Università Ca'Foscari Venezia

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Final Thesis

Improving the Optimization of a Hedge Fund portfolio in times of financial crisis

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I dedicate this important milestone to my family,
in particular to my maternal grandmother and my parents,
who taught me from an early age that
with perseverance
it is possible to overcome any obstacle

Abstract

The last two years have been characterized by geopolitical events and tensions which have caused anomalous fluctuations in the financial markets. The high uncertainty surrounding the global economic outlook has made investors risk-averse, focused mainly on preserving capital and neglecting the pursuit of high returns.

Nonetheless, downturns resulting from a recession can represent advantageous and attractive entry points for rational and patient investors, who have a long-term horizon.

The study focuses on the issue of asset allocation during financial crises and, in particular, on how it is possible to improve traditional methods in order to better face the increase in market volatility. To this end, a dynamic correlation model will be introduced, the DCC GARCH, through which it will be possible to study the spillover effect of variable volatility and rebalance the portfolio on a monthly basis using a rolling window.

A portfolio of Hedge Fund Indexes will be used as the reference investment, which have historically performed well during bear markets (especially in relation to their mutual fund counterparts).

The empirical analysis is developed through data from the HFR database between February 2010 and December 2022 and through the statistical software R.

Ultimately, the analysis will confirm that dynamic portfolio optimization through regular rebalancing captures changes in financial market volatility.

Keywords: DCC GARCH, Hedge fund, Portfolio Management, Bear Market, Asset allocation

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1) Survive and thrive during a recession

1.1) Introduction

The global economic context of the last three years was very complex and characterized by substantial uncertainty. A rare coronavirus pandemic caused severe disruption to financial and economic activity around the world, causing the start of a bear market state in mid-March 2020, with losses exceeding 30%.

Luckily, this period was very short and lasted for 33 days (until March 23, the day in which many stock indexes including the S&P 500 reached their minimum). An unusual decline ensued through which in the following five months in which the market not only recovered the previous level but went further surpassing all-time highs.

The year 2021 was also characterized by many events (for example the assault on the Capitol in the US or the increase in inflation can be mentioned); unlike what happened in the previous year, however, investors ignored external inputs (which sometimes suggested divesting and causing alterations in the stock market). It follows that throughout the year the markets were characterized by stable growth and many of the main indexes in fact had record increases, gaining from 18 to 26% (Jackson, Schmidt, 2022).

However, this balance did not last long, until February 24th 2022, the day the Russian invasion of Ukraine generated an economic storm. Indeed, the war caused a "domino effect" that involved the whole continent, with an exponential rise in inflation especially in the price of some raw materials like wheat, corn, oil, electricity and nickel. The severe losses in the Russian and Ukrainian markets have spread creating a financial contagion especially in countries that had ties to the protagonists, or geographically close to the conflict. In the study carried out by Zangari "Signs of Contagion from the Russia-Ukraine War" it was found that Credit Default Swap (CDS) spreads on sovereign bonds of Eastern European countries increased considerably up to March 10th (therefore expectations by default they have risen considerably) (Zangari, 2022). Additionally, signs of contagion have also emerged in the foreign exchange markets of the neighboring regions, where the Polish, Hungarian and Czech Republic currencies have recorded sharp depreciations against the US dollar. The euro was also affected by this event and reached the same value as the dollar on July 12th 2022 (it had not happened since 2002). The war also tainted the stock market where investors rushed to reduce their exposure to the markets. However, this reduction was not only due to the reaction of investors to this macroeconomic event, but also to public reasons which led particular types of politically exposed

institutional investors to disinvest in Russian securities.

The uncertainty of the economic context had a large impact on the prices of financial products, which presented a fluctuating trend, subject to news announcements and events. These changes have affected correlations between stocks, which are a key input to the asset allocation process as well as risk estimation. Indeed, as reported in the article "Flexible Dynamic Conditional Correlation multivariate GARCH models for asset allocation" (Billio, Caporin e Gobbo, 2006), the most experienced professionals argue that correlations increase in periods of high instability and volatility influences both the persistence and the extent of the correlation. Therefore, it becomes important to account for the time variation of correlations in portfolio returns. Indeed, a static correlation structure may not accurately measure risk, with significant impacts in terms of asset allocation and portfolio construction. In this thesis, the topic of dynamic portfolio correlation will be explored through the DCC GARCH model by Engle (2002). The empirical application carried out in the study represents a specific case, that is the one of a patient investor, who at the advent of the financial crisis caused by the Covid-19 pandemic decides to enter the market by contrasting the opposite signals from the environment. The investment will be that of a "hedge fund" type portfolio, which has historically had excellent returns during bearish periods thanks to an aggressive active investment style and the advantage of not being subject to rules. The portfolio will be composed of some of the main hedge fund indexes (data taken from the "Hedge Fund Research" website).

The thesis will be structured as follows. In chapter 1, after presenting a historical overview of financial crises and the strategies adopted, the theme of Hedge Funds will be treated.

In chapter 2, the topic of portfolio management theories will be explored, starting from the presentation of the Markowitz model (which is still the starting point of modern finance) and its criticisms. Through the latter, some of the subsequent models will then be introduced, including Tobin's Separation Theorem, the Black-Litterman model and Sharpe's Capital Asset Pricing Model, which will be used in the empirical analysis. Starting from the limits of the same, Engle's DCC GARCH model will be introduced and deepened.

Finally, in chapter 3, the empirical model will be presented, developed with the help of the statistical software R. The purpose of the study is to verify the benefits of implementing the dynamic component in the optimization of a hedge portfolio. This analysis will be carried out taking as a reference time interval the one from February 2020 to November 2022, in which numerous macroeconomic and geopolitical events took place and led to the high instability of the financial markets mentioned at the beginning of the chapter.

1.2) Active and passive strategies

Capital market theory states that the market portfolio, which is the portfolio made up of all the assets present, is the most efficient portfolio as it optimally combines risk and return. The elements that determine the choices of investors in the creation of the portfolio will therefore be their propensity for risk and the amount they want to invest (also resorting to a loan).

This type of approach is the foundation of the concept of passive (or indexed) strategy, in which the manager follows an asset allocation that aims to replicate a reference benchmark as closely as possible. The watchword of passive strategies is "Try to be the market". It follows that, by replicating the weights of the index, namely buying the shares that have done well and selling those that have lost, in fact they buy when the price is high and sell when it is low. Passive investors therefore generate a momentum-driven market, further driving up the price of assets that rise the most monthly, creating bubbles in asset prices.

However, the empirical context is much more complex than that presented in capital theory and is characterized by a high information asymmetry between the various stakeholders which consequently leads to a divergence between analyses and expectations. This "imperfect" environment creates other investment strategies, based on exploiting market flaws and mispricings to make a profit. Such methodologies classify as active and are highly sought after by investors. Indeed, active investment strategies aim to beat the market, outperforming a given benchmark, by selecting a small portion of financial products.

While passive strategies favor established companies that are in a maturity stage in their life-cycle, active managers are free to select stocks characterized by a small market capitalization but enormous growth potential due to the fact that the market attention is directed elsewhere, resulting in more insight into emerging and frontier markets. Therefore, it is easy to understand what the weaknesses of these strategies are, namely that of being more complex to manage, both in the selection and management phases, and consequently more time consuming and characterized by higher commissions and investment fund costs.

At the end of the global financial crisis, markets experienced a long bull run in asset prices and limited volatility. In the decade 2009-2019, most of the stock indexes brought significant positive performances, this means that passive strategies have been found to have good returns. The intervention of the central banks was important, using unconventional instruments of expansive monetary policy (quantitative easing). This consists in generating money (real or electronic) to buy securities with the effect of increasing their price, injecting liquidity into the system and thus lowering

the interest rate. The securities purchased can be of various types: from short-term ones (usually the subject of the first transactions) to long-term ones, from non-performing bank loans to share capital companies. This monetary policy has had positive effects on the real economy, reducing debt burdens on the public debt, supporting inflation and investments.

Figure 1: The performance of stocks and bonds over a 20-year period: from January 1999 to October 2019

Source: Lustig, Y. (2019), *A New Era of Active Management Looms*, troweprice.com, December 2019

As can be seen from Figure 1, falling interest rates have meant that since 2009 the market yield has exceeded the excess yield, resulting in a high expected return on stocks and bonds. Analyzing instead the previous period of time, it is possible to notice an opposite trend, especially in the last eight periods characterized by bearish markets.

From the study "Active vs passive fund management: which works during a crisis?" conducted by Greg Flash and Holly Lobprise, it is possible to see how powerful active strategies are in these bearish contexts (Flash, Lobprise, 2020).

The study carried out by the two analysts starts from the case in which an investor randomly selects an actively managed fund and compares its performance with the reference benchmark. By selecting the JSE All Share (ALSI) as a benchmark, it is possible to see that the probability that a random fund of that market outperforms the index is 44% (therefore an average performance lower than the reference index). This means that a short-sighted acceptance is unlikely to produce good results. However, there are funds that can outperform it, even consistently and over the long term. In

particular, this happens in the "lean" years and/or characterized by financial crises, characterized by a drop of at least 20% compared to the previous maximum.

Figure 2: Alsi returns and outperformers
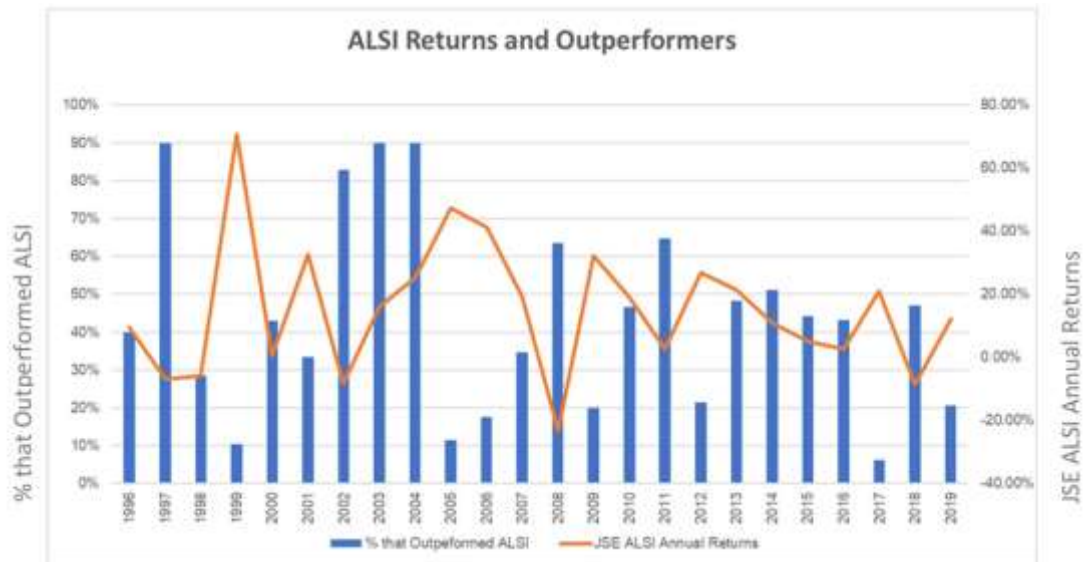
Bear Markets	ALSI (Total Return)	Actively Managed Funds (Avg)	% of Actively Managed Funds Outperformed
1997/08/07 – 1998/01/12	-27.2%	-14.4%	100%
1998/04/20 – 1998/09/11	-40.4%	-39.0%	47%
2000/01/17 – 2000/04/17	-25.9%	-20.2%	96%
2001/05/22 – 2001/09/21	-23.0%	-11.8%	97%
2002/05/22 – 2003/04/25	-33.9%	-17.9%	100%
2007/10/11 – 2008/01/23	-20.0%	-15.6%	97%
2008/05/22 – 2008/11/20	-44.8%	-34.6%	96%
2020/01/19 – 2020/03/19	-35.2%	-33.4%	56%

Source: Flash, G., Lobprise, H. (2020), *Active vs passive fund management: which works during a crisis?*, citywire.com, 8 May 2020

As can be seen from the graph and table, the average return of active equity managers has outperformed ALSI, sometimes considerably, such as during the dot-com crisis of the late 1990s and early 2000s and the real estate crisis of 2008. It is therefore evident that once there is an increase in volatility, active strategies are the ones that lead to greater gains.

The main motivation appears to be linked to the greater flexibility that allows to mitigate the risk, allocating lower percentages to the most vulnerable sectors and protecting themselves from declines,

disinvesting from underperforming securities when the risks become too high. Historically, for example, the sectors that have given the worst performances during times of recession have been automotive, consumer durables (especially domestic appliances) and travel & tourism.

This greater flexibility also allows to take advantage of short-term trading opportunities (swing trading), trying to identify where the price of an asset is likely to move next in order to capture a portion of the profit. These strategies also allow to carry out operations that offset gains for tax purposes, the so-called "collection of tax losses". This consists of selling some investments at a loss to offset the gains made from the sale of other securities at a profit, thus paying taxes only on the profit obtained, thus reducing the amount of the tax bill. This operation, in addition to reducing taxes, also frees up money so that investors can acquire new businesses that are more likely to generate positive performance.

This shows that an active style allows for more extensive trading options, offering investors the ability to choose what they want to invest in (as opposed to many mutual funds), and above all greater coverage linked to the possibility of generating positive returns even through short selling, put options and other strategies to be able to insure against losses. Indeed, the best time to start shorting is, as can be imagined, during a bear market, when most stocks in the market (or in certain sectors) are on a downward trend.

There are no exact methodologies that can predict when the market is right to invest. However, there are some tools that allow investors to understand whether the markets will follow an uptrend or a downtrend. Some tools start from technical analysis, which is the study based on the analysis of supply and demand for the security and the impact that these will have on changes in the price, volume and volatility of the securities. It contrasts with another type of analysis called "fundamental", which starts from the company's data (such as turnover, current assets, ...) to arrive at the valuation of the stock. The person who opened the door to this type of analysis was Charles Dow, founder of the Wall Street Journal and creator of the Dow Jones Industrial Index.

The first tool is that of the relative strength index (RSI), which is an indicator that allows to intercept possible oversold or overbought assets. This graph allows investors to understand which trend the markets are following in relation to the number of purchases and based on this it is possible to understand how to act in the market. For example, if the RSI of the asset rises rapidly and it is in a bear market, traders will be inclined to quickly sell the financial product in order to capture a quick profit. Conversely, if the RSI is low and it is in a bullish period, the trader will be inclined to buy the asset. Typically, during a bull market the RSI numbers are usually higher and assets remain overbought for much longer. Conversely, during a bear market the RSI tends to be low, with long oversold periods. However, the main limitation of this index emerges during events of great impact,

when the provided signals tend to become meaningless.

Another tool is the stochastic oscillator, which compares the closing price of assets in a given period with its recent highs and lows to provide an indication of the stock's momentum. If the closing is close to its maximum, it is a bullish period and therefore it will be convenient to sell the security. If, on the contrary, this is at a minimum, it will be the case of a bearish period and therefore the asset will have to be purchased. The oscillator moves between two values: 1 and 100. For values lower than 20, there will be an oversold situation and an overbought situation for values higher than 80. The strength of this oscillator is that it can generate signals during phases not characterized by a trend.

1.3)   Short selling and hedge funds

Bear periods often instill fear in investors as markets are often associated with recessions, complicated and less stable. They are characterized by a very short average life of about 9-10 months (Tzanetos, 2022), about 1/5 of the average duration of bull markets. This makes market timing particularly complicated, even for experienced investors.

Historically, the type of subjects that had the greatest performances during these periods were particular alternative investment funds called hedge funds. These funds managed to outperform their mutual fund counterparts both in the Dot-com crisis and in the real estate crisis thanks mainly to two factors: the lack of subjection to rules and the ability to be able to invest in aggressive strategies (Vikas, Boyson, Naik, 2009). These funds are characterized by active management based on the use of complex investment strategies, with the use of loans held with the aim of generating "leverage". This allows them to be able to take on financial exposures in excess of their capital base, thereby increasing their returns (or possibly their losses).

The functioning is rather simple. Prime brokers lend liquidity, credit and securities to hedge funds. Moreover, the funds are able to draw on additional financial resources also thanks to the "securitisation". As an example, according to the results obtained from the hedge fund as counterparty survey (HFACS), the average financial leverage used by hedge funds after the 2008 real estate crisis was around 2x.

In addition to loans, it is also possible to create the leverage effect through derivatives (such as futures, options and swaps) via the so-called "Synthetic leverage". The operation is quite intuitive: when the investor decides to go long through the use of derivatives, they do not buy the security even though they receive exposure to it.

The peculiarity of these funds, however, is that of the extensive use of short selling, which is used to reduce risk during periods of market decline and/or to make profits when it is believed that the value of some assets will decrease in the short term. term. It is precisely this ability to reduce risk or "Hedge" that gave the name to these funds.

Identifying stocks to sell short requires a different analysis process than analyzing to find an asset to sell with a long position. The first step is to analyze the activity behind the stock being sold short, trying to understand if it is actually overvalued. The focus of this analysis can be varied and concern not only the current situation, but also possible future scenarios. Observation elements can include, for example, the phase in which the company finds itself with respect to the life cycle, the competitors, the regulatory framework or the possible development of more efficient substitute products. The second step concerns the use of technical indicators to understand the market trend (such as the already mentioned RSI and the stochastic oscillator). Once the situation of the company, the trend of the reference sector and the other extra-sector components that can influence the investment are clear, the investor has all the necessary tools to choose whether or not to make the investment.

Shorting is a particularly risky and dangerous practice for many reasons. Firstly, since bear markets fall quickly and do not last long, it is difficult to intercept the correct moment in which to enter and exit this type of investment. While the loss of a long investment, in the worst-case scenario, corresponds to the purchase value of the asset, the risk of loss in a short sale is potentially unlimited, as if the price skyrockets the investor still has to buy it at that value in order to return the stock to its broker. Therefore, it is possible to go well below "zero".

Secondly, this could be characterized by additional fees, called "stock lending fees", or an expense related to the difficulty to find the asset. The amount paid behaves like variable interest and is determined on the basis of the trend in supply and demand for the same. Another aspect to consider is the "short squeeze" that caused the Game Stop case in January 2021. This market phenomenon is caused by a rapid increase in the share price of a security, which is present in short in the portfolio of many investors. What happens is that the increase in the price of the same pushes some short sellers to close their positions by buying back the asset. This causes the price to rally further, which in turn prompts other investors to exit the position. Therefore a cycle is created which will end when all the subjects have closed their positions. The Game Stop case caused many hedge funds that had short positions in the stock to lose billions of dollars, so much so that some funds were forced out of business altogether.

These risks highlight the importance of carrying out investigations and in-depth analysis of data and information. If done correctly, this can generate several advantages, in addition to the already mentioned one of making a profit when evaluating the decrease in the value of a stock.

An advantage is certainly that of hedging, or that of protecting the earnings of an investment, helping to reach breakeven. Take as an example a hedge fund that chooses to invest in a security X for a duration equal to T. During the period 0 → T there may happen that there is an event that causes a change in the performance of X. The hedge fund could therefore choose to compensate for this new risk by taking a short position on X, in order to close the position and gut the threat that the stock will fall further in value.

Another big advantage of short selling that hedge funds often use is related to "leveraged trading". This lies in the fact that if the fund borrows X to short it, the capital employed for the investment is not very high. So if the short selling operation is successful, the fund could generate very high profits with a relatively small amount of money.

A further tool that has become popular in recent years is that of short selling campaigns by hedge funds. These allow funds to be able to communicate actively and strategically to other investors, exposing the reasons why they believe a particular company is overvalued. First of all, in-depth investigative analyses are carried out on the companies subject to the attack; subsequently the results are disclosed in an "effective" way, persuading investors to divest in the same companies. The process of communication and persuasion is extremely important: it is not based solely on the display of information and data, but topics that aim to arouse an emotion in the public are also often used, for example dealing with ethical and environmental issues. The accusations usually concern industry-wide, product-level problems, flawed business models, unfair competition, financial errors, financial fraud or financial malpractice. The campaign allows hedge funds to reap high returns as they short sell the target company's shares before publishing their report.

Thanks to the resonance of the media, the influence of the reports becomes extremely relevant for the market value of the companies, which often are subsequently delisted, suspended from the stock exchanges or, in the worst case, go bankrupt. The research "Deploying Narrative Economics to Understand Financial Market Dynamics: An Analysis of Activist Short Sellers' Rhetoric" by professors Luc Paugam and Hervé Stolowy shows that 85% of the reports are covered by the media after publication and the companies involved can show losses significant amounts of value for more than 6 months (Paugam, Stolowy, 2020). The value of companies targeted by hedge fund campaigns declined by an average of 11.2 percentage points in the first three days, which corresponds to an average reduction of $416 million in market value. Two months after publication, the results are down by about -14.5%, while on average six months later they show a negative return of 22.6%. Cumulative average loss is shown in the figure below.

Figure 3: Average Cumulative Loss over 1 year
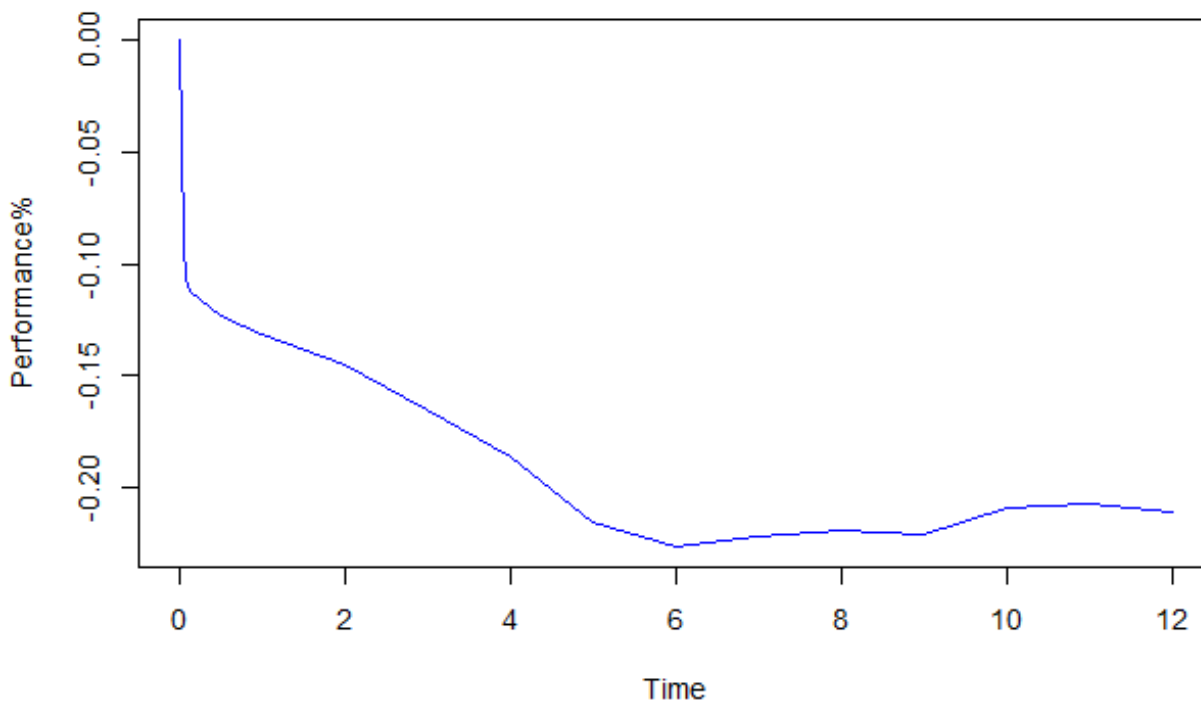
Source: the graph is built in R Studio using the data reported in «Paugam, L., Stolowy, H., Gendron, Y. (2020), *Deploying Narrative Economics to Understand Financial Market Dynamics: An Analysis of Activist Short Sellers' Rhetoric*, Contemporary Accounting Research, 15 November 2020»

Hedge funds tend to be very diverse and invest in many different strategies. As showed in figure 4, among the most famous is long-short equity, which makes up about 25% of strategies, which consists of taking long positions in stocks that can have above-average performance and shorting stocks with below-market performance. So the gain depends mostly on the ability of the fund manager to compose the portfolio.

