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All-Weather: empirical analysis of static and dynamic portfolio allocation

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Introduction

In this paper, we analyzed the All-Weather portfolio allocation, which is based on the Risk-Parity principle. The paper's objective was to demonstrate the efficiency of this portfolio and its ability to protect against any scenario. The methodology we used is based on Post Modern Portfolio Theory (PMPT) and represents an evolution of MPT (Modern Portfolio Theory). To develop the paper, we used the works of Bridgewater Associates and Gabriele Galletta, one of the first to have imported this methodology in Italy. The paper is divided into seven chapters. The first chapter deals with asset allocation; in particular, two methodologies were compared, Markowitz against Risk-Parity. In the second chapter, the limits of the Risk Parity model were tested using the popular FAANG dataset. The results we obtained are shown both graphically and in summary tables to facilitate reading. Chapter three deals with the All-Weather portfolio. We described the construction process required to obtain the famous Dalyo's portfolio, and then we discussed the combination of assets that make up the portfolio and their sensitivity in different macroeconomic contexts. We used a matrix to explain the behavior of each asset class in different scenarios. In the chapter, some criticisms were made to the traditional 60/40 portfolio, and we have demonstrated its inefficiency in economic contexts other than just "pure" growth and decline. Some considerations on Bitcoin as a defensive instrument were reported and backtested against the Gold. In chapter four, we carried out two backtests of the portfolio, the first during the crisis of 2008 and the second during the Covid pandemic of 2020 against the well-known S&P500 benchmark. In chapter five, we have described the All-Weather that best suits the European investor and its main characteristics. In chapter six, we tried to optimize our portfolio by using leverage to make it perform almost like an equity portfolio but with a third of the risk. Finally, in chapter seven, we created a dynamic portfolio allocation. The portfolio was built so that it was able to change its asset allocation every time the macroeconomic context changed while always respecting risk parity. The paper also shows how the various macro scenarios were created. In the end, the paper compares the results obtained from the two allocations, dynamic vs. static.

1 Asset Allocation

The most important decision that every investor makes, before choosing the instruments in which to invest, is certainly the asset allocation of a portfolio. Two composition methodologies will be presented below. The first follows the Markowitz principle, while the second follows the Risk Parity principle.

1.1 Markowitz

Asset allocation strategies deal with the process by which an individual decides to invest his or her resources between different investments, whether they are stocks, bonds, real estate, or otherwise. The purpose of asset allocation is none other than the optimization of an investment portfolio. Building an efficient portfolio has been the subject of various researches and theories; the problem they focus on is the ideal combination of investments that allows different investors to minimize risk and maximize overall return. The first theory of particular relevance was the portfolio selection model. See Markowitz, H. M. (1991). [7]
Markowitz poses the process of building an investment portfolio as a mathematical problem. If you invest in different and unrelated stocks, if there is a reduction in the value of one stock, this will be offset by the increase in the value of another stock, and there will be a greater level of diversification.
The fundamental hypotheses on which the Markowitz theory was built are the following:

- Investors are risk-averse and intend to maximize expected utility;
- The transaction costs and taxes are zero;
- The market is perfectly competitive;
- The time horizon is mono-periodical, from t to t + 1 and we are in the presence of N different titles

From the previous hypotheses, the Mean-Variance principle was derived, which states that, among different investment choices, the one that guarantees the highest expected return and the lowest variance / standard deviation is preferable.

1.1.1 Elements of the model: expected value, variance and covariance of a security

We define the return on a financial asset (R) as the ratio between invested capital andd profits produced by an investment. Given the security i, its rate of return in t is:

$$r_{t,i} = \frac{P_{i,t} - P_{i,t-1} + D_{i,t}}{P_{i,t-1}}$$

where P_t is the price at time t, P_{t-1} is the price at time t-1 and D_t is the dividend.

Markowitz uses two statistical indices, the expected value, and the variance. The expected value in this regime of random variables will be given by:

$$\mu_i = E\left(R_i\right) = \sum_{j=1}^{k} r_{i,j} p_{i,j}$$

The variance index allows to identify the difference between the actual return of a security and its expected return;

$$\sigma_i^2 = \mathrm{Var}(Ri) = \sum_{j=1}^{k} \left(r_{i,j} - E\left(R_i\right)\right)^2 p_{i,j}$$

Covariance measures the degree to which two variables, in this case, two securities, move together and is calculated on single pairs of securities (i,j) as the expected value of the product of the spreads from the average of the two securities. $r_i - E\left(R_j\right) e r_j - E\left(R_j\right)$.

$$\mathrm{Cov}_{i,j} = E\left\{\left(\left(r_i - E\left(R_i\right)\right)\left(r_i - E\left(R_i\right)\right)\right\}\right.$$

Alternatively, we can rewrite it this way:

$$\mathrm{Cov}_{i,j} = E\left[R_i R_j\right] - E\left[R_i\right] E\left[R_j\right]$$

We introduce the last element which is the correlation coefficient, which expresses a linearity relationship between two securities and allows us to observe how the return of one security varies as the return of another varies. It is expressed as follows:

$$\rho_{i,j} = \frac{\mathrm{Cov}_{i,j}}{\sigma_i \sigma_j}$$

where:
σ_i is the standard deviation of security i and σ_j is the standard deviation of security$j$.

From here we can observ that we can rearrange the terms in the eqaution above to obtain:

$$\mathrm{Cov}_{i,j} = \rho_{i,j} \sigma_i \sigma_j$$

where:
- x_i is the percetage of wealth inveted in security i;
- $\sum_{i=1}^{n} x_i = 1$ (no leverage);
- $x_i \geq 0$ per $i = 1, 2, \ldots, n$ (no short selling).

The correlation index has the following properties:
- If $\rho = +1$ the variables are positively correlated and there is no risk reduction;
- If $\rho < 1$ the benefit in terms of risk reduction increases as the correlation index decreases;
- If $\rho = -1$ the benefit of risk reduction is maximal;
- In case of independent variables,$\rho = 0$.

1.1.2 Portfolio return and variance as a combination of linear random variables

Once having outlined the skeleton of the Markowitz model and analyzing its main elements, we must clarify the latter, in particular return and variance. We know that a linear combination is defined as any expression like:

$$a_1 v_1 + a_2 v_2 + \cdots + a_n v_n$$

where v_i represent elements of the vector space multiplied by an arbitrary scalar a_i. We take n random variables $X_1, X_{2,\ ldots}, X_n$; then a linear combination is a new variable Y, such that:

$$Y = a_1 X_1 + a_2 X_2 + \cdots + a_n X_n$$

Hence, being a portfolio made up of n securities by the relative weights, the portfolio return R_p is a linear combination of the individual returns as random variables. Let's assume a number n of securities, with return R_i; let R_p be the random variable given by the weighted sum of the returns of the securities:

$$R_p = \sum_{i=1}^{n} R_i x_i = x_1 R_1 + x_2 R_2 + \cdots + x_n R_n$$

The expected value of a linear combination is equal to the linear combination of the expected values:

$$E\left(R_p\right) = \sum_{i=1}^{n} x_i E\left(R_i\right)$$

The variance of a linear combination is given by the following equation:

$$\sigma_p^2 = \operatorname{Var}\left(R_p\right) = E\left[R_p - E\left(R_p\right)\right]^2$$

or:

$$\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2 x_1 x_2 \sigma_{1,2}$$

hence, the variance of the portfolio formed by n securites is:

$$\sigma_p^2 = \operatorname{Var}\left(R_p\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \operatorname{Cov}_{i,j}$$

1.1.3 Constrained optimization process and efficient frontier

The mean-variance rule states that the investor will select the portfolio that will choose the highest return-risk combination among the efficient ones.
From an analytic point of view, this principle can be seen as a constrained maximization problem for the determination of optimal x; here the investor will choose the optimal combination with minimum risk, given a certain level of return, or with maximum return, given a certain level of risk.

We have to set the objective function. Given some constraints, the problem consists in finding the function to be minimized or maximized in order to observe the values of the independent variable at which the objective function reaches its minimum or maximum.

We derive the Lagrangian function as follows:

$$L(x, y, \lambda) = f(x, y) - \lambda[g(x, y) - c]$$

As we can see from this function, there are different independent variables. Consequently, to implement the process of minimization or maximization, the first partial derivatives must be calculated compared to each variable to then equal them to zero and put them to the system. Whatever the selected procedure, there is a further constraint, the sum of the weights of the individual securities must be equal to the unit, not allowing to use debt.
Taking into account what we just said we can express four optimization paths (constrained) as follows:

$$\begin{cases} \min_{x_i} \sigma_p^2 = \min_{x_i} \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{i,j} \\ E(R_p) = \sum_{i=1}^n x_i E(R_i) \\ \sum_{i=1}^n x_1 = 1 \end{cases} \quad \begin{cases} \max_{x_i} E\left(R_p\right) = \max_{x_i} \sum_{i=1}^n x_i E\left(R_i\right) \\ \sigma_i^2 = \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{i,j} \\ \sum_{i=1}^n x_1 = 1 \end{cases}$$

$$\begin{cases} \min_{x_i} \sigma_p^2 = \min_{x_i} \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{i,j} \\ E(R_p) = \sum_{i=1}^n x_i E(R_i) \\ \sum_{i=1}^n x_1 = 1 \\ x_i \geq 0 \end{cases} \quad \begin{cases} \max_{x_i} E\left(R_p\right) = \max_{x_i} \sum_{i=1}^n x_i E\left(R_i\right) \\ \sigma_i^2 = \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{i,j} \\ \sum_{i=1}^n x_1 = 1 \\ x_i \geq 0 \end{cases}$$

Markowitz noticed that, given a set of possible efficiency-variance combinations, the feasible combinations form a closed region of space, the so called efficient frontier, formed by all dominant combinations.

1.1.4 Efficient Frontier with n $>$ 2

We start defining Rs which is the return of each title, with i = A, B, ..., N. We hypothesize the absence of short selling and leverage.

As we remember from previous paragraphs, the portfolio return is a linear combination of individual returns multiplied by their respective weights.

$$E\left(R_P\right)=\sum_{i=1}^{n} x_i E\left(R_i\right)$$

the variance will be:

$$\sigma_P^2=\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \rho_{i,j} \sigma_i \sigma_j=\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \operatorname{Cov}_{i,j}$$

To calculate the portfolio variance it is necessary to calculate the covariance for each pair of securities, to simplify the calculations, we resort to the matricial notation. We build a variance-covariance matrix as follows:

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & & \cdots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}$$

From the variance-covariance matrix we know that:
- It is square and symmetrical, since the number of rows and columns is the same;
- Presents the n variance values along the diagonal, being $\sigma_{nn}=\sigma_n^2$
- The correlation terms are $\frac{n^2-n}{2}$

We can use the matrix to simplify the calculation of the variance of n titles. We can use the matrix to simplify it by making the product between vectors and matrices.
Having:
- w the column vector $(n \times 1)$ composed of the weights x_i of the n titles;
- w 'the transport vector $(1 \times n)$, which we remember to be a vector obtained by exchanging rows and columns of the previous one
- S the symmetric variance-covariance matrix $(n \times n)$;

Making the product between the above elements, we will find the standard deviation:

$$\sigma_P=\sqrt{w_{1n}^{\prime} S_{nn} w_{n1}}$$

At this point we can move on to building the efficient frontier, to identify combinations of efficient portfolios.

After having built the efficient frontier we can conclude our analysis by identifying the optimal portfolio among all combinations of risky and non-risky securities. We need to introduce a variable that can explain how the investor chooses a portfolio of securities from all the return-variance combinations found: the so called risk aversion.

Investor preferences can be represented by indifference curves. Recall that an indifference curve is a set of combinations (x, y) that guarantee the consumer, in this case, an investor, the same level of utility. We will find a series of indifference curves, each of which will represent a different level of utility for the investor.

The set of indifference curves is given by the expected utility function:

$$E[(U(x)] = E\left(R_p\right) - \frac{1}{2}\lambda\sigma_p^2.$$

with λ representing the investor's aversion to risk. Consequently, the investor will choose the portfolio that maximize the expected utility function. The indifference curves are increasing, since the higher the return the higher the risk, concave, since the higher the risk, the lower the investor's propensity to further increase it. The optimal portfolio is given by the intersection between the indifference curve with greater utility (the lowest) and the efficient frontier.

1.2 Risk Parity

After the global financial crisis in 2008, risk management has particularly become more important than performance management in portfolio optimization. The Risk Parity technique differs in its approach and methodology to traditional finance. In its simplest form, Risk Parity seeks to balance the contribution to the overall portfolio risk of each assets class that makes up the portfolio itself. A traditional 60% equity / 40% bond portfolio, which is the basis of the capital allocation of many investors, is not diversified, as 90% of the risk in this traditional portfolio is concentrated in equities since stocks have historically been three times more volatile than fixed-income securities.
Risk parity try to eliminate this weakness by building a much more diversified and balanced portfolio, ensuring that the contribution of each asset class is equal in the portfolio. Through Risk Parity it is possible to achieve allocations that give higher returns adjusted for the risk.

