $CoVaR_{\alpha}^{j}|\, median$ represents the CoVaR when institution i is at its median state. This difference captures the incremental systemic risk contributed by institution i transitioning from a normal to a distressed state.

1.4.2 Improvements for the Analysis Given by the Implementation of DeltaCoVaR

The implementation of DeltaCoVaR significantly enhances the analysis of systemic risk by providing a more granular understanding of how individual institutions contribute to the overall risk of the financial system. DeltaCoVaR allows regulators and risk managers to pinpoint which institutions are most critical to financial stability, thereby enabling more targeted regulatory interventions and risk management strategies. By quantifying the marginal impact of each institution, DeltaCoVaR facilitates more effective stress testing and scenario analysis, providing insights into potential cascading effects in times of financial distress. Furthermore, DeltaCoVaR helps in the allocation of capital reserves, ensuring that sufficient buffers are in place to absorb shocks originating from key institutions. Overall, the adoption of DeltaCoVaR leads to a more resilient financial system by promoting proactive risk management and informed policy decisions.

1.4.3 Similarities and Differences from CoVaR

DeltaCoVaR and CoVaR are both advanced risk measures that extend beyond traditional VaR to incorporate systemic risk considerations. Both metrics evaluate the risk of a financial system or portfolio conditional on the state of another institution, making them valuable tools for assessing interconnected risks. However, while CoVaR measures the risk of the financial system conditional on a particular institution being in distress, DeltaCoVaR quantifies the incremental contribution of that institution to systemic risk. In other words, CoVaR provides a static view of conditional risk, whereas DeltaCoVaR captures the dynamic change in risk due to the distress of a specific institution. This distinction makes DeltaCoVaR particularly useful for understanding the marginal effects of individual institutions on overall financial stability. Additionally, DeltaCoVaR requires more complex calculations as it involves comparing CoVaR under different states of the institution, whereas CoVaR focuses on a single conditional state. In summary, while both measures offer valuable insights into systemic risk, DeltaCoVaR provides a more detailed and actionable perspective on the contribution of individual institutions to financial stability.

This chapter delves into an exploration of the S&P sectors, as delineated by the Global Industry Classification Standard (GICS), through the lens of Exchange Traded Funds (ETFs). Specifically, it focuses on the SPDR ETFs representing 11 distinct sectors, along with SPY, encompassing the period from 2018 to 2023 at a daily frequency.

Chapter II Systemic risk Analysis

The chapter unfolds with a multifaceted approach, beginning with the download and aggregation of historical data for the identified ETFs. Subsequently, the analysis ventures into the realm of risk assessment, employing the Value at Risk (VaR) metric to gauge the potential losses within a ten-day horizon. Utilizing an empirical distribution technique, VaR calculations are conducted for each ETF as an individual portfolio, offering insights into their standalone risk profiles.

Building upon this foundation, a 100-day rolling window risk analyzer is crafted to delve deeper into the risk dynamics. By amalgamating the SPY ETF with the sector ETFs, this analyzer provides a comprehensive view of risk evolution over time. Through the lens of rolling windows, the VaR for the combined portfolio is scrutinized, providing a nuanced understanding of risk exposure within varying market conditions.

Furthermore, the analysis extends to consider the systemic risk inherent within the SPY ETF, conceptualizing it as a composite of the sector ETFs. The calculation of CoVaR (Conditional Value at Risk) offers a perspective on the interdependencies and contagion effects within the broader market ecosystem. In addition to quantitative assessments,the analysis seeks to elucidate the risk landscape through visual means. A heatmap visualization is crafted to depict the distribution of risk over the five-year period, capturing snapshots at the conclusion of each month.

Finally, the project delves into the realm of copulas, seeking to approximate the joint distribution of returns for the 11 sector ETFs. This exploration culminates in a reevaluation of portfolio risk within the copula framework, offering a holistic perspective on risk management strategies.

Incorporating the R code into my thesis is essential for several reasons. Firstly, it provides transparency and reproducibility, allowing other researchers to verify and build upon my work. Secondly, it demonstrates the practical application of theoretical concepts, bridging the gap between theory and practice. The libraries I downloaded, such as \`dplyr\` for data manipulation, \`ggplot2\` for data visualization, and \`quantmod\` for financial modeling, are integral to efficiently processing and analyzing the data. These libraries streamline complex tasks, enabling more accurate and insightful analysis, which is crucial for the robustness of my findings.

```r
library(copula)
library(dplyr)
```

```r
library(ggplot2)
library(MASS)
library(PerformanceAnalytics)
library(PortfolioAnalytics)
library(purrr)
library(quantmod)
library(quantreg)
library(RColorBrewer)
library(reshape2)
library(ROI.plugin.glpk)
library(ROI.plugin.quadprog)
library(VineCopula)
library(zoo)
```

2.1 Exploratory data analysis

2.1.1 Historical prices 2018-2023

This chunk of code retrieves historical stock price data our ETFs and the SPY from January 1, 2018, to December 31, 2023. It is important to note that the period selected includes the pandemic and energy supply crisis.

It handles potential errors using try and silent = TRUE to suppress warnings. The data is organized into a list of time series objects (P.list). Then, the sector names are defined and assigned to each ticker symbol. Adjusted closing prices are extracted from each time series and merged into a single data frame (prices). Missing values are removed, and logarithmic returns are calculated. Cumulative returns are computed and stored in a data frame (cum_returns) along with corresponding dates.

```r
tics <- c("XLC", "XLY", "XLP", "XLE", "XLF", "XLV", "XLI", "XLB", "XLR
E",
          "XLK", "XLU", 'SPY')
P.list <- lapply(tics, function(v) try(get(getSymbols(v,
                                                      from = "2018-01-
01",
                                                      to = "2023-12-31
")),
                                       silent = T))

sectors <- c("Commincation Services", "Consumer Discretionary", "Consu
mer Staples",
             "Energy", "Financials", "Health Care", "Industrials", "Ma
terials",
             "Real Estate", "Technology", "Utilities", 'SPY')

P.adj <- lapply(P.list, function(x) x[,6])
prices <- Reduce(merge, P.adj)
prices <- na.omit(prices)
```

```
returns <- log(prices/lag(prices))
returns <- na.omit(returns)
colnames(returns) <- tics

cum_returns <- cumsum(returns)
colnames(cum_returns) <- tics
cum_returns$date <- c(1:1392)
```

2.1.2 Historical prices 2006-2012

This following part also retrieves historical stock price data for the same ETFs from January 1, 2006, to December 31, 2011. Similar to the previous chunk, it handles potential errors using try and silent = TRUE to suppress warnings. The adjusted closing prices are extracted from each time series and merged into a single data frame old_prices. Missing values are removed, and logarithmic returns are calculated. Cumulative returns are computed and stored in a data frame old_cum along with corresponding dates. The time period covered by this data coincides with the global financial crisis of 2008.

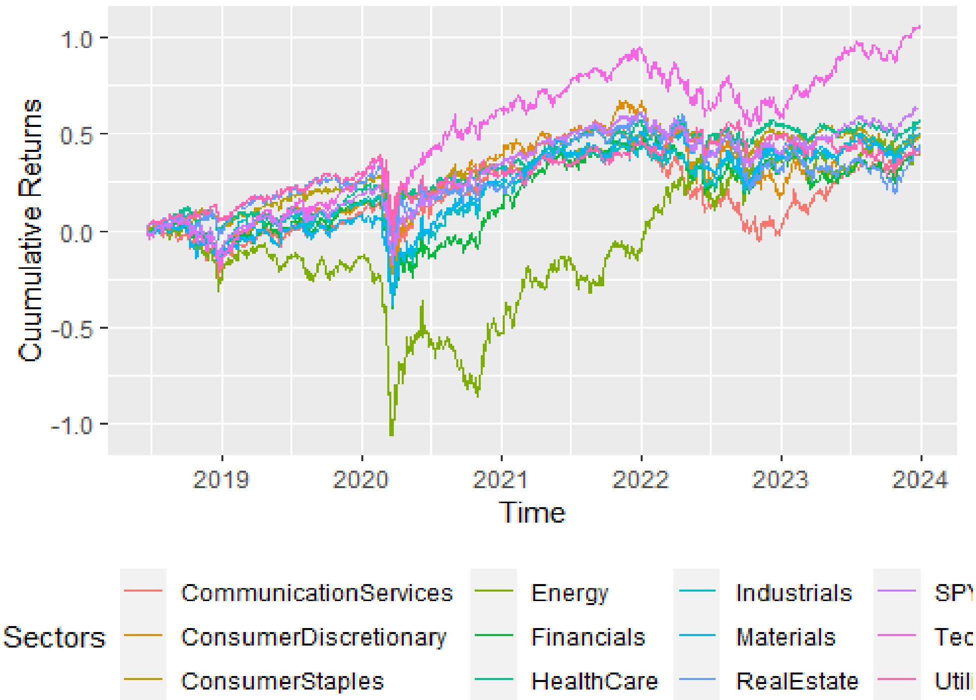
```
old_tics <- c("XLY", "XLP", "XLE", "XLF", "XLV", "XLI", "XLB", "XLK",
"XLU", 'SPY')
O.list <- lapply(old_tics, function(v) try(get(getSymbols(v,
                                                          from = "2006
-01-01",
                                                          to = "2011-1
2-31")),
                                            silent = T))

O.adj <- lapply(O.list, function(x) x[,6])
old_prices <- Reduce(merge, O.adj)
old_prices <- na.omit(old_prices)

old_returns <- na.omit(log(old_prices/lag(old_prices)))
colnames(old_returns) <- old_tics

old_cum <- cumsum(old_returns)
colnames(old_cum) <- old_tics
old_cum$date <- c(1:1510)
```

2.1.3 Prices development

These code chunks create line plots of cumulative returns for SPDR ETFs, including the SPY ETF, for two different time periods: 2018-2023 and 2006-2011. Each sector ETF is represented by a different color. The x-axis represents time, and the y-axis represents cumulative returns. The legend is positioned at the bottom of the plot.

```
ggplot(cum_returns,aes(x=index(cum_returns))) +
geom_line(aes(y=XLC, color="CommunicationServices"))+
geom_line(aes(y=XLY, color="ConsumerDiscretionary"))+
geom_line(aes(y=XLP, color="ConsumerStaples"))+
geom_line(aes(y=XLE, color="Energy"))+
geom_line(aes(y=XLF, color="Financials"))+
geom_line(aes(y=XLV, color="HealthCare"))+
geom_line(aes(y=XLI, color="Industrials"))+
geom_line(aes(y=XLB, color="Materials"))+
geom_line(aes(y=XLRE, color="RealEstate")) +
geom_line(aes(y=XLK, color="Technology"))+
geom_line(aes(y=XLU, color="Utilities"))+
geom_line(aes(y=SPY, color="SPY"))+
labs(x ="Time",
y = "Cuumulative Returns",
color="Sectors") +
 scale_x_date(date_labels= "%Y", date_breaks= "1 year") +
 theme(legend.position= "bottom" )
```

Plot 2.1 SPY sectors ETFs price movements 2018-2023



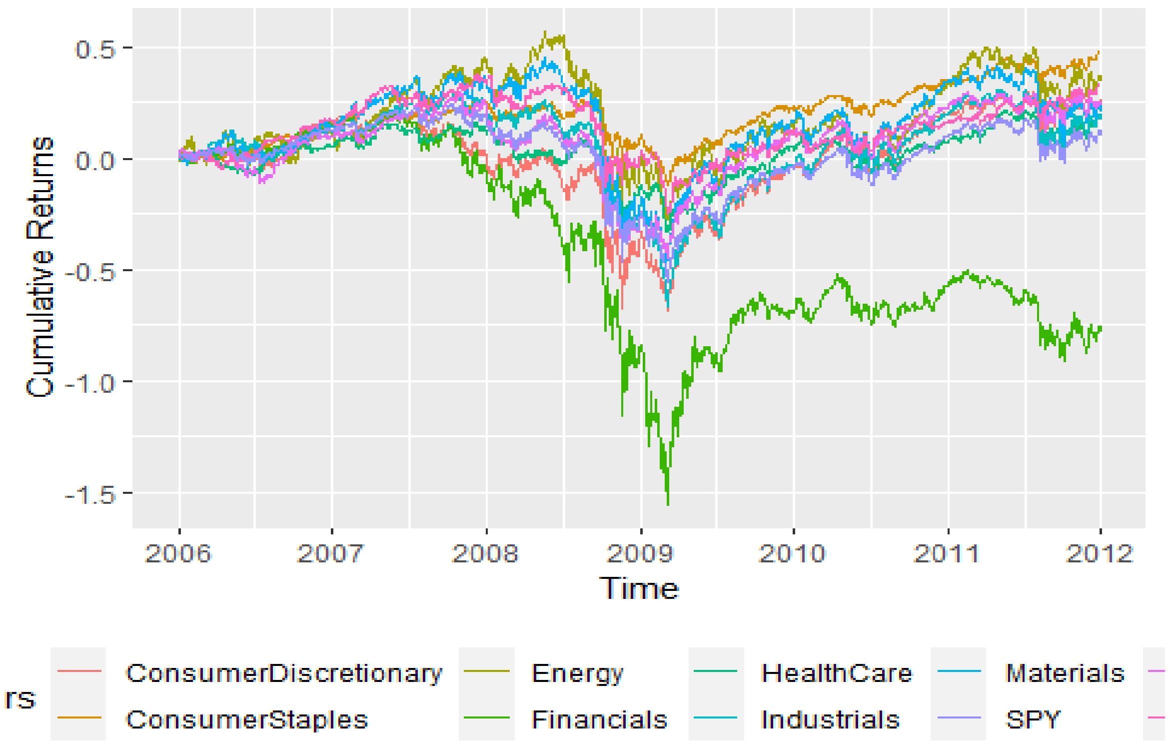
Plot 2.1 shows cumulative returns of various sector ETFs and the S&P 500 (SPY) from 2018 to 2023. Most sectors trend upwards, indicating market growth, but experienced a sharp decline in early 2020 due to COVID-19, followed by a recovery at varying speeds.

The Technology sector (XLK) has the highest cumulative returns, demonstrating robust growth, especially post-pandemic. Consumer Discretionary (XLY) and Health Care (XLV) also perform well, while the Energy sector (XLE) shows significant volatility, reflecting its sensitivity to economic conditions. Financials (XLF) and Real Estate (XLRE) show steady but moderate growth, and defensive sectors like Utilities (XLU) and Consumer Staples (XLP) exhibit stability but lower returns.

