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Analysis of Financial Independence and Retire Early Strategies

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Contents

Abstract

The thesis is focused on FIRE (Financial Independence, Retire Early) movement and the most common retirement planning guideline. The purpose of this study is to examine the validity and application of the 4% rule, which was initially proposed by William P. Bengen in his article "Determining Withdrawal Rates Using Historical Data" in 1994, in various contexts and scenarios.

The key topic addressed by this research is whether the 4% rule is still relevant and successful in today's economy, given current factors such as inflation, interest rates, and market volatility. The challenge of sustaining retirement income and achieving financial independence is getting increasingly complicated, requiring an in-depth understanding of the rule's fundamental concepts and assumptions.

Previous research has shown that if the portfolio is well-diversified and contains a mix of stocks and bonds, this strategy may offer a decent and consistent income stream over time. Some researchers, however, have criticized the rule for being overly simplistic and out of date, and they argue that it may not work in certain events, such as periods of high inflation or market downturns.

These findings are significant as they give practical insights and assistance for those pursuing financial independence and long-term retirement security. Individuals may improve their chances of attaining their financial objectives and sustaining an adequate standard of living throughout their retirement years by embracing a more sophisticated and dynamic approach to retirement planning. This study emphasizes the

importance of continued education and knowledge of the complexities and problems of retirement planning, as well as the need for more research and innovation in this sector. It is intended to contribute to continuing debates and conversations about Financial Independence, and also to assist individuals in making informed decisions about their retirement planning.

Chapter I
FIRE Movement

1.1 History

The Financial Independence Retire Early (FIRE) movement is a contemporary personal finance movement that has grown in popularity recently. The concept is based on the idea that through investing and accumulating money, one may become financially independent and use that freedom to retire early or pursue other interests. The FIRE movement's roots were the personal finance and early retirement communities that emerged in the United States in the 1990s and early 2000s. At this time, several bloggers and personal finance experts became well-known for their counsel on handling money, paying off debt, and accumulating wealth.

Even in the 1990s, supporting life costs through savings or investments was familiar. For instance, Ed and Carolyn Robinson's 1943 book *The "Have-More" Plan* is a prime example. The book offers helpful suggestions for those wanting to build a small farm and lead self-sufficient lives. It addresses various farming-related issues, such as picking the best piece of land, constructing and maintaining farm structures, rearing chickens, pigs, and other animals, and cultivating vegetables and fruits. The Robinsons offer comprehensive advice and pointers on every facet of running a farm by drawing on their experiences as small-scale farmers. The significance of living within one's means and self-sufficiency are two of the book's major topics. The Robinsons high-light the importance of diligence and thrift, and they give several instances of how they were able to cut costs by recycling and reusing goods. This book has nothing to do with traditional finance but has much in common with basic personal finance concepts.

Similar to *The "Have-More" Plan*, other writers jumped on board, and soon there were many such journals containing frugality tips. Vicki Robin and Joe Dominguez were among the writers interested in the topic, and in 1992, they published *Your Money or Your Life*, which investigates the connection between wealth, time, and happiness. The book is a manual for establishing financial independence and building a satisfying life. The authors argue that our conventional view of money, which emphasizes working hard to earn as much as possible and spending it on material possessions, is an incorrect strategy for pursuing happiness. Instead, they suggest a different approach, which entails figuring out the actual cost of our purchases in terms of the time and effort we put into earning the money to pay for them.

The book outlines a nine-step approach, which includes the following, to change the relationship with money:

1. Tracking your expenses and income to get a clear picture of your financial situation.

2. Evaluating your spending habits and identifying areas where you can cut back.

3. Calculating your true hourly wage by factoring in all the time you spend working and commuting.

4. Creating a budget that reflects your values and priorities.

5. Building an emergency fund and paying off debt.

6. Maximizing your income through career development and entrepreneurship.

7. Investing your money wisely to generate passive income.

8. Reducing your consumption and simplifying your life.

9. Achieving financial independence and using your time and resources to pursue your passions and contribute to society.

In contrast to treating money as a means to an end, the book encourages readers to see it as a tool for attaining their objectives and leading satisfying lives (Dominguez, 1993)[11].

Except for a few industry experts, practically everyone missed a crucial FI/ER development in October 1994. "Determining Withdrawal Rates Using Historical Data" by William P. Bengen is a seminal study on sustainable withdrawal rates for retirees. Years after its publication in the Journal of Financial Planning, the study has received several citations and references (Early Retirement Dude, 2017)[13]. Bengen was the first to suggest the retirement planning approach known as "SAFEMAX", more popularly known as the "4% Rule", which will be analysed specifically in a future chapter.

In 2010, blogger Pete Adeney, commonly known as *Mr. Money Mustache*, first used the word "FIRE". The FIRE movement gained popularity through Adeney's blog, which emphasised obtaining financial independence and retiring early via frugal living and wise investing. During this time, other blogs and online groups started to appear. One such group was the Reddit topic *r/financialindependence*, which allowed users to discuss their financial journeys and methods. Several bloggers and online forums solely dedicated to the concept of gaining financial independence and retiring early started to appear. These forums advised on how to save money, make sensible investments, and accumulate a nest egg to support early retirement.

While its origins in the United States, the FIRE movement has subsequently extended to other nations, with internet forums and blogs devoted to obtaining financial independence and retiring early.

1.2 Financial Independence

The first two letters of the acronym, *Financial Independence* (FI), explain the core idea of FIRE, so we must first define this concept to describe the movement's goals. Being financially independent means having enough money or assets to cover living needs for the rest of your life without having to work or depend on others. Traditionally someone who reaches this economic status manages to maintain daily expenses through passive income, a type of unearned revenue maintained or generated automatically with little or no effort.

Financial independence is seen as the ability to do anything you want whenever you want. However, this definition may need to be more generalised, given that other factors influence people's decision-making in addition to money. Nonetheless, it emphasises personal freedom, one of financial independence's most widely discussed aspects. Financial freedom is often associated with this theme, but it has a slightly different meaning. It refers to a situation when a person does not feel pressured or concerned about money when making decisions. These two terms are fairly similar and often linked; however, the fundamental distinction to be emphasised is that being financially independent does not always imply being "rich". We should first define what rich means, but it is not enough to identify a wealth threshold above which we are called rich because factors such as where you live, or your monthly expenses change the final result considerably. For instance, for many individuals, 1 million euros is enough to be considered wealthy, but when asked what it would be like to have 1 million euros in a bank account in Switzerland, Mr Rip said that there is no shame in being poor there. The definition of a wealthy individual is based on the relationship between wealth and costs; absolute wealth is meaningless.

Researchers and content producers who handle the topic of financial independence generally focus on finance, although it is not the only topic that is discussed. FIRE is a lifestyle that, to pursue, more than having a good financial strategy is needed; areas such as stoicism, materialism and the science of well-being often need to be analysed. To embrace this movement, you must examine your lifestyle and how to acquire the wealth required to maintain it. There are three main steps to carry out: decrease expenses, increase savings and invest. All three are essential to be able to build one's financial independence, and only one of the steps directly concerns the area of financial investments. The other two concern money management, decision making and personal satisfaction. Planning the perfect financial strategy, finding the best investing opportunity, or focusing all your energy on getting the best trades is pointless. The journey to financial independence typically begins with an analysis of your monthly income, lifestyle, additional spending, and ongoing expenses rather than starting to explore the world of investing.

So, it is obvious that those who want this degree of economic freedom would travel a road that, while unique to each person, has some characteristics with others. It depends on your profession, personal abilities, educational background, residence location, whether you have a family or live alone, and other factors that need to be considered. In other words, it all relies on who you are and what you want from your life. Although achieving financial independence is a goal that many people strive for, not everyone will achieve or be able to follow this route because it may involve making sacrifices or tough decisions. Furthermore, it is a long-term goal, the reward a person only experiences in the second half of their life. To achieve financial freedom is crucial to comprehend all aspects and not simply focus on the final result.

Financial independence originally meant being able to not work, but over time, it has come to represent various things. Nowadays, several forms of financial independence are available that may be suited for different situations. The classic FIRE has enough investments to meet the costs of living. This is the definition of traditional retirement and financial independence that is most frequently used. When you have enough assets to qualify as a Regular FI, you can stop working and maintain your current level of spending. In addition to this type of FIRE, there are three others that differ in various fundamental aspects in the choices of a proper financial plan.

1.2.1 Lean FIRE

Lean FIRE is the ability to retire earlier than the typical age while leading a very frugal and simple lifestyle. With this lifestyle, you have enough savings to pay your *basic expenses*. *Basic expenses* include the very least necessary to lead a minimalist life; consequently, by using Malow's pyramid to analyze human needs, the costs will only comprise those for food, transportation, healthcare, clothes, and personal hygiene. This theory is simplistic and does not account for the complexity of human motivation, but it is indicative of life with meagre costs.

In Lean FIRE, there is little space for extravagance and living in a big city or having children is considerably more difficult. Lean FIRE often requires sacrifice and significant changes in an ordinary way individuals see life and happiness. Being able to be happy and being able to do what you want while living a minimalist life is the fastest way to achieve financial independence. However, lowering the cost of living can be challenging, and in some cases, it is not possible. Nevertheless, the majority of individuals want to advance past Lean FIRE. Most individuals do not want to spend years living very simply, so they may only do so in retirement. Lean FIRE is limited to individuals who desire to start families.

1.2.2 Fat FIRE

Fat FIRE means having sufficient assets to pay for costs associated with a higher level of living than our current lifestyles. It is not required for any additional job or activities during retirement. Also, it is possible to live in some of the most expensive cities on earth without worrying about money.

To have a more pleasant and luxurious retirement, a person must accumulate more cash through higher wages, profitable enterprises, or riskier investments. Although this implies that Fat FIRE takes the longest to complete, it also offers a significant advantage that the other FIRE types do not: the most financial freedom.

The benefits during retirement are high, but this can require significant efforts during the working period, such as having an unhealthy work-life balance, spending less time with family, or making career choices due solely to the increase in earnings and not to personal values.

1.2.3 Barista FIRE

The term *Barista FIRE* describes a situation in which we have enough investments to pay a portion of our current costs and can make up the difference by working a part-time, lower-stress job. Its name describes the idea of having sufficient savings and passive income to support your lifestyle with just a few evening or weekend shifts as a barista, which is a cushy job. Barista FIRE is for people who are essentially halfway through complete financial independence but aren't sure if a full-on retirement is the best course of action.

While FIRE is often enjoyed in later life, this type allows you to instantly benefit from your assets and devote more time, even if just partially, to your passions. It is ideal for those who particularly detest their jobs and wish to quit as soon as feasible or for those who have a dream job in mind but realize that it would not be sufficient to cover their expenses on its own. This FIRE requires less effort than a regular FIRE, but it does not leave you free to choose when to stop working.

1.3 FIRE is not for everyone

Financial independence may not always mean the same thing to different people. This occurs because each person has different long-term goals and different financial aspirations. One needs sound spending practices, solid money management skills, careful planning, and investment to achieve financial independence. Financial independence is hampered by incorrect investing, impulsive spending, and wrong spending decisions (Chopra, 2022)[8].

Understanding your life objectives is crucial when taking financial decisions since your financial plan should be made to assist you in achieving those goals. Your life objectives may include things like buying a house, funding the education of your children, or retiring comfortably. Understanding these objectives will allow you to create a financial strategy tailored to your need. Each choice must include creating pertinent goals based on values, facts, and knowledge.

1.3.1 Set Financial Goals

Setting financial objectives is a crucial first step on the path to success, security, and stability. Setting life objectives improves your quality of life by bringing focus and confidence. But, by including clear financial goals in your life plan, you prepare yourself for positive results that improve several aspects of your life.

Your investment activities have direction and purpose when you have financial goals. When you are aware of the goal you are aiming towards, they make it simpler for you to make sacrifices or keep to a spending plan; they assist you in maintaining long-term attention. Moreover, putting your objectives on paper, keeping yourself accountable for your progress, and evaluating your goals regularly helps you stay on track.

People are empirically poor at identifying what their aims are; thus, it is typical for financial advisors to help their clients recognize their goals. Research made by Bond, Carlson, & Keeney (2008) [6] investigates decision makers' capacity to establish self-relevant goals for important choices. Participants consistently omitted about half of the goals they later recognized as personally meaningful. Even more unexpectedly, omitted objectives were thought to be virtually as significant as those that participants independently came up with. The research indicates that decision-makers have significant shortcomings in formulating goals for the decisions they must make based on their expertise and beliefs. You need to establish your objectives before becoming interested in and pursuing Financial Independence; not everyone has Early Retirement as their economic priority.

In 2022 Benjamin Felix surveyed 310 people who had to answer three questions centred on financial goals, the responses to the questions were then categorized to determine the most frequent objectives. The results generated 28 financial goals; the top 10 results have an average of at least 20% presence in the participants' goal lists.

Financial Goals	**Frequency**
Being financially independent - work is optional	70.3%
Feeling financially secure	61.0%
Affording travel / leisure time / experiences with family	49.4%
Maintaining physical health through sleep, diet, and exercise	43.9%
Financially supporting my community or causes that are important to me	38.7%
Finding and affording engaging hobbies	36.1%
Assisting children with education costs / early adulthood setup	27.1%
Owning a home and affording its operating costs	25.8%
Having the ability to be generous with loved ones	25.5%
Avoiding the hedonic treadmill, but not over-saving ("enough")	21.9%

Table 1: Top 10 financial goals (PWL Capital) [17]

Financial Independence, which is in first place with a frequency of 70.3%, appears to be the primary reason an individual is interested in finance and investments. These results are very favourable for the FIRE movement, but the limitations of this survey must be underlined. The 310 participants were 75% male and 25% female, and the majority of respondents were aged 25-34 (44%) and 35-44 (31%). Moreover, the survey has been proposed to the listener of the *Rational Reminder Podcast* that mainly attracts wealthier, younger, and more financially literate males than the average individual. Indeed, the results would be significantly different with a pool of individuals with average characteristics, but what can therefore be noted is that the followers of the FIRE movement are often young people with a financial culture and with a promising economic future.

Chapter II
The 4% Rule

2.1 Introduction

William P. Bengen is a well-known retired American financial advisor specializing in investment management. He has been a consultant for many years, aiding customers with asset management and retirement planning. Bengen is well recognized for his study published in 1994, "Determining Withdrawal Rates Using Historical Data", which examined historical U.S. stock and bond market data from 1926 to 1992 to establish a general guideline for allocating pension funds. The purpose of this study was clear:

> *"At the onset of retirement, investment advisors make crucial recommendations to clients concerning asset allocation, as well as dollar amounts they can safely withdraw annually, so clients will not outlive their money. This article utilizes historical investment data as a rational basis for these recommendations."*

As a sole practitioner in El Cajon, California, he wanted to develop a range of stock and bond asset allocations ideal for almost all retirement portfolios and use graphical interpretations of the data to estimate the maximum safe withdrawal rate as a percentage of beginning portfolio value. Due to the insufficient information at the time, Bengen chose an average return of 10.3% for stocks, 5.2% for bonds, and 3% for inflation for 1944 and later years. Tax expenses are also not considered in the study because of the complexity that this variable involves. States in the United States have very distinct income tax systems, and each tax bracket would have needed to be calculated differently.

2.2 Worst Case Scenario

Bengen decided to concentrate his research on the *Worst Case Scenario*, considering the most severe or catastrophic possibility while constructing the portfolio and making estimates. The worst case is presented when the stock market experiences significant declines since the study was conducted on portfolios that show the presence of stocks and bonds in varying percentages.

Before 1994, the year the report was published, three significant financial crises had previously occurred, the Great Depression (1929–1939), the recession of 1937, and the 1973 oil crisis.

Period	**Return of common stocks**	**Return in-term bonds**	**Inflation**
Great Depression (1929–1939)	-61%	10.5%	-15.8%
Recession (1937–1941)	-33.3%	16.7%	10.5%
Oil crisis (1973–1974)	-37.2%	10.6%	22.1%

Table 2: Major Financial Events before 1994 [4]

Paying attention to these events is crucial due to the potential effects they may have on investor portfolios and purchasing power. In the research, Bengen assigns the occurrences three distinct effect levels while referring to them in astronomical terms. The crisis of 1929, referred to as the "Little Dipper" in the article, had the least significant effects because, despite the dramatic decline in stock prices, the dollar's purchasing power did not only not decline but rather gained significantly.

The 1937 recession, called the "Big Dipper", saw low inflation and increased bond yields. As a result, even while its impact on portfolios was significant and was less severe than that of the Little Dipper, it was still critical, given how closely it followed it.

The most severe recession was the oil crisis, indeed called the "Big Bang", during which investors suffered significant paper losses in their portfolios, and the purchasing power of what was left was much diminished, making it a terrifying period for them.

It is crucial to take into account historical inflation and year-to-year changes in investment returns in addition to average returns. According to Larry Bierwirth's article *Investing for Retirement: Using the Past to Model the Future*, extreme situations like the Great Depression or a recession like the one that occurred in 1973–1974 can have long-term repercussions that are beyond the average. These occurrences must be taken into account while planning for retirement.

For instance, if an investor considers the stock market's 30-year average return, they could believe that they can anticipate a constant rate of return in the future. The year-by-year returns, however, may reveal that there were times of considerable losses, such as in the early 2000s dot-com bubble or the 2008 financial crisis. These losses should be taken into account while planning since they may have a big impact on retirement funds.

Moreover, financial advisers should inform their customers about these developments and how they can affect their retirement funds. Clients may then possibly make better-informed judgments and make necessary adjustments to their retirement plans[5].

2.3 Analysis

The analysis aims to determine the appropriate withdrawal value that will enable an investor to maintain his assets long enough to sustain the retirement period. He did this by comparing the performance of various portfolio structures and using different withdrawal rates. The analysis is performed by following the portfolio value throughout the first 50 years after the start of the retirement period. The process is repeated for all years from 1926 to 1976, with a time-lapse of 50 years that includes all three of the 1900s' severe recessions. Bengen chose this approach because it was simple to communicate to clients whose primary concern is surviving retirement without running out of money and whose secondary concern is building wealth for their descendants.

To explain how the value of the portfolio and the quantity of money withdrawn are determined and how inflation is applied, I will make an example of a portfolio with an initial value of 1 million dollars, composed of 50% stocks and 50% of Long-Term U.S. Government Bonds.

Let us assume that 4% of the portfolio is needed to support living expenses for the entire first year of retirement, for example $40'000. Observe that the initial amount is not withdrawn from the invested portfolio, therefore in this example, the total liquidity available at the beginning of the year is $40'000 in addition to the 1 million dollars invested; in this way, the expenses are equivalent to 3.85% of the liquidity net worth [1]. In

[1] Liquid net worth is the amount hold in cash or cash equivalents after all liabilities are deducted from

this study, the withdrawal rate is always related to the portfolio's value and not to the liquidity net worth.

After one year, the portfolio's value has changed, and it is necessary to rebalance its composition and withdraw the necessary amount for the following year's living expenses. Assuming that the shares have appreciated by 10% and the bonds by 4%, the portfolio's value is $1'070'000 of which $550'000 of shares and $520'000 of bonds. The yearly cost of living increases to $41,200 due to inflation, which we assume will be 3%, and that must be taken out of the portfolio, which decreases to $1,028,800.

It should be noted that in this case, the living costs are no longer precisely 4% of the portfolio but 4.0047% because the variation in returns and inflation leads to a different ratio between the quantity to be withdrawn and the portfolio. The portfolio is rebalanced to a 50/50 allocation at the beginning of the second year, with bonds and equities each starting the year with a value of $514'400.

During the second year, let's assume that stocks have grown by 7% and bonds by 4%, while inflation reaches 4%. In this case, the portfolio grows to a value of $1'085'384 given by $550'408 from stocks and $534'976 from bonds, while the living expenses grow up to $42'848 which after being withdrawn just $1'042'536 remained invested. This time the withdrawal rate is equal to 4.11% of the portfolio at the end of the year and equal to 4.16% of the value at the beginning of the year. After rebalancing, the method is continually repeated regardless of inflation and market changes.

I re-performed all the analyzes and graphs reported in this chapter based on the same methods and data of the Bengen study.

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liquid assets. It includes stocks, which are seen as liquid assets because they can easily be bought and sold.

2.3.1 50/50 Stock/Bond Allocation

The first analysis is performed on a portfolio made up of 50% common stocks and 50% of intermediate-term treasury notes. Performance evaluation is measured on the longevity that the portfolio manages to achieve before the investments are extinguished. In the graphs shown, the vertical axis measures longevity, and the horizontal axis the starting year from which retirement begins. For example, the first vertical bar measures the longevity of a portfolio whose owner begins retirement on January 1, 1926. Withdrawals are always performed at the end of the year and adjusted for inflation, exactly as explained in 2.3 Analysis.