Another very popular strategy (about 17%) is that of fixed income, which consists in holding similar fixed-income securities characterized by a strong historical correlation. Since the securities have an interest rate correlation, it follows that an increase in interest rates will have a positive influence on the short position and a negative one on the long position. A typical operation of this strategy is to go long on a bond and short on another with similar duration to offset the effects due to changes in interest rates. The strategy is therefore not based on creating a profit based on intercepting the future trend of the market but will focus on the relationship between positions in relation to the optimal gap estimated by investors, trying to neutralize changes in interest rates.

Other less popular strategies are based for example on opportunities deriving from price inefficiencies

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deriving from extraordinary operations (such as acquisitions, spin-offs, restructurings, share buybacks, hostile takeovers,…), on trend-following through the purchase of futures, on 'purchase of financial products in emerging markets,… A brief summary of these strategies can be seen in Figure 4.

Figure 4: Summary of hedge fund strategies
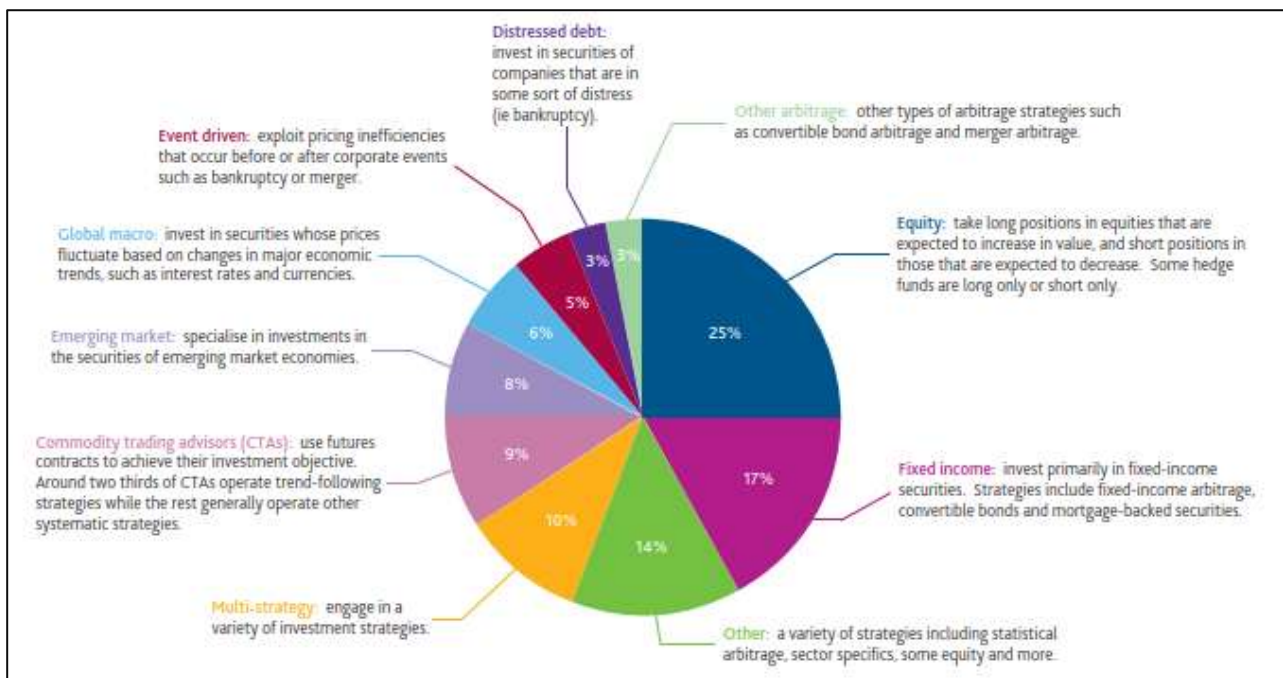
Source: Barclayshedge.com, "Databases" section, reports relating to the third quarter of 2017

Another type of fund is the quant (or quantum fund), which has experienced exponential growth in the last decade (since 2010 an average of 9.5% per year, against 6.4% in the sector) (Barclayhedge and Barclays, 2017). These new investments have prompted managers to continuously innovate management methodologies. One of the most innovative is the implementation of unsupervised machine learning systems, real artificial intelligences in which the computer identifies complex processes and patterns in the data without any manual guidance. The strategy of these funds is based on a set of rules regarding the analysis of past patterns and present data trends. These funds usually adopt a very high leverage (around 5x or 6x) to make the most of the price anomalies between the assets. Another feature of these funds is that they operate in "liquid assets", which allow easier asset management in market crisis situations. There are also residual less liquid assets, such as fixed income securities or emerging market stocks.

1.4) Hedge fund indexes

Historically, hedge funds have been exclusive funds, accessible only to high net worth individuals or by institutional investors such as endowments or pension funds. The popularity of this last type of fund has grown particularly in the last decade, reaching high shares of total assets in some countries. An example of this is Great Britain, whose hedge fund activity exceeds 7% (barclayshedge.com) while in the US it is around 5.9% (fundmap.com). This is due to the fact that these individuals use a "barebell strategy", a strategy that consists of finding a balance between risk and return, balancing between high risk-return and low-risk investments. The goal of the strategy is to reduce risk without decreasing overall return. In this case, the riskiest assets are hedge funds, while the least risky ones are bonds, held to cover pension liabilities.

Hedge fund indexes have been introduced since the late 1990s, through which even investors with medium-low capital can have access to this type of investment.

Index strategies have marked a major transformation in hedge fund investing, as they offer the advantage of professional management and diversification. These products also have the added benefit of helping the investor manage the risks associated with individual hedge funds; therefore if a single fund is faced with management or trading problems, the impact on the entire portfolio is not relevant.

Hedge fund indexes deviate significantly from the traditional definition of an index, which is that of a barometer for a particular market, sector or asset class, which allows it to serve as a point of reference for monitoring performance. A hedge fund index, in contrast, captures the returns of a broad universe of hedge funds, making it an alternative benchmark for measuring the performance of a given hedge fund.

Hedge fund indexes allow to capture attractive returns over the long term, in both bull and bear markets, ensuring a never significantly underperformance of the market.

Below is a comparison between some of the main equity indexes (S&P 500, Nasdaq and Dow Jones) and some hedge fund indexes (Fund of funds, equity hedged, emerging markets); the input data was taken from the MarketWatch website, from HFR and from YCharts. The graph was created assuming an investment of 100 dollars on January 1 1992 for each financial product.

Figure 5: Performance analysis Stock vs Hedge indexes
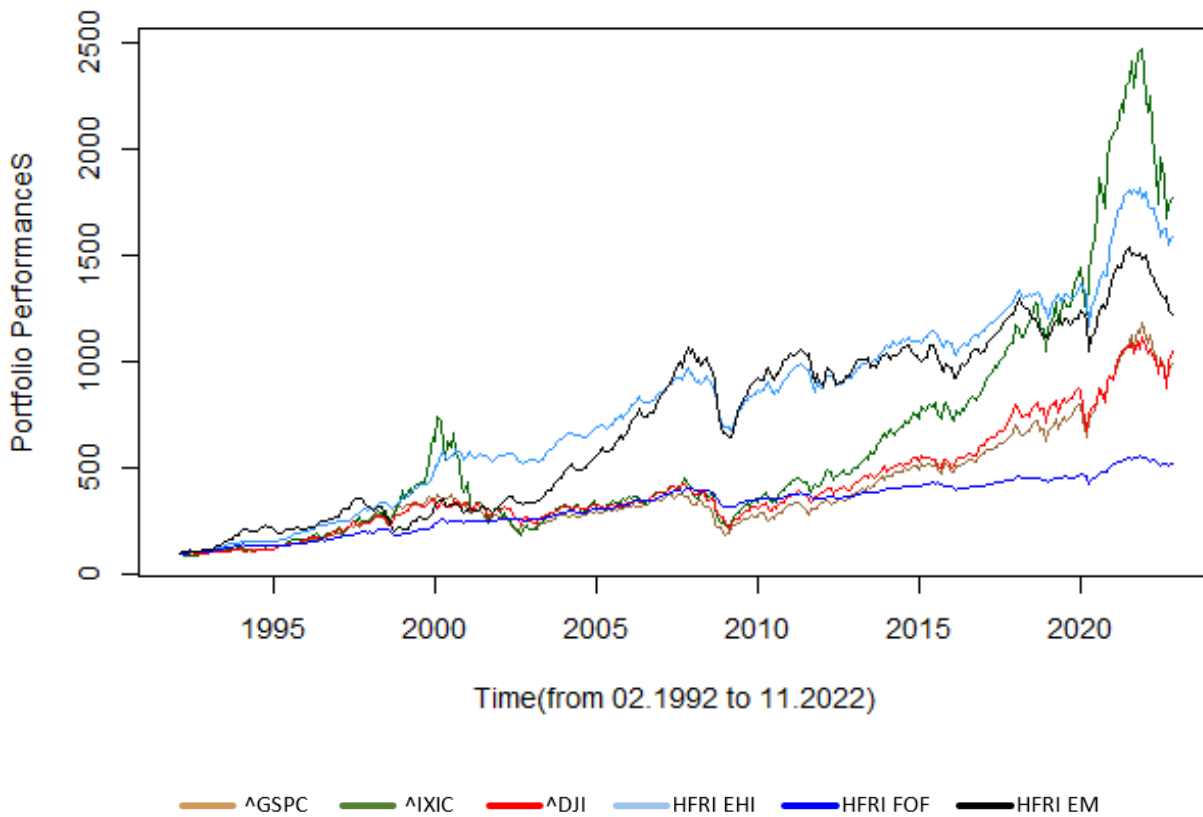
Source: the graph is built using the data from the sites of MarketWatch, HFR and YCharts

Until about 1998, the yields of the 6 indexes were almost identical; the only index that appears to generate a higher profit is the emerging markets hedge index (HFRI EM).

This "equilibrium" breaks with the first bear market, namely the dot-com bubble (24/03/2000–21/09/2001); on March 10th 2000 the Nasdaq (^IXIC) indeed reached its maximum point (5132.52 points) to then lose more than 10% of its value in one week. In this first bearish period, shown in the graph, it is possible to see how the HF indexes were the ones that coped better with the period, managing not only to preserve their value but also to increase it. In particular, the following years showed a great growth in the value of equity hedged investments (HFRI EHI) and emerging markets, which reached 1000 dollars shortly before the bursting of the real estate bubble (ten times more than the value of the initial investment).

The second bearish period (09/10/2007–20/11/2008) does not substantially change the trend of the relationship of the indexes, which decrease by an average percentage of 30-40% in a few months. The index that turned out to be more stable during the subprime mortgage crisis is the HFRI FOF (funds of funds) while the other two HF indexes have had the greatest decline in absolute terms.

During the long bull period following the Great Recession, the Nasdaq once again became one of the highest-yielding indexes. The third short bearish period caused by the Covid-19 pandemic did not alter the balance between the index reports, which soon returned to an upward trend. In this subsequent period, the Nasdaq experienced exponential growth, managing to reach the level of $2500 on November 19th 2022 (which corresponds to 25 times more than the original value of the investment).

The fourth and final bearish period began in the first quarter of 2022, in which the world economy was rocked by a mix of events such as a sudden monetary tightening, China's standstill due to its zero-Covid policy, but above all the invasion of Ukraine, which contributed to the generation of an energy shock and increased inflation. In this period, also due to the huge increase in the losses of giants such as Meta and Amazon.com, the Nasdaq experienced a dot-com 2.0 effect, losing to date (October 7, 2022) about 30% of its value.

It can be concluded that hedge fund indexes have performed well over the past three decades, but more importantly, they have been better able to cope with increased volatility during bear markets. However, it should be noted that commissions and management expenses must be deducted from the return of the HF indexes, which narrow the gap with the equity indexes.

2. Models of analysis and selection of the optimal portfolio

2.1. Markowitz's revolution

When an individual makes investment choices, he usually tends to select more than one stock to better diversify the portfolio. This allows, especially in the event that the financial products are not correlated to each other, to reduce the overall risk of the investment. The quality of the financial products, as well as, the level of portfolio diversification, depends on the investor's propensity to tolerate uncertainty (the so-called "risk appetite").

This concept, that seems to be taken for granted nowadays, was introduced for the first time in 1952 by the US economist Henry Markowitz in a scientific article called "Portfolio Selection". This work earned him the Nobel Prize in 1990 for his pioneering contributions in the field of financial economics.

Markowitz's study starts from the analysis of the historical performance of financial products in order to be able to make expectations on future ones and implement the portfolio choice.

The central theme of Markowitz's theory is diversification, that is which stocks to buy among those present in the market to maximize returns and minimize risk (or volatility). The benefit of diversification allows the overall volatility of the portfolio to be reduced, which will be lower than the average volatility of individual stocks.

A second theme is linked to the fact that in building a portfolio there are some combinations of securities that are undoubtedly better than others, which allow for the best risk-return ratio to be achieved.

The third and final issue concerns the subjective side: once the various efficient combinations have been identified, the model also indicates which is the most suitable for the investor on the basis of his risk appetite.

The model is based on five assumptions:

1. The time horizon is one-period

This means that the investor selects the allocation of securities in the portfolio only once at the beginning of the investment and maintains it for the entire duration of the same, without making any rebalancing.

2. Investors select portfolios based on expected return and risk

The expected return $E(R_p)$ is defined by Markowitz as the average of the possible returns of a security,

weighted by the probability that each return occurs.

As for expected risk, this is defined as the standard deviation of the security's return. Unlike yield, portfolio risk is not the average of the stock's risks. This is due to the fact that when calculating the portfolio variance it is consider the variance of the sum of n random variables.

3. Investors are risk averse

It follows that the subject, considering the risk as something unpleasant, will make a greater risk correspond to a greater return.

4. The market is in a form of perfect competition

This means that subjects are unable to influence the price of financial assets.

5. The market is free of transaction costs and taxes

The most interesting hypothesis that deserves further study is certainly number 2, which is the starting point for understanding the asset allocation process. In particular, it is possible to define the expected return of the portfolio with the following equation:

$$2.1) \ E(R_p) = w_1 * E(R_1) + w_2 * E(R_2) + w_3 * E(R_3) + \cdots + \ w_n * E(R_n)$$

Where $\sum_{i=1}^{n} w_i = 1$.

If the hypothesis of short selling is accepted, w_1 can also assume a negative sign.

It follows that the standard deviation is given by the following formula:

2.2)

$$\sigma_p = \sqrt{w_1 * \sigma_1^2 + w_2 * \sigma_2^2 + w_3 * \sigma_3^2 + \cdots + \sigma_n^2 + 2cov(x_1, x_2) + 2cov(x_1, x_3) + \cdots}$$

The covariances in this formula are extremely important as they correspond to the so-called "benefit of diversification". In fact, if the securities are not perfectly correlated, the standard deviation of the portfolio will be less than the weighted average of the standard deviation of the individual securities. Consequently, the risk will decrease as the number of stocks increases if they are sufficiently "uncorrelated".

By analyzing the problem through analytical geometry, all the possible mean-variance combinations in an area delimited by a frontier can be drawn on a Cartesian plane.

Take for example a portfolio composed of two assets, marked in the expected return-standard

deviation graph below as "a" and "b".

Figure 6: Efficient frontier - Markowitz Portfolio Theorem
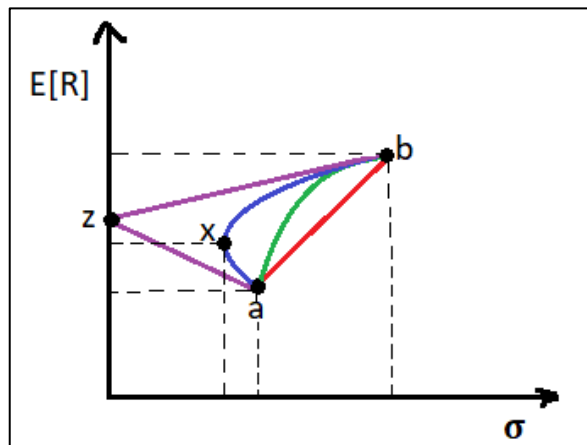
If portfolio risk and return both followed the weighted average logic, the portfolios that could be constructed by varying the proportions of the two securities would be positioned on the red line, also called the "efficient frontier". However, since the portfolio standard deviation is less than the weighted average, the portfolio curve will likely be the one shown in green.

Depending on the level of correlation of the two stocks, even less risky combinations can be achieved; indeed, if the securities are negatively correlated, there could be portfolios whose risk is even lower than the risk of the less risky asset. This is the case of the efficient frontier represented by the blue line, where some portfolios are further to the left of security a. The more the securities are negatively correlated, the more the curve extends to the left; the extreme case is represented by the purple curve, which corresponds to the case in which the two stocks have a perfect negative correlation. Since every movement in one security is mirrored equally and oppositely by the other, the portfolio will be risk-free.

The mean-variance principle derives from the second and third hypotheses according to which, between two investment strategies, the one that generates the highest return for the same level of risk or the one that generates the least risk for the same level of return is preferable.

For the identification of the optimal portfolio, however, the efficient frontier is not sufficient but it is useful to introduce the concept of risk appetite. Indeed, subjects can be divided on the basis of this factor into three categories: risk lover, risk neutral and risk adverse.

Investor preferences are represented by curves called "indifference" curves which encompass the set of risk-return combinations which guarantee the same level of utility or satisfaction to the subject. Since investors are risk averse (hypothesis 3), these curves will have a positive slope and different positions based on how much the individual investor is willing to take risks. For example, the

indifference curves of a very risk averse subject will be more to the left (like those indicated in green in the graph), while those of an investor who wants a higher return and is willing to take more risks for it will be more right (in orange).

Figure 7: Indifference curves - Markowitz Portfolio Theorem
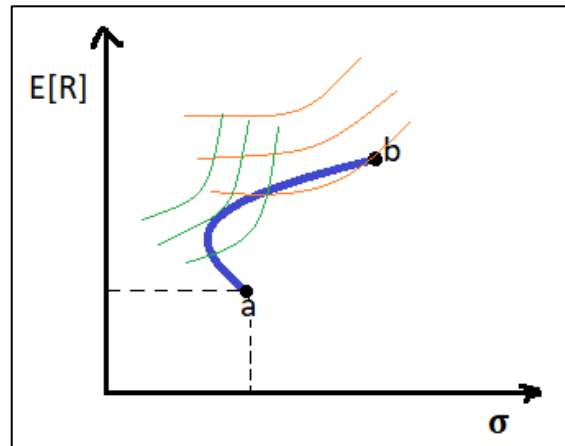
The set of indifference curves is given by the following function:

$$_p^2 \lambda * \sigma *_p) - 0{,}5R(E =)_p U(2.3) \ E$$

In which λ represents the subject's aversion to risk. The chosen portfolio will be the one present at the point of tangency between the efficient frontier and the highest indifference curve.

It is not always easy to identify the investor's degree of aversion to risk as it is a factor that is difficult to quantify, varies over time and is conditioned by numerous variables (think, for example, of the investor's knowledge, his objectives, financial stability, market trends, etc).

Therefore, Markowitz's significant contribution was not only to introduce the benefit of diversification understood as "volatility", but also to use the investor's risk appetite as a criterion for identifying the portfolio.

2.2. Criticisms of the theory

Markowitz's theory was the "beaver" for asset allocation and in fact is the basis of subsequent investment theories and, more generally, of modern finance.

However, the theory has many weaknesses that limit its value. Although of considerable scientific importance, the study does not show great practical utility and in fact the use by investors is extremely limited. The assumptions on which the model is based are highly and not likely to apply in the real

world (a criticism that in part affects most future asset allocation theories).

The model does not offer results with a high degree of reliability since it overestimates assets with a high expected return, low risk and low correlation with other securities. It follows that the Markowitz optimal Portfolio will be composed of a small selection of assets.

Other problems include, for example, the absence of perfect rationality among investors, who often make investments that do not respect the risk-return ratio or the presence of information asymmetries that do not allow all subjects to have access to all the data available in the market. Other unlikely assumptions are unlimited access to capital, the absence of efficient markets or the absence of taxes and transition costs.

With regard to the model itself, an important criticism concerns the function linked to the indifference curves, which is different from individual to individual since it depends on the subjective risk aversion λ; this certainly makes the comparison between the choices of different individuals very complicated. In addition to not being reliable in the results, the model presents some problems in terms of temporal instability: each slight variation in the model inputs results in a composition of the optimal portfolio that is very different from the previous one.

2.3. CAPM and subsequent development

A development to Markowitz's theory was given by James Tobin in 1958 who, starting from the concept of diversification introduced by the US economist, introduced a risk-free rate R_f to replace the indifference curves. This makes possible to overcome one of the previously mentioned limitations of the US compatriot theory, that is the estimate of risk aversion λ.

The risk-free rate is characterized by a certain return and zero risk $\sigma_{R_f} = 0$. Therefore, it is placed in the Cartesian graph on the abscissa axis.

In the real world, a proxy for the risk-free rate is, for example, the 10-year Italian bond: this is considered a risk-free rate as the probability that the Italian government defaults and is unable to pay the return to the investor it is really small, so the risk is considered negligible.

 The new optimal (or tangency) portfolio is therefore obtained through the intersection of the efficient frontier and the "capital market line" (CML), which is the half-line that originates in the risk-free rate.

Figure 8: Tangency Portfolio - Tobin's Separation Theorem
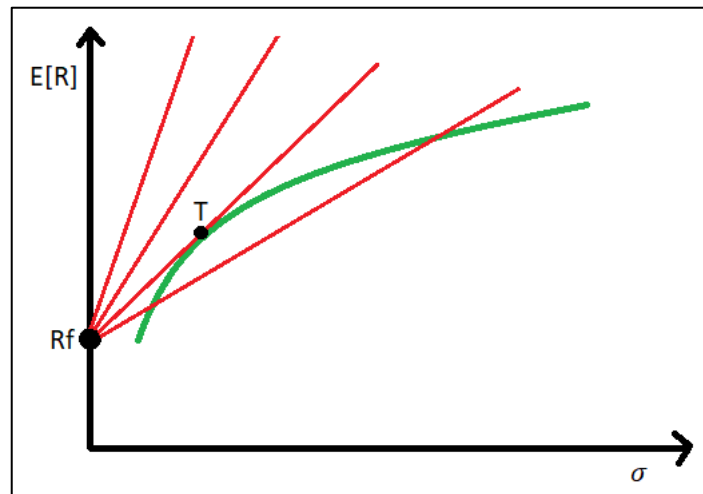
The capital market line represents portfolios that perfectly combine risk and return. The capital market line equation is as follows:

$$2.4) \ R_p = r_f + \frac{R_T - R_f}{\sigma_T} * \sigma_p$$

Where R_T is the market return, σ_T the standard deviation of market returns and σ_p the standard deviation of portfolio returns.

Tobin argues that investors should, once they have found the optimal portfolio T, leverage up or down to achieve the desired level of volatility. What has been obtained is a process with two independent phases (hence the name "Separation Theorem"): in the first part the composition of the risky part of the portfolio is chosen, while in the second the amount of financial leverage to use.

An evolution of Tobin's theorem is the Capital Asset Pricing Model (CAPM) by William Sharpe (1964) who demonstrated that the optimal portfolio present at point T is the market portfolio, which is characterized by the highest achievable Sharpe ratio.

$$2.5) \ Sharpe \ Ratio = \frac{R_P - R_f}{\sigma_P}$$

In particular, the CAPM studies the relationship between two components: the first factor is the expected return, while the second is the intrinsic risk of the market (or systematic risk), that is that unpredictable risk that can be avoided. This risk cannot be modified through diversification and reflects the impact of various macroeconomic, geopolitical and financial factors.

These two elements are related by the formula:

$$2.6) \; E(R_i) = r_f + \beta_i * [E(R_m) - R_f]$$

This equation introduces many interesting elements. The first is the market risk premium $[E(R_m) - R_f]$ which has a direct impact on the expected return of the financial product. The market risk premium represents the expected reward for taking the extra risk and is calculated using the difference between two components: the average return on the stock market and the risk-free.

The second element is that of beta which indicates the exposure of an asset to systematic risk. The β_i measures the expected change in the return on the security for each % change in the market return. It follows, as can be seen in equation 2.6, that the expected return of an asset will have a linear variation with its beta. If $\beta_i > 1$ then my asset will be riskier than the market and will tend to widen its movements (in fact $|r_i| > |r_m|$). Conversely, if $0 < \beta_i < 1$ then the asset will be less volatile than the market, both in the rising and falling prices. Finally, if $\beta_i = 1$ the stock will tend to move in line, while if $\beta_i < 0$ the asset will tend to move in the opposite direction to that of the market. The negative correlation of the asset with the market will lead it to have positive returns during bearish markets and negative returns during bullish moments. An example of an asset with a typically negative beta is gold (or more generally precious metals).

