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Final Thesis

Collateralized Debt Obligation

Theoretical and
Pragmatic overviews

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Introduction

The main goal of this thesis is the analysis of a relatively new financial instrument, this essay will be divided into different chapters, and this examination will be separated into two primary sections. The first will examine from a theoretical point of view the functioning of these derivatives. The second will take into consideration the empirical perspective of the pricing model and the market behavior under the stress of a crisis.

Collateralized Debt Obligation, also known as CDO was issued in the market for the first time in 1987 by the renowned investment bank Dexel Burnham Lambert Inc., evolving through time into a contagion-spreading derivative that triggers the global financial crisis. To understand the correlation between CDO behavior and worldwide crisis we need to exploit the origin of the instrument itself.

In the early years of the 2000s, a context of extreme recession occurred, and the Federal Reserve System consequentially decided to keep the interest rates as low as possible. Along with the FRS's resolution, although house prices from 1997 to 2006 increased abnormally by 85% and the trend was pervaded by peaks and deviations, the FED convinced citizens to take advantage of these rates to promote the "American Dream". Furthermore, the government offered collateral on the new securities put in circulation based on mortgages, through two agencies: Fannie May and Freddie Mac. This warranty was created for stimulating the creation of new tools, but probably also triggered less prudent behavior on the part of financial system institutions that felt safe from sanctions. In this context, the idea of the vice-chairman of the Salomon Brothers, Lewis Ranieri, helped banks and bankers to exploit this opportunity and increased tremendously earnings. Ranieri invented a new banking instrument named Mortgage-backed security (MBS), these securities are the most important member of the family of Asset-backed security, and they are issued by a specialized intermediary against a package of mortgages in which the lender had become the transferee. These mortgages and loans are contracted to finance or refinance the purchase of a residential home or another real estate, having quite a long term, 15 to 30 years are the more frequent, during which the borrower repays the debt in monthly payments.

To increase earnings, credit institutions like small regional banks, refinanced themselves by selling groups of mortgages with similar characteristics (interest rates) to specialized

institutions namely investment banks. The credit institution or the mortgage company reinvest the money made from the cooperation with the investment banks to concede more loans, this created a vicious circle that kept the earnings machine always powered.
Investment banks then grouped the mortgages into packages, against which they issued securities called Mortgage-backed securities, which they guarantee, and are proposed to the public.
This invention changed the banking and mostly the housing market the MBS allowed more people to have property house. Most of the lenders didn't take into consideration to confirm that borrowers could repay their mortgages, in many cases people got into mortgages they couldn't afford. To increase their earnings the target of lenders and intermediaries was to grant more and more loans, even without checking the economic possibility of the borrowers, the main example is immigrants and workers with huge liquidity but no guarantees.
Since all kinds of investors owned mortgage-backed security when the asset bubble burst in 2007 everyone took enormous losses, especially pension funds which are investment pools that pay for workers' retirements, these companies reduce pension fund risk by relying on non-risky investments and MBS were considered very secure by rating agencies.

As has been explained in the previous part, one of the causes that triggered this crisis were toxic loans and mortgages, moreover, credit and investment banks created a market worth hundreds of millions of dollars based exactly on these types of financial instruments.
Considering the massive amount of loans that had been granted until 2005/6, the number of citizens that actually could afford to repay the debt was meagre. Financial institutions were greedy for more loans, so they started to put pressure on smaller credit banks to take into consideration a different bracket of the population. This group of citizens generally was unable to provide any kind of guarantees, so a new category of loans was created, the subprime one.
To fully understand the mechanisms of subprime, it is necessary to explain mechanisms that regulate prime loans; prime essentially means the economic availability of the borrower (amount of money in the bank account or the money relative to the loan that the borrower has taken out), the more likely the debtor is going to keep making the interest payments, the more prime he/she becomes. Contrarywise subprime means that the bank account or the monthly income is not enough to satisfy the payments of the loan.

Fannie May use three criteria to qualify a prime borrower:

- FICO score of 620 or more, this index was created by Fair Isaac Corporation, it determines the creditworthiness of a borrower by considering five areas: credit history's length, types of credits used, level of indebtedness, payment history and new credit accounts.
- Amount of deposit the borrower puts down, if the amount exceeds 10%, he/she can be considered a prime borrower.
- Servicing of income in comparison with the loan, if 45% or less is taken of the borrower's income to service the loan, he/she can be qualified as prime.

Subprime loans are riskier than normal and prime loans, in the case of home loans the most proposed type of subprime are "2/28 loans", which have the first two years of fixed rate, then for the next 28 ones it becomes an adjustable rate, and it fluctuates based on an index plus a margin. Another type of subprime home loan is an "interest-only loan", this form is characterized by the obligation of refunding enough payments to cover only the interest, and not having to pay anything against the principal. Interest-only can be attractive because of their flexibility, nevertheless, the risk is that if nothing is paid against the principal the amortization process never begins.

Subprime was created not only for home loans but also for credit card debts, very high interest and penalty rates make these loans subprime, this is caused by the large risk attached to the borrower.

As the demand for new mortgages by the investment banker increased constantly, immigrants, unemployed people, and low-wage workers become the new type of borrower designed for subprime loans and mortgages.

Citizens were not obliged to take on these mortgages but in case of inability of repaying the debt, their responsibilities were few: they lose the advance fee usually very low or even absent, and to clear the debt to the bank it was enough to leave the house without paying any penalties. Inevitably when a subprime borrower defaulted on the mortgage, which is owned by the bankers, he/she transformed monthly payments in an asset (house), when there were only a few foreclosures, for the financial institution it was not a big deal because, with the increasing trend of real estate's price, the asset can be put on sale directly by the bank. As more and more monthly payments turned into houses the real estate market was pervaded by properties on sale, creating more supply than demand and

housing prices weren't rising again, in fact, they plummeted. The precipitation of the prices created an issue for the homeowners still paying their mortgages, as all the houses in the markets went up for sale, the value of their homes decreased so they stop makes payments even though they can afford to and left (without penalties), instead of paying a mortgage that value more than twice the current value of their property. Investments banks at the beginning of 2006 instead of having a constant flow of monthly payments to satisfy the leverage, held millions of worthless houses in all the U.S.

This was already enormous economic damage, but it was accentuated by the lower interest rates applied in 2001.

At the end of the dot-com bubble burst at the end of 2000, the Federal Reserve chairman Alan Greenspan decided to lower the interest rates to keep the economy strong, the level of these rates was only about 1%. Investors traditionally buy Treasury Bills that are short-term U.S. government debt obligations stopped buying them. These obligations were considered exceptionally secure and to be of very high credit quality because they are baked by the Treasury Department, with the return on investments reduced at 1% the advantages of the Treasury Bills didn't attract any investors. On the other hand, a very low-interest rate permitted the banks of Wall Street to obtain a huge quantity of money from not only the Fed for 1%, but also added to general surpluses from Japan, China and the Middle East, this cheap credit made borrowing money very easy and convenient for the banks causing a huge issue with leverage.

Leverage is the ability to reinvest borrowed capital as a funding source when increasing firms' assets to generate returns on risk capital.

Investors use financial leverage to multiply their buying power in the market, and for banks, leverage means borrowing money to amplify the outcome of deals. In 2004 the CEO of Goldman Sachs, Henry Paulson, helped lobby the Security and Exchange Commission to lift the limits on leverage, in doing so he allowed banks to have high leverage.

A high ratio of leverage is risky but profitable for these institutions, till the prices of the houses were increasing, banks could earn an enormous quantity of money maintaining high financial leverage, when the prices plummeted banks couldn't have enough capital to absorb losses: most of them declared bankrupt.

As explained above, CDOs played a primary role in the transformation from the American housing market's collapse into a catastrophic financial failure and a global credit crunch.

Responsible for a loss of 542$ billion, CDO performed poorly during the crisis due to the inclusion of low-grade quality collateral originating in 2006 and 2007. Credit and investment banks created a market worth hundreds of millions of dollars based exactly on these types of instruments. An enormous boost to the responsibility of this and many other financial instruments was given by the fallacy and sometimes egoistic opinion of the credit agencies.

To value, the riskiness nature of a bond or a stock determined agencies helped investment banks, hedge funds and general investors give them pieces of advice and opinions. The evaluation of the so-called credit agencies ranges from AAA to CCC, each rating has a different Spread and most important default rate and the financial instrument rated by these agencies consists of government bonds, preferred stock, corporate bonds, and collateralized securities such as MBSs and CDOs.
Short-term and long-term are rated differently, the first one is focused on the ability of the security taken into consideration to perform by observing the company's financial and industry performance conditions, and the long-term ratings scrutinize the company's ability to meet the responsibilities concerning the securities issued. Credit rating agencies can also evaluate the creditworthiness of a country or an entity, this rating gives insights to investors into the level of risk associated with investing in the debt of a certain country, credit rating agencies evaluate the economic environment and political risk and giving a high rating is crucial for the development of the country that is accessing the international bond market.
The third chapter is entitled "Credit Agencies compromised" due to the misleading rating given by the major and most powerful credit rating agencies on Wall Street, they evaluate highly risky investments such as Collateralized Debt Obligation based on subprime loans.

Moody's Corporation (MCO), Standard & Poor's (S&P) and Fitch Ratings control 95% of the global market together, spreading risky instruments all around the world, these companies earned billions of dollars in exchange for giving a high rating to risky securities. The increment of highly rated instruments exploded counting from a limited number to thousands of them, hundreds of billions of dollars per year have been rated by these agencies.

Moody the biggest of the rating agencies has increased its profit massively even during the crisis, earnings almost quadrupled due to an enormous quantity of AAA securities rated by the company in exchange for gigantic payments from financial institutions.

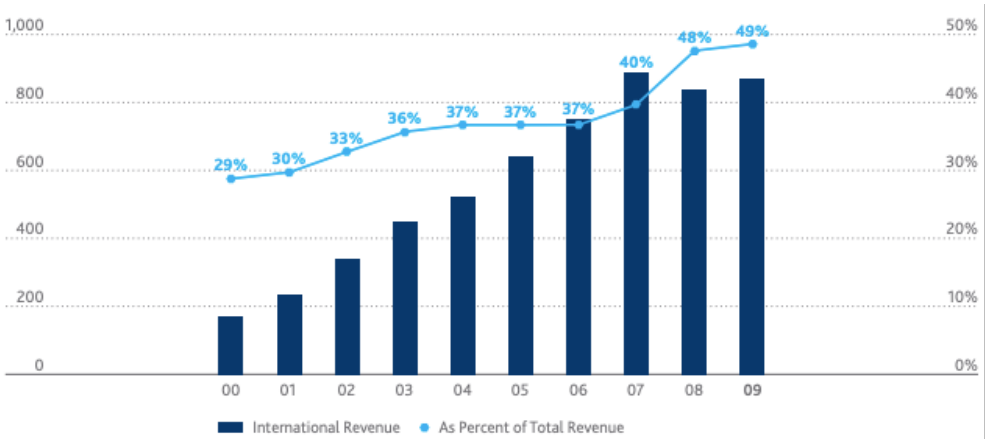

Figure 1: Moody's total revenues in millions

Moody rates companies' and countries' debt, and the grade of the investment range from Aaa which indicates the most solid investment available in the market to Baa3 which expresses the ability of the debtor of repaying the short-term debt. Below the Baa3 are ratings that indicate the high-yield and high-risk investments (from Ba1 to C).

Standard & Poor's and Fitch as all the other minor agencies have similar credit rating grades to Moody's ones, from AAA to BBB- the investment can be considered very secure since the debtor's ability to repay is not threatened. BB+ to D are considered speculative investments, and D-rating is the worst scenario possible with the extremely high possibility of a potential default.

Moody's, S&P and Fitch had the power to stop the funding to risky borrowers by giving fair valuations to risky financial instruments and companies. The "Big Three" gave an A2 rating to Bear Stearns months before it failed, and an A2 to Lehman Brothers within days of failing, AIG was given the AA rating likewise Lehman Brothers just a few days before the bailout. Citigroup, Merrill Lynch, Fannie Mae, and Freddie Mac were given investment-grade ratings. Along with high ratings for failing or failed companies, Wall Street's rating agencies continued to give Mortgage-Backed Securities triple-A ratings, convincing investors all over the world that these investments were very safe with little to no risk.

Ratings of foreign debt for European Countries were not maintained high as many companies in the U.S., quite the opposite, the big rating agencies downgraded the grade of several countries, the ones that were devalued the most were Greece and Portugal.