Figure 1: 1^{st} year withdrawal as a percentage of starting portfolio value: 3%

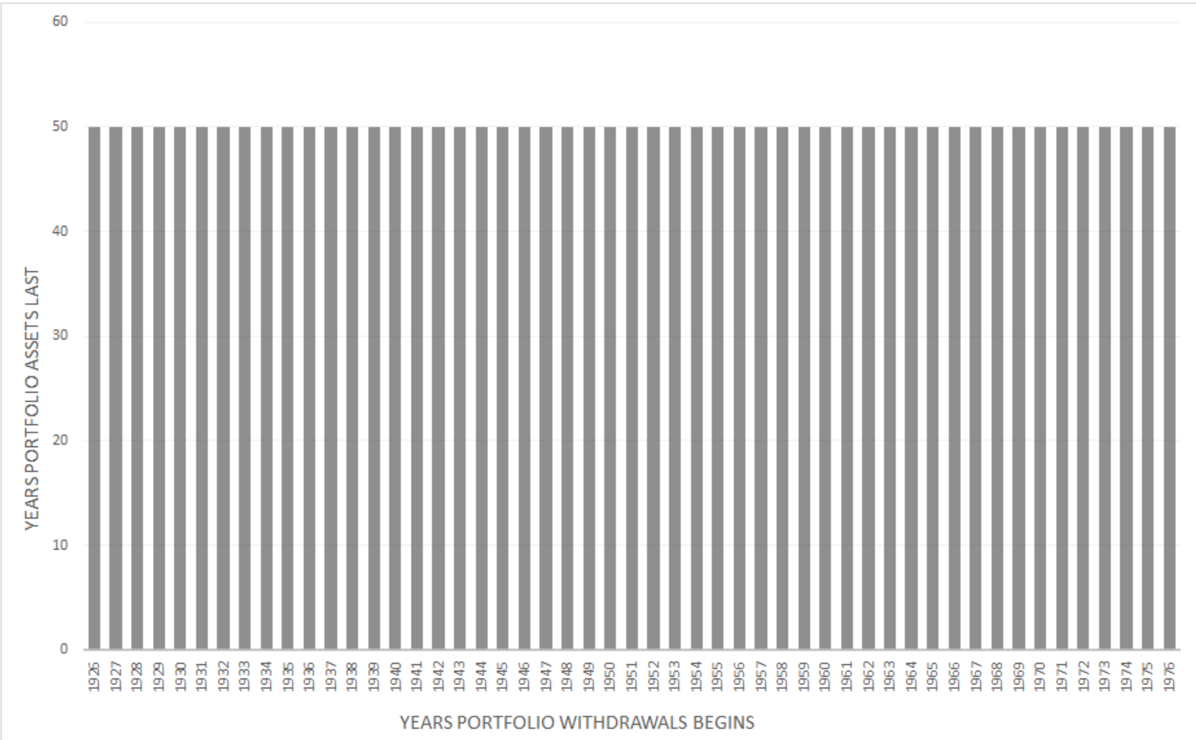
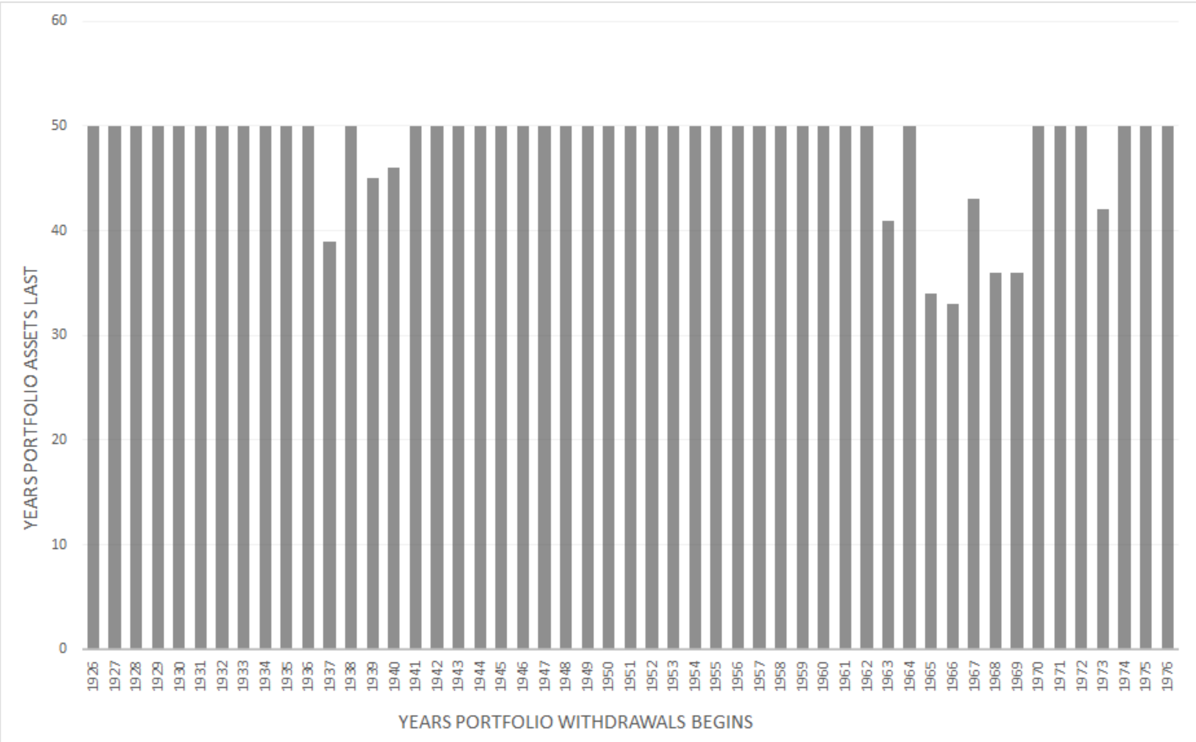


Figure 1 demonstrates that all investors could hold at least 50 years' worth of inflation-adjusted withdrawals from their portfolios, regardless of the year they started their retirement. Moreover, the portfolio's value would probably have lasted for a longer period; nevertheless, the study ends analyzing at a maximum of 50 years of retirement because most people do not desire to retire for longer than that.

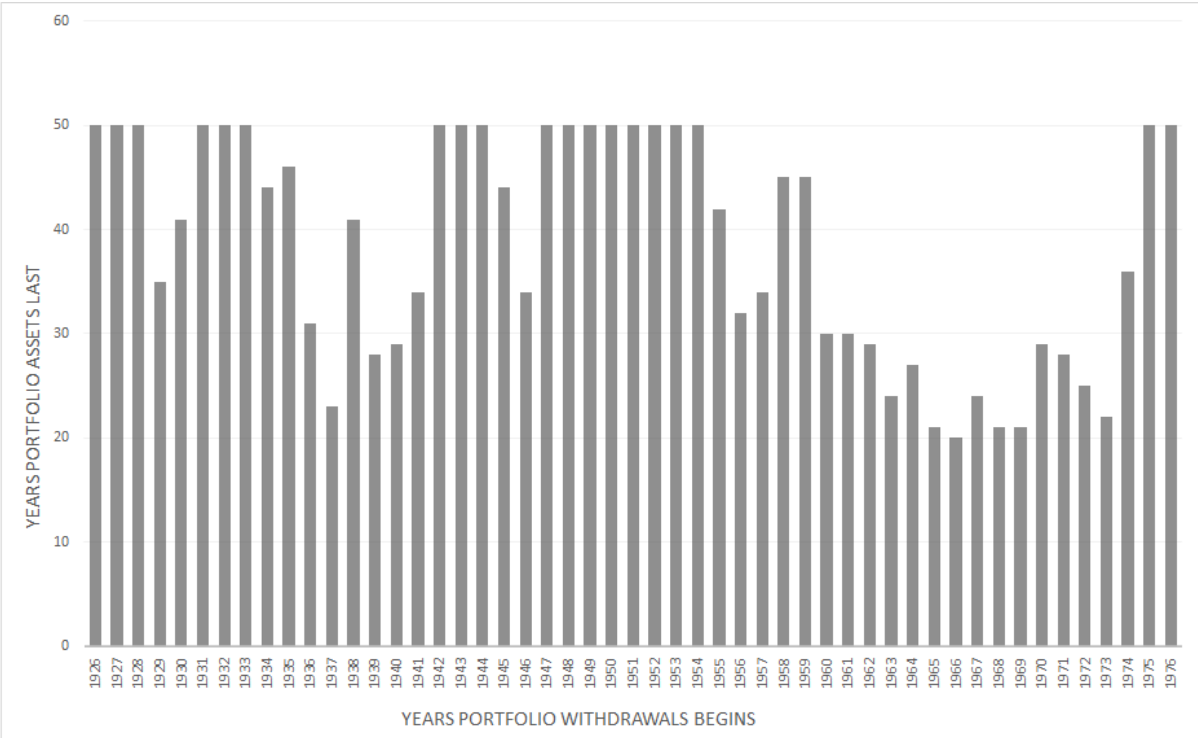
The following graphic, which still analyses a portfolio with a 50/50 stock/bond allocation but with a 4% withdrawal rate, is more interesting because it shows different results over the years.

Figure 2: 1^{st} year withdrawal as a percentage of starting portfolio value: 4%



In Figure 2 the majority of portfolios continue to last up to 50 years, but some have lower longevity. All of them have at least 30 years of retirement which is a positive result given that today the average retirement period in the U.S. is lower than 20 years[2]. The lower bar is in 1966 with a portfolio that didn't last more than 33 years. It is already clear from this graph which years were most negatively impacted by the significant stock market collapses of the 20^{th} century.

Figure 3: 1^{st} year withdrawal as a percentage of starting portfolio value: 5%



[2]The average retirement age in the U.S. is 64 years old, with the average retirement age across all states spanning from 61 to 67 years old [18] and life expectancy in the U.S. was 77 in 2020 [34]. The average retirement period, therefore, lasts less than 20 years.

Figure 3 (5% withdrawal rate) shows, exactly as expected, that as the rate of withdrawal rises, more and more portfolios fail to reach a satisfactory number of years. The majority continues to have a durability of more than 30 years with this percentage, but many portfolios, such as those that begin in the second half of the 1960s, barely make it to 20 years, which is generally not enough for the lifetime of a retired.

Bengen's last graph of the 50/50 allocation is based on a withdrawal rate of 6% in which the majority of portfolios do not reach the threshold of 30 years of longevity; increasing this rate would be pointless because someone entering retirement at rates like 7% or 8% would quickly exhaust their assets.

These results remark the impact of inflation, which affects the amount of assets that must be withdrawn each year.

Assuming a retirement goal of at least 30 years, all the 3% and 4% withdrawal rates portfolios would exceed this threshold. Instead, the graph with a rate of 5% shows a higher chance of consuming all the assets before reaching the goal; just 34 out of 51 have longevity longer than 30. With 6%, things worsen since just 20 portfolios, or 40% of the total, satisfy expectations.

Whether retiring in 30 years or what withdrawal rate to apply depends on personal preferences; some people set a 40-year retirement as their minimum objective, while others won't accept anything less than a 6% annual withdrawal from their assets. Knowing your desires or those of the client to whom you are suggesting a financial plan is essential.

For example, an investor, who wants to be certain that he/she will be financially secure for the next 30 years after retiring at age 60 and with yearly costs of about $50,000, should have beginning assets of $1,250,000 by using a 4% withdrawal rate.

Bengen wrote this article intending to assist his clients in selecting the optimum asset allocation and the appropriate withdrawal strategy depending on their needs.

2.3.2 Different Asset Allocation

All the analyzes carried out so far have been performed with a portfolio made up of 50% stocks and 50% bonds, although this was only an initial assumption made to simplify the computations. Bengen not only performed the study with this composition but produced the analyzes for various stock/bond distribution ratios.

Although stocks have traditionally been more profitable, excessive exposure to them during times of collapse can seriously harm the portfolio, which, when combined with the yearly necessity to withdraw a portion of assets, can result in the total consumption of funds. On the other hand, bonds provide lower yields that are safer but cannot extend the portfolio's durability; therefore, an investor's overexposure to the bond market is also unfavourable.

Bengen conducted the same study as in section 2.3.1 50/50 Stock/Bond Allocation for stock exposure of 0%, 25%, 50%, 75%, and 100%. Moreover, this time it has also included the withdrawals rate of 7% and 8%. The exposure to stocks was chosen arbitrarily, and the purpose of this step is to compare the results as the allocation changes. Finding the right allocation for a client will depend on their specific investment goals, risk tolerance, and financial situation.

Figure 4: Minimum Number of Years Withdrawals Will Last Assuming Worst Case

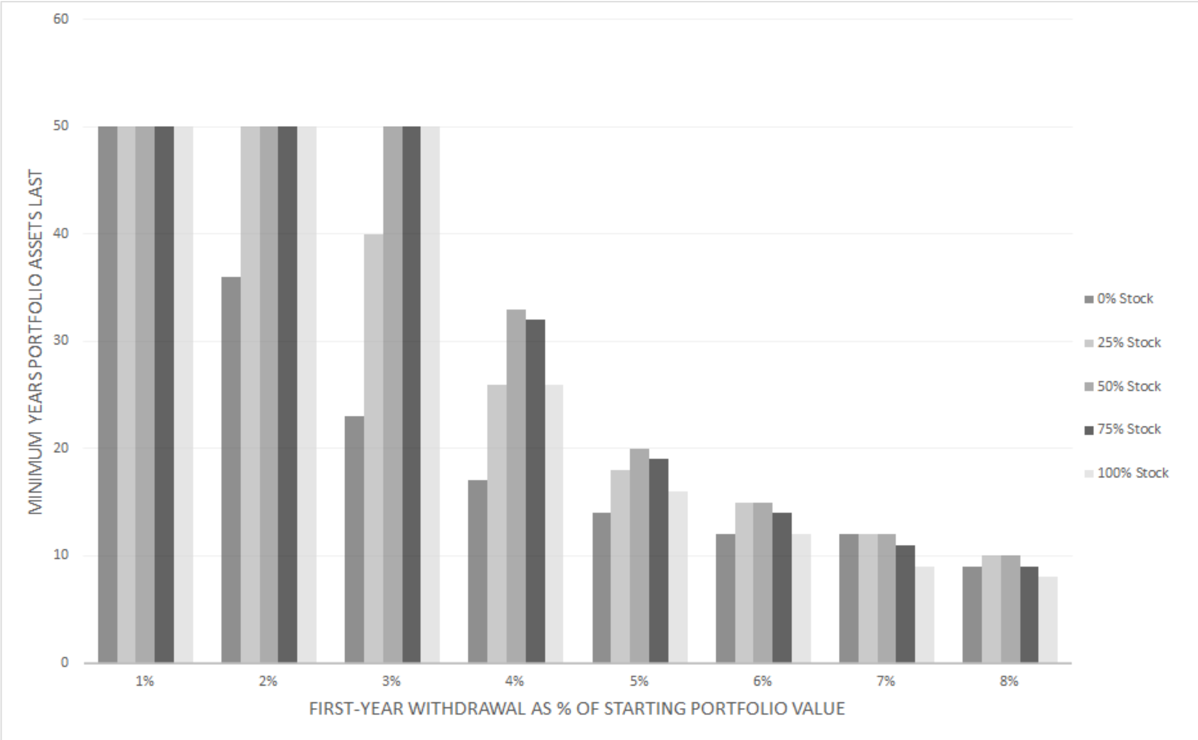


Figure 4 demonstrates the importance of stock holdings in a portfolio by showing how poorly the portfolios with only bonds perform compared to the others in the 2% withdrawal rate analysis. The 25% holding of stocks also had the lowest bars confirming that the return of stocks is needed for the long term.

The complete exposure to equity on the other hand doesn't give the best results, instead portfolios with 50% and 75% of stocks perform the best starting at a withdrawal rate of 4%. Longevity decreases more and more as the withdrawal rate rises, with percentages of 7% and 8% is only a few years.

It appears from Figure 4 that there is not much of a difference between 50% and 75% holdings, therefore the risk of having a larger stock concentration does not provide an additional benefit. However, the graph only displays the worst-case scenario, which is used as an evaluation parameter, rather than detailing the performance of the portfolios.

Figure 5: 1^{st} year withdrawal as a percentage of starting portfolio (75% Stock) value: 4%

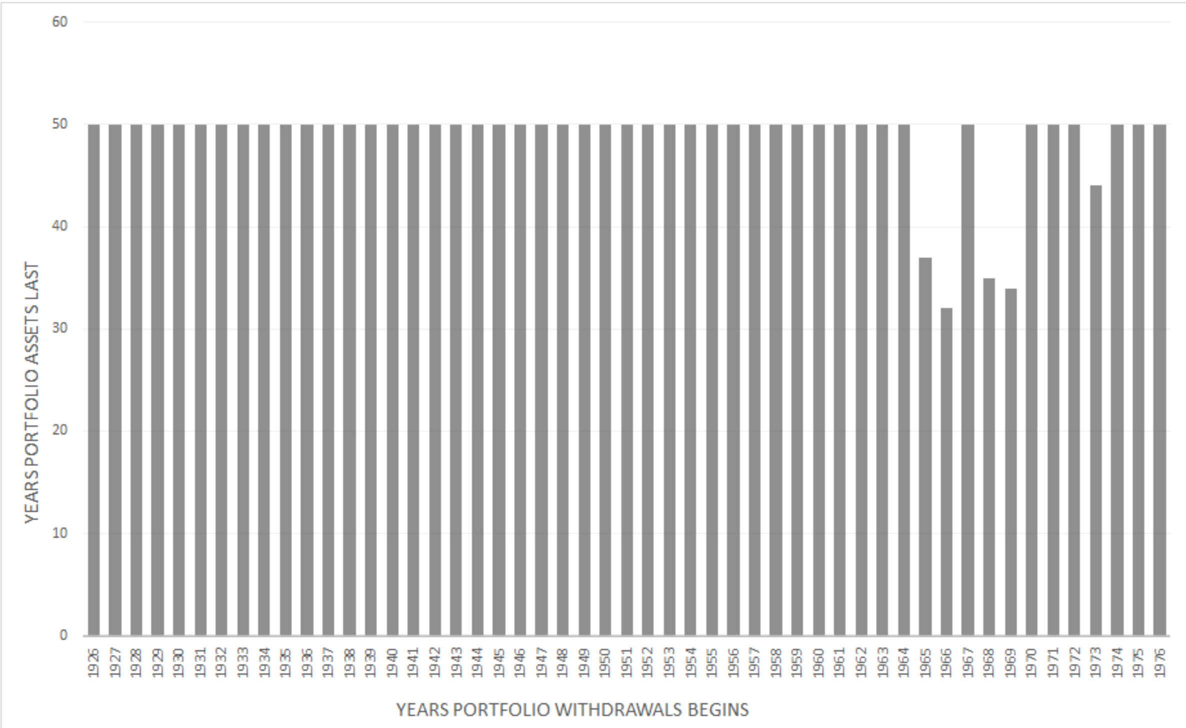


Figure 5 shows the same analysis as Figure 2, but for a portfolio that contains 75% stocks. A direct comparison of the two graphs reveals that the 50/50 allocation performs worse since 9 portfolios do not reach 50 years, as opposed to 5 of this last analysis. The only differences are in portfolio years 1966, which is reduced from 33 to 32, and 1969, which is reduced from 36 to 34. The longevity of every other year in the scenario is greater or equal. The 75/25 portfolio appears to be the best based on these results; therefore, Bengen repeats the research using the 5% withdrawal rate to check for confirmation. Even in the 5% case, results are better for most years, but on the other hand the minimum portfolio has a longevity of 19 years instead of the 21 of the 50/50 allocation case.

It can therefore be deduced how the capacity of stocks to bounce back from stock market corrections more quickly than bonds allows the longevity of a greater number of portfolios to be extended. The benefit of increasing stock holdings for a retiree is obvious, but increasing it beyond 75% becomes counterproductive.

Bengen says he performed the same analyses with higher than 75% allocation of stocks and that the results showed portfolio deterioration despite significant wealth accumulation during periods of growth. Furthermore, the minimum reached by these analyses, visible in Figure 4, is further lower. As specified in the introduction, the objective is to find a strategy that allows you to keep your investments balanced even through periods of a severe crisis; thus, making an allocation higher than 75/25 is unfavourable.

2.3.3 Ending Value of Portfolios

In addition to the portfolio's longevity, another variable analyzed is the final wealth that an investor accumulates over time. Several of Bengen's clients also wanted to leave their spouses or children a sizable inheritance by avoiding depleting all their assets during retirement. This variable is also relevant to persons who wish to leave no inheritance, given the personal interest in using up all investments before retirement.

