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ESG Integration into the Investment Process

Supervisors
Ch. Prof. Diana Barro
Ch. Prof. Michele Costola

Graduand
Dario Carniello
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867379

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Abstract

Since Markowitz's Portfolio Theory the financial world has been described by two variables: return and risk. According to traditional finance, a rational investor should take a financial decision based only on these two variables. However, once again, reality makes economic theory obsolete. Empirical evidence shows that investors are not only concerned about how much money they will gain and how much money they are going to risk: the sustainability dimension has a profound impact on their investment decision. Financial theory must take into consideration a third sustainability-related variable. The most common and widest used proxy variable for sustainability is the ESG score. The contribution of this Master's Thesis is fourfold: first, it discusses the foundations of sustainable finance, trying to give a definition to the concepts of sustainable and responsible finance, corporate social responsibility, sustainable and responsible investing, and ESG score; second, it performs a literature review of the most influential studies on ESG integration into the investment process; third, it develops a method – a multi-goal tracking error model – that theoretically avoids previous models' limitations; fourth, it provides a practical application of the model using a sample of the 50 companies included in the EURO STOXX 50® Index for a 20-years time period.

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Introduction

In recent years, the world has become increasingly aware of the negative impact that businesses and the economy as a whole may have on society and the environment. The consciousness of the need for change has brought governments to sign international agreements and people to prioritize their ethical values also within the realm of economic decisions. The financial world is no exception: the sector of sustainable and responsible finance takes off.

The first chapter of this Master's Thesis presents the recent global trends within the spheres of governmental decisions and actions toward sustainable development. Then it defines the two interlinked and dependent topics of sustainable and responsible finance and ESG score.

In a nutshell, the ESG score, being a measure that quantifies the impact of a security on society and the environment, is the most common and intuitive way to include the dimension of sustainability into an investment decision. The second chapter is developed on the topic of ESG integration: first, the most common way of incorporating the sustainability element into the allocation process is described; second, a literature review on the mathematical models whose objective is the optimization along the three dimensions of return, risk, and sustainability is performed.

In the third chapter, the limitations of the previously described model are presented paving the way for the formulation of a different type of model from the one encountered in the literature review. A multi-goal tracking error model is proposed as the most suitable approach in order to integrate the ESG dimension into the investment process. The versatility of this type of model enables investors to design a tool appropriate for an investor's specific preferences. Finally, in the last chapter, the developed models are tested, and some statistical analyses are performed on the portfolios that constitute the output of previous optimizations

1 Sustainable Finance and ESG

1.1 Global Trends

The world's economy as currently constructed has proven to be unsustainable and its shortcomings can be seen both in the environmental and social dimensions. In this section, we are going to present some examples and give some data on past and future negative impacts, along the three dimensions of ESG, on the way humans conduct business.

Starting from the environmental sphere, our recent history is full of infamously known anthropogenic disasters. Some examples are the deepwater oil spill caused by the explosion of BP plc's oil rig in the Gulf of Mexico in 2010, and the record wildfires in Brazil's Amazon Rainforest, which were linked by the Guardian to JBS S.A. (Campos, A., et al. 2020), the largest meat processor in the world. However, the most impactful consequences of human negligence are subtle and less visible.

The 2021 report by the Intergovernmental Panel on Climate Change (IPCC), which studies the effects and levels of emissions of greenhouse gases (GHG), draws a scary picture for the future of our planet. According to this study, "there is a fifty-fifty chance that global warming will exceed 1.5°C in the next two decades, and unless there are immediate, rapid and largescale reductions in GHG emissions, limiting warming to 1.5°C or even 2°C by the end of the century will be beyond reach" (UN 2021). Data coming from a European Union's publication on anthropogenic emissions (Crippa, M., et al. 2021) of GHG from 1970 to 2020 are presented in figure 1: leaving aside the impact of 2020, when the Covid-19 pandemic dramatically affected our lives instantaneously changing habits and consumption, a steadily growing trend is displayed for the emissions produced by every economic sector. The energy sector stands first both in terms of absolute value and relative increase with respect to other sectors: indeed, energy demand is one of the key drivers of GHG emissions. If we consider that since estimates forecast a 4.6% increase in 2021, which is above pre-Covid-levels (IEA 2021), and if we add that 2022's political events forced countries, such as Italy, to return to utilizing coal-fired power plants, stopping or slowing down the energy transition in place, the environmental outlook for the comings years seems alarming.

Moving to the social dimension, it would be impossible and beyond the scope of this work to list the impact that the world's economies have on society. However, it is clear that social injustice is present in, if not related to, the current ways of conducting business. To back up these thoughts, statistics related to discrimination in the corporate world, which represents a significant part of the overall workforce, are going to be presented.

Even if many improvements have been achieved throughout the years, gender-related discrimination is still eradicated in the work environment. A 2022 report by the World Economic Forum (WEF) shows that gender gaps are still considerable and, in some instances, widening: data shows employment losses have been significantly worse during the Covid-19 pandemic for women than for men. This adds to a historic situation of differences in the unemployment rate, which currently (the last available observation is the fourth quarter of 2021) sees a global unemployment rate of 6.5% for men and 7.8% for women. Another astonishing datum related to this topic is the relative presence of women in roles at the top of hierarchical

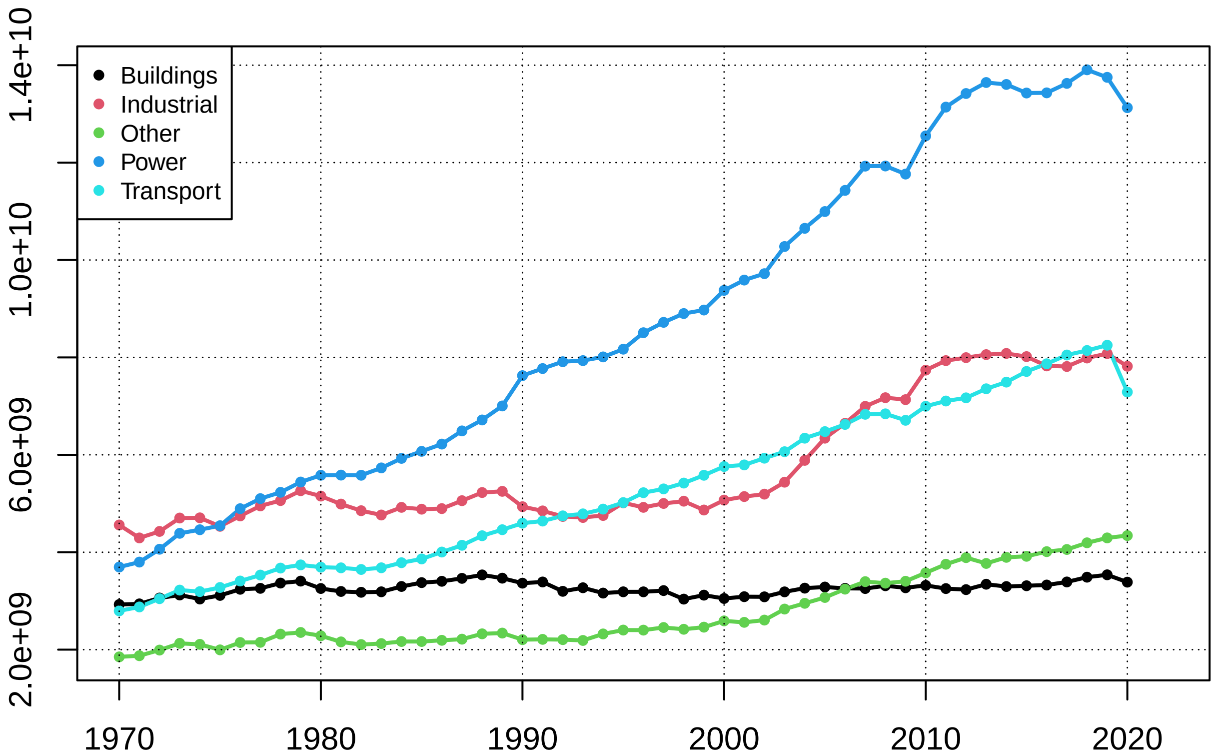

Figure 1: Global greenhouse gas emissions in $GtCO_2e$ produced by economic sector over the period between 1970 and 2020 (source: Crippa, M., et al. 2021)

structures: figure 2 presents the percentage of women and men in leadership roles divided by industries. According to these data, the sectors where gender discrimination seems to be absent are the education and non-governmental institutions, instead, the most differences occur in the energy, manufacturing, and infrastructure industries, where women hold only 16% of the top roles within organizations (WEF 2022).

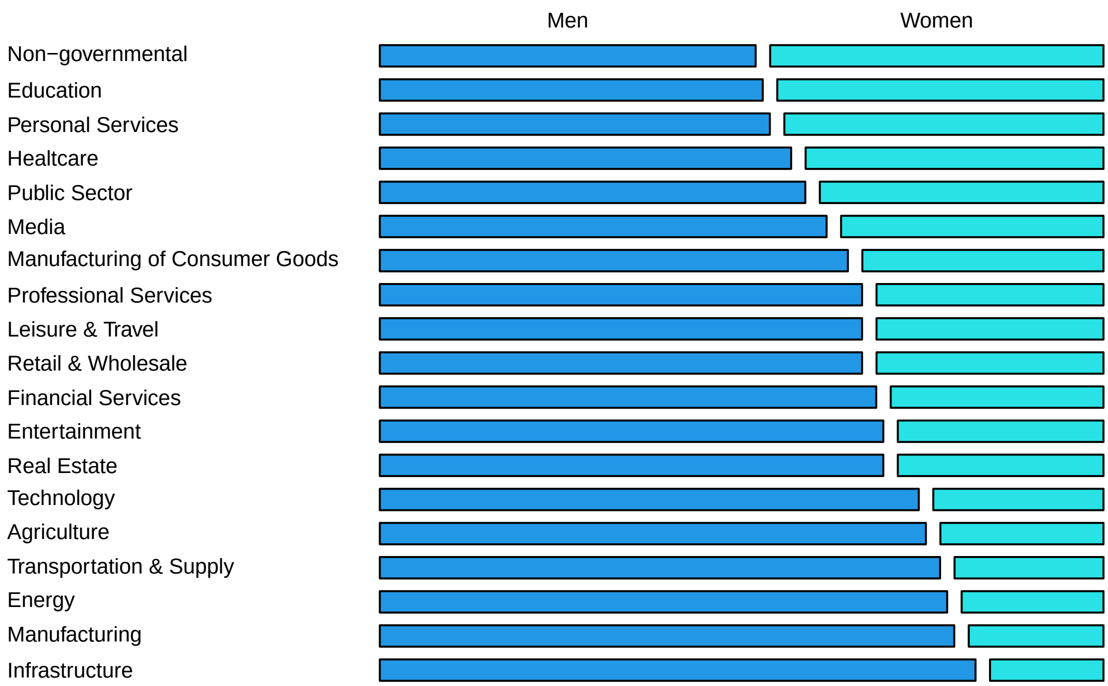

Figure 2: Percentage of men and women in leadership roles divided by industries (source: WEF 2022)

Another critical topic is race-related discrimination within the workplace, which has gained momentum for change in the last few years especially thanks to the “Black Lives Matter” movement. These injustices are difficult to monitor and generalize on a global basis because they strictly depend on a country’s ethnic composition, history, culture, etc. However, we can use data on the racial disparity in the U.S. corporate world to show this issue is present and far from a solution. The U.S. economy represents a well-suited sample for two reasons: first, statistics and research are abundant; second, American society is much more spread across different races, such as Whites, Blacks, Asians, and Hispanics, compared to other countries.

In the last two decades, the US workforce has seen socio-demographic changes in its composition: the Hispanic group has increased by 7%, the Asian increased by 2%, the White has decreased by 5%, and the Black has remained unchanged (Nelson and Vallas 2021). We can notice that the percentage increase of a group relative to the total has not been followed by a homogeneous distribution of the group across occupational categories, nor has the segregation of the Black group lessened. Therefore, the American labor market shows a significant difference between the presence of racial groups across sectors and job levels.

For instance, the business sectors where racial segregation is the most evident are business services, construction, printing and publishing, and furniture manufacturing (Nelson and Vallas 2021).

Differences in sector representations and leadership levels imply wage disparities across racial groups. According to the data published by the U.S. Bureau of Labor Statistics for the first quarter of 2021, the median weekly earnings of a full-time worker is $1006 for the White group, $799 for the Black, $1286 for the Asian, and $750 for the Hispanic (Nelson and Vallas 2021). These marked differences between races both in terms of salaries earned and positions held within the organizations are clear symptoms of social injustice. Even if the sample from which the data were obtained represents only the U.S., we can confidently state racial inequality is an issue widespread across most countries and most organizations around the globe.

We saw evidence of the presence of gender and racial discrimination within the workplace, however, these are only a few of the injustices some workers face during their working hours. Some types of discrimination are obvious and visible, such as age discrimination, particularly in the hiring process, others are more subtle and, thus, less considered by the public opinion and not addressed by laws: examples of these phenomena are bullying, social exclusion, and prejudice.

Finally, we focus on the governance-related dimension. Corporate governance can be briefly presented as the set of activities, rules, and processes in place with the aim of balancing the interests, financial and non-financial, of all the company's stakeholders. These parties are often the first victims of a malicious way of conducting business. Poor management decisions and frauds have a profound impact on workers, clients, suppliers, investors, etc. These episodes are far from rare, indeed, only in 2020 the list of infamous frauds is long. Some of the most notable examples, which are also representative of three out of four of the biggest economies in the world, are the following: Nikola Corporation, an American manufacturer of heavy-duty commercial vehicles with innovative energy solutions, misrepresented its results in terms of technological innovation, determining a decrease of the stock price around 90%; Wirecard AG, a young technological corporation belonged to the DAX 30 index and considered one of the gems the German economy, declared bankruptcy after €1.9 billion were found missing, causing the unemployment of 5100 people and losses of billions of euros to investors; Luckin Coffee Inc., a Chinese unicorn start-up in the coffee chain sector, was accused of fraud for inflating revenues for $310 million, delisted by the Nasdaq stock exchange, and fined $180 million by the SEC.

1.2 International Agreements and Sustainable Development

In the last few years, people have become more and more conscious of the issues described above. The safeguard of the environment, the social disparities, and the unconcern to take care of stakeholders' interests have been severely scrutinized topics by the public. Actions have been taken at governmental, corporate, and individual levels: the main responses undertaken by governments and public authorities, such as treaties and future commitments, are discussed in this section; the diffusion and the adoption of the concept of social and responsible finance between institutional and retail will be the topic of the next section.

In economic theory, environmental spoiling, pollution, and global warming fall under the concept of *negative externality*: the situation that occurs when part of the cost of an activity, such as the production of a good, is not borne by the party which is benefiting from the activity or it is shared with unknown or unwilling third parties. For this reason, to combat environmental problems, which are transboundary in nature and affect all countries, measures are adopted at an international level. The main instruments used are treaties and agreements between nations. The main multilateral environmental agreements are the following:

- Convention on International Trade in Endangered Species of Wild Flora and Fauna (CITES)
- The Montreal Protocol on Substances That Deplete the Ozone Layer
- Convention on Biological Diversity (CBD)
- Kyoto Protocol
- Paris Agreement

The Kyoto Protocol and the Paris Agreement share the objective of containing GHG emissions. Moreover, it is of particular interest for the sake of this work the specific objective of the Paris Agreement, which is developed in Article 2.

"This Agreement, in enhancing the implementation of the Convention, including its objective, aims to strengthen the global response to the threat of climate change, in the context of sustainable development and efforts to eradicate poverty, including by:

a. Holding the increase in the global average temperature to well below 2°C above pre-industrial levels and pursuing efforts to limit the temperature increase to 1.5°C above pre-industrial levels, recognizing that this would significantly reduce the risks and impacts of climate change;

b. Increasing the ability to adapt to the adverse impacts of climate change and foster climate resilience and low greenhouse gas emissions development, in a manner that does not threaten food production; and

c. *Making finance flows consistent with a pathway towards low greenhouse gas emissions and climate-resilient development" (UN 2015).

We focus on this article for three reasons, which give us the possibility to expand some concepts at the foundation of Sustainable Finance. First, the Paris Agreement sets the maximum increase in temperature, compared to pre-industrial levels, that would not severely threaten the environment and contain the associated risks. This target helps regulators to emanate laws in coordination, analysts and scientists monitor the improvements or the retrogressions of international efforts, and investors effectively direct capitals, to achieve the stated objectives.

Second, it introduces the concept of *sustainable development*: for instance, actions and investments undertaken to achieve the temperature target must not be a threat to food production. In 1987, a Report by the UN gave the first definition of sustainable development as "development that meets the needs of the present generation without compromising the ability of future generations to meet their own needs" (UN 1987). In other words, it is a principle on how to organize human activities, seeking innovation and improvement of quality of life but taking care of the resources used and the ecosystem on which innovation and life quality depend too. From these ideas, the Sustainable Development Goals (SDGs) were established in 2015 by the United Nations General Assembly to be achieved by 2030. The 17 goals belonging to the Agenda 2030 are grouped in the following image.

Figure 3: The 17 SDGs of the United Nations 2030 Agenda

As we can notice, every social and environmental issue that we touched on in the previous section is included in one of the SDGs.

Third, Article 2 of the Paris agreement states that finance should be one of the tools to be used to low GHG emissions and sustainable development. Finance and, thus, investments assume a central role in the preservation of the environment and in the support of social issues: we can talk about *sustainable and responsible finance*.

1.3 Sustainable and Responsible Finance

The concept of *sustainable and responsible finance* (SRF) is commonly used as a synonym for *sustainable and responsible investing* (SRI), however, for the purpose of this paper, SRI is a subset of SRF. Even though their driving objectives may be similar, on one hand, SRI includes all the financial activities performed by investors, the parties that lend capital. On the other hand, SRF combines both SRI and another set of activities, called *corporate social responsibility* (CSR), executed by investees, the business entities in which the funds are transferred. In this section, we are going to discuss the trends and the history of SRF in general and, then, we are going to give the specific definitions of CSR and SRI.

1.3.1 Trends and History

Social media and the internet have made environmental and social issues more visible and difficult to hide by malicious parties. People and, in particular, millennials, have become more sensible and aware of these types of issues. According to the last available Morgan Stanley report on trends of SRI, we see that, from their sample of individual investors, 79% are interested in the topic, in particular, among millennials, the percentage raises to 99%. Concerns about climate change are the main driver of attention: specifically, 64% of investors are concerned about environment-related issues, such as global warming and GHG emissions, and 88% between millennials (MS 2021).

Awareness about SRF has grown in the last decades not only between retail and professional investors. Figure 4 shows the trends for the searches on topics related to ESG and Sustainable Finance on Google from 2004 (Google, n.d.), where 100 corresponds to the highest monthly search volume during the period, and values in other months are calculated proportionally to the maximum. If we consider these trends as a proxy for people's interest, we deduce that, even if sustainable and responsible finance has foundations rooted long back in history, the public attention on the subject has never been higher.