Figure 9: Comparison between an ETF built on the price of gold and the S&P 500
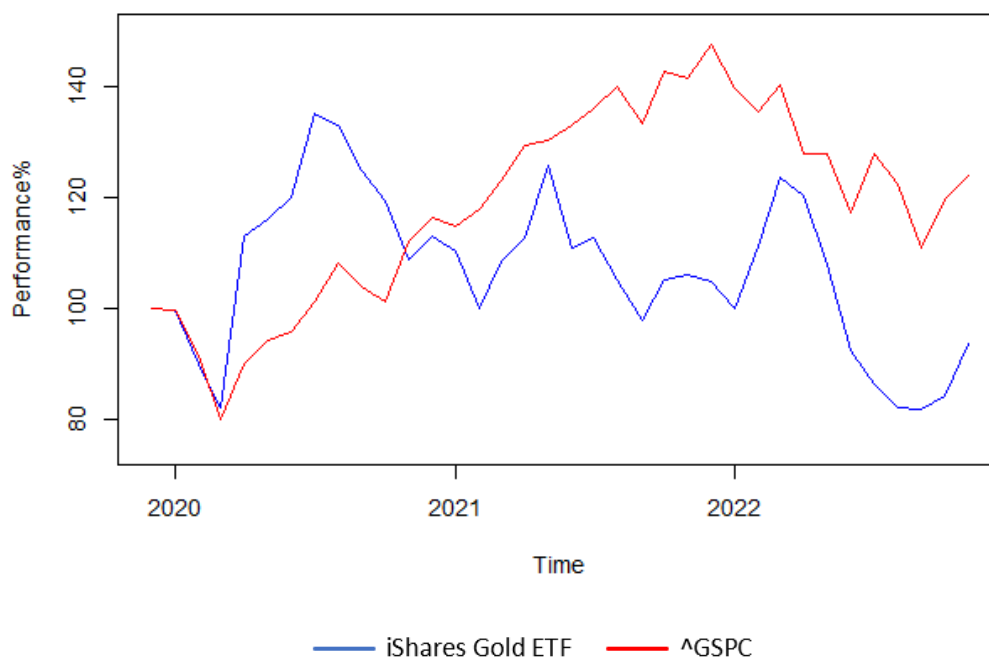
Source: the graph is built using the data from the sites of MarketWatch

As it is possible to see from the graph both during the advent of the Covid-19 pandemic and during the outbreak of the war in Ukraine, gold not only managed to have a less volatile return than the market, but it even managed to grow worth. A study by Roshan Gautam and Yoochan Kim (published on 23 May 2022) showed that for every 1% daily increase in Covid-19 cases, it was attributed a 0.2885% increase in the price of gold.

It is possible to represent the equation of the CAPM model as a line, exactly like the capital market line in Figure 9, even if in the latter method the y-axis measures beta. This half-line, called the Security Market Line, indicates for any beta level what expected return the asset should have according to the CAPM model.

This is fact a tool to check if a security is listed incorrectly in the market. Take for example the stock of Amazon.com, Inc whose Beta as of December 6 2022 is 1.28. Taking the historical average value of the S&P 500 as a proxy for the calculation of the expected market return (calculated considering the last 20 years) and the US bond as the risk-free rate, the following result is obtained: $E(R_{AMZN}) = 0.038 + 1.28 * (0.11 - 0.038)$=13.02%. Since the value of Amazon's effective average return over the last 20 years is 72%, as a result the asset is undervalued because for the same level of beta there is a much higher level of return.

Graphically speaking, all shares found above the Security Market Line are undervalued, conversely they can be considered overvalued.

Contrary to the Markowitz model, the Sharpe model is still widely used because in addition to being very simple to estimate, it is an excellent tool for evaluating future expectations and also allows for easy comparisons between different market alternatives. The CAPM allows the investor to make judgments about how the portfolio has been constructed in relation to, for example, which assets have significantly increased the risk of the portfolio.

The fundamental concept of the Sharpe model is the possibility of obtaining higher returns than those of the market by assuming a higher risk (beta). Such message has greatly contributed to the increase in the use of indexing or assembling portfolios that represent a given market.

A limitation of the Markowitz model (which also partially characterizes the CAPM) is that of the use of historical prices as an input to the portfolio optimization process. Indeed, the model presents an instability linked to its strong sensitivity to the inserted historical inputs; this factor could cause a high variability over time of the weights defined through the asset allocation, as well as the possibility of assuming extreme values.

A model that improves this limit of the mean-variance model is the one introduced by Fisher Black and Robert Litterman, who try to find an agreement between the market's expectations and those of the investor, linked for example to information that is not yet in the domain public. Another point of improvement is that B&L theory does not assume that past expected returns will continue to play out in the future.

The model is characterized by two distinct distributions of returns (and consequently two variances): one that follows market expectations and one that follows those of the investor. If the means and the variances coincide it would mean that the investor will invest in the market portfolio (for example in a highly diversified index).

One of the innovative aspects of the theorem lies in the fact that it is based on a Bayesian methodology, which combines the two types described above (market expectations and investor opinions).

The hypothesis underlying the model is that the security allocation process should be proportional to the market values of the assets, to then be modified using the basis of how much the opinions (or views) on the expected returns considered by the investors differ from those of the market (a kind of "desired" allocation). If there are two assets in the portfolio (X and Y) and the subject believes that stock X will yield better than Y, the opinion can be represented by the following equation:

$$2.7) \ V = E(R_X) - E(R_Y)$$

Where V represents the amount that the subject expects to receive more from the first title.

An important observation concerns the fact that, even if the opinions concern only some securities in the portfolio, the consequences of the same will fall on all the components.

Views are called "absolute" if they express the opinion that the yield of a financial product will be higher or lower than its current yield. They are "relative" if instead they make a comparison between the future return of two different assets.

To calculate the excess returns, a procedure is used that is the opposite of the one previously analyzed for the Markowtiz model and for the CAPM. In particular, the values of the weights of the securities are set as optimal, which are used to solve the equation which has the excess returns as unknown. The methodology is therefore based on maximizing the utility function with respect to the weights:

$$2.8) \ U = w * \pi - \frac{\lambda}{2} * \omega * w^2$$

Where π represents the vector of equilibrium excess returns, λ the investor's risk aversion and ω the variance-covariance matrix.

With regard to the subjective component, the investor will have to measure his degree of certainty regarding the forecasts. There are several methods for calculating this factor; the most common method uses the equation $\tau = \frac{1}{n}$, where n represents the number of observations relating to the distribution of expected returns. Alternative methods are, for example, to use the standard deviation of the probability distribution, or to choose it arbitrarily in a range from 0 to 1 based on the level of certainty that the investor has in relation to his views. The lower the value of τ, the more the portfolio will tend to be similar to the market one.

An innovative aspect introduced by Black & Litterman is also that of the periodic rebalancing of the portfolio through the level of τ. Indeed, the investor could, over time, have external inputs that can cause decreases or increases in the level of security relating to some or all views; a rebalancing of the portfolio will therefore be necessary, which would make it possible to generate a better excess return.

The presence of a periodic rebalancing is essential during a bear market to restore the risk-reward ratio. Indeed, during a market downturn, the products in the portfolio will tend to appreciate or depreciate more rapidly, misaligning the same with the predefined asset allocation. It therefore becomes necessary to periodically adjust the mix of asset classes to new market conditions.

The absence of periodic optimization represents a weakness of Sharpe and Markowitz's theory, which will be the result of in-depth analysis in this study. In fact, in the model of the American economist, the investor is relegated to being only a spectator of the market dynamics, unable to deal with the different situations of the portfolio through the control and adjustment of the weights of the securities.

2.4. Financial contagion and DCC GARCH

In the last thirty years, numerous financial crises have followed one another which have led to consequences both in the local context and in the global one. Examples include the Tequila Crisis of 1994, the South East Asian Crisis of 1997, the Russian Fever of 1998, the Brazilian Crisis of 1999, the Dot-Com Bubble of 2000, the Argentine Crisis of 2001, the Financial crisis of 2008, the Sovereign Debt Crisis of 2011, the Covid-19 pandemic of 2020 and finally the current outbreak of the War in

Russia of February 2022.

These crises, very different from each other both in type and in the cause of the event, are united by the fact that although they occur at a regional level, they have then expanded causing effects to the entire global economy. This phenomenon (called "financial contagion") has not only concerned a transmission between countries that have direct macroeconomic links, but also to states that have no apparent affinity.

Indeed, the definition of financial contagion has been the subject of extensive debate. Three definitions can be listed:

1. "Contagion is the transmission of global or local shocks between nations via economic fundamentals (spillover effect)" (Calvo, Reinhart, 1996) (Pristker, 2000).

   From this definition it is possible to deduce that financial contagion is not only present in cases of financial crises but also in the absence of them, since this is a manifestation of mutual dependence between regions.

2. "Contagion is the transmission of global or local shocks between nations through mechanisms that do not involve the fundamentals of the affected nations" (Claessens, Dornbusch, Park, 2001) (Masson, 1998).

   Based on this second definition, it would seem that contagion is determined by the irrationality of investors and their aversion to risk.

3. "Contagion occurs when the correlation between the prices of financial assets of nations, during a period of crisis, changes from that which occurs during periods of stability, and this is due to changes in the parameters of the underlying model" (Forbes, Rigobon, 2000).

According to this last hypothesis it is possible, for example, to intercept this phenomenon by observing the correlation between the markets. Indeed, as report in the article "Flexible Dynamic Conditional Correlation multivariate GARCH models for asset allocation" (Billio, Caporin, Gobbo, 2006), the most experienced professionals argue that correlations increase in periods of high instability and volatility influences both the persistence and the extent of the correlation.

This last aspect is the one that will be considered in this thesis as it manages, unlike previous studies, to reduce portfolio risk during financial crises.

In the literature, the first economist who spoke of the interdependence of financial product markets was the aforementioned Sharpe (1964) who argued that asset returns are determined by a set of

common factors and by idiosyncratic factors which represent an unrelated risk diversifiable. However, the first empirical studies concerning contagion were carried out by Grubel and Fadner (1971). The phenomena that occurred in the 1990s (mentioned above) were the result of numerous studies on the presence of possible financial contagion. The first was by Calvo and Reinhart (1996) who studied the Tequila Crisis using correlation coefficients and concluded that there was financial contagion between Latin America and Asia. Baig and Goldfajn (1999) studied the subsequent crisis (Southeast Asia) confirming the presence of financial contagion between the markets of Malaysia, Indonesia, the Philippines and South Korea. Caramazza, Ricci and Salgaredo (2000) identified common factors in the presence of indirect effects of the financial channel through common creditors in the Mexican, Asian and Russian crises.

Many economists have analyzed the impacts related to the real estate crisis of 2008. It is possible to cite for example Luchtenberg and Vu (2015) who argued in their empirical work that the US market influenced all the markets of North America, Europe and Asia with the exception of the Chinese, Japanese and German.

Therefore, the study of the dynamism of the relationships between assets in times of financial instability becomes of fundamental importance. Multivariate GARCH models (or MGARCH models) are one of the most effective tools for modeling the co-movement of time series.

However, before introducing these tools, it is helpful to first understand what an ARCH and a GARCH are. These models are especially useful when the goal of the study is to analyze and predict volatility.

An ARCH (AutoRegressive Conditional Heteroskedasticity) is a model used in history analysis to describe a changing variance. It was presented for the first time in 1982 by Engle, probably the greatest scholar of methods of analysis of economic-financial historical series. As the acronym says, it is an autoregressive model (therefore depending on its past) characterized by conditional heteroscedasticity (variance not constant) (the variance depends on past information). The ARCH(p) process is defined by the following formula:

$$2.9) \quad a_t = \sqrt{\omega + \sum_{j=1}^{q} \alpha_j * a_{t-j}^2 v_t}$$

Where the empirical parameters ω and α have the following conditions $0<\omega$ and $0<\alpha$ (so that the quantities under the root are always positive) and $\alpha<1$ (stationarity condition). The variance of the

time series is therefore a linear combination of the variance of the elements preceding the series itself.

The GARCH models (Generalized AutoRegressive Conditional Heteroskedasticity Model) are nothing more than a development of the Engle model, which manages to approximate higher order ARCH models through the use of a smaller number of parameters (advantage of parsimoniousness).

The ARCH model described above is appropriate when the variance of the error in a time series follows the trend of an AR(p). However, if an ARMA(p,q) model is assumed for the variance of the error, it is possible to obtain a GARCH(p,q).

This model was developed by Bollerslev in 1986, who did his Ph. D. under Engle on the same topic.

In a GARCH(p, q) model for conditional variance, forecasts depend on a (non negatively) weighted sum of past squared residuals (from some conditional mean function model) and past forecasts. The formula is as follows:

$$2.10) \; \sigma_t = \sqrt{\omega + \sum_{i=1}^{q} \alpha_i * a_{t-i}^2 + \sum_{i=1}^{p} \beta_j * \sigma_{t-j}^2}$$

Also in this model the parameters ω , α and β are >0 (for each "i" and "j") and $\sum_{i=1}^{q} \alpha_i < 1$ and $\sum_{i=1}^{p} \beta_j < 1$ (covariance stationarity).

When one univariate time series impacts another univariate time series then it is known as Multivariate Analysis.

MGARCHs are designed to understand the relationship between volatility and co-volatility of several univariate variables.

A first model was developed by Bollerslev in 1990, who proposed a GARCH model consisting of a number n of univariate GARCH models, related to each other by a matrix ρ of constant conditional correlation. It follows that the conditional covariances are proportional to the product of the corresponding conditional standard deviations. This restriction solves one of the main problems of the multivariate GARCH literature, namely the complexity of the estimation due to the large number of parameters. This model, called a Constant Conditional Correlation (CCC GARCH Model) is defined by the following formula:

$$2.11) \; H_t = D_t * R * D_t$$

Where H_t represents the conditional variance matrix, R is a symmetric positive matrix (with $\rho_{ii}=1$,

$\forall_i$) containing the constant conditional correlations ρ_{ij}. The matrix is symmetric and contains $\frac{k*(k-1)}{2}$ elements:

$$R = \begin{vmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} & \cdots & \rho_{1n} \\ \rho_{12} & 1 & \rho_{23} & \rho_{24} & \cdots & \rho_{2n} \\ \rho_{13} & \rho_{23} & 1 & \rho_{34} & \cdots & \rho_{3n} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \rho_{n-1,n} \\ \rho_{1n} & \rho_{2n} & \rho_{3n} & \cdots & \rho_{n-1,n} & 1 \end{vmatrix}$$

D_t is the conditional standard deviation which has the form of a diagonal square matrix $\sqrt{\sigma_{11t} * \cdots * \sigma_{nnt}}$; this can be represented as:

$$D_t = \begin{vmatrix} \sqrt{\sigma_{11,t}} & 0 & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{\sigma_{22,t}} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \sqrt{\sigma_{33,t}} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \sqrt{\sigma_{44,t}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \sqrt{\sigma_{nn,t}} \end{vmatrix}$$

R is definite positive, while H_t is positive if and only if all conditional variances are positive.

The CCC GARCH, although it represents an important development of multivariate modeling, has a significant limitation: in the real world the hypothesis that conditional correlations are constant is unrealistic. Engle in 2002 proposes a generalization of the Bollerslev model making the conditional correlation matrix time dependent; this model is called Dynamic Conditional Correlation GARCH (DCC).

This model allows, through dynamic conditional variances and correlations, to take into account the dynamic effect of financial contagion. In particular, the model has been used by many economists to estimate the contagion effects of the US subprime crisis on international stock markets. For example, one can cite the contribution "Contagion Effects of the U.S. Subprime Crisis on International Stock Markets" by Inchang Haeuck, Tong Suk Kim and Francis Hwang In, who not only found financial contagion in emerging markets, more sensitive to shocks due to their underdeveloped markets and high public deficits, but even in developed markets (Haeuck, Kim, Hwang, 2010).

DCC GARCH is applied to capture the degree of volatility correlation changes or spillover between two or more variables. The aim of the model is to study the interdependence in volatility of one variable on another. Market integration is represented by a conditional correlation movement that varies over time in nature. The model is nothing but a non-linear combination of univariate GARCH

models in which the multivariate conditional variance is estimated through the univariate GARCH model of each market.

The equation through which the model is expressed is similar to that of CCC GARCH Model:

2.12) $H_t = D_t * R_t * D_t$

Where H_t represents the conditional correlation estimator, which has the form of a square diagonal matrix of conditional variances at time. D_t is the conditional standard deviation which has the same diagonal square matrix form as the one represented above.

The elements constituting the matrix D_t can also be estimated by processes which are not standard GARCH(p,q) but also by any GARCH process with normally distributed errors and satisfying the conditions of stationarity and non-negativity. Also the number of lags need not be the same for all univariate series.

R_t is the conditional correlation matrix of the standardized disturbances $\epsilon_t \sim N(0, R_t)$. The matrix is symmetric and has the following form:

$$R_t = \begin{vmatrix} 1 & \rho_{12,t} & \rho_{13,t} & \rho_{14,t} & \cdots & \rho_{1n,t} \\ \rho_{12,t} & 1 & \rho_{23,t} & \rho_{24,t} & \cdots & \rho_{2n,t} \\ \rho_{13,t} & \rho_{23,t} & 1 & \rho_{34,t} & \cdots & \rho_{3n,t} \\ \rho_{14,t} & \rho_{24,t} & \rho_{34,t} & 1 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \rho_{n-1,n,t} \\ \rho_{1n,t} & \rho_{2n,t} & \rho_{3n,t} & \cdots & \rho_{n-1,n,t} & 1 \end{vmatrix}$$

One of the advantages of DCC models is that correlation matrices are easier to handle than covariance ones.

To ensure that the above requirements are met in Engle's model, R_t is presented as:

2.13) $R_t = Q_t^{*-1} * Q_t * Q_t^{*-1}$

2.14) $Q_t = (1 - \propto - \beta) * \bar{Q} + \alpha * \epsilon_{t-1} * \epsilon_{t-1}^T + \beta * Q_{t-1}$

With $\propto$ and β scalar parameters that implies that the estimators obtained in the DCC GARCH are dynamic and time-varying; $\bar{Q}_t$ represents the unconditional variance matrix relating to the standardized errors ϵ_t, and can be estimated through the equation $\bar{Q}_t = \frac{1}{T}\sum_{t=1}^{T} \epsilon_t * \epsilon_t^T$.

So, the correlation process is driven by only two parameters $\propto$ and β. It is specified that the number of parameters does not depend on the number of model variables: $\propto$ and β control the time evolution of all the conditioned correlations regardless of the number of historical series k involved. This is indeed a great advantage as the models are relatively simple to estimate.

$\propto$ measures the short run volatility impact, which means the persistence of the standardized residuals for the previous period. β measures the lingering effect of a shock impact on the conditional correlation, which is the persistence of the conditional correlation process. The sum of these parameters must satisfy the following condition $0<\propto+\beta<1$, which indicates that the conditional correlation in the models are not constant over time.

It can be seen that the equation of Q_t is similar to that of a GARCH(1,1) process; Q_t is a positive definite matrix, while Q_t^{*-1} is a normalization matrix. The dynamic dependence of the correlations is controlled by the parameters $\propto$ and β.

The model is divided into two steps. In the first step, the series of all univariate GARCH variables is estimated; in the second phase the correlation between the variables is estimated.

The procedure is therefore as follows:

- A VAR_P model is used to estimate the conditional mean of the time series of returns (μ_t) while $\hat{a}_t = z_t - \mu_t$ are the residuals (where z_t is a multivariate time series of returns);

- GARCH univariate volatility models are applied to each component of the series;

- The standardization is carried out through $\epsilon_t = \frac{\hat{a}_{it}}{\sqrt{\sigma_{nn,t}}}$ ($\epsilon_t = \epsilon_{1t,...,}\epsilon_{kt})'$ and the DCC model is adapted to ϵ_t .

It must be specified that the conditional distribution of ϵ_t can be both that of a multivariate standard Gaussian and that of a multivariate standard t-Student with n* degrees of freedom.

It can be concluded that the main advantage of the DCC GARCH is related to the fact that this allows modeling not only the variance and covariance, but also the flexibility.

In addition to the DCC model, there are other models that analyze the volatility spillover between multiple markets or assets, such as for example the BKK GARCH proposed by Engle and Kroner in 1995. This model is a multivariate GARCH model which estimates the conditional mean function and volatility conditional function of high dimensional relationships. It is used to test the volatility spillover between the multi-market segments that is estimated by maximum likelihood.

The model is specified by the following equation:

$$v(t-1)v(t-1)'D'H(t-1)B + D'A + B')1 - t - 1)U(t(' + a'UCC = {}_t 2.15) H$$

Where H indicates the time varying variance and covariance matrix of the time series variables, the U is instead the matrix of the residuals from the mean equation, A-B-C-D are mean coefficient matrices.

The BKK and DCC GARCH models are very similar; the DCC is usually used to predict conditional correlations, while the BEKK conversely is used to predict conditional covariances. However some theoretical studies (such as the article "Do We Really Need Both BEKK and DCC? A Tale of Two Covariance Models") reported that the DCC model is empirically preferred due to the fact that the BKK suffers from so-called "curse of dimensionality" (Caporin, McAleer, 2012). This phenomenon implies that the fully parameterized conditional covariance model has the number of parameters increasing by order greater than the number of resources, with growth of order $O(k^2)$. This "curse" is actually common to both models; however, it is possible to apply the so-called "targeting" to the DCC model, through which, if certain conditions are met, it is possible to use a sample variance estimator to reduce the number of intercept parameters and, consequently, make the estimation easier also for large cross-sectional dimensions.

3. Empirical analysis

3.1. Model data and inputs

After having introduced the context and the elements covered by the study, it is possible to move on to the empirical part of the thesis, where the advantages of implementing a dynamic allocation scheme in the portfolio will be presented.

For the purposes of carrying out this verification, this study considers a specific case, namely that of a patient investor who, at the advent of the financial crisis caused by the Covid-19 pandemic, decides to enter the market, countering the opposite signals from the environment. The portfolio that the individual will decide to use is that of a hedge fund portfolio, which has historically had excellent returns in bearish periods.

The CAPM mean-variance approach will be used as a portfolio optimization method; this assumes that the covariance matrix is constant, or rather that the investor "looks forward one period". However, investment decisions can also be made by analyzing multiple future time periods: these generate dynamic optimization models.

Such models assume that the variance-covariance matrix is time-varying; it follows that the weights of the portfolio must be adjusted according to the modifications of the optimization parameters, so as to obtain a real-time update. For this purpose the DCC GARCH model will be used, which will allow rebalancing at regular intervals.

The data used in the study was taken from the Hedge Fund Research website and is monthly. The sample will be divided into two time intervals: the first is between February 2010 and February 2020, the second from the latter date until December 2022. The data referring to the first period are used to adapt the model, estimating all the parameters necessary for the formation of the portfolio which optimizes the investor's utility function. Regarding the second period, the data obtained is used to adjust and rebalance the portfolio on a monthly basis.

Before starting the application part on the model, it is advisable to understand which indexes the investor must select through a preliminary analysis.

For this purpose, an analysis was carried out among ten of the major hedge fund indexes, in order to select the best assets to include in the portfolio. The indexes to be analyzed are the following:

- HFRIEHI: this index adopts an equity hedge strategy, so it is mainly composed of HFs holding

both long and short positions in equities and equity derivatives. Investment decisions are the result of a variety of both quantitative and fundamental techniques; strategies are usually broadly diversified and can vary widely in terms of net exposure levels, leverage, holding periods, market capitalization concentrations and portfolio valuation ranges.

- HFRISEN: is a sector index with an equity hedge strategy, which however provides a primary focus in the energy and basic materials area

- HFRIHLTH: this is also a sector index with an equity hedge strategy, which has a primary focus on the Healthcare sector

- HFRITECH: is a sector index with an equity hedge strategy, which has a primary focus on the Technology sector

- HFRIEDI: this index adopts an "Event-Driven" strategy, which means that it is composed of funds that maintain positions in companies currently or potentially involved in corporate transactions of a large scale and variety, such as for example mergers, restructurings, financial difficulties, public offers of purchases, share buybacks, debt swaps, security issues or other capital structure adjustments.

- HFRIMI: this index adopts a "macro" strategy, that is one based on the movements of macroeconomic variables and the impact these have on the equity, bond, hard currency and commodity markets. As is the case with EHs, the managers of these funds use a wide variety of both quantitative and fundamental analyses; the periods of detention are both long-term and short-term.

