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Tesi di Laurea

Portfolio Optimization:

An Introduction to Higher Order Moments

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Abstract

The basic approach when it comes to portfolio optimization is the Markovitz[1] approach, which uses the first two moments to build the efficient frontier for the asset allocation. This method requires low computational power and supplies a good accuracy of the results. However, the fact that it only accounts for the first two moments of the distribution, it may create some limitations or bias to the model which could be fixed by including higher order moments like skewness and kurtosis. This paper explores the possible improvements to portfolio construction techniques that the inclusion of skewness could bring. The main analysis has been performed using the daily and monthly returns of 205 S&P500 components, ranging from 1990 to 2020, with a three-year rolling window approach for the selection of the stocks included in each portfolio. The purpose of the rolling window is to compute the daily skewness of all stocks and then divide them into nine groups based on the monthly average obtained. Groups are flexible, meaning that the stocks inside each group are free to move from one group to another from month to month and the first group is the left skewed portfolio while the last group represents the right skewed one. The portfolio techniques implemented are the naïve approach and, from the Markovitz optimization, the minimum variance and the maximum Sharpe portfolios. The cumulative returns obtained from the portfolios were compared to the SPY benchmark and not only did almost all the portfolios outperform the benchmark, the two best performing portfolios were the equally weighted ones. If we focus only on the cumulative returns from 2008, the naïve right-skewed portfolio is the superior one due to the drastic decrease in performance from the left skewed. Furthermore, the Markovitz optimization seems to provide better results in the 2008 analysis in comparison to the cumulative returns over the whole period. What this study will demonstrate is that the higher moments are necessary for portfolio optimization, and they can enhance the performances of the Markovitz approach that does not account for skewness as an input by itself.

[1] Markowitz, H. (1952) Portfolio Selection. The Journal of Finance, Vol. 7, No. 1. March. 1952.

Introduction

Portfolio optimization is a crucial part of asset management since, after selecting the best assets to which allocate his resources, the investor needs to optimize the portion of wealth assigned to each asset. Economists have provided diverse ways to optimize a portfolio, based on the individual needs of each type of investor. The first step is to decide between an active or a passive asset allocation strategy, with the first case including frequent rebalancing and analysis while the second one is focused on a long-term horizon and less frequent portfolio rebalancing. Then, the next step will involve the choice of the algorithm to use for optimizing the weights assigned to each asset, the most recent techniques involve Machine Learning. However, one of the most famous optimization techniques when it comes to portfolio allocation is the Markovitz approach. This approach consists of creating an efficient frontier of portfolios exploring the effects on portfolios' performances of using different weights on the same assets' selection. The simplicity of this approach is sometimes what makes professional investors look for models that incorporate more data and characteristics of stocks. This paper explores a possible improvement of the model by adding a third moment (on top of mean and variance) into the optimization process, which is skewness. Stocks are assumed to follow a Gaussian distribution but, for most stocks, this is true only in a long-term horizon while in the short-term they are often skewed. However, the implementation of skewness in the efficient frontier estimation would significantly increase the complexity of computations and, consequently, the cost of estimation. This side-effect has been overcome by using skewness as a criteria for the stock selection in the early stage of the process. The selection process is dynamic since it is based on a three-year rolling window that divides stocks into nine groups, where the first group is the left-skewed portfolio and the ninth is the right-skewed portfolio. Stocks can move in and out of portfolios freely from month to month, since the only criterion for grouping is the skewness coming out of the rolling window. After building the portfolios, their performance has been tracked using the returns of the next month and three different optimization techniques have been applied to both portfolios. The first technique was the basic equally weighted (naïve), then Markovitz optimization selecting both the minimum volatility and the maximum Sharpe Ratio portfolios. The analysis continues by regressing portfolios' returns on the Fama & French risk factors to examine which factors can explain the excess returns obtained. In this way, it would be possibly to capture relevant risk factors common to most of the skewed groups and develop an investment strategy aimed at exploiting that factor for better portfolio performances.

Data Description

After downloading the list of components of the S&P500, I downloaded their daily prices from January 1990 to December 2020 using RStudio. Then, I subset the data to keep only the stocks with no missing data and, in the end, only 205 stocks satisfied the criterion and were picked for the analysis. Table 1 shows the selected stocks and the sector in which they operate. The dominant sectors are Financial Services, IT, and Industrials, which are also the predominant sectors in the economy. The diverse sector of the stocks is a source of natural diversification for the portfolios created in the analysis.