The first appearance of SRF occurred in the 18th century: in 1758, the Philadelphia yearly meeting of Quakers prohibited the ownership of slaves or any trade of humans. This was the first step toward a new way of approaching finance: investment did not have to be in conflict with ethical values. Following this principle, they later divested money from weapon manufacturers, alcohol producers, and gambling businesses. A few years later, John Wesley, the founder of the Methodist movement, with his sermon "The Use of Money", helped to shape this new ethical approach to finance, stating that money at the expense of life or by losing our souls (Heaps 2021).

In 1928, the first experiment of SRF with structured products took place: Philip Carret launched the Fidelity Mutual Trust, which was later recalled by the Pioneer Fund, the first publicly offered sustainable and responsible investment fund (Heaps 2021). The interest in SRF made a big leap during the 70s when "the Vietnam War had grown more complicated for the general population and socially minded investors" (Townsend 2020). The straw that broke the camel was the utilization, for a 5-year period and over 10% of South Vietnam's surface, of "Agent Orange", a combination of toxins designed to kill forests' flora and fauna. In order to provide an option to avoid investment in companies that were related to the production of this chemical weapon, the Pax World Balanced Fund was launched in 1971. Other similar initiatives followed in the same period: the First Spectrum Fund was established in 1972 and the Dreyfus Third Century Fund in 1972 (Townsend 2020).

In the 80s, after the infamously known environmental disasters, such as the massive Exxon Valdez oil spill, the entire world's attention was for the first time directed towards the environment and climate change. As a consequence, the United States Sustainable Investment Forum (US SIF) was established in 1984: this association has led the industry and shaped the

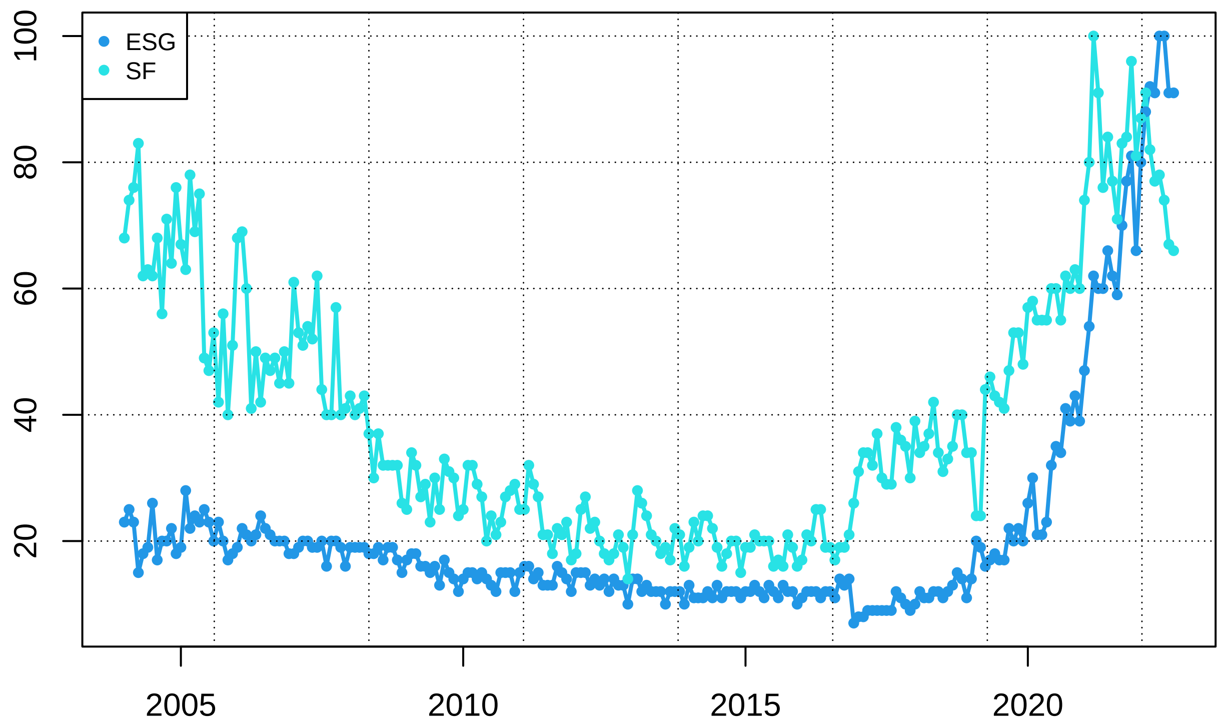

Figure 4: Search volume of the words "ESG" and "Sutainable Finance" on Google during the period between 20004 and 2022, where 100 corresponds to the maximum volume reach within the period and the other observations are computed accordingly (source: Google Trends)

principles of SRF, paving the way for the emergence of a new industry. First, the Nouvelle Strategies Fund was launched in France in 1983, the Friends Provident Stewardship Fund in the UK in 1984, and the Sanpaolo Azionario Internazionale Etico in Italy in 1997. Second, the Dow Jones Sustainability Index, the first global sustainable and responsible index, was released in 1999 and followed by Ftse4Good in 2001. Third, the first agencies, such as KLD, Sustainalytics, and Asset4, released ratings related to the dimensions of sustainability and responsibility. As of today, sustainable and responsible assets have reached $35.3 trillion (GSIA 2020) and, as shown in figure 5, which represents the evolution of the U.S. market measured in trillions of dollars, this type of investment has increased at an exponential pace (SIF 2020).

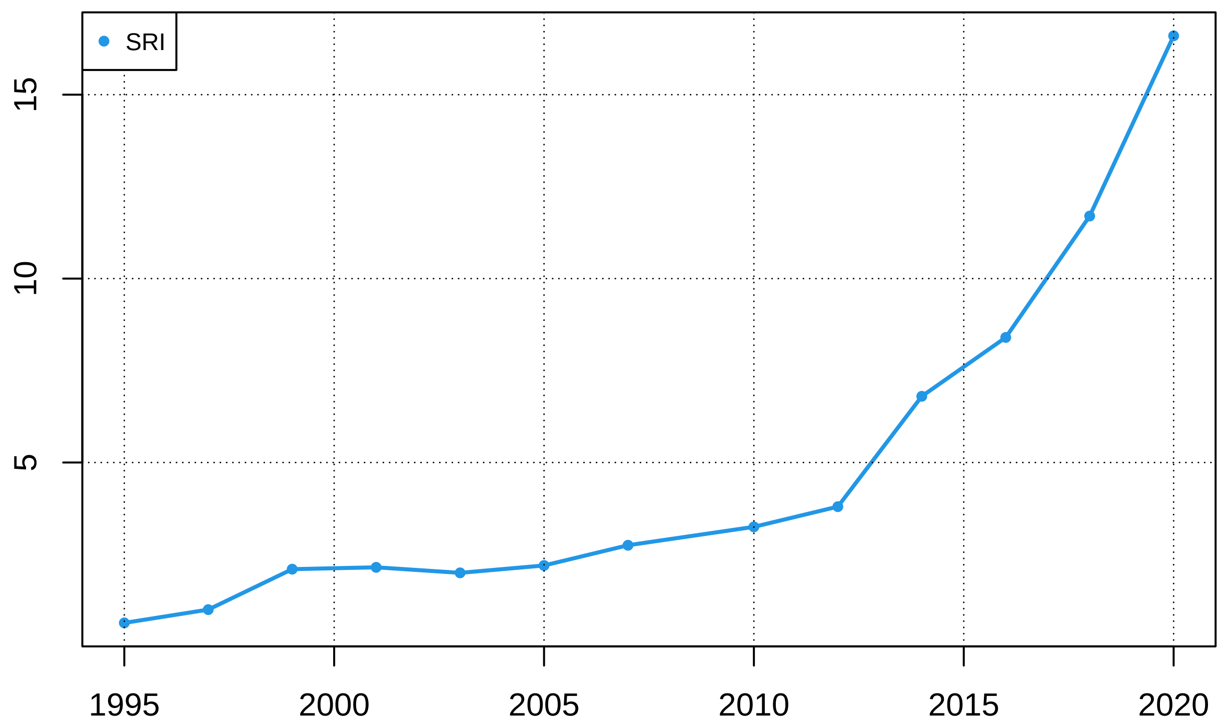

Figure 5: Value of sustainable investments in the United States during the period between 1995 and 2020 expressed in trillion of dollars (source: SIF 2020)

The data we just presented refer all to the SRI side of SRF: the part that deals with how money is invested in a sustainable and responsible way. For years, it has been studied that "SRI investors push the industry, they are not pulled" (Townsend 2020). In other words, the ethical values of investors, and society at large, shape accordingly corporate's culture and way of conducting business: CSR. One tangible effect of SRI pressures on companies is the level of sustainability reporting: the disclosures, often on a voluntary basis, toward outsiders of information related to environmental and social aspects of the business. A KPMG report shows that, in 2020, 80% of the companies in the sample considered, a set of 5,200 companies that comprises the top 100 corporations by revenue in each of the 52 countries and jurisdictions considered, reported on sustainability. As shown in figure 6, this is the maximum of a trend that is steadily growing year after year (KPMG 2020b).

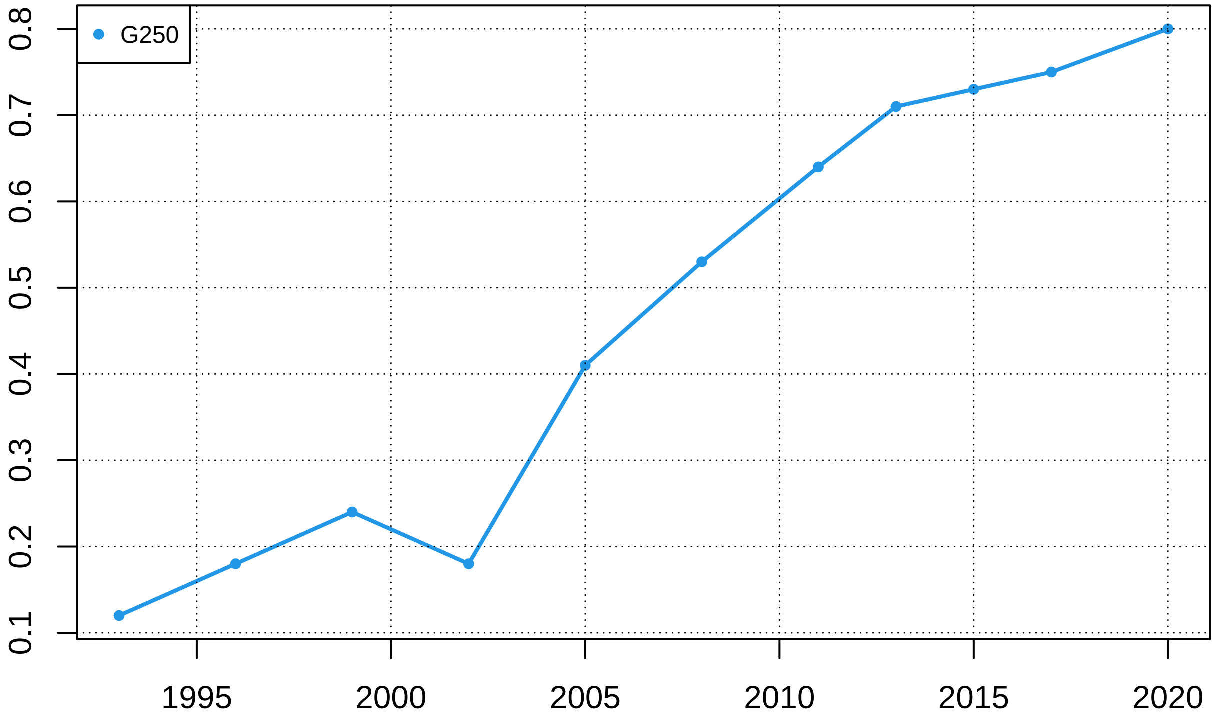

Figure 6: Percentage of the corporations, from a sample of 5200 comprising the top 100 companies by revenues in each of the 52 countries considered, that release reports on sustainability during the period between 1993 and 2020 (source: KPMG 2020b

1.3.2 Corporate Social Responsibility

"Despite its long history no consensus has been developed among the industry participants, academics or other interested parties" on a common definition of CSR (Sheehy 2015). Following the reasoning of the scholar, the creation of a distinctive meaning of CSR is a complex task, for the following reasons: CSR affects and is affected by ecological, societal, and economic dimensions, which are shaped by complex interacting systems; an organization's CSR is the opinion whether the activities performed by the organization itself are socially responsible, as a consequence, each entity has the incentive to shape the idea of CSR in a way that is compatible to their existing culture and values; many definitions of CSR have been given by academics reflecting different and contrasting perspectives; the dialogue around CSR has been shaped by different political and economic ideologies; in many countries, governments have used CSR as a political tool to affect desired change in a less costly manner than traditional means, such as laws and regulations, however, in order to nudge national enterprises to direct their efforts on the subject, the definition of CSR has been accordingly adapted.

Understood that the meaning of CSR depends on a number of specific factors, we can find an institutional definition provided by the EU Commission. It defines CSR as "the responsibility of enterprises for their impacts on society”, then, it adds that “to fully meet their corporate social responsibility, enterprises should have in place a process to integrate social, environmental, ethical, human rights and consumer concerns into their business operations and core strategy in close collaboration with their stakeholders, with the aim of: (i) maximizing the creation of shared value for their owners/shareholders and for their other stakeholders and society at large; (ii) identifying, preventing and mitigating their possible adverse impacts” (Commission et al. 2011). We can notice that, according to the EU commission, enterprises, including both profit and non-profit organizations, have the responsibility to act sustainably and responsibly: the corporation’s objective is not limited to maximizing value for shareholders but also consists of maximizing value for stakeholders and the society at large. Even if the advantages of this view relative to an only value-maximizing entity seem obvious at a first glance, academics have not agreed on which way of conducting business should be preferable yet.

The traditional economic perspective on CSR can be perfectly expressed by a sentence that appeared in the New York Times in 1970: “there is one and only one social responsibility of business—to use its resources and engage in activities designed to increase its profits so long as it stays within the rules of the game, which is to say, engages in open and free competition without deception fraud” (Friedman 1970). Recalling the same reasoning provided by Friedman, let us consider the case of a corporation. A publicly listed company does not have a social responsibility because ethics is a concept related to the consciousness of individuals. Therefore, CSR is embodied by the people who are in charge of taking decisions within the organization: corporate executives. Managers are agents of shareholders, employed to serve owners’ best interest, thus, whenever they act according to some sort of social responsibility which is in contrast with their mandate, they are breaching their contractual relationship with their principal.

In addition, executives that behave following CSR principles, at the same time, are effectively imposing taxes and deciding how to use the proceedings. Let us consider the three following examples. If, in an inflationary environment, a manager decides not to increase products’ prices to contribute to the fight against inflation, even though a price raise would be in the best interests of shareholders, she is effectively deciding how to spend owners’ money. If a manager decides to undertake an expenditure program, which is reflected in a product’s price increase, to reduce pollution to a level that is beyond the legal requirement and the economic optimum, she is effectively deciding how to spend customers’ money. Finally, if a manager hires long-unemployed workers instead of highly-qualified professionals, to reduce poverty and social inequalities, she is spending more-experienced workers’ money. To sum up, in all instances, the executives would decide how to employ someone else capital without their permission, a process that should only be exercised and regulated by the government because of the need for an underlying elaborate set of institutions (Friedman 1970).

If Friedman’s was the prevailing school of thought, the EU Commission’s definition of CSR would not make any sense. In fact, in more recent times, the dominant point of view can be summarized by Freeman’s *Stakeholder Theory*: not only shareholders, but all stakeholders, composed of owners, managers, members of the local community, customers, employees, suppliers, etc., have a moral on the corporation for the fact that the business entity has the capacity to produce good or harm to all of them. This is a two-way relationship: a corporation influences the life quality of stakeholders, but the latter are essential for the survival of the business. Therefore, whenever a decision has to be reached, the point of view of all the interested parties not only should but must be taken into consideration for the good of the company (Freeman 2016). For instance, if executives behave in a way that hurt employees, they may induce the start of strikes and protests, or if they do not consider policies of pollution reduction, they may cause insurrections of local groups and authorities.

1.3.3 Social and Responsible Investing

"Socially responsible investment (SRI) – sometimes termed ethical investment – refers to the practice of integrating social, environmental, or ethical criteria into financial investment decisions" (Cowton and Sandberg 2012). Expanding this statement, SRI is an investment practice that includes new factors within the investment process: other variables should be considered other than the risk and the return. Traditionally, established an investable universe and estimated the specific variable for each asset belonging to this set, the portfolio optimization process consists of two separate steps: (i) the *efficient frontier*, the set of achievable portfolios that offer the best return for each level of risk or equivalently the lowest risk for each level of return, has to be found; (ii) the portfolio lying on the frontier which satisfies a specific level of risk-aversion, specific to each investor, is picked (Markowitz 1952). Generally speaking, SRI changes the way the first step of the traditional optimization process is conducted: the investor set of preferences is not limited to the risk-return dichotomy but includes also social, environmental, and ethical considerations.

From this new way of approaching investing, two problems arise: first, how to incorporate individual preferences belonging to the realm of ethics into a mathematical optimization problem, in other words, how to translate into quantitative terms an investor's qualitative inclinations; second, solved the first problem, how to include the newly defined variables into the optimization process, specifically the calculation of the efficient frontier. The answer to the first problem is ESG scores, which are discussed in the next section, instead, trying to answer the second problem is the purpose of this Master's Thesis.

1.4 ESG Scores

Specialized rating agencies value companies' performance along the three dimensions of sustainability through ESG scores, also called ESG factors. ESG stands for "environmental, social, and governance": the three conventionally accepted dimensions of sustainability. The subtopics that are included within the environmental, social, and governance dimensions are presented in the following table (CFA 2022).

Table 1: subtopics that are included within the environmental, social, and governance dimensions (source: CFA 2022

Environmental	Social	Governance
Climate change and carbon emissions	Customer satisfaction	Board composition
Air and water pollution	Data protection and privacy	Audit committee structure
Biodiversity	Gender and diversity	Bribery and corruption
Deforestation	Employee engagement	Executive compensation
Energy efficiency	Community relations	Lobbying
Waste management	Human rights	Political contributions
Water scarcity	Labor standards;	Whistleblower schemes

1.4.1 Data Providers

The primary role of ESG scores is to serve as a quantitative measure to be included in the investment process: the types of inclusion are the topic of the next chapter. PRI reports that 3038 investors representing over $100 trillion worth of assets have committed to integrating ESG information into their investment decisions (PRI 2020). Therefore, ESG rating agencies have acquired a powerful and influential role, since their judgment has a direct impact on investment choices and flows of capital into the real economy.

The industry of ESG data and data providers is large and in rapid expansion: it was estimated that there are more than 600 products, such as ESG data, ratings, and rankings, from over 150 organizations (Hawley 2017). However, the biggest player in the industry, whose data are often more frequently used in academic research are 7: KLD, Sustainalytics, Moody's ESG (previously Vigeo-Eiris), S&P Global (previously RobecoSAM), Refinitiv (previously Asset4), MSCI, FTSE, RepRisk, and ISS Environmental & Social QualityScore. Table 2 shows each organization's country of origin and rating scale.