CDO: Theoretical Overview

In the first section of this thesis, the scrutiny will cover the theoretical analysis of CDO. Initially, the focus will be on a general definition of derivatives and then it will shift to a more in-depth analysis of the mechanisms and features of CDOs. Along with the description of the various types of Collateralized Debt Obligations, there will be a paragraph on the structure of this derivative.

2.1 Derivatives

The concept at its essence of a derivative is very simple, the problem is that encompasses the enormous world of financial instruments each derivative is slightly different although it does have a single binding element that makes them similar to each other.
The definition of a derivative is a security whose price is dependent upon or derived from one or more underlying assets. A derivative is a contract between two or more parties, the value is defined by the fluctuation of the values of the underlying assets, which can consist of commodities, stocks, currencies, bonds, interest rates and market indexes. Warren Buffett gave derivatives a definition that can express all the danger that these products may produce, "derivatives are financial weapons of mass destruction, carrying dangers that, while low talent, are potentially lethal".
As mentioned above each derivative is different but they can be categorized into four major groups: future, forward, option and swap.

- Futures:
  This contract obliged a party to sell or buy underlying assets in the future, but at a pre-determined price. Future contracts can trade physical commodities or other financial instruments, but it is fundamental to detail the quantity of the exchanged assets. Generally, futures contracts are used by two classes of investors: hedgers and speculators, buyers or suppliers of underlying assets hedge the price at which the asset is sold or purchased, at the same time traders or speculators can try to make a profit from the price change, however, the principal goal of a future contract is to protect the parties from price volatility. These contracts are

standardized to facilitate trading on a special market, where a disparate range of commodities futures, options on futures and index futures contracts can be traded; this market is called Futures exchange.

- Forwards:
  Forwards contracts also called simply forwards are comparable with futures, the difference is that these contracts do not trade on an exchange only Over the Counter. OTC is the process of how securities can be traded for companies and general investors that are not listed on a formal exchange, these contracts are traded via a broker-dealer network that is opposed to on a formal exchange.
  The colossal size and unregulated nature make forwards easier to trade and customize, nevertheless not being in a centralized marketplace implies that the susceptibility to a series of defaults is very high. Other risks involve are forwards are settled on the settlement date and are not market-to-market as futures, or the divergences between the forward rate and the spot rate, in this case, the financial institution that creates the forward is highly exposed to a risk of default.

- Options:
  Options contracts are identical to futures, but the purpose is to facilitate the potential transaction of underlying assets.
  Buyers of these options have the right but not the obligation to exercise the option to purchase or sell the assets every time before the expiration date.

- Swaps:
  Swaps are a special type of derivative used to exchange a cash flow with another. The main "assets" used in swaps are interest rates, credit, foreign exchange, equities, and mortgages. Swaps can also be used to trade credit risks, in fact in the subprime crisis, the main underlying asset traded with swaps was the default risk on a mortgage or a loan, with the famous Credit Default Swap.

A characteristic associated with all these categories of derivatives is the tradable nature. The derivatives market was growing at an incredible speed before and during the financial bubble that burst in 2007, banks and financial institutions traded $ billions in derivatives without being interested in what was underlying, the only concern of these traders was

the fluctuation of the price of these contracts. Traders could gamble on virtually everything, they could bet on the rise or the fall of the price of oil, or the fluctuation of the exchange rate of currencies, prices of oil, and even the weather, despite this, economists and bankers claimed they made the markets safer with derivatives, but instead, they made it unstable. Wall Street's banks used leverage (hundreds of millions of dollars borrowed) to use large quantities of these derivatives creating in the late 1990s a 50 trillion dollars unregulated market.
The usage of leverage implied that when the price of the contract rises the profits for banks were immense, on the other hand when the value of these derivatives plummet the losses could be massive, creating a huge ticking bomb.

2.1.1 CDOs

It has been already explained that hundreds of thousands of mortgages were bought by investment banks from credit banks, now the tools used by these institutions to create a profit from a huge number of mortgages will be further analyzed. Investment banks that had regrouped and packed an enormous amount of loans now have to offer them to the investors for making profits and reduce the risk of default attached to them.
To present these loan packs to public investors the instrument used by investment banks are mainly Mortgage-Backed Security and a new instrument called Collateralized Debt Obligation which is a specific category of Asset-Backed Security.
An Asset Backed Security is insurance on an investment, such as a bond or a note, which is collateralized by a pool of assets, the most common ones are loans, credit card debt, receivables, and leases. The major difference between MBSs and CDOs is that the second ones aside from mortgages a wide range of debt can be included: not only the already mentioned credit card debt but also students' debt and corporate debt.

A Collateralized Debt Obligation is a structured financial instrument backed by a pool of assets particularly loans for instance mortgages and car loans that a commercial bank approved and sold to a Special Purpose Entity (SPE). SPEs companies most of the time are bankruptcy-remote, meaning that the delinking of the entity from the originator's credit risk grants a minimum impact in the case of default on the SPE. To the originator are guaranteed servicing, hedging and administration fees but otherwise have no claim on the cash flow of the asset. The SPE usually an investment bank then repacked these loans

to create a financial instrument as an investment called CDO, which is then sold to public investors. The structure of the CDO is linear, the principal and the interest payments linked with the loan are redirected to the investor on the pool meanwhile the collateral is composed of the promise of repayment of the loan in the pool, this is the reason why it is called collateralized.
The main feature that makes these instruments extremely attractive for SPV “Special Purpose Vehicle” a synonym of SPE is the fact that if the underlying assets fail (in the case of no payments for the loan), most of the risk is transferred from the bank to the investors such as pension or hedge funds. CDOs are the financial instrument that represents in the best possible way the originate-to-distribute model they move credit risk from banks to investors rather than holding the assets generated in the balance sheet. This model permits banks to the relieve balance sheet from legislation-imposed capital constraints, indeed commercial institutes that provide loans are obliged to keep part of the capital in case of loan failure.
In addition, banks started to sell CDOs due to the reaction chain they create, through the funds it receives by selling these products a bank manages to make new loans and so acquire new clients.

During the housing boom, every bondholder was satisfied with the regular monthly payment of each mortgage, whenever one or more mortgage holders failed their mortgage and stop paying, the dangerousness of these products becomes visible.

2.1.2 CDOs framework

Collateralized Debt Obligation like many other financial instruments is subdivided into various levels. Let's assume that a bunch of loans are packaged together (from the safer AAA-rated to B-rated, the riskier ones). The final product has a credit rating of BBB along with a percentage reward for the risk attached to it. Investors in this situation will consider the credit rating and the benefit inadequate thus the product is not safe enough. Instead of issuing identical bonds linked to several sub-stratal assets, banks try to satisfy the different risk appetites of investors and created the so-called "tranches".
Each tranche is differently rated, the one rated AAA is generally called Senior Tranche and has a lower return however the risk of default attached to it is almost insignificant.

On the contrary, Junior Tranche generates more than X% but the rating related to the risk is very low commonly B. Alongside Senior and Junior tranches there are Mezzanine ones that obtain an average rating with a return that is consistent with the initial X%.

To explain how the risk is related to this obligation, it is necessary to visualize figuratively the CDO as a pyramid where the safer tranches are on the top and the riskier on the bottom as we can see in Table 1. Cash flows from mortgages' interests are used as payment to CDO's Senior Tranche holders first because they have the first claim on collateral, then Mezzanine Tranche holders get paid and lastly Junior Tranches holders received their reward.

Table 1: Structure of a hypothetical CDO

Debt	***Maturity***	***Rating***	***Yield***	***Price***	***Coupon***	***Tranche***
Senior	Feb 2006	AAA	5.27%	115.32	5.65%	84.52%
Senior	Oct 2006	A	5.44%	118.24	5.88%	6.48%
Mezzanine	Jan 2007	BBB	5.63%	102.27	7.11%	4.68%
Junior	Jan 2007	B	5.79%	111.18	9.67%	4.32%

In 2006/07 lot less interest was being paid for the enormous number of subprime mortgages conceded years before. The drastic reduction in the size of cash flow used by banks as a reward to CDOs implied the creation of a whole class of Tranche holders that were not getting paid.
Senior Tranche holders having the first claim right and the possession of the safer products received the return, what remains of the resources were used to pay the Mezzanine Tranche holders the money from interests wasn't enough to pay Junior Tranche holders.

This mechanism has two faces, and it must be mentioned. The pro side is the clear diversification of the risk, but on the con side, there is the concern concerning how

dangerous for the global economy it can be, to have complex products such as these CDOs on top of another already complicated financial instrument.
The situation became even more intricate and unstable when other asset managers decided to create the so-called CDO-squared. These secondary CDOs were structured in the same way as the primary Collateralized Debt Obligation but the CDO-squared Senior Tranche was composed of the Mortgage-Backed Securities that belong to the original CDO's riskier tranches.
The creation of even more complex financial products was justified by the fact that even if the risk attached to all the tranches (also Senior ones) was very large, the whole product was considered diversified, and the rating given by agencies was AAA or AA4 as the original assets.
In addition to the high credit rating, another justification was the important interest generated by the riskier assets; hundreds of millions of dollars have been invested in these CDO-squared. These products were handled by CDO managers that decided to offer them to people all over the world, knowing that these investments were not paying anything and were liable never to pay anything in the context of housing market failure.
The result of millions of defaults on mortgages is the crush of the entire world financial investment market, with $ billions of losses.

2.2 CDOs structure and CDO types

The scheme integrated below represents the structure of CDOs, the main differences, and the reason why all these kinds of financial instruments are related in one way or another.

Starting with a bottom-up approach we observe the similarities and discrepancies between Cash-Flow and Market Value CDOs[1]. Then distinguishing two main profit origins such as Arbitrage or Asset Removal from the Balance Sheet. Concluding with the explanation of three macro-categories of Collateralized Debt Obligations.

[1] Both are subgroups of cash CDO the first macro classification of CDO.

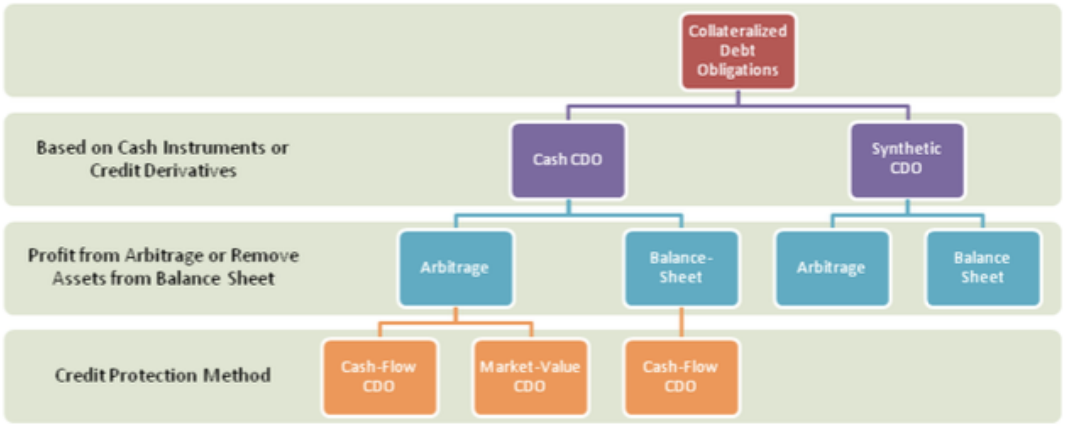

Figure 2: CDOs structure and CDOs types.

2.2.1 Credit Protection Method

Earlier in this essay, we observed that CDOs can be subdivided according to their underlying asset, a further classification can be made also considering the leverage structure. Notably, we can observe the source of funds for the payments of principal and interest.

Two main categories identified by this categorization are Cash Flow and Market Value CDO.

- Cash Flow CDO:

  The structure of this product is analogous to every other CDO. A series of assets are pooled and repackaged together in a derivative. To redistribute the risk the framework is composed of tranches, and finally, it is sold to investors that receive a distinctive return linked to the risk. In a cash flow CDO, interest and principal are paid to the tranche holder by the cash flow of the underlying asset and these payments are fully covered by the underlying pool.

  Cash flow CDO's structure spans over three main periods and is based on the waterfall scheme which will be explored in more detail later. The first time-interval is called the ramp-up period and it consists of the assembling phase of the portfolio, the second or reinvestment period in which principals' incomes are reinvested into new collaterals, and finally, the winding-up period which is the

ending step when the procedure is completed, the collateral pool matures, and the entirety of the obligations are fully repaid to the investor.