Most sectors follow the general trend of the SPY, with Technology outperforming and Energy fluctuating more. This graph highlights the varying performance of sectors, the significant impact of COVID-19, and provides insights into market dynamics and sector-specific risks and opportunities. The graph shows cumulative returns for various sector ETFs and the S&P 500 (SPY) from 2006 to 2012, providing a comparison to the 2018-2023 period.

```
ggplot(old_cum, aes(x= index(old_cum)))+
geom_line(aes(y=XLY,color= "ConsumerDiscretionary"))+
geom_line(aes(y=XLP, color="ConsumerStaples"))+
geom_line(aes(y=XLE, color="Energy"))+
geom_line(aes(y=XLF, color="Financials"))+
geom_line(aes(y=XLV, color="HealthCare"))+
geom_line(aes(y=XLI, color="Industrials"))+
geom_line(aes(y=XLB, color="Materials"))+
geom_line(aes(y=XLK, color="Technology"))+
geom_line(aes(y=XLU, color="Utilities"))+
geom_line(aes(y=SPY, color="SPY"))+
labs(x ="Time",
y = "Cumulative Returns",
color="Sectors") +
scale_x_date(date_labels="%Y", date_breaks="1 year") +
theme(legend.position= "bottom" )
```

Plot 2.2 SPY sectors ETFs price movements 2006-2011



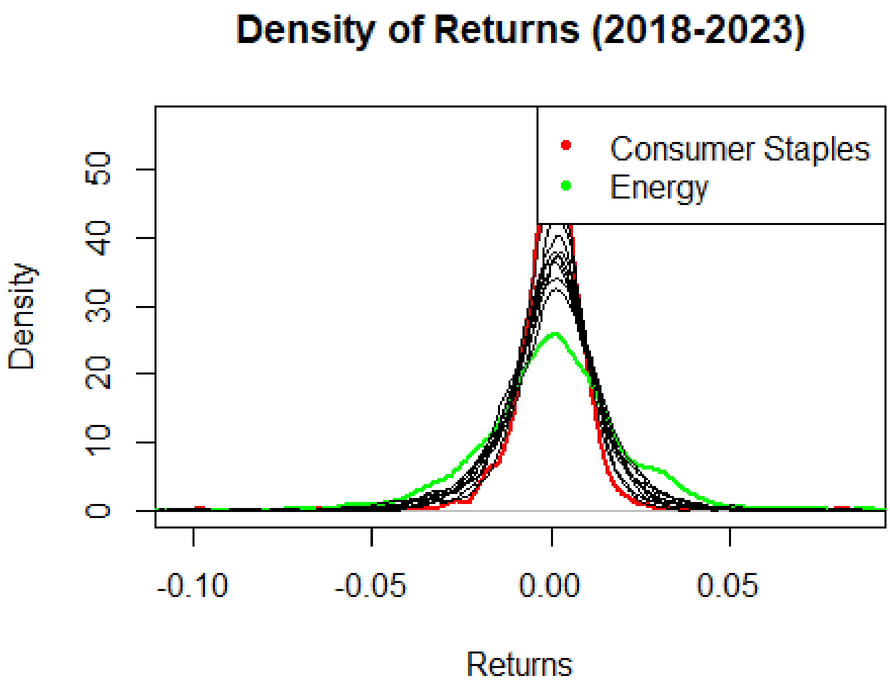
Plot 2.2 highlights the severe impact of the 2008 financial crisis, with a significant drop across all sectors, particularly in Energy (XLE), and a slow recovery starting in 2009. In contrast, the 2018-2023 graph shows a dip during the COVID-19 pandemic in early 2020, followed by a swift recovery, with Technology (XLK) performing exceptionally well. While both periods experienced downturns and recoveries, the financial crisis had a more prolonged impact, especially on the Energy sector, whereas the COVID-19 period saw a quicker rebound, driven by the Technology sector. This comparison underscores the different economic challenges and recovery patterns faced by the markets.

2.1.4 Prices distribution

```
plot(density(returns$XLP), col = 'red', lwd = 2,
     main = 'Density of Returns (2018-2023)', xlab = 'Returns')
lines(density(returns$XLY))
lines(density(returns$XLC))
lines(density(returns$XLE), col = 'green', lwd = 2)
lines(density(returns$XLF))
lines(density(returns$XLV))
lines(density(returns$XLI))
lines(density(returns$XLB))
lines(density(returns$XLRE))
lines(density(returns$XLK))
lines(density(returns$XLU))
legend('topright', c('Consumer Staples', 'Energy'), col = c('red', 'gr
```

```
een'),
pch = 20)
```

Plot 2.3 SPY sectors ETFs prices distributions 2018-2023



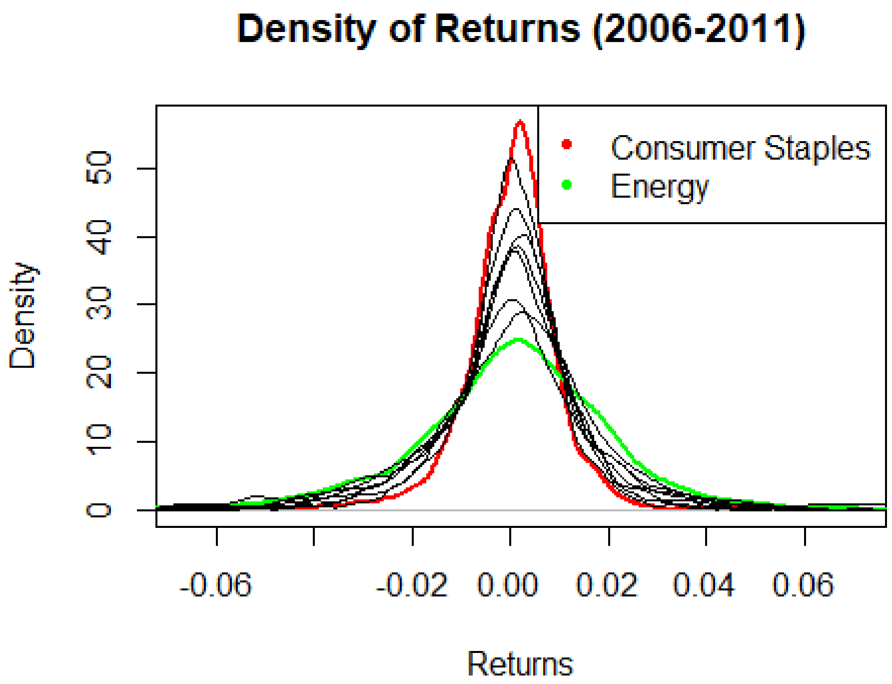
Plot 2.3 displays the density distributions of daily returns for sector ETFs and the S&P 500 (SPY) from 2018 to 2023, with Consumer Staples and Energy sectors highlighted in red and green, respectively. The distributions show that most returns are concentrated around zero, indicating that daily price changes were typically small.

The peak of the density curves, especially for Consumer Staples, is higher and narrower, suggesting lower volatility and more consistent returns. In contrast, the Energy sector's distribution is wider and flatter, indicating higher volatility with a greater spread of returns. This difference in distribution shapes signifies that the Energy sector experienced more frequent and larger price fluctuations compared to the Consumer Staples sector.Statistically, the density of returns provides insight into the risk profile of each sector. The sharp peak and thin tails of the Consumer Staples sector imply lower risk and less extreme return events. Conversely, the broader distribution of the Energy sector suggests higher risk with a greater probability of extreme returns.

```
plot(density(old_returns$XLP), col = 'red', lwd = 2,
     main = 'Density of Returns (2006-2011)', xlab = 'Returns')
```

```
lines(density(old_returns$XLY))
lines(density(old_returns$XLE), col = 'green', lwd = 2)
lines(density(old_returns$XLF))
lines(density(old_returns$XLV))
lines(density(old_returns$XLI))
lines(density(old_returns$XLB))
lines(density(old_returns$XLK))
lines(density(old_returns$XLU))
legend('topright', c('Consumer Staples', 'Energy'), col = c('red', 'gr
een'), pch = 20)
```

Plot 2.4 SPY sectors ETFs prices ditributions 2006-2011



Plot 2.4 shows the density distributions of daily returns for sector ETFs and the S&P 500 (SPY) from 2006 to 2011, focusing on Consumer Staples (red) and Energy (green). The Consumer Staples sector has a pronounced peak around zero, indicating low volatility and stable returns, similar to the 2018-2023 period. The Energy sector displays a broader distribution, signifying higher volatility and a wider range of returns.

Comparing the two periods, Consumer Staples consistently show stability with narrow peaks, while the Energy sector is volatile in both but exhibits a slightly narrower distribution in 2006-2011 compared to 2018-2023. This suggests the Energy sector's volatility increased in the latter period. Overall, this highlights the stability of Consumer

Staples and the fluctuating nature of the Energy sector, emphasizing the importance of sector-specific risk profiles for risk management and portfolio diversification.

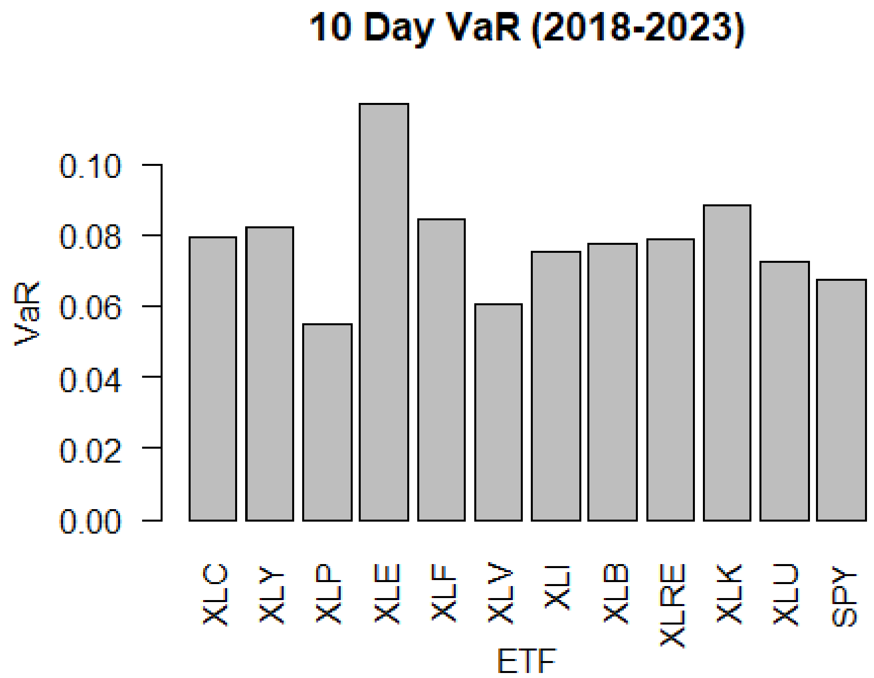
2.2 VaR

This code defines the function va_r to calculate the Value at Risk (VaR) using the normal distribution approach. The function takes a vector of returns as input and computes the 5% VaR for a 10-day period based on the mean and standard deviation of the returns. After defining the function, it is applied to the returns for both time periods (2018-2023 and 2006-2011) using sapply. The resulting VaR values are stored in var_10 and old_var10 respectively.

Finally, bar plots are generated to visualize the VaR values for each time period.

```
va_r <- function(x){
  mean <- mean(x)
  sd <- sd(x)
  var <- qnorm(.05, mean, sd * sqrt(10))
  return(var)
}

var_10 <- sapply(returns, va_r)
var_10 <- abs(var_10)
var_10

##        XLC        XLY        XLP        XLE        XLF        XLV
XLI
## 0.07935207 0.08215696 0.05464986 0.11671612 0.08465812 0.06034258 0
.07549508
##        XLB       XLRE        XLK        XLU        SPY
## 0.07762757 0.07888750 0.08839759 0.07253396 0.06750114

barplot(var_10, ylab = "VaR" , xlab = "ETF" ,
 las=2)
title('10 Day VaR (2018-2023)')
```

Graph 2.1 10 day VaR 2018-2023



Graph 2.1 illustrates the 10-day Value at Risk (VaR) for various sector ETFs and the S&P 500 (SPY) for the dataset ranging from 2018 to 2023.

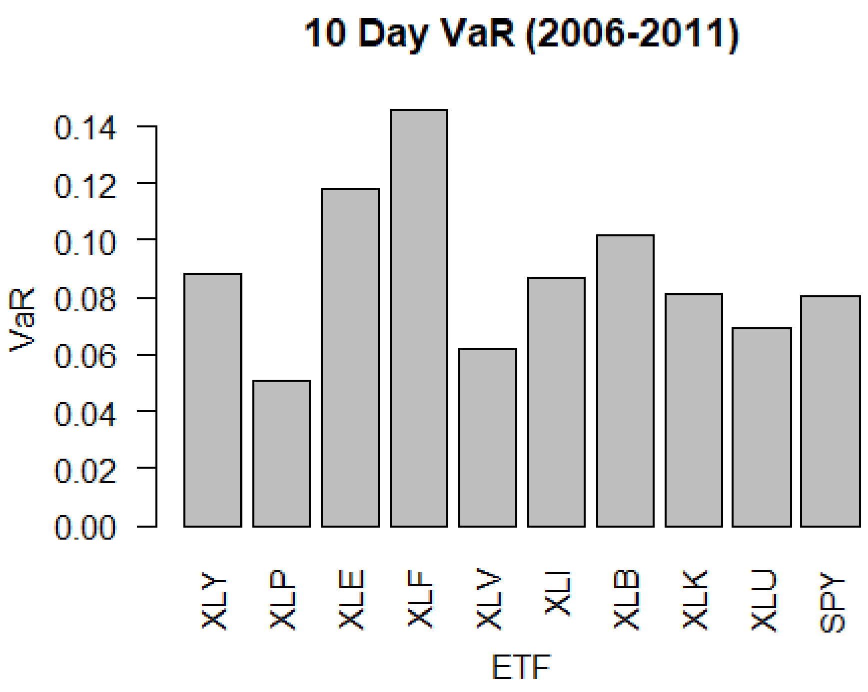
Notably, the Energy sector (XLE) exhibits the highest 10-day VaR, exceeding 0.10, which indicates that it has the greatest potential for significant losses over a short period. This high VaR reflects the sector's high volatility and sensitivity to external factors such as geopolitical events and fluctuations in oil prices. In contrast, sectors like Consumer Staples (XLP) and Utilities (XLU) show relatively lower VaR values, underscoring their role as defensive sectors. These sectors tend to have lower volatility and offer more stable returns even during market downturns, due to the consistent demand for their essential products and services.

The Financials (XLF) and Real Estate (XLRE) sectors display moderate VaR values, suggesting a balanced level of risk. Their performance is often influenced by market interest rates and economic cycles, which contribute to their moderate volatility. The S&P 500 (SPY) itself shows a VaR value that is relatively balanced compared to individual sectors, reflecting its diversified nature.