Table 2: Country of origin and rating scale utilized by the 7 major ESG ratings providers

Agency	Country	Scale
KLD	US	from -1 to +1
Sustainalytics	Netherlands	from 0 to 100
Moody's ESG	US	from 0 to 100
S&P Global	US	from D- to A+
Refinitiv	Switzerland	from 0 to 100
MSCI	US	from CCC to AAA
FTSE	UK	from 0% to 100%
RepRisk	Switzerland	from D to AAA

Agency	Country	Scale
ISS	Germany	from 0 to 10

Even if ESG scores have become a fundamental part of the investment process and an entire industry has flourished around the topic, as of today, the absence of uniformity between data from different providers limits the investors' ability to benefit from this type of information and of academics to achieve coherent results.

1.4.2 Scores Divergence

A few years ago, in the Wall Street Journal appeared an article with the provocative title "Is Tesla or Exxon More Sustainable? It Depends Whom You Ask" (Mackintosh 2018). The journalist used the ESG scores of some of the famous American stocks, to highlight the divergence of opinions between rating agencies. The most emblematic example is Tesla, which at the period had a combined ESG score at the top of the industry according, if we were looking at the ratings provided by MSCI, or the worst carmaker in the world, if by FTSE, or average, if by Sustainalytics.

The situation becomes incredibly surprising if we look at correlations between the ESG scores of different rating agencies. A paper by (Gibson Brandon, Krueger, and Schmidt 2021), which studies the divergence of the ESG scores from 7 different organizations over a period of 7 years, found that the average correlation is 0.45. However, we should say that there were some cases where variables seemed to be more correlated: for instance, the combined ESG score for Sustainalytics, Refinitiv, and Bloomberg had a correlation of around 0.75. These results show a high degree of divergence if we make the comparison with credit default ratings: the correlations between credit default ratings issued by Moody's Investors Service and Standard & Poor's were estimated to be 0.99 (Berg, Koelbel, and Rigobon 2019).

The absence of coherence between ESG scores is not only a quantitative curiosity but has tangible implications on the overall SRF industry. Following the reasoning of Berg, Koelbel, and Rigobon (2019), this type of divergence has five main drawbacks. First, it creates doubts about the reliability of these measures and, thus, makes difficult the achievement of their primary purpose: assessing the ESG performance of a company, fund, etc. Second, it decreases companies' interest in improving their ESG performance, since a change may return conflicting results from the ratings of different agencies. Third, it makes it less likely that market prices incorporate ESG information: these scores should influence assets' prices according to investors' preferences, but the absence of reliability tends to extinguish this relationship. Fourth, it eliminates the possibility to reward corporate top executives with a compensation plan that is directly linked to the company's ESG performance. Fifth, it lessens the validity of the research around the topic, since empirical results are strictly linked to the sample period and source considered. Now, let us discuss the motivations behind the divergence of ESG ratings.

The causes of this lack of consistency can be divided into two groups: the first set is composed of 3 quantitatively identifiable and objective causes, and the second set of 4 more subtle reasons related to unconscious human behavior, a topic studied in Behavioral Finance. The first cause within the first group is *scope divergence*, which refers to the instance when two ESG scores are constituted by different attributes. The WSJ's article cited above gives us an example: while MSCI considers as one of the attributes of the ESG score of car manufacturers the emissions produced by their products, FTSE does not consider that attribute but the one related to the emissions produced by the factories, yielding two completely different results. The second cause is *measurement divergence* which refers to the fact that two attributes may be calculated through different models. For example, for the calculation of the score related to GHG emissions, which is a relevant part of the environment-related score, a rating agency considers methane as a GHG, while another considers only carbon dioxide. If the company evaluated releases a large quantity of methane, it has a relevant impact on its ESG performance. The third cause is *weight divergence*, which takes place when, in the calculation of an ESG score, two agencies give different weights to the same attribute. Empirical results suggest that measurement divergence explains 56% of the overall ESG rating divergence, scope divergence 38%, and weight divergence 6% (Berg, Koelbel, and Rigobon 2019).

Behavioral Finance's arguments may be used to explain the problem from a different perspective. Substantial evidence of the presence of a *rater effect*, also known as *halo effect*, the tendency of an agency to high assess a company in all categories, has been found (Berg, Koelbel, and Rigobon 2019). In addition, it has been found evidence of other three behavioral causes for rating divergence: *company size bias*, the tendency to give a higher rating to top-tier companies for market value; *geographic bias*, the propensity to evaluate a company's ESG performance based on the country or jurisdiction where it is headquartered; *industry-sector bias*, the habit to not capture adequately industry-specific risks and characteristics, unfairly evaluating companies within one business sector relative to another (Doyle 2018).

2 ESG Integration

2.1 SRI and Financial Performance

As mentioned in the previous chapter, the last available information shows that globally the value of social and responsible investments is around $35.3 trillion, which means that 35.9% of total assets under management are ethical (GSIA 2020). Recalling the trend in figure 5, the growth spurt of the industry in the last few years is astonishing: during the 2016-2020 period, SRI increased by 54.3% compared to a mere 6.8% increase in traditional assets. The reasons for this difference must lay within the change in investors' values and preferences: the acquired new awareness of social and environmental issues has played a pivotal role. However, scholars suggest that SRI may be superior in performance to traditional investments and, according to this assumption, even investors that do not derive any utility from ethical assets are attracted by the potential of a generous return.

The question of whether SRI is superior in performance to traditional investing has obsessed academics since the 70s when the concept of SRI was first coined. In figure 7 we see the estimation performed by Friede, Busch, and Bassen (2015) of the number of cumulative studies on the relationship between ESG and corporate financial performance during the period between 1970 and 2015. In the same paper, it is calculated that, given a sample of more than 2000 academic researchers, 89.3% of them show a non-negative relationship and 48.2% show positive results (Friede, Busch, and Bassen 2015). In other words, the vast majority of the academic literature agrees on the fact that SRI is superior or at least non-inferior to traditional investments based on performance considerations only.

We can conclude that empirical evidence shows that SRI does not sacrifice monetary returns in favor of the achievement of ethical purposes, rather, many scholars suggest the presence of a positive relationship between sustainability attributes and financial performance. In this regard, it is particularly interesting the connection between ESG-related news and stock price movements. Evidence from event studies shows negative ESG news tends to decrease stock prices and positive news tends to increase them. "Capital markets are clearly penalizing firms whose social costs exceed their private costs. Precisely why they do this is not clear from the studies: presumably capital markets are anticipating that firms will at some point be forced to pay for the excess of private over social costs, or at least for some part of this. They are therefore treating this excess, or some part of it, as a liability to the firm" (Heal 2005). Another possible explanation of this type of behavior may be the fact that investors' preferences are described by a utility function that includes ethical dimensions. Therefore, a change in ESG-related attributes of a company modifies investors' preferences.

Now, we are going to try to explain the existence of a positive relationship between sustainability attributes and financial performance. Part of the motivations belongs to the sphere of CSR and part pertains to the sphere of SRI: on one hand, a social and responsible investee can impact its stock price, on the other hand, investors' approach to SRI must have an influence on prices. Starting from the CSR side, a paper by Heal (2005) lists the contributions of effective CSR programs that may have a positive impact on the company's stock performance: (i) reducing risk, (ii) reducing waste, (iii) improving relations with regulators, (iv) generating

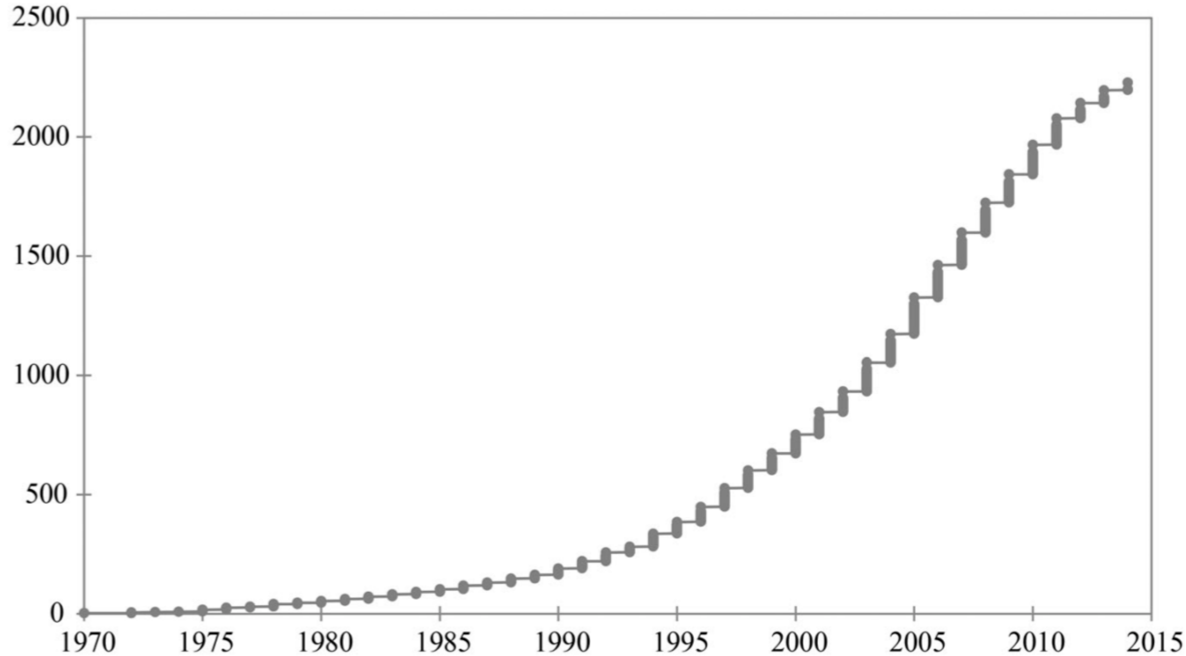

Figure 7: Estimated cumulative number of empirical studies on the relationship between ESG or corporate social responsibility and financial performance during the period between the 1970 and 2015 (source: Friede, Busch, and Bassen 2015)

brand equity, (v) improved human relations and employee productivity, and (vi) lower cost of capital. Now, we are going to present the reasoning of (Heal 2005).

First, a well-structured and implemented CSR program serves also as a tool for conflict avoidance. Corporations' disputes with other groups of society can be incredibly costly, especially with aggressive NGOs, in terms of direct consequences, such as legal expenses, fines, and reputational damage, and indirect ones, such as earnings and share price depression. An emblematic example is the Monsanto Company, an American giant working in the agricultural biotechnology sector and recently acquired by Bayer AG, which has been frequently pointed out for controversial activities by organizations, such as Greenpeace, and has been forced to pay several million in fines. For instance, during the Vietnam war, Monsanto Company was the producer of the already discussed Agent Orange. An organization implementing an effective CSR program does not pay litigation costs from conflicts with stakeholders and, thanks to a good reputation, may secure part of the market share of its less judicious competitors.

Second, CSR may reduce companies' waste. The paper suggests that a company such as Dow Inc., which is one of the biggest producers in the world, implementing a strict program to avoid the loss of chemicals to the environment, would, at the same time, improve its ethical standards and reduce product waste. Possible savings from improved management and processing are often overlooked because of their nature of non-monetary costs. A sustainable and responsible organization may benefit from savings in these hidden costs and, thus, improve its performance relative to its non-sustainable peers.

Third, regulators are often inclined to take decisions in favor of a company with a strong reputation rather than in favor of a company with a history of social and environmental misconduct. Indeed, a solid CSR program makes lobbying pressure more effective, which may constitute a significant advantage in heavily regulated industries. These kinds of practices are routine in the oil and gas industry: for example, BP plc delayed and forced the European Commission to revise a decision on GHG emissions that would have negatively impacted its business (Abnett and Nasralla 2021).

Fourth, CSR may not be limited as a tool for conflict avoidance but also used as a competitive advantage, if properly marketed to the public. This has been the strategy of Starbucks Corporation, the most famous chain of coffeehouses in the world. The company has developed a strong reputation of sustainability around its brand through some thoughtful decisions: in 2000, it started the sale of fair trade products and, recently, has started the purchase of coffee through the Fair Trade NGO, to guarantee fair living standards for coffee-growers communities; in 2008, it entered into a partnership with Conservation International, an NGO with the objective of preserving the global biodiversity, to protect the natural environment within the coffee-producing community. Therefore, Starbucks Corporation has positioned itself as the industry leader in social and environmental practices, expecting that customers will positively discriminate in favor of the responsible organization or, at least, avoid non-sustainable peers.

Fifth, sustainable organizations have an advantage in recruiting and maintaining their workers. A survey on the career preferences of a sample of European and American MBAs shows that "reputation-related attributes of caring about employees, environmental sustainability, community/stakeholder relations, and ethical products and services are important in job choice decisions" (Montgomery and Ramus 2003). Employees are willing to accept a lower salary in order to work for organizations with a strong reputation. In addition, according to the *efficiency wage theory* (Marshall 1890), a pay rise increases employees' productivity, offsetting costs with a boost in output production. Therefore, a socially responsible organization is able to acquire the best talent in the job market and achieve a greater level of productivity with respect to its competitors.

Sixth, SRI investors typically exclude from their assets universe all the companies that are related to controversial practices and focus on the best performers in sustainability matters. Therefore, only organizations with high-level sustainability standards benefit from being the target of both SRI and non-SRI investors. For this reason, ethical corporations have, everything equal, a lower cost of capital than non-ethical peers, which find it more difficult to raise equity capital.

Focusing on the group of reasons related to the SRI side that increase the financial performance of a responsible asset, we find two related motivations. Socially responsible investors show a higher degree of loyalty, which translates into fewer withdrawals of funds during economic downturns and higher injections during periods of economic expansion. "This then translates into higher profitability and reduces the sensitivity of responsible businesses against systematic risk and economic downturns" (Omura, Roca, and Nakai 2021). In other words, investor loyalty makes responsible investments less sensible to the economic cycle, which helps their financial performance in terms of risk-return trade-off.

Coherently with the anecdotal evidence of investors' loyalty towards ethical investments, the Bollen (2007) finds that the monthly volatility of cash flows is lower in SRI funds than in conventional funds. To achieve these results, the author develops 3 different hypotheses to be tested, we summarize his reasoning below.

The efficient market hypothesis (EMH) implies that mutual fund investors should not be concerned with past performance but only with fund expenses because the average risk-adjusted performance of the fund is a reflection of just the fund expense and the fund value variations are random (Jensen 1968). In a world where the EMH hold, the observed mutual fund cash flows, which are independent of change in fund expenses, do not seem to make sense. Using concepts of behavioral finance, this phenomenon may be explained by the *representative heuristic*, "the subjective probability of an event, or a sample, is determined by the degree to which it: (i) is similar in essential characteristics to its parent population; and (ii) reflects the salient features of the process by which it is generated" (Kahneman and Tversky 1972). In our context, it means that mutual fund investors do not consider their prior expectations and opinions, basing their decision on the most recent results: directing funds toward the best performers and withdrawing funds from the worst ones. Another possible explanation, proposed by Berk and Green (2004) and others, is the fact that the mutual funds' cash flows-performance relationship is due to an update of investor beliefs on the fund's expected future return.

Mutual funds' inflow and outflows of capital are determined by the change in investors' expectations, which are in contradiction with EMH, and by other observable factors, such as liquidity needs, change in consumption levels, or change in the exposure to attributes different from risk and return. It is natural that the study of cash flows volatility takes into consideration the net effects, without associating the cash flows with their end. That being said, the 3 hypotheses are articulated as follows:

- "The flow-performance relation and fund flow volatility of SR funds is equal to that of conventional funds
- The flow-performance relation of SR funds is stronger than that of conventional funds
- The flow-performance relation of SR funds is weaker than that of conventional funds, and the fund flow volatility of SR funds is lower than that of conventional funds" (Bollen 2007)

The first hypothesis respects the EMH and the assumption that an investor utility function is determined solely by the first two moments of the mutual fund's returns distribution. The second hypothesis is coherent with a modified utility function that includes a variable representing the sustainability dimensions of the fund over the conventional expected return and variance variables. The third hypothesis depends on two assumptions. First, investors' preferences are described by a utility function constituted of additive attributes: where the attributes are expected return, variance, and the sustainability dimension. Additivity, which makes the problem more tractable from a mathematical point of view, assumes independence between variables. Second, at least part of the SRI capital is invested with a long-term perspective: some examples of investors that may invest in ethical mutual funds with a long-term time horizon are charitable foundations and universities. Finally, as anticipated above, the results from hypotheses testing show that mutual funds' cash flows have significantly lower monthly volatility for ethical funds compared to conventional ones, supporting the third proposed hypothesis (Bollen 2007).

2.2 SRI Strategies

In the previous chapter, we presented the concept of SRI, which we can briefly define as "a long-term oriented investment approach which integrates ESG factors ... in order to better capture long term returns for investors, and to benefit society by influencing the behavior of companies" (Eurosif 2018). However, reaching an agreement on what is sustainable and what it is not is still an unresolved task: the definition of SRF and SRI are vague and create some grey areas, which may be considered sustainability related depending on the observer. The European Sustainable and Responsible Investment Forum (Eurosif), with the support of the European Commission, worked on the production of a common taxonomy, introducing a definition of SRI in 2016 and classification of SRI approaches in 2012. In table 3 we can see that Eurosif's categorization of strategies is aligned with other already implemented frameworks.

Table 3: Eurosif's classification of SRI strategies compared to other international accepted frameworks (source: Eurosif 2018)

Eurosif	GSIA-equivalent	PRI-equivalent	EFAMA-equivalent
Exclusion of holdings from investment universe	Negative/ exclusionary screening	Negative/ exclusionary screening	Negative screening or Exclusion
Norms-based screening	Norms-based screening	Norms-based screening	Norms based approach (type of screening)
Best-in-Class investment selection	Positive/ best-in-class screening	Positive/ best-in-class screening	Best-in-Class policy (type of screening)
Sustainability themed investment	Sustainability-themed investing	Sustainability-themed investing	Thematic investment (type of screening)
ESG integration	ESG integration	Integration of ESG issue	–
Engagement and voting on sustainability matters	Corporate engagement and shareholder action	Active ownership and engagement (three types): • Active ownership Engagement • (Proxy) voting • Shareholder resolutions	Engagement (voting)

Eurosif	GSIA-equivalent	PRI-equivalent	EFAMA-equivalent
Impact investing	Impact/community investing	–	–

It is interesting to compare that figure 8 from the Eurosif 2018 report showing an overview of the value of the European asset under management belonging to each SRI strategy and figure 9 from a private survey (KPMG 2020a). In the latter, given a sample of hedge fund managers and institutional clients representative of $6.25 trillion of assets under management, it is shown which types of SRI strategy organizations adopt. The two figures are in line apart from two aspects. First, in figure 9 norm-based screening is not considered, strengthening the argument for the need for a common framework. Second, the strategy related to ESG integration is more relevant in the second figure: this may be caused for the fact that ESG integration has increased its relative importance from 2018 to 2020 and that this type of strategy may be more implemented in the rest of the world (especially the U.S.) than in Europe. Now, the SRI strategies are going to be briefly presented.