- HFRIRVA: this index adopts a strategy based on the analysis of valuation discrepancies in the relationship between multiple securities. In particular, the manager exploits price differentials by buying and selling different financial products simultaneously. As in equity hedges and macros, the managers in these strategies use fundamental and quantitative methods to establish their strategies and identify investment opportunities.

- HFRIFOF: This index aims to achieve a broadly diversified portfolio and appropriate asset allocation with investments in a variety of fund categories (a true diversified portfolio of funds). The aim is to obtain better exposure with lower risks than investing directly in financial products. The FOF manager has discretion in choosing which strategies to invest in for the portfolio.

- HFRIEM: this index is based on a geographical strategy based on investment in emerging markets. In particular, the managers mainly invest long in corporate or sovereign debt securities

of developing countries such as Africa, Asia (excluding Japan), Latin America, the Middle East, Russia and Europe Oriental. The weightings between the various regions are carried out on the basis of the country risk and the future prospects of the various areas.

- HFRICRDT: this index is based on strategies traded mainly in the credit markets. It is an aggregation of different types of analysis methodologies, mainly relating to the Event-Driven and Relative Value HF indexes. Among these it is possible to mention credit arbitrage and fixed income convertible arbitrage strategies, distressed/Restructuring strategies, strategies based on Fixed Income assets and fixed income.

Before determining the efficient frontier it is therefore necessary to obtain some indicators through which it is possible to select the most suitable indexes for the analysis.

Table 1: Preliminary analysis of the HF indexes

	Average corr	Sharpe
HFRISEN	0.52	- 0.02
HFRIHLTH	0.48	0.31
HFRITECH	0.52	0.35
HFRIEHI	0.32	0.20
HFRIFOF	0.30	0.21
HFRIEM	0.35	0.11
HFRIEDI	0.61	0.29
HFRIFWI	0.61	0.25
HFRIMI	0.30	0.10
HFRIRVA	0.60	0.49
HFRICRDT	0.61	0.48

Source: HFR website, data from February 2010 to February 2020

Where darker colors denote more interesting elements.

The Sharpe ratio represents the measurement of the portfolio's performance from the point of view of the return on securities (net of the risk-free rate), in relation to the risk (standard deviation).

As can be seen from the table described above, the Sharpe ratio of HFRISEN is negative since it has a negative average return value (it should be noted that a risk-free rate =0 was chosen for convenience). The indexes with the highest Sharpe are HFRIRVA and HFRICRDT, with a value close to 0.5.

The average correlation was selected as the second indicator in order to be able to make selections in order to minimize the variance of the portfolio: a lower minimum average correlation is synonymous

with better portfolio diversification. From this second factor it is possible to select the indexes HFRIEHI, HFRIFOF and HFRIMI which have an average value of 0.3.

3.2. Data analysis and diagnostic checks

It is therefore possible to move on to the application part of the thesis, in which the data described in the previous paragraph will be used in order to analyze the benefit of a dynamic portfolio rebalancing. It should be noted that the calculations carried out from here on were carried out using the R Studio software and are reported in the Appendix 1 section.

After calculating the returns of the indexes, it is possible to proceed with the analysis of the characteristics of the data for the series of returns. In this regard, the variance-covariance matrix and the correlation matrix were calculated.

Table 2: Variance-Covariance matrix of HF portfolio

	HFRIEHI	HFRIFOF	HFRIMI	HFRIRVA	HFRICRDT
HFRIEHI	0.4102	0.2068	0.0080	0.0248	0.0395
HFRIFOF	0.2068	0.1237	- 0.0066	0.0095	0.0183
HFRIMI	0.0080	- 0.0066	0.1552	0.0366	0.0357
HFRIRVA	0.0248	0.0095	0.0366	0.0742	0.0750
HFRICRDT	0.0395	0.0183	0.0357	0.0750	0.0811

Source: HFR website, data from February 2010 to February 2020

Table 3: Correlation matrix of HF portfolio

	HFRIEHI	HFRIFOF	HFRIMI	HFRIRVA	HFRICRDT
HFRIEHI	1.0000	0.9180	0.0315	0.1421	0.2166
HFRIFOF	0.9180	1.0000	- 0.0474	0.0991	0.1831
HFRIMI	0.0315	- 0.0474	1.0000	0.3409	0.3184
HFRIRVA	0.1421	0.0991	0.3409	1.0000	0.9672
HFRICRDT	0.2166	0.1831	0.3184	0.9672	1.0000

Source: HFR website, data from February 2010 to February 2020

From the first table it is not possible to comment much on the portfolio as finding two indexes with a high or low covariance is not a useful parameter if used alone. It can only be noted that the HFRIEHI has a much higher variance than that of the other indexes, a figure that is even more relevant taking into account that previously it was seen that it was one of the indexes that had the lowest average correlation.

More interesting is certainly the correlation matrix, in which it is possible to see first of all that almost all the values are positive (the only negative value is that between the Fund of Funds and the macro HFI, which is negligible anyway). The most correlated indexes are HF equity hedge and HF FOF which have in common the fact that they invest in the same equity class (entirely in the case of the first, predominantly in the case of the second) and that they are both funds in which the managers mainly make use of fundamental analysis , selecting the best investment opportunities with respect to market prices. Two other particularly correlated indexes are HFRIRVA and the HFRICRDT: the positive and high correlation between these two indexes was quite obvious as the HFRICRDT is largely based on sub-strategies of the Relative Value index (such as fixed income - Asset Backed Index, Fixed Income-Convertible Arbitrage Index, Fixed Income-Corporate Index, Multy-Strategy Index).

If it is excluded the possibility of having negative weights (so without short selling), proceeding with the construction of the efficient frontier it is possible to obtain:

Figure 10: Efficient frontier and equally weighted portfolio

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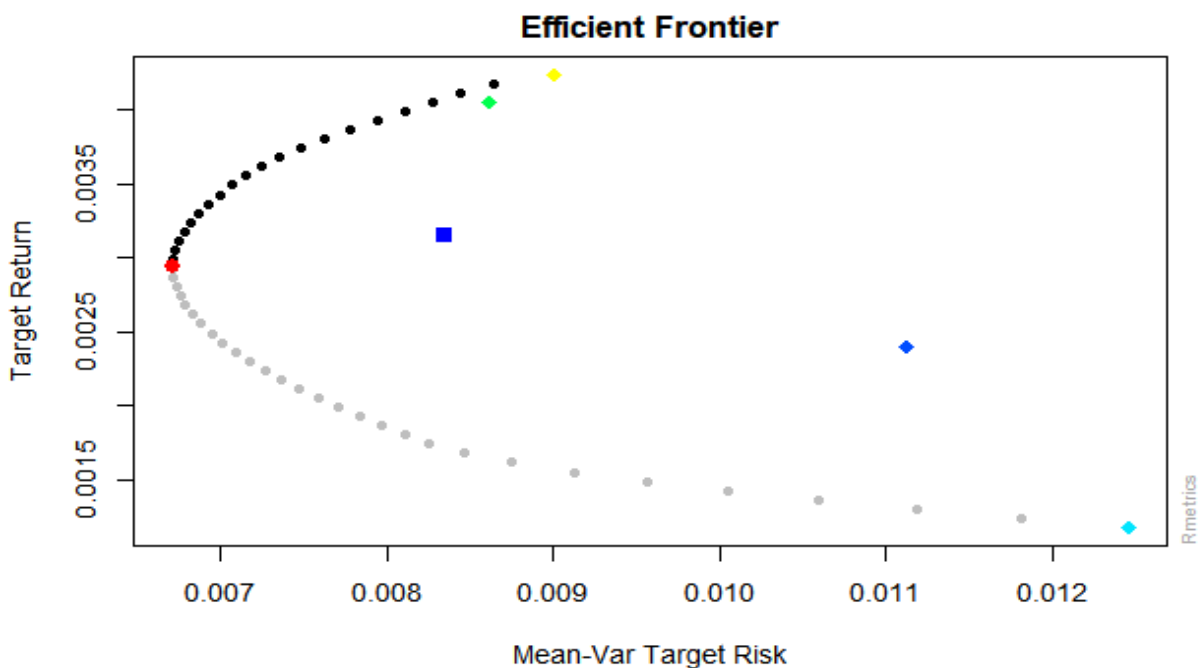
Where the red point represents the global minimum variance portfolio, the colored rhombuses represent the positioning of each asset according to their Risk/Return and the blue square represents the equal weights portfolio. As was imaginable to think, the efficient frontier is better than single assets and the equally weighted portfolio because it is the output of an optimization process. The yellow diamond representing the HFRICRDT index is approaching the efficient frontier, which will most likely constitute a very high percentage of the weight in the portfolio due to a high level of risk.

Moving on to the analysis of the weights, it is possible to obtain that in the minimum variance Portfolio (identified in the point [0.0066; 0.0014] these are:

Figure 11: Analysis of the minimum variance portfolio
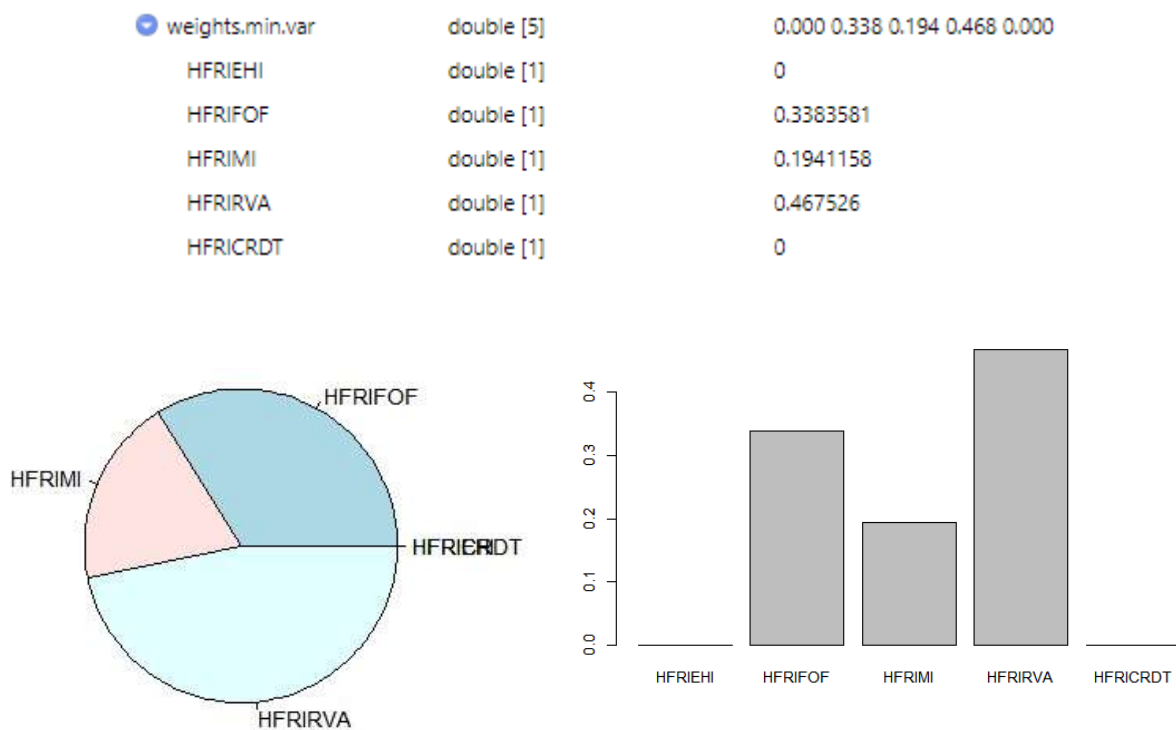
From the analysis carried out previously, it was foreseeable that the minimum variance portfolio had the HFRIMI and HFRIRVA indexes which had a low average correlation. Less obvious was the presence of the HF Relative Value index, which, despite having a high average correlation (especially with the HFRICRDT index), has the lowest standard deviation (as can be seen from the variance-covariance matrix).

It is also interesting to analyze the following graph, from which it is possible to observe the trend of the weights as the standard deviation increases (it should be noted that the x axis represents the points of the efficient frontier following zero, or rather following the Minimum Variance Portfolio).

Figure 12: Weights variation (without short selling)
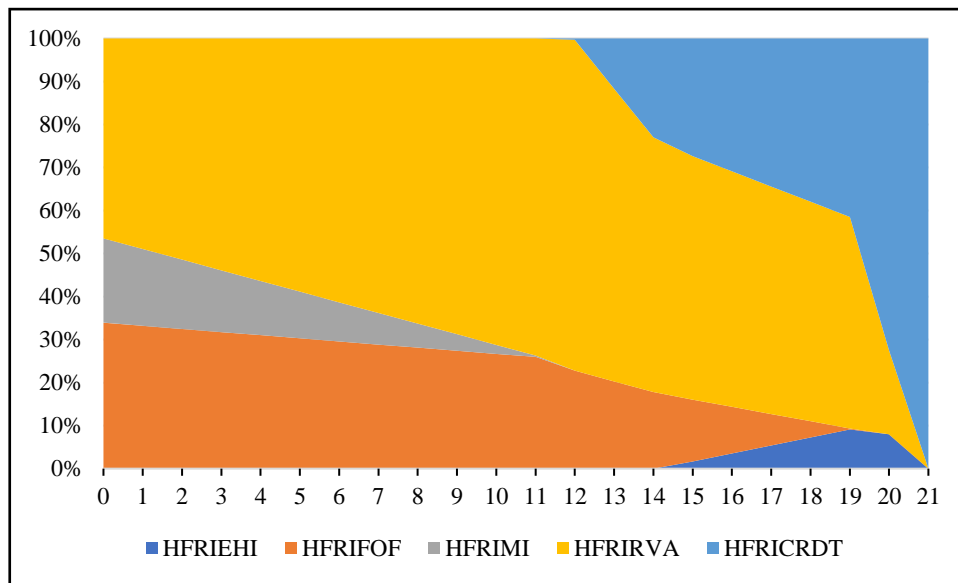
As can be seen, as the risk increases, the three indexes that make up the minimum variance portfolio decrease more and more. From point 15 onwards, the two main indexes that make up the portfolio are those that had the highest Sharpe Ratio; the HFRIEHI timidly enters point 15 and then disappears in point 21, reaching a maximum value of 9.11% in point 19. From point 21 onwards, the portfolio consists almost exclusively of the HFRICRDT asset (which agrees with the comment of the efficient frontier).

If the positivity constrain of the weights is removed (so if short selling is admitted), the following efficient frontier is obtained:

Figure 13: Comparison between the efficient frontier with and without short selling
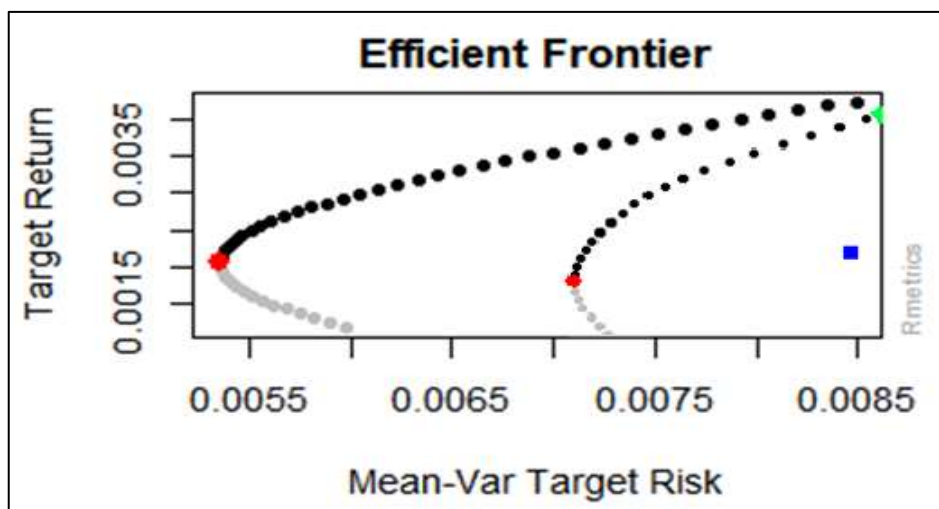
The new efficient frontier, presented in the graph with the larger dots, is higher than the one calculated

with the short selling constraint; this was to be expected as short selling allows for greater portfolio differentiation. The global minimum portfolio in this case is identified in the point [0.0053; 0.0016], thus presenting a significant improvement in the risk-return ratio. This is found more to the left as it is characterized by a lower correlation between assets (and therefore to maximize the effects of diversification).

As in the previous case, it is interesting to analyze the changes in the weights as one moves away from the zero point (that is the minimum variance portfolio).

Figure 14: Weights variation (with short selling)
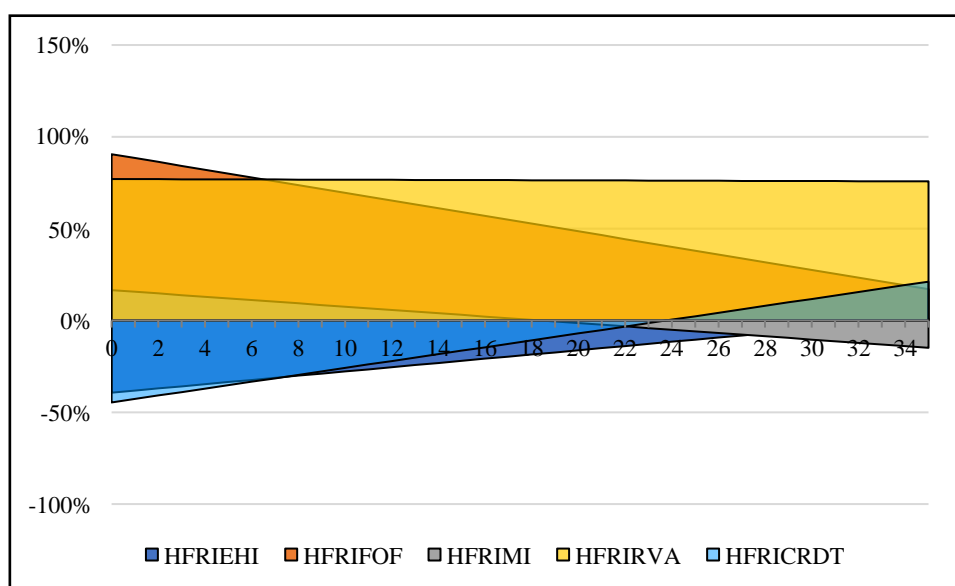
As can be seen from the graph, the HFRIEHI and HFRICRDT indexes (blue and light blue area) have negative weights for many of the points following the global minimum portfolio of the efficient frontier. The HF relative value index is even more important in the portfolio, consistently covering between 76% and 78%. Basically, the main change in the composition of the portfolio that occurs as risk increases is the progressive replacement of the HF Funds of Funds with the two indexes that initially show negative weights (HFRIEHI and HFRICRDT).

It is also possible to visualize the graph of the efficient frontier through a Monte Carlo simulation, which allows to calculate a series of possible realizations of the HF portfolio through a probabilistic model that assigns the value of the weights randomly.

Figure 15: Efficient frontier with a Monte Carlo simulation
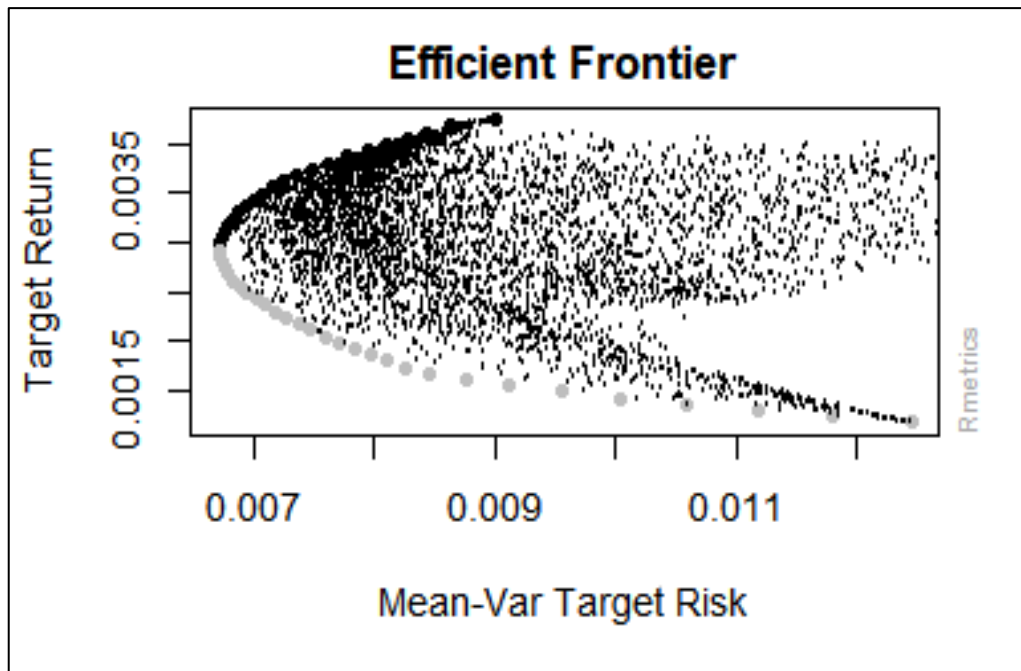
As it is possible to see there are several thousand portfolios within the efficient frontier, which, being located under the curve, are considered sub-optimal (therefore it will always be better to choose the realizations that coincide with the frontier, as they represent the highest return for each level of risk).

It is possible to proceed with the analysis of some descriptive statistics relating to the series of returns.

The average Skewness and Kurtosis are as follows:

Table 4: Skewness and Kurtosis analysis

	HFRIEHI	HFRIFOF	HFRIMI	HFRIRVA	HFRICRDT
Skewness	-0.5699009	-0.6787536	0.09064997	-0.6511869	-0.6083743
Kurtosis	0.8548134	0.4795859	-0.02802581	0.356992	0.452555

As it is possible to see all the series present a non-normality, with a generally negative skewness (with the exception of the HFRIMI which is very close to zero) and a kurtosis always higher than zero (note that the R Studio function used subtracts 3 for obtain the Kurtosis coefficient). Also in this case the only exception is found HFRIMI which value is even negative and is very close to zero of the normal.

It is possible to confirm these results also by analyzing the results of the Jarque-Bera test, through which it is possible to verify the hypothesis of normality:

Table 5: Jarque-Bera test

Jarque Bera	HFRIEHI	HFRIFOF	HFRIMI	HFRIRVA	HFRICRDT
X-squared	10.9760	10.9830	0.1702	9.6278	8.9653
p-value	0.0041	0.0041	0.9184	0.0081	0.0113

As is possible, with the exception of HFRIMI, all indexes tests have the result of rejecting the null hypothesis that the data are normally distributed. In the HFRIMI, on the other hand, having a test statistic of 0.1702 and a p-value of 0.9184, it is the case of "do not reject" the null hypothesis (therefore the series appears to be normally distributed, confirming what had been anticipated in the previous analysis).

It is possible to move on to the study of the stationarity of the return series to verify that they can be used directly for the analysis without requiring further transformations. First of all let's analyze the graphs of the return series.