- Market Value CDO:
  Equivalently to the first kind of Cash Flow one Market Value CDO hold the same risk-to-return scheme and the procedure consists of three phases (ramp-up, reinvestment and winding-up). There are however some differences that distinguish them, the first to pop up is the maximum level of liabilities that this kind of CDO can issue. This quantity is established by an advanced rate for each asset of the pool and simultaneously the collaterals are regularly marked to market. In the case in which the liabilities are superior to the advanced rates, the manager of the CDO must re-establish a sort of equilibrium selling collaterals and paying notes.
  A further distinction between Cash Flow and Market Value CDOs is the role and the implication of the manager on the performance of the CDO itself. In the first type, the manager is not involved beyond controlling the credit quality of the portfolio. In the case of a Market Value CDO, its performance is directly correlated with the ability of the manager to operate trading and conduct transactions with securities.

2.2.2 Arbitrage or Asset Removal

As we explain earlier in this thesis there are several reasons why CDOs are such an appealing financial product for investment and commercial banks. The primary motives are related to risk redistribution, balance sheet relief and more than other profits. CDOs as we will see can fit disparate causes for every institution.

For commercial banks, the regulatory capital requirements are very strict, and issuing CDOs serves to achieve these requirements or to make capital available for lending purposes. Another pressing factor that leads commercial banks, which are lending institutions, to propose such a product is the necessity to release the institution from credit risk.

Investment banks however behave differently and CDOs are created only for for-profit intents. Acquiring assets from commercial banks and repackaging them into CDOs is an optimal approach to earn fees on the transaction. The method adopted by investment banks is called arbitrage.

2.2.3 Cash Instrument or Credit Derivatives

We have observed the distinction between Cash Flow and Market Value CDOs with the different leverage structures and the role of the manager, then it has been described the reasoning behind the issuing of this product made by investment and commercial banks, now we will portray the most crucial classification in which CDOs are labelled.
This distinction is made by examining the funding for interests and payments aspects along with credit rating pricing and credit risk weighting for a determinate sort of CDO.

- *Cash CDOs*

  Starting from the most typical and popular CDO present in the market the Cash Collateralized Debt Obligation (Figure 3). A cash CDO scheme is a chain of transactions that must be made before the assets are indeed qualified to be sold to public investors.

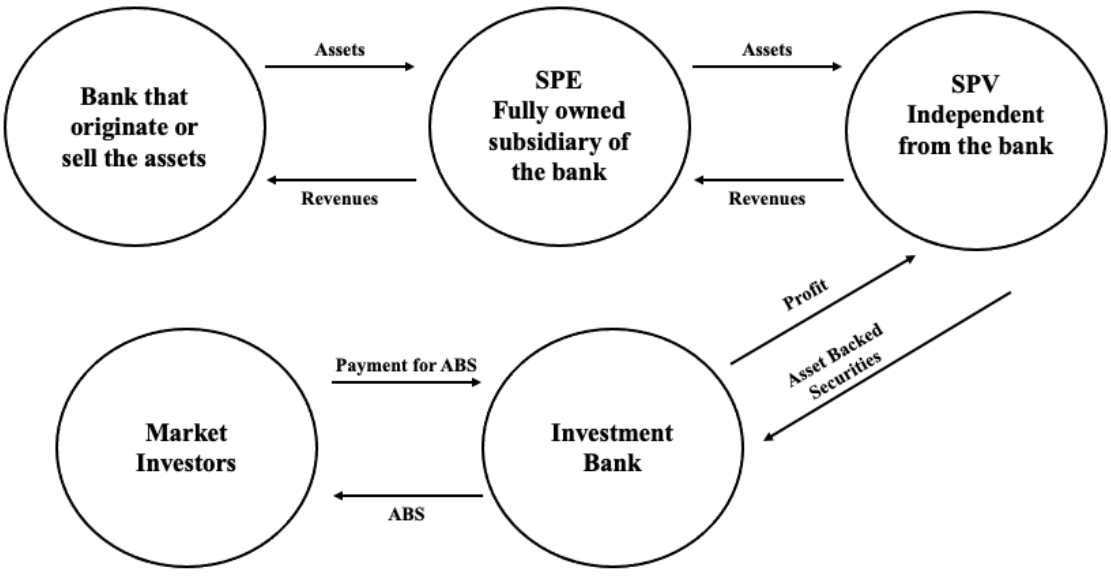

Figure 3: Cash CDOs structure.

The first transaction is made by a Special Purpose Entity; this first SPE, through a true sale, buys the assets such as loans, mortgages and credit card debts from a bank that transfers the ownership of these assets to the SPE. This Entity is a bankruptcy-remote subsidiary of the bank that originate and sell the assets but is still fully owned by the institution itself. A second SPV[2] completely independent from the selling bank to acquire the assets of the first SPE with a true sale. To distribute the liabilities to the public the SPV issue CDOs and sell them on the market. Investors all over the world can now take possession of these CDOs and for a specific term, the underlying collateral generates the payment of principals and interests. The credit risk is divided between the trances in the opposite order of seniority, instead, the profits are subdivided in seniority order.

- *Synthetic CDOs*

  A more complex financial instrument is the synthetic CDO (Figure 4), and it differs from the Cash one for some aspects. Firstly, a synthetic CDO achieves credit exposure not through asset acquisition but through derivatives contracts.
  Using a credit default swap, a Synthetic CDO is not required holding cash assets like bonds or loans. The scheme of a synthetic CDO is divided into three phases, the first is the issuing of notes to investors, then profits are reinvested in risk-free securities and finally starting several CDSs take the part of protection seller. Similarly, to a Cash CDO, the risk of default is spread among the tranches however in a Synthetic CDO the investors collect the payments from the swap premiums.

[2] The second Special Purpose Entity is the issuer of the debt or Asset-Backed Security ABS.

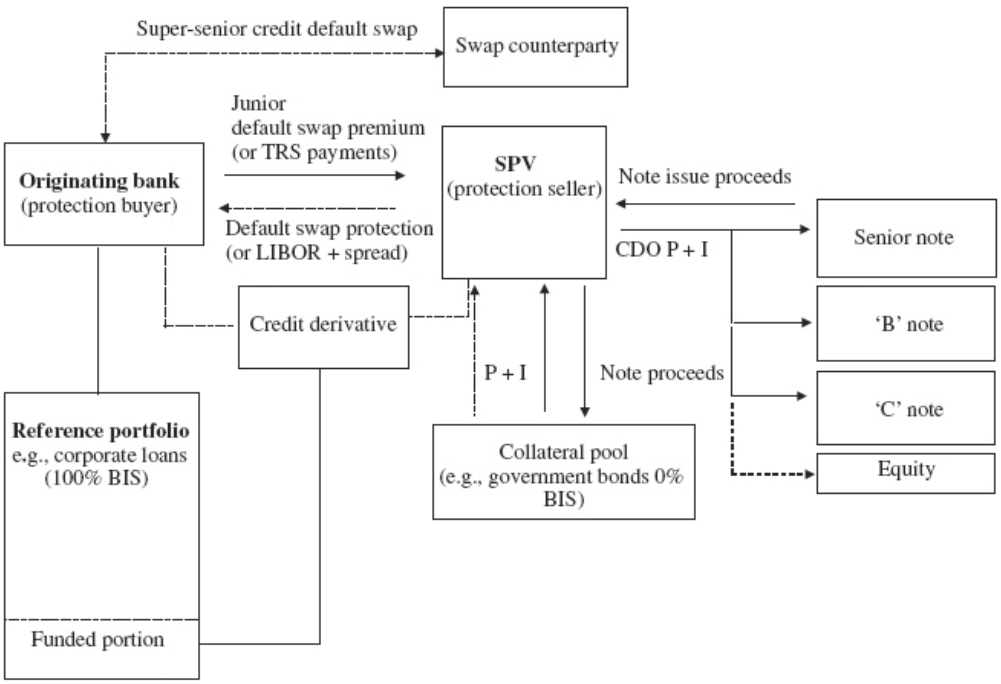

Figure 4: Generic Synthetic CDOs structure.

As mentioned above the core method to gain credit exposure is entering into credit default swap deals. The SPE signs such a contract where the insured products are the underlying assets of the CDO, it implies that in the case of a credit event the SPV and investor who acquire its obligations, are obliged to cover this liability. To prevent this exposure SPE reinvest the profits by issuing high-rating products that are used as collateral for the bank's obligations and the payments of the notes. In the event of default, all investors must absorb all the loss but if the CDO reaches maturity successfully the Credit Default Swap is concluded and the SPV can repay the investors through the liquidation of the collaterals.

Analyzing the Credit Default Swap contract in a more detailed manner we can observe that the payments for the swap counterparty are derived from the reserve account. The difference in the interest rate paid by the buyer and the potential payout that the seller of the CDS would be required to make in the event of a default is fixed over time and it is paid on the tranche principal to protect for eventual loss on that specific spectrum. The structure is slightly different if the tranche is an equity one, in this case, the protection buyer must pay a specific percentage of the tranche's principal both upfront and yearly a percentage that is not over 5%. The protection buyer is required to pay fixed annual payments and an upfront payment, considering whether different tranches' seniority upfront fees

are required or not, in the case of senior tranches they are not mandatory although for junior tranches they are compulsory due to the high risk of default. A significant difference between Cash CDO and Synthetic CDO is the initial capital allocation when the investor takes part in the deal, for the synthetic CDO senior investors' allocation is not present however if the portfolio's loss reaches senior tranches, investors must cover the exposure with payments to the CDO.

In the financial market, there are several subgroups of synthetic CDOs: partially funded synthetic CDOs, fully funded synthetic balance sheet CDOs, and fully synthetic or unfunded CDOs.

- *Hybrid CDOs*

  This typology of CDO is more uncommon than the CDOs explained above due to its nature. As the name suggests in fact, this is a combination of the previous Synthetic and Cash CDOs, indeed Hybrid CDOs contain both cash assets and swaps. A portion of the profit derived from the tranches is invested in cash assets and the rest is held in the case of payments of credit default swaps. Hybrid CDOs receive payments with three approaches, the first is the CDO premiums, the second is the return of cash assets and the final one is the reserve account held for CDS payments.

2.3 CDOs features

To have a more focused view of this complex financial instrument we should investigate the features that characterize it. Firstly, the scrutiny will be on the waterfall scheme, as we already mentioned during the examination of Cash CDOs this is the scheme on which interests and principal allocations are based on.
A second paragraph then will concentrate on credit enhancement which is a strategy for bettering the credit risk position of a business or an investment.

2.3.1 Waterfall scheme

The waterfall scheme as the name suggests is the structure according to which the cash flow of the underlying assets is assigned to each tranche. We must make a distinction between two schemes, the structure for the distribution of the principal payments and the one for the interests' allocations. Commonly the principal payments agreed for the tranches decrease with a higher grade of seniority. A different approach is used with regard to interest payments derived from the pool of assets. The focal point is the order, contrarily to the principal allocations, the higher-rated tranches (AAA-rated) are the ones which firstly are getting promised the payments on the outstanding principal. As the name suggests following a top-down approach remaining tranches get their payments too.

This procedure can mutate depending on the type of CDO, and characteristics of the underlying assets, indeed it is not a rigid pattern. In this thesis, we are going to analyze the waterfall scheme for a Cash CDO, notwithstanding we still need to make a distinction between return on interest and principal.