In his article, Bengen examines a client's dollar value after 20 years of retirement. The study is carried out on portfolios made up of 50%, 65%, and 75% stocks based on the findings of section 2.3.2. Moreover, given the analyses in section 2.3.1, the overall initial withdrawal rate used is 4%. The initial value of the analyzed portfolios is $500'000.

Bengen only allows for the development of the portfolio to last for 20 years in this stage of the research; however, in this section, I will examine the values of the portfolio after 30 years of retirement. Verifying the value reached after as many years as feasible is more appropriate if the objective is to determine the potential final value of an inheritance. For most investors, 30 years is a suitable time frame for retirement, and we have previously confirmed that all portfolios from 1926 to 1976 with the mentioned characteristics have a longevity of more than 30 years.

Figure 6: Value of Portfolio 50/50 Allocation at the End of 30 Years

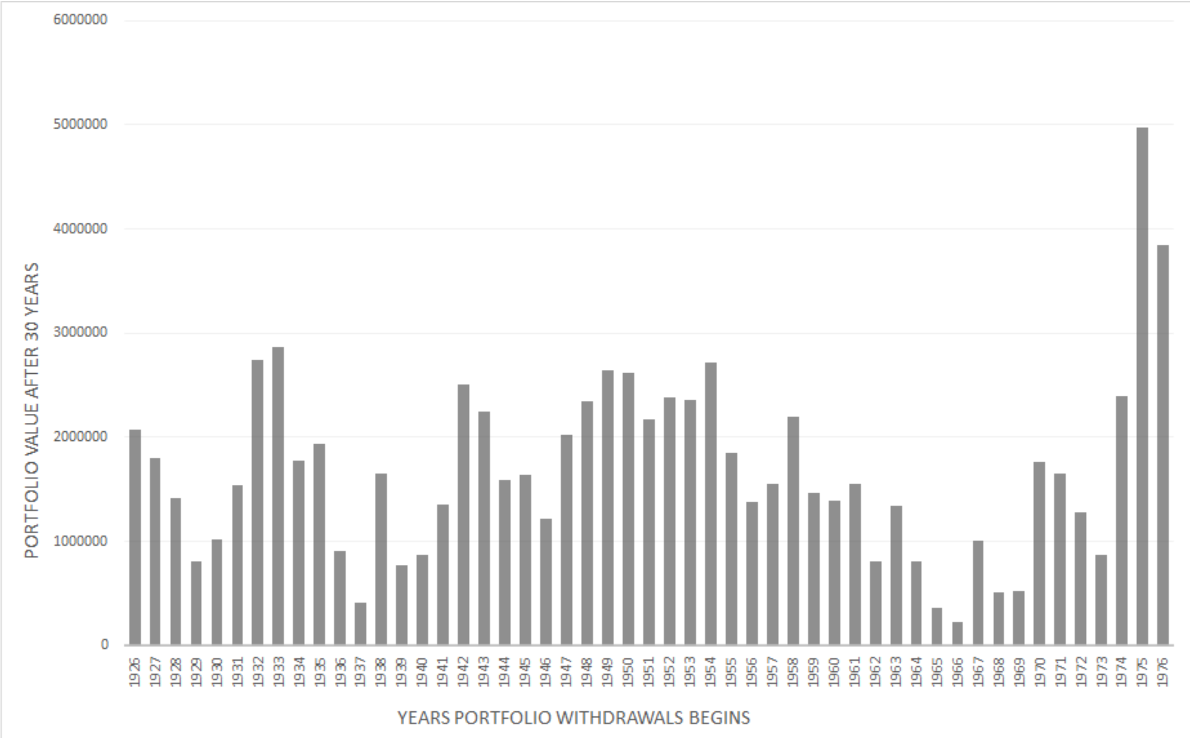


Figure 6 shows the 50-stock portfolio's final value after 30 years of withdrawal. Similar to Figure 2, in the graph are visible market witnessed significant drops, with 1966 having the lowest minimum of $227,000 in that year. The portfolios with the lower bars do not last much more than 30 years and do not leave a significant heritage. Instead, most years' outcomes are over a million dollars, and 17 portfolios reach earnings of over two million.

Figure 7: Value of Portfolio 75/25 Allocation at the End of 30 Years

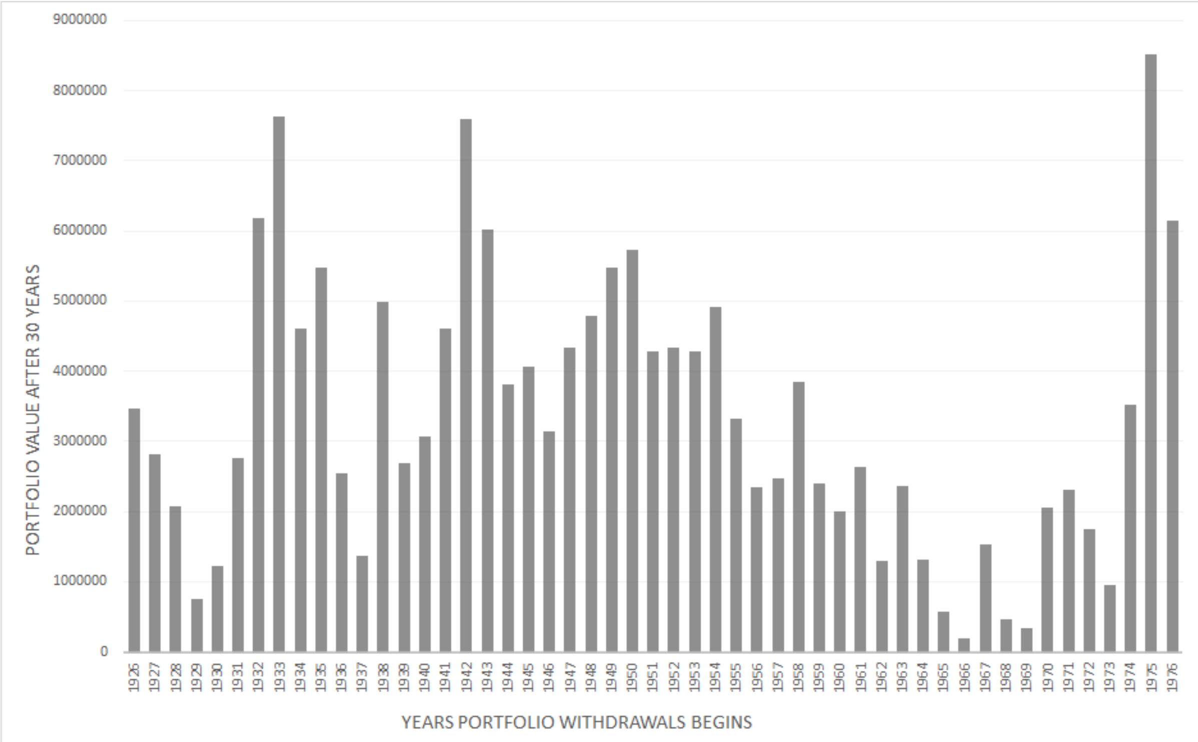


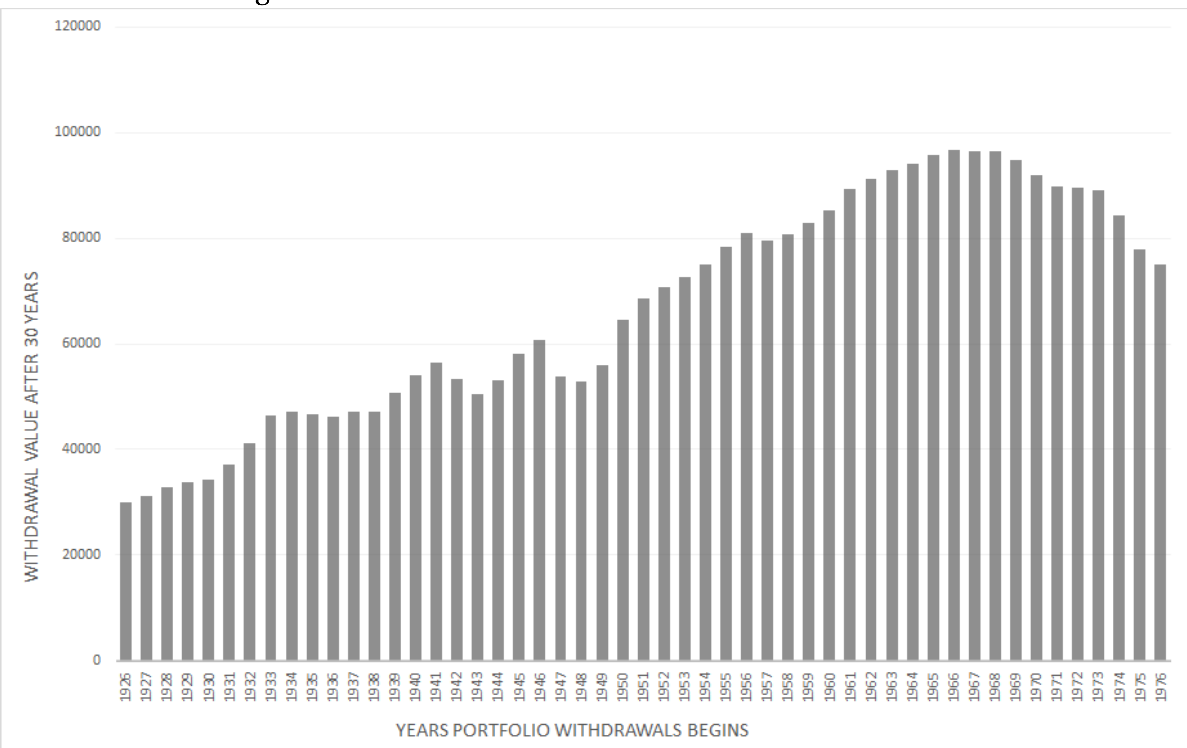
Figure 7 demonstrates an additional rise in portfolio values, which increase by an average of 65.8% when compared to the 50/50 allocation.

Analysis of final portfolio values is essential to estimate inheritances and calculate how much money will be left over if life expectancy is not more than 30 years. In the case of the 75/25 portfolio, many investors would have a net worth of more than $3 million after 30 years of retirement, always assuming a beginning value of $500'000. Very likely, many of these investors would like to spend their wealth more for personal interest and have no intention of leaving such a sizable inheritance.

An essential factor to be looked at concerning these graphs is the value of the withdrawal after 30 years of inflation. With a portfolio size of $500'000 and an initial withdrawal rate of 4%, the initial withdrawal is $20’000. Figure 8 shows the value of the withdrawal, regardless of the portfolio’s composition, after 30 years of retirement. It is evident from the graph that in the 20s and 30s, inflation was low or even negative; instead, all the portfolios from 1966 onwards have more than quadrupled the value of the withdrawal in 30 years of inflation.

For this reason portfolio like the one from 1968, which after 30 years still has half a million dollars in value and a withdrawal of $100’000, cannot maintain retirement for more than five years.

Figure 8: Value of Withdrawal at the End of 30 Years



2.4 Data Used

The article on which the 4% rule is based was written in 1994, and the data available then was limited to complete an accurate analysis. Bengen used as a source of data *Stock, Bonds, Bills and Inflation: 1992 Yearbook* of Ibbotson Associates [20], an investment management company which *Yearbooks* provides historical performance data of U.S. asset classes. This database of information—also known as SBBI—has been available online for decades since the first series of data published by professors Roger G. Ibbotson and Rex A. Sinquefield on common stocks, long-term government and corporate bonds, Treasury bills, and inflation.

The study is performed for all years, starting from 1926 to 1976, and each analysis reaches 50 years of evaluation. This means that, for example, for the year 1976, data up to 2026 would be needed, which is still impossible today.

The data source used in the study goes up to 1992, and therefore for all calculations performed after that year, Bengen used the average value of stocks (10.3%), bonds (5.2%) and inflation (3%). This approach calculates the value and longevity of portfolios since 1967 with less than half the real data needed. Today we know that all these portfolios would have faced two significant crises, the Dot-com bubble and the 2008 financial crisis.

The composition of the portfolios analyzed is divided into common stock and U.S. Intermediate-Term Notes, while inflation is measured by a Consumer Price Index. In The Journal of Business of 1976 Vol.49 [22] Ibbotson measures *common stocks* with the Standard and Poor's index, indeed over the years, the expression *common stock* is replaced by Large-Capitalization Stocks.

Standard and Poor's published a quarterly dividend series in addition to its Composite Index, which excludes dividends available only in the form of four-quarter moving totals. Dividends were then calculated:

$$\overline{D}_{m,t} = \sum_{j=t-3}^{t} D_{m,j} \tag{1}$$

Any $D_{m,t}$ can be solved recursively by using $\overline{D}_{m,t}$ for all t and four consecutive quarterly dividends.

$$D_{m,t} = D_{m,t+4} + \overline{D}_{m,t+3} - \overline{D}_{m,t+4} \tag{2}$$

Using common stocks designated as m, monthly returns are calculated by:

$$R_{m,t} = \frac{P_{m,t} + D_{m,t}}{P_{m,t-1}} - 1 \tag{3}$$

Where $P_{m,t}$ is the value of the S&P Composite Index at the end of month t; $R_{m.t}$ is the total return on common stock during month t; and $D_{m.t}$ is the expected dividends received during month t and reinvested at the end of month t.

Ibbotson and Sinquefield measured inflation using the Consumer Price Index (CPI), the rate of change in the prices of consumer goods. The CPI is an indicator structured by the U.S. Bureau of Labor Statistics of how prices for a market basket of consumer goods and services have changed on average over time for urban consumers, a selected consumer group that does not include all consumers in the U.S. economy. Nonetheless, it is a valid indicator for inflation which monthly rates of change are:

$$I_t = \frac{V_{I,t}}{V_{I,t-1}} - 1 \tag{4}$$

Where I_t is the inflation in time t and $V_{I,t}$ is the value of the CPI in time t.

The article written by Bengen is based on data published in 1993, but since I could not find the same Yearbook, I used the SBBI database of 2016 [21] (Appendix A) to perform calculations and create graphs. To adhere to the article *Determining withdrawal rates using historical data*, I used the data up to 1992 and kept the exact approximations used by Bengen.

2.5 Conclusions

An investor getting ready to retire must first decide what starting withdrawal rate best meets his needs by considering his total portfolio and life expectancy. Bengen verified that 4% of the portfolio is an adequate rate for those who intend to retire around the age of 60; 30 years should be enough to give the customer the security to not run out of funds.

The withdrawal rate is only used to determine the initial value that covers the pensioner's living expenses; after the first year, this value is adjusted according to inflation. An investor, dissatisfied with only 4%, who wants to increase the withdrawal rate takes a high risk. By increasing only up to 5%, as shown in Figure 4, there is a real possibility of exhausting the funds after 20 years; with even higher percentages, the risk is so high that it becomes a gamble.

Finally, an investor must assess the portfolio's proper allocation; as shown in 2.3.2, a proportion of shares between 50% and 75% must be used to create a lasting portfolio. Values lower than 50% would not be sufficient to support the portfolio, while values above 75% would be unfavourable in times of financial crisis.

This strategy is very effective if the retirees adhere to the original plan; therefore, they withdraw only the amount calculated annually based on inflation and do not apply changes to the portfolio's allocation. If it is needed to withdraw more money from the portfolio than what was suggested, an analysis would need to be repeated to determine the effect it would have on the longevity of the funds. If an investor decides to make changes to the initial allocation after high periods of volatility, such as increasing the percentage of bonds for fear of a market collapse, he/she would risk significantly damaging the portfolio's durability.

Based on the performance of the 4% rule Bengen divides retirees into three distinct groups. The first group, which he refers to as the "Black Hole" group (in reference to the field of astronomy that fascinated him), concerns investors who find themselves in the worse scenario. This group includes those who began their retirement period in the years preceding a major financial crisis and see their portfolio in sharp decline after a few years. *Black hole* retirees may want to change their asset allocation for fear of losing more money, and the current analyses would also point to portfolio longevity that is significantly lower than anticipated at the beginning of the plan. The best course of action in these situations is to stick to the initial strategy, and the only other option to see an improvement in the short term is to lower the withdrawal amount by cutting back on yearly costs.

The second group is called the "Asteroids", investors with a portfolio that matches initial expectations. This group includes, for example, those who started in 1942-1946 or 1959-1960. In this group, it is simpler to stick to the original strategy. Because of the prior accumulation, even if stocks decline in the final stages of retirement, there would be no concern about losing all assets.

They are the best group to handle from a financial advisor's perspective since strategy matches expectations, and clients do not need to question their original decisions.

The last group, the "Stars", is composed of fortunate investors who retired early when the stock market was booming, such as in 1949, the 1950s, 1975–1976, or even 1982. Their situation is entirely the reverse of the "Black Hole" group, their wealth expanded exceptionally quickly when they first retired, and they are inclined to raise their withdrawals and stock market allocation. Both actions may compromise their retirement.

Bengen made an example of how changing the initial values could harm the final result even in the case of a *Star*. Suppose a client starts retirement in 1958 with a portfolio of $500'000 and a withdrawal of 4%. Over the course of the next decade, he/she manages to double his wealth, reaching one million dollars, while the withdrawal has had a slight growth. As a result, he/she finds withdraws just 2.3% of the portfolio each year and chooses to raise the value again to 4%, almost doubling the total amount. Unfortunately, the years that followed were not as favourable, and by the end of 1974, he/she was left with a value of $777'000 and a high inflation rate, which increased the yearly withdrawal rate to 8%. Even in this scenario, a customer's fear may result in poor decisions, and furthermore, he/she could refuse to lower their lifestyle costs back to the original value.

Thus, it is necessary to caution the *Star* investors against significantly modifying their asset allocation or withdrawal plan. Although a little rise in withdrawals should not destroy the retirement plan, clients need to be aware that more returns made now may be necessary to offset losses in the future.

In conclusion, as a result of this study, many middle-class people now believe they can retire early on their savings. According to Bengen, the suggested amount for an individual's yearly retirement expenses is 4% of the portfolio or less. The portfolio's allocation is very simple; at the client's option, it can range from 50% to 75% in stock and the remaining share in Intermediate-Term Bonds.

Individuals, using the 4% rule in a planned approach and aware of their annual income and spending, may theorize the portfolio value they need to retire early and their retirement date. The rule may seem simplistic, but Bengen's thesis and calculations have been replicated and verified numerous times. This simplicity has made the study very famous, and today any blog or content creator dealing with financial independence uses the 4% rule as one of the reference points.

This study is not void of criticisms and limitations; the calculations are mainly based on average values and not on accurate data; furthermore, the fees and costs of the transactions are not considered. Bengen does not even consider the taxes owed on capital gains, but this issue may be resolved by including tax expenses in the total withdrawal amount. Moreover, since it is focused on the U.S., the analysis is unreliable from the perspective of any other global investor. Some have recently questioned the 4% rule's continued applicability; they note the poor projected returns from stocks in light of their high values, and fixed-income assets have lower expected yields.

Although criticisms are valid, the 4% rule has been shown to be dependable in various challenging circumstances. The analysis was carried out over a period facing several financial crises where sharp market downturns challenged all tested portfolios.

With this study, the research for the most effective strategy to achieve financial independence did not end; even Bengen presented new variations and alternative interpretations of the rule. Overall the 4% rule is still a reliable approach to seeking early retirement for any blog or content creator interested in the FIRE.

Chapter III
4% Rule Modern Analysis

3.1 Introduction

Nowadays, few studies are written on financial independence and the 4% rule, and they are all from the perspective of Americans who invest in the U.S. stock market and U.S. treasury bonds. For any investors who reside outside of the United States, they could be more reliable. A Japanese investor, for instance, should look at Yen inflation and the history of the USD/JPY exchange rate before making any consideration because historical inflation data changes depending on the reference currency. The same holds for every country whose official currency is not the dollar, making the analyses thus far meaningless for most people.

Let us take as an example currencies that have existed for more than a century such as the Yen, the Pound and the Swiss franc. From 1926 to the present day, the dollar's annual inflation rate has an average of 3%; instead, currencies such as the Yen and the Pound have more excellent annual inflation rates (around 4.1%), while the Swiss franc has had a lower average, 1.82%. Inflation reduces portfolio longevity by increasing annual withdrawals; only from this observation can we conclude, for example, that an investor living in Switzerland who applies the 4% rule by investing in American markets achieves better results than those identified by Bengen due to lower inflation.