Figure 16: Return series analysis
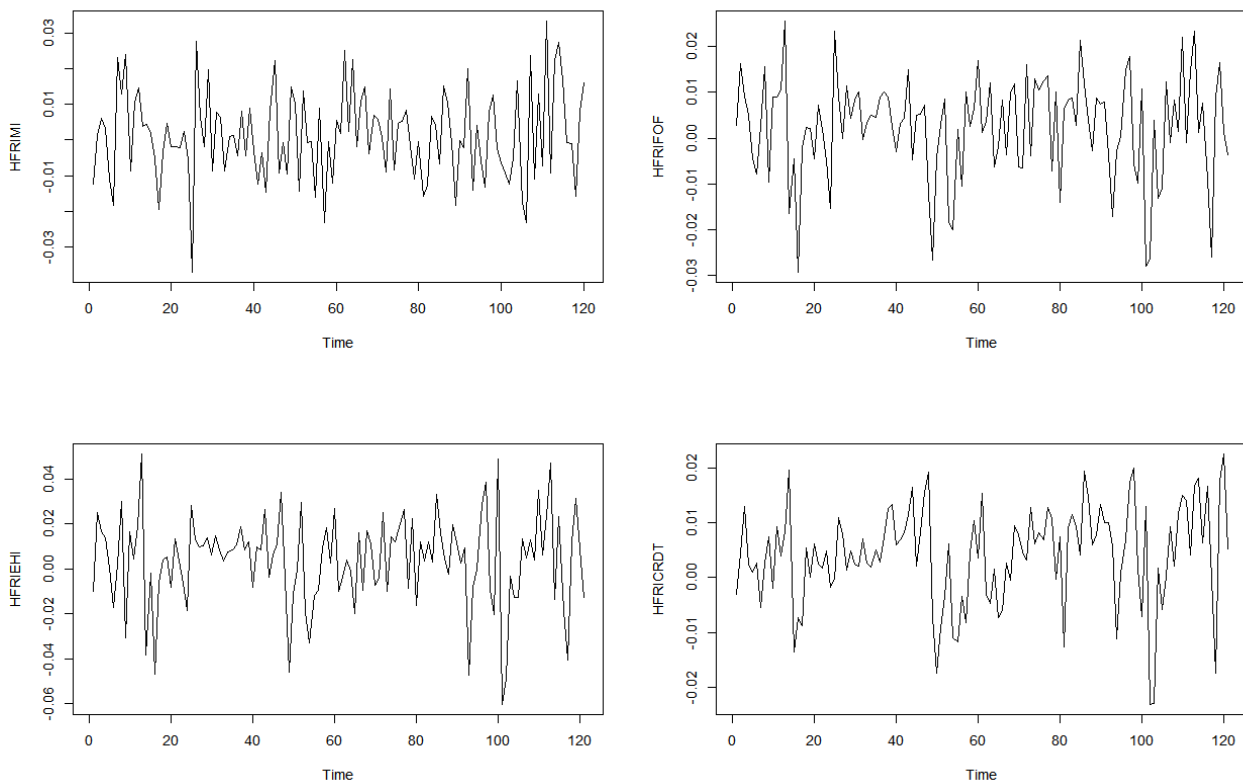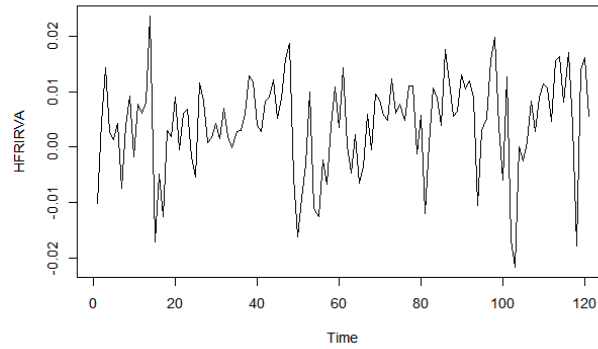
From the graphs it is possible to notice that in none of the 5 return series there is neither a constant (the average of the fluctuations is always around zero) nor a time trend (as there is no positive or negative trend in the series but the occur around a constant that can be approximated to zero).

To this end, some unit root tests are performed, including the Augmented Dickey-Fuller (ADF), the Philipps-Perron (PP) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS).

Table 6: Unit root tests analysis

	HFRIEHI	HFRIFOF	HFRIMI	HFRIRVA	HFRICRDT
ADF stat	- 5.0077	- 4.6518	- 4.6616	- 4.6754	- 4.2396
p-value	0.01	0.01	0.01	0.01	0.01
PP stat	- 114.1600	- 103.4700	- 137.7900	- 89.0660	- 84.0920
p-value	0.01	0.01	0.01	0.01	0.01
KPSS stat	0.035118	0.043189	0.06579	0.1852	0.16429
p-value	0.01	0.01	0.01	0.01	0.01

Assuming a significance level of 5%, the ADF Stat is equal to -1.94 (since it was possible to see from the previous analysis that this does not have a time trend or a constant. Net of this it is possible to conclude that one must reject the null hypothesis, as all test statistics are more extreme than the critical value and the p-values are lower than the significance level (5%). This implies that the series do not have a unit root (which means that the data is stationary).

The table also includes the Phillips-Perron and the Kwiatkowski-Phillips-Schmidt-Shin which confirm the result previously obtained with the ADF test. The fact that all three of these tests had the same result is not obvious: indeed it can happen that for some tests the data show non-stationarity and in another the same series turns out to be stationary. One of the reasons of these different results is for example the presence of structural breaks, which can alter the functioning of the ADF test.

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If, on the other hand, the test revealed that the series was not stationary, this would have required the use of a transformation that made the data become stationary, by taking the log returns and first difference.

Another preliminary test that is useful to analyze is the Ljung-Box test, which allows to check whether there is autocorrelation in a time series. In particular, in this test the null hypothesis predicts that the series is distributed in an uncorrelated way, while the alternative hypothesis predicts on the contrary that these show a serial correlation.

Table 7: Ljung–Box test

Ljung-Box test	HFRIEHI	HFRIFOF	HFRIMI	HFRIRVA	HFRICRDT
X-squared (lag=20)	14.282	16.629	16.17	24.621	35.18
p-value (lag=20)	0.8159	0.6769	0.706	0.2163	0.01917

The test was performed with lag=20; as it is possible to see from the analysis of the X-Squared and of the p-values it is possible to notice that for all the indexes with the exception of the HFRICRDT it is not possible (fail) to reject the null hypothesis so the series are iid (independent and identical distributed).

This result is quite anomalous since the object of the analysis is that of financial products, but plausible since the data are monthly and therefore there could be even considerable variations. So the series, with the exception of the Credit Index, do not have a serial correlation.

It is also possible to analyze the presence of volatility clusters by re-testing with squared returns:

Table 8: Ljung–Box test (r^2)

Box-Ljung test	HFRIEHI^2	HFRIFOF^2	HFRIMI^2	HFRIRVA^2	HFRICRDT^2
X-squared (lag=20)	21.561	17.932	15.512	26.271	47.281
p-value (lag=20)	0.3648	0.5919	0.7464	0.157	0.0005362

Also in this case the results are compromised by the monthly data; the test highlights the absence of the presence of autoregressive conditional heteroskedasticity.

Further investigation of the autocorrelations in returns may be needed therefore the ACF and PACF are analyzed.

To this end, an analysis was performed to find the autocorrelation (p) and the moving average (q)

lags using the auto.arima function. The procedure is as follow: the first step is determining the optimal lags, so they are used to compute adjusted returns and residual time series from the average model. The residual time series are calculated as the difference between returns and fitted returns. So residuals in that time series model are what is left after fitting the model. They are very helpful in checking whether the model has adequately captured the information in the data or not.

It is possible to move on to the "diagnostic check", analyzing the graphs relating to the autocorrelogram (ACF) and partial-autocorrelogram (PACF) to view the analysis of the autocorrelation of the residuals from the optimal ARMA(p,q) model.

Figure 17: Analysis of ACF and PACF
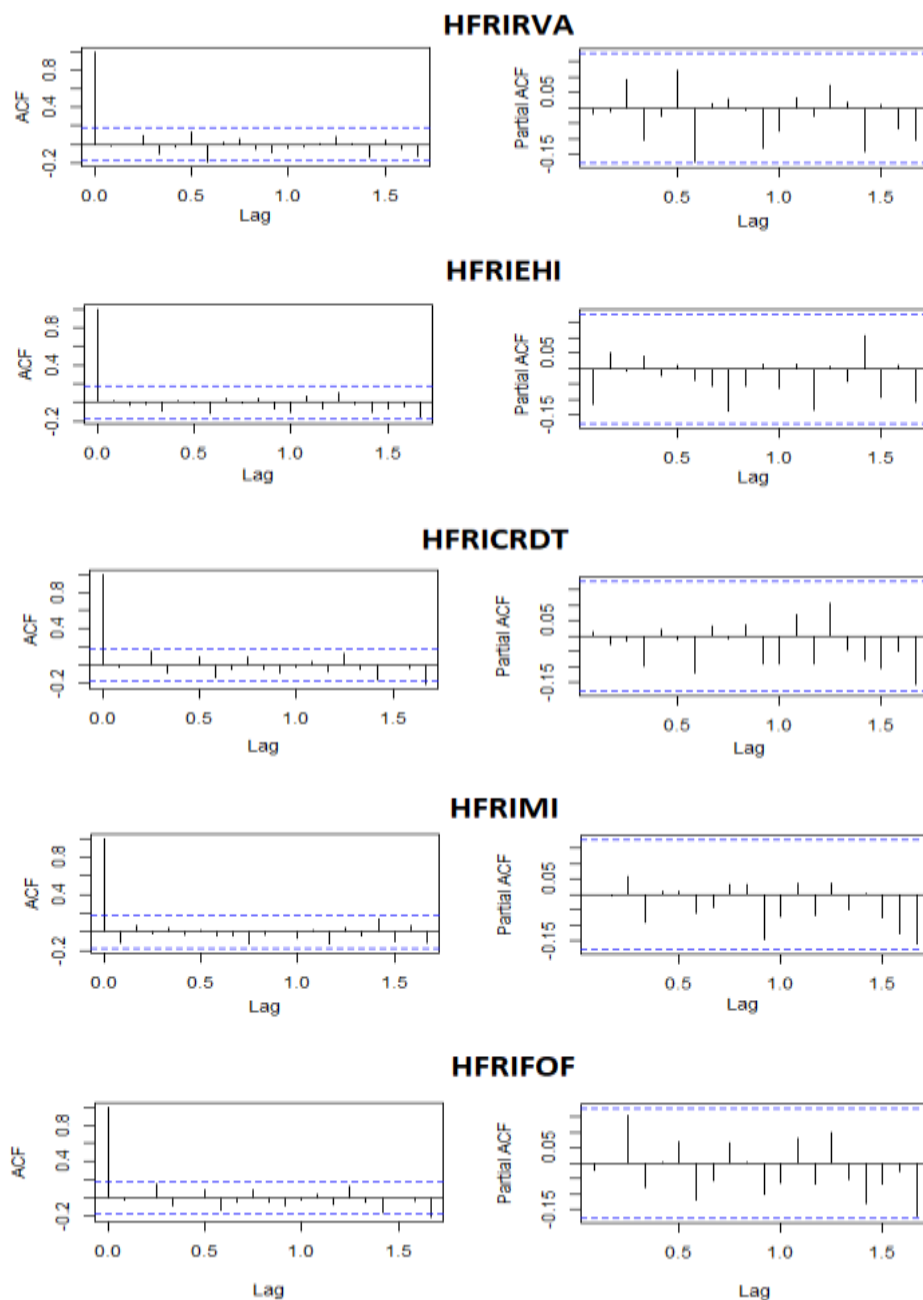
None of the time series of the residuals appears to present autocorrelation as there is no peak outside the statistically insignificant area; therefore the residuals are independent.

A second check is to verify that the residuals are normally distributed and if they have constant mean at zero.

Figure 18: Analysis of the residuals
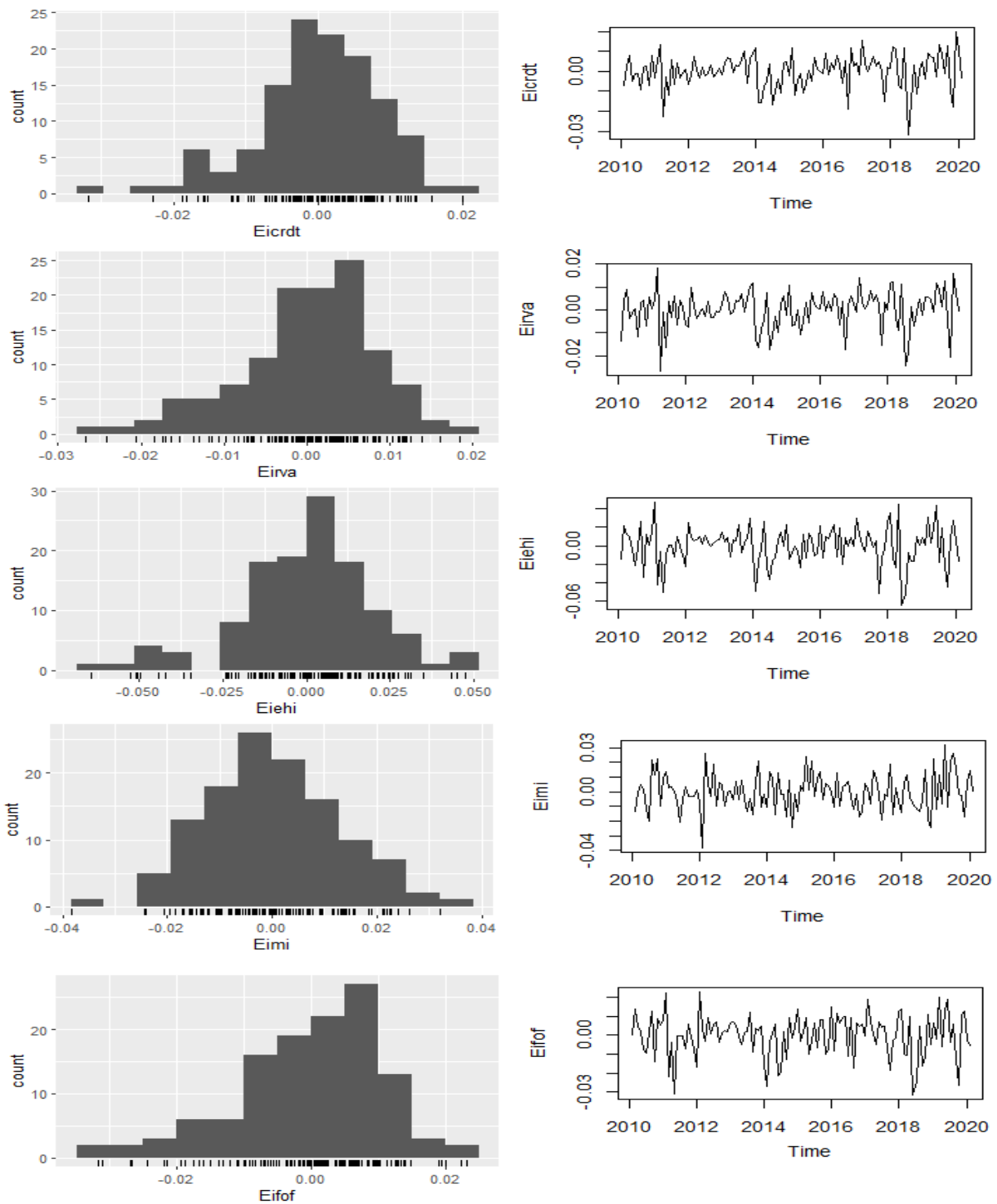
In the graphs on the left it is possible to see that the residuals are in all cases bell shapes and many of which show a fairly normal distribution. Some of these graphs like HFRIRVA and HFRIFOF look slightly left skewed, however this could be due to the fact that there are only 121 observations in the analysis. From the graphs on the right, however, it is possible to see that the residuals show a constant mean at time zero.

Therefore, from the various tests it is possible to conclude that the return series are ready for the use of the DCC GARCH model.

3.3. Application of the DCC GARCH

Once the time series has been analyzed, it is therefore possible to move on to the introduction of the dynamic component by analyzing the interdependence between the volatility of one variable and another. For this purpose a DCC GARCH is performed, a model that allows modeling directly the variance and the covariance but also its flexibility, using only two parameters (alpha and beta).

The first step is to study the spillover effect between the five indexes. To this end the volatility of the times series is modeled using a GARCH(1,1) model with a multivariate normal distribution, while the mean equation has been fitted with an ARMA(0,0).

The parameters that will then be used to estimate the variance-covariance matrix and the time-varying correlation processes are then calculated.

The estimated values for the DCC are shown below.

Figure 19: Output of the DCC GARCH model

```
Distribution         : mvnorm                                      t value Pr(>|t|)
Model                : DCC(1,1)         [HFRIEHI].mu     4.7169e+00 0.000002
No. Parameters       : 32               [HFRIEHI].omega  1.1403e+00 0.254168
[VAR GARCH DCC UncQ] : [0+20+2+10]      [HFRIEHI].alpha1 1.3679e+00 0.171332
No. Series           : 5               [HFRIEHI].beta1  7.1576e-01 0.474143
No. Obs.             : 123              [HFRIFOF].mu     1.7054e+00 0.088126
Log-Likelihood       : 2065.073         [HFRIFOF].omega  8.2128e-01 0.411487
Av.Log-Likelihood    : 16.79            [HFRIFOF].alpha1 1.0000e-06 0.999999
                                        [HFRIFOF].beta1  1.7578e+02 0.000000
Optimal Parameters                      [HFRIMI].mu      1.0133e+00 0.310930
-----------------------------------     [HFRIMI].omega   2.5079e-01 0.801978
               Estimate  Std. Error     [HFRIMI].alpha1  3.3125e-02 0.973575
[HFRIEHI].mu    0.006845   0.001451      [HFRIMI].beta1   8.6654e+02 0.000000
[HFRIEHI].omega 0.000151   0.000133      [HFRIRVA].mu     5.4774e+00 0.000000
[HFRIEHI].alpha1 0.827404  0.604855      [HFRIRVA].omega  8.4715e-01 0.396910
[HFRIEHI].beta1 0.171596   0.239741      [HFRIRVA].alpha1 0.0000e+00 1.000000
[HFRIFOF].mu    0.001663   0.000975      [HFRIRVA].beta1  1.4207e+03 0.000000
[HFRIFOF].omega 0.000001   0.000001      [HFRICRDT].mu    5.2434e+00 0.000000
[HFRIFOF].alpha1 0.000000  0.002695      [HFRICRDT].omega 1.0853e+00 0.277801
[HFRIFOF].beta1 0.999000   0.005683      [HFRICRDT].alpha1 0.0000e+00 1.000000
[HFRIMI].mu     0.001186   0.001170      [HFRICRDT].beta1 9.4260e+02 0.000000
[HFRIMI].omega  0.000000   0.000001      [Joint]dcca1     1.1000e-05 0.999991
[HFRIMI].alpha1 0.000041   0.001240      [Joint]dccb1     1.1745e+01 0.000000
[HFRIMI].beta1  0.998054   0.001152
[HFRIRVA].mu    0.003542   0.000647      Information Criteria
[HFRIRVA].omega 0.000001   0.000001      --------------------
[HFRIRVA].alpha1 0.000000  0.012767
[HFRIRVA].beta1 0.999000   0.000703      Akaike        -33.058
[HFRICRDT].mu   0.003673   0.000701      Bayes         -32.326
[HFRICRDT].omega 0.000001  0.000001      Shibata       -33.159
[HFRICRDT].alpha1 0.000000 0.013983      Hannan-Quinn  -32.761
[HFRICRDT].beta1 0.999000  0.001060
[Joint]dcca1    0.000000   0.000002      Elapsed time : 11.10317
[Joint]dccb1    0.897983   0.076457
```

In the left column it is possible to see for each index the mu (overall mean), omega (constant), alpha (which represents the ARCH model), beta (which represents the GARCH model), dcca1 (that is the short run volatility impact) and dccb1 (that is the long run volatility impact). From the column on the right it is possible to see through the p-values that some of the parameters are not significant.

What is important to verify is that the beta is significant: this is the case in all indexes except the HFRIEHI. Another thing that is important to check is the significance of dcca1 and dccb1: dcca1 is not significant and this means that there is no short run spillover effect of volatility. On the other hand, analyzing the dccb1 it will be found that this is instead significant and therefore there is a long run spillover of volatility. The estimates of these coefficients are positive and if dcca1 and dccb1 are added, the value is less than 1, therefore the estimates are also below the threshold limit value. This is proof that there is a dynamic conditional correlation between the five indexes.

It is possible to proceed with the estimation of the correlation and variance-covariance matrix which will be different in the different time periods which will be useful for calculating the dynamic efficient frontier. The optimization of the portfolio that is the subject of this study is based on the division of

the time span into two sub-periods: the first that has been analyzed up to now goes from February 2010 to February 2020 and the second from February 2020 to December 2022. And it is thanks to the data of this second period of time that all the parameters necessary for the formation of the optimal dynamic portfolio are estimated.

Therefore, it is possible to proceed with the estimation of the correlation and variance-covariance matrix which will be different in the different time periods which will be useful for calculating the dynamic frontier. The estimation of these matrices will be carried out through the use of a rolling window, that is an interval of fixed size made up of 121 data (10 years) which however will move at each rebalancing of a month. It follows, for example, that for the purposes of estimating the matrix of differences1 using the DCC GARCH, the input data from March 2010 to March 2020 will be used, for the second from April 2020 to April 2022 and so on. Since between 2010 and 2018 the data did not show significant changes that could influence the window, it has been chosen to use the 10-year interval.

The first portfolio was then constructed using historical data up to February 2020; following this interval, a monthly rebalancing process is programmed in which the weights of the portfolio are updated and a new efficient frontier is consequently generated (34 total rebalances). What is obtained after all the related portfolio' updates is as follows.

Figure 20: Changes of the efficient frontier over time – 2020
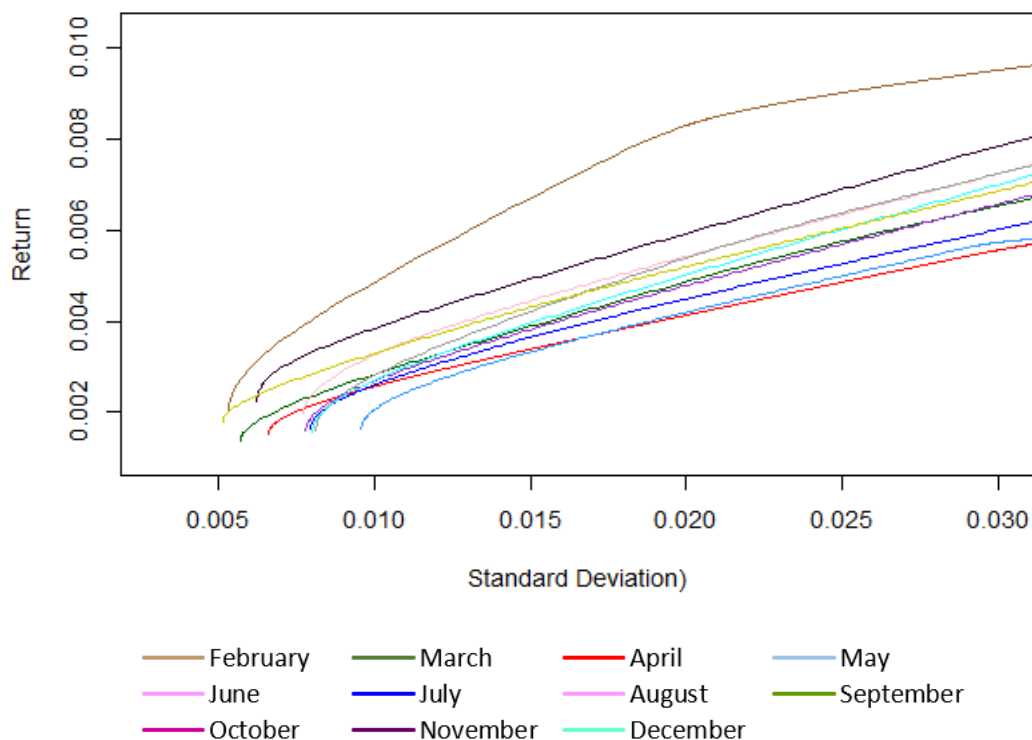
As can be seen there is a rather high degree of instability in efficient frontiers. The initial frontier of the investor (that is the one built on the day of the advent of the pandemic) stands above the others. This was to be expected as in March 2020 the worsening of the pandemic led to the fall of the markets. As a consequence, the new efficient frontiers will be located lower down and will be characterized by a lower return for the same level of standard deviation.

After April 2020 there is an increase of the efficient frontier, as a consequence of the end of the bear market due to the Covid-19 pandemic (which ended on April 7th). The frontier progressively only to then have a new slight relapse following the second wave of viruses. It is also interesting to analyze the shift of the global minimum portfolio, which initially falls and moves further to the right. This is not new to us, as it confirms what was said in the article "Flexible Dynamic Conditional Correlation multivariate GARCH models for asset allocation" (Billio, Caporin e Gobbo, 2006): correlations increase positively in periods of high volatility and that both the magnitude and persistence of the correlation are affected by risk.