Return on Interest	**Return on Principal**
Interests on the underlying assets	Principal's prepayments from asset's portfolio
Payments from hedge counterparties	Inflow from assets sales
Interests on cash in eventual reserve account [3]	Trading proceeds
	Payments at maturity
	Recoveries on default assets

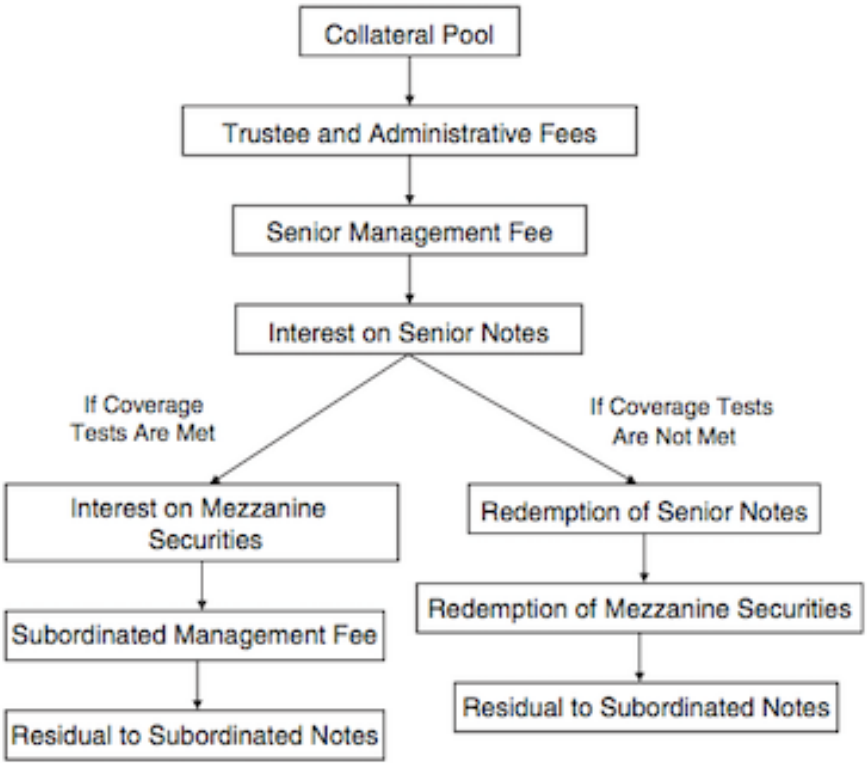

Figure 5: Interest waterfall scheme for Cash CDO.

[3] Possible reserve accounts are financed from future interest gains to secure the interest payments for tranches rated above equity ones.

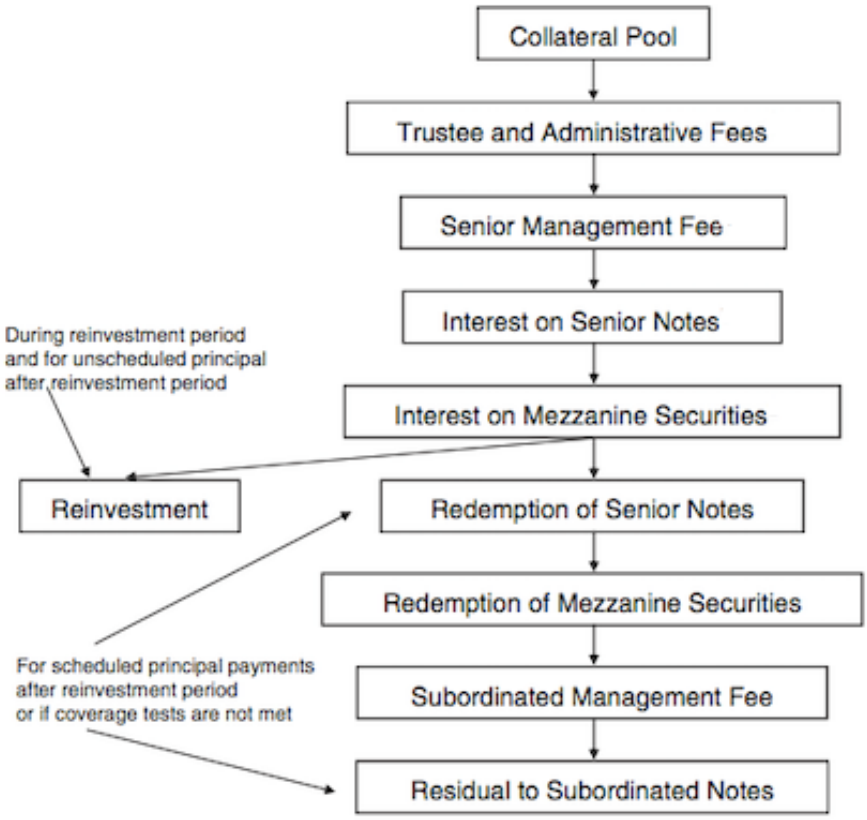

Figure 6: Principal waterfall scheme for Cash CDO.

Observing the two procedures the discrepancies are noticeable but the starting point for both schemes is the distribution of the fees. Various fees must be paid before the cash flow can be distributed through tranches with seniority orders. The first tranche to be paid is the most senior one then the second most senior one and following the same scheme the last one to be considered is the equity tranche.

A crucial passage of the waterfall scheme is the so-called Coverage test. This assessment is conducted to control whether the most senior tranche has been paid completely. Coverage tests pass if the superior tranche is entirely compensated and the progress to the next tranche starts to be paid off. These tests are operated to guarantee both secure CDOs' rated debt tranches and retain coverage levels and appropriate collateralization. Coverage tests most often included par value (or collateralization) tests and interest coverage tests.

- Par value tests are designed to maintain the lowest possible ratio of collateral portfolio amount to the par amount of debt tranches.
- Interest coverage tests instead are intended for the oversight of the ratio of the collateral portfolio's interest profit to coupon payable on debt tranches.

In case of lack of success of the coverage test, interests are not paid to the sub-tranches and the principal on senior tranches is instead paid down. The principal continues to pay down senior tranches until the tests are satisfied, still in reinvestment occasion in case the violation of the tests. The scheme may become complicated if the number of defaults from underlying assets increment.

2.3.2 Credit enhancement

Credit enhancement or credit support is a set of techniques that grant protection against losses in case of stressed scenarios. This risk-reduction procedure is a financial barrier that protects securities backed by a pool of assets and permits them to absorb losses derived from the default of loans or mortgages (Mason, 2008). It is important to understand that credit enhancement cannot be interpreted as a conversion method from a scarce investment to a profitable one, indeed this is an aid against potential losses. Credit support is designed for several purposes, from risk mitigation in structured financial instruments to partnership transactions and financing.

Four principal methodologies are used to protect an investment, and it is not uncommon that more than one type is adopted simultaneously.

- Subordination

  Also known as tranching is the technique in which the risk and the losses are divided and allocated following a specific order. Every layer of the investment has an exact percentage of risk attached to it, junior layers (or tranches) perform as a credit enhancement for the more senior ones. As already explained in a previous part the principal losses on the underlying assets hit junior tranches first, equivalently an interest shortage will affect the most junior layers first.

- Overcollateralization

  In this case, credit protection is provided by the fact that the aggregate face value of the underlying assets is larger than the par value of the securities backed by the

assets. The greater value permits the observation of the payments of the principal and interests on the investment even in the event of default or late payments on the underlying loans.

- Excess Spread

  The difference between the coupons on the underlying collaterals and the coupon on the issued security generates additional revenues. These extra funds are used to perform coupon payments even if underlying loan payments are late or default.

The use of a combination of techniques is not uncommon as we can observe in this scheme.

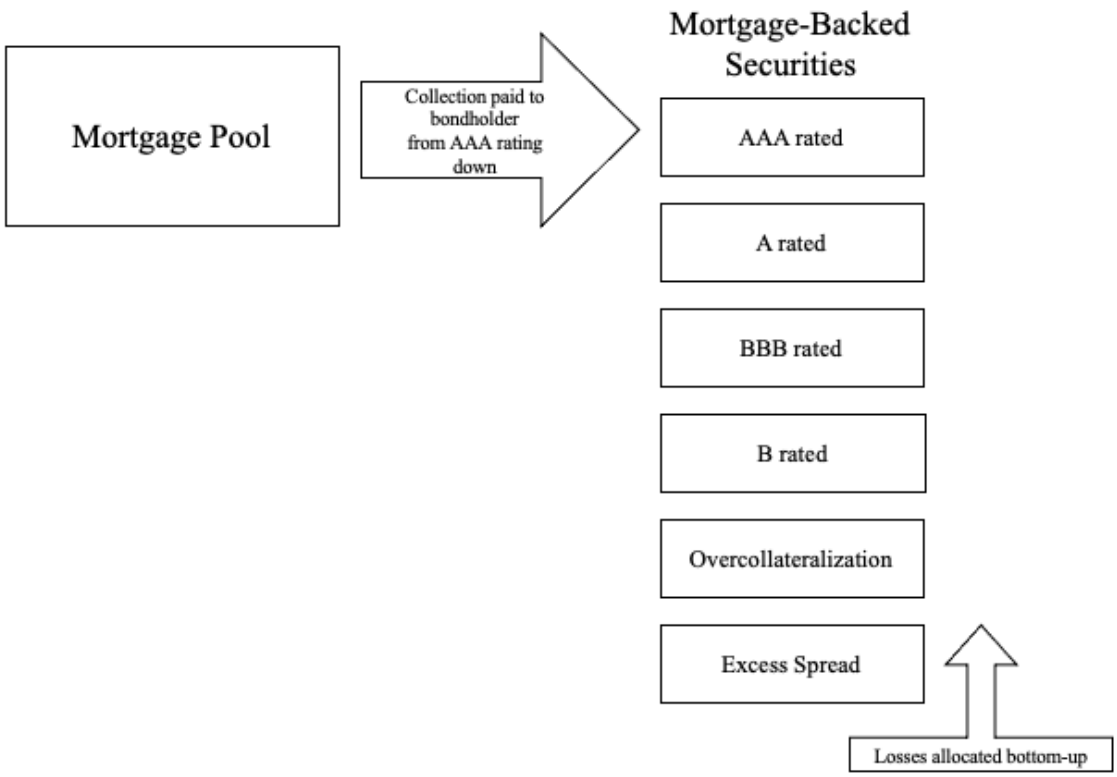

Figure 7: Credit Enhancement on a Securitization.

Indeed, if losses on the pool of mortgages begin to appear, those are firstly allocated to excess spread to mitigate or absorb them. If the losses are larger than the applicable excess spread, the allocation would pass to over-collateralization.

In the event of losses greater of the combination of over-collateralization and excess spread the first tranche to be hit would be the most junior one. Time after the allocation of losses on the most junior layer, the excess spread is used to restore over-collateralization.

Consider now a hypothetic investment in which the pool's performance suggests a 35% credit enhancement to support an AAA rating, that specific rating cannot be obtained if the investment has not a 35% more collateral exceeding the par value of the securities issued. In a hypothetical collateral pool of $5 million, the issue of only $3.25 million AAA-rated securities is permitted. If the collateral has a scarce performance provoking losses of a 20%, still a 15% to cover losses from additional defaults. We can affirm that this investment has $5 million in collaterals to support $3.25 millions of AAA-rated securities.

The important point to highlight is the concept that credit enhancement is not the transformation of poor investments into strong ones every method or combination of methods accumulates additional resources for securities or investments that would be available from the underlying asset. With this barrier in the case the pool encounters losses, credit enhancement still grants the payments of the bonds. Providing a safety net credit protection increases the capacity of senior bonds to receive the entire repayment of principal and interest.

CDO: practical overview

So far, we observe CDO features and characteristics following a theoretical path, in particular analyzing the differences between some kinds of CDOs and their framework. In this second part, the object of analysis will be not merely theoretical but practical topics and reasonings will be explored.

The pricing model is the first topic to be considered as it is essential to have a deeper understanding of the financial instrument taken into analysis. The pricing of a CDO is determined by the creditworthiness of the underlying assets, as well as the structure of the CDO itself.
CDO pricing models typically use a combination of historical data, credit ratings, and market conditions to estimate the probability of default for the underlying assets. This information is then used to calculate the expected loss for each tranche of the CDO, which determines the price of the tranche.
The pricing of CDOs can be complex and depends on the specifics of the underlying assets and the tranche structure. It is important to note that the pricing of CDOs has been a central issue in the financial crisis of 2007-2008, as the complexity of the securities made it difficult for investors to understand their true value.

Then the focus will shift to the reaction and the behavior of the derivatives (especially CDOs) market to a crisis such as the financial crisis of 2007 and the Covid-19 crisis of 2020.
The market for Collateralized Debt Obligations (CDOs) was a significant contributor to the financial crisis of 2007-2008. CDOs are securities that are backed by a pool of loans or other debt obligations. In the years leading up to the crisis, there was a large increase in the issuance of CDOs, particularly those that were backed by subprime mortgages. These mortgages were given to borrowers with weaker credit, and as the housing market began to decline, many of these borrowers were unable to make their mortgage payments. This led to a large number of defaults and foreclosures, which in turn caused significant losses for the holders of the CDOs.
During 2020, there was a sudden and severe economic downturn due to the COVID-19 pandemic. The CDO market has been affected by this downturn, as the loans and debt

obligations that make up the collateral for these securities have been impacted by the economic conditions. The CDO market has not been as significant a factor in this crisis as it was in the financial crisis of 2007-2008, but it has still been affected by the overall economic downturn.