Sector	Stocks
Financial Services	AXP, RF, RJF, FITB, WRB, AFL, C, L, AIG, AJG, TFC, JPM, TROW, MMC, ZION, BRO, SPGI, GL, AON, WFC, KEY, BAC, USB, LNC, MTB, SCHW, BEN, SIVB, CINF, CMA, TRV, HBAN
Real Estate	HST, PEAK, UDR, VNO, WY, WELL, FRT
Consumer Staples	HSY, ADM, TAP, K, MKC, CL, WBA, CAG, SYY, MNST, CHD, GIS, MO, KMB, HRL, CLX, WMT, TSN
Information Technology	JKHY, ADI, ADP, AAPL, TXN, TYL, TER, KLAC, AMAT, AMD, ADSK, WDC, ADBE, MSI, KO, KR, INTC, MSFT, MU, HPQ, ORCL, SWKS, IBM, NLOK, LRCX
Health Care	MDT, HUM, UHS, WST, JNJ, CI, TFX, UNH, SYK, AMGN, ABMD, MRK, BAX, LLY, TMO, BDX, VTRS, ABT
Energy	COP, SO, AEP, BKR, HAL, VLO, HES, CVX, OXY, EOG, APA, MRO, XOM, OKE, SLB, WMB
Consumer Discretionary	GPC, NKE, PVH, BBWI, HAS, MGM, F, LEN, ROST, TGT, PHM, HD, TJX, NVR, NWL, WHR, LOW, VFC, MCD
Industrials	LUV, FAST, TT, EFX, NDSN, EXPD, J, TXT, NOC, JBHT, WM, ROK, ROL, SWK, ALK, AME, NSC, MMM, GWW, EMR, UNP, LHX, GD, GE, RTX, CTAS, AOS, LMT, JCI, HON, ITW, FDX, MAS, RHI, SNA
Materials	IFF, IEX, VMC, BLL, FMC, SEE, NUE, MOS, APD, IP, SHW, NEM, AVY
Communication	T, VZ, CMCSA, EA, IPG, LUMN, OMC
Utilities	XEL, EIX, ED, ES, WEC, LNT, EVRG, ATO, NEE, ETR, NI, EXC

Table 1. Selected stocks and their sectors.

Skewness

The skewness is a measure of the asymmetry of a random variable's probability distribution around its mean. Its value can be negative, zero, positive or undefined. When a random variable has zero skewness, the weight assigned to its tails is well balanced and equally distributed, and its mean, median and mode coincide. A positive value of the skewness means the distribution is right skewed (its right tail is longer than its left one), a value between -0.5 and 0.5 is said to be symmetrical, while a negative skewness value means the distribution is left skewed. This measure is relevant especially when dealing with financial assets because it provides insights on how often an asset has a positive/negative return or, when its returns deviate from the mean, in which side of the distribution they might fall. In this way it is possible to estimate the risk associated with the asset in more accurate way, pricing it appropriately. For example, a right skewed stock is associated to higher risk because the probability of positive returns is way lower compared to the one of negative returns and, for this reason, the reward for bearing the risk of buying it must be higher than a normally distributed stock. On the other side, a left skewed stock looks more profitable to investors and thus it requires a lower yield to attract investors. For all these reasons, the incorporation of skewness into the asset selection and the portfolio optimization process could provide an improvement of the performances and an allocation tailored to each investors' needs.

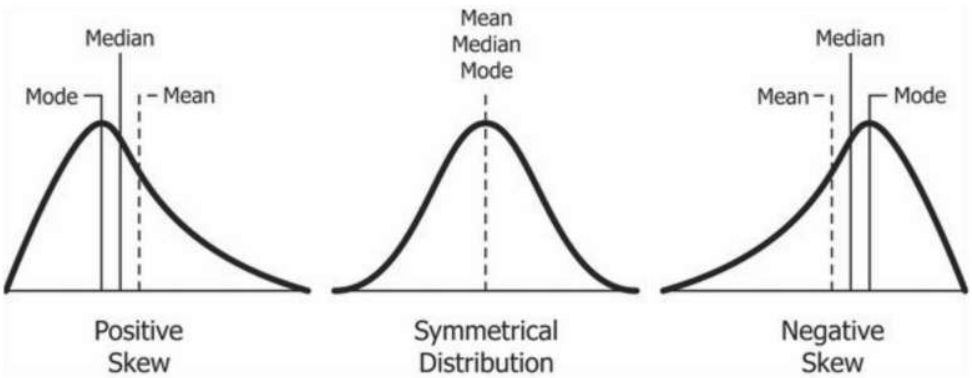

Figure 1: Right skewed, normally distributed and left skewed distributions, respectively.