As we can see from Figure 1, Risk-Parity focuses more on allocation of risk rather than allocation of capital. The minimum variance portfolio tries to minimize variance (with the disadvantage that few assets contribute more to risk) the risk parity portfolio seeks to constrain each asset to contribute equally to the risk (volatility) of the entire portfolio.

1.2.1 Solving the Risk Parity portfolio (RPP)

Vinícius, Z. & Palomar, D.P. (2019)[13] explained in their article a methodology for obtaining the weights of a portfolio that follows the logic of Risk-Parity. The method they used is presented below:
From Euler's theorem, the volatility of the portfolio $\sigma(\mathbf{w}) = \sqrt{\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}}$ can be decomposed as

$$\sigma(\mathbf{w}) = \sum_{i=1}^{N} w_i \frac{\partial\sigma}{\partial w_i} = \sum_{i=1}^{N} \frac{w_i(\boldsymbol{\Sigma}\mathbf{w})_i}{\sqrt{\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}}}$$

The risk contribution (RC) from the i th asset to the total risk $\sigma(\mathbf{w})$ is defined as

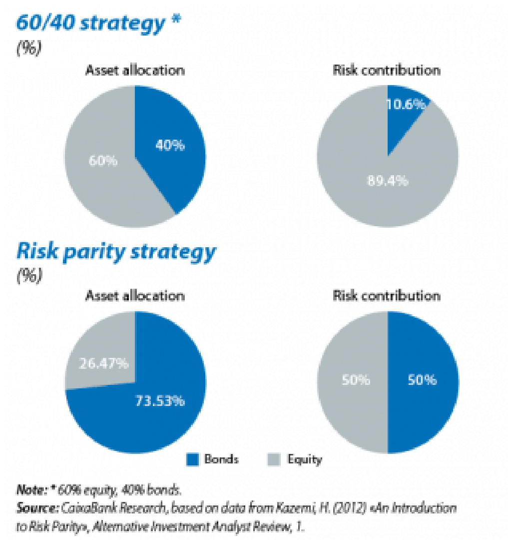

Figure 1: Capital contribution versus Risk contribution.

$$\mathrm{RC}_i = \frac{w_i(\boldsymbol{\Sigma}\mathbf{w})_i}{\sqrt{\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}}}$$

which satisfies $\sum_{i=1}^{N} \mathrm{RC}_i = \sigma(\mathbf{w})$
The relative risk contribution (RRC) is a normalized version:

$$\mathrm{RRC}_i = \frac{w_i(\boldsymbol{\Sigma}\mathbf{w})_i}{\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}}$$

so that $\sum_{i=1}^{N} \mathrm{RRC}_i = 1$
The risk parity portfolio (RPP) attemps to "equalize" the risk contributions:

$$\mathrm{RC}_i = \frac{1}{N}\sigma(\mathbf{w}) \quad \text{or} \quad \mathrm{RRC}_i = \frac{1}{N}$$

More generally, the risk budgeting portfolio (RBP) attempts to allocate the risk according to the risk profile determined by the weights $\mathbf{b}$ (with $\mathbf{1}^{\mathbf{1}}\mathbf{b} = 1$ and $\mathbf{b} \geq \mathbf{0}$):

$$\mathrm{RC}_i = b_i\sigma(\mathbf{w}) \quad \text{or} \quad \mathrm{RRC}_i = b_i$$

In practice, one can express the condition $\mathrm{RC}_i = \frac{1}{N}\sigma(\mathbf{w})$ in different equivalent ways such as

$$w_i(\Sigma\mathbf{w})_i = w_j(\Sigma\mathbf{w})_j, \quad \forall i, j$$

Diagonal formulation

Assuming that the assets are uncorrelated, i.e., that $\boldsymbol{\Sigma}$ is diagonal, and simply using the volatilities $\boldsymbol{\sigma} = \sqrt{\text{diag}(\boldsymbol{\Sigma})}$, one obtains

$$\mathbf{w} = \frac{\boldsymbol{\sigma}^{-1}}{\mathbf{1}^T \boldsymbol{\sigma}^{-1}}$$

or, more generally,

$$\mathbf{w} = \frac{\sqrt{\mathbf{b}} \odot \boldsymbol{\sigma}^{-1}}{\mathbf{1}^T \left(\sqrt{\mathbf{b}} \odot \boldsymbol{\sigma}^{-1} \right)}$$

However, for non-diagonal Σ or with other additional constraints or objective function terms, a closed form solution does not exist and some optimization procedures have to be constructed. The previous diagonal solution can always be used and is called naive risk budgeting portfolio.

Vanilla convex formulation

Suppose we only have the constraints $\mathbf{1}^T\mathbf{w} = 1$ and $\mathbf{w} \geq \mathbf{0}$. Then, after the change of variable $\mathbf{x} = \mathbf{w}/\sqrt{\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}}$, the equations $w_i(\boldsymbol{\Sigma}\mathbf{w})_i = b_i\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}$ become $x_i(\boldsymbol{\Sigma}\mathbf{x})_i = b_i$ or, more compactly in vector form, as

$$\boldsymbol{\Sigma}\mathbf{x} = \mathbf{b}/\mathbf{x}$$

with $\mathbf{x} \geq \mathbf{0}$ and we can always recover the portfolio by normalizing: $\mathbf{w} = \mathbf{x}/\left(\mathbf{1}^T\mathbf{x}\right)$

At this point, one could use a nonlinear multivariate root finder for $\boldsymbol{\Sigma}\mathbf{x} = \mathbf{b}/\mathbf{x}$. For example, in R we can use the package rootSolve.

With the goal of designing risk budget portfolios, Spinu proposed to solve the following convex optimization problem:

$$\underset{\mathbf{x} \geq 0}{\text{minimize}} \quad \frac{1}{2}\mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x} - \sum_{i=1}^{N} b_i \log\left(x_i\right)$$

where the portfolio can be recovered as $\mathbf{w} = \mathbf{x}/\left(\mathbf{1}^T\mathbf{x}\right)$

Indeed the risk budgeting equation $\boldsymbol{\Sigma}\mathbf{x} = \mathbf{b}/\mathbf{x}$ corresponds to the gradient of the convex function $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x} - \mathbf{b}^T\log(\mathbf{x})$ set to zero:

$$\nabla f(\mathbf{x}) = \boldsymbol{\Sigma}\mathbf{x} - \mathbf{b}/\mathbf{x} = \mathbf{0}$$

Thus, a convenient way to solve the problem is by solving the following convex optimization problem:

$$\underset{\mathbf{x} \geq 0}{\text{minimize}} \quad \frac{1}{2}\mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x} - \mathbf{b}^T\log(\mathbf{x})$$

which has optimality condition $\boldsymbol{\Sigma}\mathbf{x} = \mathbf{b}/\mathbf{x}$.

Such solution can be computed using a general-purpose convex optimization package, but faster algorithms such as the Newton method and the cyclical coordinate descent method, are implemented in this package.

2 Empirical Analysis with fully equity portfolios

This section will present the empirical analysis using the two allocation methods, the Markowitz allocation with the Tangency Portfolio and the Risk Parity allocation described in the previous chapter applied to fully equity portfolios. To perform all the necessary calculations and backtest the two strategies, we used the R Studio Software[1]. All the packages that we installed will be listed later.

2.1 Data used

As the first thing, we chose our basket of stocks, we opted to choose the popular FAANG dataset which encompasses the shares of Facebook, Apple, Amazon, Netflix, and Google. At this time we have not given too much importance to diversification which will be dealt later with another dataset. The historical series of share prices listed above were downloaded using the stockDataDownload [2] function from the portfolioBacktest package in the period 2014-01-01 to 2021-08-25. Once the returns were made, we then created two functions: *riskParity* and *maxSharpeRatio* to which we passed the downloaded data. The first function, which made use of the *riskParityPortfolio* package, returned a portfolio with the weights distributed according to the logic of risk parity in which each asset contributed equally to the portfolio risk. The second function returned the portfolio with the greatest expected value and the least variance, thus following the Markowitz logic.

We get the following weights by executing the two functions with our FAANG dataset.

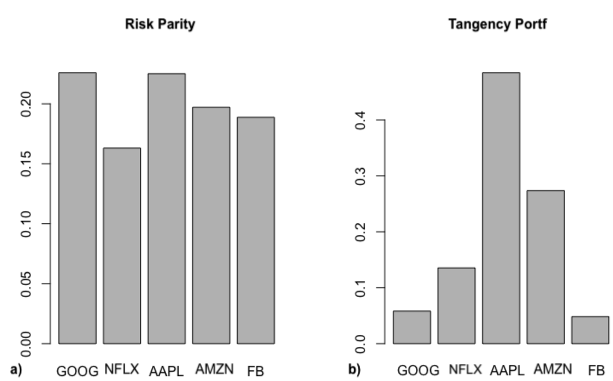

Figure 2: Weight allocation.

From figure 2.(a) It can be seen that, being companies operating in the same sector, their weight within the portfolio will be similar as they will have a similar risk, while in figure

[1]R is a language, or an environment, for data analysis and visualization. RStudio is an integrated development environment (IDE) for R. It includes a console, syntax-highlighting editor that supports direct code execution, as well as tools for plotting, history, debugging and workspace management.

[2]This function is used to download stock data from the internet. It will return 6 time-series objects of the same dimensions named 'open', 'high', 'low', 'close', 'volume', 'adjusted' and 'index'

2.(b) more weight is allocated to the security, in this case Apple, that dominates the others.

2.2 Empirical Results

Once the portfolio is built, it is necessary to periodically rebalance it because, for sure, one asset class will prevail over another. If we do not rebalance it, we will break away from our initial allocation. We, therefore, decided to analyze our FAANG dataset over the seven years and rebalance it every three months to see how the two allocation strategies behaved: Risk Parity vs. Tangency portfolio.
To perform the calculus, we have used the portfolioBacktest[3] package; we have also included transaction costs, which are the costs that arise every time the investor buys or sells an asset to rebalance the portfolio. The transaction costs have been estimated at 0.5% for the sale of an asset and 0.5% for purchasing an asset.
Figure 3 shows that the risk parity portfolio, red line, always outperforms the tangency portfolio, blue line, and drawdowns have a lower intensity. .

The figure 3 shows that the risk parity portfolio, red line, always outperforms the tangency portfolio, blue line, and drawdowns have a lower intensity.

Figure 3: Comparison Results.

As the Table 1 shows, the risk parity methodology applied only to assets performs better than the Markowitz allocation. The Sharpe ratio is significantly higher showing greater efficiency, the max drawdown is lower as well as the volatility confirming the lower riskiness of this portfolio.

[3]Automated backtesting of multiple portfolios over multiple datasets of stock prices in a rolling-window fashion.

	Risk Parity portfolio	Tangency portfolio
Sharpe ratio	1.420412e+00	9.805226e-01
max drawdown	3.062254e-01	3.544405e-01
annual return	3.530426e-01	2.918230e-01
annual volatility	2.485494e-01	2.976198e-01
VaR (0.95)	2.560722e-02	2.991502e-02
CVaR (0.95)	3.771379e-02	4.465906e-02

Table 1: Performance comparison between the two allocations.

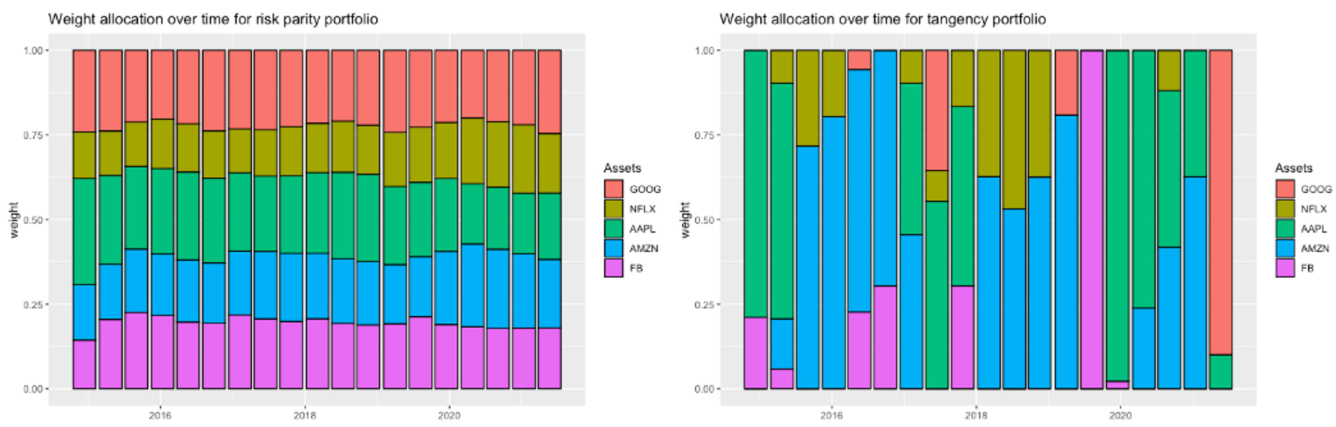

Figure 4: Rolling window weights allocation.

