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On The Impact of Labor Market and Financial Frictions within the Smets-Wouters DSGE Model

Supervisor
Ch. Prof. Pietro Dino Enrico Dindo

Graduand
Marco Urbani
Matriculation Number 882444

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Marco Urbani

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Abstract

Dynamic Stochastic General Equilibrium (DSGE) models have gained popularity as a fundamental theoretical framework for comprehensive analysis of business cycle fluctuations. They constitute a workhorse of modern dynamic macroeconomics both for policy makers and academics striving to fine-tune increasingly more sophisticated models which would be able to capture empirical regularities and to solve persistence and co-movements puzzles of aggregate quantities as well. As such, the present thesis lays down a medium-scale DSGE model featuring, essentially, monopolistic competition in both the labor and goods market as the modelling platform to allow for non-trivial interaction among overlapping frictions. A search friction and a hybrid form of rigidity, alongside with other sources of imperfections, are used to endogenize part of the shock structure of the Smets and Wouters model (Smets, F., and Wouters, R., 2003. An Estimated Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association* Vol. 1, No. 5: 1123-1175); a financial friction is then added to create an additional internal propagation mechanism for structural shocks. Moreover, the general model naturally spawns a family of smaller models maintaining just one friction, and controlling for the other sources of imperfections: by removing all the inefficiencies, we further extract a benchmark model displaying full price flexibility and imperfect competition as the sole non-walrasian feature. Models' economies are then calibrated and impulse responses of selected aggregate variables are analysed: simulated samples are compared with each other and, in particular, are used to mimic the dynamics of the benchmark NK model but reducing exogeneity in a substantial way.

Keywords: General Equilibrium, Frictions, DSGE, Business Cycle, Structural Shocks

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1 Introduction: from RBC to DSGE

Since the publication on the seminal paper 'Time to Build and Aggregate Fluctuations' by Finn Kydland and Edward Prescott (1985), the idea that business cycle fluctuations have to be studied within a general equilibrium framework has established as a cornerstone for modern dynamic macroeconomics; alongside the enhancement of general equilibrium analysis, the paper lays down at least two other groundbreaking ideas. Firstly, it states that cyclical fluctuations of economic activity around a deterministic long-run trend are consistent with growth theories and are embedded in the sustained growth process in which the growth rate of aggregate quantities, mainly GDP per capita as a proxy measure for standard of living, is driven by exogenously determined technical change. Secondly, it points out the necessity to go far beyond the qualitative comparison of models to time series data by exploiting calibration tecniques in order to make the model generate dynamic samples for variables of interest, according to the specification of the model itself.

As regards the reconciliation of business cycle theory with growth theory, Prescott and Kydland claim that the fact that, among other variables, employment and output display fluctuations is not a puzzling result and does not prove wrong growth models predicting balanced growth path of per capita quantities as it is in Solow (1956) and Ramsey(1927), at least once some source of stochasticity is introduced in the stantard neo-classical growth model. That is, once the dynamic path of variables has fluctuated far away from the stationary long-run equlibrium, then it is supposed to catch it again, as the internal propagation mechanism of the stationary shock exhausts.

As regards the calibration tecniques, the paper is concerned with matching unconditional second moments of data by extracting individual and behavioral parameters of optimizing agents both from microeconometric literature and from model-implied steady-state relationships; the fit turns out to be remarkably good despite the simplicity of the model. Actually, the main findings of real business-cycle theorists (for instance, the percistency of most aggregate variables, pro-ciciclicality of most macro variables except for real wages, high volatilty of investment relative to the volatility of output, low volatility of consumption with respect to the volatility of output) are almost accepted as stylized facts by academics.

What authors do in their seminal work is augmenting a neoclssical growth model in two directions. First thing, they posit a utility function which is increasing and concave with respect to both consumption and leisure in order to make the model produce employment dynamics; secodndly, the CRS Cobb-Douglas production function embodies a technological shock as a source of fluctutations. The resulting model is stark in its semplicity and simultaneosly powerful in its capability of matching unconditional second moments of data.

A clear implication of the capability of the model of matching the main characteristics of US time series data is that it could in principle serve as a policy instrument: that is, it articulates an artificial economy to be used to study the effects of both optimal fiscal and monetary policy [1], to run counterfactuals and to assess the effect on the overall empirical performance of the model when it is twisted in its baseline specification.

In spite of its remarkable simplicity, the baseline RBC model is affected by serveral shortcomings. Two weak-

[1]Truthfully, baseline RBC model does not provide any meaningful insight on monetary policy as clearing (fully-flexible) prices of goods and factors are fully informative of money supply, which turns out to be neutral; monetary innovations are suddenly absorbed.

nesses are of the order of modelling choises, a couple of others undermines its empirical predictive power.
As regards modelling choices, first real business-cycle theories undeniably attributes the greatest part of the deviation of several variables from their long-run equilibrium to technological shock as modelled by specifying a stochastic process for the aggregate productivity. While there is general consensus among economic theorists that producivity, along side demographic forces and capital accumulation, drives long-run growth and the general improvement of average standards of living as well, the argument might be invalid for the fluctuations over a short-run horizon. Arguably, the notion of productivity shock embodies the idea of technological regress in case of a negative shock, which is not consistent with the fact that aggregate productivity is the main driver of sustained growth over a long-run horizon. Secondly, aggregate productivity is measured by TFP which in Solow growth accounting exercise (1957) is the part of per capita output growth which is not explained neither by physical capital growth nor by labor force growth. That is, it explains so called Solow residual : as it is pointed out by Gali (1996), Solow residual is just a measure of our uncertainty about driving forces of GDP growth net of the share of output growth explained by capital growth and the fraction explained by labor force growth.
As far as empirical predictive power is concerned, then it is worth stressing that RBC models match time series data by assuming a labor supply elasticity with respect to real wages that is higher than the values estimaded by microeconometric literature; secondly it fails to produce observed weak cross-correlation between wages and employment.

As a direct consequence, the work presented in this thesis moves from recognizing the methodological impact of RBC research program. Real business cycle model does not lead to a standard specification on how to model aggregate fluctuations, as it was believed to do by its proponents; rather, it pins down a full methodological revolution which has deeply changed the way we model short-run macroeconomic phenomena, and, mainly, business cycle fluctuations. Its main advantage is twofold.
Since the contribution of RBC literature is methodological ex post, we now stress methodological advantages associated with this theortical framework, as it informs recent formulations such as DSGE models.
First of all, it provides a self-contained and fully articulated theoretical framework in which the economy is modelled from the bottom-up. Since primitives of the model are specified in terms of preferences and technological frontier, then the model allows for welfare analysis as utility losses and gains associated with different policies can be calculated in closed form expressions[2].
Secondly, this theoretical framework is immune to Lucas critique (1976) which stems directly from Keynesian consensus breakdown. Since this way of modelling allows for atomistic agents forming expectation rationally conditional on complete information sets about the current state of the economy, then aggregate relationships among variables are derived from first principles. Therefore, they can be used to do policy analysis as they does not incur in the risk of being obsolete once the anticipated policy intervention is introduced, as structural macroeconometric relations of old keynesian models do because of the fact that the representative agent adjust its, say, consumption behavior with respect to the upcoming, for instance, tax-increase policy; this indeed happens due to the fact that agents

[2]Again, this does not happens in the baseline RBC model as competitive equilibrium is Pareto optimal and any policy intervention is found to be inherently distortionary as it deviates from the planner's problem equilibrium allocation

are supposed to form rational expectation so that their own knowledge about the current structure of the economy overlaps the model-implied structure, with no forecast error whatsoever.
Given such methodology, real business cycle models have been extended in a wide array of directions to include additional sources of stochasticity and imperfections.
As regards exogenous stochastic disturbances, money neutrality implied by the baseline specification has been attacched by introducing nominal rigidity and inertial behavior of prices; therefore monetary shocks came into the fore as one of the major sources of business cycle fluctuations alongside TFP shocks.
As regards real imperfections, the ratio beyond introduction of them relies in fine-tuning empirical performance of this family of models and make it able to produce impule responses comparable to VAR-implied impulses. Just to cite some of them, habit formation, adjustment costs, variable utilization of production factors, and so on.
The new synthesis of this ambitious research programme has lead to the construction of the first DSGE models, which encompass elements from the real business-cycle tradition, such as productivity shocks, and non-walrasian elements from the keynesian traditions, such as nominal rigidities, monetary shocks, real imperfections and monopolistic competition.

The present thesis enters precisely at this point. It starts out by negleting methodological critique carried out against rational expectation general equilibrium approach, though it has spawn florishing strands of research [3]. We do not make this choice because we think that what falls outiside the domain of rational expectation framework is doomed to fail, but because we focus on a particular research issue that has to be addressed within the domain of rational expectation DSGE models.
In particular, this class of models is based upon a quite complex structure containing multiple sectors even in their baseline formulation (household, firms, labor union, aggregator sector). Each sector is hit by one or more exogeneous shock which creates short-run volatility in the time series generated by the model. This actually means that fluctuations remains unexplained. What is inside the model in terms of rigidities, imperfections and other non-walrasian features contributes to amplify and propagate the effect of the arrival of shocks, but the shocks theirselves just hit randomly the dynamic system pushing it away from its steady state; how far away is determined by both the magnitude of the stochastic innovations of the shocks and by the internal propagation mechanism of the model.
The present thesis is aimed at reducing the exogenous shock structure of a benchmark DSGE model, namely the Smets-Wouters DSGE model, which features ten structural shocks. This is done by magnifying the internal propagation mechanisms of both the productivity shock and monetary shocks as the unique sources of business-cycle fluctuations through the introduction of frictions in the labor market, in the capital market and in the goods market. That is, the labor market friction is a search friction affecting the wage determination. The capital market friction takes the form of a financial accellerator mechanism. The goods market friction gives rise to an alternative price setting behavior of firms with another form of rigidity, namely a sticky information rigidity. Further, the alternate introduction of frictions spawns a battery of models that twist the SW baseline specification by adding

[3]See ABM macroeconomics, for instance; rational expectation and forward looking processes are substituted with backward looking ones. For a compendium, see Delli Gatti (2005).

up just one friction at a time. The introduction of frictions reduces the exogeneity of the benchmark NK model without altering its business cycle properties and the characteristics of the SW-implied time series. This naturally comes at a cost of a reduction of analytical tractability of the model.

Therefore, the present thesis is organized as follows. Section 2 presents the literature related to DSGE models in general and to the number of frictions we embed into the SW model. Section 3 provide a full analytical exposition of the Smets-Wouters model, by even focusing on its weaknesses and possible solutions to them. Section 4 is devoted to the introduction of the hybrid pricing friction in the goods market. Section 5 presents the search friction and the bilateral wage setting device. Section 6 focuses on the financial accellerator mechanism. Section 7 provides an overview on models we are dealing with to have a snap on how household sector, labor arket, goods arket and capital market are structured in each specification. Setion 8 is devoted to the comparison of the various model through the analysis of unconditional second moments and impulse response analysis. Section 9 concludes.

2 Related Literature

The present thesis relies on a grat bulk of work carried out in relation to the analysis of business-cycle fluctuations and business-cycle empirical regularities as the main drivers. Since causes of cyclical fluctuations of economic activity have been indentified in a wide variety of sources, each source as an object of analysis has spawned a specific field of literature. Provided that the present thesis lays down a full DSGE model augmented to include a number of interacting and contemporaneous frictions, and that DSGE itself as a modelling tool for studying business cycle fluctuations and policy implications of cyclical swings of economic activity synthetizes the preceding business-cycle related strands of research, we think it is worth highlighting the roots of the model we build.

2.1 DSGE

Our benchmark model is the Smets and Wouters DSGE model, which alongside a number of related similar models such as Christiano, Eichenbaum, and Evans (2005) and Blanchard and Gali, (2007) is the workhorse of modern dynamic representative agent rational expectation macroeconomics. It displays a sophisticated shock structure encompassing both supply-side exogenous shocks, especially productivity shock, and demand-side shocks such as monetary shocks.

The idea that cyclical fluctuations of aggregate quantities are caused mostly by productivity shocks roots back to the work of Prescott/Kydland(1985, 1986) and Hansen (1985). Once they had applied a stationarity-inducing transformation to time series data in order disentangle their trend component from the cyclical one with the so-called Hoddrick-Prescott filter, they substantially augment deterministic growth model with exogenous technical change by including a two period utility function in both consumption and leiure and specifiying a stochastic exogenous stationary process for the aggregate productivity. After calibrating the model, they noticed that simulated series were able to mimic the dynamics empirical post-war time series dynamics for US data and that model-implied unconditional second moments were close to empircial unconditional second moments of some variable of interest. For an overview of the early stage real business cycle research program, see King, Plosser, and Rebelo (1988), and

Rebelo (2005).

While real business cycle tradition attributes the great part of oscillations of economic activity around the deterministic trend to productivity shifts, the idea has been questioned by New Keynesian tradition, stressing instead the relative importance of nominal imperfections such as incomplete nominal adjustments of prices and wages to monetary innovations as well as adjustment costs in shaping inertial behavior of nominal variables. For a review, see Mankiw (2001); Ball, Mankiw and Romer (1988); Mankiw (1990); Ball and Mankiw (1994) and Gali (2018). There exist essentially two ways of modelling incomplete nominal adjustments, according to the source of rigidity. Both the ways are featured in the model we arrive to through the obliged step of SW.

Firstly, an important source of money non-neutrality lies in the wage stickness as in Taylor (1980) and Fischer (1977): they basically point out that staggered re-negotiation of wage contracts is not synchronous with monetary policy decisions, especially in low inflation periods; since wages are changed less frequently than the money stock is changed thorugh the nominal interest rate as the policy instrument, then this staggering is able to affect short-run behavior of output. With respect to the estabilished tradition, SW model does not endogenize wage staggering by assuming the existence of finite-horizon contracts as it would cast additional complexity into the general equilibrium analysis, but just assumes that wages are periodically updated according to the arrival of a random signal.

Further, money non-neutrality is even ensured by inertial behavior of prices. Even though there are plenty of ways to model price stickness, the fact the representative firm takes pricing decision calls for monopolistic competition as it is laid down by Dixit/Stiglitz (1977) and Blanchard and Kiyotaki (1988). Moreover, the introduction of monopolistic competition naturally embeds in the general equilibrium model a multi-layer structure which makes it relatively easy to add up multiple frictions. Once the monopolistic competition is estabilished as the modelling platform, then price stickness might results from near-rational behavior of agents as in Akerlof and Yellen (1985): since the losses from non-optimizing behavior for the agent are second order relatively to the monetary innovation, then the agents might prefer to be stuck in a sub-optimal behavior which does not significantly affect their welfare but which displays significant impact on the short-run behavior of aggregate output. What is more, price stickness might even results from the presence of so-called menu costs as in Mankiw (1985) and Ball (1990): given a proper degree of real rigidity in price setting behavior of firms or even in th labor market, firms might not find it incentive compatible to pay up the otherwise neglectible 'menu cost' to adjust nominal price; while the individual loss of so doing is second order, it can induce first-order welfare losses. Finally, it is worth stressing that nominal price stickness can even derive from the presence of quadratic adjustment costs as in Rotemberg (1982): he builds up a monopolistic model in which the representative firm chooses its price optimally, subject to a market demand for partially substitable goods and a costly price adjustment constraint capturing the degree of real rigidity; the actual difference with near-rational model is given by the fact that in the Rotemberg model the representative firm behaves optimally; the real difference with repect to menu-cost models lies in the fact that the rational monopolist in the Rotemberg model has not an exogenous menu cost used to evaluate welfare effect of incomplete adjustment, but it has a well-behaved cost function to cast in the constrained maximization problem. Again, SW model sums up a large amount of literature on price stickness by specifying a stochastic process for the arrival of an exogenous signal on a mass of good-producing firms which allows them to re-optimize prices; the prices of non-reoptimizing firms are consequently mantained at the previous period level as in Calvo (1983).

Therefore, SW model condenses in a tractable manner previous schools of thoughts identifying as the major driver of business-cycle fluctuation of economic activity, respectively, supply-side productivity shock and demand-side monetary shocks as identified by the innovations in the money supply. Further, it adds up a number of real imperfetions to fine tune empirical capability of the model to match VAR-implied impulse responses such as habit formation, variable capital utilization and adjustment costs for investment.

As regards twists in the utility function, habit formation in the consumption process as in Abel (1990) and Boldrin, Christiano and Fisher(2001) is introduced originally to provide an explanation to equity premium puzzle (ie. with external habit formation the model is consistent asset returns facts); though, the introduction of habits is able to produce hump-shaped response of consumption to both monetary and productivity shocks. Actually, marginal utility of current consumption turns out to be dependent on either aggregate or individual lagged consumption according to the type of habit formation introduced in the model, or even on both the types; as such, aggregate consumption equation might generate quite complex dynamics as current consumption depends both on past periods consumption and on expected consumption for future periods.

As regards variable capital utilization, it is introduced in the SW model as a form of factor hoarding, as in Burnside and Eichnbaum (1994) do to enhance internal propagation mechanism of a three-shocks model and improve the capability of the baseline RBC model of matching selected features of time series data.

As regards adjustment costs for investment, it is introduced in the SW model in order to capture physical loss of capital throughout the investment process. In the absence of any phisical adjustment cost, infact, physical capital stock adjusts from one period to the other without virtually any transformation cost as representative agent takes investment decisions such that the resulting level of physical capital is optimal; as in Ayashi (1982), adjustment costs are intoduced to model additional forms of real rigidities related to implicit costs in the investment process.

2.2 Hybrid Rigidity

The approach to model hybrid forms of stickness has been initiated by Gali and Gertler (1999) to test empirical performance of NKPC against a generalized inflation dynamics that nests the Calvo-based NKPC as a particular case; this has been done by assuming some form of heterogeneity in the price setting behavior throughout the mass of intermediate good producers in a baseline DSGE. Further a generalization of the PC relying on a number of pricing regimes in the mass of firms has been proposed by Coibion and Gorodnichenko (2011): again, their specification nests the NKPC as a particular case in which all the firms are subject to Calvo-type incomplete nominal adjustments. In our specification, we just focus on a dual price setting regime: therefore, a fraction of firms are subject to Calvo-rigidities while the remaining mass is subject to sticky information proposed as a form of rigidity in a seminal paper by Mankiw and Reis (2002) in order to account empirical inefficiencies of the NKPC inflation dynamics. Reis (2006) analyze incentive compatibility problem of the agent who rationally choose to be inattentive, aggregates this form of quasi-rational behavior over a mass of firms and analyze the effects of this form of staggered price adjustment on inflation dynamics. For an empirical comparison of 'hybrid' models of inflation see Duport and Kitamura (2010). Trabndt (2007), Klenow (2007) build medium-scale monetary DSGE models to account for different regimes of price setting both in a time-dependent framework and in a state-dependent pricing regime. Even if firms are subject to the staggered arrival of information, they still mantain complete information sets; for the effect of imperfect common knowledge of price process on real variables, see Woodford (2003) which build a general equilibrium model featuring informational friction in term of imperfect information.

2.3 Labor Market Friction

The introduction of labor market frictions in DSGE model has been initiated to enable this family of model to provide a characterization of labor market alternative to the monopolistic labor market in order to account for the observed degree of wage stickness; they are introduced to desrcibe unemployment facts as well. For a first trial to account for the weak correlation between labor supply as measured in hours and wages, see Christiano and Eichenbaum (1992) which introduce a government consumption shock to influcene labor market dynamics. Alternative wage setting devices as a result of a search and matching process have been introduced by Mortensen and Pissarides (1994) and casted into a general equilibrium model by Adolfatto (1996) which replicate real business cycle stylizeed facts by augmenting the neoclassical labor market embedded in stochastic growth models with a search friction; Merz (1995) finds that two-sided search and matching mechanism creates a significant internal propagation mechanism for real business cycle models. For full-fledged general equibrium model featuring search and matching fricitons along side a number of forms of real rigidities, see Trigari (2006), Knell (2014), and Christoffel, Kuester and Linzert (2006).

2.4 Financial Friction

Financial frictions have been added in order to asses whether imperfections in financial market are able to magnify internal propagation mechanisms of exogenous structural shocks in baseline NK models. Basically, there are two main ways to introduce financial frictions in general equilibrium models. The first approach places the friction in the lending channel by casting an agency problem between the lender who have to monitor the borrower, and the borrowing firms which is somehow contrained in its funding capacity; the second approach places the financial friction in the borrowing channel, whether the borrower is the household sector or another financial intermediary, due to information asymmetries.

As regards the first approach, the seminal paper is Bernanke, Gertler and Gilchrst (1999), who build a NK model where the presence of an idiosyncratic disturbance at the entrepreneurial level rises the need for the bank to monitor the borrower; as such, when external funding is needed to finance capital acquisition at the entrepreneurial level, an external finance premium is charged on the safe loan rate to hedge againt the negative shock. For a specification of the BGG-type of friction in a full DSGE model displaying real rigidities, see Christensen and Dib (2008); Merola (2018) casts a BGG friction into the Smets and Wouter model.

As regards the second approach, Gertler and Karadi (2011) assume that the bank, while perfectly monitoring the gross capital return of entrepreneurial sector, is constrained in acquiring assets as it might have an incentive to divert part of their assets to its stakeholders. Gertler and Kiyotaki (2010) adds up to this theoretical framework occasionally binding liquidity constraintsw hich limit bank's ability to obtain funds from depositors.

For alternative approaches, Gerali (2010) builds a model with monopolistic competition in the banking sector as banks are able to set different rates on deposits and loans and are subject to both credit constraints in the lending channel and borrowing constraints. Meh and Moran (2009) develop a general equilibrium model in which investors are not able to monitor directly entrepreneurial sector so they deposit funds to banks and delegate their monitoring task; since banks might not assolve to the monitoring function properly, this give rise to a moral hazard problem. Pietrunti (2017) builds a DSGE model with occasionally binding constraint's on bank capital and study non-linearities associated with the financial frictions.

3 Smets and Wouters Model

The Smets and Wouters model is a full-fledged medium scale DSGE that has estabilished as a workhorse of modern dynamic orthodox (representative agent and rational expectation) macroeconomics. It has been designed and estimated by using full information bayesian tecniques in order to capture cyclical properties of time series data and analyze business cycle empirical regularities as well. It is a dynamic model as it aims to characterize transitional dynamics of a number of aggregate variables and their dynamic response to a variety of shocks hitting the dynamic economic system it fully lays down. Further, it displays the features of a general equilibrium system as the model economy is structured on multiple layers and does not abstract from the interactions of representative agents populating each layer.[4] It is a medium scale model as it provides the representation of a closed economy, meaning that the resource constraint of the entire system does not encompass net exports. Moreover, it is a stochastic model as authors cast in the general structure of the economy multiple sources of exogenous volatility which are able to condition growth trajectories of aggegate quantities. Along the line of Real Business Cycle (RBC) research programme, SW model relies on a notion of stationary equilibrium in which all the aggregate per capita variables exhibit balanced growth dynamics, absent any time trend on the stochastic processes governing the time evolution of these quantities [5]: though, dynamic path of aggregated quantities is perturbed by the arrival of multiple exogenous stationary shocks creating transitional dynamics, as variables returns to their steady state. In this connection, authors create a 'toy' laboratory which, by its own nature, is suitable to identify exogenous sources of business cycle fluctutations, provided that the strict assumption that the economy is stuck on a stationary sub-optimal equilibrium is satisfied[6]; this way, the fact that the time path of a number of control variable is linked to the volatility of innovations of persistent structural shocks creates scope for policy analysis, especially interest rate regimes switching and in general monetary policy issues the model has been primarily deviced to address. In fact, the arrival of exogenous impulses provides a snap of the direction, the magnitude and the persistence of the responses of variables to estimated shocks; since the responses to unobservable shocks pin down the functioning of the entire model economy, it turns out to be in principle possible to mitigate the impact of negative shock by controlling for a bunch of policy variables.

Turning now our attention to the distinctive features of the model economy, Smets and Wouters exploit monopolistic competition as a modeling platform to incorporate two groups of imperfections. Firstly, both the labor and goods

[4] Here the notion of general equilibrium refers purely to the capability of the model of representing a full multisectorial and multi agent economy. In other words, it lacks of a welfare characterizatiom and has not to be traced back to the general equilibrium in a walrasian sense: that is, since the model displays multiple sources of imperfections even in its baseline two-rigidities specification, first welfare theorem does not hold and the competitive equilibrium for the SW economy is not Pareto-efficient.

[5] For a non-stationary version of the Smets and Wouters model, see Smets and Wouters (2007); in the paper they estimate the same model on US data with the real difference that there are time trends on the production side of the economy

[6] The suboptimality of the equilibrium is given by the combination of both nominal and real rigidities

markets exhibit nominal rigidities: the presence of incomplete nominal adjustments constitutes the theoretical ground for the existence of an upward sloping aggregate supply curve and ensures the money non-neutrality in the short run. Obviously, sluggish price movements and persistence af th aggregate price index rules out any form of monetary policy irrelevance, even with rational expectations and perfect foresight of fully rational representative agents. Secondly, both household sector and goods market display some form of real rigidities: namely, consumption process is affected by habit formation in order to create the empirically observed persistence of consumption process; utilization of physical capital is variable and there are quadratic convex adjustment cost of investment decisions. Moreover, the model economy is potentially hit by ten orthogonal gaussian simultaneous shocks: five shocks are related to the primitives of the model, namely preferences and technology (preference shock, labor supply shock, the shock to investment cost function and the government spending shock); there are three 'cost push' shocks driving the price process (wage mark-up shock deriving from monopolistic competition in the labor market, price mark-up shock deriving from monopolistic competition in the good market, a risk premium shock deriving from investment decision of the agent); lastly, two monetary shocks (a shock to the inflation target and a purely monetary shock to nominal interest rate set by the monetary authority).

We now provide a complete exposition of the Smets and Wouters (2003) model in its non linear form. In laying out the details of the full model economy, we follow a 'block' approach: since the model is based on a multi-layer structure, we characterize in turn optimal choices of each layer by mirroring it with the optimal choises of the agent of a hypothetic non-observable flexible economy counterpart, at least for the production and labor market side of the model economy.

3.1 Household

Time is discrete and runs over an infinite horizon[7]. In the model economy there exists a unit mass of households $i \in [0,1]$. Each household enjoys a time endowement which is normalized to 1; time is allocated among leisure, $1-N_t(i)$ which delivers well-being to the household, and labour, $N_t(i)$, which yields disutility instead. Instantaneous felicity depends on consumption, $C_t(i)$ net of disutility deriving from supplying labor services, $1-N_t(i)$.[8]. As such, period utility functional for the household reads as follows:

$$U_t^i(C_t(i), N_t(i)) = u_t^i(C_t(i)) - g^i(N_t(i)) \tag{1}$$

It turns out to be the difference of the utility, u, yielded by the consumption of the unique aggregate consumption good and the disutility, g, the household gets from supplying labor services and dissipating time endowement in non-leisure activities.

Utility function is consistent with the following class of utility functions used in dynamic models:

[7] This is the standard timing assumption adopted in a large family of dynamic models in order to capture some degree of altruism among generation; further, setting the concave optimization problems in an infinite horizon time span greatly simplifies mathematical analysis as it prevents complications associated with finite horizon concave problem and overlapping generations.

[8] As it is common in business cycle literature and in growth theory, standard consumption-based utility is augmented to a two-input function to provide a description of labor market. See King and Rebelo (1999) for a survey of the differences and links between neoclassical growth models and real business cycle literature.

$$\mathcal{U} = \left\{u \in \mathcal{C}^2 \mid u : \mathbb{R}_+ \to \mathbb{R}_+, u'(\cdot) > 0, u''(\cdot) < 0\right\}$$

Utility functions belonging to this set rest on certain restrictions on marginal utility the household derives from consumption: since utility functions is increasing and concave, marginal utility increases in the full domain but at an increasingly lower rate. Similarly, disutility associated with supplying labor hours belongs to the following set:

$$\mathcal{G} = \left\{g \in \mathcal{C}^2 \mid g : \mathbb{R}_+ \to \mathbb{R}_+, g'(\cdot) < 0, g''(\cdot) > 0\right\}$$

In other words, as labor supply increases, household get an increasingly higher disutility from working.

To sum up, for a generic time instance t, representative household consumes and supplies labor intetermporally according to:

$$U^i\left(\{C_\tau(i)\}_{\tau=0}^{\infty}, \{N_\tau(i)\}_{\tau=0}^{\infty}\right) = \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{t+\tau} U^i_{t+\tau}(C_{t+\tau}(i), N_{t+\tau}(i)) \tag{2}$$

where β is the subjective discount rate at which each household discounts exptected stream of upcoming consumption; it is assumed to be the same throughout the unit mass of consumers. Further, $\mathbb{E}_t$ is the rational expectation operator capturing the idea that the household takes consumption and labor rational decisions conditional on a complete information set about the current state of the economy; such decisions are therefore optimal by assumption as cognitive cost of acquiring, processing and interpreting information is assumed to be neglectible and does not create scope for judgement mistakes and forecast errors[9]. Preferences are assumed to be self-regarded, meaning that we can get rid of the i index used both in the utility/disutility functions and in the the arguments of such functions.