Figure 9: Inflation Index of: CHF, USD, GBP, JPY

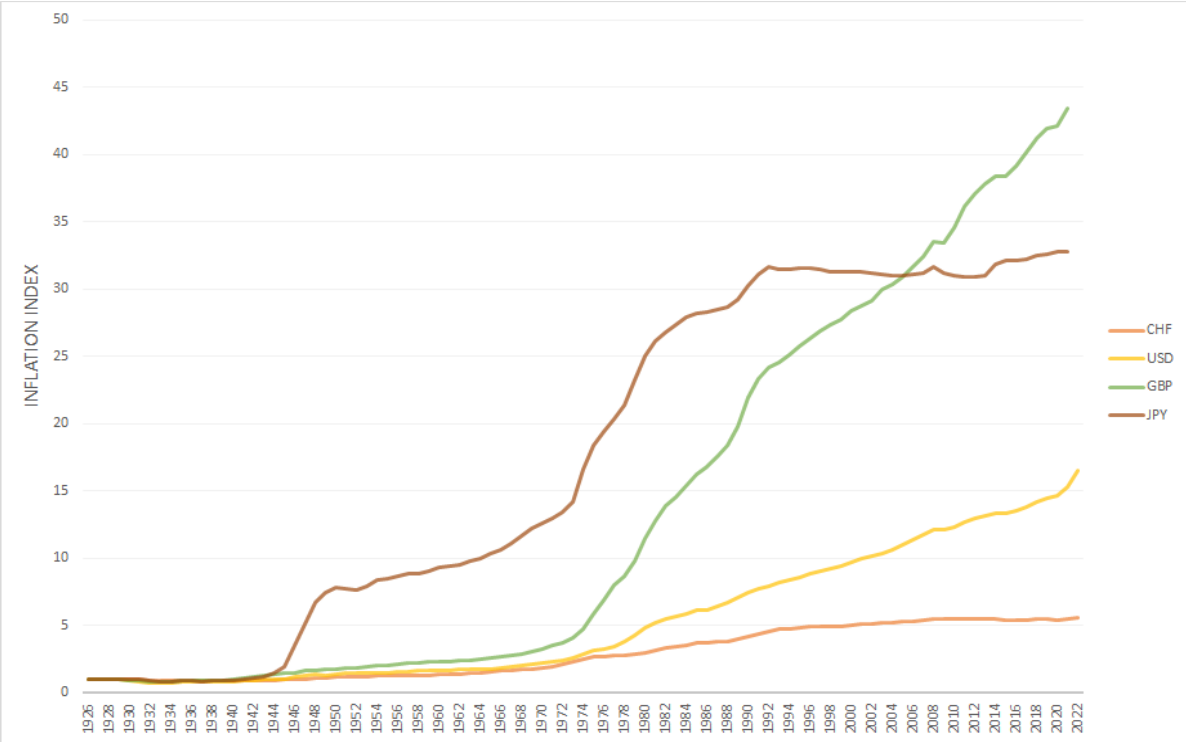


Figure 9 shows the Consumer Price Index trend value for Switzerland, the United States, England, and Japan. The purchasing power of the Pound and Yen has declined significantly over the previous century, particularly after the 1973 oil crisis. However, the inflation rate does not remain constant over time; for example, the Yen has only experienced 0.18% inflation over the previous 30 years, implying that under the 4% rule, yearly withdrawals would have been nearly constant.

Moreover, since the study is based on portfolios of stocks and American bonds, it works with returns based on the dollar's value. As a result, if foreign investors want to do the same analysis from their perspective, they must consider the exchange rate of the reference currency with the dollar. The value of the exchange rate can further raise the return on investment if the dollar's value increases with respect to the local currency; on the other hand, the opposite can happen when the dollar's value falls.

Year	CHF	GBP	JPY
1926	5.178	0.206	4.433
1936	3.313	0.265	5.028
1946	4.280	0.248	361.100
1956	4.286	0.357	360.000
1966	4.326	0.358	360.000
1976	2.500	0.556	296.552
1986	1.799	0.662	168.520
1996	1.236	0.667	108.779
2006	1.254	0.543	116.299
2016	0.985	0.741	108.793
2022	0.955	0.808	131.498

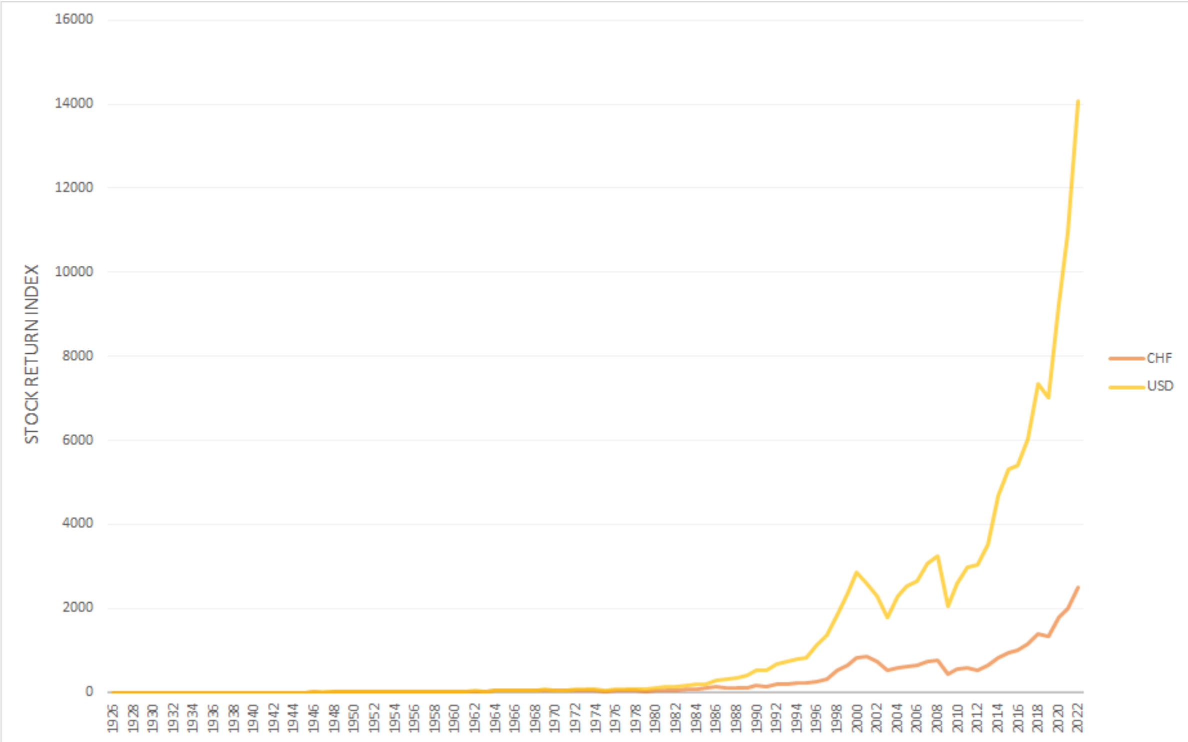
Table 3: The value of a dollar from 1926 to the present in CHF, GBP, JPY

Table 3 shows that British and Japanese investors exposed to the American market have obtained a further advantage over time thanks to the change in the exchange rate. In particular, the Japanese yen lost much value after the Second World War, a dollar in 1946 is worth almost 72 times more than 10 years earlier. The Swiss case, on the other hand, is the opposite, given that the dollar has lost 81.5% of its value over the last century against the Swiss Franc, thus making returns smaller on the American markets.

According to Ibbotson data, the return on American stocks was 236.6% from 1926 to 1936; therefore, a $1 investment at the beginning of 1926 would be worth $2.37 in 1936. Swiss investors who opted to invest in American stocks in 1926 would have had an initial exchange rate of 5,178 CHF for each dollar; in 1936, they would end up with $2.37 for every dollar invested, which at the current rate would be 7.84 CHF. This shows that the Swiss francs invested in 1926 in the American stock market returned 151.4%

after 10 years, 85% less than the return in dollars.

Figure 10: Stock return index in USD and CHF



From Figure 10, it is evident how the impact of the exchange rate can completely change the result of the analyzes carried out in the second chapter. Due to the compound return of stocks, an investment in dollars made in 1926 today would be worth 11535.39% more, while the same investment valued in CHF would be "only" grown by 2127.42%.

3.2 The Swiss Case

This section reproduces the same analysis by Bengen in *Determining withdrawal rates using historical data*, but on portfolios of Swiss retirees who invest in the stock and bond American markets. The only variables to be included are Swiss inflation and the historical USD/CHF exchange rate. The two additional factors have opposing effects: lower inflation has less influence on portfolio longevity, but the exchange rate, which loses value with time, diminishes the effect of the return on investment.

The analysis is carried out on portfolios that begin the withdrawal period from 1926 to 1976, always maintaining the same initial sock and bond allocation. It is performed using the same data database as the second chapter so that it can be compared directly with the results already carried out. From a Swiss point of view, the withdrawal amount is adjusted every year not only according to Swiss inflation but also according to the current exchange rate.

For example, assuming a Swiss retiree needs 100'000 francs to cover annual expenses and that the first year USD/CHF equals 5, then the dollar value needed to be withdrawn from his portfolio is $20'000. If the following year, Swiss inflation was 3% and the new exchange rate equals 4, then he/she will need to withdraw 103'000 CHF from the portfolio, which is $25'750. A sharp drop in the dollar value significantly increases the withdrawal amount from the portfolio to support living expenses. The withdrawal value measured in CHF grows only based on inflation, while that measured in USD varies considerably also based on the exchange rate.

Figure 11: 1^{st} year withdrawal as a percentage of starting portfolio (75% Stock) value: 4%. Based in Switzerland

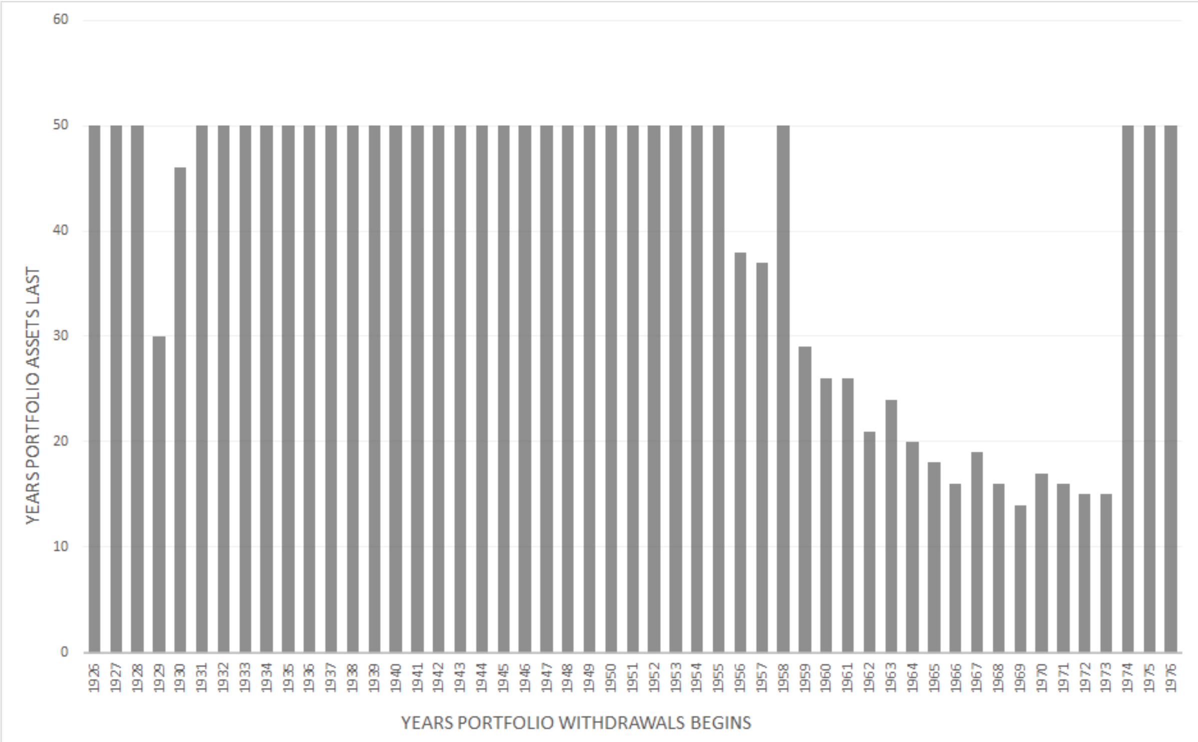


Figure 11 reveals results that are entirely different to those in Figure 5; both graphs assess the longevity of portfolios with 75% stock and a 4% initial withdrawal. It is evident how the same methodology that works for the American case has not had the same results in the Swiss case; the shift in the exchange rate has collapsed the longevity of a substantial portion of the portfolios. There is a sharp decline in 1929, which barely surpass 30 years of longevity, and the second begins in 1955 with a constant decrease (except in 1958) that ends before the final three portfolios. Overall, only 35 out of 51 portfolios exceeded the threshold of 30 years of longevity, which instead, in the American case, was exceeded for all years. Furthermore, just 25 of the portfolios with less than 30 years of longevity exceeded 20 years, and the minimum was achieved in 1969 with only 14 years of longevity.

All the portfolios of the 60s achieved low performances due to the USD/CHF exchange rate that fell by 61.1%, from 4.3105 to 1.67571, between 1970 and 1980. In just one decade, the movements in the dollar's value and the stocks drop due to the 1973 oil crisis have collapsed the value of portfolios. For an American, the value of stocks in 1973 had fallen by 15% while from the point of view of the Swiss franc, the decline was double. Moreover, in these years, Swiss inflation has reached an all-time high, having an average of 7.69% between 1971 and 1975.

The dramatic effect of the exchange rate and inflation is visible as the withdrawal amount in dollars increases. Consider the Swiss who begin their retirement period in 1973 by investing in the American market and who require $40'000 (i.e. 126'593.2 CHF) for living expenses in the first year and whose withdrawal rate represents 4% of the portfolio. After only one year, the value required to cover living expenditures goes from $40'00 to $52'494.88, while the portfolio value falls from $1'000'000 to $848'855.12. It implies that the ratio between withdrawal and portfolio goes from 4% in the first year to 6.18% in the following year. At the end of 1975, the portfolio will be worth $631'198.06 and the withdrawal amount $61'213.05, therefore 9.69%. With such high withdrawal rates after only a few years of investing, it is not unexpected that the portfolio has a longevity of under two decades. All these calculations were carried out with the dollar as the reference currency, but the result would remain unchanged if they were carried out with the Swiss franc as the reference point.

Figure 11 shows a stock allocation of 75%; however, even though the exposure to equities is reduced or increased, the results are still negative. With the 3% initial withdrawal rate, better results are obtained, but they are still far from the study carried out by Bengen. His conclusions no longer hold up in the Swiss case; 4% is not a safe withdrawal rate given that only 68.62% of the portfolios exceed 30 years of longevity.

This analysis was carried out on a Swiss investor exposed to the American markets, thus also subjecting him to the American currency. This significant volatility may be avoided by investing directly in Swiss equities, but because the first Swiss market index was established in 1988, a direct comparison with these results is impossible.

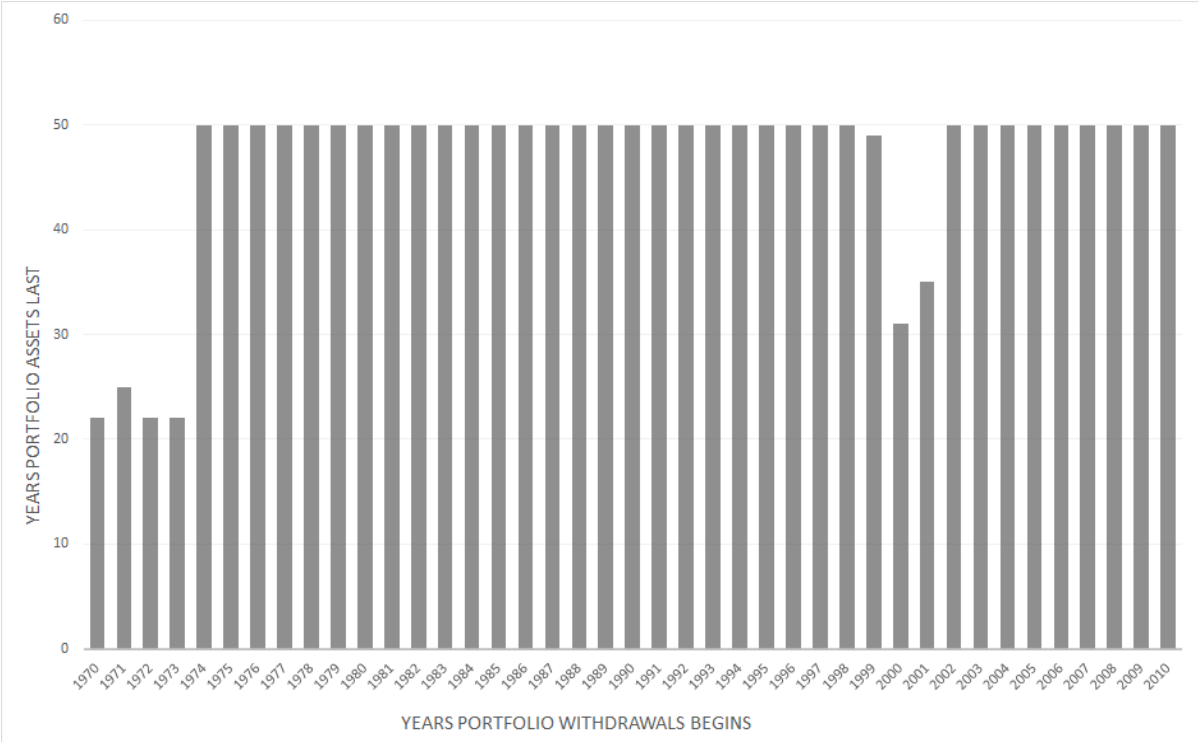
A more recent analysis can be performed to verify if the 4% rule remains applicable to portfolios that are not primarily invested in American markets. Because the Swiss index lacks sufficient historical data to conduct a significant analysis, the MSCI World index was chosen for the equity component of the portfolios (for reasons explained further in 3.3). For the bond allocation has been used Swiss government with 10 years of maturity bonds that pay coupons annually.

The results shown in Figure 12 range from 1970, the year of establishment of the MSCI World, to 2010, so that even the latest portfolios analyzed have at least more than a decade of accurate data used.

Also, in this case, in the early 70s, the longevity is considerably lower than the rest of the portfolios that exceed at least 30 years. The years from 1970 to 1976 align with the results in Figure 11, demonstrating that even if the stock index and government bonds are changed, the 4% rule in that period does not give good results.

Overall, the results are positive; only 4 of the 41 portfolios analyzed do not exceed 30 years of longevity.

Figure 12: 1^{st} year withdrawal as a percentage of starting portfolio (75% Stock) value: 4%. Swiss market



3.2.1 Other Countries' Case

The Swiss case reveals how the analyses performed on the 4% rule do not stand up in any scenario; instead, opposite results would be obtained when considering other reference nations such as Great Britain or Japan.

It is sufficient to compare the annual withdrawal change to verify if the performances of a foreign portfolio invested in the American market owned by a foreign resident outperform those found by Bengen. If inflation is the only element that influences an American's withdrawal, in the case of a resident abroad, in addition to national inflation, the exchange rate also impacts. Low inflation and dollar appreciation minimize or even decrease the amount to be withdrawn from the portfolio each year.