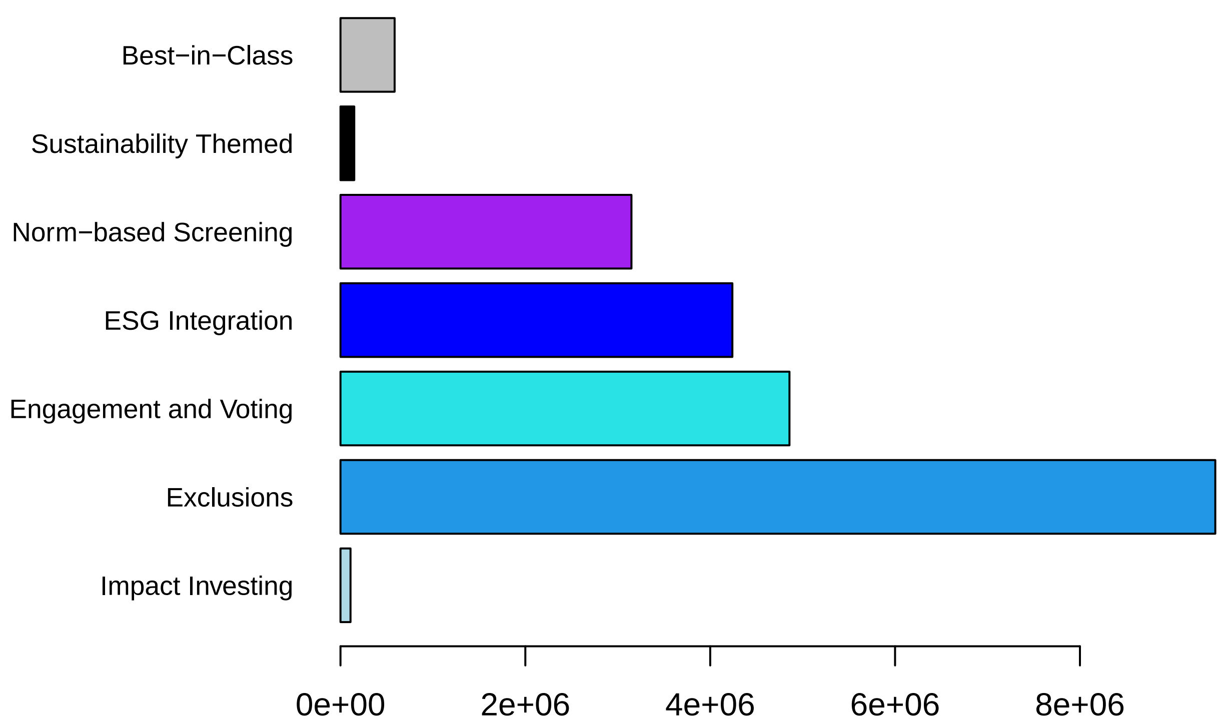

Figure 8: Value of the European asset under management belonging to each SRI strategy expressed in billion of euros (source: Eurosif 2018)

- The *Best-in-class* is an SRI strategy that, given some specified non-monetary performance measurements, such as ESG scores, creates an investment set with the best performers in each industrial sector.

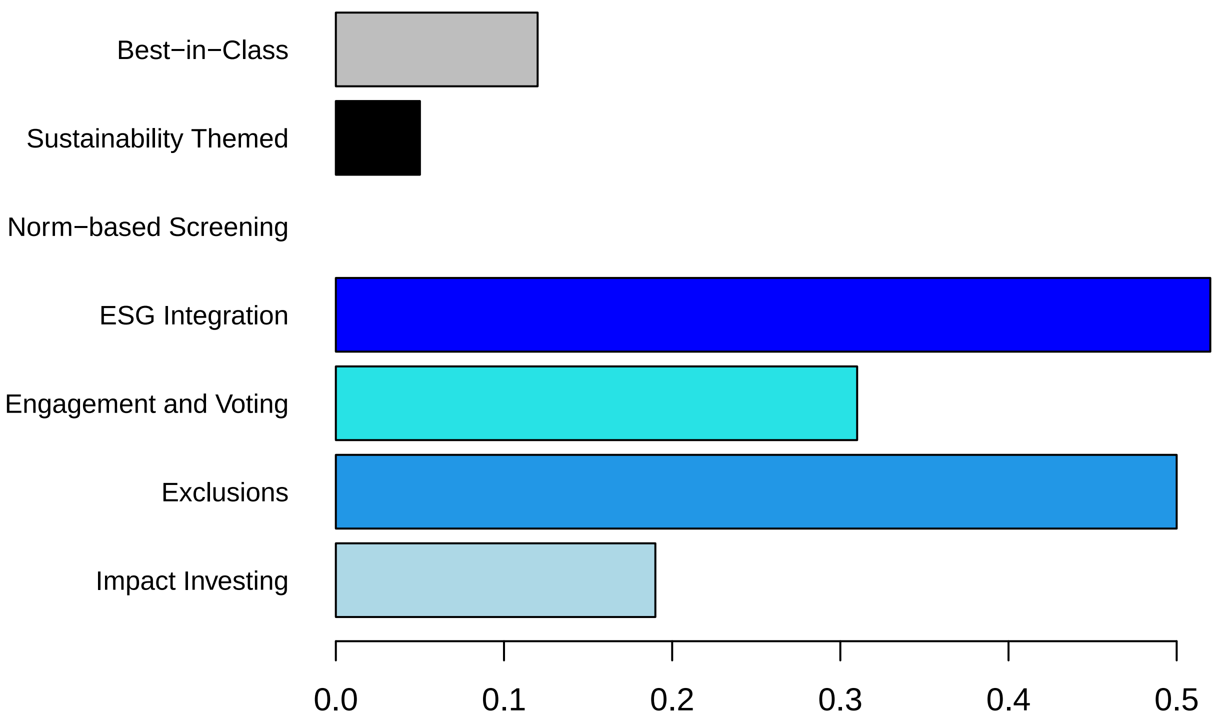

Figure 9: Percentage of asset managers within a sample representing 6 trillion dollars that adopt each SRI strategy (source: KPMG 2020a)

- Single- or multi-themed funds adopt the *sustainability themed* strategy that consists of a selection of assets that are related to one or more sustainability themes. For example, the stock of a solar panel manufacturer is the perfect candidate for a fund whose sustainability theme is renewable energies. In figure 10, the distribution of sustainability themes in Europe is shown (Eurosif 2018).

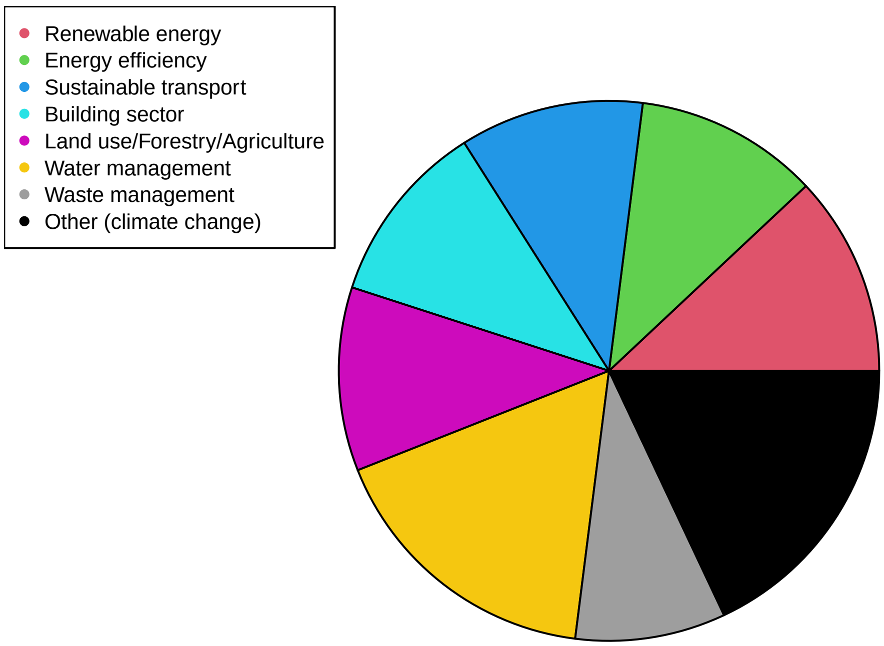

Figure 10: Sustainability themed strategy's distribution of themes (source: Eurosif 2018)

- *Norm-based screening* is an SRI strategy that allows investors to discriminate against assets not complying with international standards and norms. Examples of these are international agreements and guidelines related to the spheres of environmental protection, human rights, labor standards, and anti-corruption principles. The distribution of the main norms utilized in this type of strategy is displayed in figure 11 (Eurosif 2018).

- Through the *engagement and voting* strategy, shareholders try to affect change within a corporation. They may start a dialogue with the top management or exploit their voting rights to replace the management with someone more in line with the sustainability priorities.

- The *exclusions* strategy is based on the application of a negative screen on the investable universe. In this sense, norm-based screening is a particular type of exclusions strategy where the negative screen is given in compliance with some standards and norms. The most common criterion used to discriminate through this strategy is the association with a controversial business sector, such as weapons, tobacco, animal testing, gambling, etc.

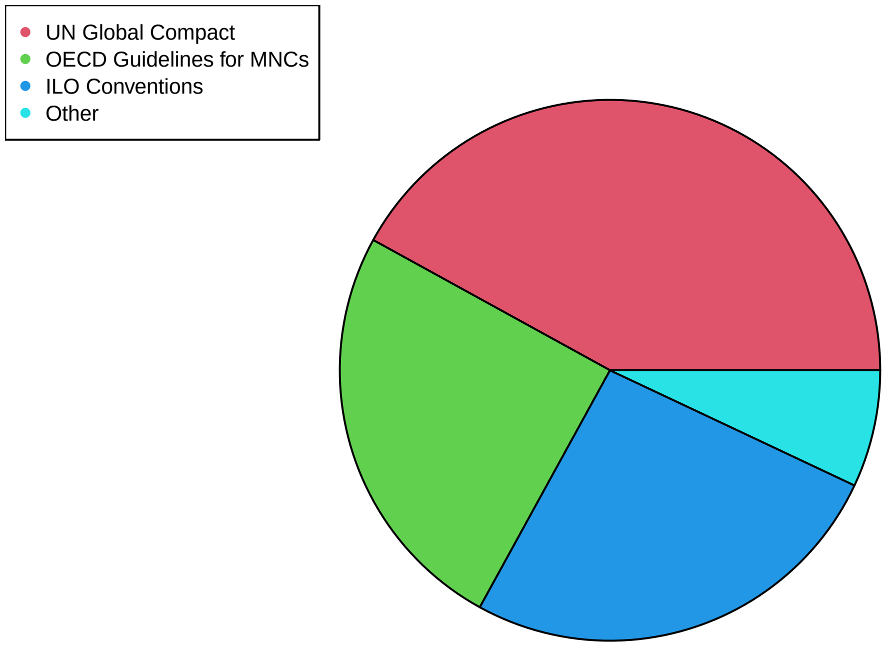

Figure 11: Norm-based screening strategy's distribution of standards (source: Eurosif 2018)

- *Impact investing* is an SRI strategy that satisfies 3 key requirements: (i) intentionality, (ii) additionality, and (iii) measurement. Intentionality is the willingness of an investor to affect sustainable change and have an impact on the social and environmental dimensions. Additionality is the ability to affect an impact that goes beyond the mere transmission of capital. Finally, measurement is the ability to show through quantitative scales the impact achieved by this type of strategy on the social and environmental dimensions.

There is a passionate debate between academics and practitioners on which types of investment practices fall and which do not into the category of *ESG integration* strategy. For now, we define an ESG integration strategy as an investment process that includes the adoption of ESG scores. In the next section, we perform a literature review of the works that we think have mostly contributed to shaping opinions on the topic.

2.3 Literature Review on ESG Integration

The problem of incorporating ESG considerations into the portfolio selection process is addressed in different ways across academics and practitioners. The easiest and first-developed methodology is screening. Portfolio managers use positive or negative screens on the investment universe: in the case of negative screens, eliminating from the investment's basket the worst ESG-performing stocks, in the case of positive screens, instead, picking the best-in-class assets. The question of whether screens have an impact on portfolio performance has been a debate among scholars for years. In particular, we can say that the conclusion differs based on subjective criteria, such as the choice of the performance measure and the available or chosen sample data. Now, we present some papers that focus on ESG screening.

Auer (2016) suggests that there exist 3 possible schools of thought about the relationship between a high level of sustainability, which may be represented by a company's ESG score relative to its industry, and financial performance (i.e. dividend yield plus capital gain), and, thus, about the effect of ESG-negative-screens:

- stocks characterized by a high ESG score are more profitable and financially stronger than less-responsible firms for the reason discussed by Heal (2005);
- excluding firms with low ESG ratings from the investable universe may negatively affect portfolio returns for 2 reasons: (i) excluded firms, avoiding costs related to sustainability matters, are more profitable at leat in the short-term, (ii) ethical investors impose themselves a constraint on the choice of assets, thus, reducing the benefits from diversification;
- financial performance is independent of ESG performance because the market does not price the sustainability dimension, implying that any relationship exists (Auer 2016).

To assess which of the 3 hypotheses better represents reality, using the set of stocks included in the Eurostoxx 600, the author builds equally-weighted portfolios of (i) only the ESG-rated stocks, (ii) the 95%, (iii) the 90%, (iv) the 85%, and (v) the 80% best ESG-performing assets. This paper concludes that negative screens excluding non-rated stocks have a significant impact on performance but a stricter selection neither adds nor destroys portfolio value. On the other hand, "because of a loss of diversification, positive screens can cause portfolios to underperform the benchmark" and "this implies that investors should concentrate on eliminating the worst firms" (Auer 2016).

Using a dataset of all stocks from developed countries or emerging markets, Verheyden, Eccles, and Feiner (2016) concludes that a portfolio investing in the period from 2004 to 2012 would have faced significant benefits in terms of return and risk from the application of a negative screen with a 10% threshold (equivalent to a 90% positive screen). In addition, in accordance with the results found by Bollen (2007), Omura, Roca, and Nakai (2021) show that socially responsible stock indexes outperform, both in terms of risk and return, market indexes during the Covid-19 pandemic outbreak.

The integration of ESG scores into the investment process through positive/negative screens has 3 relevant drawbacks. First, assets' weights are subjectively determined: Auer (2016) construct equally weighted portfolios, Verheyden, Eccles, and Feiner (2016) build portfolios based on assets' market capitalization, and Omura, Roca, and Nakai (2021) leave the decisions to the index providers. Second, the screening threshold is a discretionary decision: the choice of one value with respect to another may have a profound impact on the portfolio's performance. Auer (2016) show that, for the sample considered, a negative screening threshold close to zero may have a positive or neutral impact up to the point where portfolio value is destroyed. Third, for every type of screen or threshold applied in the investment process, the investable universe is reduced, implying a lower portfolio diversification a priori. To overcome these issues, other approaches that include the ESG factors in the portfolio selection process have been studied.

The main challenge of incorporating ESG considerations into the investment process is to generalize asset allocation avoiding to rely on some discretionary parameters, such as the screening threshold, that make the investment process non-reproducible and inefficient. One elegant course of action is to expand the classical mean-variance portfolio optimization to a mean-variance-sustainability (ESG is a proxy for the level of sustainability) optimization. First, let us briefly recall the main concept related to traditional portfolio optimization (Markowitz 1952).

Modern Portfolio Theory begins with the *Portfolio Selection* article (Markowitz 1952). The paper's groundbreaking insight is to look at assets' risk-return profiles in a portfolio context rather than individually. In addition, Markowitz introduces the concept of efficient portfolio: a basket of assets that maximizes its expected return for each risk level (expressed with the portfolio's standard deviation) or a basket that minimizes the risk for each return level.

As discussed in Markowitz's paper, we can deal with the process of portfolio optimization looking from two different but equivalent perspectives. First, we may find the portfolio that minimizes the risk for a specified return level, performing a quadratic optimization with linear constraints, or we may maximize the return for every risk level, performing a linear optimization with quadratic constraints. This happens to be true for the fact that the portfolio return $\bar{r}$ is defined by the scalar product of the $(N \times 1)$ weight vector ω and the $(N \times 1)$ vector μ, which is composed by the expected return of the N assets within the investable universe, and that the portfolio risk is measured by the portfolio variance σ_W^2, which is defined as

$$\sigma_W^2 = \omega' \Sigma \omega \tag{1}$$

where Σ denotes the $(N \times N)$ positive semi-definite variance-covariance matrix of the assets' returns. Following the first approach, thus, the optimization problem can be written as:

$$P = \arg\min_{\omega} \sigma_W^2 = \omega' \Sigma \omega \tag{2}$$

$$\omega' \mu = \bar{r} \tag{3}$$

$$\omega' i = 1, \tag{4}$$

where i is the $(N \times 1)$ vector of ones.

The analytic solution to Markowitz's optimization problem is developed by Roy (1952), even if the most frequently cited paper about this topic is Merton (1972), and, according to this function, the weight vector of a minimum variance portfolio for a target return is given by

$$\omega^* = \bar{r}\omega_0^* + \omega_1^*, \tag{5}$$

with

$$\omega_0^* = \frac{1}{d}(c\Sigma^{-1}\mu - b\Sigma^{-1}i), \tag{6}$$

$$\omega_1^* = -\frac{1}{d}(b\Sigma^{-1}\mu - a\Sigma^{-1}i). \tag{7}$$

The portfolio standard deviation is given by

$$\sigma = \sqrt{\frac{1}{d}(c\bar{r}^2 - 2b\bar{r} + a)}, \tag{8}$$

with $a = \mu\Sigma^{-1}\mu$, $b = \mu\Sigma^{-1}i$, $c = i\Sigma^{-1}i$, and $d = ac - b^2$.

Having the formula to compute the frontier, we need a point that helps to discriminate between the efficient (upper) part of the frontier and the inefficient (lower) one – the lower part is inefficient because for every one of its points it exists for sure another point (portfolio) with the same risk but higher expected return. This is the global minimum variance portfolio, whose weights are given by

$$\omega^*_{GMV} = \frac{\Sigma^{-1}i}{i'\Sigma^{-1}i}. \tag{9}$$

This point is the apex of the frontier and, as such, it is the left-most point achievable with a combination of risky-assets.