To put stucture on the utility functional, we assume $u(C_t(i))$ is a generalized CRRA displaying habit formation, augmented by a shock parameter:

$$u(C_t(i), D_{t-1}(i); \epsilon_t^B) = \frac{\epsilon_t^B}{1-\sigma_c}(C_t(i) - D_{t-1}(i))^{1-\sigma_c} \tag{3}$$

σ_c is the coefficient of relative risk aversion, or the inverse of the intertemporal elasticity of substitution capturing the degree (ie. the curvature of utility function) at which the representative household is willing to substitute current consumption for future consumption in response to changes in the current state of the economy. The utility function displays generalized habit formation:

[9]Rational expectation assumption can be twist in a variety of ways. While there exists just one way to adopt full rationality, plenty of ways are available to be boundedly rational. Among the large amount of behavioral literature on NK models with behavioral foundations, see Gabaix(2020) for modelling myopic behavior by expressing rational expectation in a reduced form, and Milani (2008) for introducing learning process in expectation formation.

$$D_{t-1}(i) = hC_{t-1}(i)^d (C_{t-1}^{ag})^{1-d}$$

with $C_{t-1}(i)$ being the previous period individual consumption of the household, and C_{t-1}^{ag} an aggregate previous period consumption index; h is the scale parameter and d is the elasticity of habit formation with respect to past period individual consumption. Actually, the oscillations of d parameter determines the relative strenght of internal (referred to agents' own previous consumption) habit formation to external (referred to previous aggregate consumption) habit formation. While Smets and Wouters assumes $d = 0$ to pin down a complete external habit formation specification, we assume $d = 1$ to have fully internal habit formation; this creates non trivial dynamics for consumption as it makes marginal utility dependent on both forward and backward lagged consumption. Note that the distinction between internal and external habit formation in the utility function allows to introduce a first distinction between models we are going to use in assessing business cycle properties associated with the introduction of frictions. In other words, if $d = 0$, we are in a Smets-Wouters baseline speficication (sw), while for $d = 1$, we are in Smets-Wouters with internal habit formation specification ($swih$).
ϵ_t^B is an AR(1) shock to consumption related utility:

$$\epsilon_t^B = \rho_B \epsilon_{t-1}^B + \eta_t^B \quad \text{with } \eta_t^B \sim \mathcal{WNN}(0, \sigma_B^2) \tag{4}$$

As regards the disutility of devoting time endowement to non-leisure activities, disutility of labor is a CRRA, again augmented by a shock parameter:

$$g(N_t(i); \epsilon_t^N) = \frac{\chi \epsilon_t^N}{1 + \sigma_N} (N_t(i))^{1+\sigma_N} \tag{5}$$

χ is a scale parameter for the labor supply; σ_t^N is the inverse of the Frisch elasticity of labor supply with respect to real wage.
ϵ_t^N is an AR(1) stationary process:

$$\epsilon_t^N = \rho_N \epsilon_{t-1}^N + \eta_t^N \quad \text{with } \eta_t^N \sim \mathcal{WNN}(0, \sigma_N^2) \tag{6}$$

Decision problem is constrained by the availability of resources. The following equation determines the budget constraint of the household:

$$C_t(i) + I_t(i) + b_t \frac{B_t(i)}{P_t} + \int_{\Omega} q_t(\omega) a_{t,i}(\omega) d\omega \leq \Upsilon_t(i) + \frac{B_{t-1}(i)}{P_t} + \Pi_t(i) - T_t \tag{7}$$

with real total income $\Upsilon_t(i)$ given by

$$\Upsilon_t(i) = w_t(i)N_t(i) + r_t^K u_t(i)\bar{K}_{t-1}(i) - \Psi(u_t(i))\bar{K}_{t-1}(i) + A_t(i) \tag{8}$$

All the variables are expressed in real term, with the aggregate price index given by P_t. Several insights can be extracted by the budget constraint. Household detains its financial wealth in one-period riskless bond with price b_t with gross nominal return of bond $R_t = 1/b_t$. Total real income is additively given by: real labor income $w_t(i)N_t(i)$, $w_t(i)$ being the wage level for the agent i; return on the real utilized capital stock $r_t^K u_t(i)\bar{K}_{t-1}(i)$ net costs associated with a particular degree of utilization $\Psi(u_t(i))\bar{K}_{t-1}(i)$; dividends $\Pi_t(i)$ deriving from owning shares of intermediates good firms which makes profits by exploiting some degree of market power due to monopolistic competition, and net cash flow income $A_t(i)$ deriving from partecipating to a market in which Arrow securities are traded [10]. Representative household owns the stock of physical capital $\bar{K}_t(i)$, utilize it to turn it into installed capital services at rate $u_t(i)$, as variable capital utilization entails quadratic costs $\Psi(u_t(i))$. The law of motion for capital given by

$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + I_t \left[1 - S\left(\frac{I_t}{I_{t-1}}; \epsilon_t^I\right)\right] \tag{9}$$

where δ is the constant time-independent depreciation rate of capital, and S is the phisical adjustment cost funtion for investment. In order to create additional degree of real rigidity, the S function does not depend on the level of investment at period t, but on the change in investment level among two subsequent periods.
ϵ_t^I is an AR(1) shocks to current period investment[11]:

$$\epsilon_t^I = \rho_I \epsilon_{t-1}^I + \eta_t^I \quad \text{with } \eta_t^I \sim \mathcal{WNN}(0, \sigma_I^2) \tag{10}$$

To complete the characterization of unknowns, we assume that the utilization rate is 1 at the steady state, $u = 1$, and that the variable capital utilization cost function is such that $\Psi(1) = 0$, meaning that the imperfection associated with variable capital utilization fades out in the long run; the elasticity of utilization costs with respect to utilizaton rate $\psi = \frac{\Psi''(1)}{\Psi'(1)}$ evaluated at the steady state continues to influence the dynamic properties of the model, as it will become soon clear by linearizing optimality conditions. Similarly, we assume that $S(1) = S'(1) = 0$, meaning that the imperfection related to adjustment costs of investments fades out in the steady state, while $S''(1) = \Delta$ keeps influencing dynamic properties of the model. Just for the sake of completness, functional forms that satisfy such properties are

[10] Implcitely, we assume there exists a complete market for Arrow securities: agents can fully insure against both aggregate and idiosyncratic risk across times and among states of nature $\omega \in \Omega$. This implies that in a competitive equilibrium household takes the same allocative choices on consumption, worked hours, bond holdings and capital. For the effects of incomplete markets within DSGE framework, see Kaplan, Moll, and Violante (2018).

[11] Note that the exogenus shock to investment enters directly the convex cost function; that is, a positive shock to the S function implies a negative shock to investment as it rises physical adjustment costs, and vice versa.

$$\Psi(u_t(i)) = \frac{r^K}{\psi}\left[\exp\left(\psi(u_t(i)-1)\right)-1\right] \tag{11}$$

and

$$S\left(\frac{I_t}{I_{t-1}};\epsilon_t^I\right) = \frac{\Delta}{2}\left[\frac{\epsilon_t^I I_t}{I_{t-1}}-1\right]^2$$

Given the dynamic utility fucntional (2), given the budget constraint (7), given real financial wealth (8) and the law of motion for physical capital (9), representative household controls for C_t, B_t, K_t, I_t, u_t and solve the following dynamic optimization problem

$$\max_{\{C_{t+\tau}(i),B_{t+\tau}(i),K_{t+\tau}(i),I_{t+\tau}(i),u_{t+\tau}(i)\}_{\tau=0}^{\infty}} \quad \mathbb{E}_t\sum_{\tau=0}^{\infty}\beta^{\tau}\left[\frac{\epsilon_{t+\tau}^B}{1-\sigma_c}(C_{t+\tau}-hC_{t+\tau-1})^{1-\sigma_c} + \frac{\chi\epsilon_{t+\tau}^N}{1+\sigma_N}(N_{t+\tau}(i))^{1+\sigma_N}\right] \tag{12}$$

$$\text{s.t.}\quad C_{t+\tau}+I_{t+\tau}+\frac{b_{t+\tau}B_{t+\tau}(i)}{P_{t+\tau}}+\int_{\Omega}q_{t+\tau}a_{t+\tau,i}(\omega)d\omega \leq \Upsilon_{t+\tau}(i)+\frac{B_{t+\tau-1}(i)}{P_{t+\tau}}+\Pi_{t+\tau}-T_{t+\tau} \tag{13}$$

$$\Upsilon_{t+\tau}(i) = w_{t+\tau}(i)N_{t+\tau}(i)+r_{t+\tau}^K u_{t+\tau}(i)K_{t+\tau-1}(i)-\Psi(u_{t+\tau}(i))K_{t+\tau-1}(i)+A_{t+\tau}(i) \tag{14}$$

$$\bar{K}_{t+\tau} = (1-\delta)\bar{K}_{t+\tau-1}+I_{t+\tau}\left[1-S\left(\frac{\epsilon_t^I I_{t+\tau}}{I_{t+\tau-1}}\right)\right] \tag{15}$$

By focusing on two subsequent time periods, namely $\tau = 1$, due to the recursive nature of the dynamic optimization problem, FOCs are:

$$(\partial C_t)\quad (C_t-hC_{t-1})^{-\sigma_c}-\beta h\mathbb{E}_t\left(C_{t+1}-hC_t\right)^{-\sigma_c} = \lambda_t \tag{16}$$

$$(\partial B_t)\quad \beta\mathbb{E}_t\Lambda_{t,t+1}\frac{R_t}{\pi_{t+1}} = 1 \tag{17}$$

$$(\partial \bar{K}_t)\quad Q_t = \beta\mathbb{E}_t\Lambda_{t,t+1}\left[Q_{t+1}(1-\delta)+r_{t+1}^K u_{t+1}-\Psi'(u_t)\right] \tag{18}$$

$$(\partial I_t)\quad Q_t\left[1-S\left(\frac{I_t}{I_{t-1}}\right)-\frac{\epsilon_t^I I_t}{I_{t-1}}S'\left(\frac{I_t}{I_{t-1}}\right)\right]+\beta\mathbb{E}_t Q_{t+1}\Lambda_{t,t+1}\left[\frac{I_{t+1}}{I_t}\frac{\epsilon_{t+1}^I I_{t+1}}{I_t}S'\left(\frac{\epsilon_{t+1}^I I_{t+1}}{I_t}\right)\right] = 1 \tag{19}$$

$$(\partial u_t)\quad r_t^K = \Psi'(u_t) \tag{20}$$

Equation (16) pins down the equality between shadow price of consumption λ_t with marginal utility of consumption augmented to include internal habit formation; note that the h parameter controls for the strenght of internal habit, as long as the condition reduces to a standard CRRA FOC as h approaches 0.

Equation (17) is the Euler equation describing consumption/saving behavior of representative household and it basically determines intertemporal allocation of consumption as a function of the return of investment in the riskless security: as the interest rate R_t increases, the household finds it profitable to save, thus deferring resources from current period consumption to subsequent period consumption. Infact, an increase in the gross nominal interest rate increases marginal utility of current consumption, meaning that current consumption decrease. Similarly, an increase in the interest rate decrease marginal utility of next period consumption, so that the effective next period consumption increases. Moreover, the equation lays down a positive relation between current interest rate and future inflation π_{t+1}.

Equation (18) establish a Q relation for the market value of capital: curennt value of installed unit of physical capital positively depends on discounted expected future value of installed undepreciated capital, gross return on utilized capital net of variable utlization costs.

Equation (19) is a description of the dynamics of investment augmented to include physical adjustment costs. Note that without quadratic convex costs the shadow price of capital would have been equal to the shadow price of consumption [12], just reducing to unity the market value of capital in term of consumption goods, $Q_t = 1$; this actually happens at the steady state due to the assumed properties of the adjustment cost function.

Equation (20) equates rental rate of capital to marginal utilization costs: as rental rate increases it becomes more profitable to utilize physical capital intensively.

3.2 Labor market and wage setting

Labor market in Smets and Wouters model displays monopolistic competition and nominal stickness: monopolistic competition is modelled through the theoretical framework introduced by Dixit and Stiglitz (1977); nominal rigidities reduces to an augmented form of Calvo wage stickness (Erceg, 2000). Therefore, labor market is a two-layers market as it features a labor union aimed at aggregating labor demand by exploiting a CES aggregator technology, and again the household sector which takes aggregate labor demand as given and unilaterally chooses real wages subject to Calvo type incomplete nominal adjustments.

First of of all, we characterize optimal decision problem of the labor packaging sector. Labor supply is aggregated according to the following aggregator technology:

$$N_t = \left(\int_0^1 N_t(i)^{\frac{\gamma_{w,t}}{1+\gamma_{w,t}}} di \right)^{1+\gamma_{w,t}} \tag{21}$$

where $\gamma_{w,t}$ is the time dependent mark-up over real wages which capture some degree of imperfect substitability among labor services provided by households. Moreover the previous equation is the constraint of the optimization problem of the representative labor union:

[12] Lagrange multiplier associated with the budget constraint would have been equal to the Lagrange multplier associated with the law of motion for capital, $\lambda_t = \mu_t^K$

$$\max_{\{N_t, N_t(i)\}} \quad W_t N_t - \int_0^1 w_t(i) N_t(i) \tag{22}$$

The previous static maximization problem yields the following labor demand schedule which is going to constraint the household wage setting problem, subject to Calvo type incomplete nominal adjustment:

$$N_t(i) = \left(\frac{w_t(i)}{W_t}\right)^{-\frac{1+\gamma_{w,t}}{\gamma_{w,t}}} \tag{23}$$

Moreover, we can exploit the zero profit condition of the competitive representative labor union to obtain the aggregate wage equation:

$$W_t = \left(\int_0^1 w_t(i)^{-\frac{1}{\gamma_{w,t}}} di\right)^{-\gamma_{w,t}} \tag{24}$$

We now turn our attention to the monopolistic wage setting decision of the household. It basically takes labor demand as given and chooses wage that maximizes the difference between labor income and the disutility it gets from supplying labor services. In order to introduce nominal rigidity, the unit mass of wage setters household is split in two parts according to the realization of the Poisson process governing the arrival of the Calvo signal: there exists a mass $1-\xi_w$ which receives the Calvo signal and is able to reset its wage; the remaining fraction ξ_w does not receive the signal to reoptimize and set the wage by adopting the following rule-of-thumb behavior:

$$w_t(i) = \pi_{t-1}^{\iota_w} w_{t-1}(i) \tag{25}$$

It means that the not-reoptimizing household indexes real wage to past period inflation, withe the strenght of wage indexation given by the parameter ι_w. Note that this parameter can be interpreted as a degree of real rigidity in the wage process: as ι_w approaches 1, there is complete idexation to past period inflation, while we get full nominal rigidity in the case ι_w approaches 0. It follows that for the household that adjusts wage in t and does not reoptimize up to a generic τ-horizon, we can recursively iterate on the rule of thumb for not-adjusted wage so as to obtain:

$$w_{t+\tau}(i) = \prod_{k=1}^{\tau} \pi_{t+k-1}^{\iota_w} w_t(i)$$

Therefore, given the labor demand (23), representative household adjust wages in order to maximize the expected discounted stream of benefits net of costs associated with supplying labor services. The household optimization problem reads

$$\max_{\{w_t^*(i)\}} \quad \mathbb{E}_t \sum_{\tau=0}^{\infty} (\xi_w \beta)^\tau \left[\lambda_{t+\tau} w_t^*(i) X_\tau^w N_{t+\tau}(i) - \frac{\chi \epsilon_t^N (N_{t+\tau}(i))^{1+\sigma_L}}{1+\sigma_L} \right] \tag{26}$$

$$\text{s.t.} \quad N_{t+\tau}(i) = \left(\frac{w_{t+\tau}(i)}{W_{t+\tau}} \right)^{-\frac{1+\gamma_{w,t+\tau}}{\gamma_{w,t+\tau}}} N_{t+\tau} \tag{27}$$

$$w_{t+\tau}(i) = X_\tau^w w_t^*(i) \tag{28}$$

with

$$X_\tau^w = \begin{cases} 1 \text{ if } \tau{=}0 \\ \prod_{k=1}^{\tau} \pi_{t+k-1}^{\iota_w} \text{ if } \tau{=}1,.. \end{cases}$$

The dynamic problem yieds the following optimality condition for the real wage:

$$\frac{w_t^*}{P_t} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\xi_w \beta)^\tau \left(\prod_{k=0}^{\tau} \frac{\pi_{t+k-1}^{\iota_w}}{\pi_{t+k}} \right) \frac{N_{t+\tau}(i) \lambda_{t+\tau}}{1+\gamma_{w,t}} = \mathbb{E}_t \sum_{\tau=0}^{\infty} (\xi_{w,t} \beta)^\tau N_{t+\tau}(i) \chi \epsilon_{t+\tau}^N N_{t+\tau}^{\sigma_N}(i) \tag{29}$$

with $\gamma_{w,t}$ giving a wage mark-up iid shock(a cost push shock driving inflation process through the channel of marginal costs)

$$\gamma_{w,t} = \eta_t^w, \quad with \quad \eta_t^w \sim (0, \sigma_{\gamma_w}^2) \tag{30}$$

Since the unit mass of household has been splitted in two sub-masses according to the realizations of the Calvo signal, we augment the aggregate wage equation in rder to account for the presence of nominal rigidities in wage setting process. Therefore,

$$W_t = \left[\int_0^{\xi_w} \pi_{t-1}^{\iota_w} w_{t-1}(i)^{-\frac{1}{\gamma_w}} di + \int_{\xi_w}^1 w_t^*(i)^{-\frac{1}{\gamma_w}} di \right]^{-\gamma_w} \tag{31}$$

By focusing on a symmetric equilibrium, we get a relation characterizing the aggregate behavior of real wages:

$$W_t = \left[\xi_w (\pi_{t-1}^{\iota_w} W_{t-1})^{-\frac{1}{\gamma_w}} + (1-\xi_w)(w_t^*)^{-\frac{1}{\gamma_w}} \right]^{-\gamma_w} \tag{32}$$

3.2.1 Wage setting in a flexible economy

Assume $\xi_w = 0$. Without nominal rigidities, household problem is static and consists in maximizing the difference between the actual wage charghed by household and the cost in terms of leisure associated with supplying labor services. The decision problem reads:

$$\max_{\{\tilde{w}_t\}} \quad [\lambda_t \tilde{w}_t(i) - w_t^{disut}(i)]N_t(i) \tag{33}$$

$$\text{s.t} \quad N_t(i) = \left(\frac{\tilde{w}_t(i)}{w_t}\right)^{-\frac{1+\gamma_w}{\gamma_w}} N_t \tag{34}$$

with

$$w_t^{disut} = -\frac{\chi \epsilon_t^N (N_t(i))^{1+\sigma_N}}{1+\sigma_N} \tag{35}$$

derived from the household first order condition with respect to $N_t(i)$. The maximization problem equates the wage to a mark-up over the ratio between marginal leisure cost associated with worked hours and marginal utility of consumption:

$$\tilde{w}_t(i) = (1+\gamma_w)\frac{w_t^{disut}(i)}{\lambda_t} \tag{36}$$

3.3 Goods market and price setting

Likewise the labor market, Smets and Wouters model displays a monopolistic goods market and incomplete nominal adjustment. In order to account for monopolistic competition, authors introduce a two-layers structure: there is a representative final good producers which aims to package intermediate good output by using CES aggregator and provides intermediate good demand to firms; further, there is a unit mass $j \in [0,1]$ of intermediate good producers which takes labor demand as given and sets price subject to Calvo rigidities.

We begin by characterizing opimal choice of final good producers. First of all, we specify the following aggregation technology for the intermediate good output:

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{1}{1+\gamma_{p,t}}}\right)^{1+\gamma_{p,t}} \tag{37}$$

where $\gamma_{p,t}$ is the price mark-up representing the degree of imperfect substitability among intermediate goods. As in the labor market, the previous equation is the constraint we plug into the objective of the following optimization problem for the final good producer:

$$\max_{\{Y_t, y_t(j)\}} \quad P_t Y_t - \int_0^1 p_t(j) y_t(j) dj \tag{38}$$

The optimization problem yields the following demand schedule for intermediate good output:

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\frac{1+\gamma_{p,t}}{\gamma_{p,t}}} Y_t \tag{39}$$

moreover, we can exploit zero profit condition of the competitive representive final good producer to obtain the following equation for the aggregate price level:

$$P_t = \left(\int_0^1 p_t(j)^{-\frac{1}{\gamma_{p,t}}} dj\right)^{-\gamma_{p,t}} \tag{40}$$

We now focus on price setting of intermediates. Intermediate output is produced with a Cobb-Douglas CRS production technology, augmented to include a fixed cost Φ:

$$y_t(j) = \epsilon_t^Z K_t(j)^\alpha N_t(j)^{1-\alpha} - \Phi \tag{41}$$

with

$$K_t(j) = u_t(j) \bar{K}_{t-1}(j) \tag{42}$$

and ϵ_t^Z being the AR(1) process for the productivity

$$\epsilon_t^Z = \rho_Z \epsilon_{t-1}^Z + \eta_t^Z \quad \text{with } \eta_t^Z \sim \mathcal{WNN}(0, \sigma_Z^2) \tag{43}$$

Right before setting prices, firms choose the bundle of both capital and labor that minimizes total costs of prodution by solving the cost minimization problem. The nominal margnal cost function [13]:

[13]Nominal costs are equal for each firm belonging to the mass of intermediate goods producers.

$$MC_t = \frac{(r_t^K)^\alpha w_t^{1-\alpha}}{\epsilon_t^Z}(\alpha^{-(1-\alpha)}(1-\alpha)^{-\alpha})$$

Nominal rigidity is now introduced in order to account for the inertial behavior of prices[14]. Unit mass of intermediates is split in two parts. A fraction $1-\xi_p$ receives the calvo signal to reset price and chooses the price level in order to optimize the discounted flow of expected profits. The remaining mass ξ_p does not receive the calvo signal to adjust price; the rule of thumb behavior results in the following backward looking relation:

$$p_t(j) = \pi_{t-1}^{\iota_p} p_{t-1}(j) \tag{44}$$

Non-optimizing firms basically index their price to previous period inflation, the strenght of indexation given by ι_p parameter. Likewise the labor maket, as ι_p approaches 1, there is compelte real rigidity while we have complete nominal rigidity as ι_p approaches 0. The previous relation implies that for a generic reoptimizing firm which does not expect to receive the Calvo signal up to a generic time-τ horizon, pricing rule is the following:

$$p_{t+\tau}(j) = \prod_{k=1}^{\tau} \pi_{t+k-1}^{\iota_p} p_t(j)$$

Therefore, the contrained optimization problem for the Calvo firm consists in choosing the price level so that profits are maximized, subject to incomplete nominal adjustment:

$$\max_{\{p_t^*(j)\}} \quad \mathbb{E}_t \sum_{\tau=0}^{\infty} (\xi_p \beta)^\tau \Lambda_{t,t+\tau} y_{t+\tau}(j) \left[\frac{p_t^*(j)}{P_{t+\tau}} X_\tau^p - mc_{t+\tau} \right] \tag{45}$$

$$\text{subject to} \quad y_{t+\tau}(j) = \left(\frac{p_{t+\tau}(j)}{P_{t+\tau}} \right)^{-\frac{1+\gamma_{p,t+\tau}}{\gamma_{p,t+\tau}}} Y_{t+\tau} \tag{46}$$

$$p_{t+\tau}(j) = X_\tau^p p_t^*(j) \tag{47}$$

with

$$X_\tau^p = \begin{cases} 1 \text{ if } \tau\text{=0} \\ \prod_{k=1}^{\tau} \pi_{t+k-1}^{\iota_p} \text{ if } \tau\text{=1,..} \end{cases}$$

The firm problem leads the following optimality condition:

$$\frac{p_t^*(j)}{P_t} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\xi_p \beta)^\tau \Lambda_{t,t+\tau} y_{t+\tau}(j) \left(\prod_{k=0}^{\tau} \frac{\pi_{t+k-1}^{\iota_p}}{\pi_{t+k}} \right) = \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} y_{t+\tau}(j)(1+\gamma_{p,t+\tau}) mc_{t+\tau} \tag{48}$$

[14]See Blinder (1991) for an empirical assessment on price stickness; author conducts a survey to identify the actual degree of nominal rigidity in a number of industrial sectors.

The previous equality basically pins down a relation between the price charged by the monopolistic firm and a mark-up over a weighted average of future expected marginal costs, with the weighted given by the arrival of Calvo probabilities; moreover, price mark-up is assumed to follow the process:

$$\gamma_{p,t} = \eta_t^p \quad with \quad \eta_t^p \sim (0, \sigma_{\gamma_p}^2) \tag{49}$$

Since the unit mass of firms is splitted in two sub-masses to introduce Calvo rigidities, then we can augment the aggregate price equation. It therefore reads:

$$P_t = \left(\int_0^{\xi_p} \pi_{t-1} p_{t-1}(j)^{-\frac{1}{\gamma_p}} dj + \int_{\xi_p}^1 p_t^*(j)^{-\frac{1}{\gamma_p}} dj \right)^{-\gamma_p} \tag{50}$$

If we focus on a symmetric equilibrium, we get the following relation which fully characterizes the behavior of aggregate price:

$$P_t = \left[\xi_p (\pi_{t-1}^{\iota_p} P_{t-1})^{-\frac{1}{\gamma_p}} + (1-\xi_p)(p_t^*)^{-\frac{1}{\gamma_p}} \right]^{-\gamma_p} \tag{51}$$

3.3.1 Price setting in a flexible economy

We assume $\xi_p = 0$. Without nominal rigidities, maximization of profits is static:

$$\max_{\{\tilde{p}_t(j)\}} \quad [\tilde{p}_t(j) - mc_t]\, y_t(j) \tag{52}$$

$$\text{s.t} \quad y_t(j) = \left(\frac{\tilde{p}_t(j)}{P_t} \right)^{-\frac{1+\gamma_p}{\gamma_p}} Y_t \tag{53}$$

Actually, since there is no inflation process as price is flexible, mark-up over price is no longer time dependent; the maximization problem reduces to charging a constant mark-up over the marginal cost

$$\tilde{p}_t(j) = (1+\gamma_p) P_t mc_t \tag{54}$$

3.4 Fiscal and Monethary autorithy

The Smets and Wouters model assumes there is a governent financing public consumption expanditures through either lump-sum taxtation, $T_t = G_t$ or even by issuing debt[15].

The government budget constraint is the following:

[15] Ricardian Equivalence indeterminacy argument holds in this model economy

$$G_t + \frac{B_{t-1}}{\pi_t} = T_t + \frac{B_t}{R_t} \tag{55}$$

with

$$G_t = \bar{G}\epsilon_t^G \tag{56}$$

with $\bar{G}$ giving the government spending share on output.
ϵ_t^G is the government spending exogenous shock, and it follows an AR(1) process:

$$\epsilon_t^G = \rho_G \epsilon_{t-1}^G + \eta_t^G \quad \text{with } \eta_t^G \sim \mathcal{WNN}(0, \sigma_G^2) \tag{57}$$

Further, there exists a monetary authority devoted to the stabilization of inflation which sets the nominal interest rate on the riskless bond according to the following generalized Taylor equation[16]:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R} \left(\left(\frac{\pi_t}{\overline{\pi}}\right)^{r_\pi} \left(\frac{Y_t}{Y_t^{flex}}\right)^{r_Y}\right)^{1-\rho_R} \left(\frac{Y_t/Y_{t-1}}{Y_t^{flex}/Y_{t-1}^{flex}}\right)^{r_{\Delta y}} \epsilon_t^R \tag{58}$$

where $\overline{\pi}_t$ follows an AR(1) process

$$\overline{\pi}_t = \rho_{\overline{\pi}} \overline{\pi}_{t-1} + \eta_t^{\overline{\pi}} \quad \text{with } \eta_t^{\overline{\pi}} \sim \mathcal{WNN}(0, \sigma_{\overline{\pi}}^2) \tag{59}$$

The interest rate setting rule makes it clear that the monetary authority takes into account a number of factors. Firstly, the monetary authority considers the deviation of the past period nominal interest rate from its steady state value, R_{t-1}/R^*. It also takes into account the deviation of current inflation from the inflation objective, $\pi_t/\overline{\pi}_t$. Finally, it considers the output gap, Y_t/Y_t^{flex} where Y_t^{flex} is the frictionless output, namely the output produced by a frictionless economy without nominal rigidities. The central bank accounts for dynamic factors as well. It examines, in particular, output gap dynamics, $(Y_t - Y_t^{flex})/(Y_{t-1} - Y_{t-1}^{flex})$.
In particular, output gap is defined as the difference between the observed (frictional) output and potential unobservable (frictionless) output the monetary authority would be eventually able to observe in the absence of any form of nominal rigidity. From a welfare standpoint, it is notewhorty that the frictionless output is not the Pareto-efficient output that would rise in a competitive equilibrium for an economy working under perfect competition by the first welfare theorem. It is indeed an inefficient output and it does not correspond to the allocation we would have by setting the model as a planning problem without phisical adjustment costs for investment and variable capital utilization: in the frictionless economy money is indeed neutral as both wages and prices are not sticky, but still

[16] See Clarida, Gali, Gertler (1999) for a theoretical investigation of welfare implications of different monetary rules and different regimes of monetary policy.

there are sources of real imperfections. Moreover, both the labor and good market are monopolistic, meaning that the degree of market power allows households and firms to charge a mark-up over, respectively, the ratio between marginal disutility of work and marginal utility for the household and over the marginal cost for the firm. This creates a subsequent deadweight loss.

3.5 Resource Constraint

The model is closed by aggregating over state variables and specifying the resource constaint.
For competitive capital market, capital demand equals capital supply:

$$K_t \equiv \int_0^1 K_t(j)dj = \int_0^1 K_t(i)di \tag{60}$$

For monopolistic labor market, market clearing condition is the following:

$$N_t \equiv \left(\int_0^1 N_t^{\frac{\gamma_{w,t}}{1+\gamma_{w,t}}}(i)di\right)^{1+\gamma_{w,t}} = \int_0^1 N_t(j)dj \tag{61}$$

where LHS is the labor supplied by labor union which aggregates over households specific labor supply by using CES aggregaation technology, and RHS is the aggregatated firm specific labor demand. As such, real dividends can be stated as

$$\Pi_t = \int_0^1 y_t(j)dj - W_t N_t - r_t^K K_t = Y_t - W_t N_t - r_t^K K_t \tag{62}$$

By firstly integrating the budgest constraint over the unit mass of household and then plugging into it the expression for real dividend, we obtain the economy-wide agrgegate resource constraint

$$C_t + I_t + G_t + \Psi(u_t)K_t = Y_t \tag{63}$$

Bond market clears

$$B_t = 0 \tag{64}$$

Arrow securities are in zero net supply

$$\forall \omega \in \Omega, \quad \int_0^1 a_t(i)di = 0 \tag{65}$$

3.6 Equilibrium

Given a monetary policy rule $\{R_t\}_{t=0}^{\infty}$ and a fiscal policy rule $\{G_t, T_t\}_{t=0}^{\infty}$, a competitive equilibrium for the Smets-Wouters economy is a dynamic stochastic allocation $\{C_t, B_t, I_t, \{N_t(i,j)\}_{i,j\in[0,1]}, \{\bar{K}_{t-1}(i)\}_{i\in[0,1]}, \{u_t(i)\}_{i\in[0,1]}, \{K_t(i,j)\}_{i,j\in[0,1]},$ $\{y_t(j)\}_{j\in[0,1]}, N_t, K_t, Y_t, \Pi_t\}_{t=0}^{\infty}$ and a state-contingent price system $\{b_t, r_t^K, \{w_t(i)\}_{i\in[0,1]}, \{p_t(j)\}_{j\in[0,1]},$ $w_t, P_t\}_{t=0}^{\infty}$, such that:

-Household i maximize intertemporal expected utility (12) subject to the budget constraint (13) and law of motion for physical capital (15);

-Household i maximize the expect intertemporal gain deriving from supplying labor services (26) subject to labor demand (27) and Calvo adjustments of wages (28);

-Labor union maximize profits (22) subject to CES aggregation technology for individual labor services (21);

-Intermediate good firm j maximizes profits (45) subject to intermediate good demand (46) and Calvo adjustments of prices (47);

-Final good producer maximize profits (38) subject to CES aggregation technology for intermediate goods (37);

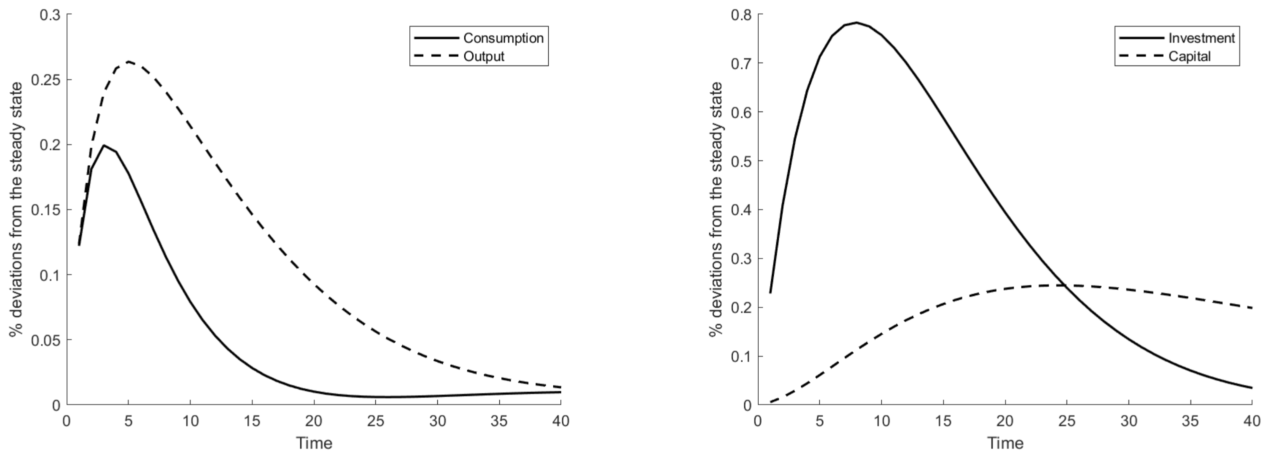
-Markets clear;

3.7 Results

In this setion we sum up some results of the SW model. In particular, we provide insights on impulse responses, deferring the analysis of unconditional second moments to the models' comparison section.