Figure 13: The growth of withdrawal from 1926 to 2022 in Switzerland, United States, Great Britain and Japan

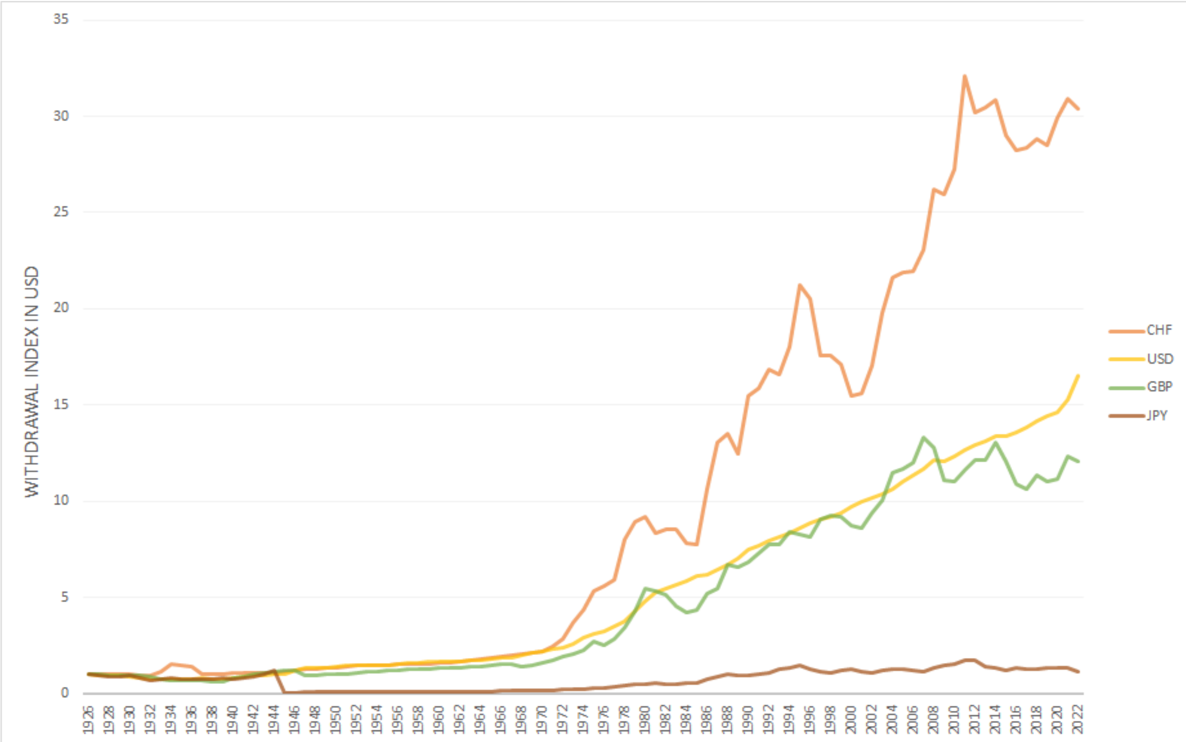


Figure 12 shows why the Swiss example does not have the same performance as the U.S. case; in Switzerland, the cost of living in dollars has increased by 30 times in 100 years, while in the United States, by 16 times. In Great Britain, the growth in the cost of living in dollars does not exceed that of the United States; therefore, an analysis of the longevity of the portfolio of an English citizen would give better results. The cost of living in Japan measured in dollars has almost flattened as the dollar appreciated 70 times against the yen after WWII, but the yen has recovered since 1970.

In 2017 Professor Wade D. Pfau, in his book *How Much Can I Spend in Retirement?: A Guide to Investment-Based Retirement Income Strategies* analyzed the withdrawal rate with historical data of 19 other developed market countries. Of these countries, only Canadian portfolios achieved 30 years of longevity safely, while for the other 18 cases, the chance of success varies from 8% to 62%. On the other hand, the global market would have been able to sustain an initial withdrawal of 3.5%[31]. These results confirm the limited significance that the 4% rule has for non-US people, but they also point out how the returns of American stocks during the 20^{th} century have been exceptional. It is also wrong to expect that historical data reflect perfectly the future; following returns of the American market may not necessarily satisfy the same growth that occurred in the last century.

3.3 Analysis

The analyzes carried out up to now have all been performed for the years ranging from 1926 to 1976, the period analyzed in *Determining withdrawal rates using historical data*. This period dated back more than 50 years ago; therefore, an analysis carried out over the last 50 years is necessary to verify the applicability of the 4% rule. Also, comparisons were always made between portfolios invested in U.S. stocks and bonds. In this chapter, the analysis will focus on a recent period of years and will also be performed on portfolios invested in bonds and stock indexes other than the U.S. treasury notes and the S&P 500.

In this section, the portfolio used is indicated for a general retiree and does not indicate a specific country or geographic area. The goal is to conduct a generally valid analysis for any investor who intends to retire early. Furthermore, it will no longer be carried out to verify which would have been the best withdrawal in the past decades but to try to identify the best strategy for retiring based on the data we have.

A retiree's portfolio should be structured for a passive investing strategy, as the only changes to be made are annual rebalancing. Passive investing also does not require active knowledge of the financial market and the delegation of portfolio management to a professional investor. Overall, passive investing offers a simple and low-cost approach because a passive portfolio does not require as much time and resources as an active one, which translates to lower fees for investors. Therefore, the best choice is to invest in an index that tracks the market because it is more consistent in returns and less prone to market volatility.

A well-diversified portfolio should not only be exposed to single-country equities but also invested in international equities. Home bias is widespread among investors; it is the tendency to own many stocks in one's own country and less foreign equity, thereby reducing international diversification. Many investors are satisfied with diversification only across sectors, so an index like the S&P 500 is already properly balanced. Modern portfolio theory shows that the risk of an Investment Portfolio is determined by the correlation between its assets and not by their average riskiness of them. Diversifying a portfolio across imperfectly linked assets helps investors improve anticipated returns while decreasing risk or decreasing risk while raising expected returns. According to Markowitz, diversification is the only optimization in investing because raising projected returns necessitates taking on greater risks[27], and so cutting off international stocks from a portfolio is counterproductive.

Many investors consider international markets to be highly correlated, but this correlation is only present in the short run and not in the long run, as shown by Viceira, L., Wang, Z. (2018)[39].

> *"....global stock return correlations has increased the short-run risk of globally diversified equity portfolios in the 2000-2016 period relative to the preceding 1986-1999 period. However, long-run global equity portfolio risk has not increased, optimal long-horizon portfolios are as globally diversified and invest in equities as much as in the preceding period, and the expected utility of long-horizon investors from holding global equity portfolios has if anything increased."*

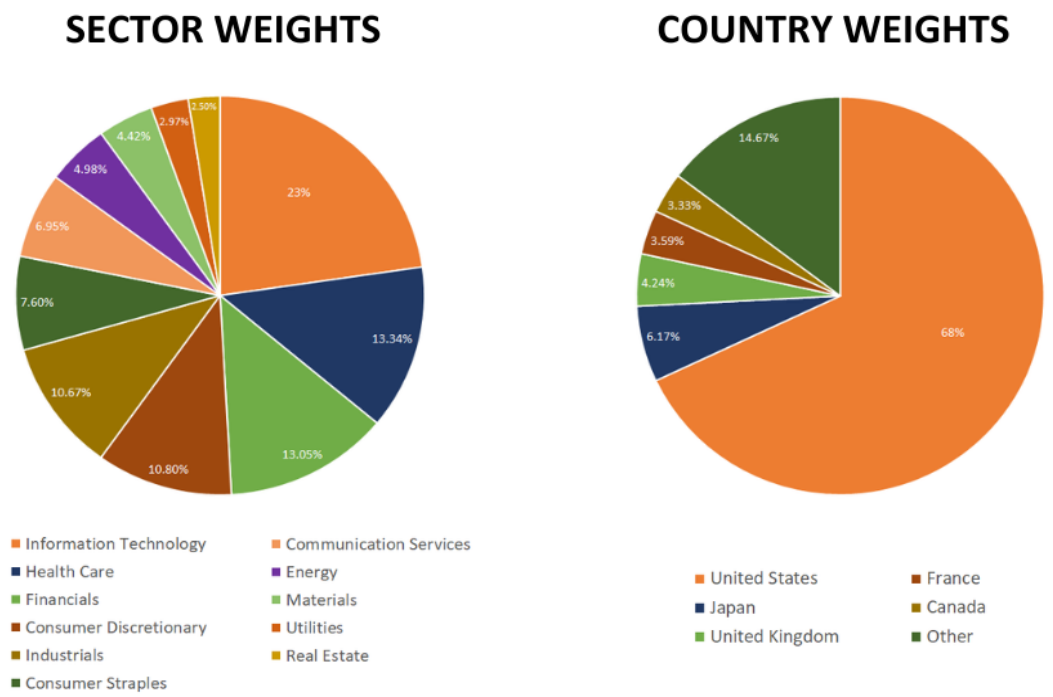
This implies that the benefits of international diversification have not diminished for long-term investors; long-term returns are more important to wealth creation and wealth destruction, and markets do not tend to crash simultaneously; global equity diversification protects investors from holding concentrated positions in countries with low long-term returns.

International diversification is essential to portfolio construction both theoretically and empirically. The past success of a single market, such as the American one, provides security. However, the returns' expectations are reflected in current prices, making it less likely that the future will reflect the past because we cannot predict which markets will be most successful in terms of investment returns[16].

For the analysis of portfolios in this chapter, it has been used the MSCI World, a capitalization-weighted stock market index with 1'508 members from throughout the world, serves as a standard benchmark for 'global' equity funds. Since 1969, the MSCI

World Index has been evaluated in U.S. dollars, Euros, and local currencies, making it easier and cheaper for people outside the United States to invest in it. Moreover, this index was established more than 50 years ago, while the European national indices were mainly founded in the second half of the 1980s. MSCI World is a capitalization-based index, and so the most significant percentage is occupied by U.S. equities, which takes 68% of the total weight.

Figure 14: MSCI World general information[29]



The debt securities element of the portfolio aims to balance the high volatility of the equity portion while also providing a certain level of capital preservation. The ideal bonds for a long-term retirement plan are those with the lowest risk and, therefore, with an investment grade. In the analysis of this section were used 10-year U.S. government bonds were the same chosen by Bengen in the previous chapter. The American bonds were maintained because historical data are available, and they have always received a grade better than AA[37]. Moreover, using bonds that have almost always obtained the strongest grade has many advantages. Bonds with a high grade are considered to be the safest and most secure bonds available, any investor in the world can easily acquire them, and they are easier to sell than lower-rated bonds; this makes them a more attractive option for retirees who may need to sell their bonds. Furthermore, if an investor wanted to conduct a similar analysis with AAA bonds from other countries, he would obtain similar results given the low spread between bonds with the same grade.

The last component to carry out the analysis is inflation which varies according to the country to be analysed. In this chapter, we will use the inflation of the U.S., U.K., Switzerland, Italy, France, and Germany.

3.3.1 Time Series Models

In order to analyze a retirement plan that starts at the beginning of 2023, one must rely on more than just the average values of the historical data of the last few decades, but it is necessary to create forecasting models that simulate possible future values. Bengen, in his study, used the mean value as a replacement for missing data, which was only a minor part of the data he needed. In this case, the totality of the data is missing, and therefore the average value is not sufficient to perform an accurate analysis; instead, it will be used time series models that are statistical models used to evaluate and forecast time series data. The primary goal of time series modelling is to discover underlying patterns, trends, and correlations in data and use them to forecast future values of the variable. This analysis aims to find a model that allows not only to identify the trend of the data series but especially its volatility.

With the expected value of returns on stocks and bonds, and the expected inflation, the best initial withdrawal rate could be calculated directly without the need for any analysis, but it would be a strategy that does not take market volatility into account. A good retirement plan must consider that there are periods of high growth and periods of crisis, so the variance of historical data must be addressed.

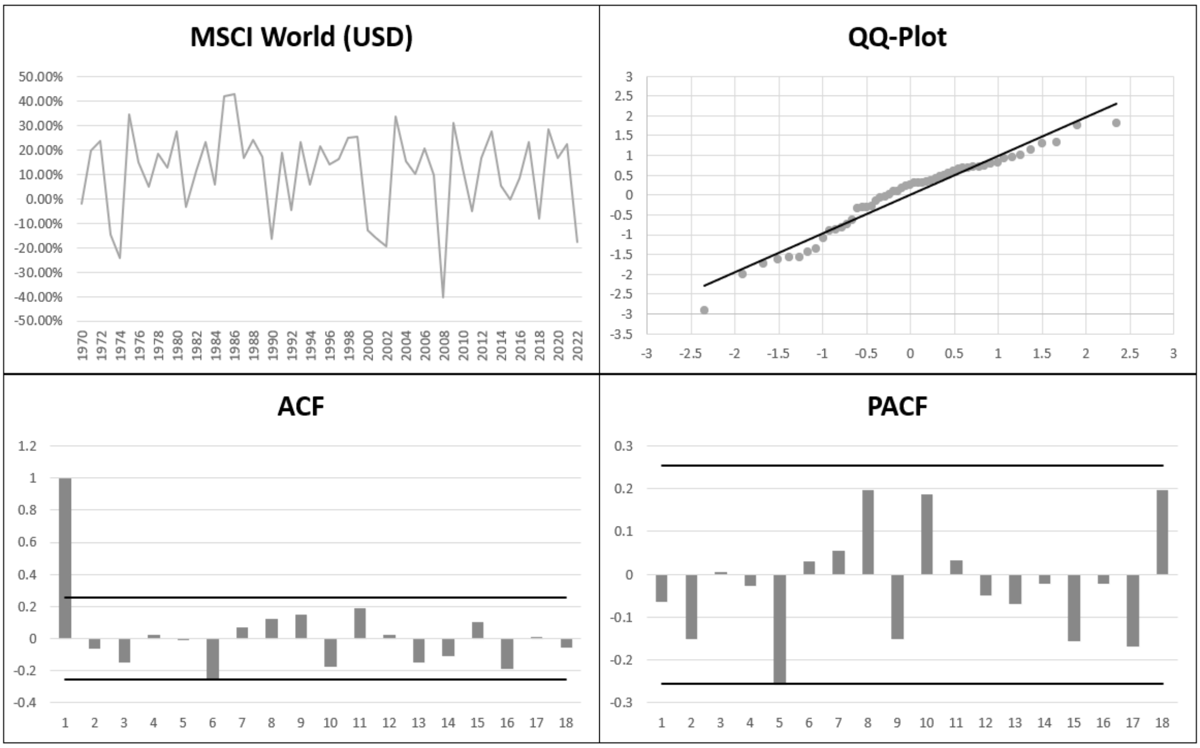
To develop the appropriate model, start by examining the data to identify any trends, seasonal patterns, variance and mean stationarity. If the time series is stationary, ARIMA models should be the first to be considered by identifying the order of the model parameters based on the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the time series data. These functions offer information on the correlation between time series data at various delays, which can be used to determine the order of the autoregressive (AR) and moving-average (MA) terms in an ARIMA model. To evaluate the model to use, the average absolute difference between the actual and anticipated values of the time series must be minimized. Moreover, to compare different model, there are several value to look at, such as the log-likelihood value or the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

When analyzing time series models, it is necessary to select the proper measurements based on the specific context and goals of the analysis. A model that performs well in one metric may perform poorly in another; therefore, it is important to under-stand the model's accuracy and effectiveness. In this analysis, it is not essential to identify the future returns in the coming years precisely because the objective is to evaluate the various scenarios that a retiree's portfolio could face in the next 50 years. No model can say with a high probability what the returns of stocks and bonds, and the level of inflation, will be like in 50 years.

To run a retirement portfolio simulation, the time series models of MSCI World, 10-year U.S. government bonds, and inflation are needed.

Figure 15 shows the results of the MSCI World returns measured in U.S. Dollars.

Figure 15: Output of MSCI World (USD)



The graph shows that the MSCI World series is stationary; this observation is further supported by conducting a unit root test, such as the commonly used Augmented Dickey-Fuller test. The use of this statistical test in the current situation provides a p-value of 0.02788, which confirms the series' stationarity. Furthermore, a quantile-quantile plot (QQ-plot) demonstrates that the time series data is highly consistent with a normal distribution. Further examination of the series using the autocorrelation function (ACF) and partial autocorrelation function (PACF) reveals that neither an autoregressive nor a moving average component is required to build an effective model for this time series.

The model found for the MSCI World returns is an ARIMA (0,0,0) with a statistically significant mean of 0.1099 and ϵ_t as the error term. The mean and the variance of the model coincide with the ones of the data. The ARIMA(0,0,0) model is typically used to analyze and forecast time series that have no trends or seasonality and in which the mean and the variance are constant over time.

$$Y_t = 0.1099 + \epsilon_t \qquad \epsilon \sim N(0, 0.03087)$$

This model is the one with the highest log-likelihood and the lowest AIC and BIC. The normality of the error term is verified with the Shapiro-Wilk test.

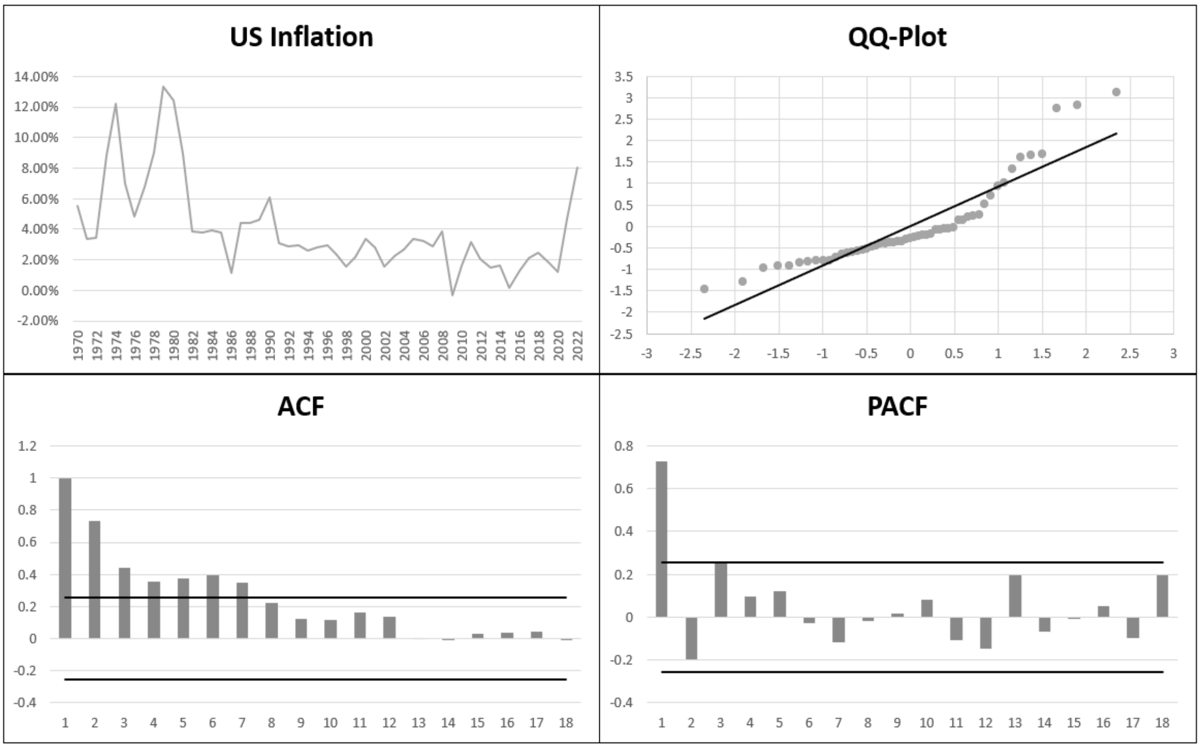
The 10-year U.S. government bonds (X_t) model is calculated with the same process and with data from 1970 to 2022.

$$X_t = 0.0663 + \eta_t \qquad \eta \sim N(0, 0.00529)$$

The best model for bonds is an ARMA (0,0,0) with a mean of 0.0663 and a variance of 0.00529. The mean and the variance of the model coincide with the ones of the data.

The model for U.S. inflation (I_t) gave different statistical results, as shown in Figure 16.

Figure 16: Output of U.S. Inflation



The ACF and PACF functions suggest the presence of AR and MA parameters, while the QQ-Plot does not suggest a normal distribution of the data.

$$I_t = 0.5698 I_{t-1} + 0.4461 \xi_{t-1} + 0.01811 + \xi_t \qquad \xi \sim N(0, 0.0003703)$$

The best-fitting model for inflation is an ARMA (1,0,1) with a mean of 0.042099 and a variance of 0.000936. The mean and the variance of the model differ slightly from the ones of the data mean of 0.0401 and a variance of 0.000892. The errors are normally distributed as confirmed by the Shapiro-Wilk test.

Through these three models, 50 years of data can be simulated to verify the trend of a portfolio starting from 2023. The last thing to consider before performing the analysis is the correlation between the models because stock, bond, and inflation are all correlated since they are all affected by the economy's general health and performance.

Data generation is, therefore, also dependent on the correlation between the estimated residuals of the models.

	ϵ_t	η_t	ξ_t
ϵ_t	1	0.03660276	-0.3376415
η_t	0.03660276	1	-0.2584375
ξ_t	-0.3376415	-0.2584375	1

Table 4: Correlation between residuals

3.3.2 Alternative to ARIMA Models

Other models besides ARIMA can be used to carry out the analysis, such as VAR Models and State Space Models.

VAR (Vector Autoregression) Models are a class of multivariate time series models that helps analyse and forecast the dynamic relationships among multiple variables. They are a generalisation of the Autoregressive (AR) Model by allowing for multivariate time series. This type of model could replace ARIMA Models because they are particularly useful when analysing time series data that exhibit correlation, such as in this analysis composed of Stocks, Bonds and Inflation.

The second alternative is State Space Models, a class of statistical models that are used to describe how systems behave across time. They are frequently used in engineering, economics, and other professions where complex system behaviour must be understood and planned.