3.1 CDOs Pricing model

The pricing of CDOs is an important consideration for investors, as it affects the returns they can expect from the investment. The structure of a CDO can be divided into different tranches, each with a different level of risk and expected return. The highest-rated tranches, known as senior tranches, have the lowest risk of default and are typically the most expensive. On the other hand, the lower-rated tranches, known as junior or mezzanine tranches, have a higher risk of default but offer a higher expected return.
The pricing of CDOs is based on the credit spread, which is the difference between the yield on a risk-free investment, such as a Treasury bond, and the yield on a risky investment, such as the CDO. The credit spread represents the compensation that investors demand taking on additional risk. The wider the credit spread, the higher the risk associated with the CDO and the lower the price.
In addition to credit spread, the pricing of CDOs can also be influenced by the credit ratings assigned to the underlying assets and the CDO tranches. Rating agencies such as Moody's and Standard & Poor's provide independent evaluations of the creditworthiness of the assets and tranches, which can affect their perceived risk and expected return. It is important to note that CDO pricing can also be affected by market conditions and changes in interest rates. During periods of economic growth and low-interest rates, CDOs may be in high demand and have higher prices. Conversely, during periods of economic uncertainty and rising interest rates, CDO prices may fall.
In summary, the pricing of CDOs is a complex process that involves several factors, including credit spread, credit ratings, and market conditions. Understanding these factors is important for investors who are considering investing in CDOs.

To calculate the price of CDO tranches, it is necessary to model the expected defaults in the underlying pool of assets, this is due to the influence and the impact of default on CDO payments. Especially one factor that needs to be considered in a detailed manner, is

the default correlation, which indicates if an asset, a borrower, or an obligor defaulting on its debt has been affected or not by another default of the same class. Simplifying it is the measure of the likelihood that two underlying assets (in the case of CDOs) will default concurrently or independently.

We can distinguish two principal approaches for the pricing of CDOs tranches, bottom-up and top-down model:

1. Bottom-up approach:
    - This approach is more data-intensive and involves analyzing each underlying asset in the CDO portfolio, such as individual loans or bonds.
    - It considers the creditworthiness and performance of each individual asset, as well as the structure of the CDO and its tranches.
    - This approach provides a more accurate and precise estimate of the CDO's value but requires significant time and resources to gather and analyze all of the necessary data.

2. Top-down approach:
    - This approach is quicker and requires less information than the bottom-up approach.
    - It involves analyzing macroeconomic factors such as interest rates and credit spreads to determine the overall credit quality of the CDO portfolio.
    - This approach provides a high-level estimate of the CDO's value but may not be as precise as the bottom-up approach, especially if the underlying assets in the CDO portfolio are not homogeneous.

In conclusion, both the bottom-up and top-down approaches are used in CDO pricing, with the choice of approach depending on various factors such as the size and complexity of the CDO, the available data and resources, and the intended use of the valuation. It's important to keep in mind that both approaches have their limitations, and no single approach is universally better than the other.

3.1.1 Gaussian Copula

The Gaussian copula model was introduced by David X. Li in 2000 as a method for modelling the dependence between variables. The Gaussian copula model is based on the idea that the dependence structure between variables can be separated into two components: the marginal distributions and the copula. The marginal distributions describe the behavior of each variable individually, while the copula describes the dependence structure between variables. The Gaussian copula model assumes that the copula between variables is a Gaussian distribution, which leads to a simple mathematical form for the model (Watts, 2016).

The Gaussian copula model has been widely used in finance to model the dependence structure between different financial assets, such as stocks and bonds, and price financial derivatives, such as credit default swaps. In this context, the Gaussian copula is used to model the dependence between the default times of different assets.

However, the limitations of the Gaussian copula model were highlighted during the 2008 financial crisis, when it was found that the model failed to accurately capture the dependence structure in financial markets. This was due to the assumption that the copula between variables is Gaussian, which may not always be the case in real-world financial data. As a result, alternative copula models, such as the t-copula and the vine copula, have been developed to address the limitations of the Gaussian copula.

Firstly, we define what a copula is and what is derived from. Starting from the definition of the joint distribution C, given n uniform random variable $U_i, \dots, U_n$ (Watts, 2016)

$$C(u_1, \dots, u_n, \rho) = P(U_1 \leq u_1, \dots, U_n \leq u_n)$$

Copula is a function that joins the univariate marginal distribution to the multivariate distribution. This is possible thanks to a property called Sklar's Theorem.[4] It states that any continuous multivariate cumulative distribution function (CDF) can be decomposed

[4] The theorem is a result in multivariate statistics named after American statistician Abraham Sklar. It states that if $F(x_1, x_2, \dots, x_n)$ is a joint multivariate distribution function with univariate marginal distribution functions $F(x_1), F(x_2), \dots, F(x_n)$, then there exists a copula function such that $F(x_1, x_2, \dots, x_n) = C(F(x_1), F(x_2), \dots, F(x_n))$.

into the distribution of its marginals and a copula, which captures the dependence structure between the variables (Watts 2016).

Examined with a general introduction to copulas now we can analyze Gaussian Copula: Given Φ as the distribution function of the one-dimensional standard normal distribution and Φ_Σ^n the distribution function of the n-dimensional standard normal distribution with a positive define correlation matrix Σ, we can define the n-dimensional Gaussian Copula as (Watts, 2016)

$$C_\Sigma^\Phi(u_1, u_2, \dots, u_n) = \Phi_\Sigma^n(\Phi - 1(u_1), \dots, \Phi - 1(u_n))$$

In order to study the one-factor Gaussian Copula it is compulsory to analyze the one factor copula model that is the statistical model used as fundamentals for the default correlation. Consider a portfolio of N obligors with the marginal probabilities of default known for each one of them. Let t_i be the time of default for i^{th} obligor and $Q_i(t)$ the cumulative default probability function if obligor i will default before time t.
Determining a random variable $x_i(1 \leq i \leq N)$ we can assess the one factor copula model (Elizalde, 2005)

$$x_i = \rho_i Y + \sqrt{1 - \rho^2} \cdot \varepsilon$$

Some consideration about this formula. X_i can be considered as the default variable for i^{th} obligor, in fact, lower is X_i sooner the default is likely to rise. X_i has two stochastic elements:

- Y the systematic risk factor that gives the description of universal the market risk.
- ε idiosyncratic risk factor, specific for each obligor.

Both Y and ε have independent zero-mean and unit-variance distribution and the correlation with the markets is represented by ρ_i. It is necessary to highlight a central assumption, ρ is equal for every couple of obligors.[5]

[5] Thanks to homogeneity of the assets in the pool the correlation between each pair of obligors is flat.

To construct the default correlation, we need to map variables as t_i to more stable variables as x_i through the factor copula model then find the default correlation among variables. Given F_i the cumulative density function of x_i and H the cumulative density function of ε_i, the mapping of $x_i = x$ to $t_i = t$ we have:

$$x = F_i^{-1}[Q_i(t)]$$

or

$$t = Q_i^{-1}[F_i(t)]$$

Starting from the one factor copula model if and only if Y's and ε's follows a standard distribution then it generates the Gaussian copula. Given a portfolio of reference asset constructed by N obligors we recall the model (Elizalde, 2005)

$$x_i = \rho_i Y + \sqrt{1-\rho^2} \cdot \varepsilon$$

Where x_i is a random variable with $x_i \sim N(0,1)$ $i = 1,2,\dots,N$.
Starting from the default times $t = Q_i^{-1}[F_i(t)]$ where t_i, $i = 1,2,\dots,N$ that are modelled from a Gaussian vector $X = (x_1, x_2, \dots, x_N)$ with $X \sim N_N(0,\Sigma)$, we have the case for the Gaussian copula:

$$t = \mathrm{Q}_i^{-1}[\Phi_i(x)]$$

With:

- Q_i the cumulative density function of t_i, and Q_i^{-1} the inverse function of Q_i
- Φ_i the cumulative density function of x_i

Then we have:

$$x = \Phi_i^{-1}[Q_i(x)]$$

The cumulative probability of the i^{th} default with time t conditional to Y is *(*Elizalde, 2005*)*:

$$P(t_i < t \mid Y = y) = P(x_i < x \mid Y = y)$$

$$= P\left(\varepsilon < \frac{x-\rho Y}{\sqrt{1-\rho^2}}\middle| Y = y\right) = \Phi\left[\frac{\Phi_{-1}[Q(t)]-\rho Y}{\sqrt{1-\rho^2}}\right] = p_i(y)$$

The assumptions for this model are the following:

1. All defaults for all obligors over the same time period have the same intensity
2. Same pairwise correlation
3. Portfolio is equally weighted, meaning that all shares in the portfolio are equal

3.1.2 Student-t Copula

The Student-t copula is a generalization of the Gaussian Copula. This copula extends the Gaussian copula by allowing for heavier tails, which means that it can better capture extreme events or tail risk. This makes the Student-t copula a popular choice for modelling financial returns, where the heavy tails of returns distributions can have a significant impact on risk measures and portfolio performance.

The dependence structure of the Student-t copula is specified by its degrees of freedom parameter. The larger the degrees, the more similar the Student-t copula is to the Gaussian copula, and the smaller the degrees, the more the copula allows for heavy tails and extreme events. This allows the user to model the dependence structure of the data according to their knowledge or preference.

In addition to modelling dependence, the Student-t copula can also be used to model the marginal distributions of each variable. The choice of marginal distribution will depend on the characteristics of the financial data, such as skewness and kurtosis.

Overall, the Student-t copula is a useful tool for modelling the dependence structure of financial data, as well as for portfolio optimization, risk management, and other applications in finance and economics.

Given a vector $V = (V_1, V_2, \dots, V_n)$ that follows a Student-t distribution with v degrees of freedom in a symmetric case we have $V_i = \sqrt{W}X_i$ where (Burtschell et al., 2008)

$$X_i = \rho V + \sqrt{1-\rho^2}\overline{V_i}$$

- $V, \overline{V_i}$ independent Gaussian random variables

- W is independent from $(X_1, \dots, X_n)$ and inverse Gamma distribution with parameters equal to $\frac{v}{2}$

It is important to observe that the covariance between V_i and V_j, $i \neq j$ is $\frac{v}{v-2}\rho^2$ for $v > 2$. Considering also t_v the distribution function of the standard univariate Student-t being the univariate cumulative density function of the V_i's we have $\tau_i = F_i^{-1}\big(t_v(V_i)\big)$. Conditionally on (V, W) default times are independent and given by (Burtschell et al., 2008)

$$p_t^{i|V,W} = \Phi\left(\frac{-\rho V + W^{-\frac{1}{2}} t_v^{-1}\big(F_i(t)\big)}{\sqrt{1-\rho^2}}\right)$$

The Student-t has equal coefficients for upper and lower tail dependence and they are given by (Burtschell et al., 2008)

$$2t_{v+1}\left(-\sqrt{v+1} \times \sqrt{\frac{1-\rho^2}{1+\rho^2}}\right)$$

An important point is that for $\rho = 0$ we no dot have tail independence, indeed tail dependence is always present for whether values assumed by ρ and v.

3.1.3 Double-t Copula

The double-t copula is constructed as a scale mixture of Gaussian and Student's t distributions. This combination allows for the modeling of both the central and tail dependence structures in the data. The parameters of the double-t copula can be estimated from data, which can provide insight into the dependence structure of the data and improve risk management. It has been used for the first time in the early 2000's for the pricing of CDOs (Hull and White 2004).

Overall, the double-t copula provides a flexible and powerful tool for modeling the dependence structure in multivariate data, and has applications in finance, insurance, and other fields were data exhibit heavy tails and non-linear dependencies.

Given a latent vector $(V_1, \dots, V_n)$ that models the default times, we can define the latent variables as (Burtschell et al., 2008)

$$V_i = \rho \left(\frac{\nu - 2}{\nu}\right)^{\frac{1}{2}} V + \sqrt{1-\rho^2} \left(\frac{\bar{\nu} - 2}{\bar{\nu}}\right)^{\frac{1}{2}} \bar{V}_i$$

- $V, \bar{V}_i$ are independent random variables that follow Student-t distribution with ν and $\bar{\nu}$ degrees of freedom
- $\rho \geq 0$

Student-t distribution is not stable under convolution so both V and $\bar{V}_i$ follow the Student distribution however V_i's do not (Burtschell *et al.*, 2008). We can affirm the Copula that associated with $(V_1, \dots, V_n)$ is not a Student-t copula but it is distinct from the previous one.