As regards impulse responses, note that the SW model displays ten structural shocks, meaning that it is able to generate a large set of responses of aggregate variables, namely ten responses for each variable. As we have already pointd out, the aim of the present thesis is to reduce the shock structure of the SW model and simultaneosly leaving untouched its business cycle properties. Since we end up having a DSGE model including four shocks (for the supply side, the productivity shock; for the demand side, government spending shock, monetary shock and inflation target shock), we just focus here on the responses to both productivity shock and monetary shock of a number of selected variables (output, consumption, investment, capital, employment, labor demand, rental rate of capital, wages, inflation and interest rate).

We begin by characterizing impule responses to aggregate productivity shock.

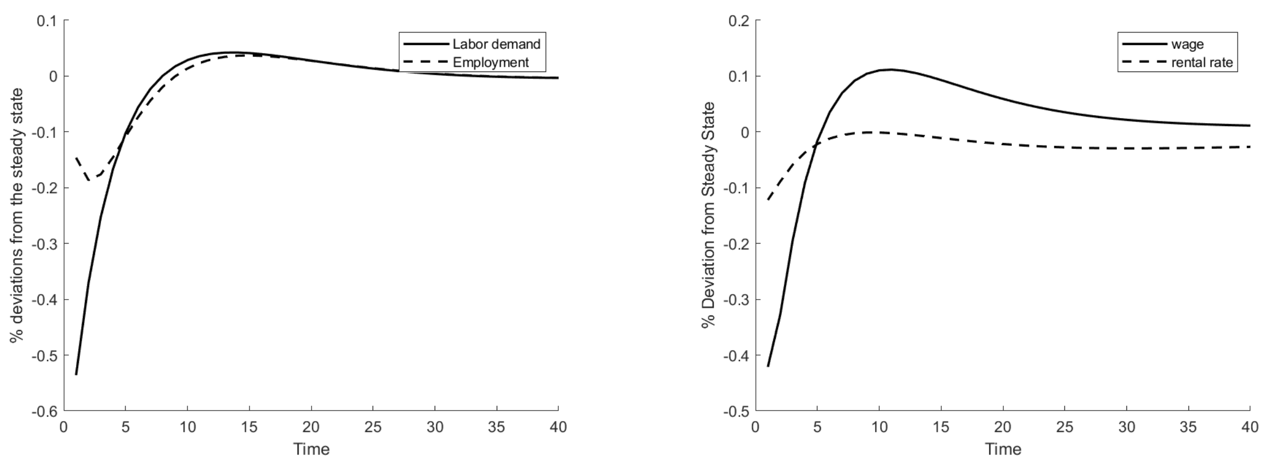


(a) IRFs of output and consumption

(b) IRFs of investment and capital stock

Figure 1: SW IRFs for output, consumption, investment and capital to technology shock

Figure 1a plots 40-periods impulse responses to AR(1) technology shock for output and consumption. As it is consistent with stylized facts about the impact of positive technology shock, output and consumption rise. Figure 1b plot impulse responses for investment and installed capital stock (ie. physical capital rescaled by the variable utilization rate). Investment is positively hit by the arrival of the technology shock and the magnitude of the response is larger than output and consumption. Capital displays an acyclical behavior as it first rise then it does not return suddenly to steady state value.



(a) IRFs for labor demand and employment

(b) IRFs for wage and rental rate

Figure 2: SW IRFs for labor demand, employment and factor prices to technology shock

Figure 2a plots impulses of employment and labor demand. While labor demand fall is more prononced as a conseqence of the productivity shock, then employment display a short run fall before settling on the steady state in the medium-run. Figure 2b displays the impulses of factor prices. As we note, both real wages and rental rate of capital display acyclical behavior. The fact that real wage follows a somehow umpredictable pattern is consistent with the presence of nominal rigidities.

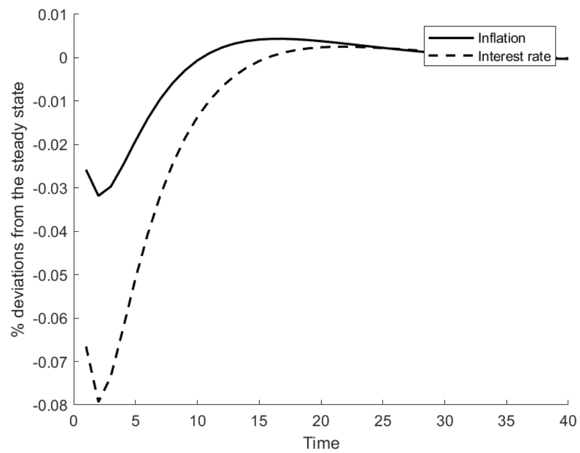


Figure 3: SW IRFs for interest Rate and inflation to technology shock

Figure 3 plot impulse responses of inflation and interest rates. Since the productivity shock make marginal cost fall, provided that marginal costs are the main driver for inflation dynamics, then inflation display a sudden fall in the short run. As a monetary policy response, interest rate falls as well creating an additional effect on output and real variables as well.

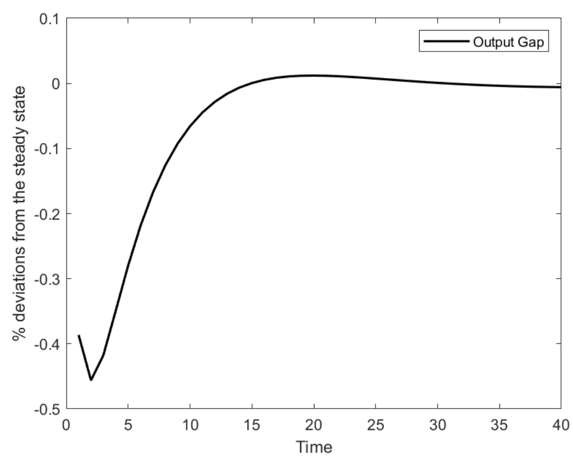
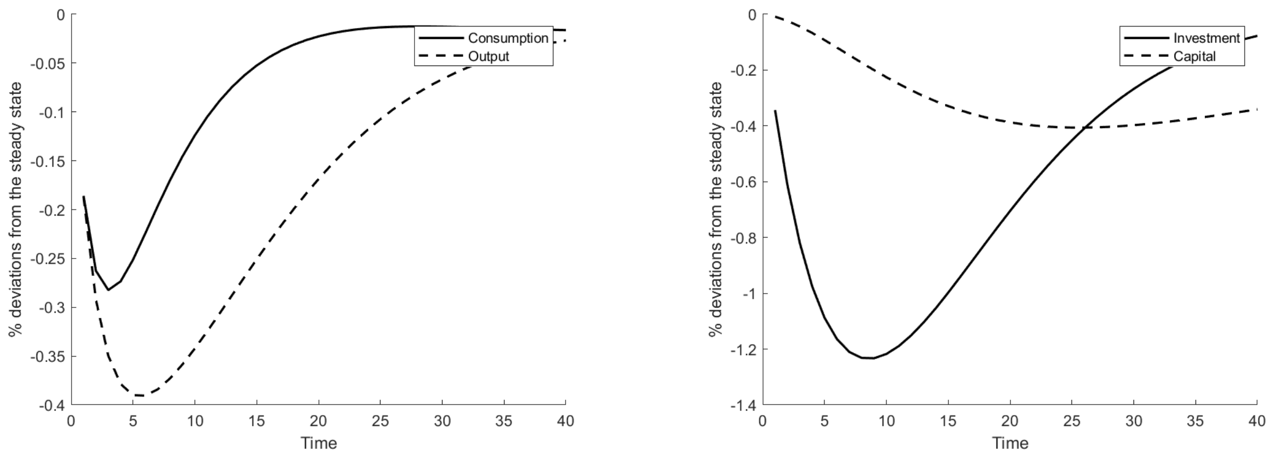


Figure 4: SW IRF for output gap to technology shock

Figure 4 plot the response of output gap to productivity shock; the gap between potential and fricitonless output decreases after a productivity boost , then it returnes to its steady state level. Actually, note that the productivity shock is active even in the underlying frictionless economy, meaning that the difference is still associated with the presence of nominal rigidities; though, the fact that output gap decline in the the short run suggest that the positive effect of the technology shock is propagated in the frictionless economy more easiliy than it is in the frictional one. We now focus on impulse responses to a positive monetary shockn, namely an increase in the nominal interest rate.

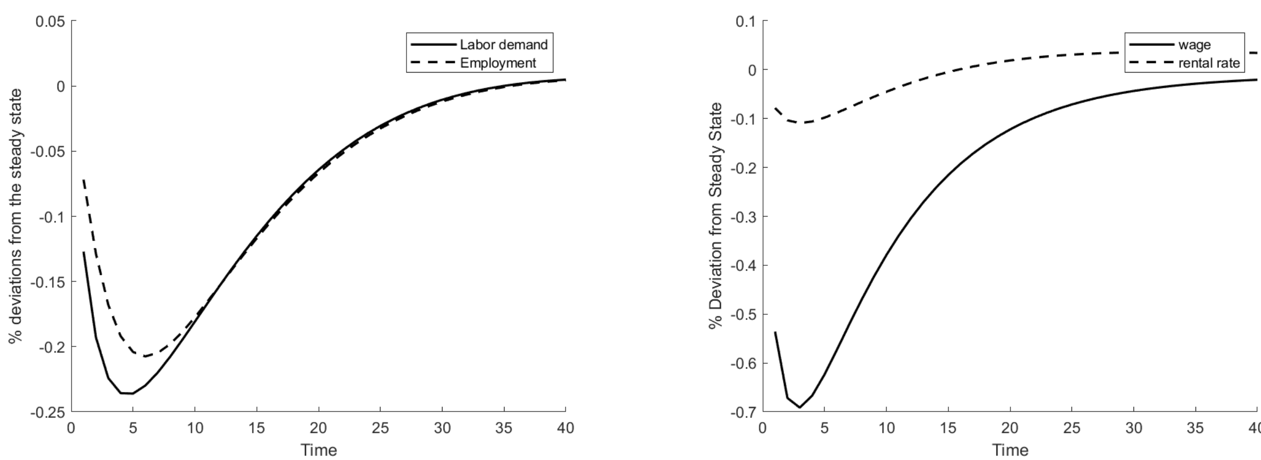


(a) IRFs of output and consumption

(b) IRFs of investment and capital stock

Figure 5: SW IRFs of output, consumption, investment and capital stock to monetary shock

Figure 5a and 5b plot impulse responses for output, consumption, investment and capital stock to the IID monetary shock. The temporary increase in the nominal interest rateproduces a hump-shaped fall in output, consumption and investment; this is indeed consistent with the general consensus on contractionary monetary policy. Again, capital stock shows up a substantial acyclicality.



(a) IRFs of labor demand and employment

(b) IRFs of wage and rental rate

Figure 6: SW IRFs of labor demand, employment and factor prices to monetary shock

Figure 6a focuses on the responses of labor demand and emoloyment: as a result of the monetary shock, employment fall and labor demand as well; their dynamic trajectory is quite synchronized. Figure 6b plots responses of rental rate and wages. As a result of a decrease in the interest rate, wages and rental rate goes down, though some acyclicality is given by the presence of nominal rigidities.

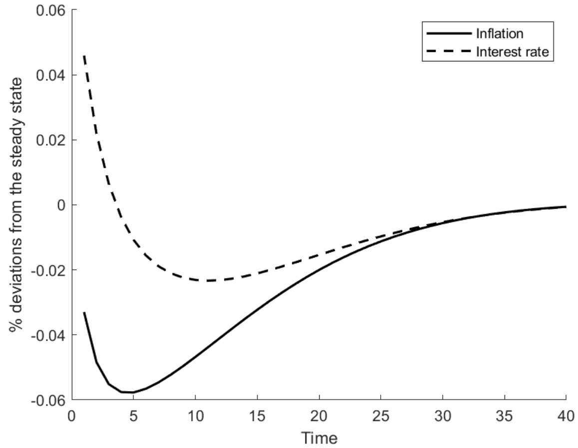


Figure 7: SW IRFs for interest rate and inflation to monetary shock

Figure 7 plots the responses of nominal variables. As a result of the monetary shock, interest rate goes un and then slowly returns to its setady state value while inflation fall due to the slowdown of economic activity.

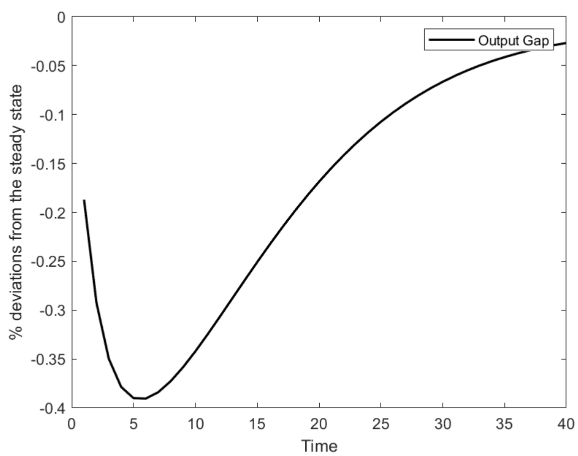


Figure 8: SW IRF for output gap to monetary shock

Figure 8 plots the impulse response of output gap to the arrival of a positive monetary shock. Since the underlying flexble economy is immune to monetary innovation due to full-price flexibility and the absence of any form of nominal rigidity, then the IRF of output gap is numerically equivalent to the IRF of output to a purely monetary shcok.

3.8 Critical Assessment of the SW Model: Weaknesses

As it results from the previos exposition, Smets and Wouters articulate an artificial economy which is suitable for running policy experiments, as it encompasses a number of sectors for the real side, and price processes for the nominal side. Implications of such experiments could even be considered as policy driving criterion for decision makers, if the underlying model economy would rest on a sound theoretical ground, regardless its capability of matching in-sample features (for instance, unconditional second moments) of empirical data. Though, SW model is affected by multiple shortcomings undermining its overall credibility and enforcability as a policy instrument tool. In this conncetion, there are three orders of weaknesses associated with the Smets and Wouters model. Noteworthy, the first two classes of weaknesses are generalizable to the whole family of DSGE-NK models, at least until very recent and not universally accepted developments in the field. The first class concern rational expectation formation and representative agent assumption which is pursued firstly for analytical tractability: DSGE models leave aside any form of heterogeneity and strategic interaction among agents. The second class concerns the fact that NK models rest on the assumption that the economy settles on a long run steady state equilibrium, perturbed by the arrival of stochastic shocks of various magnitude: as a modelling choice, out-of-equilibrium dynamics and non-linearities are completely neglected. We focus here on the third class of weaknesses related especially to the model presented in the previous section. It concerns the lack of descriptive realism of the functioning of the economic system accompanied by a great mass of exogeneity present in the general structure of the model: while this modelling choice does not alter the capability of the model to capture business cycle empirical regularities in terms of unconditional second moments, it directly undermines the capability of it of serving as a policy tool to essentialy mitigate detrimental effect of downturns of economic activity; infact, sources of fluctuations are not explained by the model but their existence is just assumed.

Firstly, consider the labor market. In SW model, labor market displays monopolistic competition and the wage is unilaterally set by household against the demand of the labor packaging technology: while the existence of a labor union mantain a degree of descriptive realism, the presence of market power only on the household side is not consistent with labor markets functioning at all. As a direct consequence, modelling incomplete nominal adjustment of real wages by using Calvo framework, as if households acted as price-setting firms, is purely a modeling choice which sacrifices realism for purely empirical tractability of the resulting wage equation. Actually, real wage dynamics are fully described by a NK Wage-Phillips curve augmented with indexation to past period inflation in order to capture some degree of backward-looking behavior of agents. Weights among empirical tractability and descriptive capability of the model are assigned unequally, and this limit modeling choice is not balanced by the fact that it brings any particular empirical precision as it leads the complications associated with NK-types price/wage dynamics, especially in term of persistence and emprical validity of impulse responses to structural shocks. What is more, Calvo types adjustment and the resulting NKPC were originally deviced to model price and not wage dynamics. Further, SW model does not produce unvoluntary equilibrium unemployment but just voluntary one as in the RBC tradition.

Next, we focus on goods market. Calvo adjustment is more appropriate than it is for modelling labor market as nominal stickness comes primarily from inertial behavior of prices. According to a well estabilished tradition focusing on the idea 'menu costs', changing prices might not be incentive compatible for the indiviual firm; though, inertial behavior (ie. unwillingness to pay menu costs) might have first order effect from a welfare standpoint, and it might display a non neglectible business cycle effect. Incomplete adjustment of prices can even arise from "near-rational" behavior of firms who find it costly to acquire, interpret and process continuously all the necessary information to re-adjust prices following innovation in the stochastic process governing money supply. Regardless of the sources of stickness, price are indeed rigid. Though, SW model just assumes there exists one type of rigidity: according to a great amount of empirical studies, there are different types and different degree of rigidities. The cohexistence of heterogeneous forms of rigidities in the unit mass of price-setter firms make it unlikely that the behavior of the aggregate price index is fully characterized by just one type of stickness, namely Calvo stickness; moreover, inflation/unemployment dynamics associated to the Calvo stickness through NKPC-type of relations are counterfactual (Fuhrer and Moore, 1995; Ball, 1995).

To continue, SW model completely neglects financial market. It is just a veil with the role of reducing informtion asymmetries between borrowers and lenders: it basically channels savings to investment. Representative household produce physical capital and rent it to intermediate good firms at rate r_t^K; capital market is competitive and the only source of turbolence is an *iid* exogenous shock added to the log-linearized version of the Tobin's Q relation. In spite of this, deep relation between so-called "real economy" and financial economy is almost a stylized truth of modern macroeconomics, and synchronization between financial cycle and business cycle is confirmed by a large amount of empirical studies. Perfect competition is removed from both the labor and good market as modelling nominal rigidities naturally calls for monopolitic competition in order to allow firms to take non-trivial pricing decisions. Similarly, descriptive realism dictates the elimination of walrasian capital markets: this would imply twisting radically the structure of the artificial economy to allow for the presence of a banking sector, and introducing frictions to endogenize SW structural shock to equity premium.

Lastly, SW model embodies a substantial mass of exogeneity: there are ten orthogonal shocks hitting litterally all sectors of the economy. This implies that the gratest part of fluctuations the model is able to produce remains exogenous and does not depend by the structure of the model. We have already claimed that the presence of such exogeneity invalidates the capability of the model of serving as a policy instrument. In this connection, we argue that it would be a substantial improvement for the SW model to have its empirical performance untouched by cutting down on its multiple-shocks-related exogeneity.

3.9 Possible Solutions

We address possible solutions to the various shortcomings of the SW model by considering the weaknesses we have highlighted in the previous section separately. The proposed solutions are motivaded essentially by the need of descriptive realism. We proceed by introducing additional frictions respectively in the labor market, in the goods market and in the capital market: structural modifications are achieved by substituting or even eliminating nominal Calvo rigidities for the first two blocks and by twisting the general structure of the economy for the last one. All modifications we carry on imply the introduction of a new friction.

As regards labor market, we twist the general structure of the model by introducing a search friction: this basically means that wage setting process is no longer monopolistic but it becomes bilateral. Additionally, the introduction of a search and matching process before the bilateral bargaining among searching firms and potential workers takes place requires augmenting the structure of the labor market. If there are are search and matching dynamics, then the labor market must be made up of a pool of employed agents, and a pool of unemployed ones as well. The flows between the two states are governed by endogenous probabilities.

As regards goods market, we do not twist the structure of the model but just modify it by augmenting the form nominal rigidities. That is, we still deal with a unit mass of price-setting firms. Though, we reduce tha mass of intermediate good producers that set price subjet to Calvo rigidities and we assume that the remaining fraction adopts a form of accidental rational inattention: this does not mean that firms choose deliberately not to acquire ad process new information due tue transactional costs as in Sims (2002), but that a process for the arrival of information is specified similarly to the process for the arrival of the Calvo signal. Firms that does not receive the information signal keep maximizing expected discounted profits conditional on a compelte but outdated information structure. Still, expectations are rational but they are not formed conditional on the current state of the economy, but on a previous period state. As a result, aggregate price level does not only account for Calvo rigidities, but even for sticky information rigidities. Therefore, inflation dynamics is just partially described by the NKPC; it relies on a non-trivial system of expectations.

As regards capital market, we twist the structure of the model. First of all, we decentralize investment decision as in Christiano, Motto, Rostagno (2010) to a sector, namely capital good producers, devoted to the production of capital good. Then, we introduce an entrepreneurial sector which raise funds for the acquisition of phisical capital both from its net worth and by issuing a debt from a representative bank a la Bernanke, Gertler and Gilchrist. Since entrepreneurial sector is subject to an idiosyncratic shock which could in principle affect the capability of the entrepreneur to pay the loan back, the bank charges an external finance premium on the baseline deterministic loan rate. If the idiosyncratic shock was absent, then the loan rate would be equal to the deposit rate and the banking sector collappses to the "veil" it is in SW.

The introduction of multiple friction in the baseline specification lays down a family of model, if it is done by considering one friction at a time. Actually, we have a *searchmodel* featuring search and matching friction in the labor market and the SW goods market: this is supposed to reduce or even eliminate the impact of exogenous labor supply shock and wage mark-up shock. We then have the *hybridmodel* featuring the dual form of rigidity in the goods market and SW labor market: this is done mainly to mitigate the impact of the price mark-up shock on the inflation process. Since the SW model is structure on multiple layers, then frictions can be interacted without conditioning analytical tractability of the model. So, we just add the financial friction to a $fullmodel$ featuring the structural properties of both *searchmodel* and *hybridmodel*, with the augmentation given by the frictional capital market: it is supposed to lower the impact of the previous exogenous shock. Further, it should endogenize the impact of investment shock and equity premium shock.

4 Goods Market: the Hybrid Stickness Friction

In the hybrid stickness model we augment the baseline SW specification of the aggregate price process. The reason for this augmentation is twofold.

Along side a well estabilished tradition devoted to enrich production sector in DSGE models with some degree of heterogeneity in the intensity, the frequency and, mainly, the form of incomplete nominal adjustments, we introduce a dual form of price stickness. In so doing, we get a more micro-founded version of the aggregate price equation, as it reflects inertial behavior of prices deriving from two types of adjustments and expectation formation.

Secondly, we end up having an inflation dynamics which is not fully described by Calvo-type staggered price setting and by the resulting NKPC. In fact, in most DSGE models, inflation process is fully characterize by NKPC-types of dynamics which only stems from Calvo rigidities: truthfully, Calvo model is based on the specification of a stochastic process for the arrival of Calvo signal to reset prices; but it leaves some degree of freedom to the modeler in characterizing the behavior of the mass of firms which does not receive the calvo signal. This degree of freedom can in principle spawn a large family of NKPCs with augmentation with respect to the baseline version given by a peculiar rule-of thumb behavior which is assigned to non-reoptimizing firms. In a generalized specification, inflation dynamics in DSGE models are approximated by the following equation in log-linear form:

$$\hat{\pi}_t = \mathbb{E}_t \hat{\pi}_{t+1} + f(\boldsymbol{\theta}) MC_t + z(\boldsymbol{\theta}) \hat{\pi}_{t-k} \tag{66}$$

where $\boldsymbol{\theta}$ is a n-dimensional vector of structural parameters for the model economy, MC_t are the marginal costs from the cost minimization problem of the representative Calvo-sticky firm, and $\hat{\pi}_t$ is the inflation level among two subsequent time periods. What is common to all kind of specifications is their forward-looking trait: current period inflation depends positively on current period rational expectation for future inflation and increases with marginal costs rescaled by a non linear function of structural parameters[17]. This form of anchoring has been longly investigated empirically. Despite its empirical tractability, it leads to counterfactual dynamics for the inflation/employment trade-off embadded in the specification, through the channel of marginal costs and output gap as well. DSGE models have been primarily deviced to investigate interal propagation mechanisms of structural stochastic shocks and in particular monetary policy trasmission (ie. the effect on real variables and asset prices of a monetary shock in terms of an interest rate cut in the case of expansionary monetary policy and an interest rate increase for the case of contractionary monetary policy). The fact that these models rely on an inflation process that is not empirically plausible, undermines their primary policy-serving function. Therefore, we address the issue of expectation formation by casting in the NKPC rational expectations for current period inflation formed in a previous period, alongside current period rational expectations for future period inflation. We do so by resorting to the sticky information framework.

Still, we deal with complete information sets: it would be interesting to model some form of imperfect monitoring of the stochastic process governing money supply or impefect foresight about the aggregate price level. Moreover,

[17] $\boldsymbol{\theta}$ vector often reduces to a two-dimensional vector including the disount rate β and the frequency of arrival of the Calvo signal ξ_p.

we still use time-dependent framework because of its capability to deliver closed form expression for aggregate variables[18].

We now present the hybrid price stickness model. Since the general structure of the economy stays unchanged with respect to our benchmark SW model, we just focus on the goods market and price setting behavior of firms.

4.1 Hybrid Model: Price Setting with Hybrid Stickness

There is still a unit mass of intermediate good producers indexed by j. Still, they minimize costs and marginal cost function is the same thorughout the whole set of intermediates. A regards price setting, we now split the unit mass in two parts $\{J_n\}_{n=1,2}$ such that $J_1 + J_2 = 1$. As such, the fraction J_1 is subject to Calvo-type incomplete nominal adjustment of price. The remaining fraction J_2 is subject to the staggered random arrival of information. In other words, since acquiring, processing and exploiting new information (ie. taking production and pricing decisions conditional on a complete and updated information set) embodies non-neglectible transactional costs for this mass of firms, we assume that just a fraction of sticky information firms is willing to do so without paying up considerable costs. The other fraction finds it incentive compatible to keep staying stuck on the same information set for multiple periods and not to update their knowledge about current state of the economy. Therefore, sticky information assumption is primarily an assumption about the importance of transactional (in this case, informational) costs and how they are able to create inefficiencies in the production process.

To formalize, we proceed by splitting the unit mass in types, and we characterize the aggregate price level for the hybrid stickness economy.

The unit mass of intermediate good firms is characterized by the following type structure:

$$\text{Types} = \mathcal{T} = \{n_j : j = \text{calvo}, \text{si}\} \tag{67}$$

with *calvo* indicating the presence of Calvo rigidities, and *si* indicating the presence of sticky information rigidities. As a consequence, the aggregate price level obtained by the zero-profit condition of final good producers for this economy is augmented with respect to the aggregate price level of the SW economy, as only a fraction J_1 of firms is Calvo-type. Therefore, the aggregate price level reads:

$$P_t^{-\frac{1}{\gamma_p}} = \sum_{n_j \in \mathcal{T}} \int_{n_j} p_t^{-\frac{1}{\gamma_p}}(j)\, dj = \int_{n_{\text{calvo}}} p_t(j)^{-\frac{1}{\gamma_p}}\, dj + \int_{n_{\text{si}}} p_t(j)^{-\frac{1}{\gamma_p}}\, dj \tag{68}$$

[18] State-dependent framework as in Caballero and Engel (1993) is likely to represent a more emirically plausible tool for modelling incomplete nominal price adjustment as it does not exogenously assumes that price changes just happen randomly, but it endogenizes the frequency of price adjustments by focusing on an unobservable distribution of deviation of prices for a cross section of heterogeneous firms from their target (desired) price. Given trigger upper and lower bounds to activate price adjustments, as a firm's own price hits one of the bounds, it is immediately reset. It follows that the magnitude and frequency of adjustments is monotonic in the magnitude of the monetary innovation which acutally changes the cross sectional distribution of price deviations from their level (Gertler and Leahy, 2008); the model might even lead some form on money neutrality under certain restriction of the form of the stochastic process for money supply (Caplin and Spulber, 1987).