Figure 21: Changes of the efficient frontier over time - 2021
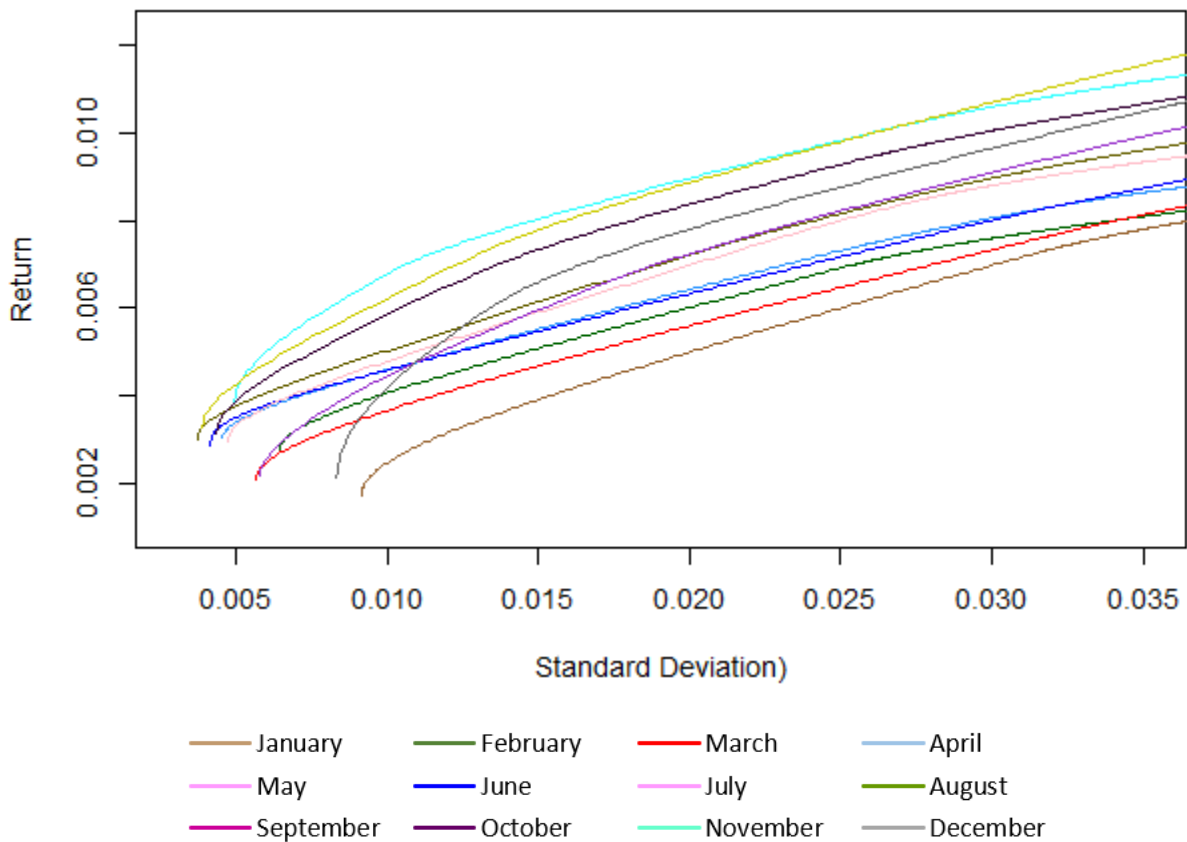
It is therefore possible to move on to the analysis of the year 2021. After the first quarter of the year, which remains in line with the latest updates of 2020, there is a significant rise in the frontier mainly linked to a considerable increase in HFRIEHI and HFRIFOF. The two indexes in question, as it has been possible to observe from the analyses carried out previously on historical data, are characterized by a high correlation; their increase probably derives from the introduction of the vaccine and from the fiscal stimulus measures which helped to offset the concerns related to the previous increase in inflation. The situation remains quite stable for the rest of the year in terms of border positioning, except for some shifts in the GMV portfolio (especially in December 2021).

Figure 22: Changes of the efficient frontier over time – 2022
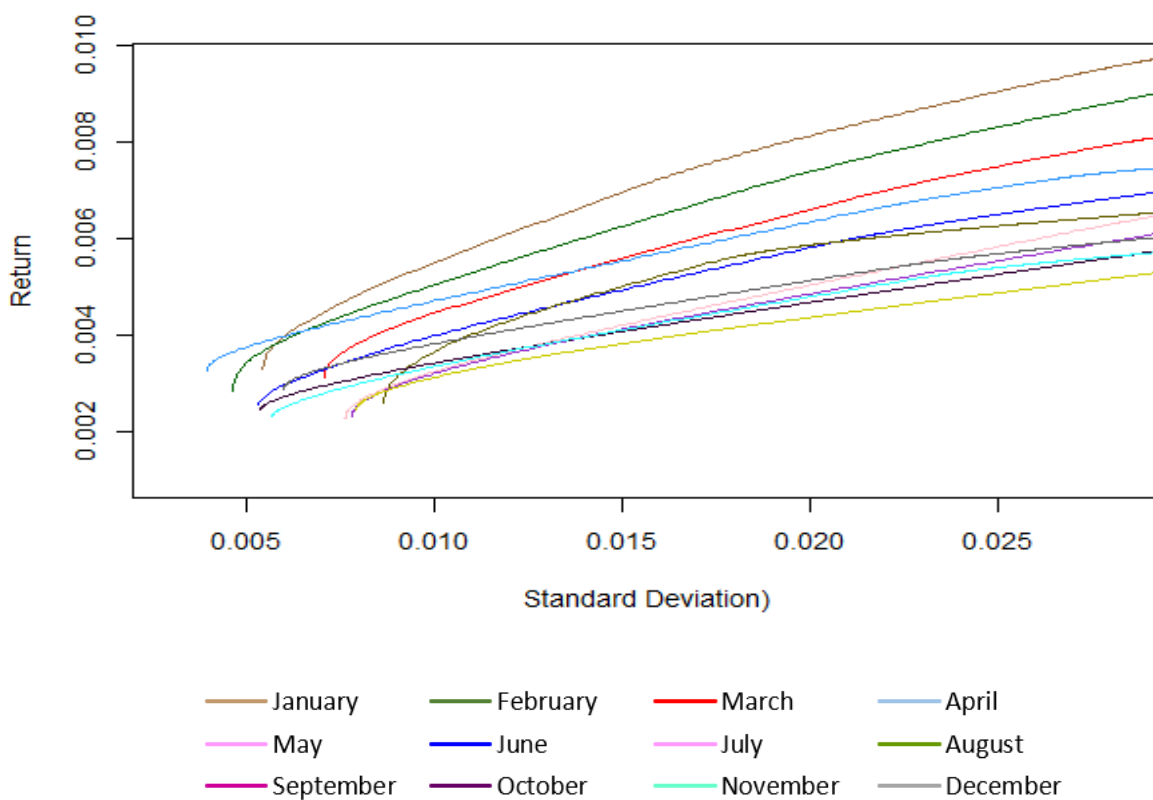
Finally, the efficient frontiers relating to the year 2022 are reported. The year in question was characterized by some macroeconomic events that shook the financial markets, first and foremost the war in Russia, which consolidated and amplified some of the critical issues that had already appeared in the second half of 2021, including the rise in the prices of energy and food raw materials, the appearance of new obstacles to the functioning of the value chains and the supply of some raw materials, as well as the increase in volatility on the financial markets. As a consequence, the efficient frontier has progressively declined. In particular there was a sharp drop in August 2022, mainly caused by the drops linked to an increase in inflation (these are months in which the Euribor rate

increases considerably, returning positive after seven years). These are months characterized by "wild" and disorienting swings in stocks, bonds, commodities and currencies (note that the last three curves have shifted more to the right than the previous ones).

It is therefore possible to move on to the heart of the study where the advantages of periodic portfolio rebalancing will be analyzed. Using the indexes analyzed until now, it is possible to proceed with the creation and optimization of six portfolios (without any long constrain), in order to compare their performances:

1. Equally weighted portfolio, that is that portfolio whose components will all have the same percentage weight (in the case of the study since there are 5 assets, each constitute 20% of the capital);

2. Efficient portfolio (or benchmark portfolio), that is a portfolio that follows the performance of a representative benchmark. In this study, the Hedge Fund World Index will be used (HFRIWI);

3. A static portfolio built with the weights corresponding to the global minimum variance (GMV) at time zero;

4. A static portfolio built with the weights corresponding to the maximum Sharpe at time zero (CAPM asset allocation method);

5. A dynamic portfolio built using the GMV of each efficient frontier calculated using the DCC GARCH;

6. A dynamic portfolio built using the maximum Sharpe of each efficient frontier calculated using the DCC GARCH.

The weights of the dynamic portfolios have had the range [-0.5; +1.0] as their position, so it is not possible to go short by more than -50% and long by more than 100%. The trend of the weights relating to portfolio 6 is as follows:

Figure 23: Change over time of the weights related to the DCC GARCH Portfolio (using CAPM allocation method)
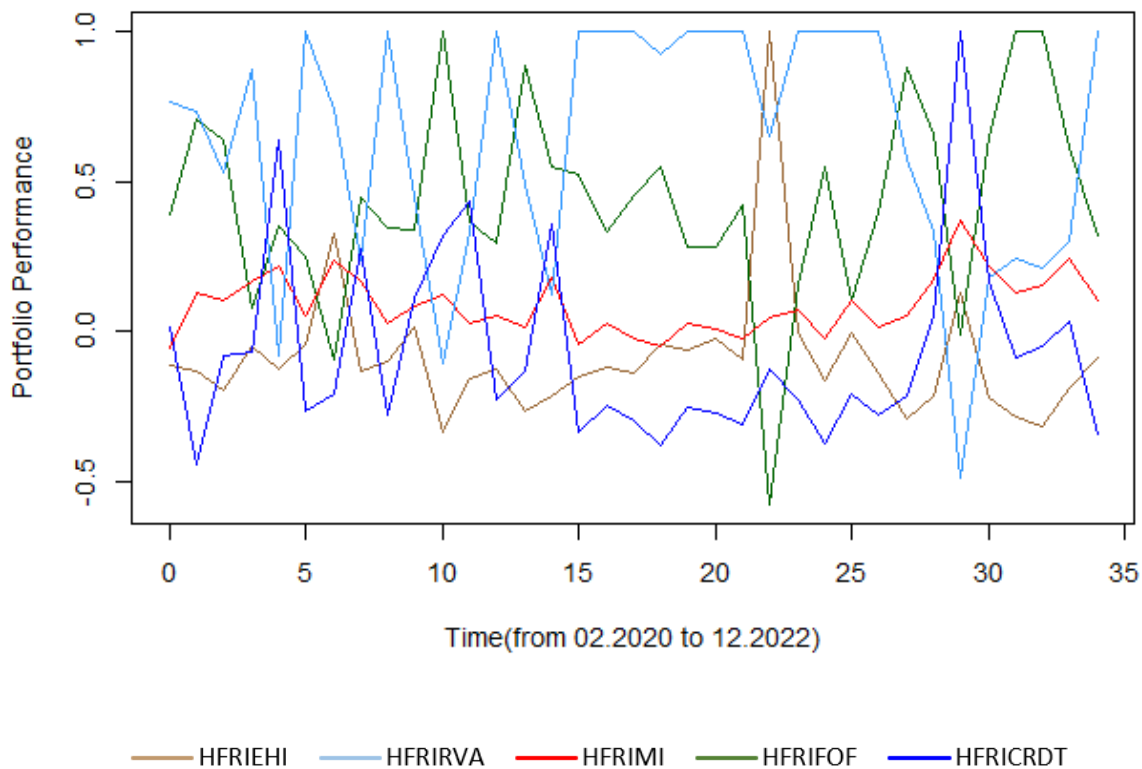
The initial weights are those that coincide with the static portfolio (number 4); the portfolio is mainly composed of the HF Relative Value and Fund of Funds indexes which together cover 114.96% of the portfolio.

Speaking instead of the subsequent weights output of the dynamic rebalancing, in light blue it is possible to see the HFRIRVA which starts from a value of 76.11% and remains quite high except for some ups and downs during the first year due to the effects of market volatility caused by the pandemic. Another significant fall occurred in August 2022, due to the increase in inflation and the Euribor mentioned in the previous analysis. Overall, the index covered 62.89% of the portfolio, almost 20% more than the average weight of the second highest index HFRIFOF which covers an average of 43.18%. Also this index (in the graph it is the one in dark green) shows a good speed in changing the weight in the portfolio; the greatest ones are found in balancing number 23 (which corresponds to January 2022, the month in which the index was characterized by a sharp decline in the stock market (of which the index is largely composed). This decrease is due to the ever greater increase of consumer price inflation, which during that month reaches annualized highs in four decades, and by the decisions made by the Federal Reserve to combat it. Furthermore, as early as this month there was war between Russia and Ukraine, with the consequent threat from of possible

sanctions against the Russian state in case of invasion. This has further contributed to the increase in the instability of the financial markets. In red it is possible to see the HFRIMI which remains stable throughout the time frame, without having particular increases or decreases (covers on average 8.94% of the portfolio). The last two indexes HFRICRDT and HFRIEHI are almost always in short and indeed cover on average resp actually -6.44% and -8.59% of the portfolio. The trend of the HFRIEHI is very unusual as, despite having a high correlation with the Fund of Funds, this very often turns out to have an opposite trend in almost the entire interval under analysis. The HFRICRDT, on the other hand, historically has a high positive correlation with the Relative Value index; this analysis also does not find much correspondence in the dynamic analysis of portfolio weights.

The trend of the weights relating to portfolio 5 (DCC global minimum variance portfolio) is as follows.

Figure 24: Change over time of the weights related to the DCC Global Minimum Variance portfolio
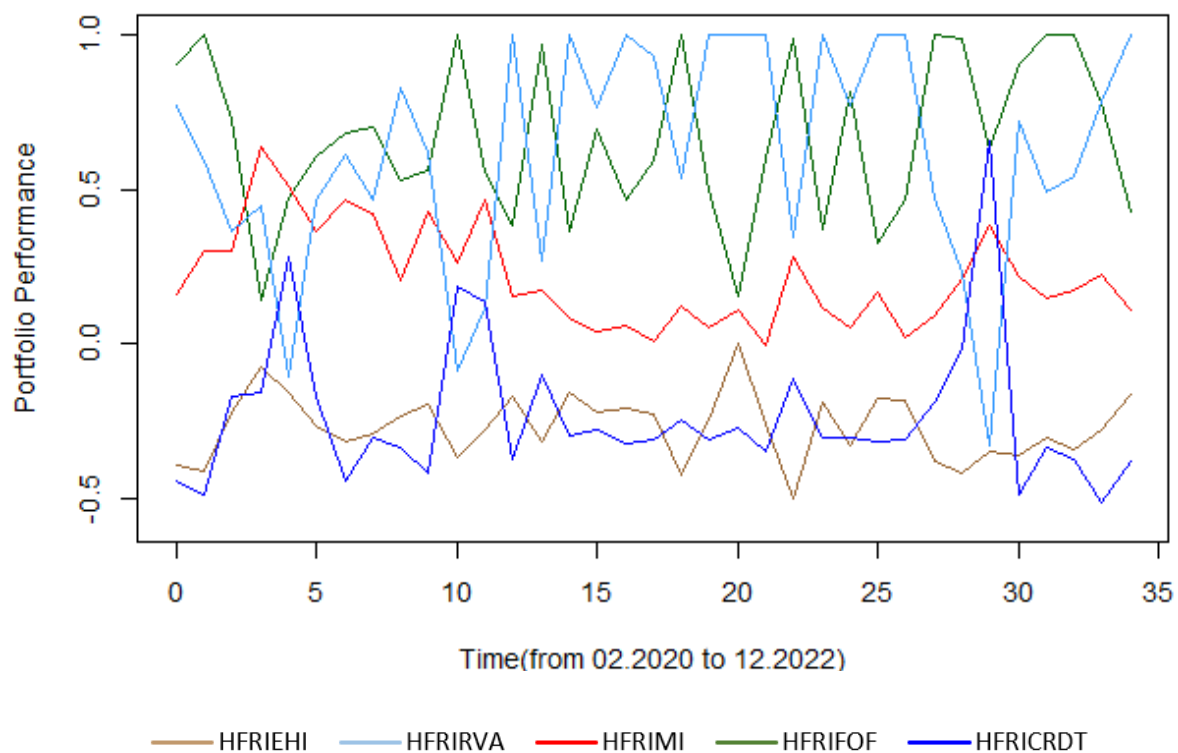
The weights appear to have more "extreme" average values than the correspondents of portfolios number 4 and 6, so both in the static portfolio and in the dynamic one. In particular, in the static portfolio, the Relative Value Index and the Fund of Funds which covered 114.96% now moved to

167.24%; this is balanced by the equity and credit index (-83.61% overall).

Even in the dynamic portfolio these values are on average more extreme. The analysis of the portfolio is almost identical to that carried out previously in portfolio 6. The weights of the indexes show, as normally expected, less variability over time. Also in this case there is a big change near August 2022 and the reasons are the same as mentioned in the previous analysis.

For an investment of 100k euros in each of the six portfolios at zero time, these will perform as in the following graph between February 2020 and December 2022 (one time unit corresponds to one month):

Figure 25: Comparison between the performance of the various portfolios identified
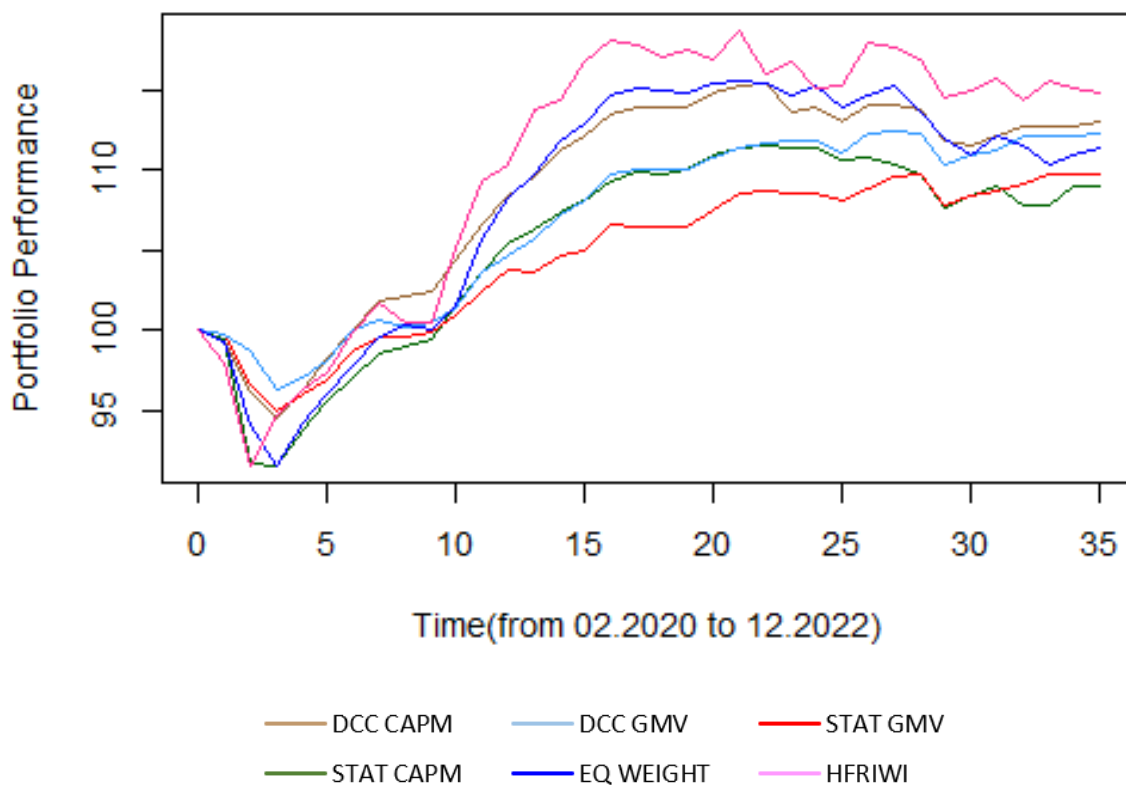
As can be seen from the graph, the portfolios that seem to suffer the least impact from the Covid-19 pandemic are the two DCC portfolios (in particular, the one calculated with the minimum global variance) and the static GMV portfolio. After the end of this first bear market, the two portfolios that seem to present the best performances are the benchmark portfolio and the DCC maximum Sharpe portfolio, which confirm their good stability also in period 8 which coincides with the start of the second wave of the pandemic. Subsequently, the equally weighted portfolio will also perform better

than the Dynamic Sharpe Portfolio; the two DCC portfolios become competitive again since period 25 following the increase in financial market volatility and instability caused by the Russia-Ukraine conflict. This demonstrates how effective dynamic rebalancing is during periods of high volatility.

Beyond the general comparison, what is interesting to note for the purposes of this study is that the two DCC portfolios have always beaten the corresponding static portfolios in each time interval. The light blue line (dynamic GMV) has always outperformed the red line (static GMV) and this gap has remained fairly constant over time. The divergence between dynamic and static is even more visible in the Maximum Sharpe portfolio where in some periods it is close to 8 thousand euros.

Another comparison concerns the average volatility between static and dynamic portfolios.

Table 9: Comparison of volatility in portfolios

MONTHS	DCC CAPM	STATIC CAPM	DCC GMV	STATIC GMV	EQUALLY	HFRIWI
STD DEV PORT	0.011789	0.0167102	0.0091566	0.0095427	0.0165671	0.0202971

As can be seen from the table, the advantage of dynamic rebalancing is considerable both in the CAPM portfolio and in the GMV.

In the analysis carried out in Figure 25 it was possible to see that the benchmark portfolio HFRIWI outperformed both DCC portfolios in terms of performance 34 months after the initial investment. However, the table also shows that the World Index has 221.6% more standard deviation than the dcc global minimum variance portfolio and 172.2% more than the DCC CAPM.

It is therefore possible to conclude that the results of the empirical analysis are positive, as the purpose of the study was to be able to capture the changes in the riskiness of the indexes in a context of financial market instability, improving the asset allocation of the portfolio. The aim was achieved as both through the quantitative analysis and through the graphic analysis it was possible to ascertain the positive effects that the dynamic rebalancing brought about in terms of investor protection towards downwards.

However, it is necessary to mention an aspect that has not been considered in the analysis until now: transaction costs. The empirical analysis showed that DCC portfolios actually lead to a more competitive risk/reward ratio; these better performances may disappear as the various fees are presented. It therefore emerges that for an analysis more likely to the real context, there must be a compromise between the benefits and costs of portfolio rebalancing.

Conclusions

This study aimed to improve the optimization of a portfolio in a context of financial market instability, characterized by numerous events that generate an alternation of bullish and bearish periods.

This improvement, therefore, had the aim of dealing with the high instability of the markets and the limits of traditional asset allocation models (such as the CAPM which was the subject of the empirical part) linked in particular not to taking into account any variances and correlations time-varying returns (thus assuming a constant/static structure). This limit is particularly relevant in these contexts of high instability as it does not allow to provide an accurate measure of the variance and, consequently, it does not allow to measure the risk accurately.

For this purpose, a portfolio of Hedge Fund Indexes was used as a reference investment. This choice was not taken by chance: these, as it is possible to see in the analyzes carried out in chapter 1, have had a good return during past bear markets, especially in relation to their mutual fund counterparts. This is due to the fact that they are changeable investments, which use the short sell during down markets in order to protect themselves against downturns and to obtain positive returns.

Despite the fact that Hedge Funds are investments themselves, the dynamics are the variance and variability of the financial time series over time, with impacts in terms of portfolio construction and risk measurement.

Therefore, a DCC GARCH was introduced which made it possible to study the spillover effect of volatility that varies over time as well as, through a monthly rolling window, to create dynamic portfolios.

The results of the empirical application show that both dynamic investments (GMV and CAPM) show better performances after 34 months than their static portfolios. However, what is even more important to note is the lower volatility of dynamic portfolios during the most "stressed" and volatile periods.

It is therefore possible to conclude that the empirical analysis shows that a dynamic optimization of the portfolio through periodic rebalancing guarantees to capture the changes in the riskiness of the indexes and therefore to improve the asset allocation during periods of market instability.

Furthermore, carrying out the empirical part, some points for improvement were found which would allow both to make the evaluation carried out more reliable and to deepen the theme of dynamic models.

An improvement of this study concerns the use of daily data and not monthly data. However this was not possible as the HFRI site does not provide data with a frequency other than the monthly one. This would certainly lead to a better ARCH effect and a better evaluation of the advantage of the dynamic correlation; the absence of high frequency data makes it more difficult to draw reliable conclusions from the results of the empirical study.