In the context of mean-variance portfolio optimization, the assets considered are described by risk and expected return parameters: they are all risky-asset. Also including a risk-free asset, a financial instrument with a deterministic return r_f, the problem of calculating the set of optimal portfolios changes. Let us recall that, considering a set of only risky assets, a rational investor would choose one of the portfolios belonging to the efficient frontier according to her risk-return preferences. With the inclusion of the risk-free asset, the set of optimal portfolios is represented by the half-line that intersects r_f for a zero risk level and that is tangent to the efficient frontier. In addition, we know that the tangency point represents the portfolio with the highest Sharpe ratio, which is defined as

$$S = \frac{\bar{r} - r_f}{\sigma_W}, \tag{10}$$

where

$$(\bar{r}$$

is the expected return of a portfolio and σ_W its standard deviation. This set is called the capital market line (CML), which can be interpreted as a combination of the risk-free asset and the tangency portfolio, also called the market portfolio. Recalling that this is the highest Sharpe ratio portfolio, the optimization problem to obtain portfolio weights becomes the following:

$$P = \arg\max_{\omega} S = \frac{\bar{r} - r_f}{\sigma_W} \tag{11}$$

$$\sigma_W = \sqrt{\omega'\Sigma\omega} \tag{12}$$

$$\omega'\mu = \bar{r} \tag{13}$$

$$\omega' i = 1 \tag{14}$$

The solution of this problem has the following nice expression:

$$\omega^*_{TAN} = \frac{\Sigma^{-1}(\mu - r_f i)}{i'\Sigma^{-1}(\mu - r_f i)}. \tag{15}$$

Starting from this framework, Drut et al. (2010) introduces linear constraints into the mean-variance optimization problem, focusing on the implications that the consideration of an ESG score threshold has on portfolio's performance. In more detail, the author starts from a classical utility function (Von Neumann and Morgenstern 1944), which represents the expected utility from a portfolio's random returns R_P as follows:

$$E[U(R_P)] = U(\bar{r}) + \frac{1}{2}U''(\bar{r})\sigma^2_W + e(\sigma^2_W). \tag{16}$$

As shown by Feldstein (1969), maximing this utility function is equivalent to performing the following optimization problem, under the assumptions that the utility function is quadratic and assets' returns follow a multivariate normal distribution:

$$P = \arg\min_{\omega} \bar{r} - \frac{\lambda}{2}\sigma^2_W, \tag{17}$$

$$\text{s.t. } \omega'\mu = \bar{r}, \tag{18}$$

$$\omega' i = 1. \tag{19}$$

Then, the sustainability dimension is included into the problem through the portfolio's ESG score

$$\nu_W = \omega'\nu$$

where ν is the vector of assets' ESG scores:

$$P = \arg\min_{\omega} \bar{r} - \frac{\lambda}{2}\sigma^2_W, \tag{20}$$

$$\text{s.t. } \omega'\mu = \bar{r}, \tag{21}$$

$$\omega' i = 1, \tag{22}$$

$$\nu_W \geq \nu_0. \tag{23}$$

There exists an analytic solution to the problem, assuming that the portfolio's ESG score is a linear function of portfolio's expected return:

$$\nu_W = \delta_0 + \delta_1 \bar{r}, \tag{24}$$

where the two coefficient δ_0 and δ_1 of the linear equation are a combination of μ, Σ, and ν. Looking at the analytic solution, it can be shown that the efficient frontier with respect to the ESG threshold can assume 4 different shapes: (i) equal to the traditional mean-variance; (ii) the lower part under the minimum variance portfolio is penalized, implying that there is a cost for low risk-averse investors only; (iii) only the upper part under the minimum variance portfolio is penalized, implying that there is a cost for high risk-averse investor only; (iv) the whole frontier is inferior to the conventional one, negatively impacting all investors.

On one hand, this paper offers an original explanation of the relationships between the financial performance of conventional and ethical assets: sustainable portfolios may be more costly depending on investors' risk aversion and single assets' risk and return. On the other hand, the ESG dimension is not automatically handled by the allocation process but is taken into account through a discretionary threshold, similar to what happens with a positive/negative screening procedure.

Utz et al. (2014) expand Drut et al. (2010) optimization to include the portfolio's ESG score ν_W not as a threshold but as a variable to be maximized. The construction of this tri-criterion problem allows the authors to surpass the limitations embedded in ESG portfolio screens, avoiding investors' subjectivity on the ESG threshold, including the entire investable universe without diminishing diversification a priori, and giving an objective tool based on strong theoretical foundations. In particular, an investor's utility function must incorporate a sustainable dimension as follows:

$$\Phi(\omega, \sigma_W^2, \bar{r}, \nu_W, \lambda_r, \lambda_\nu) = -\sigma_W + \lambda_r \bar{r} + \lambda_\nu \nu_W, \tag{25}$$

where λ_r is the risk tolerance of an investor regarding expect return and λ_ν is the risk tolerance regarding ESG score. "Varying the risk tolerance pair" (λ_r, λ_ν) "over the nonnegative portion of" $\mathbb{R}^2$ "(which we call parameter space) and maximizing $\Phi(\omega, \sigma_W^2, \bar{r}, \nu_W, \lambda_r, \lambda_\nu)$ causes expected utility to yield, in contrast to the nondominated frontier, a nondominated surface in variance-expected return-ESG space" (Utz et al. 2014). Figure 12 shows a graphical representation of the result of this tri-criterion optimization.

The main disadvantage of this approach is the fact that it has not been found an analytic solution to the problem. Utz et al. (2014) use an algorithm developed by Hirschberger et al. (2013) that computes the tri-criterion efficient frontier exactly: this algorithm is called CIOS (Custom Investment Objective Solver).

Gasser, Rammerstorfer, and Weinmayer (2017) try to solve the limitations encountered by Utz et al. (2014) avoiding to directly compute the tri-criterion efficient frontier, which has been shown not to be analytically tractable. Including a risk-free asset into the analysis, they develop a model that is able to derive the capital allocation plane (CAP), the tri-dimension equivalent of the bidimensional capital market line. Assuming a linear utility function, all the possible portfolios belonging to the CAP, represented in figure 13, can be derived as a combination of the risk-free asset and 3 corner portfolios: (i) the portfolio that minimizes risk (i.e. the minimum variance point), (ii) the conventional mean-variance optimizing portfolio (i.e. the portfolio with the highest achievable Sharpe ratio), and (ii) the portfolio that maximizes the social factor relative to the risk.

Pedersen, Fitzgibbons, and Pomorski (2021) transform the tri-criterion problem into a bi-criterion one. The authors build a new type of efficient frontier: for every level of ESG-score, the maximum Sharpe ratio achievable is computed. This line represents the portfolios that a rational investor would pick for each level of ESG preference and is drawn in figure 14.

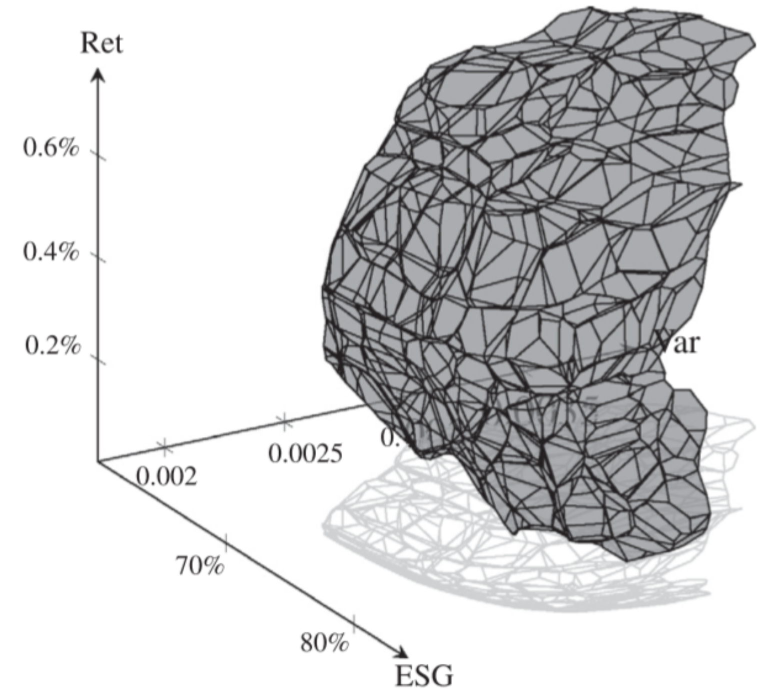

Figure 12: "A tri-criterion nondominated surface. Note that while a nondominated frontier is composed of a connected collection of parabolic segments, a nondominated surface is composed of a connected collection of paraboloidic “platelets.” The surface shown is from a tri-criterion problem with n = 50 securities and has 1460 platelets" (source: Utz et al. 2014)

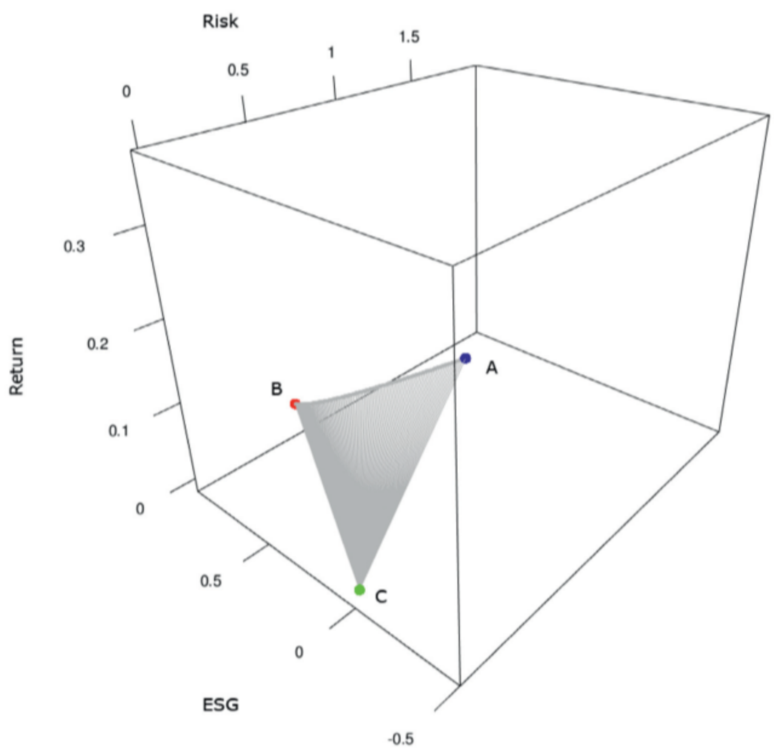

Figure 13: Representation of the capital allocation plane which is the set composed by all possible combinations of the 3 corner portfolios (i.e. the portfolio that minimizes risk, the conventional mean-variance optimizing portfolio, and the portfolio that maximizes the social factor relative to the risk) and the risk-less asset (source: Gasser, Rammerstorfer, and Weinmayer 2017)

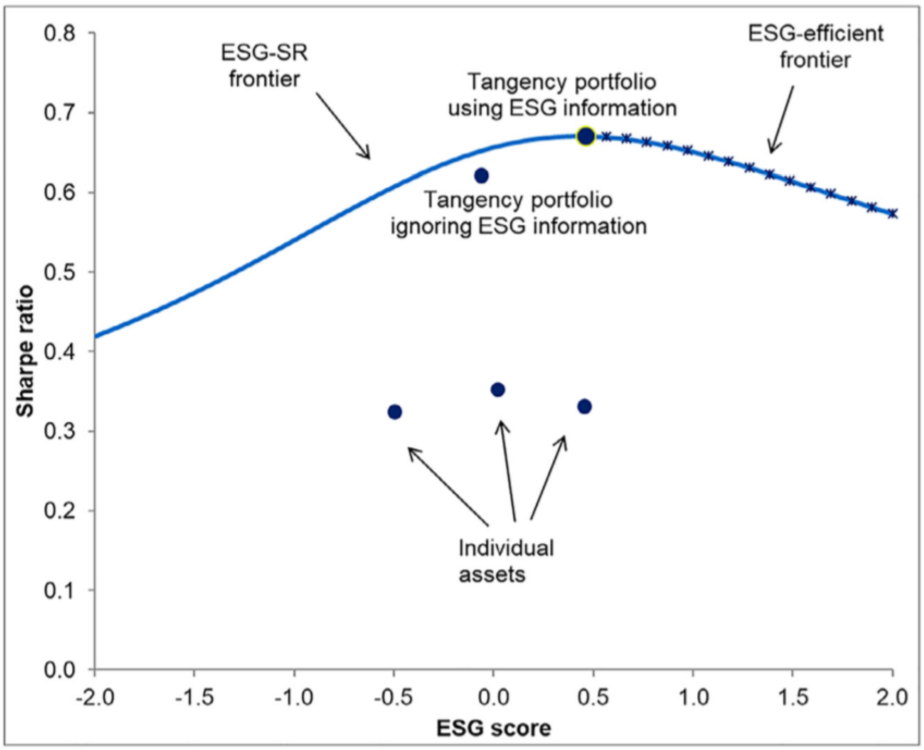

Figure 14: Representation of the ESG-Sharpe ratio frontier "that is the maximum Sharpe ratio (on the y-axis) that can be achieved for all portfolios with a given ESG score (on the x-axis). The peak of the ESG-SR frontier is the Sharpe ratio (SR) of the standard tangency portfolio. Investors who care about both SR and ESG should choose a frontier portfolio to the right of this portfolio, on the ESG-efficient frontier" (source: Pedersen, Fitzgibbons, and Pomorski 2021)

As we can see, the so-called ESG efficient frontier developed by Pedersen, Fitzgibbons, and Pomorski (2021) is humped shaped. This occurs for the fact that the maximum Sharpe ratio characterizing a portfolio belonging to the conventional efficient frontier is the highest achievable Sharpe ratio of a portfolio of the available assets and, thus, for any level of ESG score. Therefore, for any other ESG threshold a lower Sharpe ratio must be found by construction.

Schmidt (2022) develops a process that builds portfolios maximizing the Sharpe ratio for a fixed PESGV (portfolio ESG value), which is the weighted average of stocks' ESG values. Some particular aspects of this paper are that the variance-covariance matrix is constructed using partial correlations between stocks, only long positions are allowed, and the ESG tilted Sharpe ratio, a performance measure that takes into account also the ESG component, is proposed.

De Spiegeleer et al. (2021) compares the risk and return characteristics of different portfolios derived by 3 different methodologies including the ESG components. All the methods derive an efficient frontier of the investable universe but they are structured in slightly different ways: (i) constrained mean-variance portfolios are portfolios obtained by the minimization of the variance constrained on a fixed expected return and an ESG-score (the weighted average of single stocks' ESG-scores) lower or equal than a threshold; (ii) green-variance portfolios are portfolios obtained minimizing the variance constrained to a fixed ESG-score; (iii) most similar constrained portfolios are portfolios derived by reallocating the mean-variance portfolio as little as possible to make it comply with sustainability targets. For all the 3 approaches, the authors perform a Michaud optimization, which they consider to be more reliable when the parameters have to be estimated from a data set of historical data.

3 Model Presentation

In this chapter, we are going to present some models that integrate the ESG dimension avoiding the limits encountered by the works discussed above. To sum up, the easiest and most intuitive way to incorporate ESG score into the investment process is through the use of a positive or negative screen, but this methodology requires investors to use their discretion both on the screening threshold and on the assets' allocation rule. In order to dispose of a more general methodology, some scholars have proposed to expand the traditional mean-variance portfolio optimization theory to a tri-criterion mean-variance-ESG score portfolio optimization. The solution to this problem is not a close formula, thus, even if an approximate solution can be found thanks to the algorithm developed by Hirschberger et al. (2013), many have tried to reformulate the problem in order to make the solution more intuitive and analytically tractable. In this Master's Thesis, it is suggested the utilization of tracking error models to address the problem of incorporating sustainability considerations within the investment process.

Let us consider the case of a social and responsible investor. Supposing she is a rational individual, we can firmly suppose that she is concerned about how much money she will make from the investment (the portfolio's expected return), how much of her investment is at risk (the portfolio's risk), and how much positive impact on society and the environment she has produced through her investment (proxied by portfolio's ESG score). Our investor needs to find the portfolio optimizing model that best fits her needs: a comprehensible and reproducible system that returns the assets' weights to be allocated according to the investor's preferences. At a first glance, the answer is to find the investments basket that maximizes the 3 performance dimensions with respect to the investor's risk aversion relative to the expected return and the risk aversion relative to the ESG score. Even if formulated in different ways, as shown in the previous chapter, the models proposed by Drut et al. (2010), Utz et al. (2014), Gasser, Rammerstorfer, and Weinmayer (2017), Pedersen, Fitzgibbons, and Pomorski (2021), Schmidt (2022), and De Spiegeleer et al. (2021) share this objective.

In the case our investor decides to utilize one of the models above to derive her optimal asset allocation, she will face some issues. The first limitation of this approach is related to the portfolio's performance evaluation. Generally, investors evaluate whether the performance of their holdings is coherent with their initial expectation on portfolio expected return and volatility on a monthly or annual basis. However, the stochastic nature of financial assets prevents investors from correctly inferring portfolios' characteristic variables in the short term. For this reason, tracking error volatility has been introduced as a tool for portfolios' performance evaluation.

Due to the element of randomness in portfolios' return, we cannot evaluate financial holdings only on their historical performances, especially for a small sample period, but we must assess their behaviors relative to a benchmark. For instance, let us consider a householder holding shares of an active mutual fund investing in the American stock market. Presumably, she chose this fund because she is persuaded that the fund manager can systematically outperform the market. Therefore, to assess whether her opinions on the mutual fund's manager are correct, she needs a benchmark, which may be represented by the S&P500

Index, and a quantitative measure that evaluates the performance of an asset relative to the benchmark. Roll (1992) proposes to use the volatility of tracking error (TEV). Our investor should desire a TEV as low as possible for the fact that, if it existed, the perfect mutual fund would systematically outperform the market by a constant amount net of the fund's expenses, implying a zero TEV.

The second problem deriving from the utilization of previously described ESG integrating models is the fact that they do not consider the stochastic nature of ESG scores, including these data as deterministic variables. As we will observe in the next chapter, ESG scores embed a random element and should be described by a stochastic process. Facing this characteristic, traditional models become insufficient, we must consider the dynamic of ESG scores within the optimization problem. This reason, together with the necessity of a tracking error measure to evaluate portfolio performance, suggests to us that a multi-goal tracking error model is the suitable model to address the need of integrating the sustainability dimension into the investment process.

Another aspect that we want to incorporate in our model is the short-selling constraint. We do not want to allow the model to return negative weights for 2 reasons: first, it is often technically unfeasible for investors to undertake short positions on single stocks, thus, allowing for this instance, we would impair the usefulness of our results; second, the possibility of taking short positions may create ethical conundrum for our socially responsible investor: it may occur the event in which the model, that performs an optimization along the 3 discussed dimension, induces her to short sell a company with high ethical standards. Therefore, she will face the conflicting situation where it would be rational to take a position against an ethical organization in order to reach the highest possible impact on the society and the environment.

The rest of the chapter is structured in the following sections: first, static and dynamic tracking error problems will be presented, then, specific multi-goal tracking error ESG incorporating models will be introduced. The data collection and the empirical implementation of the models is developed in the next chapter.

3.1 Tracking Error Models

A tracking error model conventionally consists in an optimization problem of a quantifiable measure, called the tracking error, subject to some specific restrictions. In a financial context, the most common type of tracking error model is the minimization problem whose solution are the weights of the portfolio that is characterized by the smallest possible deviations from the benchmark. In the case all its components are included in the investable universe considered the solution is the benchmark itself. Otherwise, if we restrict the number of assets, the minimization problem returns the portfolio, composed of a subset of the benchmark's components, that is the best replicant of the benchmark itself. The first case represents an example of a full replication strategy, instead, the latter is an instance of a partial replication (Barro and Canestrelli 2009). Tracking error models are a quantitative methodology particularly useful for replicating and hedging investment strategies. Before diving into the description of the different types of models, we need to present how tracking error (TE) measures may be constructed. In order to do so, we are going to follow the reasoning and mathematical notation developed in Barro and Canestrelli (2009).