Overall, the graph highlights the varying risk profiles of different sectors, with Energy being the most volatile and Consumer Staples and Utilities being the least.

```
old_var10 <- sapply(old_returns, va_r)
old_var10 <- abs(old_var10)
old_var10

##        XLY        XLP        XLE        XLF        XLV        XLI
XLB
## 0.08830299 0.05053169 0.11804398 0.14545223 0.06187655 0.08657441 0
.10166708
##        XLK        XLU        SPY
## 0.08094485 0.06924837 0.08039513

barplot(old_var10, ylab = "VaR" , xlab = "ETF" ,
 las=2)
title('10 Day VaR (2006-2011)')
```

Graph 2.2 10 day VaR 2006-2011



Graph 2.2 gives us a comparison with the previous graph, depicting the 10-day Value at Risk (VaR) for various sector ETFs and the S&P 500 (SPY) between 2006-2011.

During 2006-2011, the Financials sector (XLF) had the highest VaR, over 0.14, reflecting the volatility of the financial crisis. In contrast, the 2018-2023 period saw the Energy sector (XLE) with the highest VaR, over 0.10, indicating a shift in risk dynamics. Consumer Staples (XLP) and Utilities (XLU) consistently show lower VaR values in both periods, but with slightly higher values in 2018-2023, indicating increased perceived risk.

The S&P 500 (SPY) shows relatively balanced VaR in both periods, though higher in 2006-2011, reflecting the financial instability of that time. This comparison highlights the changing nature of sector-specific risks, with Financials being most volatile during the crisis and Energy becoming riskier in recent years.

2.3 VaR rolling window

The following section computes the 100-day rolling window Value at Risk (VaR) for each sector ETF and the SPY ETF for two different time periods: 2018-2023 and 2006-2011. For the first plot (2018-2023), the rolling window VaR values for each sector ETF and SPY are calculated using the rollapply function and stored in roll. The resulting VaR values are then plotted against the window number, with each sector ETF represented by a different color. The legend is positioned at the bottom of the plot.

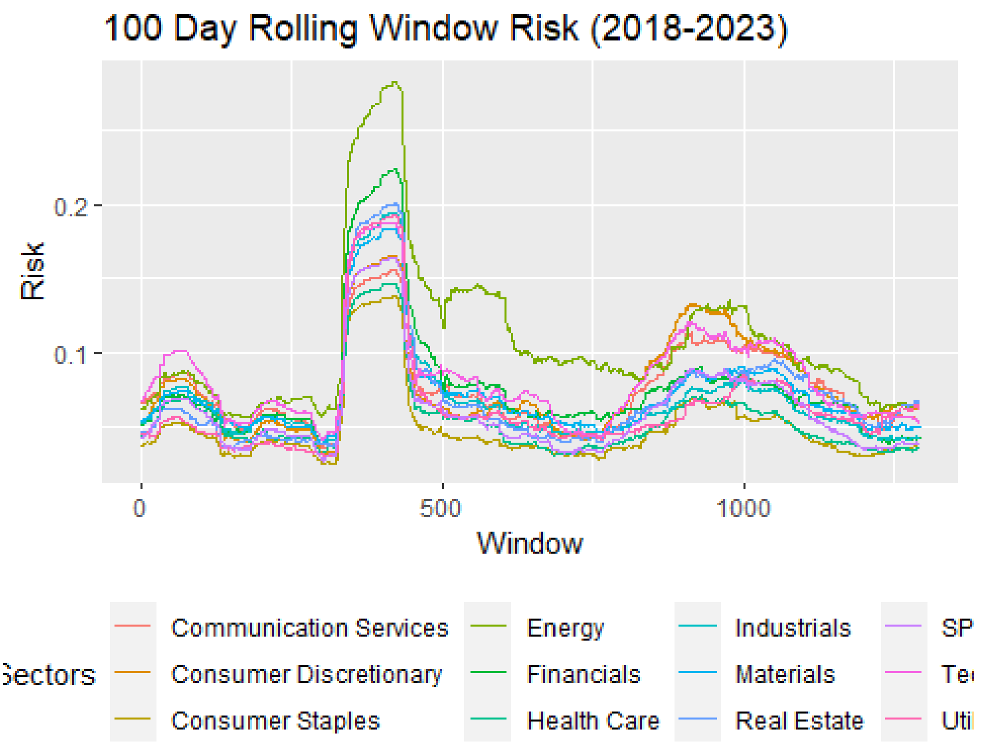
For the second plot (2006-2011), the same procedure is followed, with the rolling window VaR values stored in old_roll and plotted against the window number. Again, each sector ETF is represented by a different color, and the legend is positioned at the bottom.

These plots visualize the changing risk levels over time for each sector ETF and the broader market represented by the SPY ETF, providing insights into the evolving risk profiles of these assets over the respective time periods.

```
roll <- rollapply(returns, width = 100, FUN = function(x) va_r(x))

roll <- na.omit(roll)
roll <- abs(roll)
roll$window <- c(1:1293)

ggplot(roll, aes(x = window)) +
geom_line(aes(y = XLC, color = "Communication Services")) +
  geom_line(aes(y = XLY, color = "Consumer Discretionary")) +
  geom_line(aes(y = XLP, color = "Consumer Staples")) +
  geom_line(aes(y = XLE, color = "Energy")) +
  geom_line(aes(y = XLF, color = "Financials")) +
  geom_line(aes(y = XLV, color = "Health Care")) +
  geom_line(aes(y = XLI, color = "Industrials")) +
  geom_line(aes(y = XLB, color = "Materials")) +
  geom_line(aes(y = XLRE, color = "Real Estate")) +
  geom_line(aes(y = XLK, color = "Technology")) +
  geom_line(aes(y = XLU, color = "Utilities")) +
  geom_line(aes(y = SPY, color = "SPY")) +
  labs(x = "Window",
       y = "Risk",
       title = "100 Day Rolling Window Risk (2018-2023)",
```

```
      color = "Sectors") +
  theme(legend.position = 'bottom')
```

Plot 2.5 VaR Rolling window 2018-2023



Plot 2.5 shows the price movements of the different sectors, highlighting periods of volatility and stability. A notable spike in risk occurs around the 500th window, corresponding to early 2020 and the onset of the COVID-19 pandemic. The Energy sector (green line) experiences the highest volatility during this period, reflecting its sensitivity to global disruptions.

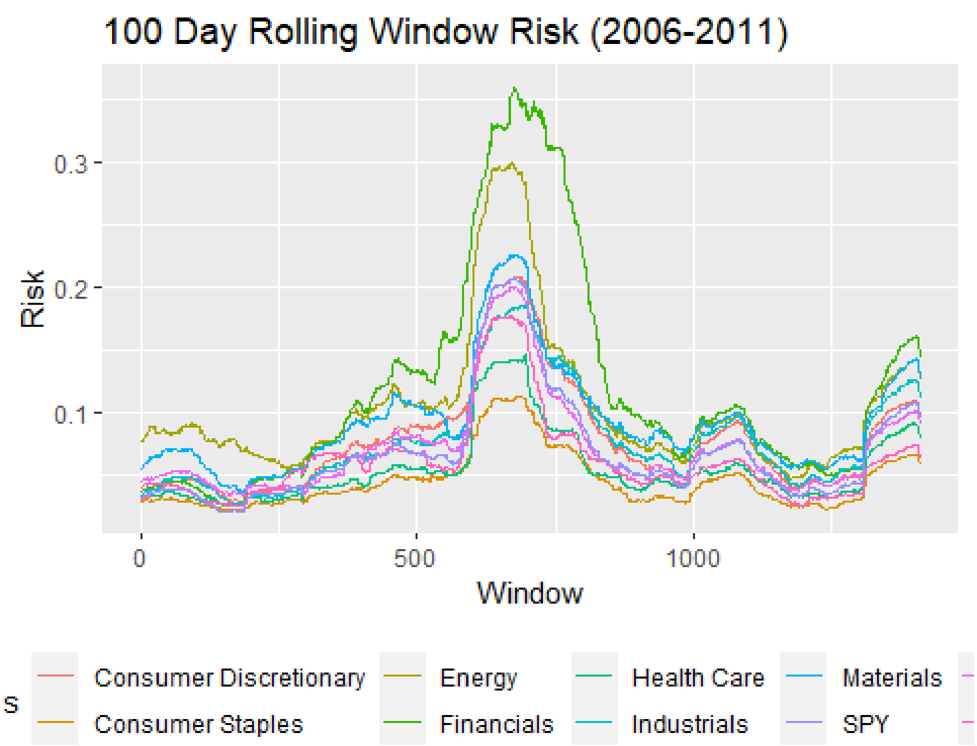
After this peak, risk gradually declines across all sectors, although Energy remains relatively more volatile. Minor subsequent spikes suggest responses to other economic events. Consumer Staples (red line) and Utilities (light purple line) consistently show lower risk, emphasizing their stability as defensive sectors.

Overall, the plot captures the dynamic nature of sector-specific risks, underscoring the importance of monitoring rolling risk measures to manage market volatility effectively.

```
old_roll <- rollapply(old_returns, width = 100, FUN = function(x) va_r
(x))

old_roll <- na.omit(old_roll)
old_roll <- abs(old_roll)
```

```r
old_roll$window <- c(1:1411)

ggplot(old_roll, aes(x = window)) +
  geom_line(aes(y = XLY, color = "Consumer Discretionary")) +
  geom_line(aes(y = XLP, color = "Consumer Staples")) +
  geom_line(aes(y = XLE, color = "Energy")) +
  geom_line(aes(y = XLF, color = "Financials")) +
  geom_line(aes(y = XLV, color = "Health Care")) +
  geom_line(aes(y = XLI, color = "Industrials")) +
  geom_line(aes(y = XLB, color = "Materials")) +
  geom_line(aes(y = XLK, color = "Technology")) +
  geom_line(aes(y = XLU, color = "Utilities")) +
  geom_line(aes(y = SPY, color = "SPY")) +
  labs(x = "Window",
       y = "Risk",
       title = "100 Day Rolling Window Risk (2006-2011)",
       color = "Sectors") +
  theme(legend.position = 'bottom')
```

Plot 2.6 VaR rolling window 2006-2011



Plot 2.6 provides a detailed view of risk dynamics during the 2006-2011 period, which includes the global financial crisis.

A significant spike in risk is observed around the 500th window, corresponding to the peak of the financial crisis in 2008-2009. The Financials sector (yellow line) and the

Energy sector (green line) exhibit the highest levels of risk, surpassing 0.3 and 0.2 respectively, reflecting the severe impact of the crisis on these sectors. This heightened risk persists for a considerable duration before gradually declining.

In comparison to the 2018-2023 graph, both periods show major spikes in risk: early 2020 for the COVID-19 pandemic and 2008-2009 for the financial crisis. However, the magnitude of risk during the financial crisis is notably higher, particularly in the Financials sector, which indicates more severe and prolonged market stress compared to the COVID-19 period.

Consumer Staples (blue line) and Utilities (light purple line) consistently show lower risk across both periods, underscoring their defensive nature. However, the overall risk levels are higher during the financial crisis, even for these stable sectors, reflecting the widespread impact of the economic downturn.

Overall, the 2006-2011 plot highlights the extreme volatility and elevated risk during the financial crisis, with the Financials and Energy sectors being most affected. The comparison with the 2018-2023 period underscores the varying impact of different crises on sector-specific risks, emphasizing the unique severity of the financial crisis on market stability.

2.4 CoVaR

The following code chunks define two functions, `co_var` and `co_var2`, which are used to calculate the Conditional Value at Risk (CoVaR) for different sector ETFs in relation to the SPY ETF. The CoVaR metric is a measure of systemic risk, quantifying the risk spillover from one asset or sector to the broader market.

In the `co_var` function, a vector `x` representing the returns of a particular sector ETF is taken as input. A quantile regression is performed using the `rq` function from the `quantreg` package, with the SPY ETF returns as the response variable and the sector ETF returns (`x`) as the explanatory variable. The quantile regression is performed at the 95th percentile, fitting the regression line to the upper 5% quantile of the SPY returns. The coefficients of the quantile regression, `beta_0` (intercept) and `beta_1` (slope), are extracted from the regression output. The 95th percentile of the SPY returns (`x_95`) is calculated using the `quantile` function. The CoVaR for the given sector ETF is computed as `beta_0 + beta_1 * x_95`, representing the Value at Risk (VaR) of the SPY ETF at the 95th percentile, conditional on the sector ETF returns being at their 95th percentile. The calculated CoVaR value is returned by the function.

The co_var2 function follows a similar structure but is applied to the old_returns data, which corresponds to the 2006-2011 time period. This function allows the calculation of CoVaR for the same sector ETFs during the global financial crisis period. By calculating the CoVaR for each sector ETF, the analysis can assess the systemic risk contributions of different sectors and their impact on the overall market during different time periods.

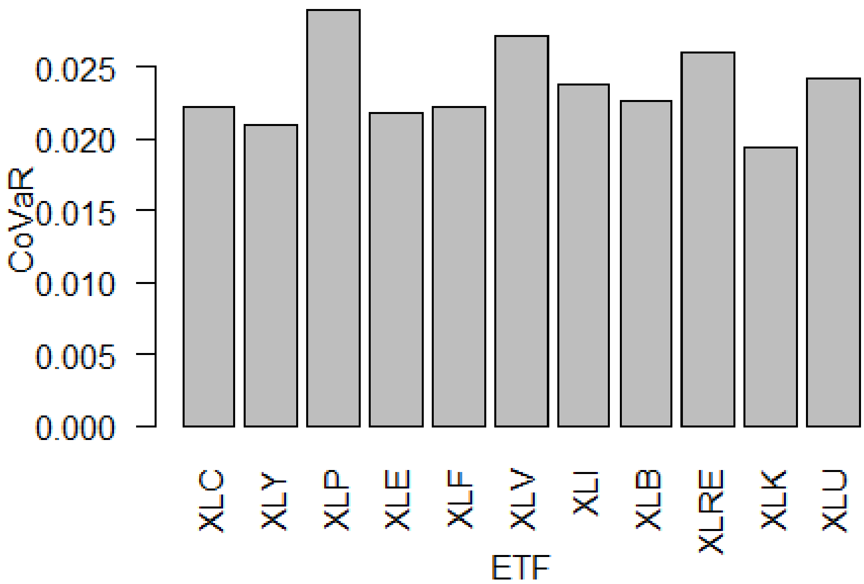
```
co_var <- function(x) {
  qr <- rq(SPY ~ x, data = returns, tau = 0.95)
  beta_0 <- coef(qr)[1]
  beta_1 <- coef(qr)[2]
  x_95 <- quantile(returns$SPY, 0.95)
  covar <- beta_0 + beta_1 * x_95
  return(covar)
}

co_var2 <- function(etf) {
  qr <- rq(SPY ~ etf, data = old_returns, tau = 0.95)
  beta_0 <- coef(qr)[1]
  beta_1 <- coef(qr)[2]
  x_95 <- quantile(old_returns$SPY, 0.95)
  covar <- beta_0 + beta_1 * x_95
  return(covar)
}
```