A second point for improvement concerns the fact that in this thesis only the context of financial markets characterized by high volatility has been examined (since this was the context present in the premise of the thesis). However, it would be interesting to compare the performance of the DCC model during more "calm" periods, characterized by a long-lived bull market.

The third and final point of improvement concerns the fact that the study used only the DCC GARCH as a dynamic correlation model. It might be interesting to examine the results of multiple models and their performance, such as the VEC and BEKK models mentioned in chapter 2 that directly model the conditional covariance matrix, or study a possible extension of Engle's model, such as the Flexible Dynamic Conditional Correlation GARCH (FDCC) presented in the article "Flexible Dynamic Conditional Correlation multivariate GARCH models for asset allocation" (Billio, Caporin e Gobbo, 2006).

Appendix (R Code)

```r
#Load libraries

library(quadprog)

library(ggpubr)

library(openxlsx)

library(fPortfolio)

library(timeSeries)

library(quantmod)

library(dplyr)

library(PortfolioAnalytics)

library(PerformanceAnalytics)

library(ggplot2)

library(DAAG)

library(tseries)

library(moments)

library(FinTS)

library(zoo)

library(rugarch)

library(e1071)

library(rmgarch)

library(moments)

library(forecast)

library(DEoptim)

library(ROI)


#Figures in chapter1

plot(Indexes_confronto$Data,
Indexes_confronto$IXIC,col='white',xlab="Time(from
02.1992 to 11.2022)",ylab="Portfolio PerformanceS")

lines(Indexes_confronto$Data,
Indexes_confronto$GSPC, col='#996633', lwd=1)

lines(Indexes_confronto$Data,
Indexes_confronto$IXIC, col='darkgreen', lwd=1)

lines(Indexes_confronto$Data, Indexes_confronto$DJI,
col='red', lwd=1)

lines(Indexes_confronto$Data,
Indexes_confronto$HFRIEHI, col='#3399FF', lwd=1)

lines(Indexes_confronto$Data,
Indexes_confronto$HFRIFOF, col='blue', lty=1)

lines(Indexes_confronto$Data,
Indexes_confronto$HFRIEM, col='black', lty=1)

plot(figure$Months, figure$`Average Cumulative Loss
over        1        year`,        type="l",
col='blue',xlab="Time",ylab="Performance%")


#Figures in chapter2

plot(Gold_Etf$Date,    Gold_Etf$compl,    type="l",
col='white',xlab="Time",ylab="Performance%")

lines(Gold_Etf$Date,  Gold_Etf$`iShares  Gold  etf`,
col='blue', lwd=1)

lines(Gold_Etf$Date,  Gold_Etf$`^GSPC`,  col='red',
lwd=1)


#Empirical Analysis

#0 Organize the data

HFRIEHI=select(returns,HFRIEHI)

HFRIFOF=select(returns,HFRIFOF)

HFRIMI=select(returns,HFRIMI)

HFRIRVA=select(returns,HFRIRVA)

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HFRIMI1=select(returns1,HFRIMI)

HFRIRVA1=select(returns1,HFRIRVA)

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HFRIFOF2=select(returns2,HFRIFOF)

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HFRICRDT2=select(returns2,HFRICRDT)
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HFRIRVA32=select(returns32,HFRIRVA)
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HFRIRVA33=select(returns33,HFRIRVA)
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return.matrix22=as.timeSeries(returns22)
return.matrix23=as.timeSeries(returns23)
return.matrix24=as.timeSeries(returns24)
return.matrix25=as.timeSeries(returns25)
return.matrix26=as.timeSeries(returns26)
return.matrix27=as.timeSeries(returns27)
return.matrix28=as.timeSeries(returns28)
return.matrix29=as.timeSeries(returns29)
return.matrix30=as.timeSeries(returns30)
return.matrix31=as.timeSeries(returns31)
return.matrix32=as.timeSeries(returns32)
return.matrix33=as.timeSeries(returns33)
return.matrix34=as.timeSeries(returns34)
x.Return=ts(returns,start=c(2010, 2), end=c(2020, 2),
frequency=12)
return.HFRIEHI=as.timeSeries(HFRIEHI)
return.HFRIFOF=as.timeSeries(HFRIFOF)
return.HFRIMI=as.timeSeries(HFRIMI)
return.HFRIRVA=as.timeSeries(HFRIRVA)
return.HFRICRDT=as.timeSeries(HFRICRDT)


#1 Calculating the efficient frontier
#1.1 Long only constraint
efficient.frontier1=portfolioFrontier(return.matrix,
constraints = "LongOnly")
plot(efficient.frontier1, c(1,2))
portfolio.min.var=minvariancePortfolio(return.matrix,
constraints ="LongOnly")
weights.min.var1=getWeights(portfolio.min.var)
weights.min.var1
barplot(weights.min.var1)
pie(weights.min.var1)
frontierWeights=getWeights(efficient.frontier1)
frontierWeights
risk_return<-frontierPoints(efficient.frontier1)
```

```
cor_matrix<-cor(return.matrix)

cor_matrix

cov_matrix<-cov(return.matrix)

cov_matrix
```

#1.2 Introduce Short Selling

```
Spec=portfolioSpec()

setTargetRisk(Spec)=.12

setSolver(Spec) = "solveRshortExact"

constraints1<-c("minW[1:length(tickers)]=-
1","maxW[1:length(tickers)]=.60","Short")

efficient.frontier2=portfolioFrontier(return.matrix,
Spec, constraints = constraints1)

plot(efficient.frontier2, c(1,2,3,4,5))

weightsShort=getWeights(efficient.frontier2)

weightsShort

portfolio.min.var2=minvariancePortfolio(return.matrix,
Spec, constraints = constraints1)

weights.min.var2=getWeights(portfolio.min.var2)

plot(efficient.frontier1, c(1,7))
```

#2 Descriptive statistics

#2.1 Normality tests

```
skewness(return.HFRIRVA)

skewness(return.HFRIMI)

skewness(return.HFRIEHI)

skewness(return.HFRIFOF)

skewness(return.HFRICRDT)

skewness(return.matrix)

kurtosis(return.HFRIRVA)

kurtosis(return.HFRIMI)

kurtosis(return.HFRIEHI)

kurtosis(return.HFRIFOF)

kurtosis(return.HFRICRDT)

kurtosis(return.matrix)

jarque.bera.test(return.HFRIRVA)

jarque.bera.test(return.HFRIMI)
```

```
jarque.bera.test(return.HFRIEHI)

jarque.bera.test(return.HFRIFOF)

jarque.bera.test(return.HFRICRDT)
```

#2.2 Unit root tests

```
plot(return.HFRIRVA)

plot(return.HFRIMI)

plot(return.HFRIEHI)

plot(return.HFRIFOF)

plot(return.HFRICRDT)

adf.test(return.HFRIRVA)

adf.test(return.HFRIMI)

adf.test(return.HFRIEHI)

adf.test(return.HFRIFOF)

adf.test(return.HFRICRDT)

pp.test(return.HFRIRVA)

pp.test(return.HFRIMI)

pp.test(return.HFRIEHI)

pp.test(return.HFRIFOF)

pp.test(return.HFRICRDT)

kpss.test(return.HFRIRVA)

kpss.test(return.HFRIMI)

kpss.test(return.HFRIEHI)

kpss.test(return.HFRIFOF)

kpss.test(return.HFRICRDT)
```

#2.3 Ljung-Box tests

```
x.HFRIRVA=ts(HFRIRVA,start=c(2010,          2),
end=c(2020, 2), frequency=12)

Box.test(HFRIRVA,lag=20,fitdf=0,    type   =   "Ljung-
Box")

x.HFRIMI=ts(HFRIMI,start=c(2010, 2), end=c(2020,
2), frequency=12)

Box.test(x.HFRIMI,lag=20,fitdf=0,    type   =   "Ljung-
Box")

x.HFRIEHI=ts(HFRIEHI,start=c(2010, 2), end=c(2020,
2), frequency=12)
```

```
Box.test(x.HFRIEHI,lag=20,fitdf=0, type = "Ljung-Box")

x.HFRIFOF=ts(HFRIFOF,start=c(2010,           2),
end=c(2020, 2), frequency=12)

Box.test(x.HFRIFOF,lag=20,fitdf=0, type = "Ljung-Box")

x.HFRICRDT=ts(HFRICRDT,start=c(2010,          2),
end=c(2020, 2), frequency=12)

Box.test(x.HFRICRDT,lag=20,fitdf=0, type = "Ljung-Box")

x.HFRIRVA2=x.HFRIRVA^2

x.HFRIEHI2=x.HFRIEHI^2

x.HFRIFOF2=x.HFRIFOF^2

x.HFRICRDT2=x.HFRICRDT^2

x.HFRIMI2=x.HFRIMI^2

Box.test(x.HFRIRVA2,lag=20,fitdf=0, type = "Ljung-Box")

Box.test(x.HFRIMI2,lag=20,fitdf=0, type = "Ljung-Box")

Box.test(x.HFRIEHI2,lag=20,fitdf=0, type = "Ljung-Box")

Box.test(x.HFRIFOF2,lag=20,fitdf=0, type = "Ljung-Box")

Box.test(x.HFRICRDT2,lag=20,fitdf=0, type = "Ljung-Box")


#2.4 Diagnostic

auto.arima(x.HFRIRVA)

auto.arima(x.HFRIMI)

auto.arima(x.HFRIEHI)

auto.arima(x.HFRIFOF)

auto.arima(x.HFRICRDT)

modelHFRIRVA=arima(x.HFRIRVA,order = c(0,0,1))

modelHFRIMI=arima(x.HFRIMI,order = c(0,0,0))

modelHFRIEHI=arima(x.HFRIEHI,order = c(0,0,0))

modelHFRIFOF=arima(x.HFRIFOF,order = c(0,0,1))

modelHFRICRDT=arima(x.HFRICRDT,order        =
c(0,0,1))

Eirva=residuals(modelHFRIRVA)

Eimi=residuals(modelHFRIMI)

Eiehi=residuals(modelHFRIEHI)

Eifof=residuals(modelHFRIFOF)

Eicrdt=residuals(modelHFRICRDT)

acf(Eirva)

acf(Eimi)

acf(Eiehi)

acf(Eifof)

acf(Eicrdt)

pacf(Eirva)

pacf(Eimi)

pacf(Eiehi)

pacf(Eifof)

pacf(Eicrdt)

plot(Eirva)

plot(Eimi)

plot(Eiehi)

plot(Eifof)

plot(Eicrdt)

Box.test(Eirva,lag=20,fitdf=1, type = "Ljung-Box")

Box.test(Eimi,lag=20,fitdf=0, type = "Ljung-Box")

Box.test(Eiehi,lag=20,fitdf=0, type = "Ljung-Box")

Box.test(Eifof,lag=20,fitdf=1, type = "Ljung-Box")

Box.test(Eicrdt,lag=20,fitdf=1, type = "Ljung-Box")

Box.test(Eirva^2,lag=20,fitdf=1, type = "Ljung-Box")

Box.test(Eimi^2,lag=20,fitdf=0, type = "Ljung-Box")

Box.test(Eiehi^2,lag=20,fitdf=0, type = "Ljung-Box")

Box.test(Eifof^2,lag=20,fitdf=1, type = "Ljung-Box")

Box.test(Eicrdt^2,lag=20,fitdf=1, type = "Ljung-Box")

gghistogram(Eirva)

gghistogram(Eimi)

gghistogram(Eiehi)

gghistogram(Eifof)

gghistogram(Eicrdt)


#3 DCC Garch

#3.1 Study the spillover effect
```

```r
model1=ugarchspec(mean.model=list(armaOrder=c(0,
0)),variance.model=list(garchOrder=c(1,1),model="sG
ARCH"),distribution.model="norm")

modelspec=dccspec(uspec=multispec(replicate(5,mode
l1)),dccOrder=c(1,1),distribution="mvnorm")

modelfit1=dccfit(modelspec,data=data.frame(HFRIEH
I1,HFRIFOF1,HFRIMI1,HFRIRVA1,HFRICRDT1))

modelfit1

correlation1=rcor(modelfit1)

dim(correlation1)

corr.1=correlation1[,,dim(correlation1)[3]]

corr.1

coEHIFOF=correlation1[2,1,]

coEHIIMI=correlation1[3,1,]

coEHIIRVA=correlation1[4,1,]

coEHICRDT=correlation1[5,1,]

coFOFIMI=correlation1[3,2,]

coFOFIRVA=correlation1[4,2,]

coFOFCRDT=correlation1[5,2,]

coIMIIRVA=correlation1[4,3,]

coIMICRDT=correlation1[5,3,]

coIRVACRDT=correlation1[5,4,]

par(mfrow=c(4,1))

plot.ts(coEHIFOF)

plot.ts(coEHIIMI)

plot.ts(coEHIIRVA)

plot.ts(coEHICRDT)

par(mfrow=c(4,1))

plot.ts(coFOFIMI)

plot.ts(coFOFIRVA)

par(mfrow=c(4,1))

plot.ts(coFOFCRDT)

plot.ts(coIMIIRVA)

plot.ts(coIMICRDT)

plot.ts(coIRVACRDT)

covariance1=rcov(modelfit1)

dim(covariance1)

cov.1=covariance1[,,dim(covariance1)[3]]

cov.1

covEHIFOF=covariance1[2,1,]

covEHIIMI=covariance1[3,1,]

covEHIIRVA=covariance1[4,1,]

covEHICRDT=covariance1[5,1,]

covFOFIMI=covariance1[3,2,]

covFOFIRVA=covariance1[4,2,]

covFOFCRDT=covariance1[5,2,]

covIMIIRVA=covariance1[4,3,]

covIMICRDT=covariance1[5,3,]

covIRVACRDT=covariance1[5,4,]

plot.ts(covEHIFOF)

plot.ts(covEHIIMI)

plot.ts(covEHIIRVA)

plot.ts(covEHICRDT)

plot.ts(covFOFIMI)

plot.ts(covFOFIRVA)

plot.ts(covFOFCRDT)

plot.ts(covIMIIRVA)

plot.ts(covIMICRDT)

plot.ts(covIRVACRDT)

modelfit2=dccfit(modelspec,data=data.frame(HFRIEH
I2,HFRIFOF2,HFRIMI2,HFRIRVA2,HFRICRDT2))

correlation2=rcor(modelfit2)

dim(correlation2)

correlation2[,,dim(correlation2)[3]]

covariance2=rcov(modelfit2)

dim(covariance2)

cov.2=covariance2[,,dim(covariance2)[3]]

modelfit3=dccfit(modelspec,data=data.frame(HFRIEH
I3,HFRIFOF3,HFRIMI3,HFRIRVA3,HFRICRDT3))

correlation3=rcor(modelfit3)

dim(correlation3)

correlation3[,,dim(correlation3)[3]]

covariance3=rcov(modelfit3)

dim(covariance3)
```

cov.3=covariance3[,,dim(covariance3)[3]]

modelfit4=dccfit(modelspec,data=data.frame(HFRIEHI4,HFRIFOF4,HFRIMI4,HFRIRVA4,HFRICRDT4))

correlation4=rcor(modelfit4)

dim(correlation4)

correlation4[,,dim(correlation4)[3]]

covariance4=rcov(modelfit4)

dim(covariance4)

cov.4=covariance4[,,dim(covariance4)[3]]

modelfit5=dccfit(modelspec,data=data.frame(HFRIEHI5,HFRIFOF5,HFRIMI5,HFRIRVA5,HFRICRDT5))

correlation5=rcor(modelfit5)

dim(correlation5)

correlation5[,,dim(correlation5)[3]]

covariance5=rcov(modelfit5)

dim(covariance5)

cov.5=covariance5[,,dim(covariance5)[3]]

modelfit6=dccfit(modelspec,data=data.frame(HFRIEHI6,HFRIFOF6,HFRIMI6,HFRIRVA6,HFRICRDT6))

correlation6=rcor(modelfit6)

dim(correlation6)

correlation6[,,dim(correlation6)[3]]

covariance6=rcov(modelfit6)

dim(covariance6)

cov.6=covariance6[,,dim(covariance6)[3]]

modelfit7=dccfit(modelspec,data=data.frame(HFRIEHI7,HFRIFOF7,HFRIMI7,HFRIRVA7,HFRICRDT7))

correlation7=rcor(modelfit7)

dim(correlation7)

correlation7[,,dim(correlation7)[3]]

covariance7=rcov(modelfit7)

dim(covariance7)

cov.7=covariance7[,,dim(covariance7)[3]]

modelfit8=dccfit(modelspec,data=data.frame(HFRIEHI8,HFRIFOF8,HFRIMI8,HFRIRVA8,HFRICRDT8))

correlation8=rcor(modelfit8)

dim(correlation8)

correlation8[,,dim(correlation8)[3]]

covariance8=rcov(modelfit8)

dim(covariance8)

cov.8=covariance8[,,dim(covariance8)[3]]

modelfit9=dccfit(modelspec,data=data.frame(HFRIEHI9,HFRIFOF9,HFRIMI9,HFRIRVA9,HFRICRDT9))

correlation9=rcor(modelfit9)

dim(correlation9)

correlation9[,,dim(correlation9)[3]]

covariance9=rcov(modelfit9)

dim(covariance9)

cov.9=covariance9[,,dim(covariance9)[3]]

modelfit10=dccfit(modelspec,data=data.frame(HFRIEHI10,HFRIFOF10,HFRIMI10,HFRIRVA10,HFRICRDT10))

correlation10=rcor(modelfit10)

dim(correlation10)

correlation10[,,dim(correlation10)[3]]

covariance10=rcov(modelfit10)

dim(covariance10)

cov.10=covariance10[,,dim(covariance10)[3]]

modelfit11=dccfit(modelspec,data=data.frame(HFRIEHI11,HFRIFOF11,HFRIMI11,HFRIRVA11,HFRICRDT11))

correlation11=rcor(modelfit11)

dim(correlation11)

correlation11[,,dim(correlation11)[3]]

covariance11=rcov(modelfit11)

dim(covariance11)

cov.11=covariance11[,,dim(covariance11)[3]]

modelfit12=dccfit(modelspec,data=data.frame(HFRIEHI12,HFRIFOF12,HFRIMI12,HFRIRVA12,HFRICRDT12))

correlation12=rcor(modelfit12)

dim(correlation12)

correlation12[,,dim(correlation12)[3]]

covariance12=rcov(modelfit12)

dim(covariance12)

cov.12=covariance12[,,dim(covariance12)[3]]

```
modelfit13=dccfit(modelspec,data=data.frame(HFRIE
HI13,HFRIFOF13,HFRIMI13,HFRIRVA13,HFRICR
DT13))

correlation13=rcor(modelfit13)

dim(correlation13)

correlation13[,,dim(correlation13)[3]]

covariance13=rcov(modelfit13)

dim(covariance13)

cov.13=covariance13[,,dim(covariance13)[3]]

modelfit14=dccfit(modelspec,data=data.frame(HFRIE
HI14,HFRIFOF14,HFRIMI14,HFRIRVA14,HFRICR
DT14))

correlation14=rcor(modelfit14)

dim(correlation14)

correlation14[,,dim(correlation14)[3]]

covariance14=rcov(modelfit14)

dim(covariance14)

cov.14=covariance14[,,dim(covariance14)[3]]

modelfit15=dccfit(modelspec,data=data.frame(HFRIE
HI15,HFRIFOF15,HFRIMI15,HFRIRVA15,HFRICR
DT15))

correlation15=rcor(modelfit15)

dim(correlation15)

correlation15[,,dim(correlation15)[3]]

covariance15=rcov(modelfit15)

dim(covariance15)

cov.15=covariance15[,,dim(covariance15)[3]]

modelfit16=dccfit(modelspec,data=data.frame(HFRIE
HI16,HFRIFOF16,HFRIMI16,HFRIRVA16,HFRICR
DT16))

correlation16=rcor(modelfit16)

dim(correlation16)

correlation16[,,dim(correlation16)[3]]

covariance16=rcov(modelfit16)

dim(covariance16)

cov.16=covariance16[,,dim(covariance16)[3]]

modelfit17=dccfit(modelspec,data=data.frame(HFRIE
HI17,HFRIFOF17,HFRIMI17,HFRIRVA17,HFRICR
DT17))

correlation17=rcor(modelfit17)

dim(correlation17)

correlation17[,,dim(correlation17)[3]]

covariance17=rcov(modelfit17)

dim(covariance17)

cov.17=covariance17[,,dim(covariance17)[3]]

modelfit18=dccfit(modelspec,data=data.frame(HFRIE
HI18,HFRIFOF18,HFRIMI18,HFRIRVA18,HFRICR
DT18))

correlation18=rcor(modelfit18)

dim(correlation18)

correlation18[,,dim(correlation18)[3]]

covariance18=rcov(modelfit18)

dim(covariance18)

cov.18=covariance18[,,dim(covariance18)[3]]

modelfit19=dccfit(modelspec,data=data.frame(HFRIE
HI19,HFRIFOF19,HFRIMI19,HFRIRVA19,HFRICR
DT19))

correlation19=rcor(modelfit19)

dim(correlation19)

correlation19[,,dim(correlation19)[3]]

covariance19=rcov(modelfit19)

dim(covariance19)

cov.19=covariance19[,,dim(covariance19)[3]]

modelfit20=dccfit(modelspec,data=data.frame(HFRIE
HI20,HFRIFOF20,HFRIMI20,HFRIRVA20,HFRICR
DT20))

correlation20=rcor(modelfit20)

dim(correlation20)

correlation20[,,dim(correlation20)[3]]

covariance20=rcov(modelfit20)

dim(covariance20)

cov.20=covariance20[,,dim(covariance20)[3]]

modelfit21=dccfit(modelspec,data=data.frame(HFRIE
HI21,HFRIFOF21,HFRIMI21,HFRIRVA21,HFRICR
DT21))

correlation21=rcor(modelfit21)

dim(correlation21)

correlation21[,,dim(correlation21)[3]]

covariance21=rcov(modelfit21)
```

dim(covariance21)

cov.21=covariance21[,,dim(covariance21)[3]]

modelfit22=dccfit(modelspec,data=data.frame(HFRIE HI22,HFRIFOF22,HFRIMI22,HFRIRVA22,HFRICR DT22))

correlation22=rcor(modelfit22)

dim(correlation22)

correlation22[,,dim(correlation22)[3]]

covariance22=rcov(modelfit22)

dim(covariance22)

cov.22=covariance22[,,dim(covariance22)[3]]

modelfit23=dccfit(modelspec,data=data.frame(HFRIE HI23,HFRIFOF23,HFRIMI23,HFRIRVA23,HFRICR DT23))

correlation23=rcor(modelfit23)

dim(correlation23)

correlation23[,,dim(correlation23)[3]]

covariance23=rcov(modelfit23)

dim(covariance23)

cov.23=covariance23[,,dim(covariance23)[3]]

modelfit24=dccfit(modelspec,data=data.frame(HFRIE HI24,HFRIFOF24,HFRIMI24,HFRIRVA24,HFRICR DT24))

correlation24=rcor(modelfit24)

dim(correlation24)

correlation24[,,dim(correlation24)[3]]

covariance24=rcov(modelfit24)

dim(covariance24)

cov.24=covariance24[,,dim(covariance24)[3]]

modelfit25=dccfit(modelspec,data=data.frame(HFRIE HI25,HFRIFOF25,HFRIMI25,HFRIRVA25,HFRICR DT25))

correlation25=rcor(modelfit25)

dim(correlation25)

correlation25[,,dim(correlation25)[3]]

covariance25=rcov(modelfit25)

dim(covariance25)

cov.25=covariance25[,,dim(covariance25)[3]]

modelfit26=dccfit(modelspec,data=data.frame(HFRIE HI26,HFRIFOF26,HFRIMI26,HFRIRVA26,HFRICR DT26))

correlation26=rcor(modelfit26)

dim(correlation26)

correlation26[,,dim(correlation26)[3]]

covariance26=rcov(modelfit26)

dim(covariance26)

cov.26=covariance26[,,dim(covariance26)[3]]

modelfit27=dccfit(modelspec,data=data.frame(HFRIE HI27,HFRIFOF27,HFRIMI27,HFRIRVA27,HFRICR DT27))

correlation27=rcor(modelfit27)

dim(correlation27)

correlation27[,,dim(correlation27)[3]]

covariance27=rcov(modelfit27)

dim(covariance27)

cov.27=covariance27[,,dim(covariance27)[3]]

modelfit28=dccfit(modelspec,data=data.frame(HFRIE HI28,HFRIFOF28,HFRIMI28,HFRIRVA28,HFRICR DT28))