There are many ways to compute the skewness, the method used in the analysis is the Fisher's moment coefficient formula. The formula uses the mean and standard deviation of the distribution to estimate how much each observation deviates from normality.

$$\mu_3 = E\left[\left(\frac{x-\mu}{\sigma}\right)^3\right] = \frac{\mu^3}{\sigma^3} = \frac{E[(x-\mu)^3]}{\left(E[(x-\mu)^2]\right)^{3/2}} \tag{1}$$

Equation 1 shows the Fisher equation for skewness, where the first two moments of the distribution (mean and variance) are used to compute the third moment. The target stock for the analysis were the most skewed stocks in both directions so, after dividing them in nine groups after each rolling window, the left skewed stocks portfolio was made of the stocks with the most negative skewness value and the right skewed portfolio with the most positive skewness value stocks. It's interesting to notice how the average skewness value of the portfolios changed throughout the analysis, as captured by Figure 2.

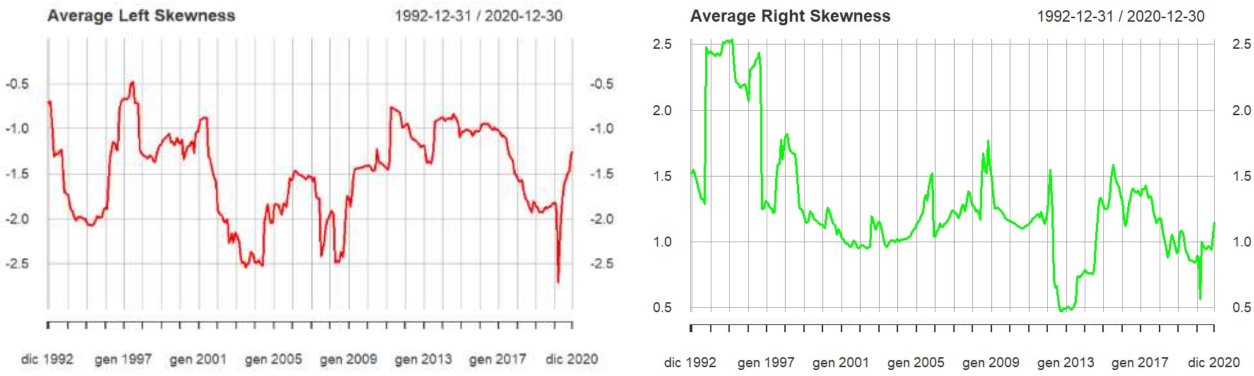

Figure 2: These graphs show the changes in the average skewness value of the set of stocks selected to build the portfolios throughout the period under analysis.

Table 2 highlights the statistics of both time series. We can see that the values are in line with the theoretical definition of positive and negative skewness and the volatility is substantial in both cases. It's worth noticing that the volatility occurs in specific economic conditions and the direction of the movement tells a story. The average skewness value, in both directions, during the dot-com bubble decreased toward zero because the stock market was experiencing a bull phase and this reduces the probability of extreme values. After the bust of the bubble, the average left skewness of the portfolio moved closer to zero while the right skewed one increased. Both these results are consistent with the economic phase that financial market was experiencing since, during a bull phase the probability of positive returns is consistently higher than during a bearish phase. At the same time, when a bubble bust the economic turmoil and volatility increase and, consequently, the probability of consistent negative returns follows the same pattern. The same scenario occurs before and during the 2008's financial crisis where we can observe a decrease in the average left skewness from 2005 until 2007 and then a significant drop in the 2008-2009 period. On the other hand, the average right skewness drops in 2005 and then slowly increases until 2009. This cycle repeats itself in every financial crisis in the period under analysis, suggesting that skewness may be an early indicator of an imminent financial crisis.

	Left Skewed	*Right Skewed*
Mean	-1.49	1.28
Standard Deviation	49.52%	44.92%

Table 2: Mean and Standard Deviation of the average left and right skewness value of the stocks building the right and left skewed portfolios.