For the Calvo mass, a fraction $1-\xi_p^{hyb}$ receives the Calvo signal to adjust price; the remaining mass ξ_p^{hyb} does not receive the signal and adopts as a rule-of-thumb behavior the indexation of price to past period inflation, with variable degree of price indexation. So, price level for Calvo firms is:

$$\int_{n_{\text{calvo}}} p_t(j)^{-\frac{1}{\gamma_p}}\, dj = \int_0^{\xi_p^{hyb}} \left(\pi_{t-1}^{\iota_p} p_{t-1}(j)\right)^{-\frac{1}{\gamma_p}} dj + \int_{\xi_p^{hyb}}^{J_1} \left(p_t^*(j)\right)^{-\frac{1}{\gamma_p}} dj \tag{69}$$

meaning that, in a symmetric equilibrium, the aggregate price level for the calvo mass is

$$\left[P_t^{calvo}\right]^{-\frac{1}{\gamma_p}} = \xi_p^{hyb}\left(\pi_{t-1}^{\iota_p} P_{t-1}\right)^{-\frac{1}{\gamma_p}} + (J_1 - \xi_p^{hyb})(p_t^*)^{-\frac{1}{\gamma_p}} \tag{70}$$

As regards the sticky information firms, the characterization of the frequency of adjustment follows closely the one adopted for the calvo model and it reduces to a specification of the stochastic process governing the arrival of information. The real difference consists in explaining the effect of the informational stickness: while in the Calvo model the rigidity affects the capability of firms of continuously optimizing price, in the sticky information model it affects the timing of expectation formation. To lay down the structure of the model, the fraction of sticky information firm is J_2. We assume that a mass $J_2 - \phi$ receive the signal to update their information structure and takes transactional costs for this update as neglectible; so they maximize the expected discounted stream of profits conditional on an updated information structure. The remaining mass ϕ does not receive the infomation signal. Instead of adopting a rule-of-thub behavior they keep maximize expected profits but they do so conditional on an outdated information structure: information set is still complete but it is not based on the current state of the economy but on a past period state. The aggregate price level for sticky information firms is:

$$\int_{n_{\text{si}}} p_t(j)^{-\frac{1}{\gamma_p}}\, dj = (J_2 - \phi)\sum_{k=0}^{\infty} \phi^k \int_{J_1}^{1} \left[p_{t|t-k}^{si}(j)\right]^{-\frac{1}{\gamma_p}} dj \tag{71}$$

meaning that, in a symmetric equilibrium, the level of price reads:

$$\left[P_t^{si}\right]^{-\frac{1}{\gamma_p}} = (J_2 - \phi)\sum_{k=0}^{\infty} \phi^k \left[p_{t|t-k}^{si}\right]^{-\frac{1}{\gamma_p}} \tag{72}$$

Since we have already provided details for the optimal price decision of the Calvo firm, we now proceed with the characterization of the optimal price decision for a representative sticky infromation firm that has received the information signal k periods ago. The decision problem is a static profit optimization problem for a monopolistic market, with the demand given by final good aggregator. Therefore, the optimization problem is the following:

$$\max_{\{p_t^{si}\}} \quad \mathbb{E}_{t-k} y_t(j) \left[p_t^{si}(j) - mc_t \right] \tag{73}$$

$$\text{s.t} \quad y_t(j) = \left(\frac{p_t^{si}(j)}{P_t} \right)^{-\frac{1+\gamma_p}{\gamma_p}} Y_t. \tag{74}$$

The optimality condition for the sticky information firm in a symmetric equilibrium reads:

$$\mathbb{E}_{t-k} \left[p_t^{\text{si}}(j) \right] = (1 + \gamma_p) \mathbb{E}_{t-k} P_t mc_t \tag{75}$$

Note that the previous equation is a standard optimality condition for a price-setting firm which charges a constant mark-up over the marginal costs. The real difference with a flexible economy lies in the information structure conditioning: while in a flexible economy, current period price and marginal costs belong to the information set of the producer, in the sticky infromation economy they don't have this property.
Aggregate price level for the sticky information mass is the following:

$$P_t^{si} = (1 + \gamma_p)(J_2 - \phi) \sum_{k=0}^{\infty} \phi^k \mathbb{E}_{t-k}(P_t mc_t) \tag{76}$$

We are now able to obtain the aggregate price level for this rational expectation model economy with two types of rigidities in the goods market, which condition pricing decisions:

$$P_t^{-\frac{1}{\gamma_p}} = \xi_p^{hyb} (\pi_{t-1}^{\iota_p} P_{t-1})^{-\frac{1}{\gamma_p}} + (J_1 - \xi_p^{hyb})(p_t^{calvo})^{-\frac{1}{\gamma_p}} + (J_2 - \phi) \left(\sum_{k=0}^{\infty} \phi^k p_{t|t-k}^{si} \right)^{-\frac{1}{\gamma_p}} \tag{77}$$

In order to deal with a finite state-space, we arbitrarily truncate the infinitely backward expectations to a 1-period backward lag: this arbitrary assumption ensures that firm does not have an outdated information structure for more than a period. Further, the previous specification of the aggregate price level makes it clear that some firms behave as if they were in a flexible economy: whenever $k = 0$, a fraction $J_2 - \phi$ just charges a mark-up over the marginal cost. This is justified by a number of empirical study focusing on the fact that in some markets prices are close to be flexible and exhibits even daily frequency of adjustment.
This complete the exposition of the hybrid price stickness model: additional feauteres on the characterization of the goods market are supposed to lower the impact of price mark-up shock on the inflation process in the baseline specification. For the full characterization of the model, refer to the baseline SW specification laid down in section 3: all the blocks of the model are the same, expect for the price setting process which leads to an augmented aggregate price equation.

5 Labor Market: the Search Friction

This section provides a alternative characterization of the labor market with respect to the baseline SW model. Changes to the baseline specifications are both structural and theoretical.

As regards structural changes, we twist the structure of SW model by eliminating monopolistic competition and unilateral wage setting. In the SW model there is a labor union which aggregates differentiated labor services and provide labor demand to the wage-setting household. In this specification labor market is made up by both unemployed and employed agents and there are flows between the two states governed by endogenous probabilities; wage is bilaterally set by the household and the hiring firm according to a bilateral bargaining process we are going to explain later on.

As regards theoretical changes, dynamically modelling labor market as the interplay between agents switching from unemployment state to employment one and the opposite according to incentive compatibility of each state change, creates scope for so called frictional equilibrium unemployment. In SW model unemployment is just structural as it is determined by non-clearing wages due to the market power enjoyed by household; it derives from the specific structure of labor market. In the present model, a search-and-matching process alongside an exogenous disruption of hired labor force is introduced: when labor force is disrupted, resulting unemployed agents may find it incentive compatible to be stuck in the unemployed pool as we assume the existence of exogenously determined unemployment benefits and they may not enter the matching process.

The introduction of the search friction in a general equilibrium DSGE model displays an impact on firms' marginal cost which turns out to be dependent on search-matching related parameters. They are higher with a wage level produced by search-matching fictions than they are in a flexible economy with non-clearing flexible wages; this leads a non-neglectible impact of inflation dynamics as pinned down by the NKPC (Calvo sticky prices).

Since the structure of the general equilibrium search model follows closely the one in SW, we just characerize search-matching process in the labor-market and we arrive to a wage equation which turns out to be dependent of marginal payoffs agents get from being stuck in one state rather than the other and on structural searching probabilities as well. On balance, the wage equation is more microfounded than the one provided by SW as it relies on a complete characterization of the labor market.

5.1 Search Model: Household

Before digging into the details of wage setting process, we must stress that the introduction of labor market flows create some form of hetereogeneity in the household sector. There is a unit mass of household indexed by i, as in the baseline SW specification. But in the search model households can find themselves in two possible states.

There is a mass I_1 of emloyed households. If the household is employed, then it behaves like the SW household: it consumes, takes investments/saving decisions; it gets labor income from supplying hours of labor services; it gets capital income net of utilization costs by producing new capital good and rent it in the competitive capital market. Moreover, it attaches utility to leisure, meaning that it displays a two-input utility function, as in the SW model.

The remaing $1 - I_1$ mass of household is made up of unemployed agents. If the household in unemployed, then

it has the same control variables as in the SW mode: though, it does not supply labor services in order to get labor income but it just has exogenously determined unemployment benefits. Further, devoting the whole time endowement to leisure activities, does not yield any additional utility, meaning that it has incentive to begin the search process in order to switch from the unemployment pool to employment one.
To formalize the type structure of the household sector, we have:

$$\mathcal{T}_i = \{i \in [0,1] : i = \textit{employed}, \textit{unemployed}\} \tag{78}$$

Period utility function reads:

$$U_t(i \mid i \in \mathcal{T}_i) = \begin{cases} u_t(C_t(i)) - g_t(N_t(i)) & \text{if } i \in \textit{employed} \\ u_t(C_t(i)) & \text{if } i \in \textit{unemployed} \end{cases}$$

Similarly, financial wealth turns out to be dependent on the type of the household:

$$\Upsilon_t(i \mid i \in \mathcal{T}_i) = \begin{cases} w_t(i)N_t(i) + r_t^K u_t(i)\bar{K}_{t-1}(i) - \Psi(u_t(i))\bar{K}_{t-1}(i) + A_t(i) & \text{if } i \in \textit{employed} \\ u.b + r_t^K u_t(i)\bar{K}_{t-1}(i) - \Psi(u_t(i))\bar{K}_{t-1}(i) + A_t(i) & \text{if } i \in \textit{unemployed} \end{cases}$$

Regardless of the state the household is stuck into, the mazimization problem is the same for the both of the states as labor supply is not a control variables neither in the monopolistic wage setting problem of the SW model economy, nor in the bilateral wage setting process.

5.2 Search Model: Labor Market

5.2.1 Flows

In this setion we are going to endogenize the transition probabilities from one state to the other as they directly enter the wage equation, linking it with the degree of market tightness.
Firstly, we assume that aggregate employment, N_t, as measured in worked hours evolves according to the following law of motion:

$$N_t = \int_0^{I_1} N_t(i)di = (1-\Phi)\int_0^{I_1} N_{t-1}(i)di + m_t(v_t, u_t) \tag{79}$$

where Φ is an exogenous separation rate capturing the degree of structural job disruption; moreover, $m_t(v_t, u_t)$ is a mapping from the vacancies, v_t, and unemployed agents, u_t, to the number of matches m_t.
To put some structure, we assume the matching technology is a constant return to scale technology as it makes it simple to express transition probabilities as a function of the market tightness parameter $\theta_t = v_t/u_t$.

$$m_t(v_t, u_t) = u_t^{\Gamma} v_t^{1-\Gamma} \tag{80}$$

The previos function can be considered the labor market counterpart of the CRS Cobb-Douglas technology. Therefore, Γ parameter defines input share on the number of matches: Γ is the elasticity of matches with respect to uneployment whereas $1-\Gamma$ is the elasticity of the number of matches with respect to the number of vacancies
We now define transition probabilities by rescaling the matching technoloogy in the following way:

$$q_t(\theta_t) \equiv \frac{u_t^{\Gamma} v_t^{1-\Gamma}}{v_t} = \theta_t^{-\Gamma} \tag{81}$$

$$s_t(\theta_t) \equiv \frac{u_t^{\Gamma} v_t^{1-\Gamma}}{u_t} = \theta_t^{1-\Gamma} = \theta_t q_t(\theta_t) \tag{82}$$

where $q_t(\theta_t)$ is the vacancy filling rate for the searching firm; it is decreasing in the market tightness, $q_t'(\theta_t) < 0$. Instead, $s_t(\theta_t) = \theta_t q_t(\theta_t)$ is the job finding rate for the searching worker; it is increasing in the labor market tightness, $s_t'(\theta_t) > 0$.

5.2.2 Wage Setting

In this section we characterize bilateral bargaining of wages between the unemployed worker and the searching firm. This happens at time t once the match is formed in the same time period.
First of all, we define the payoffs the agent gets from switching from one state to the other by making use of recursive Bellmann equations. This approach allows us to describe the agent choise in a two period setting, and then to make use of the payoff each agent gets from each state to extract relative surpluses that are generated from one state to the other[19]. Since we have two states and two representative agents, namely a household and a firm, we have a total of four Bellman equations charactrizing state dynamics.
For the hiring firm Bellman equations are:

$$V_t = -\frac{k}{\lambda_t} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left\{ q_t(\theta_t)(1-\Phi) J_{t+1} + (1 - q_t(\theta_t)) V_{t+1} \right\} \tag{83}$$

$$J_t = y_t(j) - w_t N_t(j) - r_t^K K_t(j) + \beta \mathbb{E}_t \Lambda_{t,t+1} \left\{ (1-\Phi) J_{t+1} + \Phi V_{t+1} \right\} \tag{84}$$

where κ/λ_t is a fixed cost associated with opening a vacancy rescaled by marginal utility of consumption, λ_t.
For the worker, Bellman equations read:

$$U_t = u.b + \beta \mathbb{E}_t \Lambda_{t,t+1} \left\{ s_t(1-\Phi) W_{t+1} + (1 - s_t + s_t \Phi) U_{t+1} \right\} \tag{85}$$

$$W_t = w_t(i) N_t(i) - \frac{g(N_t(i))}{\lambda_t} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left\{ (1-\Phi) W_{t+1} + \Phi U_{t+1} \right\} \tag{86}$$

[19] For a detailed mathematical presentation of dynamic programming methods, including existence and uniqueness theorems associated with concave programming problems, the reference is Stokey, Lucas and Prescott (1989) and Ljiungqvist and Sargent (2012).

where $u.b$ are the exogenously determined unemployment benefits deriving from being unemployed.
Provided that the transition from unemployment to employment and vice versa is ruled by the matching process, then we must identify surplus measures agents gets from switching from one state to the other. Then, agents are assumed to bargain over wage by maximizing joint weighted surplus from existing employment relations, with the relative weight in the bargaining process given by their bargaining strenght.
In particular, we assume the RTM framework in modelling bilateral wage bargaining as it does not alter firms' costs minimization problem. In other words, representative firm unilaterally choose both labor and capital to minimize marginal costs given the available technology frontier as in the baseline SW model; given the cost-minimizing amount of hours, it engages in the wage bargaining problem. This means that in this framework worked hours are an allocative signal for wages. In particular, from cost minimization problem we know that for a symmetric equilibrium:

$$(1-\alpha)\frac{y_t}{N_t} = w_t \tag{87}$$

That is, real marginal product of labor is equated real wage; note that the previous equation does not imply the equivalence with the ratio between the marginal disutility associated with supplying labor sevices and the marginal utility of consumption, ie. $g'(N_t(i))/\lambda_t(i)$ as it would happen in a purely neoclassical labor market we find in growth models with exogenous technical change or in real business cycle theory. Actually, this happens due to the presence of an alternative wage setting device.
In characterizing wage bargaining process, we firstly lay down surplus measures.
For the firm we have:

$$J_t - V_t = y_t - w_t N_t - r_t^K K_t + \mathbb{E}_t \beta \Lambda_{t,t+1}(1-\Phi)J_{t+1} \tag{88}$$

while for the worker,

$$W_t - U_t = w_t N_t - \frac{g(N_t)}{\lambda_t} - u.b + \mathbb{E}_t \beta \Lambda_{t,t+1}(1-\Phi)(1-s_t)(W_{t+1} - U_{t+1}) \tag{89}$$

where the both of the surpluses depends on wages.
The bilateral maximization problem of weighted joint surplus is the following:

$$\max_{\{\bar{w}_t\}} \quad (W_t - U_t)^{\nu}(J_t - V_t)^{1-\nu} \tag{90}$$

Optimality with respect to wage implies the following first order condition:

$$\nu(W_t - U_t)^{\nu-1}(J_t - V_t)^{1-\nu}\frac{\partial(W_t - U_t)}{\partial\bar{w}_t} = -(1-\nu)(W_t - U_t)^{\nu}(J_t - V_t)^{-\nu}\frac{\partial(J_t - V_t)}{\partial\bar{w}_t} \tag{91}$$

Define partial derivatives of the surpluses with respect to real wage as:

$$\mu_t^{J-V} \equiv -\frac{\partial(J_t - V_t)}{\partial\bar{w}_t} = N_t \tag{92}$$

$$\mu_t^{W-U} \equiv \frac{\partial(W_t - U_t)}{\partial\bar{w}_t} = \frac{N_t}{1-\alpha}\left[\frac{mrs_t}{\bar{w}_t} - \alpha\right] \tag{93}$$

where μ_t^{J-V} is the firm's net marginal benefit associated with an increase of real wages and μ_t^{W-U} is the worker's net marginal benefits associated with an increase in the real wage; mrs_t is the ratio between the marginal disutility the agent get from worked hours, $g'(N_t)$ and the marginal utility of consumption, λ_t.
Further, define expectational terms in the recursive Bellman formulation of the surpluses as:

$$f^{J-V} = \mathbb{E}_t\beta\Lambda_{t,t+1}(1-\Phi)J_{t+1} \tag{94}$$

$$f^{W-U} = \mathbb{E}_t\beta\Lambda_{t,t+1}(1-\Phi)(1-s_t)(W_{t+1} - U_{t+1}) \tag{95}$$

After algebraic manipulation, we obtain the following expression for labor income:

$$\bar{w}_t N_t = \zeta_t(y_t - r_t^K K_t + f^{J-V}) + (1-\zeta_t)(\frac{g_{N_t}}{\lambda_t} + u.b - f^{W-U}) \tag{96}$$

with

$$\zeta_t = \frac{\nu\mu_t^{W-U}}{\nu\mu_t^{W-U} + (1-\nu)\mu_t^{J-V}} \tag{97}$$

$$1-\zeta_t = \frac{(1-\nu)\mu_t^{J-V}}{\nu\mu_t^{W-U} + (1-\nu)\mu_t^{J-V}} \tag{98}$$

where the last two equations are the share of net marginal benefits associated with an increase in real wage of, respectively, the worker and the firm. That is, $\nu\mu_t^{W-U} + (1-\nu)\mu_t^{J-V}$ is the total net marginal benefits deriving from the increase in real wage. Over it, worker's relative share is $\nu\mu_t^{W-U}$ while firm's relative share is $(1-\nu)\mu_t^{J-V}$.
Rearranging, we get the following wage equation:

$$\bar{w}_t = \zeta_t\left(\frac{mpl_t}{1-\alpha} - r_t^K\frac{K_t}{N_t} + \frac{\kappa\theta_t}{\lambda_t N_t}\right) + (1-\zeta_t)(\frac{mrs_t}{1+\sigma_N} + \frac{u.b}{N_t}) + \zeta_t(1-s_t)\frac{\kappa}{\lambda_t q_t N_t}\left(1 - \frac{1-\zeta_t}{\zeta_t}\frac{\zeta_{t+1}}{1-\zeta_{t+1}}\right) \tag{99}$$

Wages are therefore a weighted average between firm revenues and the workers disutility associated with supplying labor services; moreover in RTM framework weights not only depnd on the bargaining strenght ν, but even on the wage reallocative effect as capture by net increase in agent's surpluses as a response to real wage.

This completes the exposition of the search model.

We stress a couple more conclusions. One is internal to the relative strenght of this RTM wage setting device with respect to other form of wage bargaining. The other one focus on the strenght of search models with respect to models hinging on a walrasian labor market.

As regards the first point, it is noteworthy that the most commonly used wage setting device in search moedels is the so-called "Nash Bargaining". It differs from RTM as firms and workers bargain over both wages and hours; this implies that hours are not allocative for wages and that marginal product of labor is equated to the marginal rate of substitution, as it happens in neoclasscal labor market. Further, Nash bargaining framework is subject to the so called Shimer critique (2005) pointing out the lack of capability of the search model with Nash baraginig of generating the business cycle percistency of unemployment fluctuations in response to productivity shocks.

As regards the second points, SW model enriched with search matching friction is able to generate frictional unemployment: while this is indeed an improvement from the standpoint of descriptive realism, it must be also considered an improvement from the modelling point of view if the empirical performance of the model stays unaltered.

For the full exposition of the search model, refer to the structure laid down in 3: the search model is the same as in our benchmark SW specification, except for the wage setting process we have already characterize.

6 Capital Market: the Financial Accellerator Friction

In this section we depart from the characterization of capital market SW model uses by intoducing a financial accellerator mechanism à la Bernanke, Gertler and Gilchrist to define capital demand. In so doing we twist the baseline SW specification both structurally and from the point of view of modelling choices.

As regards the structure of the environment of the model economy, introducing a financial friction in the form of financial accellerator mechanism implies firstly decentralizing investment decisions and capital good production. As such, capital demand is defined between the interaction among three sectors. Capital good producers are devoted to investment and the production of physical capital; then we have an entrepreneurial sector which just transforms physical capital into installed capital services to trade with intermediate goods producers and raise funds by issuing a debt contract; finally, a representative banck that raise funds from deposits of households and give loan to te entrepreneurial sector in order to make it capable to acquiring physical capital. The rest of the model economy is the same.

As regards modelling choices, we articulate the capital market in order to remove perfect competition assumption[20] and to introduce a friction in the lending channel in the form of external finance premium the bank charges in order to hedge against idiosyncratick shocks.

The reason behind the introduction of the financial friction is twofold and we think it enriches SW baseline specification.

Firstly, imperfect functioning of financial markets has been proven empirically to display a significant effect on real economy; therefore, embedding a financial market in comprehensive models like DSGE models should not be regarded as an augmentation but just as a starting point.

Secondly, financial friction creates scope for additional propagation mechanism and amplifies businesse cycle fluctuations through the channel of marginal costs. In fact, the price of capital in this model does not clear capital market but embeds asymmetries in the lending channel, meaning that marginal costs are higher than they would be within a competitive capital market. Adding the effect of price stickness, this reflects on the inflation dynamics. Ultimately, as it is common in DSGE literature, since the frictions as a whole are embedded in inflation dynamics through the channel of marginal costs, frictional economies displays different degree of responsivness to monetary shocks according to the friction concerned.

We now provide an exposition of the financial accellerator mechanism. Since the general structure of the economy is the same as the one laid down for the SW model except for caital markets, though augmented with both the search friction and the hybrid form of stickness, we just focus on the characterization of capital markets.

6.1 Financial Accellerator: Household

Since invesment decisions are allocated upon a sector specifically devoted to the production of physical capital, namely capital good producers, representative household no longer gets capital income deriving from renting installed capital services to intermediate goods producers. This implies that the real financial wealth for the household

[20] Marginal product of capital equates its gross return as in real business cycle literature, and even in SW model with this relation being modified by the presence of real imperfections such as variable capital utilization and physical adjustment cost in investment process.

i in the model with frictional capital market reads:

$$\Upsilon_t(i) = w_t(i)N_t(i) + A_t(i) \tag{100}$$

As a direct consequence, the definition of a new financial wealth implies that the household no longer controls for variables related to caputal accumulation in the dynamic optimization problem, namely $\bar{K}_t(i)$, $I_t(i)$ and $u_t(i)$.

6.2 Financial Accellerator: Capital Good Producers

We begin our exposition by descriping the optimal investment of capital goods producers, providing a capital supply to the entrepreneurial sector. They invest to produce physical capital and sell it to entrepreneural sector at price Q_t; moreover they buy back undepreciated not utilized capital at the same price. So, capital trades among capital good producers and entrepreneurs is competitive.
The maximization problem reads as follows[21]:

$$\max_{\{I_t\}} \quad Q_t\bar{K}_{t+1} - Q_t(1-\delta)\bar{K}_t - I_t \tag{101}$$

$$\text{s.t} \quad \bar{K}_{t+1} = (1-\delta)\bar{K}_t + I_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] \tag{102}$$

The maximization problem delivers the following optimality condition

$$I_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}}S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta\mathbb{E}_t Q_{t+1}\left[\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right] = 1 \tag{103}$$

which is equivalent to the capital supply we have in SW model, except for the fact that the expectational term is not rescaled by the bi-periodal stochastic discount factor $\Lambda_{t,t+1}$.

6.3 Financial Accellerator: Entrepreneurs and Banks

We characterize optimal behavior of entrepreneurs and banks jointly as the capital demand will depend on the friction on the lending channel.
There exists a continuum of entrepreneurs indexed by e. Entrepreneurs buy physical capital from capital good producers at price Q_t, they turn it into installed capital service and rent it to intermediate good producers for prouction at rate r_t^K; further, they sell undepreciated non-utilized physical capital back to capital good producers. So, profits of the entrepreneurs read:

[21]Note that in this framework we are assuming a one period delay between investment and the production of physical capital stock.

$$\Pi_{t+1}(e) = r^K_{t+1}\omega^e u_{t+1}(e)\bar{K}_{t+1}(e) + (1-\delta)Q_{t+1}\omega^e \bar{K}_{t+1}(e) - \Psi(u_{t+1}(e))\bar{K}_{t+1} \tag{104}$$

where ω^e is the idiosyncratic disturbance to entrepreneur's e stocked capital, such that $\mathbb{E}\{\omega\} = 1$.
Representative entrepreneur finances its capital acquisition partially with its own net worth, partially by issuing debt from the bank. In particular, external funds need is given by:

$$B_{t+1}(e) = Q_t\bar{K}_{t+1}(e) - EQ_{t+1}(e) \tag{105}$$

meaning that the external finance need, $B_{t+1}(e)$ is equivalent to the difference betwee market value of capital assets, $Q_tK_{t+1}(e)$ and equity, $EQ_{t+1}(e)$ Also, the optmal debt contract is defined by a cut-off value $\bar{\omega^e}$ for the idiosyncratic disturbance. It satisfies:

$$\bar{\omega}^e Q_t(1+R^K_{t+1})\bar{K}_{t+1}(e) = (1+Z_{t+1})B_{t+1}(e) \tag{106}$$

where Z_{t+1} is the non-default gross loan rate. Moreover,

$$(1+R^K_{t+1}) = \frac{u_{t+1}(e)r^K_{t+1} + (1-\delta)Q_{t+1} - \Psi(u_{t+1}(e))}{Q_t} \tag{107}$$

is just a way to endogenize return on capital by linking it with the profits of entrepreneurial sector. It follows that

$$\bar{\omega}^e = \frac{1+Z_{t+1}}{1+R^K_{t+1}}\left(\frac{Q_t\bar{K}_{t+1}(e) - EQ_{t+1}(e)}{Q_t\bar{K}_{t+1}(e)}\right) \tag{108}$$

If $\omega^e < \bar{\omega}^e$, then the entrepreneur is not able to pay the loan back; therere the bank pays up an auditing cost μ which is a fraction of entrepreneur's revenues and keep the remaining part for itself.
If $\omega^e > \bar{\omega}^e$, then the entrepreneur gets a positive shock and it is able to pay the loan back.
We are now going to characterize the optimal loan contract from the perspective of the bank. The bank concedes the loan if the return it receives from issuing the loan contract equals in equiibrium the opportunity cost of so doing; therefore:

$$[1-F(\bar{\omega}^e)]\,(1+Z_{t+1})\,B_{t+1}(e) + (1-\mu)\int_0^{\bar{\omega}^e} \omega^e dF(\bar{\omega})Q_t\left(1+R^K_{t+1}\right)\bar{K}_{t+1}(e) = (1+R_{t+1})\,B_{t+1}(e) \tag{109}$$

where LHS is the gross return on the loan contract, $1-F(\bar{\omega}^e)$ being the mass of non-defaulting entrepreneurs; RHS is the opportunity cost of doing it. It pins down the gross return as a function of the cut-off value $\bar{\omega}^e$: on the one hand, an increase in the cut-off value increases the non-default payoff of the bank as giving the loan embodies a degree of risk which pushes up the loan rate; as a direct consequence, on the other hand, a rise in the cut-off value rises the default probabilities and makes it more likely for the bank itself to pay up the auditing cost which lower the default payoff.

In particular, by exploiting the definition of the cut-off value, we can express the optimal loan contract equating expected return of the loan to the opportunity cost for the bank of giving funds to the entrepreneurial sector as:

$$\{[1-F(\bar{\omega}^e)]\,\bar{\omega}^e+(1-\mu)\int_0^{\bar{\omega}^e}\omega^e dF(\omega)\}Q_t\left(1+R_{t+1}^K\right)\bar{K}_{t+1}(e)=(1+R_{t+1})\left(Q_t\bar{K}_{t+1}(e)-EQ_{t+1}(e)\right) \tag{110}$$

Define now

$$G(\bar{\omega}^e)=\int_0^{\bar{\omega}^e}\omega^e dF(\omega^e) \tag{111}$$

and

$$\Gamma(\bar{\omega}^e)=\bar{\omega}^e\left[1-F(\bar{\omega}^e)\right]+G(\bar{\omega}^e)\quad\text{with}\quad F(\bar{\omega}^e)=\int_0^{\bar{\omega}^e}dF(\omega) \tag{112}$$

as the share of entrepreneurial earnings received by the bank before paying the monitoring cost. We are the able to express optimal contract condition as

$$\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right]\frac{1+R_{t+1}^K}{1+R_{t+1}}\frac{Q_t\bar{K}_{t+1}}{Q_t\bar{K}_{t+1}-EQ_{t+1}}=1 \tag{113}$$

with $\Gamma(\bar{\omega})-\mu G(\bar{\omega})$ denoting the same share of entrepreneurial profits net of monitoring costs.

We now turn to time entrepreneur profits:

$$\Pi_{t+1}(e)=\int_{\bar{\omega}}^{\infty}\left(\omega\left(1+R_{t+1}^K\right)Q_t\bar{K}_{t+1}(e)-\bar{\omega}\left(1+Z_{t+1}\right)B_{t+1}(e)\right)dF(\omega) \tag{114}$$

which reduces by the cut off value expression to

$$\Pi_{t+1}(e)=\int_{\bar{\omega}^e}^{\infty}\left(\omega^e-\bar{\omega}^e\right)dF(\omega)\left(1+R_{t+1}^K\right)Q_t\bar{K}_{t+1}(e) \tag{115}$$

Note that

$$\int_{\bar{\omega}^e}^{\infty} (\omega^e - \bar{\omega}^e)\, dF(\omega^e) = \int_{\bar{\omega}^e}^{\infty} \omega^e dF(\omega^e) - \bar{\omega}^e \int_{\bar{\omega}^e}^{\infty} dF(\omega^e) = 1 - G(\bar{\omega}^e) - \bar{\omega}^e\, [1 - F(\bar{\omega}^e)] = 1 - \Gamma(\bar{\omega}^e) \tag{116}$$

where $1 - \Gamma(\bar{\omega})$ is the share of entrepreneurial earnings reteined by the entrepreneurs. So,

$$\Pi_{t+1}^{ent} = [1 - \Gamma(\bar{\omega}^e)] \left(1 + R_{t+1}^K\right) Q_t \bar{K}_{t+1} \tag{117}$$

Therefore, in a symmetric equilibrium the optmimization problem for the entrepreneur reduces to the choice of K_{t+1} and $\bar{\omega}$ so as to solve:

$$\max_{\{\bar{K}_{t+1}, \bar{\omega}^e\}} \quad \mathbb{E}_t\{[1 - \Gamma(\bar{\omega}^e)] \left(1 + R_{t+1}^K\right) Q_t \bar{K}_{t+1}\} \tag{118}$$

$$\text{s.t} \quad [\Gamma(\bar{\omega}^e) - \mu G(\bar{\omega}^e)]\, \frac{1 + R_{t+1}^K}{1 + R_{t+1}} \frac{Q_t \bar{K}_{t+1}}{Q_t \bar{K}_{t+1} - EQ_{t+1}} = 1 \tag{119}$$

with the optimal debt contract constraint (the one equating expected return of the loan to the opportunity cost of issuing it) pinning down a full set of state-contingent constraints according to the realization of the idiosyncratic shock.