IIn Figure 4, we can notice that the asset composition in the first portfolio is more balanced. In contrast, the tangency portfolio for some periods does not use all the asset classes that make up the portfolio. Not constantly using and rebalancing all the securities in the portfolio may reduce costs.

Does risk parity always perform better than the tangency portfolio?

To answer this question, we back-tested a more extensive and more diversified equity dataset than FAANG. The datasets we used were created by sampling 50 shares for each dataset randomly from the S&P 500 index. The time window provided by the dataset went from 2016 to 2017, and the rebalancing was always performed every 90 days. We have repeated the procedures seen above and included the transaction costs.

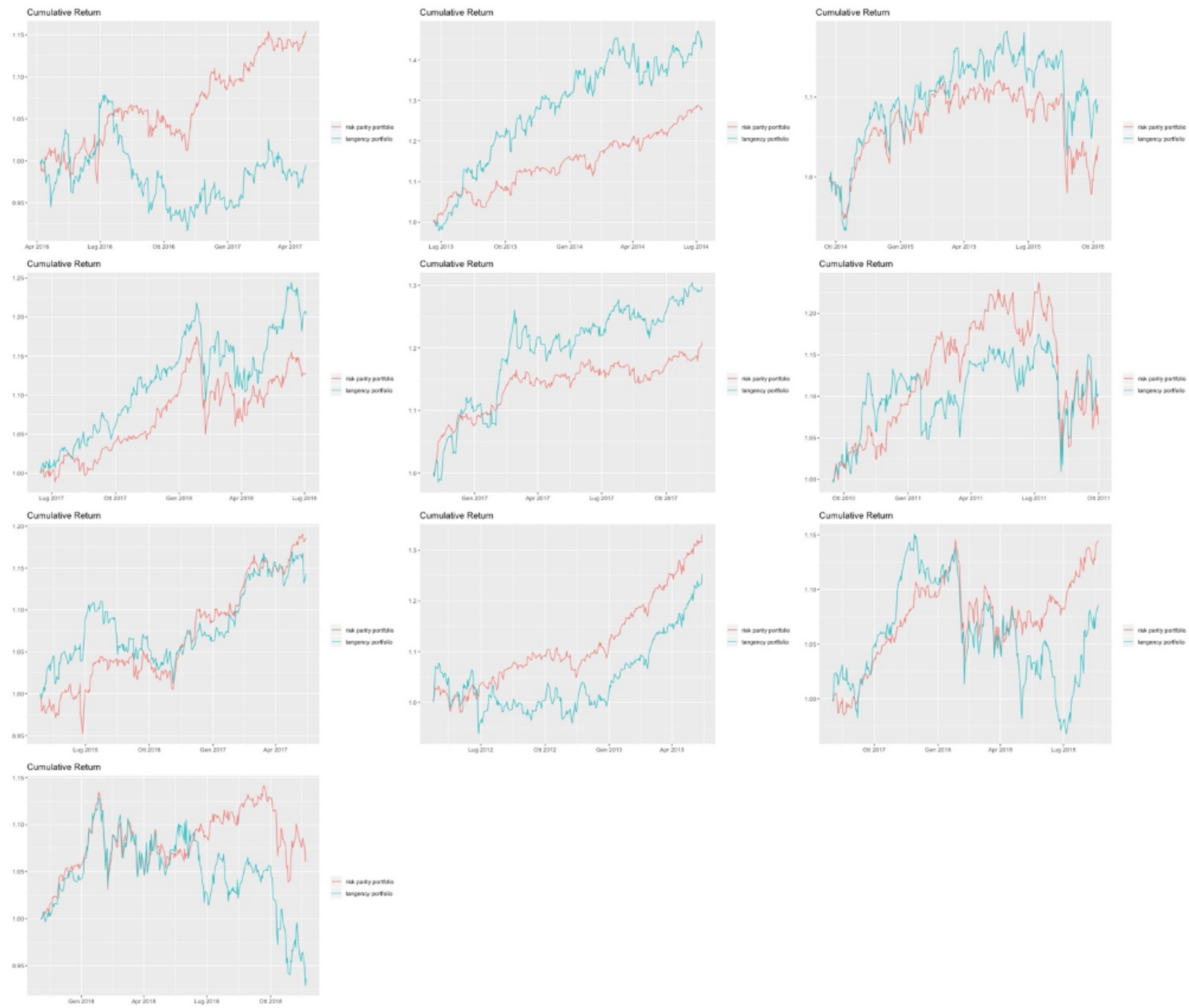

Figure 5: Risk Parity Vs Tangency Portfolio comparison over 10 datasets.

The results are illustrated in Figure 5. We can observe that the cumulative returns of the equity portfolio that follow the risk parity logic beat the tangency portfolio in 60% of cases. In addition, risk parity portfolios tend to move more homogeneously without sudden changes of direction that occur in the tangency portfolio; this might happen because the second portfolio often does not use the entire universe of securities available.

2.3 Transaction Costs

Often investors ask themselves how often they need to rebalance their portfolios to preserve the returns gained. In the following paragraph, we will analyze the optimal rebalancing period that allows us to obtain the highest returns for our portfolio. We will see how the two allocation methods respond based on different rebalancing periods. Selling and purchasing costs were kept at 0.5% per transaction. We first considered the FAANG dataset, and then some datasets were created randomly by extracting shares from the S&P500.

We chose a vector that contained the number of days to pass until the next rebalancing. We calculated the portfolio performance for each value contained in the vector and printed the results.

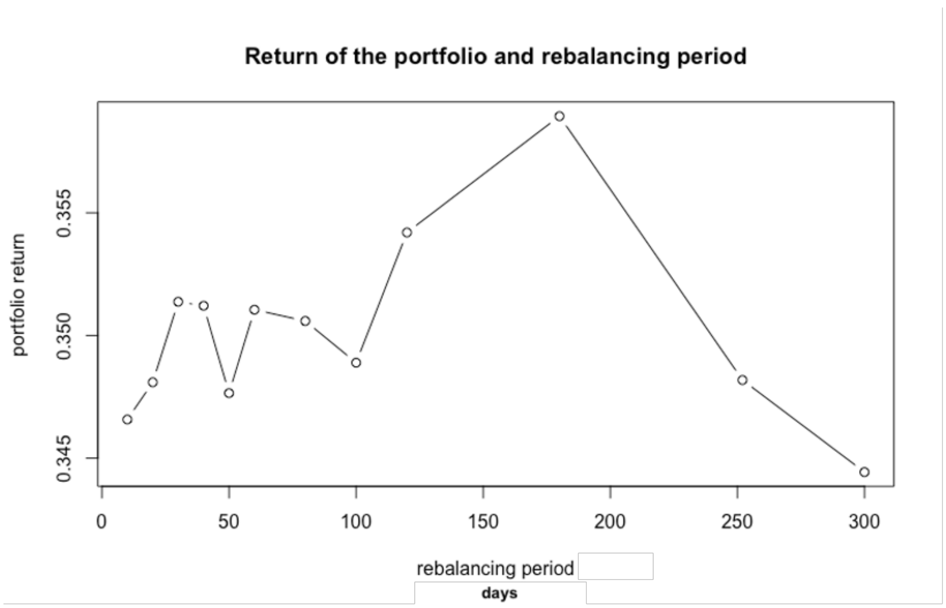

Figure 6: Risk Parity portfolio.

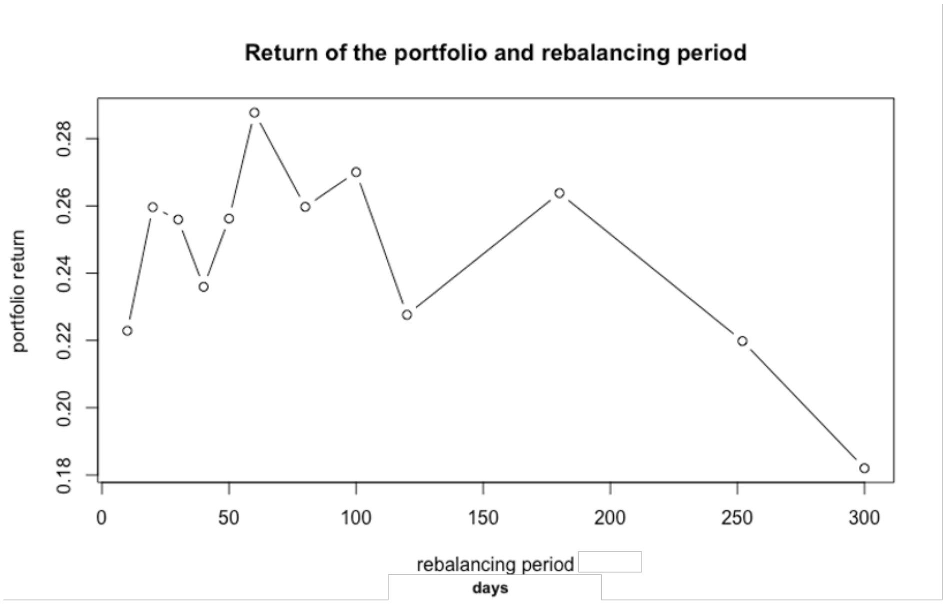

Figure 7: Tangency Portfolio.

Figures 6 and 7 show the relationship between portfolio return and the rebalancing period for the FAANG dataset. For the risk parity method, the returns increase if the rebalancing frequency decrease, with a maximum return (35.8% annualized) realized at 180, i.e., every six months.
For the tangency portfolio, a higher rebalancing frequency leads to better per- performance, with a maximum return of (28.7% annualized) obtained at 60, a rebalancing every two months. We found similar results using the second dataset, which is more diversified. For each dataset analyzed among the ten available, the risk parity method tends to rebalance less frequently than the tangency portfolio.
We recalculated the performance table using the optimal rebalancing period for each methodology, 180 days for risk parity, and 60 days for the tangency portfolio. The results obtained are the following:

In Table 2, we can observe that the former portfolio continues to dominate; however, the latter portfolio improves its performance in all four fields.

	Risk Parity optimized	Tangency optimized
Sharpe ratio	1.439330e+00	9.747950e-01
max drawdown	3.103118e-01	3.553224e-01
annual return	3.589367e-01	2.877285e-01
annual volatility	2.493776e-01	2.951682e-01

Table 2: Performance comparison between the two allocations after optimization.

3 Risk Parity applied to All-Weather

If the rule according to which there is a consistent long-term relationship between risk and return, as shown in Figure 8, is always valid, the asset classes with higher risk give a higher return. If the Securities Market Line (the curve that in the CAPM combines the expected return of asset classes versus their risk) has a constant slope, then the asset's risk-adjusted return is constant.

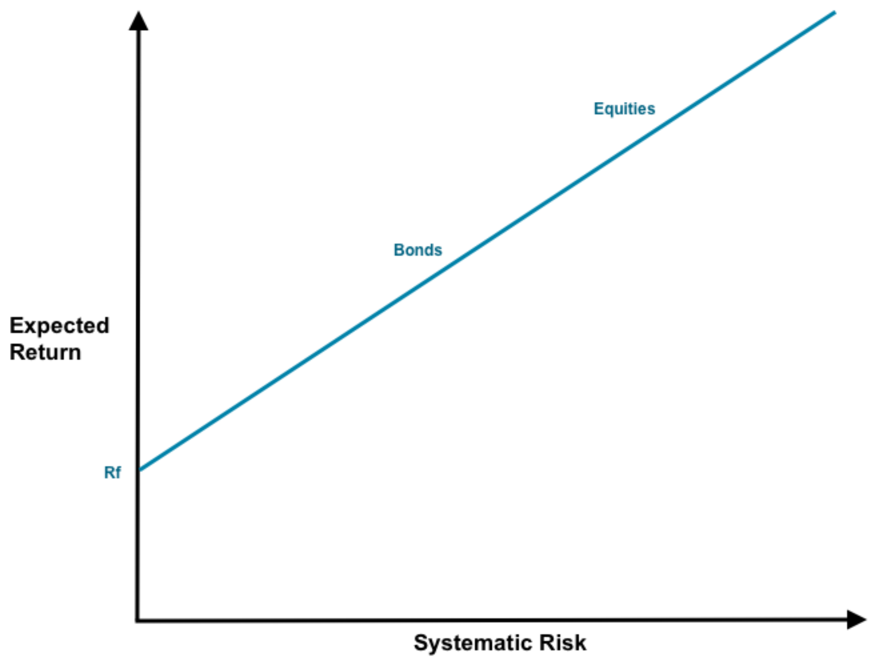

Figure 8: Security Market Line.

In other words, if the expected returns on equities and fixed income securities were adjusted for their respective levels of risk, the expected excess return on equities would be equal to the expected excess return on bonds. The exciting element is that classical economic theory supports this architecture on which Risk Parity is based. If an asset class existed capable of returning higher risk-adjusted returns, an excess of demand from the same asset class and free-market forces would remove the advantage. If every asset in which you can invest your capital has a risk-adjusted return equal to the other asset classes, a balanced and diversified portfolio that contributes to the overall risk in equal measure is more efficient than a traditional portfolio 60 / 40[4] or other types of similar portfolios.