Analysis

The dataset for this analysis includes the daily returns of S&P500's components ranging from 1990-01-01 to 2020-12-31, for a total of 30 years. Out of all the S&P500 components only 205 had data for the period covered by the analysis and have all been included in the dataset. The first step of the analysis is to compute the daily skewness of each stock using a rolling window of three years, then the daily skewness that has been converted into monthly average. This criterion has been used to split the stocks into nine groups each month with the first group representing the left skewed stocks and the last group representing the right skewed stocks. The stocks of those groups have been included into the left skewed and the right skewed portfolio respectively, computing the returns of the next month. As shown in DeMiguel et al. (2009)[2], the naïve portfolio provides solid performances even when compared to optimized portfolios, for this reason I performed the analysis using three different type of portfolios: equally weighted (naïve), minimum variance and maximum Sharpe ratio portfolios for both the left and right skewed stocks. The last two portfolios have been optimized using the mean-variance approach; after building the efficient frontier with a grid of target portfolio returns, the weights that minimized the volatility and those that maximized the Sharpe Ratio were then selected.

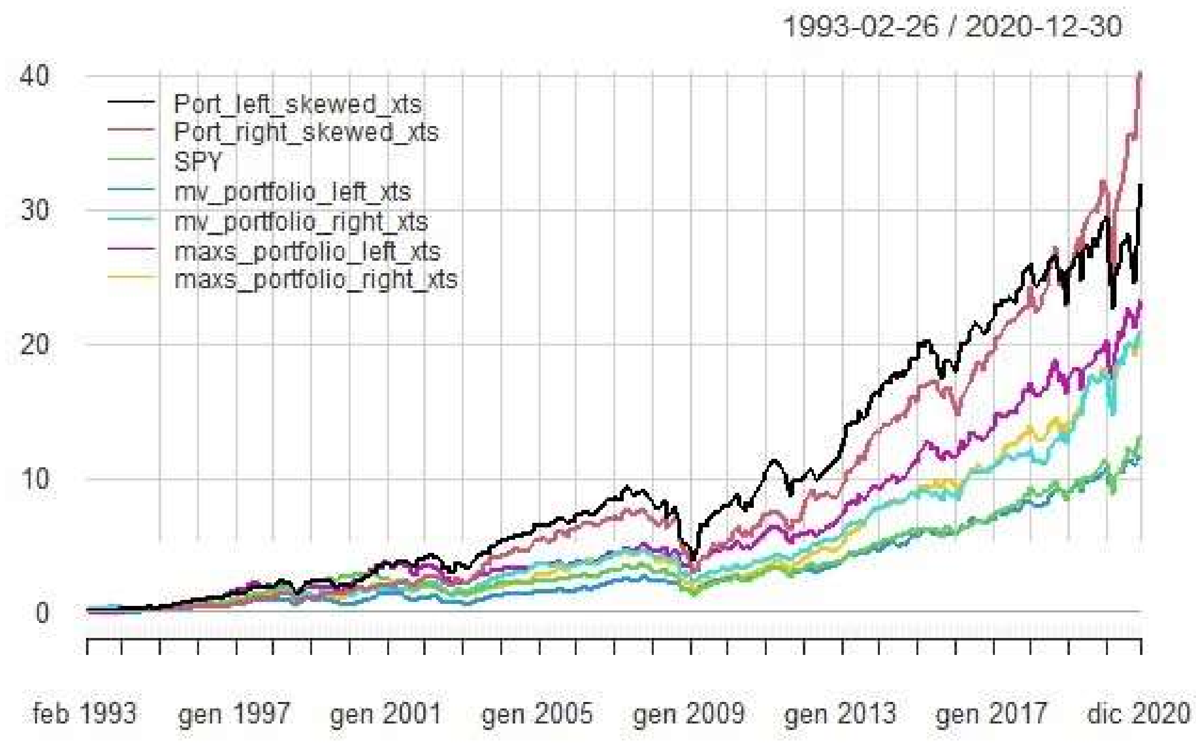

Figure 1: Cumulative returns of all the portfolios throughout the entire period under analysis.