correlation28=rcor(modelfit28)

dim(correlation28)

correlation28[,,dim(correlation28)[3]]

covariance28=rcov(modelfit28)

dim(covariance28)

cov.28=covariance28[,,dim(covariance28)[3]]

modelfit29=dccfit(modelspec,data=data.frame(HFRIE HI29,HFRIFOF29,HFRIMI29,HFRIRVA29,HFRICR DT29))

correlation29=rcor(modelfit29)

dim(correlation29)

correlation29[,,dim(correlation29)[3]]

covariance29=rcov(modelfit29)

dim(covariance29)

cov.29=covariance29[,,dim(covariance29)[3]]

modelfit30=dccfit(modelspec,data=data.frame(HFRIE HI30,HFRIFOF30,HFRIMI30,HFRIRVA30,HFRICR DT30))

correlation30=rcor(modelfit30)

```
dim(correlation30)

correlation30[,,dim(correlation30)[3]]

covariance30=rcov(modelfit30)

dim(covariance30)

cov.30=covariance30[,,dim(covariance30)[3]]

modelfit31=dccfit(modelspec,data=data.frame(HFRIE
HI31,HFRIFOF31,HFRIMI31,HFRIRVA31,HFRICR
DT31))

correlation31=rcor(modelfit31)

dim(correlation31)

correlation31[,,dim(correlation31)[3]]

covariance31=rcov(modelfit31)

dim(covariance31)

cov.31=covariance31[,,dim(covariance31)[3]]

modelfit32=dccfit(modelspec,data=data.frame(HFRIE
HI32,HFRIFOF32,HFRIMI32,HFRIRVA32,HFRICR
DT32))

correlation32=rcor(modelfit32)

dim(correlation32)

correlation32[,,dim(correlation32)[3]]

covariance32=rcov(modelfit32)

dim(covariance32)

cov.32=covariance32[,,dim(covariance32)[3]]

modelfit33=dccfit(modelspec,data=data.frame(HFRIE
HI33,HFRIFOF33,HFRIMI33,HFRIRVA33,HFRICR
DT33))

correlation33=rcor(modelfit33)

dim(correlation33)

correlation33[,,dim(correlation33)[3]]

covariance33=rcov(modelfit33)

dim(covariance33)

cov.33=covariance33[,,dim(covariance33)[3]]

modelfit34=dccfit(modelspec,data=data.frame(HFRIE
HI34,HFRIFOF34,HFRIMI34,HFRIRVA34,HFRICR
DT34))

correlation34=rcor(modelfit34)

dim(correlation34)

correlation34[,,dim(correlation34)[3]]

covariance34=rcov(modelfit34)
```

```
dim(covariance34)

cov.34=covariance34[,,dim(covariance34)[3]]

return.matrix=as.timeSeries(returns)


#3.2 Find all the efficient frontiers

covariance<-cov(return.matrix)

covariance

print(covariance)

n <- ncol(covariance)


eff.frontier <-  function (returns, covariance,

                short = "no",

                max.allocation        =        NULL,
risk.premium.up = .5,

   risk.increment = .000

        ) {


 Amat <- matrix (1, nrow = n)

 bvec <- 1

 meq <- 1


 if (short == "no") {

  Amat <- cbind(1, diag(n))

  bvec <- c(bvec, rep(0, n))

 }


 if (!is.null(max.allocation)) {

  if (max.allocation > 1 | max.allocation < 0) {

    stop("max.allocation must be greater than 0 and less
than 1")

  }

  if (max.allocation * n < 1) {

    stop("Need to set max.allocation higher; not enough
assets to add to 1")

  }

  Amat <- cbind(Amat, -diag(n))

  bvec <- c(bvec, rep(-max.allocation, n))
```

```r
    }

    loops <- risk.premium.up / risk.increment + 1

    loop <- 1


    eff <- matrix(nrow = loops, ncol = n + 3)

    colnames(eff) <-

      c(colnames(returns),    "Std.Dev",    "Exp.Return",
"sharpe")


    for (i in seq(from = 0, to = risk.premium.up, by =
risk.increment)) {

      dvec <-

        colMeans(returns) * i # This moves the solution up
along the efficient frontier

      sol <-

        solve.QP(

          covariance,

          dvec = dvec,

          Amat = Amat,

          bvec = bvec,

          meq = meq

        )

      eff[loop, "Std.Dev"] <-

        sqrt(sum(sol$solution * colSums((

          covariance * sol$solution

        ))))

      eff[loop, "Exp.Return"] <-

        as.numeric(sol$solution %*% colMeans(returns))

      eff[loop, "sharpe"] <-

        eff[loop, "Exp.Return"] / eff[loop, "Std.Dev"]

      eff[loop, 1:n] <- sol$solution

      loop <- loop + 1

    }


    return(as.data.frame(eff))

}
```

```r
Rf=0.001

effshort0 <-

  eff.frontier(

    returns = return.matrix, covariance=covariance,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

efflong0 <-

  eff.frontier(

    returns = return.matrix, covariance=covariance,

    short = "no",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff1 <-

  eff.frontier(

    returns = return.matrix1, covariance=cov.1,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

)

eff2 <-

  eff.frontier(

    returns = return.matrix2, covariance=cov.2,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff3 <-

  eff.frontier(

    returns = return.matrix3, covariance=cov.3,
```

```
    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff4 <-

  eff.frontier(

    returns = return.matrix4, covariance=cov.4,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff5 <-

  eff.frontier(

    returns = return.matrix5, covariance=cov.5,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff6 <-

  eff.frontier(

    returns = return.matrix6, covariance=cov.6,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff7 <-

  eff.frontier(

    returns = return.matrix7, covariance=cov.7,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001
```

```
  )

eff8 <-

  eff.frontier(

    returns = return.matrix8, covariance=cov.8,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff9 <-

  eff.frontier(

    returns = return.matrix9, covariance=cov.9,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff10 <-

  eff.frontier(

    returns = return.matrix10, covariance=cov.10,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff11 <-

  eff.frontier(

    returns = return.matrix11, covariance=cov.11,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff12 <-

  eff.frontier(

    returns = return.matrix12, covariance=cov.12,
```

```
    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff13 <-

  eff.frontier(

    returns = return.matrix13, covariance=cov.13,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff14 <-

  eff.frontier(

    returns = return.matrix14, covariance=cov.14,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff15 <-

  eff.frontier(

    returns = return.matrix15, covariance=cov.15,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff16 <-

  eff.frontier(

    returns = return.matrix16, covariance=cov.16,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001
```

```
  )

eff17 <-

  eff.frontier(

    returns = return.matrix17, covariance=cov.17,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff18 <-

  eff.frontier(

    returns = return.matrix18, covariance=cov.18,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff19 <-

  eff.frontier(

    returns = return.matrix19, covariance=cov.19,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff20 <-

  eff.frontier(

    returns = return.matrix20, covariance=cov.20,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff21 <-

  eff.frontier(

    returns = return.matrix21, covariance=cov.21,
```

```r
    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff22 <-

  eff.frontier(

    returns = return.matrix22, covariance=cov.22,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff23 <-

  eff.frontier(

    returns = return.matrix23, covariance=cov.23,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff24 <-

  eff.frontier(

    returns = return.matrix24, covariance=cov.24,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff25 <-

  eff.frontier(

    returns = return.matrix25, covariance=cov.25,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001
```

```r
  )

eff26 <-

  eff.frontier(

    returns = return.matrix26, covariance=cov.26,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff27 <-

  eff.frontier(

    returns = return.matrix27, covariance=cov.27,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff28 <-

  eff.frontier(

    returns = return.matrix28, covariance=cov.28,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff29 <-

  eff.frontier(

    returns = return.matrix29, covariance=cov.29,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff30 <-

  eff.frontier(

    returns = return.matrix30, covariance=cov.30,
```

```
    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff31 <-

  eff.frontier(

    returns = return.matrix31, covariance=cov.31,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff32 <-

  eff.frontier(

    returns = return.matrix32, covariance=cov.32,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff33 <-

  eff.frontier(

    returns = return.matrix33, covariance=cov.33,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001

  )

eff34 <-

  eff.frontier(

    returns = return.matrix34, covariance=cov.34,

    short = "yes",

    max.allocation = 1.0,

    risk.premium.up = .5,

    risk.increment = .001
```

```
  )

write.xlsx(effshort0,"/Users/Niccolo/Desktop/effshort0.xlsx",asTable=TRUE)

write.xlsx(efflong0,"/Users/Niccolo/Desktop/efflong0.xlsx",asTable=TRUE)

write.xlsx(eff1,"/Users/Niccolo/Desktop/eff1.xlsx",asTable=TRUE)

write.xlsx(eff2,"/Users/Niccolo/Desktop/eff2.xlsx",asTable=TRUE)

write.xlsx(eff3,"/Users/Niccolo/Desktop/eff3.xlsx",asTable=TRUE)

write.xlsx(eff4,"/Users/Niccolo/Desktop/eff4.xlsx",asTable=TRUE)

write.xlsx(eff5,"/Users/Niccolo/Desktop/eff5.xlsx",asTable=TRUE)

write.xlsx(eff6,"/Users/Niccolo/Desktop/eff6.xlsx",asTable=TRUE)

write.xlsx(eff7,"/Users/Niccolo/Desktop/eff7.xlsx",asTable=TRUE)

write.xlsx(eff8,"/Users/Niccolo/Desktop/eff8.xlsx",asTable=TRUE)

write.xlsx(eff9,"/Users/Niccolo/Desktop/eff9.xlsx",asTable=TRUE)

write.xlsx(eff10,"/Users/Niccolo/Desktop/eff10.xlsx",asTable=TRUE)

write.xlsx(eff11,"/Users/Niccolo/Desktop/eff11.xlsx",asTable=TRUE)

write.xlsx(eff12,"/Users/Niccolo/Desktop/eff12.xlsx",asTable=TRUE)

write.xlsx(eff13,"/Users/Niccolo/Desktop/eff13.xlsx",asTable=TRUE)

write.xlsx(eff14,"/Users/Niccolo/Desktop/eff14.xlsx",asTable=TRUE)

write.xlsx(eff15,"/Users/Niccolo/Desktop/eff15.xlsx",asTable=TRUE)

write.xlsx(eff16,"/Users/Niccolo/Desktop/eff16.xlsx",asTable=TRUE)

write.xlsx(eff17,"/Users/Niccolo/Desktop/eff17.xlsx",asTable=TRUE)

write.xlsx(eff18,"/Users/Niccolo/Desktop/eff18.xlsx",asTable=TRUE)

write.xlsx(eff19,"/Users/Niccolo/Desktop/eff19.xlsx",asTable=TRUE)

write.xlsx(eff20,"/Users/Niccolo/Desktop/eff20.xlsx",asTable=TRUE)
```

```
write.xlsx(eff21,"/Users/Niccolo/Desktop/eff21.xlsx",a
sTable=TRUE)

write.xlsx(eff22,"/Users/Niccolo/Desktop/eff22.xlsx",a
sTable=TRUE)

write.xlsx(eff23,"/Users/Niccolo/Desktop/eff23.xlsx",a
sTable=TRUE)

write.xlsx(eff24,"/Users/Niccolo/Desktop/eff24.xlsx",a
sTable=TRUE)

write.xlsx(eff25,"/Users/Niccolo/Desktop/eff25.xlsx",a
sTable=TRUE)

write.xlsx(eff26,"/Users/Niccolo/Desktop/eff26.xlsx",a
sTable=TRUE)

write.xlsx(eff27,"/Users/Niccolo/Desktop/eff27.xlsx",a
sTable=TRUE)

write.xlsx(eff28,"/Users/Niccolo/Desktop/eff28.xlsx",a
sTable=TRUE)

write.xlsx(eff29,"/Users/Niccolo/Desktop/eff29.xlsx",a
sTable=TRUE)

write.xlsx(eff30,"/Users/Niccolo/Desktop/eff30.xlsx",a
sTable=TRUE)

write.xlsx(eff31,"/Users/Niccolo/Desktop/eff31.xlsx",a
sTable=TRUE)

write.xlsx(eff32,"/Users/Niccolo/Desktop/eff32.xlsx",a
sTable=TRUE)

write.xlsx(eff33,"/Users/Niccolo/Desktop/eff33.xlsx",a
sTable=TRUE)

write.xlsx(eff34,"/Users/Niccolo/Desktop/eff34.xlsx",a
sTable=TRUE)

eff.optimal.point1    <-    eff1[eff1$sharpe    ==
max(eff1$sharpe),]


ealred  <- "#7D110C"

ealtan  <- "#CDC4B6"

eallighttan <- "#F7F6F0"

ealdark  <- "#423C30"

mvwHFRIEHI=-0.3919

mvwHFRIFOF=0.9024

mvwHFRIMI=0.1636

mvwHFRIRVA=0.7701

mvwHFRICRDT=-0.4442

statwHFRIEHI=-0.1093

statwHFRIFOF=0.3885
```

```
statwHFRIMI=-0.0559

statwHFRIRVA=0.7611

statwHFRICRDT=0.0156

check1=mvwHFRIEHI+mvwHFRIFOF+mvwHFRIMI
+mvwHFRIRVA+mvwHFRICRDT

check2=statwHFRIEHI+statwHFRIFOF+statwHFRIM
I+statwHFRIRVA+statwHFRICRDT

check1

check2


ggplot(eff1, aes(x = Std.Dev, y = Exp.Return)) +
geom_point(alpha = .1, color =

                          ealdark) +

  geom_point(

    data = eff.optimal.point1,

    aes(x = Std.Dev, y = Exp.Return, label = sharpe),

    color = ealred,

    size = 5

  ) +

  annotate(

    geom = "text",

    x = eff.optimal.point1$Std.Dev,

    y = eff.optimal.point1$Exp.Return,

    label = paste(

      "Risk: ",

      round(eff.optimal.point1$Std.Dev * 100, digits = 3),

      "\nReturn: ",

      round(eff.optimal.point1$Exp.Return * 100, digits =
        4),

      "%\nSharpe: ",

      round(eff.optimal.point1$sharpe * 100, digits = 2),

      "%",

      sep = ""

    ),

    hjust = 0,

    vjust = 1.2

  ) +
```

```r
  ggtitle("Efficient Frontier\nand Optimal Portfolio") +
  labs(x = "Risk (standard deviation of portfolio variance)", y =

                                   "Return") +

  theme(

    panel.background = element_rect(fill = eallighttan),

    text = element_text(color = ealdark),

    plot.title = element_text(size = 24, color = ealred)

  )
```

#3.3 Plot the results

```r
plot(X2020$RY,X2020$SDX,col='white',xlab="Standard Deviation)",ylab="Return")

lines(X2020$R0, X2020$SD0, col='#996633', lwd=1)

lines(X2020$R1, X2020$SD1, col='darkgreen', lwd=1)

lines(X2020$R2, X2020$SD2, col='red', lwd=1)

lines(X2020$R3, X2020$SD3, col='#3399FF', lwd=1)

lines(X2020$R4, X2020$SD4, col='blue', lty=1)

lines(X2020$R5, X2020$SD5, col='pink', lwd=1)

lines(X2020$R6, X2020$SD6, col='666000', lwd=1)

lines(X2020$R7, X2020$SD7, col='#9933CC', lwd=1)

lines(X2020$R8, X2020$SD8, col='#330033', lwd=1)

lines(X2020$R9, X2020$SD9, col='#33FFFF', lty=1)

lines(X2020$R10, X2020$SD10, col='#CCCC00', lty=1)

plot(X2021$RY,X2021$SDX,col='white',xlab="Standard Deviation)",ylab="Return")

lines(X2021$R11, X2021$SD11, col='#996633', lwd=1)

lines(X2021$R12, X2021$SD12, col='darkgreen', lwd=1)

lines(X2021$R13, X2021$SD13, col='red', lwd=1)

lines(X2021$R14, X2021$SD14, col='#3399FF', lwd=1)

lines(X2021$R15, X2021$SD15, col='blue', lty=1)

lines(X2021$R16, X2021$SD16, col='pink', lwd=1)

lines(X2021$R17, X2021$SD17, col='#666000', lwd=1)

lines(X2021$R18, X2021$SD18, col='#9933CC', lwd=1)

lines(X2021$R19, X2021$SD19, col='#330033', lwd=1)

lines(X2021$R20, X2021$SD20, col='#33FFFF', lty=1)

lines(X2021$R21, X2021$SD21, col='#CCCC00', lty=1)

lines(X2021$R22, X2021$SD22, col='#666666', lty=1)

plot(X2022$RY,X2022$SDX,col='white',xlab="Standard Deviation)",ylab="Return")

lines(X2022$R23, X2022$SD23, col='#996633', lwd=1)

lines(X2022$R24, X2022$SD24, col='darkgreen', lwd=1)

lines(X2022$R25, X2022$SD25, col='red', lwd=1)

lines(X2022$R26, X2022$SD26, col='#3399FF', lwd=1)

lines(X2022$R27, X2022$SD27, col='blue', lty=1)

lines(X2022$R28, X2022$SD28, col='pink', lwd=1)

lines(X2022$R29, X2022$SD29, col='#666000', lwd=1)

lines(X2022$R30, X2022$SD30, col='#9933CC', lwd=1)

lines(X2022$R31, X2022$SD31, col='#330033', lwd=1)

lines(X2022$R32, X2022$SD32, col='#33FFFF', lty=1)

lines(X2022$R33, X2022$SD33, col='#CCCC00', lty=1)

lines(X2022$R34, X2022$SD34, col='#666666', lty=1)

dccweights_HFRIEHI=select(dcc_weights,HFRIEHI)

dccweights_HFRIFOF=select(dcc_weights,HFRIFOF)

dccweights_HFRIMI=select(dcc_weights,HFRIMI)

dccweights_HFRIRVA=select(dcc_weights,HFRIRVA)

dccweights_HFRICRDT=select(dcc_weights,HFRICRDT)

ret_HFRIEHI=select(returns_2020_2022,HFRIEHI)

ret_HFRIFOF=select(returns_2020_2022,HFRIFOF)

ret_HFRIMI=select(returns_2020_2022,HFRIMI)
```

ret_HFRIRVA=select(returns_2020_2022,HFRIRVA)

ret_HFRICRDT=select(returns_2020_2022,HFRICRDT)

dcc_ret_HFRIEHI=dccweights_HFRIEHI*ret_HFRIEHI

dcc_ret_HFRIFOF=dccweights_HFRIFOF*ret_HFRIFOF

dcc_ret_HFRIMI=dccweights_HFRIMI*ret_HFRIMI

dcc_ret_HFRIRVA=dccweights_HFRIRVA*ret_HFRIRVA

dcc_ret_HFRICRDT=dccweights_HFRICRDT*ret_HFRICRDT

dcc_portfolio=dcc_ret_HFRIEHI+dcc_ret_HFRIFOF+dcc_ret_HFRIMI+dcc_ret_HFRIRVA+dcc_ret_HFRICRDT

Wi=1/5

equallyHFRIEHI=(ret_HFRIEHI-Rf)*Wi

equallyHFRIFOF=(ret_HFRIFOF-Rf)*Wi

equallyHFRIMI=(ret_HFRIMI-Rf)*Wi

equallyHFRIRVA=(ret_HFRIRVA-Rf)*Wi

equallyHFRICRDT=(ret_HFRICRDT-Rf)*Wi

equally_portfolio=equallyHFRIEHI+equallyHFRIFOF+equallyHFRIMI+equallyHFRIRVA+equallyHFRICRDT

statHFRIEHI=(ret_HFRIEHI)*statwHFRIEHI

statHFRIFOF=(ret_HFRIFOF)*statwHFRIFOF

statHFRIMI=(ret_HFRIMI)*statwHFRIMI

statHFRIRVA=(ret_HFRIRVA)*statwHFRIRVA

statHFRICRDT=(ret_HFRICRDT)*statwHFRICRDT

stat_portfolio=statHFRIEHI+statHFRIFOF+statHFRIMI+statHFRIRVA+statHFRICRDT

mvHFRIEHI=(ret_HFRIEHI)*mvwHFRIEHI

mvHFRIFOF=(ret_HFRIFOF)*mvwHFRIFOF

mvHFRIMI=(ret_HFRIMI)*mvwHFRIMI

mvHFRIRVA=(ret_HFRIRVA)*mvwHFRIRVA

mvHFRICRDT=(ret_HFRICRDT)*mvwHFRICRDT

mv_portfolio=mvHFRIEHI+mvHFRIFOF+mvHFRIMI+mvHFRIRVA+mvHFRICRDT

dccmvweights_HFRIEHI=select(dcc_mv_weights,HFRIEHI)

dccmvweights_HFRIFOF=select(dcc_mv_weights,HFRIFOF)

dccmvweights_HFRIMI=select(dcc_mv_weights,HFRIMI)

dccmvweights_HFRIRVA=select(dcc_mv_weights,HFRIRVA)

dccmvweights_HFRICRDT=select(dcc_mv_weights,HFRICRDT)

dcc_mvret_HFRIEHI=dccmvweights_HFRIEHI*ret_HFRIEHI

dcc_mvret_HFRIFOF=dccmvweights_HFRIFOF*ret_HFRIFOF

dcc_mvret_HFRIMI=dccmvweights_HFRIMI*ret_HFRIMI

dcc_mvret_HFRIRVA=dccmvweights_HFRIRVA*ret_HFRIRVA

dcc_mvret_HFRICRDT=dccmvweights_HFRICRDT*ret_HFRICRDT

dcc_mv_portfolio=dcc_mvret_HFRIEHI+dcc_mvret_HFRIFOF+dcc_mvret_HFRIMI+dcc_mvret_HFRIRVA+dcc_mvret_HFRICRDT

c <- data.frame(dcc_portfolio = dcc_portfolio,

    equally_portfolio = equally_portfolio,

    stat_portfolio = stat_portfolio,

    mv_portfolio = mv_portfolio,

    dcc_mv_portfolio = dcc_mv_portfolio,

    HFRIWI = HFRIWI

)

c1 <- data.frame(dcc_weights)

c2 <- data.frame(dcc_mv_weights)

colnames(c) <- c("dcc_portfolio", "equally_portfolio", "stat_portfolio", "mv_portfolio","dcc_mv_portfolio", "HFRIWI")

b <- data.frame(c(100, 100, 100, 100, 100, 100), row.names = c("dcc_portfolio", "equally_portfolio", "stat_portfolio", "mv_portfolio","dcc_mv_portfolio", "HFRIWI"))

df <- data.frame(dcc_portfolio = c(100),

    equally_portfolio = c(100),

    stat_portfolio = c(100),

    mv_portfolio = c(100),

    dcc_mv_portfolio = c(100),

```
        HFRIWI = c(100)
        )


for(i in 1:nrow(c)){
  df[i+1,] <- df[i,]*(1+c[i,])
}
x=0:35
x1=0:34
plot(x,df$HFRIWI,col='white',xlab="Time(from
02.2020 to 12.2022)",ylab="Portfolio Performance")

lines(x, df$dcc_portfolio, col='#996633', lwd=1)

lines(x, df$stat_portfolio, col='darkgreen', lwd=1)

lines(x, df$mv_portfolio, col='red', lwd=1)

lines(x, df$dcc_mv_portfolio, col='#3399FF', lwd=1)

lines(x, df$equally_portfolio, col='blue', lty=1)

lines(x, df$HFRIWI, col='#FF3399', lty=1)

plot(x1,c1$HFRIFOF,col='white',xlab="Time(from
02.2020 to 12.2022)",ylab="Portfolio Performance")

lines(x1, c1$HFRIEHI, col='#996633', lwd=1)

lines(x1, c1$HFRIFOF, col='darkgreen', lwd=1)

lines(x1, c1$HFRIMI, col='red', lwd=1)

lines(x1, c1$HFRIRVA, col='#3399FF', lwd=1)

lines(x1, c1$HFRICRDT, col='blue', lty=1)

plot(x1,c1$HFRIFOF,col='white',xlab="Time(from
02.2020 to 12.2022)",ylab="Portfolio Performance")

lines(x1, c2$HFRIEHI, col='#996633', lwd=1)

lines(x1, c2$HFRIFOF, col='darkgreen', lwd=1)

lines(x1, c2$HFRIMI, col='red', lwd=1)

lines(x1, c2$HFRIRVA, col='#3399FF', lwd=1)

lines(x1, c2$HFRICRDT, col='blue', lty=1)
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