The default times are given by $\tau_i = F_i^{-1}\big(H_i(V_i)\big)$ for $i = 1, \dots, n$, in which we observe H_i dependent on ρ that is the distribution function of V_i (Burtschell et al., 2008)

$$p_t^{i|V} = t_{\bar{\nu}} \left(\left(\frac{\bar{\nu}}{\bar{\nu} - 2}\right)^{\frac{1}{2}} \frac{H_i^{-1}\big(F_i(t)\big) - \rho \left(\frac{\nu - 2}{\nu}\right)^{\frac{1}{2}} V}{\sqrt{1-\rho^2}} \right)$$

3.1.4 Stochastic Correlation

Stochastic correlation is one of the most straight forwarded extension of the Gaussian Copula model, it was created to match "correlation smiles" in the derivatives market especially CDO one (Andersen and Sidenius 2005, Schloegl 2005).

Defining the latent variable as (Burtschell *et al.*, 2008)

$$V_i = B_i(\rho V + \sqrt{1-\rho^2}\overline{V}_i + (1-B_i)\left(\beta V + \sqrt{1-\beta^2}\overline{V}_i\right)$$

For $i = 1, \dots, n$ where:

- B_i are Bernoulli random variables
- V and $\overline{V}_i$ are standard Gaussian random variables
- ρ and β are correlation parameters such $0 \leq \beta \leq \rho \leq 1$

The above written model is a convex sum of the Gaussian copula that contains a mixing distribution over factor exposure (Burtschell *et al.*, 2008).
Recasting the model such that:

$$V_i = (B_i\rho + (1-B_i)\beta)V + \sqrt{1-(B_i\rho + (1-B_i)\beta)^2}\overline{V}_i$$

ρ is the factor exposure with probability p and β with the correlation $1-\rho$. V_i's marginal distributions are Gaussian, and we can define the default dates as:

$$\tau_i = F_i^{-1}\big(\Phi(V_i)\big) \qquad i = 1, \dots, n.$$

At the same time, we can delineate the default probabilities knowing that default times are independent unconditionally on V (Burtschell *et al.*, 2008)

$$p_t^{i|V} = p\Phi\left(\frac{-\rho V + \Phi^{-1}\big(F_i(t)\big)}{\sqrt{1-\rho^2}}\right) + (1-\rho)\Phi\left(\frac{-\beta V + \Phi^{-1}\big(F_i(t)\big)}{\sqrt{1-\beta^2}}\right)$$

This model is conceived as a mixture of Gaussian copulas that includes combinations of correlations (Burtschell *et al.*, 2008). We can observe that the tail dependence coefficient is:

- 0 if $\beta \leq \rho < 1$
- ρ^2 if $\beta < \rho = 1$
- 1 if $\beta = \rho = 1$

3.1.5 Default and Premium legs

Starting from the previous Copulas analyzed in the previous part we can notice that by increasing certain parameters the result is the increment of the dependence among default times. These parameters would be:

- ρ for the Gaussian copula
- ρ for the Student-t copula
- ρ for Double t copula
- ρ , β or p for the stochastic correlation model

Premiums for CDO tranches, either equity or senior ones, with an attachment point of zero or a detachment point of 100% are monotonic with respect to dependence parameters. We must introduce the concept of attachment and detachment point:

- The attachment point refers to the point at which the tranche starts to bear losses from the underlying portfolio of debt obligations. In other words, the attachment point determines when the tranche becomes "at risk" of incurring losses. The higher the attachment point, the lower the risk of the tranche, and vice versa.
- The detachment point, on the other hand, refers to the point at which the tranche has fully absorbed all losses from the underlying portfolio and can no longer suffer any further losses. The detachment point determines the maximum loss that can be absorbed by the tranche. The higher the detachment point, the higher the risk of the tranche and the higher the expected return, and vice versa.

Taking into exam the Gaussian copula case, the premiums of equity tranche diminish according to the correlation parameters. This property grants the singularity of the correlations regardless of CDO's maturity or marginal distribution of default times.
The expected loss for a given portfolio is the sum of the expected obligors losses and is constant in function of the correlation structure. Also called base expected loss is the expected loss for a tranche with detachment points K is given by:

$$E\big[\min\big(K, L(t)\big)\big]$$

Where $L(t)$ is the aggregate loss for time t. The present value of the default leg of an equity tranche that contain a discounted average of such expectations (Laurent and Gregory, 2005), it decreases when he correlation parameters increment (ρ).

To conclude the analysis, we will consider the impact of the premium leg of a CDO tranche on the dependence parameter. We recall that the premium paid is proportional to the nominal value of the CDO tranche. As a result, we find that the value of the premium leg increases with the correlation parameter. At the same time, the value of the default leg decreases, causing the value of the buy protection (in a synthetic CDO) on an equity tranche to decrease as the correlation parameter increases. This conclusion also applies to the dependence parameters of the stochastic correlation, Student-t.
The value of the default on a senior tranche with an attachment point of K always increases as the correlation parameter increases. The premium paid on the senior tranche is proportional to its nominal value, which is more complex than that of an equity tranche.

3.2 CDO market during crisis

So far, we have analyzed CDOs in an instrument-detailed way, in this last chapter the main objective is to understand in a more macro-sectorial approach. It is interesting to observe the reactions and the behavior of the derivatives market (particularly the CDOs market) to the two most recent and rigid crises that occurred.

The First taken into the exam is the renowned Sub-prime crisis, the global financial crunch that has gripped all kinds of economic aspects for several years. The perpetrators of this speculative bubble are now renowned, but it is important to emphasize once again the damage caused by this veritable financial catastrophe. From commercial banks to investment banks to the big rating agencies, the profits made through a system that was not fully supervised allowed these institutions to feed a derivatives market 10 times larger than the asset market on which they relied.

Later we will try to analyze the consequences of a second crisis that occurred a few years ago and drags its consequences to this day. Covid-19 had several repercussions in a great many areas: health and social, but also at the economic and investment level. The reluctance in social contacts due to the pandemic is now also reflected at the financial level, investors are now hesitant to make a dent in their savings and are becoming increasingly risk-averse, this leads to a fossilization of the public and private investment sector.
The effects of this new crisis are perhaps less obvious than the aforementioned crisis perhaps this is due to the shorter time since the peak of the emergency. Nevertheless, we should not underestimate them as they can be a source of study and useful in predicting as well as anticipating and countering further economic crises derived from emergencies unrelated to the world of finance.

We will notice differences between the two market reactions obviously due to the different nature of the two events, but there are also similarities they share.

3.2.1 The sub-prime crisis

The CDO market was one of the key drivers of the subprime mortgage boom in the early 2000s.

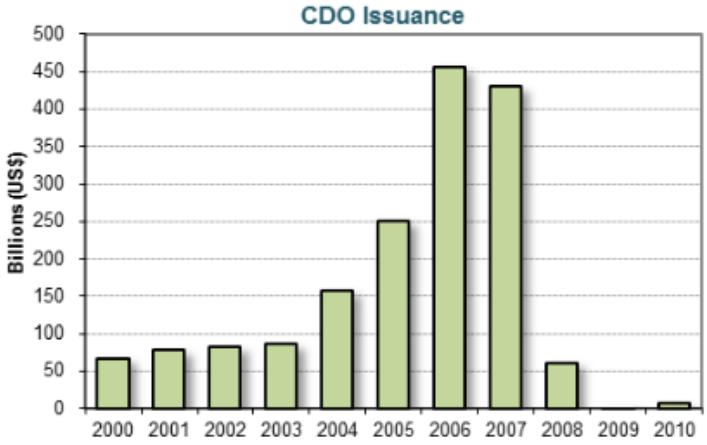

Figure 8: CDOs Issuance in the first decade of 2000's. Source: SIFMA

Because CDOs and other dangerous financial instruments were rated highly by credit rating agencies, they were considered low risk and many investors, including pension funds and insurance companies, invested heavily in them. Indeed, we can observe from Figure 3.2.1 (1) the sharp increase in demand for CDOs until 2007, where the drop was justified by the impending crisis and the realization by investors of their not-so-safe nature of them (Kyle & Russell, 2012).

The subprime mortgage crisis exposed the underlying weakness of the CDO market and the opaque nature of this derivative. As defaults and foreclosures on subprime mortgages rose, the value of the mortgage-backed securities (MBS) that many CDOs were tied to declined, leading to significant losses for CDO investors. This in turn caused a chain reaction throughout the financial system, as many banks and financial institutions held large amounts of CDOs on their balance sheets. The declining value of these assets contributed to the collapse of some of the largest banks and financial institutions in the world, leading to the 2008 financial crisis.

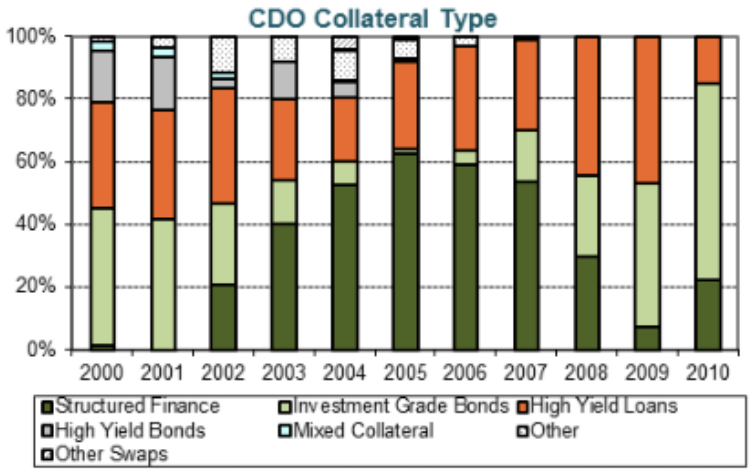

Figure 9: CDOs Collateral Type shift. Source: SIFMA

Another factor to consider is the shift of the collateral that backed the CDO in the early 2000s (Kyle & Russell, 2012). As we notice from the mixture of high-yield loans and investment-grade bond the conversion to structured finance is gradual but considerably sharp, this is precisely due to the fact that the "hunger" for-profit overcome the risk aversion misled by the corrupted rating agencies' opinions.

Overall, the reaction of the CDO market to the subprime mortgage crisis in 2007 was a major factor in the global financial crisis of 2008. The market's failure was due to a combination of factors, including poor underwriting standards, conflicts of interest between financial institutions, and the over-reliance on credit rating agencies to assess risk.

The subprime mortgage crisis and the subsequent reaction of the CDO market had far-reaching consequences for the global economy. In addition to the failures of many banks and financial institutions, the crisis led to a significant contraction in credit markets and a sharp decrease in economic activity. One of the key outcomes of the crisis was a significant reduction in the availability of credit, which harmed economic growth and made it more difficult for businesses and individuals to access financing. The crisis also led to a decline in consumer confidence and a reduction in spending, which had a further impact on the economy.

Another major outcome of the crisis was a significant increase in government debt, as many countries resorted to fiscal stimulus measures to support their economies. This has

led to concerns about the sustainability of government debt levels in many countries and the potential for future financial instability.

Finally, the crisis has led to a renewed focus on the need for effective regulation and oversight of the financial system. In addition to the regulations introduced in many countries in the aftermath of the crisis, there has been a growing recognition of the need for international cooperation and coordination in the regulation of financial markets.
As a result, many countries introduced new regulations and reforms to increase stability in financial markets and prevent similar crises from happening in the future. The Dodd-Frank Wall Street Reform and Consumer Protection Act was passed in the US in 2010 to address some of the underlying causes of the crisis, such as the lack of oversight in the subprime mortgage market and the over-reliance on credit rating agencies.

In addition, the crisis led to a renewed focus on the role of central banks in providing stability to financial markets. Central banks in many countries, including the US Federal Reserve, intervened to provide support to the financial system and prevent a complete collapse of the global economy.
The impact of the subprime mortgage crisis and the reaction of the CDO market was felt by individuals and communities around the world, as many people lost their homes, jobs, and savings this global financial crunch had a profound impact on the global economy and its aftermath continues to be felt today. In addition to the immediate consequences of the crisis, such as high levels of unemployment and reduced economic activity, the crisis also led to several long-term changes in the financial system. The crisis serves as a reminder of the importance of maintaining stability and trust in financial markets, and the need for effective regulation and oversight to prevent similar crises from happening in the future.