A state space model is made up of two parts: the state equation and the observation equation. The observation equation ties the system's state to the observed data, whereas the state equation defines how the system's state changes through time. The state equation describes how the system evolves from one-time step to the next, and the observation equation describes how the system's state is related to the measurements or observations available at each time step.

$$State \quad Space \quad Equation: \quad \dot{x}_t = Ax_t + Bu_t + w_t$$

$$Observation \quad Equation: \quad y_t = Cx_t + Du_t + v_t$$

In the State Space Equation, $\dot{x}_t$ is the time derivative of the state vector x_t; A is the state transition matrix describing how the system evolves over time, B is the input matrix describing how the input impacts the system, u_t is the input vector, and w_t is the state noise process.

While in the Observation Equation, y_t is the observation vector containing the variables that can be observed at time t, C is the observation matrix describing how the state variables relate to the observed variables, D is the input-output matrix describing how the input affects the observed variables, u_t is the input vector, and v_t is the measurement noise process. u_t reflects any external impacts on the system being represented that are known or controlled and may alter the state variable x_t.

State space models are a general class of models that describe the behaviour of a wide range of systems throughout time. ARIMA models can be considered as a specific case or a subset of state space models.

Let us take as an example an ARIMA(1,0,1) such as the one that fits the U.S. inflation. The general equation for an ARIMA (1,0,1) is:

$$Y_t = \mu + \phi Y_{t-1} + \theta \epsilon_{t-1} + \epsilon_t$$

Write the state space equation and the observation equation where X_t = state variable vector at time t, Y_t = observed value at time t, ϵ_t = error term at time t, q_t = process noise at time t.

$$State \quad Vector: \quad x_t = [Y_{t-1}, \quad \epsilon_{t-1}]$$

$$State \quad Space \quad Equation: \quad X_t = Ax_{t-1} + Bu_t + q_t$$

$$Observation \quad Equation: \quad Y_t - \mu = CX_t + Du_t + \epsilon_t$$

It can be written in a state space model form by making the matrices A, B, C, and D explicit. In the U.S. inflation model, no input vector is used, and therefore the matrices B and D are equal to 0.

Matrix A defines how the system evolves over time, and matrix C defines the relationship between the state variables and the observed variables.

$$A=\begin{bmatrix} \phi & 0 \\ 0 & \theta \end{bmatrix} \qquad C=\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad q_t \sim N(0, Q)$$

Where Q is a process noise covariance matrix.

3.3.3 Modern Analysis on U.S. case

Using the data generated by the models, it is possible to simulate the retirement expectations of individuals who aspire to achieve financial independence and retire early (FIRE) by relying on passive income streams for the remainder of their lives, beginning in 2023. To ensure consistency across all portfolios studied, the investments will be allocated to MSCI World and 10-year U.S. Government bonds (rated AAA), while inflation data will be tailored to each respective country. The simulations performed for each analysis will be on 10'000 portfolios in order to obtain statistically significant results.

Unlike the study by Bengen, the results only concern portfolios whose withdrawals begin in 2023. The analytical approach adheres to the same method employed by Bengen, whereby the initial withdrawal rate adjusts to account for inflation. The evaluation yardstick will be given by the distribution of the longevity of the 10'000 simulations.

The first case studied involves U.S. residents with portfolios 50/50 (stock/bond) and an initial withdrawal rate of 4%.

Figure 17: Empirical CDF and Empirical PDF of 10’000 portfolios based in U.S.

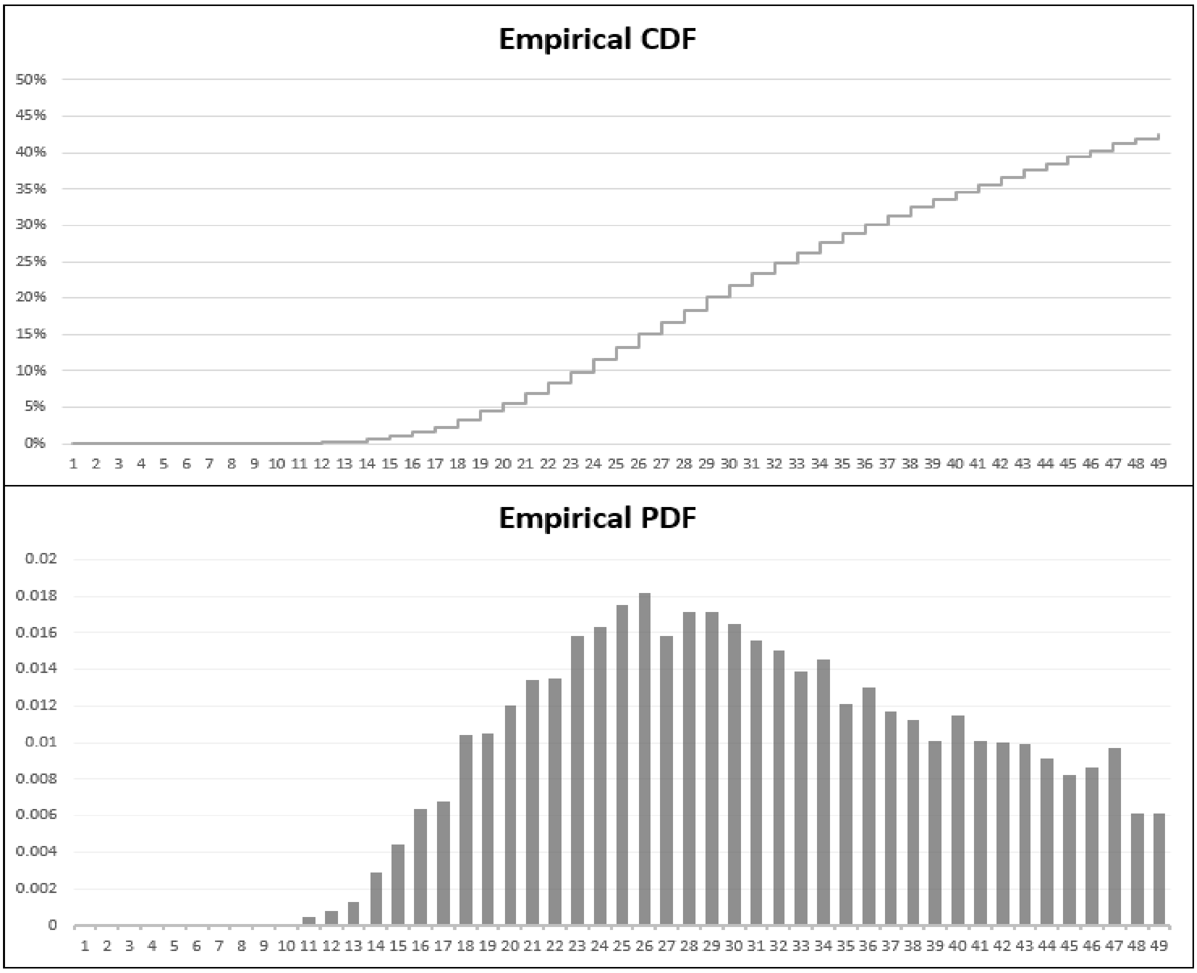


Figure 17 shows the Empirical Cumulative Distribution Function (CDF), a mathematical function that describes the probability distribution of a random variable that, in this case, is the longevity of the portfolios of the 10’000 simulations performed. The CDF shown is a discrete CDF given that it can be represented graphically as a step function, where the horizontal axis represents the longevity, and the vertical axis represents the cumulative probability.

The Empirical Probability Distribution Function shows the probability that the portfolio lasts a specific number of years. The graph does not show the probability for the longevity of 50 years or more since it is clearly higher than the others (57.63%) and, therefore, would have made the graph unreadable.

The table below summarizes the cumulative probabilities.

Longevity	**Probability**
$L \leq 10$	0.01%
$L \leq 20$	5.61%
$L \leq 30$	21.73%
$L \leq 40$	34.59%
$L \geq 50$	57.63%

Table 5: Empirical Cumulative Probability (US). Portfolio 50/50, 4% withdrawal rate

The results shown in the table do not give credence to the 4% rule. Only 57.63% of the portfolios reach or exceed 50 years of longevity, as opposed to the 90% found by Bengen in his study. Furthermore, the portfolios that do not exceed 30 years of longevity should be a very low percentage; instead, from the simulation, they appear to be 21.73%. This places a high risk on the part of the retirees of depleting their assets before their death.

To validate the 4% rule with high confidence, one should expect at least 95% success for portfolios to exceed threshold longevity. From the results obtained only if the retirement expectation is at least 20 years, with a success rate of 94.39%, it would be recommended to apply this strategy; otherwise, with higher expectations, there is a greater possibility of failure.

For example, Bengen, in his study, sets a minimum threshold of 30 years of longevity, given that his article was aimed at helping financial advisors in assisting their customers when they desire to retire. From a FIRE perspective, the minimum threshold could be higher since many within this movement aim for financial independence well before the age of 60.

The performance of the portfolios can improve by changing their allocation between stocks and bonds or by reducing the initial withdrawal rate.

The table below summarizes the results of the various analyzes performed. The only change made is on the allocation with the same initial withdrawal rate.

Longevity	**Probability for each allocation (stocks/bonds)**				
	(35/65)	**(50/50)**	**(65/35)**	**(75/25)**	**(100/0)**
$L \leq 10$	0%	0.01%	0.02%	0.11%	0%
$L \leq 20$	5.02%	5.61%	6.83%	8.17%	10.35%
$L \leq 30$	25.09%	21.73%	21.31%	21.81%	45.76%
$L \leq 40$	41.73%	34.59%	32.3%	31.46%	67.92%
$L \geq 50$	48.96%	57.63%	61.42%	63.56%	21.37%

Table 6: Empirical Cumulative Probability (US). 4% withdrawal rate

As confirmed by Bengen, the best results are obtained with allocations ranging from 50% to 75% of stocks; in cases where this component is smaller there is a lower probability for high longevities. The worst case, however, is given by the total exposure to stocks in which only 21.37% exceeds 50 years of longevity.

The difference in the allocation between 50%, 65% and 75% can be seen in the progressive increase in the risk of obtaining very low performances (a progressive increase of $L \leq 20$) and, at the same time, the increase in the probability of achieving high results (progressive decrease of $L \leq 40$). The probability of not reaching 30 years of longevity remains constant at around 21.6% .

The results confirm the best allocation range, which can vary according to risk preferences, but are less satisfactory than the results from 1926 to 1976. The withdrawal rate of 4% is excessively high since even with the greatest allocations; there is a probability of 21.6% to exhaust the assets within 30 years. Although 79.4% success appears to be an excellent probability, it is only for some people; only risk-lovers would accept this probability of success given the high cost involved in the case of failure.

Longevity	**Probability for each allocation (stocks/bonds)**		
	(50/50)	**(65/35)**	**(75/25)**
$L \leq 10$	0%	0%	0.01%
$L \leq 20$	2.59%	3.5%	4.31%
$L \leq 30$	12.69%	13.76%	14.11%
$L \leq 40$	23.70%	22.94%	22.59%
$L \geq 50$	69.62%	71.46%	72.2%

Table 7: Empirical Cumulative Probability (US). 3.5% withdrawal rate

Table 7 shows the results by decreasing the initial withdrawal rate to 3.5%.

Longevity	Probability for each allocation (stocks/bonds)		
	(50/50)	**(65/35)**	**(75/25)**
$L \leq 10$	0%	0%	0%
$L \leq 20$	0.79%	1.25%	1.19%
$L \leq 30$	6.21%	7.72%	8.31%
$L \leq 40$	14.23%	14.56%	14.78%
$L \geq 50$	80.22%	80.95%	80.06%

Table 8: Empirical Cumulative Probability (US). 3% withdrawal rate

Table 8 shows the results by further decreasing the initial withdrawal rate to 3%. In both tables, clear improvements can be seen from Table 6. With a withdrawal rate of 4%, there is an 80% chance of exceeding 30 years of longevity, while with a withdrawal rate of 3%, one has the same probability of exceeding 50 years of longevity.

It is evident that for people in their 30s or 40s who want to go into early retirement, using the 4% withdrawal rate is decidedly more risky.

3.3.4 Modern Analysis of European cases

To carry out the same analysis on European countries it is necessary to translate the returns of stocks and bonds into the national currencies; therefore, also, the models that describe the returns must be modified. Inflation is calculated on the Price Consumer Index of the country analysed.

From the point of view of a Swiss resident, the models found are:

$$Y_t = 0.0832 + \epsilon_t \qquad \epsilon \sim N(0, 0.03754)$$

$$X_t = 0.0414 + \eta_t \qquad \eta \sim N(0, 0.01586)$$

$$I_t = 0.7273 I_{t-1} + 0.2250 \xi_{t-1} + 0.006326 + \xi_t \qquad \xi \sim N(0, 0.000207)$$

Where Y_t are stocks' total return, X_t are bonds' total return, and I_t is inflation. As already shown in 3.2.1, the Swiss franc has lower returns on stocks and bonds than the U.S. dollar.

From the point of view of an English resident, the models found are:

$$Y_t = 0.1255 + \epsilon_t \qquad \epsilon \sim N(0, 0.03695)$$

$$X_t = 0.304 X_{t-1} + 0.058255 + \eta_t \qquad \eta \sim N(0, 0.01454)$$

$$I_t = 0.3751 I_{t-1} + 0.9221 \xi_{t-1} + \xi_{t-2} + 0.033744 + \xi_t \qquad \xi \sim N(0, 0.000531)$$

From the point of view of Euro-based countries there should be no difference in the projections of the returns of stocks and bonds since they are measured with the same currency, but this means reducing the analysis of the models to only the last 22 years of data. MSCI World data starts from 1970, but data from 1970 to 2000 are different for countries in the Eurozone and therefore, by using them, the models would be unique for each country.

The models on the returns of MSCI World and bonds based in Euro are:

$$Y_t = 0.0737 + \epsilon_t \qquad \epsilon \sim N(0, 0.0388)$$

$$X_t = 0.0278 + \eta_t \qquad \eta \sim N(0, 0.00688)$$

The models found have a significantly lower average than all other models due to the lack of data from 1970 to 2000, in which returns were much higher. The MSCI World in U.S. Dollar has averaged returns of 10.99% over the past 50 years and only 7.85% over the past 22 years. The total returns on bonds have been 6.66% on average in the last 50 years and a half in the last 20 years, just 3.28%.

An analysis performed with these models cannot be directly compared with the analyses performed on the rest of the models found.

The Italian, French and German inflation models are:

$$(IT) \qquad I_t = 0.7996 I_{t-1} + 0.0041901 + \xi_t \qquad \xi \sim N(0, 0.0004147)$$

$$(FR) \qquad I_t = 0.707 \xi_{t-1} + 0.0168 + \xi_t \qquad \xi \sim N(0, 0.0001037)$$

$$(DE) \qquad I_t = 0.7894 \xi_{t-1} + 0.0189 + \xi_t \qquad \xi \sim N(0, 0.0002046)$$

The data obtained from the analyzes are summarized in the following tables:

Longevity	Probability for each country				
	UK	**Switzerland**	**Italy**	**France**	**Germany**
$L \leq 10$	0.32%	0.11%	0.06%	0.01%	0%
$L \leq 20$	11.1%	12.31%	15.64%	6.79%	8%
$L \leq 30$	25.63%	32.95%	40.82%	30.85%	33.52%
$L \leq 40$	35.34%	47.34%	56.17%	50.24%	53.53%
$L \geq 50$	58.57%	44.11%	35.81%	39.23%	36.07%

Table 9: Empirical Cumulative Probability with portfolios (50/50) and 4% withdrawal rate

Longevity	Probability for each country				
	UK	**Switzerland**	**Italy**	**France**	**Germany**
$L \leq 10$	0.1%	0.01%	0.03%	0%	0%
$L \leq 20$	6.3%	6.71%	8.88%	2.8%	3.27%
$L \leq 30$	18.45%	24.08%	29.23%	18.19%	20.25%
$L \leq 40$	27.66%	37.72%	45%	34.85%	38.9%
$L \geq 50$	66.67%	53.91%	46.36%	54.13%	49.34%

Table 10: Empirical Cumulative Probability with portfolios (50/50) and 3.5% withdrawal rate

Longevity	Probability for each country				
	UK	**Switzerland**	**Italy**	**France**	**Germany**
$L \leq 10$	0.03%	0%	0%	0%	0%
$L \leq 20$	3.89%	3.22%	4.55%	0.81%	1.13%
$L \leq 30$	12.27%	15.02%	20.58%	8.88%	10.6%
$L \leq 40$	19.72%	27.2%	35.04%	21.53%	24.29%
$L \geq 50$	74.85%	64.55%	55.59%	68.38%	64.89%

Table 11: Empirical Cumulative Probability with portfolios (50/50) and 3% withdrawal rate

The data reported in Tables 9, 10 and 11 only concern portfolios composed of half stocks and half bonds, while all the results from the analysis are written in Appendix C.

The results for the U.K. and Switzerland are worse than those for the U.S. in Tables 6, 7 and 8 due to inflation in the English case and due to the low yields in Swiss francs in the Swiss case.

Italy, France, and Germany cannot be directly compared to U.S., U.K. and Switzerland because the time frame used to create the models is different. Germany's results are very similar to France's results but slightly poorer. The worst outcomes were observed in Italy, with only a 55.59% probability of exceeding 50 years of longevity with a 3% of withdrawal rate.

3.4 Withdrawal Growth Ratio

It is clear from the study in 3.3.4 that the results for the U.S., the U.K., and Switzerland are considerably different from those for the Eurozone member nations. This is due to the difference in the period used to develop the ARIMA models.

Stocks and bonds have averaged 10.99% and 6.63% (in U.S. dollars) during the past 50 years, respectively, whereas they have averaged 7.85% and 3.28% over the previous 20 years. Inflation rates in Italy, France, and Germany have been 6.13%, 4.08%, and 2.73% correspondingly during the past 50 years, but just 2.09%, 1.61%, and 1.8% over the past 20. Returns on stocks got 28.57% lower in the last 20 years, and returns on bonds decreased by 50.52%; on the other hand, inflation decreased by 65.84% for Italy, 60.5% for France and 34% for Germany.

Since the average inflation has decreased considerably, in addition to the average returns, why have the countries in the Eurozone not had the same performance as the other countries analysed? The answer is found in the ratio between the annual withdrawal growth and the total portfolio growth, which I named the Withdrawal Growth Ratio.

The portfolios with the greatest longevity are those whose annual withdrawal remains constant or decreases in proportion to the portfolio. For example, if the ratio between withdrawal and portfolio in the first year, that is, the initial withdrawal rate, is 4%, then to ensure that the portfolio lasts over time, this value must remain constant or decrease.

Let's call *WGR* the Withdrawal Growth Ratio, W_g the withdrawal growth and P_g the portfolio growth:

$$WGR = \frac{W_g}{P_g} \tag{5}$$

W_g is defined just by inflation i:

$$W_g = 1 + i \tag{6}$$

Instead P_g is defined by:

$$P_g = (1 + s) \cdot A_s + (1 + b) \cdot A_b - WR \cdot (1 + i) \tag{7}$$

Where s is the stocks return, A_s is the stocks allocation, b is the bonds return, A_b is the bonds allocation, and WR is the current Withdrawal Rate. The Withdrawal Rate is the ratio between the current yearly expenses and the current total portfolio.

The WGR value indicates how the initial Withdrawal Rate changes from year to year:

$$WGR = \begin{cases} 1 & WR \quad remains \quad constant \\ < 1 & WR \quad decreases \\ > 1 & WR \quad increases \end{cases}$$

So to calculate next year's Withdrawal Rate, just multiply the current WR with the expected *WGR*. The goal is to achieve the lowest possible *WGR* to reduce annual expenses to a small fraction of the portfolio. Otherwise, if the *WGR* is greater than 1, then the average portfolio will be eroded over time.

This ratio can be used as an indicator of the goodness of a strategy based on the 4% rule. For example, using data from the Italian case in which the expected return of stocks is 7.38%, the expected return of bonds is 2.8%, and the expected inflation is 2.09%, if we apply an initial withdrawal rate of 4% to a portfolio (50/50), the *WGR* is equal to 1.0107 and therefore the *WR* of the following year will be 4.04%.

Thanks to the *WGR*, we can state that applying the 4% rule to a portfolio (50/50) in Italy, considering the average returns and inflation over the last 20 years, is not a winning strategy given that, on average, the portfolio grows less than the expenses.

The *WGR* indicator is a good starting point, but it cannot be the only reference point for building a strategy; it does not consider the variance of returns and inflation. Indeed by applying only the *WGR*, the winning strategy would always have portfolios with complete exposure to stocks since only the average is considered, but as already demonstrated, complete exposure to stocks is not advantageous.