[4]The "60/40 portfolio" is formed by 60% allocation to equities with the intention of providing capital appreciation and a 40% allocation to fixed income to potentially offer income and risk mitigation.

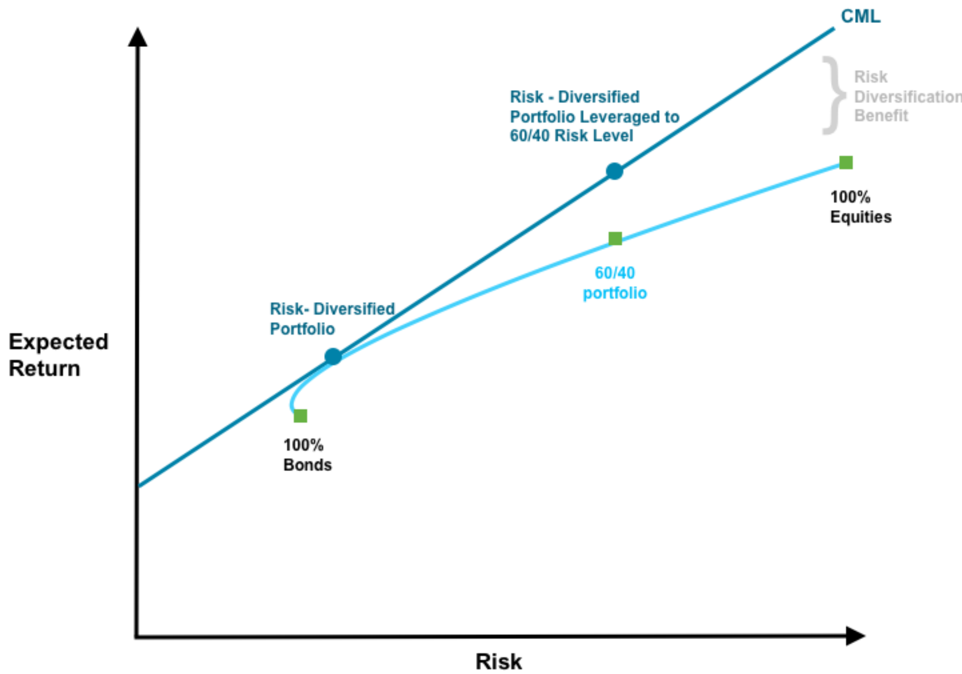

Figure 9: Benefit of Risk Diversification.

This greater efficiency can then be increased thanks to the use of financial leverage. By manipulating the individual assets that make up the portfolio, it is possible to improve the overall return while keeping the risk constant. As shown in Figure 9, starting from a classic 60/40 portfolio, a more profitable portfolio is achieved, with greater efficiency on the corresponding Capital Market Line, with the same risk.
This is a fundamental aspect of the strategy that we will implement, especially when dealing with the management of an All-Weather portfolio. In such a portfolio, the assets are selected based on their diversification advantages and are manipulated upwards or downwards to reach the target volatility. This construction process creates an ideal environment for systematically collecting portfolio gains through rebalancing or taking advantage of over-threshold price reductions on other assets. This process, in other words, allows you to take advantage of over-bought and over-sold phenomena on assets resulting in higher potential returns over the long term.

3.1 Portfolio Design

When designing a portfolio, we want to do it with the lowest possible risk. Considering that the portfolio's overall return will be given by the weighted average of the returns of the asset classes that make up the portfolio, it will be necessary to choose the basic allocation by dividing the asset classes so that they will give us the return we have chosen. The basic components will be:

- risk-free return;
- market beta, or the excess returns of each asset over the risk-free return
- Jensen's alpha, or the manager's extra return

Dalio.R (2005)[2] remind us in his article that the first step in engineering a portfolio to achieve the desired return is deciding how much of the return should come from the market beta and how much from Jensen's Alpha. Unfortunately, this aspect and this analysis are often not easily predictable, as they are difficult to quantify. A more qualitative approach will therefore be required. We know that the market betas are generally known a priori, over long time horizons, and quite reliable. At the same time, the returns deriving from Alpha can also vary a lot but are not very stable and dependable. For example, assuming a risk-free return of 1%, the returns from market betas over a 20-year time horizon could be as follows:

- Global Diversified Stocks: 5,5%
- US Stocks: 10%
- Gold: 5,5%
- Global bond LT: 5,5%
- Global corporate bond: 4%

The average return derived from the market betas of the portfolio, weighted by the weights of the same, could, for example, be 6.5%.
The range of returns from negative Alpha is enormous because, in this zero-sum game, exceptional managers can produce excellent Alpha while underperforming managers can produce even more negative Alpha. Based on these considerations, our approach envisages an initial allocation that favors market betas and then moderately adds Alpha thanks to periodic portfolio adjustments while anchoring ourselves to automatic behaviors and not being dependent on subjective assessments. These will be a consequence of the Risk Parity as a Portfolio management methodology.

We now have to decide the composition of our asset allocation to recreate a portfolio that respects the basic principles we set ourselves, that is, to be immune to short-term economic and financial shocks and to perform and create value in any economic context. Following this approach, All Weather's asset allocation is recreated, designed to work well in different economic environments, and oriented to produce a return similar to equities but with considerably less risk. Furthermore, using leverage makes it possible to build allocations with higher returns on the shares for the same risk assumed.
Figure 10 shows All-Weather returns since 1970 modified for the same level of risk as conventional 60/40 asset allocation (equities/bonds).

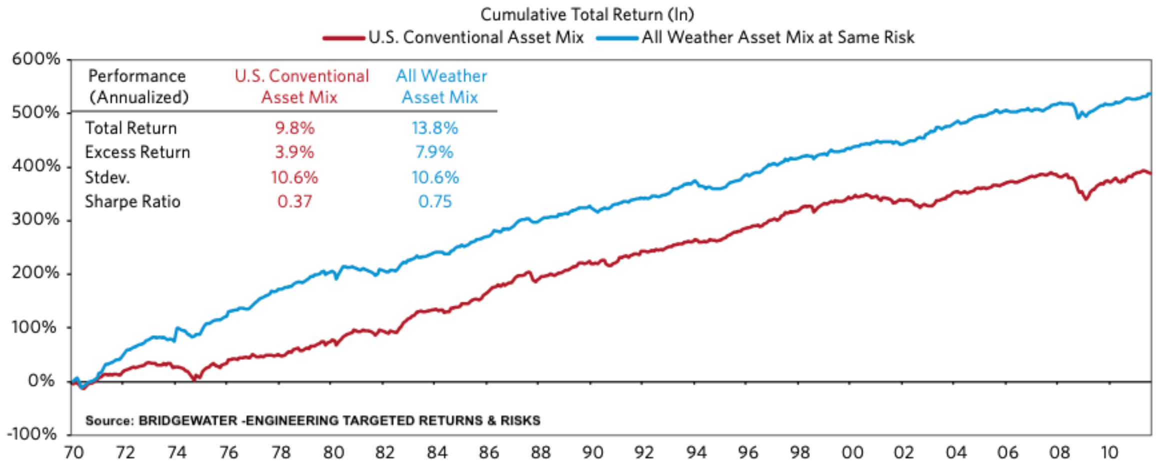

Figure 10: All Weather asset mix run at the same risk as the conventional portfolio Cumulative Total Return (ln).

The All-Weather asset mix (the blue line) would have produced an additional 300-400 basis points of return per year at the same risk level as the conventional portfolio (the red line).

3.2 Asset Mix choice

The long-term risk of holding a portfolio concentrated in stocks, or any other asset is far more tremendous than most investors realize and too great to bear. Hence, the need to build long-term investments immune to economic changes.

Which combination of assets has the best chance of delivering good returns over time in all economic environments?

Before answering this question, let's look at the mistakes that are made by traditional approaches. The traditional investment approach is often based on a very high allocation to equities, a concentration of risk in this asset class to generate higher long-term returns (as seen in the previous paragraphs). A conventional portfolio has over half of its funds, and about 90% of its risk in equities, this type of portfolio remains highly correlated with the equity markets, with even lower returns. [9]
This approach has a severe flaw: if the portfolio is characterized by an intense concentration in a single type of asset, this approach carries with it a significant risk of poor long-term returns caused by the fact that each asset is subject to poor performance, which can last for a decade or more, due to a prolonged change in the economic environment.
Think of the lost decade of stocks between 2000 and 2008, the 1970s characterized by hyperinflation, or the 1930s.
A portfolio investment designed for the long term should be neutral to changes in the eco-

nomic environment. This means that short-term risks can cancel over time, allowing an investor to achieve the desired long-term returns.
Even if the asset management industry continues to repeat how, in the long term, markets are always growing, it does not consider that an economic depression, which affects not only emerging countries but also developed countries, is an eventuality that happens often, so often that it is practically certain that in our life we will live at least one period of this kind, where the shares will return poor returns for a considerably long time.
Even modern economic theories have consolidated a basic orientation according to which an under-utilization of the factors of production (labor and capital) is a condition that can be persistent, such as creating sub-optimal but stable sub-balances of the economic system. A lower level of production and employment than what equilibrium could be (with full use of resources) is not only valid in the short term but becomes structural, without the normal market forces being able to intervene. Cooming back to the initial question, the answer is not trivial. Many factors affect it and depend on assumptions related to volatility and correlations between asset classes that, unfortunately, change over time. Furthermore, most risk measures do not adequately reflect the risk of being in pro-longed adverse environments, producing meager returns for a long time. Starting from these assumptions, we built an allocation based on two fundamental concepts:

- Asset classes outperform liquidity over time.
- Asset prices discount future economic scenarios.

Point number 1 is a fundamental assumption of the financial system we live in: when you decide to invest the liquidity owned, you decide to transfer this liquidity to someone who needs it today. For this transfer, our borrower will have to pay a premium for the liquidity lent, depending on the risk we take on by lending this liquidity. This risk premium will be positive, it may become negative under conditions of high uncertainty in the short term, but it will always be positive in the long run.
In point number 2, we want to emphasize that the price of any assets reflects the discounted value of the expected future cash flows. These expected cash flows and the discount rate incorporate expectations about the future economic environment, such as the level of inflation, earnings growth, the likelihood of business default, and so on. [8] As the environment and expectations change, the price of assets will change as well.
Given these two structural elements, the returns of any asset will have two main factors: the accumulation and changes in the risk premium and unexpected changes in the economic environment, which will affect cash flows. Therefore, the objective of strategic asset allocation becomes very clear: to collect the risk premium as consistently as possible, minimizing the risk due to unexpected changes in the economic environment. This simple model based on such simple concepts shows why conventional asset allocations based only on equities and bonds are limiting and unable to grow in any economic context.
Stocks discount future paths of earnings growth and grow more when companies' profits and earnings are more robust than expected. Bonds remunerate capital adequately when interest rates fall unexpectedly due to unforeseen economic weakness (but they will do so

less in the world of zero interest rates). In other words, these asset classes have had opposite sensitivities to growth surprises. But they have the same sensitivity to inflationary shocks. In an environment with high inflation rates, both asset classes will suffer. In the analysis of economic cycles, investors give less and less weight to inflation and, especially, to inflationary surprises: it is not the expected inflation that is worrying, but real inflation higher than expected. In this context, inflation is always of little concern as investors are fooled by the fact that their investments are doing substantially well, observing and paying attention only to their nominal growth. In times of high inflation, it is not difficult to find, as happened in the 1970s, government bonds that pay double-digit interest, with the capital revalued, with almost no risk, in a few years. Unfortunately, that revaluation is only imaginary, as the real increase in capital is zero, if not even negative. In times of high inflation, prices always grow faster than the central bank's nominal rate.
The dizzying rise in prices is the first sign of an economic recession. A market mechanism in which the growth of prices and wages, due to various factors, leads to a contraction of demand (due to ever-increasing prices), then of supply and finally of employment, creating an economic recession that, without the intervention of the Policy Makers, could become a depression. That's why central banks give very important weight to inflation control.

3.3 Asset Classes sensibility in different economic cycles

The financial engineering behind All Weather is to design a portfolio that is insensitive to future shocks. This will be possible thanks to understanding the relationship between asset classes and their variation in different economic contexts. To build this type of portfolio, we can use an exemplification: split the possible economic scenarios into four different environments and, based on the analysis of the back-tests carried out, understand the behavior of the asset classes in each context.
The economic contexts in which we could find ourselves are the following:

- Growth
- Recession
- Deflation
- Inflation

Each of these environments can mix with at least one other environment to generate periods of growth and inflation (called successful' reflation', which however can degenerate into hyper-inflation), or periods of moderate or zero growth, with falling prices (stag-deflation), which could degenerate into a depression.