3.2.2 The outbreak of Covid-19

During the COVID-19 pandemic, the Collateralized Debt Obligation market experienced significant stress and turmoil. The lockdowns and economic downturn caused by the pandemic led to widespread defaults among companies, particularly in industries such as travel, hospitality, and retail. This in turn led to a decline in the value of the underlying assets in many CDOs, causing a decrease in their market value. Additionally, the sudden increase in market volatility and uncertainty caused by the pandemic led many investors to sell off their CDO holdings, further exacerbating the decline in market value. As a result, the CDO market faced significant challenges during the COVID-19 crisis, with many firms struggling to survive.

The CDO market is closely tied to the credit market, as CDOs are essentially a way of pooling together and repackaging individual debt obligations into a single security. When a significant number of the underlying debt obligations default, it can have a cascading effect on the value of the CDO. During the COVID-19 crisis, many companies in a variety of industries were unable to meet their debt obligations, leading to a wave of defaults and exacerbating the already fragile state of the CDO market.
Furthermore, the sudden and unexpected nature of the pandemic created significant uncertainty and volatility in financial markets, leading many investors to sell off their holdings in CDOs and other riskier assets. This further pressured the CDO market, causing prices to drop even further.

The CDO market also faced challenges from a liquidity perspective. Due to the complexity of CDOs and the fact that they are often made up of many individual debt obligations, they can be difficult to value and trade, particularly during periods of market stress. This lack of liquidity can aggravate declines in the market value of CDOs, as investors are less able to sell their holdings when they need to.
In conclusion, the COVID-19 crisis had a significant impact on the CDO market, leading to widespread defaults, declines in market value, and challenges related to liquidity. The overall state of the market has improved since the initial shock of the pandemic, but the CDO market is still facing headwinds and uncertainty as the world continues to grapple with the aftermath of the crisis.

It's worth noting that the decline in the CDO market during the COVID-19 crisis has implications beyond just the investors in those securities. CDOs are often used as a source of financing for companies and play an important role in the broader credit market. The decline in the value of CDOs can make it more difficult for companies to access financing, potentially leading to additional defaults and further damaging the broader economy.
Additionally, the decline in the CDO market may have deeper implications for the financial system as a whole. CDOs and other complex financial products played a role in the financial crisis of 2008, and a decline in the CDO market could raise concerns about the stability of the financial system and potentially trigger a wider sell-off in riskier assets.
In response to the challenges faced by the CDO market during the COVID-19 crisis, regulators and policymakers have taken steps to provide support and stability to the market. For example, central banks have implemented measures such as low interest rates and asset purchases to provide liquidity and support to the market. Additionally, regulators have taken steps to address specific challenges faced by the CDO market, such as improving the transparency and disclosure of information about the underlying assets in CDOs. However, the response to the pandemic in the two markets differed somewhat due to differences in the regulatory and economic environments:

- In Europe, the regulatory environment for the CDO market was already more restrictive before the pandemic, which made it more challenging for market participants to navigate the downturn. For example, the European Market Infrastructure Regulation (EMIR) required clearinghouses to increase the amount of collateral they hold against the default of their counterparties, which reduced the available capital for the market. Additionally, the EU's Capital Requirements Regulation (CRR) required banks to hold more capital against their exposures to the CDO market, which reduced their ability to participate in this market.

- In the US, the CDO market was more developed and had a larger investor base compared to Europe, which allowed for a more rapid response to the pandemic. The US government's CARES Act provided stimulus to the economy, which helped to stabilize the market, and the Federal Reserve's actions to support the financial system provided additional liquidity to the market. The market participants in the US also had access to a more diverse range of instruments and

strategies to manage their risks, which allowed them to respond more effectively to the pandemic.

However, despite the differences in the regulatory environment, both the European and US CDO markets were affected by the pandemic and have seen reduced activity levels. Investors have become more cautious and selective, focusing on high-quality assets and avoiding those with lower credit ratings or those that are more sensitive to economic cycles. Additionally, the market has become more fragmented, with smaller and more specialized players emerging to fill the gaps left by larger participants.

Overall, the CDO market continues to face challenges as a result of the COVID-19 crisis, but efforts are being made to provide support and stability to the market in order to minimize the damage to all aspects of economy. Additionally, the rollout of vaccines and the gradual reopening of economies has led to a reduction in uncertainty and increased confidence in the economic outlook, which has further supported the recovery of the CDO market.

As we have seen there are many aspects shared by both reactions and market behavior due to two profoundly different crises. The effects are similar, especially considering the collapse in the value of CDOs and the consequences that these two emergencies have had on the urgency for regulators to step in to make the world of derivatives more transparent and accessible.
However, we should not forget the effects that these two crises have had on society and on the way investors/consumers think about a particular financial product. The reluctance with which certain products are treated is due precisely to the dangerousness of these products if treated unwisely and the harmfulness that these two catastrophes have brought to the surface.

Evolution of CDOs analyses

In this chapter, we will look at the literature concerning CDOs and many of their facets. One can immediately notice a rapid change of opinion regarding these financial instruments if one looks at pre-2007 articles or papers compared to studies conducted after the crisis. In fact, in pre-crisis years, authors followed the line to praise their innovativeness and flexibility.
Subsequently, however, almost all the articles reviewed emphasize its risk factor and its potential contagiousness to other markets or sectors. This is because one of the factors most responsible for the Sub-Prime crisis is precisely CDOs.

Starting from an article published in the early years of the 2000s the analysis is focused on the risk and valuation of CDOs. The authors focusing on the Cash-CDOs for the less invasive role of the CDO manager, show the effect of the correlation and prioritization on market valuation, risk of CDOs and the diversity score (a measure of the risk of CDO collateral used by credit rating agencies). To illustrate them Duffie and Gârleanu (2001) used a jump-diffusion setting for correlated default intensities. The impact of the joint distribution of underlying collateral securities' default risk on the valuation of CDO tranches was revealed to be the main problem. The focus then shifts to the effectiveness of other computational methods for the diversity scores.
The conclusion drawn by the authors mainly concerns the weak availability of empirical data regarding correlation, and actual or neutral risk of default.
Already from the beginning, the lack of transparency in the market and the scarce availability of data seems to be a problem for the literature, this issue will be encountered and reported again in the future, especially with the crisis approaches.

Through the years, however, the curiosity for these relatively new instruments strikes financial experts. An article with a notoriously positive delineation regarding CDOs is certainly that of Choudhry and Fabozzi (2003). This paper focuses on the economic drivers behind the origination of CDOs and inspects their structure. Using the data from two real transactions Choudhry and Fabozzi consider the application of CDO on economic capital management.

These two transactions were issued in the U.S. and Asian markets during 2001 and 2002, they represent how the innovative technique of securitization can reduce credit risk for a bank. In addition, the authors show how portfolio managers can utilize CDOs to provide access to the corporate markets and supply returns to investors. The results highlight that banks have new tools for greater flexibility in managing credit risk and regulatory capital requirements. They expect also new studies due both to the nature of these transactions and to the more liquid essence of the synthetic corporate market than the cash market.
As we can observe the characteristics of CDOs and their market attracted banks and other institutions, so the literature sought to understand why they were so appealing by analyzing their structure, which up to that point was unclear to the public due to lack of straightforward information.

However, studies on CDOs peaked in both quantity and comprehensiveness of analysis mainly in the years before the subprime crisis or even during the years when the emergency was at its peak. Taking as an example the study of Franke and Krahnen (2007), this paper targets the design of CDO-transaction and the impact provoked by default risk exposure of the originating entity. Risk effects are assessed as the impact on the bank's default losses and its stock beta. This article calls attention to the removal of adverse selection and moral hazard problems in CDO-transaction by the first loss position of the bank.
Through a simulation exercise, the authors demonstrate both the impact of securitization and reinvestment on a bank's default risk and the repercussion of default correlation on a bank's risk exposure. In the case of the adoption of securitization by a bank with the aim of expanding its loan business its default risk (stock beta) increments, this is demonstrated by the empirical results. Finally, they drew some conclusions about the consequences of securitization on the financial market and they point up that this technique provides a new dynamic mechanism to combine the advantages of bank and market-based financial systems.
The shift in the target audience of articles pertaining to every aspect of CDOs is evident, now the risk component and its contagiousness take center stage, this is of course due to the hints of the crisis that already in the very early months of 2007 struck experts.

Having by now realized the danger of CDOs if not properly regulated the analyses done by scholars focused on the repercussions and correlations between various markets and collateralized debt obligations.

The authors, Mason and Rosner (2007), incentre this article on the changes experienced by MBS and CDO market in the years leading up to the subprime crisis. The report focuses on the structural changes due to the increase in CDO, loosening of lending standards and loan mitigation practices and how these innovations create a risky environment for investors. The results of the analyses after demonstrating the link between mortgage, MBS and CDO show how the U.S. housing market struggles. Another fundamental point featured in this article is the chain reaction engaged by a decrease in housing starts through the weakening of CDO issuance will result in economic imbalances that could lead to economic difficulties.

The weaker issuance of CDOs, as demonstrated by the article is directly correlated to the difficulties of housing starts, this factor made it clear that it was imperative to thoroughly study all correlations that concatenated CDOs with other markets.

The beginning of a more empirical approach in the analyses demonstrates a deeper understanding of the subject. Articles begin to be written in which the structure, characteristics, and functions of CDOs are now taken for granted, and then move on to illustrate the methodology by which these are evaluated. The paper of Rosen and Saunders (2007) presents a practical CDO valuation framework, it is entirely based on multi-factor credit models along with weighted Monte Carlo techniques. Rosen and Saunders demonstrated the superiority of the multi-factor models and utilized Generalized Linear Mixed Models to define the models and assess their impact. The article then expands to find a numerical solution to the inverse problem of the implication of the factors' joint distribution from market prices, the solution was to create first discrete scenarios on the factors. For the pricing of CDOs, this article observed that the absolute values of the factor weights are less minor than the relative differences in concentration that are defined, in addition, they noticed that other models used for the pricing of bespoke portfolios integrate information derived from the multi-factor model.

Finally, the authors simultaneously focus on both the static version of the model and use the framework also the more dynamic models.

The authors hope that future work will study in more depth the numerical implementation of the dynamic models and their potential.

As the severity of the crisis grows, one of the topics that enter predominantly into studies in the literature is the search for the culprits and triggers of the downturn.
This paper (Mason and Rosner, 2007) is focused on the role played by rating agencies in the adversities of the MBS and CDO market. Highlighting the fact that rating agencies must face conflict that affects the choice for the measure of risk, in addition, these agencies became part of the underwriting team leading to even more conflict. The authors concentrated on the difference between a product like CDO or RMBS and corporate debt. The result obtained by the authors indicates that inefficiencies and conflict in the rating system of financial products are the cause of U.S. investor discounts for the national market. The blameworthiness of rating agencies is and will be emphasized by various studies especially in having diverted investors by entering into various conflicts of interest. This trend will continue for several years following the peak of the crisis while fading only in analyses farther back in time.

Even though the common and most shared thought came close to a critique, a few articles that continued to appreciate and praise CDOs were published in the early years of the housing bubble. However, the need for more thorough and transparent regulation, especially of derivatives markets, is emphasized. This paper (Lucas *et al.*, 2007) can be located in the series of studies that investigate the risk transfer vehicle from the financial to the non-financial sector. This article pivots on CDOs focusing on synthetic collateralized debt obligations. In addition, the authors highlight the apprehension of regulators about CDOs and CDSs, which have an overall positive judgement sustaining that new tools to mitigate, diversify and hedge banks' risk are helpful for the financial system to become more stable and efficient. The component of risk contagion from financial to non-financial sectors is a cornerstone of the literature of the period. Used as a warning both before and during the crisis, it had no effect on the preoccupation of large investment banks that suffered its effects

Many papers focused on the pricing of CDO tranches by analyzing different methodologies to find the best one, a technique that is able to price the various levels even with the paucity of available data.
This article (Cousin and Laurent, 2008) is a review of the pricing for synthetic CDO tranches from the point of view of factor models. This paper includes analyses on copulas, multivariate Poisson and similar intensity models. An important point for the study is the role of the distribution of conditional probabilities that is strictly related not only to CDOs' tranche prizing but the distribution of large portfolios.
However, the authors illustrate several open questions that still need to be dealt with, the first is the calibration of CDO tranche quotes with different maturities and the same parameters, secondly, it is difficult to choose between a non-parametric approach and a specific parametric model, finally, deal with heterogeneity between names or linking factors related to geographical region or sectors.