A TE measure quantifies how much a portfolio converges to the benchmark or, equivalently, how much the first is able to replicate the other. Therefore, it is straightforward to define the TE as a distance measure between the portfolio and the benchmark returns. Let us denote portfolio weights as x_t, the vector of returns on assets as r_t, and the benchmark returns as y_t at time t. The TE over the period $[t_0, T]$ may be quantified by the α-norm of the vector of deviations:

$$TE(t_0, T) = (\sum_{t=t_0}^{T} |r_t' x_t - y_t|^{\alpha})^{\frac{1}{\alpha}}. \tag{26}$$

We can adjust the value of α in order to utilize a different type of distance measure. For instance, considering an α equal to 1 implies using the absolute deviation:

$$TE(t_0, T) = \sum_{t=t_0}^{T} |r_t' x_t - y_t|. \tag{27}$$

Instead, the 2-norm corresponds to using the Euclidean distance:

$$TE(t_0, T) = \sqrt{\sum_{t=t_0}^{T} (r_t' x_t - y_t)^2} = ||r_t' x_t - y_t||. \tag{28}$$

Finally, the ∞-norm is equivalent to the maximum absolute deviation during the period:

$$TE(t_0, T) = \max_t |r_t' x_t - y_t|. \tag{29}$$

We should notice that the α-norm is a symmetric distance measure. In our context, this translates to the fact that the TE considers equivalently both positive and negative deviations of the portfolio's returns relative to the benchmark. However, there are cases in which positive deviations are not something to be avoided but something to be desired. For this reason, asymmetric TE measures have been introduced.

Dembo and Rosen (1999) introduces the *regret*, which measures the underperformance of the portfolio relative to the benchmark. Given the vector of negative deviations of portfolio returns relative to benchmark returns $d_t^- = (r_t' x_t - y_t)^- = \min(0, r_t' x_t - y_t)$. We may define an asymmetric TE measure as the α-norm of the vector of negative deviations over the period $[t_0, T]$:

$$TE(x; t_0, T) = (\sum_{t=t_0}^{T} |d_t^-|^{\alpha})^{\frac{1}{\alpha}}. \tag{30}$$

Therefore, an α equal to 1 constitutes a TE that measures total negative deviations in absolute terms:

$$TE(x; t_0, T) = \sum_{t=t_0}^{T} |d_t^-|. \tag{31}$$

On the other hand, an α equal to 2 returns the Euclidean distance of negative deviations:

$$TE(x; t_0, T) = \sqrt{\sum_{t=t_0}^{T} (d_t^-)^2} = ||d_t^-||. \tag{32}$$

Finally, when approaching the α to infinity, the TE is represented the maximum negative deviation:

$$TE(x; t_0, T) = \max_t |d_t^-|. \tag{33}$$

In general, given the vector of negative deviations of portfolio returns relative to benchmark returns d_t^- and the vector of positive deviations $d_t^+ = (r_t' x_t - y_t)^+ = \max(0, r_t' x_t - y_t)$, and the parameter λ, we can define an asymmetric tracking error as a weighted average between positive and negative deviations:

$$TE(x; t_0, T) = (\sum_{t=t_0}^{T} |d_t^-|^\alpha)^{\frac{1}{\alpha}} + \lambda(\sum_{t=t_0}^{T} |d_t^-|^\alpha)^{\frac{1}{\alpha}}. \tag{34}$$

To sum up, TEs measure the distance between two vectors, they can be found to be symmetric or asymmetric measures depending on the purpose of the model in which they are embedded. For instance, if we are within the sphere of passive mutual funds, which are evaluated on how close they perform relative to a benchmark, a symmetric measure should be preferred, instead, if we consider a fund following an active strategy and, thus, evaluated on how consistently it outperforms a benchmark, an asymmetric TE should be preferable. In both cases, a TE measure is included in a minimization model that returns the optimizing portfolio weights. Before diving into the description of ESG integrating TE models, we first distinguish them into static, or single-stage, and dynamic, or multi-stage, models.

3.1.1 Static TE models

One general possible formulation of a static historical TE model is the following:

$$\min_{x^*} (\sum_{t=t_0}^{T} |r_t' x - y_t|^\alpha)^{\frac{1}{\alpha}}, \tag{35}$$

where the solution x^* to the minimization problem is the vector of portfolio weights that minimize the distance between portfolio and benchmark returns in the interval $[t_0, T]$. The time interval refers to an already passed moment in time, therefore, assets and benchmark returns are realized and deterministic. Usually, static historical TE models return an optimized portfolio that is kept unchanged for the subsequent time period. For example, Roll (1992) formulates a special type of historical TE model that finds the portfolio that minimizes the TEV for the period considered. Rudolf, Wolter, and Zimmermann (1999) develops some specific formulations of the minimization problem above. Fixing α equal to 1, they achieve the following minimization problem, which they called Mean Absolute Deviation (MAD):

$$\text{MAD} : \min_{x^*}(\sum_{t=t_0}^{T} |r_t' x - y_t|), \tag{36}$$

instead, using an α that tends to infinity, the define MinMax model:

$$\text{MinMax} : \min_{x^*}(\max_{t} |r_t' x - y_t|). \tag{37}$$

In addition, Rudolf, Wolter, and Zimmermann (1999) present the asymmetric counterparts of the MAD and MinMax models, the mean absolute downside deviation (MADD) and the downside MinMax (DMinMax) models, which are fromulated as follows:

$$\text{MADD} : \min_{x^*}(\sum_{t=t_0}^{T} |d_t^-|), \tag{38}$$

$$\text{DMinMax} : \min_{x^*}(\max_{t} |d_t^-|). \tag{39}$$

Remaining under the set of static models, scenario-based TE models (Dembo and Rosen 1999) are an evolution of historical TE models. Financial asset prices may be described by stochastic processes, thus, historical data represents only one of the infinitely many possible realizations of the process. A scenario approach enables us to include into the optimization as many realizations of the underlying dynamic as we wish. Moreover, we may include any subjective view on the future into the random process that we use to simulate the scenarios' realization: for this reason, static scenario TE models are considered forward-looking relative to static historical TE models (Barro and Canestrelli 2009).

Given the set of scenarios $s = 1, .., S$ characterized by a probability π_s and given the generic TE measure $TE(t_0, t; s)$ under scenario s, a static scenario TE model is formulated as follows:

$$\min_{x^*} \sum_{s=1}^{S} \pi_s TE(t_0, t; s). \tag{40}$$

If we quantify the TE as the α-norm of the vector of deviations, then the problem becomes the following:

$$\min_{x^*} \sum_{s=1}^{S} \pi_s (\sum_{t=t_0}^{t} |r_t' x - y_t|^{\alpha})^{\frac{1}{\alpha}}. \tag{41}$$

As developed in Dembo and Rosen (1999), using an α equal to 1 is equivalent to minimizing the expected absolute deviation of portfolio returns relative to the benchmark: this is the MAD model coupled with a scenario approach. Similarly, approaching the α towards infinity returns the scenario version of the MinMax model. In addition, a static scenario TE model can be constructed using an asymmetric TE measure.

3.1.2 Dynamic TE models

Historical TE models start from a sample of historical observations to compute what would have been the best-performing portfolio in those conditions. In our context, the best performing portfolio is not the one realizing the best return, but the one that optimizes the model's optimization process given a set of conditions. A single-stage scenario TE model starts from the generation of many simulations: these are the possible future realizations on which the best portfolio is computed. In this second approach, what really matters to the optimization process is the starting point at time t_0 and the final state of the underlying dynamics at time T: everything that occurs in the middle, at time t for $t_0 < t < T$, is not taken into account in this type of model formulation. In order to do so, we need to introduce dynamic TE models following the reasoning of Barro and Canestrelli (2009).

A dynamic TE model is characterized by an optimization process that spans the time and scenario dimensions. Let us consider the evolution of the underlying dynamics through a 3-period binary tree as shown in figure 15.

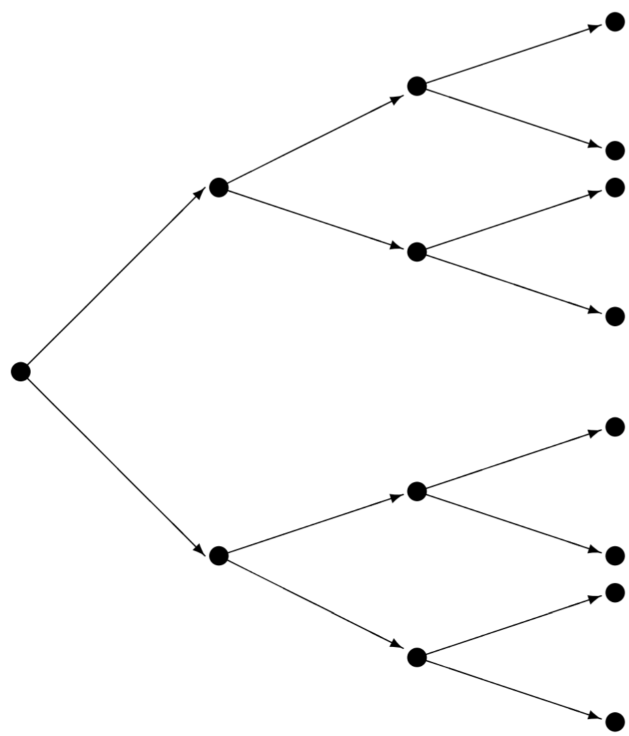

Figure 15: 3-period binary tree structure for scenario-based simulations (source: Barro and Canestrelli 2009)

Within this framework, a scenario corresponds to a path over the time interval $[t_0, T]$. Therefore, a 3-period binary tree spans over the $[0, 3]$ time interval and defines $2^3 = 8$

scenarios. Each scenario has an associated probability π_s.

A generic dynamic TE model over the period $[t_0, T]$ can be defined as follows:

$$\min_{x^*_{1s},\dots,x^*_{Ts};s=1,\dots,S} \sum_{s=1}^{S} \pi_s \sum_{t=t_0}^{T-1} [\varphi(x_{t+1s}, y_{t+1s}) + \gamma\phi(x_{ts}, x_{t+1s})], \tag{42}$$

where $\varphi(x_{ts}, y_{ts})$ represents a generic TE measure at time t and under scenario s, $\phi(x_{ts}, x_{t+1s})$ is function that penalize portfolio rebalancing in two successive periods, and γ is a penalizing parameter.

In order to take into account the time preference dimension or the decreasing reliability of forecast for periods distant int he future, we may add the sequence of weights $\beta_t = \beta_{t_0}, .., \beta_T$ into the model:

$$\min_{x^*_{1s},\dots,x^*_{Ts};s=1,\dots,S} \sum_{s=1}^{S} \pi_s \sum_{t=t_0}^{T-1} [\beta_{t+1}\varphi(x_{t+1s}, y_{t+1s}) + \gamma\phi(x_{ts}, x_{t+1s})]. \tag{43}$$

One possible specification of the previous TE model, which is developed by Barro and Canestrelli (2009), specifies $\varphi(x_{ts}, y_{ts}) = (r'_{ts}x_{ts} - y_{ts})^2$ and $\phi(x_{ts}, x_{ts}) = (x_{t+1s}, x_{ts})'(x_{t+1s}, x_{ts})$. In addition, we add the necessary constraints to take into account that no short selling is allowed, the sum of portfolio weights must sum to 1, and all the scenarios should start from the same initial point.

$$\min_{x^*_{1s},\dots,x^*_{Ts};s=1,\dots,S} \sum_{s=1}^{S} \pi_s \sum_{t=t_0}^{T-1} [\beta_{t+1}(r'_{t+1s}x_{t+1s} - y_{t+1s})^2 + \gamma(x_{t+1s}, x_{ts})'(x_{t+1s}, x_{ts})] \tag{44}$$

$$\text{s.t. } x_{ts} \geq 0 \tag{45}$$

$$\sum_{i=1}^{n} x_{its} = 1 \tag{46}$$

$$x_{0s} = x_0, \forall s \tag{47}$$

This is a dynamic TE model that penalizes portfolio rebalancing in a straightforward manner. If we want to explicitly quantify the impact of transaction costs, we need to redefine the functions of TE measure and rebalancing penalization, introducing two specific variables: the value of assets purchased a_{ts} and the value of assets sold v_{ts} at time t under scenario s, where transaction costs are expressed as a constant κ percentage of traded value. In addition, instead of calculating the TE of a portfolio's returns relative to benchmark returns, we consider the TE between the value of the portfolio, quantified as $1'z_{ts} = (z_{1ts}, ..., z_{n+1ts})$, and the value of the benchmark y_{ts} at time t under scenario s. The formulation of this model is the following (Barro and Canestrelli 2009):

$$\min_{a^*_{0s},\dots,a^*_{T-1s};v^*_{0s},\dots,v^*_{T-1s};s=1,\dots,S} \sum_{s=1}^{S} \pi_s \sum_{t=t_0}^{T-1} [(1'z_{t+1s} - y_{t+1s})^2 + \gamma\kappa 1'(a_{ts} + v_{ts})] \tag{48}$$

$$\text{s.t. } z_{it+1s} = (1 + r_{it+1s})[z_{its} + a_{its} - v_{its}] \;\; i = 1, ..., n \tag{49}$$

$$z_{n+1t+1s} = (1 + r_{n+1t+1s})[z_{n+1ts} - \sum_{i=1}^{n}(1 + \kappa)a_{its} + \sum_{i=1}^{n}(1 - \kappa)v_{its}] \tag{50}$$

$$z_{its} + a_{its} - v_{its} \geq 0 \;\; i = 1, ..., n \tag{51}$$

$$z_{n+1ts} - \sum_{i=1}^{n}(1 + \kappa)a_{its} + \sum_{i=1}^{n}(1 + \kappa)v_{its} \geq 0 \tag{52}$$

$$a_{ts} \geq 0, \;\; v_{ts} \geq 0 \tag{53}$$

$$z_{0s} = z_0 \;\; t = 0, ..., T \;\; s = 1, ..., S. \tag{54}$$

The constraints of this optimization process ensure the same initial portfolio composition for each scenario, the self-financing attribute for the portfolio, the prohibition of short selling, and the impossibility of anticipativity occurrence.

3.2 Model Formulation

Let us recall the example of a socially responsible investor. Supposing she is a rational human being, she must be interested in how much money is expected to be gained on her initial investment (i.e. expected return), how much capital is likely to be lost (i.e. risk), and how much her investment positively impacts society and the environment (i.e. ESG score). Ideally, she must find a portfolio that minimizes risk, while maximizing return and the ESG dimension. However, as we discussed before, the current state-of-the-art tri-criterion portfolio optimization models fall short in including the stochastic element of ESG score and do not provide a straightforward performance evaluation methodology. A multi-goal TE model surpasses these limitations giving the possibility to include the dynamics of ESG scores and providing a benchmark to which to compare the portfolio performance in terms of return, risk, and sustainability. Therefore, this chapter aims to develop some multi-goal TE error models that include the ESG dimension.

We start with the formulation of a historic, thus static, multi-goal TE model. Let us denote the weights of a portfolio constituted by N risky assets as x_t, the vector of returns on assets as r_t, the benchmark return as y_t, the vector of ESG returns on assets as s_t, and the benchmark ESG return as z_t at time t. These variables are calculated as following:

$$r_{i\,t} = \frac{p_{i\,t}}{p_{i\,t-1}} - 1, \tag{55}$$

$$y_t = \frac{p_{b\,t}}{p_{b\,t-1}} - 1, \tag{56}$$

$$s_{i\,t} = \frac{e_{i\,t}}{e_{i\,t-1}} - 1, \tag{57}$$

$$z_t = \frac{e_{b\,t}}{e_{b\,t-1}} - 1, \tag{58}$$

where p_{it} is the price of asset i, p_{bt} is the price of the benchmark, e_{it} is the ESG score of asset i, and e_{bt} is the ESG score of the benchmark at time t. We construct the following historic multi-goal TE model:

$$\min_{x_t*} (\sum_{t=0}^{T} |r_t' x_t - y_t| + \sum_{t=t_0}^{T} (\beta^+ |g_t^+| + \beta^- |g_t^-|) \tag{59}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} x_{it} = 1 \tag{60}$$

$$g_t^+ = (u_t' x_t - z_t)^+ = \max(0, u_t' x_t - z_t) \tag{61}$$

$$x_{it} \geq 0 \tag{62}$$

$$g_t^- = (u_t' x_t - z_t)^- = \min(0, u_t' x_t - z_t) \tag{63}$$

$$i = 1, ..., n \quad t = 1, ..., T, \tag{64}$$

where β^+ is the weight for positive deviations of the portfolio ESG returns relative to the benchamrk and β^- is the weight for negative deviations.

This is a static TE model that combines two different TE models and their objectives to achieve a unique multi-goal solution: a MAD model is used for the minimization of price returns deviation and a MADD for the ESG returns deviations. We think this formulation is the most appropriate for a socially responsible investor that seeks to replicate the risk-return profile of a benchmark, while trying to capture the highest possible portfolio ESG level.

The previous model can be easily reformulated to include a forward-looking perspective through a scenario-based approach. Given the set of scenarios $s = 1, .., S$ characterized by a probability π_s, we construct the following scenario multi-goal TE model:

$$\min_{x_t*} (\sum_{s=1}^{S} \pi_s |r_s' x_s - y_s| + \sum_{s=1}^{S} (\beta^+ |g_s^+| + \beta^- |g_s^-|) \tag{65}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} x_{is} = 1 \tag{66}$$

$$x_{is} \geq 0 \tag{67}$$

$$g_s^+ = (u_s' x_s - z_s)^+ = \max(0, u_s' x_s - z_s) \tag{68}$$

$$g_t^- = (u_s' x_s - z_s)^- = \min(0, u_s' x_s - z_s) \tag{69}$$

$$i = 1, ..., n \quad s = 1, ..., S. \tag{70}$$

Finally, we now focus on a multi-goal dynamic TE model reformulating the model developed in Barro and Canestrelli (2014). We use the scenario tree approach discussed above: each path of the tree, which spans the interval $[t_0, T]$, corresponds to a single scenario with probabilities π_{k_t} associated with each node k_t in the path. First, let us denote $\phi_{k_t}(x_{k_t}, y_{k_t})$ as a TE measure that quantifies the distance between the value of the portfolio x_{k_t} and the value of the benchmark y_{k_t} at time t in node k_t. Second, let us denote $\xi_{k_t}(e_{k_t}, z_{k_t})$ as a TE measure between the portfolio ESG score e_{k_t} and the ESG score of the benchmark z_{k_t} at time t in node k_t.