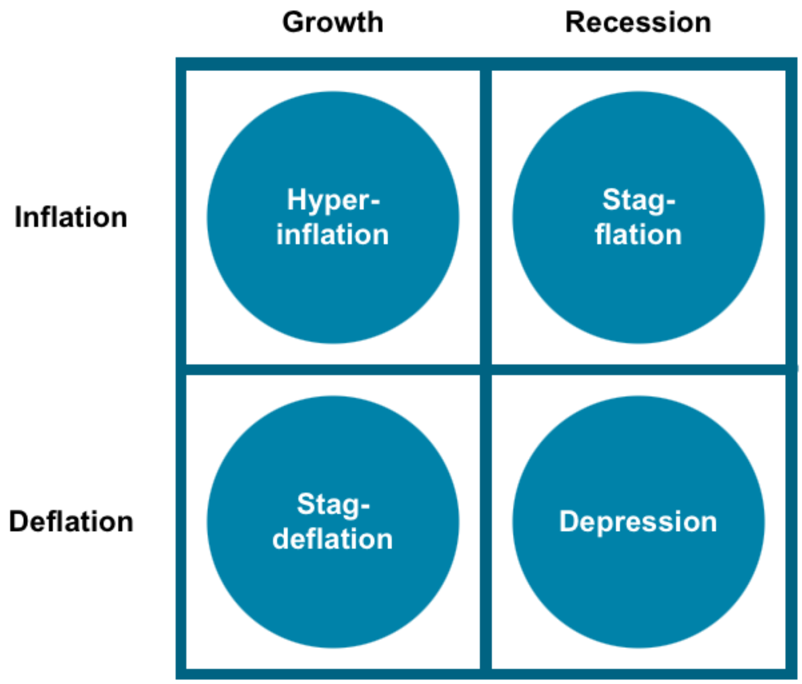

Figure 11: MSQ - Macroeconomic State Quadrant.

The quadrant just described is shown in Figure 11. It predicts that each state can be the cause and consequence of another; therefore, each general economic phase is independent, while the underlying economic phases depend on the general ones. The traditional investor has always thought of having to increase his investments in only two economic contexts, growth and recession. At the same time, deflation and inflation were secondary economic states that could derive from the former. By thinking so, the traditional investor always has and only invested in shares (growth) and nominal bonds, whose interest rates could protect him in times of recession. The advent of the ZIRP world (zero interest rate policy) uncovered the stock-bond de-correlation, back to being highly correlated and then the veil of inflation, clearly showing how these traditional allocations suffer and are not suitable in at least three out of four economic states.

We now need to understand the behaviors of asset classes in each economic state and understand what our main diversifiers are. Many underestimate the importance of gold. In a world where policymakers must print and expand their expansionary fiscal policies while keeping interest rates zero, gold plays a unique role in protecting portfolios. The 20 year Gold rally is still very modest compared to what we have seen in past reflationary periods. Given the still low global inflation levels, the need to maintain reflationary policies will per-

sist for some time. Gold is one of the few resources that can perform well even in a medium to high inflation world.
Figure 12 shows the yield of a 60/40 portfolio with 10% invested in gold. The efficiency is greater, and above all, it can hold up in times of stress (where the shares went down).

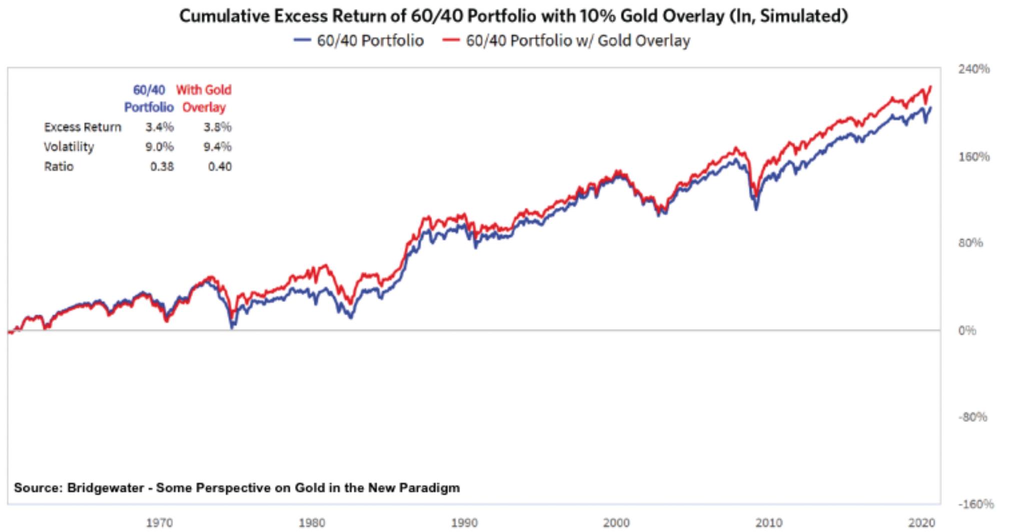

Figure 12: Bridgewater.

According to what we said before, gold is one of the most important diversification tools we know, the first tool capable of reducing our exposure to short-term shocks. Figure 13, elaborated by Bridgewater [5], clearly shows that in most of the economic states that we may observe in the future, based on the QSM model, gold returns excellent returns and stabilizes our portfolio, especially in phases in which traditional assets suffer (as mentioned inflationary crises or periods of moderate inflation).

Annualized Real Returns

	Cash	Bonds	Equities	Gold
Insufficient/Ineffective/Depression	**3.7%**	**7.2%**	**-8.6%**	**2.7%**
US 1929-1932	10.6%	13.9%	-22.0%	8.8%
JP 1994-2003	0.6%	3.7%	-3.1%	-0.2%
US 1936-1939	-0.2%	3.9%	-0.7%	-0.4%
Successful Reflation	**-1.4%**	**2.3%**	**11.9%**	**6.5%**
UK 1931-1936	1.8%	7.1%	13.5%	9.5%
US 1933-1936	-1.4%	3.9%	27.1%	14.4%
US 1940-1951	-4.7%	-2.4%	5.8%	-4.2%
UK 1947-1959	-1.6%	-1.5%	8.1%	-2.5%
US 2008-2012	-1.0%	4.6%	4.8%	15.2%
Stagflation	**-1.7%**	**-2.1%**	**-1.6%**	**21.7%**
UK 1970-1979	-2.6%	-2.5%	-1.7%	18.0%
US 1971-1979	-0.8%	-1.7%	-1.4%	25.4%

Source: Bridgewater - Some Perspective on Gold in the New Paradigm

Figure 13: Gold hedging capacity.

Times in which gold outperforms other assets tend to be periods of reflation and when we see a high money supply. The actual return on money is depreciated, and equities tend to be hurt by economic weakness, which requires such an extreme stimulating response and the possible rise of inflationary pressures, which squeeze the risk premium offered by equities and lead to an increase in valuation rates. Conversely, periods of loss for the gold tend to be when traditional portfolios do not need protection: times of strong growth and favorable inflation dynamics that do not require an extreme stimulus. In these environments, stocks, corporate bonds, HY Emerging stocks offer the best levels of return, while high-quality cash outperforms all other assets during a recession.

Given these assessments, from the MSQ quadrant, it is possible to achieve the best distribution of assets capable of making the portfolio immune to economic shocks and build the Asset Class Quadrant, see Figure 14.

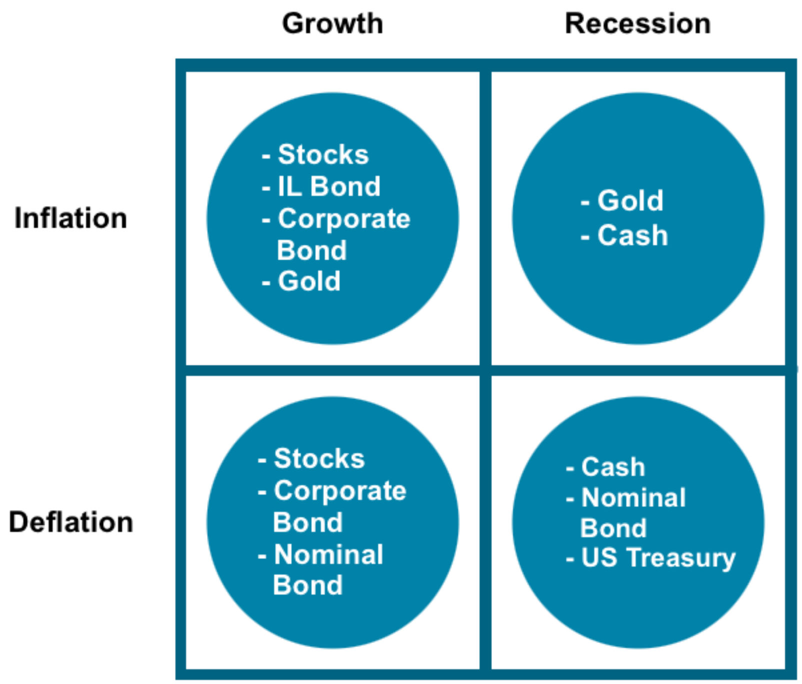

Figure 14: ACQ - Asset Class Quadrant.

The allocation seen so far has been almost qualitative, while now we need to take a further step to respect the Risk Parity.

3.4 Expectations

IIf the future was certain and the information was perfect and easily obtainable, without particular information costs, economic operators and agents would not find themselves making choices in conditions of uncertainty; therefore, there would be no risk. Traders would not produce anymore what we call "expectations" on the performance of employment, GDP, or the stock market.

Therefore, the agents' expectations are a crucial point in the world of economics and finance, as they derive from the uncertain context in which we operate, but inevitably influence the future context in which we will move up to think that the same prices on the stock exchange are practically influenced only by the expectations of the operators.

The first to formulate a complete idea of how expectations influenced stock exchanges was Keynes, putting forward the idea that stockbrokers are those who try to anticipate the anticipations of others: to explain this concept, Keynes elaborated the paradox of Beauty contest, in which participants are invited to choose the six most attractive faces out of a hundred photographs, and those who manage to pick the most popular faces are entitled to a prize. According to Keynes, contestants will not choose the most beautiful faces based on what they believe are the most beautiful faces: a beauty contest competitor, who wishes to maximize the chances of winning a prize, should think about which face is the most attractive for most people, and then make a selection based on the expectations of other operators, trying to anticipate them. This could also work on the stock exchange, where traders do not choose where to invest based on the best intrinsic investment opportunities but on future market expectations. Starting from Keynes's Beauty Contest [1], what could move the stock markets are trends in economic and financial variables above or below operators' expectations: shares grow because profits grow more than expectations, not because inflation actual is greater than expected, not because there is inflation.

3.5 The All-Weather portfolio

The All-Weather approach is similar to the Permanent Portfolio approach; see the work of Brown. H (2012)[10], as it does not treat financial and real assets based on expected return and standard deviation but treats them according to their response to economic shocks. The main difference between the two forms of asset allocation lies precisely in the fact that All Weather allocates its capital respecting risk parity. To understand how to structure our allocation, we must start from the simple matrix that exposes the level of risk/return of individual assets: Figure 16 illustrates the risks and expected returns of various asset classes.

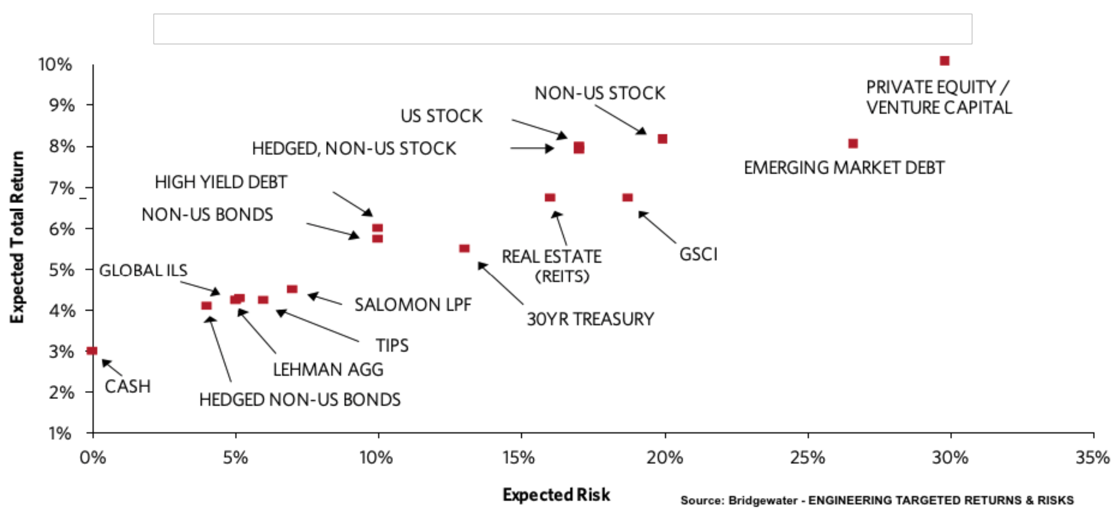

Figure 15: expected return/risk levels for various asset classes.