The cumulative returns in Fig. 1 show the performances obtained from each portfolio in the periods of 1993 through the end of 2020. The best portfolios were the naïve- with the right skewed one outperforming all the others- and the maximum Sharpe portfolios. During the 2008 financial crisis the naïve portfolios were able to

[2] DeMiguel, V., Garlappi, L., Uppal, R., 2009b. Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? Review of Financial Studies 22, 1915–1953.

outperform their peers but at a much higher volatility. In fact, despite the worse performances, the other portfolios had a stable growth during the entire period while the naïve portfolios had big jumps followed by big drops. This should be considered when choosing a portfolio due to risk aversion being a crucial aspect in the asset allocation process.

	Return	Volatility	Sharpe
Naïve Left	14.07%	17.29%	0.814097
Naïve Right	14.99%	17.72%	0.845728
Min. Volatility Left	10.06%	13.51%	0.744387
Min. Volatility Right	12.07%	13.83%	0.873145
Max. Sharpe Left	12.77%	16.00%	0.798236
Max. Sharpe Right	12.26%	15.56%	0.787763
SPY	10.66%	14.67%	0.726531

Table 1: Table summarizing the annualized performance of each portfolio.

Table 1 summarizes the annualized statistics of each portfolio. The returns column confirms the fact that the naïve right skewed portfolio is the one with the highest return, immediately followed by the other naïve portfolio. Another important insight to highlight is that almost every skewed portfolio outperformed the benchmark, except for the minimum volatility of left skewed stocks. The naïve portfolios are not only the ones with the highest returns, but also the ones with the highest volatility with a significant three percent higher volatility compared to the SPY. As mentioned in Sharpe(1998)[3], the higher the Sharpe Ratio, which is a measure of risk-adjusted returns, the better the portfolio is. In this case, surprisingly, the best Sharpe ratio is achieved by the right skewed minimum volatility portfolios. This could be due to the fact that the right skewed stocks seem to be the best performing group after reducing the volatility of the portfolio, the outcome produces the best performing portfolio overall.

[3] Sharpe, W. F. (1998). The Sharpe ratio. *Streetwise–the Best of the Journal of Portfolio Management*, 169-185.

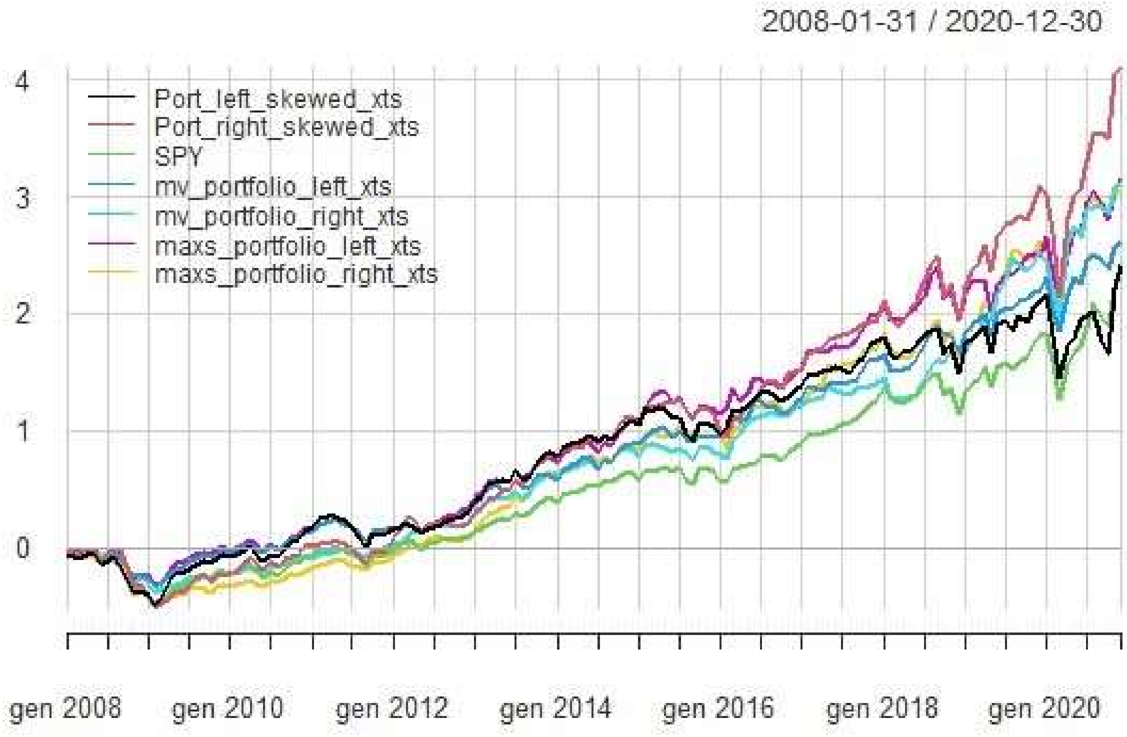

Figure 2: cumulative returns of each portfolio from January 2008 to December 2020.