These CoVaR values provide insights into the systemic risk contributions of different sectors to the broader market during the respective time periods. Higher CoVaR values indicate a greater potential for risk spillover from a particular sector to the overall market, as represented by the SPY ETF. By analyzing and comparing the CoVaR values across sectors and time periods, researchers and financial analysts can gain valuable insights into the dynamics of systemic risk and the interconnectedness of different market sectors.

```
covar_results <- sapply(returns[,1:11], co_var)
covar_results

##  XLC.(Intercept)  XLY.(Intercept)  XLP.(Intercept)  XLE.(Intercept)
##       0.02227094       0.02092047       0.02894972       0.02172879
##  XLF.(Intercept)  XLV.(Intercept)  XLI.(Intercept)  XLB.(Intercept)
##       0.02223682       0.02710334       0.02370164       0.02265361
## XLRE.(Intercept)  XLK.(Intercept)  XLU.(Intercept)
##       0.02600970       0.01932586       0.02421699

covar_results2 <- sapply(old_returns[,1:9], co_var2)
covar_results2

## XLY.(Intercept) XLP.(Intercept) XLE.(Intercept) XLF.(Intercept) XLV
.(Intercept)
##      0.02594700      0.03828713      0.02421315      0.02035493
```

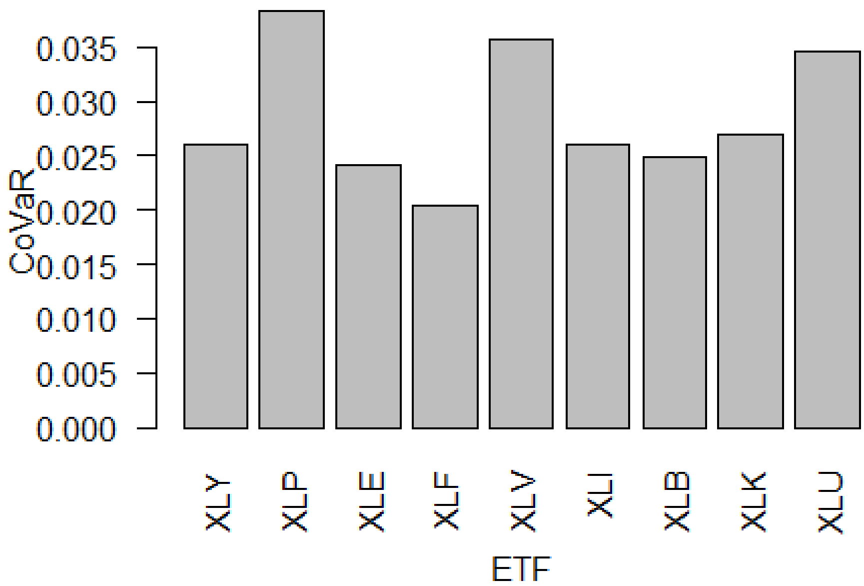
```
0.03569757
## XLI.(Intercept) XLB.(Intercept) XLK.(Intercept) XLU.(Intercept)
##       0.02604914       0.02482716       0.02700476       0.03467057

names(covar_results)<-gsub("\\.\\(Intercept\\)", "", names(covar_resul
ts))
 names(covar_results2)<-gsub("\\.\\(Intercept\\)", "", names(covar_res
ults2))
barplot(covar_results, xlab = "ETF", ylab = "CoVaR",las = 2)
```

Graph 2.3 CoVaR barplot 2018-2023




```
barplot(covar_results2, xlab = "ETF", ylab = "CoVaR",las = 2)
```

Graph 2.4 CoVaR Barplot 2006-2011



These graphs (Graph 2.3 and Graph 2.4) offer a comparison of sector-specific systemic risk contributions over two distinct periods, encompassing different market conditions.
In both periods, certain sectors, such as Consumer Staples (XLP) and Health Care (XLV), show relatively high CoVaR values, indicating their significant contributions to systemic risk. Consumer Staples and Utilities (XLU) consistently impact systemic risk despite their generally defensive nature, reflecting their consistent role in market stability.
However, the number of sectors included differs, with the first plot including 11 sectors by adding Communication Services (XLC) and Real Estate (XLRE), which are absent in the second plot. This inclusion reflects changes in sector categorization and market focus over time. Additionally, the CoVaR values in the second period (2006-2011) are generally higher compared to the 2018-2023 period, likely reflecting the heightened systemic risk during the global financial crisis compared to the more recent period.
The Energy sector (XLE) shows a moderate CoVaR value in the first plot, while it appears less prominent in the second, indicating changes in the sector's relative risk contribution over time. Notably, the Financials sector (XLF) has a higher CoVaR value in the 2006-2011 period, reflecting the financial crisis impact, whereas it shows a more moderate risk contribution in the 2018-2023 period.

Overall, these plots highlight the evolving nature of systemic risk across different sectors over time. The higher CoVaR values during the 2006-2011 period underscore the significant impact of the financial crisis, particularly on the Financials sector. In contrast, the more recent period shows a broader distribution of risk across various sectors, reflecting a more diversified risk landscape. This comparison underscores the importance of historical context in understanding sector-specific risk contributions.

2.5 DeltaCoVaR

In the subsequent section the delta_covar and delta_covar2, calculate a metric called DeltaCoVaR for different sector ETFs in relation to the SPY ETF. DeltaCoVaR measures the difference between the Conditional Value at Risk (CoVaR) at the 95th percentile and the median Value at Risk (VaR) of the SPY ETF, conditional on the sector ETF returns being at their respective quantiles.

A quantile regression is performed at the 95th percentile, fitting the regression line to the upper 5% quantile of the SPY returns. The coefficients, beta_0 (intercept) and beta_1 (slope), are extracted from the regression output.

The DeltaCoVaR is computed as (x_95 - x_50) * beta_1, which represents the difference between the CoVaR at the 95th percentile and the median VaR of the SPY ETF, conditional on the sector ETF returns being at their respective quantiles. The calculated Delta CoVaR value is returned by the function.The delta_covar2 function follows a similar structure but is applied to the old_returns data, which corresponds to the 2006-2011 time period. This function allows the calculation of DeltaCoVaR for the same sector ETFs during the global financial crisis period. By calculating the DeltaCoVaR for each sector ETF, the analysis can assess the systemic risk contributions of different sectors and their impact on the broader market during different time periods, specifically focusing on the difference between the extreme risk scenario and the median risk scenario.

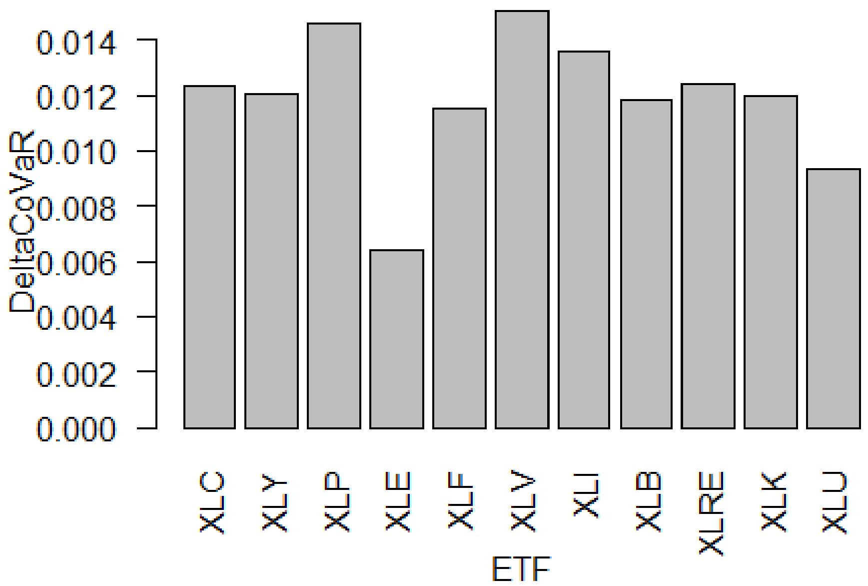
```
delta_covar <- function(etf) {
  qr <- rq(SPY ~ etf, data = returns, tau = 0.95)
  beta_0 <- coef(qr)[1]
  beta_1 <- coef(qr)[2]
  x_95 <- quantile(returns$SPY, 0.95)
  x_50 <- quantile(returns$SPY, 0.50)
  delta <- (x_95 - x_50) * beta_1
  return(delta)
}
```

```
delta_covar2 <- function(etf) {
  qr <- rq(SPY ~ etf, data = old_returns, tau = 0.95)
  beta_0 <- coef(qr)[1]
  beta_1 <- coef(qr)[2]
  x_95 <- quantile(old_returns$SPY, 0.95)
  x_50 <- quantile(old_returns$SPY, 0.50)
  delta <- (x_95 - x_50) * beta_1
  return(delta)
}
```

The Delta CoVaR values provide insights into the systemic risk contributions of different sectors to the broader market, specifically focusing on the difference between the extreme risk scenario and the median risk scenario. Higher Delta CoVaR values indicate a greater potential for risk spillover from a particular sector to the overall market, as represented by the SPY ETF, during extreme market conditions.

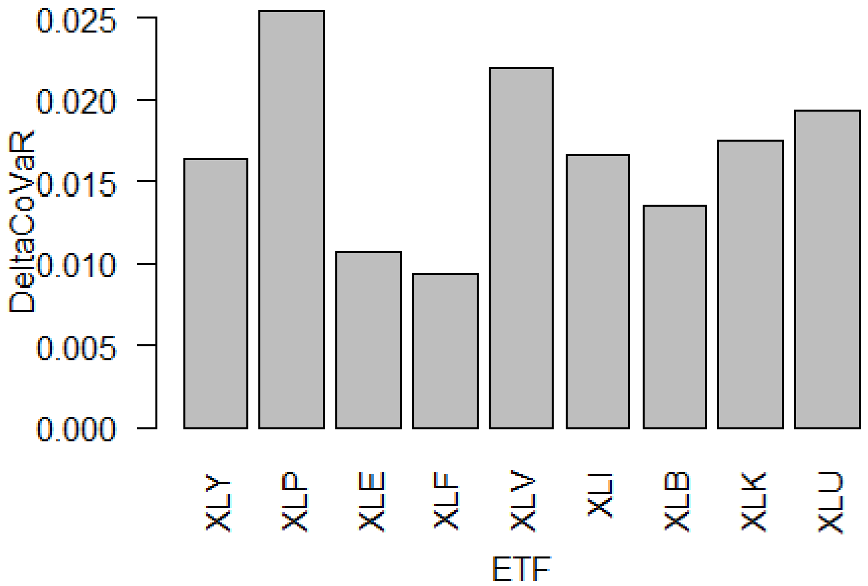
```
delta_results <- sapply(returns[,1:11], delta_covar)
delta_results

##    XLC.95%     XLY.95%     XLP.95%     XLE.95%     XLF.95%     XLV
.95%
## 0.012311279 0.012046367 0.014605446 0.006412929 0.011520509 0.01502
1775
##    XLI.95%     XLB.95%    XLRE.95%     XLK.95%     XLU.95%
## 0.013600348 0.011833472 0.012368448 0.011949523 0.009315378

delta_results2 <- sapply(old_returns[,1:9], delta_covar2)
delta_results2

##    XLY.95%     XLP.95%     XLE.95%     XLF.95%     XLV.95%     XLI
.95%
## 0.016348069 0.025371781 0.010709719 0.009349254 0.021898976 0.01665
1470
##    XLB.95%     XLK.95%     XLU.95%
## 0.013488303 0.017514361 0.019328864

names(delta_results)<-gsub("\\.95%", "", names(delta_results))
names(delta_results2)<-gsub("\\.95%", "", names(delta_results2))
barplot(delta_results, xlab= "ETF",ylab= "DeltaCoVaR",las= 2)
```

Graph 2.5 DeltaCoVaR barplot 2018-2023




```r
barplot(delta_results2, xlab= "ETF",ylab= "DeltaCoVaR",las= 2)
```

Graph 2.6 DeltaCoVaR barplot 2006-2011



The previous graphs (2.5 and 2.6) compare the systemic risk contributions of different sectors over two distinct periods, reflecting changes in market conditions and sector dynamics.In both periods, the Consumer Staples (XLP) and Health Care (XLV) sectors exhibit relatively high DeltaCoVaR values, indicating their significant contributions to systemic risk. This consistency suggests these sectors have a stable and substantial impact on the overall market, regardless of the broader economic environment.
However, there are notable differences between the two periods. The 2006-2011 graph shows generally higher DeltaCoVaR values, particularly for the Financials (XLF) sector, reflecting the heightened systemic risk during the global financial crisis.
The Energy sector (XLE) shows a moderate DeltaCoVaR value in both periods, but its relative position varies, highlighting changes in its systemic risk contribution over time.
Overall, these plots illustrate the evolving nature of systemic risk across different sectors.

2.6 Risk measures comparison

2.6.1 VaR against CoVaR

In the following chunks, the var_10 vector, which contains the 10-day VaR values for each sector ETF, is combined with the covar_results vector using the cbind function. The -12 is used to exclude the 12th element of var_10, which corresponds to the SPY ETF. The resulting matrix is then converted into a data frame using as.data.frame. So, a new column named tics is added to the data frame df1. The row names of df1 are assigned to this column using rownames(df1). This column likely represents the ticker symbols or names of the sector ETFs.