The model is an optimization of 2 distance measures with the objective of replicating the benchmark return and risk levels, while aiming to a higher ESG score. Assuming that our investor can choose among n risky and 1 risk-less asset, we must also take into account transaction costs within the model. Following the formulation developed in Barro and Canestrelli (2014), let us specify q_{ik_t}, $i = 1, .., n$ as the weight of the i-th risky asset, l_{k_t} as the weight of the risk-less instrument, $a_{k_t} = (a_{1k_t}, ..., a_{nk_t})$ as the vector of values of assets purchased at time t in node k_t, $v_{k_t} = (v_{1k_t}, ..., v_{nk_t})$ as the vector of values of assets sold at time t in node k_t, κ^+ and κ^- as the proportional transaction costs for assets' purchases and sales, $r_{k_t} = (r_{1k_t}, ..., r_{nk_t})$ as the vector of assets returns in node k_t, r_{lk_t} as the return of the risk-less asset in node k_t, $r_{k_t} = (r_{1k_t}, ..., r_{nk_t})$ as the vector of assets returns in node k_t, and $s_{k_t} = (s_{1k_t}, ..., s_{nk_t})$ as the vector of assets ESG returns in node k_t (the ESG return of the risk-less asset is zero, $s_{lk_t} = 0$, since its ESG score is constantly null), the dynamic triple-goal TE model may be constructed as follows:

$$\min_{x^*_{k_t}} \sum_{t=t_0}^{T} [\sum_{k_t=K_{t-1}+1}^{K_t} \pi_{k_t} \phi_{k_t}(x_{k_t}, y_{k_t}) + \sum_{k_t=K_{t-1}+1}^{K_t} \pi_{k_t} \xi_{k_t}(e_{k_t}, z_{k_t})] \tag{71}$$

$$\text{s.t. } \; x_{k_t} = l_{k_t} + \sum_{i=1}^{n} q_{ik_t} \tag{72}$$

$$e_{k_t} = \frac{1}{y_{k_t}} \sum_{i=1}^{n} e_{ik_t} q_{ik_t} \tag{73}$$

$$q_{ik_t} = (1 + r_{ik_t})[q_{i\,f(k_t)} + a_{i\,f(k_t)} - v_{i\,f(k_t)}] \tag{74}$$

$$e_{ik_t} = (1 + s_{ik_t}) e_{i\,f(k_t)} \tag{75}$$

$$l_{k_t} = (1 + r_{lk_t})[l_{f(k_t)} + \sum_{i=1}^{n} \kappa^+ a_{i\,f(k_t)} - \sum_{i=1}^{n} \kappa^- v_{i\,f(k_t)}] \tag{76}$$

$$a_{ik_t} \geq 0 \;\; v_{ik_t} \geq 0 \tag{77}$$

$$q_{ik_t} \geq 0 \;\; l_{k_t} \geq 0 \tag{78}$$

$$q_{i0} = \bar{q}_i \;\; l_0 = \bar{l} \tag{79}$$

$$i = 1, ..., n \;\; k_t = K_{t-1} + 1, ..., K_t \;\; t = 1, ..., T. \tag{80}$$

The first constraint represents the value of the portfolio in node k_t. The second and the third constraints show the dynamics of the risky assets and the risk-less instrument respectively from the ancestor node $f(k_t)$ at time $t-1$ to the descendent node k_t at time t, given that $K_0 = 0$. The fourth and fifth conditions deny the possibility of borrowing and short selling. Finally, the sixth condition specifies the values of the holdings in the risky and the risk-less assets at $t = 0$.

Having constructed a general multi-goal dynamic TE model, we should specify the TE measures to adopt in the optimization problem. As we discussed in this chapter, there exist different types of TE measures and the preference of one to another depends on the problem considered. In our case, we are looking from the point of view of a socially responsible investor. First, it is reasonable to assume that our investor wants to retain a portfolio with a risk-return characterization that is as close as possible to a benchmark, making its valuation straightforward. Second, since she is socially responsible, given a determined level of sustainability, it is likely that our investor desire to retain the highest portfolio's ESG score possible, minimizing negative ESG deviations while not capping the upside of positive ESG deviations. Therefore, we use a symmetric TE measure to minimize the deviations between benchmark and portfolio returns and an asymmetric TE for the comparison of ESG scores.

The TE measure that we use in our model formulation is the weighted average of positive and negative deviation proposed already introduced in this chapter (Dembo and Rosen 1999). Therefore, we specify:

$$\phi_{k_t}(x_{k_t}, y_{k_t}) = c^+[x_{k_t} - y_{k_t}]^+ + c^-[x_{k_t} - y_{k_t}]^-, \tag{81}$$

where $c^+[x_{k_t} - y_{k_t}]^+ = \max[x_{k_t} - y_{k_t}, 0] = \theta^+_{k_t}$, $c^-[x_{k_t} - y_{k_t}]^- = \max[-x_{k_t} + y_{k_t}, 0] = \theta^-_{k_t}$, and c^+ and c^- are non-negative constants;

$$\xi_{k_t}(e_{k_t}, z_{k_t}) = g^+[e_{k_t} - z_{k_t}]^+ + g^-[e_{k_t} - z_{k_t}]^-, \tag{82}$$

where $g^+[e_{k_t} - z_{k_t}]^+ = \max[e_{k_t} - z_{k_t}, 0] = \delta^+_{k_t}$, $g^-[e_{k_t} - z_{k_t}]^- = \max[-e_{k_t} + z_{k_t}, 0] = \delta^-_{k_t}$, and g^+ and g^- are non-negative constants.

The resulting dynamic multi-goal TE model is the following:

$$\min_{x^*_{k_t}} \sum_{t=t_0}^{T} [\sum_{k_t=K_{t-1}+1}^{K_t} \pi_{k_t}(c^+\theta^+_{k_t} + c^-\theta^-_{k_t}) + \sum_{k_t=K_{t-1}+1}^{K_t} \pi_{k_t}(g^+\delta^+_{k_t} + g^-\delta^-_{k_t})] \tag{83}$$

$$\text{s.t.}\ \ \theta^+_{k_t} - \theta^-_{k_t} = y_{k_t} - x_{k_t} \tag{84}$$

$$\delta^+_{k_t} - \delta^-_{k_t} = e_{k_t} - z_{k_t} \tag{85}$$

$$x_{k_t} = l_{k_t} + \sum_{i=1}^{n} q_{ik_t} \tag{86}$$

$$e_{k_t} = \frac{1}{y_{k_t}} \sum_{i=1}^{n} e_{ik_t} q_{ik_t} \tag{87}$$

$$q_{ik_t} = (1 + r_{ik_t})[q_{i\,f(k_t)} + a_{i\,f(k_t)} - v_{i\,f(k_t)}] \tag{88}$$

$$e_{ik_t} = (1 + s_{ik_t}) e_{i\,f(k_t)} \tag{89}$$

$$l_{k_t} = (1 + r_{lk_t})[l_{f(k_t)} + \sum_{i=1}^{n} \kappa^+ a_{i\,f(k_t)} - \sum_{i=1}^{n} \kappa^- v_{i\,f(k_t)}] \tag{90}$$

$$a_{ik_t} \geq 0 \ \ v_{ik_t} \geq 0 \tag{91}$$

$$q_{ik_t} \geq 0 \ \ l_{k_t} \geq 0 \tag{92}$$

$$\theta^+_{k_t} \geq 0 \ \ \theta^-_{k_t} \geq 0 \tag{93}$$

$$\delta^+_{k_t} \geq 0 \ \ \delta^-_{k_t} \geq 0 \tag{94}$$

$$q_{i0} = \bar{q}_i \ \ l_0 = \bar{l} \tag{95}$$

$$i = 1, ..., n \ \ k_t = K_{t-1} + 1, ..., K_t \ \ t = 1, ..., T. \tag{96}$$

4 Data Collection and Empirical Analysis

In this chapter, we are going to implement the multi-goal TE models developed in the previous chapter. In particular, before discussing and assessing the results of the model, we are going to briefly present the data used from a quantitative point of view. The investable universe considered in this Master's Thesis is the stocks included in the EURO STOXX 50® Index (ES50), which also constitutes the benchmark of our analyses. The price and ESG data of these risky assets are acquired from Bloomberg for the period comprised between the 31st of December 2000 and the 31st of March 2022 on a monthly basis. The choice of a monthly frequency for sample observations is due to the fact that the variability of ESG scores is very low. In fact, as we can notice in figure 16, an ESG rating provider usually revises its company's ESG valuation once every year, thus, utilizing higher frequencies does not improve the materiality of the data considered.

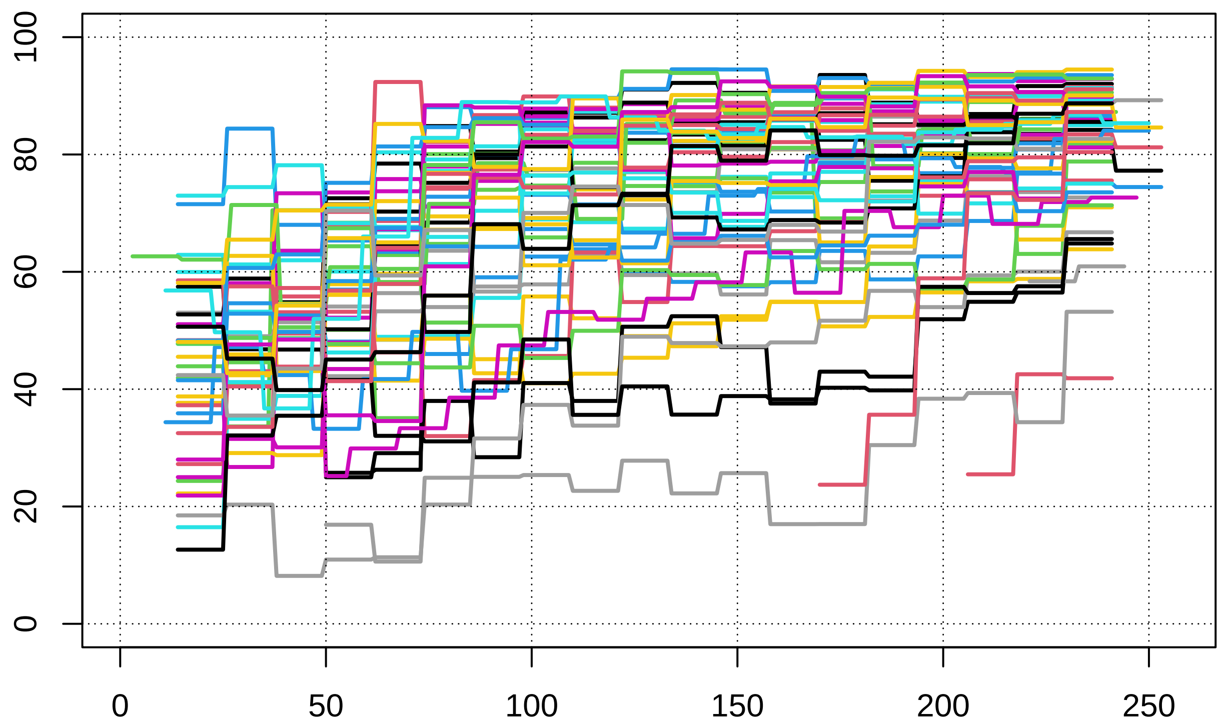

Figure 16: ESG values of the EURO STOXX 50® Index components during the period between 2000 and 2022 with a monthly frequency and a 0-100 rating scale (source: Bloomberg)

4.1 EURO STOXX 50® Index

"The EURO STOXX 50® Index represents the performance of the 50 largest companies among the 20 supersectors in terms of free-float market cap in Eurozone countries" (QONTIGO 2022). For its construction methodology, it represents at least 60% of the European free-float market capitalization. The components of the index are listed in the table below.

Table 4: EURO STOXX 50® Index components as in March 2022 (source: Bloomberg)

Name	Country	Name	Country
ANHEUSER-BUSCH INBEV	Belgium	INFINEON TECHS.	Germany
KONE ‘B’	France	MERCEDES-BENZ GROUP	Germany
AXA	France	MUENCHENER RUCK.	Germany
BNP PARIBAS	France	SAP	Germany
DANONE	France	SIEMENS	Germany
ESSILORLUXOTTICA	France	VOLKSWAGEN PREF.	Germany
HERMES INTL.	France	VONOVIA	Germany
KERING	France	CRH	Germany
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	France	FLUTTER (DUB) ENTERTAINMENT	Germany
L’OREAL	France	ENEL	Italy
LVMH	France	ENI	Italy
PERNOD-RICARD	France	INTESA SANPAOLO	Italy
SAFRAN	France	ADYEN	Netherlands
SANOFI	France	AIRBUS	Netherlands
SCHNEIDER ELECTRIC	France	ASML HOLDING	Netherlands
TOTALENERGIES	France	ING GROEP	Netherlands
VINCI	France	KONINKLIJKE AHOLD DELHAIZE	Netherlands
ADIDAS	Germany	PHILIPS ELTN.KONINKLIJKE	Netherlands
ALLIANZ	Germany	PROSUS	Netherlands
BASF	Germany	STELLANTIS	Netherlands
BAYER	Germany	BANCO SANTANDER	Spain
BMW	Germany	BBV.ARGENTARIA	Spain
DEUTSCHE BOERSE	Germany	IBERDROLA	Spain
DEUTSCHE POST	Germany	INDITEX	Spain
DEUTSCHE TELEKOM	Germany	LINDE	United Kingdom

Figure 17 displays the time-series evolution of the ES50. We can notice that the index has not recovered from the levels reached in the first years of the 2000s. The negative consequences of the dot-com bubble in 2001, the global financial crisis in 2008, the sovereign debt crisis in 2012, and the Covid-19 pandemic in 2020 are evident in terms of loss of value.

The table below shows the annualized mean, standard deviation, and higher-order moments of the log-returns of the ES50 during the period considered.

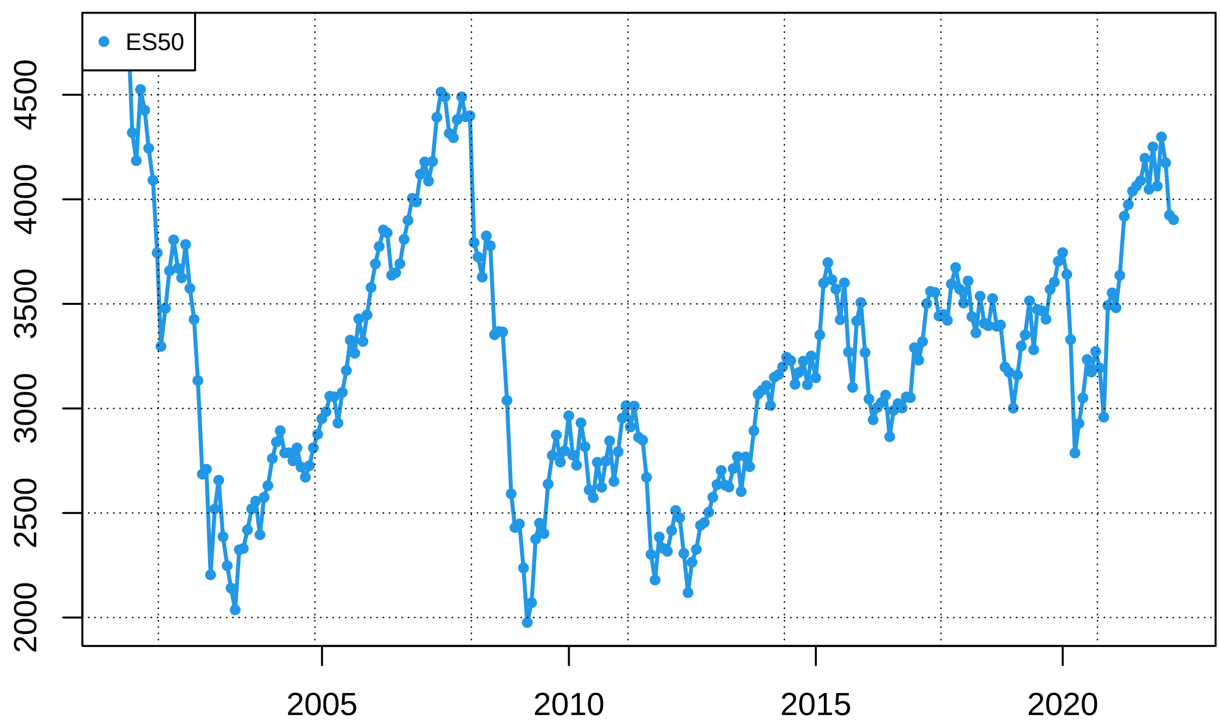

Figure 17: Time Series of the value of the EURO STOXX 50® Index during the period between 2000 and 2022 expressed in euros (source: Bloomberg)

Table 5: First four moments of the log-returns of the EURO STOXX 50® Index during the period between 2000 and 2022 (source: personal elaboration)

	Mean	St.Deviation	Skewness	Kurtosis
Index log-returns	-0.0094694	0.1863968	-0.1775529	0.365966

From these statistical measures, we deduce that the distribution of the log-returns is left-skewed and it has slightly fatter tails compared to a normal distribution. This can be clearly observed from the comparison of ES50's empirical distribution, the histograms, and the curve of a normal distribution with the same mean and variance.

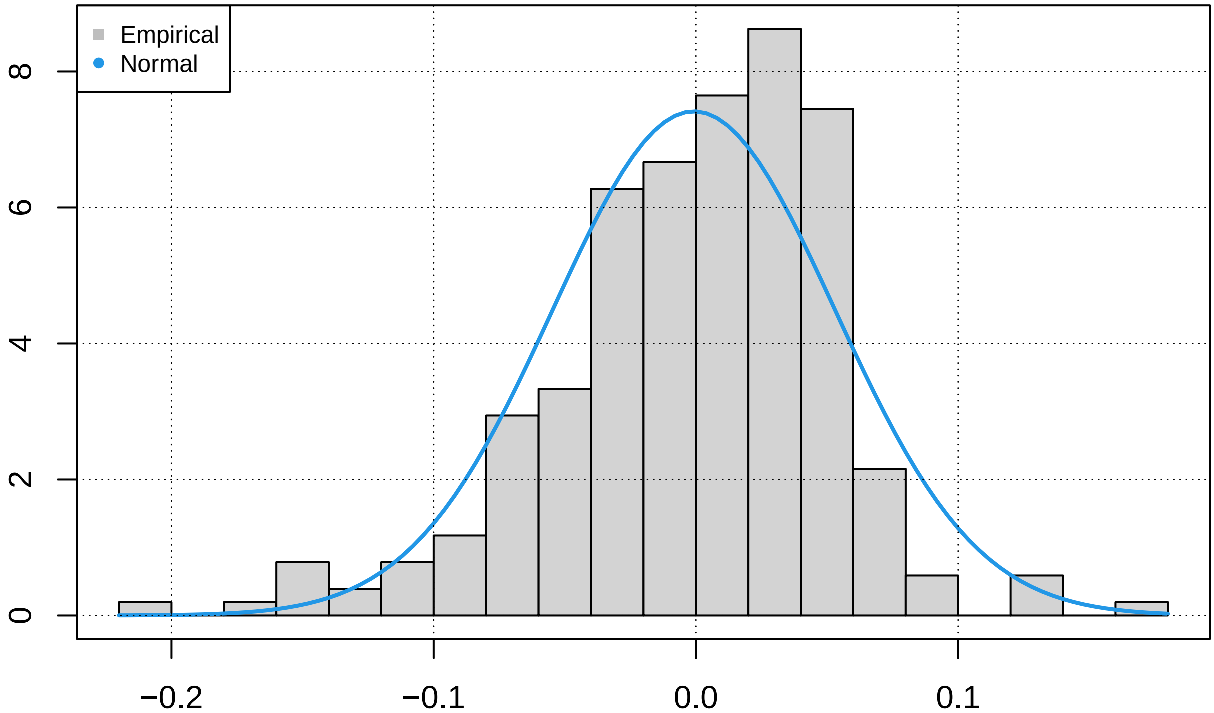

Figure 18: Comparison between the empirical distribution of EURO STOXX 50® Index log-returns and the normal distribution characterized by the same mean and standard deviation (source: personal elaboration)

4.2 ESG Score

Now we focus our attention on the ESG factors. First, we define the ESG score of the EU50 as a weighted average of individual stocks ESG scores: more precisely, the value of the ES50 ESG score $ESG_{ES50,t}$ at time t is calculated as

$$ESG_{EU50,t} = \frac{1}{CAP_{EU50,t}} \sum_{i=1}^{n} ESG_{i,t} CAP_{i,t}, \tag{97}$$

where $ESG_{i,t}$ is the the ESG score of stock i at time t, $CAP_{i,t}$ is the market capitalization of stock i at time t,

$$CAP_{ES50,t} = \sum_{i=1}^{n} CAP_{i,t}, \tag{98}$$

and n is the number of stocks included in the investable universe considered. Given the sample period considered, this specific formulation of the $ESG_{ES50,t}$ raises the problem of data scarcity. In fact, using this formula, the $ESG_{ES50,t}$ can be calculated for only 21 of the 256 observations included in the sample. Figure 19 shows the number of observations for which data are available given a number of index constituents. We have information available for all the 256 months included in the sample period only for one 1 risky asset and, as seen before, only 21 months if we consider all the stocks. We choose to consider an investable universe of 46 assets, therefore, excluding the 3 constituents of the ES50 for which fewer data are available: ADYEN, LINDE, PROSUS, and VONOVIA.