The problem leads the following optimality condition linking the capital/wealth ratio to the wedge betweeen gross return of capital and the safe rate that would pravail in absence of financial friction (ie. the premium on external financing in this case)

$$\kappa \equiv \frac{Q_t \bar{K}_{t+1}}{EQ_{t+1}} = \phi(s_t) \quad \text{with} \quad s_t = \mathbb{E}_t \frac{1 + R_{t+1}^K}{1 + R_{t+1}} \tag{120}$$

The previous equation links capital expanditures by the firm and financial condition as expressed by the wedge between the gross return on capital and the safe loan rate, and by entrepreneurial equity. Since capital expanditures by the firm are proportional to the net worth of the entrepreneur, then a rise in expected return on capital lower the default probability, meaning that the entrepreneur can get more laverage and issue more debt, with the proportionality factor given which is increasing in the expected return of capital. In other words, up to a certain point the entrepreneur is able to fully self-finance capital acquisition. When external funding is needed, then its capability to rise funds is positibvely proportional with its net worth: a positive shift in entrepreneur financial position causes the external finance premium to decline, as it directly reduces the default probability. As a direct consequence, representative entrepreneur is able to expand its capacity.

This completes the exposition of the financial friction in the capital market, conditioning capital demand of entrepreneur and linking it to macroeconomic condition. Actually, some business cycle implication of the introduction of the financial accellerator mechanism are noteworthy.

Firstly, the loan rate Z_{t+1} is supposed to adjust countercyclically. Since a rise in the expected return of capital lowers default probability (and in fact rises entrepreneur net worth, which is negatively tied the the propability of realization of the idiosyncratic shock under the cut-off value), then the loan rate decreases as a result of an improvement of the firm financial condition.

Secondly, financial friction reduces basically to an informational friction: the exsistence of extrnal finance premium is in fact endogeneously motivated by the agency problem that shapes the relationship beteen entrepreneurs and and the bank. As a consequence of the introduction of this agency problem, exogenous equity premium shock as well as investment shock are supposed to be reduced in the baseline SW specification.

7 Models Overview

In this section we summarize distinctive features of the battery of models we have built up in the previous sections by combining a number of frictions on labor market, goods market and capital market as well.

Model	**Label**	**Household**	**Labor Market**	**Goods Market**	**Capital Market**
Smets and Wouters baseline	*sw*	External habit	Calvo sticky wages	Calvo sticky prices	Perfect competition
Smets and Wouters augmented	*swih*	Internal habit	Calvo sticky wages	Calvo sticky prices	Perfect competition
Search model	*search*	Internal habit	Bilateral wage bargaining	Calvo sticky prices	Perfect competition
Hybrid model	*hyb*	Internal habit	Calvo sticky wages	Hybrid rigidity	Perfect competition
Full model	*full*	Internal habit	Bilateral wage bargaining	Hybrid rigidity	Financial accelerator

Table 1: Overview of the models

Now, we characterize the shock structure of each economy by underlying which shock the augmentation with frictions is supposed to endogenize. In fact, exogenous shocks capture some noise that equations of the model are not supposed to explain. Therefore, our trial is to explain part of this noisy exogeneity by introducing more structure on the model and more frictions.

Shocks	**Specification**	**Tipology**	**sw**	**sw_ih**	**search**	**hyb**	**full**
ϵ_t^B	Preference shock	AR(1)	•				
ϵ_t^N	Labor supply shock	AR(1)	•	•		•	
ϵ_t^I	Investment shock	AR(1)	•	•	•	•	
ϵ_t^Z	Productivity shock	AR(1)	•	•	•	•	•
ϵ_t^G	Government spending shock	AR(1)	•	•	•	•	•
$\bar{\pi}_t$	Inflation target shock	AR(1)	•	•	•	•	•
ϵ_t^R	Interest rate shock	WN	•	•	•	•	•
γ_t^p	Price mark-up shock	WN	•	•	•		
γ_t^w	Wage mark-up shock	WN	•	•		•	
γ_t^Q	Equity premium shock	WN	•	•	•	•	

Table 2: Structural shocks for the frictional economies

Table 2 makes it clear that as frictions are added to the baseline specifications, exogeneity is supposed to be reduced up to the point where we end up with a full frictional model. It just displays one supply shock, namely the productivity shock, and two demand shocks, namely the interest rate iid shock and the autoregressive shock to infaltion target, plus the government spending shock on the resource constraint, as we just focus on the three main markets without affecting government budget constraint.

Morover, we point out that in each model interast rate setting is shaped by a generalized taylor rule which accounts for both the current level and the dynamic behavior of the output gap the monetary autorithy uses to anchor monetary policy decisions. Defining output gap implies defining an underlying unobservable full flexible economy. In our system of models, the flexible economy is the economy with neither prices nominal rigidity nor wages nominal rigidity. Though, it displays some sources of imperfections such as extrnal habit formation, variable capital utilization and physical adjustment cost for investent.

Further, it is worth stressing that each flexible counterpart serving to the computation of output gap hinges on a shock structure which replicate the shock structure of the corresponding frictional economy, net of mark-up WN shocks the flexible economy is never hit by. Of course, the flexible dynamics are never conditioned by monetary policy shocks as money is neutral due to the absence of any form of incomplete nominal adjustment. So, table 3 sum up the shock structure of each flexible economy.

Shocks	**Specification**	**Tipology**	**sw**	**sw_ih**	**search**	**hyb**	**full**
ϵ_t^B	Preference shock	AR(1)	•				
ϵ_t^N	Labor supply shock	AR(1)	•	•		•	
ϵ_t^I	Investment shock	AR(1)	•	•	•	•	
ϵ_t^Z	Productivity shock	AR(1)	•	•	•	•	•
ϵ_t^G	Government spending shock	AR(1)	•	•	•	•	•
$\bar{\pi}_t$	Inflation target shock	AR(1)					
ϵ_t^R	Interest rate shock	WN					
γ_t^p	Price mark-up shock	WN					
γ_t^w	Wage mark-up shock	WN					
γ_t^Q	Equity premium shock	WN					

Table 3: Structural shocks for the flexible economies

From table 3 note that just the productivity shock and the government spending shock run throughout all the model economies as it is for the frictional counterpart. Note further that the absence of monetary shocks is due to the absence of a generalized taylor rule shaping monetary policy decisions, as the absence of nominal rigidity and the resulting price flexibility does not create scope for policy intervention.

8 Models Comparison: Results

Our aim reduces to replicate the performance of Smets and Wouters model by cutting down on the number of shocks it exploits as sources of business cycle fluctuations. While Smets and Wouters resort to a number of structural shock in order to capture empirically observed persistence and co-movements of aggregate quantities, then we start out our analysis by just assuming that empirical performance of our benchmark model which the authors estimate by using full information bayesian tecniques is consistent with VAR-based literature: that is to say, we do not aim at improving empirical performance of the SW model by refining estimation tecniques or by re-estimating it by accounting for the number of frictions we have introduced. Actually, we try to mimic the SW-implied dynamics for a variety of aggregate quantities by endogenizing structural shocks: this in turn implies that the shock structure of our battery of model is always reduced with respect to the benchmark; morover, it keeps reducing as we add frictions to the main blocks of the model economy. Also, we try to replicate business cycle properties of the SW model.

Mimicking the trajectoryies of aggregate quantities in this case reduces to geneating the same relative volatilities, the same persistence (ie. autocorrelation), the same co-movements (ie. cross-correlations)of series as well.

Replicating business cycle properties of the SW model means producing comparable impulse responses to the arrival of structural shocks. Naturally, our specifications spawn a smaller family of impulse responses with respect to the benchmark model as we deal with a minor number of shocks; as such, we are able to carry out comparison throughout all the models just for sub-set of the shock structure of SW, namely productivity shock for the supply side, and monetary shocks for the demand side.

8.1 Calibration

In this section we focus on the calibration of parameters for the models.

Table 4 presents a list of calibrated parameters for the Smets-Wouters model and specify which model they belong to. Note that all the parameters of the table are used in all the models, except for Calvo probabilities of price/wage reset. The Calvo signal for wage re-optimization is infact in the monopolistic labor market which is substituted in the *search* model by a bilateral wage setting device. The Calvo signal for price re-optimization is rescaled in the *hyb* model in order to account for the reduced mass of intermediate good firms that are subject to Calvo rigidities. Moreover, wage mark-up and partial indexation of wage parameters vanish out in the models without Calvo sticky wages as it is used in the log linearization of the Calvo-wage equation we just use in the *sw*, *swih* and *hyb* models.

Table 5 presents a list of parameters used only in the model featuring at least one friction. While the introduction of both the hybrid form of rigidity and the financial accellerator mechanism does not enlarge significantly the parameter space of the DSGE (ie. *hyb* model just leads four more parameters, while the *full* model adds two parameters), then the majority of parameters are related to the *search* model (and the *full* as well as it contains the search friction); they are used to model the flows of the labor market, as well as the bilateral bargaining process.

Table 6 presents a list of the parameters related to structural shock equations. Therefore, for autoregressive structural shock we have one autoregressive parameter and the standard deviation of the error term; for the WN shock, we just have the standard deviation of the error term. The values of the shock parameters are taken from Smets-Wouters (2003): authors estimate these shocks by using full information bayesian tecniques.

Parameter	**Specification**	**Value**	**Model**
β	Discount factor	0.99	sw, $swih$, $search$, hyb, $full$
σ_c	Coefficient of relative risk aversion	1.353	sw, $swih$, $search$, hyb, $full$
σ_N	Inverse of Frisch elasticity of labor supply	2.4	sw, $swih$, $search$, hyb, $full$
χ	Scale parameter for labor supply	1	sw, $swih$, $search$, hyb, $full$
h	Habit formation strenght parameter	0.573	sw, $swih$, $search$, hyb, $full$
δ	Depreciation rate of physical capital	0.025	sw, $swih$, $search$, hyb, $full$
α	Output elasticity with respect to utilized capital	0.30	sw, $swih$, $search$, hyb, $full$
r_{ss}	Steady state rental rate	0.016	sw, $swih$, $search$, hyb, $full$
Δ	Elasticity of investment adjustment cost	6.771	sw, $swih$, $search$, hyb, $full$
ψ	Elasticity of physical capital utilization cost	0.169	sw, $swih$, $search$, hyb, $full$
ϕ	Share of fixed cost in production	0.408	sw, $swih$, $search$, hyb, $full$
c_y	Share of private consumption on output	0.6	sw, $swih$, $search$, hyb, $full$
inv_y	Share of investment on output	0.22	sw, $swih$, $search$, hyb, $full$
g_y	Share of government public consumption on output	0.18	sw, $swih$, $search$, hyb, $full$
γ_w	Wage mark-up	0.5	sw, $swih$, hyb
ι_p	Price partial indexation degree	0.469	sw, $swih$, $search$, hyb, $full$
ι_w	Wage partial indexation degree	0.763	sw, $swih$, hyb
ξ_p	Calvo probability of price reset	0.908	sw, $swih$, $search$
ξ_w	Calvo probability of wage reset	0.737	sw, $swih$, hyb
ρ_R	Persistence coefficient for interest rate	0.961	sw, $swih$, $search$, hyb, $full$
r_π	Inflation coefficient	1.684	sw, $swih$, $search$, hyb, $full$
r_Y	Output gap coefficient	0.099	sw, $swih$, $search$, hyb, $full$
r_{Δ_y}	Output gap growth coefficient	0.159	sw, $swih$, $search$, hyb, $full$
r_{Δ_π}	Inflation growth coefficient	0.14	sw, $swih$, $search$, hyb, $full$

Table 4: Calibrated parameters for Smets-Wouters

Parameter	Specification	Value	Model
ξ_p^{hyb}	Rescaled Calvo probability of price reset	0.454	hyb, $full$
ϕ^{si}	Sticky information probability of price reset	0.375	hyb, $full$
J_1	Mass of Calvo rigid firms	0.5	hyb, $full$
J_2	Mass of sticky information firms	0.5	hyb, $full$
Φ	Exogenous separation rate	0.08	$search$, $full$
Γ	Matching elasticity with respect to unemployment	0.5	$search$, $full$
υ	Worker's share in wage bargaining	0.2	$search$, $full$
N	Steady state employent	0.2	$search$, $full$
U	Steady state unemployment	0.216	$search$, $full$
w_{ss}	Steady state wage	1.87	$search$, $full$
q	Vacancy filling rate	0.7	$search$, $full$
s	Job finding rate	0.25	$search$, $full$
ub_{ss}	Steady state unemployment benefit	0.187	$search$, $full$
$\frac{\kappa}{\lambda}$	Advertizing costs on marginal utility at steady state	0.0049	$search$, $full$
θ_{ss}	Steady state market tightness	0.36	$search$, $full$
R_{ss}^K	Steady state gross return of capital	8.21	$full$
$\phi(\frac{R_{ss}^K}{R_{ss}})/\phi'(\frac{R_{ss}^K}{R_{ss}})$	Ratio of the scaling function with its derivative	0.05	$full$

Table 5: Calibrated parameters for models with frictions

Parameter	Specification	Value	Model
ρ_B	AR coefficient for preference shock	0.855	*sw*
ρ_N	AR coefficient for labor supply shock	0.889	*sw*, *swih*, *hyb*
ρ_I	AR coefficient for investment shock	0.927	*sw*, *swih*, *search*, *hyb*
ρ_G	AR coefficient for government spending shock	0.949	*sw*, *swih*, *search*, *hyb*, *full*
ρ_Z	AR coefficient for productivity shock	0.823	*sw*, *swih*, *search*, *hyb*, *full*
$\rho_{\bar{\pi}}$	AR coefficient for inflation target shock	0.924	*sw*, *swih*, *search*, *hyb*, *full*
σ_B	St. dev of preference shock	0.336	*sw*
σ_N	St. dev of labor supply shock	3.520	*sw*, *swih*, *hyb*
σ_I	St. dev of investment shock	0.085	*sw*, *swih*, *search*, *hyb*
σ_G	St. dev of government spending shock	0.325	*sw*, *swih*, *search*, *hyb*, *full*
σ_Z	St. dev of productivity shock	0.598	*sw*, *swih*, *search*, *hyb*, *full*
$\sigma_{\bar{\pi}}$	St. dev of inflation targt shock	0.017	*sw*, *swih*, *search*, *hyb*, *full*
σ_{γ_p}	St. dev of the price mark-up WN shock	0.16	*sw*, *swih*, *search*
σ_{γ_w}	St. dev of wage mark-up WN shock	0.289	*sw*, *swih*, *hyb*
σ_Q	St. dev of equity premium WN shock	0.604	*sw*, *swih*, *search*, *hyb*
σ_R	St. dev of interest rate WN shock	0.081	*sw*, *swih*, *search*, *hyb*, *full*

Table 6: Calibrated parameters of structural shocks

8.2 Second Moments

In this section we analyze model-implied second moments. We focus on volatility of simulated series, autocorrelation to detect persistence, and cross-correlations to find out co-movements among aggregate quantities. We do so in order to assess wheter the introduction of frictions and the simultaneous elimination of structural shocks is sufficient to get business cycle properties of the bechmark SW model untouched, especially in terms of unconditional second moments.

We begin the analysis by focusing on model-implied volatilities.

Table 7 sums up the volatilities of simulated series for 10 quantities of interest in all the models considered.

First of all, note that regardless of the model we are taking into account, the introduction of a friction accompanied by the elimination of at least one structural shock creates additional volatility with respect to the baseline *sw* model for all the aggregte variables considered. Note further that the introduction of the search friction in the *search* model induces a striking difference in the volatility for all the series considered with respect to the other models. Consider then that the search friction is active even in the *full* model. The *full*-implied volatilities are indeed

higher than the volatilities of *sw* model but the difference is not so striking as it is for the *search* model; this means that the introduction of the financial friction alongside the dual rigidity regime taken from the *hyb* specification is able to mitigate the effect of the search friction.
If we consider the *swih* and the *hyb* models, note that the introduction of the dual rigidity affects negatively the volatility of the series, meaning that it is able to offset internal habit formation mechanism.
The interaction between the hybrid rigidity in the *hyb* model and the financial accellerator in the $full$ model is less linear. For instance, while the effect on output is neglectible, then the introduction of a financial friciton increases the volatility of consumption. In general, the $full$ model produces less pronounced volatilities than the *hyb* model, meaning that is closer to *sw* specification than the other models are. Actually the more stiking offsetting effect is the wage level. Consumption is an exception as the $full$ model creates additional volatility with respect to both *swih* and *hyb* specifications.

Variable	**sw**	**swih**	**search**	**hyb**	**full**
σ_Y	1.2741	2.6386	10.9565	2.2376	2.1975
σ_C	0.6857	1.4683	3.5420	1.3979	1.7564
σ_I	4.2335	12.9143	42.4442	8.9672	8.4337
σ_K	2.2569	5.7260	16.7660	3.2190	3.2101
σ_N	0.6990	1.6659	7.5623	1.3071	1.6583
σ_E	0.6285	1.4023	7.1424	0.9859	1.4273
σ_{r^K}	0.2977	0.8890	2.5351	0.7740	0.4848
σ_W	0.8660	6.7366	7.8985	6.2196	1.5566
σ_R	0.0945	0.2365	0.7362	0.2381	0.2546
σ_π	0.1791	0.5063	1.1684	0.0768	0.0191

Table 7: Volatility of simulated series

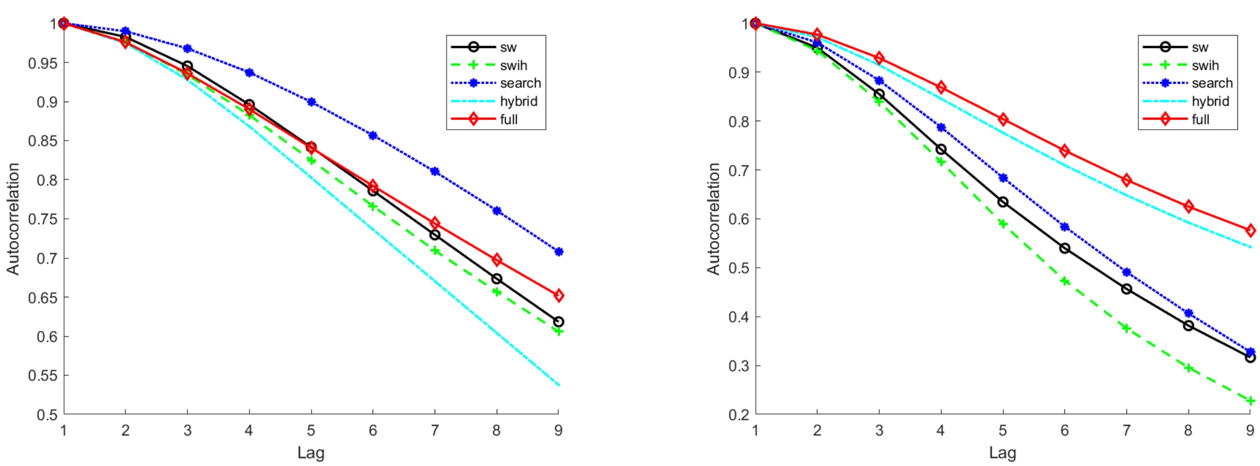
Table 8 focuses on relative volatility of simulated series. It means that each volatility is rescaled with the standard deviation of simulated output series, and so it is for all the specification considered.
Obviously, the excess volatility generated by the *search* model is reduced in a substantial way. Note further that the volatilities implied by the models are quite consistent with business cycle stylized facts; investment though is more than three times volatile than output in all the models except for the *sw* specification, meaning that the introduction of frictions creates excess volatility in the investment series. On the other hand, consumption volatility with respect to output is less than the stylized fact on business cycle dictates in all the models considered. While usually consumption is found just slightly less volatile than GDP, then it displays a half of the volatility of output. In particular, the $full$ specification moves the relative volatility of consumption close to unity, while the *sw* implied

relative volatility of consumption is fairly well replicated by the *swih* model despite the introduction of another mechanism of habit formation in the utility function.
In general, the introduction of frictions increases standard deviations of the simulated series, but the additional volatility is more self-contained if we consider relative volatilities.

Variable	**sw**	**swih**	**search**	**hyb**	**full**
σ_C/σ_Y	0.5382	0.5565	0.3233	0.6248	0.7992
σ_I/σ_Y	3.3227	4.8944	3.8739	4.0076	3.8378
σ_K/σ_Y	1.7714	2.1701	1.5302	1.4386	1.4608
σ_N/σ_Y	0.5486	0.6314	0.6902	0.5842	0.7546
σ_E/σ_Y	0.4933	0.5314	0.6519	0.4406	0.6495
σ_{r^K}/σ_Y	0.2337	0.3369	0.2314	0.3459	0.2206
σ_W/σ_Y	0.6797	2.5531	0.7209	2.7796	0.7083
σ_R/σ_Y	0.0742	0.0896	0.0672	0.1064	0.1159
σ_π/σ_Y	0.1406	0.1919	0.1066	0.0343	0.0087

Table 8: Relative volatility of simulated series

We analyze now autocorrelation of simuated series in order to draw some conclusions about the relation between the introduction of frictions and the overall capability of the models to generate the same persistence of the one implied by th *sw* specification.

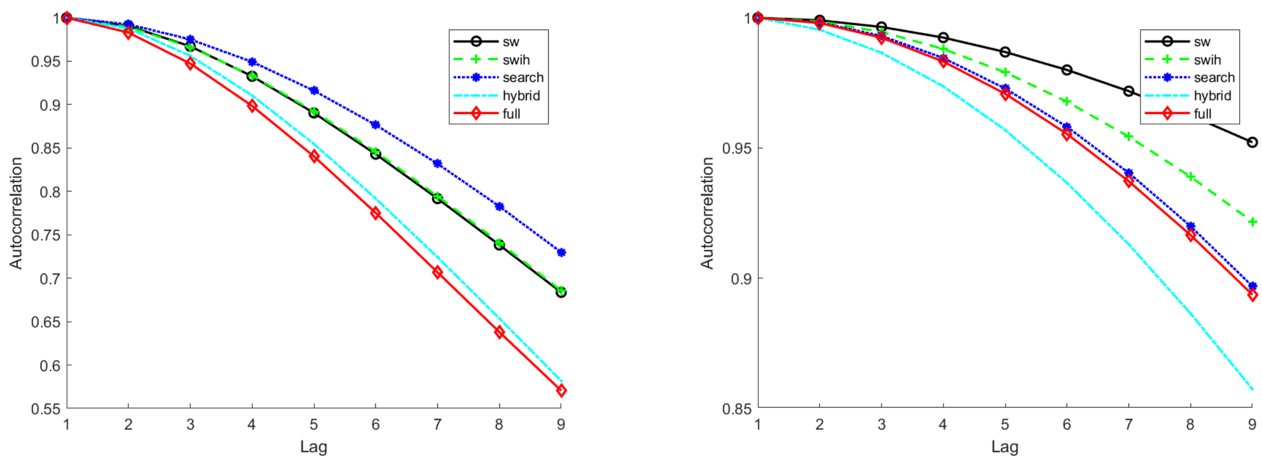


(a) output (b) consumption

Figure 9: Autocorrelations for output and consumption

As regards output, figure 9a shows that the fit between the *sw* specification and the $full$ model is remarkable.

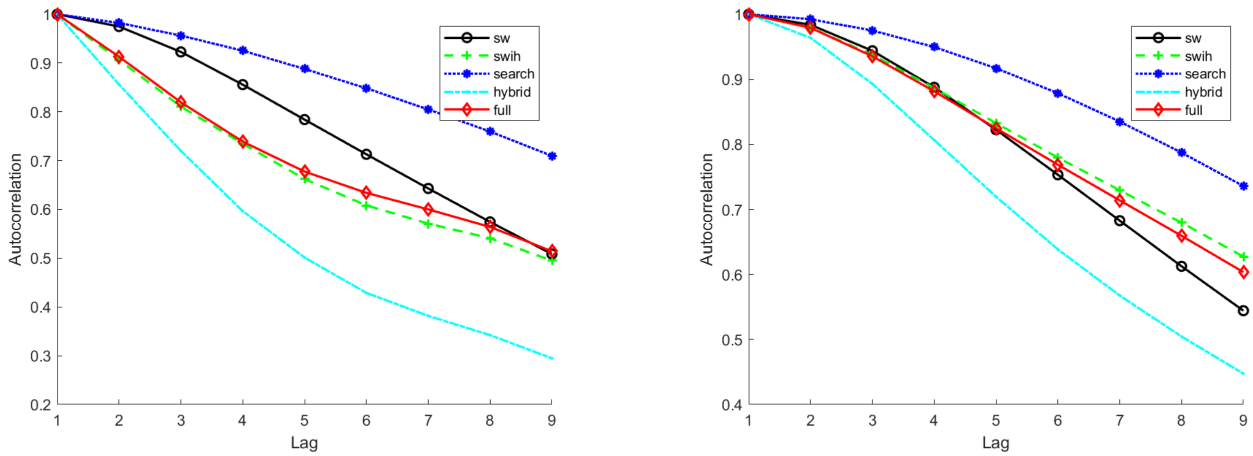
This means that the introduction of additional frictions in goods and labor market, as well as in the capital market is able to create the degree of persistence that in the SW model is ensured by the presence of autoregressive structural shocks. While the *search* model creaes additional persistence, then the introduction of financial accellerator in the $full$ specification offset this effect.
Figure 9b focuses on autocorrelation of model-implied consumption. In this case, *sw* persistence is closely replicated by the *search* model, while both the $full$ and *hyb* models create additional persistence in the series; this might be due to the presence of hybrid form of rigidity which affects inflation dynamics which in turn influences intertemporal allocation of consumption through the Euler equation, and aggregate consumption equation as well.



(a) investment (b) capital

Figure 10: Autocorrelations for investment and capital

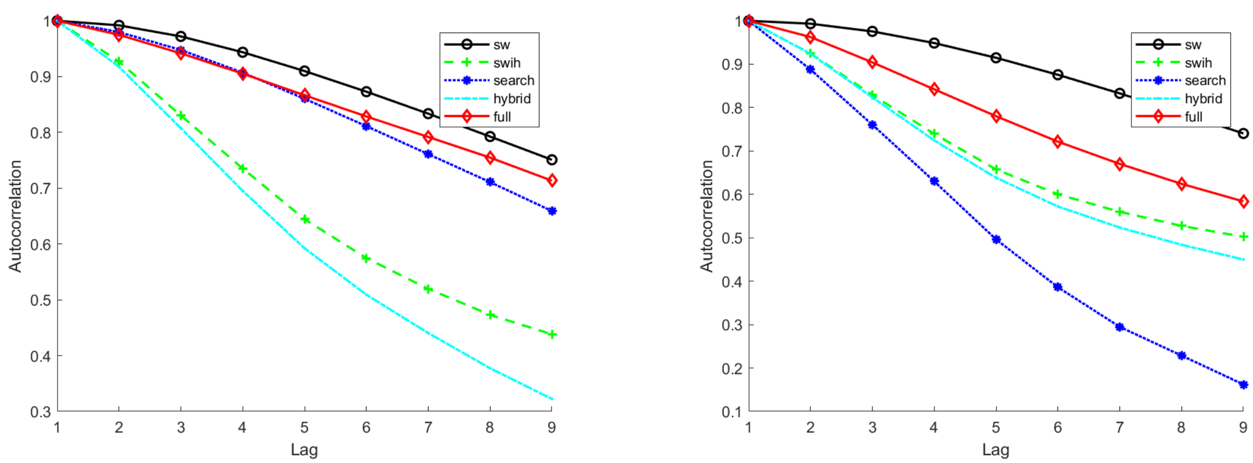
As regards investment, figure 10a shows how the introduction of internal habit formation in the *swih* model does not alter *sw* persistence of investment series. The *search* model creates additional rigidity. The *hyb* model, again, follows closely the $full$ specification due to the presence of the hybrid form of rigidity which influences investment dynamics through the market value of physical capital stock.
Figure 10b is about the persistence of utilized capital stock. While usually the introduction of the financial friction offsets the additional persistence created by the *search* model, they are able to crete the same persistence degree for utilized capital stock. In any case, the introduction of a friction weakens the persistence of the capital stock series.



(a) labor demand (b) employment

Figure 11: Autocorrelations for labor demand and emloyment

Labor demand autocorrelation is plotted in figure 11a. The *search* model creates additional persistence in the dynamics. The $full$ model replicates fairly well the autocorrelation of the labor demand implied by sw.
The analysis is the same for employment, as plotted in figure 11b. The $full$ model replicates even better the autocorrelation of sw model in terms of autocorrelation of employment with respect to the case of labor demand.

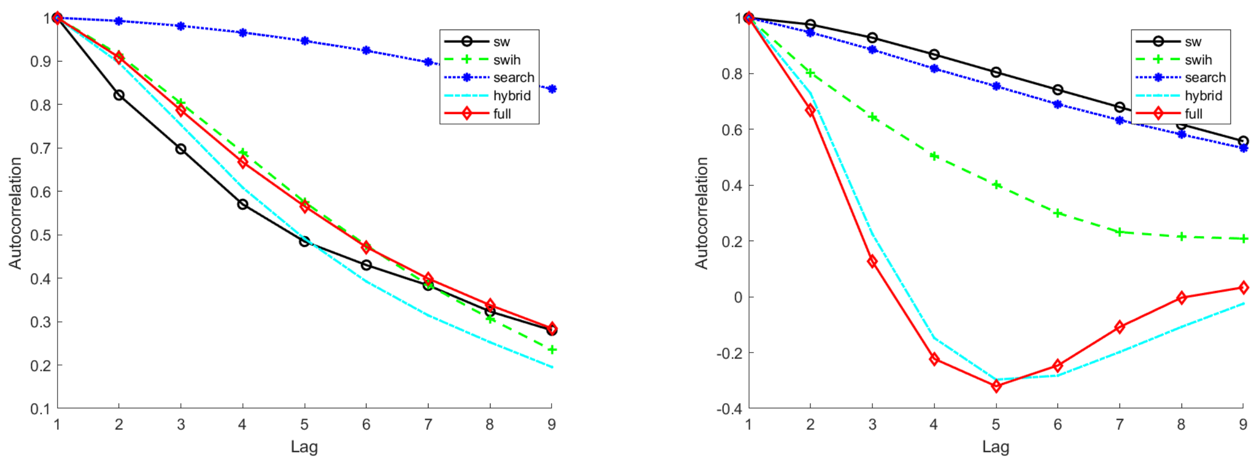


(a) rental rate (b) wage

Figure 12: Autocorrelations for factor prices

Figures 12a and 12b plot the autocorrelation of factor prices. As regards the rental rate, the hyb model and the $swih$ model reduce the persistence of the series with respect to sw. The *search* model and the $full$ one follow quite closely each other but in generale they are not able to produce the persistence of the sw model.
As regards wage, again the models with frictions are not able to produce the persistence of the sw model. The $full$ specification is the closest one though. A difference with the autocorrelation of the rental rate consists in the fact that the wage level in the *search* model does not present significant autocorrelation. This might be due to the fact that the introduction of a search friction, in any case, does not lead to any form of rigidity as it is a wage setting

device which is activated and exhausts at time t as well.