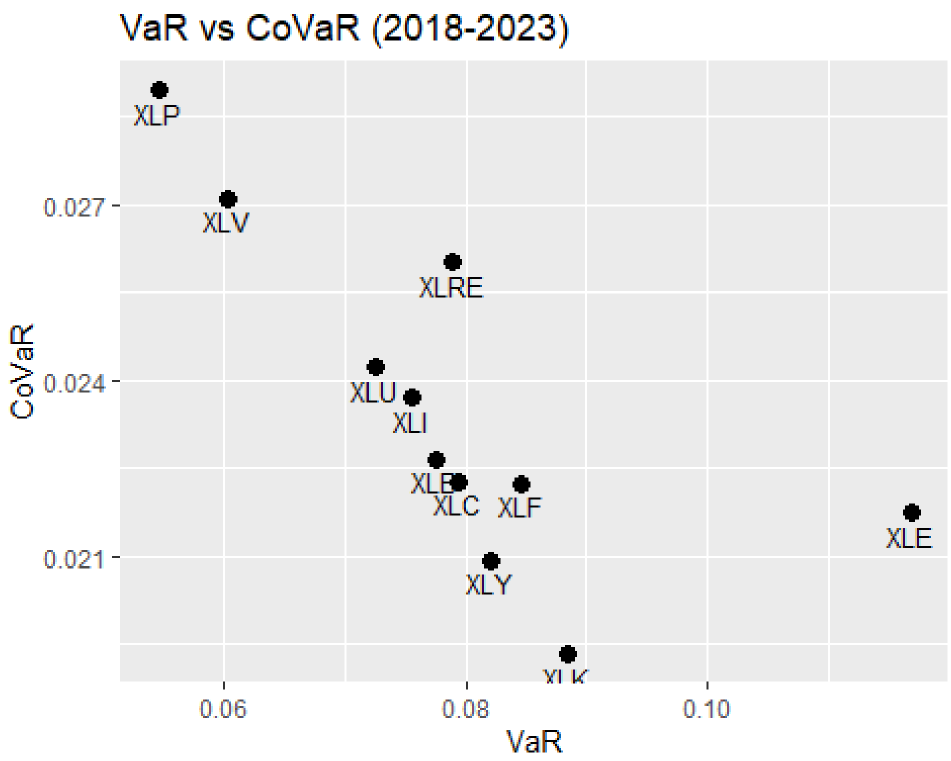
```
df1 <- cbind(var_10[-12], covar_results)
df1 <- as.data.frame(df1)
df1$tics <- rownames(df1)
```

Then we are going to see how the bond between VaR and CoVaR changes through the years. Each data point on the graph corresponds to a specific sector ETF, labeled with its ticker symbol. The graph allows for several observations and insights regarding the risk dynamics across sectors.

```
ggplot(df1, aes(x = V1, y = covar_results)) +
  geom_point(size = 3) +
  geom_text(aes(label = tics), vjust = 1.5, hjust = 0.5, size = 3.5, c
heck_overlap = TRUE) +
  labs(x = 'VaR',
```

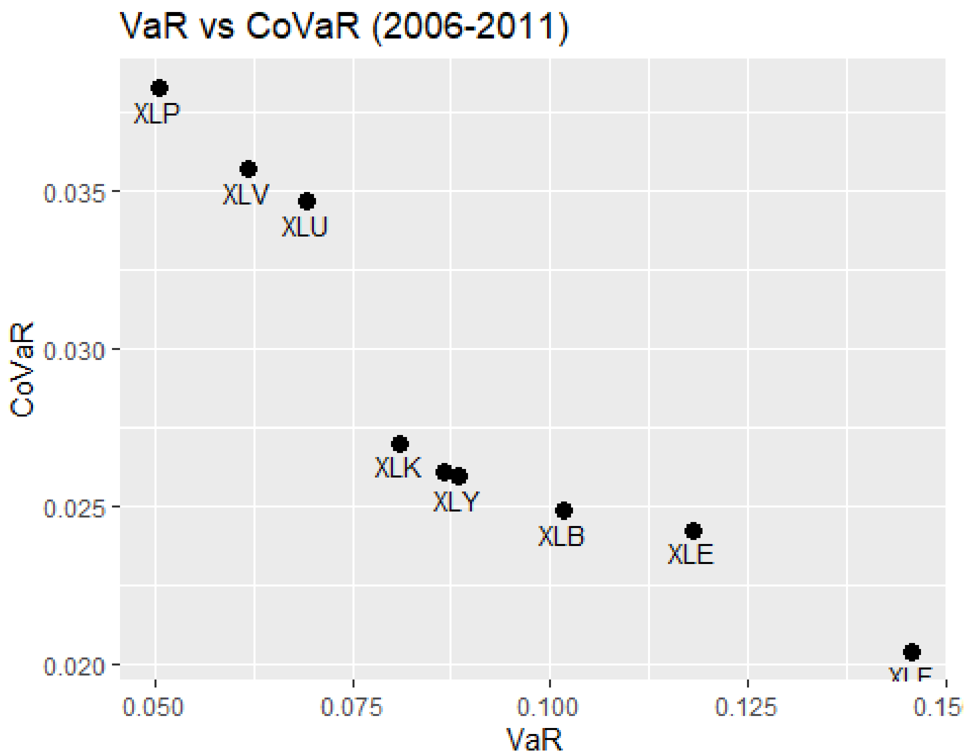
```
        y = 'CoVaR',
        title = 'VaR vs CoVaR (2018-2023)')
```

Graph 2.7




```
df2 <- cbind(old_var10[-10], covar_results2)
df2 <- as.data.frame(df2)
df2$tics <- rownames(df2)

ggplot(df2, aes(x = V1, y = covar_results2)) +
geom_point(size = 3) +
geom_text(aes(label = tics), vjust = 1.5, hjust = 0.5, size = 3.5, che
ck_overlap = TRUE) +
labs(x = 'VaR',
        y = 'CoVaR',
        title = 'VaR vs CoVaR (2006-2011)')
```

Graph 2.8



Both graphs (2.7 and 2.8) plot VaR on the x-axis and CoVaR on the y-axis, highlighting how individual sector risks (VaR) correlate with their systemic risk contributions (CoVaR).

In the 2018-2023 graph, Consumer Staples (XLP) and Health Care (XLV) have high CoVaR values despite moderate VaR, suggesting substantial systemic impact. The Energy sector (XLE), with its high VaR but lower CoVaR, shows high individual risk but less systemic impact.

The 2006-2011 graph, during the financial crisis, shows higher CoVaR values even for low VaR values. The Financials sector (XLF) stands out with the highest VaR and significant CoVaR, illustrating its central role in the crisis. The relationship between VaR and CoVaR is more pronounced, with sectors like Consumer Staples (XLP) and Utilities (XLU) showing higher CoVaR, emphasizing their systemic importance during downturns.

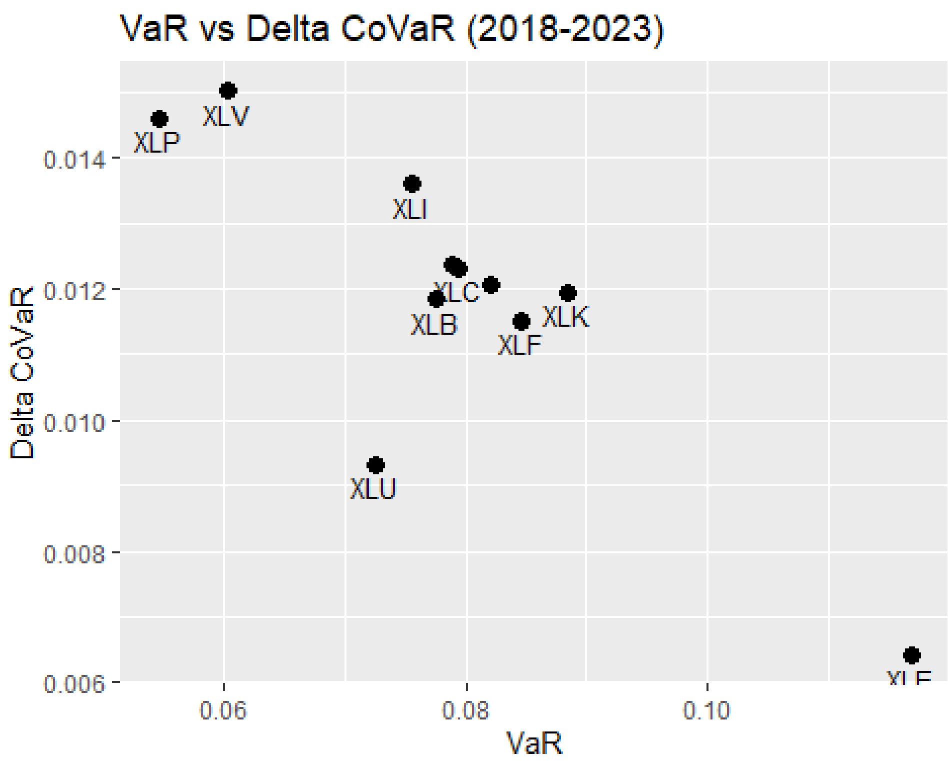
In summary, these graphs illustrate the evolving nature of sector-specific and systemic risks. The financial crisis of 2006-2011 exhibited higher risks, especially in Financials, while the 2018-2023 period shows a more diversified and slightly less intense risk landscape.

2.6.2 VaR against DeltaCoVaR

Proceeding as previously did for our VaR vs CoVaR analysis, we get the resulting data frames, df3 and df4, containing the VaR and Delta CoVaR values for the sector ETFs, along with their corresponding ticker symbols or names.

```
df3 <- cbind(var_10[-12], delta_results)
df3 <- as.data.frame(df3)
df3$tics <- rownames(df3)

ggplot(df3, aes(x = V1, y = delta_results)) +
geom_point(size = 3) +
geom_text(aes(label = tics), vjust = 1.5, hjust = 0.5, size = 3.5, che
ck_overlap = TRUE) +
  labs(x = 'VaR',
       y = 'Delta CoVaR',
       title = 'VaR vs Delta CoVaR (2018-2023)')
```

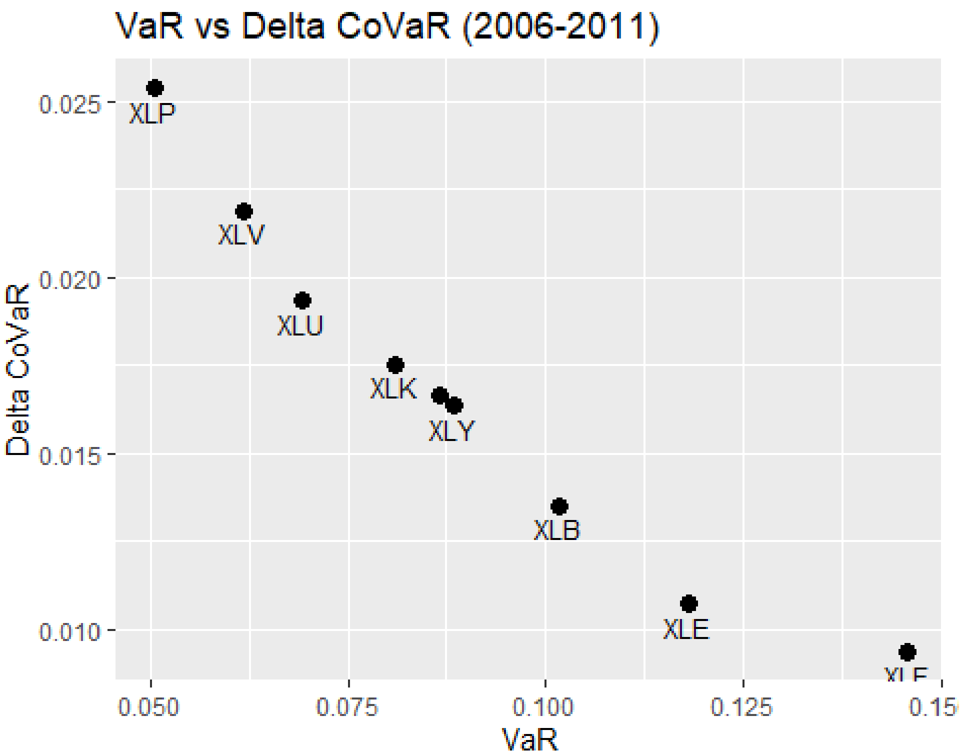
Graph 2.9




```
df4 <- cbind(old_var10[-10], delta_results2)
df4 <- as.data.frame(df4)
df4$tics <- rownames(df4)

ggplot(df4, aes(x = V1, y = delta_results2)) +
geom_point(size = 3) +
geom_text(aes(label = tics), vjust = 1.5, hjust = 0.5, size = 3.5, che
ck_overlap = TRUE) +
```

```
labs(x = 'VaR',
     y = 'Delta CoVaR',
     title = 'VaR vs Delta CoVaR (2006-2011)')
```

Graph 2.10



The absence of a straightforward correlation between VaR and Delta CoVaR can be attributed to a number of factors. VaR captures the individual risk of a sector, whereas Delta CoVaR gauges the interplay between the risk of a sector and the financial system as a whole. During periods of stress, some sectors may exhibit a high VaR but remain relatively isolated from the financial system, thereby reducing their contribution to systemic risk.

Furthermore, the structure of the market and the interconnections between sectors play a pivotal role in determining systemic risk. Consequently, the relationship between VaR and Delta CoVaR is more intricate and nonlinear.

In conclusion, the graphs demonstrate that individual risk and contribution to systemic risk are two distinct concepts, and that the relationship between them is not necessarily direct. This highlights the necessity of considering both measures for a comprehensive evaluation of financial risk.