3.4.1 Impact of the First 10 Years

For the longevity of a portfolio, the first 10 years of investment are the most important; if, during this first period, the *WR* decreases, the probability of erosion is considerably reduced.

Suppose we apply the 4% initial withdrawal rate to a portfolio (75/25) based in the

US. Considering that the expected values are 10.99% stock returns, 6.66% bonds returns and 4.01% for inflation, the *WGR* of the first year is 0.9836, therefore positive since it is less than 1.

Assuming that for the first 10 years of retirement, returns and inflation reflect the expected values, we would obtain a reduction of the *WR* from 4% to 3.29%.

Year	**Current Withdrawal Rate**	**Expected Withdrawal Growth Ratio**
2023	4%	0.9836
2024	3.93%	0.9829
2025	3.87%	0.9823
2026	3.80%	0.9816
2027	3.73%	0.981
2028	3.66%	0.9803
2029	3.59%	0.9796
2030	3.51%	0.9789
2031	3.44%	0.9782
2032	3.36%	0.9775
2033	3.29%	0.9767

Table 12: Expected Withdrawal Rate, U.S.based example

Still maintaining the same assumptions, the empirical cumulative probability also changes.

The results in Table 13 are decidedly more favourable than those in Table 6, given that the probability of eroding the portfolio before the age of 30 goes from 21.81% to 1.95%. This shows how impactful the first 10 years of investment are, considering that the returns and inflation used to perform the analysis in the first 10 years are the average of the last 50 years.

Longevity	Probability
$L \leq 10$	0%
$L \leq 20$	0%
$L \leq 30$	1.95%
$L \leq 40$	9.26%
$L \geq 50$	84.69%

Table 13: Empirical Cumulative Probability, portfolio (75/25), 4% *WR*

The same improvement would not be visible if the *WGR* was greater than 1. On the other hand, bad returns in the first decade could strongly harm the portfolio and significantly reduce the probability of reaching extended longevity.

3.5 Data Used

The data used for the models are given in Appendix B. The inflation data for the states analyzed were obtained from the national Consumer Price Indexes [23].

The data used for the fixed income securities in 3.2 were obtained with the yields of Swiss government bonds with 10 years of maturity [14] that pay coupons annually. A bond's yield is the internal rate of return that correlates the bond's price with the stream of cash flows (coupons and principal) declared to the bondholder. The total return of intermediate-term government bonds is calculated as the change in the Flat Price[3] after one year of investment. Since the flat price does not consider accrued interest, I estimated the total return one year after the bond was issued, and therefore at the first coupon payment when accrued interests are equal to zero. In the total return,

[3]A flat bond is the price of a bond that does not contain any accumulated interest. Accrued interest is the portion of a bond's coupon payment earned by the holder between scheduled coupon repayments. The price of a flat bond is known as its clean price.[7]

it is also necessary to consider the coupon received in addition to the change in the flat price.

The flat price is given by the present value of all future coupon payments over the remaining life of the bond plus the present value of the bond's face value at maturity, both discounted at the current yield to maturity.

The present value of the future coupon cash flow is given by:

$$\sum_{n=1}^{t} \frac{C}{(1+i)^n} = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + ... + \frac{C}{(1+i)^n} \tag{8}$$

Where C is the coupon value, t is the time to maturity, and i is the current yield to maturity.

Equation (8) is a convergent geometric series which can be simplified. Given the geometric series:

$$S_j = \sum_{k=0}^{j} ar^k = a + ar + ar^2 + ... + ar^{j-1} + ar^j \tag{9}$$

$$rS_j = \sum_{k=0}^{j} ar^k = ar + ar^2 + ... + ar^j + ar^{j+1} \tag{10}$$

Subtracting the second equation from the first, is obtained:

$$S_j - rS_j = a - ar^{j+1} \tag{11}$$

Given $r \neq 0$ it is possible to divide by $(1 - r)$ and obtain:

$$S_j = \sum_{k=0}^{j} ar^k = a\left(\frac{1 - r^{j+1}}{1 - r}\right) \tag{12}$$

Equation (8) can be rewritten as equation (12):

$$\sum_{n=1}^{t} \frac{C}{(1+i)^n} = C\left(\frac{1 - (1+i)^{-(t+1)}}{1 - (1+i)^{-1}}\right) - C \tag{13}$$

In equation (13), a negative C appears because equation (12) has a summation that starts with $k = 0$, while the summation of equation (13) starts with $n = 1$; it is, therefore, necessary to subtract the value of the equation when $n = 0$ that is C.

Equation (13) can be further simplified by applying basic math rules, thus reducing equation (8) to a simple expression:

$$\sum_{n=1}^{t} \frac{C}{(1+i)^n} = C\left(\frac{1 - (1+i)^{-t}}{i}\right) \tag{14}$$

The present value of the bond's face value at maturity is given by:

$$PV = \frac{PAR}{(1+i)^t} \tag{15}$$

Where *PAR* is the face value of the bond. Therefore the flat price of the bond is:

$$P_{(flat,y)} = C\left(\frac{1-(1+i)^{-t}}{i}\right) + \frac{PAR}{(1+i)^t} \tag{16}$$

The total return on Swiss government bonds is calculated by assuming that the bonds are purchased at the beginning of the year and sold the following year, precisely on the first coupon. It is important to estimate the return assuming that bonds are bought and sold yearly since a retiree must be able to withdraw enough money for the yearly expenses. Furthermore, the portfolio must be adjusted annually according to the initial allocation, implying that bonds must be sold in the secondary market rather than kept until maturity.

The total return is then calculated as follows:

$$R_y = \frac{P_{(flat,y+1)} + C}{PAR} - 1 \tag{17}$$

Where R_y is the total rate of return of a bond in year *y* after being sold the following year, $P_{(flat,y+1)}$ is the flat price of the bond at the beginning of the following year (*y+1*), and C is the coupon paid.

The total rate of return of the bond is independent of initial investment, indeed from equation (17), the PAR value can be deleted by simplifying further because the coupon is ($C = PAR \cdot i_y$) where i_y is the interest rate of the bond bought in year *y*.

$$R_y = i_y \left(\frac{1-(1+i_{y+1})^{-t}}{i_{y+1}} \right) + \frac{1}{(1+i_{y+1})^t} + i_y - 1 \tag{18}$$

In equation (18), the current yield is expressed as i_{y+1} because it is the interest rate of the same type of bond one year later. Thanks to this last equation, we can calculate the total return of bonds knowing only their interest rates at the beginning of each year. However, we must keep in mind the initial assumptions, which are that they are bonds with an annual coupon and that they are sold exactly one year after the issuance date. Bonds with different coupon frequencies require other simplifications.

Consider the following scenario: assume you acquire a 10-year Swiss bond with a yield of 5% and a face value of 1000 CHF at the beginning of 1970. This bond will be sold at the start of the next year after giving the first coupon (50 CHF). Its selling price is equal to the bond's future cash flows and therefore determined by the current yield on bonds with a similar maturity.

Assuming that on January 1 of 1971, the current yield is increased up to 5.5%, so the Flat Price=$50 \cdot \left(\frac{1-(1+0.055)^{-9}}{0.055} \right) + \frac{1000}{(1+0.055)^9} = 965.24$.

The return of the bond in 1970 is, therefore, $R_y = \frac{965.24+50}{1000} - 1 = 0.015239$, so approximately 1.52%. If the current yield had declined by 0.5% instead of growing by 0.5%, the return would have been 8.63%; if it had stayed constant, the return would have been 5% (exactly the 1970's yield). The same results can be obtained using equation (18).

Chapter IV
Conclusions

4.1 Results Analysis

In the first part of the thesis, the FIRE movement, its objectives and the various types of Financial Independence were introduced. What emerges are the reasons why a person should aim for financial freedom and that it is not a path suitable for every individual. For an ordinary person, early retirement is achievable only thanks to a long period of savings and investment, however limiting other personal goals, and this is why only some are suitable for the FIRE movement.

The most used method to verify if one's assets can support living expenses for the rest of one's life is the 4% rule. It is based on a study published in 1994 by William P. Bengen named "Determining Withdrawal Rates Using Historical Data". The study shows that anyone starting their retirement between 1926 and 1976 could have sustained their standard of living for at least 30 years, assuming that their annual expenses were at most 4% of their initial portfolio.

The 4% rule has become a well-known rule of thumb for American investors, it is essential to note that it is based on historical data from the U.S. financial markets, and its applicability to non-American investors may be limited. Due to currency exchange rates, investors might encounter additional difficulties. Exchange rate fluctuations can affect both the value of assets and the sustainability of withdrawals.

Furthermore, the period used for the analysis must be considered, given that different market conditions and diverse economic factors can substantially impact investment returns and retirement planning.

Despite its limits, this study has been very important because it prompted many individuals to achieve a portfolio capable of replicating the results reported by Bengen. This thesis has focused to verify with updated data and a new approach if the 4% rule is still valid.

The analysis was carried out on portfolios with the first withdrawal in 2023; the returns used for the calculations were generated from ARIMA models found based on the last 50 years of data for the U.S., U.K. and Switzerland and the last 22 years of data for Italy, France and Germany. The data generated by the models reflect the mean, variance and correlation between stocks, bonds and inflation of the data used. The results reported in Appendix C show the probabilities of portfolios reaching a certain longevity based on the stock/bond allocation.

The portfolio must meet minimal longevity criteria decided in advance to verify if the results obtained are satisfactory. This threshold varies from each individual's personal preferences, but for theoretical analysis can be used the threshold of the 4% rule is 30 years since it is believed to be the minimum that a retiree hopes to obtain.

Furthermore, it is necessary to indicate that the confidence level in the results is considered positive. Generally, the 95% probability of success is used to determine if the strategy is safe.

Figure 18: Probability that the portfolio reach 30 years of longevity with 4% initial withdrawal rate

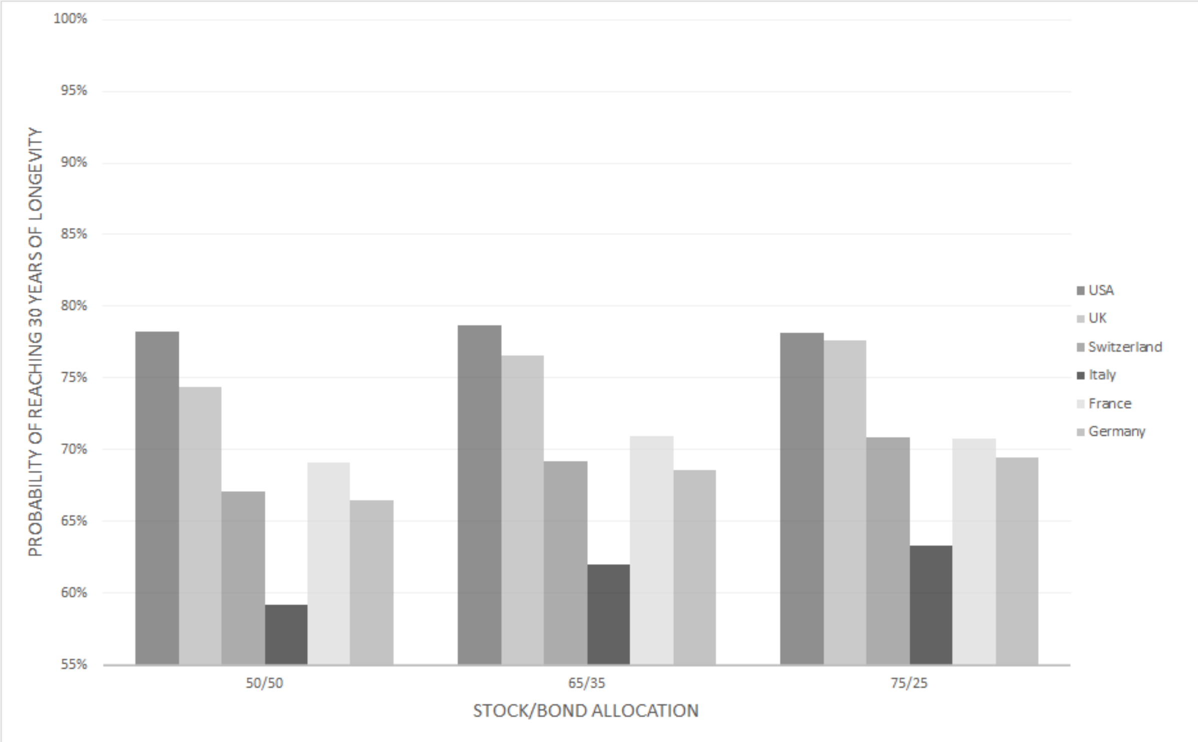


Figure 18 shows the probabilities of reaching 30 years of longevity starting with a 4% withdrawal. U.S. and U.K. have the best results; instead, Italy appears to be the riskiest country. The best result obtained is the American one, which reaches 78%. The probabilities of success increase as the bond allocation decreases.

Figure 19 shows the probabilities of reaching 30 years of longevity starting with a 3.5% withdrawal. With only 0.5 less initial withdrawal, the probabilities increased by an average of 9%. There is not much difference in the probabilities as the portfolio allocation changes.

Figure 19: Probability that the portfolio reach 30 years of longevity with 3.5% initial withdrawal rate

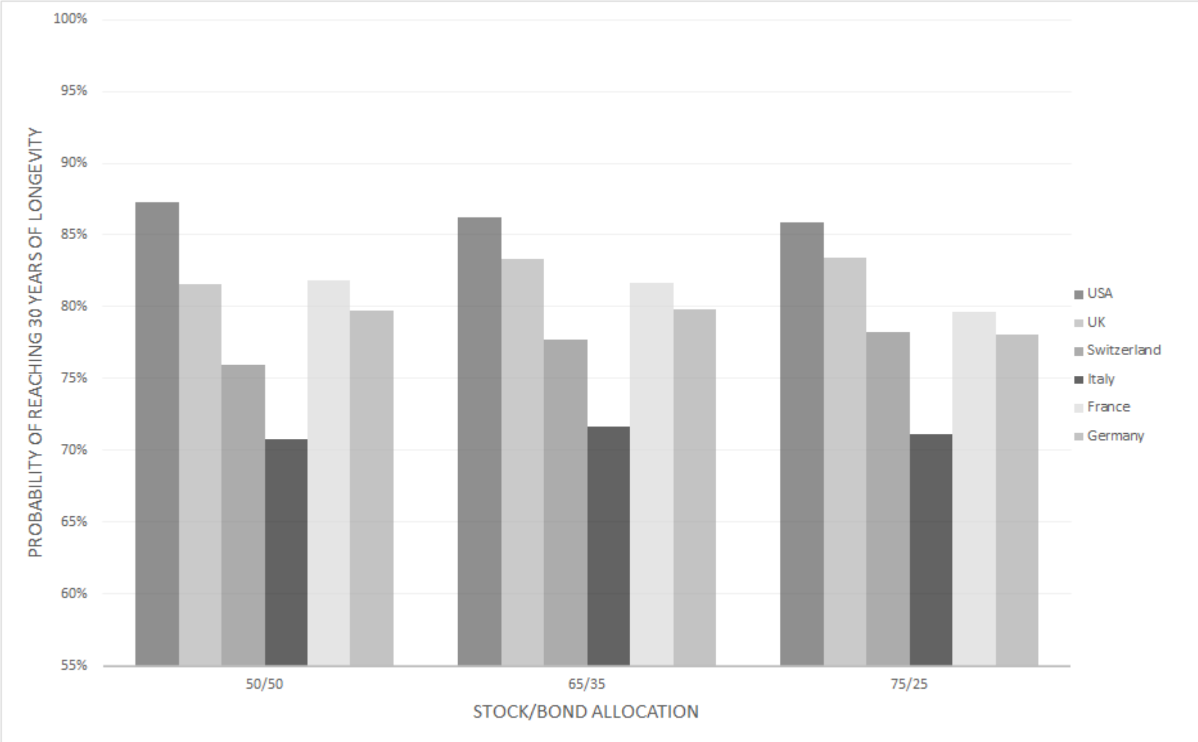


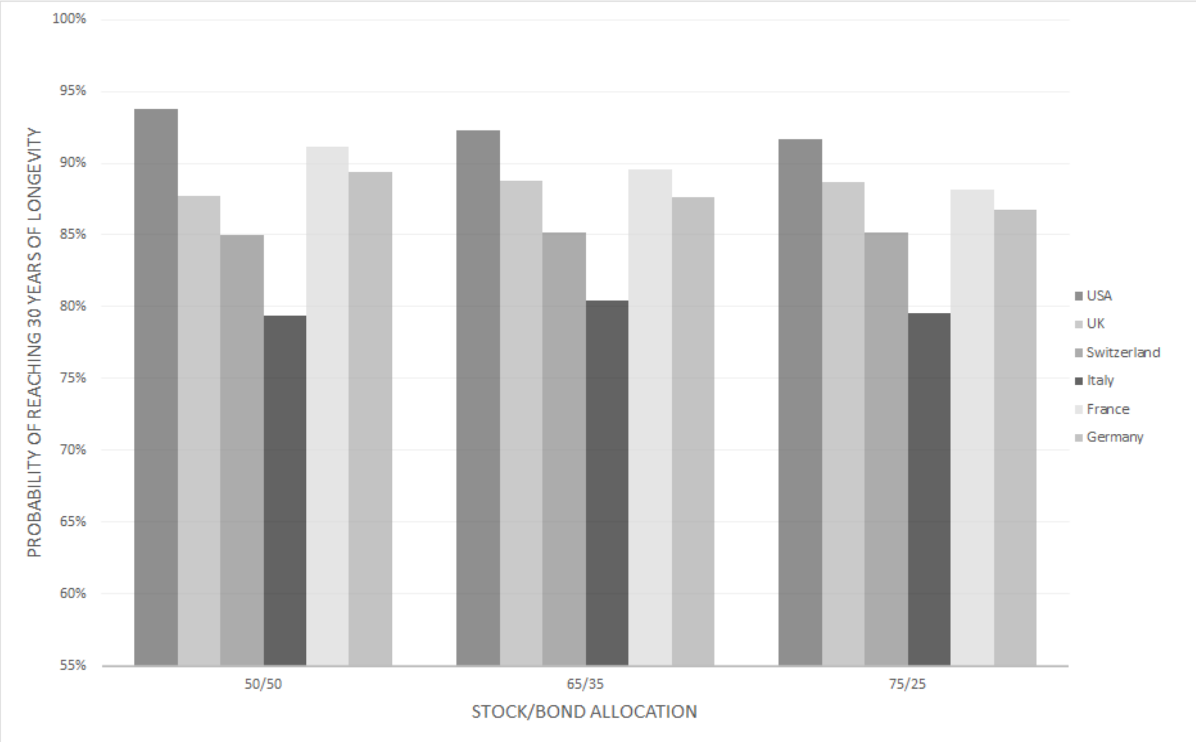
Figure 20 shows the probabilities of reaching 30 years of longevity starting with a 3% withdrawal. On average, the probability is increased by 8%, but any country reaches the 95% probability of success. Unlike the previous graphs, a slight drop in probabilities can be seen with a higher presence of stocks in the portfolio.

The three graphs show all six countries analysed, but the direct comparison must be made only between the countries that used the same data period, therefore between the U.S., the U.K. and Switzerland or between Italy, France and Germany.

Of the countries in the Eurozone, Italy is definitely the one with the worst results due to the volatility and average inflation over the last few decades.

Compared to America, U.K. and Switzerland have worse results due to very high inflation for the U.K. and insufficient returns for Switzerland.

Figure 20: Probability that the portfolio reach 30 years of longevity with 3% initial withdrawal rate



Overall, the results need to confirm the conclusions drawn by Bengen and other subsequent studies. The reasons can be found in the approach used to carry out the analysis; all the studies carried out up to now concerned only portfolios tested on historical data, of which the most famous is the "Trinity Study" carried out in 1998 and updated over the years.

The safe withdrawal rate found in these studies is always around 4%, but it is only to be considered if one assumes that future returns and inflation are not too different from past ones. Furthermore, even in these studies, the probability of reaching 30 years of longevity with an initial withdrawal of 4% does not reach 100% [9].

In conclusion, the data do not confirm the 4% rule since the failure probabilities found in this study are too high to consider 4% a safe initial withdrawal rate. From the results, it is recommended to lower this rate to 3.5% or 3%, an opinion also supported by other studies carried out in different ways in which the range from 3.3% to 4% is recommended [28].

It can be seen that although the average return is high enough to support a higher withdrawal rate, market volatility can significantly compromise the retirement plan, and for this reason, a lower initial rate is recommended.

Moreover, section 3.4.1 demonstrated how the first period after the start of retirement has a significant impact on results. In the first 10 years, a thrifty approach and good returns can ensure the retiree a very high probability of not depleting his funds.