(a) interest rate (b) inflation

Figure 13: Autocorrelations for interest rate and inflation

Figure 13a plot the autocorrelation of nominal interest rate. The *search* model induces a high degree of persistence in the series. The effect is somehow offset by the introduction of the financial friction. The persistence of the interest rate in the *sw* model is replicated fairly well by the $full$ model.

As regards autocorrelation of the inflation series, figure 13b shows the existence of some non-linearities. For instance, while the *search* model replicates quite well the *sw* autocorrelation, then *hyb* model and $full$ model follow closely each other and reduce in a substantial way the persistence of the inflation series; though, instead of decaying the autocorrlation increases up from on lag thereafter. The introduction of internal habit formation in the *swih* spefication creates a degree of persistence which is unique throughout the models and the series just lies in the midde of all the other dynamics.

We turn our attention now to contemporaneous cross-correlation among simulated series to assess the capability of the frictional models to generate the degrees of comovements the *sw* baseline specification is able to deliver.

Note from table 9 that *sw* specification delivers a high degree of comovement between output and real variables, which is particularly striking for investment, labor demand and employment. Rental rate of capital is an exception as it is substantially acyclical.

The introduction of internal habit formation decreases the comovement degree among variables, while it inverts the sign of contemporaneous cross-correlation between output and consumption. The *hyb* model relates to the *sw* baseline specification in the same way; it reduce contemporaneous cross-correlation among variables, while it reverts the sign of output correlation with interest rate.

The *search* model produces generally the same contemporaneous correlations of *sw*; while it increases the correlation with rental rate of capital, then the intoduction of the search friction weakens the corrlation between output and wage, thus inducing the a-cyclicality which is actually observed in empirical series.

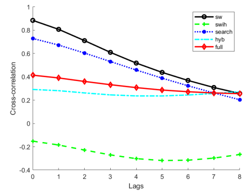
The overall effect of the $full$ model is to reduce cyclicality implied by the *sw* specification; though, the reduction of cross-correlations is less pronounced than it is in the *swih* and in the *hyb* model. Further, it is worth stressing thah the $full$ model invert the sign of the cross-correlation between outpt and nominal variables, namely interest rate and inflation; while inflation is made a-acyclical, then interest rate turns out to be counter-cyclical; this means that the introduction of friction in the $full$ specification is able to solve one of the shortcomings of real business-cycle literature consinting in failing to generate empirically observed negative relation between output and nominal interest rate.

Variable	**sw**	**swih**	**search**	**hyb**	**full**
C	0.882	−0.132	0.742	0.282	0.405
I	0.978	0.902	0.986	0.901	0.709
K	0.469	0.530	0.369	0.391	0.408
N	0.975	0.315	0.943	0.004	0.660
E	0.983	0.395	0.953	0.066	0.749
r^K	0.249	0.196	0.323	0.650	0.230
W	0.943	0.519	0.485	0.734	0.560
R	0.567	0.237	0.755	−0.335	−0.421
π	0.990	0.562	0.876	0.068	−0.027

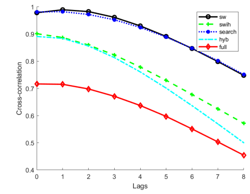
Table 9: Contemporaneous correlation of simulated series with respect to output

We now draw some insights about co-movements by focusing on cross-correlations between agrgegate quantities with lagged output. Figure 14 plots a battery of lagged cross-correlations.

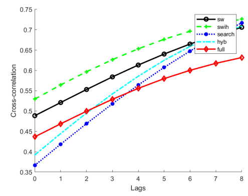
As it results from the analysis of contemporaneous cross-correlations in table 9, the $full$ model reduce the degree of co-movements and it does so even if we lag output. In fact, the trajectory of the *sw* specification and the path of $full$ model run in parallel, except for consumption, employment and labor demand. The $search$ model follows closely *sw*-implied dynamics, except for wage due to the introduction of the bilateral wage setting process. Even for the case of cross-correlations with lagged output, the $full$ model mitigates the effect of the introduction of the $search$ friction.



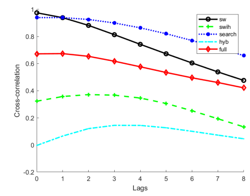
(a) Cross-correlation of consumption



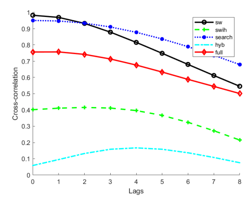
(b) Cross-correlation of investment



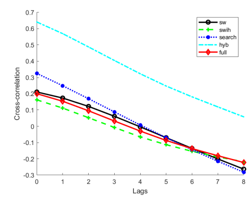
(c) Cross-correlation of capital



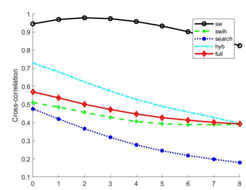
(d) Cross-correlation of labor demand



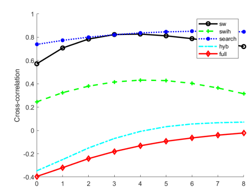
(e) Cross-correlation of employment



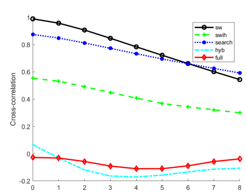
(f) Cross-correlation of rental rate



(g) Cross-correlation of wage



(h) Cross-correlation of interest rate



(i) Cross-correlation of inflation

Figure 14: Cross-correlations with lagged output

8.3 Impulse Response Analysis

In this section we focus on impulse responses of a number of both real and nominal variables to the productivity shock (ie. the aggregate productivity shock which enters the Cobb-Douglas production function) and the purely monetary shock.

We intentionally leave aside the government spending shock and the inflation target shock. As regards the government spending shock, it appears in the *full* model just because we have not focused on fiscal policy implication od the SW model and we have not introduced friction in public spending financing. As regards inflation target shock, we want to stress that inflation benchmark is fixed in european monetary union meaning that there is a time trend in the inflation process; so we are not going to investigate business cycle effects deriving from assuming a stationary process for inflation target. orevor, we rule out trend inflation from our analysis.

We begin our analysis of business cycle properties by analyzing responses to a positive productivity shock.

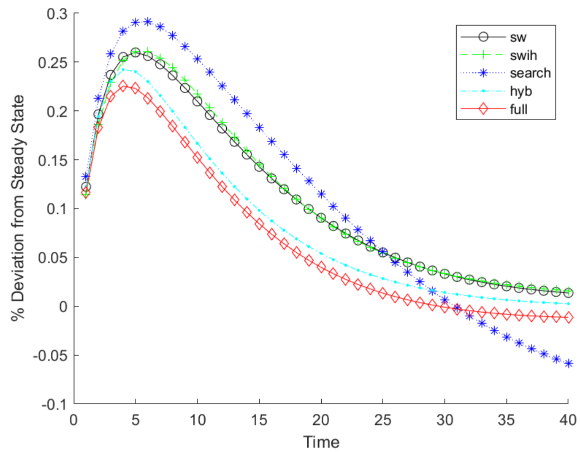


Figure 15: IRF of output to technology shock

From figure 15 note firstly that the introduction of internal habit formation does not alter the response to productivity shock as trajectories of the *sw* and *swih* models are quite close. Hump-shaped response is mantained in all models, the *search* one being an exception: while output response is higher in magnitude with respect to the other models, then the effect of the AR(1) shock fades out more quickly as the return trajectory toward the steady state present a less pronounced inflection point. Note further that in the *full* model, productivity shock displays a lower impact than in all the other model.

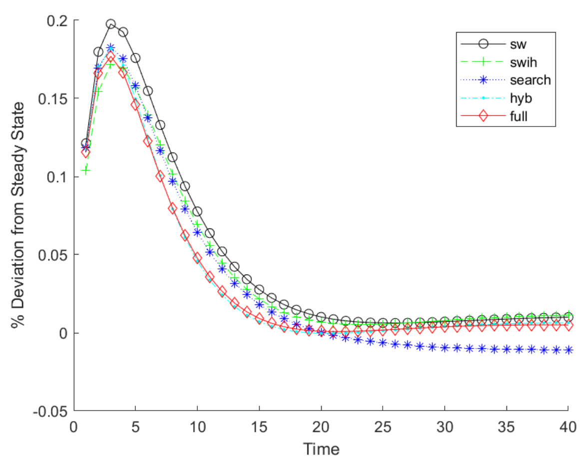


Figure 16: IRF of consumption to technology shock

As it is clear from figure 16, the introduction of a number of frictions does not alter the business cycle properties of consumption dynamics in response to a positive productivity shock; except for the *search* model, the dynamics follow closely each other both in the magnitude of the response to the shock and in the intensity of the propagation mechanism. Therefore, the introduction of multiple frictions does not affect SW-implied consumption dynamics.

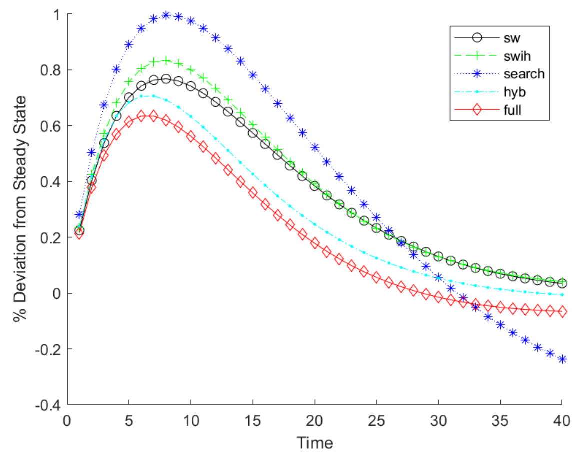


Figure 17: IRF of investment to technology shock

Figure 17 plots responses of investment. Still, it is the more volatile component of output, which is consistent with business cycle stylized facts. The magnitude of the response generated by the *search* model is 4 points larger that that one generated by the *full* model, with the *sw* model producing a response of investment which stays in the middle. As in the case of output, the introduction of a search friction negatively affects internal propagation mechanism of the productivity shock of the search model, as the effect of the shock vanishes out more quickly than it happens for the other model.

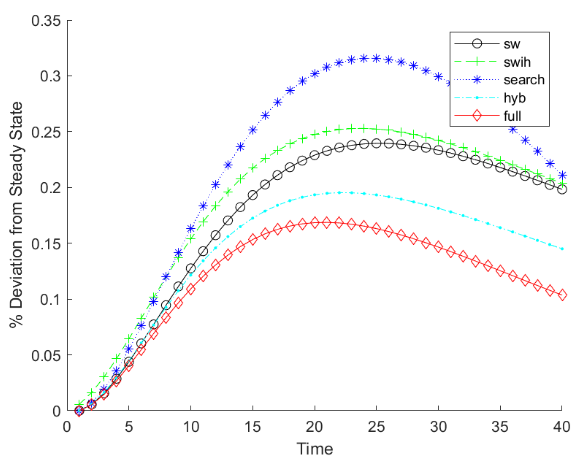


Figure 18: IRF of capital to technology shock

Figure 18 focuses on the utilized capital stock. Again, *search* model increases the magnitude of the impact of the productivity shock with respect to the baseline *sw* specification. In spite of the magnitude of the response to the shock, dynamics of utilized capital stock are quite persistent in all the models.

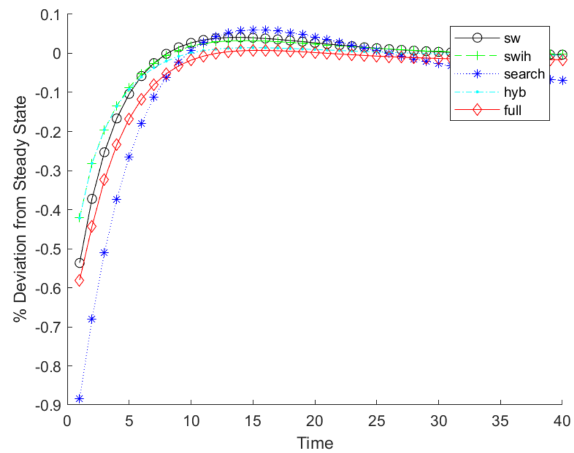


Figure 19: IRF of labor demand to technology shock

Figure 19 makes it clear that labor demand dynamics in response to a positive productivity shocks are similar regardelss of the introduction of the introduction of frictions; as such, we can conclude that the introduction of frictions doea not alter business cycle properties of labor demand in response to the productivity shock in the SW model.

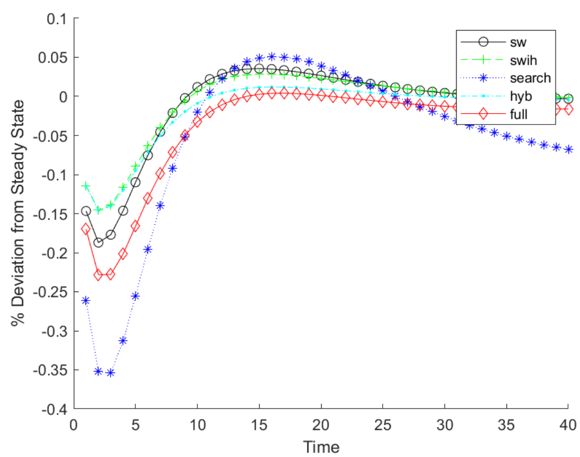


Figure 20: IRF of employment to technology shock

From figure 20, we note instead that employent responds more deeply in the *search* model than it does in all the other ones. The decrease of employent in response to productivity shock is consistent with business cycle stylized facts. Though, additional rigidity in the labor market created by a wage setting mechanism which is dependent on transition probabilities makes the fall in employment more pronunced. Again, the introduction of a search friction alters negatively the internal propagation mechanism of the SW model as the recover trajectory of emloyment is steeper.

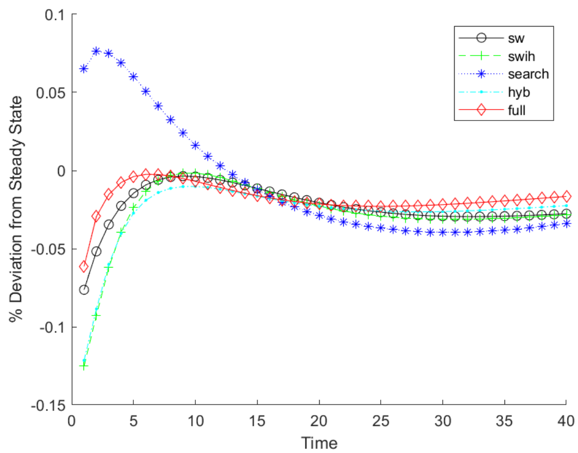


Figure 21: IRF of rental rate to technology shock

For rental rate, figure 21 shows how the technology shocks make the relative price of capital services fall toward the steady state after a spike in the very short run; in the *search* model the magnitude of the response is amplified with respect to all the other models.

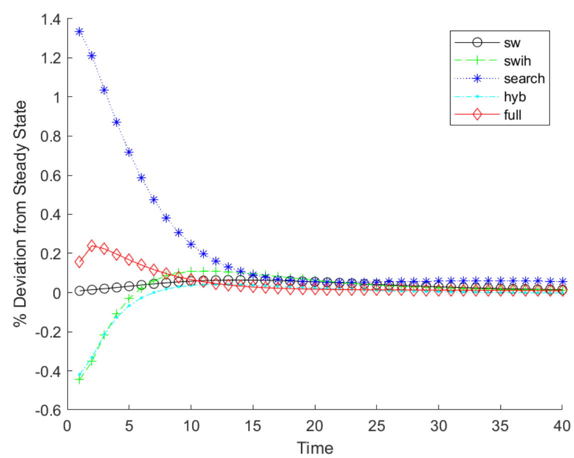


Figure 22: IRF of wage to technology shock

From figure 22 not that wages shows acyclicality with respect to technology shock. They display an immediate spike in the short run then they come back to the steady state. Again in the *search* model, wage level goes down monotonically to the steady state while the effect of the productivity shock is more pronounced than it is for the other models. Noteworthy, $full$ model displays strong internal propagation mechanism as it includes the search friction.

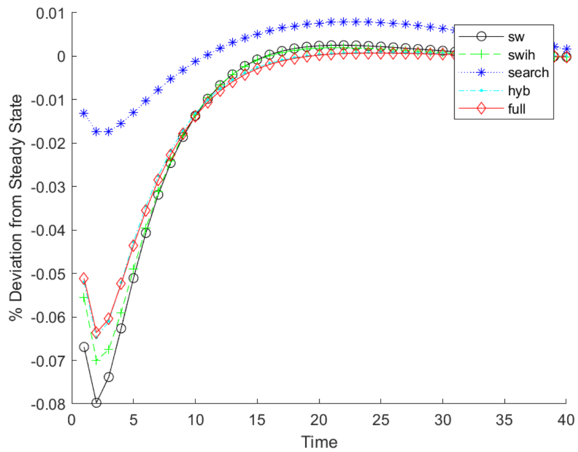


Figure 23: IRF of interest rate to technology shock

Figure 23 plots the impact of a positive technology shock on nominal interest rate set by the monetary authority; interest rate decrease in all the model with differences in the magnitude of the decrease. In the *search* model the negative effect is moderate while the higher impact is seen in the *sw* specification. The *full* model follows quite closely *sw*. The timing of the return to the steady state is equivalent in all the models.

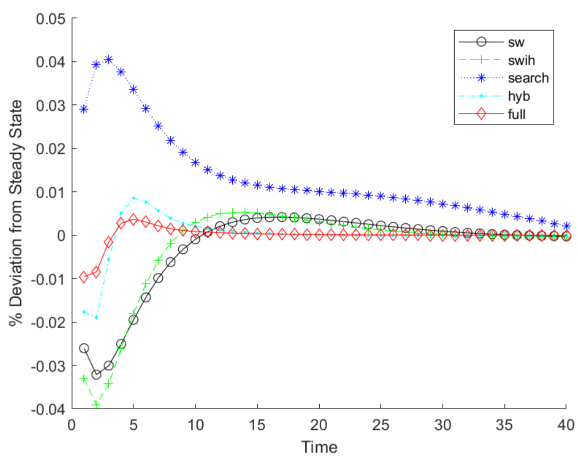


Figure 24: IRF of inflation to technology shock

From figure 24, note that the dynamics of inflation are quite complex. Except for the *hyb* model, then all the models display the hump-shaped type of response to technology shock. Both in the *sw* and in the *swih* models inflation immediately fall as a result of a technology shock; it falls in the *hyb* model too, with the difference that the impact of the shock is more self-contained both in terms of magnitude and in terms of internal propagation mechanism. In the *search* model and in the *full* model, inflation rise after a technology shock.

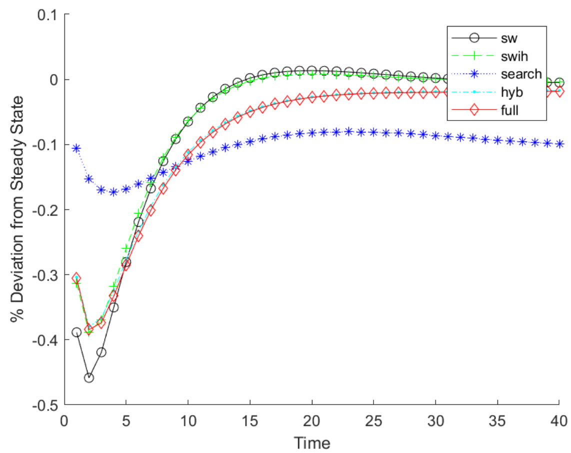


Figure 25: IRF of output gap to technology shock

Figure 25 is the response of output gap to positive productivity shock. Note that deviation from the steady state are negative in all the models, with difference in the magnitude. For the *search* model, output gap negatively deviates from its steady state just moderately, while the deviation is more pronounced in *sw* specification. The *swih*, the *hyb* and the *full* model move quite closely. In this case, the *search* model creates additional persistence in the dynamics.

The overall analysis of impulse responses to a positive technology shock gives several insight. Firstly, note that the direction of responses for real variables is consistent with the effect of a positive technical change, as it is laid down by real business cycle literature. That is, output, consumption, and investment with physical capital stock increase; employment goes down due to the technological improvement of the economy. Factor prices display some degree of acyclicality due to the presence of both nominal and real rigidities. In particular, capital demand is pinned down by the neoclassical relation equating the marginal product of capital to the rental rate; investment decisions are allocated upon the household sector, meaning that the reantal rate equates the marginal cost of variable capital utilization. Therefore, a positive technology shock shifts the capital demand up, thus increasing the rate at which household sector rent installed capital services to the intermediate good firms. By cost minimization problem, labor demand is defined by the equalization between the wage level and the marginal product of labor in all the specifications we consider; labor supply is influenced by the presence of Calvo rigidities or by the search friction in the *search* and *full* models. Therefore, for *sw*, *swih* and *hyb* models, the positive technology shock increases the labor demand, and this increase make the wage level goes up; acyclical behavior is the result of the presence of Calvo adjustments in the wage setting process. As regards interest rate and the aggregate price level (ie. inflation if we consider the rate of change), propagation channel of the positive technology shock is provided by the taylor rule which consider both the current output level and lagged one; inflation is supposed to decrease through the channel of marginal cost.

In general, note that the *search* model magnifies the response of aggregate quantities to the positive technology shcok, but it generally lacks of internal propagation mechanism as it is not able to deliver traditional hump-shaped responses of variables; this might be associated with the fact that the introduction of bilateral wage bargaining as wage setting device does not add rigidity in the labor market, but it just embdeds in the general equilibrium system another wage setting mechanism which begins and exhausts in t. The $full$ model contains the search friction but it does not deliver the same propagation dynamics: in fact, the $full$ specification preserves hump-shaped response of variables unlike the *search* one, but reduces the impact of the technology shock with respect to *sw* model. The *swih* model delivers approximately the same dynamics of the *sw* model. We must stress though that the *swih* model just introduces additional rigidity in the utility function through internal habit formation mechanism which changes the structure of marginal utility; aggregate prodictivity shock just influences consumption dynamics through the channel of marginal costs and inflation, and aggregate consumption positively depend on inflation level in the same way regardeless the type of habit formation we are dealing with. Therefore, supply-side shock as productivity shock, is not supposed to propagates through habit formation mechanism. The *hyb* model follows closely the *sw* specification and it does not delivers meaningful anomalies in the dynamics.

We now focus on impulse responses to a positive monetary shock, which actually results in a rise of nominal interest rate; therefore, a positive monetary shock appoximates a contractionary monetary policy intervention.

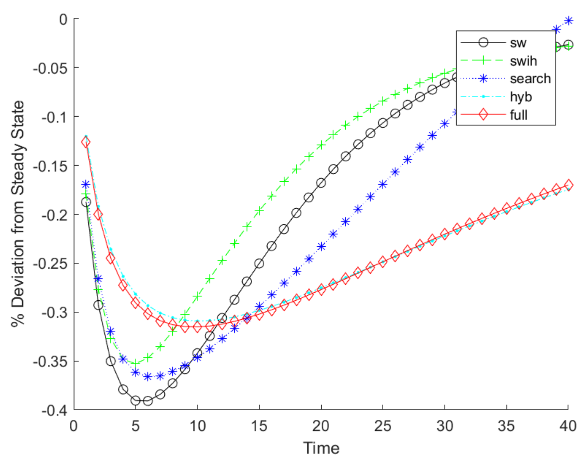


Figure 26: IRF of output to monetary shock

As a consequence of a positive monetary shock, output declines, as shown in figure 26. Magnitude and shape of the output dynamics differ though. While *sw* and *swih* mantain the traditional hump-shaped response of output to monetary shock, this shape is even more pronounced in the $full$ and *hyb* models as the internal propagation mechanism is somehow enhanced. The magnitude of the *sw* response is replicated by the *search* model which returns to the steady state more quickly than output does in the $full$ specification. Again, it seems that the introduction of the financial accellerator mechanism mitigates the search friction.

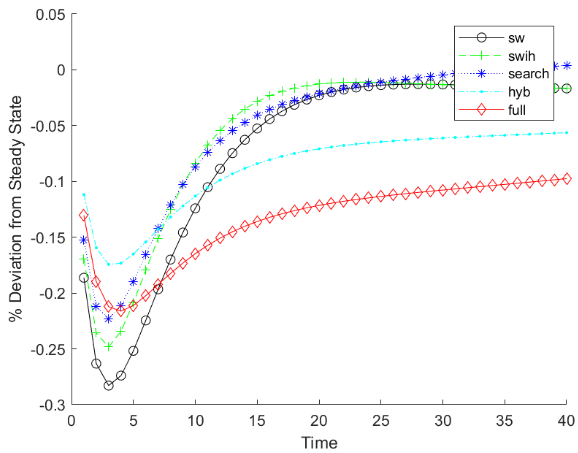


Figure 27: IRF of consumption to monetary shock

Figure 27 plots the response of consumption to an increase in the nominal interest rate. Consumption falls in all the models. While *swih* and *search* model replicates the internal propagation mechanism of *sw* specification, then *full* and *hyb* models improves it as the return traiectory towards the steady state is flatter.

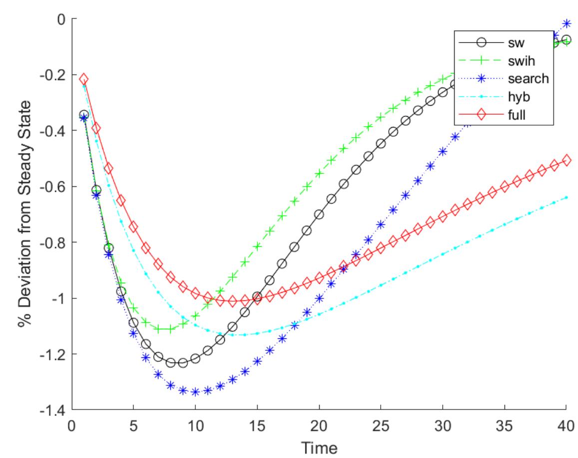


Figure 28: IRF of investment to monetary shock

Investment is the more volatile component of output as it is confirmed by figure 28 which plots the reponses of investment to positive monetary shock. The magnitude of the *sw* specification is enhanced by the *search* model and the return trajectory towards the stationary state as well. Investment in *swih* model responds more softly to positive monetary shock, as *full* and *hyb* specifications do. Though, *full* and *hyb* models improve internal propagation mechanism as the rerurn dynamics of investment is flatter. As regards the magnitude, there are 4 points of difference between the *search* model which displays the deepest decrease of investment as a result of an increase in nominal interest rate, and the *full* model which displays the most flexible response.

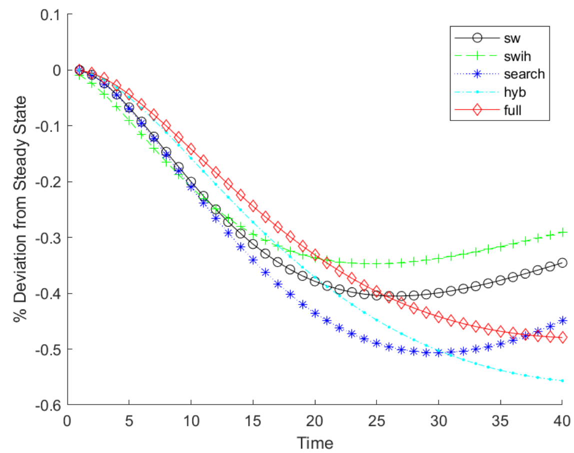


Figure 29: IRF of capital to monetary shock

From figure 29, note that utilized capital stock decreases in all the models and that it is quite acyclical. Again, while the *search* model improves the magnitude of the response of to the shock with respect to *sw*, then return to the steady state is shaped the same way. Note that *hyb* and *full* enhance internal propagation mechanism.

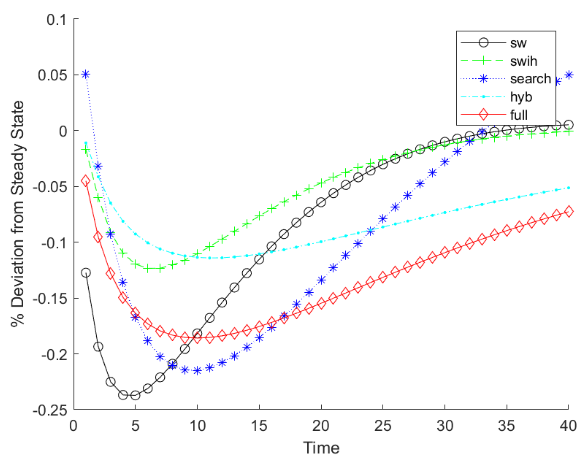


Figure 30: IRF of labor demand to monetary shock

From figure 30, note that labor demand decreases in all the model as a consequence of an increase in the nominal interest rate set by the monetary autorithy. The introduction of internal habit formation in *swih* makes the labor demand more flexible with respect to the *sw* specification. The magnitude of the response we observe in *sw* is replicated quite closely by both *search* and *full* models, with the real difference that that the introduction of the financial accellerator friction alongside the search friction in the *full* specification improves internal propagation mechanism. While the *search* model is even faster than *sw* in returning back to the stationary state, *hyb* specification is similar to the *full* one, with the difference that the magnitude of the response in *hyb* is much more self-contained.

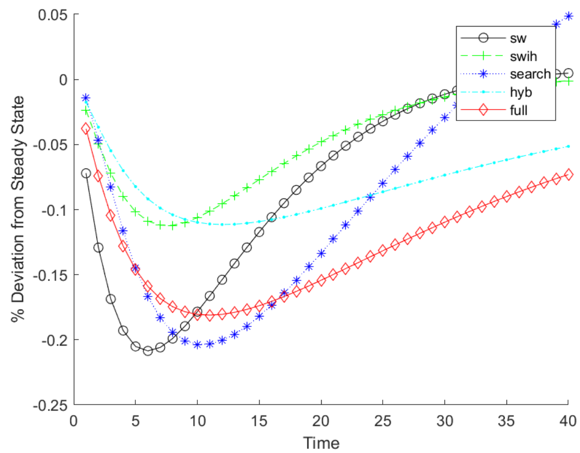


Figure 31: IRF of employment to monetary shock

Employment falls in response to a positive monetary shock as plotted in figure 31. Dynamics are quite similar to the ones of labor supply. The difference stays in the magnitude of the responses. The dynamics of the *sw* specification is in fact shifted upward by 0.5 points, and this makes the *search* model mimic quite closely *sw* speficication. All the other dynamics are the same.