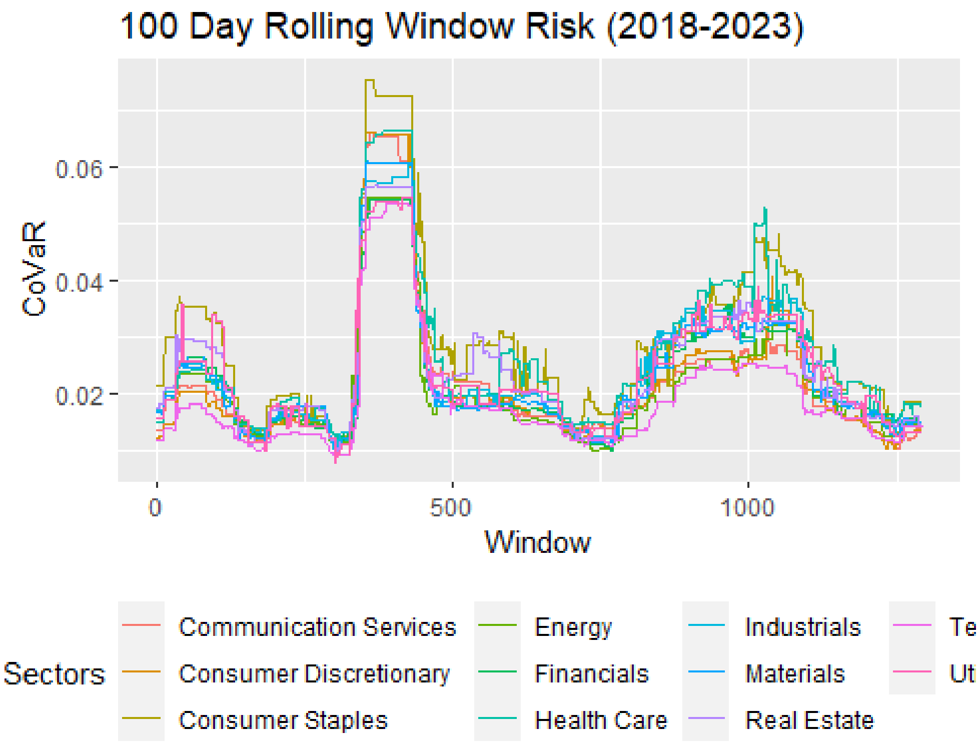
2.7 CoVaR rolling window

In the subsequent section of this paper, it was determined that the analyses would be conducted exclusively on the period between 2018 and 2023 for reasons of convenience. This decision is intended to enhance the usability of the work and reduce the volume of duplicate analyses, while maintaining a high level of methodological rigor.
The subsequent analysis concerns the construction of a moving window of CoVaR, an advanced methodology used to assess the change in systemic risk over time.
Instead of calculating CoVaR over the entirety of the analysis period, the period is divided into smaller time windows for each of which a CoVaR value is obtained. This approach allows for the observation of how systemic risk and interdependence across sectors change over time, or the rendering of a more detailed and dynamic view of market conditions.
The use of the CoVaR rolling window in our analysis is particularly advantageous for the capture of temporal changes in risk, particularly during periods of economic turbulence or financial crises. This method enables the identification of specific time intervals when systemic risk increases or decreases, thus facilitating a more profound comprehension of risk dynamics across sectors.

```
rolling_regression <- function(data) {
  rq_result <- rq(data[, 1] ~ data[, 2], tau = 0.95)
  beta_0 <- coef(rq_result)[1]
  beta_1 <- coef(rq_result)[2]
  x_95 <- quantile(data[, 1], 0.95)
  covar <- beta_0 + beta_1 * x_95
  return(covar)
}

xlc_r <- rollapply(cbind(returns$SPY, returns$XLC), width = 100,
                             FUN = rolling_regression, by.column = FAL
SE, align = "right")
xly_r <- rollapply(cbind(returns$SPY, returns$XLY), width = 100,
                             FUN = rolling_regression, by.column = FAL
SE, align = "right")
xlp_r <- rollapply(cbind(returns$SPY, returns$XLP), width = 100,
                             FUN = rolling_regression, by.column = FAL
SE, align = "right")
xle_r <- rollapply(cbind(returns$SPY, returns$XLE), width = 100,
                             FUN = rolling_regression, by.column = FAL
```

```
SE, align = "right")
xlf_r <- rollapply(cbind(returns$SPY, returns$XLF), width = 100,
                             FUN = rolling_regression, by.column = FAL
SE, align = "right")
xlv_r <- rollapply(cbind(returns$SPY, returns$XLV), width = 100,
                             FUN = rolling_regression, by.column = FAL
SE, align = "right")
xli_r <- rollapply(cbind(returns$SPY, returns$XLI), width = 100,
                             FUN = rolling_regression, by.column = FAL
SE, align = "right")
xlb_r <- rollapply(cbind(returns$SPY, returns$XLB), width = 100,
                             FUN = rolling_regression, by.column = FAL
SE, align = "right")
xlre_r <- rollapply(cbind(returns$SPY, returns$XLRE), width = 100,
                             FUN = rolling_regression, by.column = FAL
SE, align = "right")
xlk_r <- rollapply(cbind(returns$SPY, returns$XLK), width = 100,
                             FUN = rolling_regression, by.column = FAL
SE, align = "right")
xlu_r <- rollapply(cbind(returns$SPY, returns$XLU), width = 100,
                             FUN = rolling_regression, by.column = FAL
SE, align = "right")

covar_roll <- cbind(xlc_r, xly_r, xlp_r, xle_r, xlf_r, xlv_r, xli_r, x
lb_r, xlre_r, xlk_r, xlu_r)
covar_roll <- na.omit(covar_roll)
colnames(covar_roll) <- c("XLC", "XLY", "XLP", "XLE", "XLF", "XLV", "X
LI", "XLB", "XLRE", "XLK", "XLU")
covar_roll$window <- c(1:1293)

ggplot(covar_roll, aes(x = window)) +
  geom_line(aes(y = XLC, color = "Communication Services")) +
  geom_line(aes(y = XLY, color = "Consumer Discretionary")) +
  geom_line(aes(y = XLP, color = "Consumer Staples")) +
  geom_line(aes(y = XLE, color = "Energy")) +
  geom_line(aes(y = XLF, color = "Financials")) +
  geom_line(aes(y = XLV, color = "Health Care")) +
  geom_line(aes(y = XLI, color = "Industrials")) +
  geom_line(aes(y = XLB, color = "Materials")) +
  geom_line(aes(y = XLRE, color = "Real Estate")) +
  geom_line(aes(y = XLK, color = "Technology")) +
  geom_line(aes(y = XLU, color = "Utilities")) +
  labs(x = "Window",
       y = "CoVaR",
       title = "100 Day Rolling Window Risk (2018-2023)",
       color = "Sectors") +
  theme(legend.position = 'bottom')
```

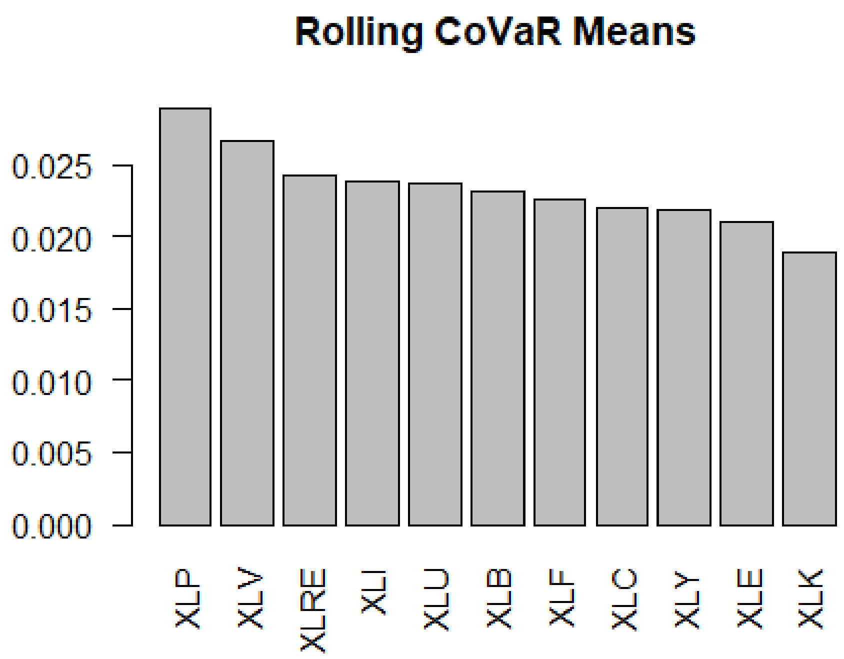
Plot 2.7



Plot 2.7 illustrates the 100-day rolling window risk across various sectors over a five-year period, from 2018 to 2023. It shows that there is a general upward trend in risk across all sectors over time. The Consumer Discretionary sector exhibits the highest level of risk consistently throughout the period, while the Utilities sector consistently demonstrates the lowest level of risk. It is important to note that without additional context, it is challenging to determine the precise factors contributing to the observed increase in risk. However, potential explanations may involve market volatility dynamics, economic uncertainties prevailing during the period, fluctuations in interest rates, and sector-specific influences. To gain a more comprehensive understanding of the data, it would be beneficial to have additional information about the source of the graph and the underlying data.

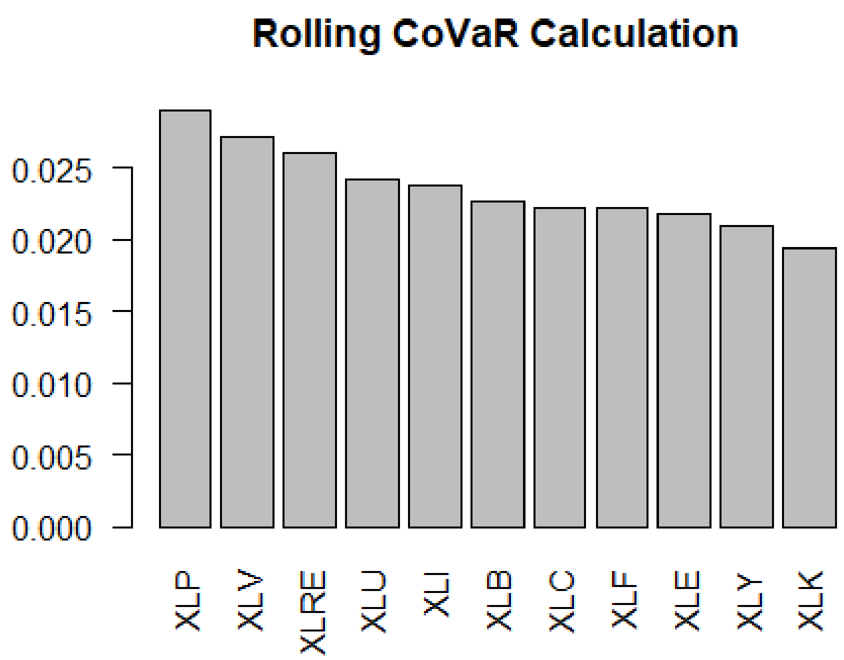
```
covar_means <- apply(covar_roll, 2, mean)
barplot(sort(covar_means[-12], decreasing = TRUE), las= 2, main = 'Rol
ling CoVaR Means')
```

Graph 2.11




```r
barplot(sort(covar_results, decreasing = TRUE), las = 2, main = 'Rolli
ng CoVaR Calculation')
```

Graph 2.12



The rolling window CoVaR barplots show the variability of systemic risk contributions across different sectors over the 100-day periods. This dynamic view helps identify periods of heightened risk and sector-specific volatility. For instance, Consumer Staples (XLP) consistently shows higher CoVaR values, indicating its substantial impact on systemic risk during various time frames. The rolling nature of this analysis captures the fluctuations in risk, offering insights into how sector-specific risks evolve in response to market conditions.

Graph 2.11, showing the mean CoVaR values, complements this by providing a summary measure of each sector's average contribution to systemic risk over the entire period analyzed. It confirms the observations from the rolling window analysis, with Consumer Staples (XLP) and Health Care (XLV) having the highest mean CoVaR values, indicating their significant and consistent impact on systemic risk. This aggregated view helps to verify the robustness of the rolling window results, ensuring that the observed patterns are not due to short-term anomalies but reflect sustained trends.

2.8 Optimized Portfolio

2.8.1 Optimized portfolio construction

The construction of a diversified portfolio using exchange-traded funds (ETFs) represents a fundamental strategy for the mitigation of sector-specific risk, as has been clearly demonstrated by our previous analyses. It has been demonstrated that various sectors exhibit distinct and specific risks, which significantly impact overall market stability and performance.

The objective of creating a multi-sector portfolio is to distribute risk across different economic segments, thereby reducing the negative effects that potential sector-specific shocks could have.

The portfolio optimization is conducted using the ROI method with the objective of maximizing the Sharpe ratio. This entails achieving an optimal balance between expected return and risk, as measured by the standard deviation of returns. To ensure disciplined risk management and effective diversification, comprehensive investment constraints have been imposed, including the prohibition of short positions and specific allocation limits for each asset.

A comparison of the optimized portfolio with SPY, an ETF that tracks the S&P 500, is essential for evaluating the efficacy of our investment strategy. SPY serves as a well-established and widely used benchmark, reflecting the overall performance of the U.S. stock market. This comparison is not merely academic; it provides practical insights into how our diversified portfolio performs in terms of risk-return relative to a passive strategy that invests across the entire market.

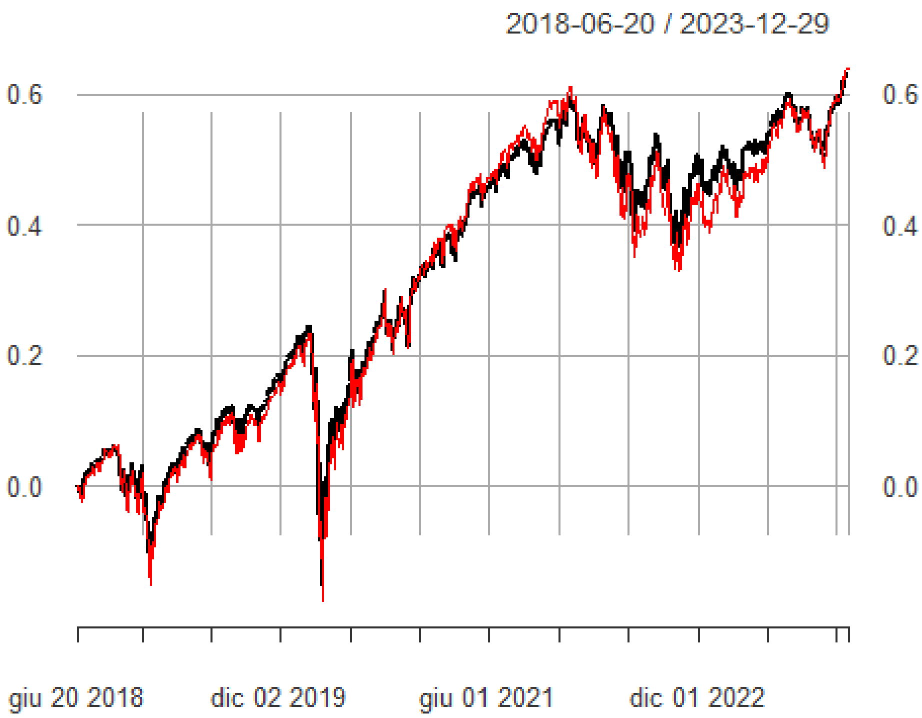
```
assets <- c("XLC", "XLY", "XLP", "XLE", "XLF", "XLV", "XLI", "XLB", "X
LRE", "XLK", "XLU")
portfolio_r <- returns[, -12]

portfolio <- portfolio.spec(assets = assets)
portfolio <- add.constraint(portfolio, type = 'full_investment')
portfolio <- add.constraint(portfolio, type = "long_only")
portfolio <- add.constraint(portfolio, type = "box", min = 0.05, max =
0.25)

portfolio <- add.objective(portfolio, type="return", name="mean")
portfolio <- add.objective(portfolio, type="risk", name="StdDev")

port.opt <- optimize.portfolio(R = portfolio_r, portfolio = portfolio,
optimize_method = "ROI", maxSR = TRUE, trace = TRUE)

weights <- pluck(.x = port.opt, 'weights')
weights_df<- data.frame(weights)
```

	weights
XLC	0.0500000
XLY	0.0500000
XLP	0.1864416
XLE	0.0500000
XLF	0.0500000
XLV	0.1635584
XLI	0.0500000
XLB	0.0500000
XLRE	0.0500000
XLK	0.2500000

The resulting optimized portfolio is 25% in XLK, 18.64% in XLP, 16.35% in XLV, and the minimum 5% weight in each other ETF.

```
port_r <- Return.portfolio(R = portfolio_r, weights = weights, geometr
ic = FALSE)

plot(cumsum(port_r), main = '')
lines(cumsum(returns$SPY), col = 'red')
```

Plot 2.8 Optimized Portfolio vs SPY(red)




```
cbind(mean(port_r), sd(port_r))

##                 [,1]       [,2]
## [1,] 0.0004521695 0.01242133

cbind(mean(returns$SPY), sd(returns$SPY))

##                 [,1]       [,2]
## [1,] 0.0004579017 0.01306532
```

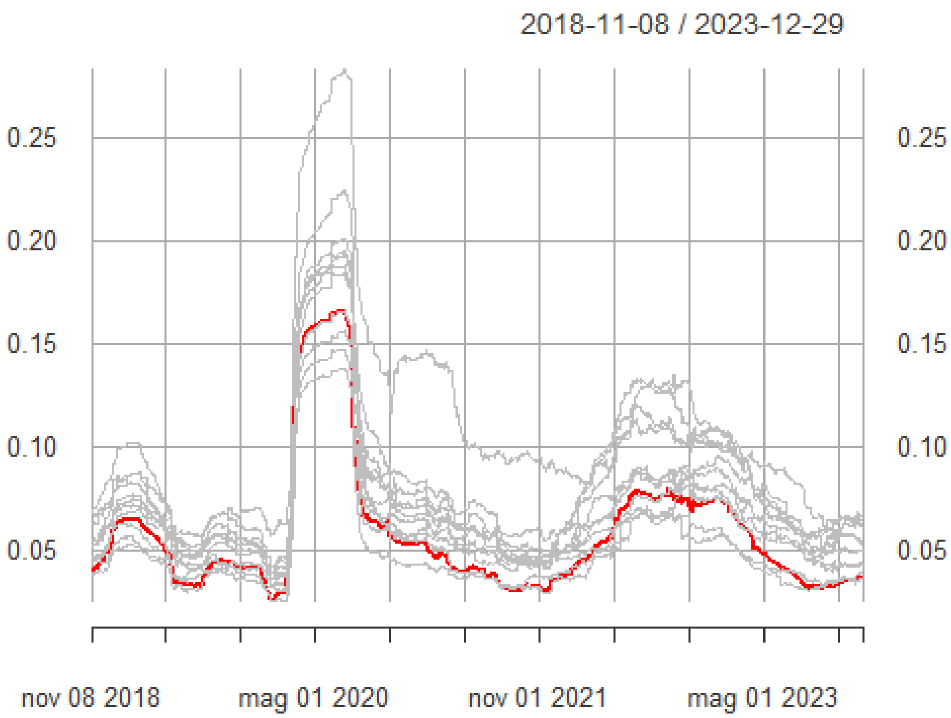
Looking at the portfolio performance, there is marginal improvement over the SPY. This is to be expected since the portfolio consists of ETFs that are all components of the SPY. The portfolio has a slightly lower return but also lower risk which results in a better Sharpe ratio.