Some authors have followed a cross-cutting line by not analyzing CDOs directly but using them as indicators and information providers. This makes us understand the pliability of these financial instruments and does not relegate them only to the role of bonds with the sole purpose of speculation.
The article of Longstaff and Rajan (2008) uses information gathered from the CDO market (the price of standardized tranches on the CDX credit index) to interpret the economic structure of default risk across firms, specifically how default event clusters.
Creating a simple multifactor portfolio credit model for CDO pricing, the authors consider three Poisson events that generate, idiosyncratic default, industrywide default and economywide default events. When considering a data set of CDX index and tranche spread the authors evaluated the model and its performance.
The result indicates that all types of risk are anticipated by the market and approximately one-third of the value of the default for the firms in the CDX index is due to events in which several firms default together.
The results once again underscore the broad correlation that CDOs have with various aspects of the market not only as risk contagiousness but also as indicators of default risk and predictors of crisis.

Despite the abundance of articles covering the U.S. market, papers on the European market have been published in significantly smaller numbers. This should not be understood as a lack of interest due to a lack of impact of CDOs on the European market, certainly, the consequences have been less than those that occurred in the U.S. nonetheless, they have been entrenched and have infected several sectors.

Scheicher (2008) centers the paper's focus on the variation of the North American CDX index and European iTraxx from 2004 to 2008. Daily data are studied through a regression analysis estimating the correlation between tranche premia and market-based measures of credit, liquidity, and rate risk. Specifically, this paper analyzed how explanatory factors have mutated starting from the credit market disruption in 2007. The results obtained stress that are sizable differences in market pricing of iTraxx and CDX tranches, especially considering that European tranche premia react weakly to the tumult with respect to U.S. tranche premia. A focal point of this article is the stronger increment of tranche in the CDX rather than in iTraxx after the start of the disruption of the market.

However, Scheicher highlights that tranche premia are importantly related to explanatory variables but still contain a solid unobservable component.

In many of the articles observed we note that despite in-depth analysis there still remains some unclear information or even components that cannot be found. This point definitely highlights the opacity of the market and the inefficiency of regulations on the ability to make key information available for market analysis.

Turning then to articles written when the crisis had just ended, we note the skepticism with which many models used in previous years to assess the quality of a CDO are analyzed. an example of this is certainly the paper of Heitfield (2009).

It is focused on the empirical difficulties in evaluating the credit quality of CDOs. Knowing the distribution losses and the collateral assets that back the obligation it is possible to assess the expected severity of CDO tranches' losses. Considering that rating agencies rely on simple copula models for the distribution of CDO collateral losses and since for more senior tranches parameters of these models are not observable Heitfiled showed the limitations of copulas accuracy and drawbacks on the evaluation of CDO tranches. The conclusion drawn by this article is that when the evaluation of risk results depending on classical techniques must be examined wot skepticism especially if the credit quality regards structured securities. To respond to the need to account for parameter uncertainty (Jorion, 1996b) the article emphasizes both the classical approach

which comprehends the adoption of best data and most effective methods available and the calculation of confidence intervals for the accuracy of those metrics and the more innovative Bayesian approach in which parameter uncertainty is ingrained in the process of computing risk metrics.
Finally, with the closure of the article, it is underlined that quantitative models used by rating agencies could lead to significant inaccuracies in default probabilities, expected losses and measures of the riskiness of CDOs.
As has been the case in the past, the responsibility of the rating agencies is emphasized more perhaps leaving out equal culpability of the inefficient and inadequate regulatory system.

Although many years have passed since the crisis, the published articles return by analyzing the past. in particular, they study the causes and perpetrators of the market collapse, also having acquired much more information than the studies carried out during the crisis they try to find a solution to the models for valuing CDOs.
Jablecki (2017) wrote this article with a double purpose. Firstly, using data from a unique dataset of more than 1000 CSO deals, the analysis focused on the performance of synthetic CDOs studying in a more detailed manner the degree and the origin of the losses that cause the downfall of the credit market in 2008. The empirical results obtained indicate that a great part of mark-to-market losses in the tranches was not correlated to epidemic credit events but reflected the high amount of credit losses in a short period. This factor was not represented effectively in pre-crisis pricing due to pricing models that still relied on Gaussian copula which is not suited for such concentration of default.
The second goal of this article is to illustrate an alternative to copula models with an analytically tractable intensity-based model of default correlation. This alternative model is based on the redefinition of systematic factors as a sequence of increasing random variables that constitute the correlation dependencies in the financial system. The result obtained by this model demonstrates the usefulness and accuracy of fitting indexes such as iTraxx and CDX and provides a decomposition of expected losses across idiosyncratic and systematic risk drivers.

The literature on CDOs is vast and covers a wide range of topics including their design, pricing, valuation, risks, and regulation as we can see from Table 2. There is a vast literature on the history and development of CDOs. This literature explores the origins of

CDOs, their evolution over time, and the reasons for their growth in popularity leading up to the financial crisis.

Most of the literature regards the design and pricing and the valuation and risk assessment of CDOs. It explores the different approaches that market participants have used to structure and price CDOs along with various techniques used to estimate the value of CDOs and to assess the risks associated with them. This literature includes theoretical models, empirical analyses, and case studies, models, and methods used in practice.

The literature with a more theoretical approach of study is the one on the regulation of CDOs explores the various regulatory frameworks that have been put in place to mitigate the risks associated with these products. The literature on the impact of CDOs on the financial crisis examines the role that these products played in the crisis and the reasons why they contributed to the severity of the crisis. This literature includes analyses of specific CDOs and the markets in which they traded, as well as more general assessments of the overall impact of CDOs on the economy. It is an important area of study for anyone interested in the financial markets and the risks associated with complex financial products.

Table 2: Reference literature

Authors	**Publication Date**	**Title and Keywords**
Darrell Duffie & Nicolae Gârleanu	January 2001	*Risk and Valuation of Collateralized Debt Obligations* Cash-CDO, correlation, prioritization, diversity score, computational method, availability data.
Moorad Choudhry & Frank Fabozzi	October 2003	*Originating Collateralized Debt Obligations for Balance Sheet Management.*

		CDOs, securitization, transactions, capital management, synthetic corporate market.
Günter Franke & Jan Peter Krahnen	January 2007	*Default Risk Sharing between Banks and Markets:* *The Contribution of Collateralized Debt Obligations.* CDO-transaction, default risk exposure, stock beta, securitization, first loss position.
Joseph R. Mason & Joshua Rosner	February 2007	*How Resilient Are Mortgage-Backed Securities* *to Collateralized Debt Obligation Market Disruptions?* MBS, CDOs, lending standards, loan mitigation, link between mortgage and CDO.
Dan Rosen & David Saunders	April 2007	*Valuing CDOs of Bespoke Portfolios with Implied Multi-Factor Models* Practical valuation, multi-factor model, Monte Carlo techniques, Generalized Linear Mixed models, inverse problem, discrete scenario.
Joseph R. Mason & Joshua Rosner	May 2007	*Where Did the Risk Go? How Misapplied d Bond Ratings Cause Mortgage-Backed Securities and Collateralized Debt Obligation Market Disruptions.* Rating agencies, CDOs, RMBS, corporate debt, conflict.

Douglas J. Lucas, Laurie S. Goodman Frank J. Fabozzi	July 2007	*Collateralized Debt Obligations and Credit Risk Transfer* Risk transfer vehicle, CDOs, CDSs, hedge risk.
Areski Cousin & Jean-Paul Laurent	January 2008	*An overview of factor models for pricing CDO tranches.* CDO pricing, synthetic CDO, factor model, multivariate Poisson, conditional probabilities, large portfolio distribution, calibration of tranches, parametric models, non-parametric models, heterogeneity.
Francis A. Longstaff & Arvind Rajan	April 2008	*An Empirical Analysis of the Pricing of Collateralized Debt Obligations.* CDO market, CDX, default risk, firms, multifactor portfolio, Poisson events, clustering of default.
Martin Scheicher	June 2008	*How has CDO market pricing changed during the turmoil?* *Evidence from CDS index tranches.* CDX, iTraxx, regression analysis, tranche premia, measure of credit, liquidity, rate risk, explanatory factors.
Erik Heitfield	January 2009	*Parameter Uncertainty and the Credit Risk of Collateralized Debt Obligations* Empirical difficulties, credit quality, tranches losses, copula model, classical approaches, Bayesian approach, default probabilities, CDOs riskiness.

Juliusz Jablecki	November 2017	*Rise and fall of synthetic CDO market: lessons learned.* CSO deals, synthetic CDO, mark-to market losses, epidemic event, Gaussian copula, concentration of default, intensity-based model, idiosyncratic risk, systematic risk.

Conclusion

Having come to the conclusion of this thesis where we have analyzed one of the most complicated financial instruments, through both theoretically and practically how a CDO is structured and how it is priced. As financial instruments whose performance is difficult to predict, the intuitive answer to these questions is no, but this, like many other derivatives, must be treated with extreme caution to yield significant income for banks and investors.

In the first chapter, we first entered the world of derivatives by explaining their components and characteristics. Going into the specifics of the CDO, we explained its structure. It being divided into tranches accommodates the demands of all types of investors from the most risk-averse to the most risk-seeking. Next, we divided CDOs following different logics, the first being the source for payment of principal and interest, subdividing between Cash Flow CDOs and Market-Value CDOs we exposed their differences such as the role of the CDO manager.

After which, we clarified the twin reasons why these instruments are extremely attractive to the market and those who manage them. Finally, we catalogued the three major families of CDOs distinguishing their characteristics and roles that are assumed by SPEs (SPVs), starting with the most popular and widely used Cash CDO then explaining Synthetic CDO with its more complex and intertwined structure, and ending with an explanation of the rarest of the three i.e., the hybrid one.

In the next chapter we explained how the cash flow of the underlying assets is allocated to each tranche for both principal and interest payments. to do this, the technique called waterfall scheme is followed, which allows the right risk-proportionate payment to be allocated to each level. We have illustrated both schemes followed by the set of techniques that grant protection against losses in s stressed scenario called credit enhancement with its various techniques.

We then moved on to the practical overview. Initially, we analyzed models for pricing CDOs, starting with the Gaussian copula and its evolutions. Li's Copula has advantages

and disadvantages that are exploited or circumvented by its evolutions such as the Student-t and double-t copula. ln the end, we analyzed the premium and default leg with its functions.

Looking at the CDO market, we explained the reactions and behavior with respect to two of the most serious crises that have occurred recently. the subprime crisis caused several shocks in the world markets most affecting the American one to which the failure of the national real estate system was directly related. many regulations came into play to cope with the crisis and avoid the future occurrence of a new crisis in the future, all of which, however, could not cope with the new crisis that hit the globe in 2020. Covid-19 with its pandemic triggered a financial and credit crisis that could not have been predicted. Once again, the market reaction relied on intervention by regulators such as the Federal Reserve for the US and the ECB for the EU.

Finally, we observe the evolution of literature from the early 2000s to these days, we observed the evolution of the thoughts and the analyses performed on CDOs and their market. We can clearly distinguish two distinct methodologies in the pre-and post-2008 line of thinking and analysis, especially whit the studies of CDO structures and for tranche evaluation.

In conclusion, CDOs are financial products that involve the pooling of various debt assets, such as mortgages, bonds, and loans, and then repackaging them into new securities that are sold to investors. The purpose of CDOs is to allow for the diversification of risk and to provide investors with access to a range of different debt assets. However, the complexity of CDOs and their underlying assets, combined with the lack of transparency, made it difficult for investors to fully understand and assess the risks involved. This was a significant contributor to the 2008 financial crisis, where many CDOs defaulted, causing significant losses for investors.

Despite these challenges, CDOs continue to play an important role in the financial system, providing a means for investors to access a range of debt assets and for companies to raise capital. However, it is important for regulators and market participants to improve the risk management and disclosure practices surrounding CDOs to ensure their stability and prevent future financial crises. This could involve increased transparency in the reporting

of the underlying assets, better risk assessment and stress testing practices, and the implementation of stricter regulations on the issuance of CDOs.

In conclusion, CDOs are complex financial instruments that pose both opportunities and challenges for investors and the financial system as a whole. While they have the potential to provide access to a range of debt assets and increase the availability of credit, it is important to continually assess and improve their risk management and disclosure practices to ensure their stability and prevent future financial crises.

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