IIn Figure 16, the more the investor moves to the right, the more the risk/return level increases. Although traders' expectations differ, most investors assume that riskier assets generally have higher expected returns. For example, most of the high-yield and high-risk asset classes, such as equities, private equity, venture capital, and real estate, have higher risks and rewards due to their built-in leverage. The average debt-to-equity ratio of companies in the S&P 500 is approximately 1: 1 or higher (company's leverage), which increases their returns and risks. Investors aiming to achieve higher returns generally rank on the right side of the previous table, exposing themselves as much as possible to equities and allocating the remaining capital to bonds. By doing this, the resulting portfolio would not be diversified. The risk-adjusted returns of the asset classes are substantially similar; the expected returns and risks of these asset classes can be made equal and adjusted to provide returns closer to target return using leverage.

The quadrant of the asset classes will be modified to consider the contribution of the risk/return of the individual assets.

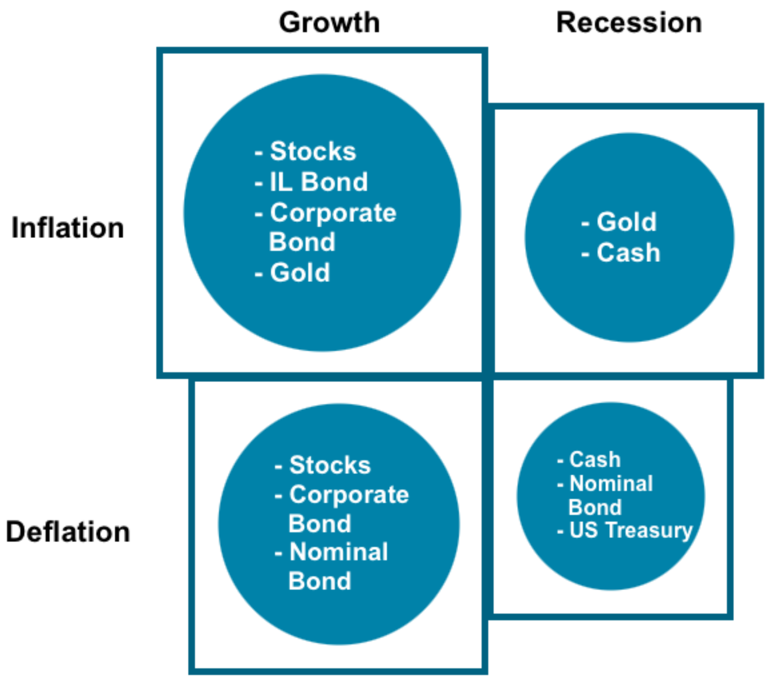

Figure 16: RACQ -Risk-Adjusted Asset Class Quadrant.

Figure 17 represents the risk-adjusted Asset Class Quadrant. The quadrants' dimensions are modified according to their level of risk/return and, therefore, their percentage in the portfolio must be inversely proportional to the risk. By allocating 45% of the capital to the left side of the asset classes and 55% to the right side, we can reach an allocation that respects Risk Parity. Starting from the results we have seen above and using the risk-adjusted asset class quadrant; we obtain an allocation that respects the Risk Parity and can represent the basis of our All-Weather Portfolio:

- 30% Stocks
- 5% Corporate Bonds
- 5% Inflation-Linked Bonds
- 30% Long Term Government Bonds
- 15% Short Term Bonds
- 15% Gold

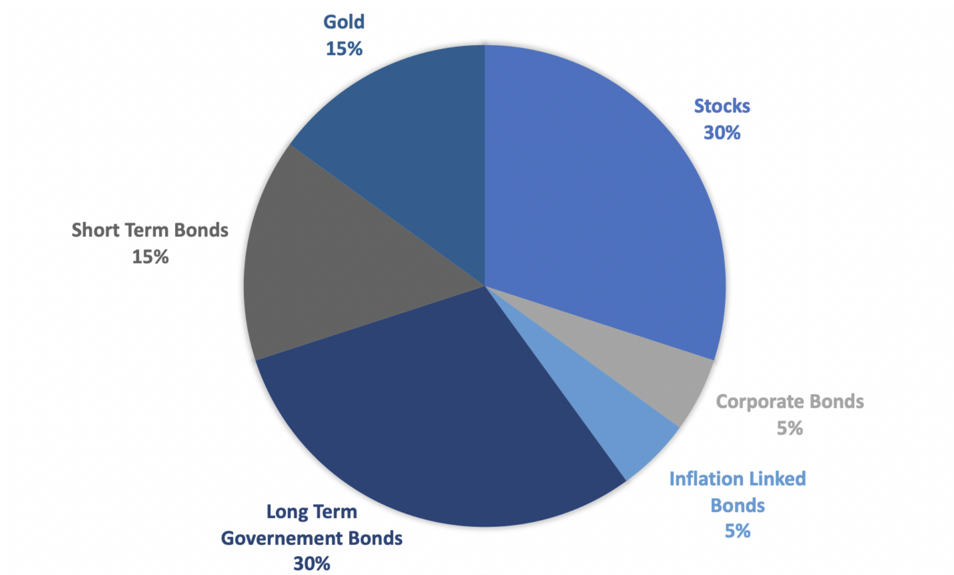

Figure 17: All-Weather.

Bonds are less risky and therefore need to be weighted more. By having an all-weather portfolio, we avoid having to make forecasts. If rates continue to fall, the government component will continue to return yield along with stocks; if rates rise with moderate reflation (rising prices), inflation-linked bonds, gold, and stocks will make our investment perform. If much higher inflation arrives, gold will continue to protect us. Given the main four assets that make up an all-weather, at least one will have to go wrong. But if we rebalance by averaging prices, we will be able to take advantage of low prices.

3.5.1 Gold Vs Bitcoin hedging capacity

As we mentioned above, gold is an important diversifier that we include in our all-weather portfolio. Today, however, bitcoin, a digital cryptocurrency, is spreading more and more and has already exceeded a trillion dollars in market cap. Many investors are starting to compare this digital asset with the well-known gold. Both assets are seen as a means of diversification in the portfolio and as a hedge against currency inflation. Yet, until recently, it was rare to see famous analysts and investors seriously compare the two assets. Bitcoin, commonly referred to as digital gold, has historically been seen as a risky speculative investment for those looking to profit in the short term. Gold, meanwhile, has always been considered a safe asset.

PROS: According to the article of Spotlight ET (2021)[12] Bitcoin is the most valuable cryptocurrency in the world, measured by market capitalization. Unlike the stock market, which is only open from Monday to Friday, the cryptocurrency exchanges are open 24 hours a day, seven days a week, allowing traders to exchange Bitcoin and other digital currencies. Another important characteristic of Bitcoin is that it has a finite supply, which means that there will be 21 million Bitcoins in circulation at any given time. If your bet on Bitcoin turns out to be positive, it will only increase in value with time.

CONS: Lee, I. & Will, D (2021)[6] in their article tells us that Gold has a thousand-year history, has a fifth of the volatility of bitcoin, and has always been the ultimate currency of central banks. Although bitcoin can be seen as the "new kid in the block" who steals market shares from gold, we must not forget that there is the possibility that bitcoin may one day cease to exist given the tough legislation. Some bitcoin derivatives have already been banned. Companies such as Facebook that have attempted to start crypto have been prevented from doing so. Gold has retained its value through the centuries. Whether bitcoin will offer the same level of longevity is highly questionable.

Let's now compare three all-weather (slightly modified) portfolios to see if adding bitcoin can stabilize the portfolio and improve performance. We will use the all-weather allocation proposed by Ray Dalio. By doing so we will not be exposed to exchange risk. To run the simulation, we used the portfoliovisualizer.com site.
Right now, we will not focus our attention on the asset allocation that will be covered shortly, but we instead focus on portfolio performance using Bitcoin.
We have composed the portfolio through the following ETFs: VTI 30%, SHY 15%, TLT 30%, SCHI 5%, TIP5%, BTC 7.5%, GLD 7.5%. Given the lack of financial instruments that replicate the trend of Bitcoin and their short time window since they were issued, we decided to include bitcoin in our portfolio physically. Three identical portfolios were created with the same asset classes except for gold which was broken down into three possible scenarios. In the first scenario, it weights 15%, in the second 7.5% together with Bitcoin, and in the third 0%, completely replaced by bitcoin.

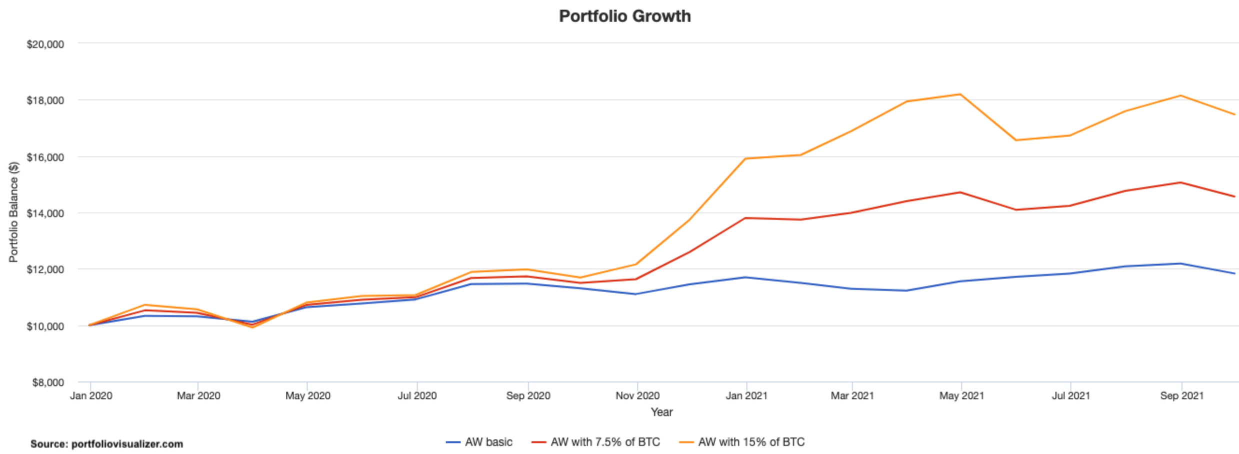

Figure 18: AW with BTC.

From Figure 18 we can see that adding BTC to the portfolio increases performance but only after November 2020, before the three portfolios were almost overlapping.

Portfolio	CAGR	Stdev	Best Year	Worst Year	Max. Drawdown	Sharpe Ratio	Sortino Ratio	US Mkt Correlation
AW basic	10.09%	7.95%	16.96%	1.17%	-4.06%	1.22	2.59	0.67
AW with 7.5% of BTC	23.96%	13.23%	38.01%	5.53%	-4.81%	1.68	4.13	0.72
AW with 15% of BTC	37.58%	20.51%	59.06%	9.88%	-8.97%	1.66	3.78	0.65

From the performance table, it can be seen that the high volatility of Bitcoin has repercussions on the entire portfolio; a 7.5% held raises the volatility by 4%, while a 15% raises the volatility by 12%, too much to be supported.
Holding 7.5% of BTC in the portfolio brings benefits in terms of performance; in fact, the Sharpe ratio and the Sortino ratio increase significantly at the expense of a slight increase in correlation with the US market.
Our analysis shows that Bitcoin can improve, but we must always remember that it is a very young asset and will need many more backtests before it can actually replace gold. Ray Dalio, in a post published on LinkedIn[3], quotes:

"It seems to me that Bitcoin has succeeded in crossing the line from being a highly speculative idea that could well not be around in short order to probably being around and probably having some value in the future. The big questions to me are what can it realistically be used for and what amount of demand will it have. Since the supply is known, one has to estimate the demand to estimate its price."

The author continues criticizing the concept of supply, even if it is limited for bitcoin, other digital currencies are not limited in the supply as new ones will be created that will compete with bitcoin, and this competition will play a fundamental role in the determination of the price of Bitcoin. Dalio points out that the new ones will replace the old ones because, after all, that's how it works. Bitcoin's modus operandi is fixed, it does not evolve over time, and probably other more innovative digital currencies will replace it

4 Empirical results with the All-Weather composition (american)

This section will compare the All-Weather (American) portfolio with the well-known S&P500 index that we will use as a benchmark. We opted for the American All-Weather to have a backtest window as long as possible. The analysis runs from 2004 to 2021. We initially thought about using the ETFs provided by Vanguard, but we excluded them as some of the ETFs that made up the portfolio did not include the 2008 crisis, so we opted to use the asset classes provided by the Portfolio-Visualizer[5], which recreates the allocation mentioned above.