2.8.2 Optimized Portfolio rolling CoVaR

```
port_roll <- rollapply(port_r, width = 100, FUN = function(x) va_r(x))
port_roll <- na.omit(port_roll)
port_roll <- abs(port_roll)

com_var <- cbind(roll, port_roll)

port_roll <- rollapply(port_r, width = 100, FUN = function(x) va_r(x))
 port_roll <- na.omit(port_roll)
 port_roll <- abs(port_roll)
```

```r
com_var <- cbind(roll, port_roll)

plot_with_lines_var<-function(){
 plot(com_var$portfolio.returns,col= "red" ,lwd= 2, main = "")
 lines(com_var$XLC,col= "grey" )
 lines(com_var$XLY,col= "grey" )
 lines(com_var$XLP,col="grey" )
 lines(com_var$XLE,col= "grey" )
 lines(com_var$XLF,col= "grey" )
 lines(com_var$XLV,col= "grey")
 lines(com_var$XLI,col= "grey")
 lines(com_var$XLB,col= "grey")
 lines(com_var$XLRE, col= "grey" )
 lines(com_var$XLK,col= "grey")
 lines(com_var$XLU,col= "grey")
 }
 plot_with_lines_var()
```

Plot 2.9 Optimized portfolio rolling VaR



The portfolio's VaR consistently stays below most individual sectors, particularly during periods of high volatility, such as the COVID-19 pandemic in early 2020.

While all sector VaRs spiked significantly, the portfolio's VaR, though increased, did not reach the same extreme levels, indicating a more resilient risk profile.

Post-pandemic, the VaR for all sectors, including the optimized portfolio, gradually declines. The portfolio maintains a lower and more stable VaR compared to individual sectors, suggesting effective risk mitigation through diversification.

This analysis confirms the success of the portfolio optimization.

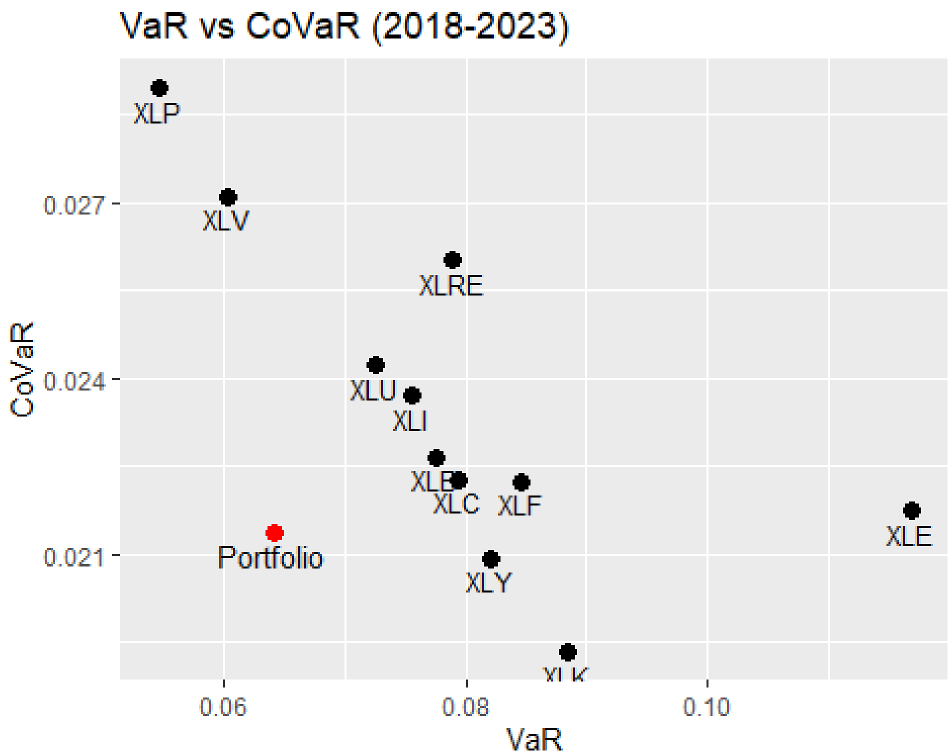
The imposed constraints, including long-only positions and specific allocation limits, contribute to a balanced risk distribution. The diversified portfolio demonstrates lower and more stable VaR than the higher and more volatile VaRs of individual sector ETFs.

Overall, the graph clearly shows that the diversified portfolio manages risk better and is more stable compared to single sector ETFs, validating the strategic approach to portfolio construction and highlighting the importance of diversification in reducing sector-specific risks.

2.8.3 Optimized portfolio VaR vs CoVaR

```
ggplot(df1, aes(x = V1, y = covar_results)) +
  geom_point(size = 3) +
  geom_point(aes(x = 0.06415, y = 0.021356), color = 'red', size = 3)
+ geom_text(label = df1$tics, vjust = 1.5, hjust = 0.5, size = 3.5, ch
eck_overlap = TRUE) +
  annotate('text', x = .064, y = .021, label = 'Portfolio') +
  labs(x = 'VaR',
       y = 'CoVaR',
       title = 'VaR vs CoVaR (2018-2023)')
```

Graph 2.13 Optimized portfolio VaR against CoVaR



Graph 2.13 displays the diversified portfolio represented by the red dot, while the sector ETFs are marked with black dots.

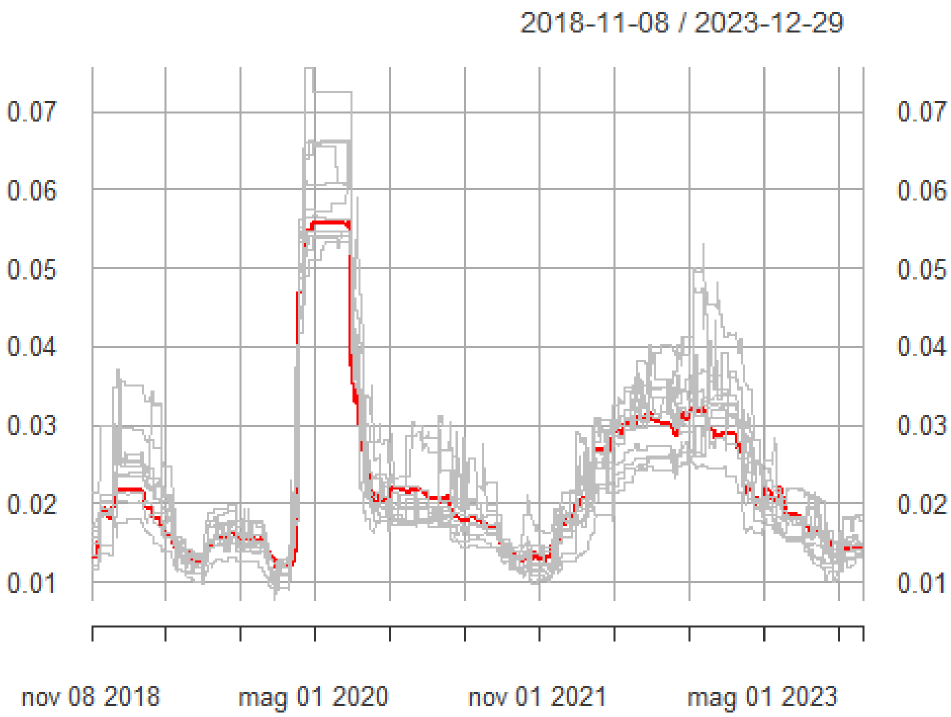
We can notice that the position of the portfolio, with a lower VaR and CoVaR compared to the sector ETFs, indicates a more favorable risk profile. Specifically, the portfolio's VaR and CoVaR are both lower than those of sectors like Consumer Staples (XLP) and Health Care (XLV), which exhibit higher values in both metrics. This suggests that the diversified portfolio effectively mitigates risk while also minimizing its contribution to systemic risk.

2.8.4 Optimized Portfolio rolling CoVaR

```
port_covar <- rollapply(cbind(returns$SPY, port_r), width = 100,
                        FUN = rolling_regression, by.column = FALSE, a
lign = "right")

port_covar <- na.omit(port_covar)
com_covar <- cbind(covar_roll, port_covar)

 plot_with_lines<-function(){
 plot(com_covar$port_covar, col= "red" ,lwd= 2, main = "")
 lines(com_covar$XLC,col= "grey" )
 lines(com_covar$XLY,col= "grey" )
 lines(com_covar$XLP,col= "grey" )
```

```
lines(com_covar$XLE,col= "grey" )
lines(com_covar$XLF,col= "grey" )
lines(com_covar$XLV,col= "grey" )
lines(com_covar$XLI,col= "grey" )
lines(com_covar$XLB,col= "grey" )
lines(com_covar$XLRE, col= "grey" )
lines(com_covar$XLK,col= "grey" )
lines(com_covar$XLU,col= "grey" )
}
plot_with_lines()
```

Plot 2.10 Optimized portfolio rolling CoVaR



As illustrated in Plot 2.9, once more, with regard to the rolling window of CoVaR, it can be observed from Plot 2.10 that the portfolio has benefited from the diversification of its constituents.

In addition to the improvement in portfolio-specific risk, there has been a concomitant reduction in systemic risk. This has enabled the portfolio to perform better than many sector ETFs in the SPY.

2.9 Copula

In financial risk management, traditional measures like Value at Risk (VaR) and Conditional Value at Risk (CoVaR) provide valuable insights into potential losses and systemic risk. However, these measures, often based on linear models, have limitations in capturing the complex dependencies between financial assets. This is where copula models come into play.

Copula models offer a sophisticated approach to understanding and modeling the dependencies between different financial instruments. Unlike linear models, copulas allow for the modeling of non-linear and asymmetric relationships, which are prevalent in financial markets. By using copulas, we can better capture the tail dependencies and extreme co-movements between assets, which are crucial for accurate risk assessment and management.

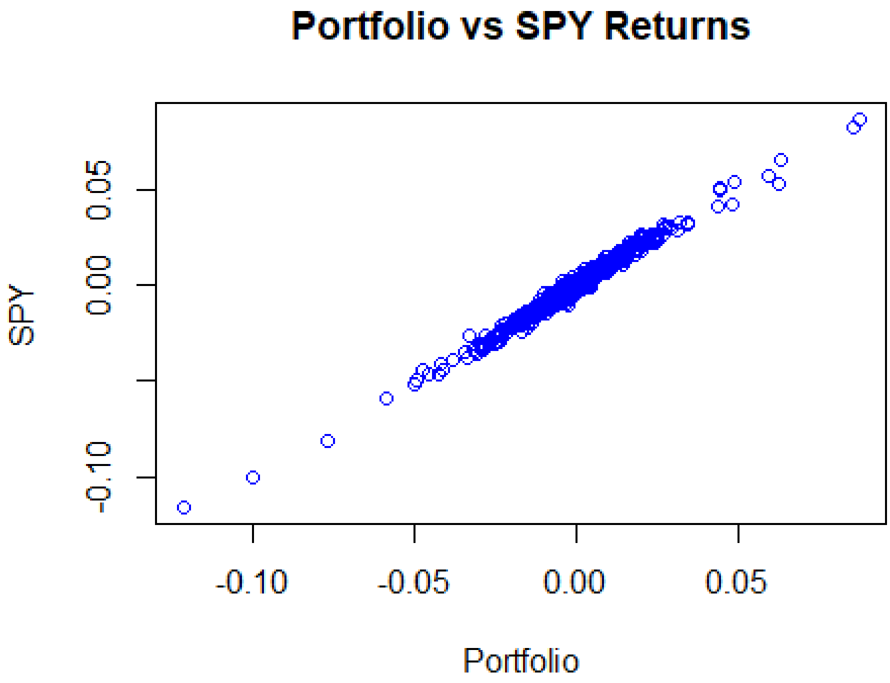
One of the significant advantages of copula models is their ability to separate the marginal distributions of individual assets from their dependency structure. This flexibility enables us to model the unique behavior of each asset while simultaneously capturing the complex interdependencies. As a result, copula models can provide a more comprehensive and accurate picture of joint risk, especially during periods of market stress when traditional models may fail.

Incorporating copula models into our analysis enhances our ability to understand the joint behavior of multiple assets and improve the robustness of our risk management strategies. By capturing the full range of dependencies, including those in the tails of the distributions, copulas offer a more nuanced and realistic assessment of risk compared to VaR, CoVaR, and other linear models. This improved risk modeling can lead to better-informed investment decisions, more effective portfolio diversification, and enhanced financial stability.