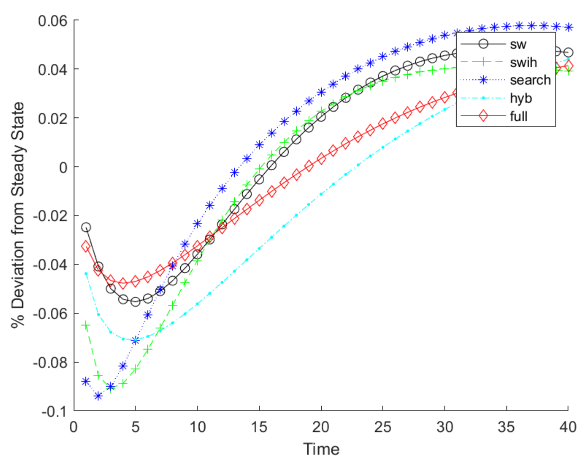


Figure 32: IRF of rental rate to monetary shock

Rental rate declines as a result of an increase in the interest rate as shown in figure 32. The $full$ model replicates quite closely *sw*, though the return towards the steady state is flatter. Moreover, *search* and *swih* enhance the magnitude of the response with respect to *sw*, while the internal propagation mechanism stays the same. Return dynamics of *hyb* and $full$ are the same, but in *hyb* the response is deeper.

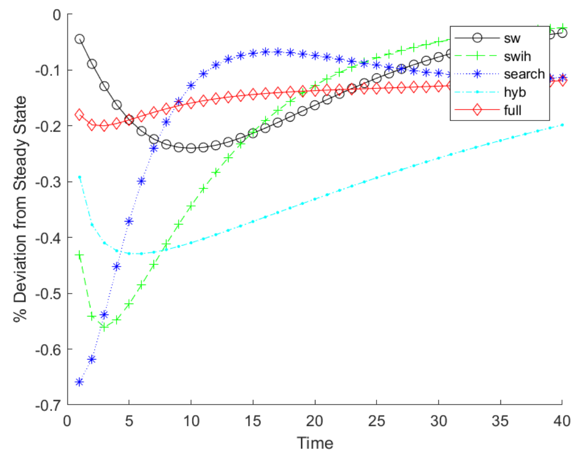


Figure 33: IRF of wage to monetary shock

Figure 33 plots the reposnse of wage. They fall in all the models but there are substantial differences in the magnitude. Note firstly that *sw* tarjectory are replicated quite closely by *full* model. In the *search* model the return trajectory is steeper than it is in all the other models. Again, it seems that the introduction of the financial friction mitigates the effect of the search friction.

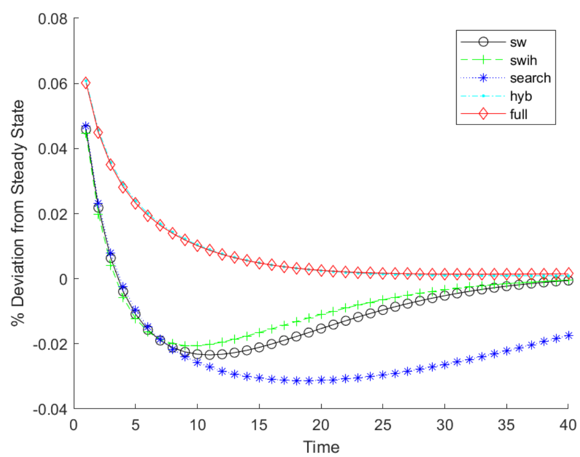


Figure 34: IRF of interest rate to monetary shock

Figure 34 displays a counterintuitive decrease in nominal interest rate as aa result of a positive moentary shock. This is consistent with Smets and Wouters (2007). In might be due to the structure of the taylor rule itself.

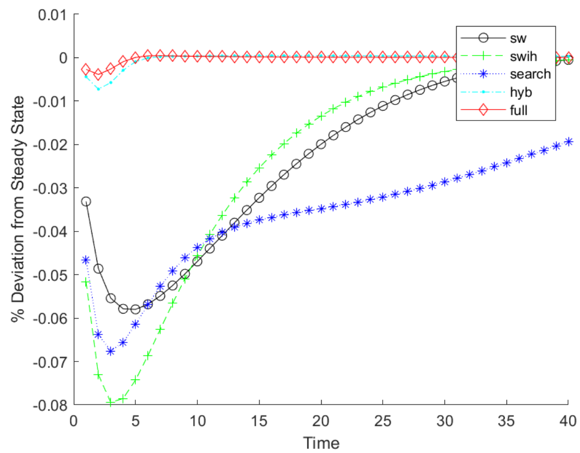


Figure 35: IRF of inflation to monetary shock

Figure 35 plots the response of inflation. As a result of a positive monetary shock, it falls meaning that this monetary policy intervention produces a generalized slow-down of economic activity. The fall is moderate in $full$ and *hyb* specification, while *sw* is quite well approximated by the *search* model. Note that the introduction of internal habit formation in the utility function in *swih* amplifies the effect of a positive monetary shock on inflation dynamics.

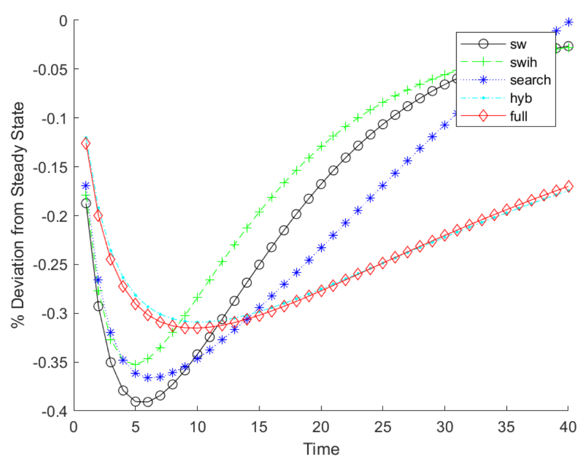


Figure 36: IRF of output gap to monetary shock

Figure 36 is about the response of output gap. In all the models it falls as output falls. This is easily explained by the neutrality argument which holds for all the flexible economies that are used as benchmark (frictionless) economies for the computation of output gap in the Taylor equation for nominal interest rate setting. Since prices are fully flexible and there are not nominal rigidities of any kind, then the purely monetary shock does not have an impact in the flexible system as monetary innovations are suddenly absorbed in the aggregate price level.

We now draw several conclusions from the analysis of impulse responses to a positive monetary shock, which translates to a contractionary monetary policy intervention.

First of all, note that the impulse responses are consistent with VAR-based literature on the effect of purely monetary shocks; see Christiano, Eichenbaum and Martins (1999). Therefore, output, consumption and investment alongside utilized capital stock fall in response to a contractionary monetary policy intervention. Factor prices fall as well. Employent and labor demand fall as a direct consequence of the general contraction of economic activity. The effect on nominal variables such as inflation are straghtforward to extract due to the backward looking behavior of agents pinned down by the inflation dynamics subject to a dual rigidity regime.

In general, the *search* models does not magnify the response to positive monetary shock with respect to the *sw* model, as it does with a positive productivity shock. The *full* model is not able to replicate business cycle properties of the *sw* model as well as it does for the supply-side shock; though, it generally enahnces internal propagation mechanism and preserves hump-shaped response fo variables.

For output and consumption, the response of the *sw* model to contractionary monetary policy intervention is not replicated by any of the specifications considered; the response of investment is more pronounced in the *search* model than it is the the *sw* one, but it lacks of internal propagation as the return trajectory to the steady state is immediate. The *search* model creates some unpredictability on the dynamic repsonse of wages which is mitigated in the *full* model by the combination of the hybrid rigidity and the financial accellerator mechanism.

As for the case of positive technology shock, the *full* model displays an offsetting effect with respect to the *search* one. Nature and causes of this effect have to be investigated more deeply; bilateral wage setting pins down a relation between profits of the representative firm, which in turn depends on its capital demand (and on the external finance premium consequently) and the wage level, as the bargained wage is exactly the one that maximize weighted joint surplus.

We stress further that the introduction of internal habit formation in the *swih* model produce the same dynamics as the *sw* one, but it increase the response of factor prices and inflation through the channel of marginal costs.

9 Conclusion

Dynamic Stochastic General Equilibrium (DSGE) models are a fundamental tool for the analysis of business cycle fluctuations. Despite their popularity, they suffer of a number of methodological shortcomings, relating mainly to their disregard for out-of-equilibrium dynamics, which is first-order with respect to other weaknesses related to estimation tecniques and empirical performance. The fact that the typical DSGE model hinges on a notion of economic dynamic system which settles on a stationary long run equilibrium and exhibits balanced growth dynamics implies that the unique way to make the dynamic system diverge from its deterministic trajectory (ie. to make the model economy create fluctuations around the deterministic long run trend) consists in hitting the evolving system itself with exogenous stochastic disturbances. Structual shocks propagates in the model economy according to their persistence degree, make aggregate quantities diverge from their trend by exploiting internal propagation mechanism, and then fade out. Therefore, what we are able to observe in sub-samples of the stochastic process of aggregate quantities is actually a business cycle fluctuation.

Though, DSGE models rely on a great mass of exogeneity to create fluctuations in the economic activity. We point out that how much exogeneity we can allow is a crucial issue in dynamic macroeconomic modelling.

Among the widespread number of DSGE model, the present thesis takes the Smets-Wouters DSGE model as a benchmark model for two reasons. Firstly, it features a number of non-walrasian traits such as internal habit formation, investent adjustment costs and variable capital utilization alongside typical DSGE modelling solutions like nominal (Calvo-type) rigidities. Secondly, it exploits ten exogenous shock to induce business cycle fluctuations. In this connection, we begin to endogenize each shock by introducing frictions in goods market, labor market and capital market. The criterion by which we assume that the introduction of a certain friction is supposed to endogenize the effect of one shock rather than the other is somehow qualitative. Exogenous shocks are noisy gaussian disturbances: so, we associate the introduction of internal habit fomation to the preference shock; the hybrid form of rigidity is linked to the price mark-up shock; the labor market friction is associated with the elimination of labor supply shock and wage mark-up shock; the financial accellerator mechanism affects the equity premium shock and the investment shock. Therefore, we extract a $full$ DSGE model with four shocks from a benchmark DSGE mdoel with ten shocks.

Then we analyze unconditional second moments and impulse responses to assess whether business cycle properties of the original model are unaltered despite the elimination of multiple sources of stochastic fluctuations. What we find is that the introduction of one friction at a time does not preserve business cycle properties of Smets-Wouters model better than the simultaneous introduction of all the frictions does. Moreover, the labor market friction alongside internal habit formation magnifies the impulses of aggregate quantities to both productivity and monetary shock but reduces the internal propagation mechanism. The $full$ model in general mitigates the effect of the $search$ model both in terms of unconditional second moments and in terms of impulse responses. In general, the $full$ specification is the closest to Smets-Wouters benchmark model in terms of dynamics.

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A Appendix A

In this section we provide log-linear approximations of optimality conditions; this means that each variable is expressed as the log-deviation of itself from its steady state value, according to the following rule:

$$\hat{x}_t = log(x_t) - log(x) \tag{A.1}$$

meaning that,

$$x_t = xexp\{\hat{x}_t\} \tag{A.2}$$

By taking first order Taylor expansion on the exponential term around zero, we get

$$x_t \approx x(1 + \hat{x}_t) \tag{A.3}$$

The Appendix A is organized as follows: the first section provides log-linear approximations for the baseline Smets-Wouters model. The following sections modify the log-linear system of Smets-Wouters model by substituting/adding up log-linear equations according to what the introdution of a particular friction dictates. This means that whenever a variable appears twice, both in the SW system and in one of the frictional system, then the frictional one has to be considered. Whenever a variable is introduced in the frictional model without already being in the SW system, then it must just be added.

Log-linear equations are used to run stochastic simulation of the models in order to obtain both simulated series of a number of aggregate quantities and impulse response functions.

A.1 Log-Linear system for Smets-Wouters (sw)

$$\lambda = \left(C - hC^{ext}\right)^{-\sigma_c} \tag{A.4}$$

$$\hat{\lambda} = \hat{\epsilon}_t^B - \frac{\sigma_c}{1-h}\left(\hat{C}_t - h\hat{C}_{t-1}^{ext}\right) \tag{A.5}$$

$$\hat{C}_t = \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}\mathbb{E}_t\hat{C}_{t+1} - \frac{1-h}{(1+h)\sigma_c}\left(\hat{R}_t - \mathbb{E}_t\hat{\pi}_{t+1}\right) + \frac{1-h}{(1+h)\sigma_c}\hat{\epsilon}_t^B \tag{A.6}$$

$$Q = 1 \tag{A.7}$$

$$\hat{I}_t = \frac{\hat{I}_{t-1}}{1+\beta} + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{I}_{t+1} + \frac{\Delta}{1+\beta}\hat{Q}_t + \hat{\epsilon}_t^I \tag{A.8}$$

$$\hat{Q}_t = \mathbb{E}_t\pi_{t+1} - \hat{R}_t + \frac{1-\delta}{1-\delta+r^K}\mathbb{E}_t\hat{Q}_{t+1} + \frac{r^K}{1-\delta+r^k}\mathbb{E}_t\hat{r}_{t+1}^K + \hat{\gamma}_t^Q \tag{A.9}$$

$$K = \frac{I}{\delta} \tag{A.10}$$

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \frac{I}{K}\hat{I}_{t-1} \tag{A.11}$$

$$w = (1+\gamma_w)\frac{w^{disut}}{\lambda} \tag{A.12}$$

$$\hat{w}_t = \hat{\epsilon}_t^N + \sigma_N\hat{N}_t - \hat{\lambda}_t \tag{A.13}$$

$$y = K^\alpha N^{1-\alpha} \tag{A.14}$$

$$\hat{y}_t = \hat{\epsilon}_t^Z + \alpha\left(\hat{K}_{t-1} + \frac{1}{\psi}\hat{r}_t^K\right) + (1-\alpha)\hat{N}_t \tag{A.15}$$

$$MC = r^\alpha w^{1-\alpha}\left(\alpha^{\alpha-1}(1-alpha)^{-\alpha}\right) \tag{A.16}$$

$$\hat{MC}_t = \alpha\hat{r}_t^K + (1-\alpha)\hat{w}_t - \hat{\epsilon}_t^Z \tag{A.17}$$

$$N = (1-\alpha)^{\frac{1}{\alpha}} w^{-\frac{1}{\alpha}} (\hat{\epsilon}_t^Z)^{\frac{1}{\alpha}} K \tag{A.18}$$

$$\hat{N}_t = \hat{K}_t + \frac{1}{\alpha}(\epsilon_t^Z - \hat{w}_t) \tag{A.19}$$

$$\hat{E}_t = \beta\mathbb{E}_t\hat{E}_{t+1} + \frac{(1-\beta\xi_e)(1-\xi_e)}{\xi_e}(\hat{N}_t - \hat{E}_t) \tag{A.20}$$

$$p = (1+\gamma_p)PMC \tag{A.21}$$

$$\hat{p}_t = \hat{MC}_t \tag{A.22}$$

$$\hat{P}_t = \xi_p(\iota_p\hat{\pi}_{t-1} + \hat{P}_{t-1}) + (1-\xi_p)(\hat{p}_t) \tag{A.23}$$

$$\hat{\pi}_t = \frac{\beta}{1+\beta\iota_p}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\iota_p}{1+\beta\iota_p}\hat{\pi}_{t-1} + \frac{1}{1+\beta\iota_p}\frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\hat{MC}_t + \hat{\gamma}_t^p \tag{A.24}$$

$$\begin{aligned}\hat{w}_t = {} & \frac{\beta}{1+\beta}\mathbb{E}_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{\pi}_{t+1} - \frac{1-\beta\iota_w}{1+\beta}\hat{\pi}_t \\ & \cdots + \frac{\iota_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w\frac{\gamma_w+(1+\gamma_w)\sigma_N}{\gamma_w}}\left(\hat{w}_t + \hat{\lambda}_t - \sigma_N\hat{N}_t + \hat{\epsilon}_t^N\right)\end{aligned} \tag{A.25}$$

$$\hat{R}_t = \rho_r\hat{R}_{t-1} + (1-\rho_r)\left[r_\pi(\hat{\pi}_{t-1} - \hat{\bar{\pi}}) + r_Y(\hat{Y}_t - \hat{Y}_t^{flex})\right] + r_{\Delta_Y}\left[\hat{Y}_t - \hat{Y}_t^{flex} - (\hat{Y}_{t-1} - \hat{Y}_{t-1}^{flex})\right] + \epsilon_t^R \tag{A.26}$$

$$\frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t + \hat{\epsilon}_t^G = \hat{Y}_t \tag{A.27}$$

A.1.1 Log-Linear autoregressive structural shocks

$$\hat{\epsilon}_t^B = \rho_B \hat{\epsilon}_{t-1}^B + \eta_t^B \tag{A.28}$$

$$\hat{\epsilon}_t^N = \rho_N \hat{\epsilon}_{t-1}^N + \eta_t^N \tag{A.29}$$

$$\hat{\epsilon}_t^I = \rho_B \hat{\epsilon}_{t-1}^I + \eta_t^I \tag{A.30}$$

$$\hat{\epsilon}_t^G = \rho_B \hat{\epsilon}_{t-1}^G + \eta_t^G \tag{A.31}$$

$$\hat{\epsilon}_t^Z = \rho_Z \hat{\epsilon}_{t-1}^Z + \eta_t^Z \tag{A.32}$$

$$\hat{\bar{\pi}}_t = \rho_{\bar{\pi}} \hat{\bar{\pi}}_{t-1} + \eta_t^{\bar{\pi}} \tag{A.33}$$

The other four shocks, namely price mark-up shock γ_t^p, wage mark-up shock γ_t^w, equity premium shock γ_t^Q and the monetary shock ϵ_t^R are iid gaussian mean-zero disturbances.

A.2 Log-Linear system for Smets-Wouters model with internal habit formation (*swih*)

$$\lambda = (1 - h\beta) C^{-\sigma_c} (1 - h)^{-\sigma_c} \tag{A.34}$$

$$\widehat{\lambda}_t = \frac{1}{1 - \beta h} \left(\hat{\epsilon}_t^B - \beta h \mathbb{E}_t \hat{\epsilon}_{t+1}^B \right) - \frac{\sigma_c}{(1-h)(1-\beta h)} \left[\hat{C}_t - h\hat{C}_{t-1} - \beta h \mathbb{E}_t (\hat{C}_{t+1} - h\hat{C}_t) \right] \tag{A.35}$$

$$\begin{aligned} \hat{C}_t = \frac{1}{1 + h + \beta h^2} \left[(1 + \beta + \beta h^2) \hat{C}_{t+1} + h\hat{C}_{t-1} - \beta h \hat{C}_{t-2} \right] - \\ \dots \frac{1 - h}{\sigma_c (1 + h + \beta h^2)} \left[\hat{\epsilon}_{t+1}^B - \beta h \hat{\epsilon}_{t+2}^B - \hat{\epsilon}_t^B - \beta h \hat{\epsilon}_{t+1}^B \right] - \frac{(1-h)(1-\beta h)}{\sigma_c (1 + h + \beta h^2)} (\hat{R}_{t+1} - \hat{\pi}_{t+1}) \end{aligned} \tag{A.36}$$

A.3 Log-Linear system for the hybrid model (*hyb*)

$$\hat{p}_{t|t-k} = \sum_{k=0}^{\infty} \phi^k \mathbb{E}_{t-k} (\hat{P}_t + \hat{mc}_t) \tag{A.37}$$

$$\hat{P}_t = \xi_p^{hyb} (\iota_p \hat{\pi}_{t-1} + \hat{P}_{t-1}) + (1 - \xi_p^{hyb}) (\hat{p}_t^{calvo}) + (1 - \phi) \sum_{k=0}^{\infty} \phi^k \mathbb{E}_{t-k} \hat{p}_{t|t-k}^{si} \tag{A.38}$$

A.4 Log-Linear sytem for the search model(*search*)

$$m = u^{\Gamma} v^{1-\Gamma} \tag{A.39}$$

$$\hat{m}_t = \hat{\epsilon}^M_t + \Gamma \hat{u}_t + (1-\Gamma)\hat{v}_t q = \frac{m}{v} \tag{A.40}$$

$$s = \frac{m}{u} \tag{A.41}$$

$$\theta = \frac{v}{u} \tag{A.42}$$

$$\hat{q}_t = \hat{m}_t - \hat{v}_t \tag{A.43}$$

$$\hat{s}_t = \hat{m}_t - \hat{u}_t \tag{A.44}$$

$$\hat{\theta} = \hat{v}_t - \hat{u}_t \tag{A.45}$$

$$n = \frac{m}{\Phi} \tag{A.46}$$

$$u = 1 - (1-\Phi)n \tag{A.47}$$

$$\hat{n}_t = (1-\Phi)\hat{n}_{t-1} + \Phi \hat{m}_t \tag{A.48}$$

$$\hat{u}_t = -\frac{n}{u}(1-\Phi)\hat{n}_t \tag{A.49}$$

$$\begin{aligned} \hat{w}_t = \frac{\nu}{1-\alpha}\left[\hat{p}_t + \hat{mpl}_t\right] + \nu s \frac{\kappa}{\lambda q w N}\left[\hat{\theta}_t - \hat{\lambda}_t - \hat{N}_t\right] + \frac{1-\nu}{1+\sigma_N} m\hat{r}s_t - (1-\nu)\frac{u.b}{wN}\hat{N}_t + \\ \cdots + (\frac{\nu}{1-\nu})(\frac{1-\alpha}{\alpha} + \frac{\kappa}{\lambda q w N})\hat{\zeta}_t - (1-s)(\frac{\nu}{1-\nu})(\frac{\kappa}{\lambda q w N})\hat{\zeta}_{t+1} \end{aligned} \tag{A.50}$$

$$\mu^{J-V} = n \tag{A.51}$$

$$\hat{\mu}^{J-V}_t = \hat{n}_t \tag{A.52}$$

$$\mu^{W-U} = \frac{N}{\alpha}\left(\frac{mrs}{w} - (1-\alpha)\right) = N \tag{A.53}$$

$$\hat{\mu}^{W-U}_t = \frac{1}{1-\alpha}(\hat{N}_t + m\hat{r}s_t - \hat{w}_t) - \frac{\alpha}{1-\alpha}\hat{N}_t \tag{A.54}$$

$$\zeta = \nu\mu^{W-U}(\nu\mu^{W-U} + (1-\nu)\mu^J)^{-1} \tag{A.55}$$

$$\nu\widehat{\mu^{W-U}_t + (1-\nu)}\mu^J_t = \frac{\nu\widehat{\mu^{W-U}_t}(\nu\mu^{W-U}) + \widehat{(1-\nu)}\mu^J_t((1-\nu)\mu^J)}{\nu\mu^{W-U} + (1-\nu)\mu^J} = \nu^2\hat{\mu}^{W-U}_t + (1-\nu)^2\hat{\mu}^J_t \tag{A.56}$$

$$\hat{\zeta}_t = \nu\hat{\mu}^{W-U}_t - \nu^2\hat{\mu}^{W-U}_t - (1-\nu)^2\hat{\mu}^J_t \tag{A.57}$$

A.5 Log-Linear system for the full model (*full*)

$$\widehat{R_{t+1}}^K = \hat{R}_{t+1} - \left[\frac{\phi\frac{R^k}{R}}{\phi'\frac{R^k}{R}}\right]\left(\widehat{EQ}_{T+1} - \widehat{Q}_t - \widehat{K}_t\right] \tag{A.58}$$

$$\widehat{R}^K_{t+1} = \frac{r^K}{R^K}\left(1 + \frac{1}{\psi}\right)\hat{r}^K_{t+1} + \frac{1-\delta}{R^K}\hat{Q}_{t+1} - \frac{r^K + 1 - \delta}{R^K}\hat{Q}_t \tag{A.59}$$

B Appendix B

In this section we provide DYNARE codes to run stochastic simulation of the models taken into account.

Each code specifies both the frictional model economy and its flexible counterpart in order to compute the output gap according to the taylor equation.

Output of each code is a 500-time-periods simulatd series for each variable declared in the *var* block and 40-time-periods IRFs for each variable declared in the *var* block to each of the shocks declared in the *varexo* block, according to the magnitude of the standard deviations declared in the *shocks* block.

B.1 Smets-Wouters(*sw*) code

```
var PI_flex R_flex lambdac_flex    c_flex tobq_flex inv_flex k_flex r_flex
y_flex n_flex w_flex EF GAP E r tobq c LAMBDAC inv y n PI PI_BAR w k R
epsaz epsab epsag epsan epsainv  one
;
varexo etaz etab etag etan etainv etaR etapibar eta_p eta_w eta_q
;

parameters xi_e gammaw calpha czcap cbeta phi_i cdelta sigmac acca
r_st ccs cinvs phi_y iotaw  xiw iotap xip  sigman rhoz rhob rhog rhon
rhoinv rhor rhopi rhodpi rho_gap rho_dgap rho_bar
;
calpha = 0.30;
cbeta = 0.99;
cdelta = 0.025;
ccs = 0.6;
cinvs = 0.22;
gammaw = 0.5;
phi_i = 6.771;
sigmac = 1.353;
acca = 0.573;
r_st = 0.016 ;
xiw = 0.737;
sigman = 2.400;
xip = 0.908;
xi_e = 0.599;
iotaw = 0.763;
iotap = 0.469;
czcap = 0.169;
phi_y = 1.408;
rhoz = 0.823;
rhob = 0.855;
rhog = 0.949;
rhon = 0.889;
rhoinv = 0.927;
rhor = 0.961;
rhopi = 1.684;
rhodpi = 0.14;
rho_gap = 0.099;
rho_dgap = 0.159;
```

```
rho_bar = 0.924;

model(linear);
0 = calpha*r_flex+(1-calpha)*w_flex-epsaz;
PI_flex = (0)*one;
LAMBDAC = epsab-(sigmac/(1-acca))*(c-acca*c(-1));
lambdac_flex = epsab-(sigmac/(1-acca))*(c_flex-acca*c_flex(-1));
c = (acca/(1+acca))*c(-1)+(1/(1+acca))*c(+1)-
      ((1-acca)/(sigmac*(1+acca)))*(R-PI)
      +((1-acca)/(sigmac*(1+acca)))*epsab;
c_flex = (acca/(1+acca))*c_flex(-1)+(1/(1+acca))*c_flex(+1)
            -((1-acca)/(sigmac*(1+acca)))*(R_flex-PI_flex)
            +((1-acca)/(sigmac*(1+acca)))*epsab;

inv = (1/(1+cbeta))*((inv(-1)+cbeta*(inv(1)))+(1/phi_i)*tobq)+epsainv;
inv_flex = (1/(1+cbeta))*((inv_flex(-1)+cbeta*(inv_flex(1)))+(1/phi_i)*tobq_flex)+epsainv;

tobq = -(R-PI(1))+((1-cdelta)/(1-cdelta+r_st))*tobq(1)+(r_st/(1-cdelta+r_st))*r(1)+eta_q;
tobq_flex = -(R_flex-PI_flex(1))+((1-cdelta)/(1-cdelta+r_st))*tobq_flex(1)
                 +(r_st/(1-cdelta+r_st))*r_flex(1);

k = (1-cdelta)*k(-1)+cdelta*inv;
k_flex = (1-cdelta)*k_flex(-1)+cdelta*inv_flex;

y = (ccs*c+cinvs*inv)+epsag;
y_flex = (ccs*c_flex+cinvs*inv_flex)+epsag;
y = phi_y*(calpha*k(-1)+calpha*(1/czcap)*r+(1-calpha)*n+epsaz);
y_flex = phi_y*(calpha*k_flex(-1)+calpha*(1/czcap)*r_flex+(1-calpha)*n_flex+epsaz);

w_flex = epsan+sigman*n_flex-lambdac_flex;
w = (cbeta/(1+cbeta))*w(1)+(1/(1+cbeta))*w(-1)+(cbeta/(1+cbeta))*PI(1)
       -((1+cbeta*iotaw)/(1+cbeta))*PI
       +(iotaw/(1+cbeta))*PI(-1)-((((1-cbeta*xiw)*(1-xiw)*gammaw))/((1-cbeta)*(gammaw+
       (1+gammaw)*sigman)*xiw))*(w-sigman*n+LAMBDAC+epsan)+eta_w;

PI = (1/(1+cbeta*iotap))*((cbeta)*(PI(1))+(iotap)*(PI(-1))+
       ((1-xip)*(1-cbeta*xip)/(xip))*(calpha*r+(1-calpha)*w-epsaz))+eta_p;


n = r*((1+czcap)/czcap)-w+k(-1);
n_flex = r_flex*((1+czcap)/czcap)-w_flex+k_flex(-1);

E = E(-1)+E(1)-E+(n-E)*((1-xi_e)*(1-xi_e*cbeta)/(xi_e));
EF = EF(-1)+EF(1)-EF+(n_flex-EF)*((1-xi_e)*(1-xi_e*cbeta)/(xi_e));
GAP = y-y_flex;

R = rhodpi*(PI-PI(-1))+(1-rhor)*(rhopi*(PI(-1)-
      PI_BAR)+rho_gap*(y-y_flex))
       +rho_dgap*(y-y_flex-(y(-1)-y_flex(-1)))
       +rhor*(R(-1)-PI_BAR)
       +PI_BAR+etaR;

epsaz = (rhoz)*epsaz(-1) + etaz;
epsab = rhob*epsab(-1) + etab;
epsag = rhog*epsag(-1) + etag;
epsan = rhon*epsan(-1) + etan;
```

```
epsainv = rhoinv*epsainv(-1) + etainv;
PI_BAR = rho_bar*PI_BAR(-1) + etapibar;
one = (0)*one(-1);
end;

shocks;
var etab; stderr 0.336;
var etaz; stderr 0.598;
var etag; stderr 0.325;
var etainv; stderr 0.085;
var etan; stderr 3.520;
var etaR; stderr 0.081;
var etapibar; stderr 0.017;
var eta_p; stderr 0.160;
var eta_w; stderr 0.289;
var eta_q; stderr 0.604;
end;

steady;
check;
stoch_simul(periods=500, irf = 40);
```