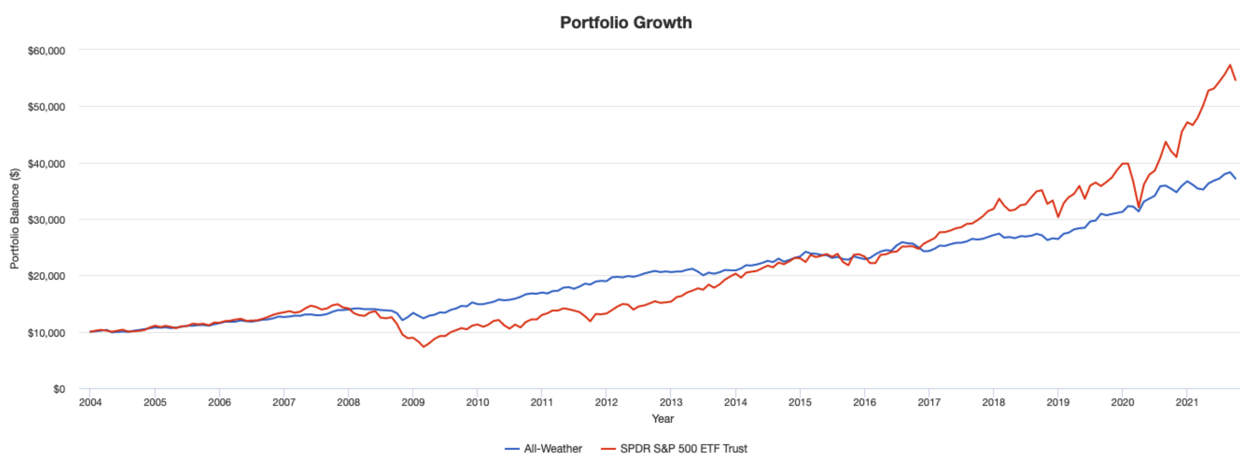

Figure 19: All-Weather (American version).

[5]Portfolio Visualizer is an online software platform for portfolio and investment analytics to help you make informed decisions when comparing and analyzing portfolios and investment products.

	CAGR	**Stdev**	**Best Year**	**Worst Year**	**Max. Drawdown**
Portfolio	7.68%	6.51%	18.24%	-4.35%	-14.78%
All-Weather	7.68%	6.51%	18.24%	-4.35%	-14.78%
SPDR S&P 500 ETF	10.04%	14.24%	32.31%	-36.81%	-50.80%

Table 3: Summary of performance measures between the S&P500 and All-Weather between 2004 and 2021.

Sharpe Ratio	**Sortino Ratio**	**US Mkt Correlation**
0.99	1.63	0.58
0.66	0.98	1.00

Table 4: Efficiency measures and market correlation.

The results are shown in the two tables above. As expected, the All-Weather portfolio has less than half the volatility of the benchmark (6.51% Vs. 14.24%) and consequently a much higher Sharpe Ratio. The drawdowns are significantly smaller. This shows that the All-Weather composition is doing precisely what it is built for, bearing every market condition. Now we are going to analyze two specific events that happened in the last twenty years that will confirm the efficiency of the All-Weather portfolio.

4.1 2008 Crisis

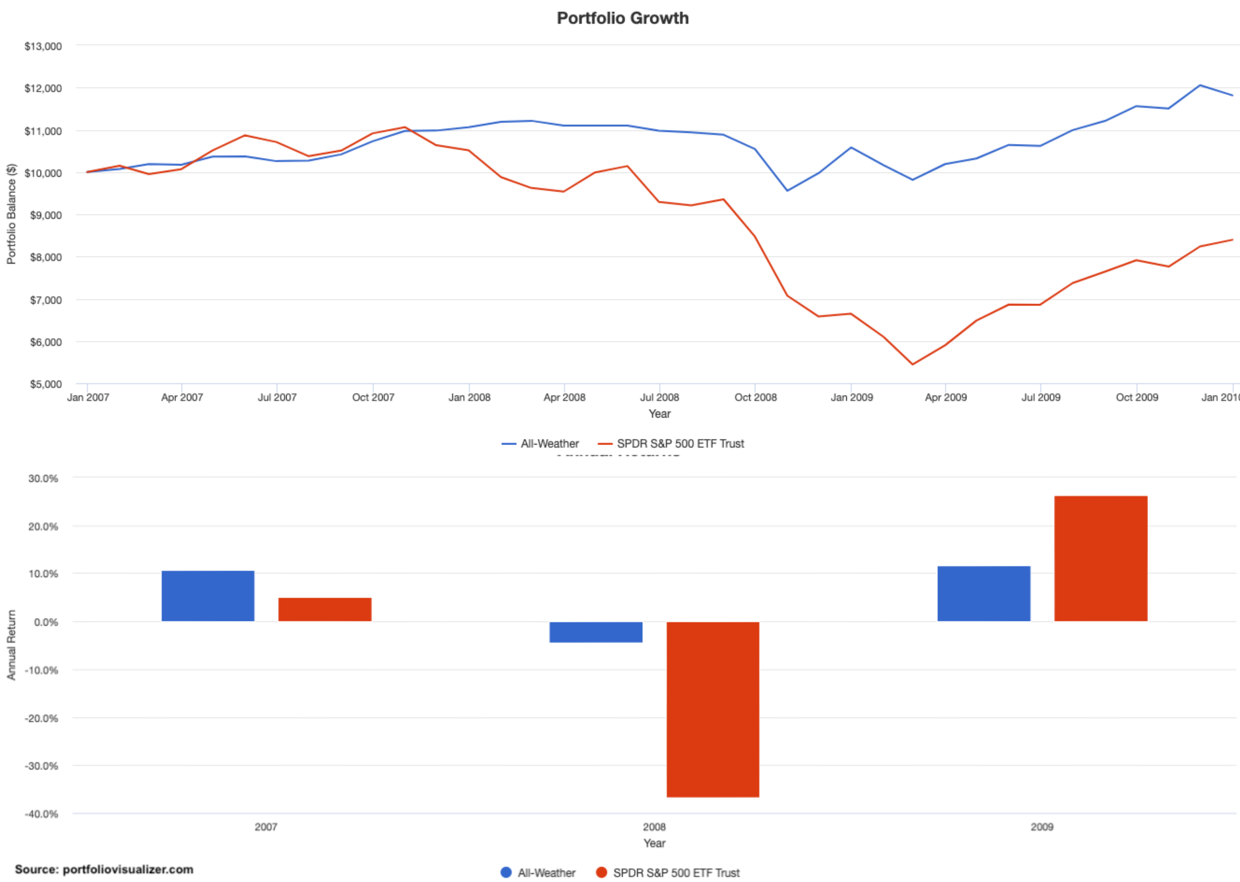

Figure 20: All-Weather during the 2008 crisis.

During the 2008 crisis, the AW portfolio demonstrated extraordinary robustness with a maximum drawdown of 12% compared to the 50% lost by the benchmark (S&P500); we also compared a portfolio 60 / 40 which recorded a drawdown of 31% during this period. Many investors do not consider how much they need to earn to break even after taking a huge loss. The table below shows the loss suffered and the gain percentage required to return to the starting point. After a 10% loss, an 11% gain is needed to break even. Things start to get worse as the losses increase. As mentioned above, the 50% drawdown lost by the S&P500 requires a 100% return to get back. Such a high return is not conceivable in just one year, while a return of 11% is far more plausible. The S&P500 took five years to grow again and ten years to reach the AW portfolio.

Percetage Loss	Percentage needed to Break Even
5	5.26
10	11
20	25
30	43
50	100
90	900
99	9900

Table 5: Loss-Recovery table.

4.2 2020 Covid Pandemic

Now let's consider the 2019-2021 interval in which the coronavirus hit the markets sharply. As Figure 21 shows us, the S&P500 recorded a drawdown of 20% compared to a 5% recorded by the AW portfolio.

Figure 21: All-Weather during 2020 pandemic.

In this case, the benchmark recovered the gap created more quickly because the markets had discounted mass vaccination. The performance indices such as Sharpe and Sortino ratio remain higher for the AW portfolio.

5 European All-Weather

The asset allocation described in Figure 18 is the American All-Weather, designed for the American investor. The European investor may find it difficult to hold this type of portfolio composition. When we invest in bonds, the most significant risk we take is given by the exchange rate. During the crisis, it is also a profit opportunity where the dollar is appreciated. We could also cover ourselves on the exchange rate neutralizing the effect of the fluctuations. Foreign exchange hedging inhibits the internal rebalancing that assets denominated in other currencies undergo, especially dollars. For this reason, we tend to avoid insuring risky assets against foreign exchange risk such as stocks.
In a European all-weather, we prefer euro-denominated securities without exchange rate risk. The all-weather portfolio also performs well during a recession as long as you agree with the definition of recession. The recession is not that market condition where everything collapses as it happened in 2008 or March 2020 or the .com bubble. The "recession" must last at least six months, two consecutive quarters of negative GDP. A strong market reversal presents itself as a liquidity crisis where all assets are sold, including gold. Immediately after the crisis, gold returns to perform well. Central banks inject more money into the system, and gold protects against currency depreciation.

What are these assets that protect us from the liquidity crises of 2002 and 2008?

Good quality cash with high rating, and the dollar. In the European version, we put treasuries in the portfolio but not in the bond component denominated in euro, but rather in the monetary component, as a cushion for market reversals and as an ally of good quality cash.

5.1 Portfolio Composition

G. Galletta (2020) [4] was the first to give a European interpretation to the All-Weather allocation. In his book he reports the following weights which would be suitable for a European investor.

- 10% US Stcks
- 15% Global Stock
- 5% China Stock
- 10% IL Bonds
- 5% Corporate Bonds
- 10% Convertible Bonds
- 5% US treasury

- 15% EU Short Term Bonds
- 5% Global Bonds
- 20% Gold

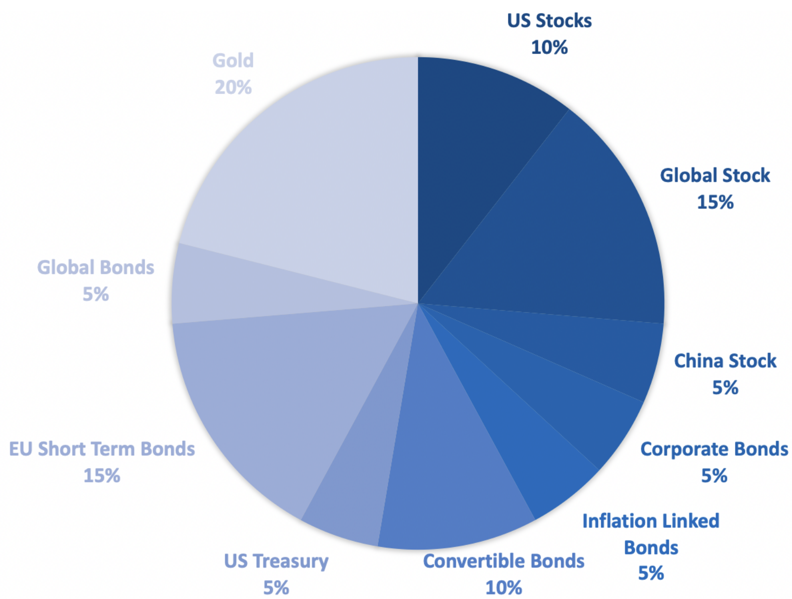

Figure 22: European AW.

There are no commodities because they are significantly correlated with the economic cycle and increase the portfolio's beta too much, while their function of protecting against inflation risk is well fulfilled by gold and inflation-linked bonds. The 20% of Gold does not increase the beta of the portfolio as its industrial use is very low and little correlated with the economic cycle (demand for industrial purpose: 12%).
The 15% of the portfolio is composed of high-quality cash that protects against market reversals. Dalio's All-Weather does not use cash but medium/short-term bonds because Dalio invests in dollars and knows that treasuries will be the most demanded asset in a crisis. The bricks can be modified, but in essence, the portfolio will remain the same.

6 Financial Leverage on All-Weather

We now come back to Figure 16 which illustrates the risk/return of various asset classes. We remember that we have to move to the right to get high returns while also increasing the risk. Using leverage, lower-yielding asset classes can be manipulated to have the same expected return and risk as equities. Figure 23 shows the expected returns and risks of the same asset classes shown in Figure 11 but manipulated to generate expected returns of 10% compared to 6.5% of the traditional portfolio.

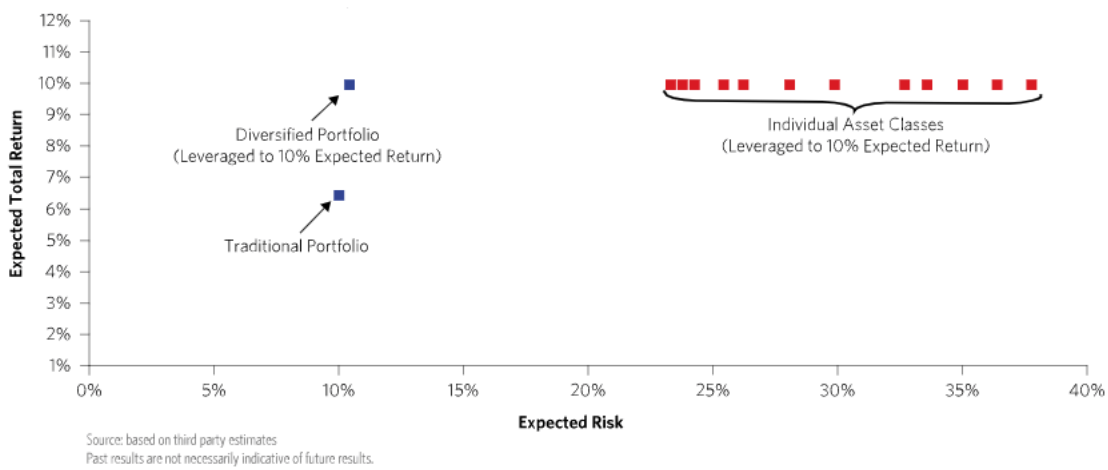

Figure 23: Expected Return/Risk Levels for Various Asset Classes and Portfolios.