Figure 2 shows the cumulative returns of the portfolios from January 2008 to December 2020. The aim of this change in the date range is to analyze if the financial crises impacted the overall performance of the portfolios. We can see that the naïve right skewed portfolio is still the best one, but the other naïve portfolio has poor performance this time, close to the SPY that is the worst performing portfolio. In the previous case, where the whole period was considered, there was not a clear difference between the performances of the two optimized portfolios while this time the Markovitz optimization is more coherent with its theoretical implementation since the maximum Sharpe portfolios are both performing better than the minimum volatility ones.

	Return	Volatility	Sharpe
Naïve Left	11.37%	19.53%	0.58
Naïve Right	14.62%	19.79%	0.74
Min. Volatility Left	10.79%	13.08%	0.83
Min. Volatility Right	11.97%	14.62%	0.82
Max. Sharpe Left	12.08%	14.59%	0.83
Max. Sharpe Right	12.01%	15.15%	0.79
SPY	10.53%	15.91%	0.66

Table 2: Table summarizing the annualized performance of each portfolio from 2008 to 2020.

In general, the returns have decreased for all portfolios while volatility increased leading to a downfall in the Sharpe ratio for most portfolios. In spite of this however, not all the Sharpe ratios decreased, in fact the Markovitz portfolios of the left skewed stocks have both increased their Sharpe ratio even though by a small amount. This change in performance is reflected in Figure 2, where there is a change in the top performing positions where the naïve left skewed portfolio is experiencing a huge decline in performances and goes from a top performer to be the last one, close to the SPY.

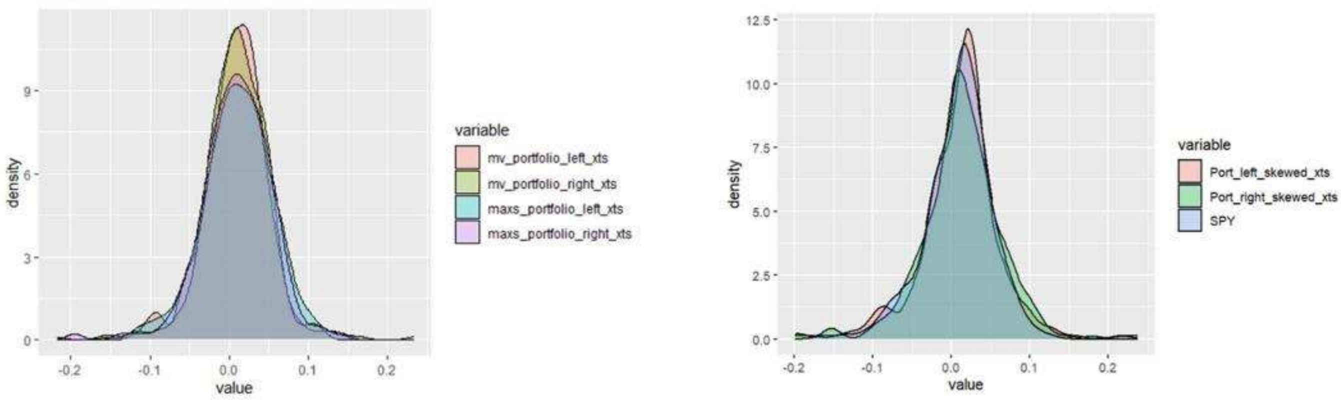

Figure 3: Density plot of each portfolio's monthly returns. The graph on the left represents the Markovitz portfolios and the one on the right the naïve portfolios and the benchmark.

Figure 3 shows the density plot of portfolio's monthly returns for the period going from 1993 to 2020. The naïve left portfolio behaves as expected while the right one is closer to a normal distribution rather than a positive skewed one. This may be due to the decreasing average value of the positive skewness during the period, which in the 90s was in line with the expectation while in the last decade it was closer to zero.