B.2 Smets-Wouters with internal habit formation (*swih*) code

```
var PI_flex R_flex lambdac_flex c_flex tobq_flex inv_flex k_flex r_flex y_flex n_flex w_flex
EF GAP_sw E_sw r_sw tobq_sw c_sw inv_sw y_sw n_sw PI_sw PI_BAR w_sw k_sw R_sw
epsaz epsag epsan epsainv one
;
varexo  etaz etag etan etainv etaR etapibar eta_p eta_w eta_q
;
parameters xi_e gammaw calpha czcap cbeta phi_i cdelta sigmac acca r_st ccs cinvs phi_y iotaw
xiw iotap xip  sigman rhoz rhob rhog rhon rhoinv rhor  rhopi rhodpi rho_gap rho_dgap  rho_bar
;
calpha = 0.30;
cbeta = 0.99;
cdelta = 0.025;
ccs = 0.6;
cinvs = 0.22;
gammaw = 0.5;
phi_i = 6.771;
sigmac = 1.353;
acca = 0.573;
r_st = 0.016 ;
xiw = 0.737;
sigman = 2.400;
xip = 0.908;
xi_e = 0.599;
iotaw = 0.763;
iotap = 0.469;
czcap = 0.169;
phi_y = 1.408;
rhoz = 0.823;
rhob = 0.855;
rhog = 0.949;
rhon = 0.889;
```

```
rhoinv = 0.927;
rhor = 0.961;
rhopi = 1.684;
rhodpi = 0.14;
rho_gap = 0.099;
rho_dgap = 0.159;
rho_bar = 0.924;

model(linear);
0 = calpha*r_flex+(1-calpha)*w_flex-epsaz;
PI_flex = (0)*one;
lambdac_sw= -(sigmac/((1-acca)*(1-cbeta*acca))*(c_sw-acca*c_sw(-1)
                      -cbeta*acca*(c_sw(1)-acca*c_sw));
lambdac_flex = -(sigmac/((1-acca)*(1-cbeta*acca)))*(c_flex-acca*c_flex(-1)
                      -cbeta*acca*(c_flex(1)-acca*c_flex));
c_sw = -(((1-cbeta*acca)*(1-acca))/(sigmac+sigmac*acca+
            sigmac*cbeta*acca^2))*(R_sw-PI_sw(1))
            +(1/(1+acca+cbeta*acca^2))*((1+cbeta*acca+cbeta*acca^2)*c_sw(1)
            +acca*c_sw(-1)-cbeta*acca*c_sw(2));
c_flex = -(((1-cbeta*acca)*(1-acca))/(sigmac+sigmac*acca+
             sigmac*cbeta*acca^2))*(R_flex-PI_flex(1))
            +(1/(1+acca+cbeta*acca^2))*(c_flex(1)-cbeta*acca*c_flex(2)+
             c_flex(1)*cbeta*acca^2+acca*c_flex(-1) + cbeta*acca*c_flex(1));

inv_sw = (1/(1+cbeta))*((inv_sw(-1)+cbeta*(inv_sw(1)))+(1/phi_i)*tobq_sw)+epsainv;
inv_flex = (1/(1+cbeta))*((inv_flex(-1)+cbeta*(inv_flex(1)))+(1/phi_i)*tobq_flex)+epsainv;

tobq_sw = -(R_sw-PI_sw(1))+((1-cdelta)/(1-cdelta+r_st))*tobq_sw(1)
                +(r_st/(1-cdelta+r_st))*r_sw(1)+eta_q;
tobq_flex = -(R_flex-PI_flex(1))+((1-cdelta)/(1-cdelta+r_st))*tobq_flex(1)
                 +(r_st/(1-cdelta+r_st))*r_flex(1);

k_sw = (1-cdelta)*k_sw(-1)+cdelta*inv_sw;
k_flex = (1-cdelta)*k_flex(-1)+cdelta*inv_flex;

y_sw = (ccs*c_sw+cinvs*inv_sw)+epsag;
y_flex = (ccs*c_flex+cinvs*inv_flex)+epsag;
y_sw = phi_y*(calpha*k_sw(-1)+calpha*(1/czcap)*r_sw+(1-calpha)*n_sw+epsaz);
y_flex = phi_y*(calpha*k_flex(-1)+calpha*(1/czcap)*r_flex+(1-calpha)*n_flex+epsaz);

w_flex = epsan+sigman*n_flex-lambdac_flex;
w_sw = (cbeta/(1+cbeta))*w_sw(1)+(1/(1+cbeta))*w_sw(-1)+
            (cbeta/(1+cbeta))*PI_sw(1)-((1+cbeta*iotaw)/(1+cbeta))*PI_sw
            +(iotaw/(1+cbeta))*PI_sw(-1)-
            ((((1-cbeta*xiw)*(1-xiw)*gammaw))/((1-cbeta)*(gammaw+(1+gammaw)*sigman)*xiw))*
            (w_sw-sigman*n_sw+lambdac_sw+epsan)+eta_w;

PI_sw = (1/(1+cbeta*iotap))*((cbeta)*(PI_sw(1))+(iotap)*(PI_sw(-1))+
             ((1-xip)*(1-cbeta*xip)/(xip))*(calpha*r_sw+(1-calpha)*w_sw-epsaz))+eta_p;

n_sw = r_sw*((1+czcap)/czcap)-w_sw+k_sw(-1);
n_flex = r_flex*((1+czcap)/czcap)-w_flex+k_flex(-1);

E_sw = E_sw(-1)+E_sw(1)-E_sw+(n_sw-E_sw)*((1-xi_e)*(1-xi_e*cbeta)/(xi_e));
EF = EF(-1)+EF(1)-EF+(n_flex-EF)*((1-xi_e)*(1-xi_e*cbeta)/(xi_e));
GAP_sw = y_sw-y_flex;
```

```
R_sw = rhodpi*(PI_sw-PI_sw(-1))+(1-rhor)*(rhopi*(PI_sw(-1)-PI_BAR)+rho_gap*(y_sw-y_flex))
       +rho_dgap*(y_sw-y_flex-(y_sw(-1)-y_flex(-1)))
       +rhor*(R_sw(-1)-PI_BAR)
       +PI_BAR+etaR;

epsaz = (rhoz)*epsaz(-1) + etaz;
epsag = rhog*epsag(-1) + etag;
epsan = rhon*epsan(-1) + etan;
epsainv = rhoinv*epsainv(-1) + etainv;
PI_BAR = rho_bar*PI_BAR(-1) + etapibar;
one = (0)*one(-1);
end;

shocks;
var etaz; stderr 0.598;
var etag; stderr 0.325;
var etainv; stderr 0.085;
var etan; stderr 3.520;
var etaR; stderr 0.081;
var etapibar; stderr 0.017;
var eta_p; stderr 0.160;
var eta_w; stderr 0.289;
var eta_q; stderr 0.604;
end;

steady;
check;
stoch_simul(periods=500, irf = 40);
```

B.3 Search model (*search*) code

```
var PI_flex R_flex c_flex lambdac_flex tobq_flex inv_flex k_flex r_flex y_flex n_flex w_flex EF
unemp_search vac_search q_vac_fil s_job_find weightW_search weightJ_search ups_search
theta_search E_search GAP_search r_search tobq_search c_search lambdac_search inv_search
y_search n_search PI_search PI_BAR_search w_search k_search R_search
epsaz epsag epsainv one
;
varexo  etaz etag etainv etaR etapibar eta_p eta_q
;

parameters xi_e  GAMMA cupsilon PHI n_steady u_steady k_n r_steady
w_steady s ub_wn cappa_clambdaq ctheta ups_w ups_ww gammaw calpha czcap
cbeta phi_i cdelta sigmac acca ccs cinvs phi_y iotaw xiw iotap xip  sigman
rhoz rhob rhog rhon rhoinv rhor  rhopi rhodpi rho_gap rho_dgap  rho_bar
;
calpha = 0.30;
cbeta = 0.99;
cdelta = 0.025;
ccs = 0.6;
cinvs = 0.22;
GAMMA = 0.5;
cupsilon = 0.2;
PHI = 0.08;
n_steady = 0.2;
```

```
u_steady = 0.816;
k_n = 22.28;
r_steady = 0.016;
w_steady = 1.87;
q = 0.7;
s = 0.25;
ub_wn = 0.5;
cappa_clambdaq = 0.007;
ctheta = 0.36;
gammaw = 0.5;
phi_i = 6.771;
sigmac = 1.353;
acca = 0.573;
xiw = 0.737;
sigman = 0.1;
xip = 0.908;
xi_e = 0.599;
iotaw = 0.763;
iotap = 0.469;
czcap = 0.169;
phi_y = 1.408;
rhoz = 0.823;
rhob = 0.855;
rhog = 0.949;
rhon = 0.889;
rhoinv = 0.927;
rhom = 0.823;
rhor = 0.961;
rhopi = 1.684;
rhodpi = 0.14;
rho_gap = 0.099;
rho_dgap = 0.159;
rho_bar = 0.924;

model(linear);
0 = calpha*r_flex+(1-calpha)*w_flex-epsaz ;
PI_flex = (0)*one;
lambdac_search = -(sigmac/((1-cbeta*acca)*(1-acca)))*(c_search-acca*c_search(-1)
                           -cbeta*acca*(c_search(1)-c_search));
lambdac_flex = -(sigmac/((1-acca)*(1-cbeta*acca)))*(c_flex-acca*c_flex(-1)-
                       cbeta*acca*(c_flex(1)-acca*c_flex));
c_search = -(((1-cbeta*acca)*(1-acca))/(sigmac+sigmac*acca+sigmac*cbeta*acca^2))*(R_search-PI_search(1))
                +(1/(1+acca+cbeta*acca^2))*((1+cbeta*acca+cbeta*acca^2)*c_search(1)
                +acca*c_search(-1)-cbeta*acca*c_search(2));
c_flex = -(((1-cbeta*acca)*(1-acca))/(sigmac+sigmac*acca+
             sigmac*cbeta*acca^2))*(R_flex-PI_flex(1))
            +(1/(1+acca+cbeta*acca^2))*((1+cbeta*acca+
            cbeta*acca^2)*c_flex(1)+acca*c_flex(-1)-cbeta*acca*c_flex(2));

inv_search = (1/(1+cbeta))*((inv_search(-1)+cbeta*(inv_search(1)))+(1/phi_i)*tobq_search)+epsainv;
inv_flex = (1/(1+cbeta))*((inv_flex(-1)+cbeta*(inv_flex(1)))+(1/phi_i)*tobq_flex)+epsainv;

tobq_search = -(R_search-PI_search(1))+((1-cdelta)/(1-cdelta+r_steady))*tobq_search(1)
                    +(r_steady/(1-cdelta+r_steady))*r_search(1)+eta_q;
tobq_flex = -(R_flex-PI_flex(1))+((1-cdelta)/(1-cdelta+r_steady))*tobq_flex(1)
                +(r_steady/(1-cdelta+r_steady))*r_flex(1);
```

```
k_search =  (1-cdelta)*k_search(-1)+cdelta*inv_search(-1);
k_flex =  (1-cdelta)*k_flex(-1)+cdelta*inv_flex(-1);

y_search = (ccs*c_search+cinvs*inv_search)+epsag;
y_flex = (ccs*c_flex+cinvs*inv_flex)+epsag;
y_search = phi_y*(calpha*k_search(-1)+calpha*(1/czcap)*r_search+(1-calpha)*n_search+epsaz);
y_flex = phi_y*(calpha*k_flex(-1)+calpha*(1/czcap)*r_flex+(1-calpha)*n_flex+epsaz);

w_flex = sigman*n_flex-lambdac_flex;
unemp_search = 1-(1-PHI)*n_search;
q_vac_fil = GAMMA*(unemp_search-vac_search);
s_job_find = (1-GAMMA)*(vac_search-unemp_search);
theta_search = s_job_find-q_vac_fil;
weightW_search = (1/(1-calpha))*((1+sigman)*n_search
                          -lambdac_search+w_search)+(calpha/(1-calpha))*(1/n_steady)*n_search;
weightJ_search = n_search;
ups_search = cupsilon*n_search-(cupsilon^2)*n_steady*n_search-
                   ((1-cupsilon)^2)*n_steady*n_search;
w_search = (cupsilon/(1-calpha))*(k_n^calpha)*(epsaz+calpha*(k_search-n_search))+
                  cupsilon*s*cappa_clambdaq*(1/(w_steady*n_steady))*(theta_search-n_search-lambdac_search)
                  +((1-cupsilon)/(1+sigman))*(sigman*n_search-lambdac_search)-(1-cupsilon)*ub_wn*n_search
                  +(cupsilon/(1-cupsilon))*(((1-calpha)/calpha)+
                  cappa_clambdaq*(1/(w_steady*n_steady)))*ups_search-
                   (cupsilon/(1-cupsilon))*(1-s)*cappa_clambdaq*(1/w_steady*n_steady)*ups_search(1);

PI_search = (1/(1+cbeta*iotap))*((cbeta)*(PI_search(1))+(iotap)*(PI_search(-1))+
                  ((1-xip)*(1-cbeta*xip)/(xip))*(calpha*r_search+(1-calpha)*w_search-epsaz))+eta_p;

n_search = (1-PHI)*n_search(-1)+PHI*(GAMMA*unemp_search+(1-GAMMA)*vac_search);
n_search = r_search*((1+czcap)/czcap)-w_search+k_search(-1) ;
n_flex = r_flex*((1+czcap)/czcap)-w_flex+k_flex(-1) ;

E_search = E_search(-1)+E_search(1)-E_search+(n_search-E_search)*((1-xi_e)*(1-xi_e*cbeta)/(xi_e));
EF = EF(-1)+EF(1)-EF+(n_flex-EF)*((1-xi_e)*(1-xi_e*cbeta)/(xi_e));
GAP_search = y_search-y_flex;

R_search = rhodpi*(PI_search-PI_search(-1))
            +(1-rhor)*(rhopi*(PI_search(-1)-PI_BAR_search)+rho_gap*(y_search-y_flex))
            +rho_dgap*(y_search-y_flex-(y_search(-1)-y_flex(-1)))
            +rhor*(R_search(-1)-PI_BAR_search)+PI_BAR_search+etaR;

epsaz = (rhoz)*epsaz(-1) + etaz;
epsag = rhog*epsag(-1) + etag;
epsainv = rhoinv*epsainv(-1) + etainv;
PI_BAR_search = rho_bar*PI_BAR_search(-1) + etapibar;
one = (0)*one(-1);
end;

shocks;
var etaz; stderr 0.598;
var etag; stderr 0.325;
var etainv; stderr 0.085;
var etaR; stderr 0.081;
var etapibar; stderr 0.017;
var eta_p; stderr 0.160;
```

```
var eta_q; stderr 0.604;
end;

steady;
check;
stoch_simul(periods=500, irf = 40);
```

B.4 Hybrid model (*hyb*) code

```
var PI_flex R_flex c_flex lambdac_flex tobq_flex inv_flex k_flex
r_flex y_flex n_flex w_flex EF mc_hyb  p_calvo_hyb p_mr_hyb
P_hyb PI_hyb z_hyb x_hyb E_hyb GAP_hyb r_hyb tobq_hyb
c_hyb inv_hyb y_hyb n_hyb PI_BAR w_hyb k_hyb R_hyb
epsaz epsag epsan epsainv  one
;
varexo etaz etag etan etainv etaR etapibar eta_w eta_q
;
parameters xi_e gammaw calpha czcap cbeta r_st phi_i cdelta sigmac acca ccs cinvs phi_y iotaw
xiw iotap J1 xip_calvo   J2 phi_mr sigman rhoz rhob rhog rhon rhoinv rhor  rhopi rhodpi rho_gap rho_dgap r
;
calpha = 0.30;
cbeta = 0.99;
r_st = 0.016;
cdelta = 0.025;
ccs = 0.6;
cinvs = 0.22;
gammaw = 0.5;
phi_i = 6.771;
sigmac = 1.353;
acca = 0.573;
xiw = 0.737;
sigman = 2.400;
J1 = 0.5;
xip_calvo = 0.454;
J2 = 0.5;
phi_mr = 0.375;
xi_e = 0.599;
iotaw = 0.763;
iotap = 0.469;
czcap = 0.169;
phi_y = 1.408;
rhoz = 0.823;
rhob = 0.855;
rhog = 0.949;
rhon = 0.889;
rhoinv = 0.927;
rhor = 0.961;
rhopi = 1.684;
rhodpi = 0.14;
rho_gap = 0.099;
rho_dgap = 0.159;
rho_bar = 0.924;

model(linear);
0 = calpha*r_flex+(1-calpha)*w_flex -epsaz;
```

```
PI_flex = (0)*one;
lambdac_hyb = -(sigmac/((1-acca)*(1-cbeta*acca)))*(c_hyb-acca*c_hyb(-1)
                      -cbeta*acca*(c_hyb(1)-acca*c_hyb));
lambdac_flex = -(sigmac/((1-acca)*(1-cbeta*acca)))*(c_flex-acca*c_flex(-1)
                      -cbeta*acca*(c_flex(1)-acca*c_flex));
c_hyb = -(((1-cbeta*acca)*(1-acca))/(sigmac+sigmac*acca+sigmac*cbeta*acca^2))*(R_hyb-PI_hyb(1))
            +(1/(1+acca+cbeta*acca^2))*((1+cbeta*acca+cbeta*acca^2)*c_hyb(1)
            +acca*c_hyb(-1)-cbeta*acca*c_hyb(2));
c_flex = +(((1-cbeta*acca)*(1-acca))/(sigmac+sigmac*acca+sigmac*cbeta*acca^2))*(PI_flex(1)-R_flex)
             +(1/(1+acca+cbeta*acca^2))*(c_flex(1)-cbeta*acca*c_flex(2)+
              c_flex(1)*cbeta*acca^2+acca*c_flex(-1) + cbeta*acca*c_flex(1));

inv_hyb = (1/(1+cbeta))*((inv_hyb(-1)+cbeta*(inv_hyb(1)))+(1/phi_i)*tobq_hyb)+epsainv;
inv_flex = (1/(1+cbeta))*((inv_flex(-1)+cbeta*(inv_flex(1)))+(1/phi_i)*tobq_flex)+epsainv;

tobq_hyb = -(R_hyb-PI_hyb(1))+((1-cdelta)/(1-cdelta+r_st))*tobq_hyb(1)
                +(r_st/(1-cdelta+r_st))*r_hyb(1)+eta_q;
tobq_flex = -(R_flex-PI_flex(1))+((1-cdelta)/(1-cdelta+r_st))*tobq_flex(1)
                +(r_st/(1-cdelta+r_st))*r_flex(1);

k_hyb =  (1-cdelta)*k_hyb(-1)+cdelta*inv_hyb(-1);
k_flex =  (1-cdelta)*k_flex(-1)+cdelta*inv_flex(-1);

y_hyb = (ccs*c_hyb+cinvs*inv_hyb)+epsag;
y_flex = (ccs*c_flex+cinvs*inv_flex)+epsag;
y_hyb = phi_y*(calpha*k_hyb(-1)+calpha*(1/czcap)*r_hyb+(1-calpha)*n_hyb+epsaz);
y_flex = phi_y*(calpha*k_flex(-1)+calpha*(1/czcap)*r_flex+(1-calpha)*n_flex+epsaz);

w_flex = epsan+sigman*n_flex-lambdac_flex;
w_hyb = (cbeta/(1+cbeta))*w_hyb(1)+(1/(1+cbeta))*w_hyb(-1)+(cbeta/(1+cbeta))*PI_hyb(1)
             -((1+cbeta*iotaw)/(1+cbeta))*PI_hyb+(iotaw/(1+cbeta))*PI_hyb(-1)-
             ((((1-cbeta*xiw)*(1-xiw)*gammaw))/((1-cbeta)*(gammaw+
             (1+gammaw)*sigman)*xiw))*(w_hyb-sigman*n_hyb+lambdac_hyb
             +epsan)+eta_w;

mc_hyb = calpha*r_hyb+(1-calpha)*w_hyb-epsaz;
p_calvo_hyb = (J1-xip_calvo)*(mc_hyb+P_hyb)+
                      (cbeta*xip_calvo)*p_calvo_hyb(1)-iotap*cbeta*xip_calvo*PI_hyb;
z_hyb = mc_hyb(1);
x_hyb = P_hyb(1);
p_mr_hyb = (J2-phi_mr)*(mc_hyb+P_hyb+phi_mr*(z_hyb(-1)+x_hyb(-1)));
P_hyb = xip_calvo*iotap*PI_hyb(-1)+xip_calvo*P_hyb(-1)+(J1-xip_calvo)*p_calvo_hyb+
             (J2-phi_mr)*p_mr_hyb+(J2-phi_mr)*phi_mr*p_mr_hyb(-1);
PI_hyb = P_hyb-P_hyb(-1);

n_hyb = r_hyb*((1+czcap)/czcap)-w_hyb+k_hyb(-1);
n_flex = r_flex*((1+czcap)/czcap)-w_flex+k_flex(-1);

E_hyb = E_hyb(-1)+E_hyb(1)-E_hyb+(n_hyb-E_hyb)*((1-xi_e)*(1-xi_e*cbeta)/(xi_e));
EF = EF(-1)+EF(1)-EF+(n_flex-EF)*((1-xi_e)*(1-xi_e*cbeta)/(xi_e));
GAP_hyb = y_hyb-y_flex;

R_hyb = rhodpi*(PI_hyb-PI_hyb(-1))
         +(1-rhor)*(rhopi*(PI_hyb(-1)-PI_BAR)+rho_gap*(y_hyb-y_flex))
         +rho_dgap*(y_hyb-y_flex-(y_hyb(-1)-y_flex(-1)))
         +rhor*(R_hyb(-1)-PI_BAR)
```

```
        +PI_BAR+etaR;

epsaz = (rhoz)*epsaz(-1) + etaz;
epsag = rhog*epsag(-1) + etag;
epsan = rhon*epsan(-1) + etan;
epsainv = rhoinv*epsainv(-1) + etainv;
PI_BAR = rho_bar*PI_BAR(-1) + etapibar;
one = (0)*one(-1);
end;

shocks;
var etaz; stderr 0.598;
var etag; stderr 0.325;
var etainv; stderr 0.085;
var etan; stderr 3.520;
var etaR; stderr 0.081;
var etapibar; stderr 0.017;
var eta_w; stderr 0.289;
var eta_q; stderr 0.604;
end;

steady;
check;
stoch_simul(periods=500, irf = 40);
```

B.5 Full model (*full*) code

```
var PI_flex R_flex c_flex lambdac_flex tobq_flex inv_flex k_flex r_flex y_flex n_flex w_flex EF
unemp_full vac_full q_vac_full s_job_full weightW_full weightJ_full ups_full
theta_full RK_full eq_full mc_full p_calvo_full P_full PI_hyb_full z_full
x_full p_mr_full E_full GAP_full r_full tobq_full c_full lambdac_full
inv_full y_full n_full PI_BAR w_full R_full k_full
epsaz epsag one
;
varexo etaz etag etaR etapibar
;
parameters xi_e GAMMA cupsilon PHI n_steady u_steady k_n
r_steady w_steady s ub_wn cappa_clambdaq ctheta ups_w
ups_ww R_k_st nu q_elast J1 xip_calvo J2 phi_mr
gammaw calpha czcap cbeta phi_i cdelta sigmac acca
ccs cinvs phi_y iotaw xiw iotap xip sigman rhoz
rhob rhog rhon rhoinv rhor  rhopi rhodpi rho_gap rho_dgap  rho_bar
;
calpha = 0.30;
cbeta = 0.99;
cdelta = 0.025;
ccs = 0.6;
cinvs = 0.22;
GAMMA = 0.68;
PHI = 0.08;
GAMMA = 0.68;
cupsilon = 0.2;
PHI = 0.08;
n_steady = 0.2;
u_steady = 0.264;
```

```
k_n = 0.66;
r_steady = 0.04;
w_steady = 0.57;
q = 0.7;
s = 0.25;
ub_wn = 0.5;
cappa_clambdaq = 0.007;
ctheta = 0.36;
ups_w = 0.35;
ups_ww = 1.40;
R_k_st = 8.21;
nu = 0.05 ;
q_elast = 0.025;
J1 = 0.5;
xip_calvo = 0.454;
J2 = 0.5;
phi_mr = 0.375;
gammaw = 0.5;
phi_i = 6.771;
sigmac = 1.353;
acca = 0.573;
xiw = 0.737;
sigman = 2.400;
xip = 0.908;
xi_e = 0.599;
iotaw = 0.763;
iotap = 0.469;
czcap = 0.169;
phi_y = 1.408;
rhoz = 0.823;
rhob = 0.855;
rhog = 0.949;
rhon = 0.889;
rhoinv = 0.927;
rhor = 0.961;
rhopi = 1.684;
rhodpi = 0.14;
rho_gap = 0.099;
rho_dgap = 0.159;
rho_bar = 0.924 ;

model(linear);
0 =  calpha*r_flex+(1-calpha)*w_flex -epsaz ;
PI_flex = (0)*one;

lambdac_full = -(sigmac/((1-cbeta*acca)*(1-acca)))*(c_full-acca*c_full(-1)-
                      cbeta*acca*(c_full(1)-c_full));
lambdac_flex = -(sigmac/((1-acca)*(1-cbeta*acca)))*(c_flex-acca*c_flex(-1)-
                       cbeta*acca*(c_flex(1)-acca*c_flex));
c_full = -(((1-cbeta*acca)*(1-acca))/(sigmac+sigmac*acca+
            sigmac*cbeta*acca^2))*(R_full-PI_hyb_full(1))
           +(1/(1+acca+cbeta*acca^2))*((1+cbeta*acca+cbeta*acca^2)*c_full(1)+
           acca*c_full(-1)-cbeta*acca*c_full(2));
c_flex = +(((1-cbeta*acca)*(1-acca))/(sigmac+sigmac*acca+
            sigmac*cbeta*acca^2))*(PI_flex(1)-R_flex)
            +(1/(1+acca+cbeta*acca^2))*(c_flex(1)-cbeta*acca*c_flex(2)+
```

```
            c_flex(1)*cbeta*acca^2+acca*c_flex(-1)+cbeta*acca*c_flex(1));

inv_full = (1/(1+cbeta))*((inv_full(-1)+cbeta*(inv_full(1)))+(1/phi_i)*tobq_full);
inv_flex = (1/(1+cbeta))*((inv_flex(-1)+cbeta*(inv_flex(1)))+(1/phi_i)*tobq_flex);

tobq_full = -(R_full-PI_hyb_full(1))+((1-cdelta)/(1-cdelta+r_steady))*tobq_full(1)+
                (r_steady/(1-cdelta+r_steady))*r_full(1);
tobq_flex = -(R_flex-PI_flex(1))+((1-cdelta)/(1-cdelta+r_steady))*tobq_flex(1)+
                 (r_steady/(1-cdelta+r_steady))*r_flex(1);
RK_full = (1/R_k_st)*(r_steady*(1+(1/czcap))*r_full+(1-cdelta)*tobq_full-
               (r_steady+1-cdelta)*tobq_full(-1));
RK_full = R_full-nu*(eq_full-(tobq_full(-1)+k_full));

k_full =  (1-cdelta)*k_full(-1)+cdelta*inv_full(-1);
k_flex =  (1-cdelta)*k_flex(-1)+cdelta*inv_flex(-1);

y_full = (ccs*c_full+cinvs*inv_full)+epsag;
y_flex = (ccs*c_flex+cinvs*inv_flex)+epsag;
y_full = phi_y*(calpha*k_full(-1)+calpha*(1/czcap)*r_full+(1-calpha)*n_full+epsaz );
y_flex = phi_y*(calpha*k_flex(-1)+calpha*(1/czcap)*r_flex+(1-calpha)*n_flex+epsaz );

w_flex = sigman*n_flex-lambdac_flex;
unemp_full = 1-(1-PHI)*n_full;
q_vac_full = GAMMA*(unemp_full-vac_full);
s_job_full = (1-GAMMA)*(vac_full-unemp_full);
theta_full = s_job_full-q_vac_full;
weightW_full = (1/(1-calpha))*((1+sigman)*n_full-lambdac_full+w_full)+(calpha/(1-calpha))*(1/n_steady)*n_f
weightJ_full = n_full;
ups_full = cupsilon*n_full-(cupsilon^2)*n_steady*n_full-((1-cupsilon)^2)*n_steady*n_full;
w_full = (cupsilon/(1-calpha))*(k_n^calpha)*(epsaz+calpha*(k_full-
             n_full))+cupsilon*s*cappa_clambdaq*(1/(w_steady*n_steady))*(theta_full-n_full-lambdac_full)
            +((1-cupsilon)/(1+sigman))*(sigman*n_full-lambdac_full)-
            (1-cupsilon)*ub_wn*n_full
            +(cupsilon/(1-cupsilon))*(((1-calpha)/calpha)+
            cappa_clambdaq*(1/(w_steady*n_steady)))*ups_full-
            (cupsilon/(1-cupsilon))*(1-s)*cappa_clambdaq*(1/w_steady*n_steady)*ups_full(1);

mc_full = calpha*r_full+(1-calpha)*w_full-epsaz;
p_calvo_full = (J1-xip_calvo)*(mc_full+P_full)+
                      (cbeta*xip_calvo)*p_calvo_full(1)-iotap*cbeta*xip_calvo*PI_hyb_full;
z_full = mc_full(1);
x_full = P_full(1);
p_mr_full = (J2-phi_mr)*(mc_full+P_full+phi_mr*(z_full(-1)+x_full(-1)));
P_full = xip_calvo*iotap*PI_hyb_full(-1)+xip_calvo*P_full(-1)+(J1-xip_calvo)*p_calvo_full
            +(J2-phi_mr)*p_mr_full+(J2-phi_mr)*phi_mr*p_mr_full(-1);
PI_hyb_full = P_full-P_full(-1);

n_full = (1-PHI)*n_full(-1)+PHI*(GAMMA*unemp_full+(1-GAMMA)*vac_full);
n_full = r_full*((1+czcap)/czcap)-w_full+k_full(-1);
n_flex = r_flex*((1+czcap)/czcap)-w_flex+k_flex(-1);

E_full = E_full(-1)+E_full(1)-E_full+(n_full-E_full)*((1-xi_e)*(1-xi_e*cbeta)/(xi_e));
EF = EF(-1)+EF(1)-EF+(n_flex-EF)*((1-xi_e)*(1-xi_e*cbeta)/(xi_e));
GAP_full= y_full-y_flex;

R_full = rhodpi*(PI_hyb_full-PI_hyb_full(-1))
```

```
          +(1-rhor)*(rhopi*(PI_hyb_full(-1)-PI_BAR)+rho_gap*(y_full-y_flex))
          +rho_dgap*(y_full-y_flex-(y_full(-1)-y_flex(-1)))
          +rhor*(R_full(-1)-PI_BAR)+PI_BAR+etaR;

epsaz = rhoz*epsaz(-1) + etaz;
epsag = rhog*epsag(-1) + etag;
PI_BAR = rho_bar*PI_BAR(-1) + etapibar;
one = (0)*one(-1);
end;

shocks;
var etaz; stderr 0.598;
var etag; stderr 0.325;
var etaR; stderr 0.081;
var etapibar; stderr 0.017;
end;

steady;
check;
stoch_simul(periods=500, irf = 40);
```