This study focuses on the probabilistic aspects of early retirement, but numbers and rules are only indicative and have no value if not matched to a person's goals and lifestyle. Retirement planning is personal, and no theoretical rule will be valid for everyone; the 4% rule or this study must be used with flexibility and understanding of the problem. Early retirement should not start from numerical calculations but from considerations of one's financial goals, future expectations and lifestyle. They may include factors such as desired travel plans, hobbies, healthcare expenses, and any other lifestyle choices that require financial support. One should evaluate their current financial situation, including assets, debts, and ongoing expenses, to determine the level of savings required for a comfortable retirement.

While studies and rules provide helpful starting points, personal goals, expectations, and lifestyle considerations should be at the forefront of retirement planning. With a flexible and personalized approach, individuals can embark on a rewarding and fulfilling early retirement journey.

4.2 Assumptions and Criticism

All studies concerning the safe withdrawal rate present critical issues due to the multitude of assumptions and the methodology used. Generally, there is a tendency to simplify a possible retirement plan to analyse the results; moreover, factors that influence the calculations should be addressed.

This study analyses the retirement plan of someone who earns solely from portfolio profits, therefore, that does not work and has no other source of income, and who spends the same amount per year adjusted for inflation. In particular, assuming that a person never has out-of-normal expenses and that inflation is the only value used to adjust yearly costs is a strong assumption that makes the case unrealistic.

Furthermore, various factors, such as transaction costs and taxation, are eluded during the evaluation of portfolios. Taxes are another challenging element to include in the analysis, especially for this study in which several countries are analysed. In addition to the complexity of the taxation systems in the various countries, it would be incorrect to count taxation as another cost to be added to the yearly expenses because it would add further difficulty in calculating the annual withdrawal.

In addition to the approach used to carry out the calculations, the data type must be discussed. In the various studies carried out on this problem, analyses have always been made using only historical data and their average. In contrast, for this study, the data were generated by ARIMA Models.

The data used to identify the models concern the last 50 years for the U.S., U.K. and Switzerland cases, but only the last 20 for Italy, France and Germany. With a short period of time, it is probable that the models found are not precise and well descriptive of future expectations; furthermore, extraordinary events such as the 2008 crisis or the 2020 Covid crisis have certainly pushed the return values down. In addition to the choice of data, it is also possible to change the class of models on which to carry out the study; in 3.3.2, alternatives such as VAR Models and State Space Models are recommended.

Overall, the limitations of the analysis are due to simplifications and data generation. A more complex and specific approach could address some of these critical issues, but to test a general rule like the 4% rule, some of these limitations are unavoidable.

4.3 Future Research

As Early Retirement and the pursuit of Financial Independence keep gaining popularity, it is essential to research to refine and improve our understanding of strategies like the 4% rule. Future research in these areas will allow more fitting retirement planning frameworks to meet the needs and challenges of early retirees. This thesis focused on the validity of the 4% rule by applying a different approach: using returns projections instead of historical data. In this analysis is assumed a portfolio with international diversification and ARIMA are the only model class used. The analysis can be performed differently using more complex models suited to the problem or by using portfolios invested in indexes and bonds of single countries.

In addition to changing the models or data, it is possible to conduct the analysis by changing the underlying approach. To avoid too many simplifications on the withdrawal value, a more variable function can be applied instead of solely adjusting the value with inflation annually. An example could be made by using the Consumption Smoothing Theory in which one's standard of life is maximized through an appropriate balance between savings and consumption over time[3]. A second possibility could be applying the Retrenchment Rule to determine the value of the withdrawal each year; in this rule, the default withdrawal each year is the prior withdrawal adjusted for inflation as in the earlier studies, but with exceptions discussed in the paper Pye (2012) [32].

Financial Independence is not solely about monetary wealth; it is also about making financial decisions in accordance with one's principles and goals in life. Individuals can approach retirement with a holistic perspective, maximizing their fulfilment and satisfaction in the post-work phase of life. Future studies may concern the philosophical and social side of the FIRE movement.

Early Retirement is involved in various areas of the economic sphere; future studies could focus on inserting insurance policies or edging risk strategies to minimize the probability of running out of funds.

Retirement is changing dramatically as freelancing, and entrepreneurship becomes more popular. Future studies could look at how these changing habits affect early retirement planning.

The field of behavioural finance examines how psychological biases influence financial decision-making. Future research could include the behavioural aspect of early retirement and how individuals perceive risk, manage their investments, and make withdrawal decisions.

In conclusion, this study can take on different variations and concerns only a part of the complex theme of Financial Independence and Early Retirement. Longevity, economic conditions, withdrawal strategies, behavioural aspects, changing retirement landscapes, and technological advancements are all areas that deserve further investigation. By undertaking future research, it will be possible to adapt retirement planning frameworks to personal needs and enable individuals to achieve long-lasting financial security and a fulfilling retirement lifestyle.

List of Tables

List of Figures

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Appendix A

Year	Large-Capitalization Stocks	Intermediate-Term Government Bonds	Inflation
1926	0.1162	0.0538	-0.0149
1927	0.3749	0.0452	-0.0208
1928	0.4361	0.0092	-0.0097
1929	-0.0842	0.0601	0.0020
1930	-0.249	0.0672	-0.0603
1931	-0.4334	-0.0232	-0.0952
1932	-0.0819	0.0881	-0.1030
1933	0.5399	0.0183	0.0051
1934	-0.0144	0.0900	0.0203
1935	0.4767	0.0701	0.0299
1936	0.3392	0.0306	0.0121
1937	-0.3503	0.0156	0.0310
1938	0.3112	0.0623	-0.0278
1939	-0.0041	0.0452	-0.0048
1940	-0.0978	0.0296	0.0096
1941	-0.1159	0.005	0.0972
1942	0.2034	0.0194	0.0929
1943	0.259	0.0281	0.0316
1944	0.1975	0.0180	0.0211
1945	0.3644	0.0222	0.0225
1946	-0.0807	0.0100	0.1816
1947	0.0571	0.0091	0.0901
1948	0.055	0.0185	0.0271
1949	0.1879	0.0232	-0.0180
1950	0.3171	0.0070	0.0579
1951	0.2402	0.0036	0.0587
1952	0.1837	0.0163	0.0088
1953	-0.0099	0.0323	0.0062
1954	0.5262	0.0268	-0.0050
1955	0.3156	-0.0065	0.0037
1956	0.0656	-0.0042	0.0286
1957	-0.1078	0.0784	0.0302

Year	Large-Capitalization Stocks	Intermediate-Term Government Bonds	Inflation
1958	0.4336	-0.0129	0.0176
1959	0.1196	-0.0039	0.0150
1960	0.0047	0.1176	0.0148
1961	0.2689	0.0185	0.0067
1962	-0.0873	0.0556	0.0122
1963	0.2280	0.0164	0.0165
1964	0.1648	0.0404	0.0119
1965	0.1245	0.0102	0.0192
1966	-0.1006	0.0469	0.0335
1967	0.2398	0.0101	0.0304
1968	0.1106	0.0454	0.0472
1969	-0.0850	-0.0074	0.0611
1970	0.0386	0.1686	0.0549
1971	0.1430	0.0872	0.0336
1972	0.1899	0.0516	0.0341
1973	-0.1469	0.0461	0.0880
1974	-0.2647	0.0569	0.1220
1975	0.3723	0.0783	0.0701
1976	0.2393	0.1287	0.0481
1977	-0.0716	0.0141	0.0677
1978	0.0657	0.0349	0.0903
1979	0.1861	0.0409	0.1331
1980	0.3250	0.0391	0.1240
1981	-0.0492	0.0945	0.0894
1982	0.2155	0.2910	0.0387
1983	0.2256	0.0741	0.0380
1984	0.0627	0.1402	0.0395
1985	0.3173	0.2033	0.0377
1986	0.1867	0.1514	0.0113
1987	0.0525	0.0290	0.0441
1988	0.1661	0.0610	0.0442
1989	0.3169	0.1329	0.0465
1990	-0.0310	0.0973	0.0611
1991	0.3047	0.1546	0.0306
1992	0.0762	0.0719	0.0290

Year	Large-Capitalization Stocks	Intermediate-Term Government Bonds	Inflation
1993	0.1008	0.1124	0.0295
1994	0.0132	-0.0514	0.0261
1995	0.3758	0.168	0.0280
1996	0.2296	0.0210	0.0293
1997	0.3336	0.0838	0.0233
1998	0.2858	0.1021	0.0155
1999	0.2104	-0.0177	0.0218
2000	-0.0910	0.1259	0.0337
2001	-0.1189	0.0762	0.0282
2002	-0.2210	0.1293	0.0159
2003	0.2868	0.0240	0.0227
2004	0.1088	0.0225	0.0268
2005	0.0491	0.0136	0.0339
2006	0.1579	0.0314	0.0323
2007	0.0549	0.1005	0.0285
2008	-0.3700	0.1311	0.0384
2009	0.2646	-0.024	-0.0036
2010	0.1506	0.0712	0.0164
2011	0.0211	0.0881	0.0315
2012	0.1600	0.0166	0.0207
2013	0.3239	-0.0368	0.0146
2014	0.1369	0.0300	0.0163
2015	0.0138	0.0179	0.0016
2016	0.1196	0.0084	0.0126
2017	0.2183	0.0279	0.0213
2018	-0.0438	0.0021	0.0244
2019	0.3149	0.0902	0.0181
2020	0.1840	0.1044	0.0123
2021	0.2871	-0.0391	0.0470
2022	-0.1811	-0.1620	0.0800

Appendix B

Year	MSCI World Total Return (USD)	GB Inflation	CH Inflation
1970	-1.98%	7.89%	3.62%
1971	19.56%	9.03%	6.57%
1972	23.55%	7.65%	6.66%
1973	-14.51%	10.58%	8.75%
1974	-24.48%	19.14%	9.77%
1975	34.50%	24.89%	6.70%
1976	14.71%	15.07%	1.72%
1977	5.00%	12.14%	1.30%
1978	18.22%	8.39%	1.03%
1979	12.67%	17.24%	3.65%
1980	27.72%	15.12%	4.02%
1981	-3.30%	12.05%	6.49%
1982	11.27%	5.41%	5.66%
1983	23.28%	5.31%	2.95%
1984	5.77%	4.58%	2.93%
1985	41.77%	5.64%	3.44%
1986	42.80%	3.75%	0.75%
1987	16.76%	3.71%	1.44%
1988	23.95%	4.55%	1.87%
1989	17.19%	5.53%	3.16%
1990	-16.52%	7.49%	5.40%
1991	18.97%	7.32%	5.86%
1992	-4.66%	2.60%	4.04%
1993	23.13%	2.37%	3.29%
1994	5.58%	2.01%	0.85%
1995	21.32%	3.03%	1.80%
1996	14.00%	2.21%	0.81%
1997	16.23%	1.73%	0.52%
1998	24.80%	1.56%	0.02%
1999	25.34%	1.11%	0.81%

Year	MSCI World Total Return (USD)	GB Inflation	CH Inflation
2000	-12.92%	0.83%	1.56%
2001	-16.52%	1.09%	0.99%
2002	-19.54%	1.62%	0.64%
2003	33.76%	1.33%	0.64%
2004	15.25%	1.57%	0.80%
2005	10.02%	2.02%	1.17%
2006	20.65%	2.86%	1.06%
2007	9.57%	2.30%	0.73%
2008	-40.33%	3.08%	2.43%
2009	30.79%	2.07%	-0.48%
2010	12.34%	3.15%	0.69%
2011	-5.02%	3.60%	0.23%
2012	16.54%	2.42%	-0.69%
2013	27.37%	1.95%	-0.22%
2014	5.50%	0.71%	-0.01%
2015	-0.32%	0.50%	-1.14%
2016	8.15%	1.79%	-0.43%
2017	23.07%	2.74%	0.53%
2018	-8.20%	2.00%	0.94%
2019	28.40%	1.31%	0.36%
2020	16.50%	0.83%	-0.73%
2021	22.35%	4.84%	0.58%
2022	-17.73%	9.24%	2.80%

Year	IT Inflation	FR Inflation	DE Inflation
1970	5.36%	4.95%	4.05%
1971	4.61%	5.99%	5.41%
1972	7.38%	6.94%	6.37%
1973	12.52%	8.48%	7.92%
1974	24.50%	15.16%	5.72%
1975	11.24%	9.63%	5.41%
1976	20.50%	9.86%	3.69%
1977	14.06%	8.98%	3.41%
1978	11.60%	9.71%	2.54%
1979	18.82%	11.79%	5.40%
1980	19.55%	13.73%	5.54%
1981	18.11%	13.89%	6.69%
1982	16.44%	9.69%	4.55%
1983	12.29%	9.29%	2.71%
1984	9.37%	6.69%	1.95%
1985	8.89%	4.70%	1.57%
1986	4.15%	2.12%	-1.00%
1987	5.19%	3.12%	1.01%
1988	5.38%	3.08%	1.88%
1989	6.28%	3.56%	3.04%
1990	6.61%	3.06%	2.74%
1991	5.88%	3.02%	5.75%
1992	4.87%	1.91%	3.33%
1993	4.38%	2.11%	4.30%
1994	4.03%	1.59%	2.45%
1995	5.57%	2.09%	1.51%
1996	2.86%	1.66%	1.49%
1997	1.90%	1.14%	2.07%
1998	1.68%	0.22%	0.36%
1999	2.11%	1.25%	1.19%

Year	IT Inflation	FR Inflation	DE Inflation
2000	2.70%	1.64%	2.00%
2001	2.37%	1.35%	1.61%
2002	2.83%	2.27%	1.14%
2003	2.50%	2.18%	1.12%
2004	2.03%	2.08%	2.22%
2005	1.99%	1.60%	1.41%
2006	1.87%	1.49%	1.39%
2007	2.61%	2.59%	3.17%
2008	2.24%	1.00%	1.13%
2009	1.02%	0.91%	0.81%
2010	1.88%	1.76%	1.31%
2011	3.29%	2.46%	1.98%
2012	2.31%	1.33%	2.04%
2013	0.66%	0.71%	1.43%
2014	0.00%	0.06%	0.19%
2015	0.09%	0.18%	0.17%
2016	0.49%	0.61%	1.50%
2017	0.90%	1.19%	1.38%
2018	1.09%	1.59%	1.56%
2019	0.49%	1.46%	1.54%
2020	-0.19%	-0.02%	-0.57%
2021	3.90%	2.75%	4.91%
2022	11.63%	5.85%	8.12%

Appendix C

4% Initial Withdrawal Rate

Longevity	**Probability for each allocation (stocks/bonds)**		
	(50/50)	**(65/35)**	**(75/25)**
United States of America			
$L \leq 10$	0.01%	0.02%	0.11%
$L \leq 20$	5.61%	6.83%	8.17%
$L \leq 30$	21.73%	21.31%	21.81%
$L \leq 40$	34.59%	32.3%	31.46%
$L \geq 50$	57.63%	61.42%	63.56%
United Kingdom			
$L \leq 10$	0.32%	0.35%	0.36%
$L \leq 20$	11.1%	10.34%	10.22%
$L \leq 30$	25.63%	23.46%	22.39%
$L \leq 40$	34.34%	32.07%	31.15%
$L \geq 50$	58.57%	63.07%	64.4%
Switzerland			
$L \leq 10$	0.11%	0.18%	0.36%
$L \leq 20$	12.31%	12.61%	12.42%
$L \leq 30$	32.95%	30.84%	29.15%
$L \leq 40$	47.34%	44.04%	41.22%
$L \geq 50$	44.11%	48.98%	52.62%
Italy			
$L \leq 10$	0.06%	0.19%	0.6%
$L \leq 20$	15.64%	16.56%	17.63%
$L \leq 30$	40.82%	37.96%	36.66%
$L \leq 40$	56.17%	51.49%	49.38%
$L \geq 50$	35.81%	41.36%	43.86%

4% Initial Withdrawal Rate

Longevity	Probability for each allocation (stocks/bonds)		
	(50/50)	**(65/35)**	**(75/25)**
France			
$L \leq 10$	0.01%	0.05%	0.14%
$L \leq 20$	6.79%	8.93%	10.61%
$L \leq 30$	30.85%	29.03%	29.21%
$L \leq 40$	50.24%	44.66%	42.61%
$L \geq 50$	39.23%	46.54%	49.62%
Germany			
$L \leq 10$	0%	0.06%	0.19%
$L \leq 20$	8%	9.81%	11.2%
$L \leq 30$	33.52%	31.38%	30.52%
$L \leq 40$	53.53%	48.03%	44.68%
$L \geq 50$	36.07%	43.72%	47.33%

3.5% Initial Withdrawal Rate

Longevity	Probability for each allocation (stocks/bonds)		
	(50/50)	**(65/35)**	**(75/25)**
United States of America			
$L \leq 10$	0%	0%	0.01%
$L \leq 20$	2.59%	3.5%	4.31%
$L \leq 30$	12.69%	13.76%	14.11%
$L \leq 40$	23.7%	22.94%	22.59%
$L \geq 50$	69.62%	71.46%	72.2%
United Kingdom			
$L \leq 10$	0.1%	0.15%	0.19%
$L \leq 20$	6.3%	5.94%	6.75%
$L \leq 30$	18.45%	16.65%	16.55%
$L \leq 40$	27.66%	24.57%	24.12%
$L \geq 50$	66.67%	70.94%	71.55%

3.5% Initial Withdrawal Rate

Longevity	Probability for each allocation (stocks/bonds)		
	(50/50)	**(65/35)**	**(75/25)**
Switzerland			
$L \leq 10$	0.07%	0.07%	0.18%
$L \leq 20$	6.71%	7.29%	7.9%
$L \leq 30$	24.08%	22.26%	21.79%
$L \leq 40$	37.72%	33.56%	32.1%
$L \geq 50$	53.91%	59.62%	60.82%
Italy			
$L \leq 10$	0.03%	0.1%	0.25%
$L \leq 20$	8.88%	10.19%	11.59%
$L \leq 30$	29.23%	28.35%	28.87%
$L \leq 40$	45%	41.58%	40.63%
$L \geq 50$	46.36%	50.48%	52.18%
France			
$L \leq 10$	0%	0%	0.05%
$L \leq 20$	2.8%	4.34%	6.03%
$L \leq 30$	18.19%	18.38%	20.33%
$L \leq 40$	34.85%	32.57%	32.15%
$L \geq 50$	54.13%	58.92%	60.75%
Germany			
$L \leq 10$	0%	0%	0.05%
$L \leq 20$	3.27%	4.78%	6.52%
$L \leq 30$	20.25%	20.22%	21.95%
$L \leq 40$	38.9%	34.94%	34.42%
$L \geq 50$	49.34%	56.31%	57.59%

3% Initial Withdrawal Rate

Longevity	Probability for each allocation (stocks/bonds)		
	(50/50)	**(65/35)**	**(75/25)**
United States of America			
$L \leq 10$	0%	0%	0%
$L \leq 20$	0.79%	1.25%	1.19%
$L \leq 30$	6.21%	7.72%	8.31%
$L \leq 40$	14.23%	14.56%	14.78%
$L \geq 50$	80.22%	80.95%	80.06%
United Kingdom			
$L \leq 10$	0.03%	0.06%	0.09%
$L \leq 20$	3.89%	3.19%	3.79%
$L \leq 30$	12.27%	11.27%	11.28%
$L \leq 40$	19.72%	17.86%	17.2%
$L \geq 50$	74.85%	77.8%	79.04%
Switzerland			
$L \leq 10$	0%	0%	0.03%
$L \leq 20$	3.22%	4.29%	4.69%
$L \leq 30$	15.02%	14.8%	14.8%
$L \leq 40$	27.2%	25.09%	24.18%
$L \geq 50$	64.55%	68.82%	70.05%
Italy			
$L \leq 10$	0%	0.06%	0.11%
$L \leq 20$	4.55%	5.53%	6.94%
$L \leq 30$	20.58%	19.59%	20.47%
$L \leq 40$	35.04%	31.97%	31.43%
$L \geq 50$	55.59%	60.02%	61.59%

3% Initial Withdrawal Rate

Longevity	Probability for each allocation (stocks/bonds)		
	(50/50)	**(65/35)**	**(75/25)**
France			
$L \leq 10$	0%	0%	0.01%
$L \leq 20$	0.81%	1.86%	2.65%
$L \leq 30$	8.88%	10.48%	11.82%
$L \leq 40$	21.53%	21.24%	21.47%
$L \geq 50$	68.38%	70.95%	71.85%
Germany			
$L \leq 10$	0%	0%	0.02%
$L \leq 20$	1.13%	2.3%	3.02%
$L \leq 30$	10.6%	12.37%	13.28%
$L \leq 40$	24.29%	23.68%	23.58%
$L \geq 50$	64.89%	67.93%	68.87%