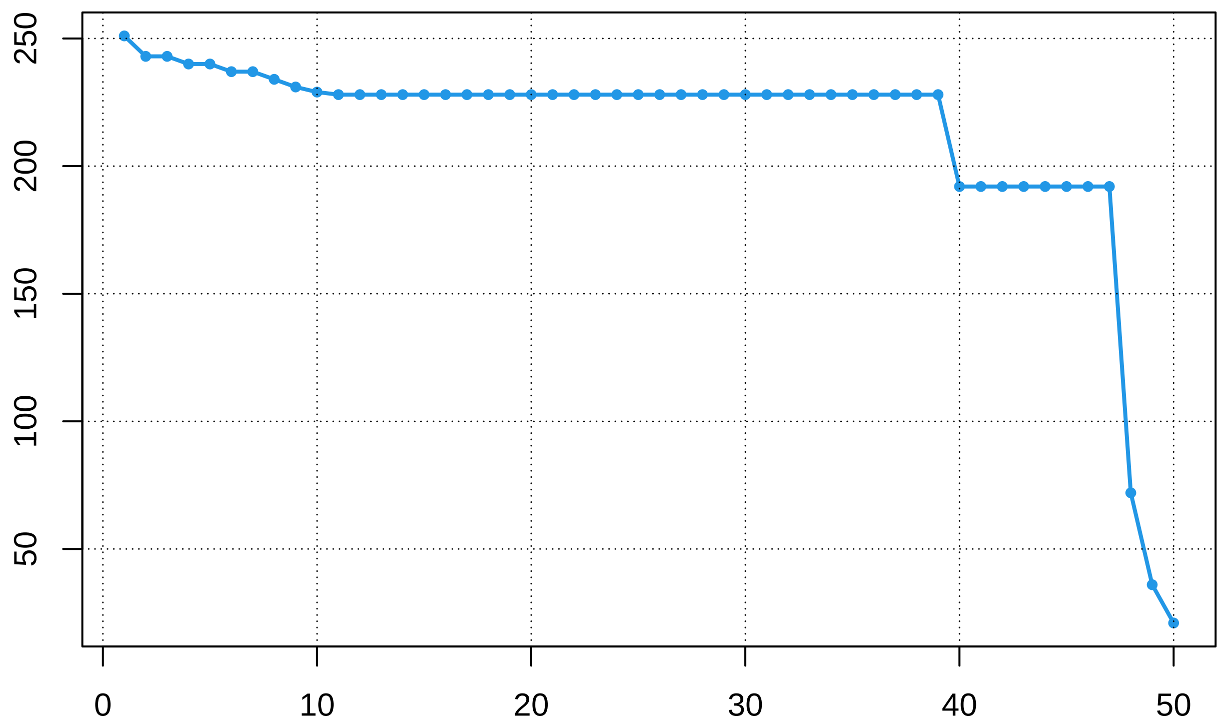

Figure 19: Number of available information depending on the number of components included: the maximum number of available information is 250 months (i.e. we consider only the stock with the largest set of available data), the lower is 21 (i.e. we do not exclude any stock from the sample) (source: personal elaboration)

Given this a priori selection imposed by the scarcity of data availability of ESG score, the value of the ES50 ESG score is proxied by the $ESG_{ES46,t}$ at time t, which is computed as follows:

$$ESG_{EU46,t} = \frac{1}{CAP_{EU46,t}} \sum_{i=1}^{46} ESG_{i,t} CAP_{i,t}. \tag{99}$$

To show that, this exclusion choice benefits our analysis by increasing the number of available observations but it does not greatly distort the correct valuation of the ES50 ESG score, let us consider figure 20 the dynamics of the weighted average ESG scores of the stock constituents of the ES50 are presented for each number of stock considered. The small black line is the ESG score calculated using all of the 50 stocks and the long green one is the ESG score computed using only one index component. We can observe that applying a small negative screen on the universe of risky assets does not profoundly change the level or the overall dynamic of the index ESG score.

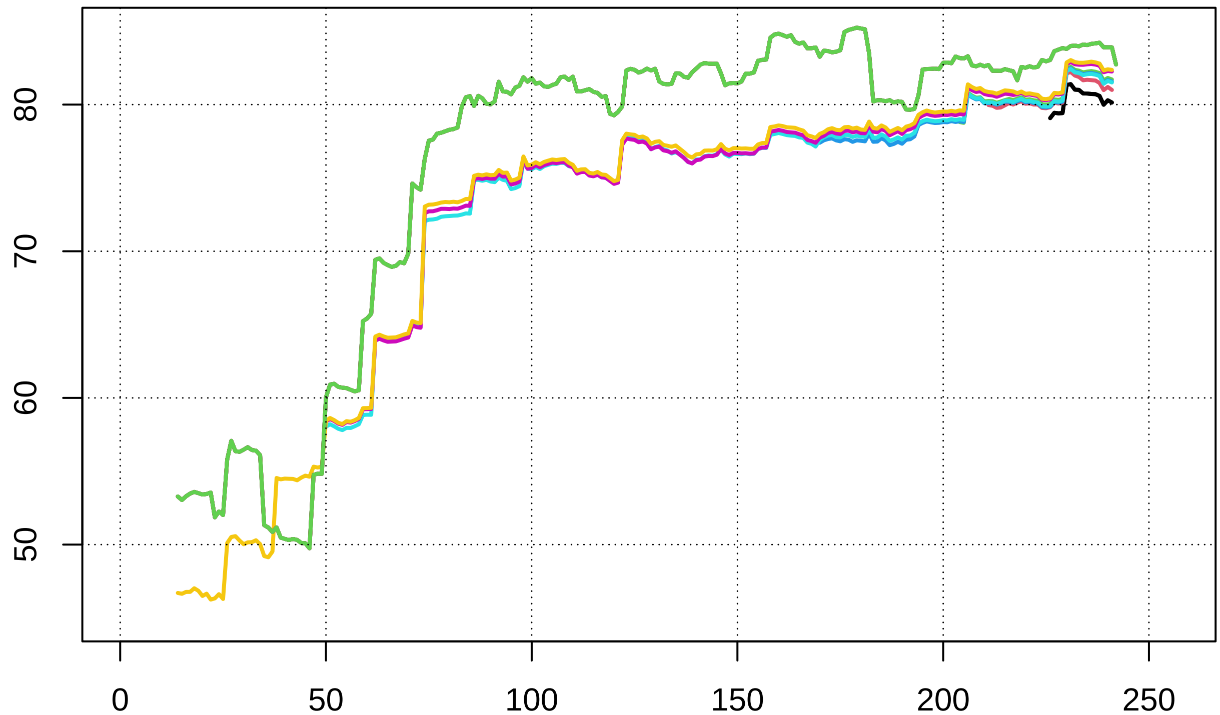

Figure 20: ESG score dynamics of EURO STOXX 50® Index calculated as the average its components ESG scores weighted on market capitalization. Each line is computed using the n components with the largest set of available data with n comprised between 1 and 50 (source: personal elaboration)

4.3 Empirical Results

In this section, we implement the models developed in the previous chapter and we discuss the results. As stated before, the monthly data (i.e. the prices and ESG score of the ES50 Index and its current constituents) collected to perform this empirical analysis span over a period of more than 20 years. However, due to the limited availability of ESG scores, in particular for the first years of the century, we were forced to exclude 4 stocks from the sample and reduce the length of the period to 15 years of monthly data. We use 157 out of the 193 available months of data for our analysis, leaving 36 observations (the last 3 years) to observe the out-of-sample behavior of our models.

We focus only on static models: the historical and scenario-based multi-goal TE model. The rationale behind this type of TE model is having a tool that returns the optimal portfolio weights. After funds are invested accordingly at the beginning of the period, the portfolio, differently from dynamic models, is not subject to rebalancing or adjustments until the end of the investment horizon. In our case, the simulation of scenarios is performed through the basic technique of historical bootstrapping.

Therefore, for the sake of our analysis, the output of a static TE model is the 46 elements vector of initial asset weights. Keeping in mind the point of view of a socially responsible investor, we are going to assess how well the optimal portfolio replicates the benchmark along the risk-return dimension and how much it is able to improve the ESG performance of the benchmark. In order to perform this performance evaluation, we use the equally weighted portfolio: at the beginning of the period, funds are evenly shared between all the assets within the investable universe.

We can perform a first raw assessment by looking at the time series of portfolio value and portfolio ESG score relative to the benchmark displayed in figure 21. In the plots of the first row, there are the dynamics of the value and ESG score of the portfolio whose initial weights are derived through the historic multi-goal static TE model developed in the previous chapter. In the second row and in the third rows, we have the values and ESG scores of the portfolio coming from the scenario multi-goal static TE model and the equally weighted portfolio respectively. Focusing on the behavior of the time series relative to the benchmark, we notice that, as we would expect, the first two portfolios present much more limited deviations from the benchmark in terms of value compared to the equally weighted portfolio. Between the first two, it seems that the historic data-based portfolio marginally better replicates the benchmark, however, they are really close and further analysis based on TE measures is necessary.

The first point of interest is the ability of the optimized portfolio to replicate the dynamic of the benchmark. In other words, we want to assess the extent to which we are able to replicate the risk and return dimension of the benchmark. If we assume that the time-series evolution of the portfolio value and ESG score can be described by a geometric Brownian motion, it means that a good replica of the benchmark is characterized by a similar expected return and standard deviation. Table 6 shows the sample average and sample standard deviation for the comparison: the portfolio derived from the historic TE model presents the closest sample average to the benchmark, instead, the equally weighted one has the closest sample

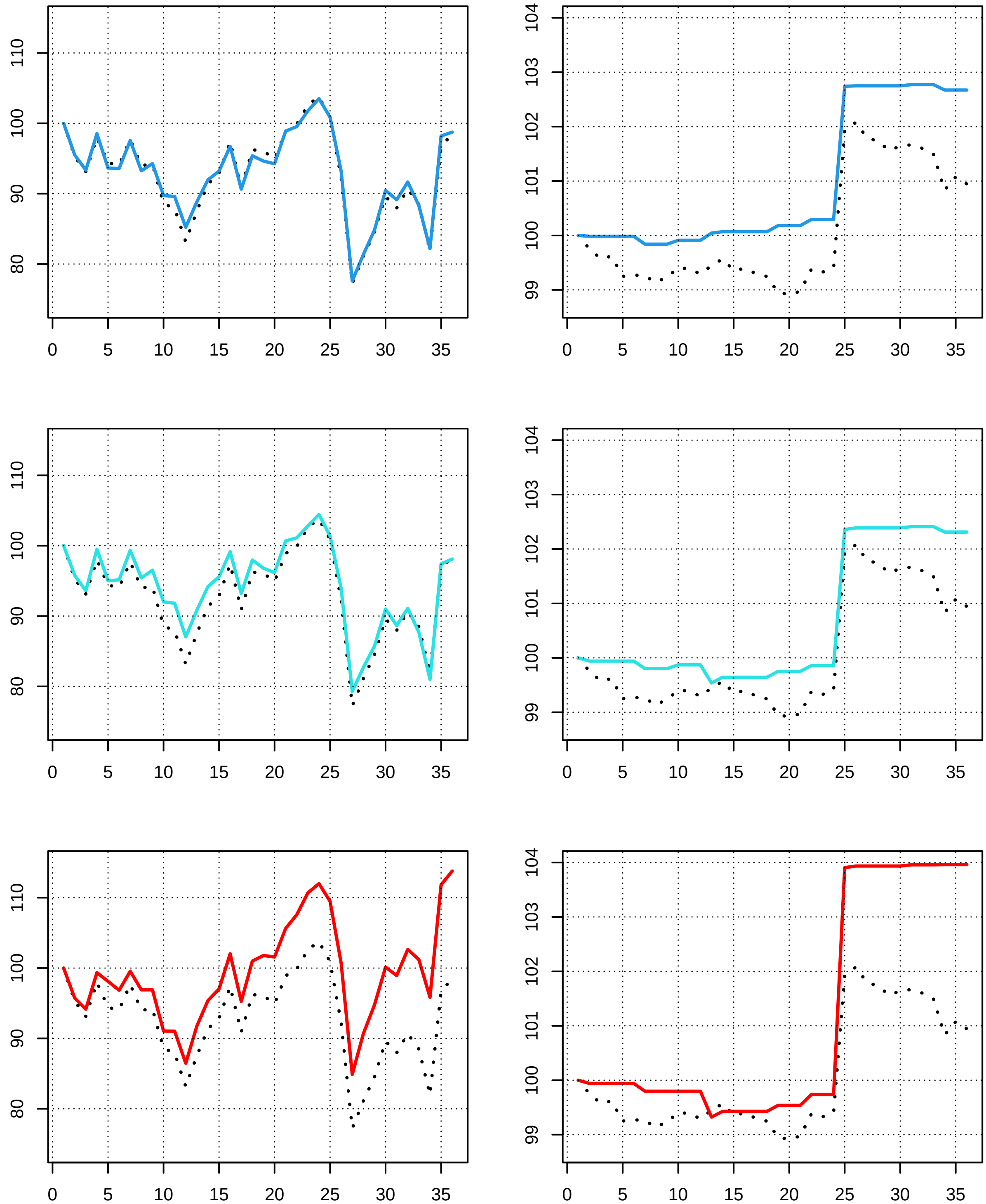

Figure 21: Time series of portfolio value and portfolio ESG score relative to the benchmark. In the first row there are the dynamics of the value and ESG score of the portfolio whose initial weights are derived through the historic multi-goal static TE model. In the second row and in the third rows, we have the values and ESG scores of the portfolio coming from the scenario multi-goal static TE model and the equally weighted portfolio respectively (source: personal elaboration)

standard deviation. However, due to the low variability of results and the small size of the out-of-sample data, this analysis does not result very informative.

Table 6: Sample average and sample standard deviation of the out-of-sample returns of the portfolios coming from the historic and the scenario TE models and the equally weighted one (source: personal elaboration)

Measure	Benchmark	Historic	Scenario	EW
Mean	0.0012261	0.0013000	0.0009130	0.0051667
Standard Deviation	0.0586739	0.0688251	0.0681529	0.0632203

The best measures to quantify the ability of the optimized portfolio to replicate the dynamic of the benchmark are TE measures. In table 7, the 2-norm (i.e. the Euclidean distance), the 1-norm, and the 1-norm for positive and negative deviation of the portfolio derived from the historic static TE model, the portfolio derived from the scenario-based static TE model, and the equally weighted one. From the perspective of our socially responsible investor, the portfolio deriving from the historic TE model is the one performing the best. At this point we are interested to find the combination of assets that returns the lowest possible deviations quantified through a symmetric TE measure: we are not interested in beating the benchmark but replicating it, thus, asymmetric measures should not be considered right now.

Table 7: 2-norm, 1-norm, and 1-norm for positive and negative deviation of the value of the portfolios coming from the historic and the scenario TE models and the equally weighted one (source: personal elaboration)

Measure	Historic	Scenario	EW
TE2	4.843196	10.834999	45.74438
TE1	22.662850	53.584343	230.01783
TE1 plus	15.076635	51.480959	230.01783
TE1 minus	7.586214	2.103384	0.00000

The second point of interest is assessing the ability of the optimized portfolio to maximize the performance of the sustainability component. In table 8, we can see that the equally weighted portfolio is the best ESG-performing in terms of average ESG returns and between the two portfolios coming from the static TE models the historic one is once again superior.

Table 8: Sample average and sample standard deviation of the out-of-sample ESG returns of the portfolios coming from the historic and the scenario TE models and the equally weighted one (source: personal elaboration)

Measure	Benchmark	Historic	Scenario	EW
Mean	0.0002796	0.0007924	0.0006907	0.0012171
Standard Deviation	0.0047508	0.0043050	0.0044564	0.0076034

Focusing on TEs, we need to consider only asymmetric measures. Even if the equally weighted portfolio has the best performance in terms of 1-norm of positive deviations, only the optimized portfolios present a zero 1-norm of positive deviations, implying that their ESG score has never underperformed the ESG score of the benchmark during the investment period under consideration.

Table 9: 2-norm, 1-norm, and 1-norm for positive and negative deviation of the ESG score of the portfolios coming from the historic and the scenario TE models and the equally weighted one (source: personal elaboration)

Measure	Historic	Scenario	EW
TE2	5.685961	4.007148	8.7281332
TE1	31.012286	20.959224	37.8409230
TE1 plus	31.012286	20.959224	37.5756946
TE1 minus	0.000000	0.000000	0.2652284

Conclusion

The statistical analysis performed in the time series of the value of the portfolios derived from a multi-goal TE model shows the potential of this type of modeling. Results in tables 7 and 9 show the superiority of the optimized model relative to the basic equally weighted portfolio in terms of replicating the financial performance of the benchmark while overperforming it from an ESG point of view.

This Master's Thesis paves the way for future implementation and different model specifications within the topic of TE models. Some possible topics for future research are: (i) the formulation of a dynamic multi-goal TE model that is able to include the sustainability dimension within the investment process, (ii) the expansion of the model to include different variables, such as market volatility in order to take into account different market conditions, and (iii) development of econometric tools in order to evolve the process of scenario simulation, in this paper, performed through the technique of historical bootstrapping.

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