The SPY itself presents a slightly negative skewness, which is in support of the fact that, on average, the positive skewness has become rarer than in the past. The same holds true for the Markovitz portfolios, with the only difference that the weight optimization has had the effect of reducing the kurtosis of the portfolio, especially for the maximum Sharpe portfolios.

An interesting statistic to analyze is how often the composition of the skewed portfolios change throughout the period, which helps us to understand if the portfolio's performance comes from few stocks that were selected consistently or if it comes from a high rotation rate. The transition matrix in Figure 4 captures the probability of a stock to change the state based on which state it comes from, with zero meaning the stock was not in the portfolio in the previous month and one meaning it was. The top left corner of the matrix represents the probability of not being included in the portfolio if the stock was not included in the previous month, while the bottom left corner represents the probability of being excluded from the portfolio after being included in the previous month. In both matrices these two probabilities are high and, considering also the other two results in the matrix, we can firmly state that the rebalancing it's always heavy since on average only two out of the twenty-three stocks composing the portfolio are kept from one month to the next.

Left	0	1
0	0.8858632	0.1141368
1	0.9016901	0.0983099

Right	0	1
0	0.8858632	0.1141368
1	0.9016901	0.0983099

Figure 4: The transition matrix of both naive portfolios, representing the probability of stocks to go from one state to another.

To understand if skewness should be considered in portfolio optimization, I computed the excess returns over the risk-free rate of the Fama and French dataset. The risk-free rate has been subtracted from the returns of both naïve portfolio, which are the best representing portfolios since their allocation does not come from optimization. A constant premium over the entire period would provide a base for further assessments on whether the premium is effectively coming from the inclusion of skewness in the asset allocation process or not.

	Average Excess Return	Standard Deviation
Left skewed	0.26%	5.05%
Right skewed	0.34%	5.21%

Table 3: Statistics for excess returns of naive portfolios over the risk-free rate from 1993 to 2020.

As expected, the right skewed portfolio's average excess return is higher than the left skewed one even though it has a slightly higher volatility. The absolute value of the excess returns is not impressively high but if we consider it under a cumulative perspective, it may be worth the effort.

The source of the excess returns is unknown, so I decided to use the Fama and French risk factors to capture what may have originated those performances. The model implemented in the analysis is the five risk factors which comprehends: SMB, HML, RMW, CMA and MKT factors.[4]

[4] SMB: Small Minus Big, HML: High Minus Low, RWM: Robust Minus Weak, CMA: Conservative Minus Aggressive, MKT: Market return.

	Average Adjusted R-squared
SMB	0.009
HML	0.027
RMW	0.019
CMA	0.152
MKT	0.273
All Factors	0.327

Table 4: Average Adjusted R-squared values from the regression of Fama and French factors on each of the 205 stocks individually.

Table 4 summarizes the average results obtained from the regression of each individual stock on the risk factors whereas the last row represents the output of a regression performed on the combination of all factors together. The results shows that the risk factors alone do not explain much of the stocks' return but, when taken together, they explain on average the 30% of returns. The enhancement obtained from aggregation works not only for risk factors but also for stocks. In fact, when we consider the portfolios instead of individual stocks, Fama and French risk factors explanatory value increase significantly as shown in Table 5.

	Adjusted R-squared
Naïve Left	0.779
Naïve Right	0.763
Min Volatility Left	0.608
Min Volatility Right	0.593
Max Sharpe Left	0.622
Max Sharpe Right	0.529

Table 5: Adjusted R-squared from the regression of Fama and French risk factors on portfolio returns.

Fama and French risk factors seems more relevant in explaining portfolios' returns rather than individual stock returns. However, their relevance decreases when Markovitz optimization is implemented, reaching a low of 52.9% for the maximum Sharpe ratio of the right skewed stocks.

Conclusion

The results from this paper suggest that the stock selection made using skewness as the main criteria seems to provide high returns consistently. However, it's not entirely clear what is causing these performances since the Fama and French risk factor explains only 53 to 78 percent of portfolio returns. A possible explanation can be found in the return distribution itself, since right skewed stocks (which seems the best performing) are riskier than normally distributed ones, the performance improvement may be a reward for bearing the risk. In addition to this, the excess return obtained by the naïve portfolios could be insufficient to justify the high rebalancing needed to keep the investment strategy consistent because transaction costs and fees may erode that excess return.

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