In this chapter, we will implement a Gaussian copula for our analysis to assess how well it fits our data, and understand how copulas can be integrated with traditional risk measures to provide a more holistic approach to risk analysis. Specifically,. By leveraging the strengths of copula models, we aim to enhance our understanding of financial dependencies and improve the accuracy and effectiveness of our risk management practices.

```
cop_data <- cbind(port_r, returns$SPY)
cop_data <- data.matrix(cop_data)
cop_data_p <- pobs(cop_data)

plot(cop_data[,1], cop_data[,2], main = 'Portfolio vs SPY Returns', xl
ab = 'Portfolio', ylab = 'SPY', col = 'blue')
```

Plot 2.11 Portfolio returns against SPY returns



First, we combine the portfolio returns (port_r) and the SPY returns (returns$SPY) into a single data frame (`cop_data`).

This data frame is then converted into a matrix to facilitate further statistical processing. The "pobs" function is applied to the matrix to obtain pseudo-observations, which are essentially the ranks of the data transformed into uniform [0,1] values. This step is crucial for preparing the data for copula modeling, which will be discussed later in this chapter.

The resulting plot displays the raw returns of the portfolio against the SPY returns, with the portfolio returns on the x-axis and the SPY returns on the y-axis.The scatter plot reveals a strong linear relationship between the portfolio returns and the SPY returns, suggesting a high degree of correlation. This is statistically significant as it indicates that the portfolio is closely tracking the performance of the SPY, which is a broad market index.

The linear trend in the scatter plot highlights that the portfolio construction follow market movements closely, which is expected for a diversified portfolio optimized for risk-return balance. However, it is also important to note the dispersion of the points around the trend line, especially during periods of extreme returns. Plot 2.11 shows a few outliers where both the portfolio and SPY returns exhibit substantial negative returns, reflecting market downturns. Similarly, there are instances of positive outliers, indicating periods of strong market performance.

This visualization serves as a preliminary step in our analysis, providing a clear indication of the dependency structure between the portfolio and the SPY.

Now, we perform a comprehensive analysis of the portfolio and SPY returns, calculating key statistical measures and fitting a copula model to understand their dependency structure. Initially, the code computes the mean, standard deviation, rate (mean divided by standard deviation), and shape (mean squared divided by standard deviation) for both the portfolio and SPY returns. These measures provide a foundational understanding of the returns distribution for both datasets.

The mean and standard deviation for the portfolio returns are calculated, followed by the computation of the portfolio's rate and shape. Similarly, the same metrics are calculated for the SPY returns. These descriptive statistics are critical in understanding the basic properties of the return distributions. For instance, the portfolio's mean and standard deviation provide insights into its average performance and volatility, respectively. The rate and shape metrics further enhance our understanding by normalizing the returns and providing a sense of the risk-adjusted performance.

We then fit a normal copula model to the pseudo-observations derived from the portfolio and SPY returns using maximum likelihood estimation (MLE). The copula model helps capture the dependency structure between the portfolio and SPY returns, going beyond simple linear correlation. The “fitCopula” function is used to fit the copula, and the summary of the fit provides key results.

```
port_mean <- mean(cop_data[,1])
port_sd <- sd(cop_data[,2])
port_rate <- port_mean/port_sd
port_shape <- ((port_mean)^2 ) / port_sd

spy_mean <- mean(cop_data[,2])
spy_sd <- sd(cop_data[,2])
spy_rate <- spy_mean/spy_sd
spy_shape <- ((spy_mean)^2 ) / spy_sd


cop_model <- normalCopula(dim = 2)
cop_fit <- fitCopula(cop_model, cop_data_p, method = 'ml')
summary(cop_fit)

## Call: fitCopula(cop_model, data = cop_data_p, ... = pairlist(method
= "ml"))
```

```
## Fit based on "maximum likelihood" and 1392 2-dimensional observatio
ns.
## Normal copula, dim. d = 2
##        Estimate Std. Error
## rho.1   0.9872          0
## The maximized loglikelihood is 2550
## Optimization converged
## Number of loglikelihood evaluations:
## function gradient
##       20       20
```

The summary indicates that the normal copula model was fit using 1392 two-dimensional observations, with the estimation based on the maximum likelihood method. The key parameter estimated is rho, which stands at 0.9872 with a standard error of 0. This high value indicates a very strong positive correlation between the portfolio and SPY returns. In copula modeling, rho represents the dependency parameter, which quantifies the strength and direction of the relationship between the two variables. The closer it is to 1, the stronger the positive dependence between the variables.Furthermore, the maximized loglikelihood value of 2550 reflects the goodness-of-fit of the copula model. A higher loglikelihood value indicates a better fit of the model to the data. The optimization process converged successfully, suggesting that the model parameters were estimated reliably.

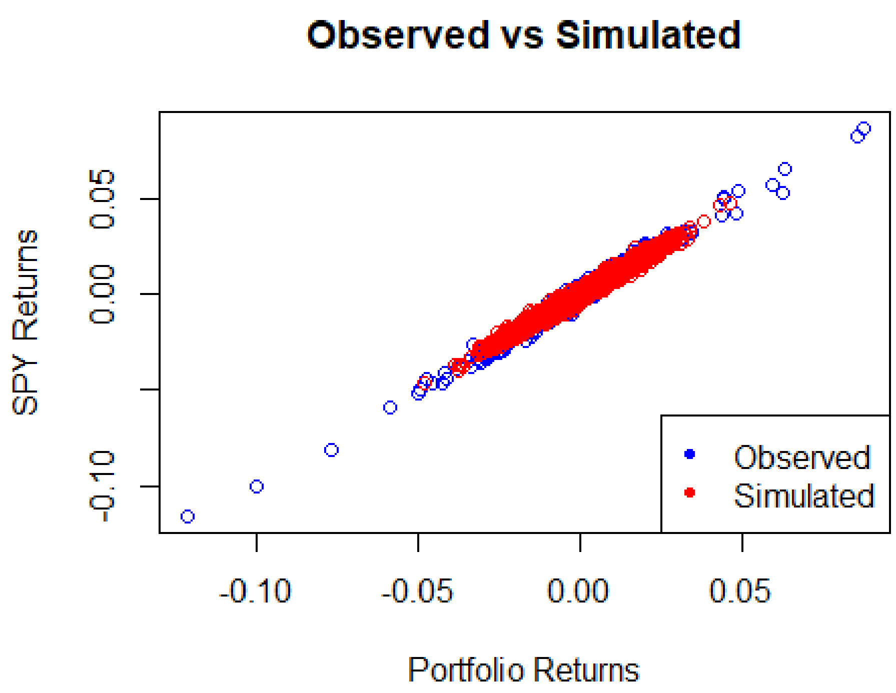
These results are statistically significant as they confirm the strong linear dependence between the portfolio and SPY returns, as initially suggested by the scatter plot. The high rho value underscores the close tracking of the portfolio with the SPY, validating the portfolio's construction strategy. The maximum likelihood estimation provides confidence in the model's robustness, enabling more accurate risk management and portfolio optimization strategies.

The rho coefficient as well as the portfolio and SPY mean and standard deviation will now be used as parameters to further tune the model and obtain a simulation of returns.

```
cop_fit <- fitCopula(normalCopula(dim = 2), cop_data_p, method = 'ml')
coef(cop_fit)

##     rho.1
## 0.9871857

rho <- coef(cop_fit)[1]
df <- coef(cop_fit)[2]

dist <- mvdc(normalCopula(param = 0.9871, dim = 2), margins = c("norm"
,"norm"),
             paramMargins = list(list(mean = port_mean, sd = port_sd),
                                 list(mean = spy_mean, sd = spy_sd)))
set.seed(3)
```

```
sim <- rMvdc(1392, dist)

plot(cop_data[,1], cop_data[,2], col = 'blue',
     main = 'Observed vs Simulated', xlab = 'Portfolio Returns', ylab
= 'SPY Returns')
points(sim[,1], sim[,2], col = 'red')
legend('bottomright', c('Observed', 'Simulated'), col = c('blue', 'red
'), pch = 20)
```

Plot 2.12



The previous code chunk undertakes the task of fitting a normal copula model to the observed returns of the portfolio and the SPY, followed by simulating new data based on the fitted model.

Initially, the "fitCopula" function fits a normal copula to the pseudo-observations of the combined returns data (cop_data_p) using maximum likelihood estimation (MLE).

Using the fitted copula parameter rho, a multivariate distribution construct (`mvdc`) is defined, specifying normal marginals with means and standard deviations calculated earlier for both the portfolio and SPY returns.

The "rMvdc" function then generates 1392 simulated data points from this multivariate distribution.

The resulting plot (Plot 2.12) compares the observed returns (in blue) with the simulated returns (in red), with the portfolio returns on the x-axis and the SPY returns on the y-axis. The close alignment of the red and blue points suggests that the simulated data closely follows the pattern of the observed data, demonstrating the effectiveness of the copula model in capturing the dependency structure.

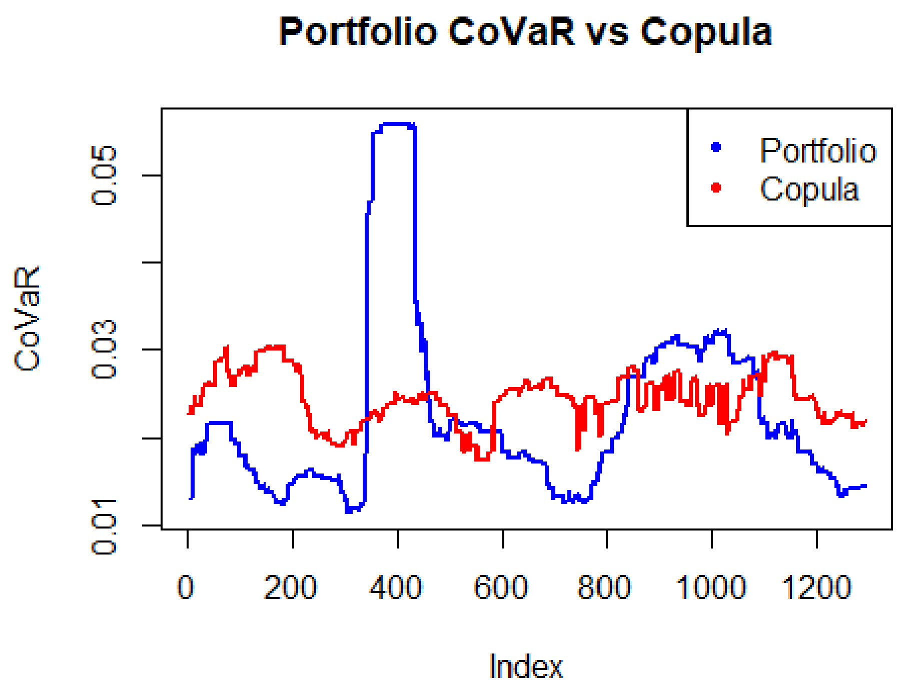
The simulation results reinforce the strong correlations hypothesis by showing that the model accurately replicates the observed data's distribution and dependency structure. The scatter plot exhibits a strong linear relationship, with the simulated points clustering tightly around the observed points, particularly in the center of the distribution. This close fit is crucial as it validates the use of the copula model for understanding and predicting the joint behavior of the portfolio and SPY returns.

The visual comparison between observed and simulated data underscores the robustness of the copula approach in financial modeling, capturing both central tendencies and the extremities of the returns distribution. This modeling technique, by accurately reflecting the underlying dependency, enhances our ability to assess risk and make informed decisions regarding portfolio management. The success of the simulation, indicated by the overlapping data points, suggests that the copula model is a reliable tool for financial analysis, offering a more nuanced understanding of the interdependencies than traditional linear models.

The observed vs. simulated plot visually confirms the model's efficacy, reinforcing the statistical significance of the copula model in capturing and predicting financial market behaviours.

```
cop_covar <- rollapply(cbind(sim[,2], sim[,1]), width = 100,
                       FUN = rolling_regression, by.column = FALSE, al
ign = "right")

plot(as.matrix(port_covar), type = 'l', col = 'blue',
     main = 'Portfolio CoVaR vs Copula', ylab = 'CoVaR', lwd = 2)
lines(cop_covar, type = 'l', col = 'red', lwd = 2)
legend('topright', c('Portfolio', 'Copula'), col = c('blue', 'red'), p
ch = 20)
```

Plot 2.13 Rolling CoVaR for Portfolio and Copula simulated data



The last part of our code performs a rolling regression analysis to compute the Conditional Value at Risk (CoVaR) for both the portfolio and the copula-simulated data, then visualizes the results. The "rollapply" function applies a rolling regression over a window of 100 data points to the simulated returns (sim) to calculate the CoVaR, stored in the variable "cop_covar". The actual portfolio CoVaR, previously calculated, is plotted in blue, while the CoVaR derived from the copula-simulated data is plotted in red.

The resulting plot (Plot 2.13) provides a visual comparison of the CoVaR over time for both the observed portfolio returns and the simulated returns from the copula model. The plot reveals that while the CoVaR values derived from the copula model (in red) generally track the same pattern as the observed portfolio CoVaR (in blue), there are notable differences in their behavior. The portfolio CoVaR exhibits higher peaks and more pronounced fluctuations, especially around the index value of 400, where a significant spike is observed. This suggests that during certain periods, the portfolio experienced more extreme co-movements with the market, leading to higher systemic risk.

In terms of statistical measures, the average CoVaR for the portfolio is 0.02255, while the average CoVaR for the copula-simulated data is slightly higher at 0.02445. This difference, though small, indicates that the copula model tends to slightly overestimate the systemic risk compared to the observed data. This overestimation could be attributed to the copula's ability to capture tail dependencies more effectively, thereby providing a more conservative risk estimate.

```
cbind(mean(port_covar), mean(cop_covar))

##             [,1]       [,2]
## [1,] 0.02255533 0.02445715
```

Conclusions

In conclusion, this project provided key insights into the systemic risk of the SPY. We observed an overall inverse relationship between VaR and CoVaR. Generally, sectors with higher individual risk had lower systematic risk. Specifically, the Consumer Staples and Healthcare sectors exhibited the lowest individual risk and highest systematic risk during the financial crises and COVID-19 periods. Conversely, the Energy sector demonstrated high individual risk and low systematic risk across both time frames. This suggests that Consumer Staples and Healthcare sectors are the most resistant to market shocks, significantly impacting the overall market. In contrast, the Energy sector, despite being highly exposed to market shocks, has a minimal effect on the overall market.

Implementing the Gaussian Copula model proved to be effective in modeling systemic risk, though it did not fully capture periods of extreme risk. The copula model showed a high degree of correlation between portfolio returns and SPY returns, which aligns with the observed data's dependency structure. However, it was less effective in capturing periods of extreme risk, highlighting a limitation of this approach. The copula model's slight overestimation of systemic risk underscores the need for further refinement. Further analysis can involve exploring different types of copula models, such as Vine Copulas, to better capture extreme events and provide a more comprehensive risk assessment. This approach can lead to more informed investment decisions and improved portfolio diversification, ultimately enhancing financial stability.

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