Behind this reasoning, the necessary condition is that the rate of return of each chosen asset class is greater than the cost of liquidity borrowed to implement leverage techniques. If this holds, the model works, and capital allocations become more efficient. Once we have chosen the asset classes adjusted to have similar returns and risks, using leverage, the main difference will be their correlations. If we leverage all asset classes to have equity-like expected returns, a diversified portfolio of these assets will have an expected return similar to stocks but with much less risk than equities or a balanced portfolio of stocks and bonds. This process makes it possible to eliminate the traditional trade-off between risk and return, which pushes the investor to concentrate a large part of his portfolio in shares. Following a traditional approach, a portfolio that combines asset classes that individually have Sharpe ratios of 0.2 to 0.3 typically produces a portfolio Sharpe ratio of approximately 0.4, with expected returns below equities, but correlated in a way high with the stocks themselves, because a large part of the portfolio is invested in stocks.

This is an essential aspect of MPT's traditional allocation, namely that portfolios built with a traditional approach will have lower expected returns than equities but will be strongly correlated with the stock market since part of the capital will be invested in actions. The PMPT approach has a better Sharpe ratio than any conventional asset or combination of assets; it can be calibrated to offer higher returns with the same risk or the same returns with less risk. As we mentioned before, the only disadvantage is to run into times when

liquidity has a higher cost than the return of some risk classes. While traditional portfolio risk is largely a function of equity risk, the risk of this portfolio is that other asset classes, on average, will underperform liquidity. This type of risk is essentially offset in the long run, and the amount of leverage needed to create this type of portfolio is typically very low. Suppose investors get used to considering leverage in a less discriminating way. In that case, they will understand that a moderately leveraged and highly diversified portfolio is significantly less risky than a non-leveraged, undiversified one. By leveraging individual assets, we increased their expected return and, therefore, the portfolio return, but the risk remained constant, as de-correlations and responses to individual economic changes have been expanded.
As shown in Figure 24, the result is the quadrant of leveraged risk-adjusted asset classes.

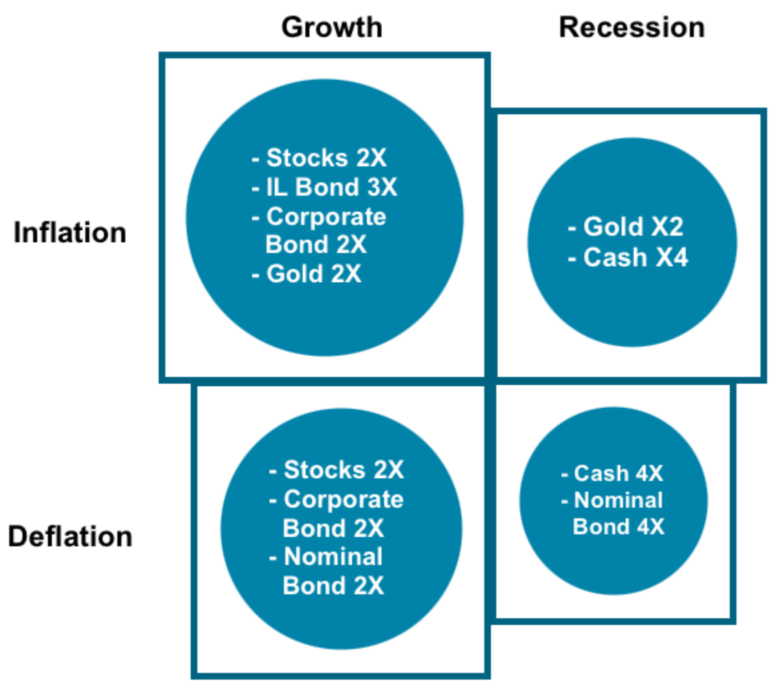

Figure 24: LRACQ - Leveraged Risk-Adjusted Asset Class Quadrant.

7 Dynamic Allocation

In the previous chapters, we have identified at least four scenarios that our portfolio could be exposed. Thanks to the literature, we identified the best-performing securities in those scenarios, See Figure 14, and decided to form our portfolio only by those securities for the time window in which that scenario occurs. If the scenario changes, our basket of securities that form the portfolio will change accordingly. Before making any computation, we need a set of rules to select the scenario we were in. Since the matrix that we are using is based on Growth/Recession and Inflation/Deflation, we decided to use two indicators: the GDP and the CPI. The Gross Domestic Product measures the monetary value of final goods and services, that is, those bought by the final user produced in a country in a given period of time. On the other hand, we have the CPI that stands for Consumer Price Index and is the instrument to measure inflation. It is used to estimate the average variation between two given periods at the prices of products consumed by households. These two indicators combined with a set of rules can select the Macro-Scenario we are in. The first thing that we did was to download the historical data for GDP and CPI on a quarterly basis. We then matched the data we downloaded with the already existing returns of our portfolio; by doing so, we were able to assign the level of inflation and growth on that particular date for each return. Figure 25 shows how the script was instructed to select the Macro-Scenario.

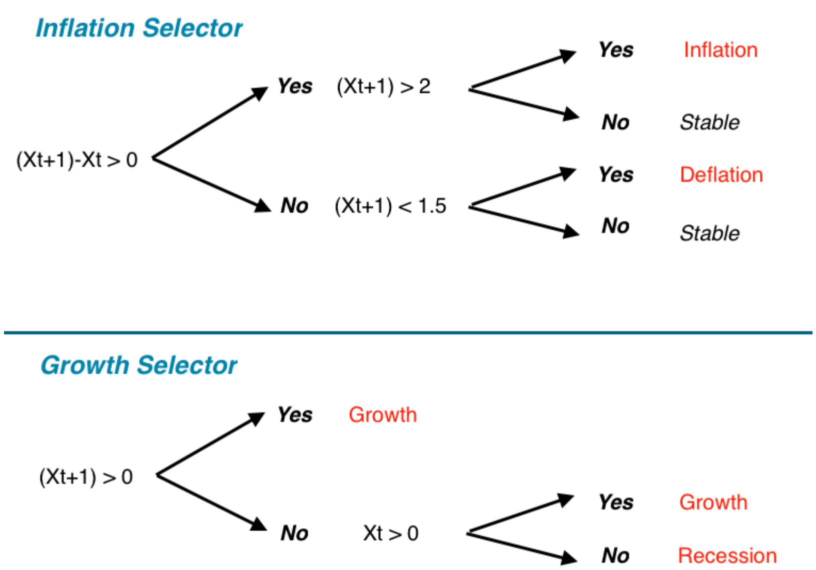

Figure 25: State Selector.

In the first part of the image, we notice the Inflation Selector. The script starts at the beginning of the time series, for convention called time (t), and checks if the value at a time (t+1) is greater than 2 (commonly known as a good value of inflation for the economy). If

the inflation is greater than 2 we have inflation. In the second case, (t+1) is not greater than (t), and we check if (t+1) is lower than 1.5, which we considered as a threshold for the Deflation case. If it's true, then we are in a Deflation phase. We also have an intermediate state, in which the increase in inflation is stable; this means that the value of (t+1) is greater than 1.5 but lower than 2.
This selection process will be repeated on each value of the series and will assign the value 1 if there is inflation, value -1 if there is deflation, and value 0 if we are in a stable phase.

In the second part of the image, we have the Growth Selector. The script starts a time (t+1) and checks if the value is greater than zero; if it's true, that means that the growing phase is still in progress. On the other hand, if the value (t+1) is lower than zero, but the previous value (t) is greater than zero, we are still in the growing phase since one-quarter of negative GDP isn't enough to declare a recession. If the value in (t) is lower than zero, then we have a Recession i.e., two consecutive negative quarters. Again, the process will be repeated on each value of the series and will assign the value 1 if there is growth and value -1 if we have two consecutive negative values.
We considered five different Macro Scenarios:

- Inflation[1] + Growth[1] = State 1
- Inflation[1] + Recession[-1] = State 2
- Deflation[-1] + Growth[1] = State 3
- Deflation[-1] + Recession[-1] = State 4
- Moderate Inflation[0] + Growth[1] = State 5

The original four, as shown in Figure 11 plus a new one, the "Hybrid Scenario." This scenario occurs when the inflation is "stable," and we have a small increase/decrease in Growth. When this happens, we decide to use the original All-Weather asset allocation. Our script was able to identify only three out of the five possible scenarios in the time period considered. See Figure 26.

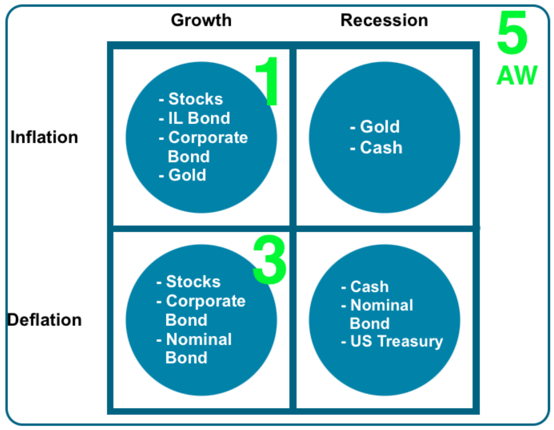

Figure 26: Scenarios used.

After identifying the portfolio states that will be used, we create three sub-portfolios, one for each state; these sub-portfolios are composed of the securities shown in Figure 14. The script then selected the time period in which each portfolio state occurred; of course, a state may recur multiple times over the period we considered. Once we obtained the matching between the portfolio states and the time length, we calculated the risk-parity allocation. The rebalancing occurred every time the portfolio state changed; it was not a fixed number of days but depended on the macroeconomic conditions. We then merged everything into one portfolio.

The performance measures of this portfolio are reported in Table 6.

We can see that the Dynamic allocation has slightly better performance with respect to the Static allocation. We managed to achieve a higher return by efficiently using the most suitable sub-portfolios for each state that occurred. Interestingly the max drawdown is the same; this makes us think that the portfolio configuration in that period was the same, the "hybrid state" (5), which indeed uses the standard All-Weather allocation. In order to have a fair comparison, we rebalanced both portfolios at the same dates, transaction costs were calculated, and the dynamic allocation suffered less since it used only a portion of all the securities. We also calculated the time period, in percentage, in which each sub-portfolio was used. Recalling Figure 26, we found that state 1 appeared 9.6% of the time, state 3 appeared 15.0% of the time, and state 5 appeared 75% of the time. The persistence of the fifth state is not strange if we consider that in the last five years, the length of our sample, the inflation kept between 1.5 and 2.5, and the growth kept growing consistently except for the Covid pandemic.

	Dynamic - Risk Parity	**Static - Risk Parity**
Sharpe Ratio	0.88519089	0.85824164
Annual Return	0.07431606	0.07009626
Annual Volatility	0.08395484	0.08167427
Max drawdown	0.1444303	0.1444303

Table 6: Summary of the performance between Dynamic allocation and Static allocation.

Conclusion

This paper aimed to understand better the Risk-Parity methodology and see if the All-Weather portfolio allocation, based on the Risk-Parity, was able to bear every type of "Risk" while giving a fair return for the investor. Empirically, we first demonstrate that the Risk-Parity methodology is more "efficient" with respect to the well-known Markowitz methodology. The performance ratios such as the Sharpe Ratio were confirmed to be consistently higher, underlying the efficiency of this type of allocation. Mind that the efficiency of the portfolio should not be confused with profitability. Even when we considered the transaction cost, the Risk-Parity methodology was still dominating. We then compared the AW Portfolio with the S&P500 in different periods, such as the 2008 crisis and the 2020 Covid pandemic. Although the AW always underperformed the S&P during the bull runs, during these airy-fairly moments, the AW managed to lose only one-third compared to the S&P and recovered much more quickly. When considering the psychological part of investments, losing 15% is less harmful than losing 50%. In the end, we tried to compose the portfolio Dynamically according to the macroeconomic scenario, and this type of allocation proved to be slightly better than the static. We can conclude by saying that the All-Weather portfolio based on the Risk-Parity allocation is a good portfolio that guarantees safeness and secure returns across years. The negative part of the portfolio has been the easy presentation given by many scholars as "the easy way to invest", where in reality, this type of portfolio requires leverage to work optimally. We all know that leverage is something that we need to handle carefully; it amplifies both positive and negative results. We conclude by confirming the capacity of the portfolio to generate consistent returns over a long time horizon despite having only one-third of the assets invested in stocks.

Appendices

A Packages Used

The packages that we used in R during our analysis and backtesting are displayed below:

```
require(quantmod)
require(PerformanceAnalytics)
require(PortfolioAnalytics)
library(ROI.plugin.glpk)
library(ROI.plugin.quadprog)

library(readxl)
library(stringi)
library(e1071)
library(timeSeries)
library(fPortfolio)
library(caTools)
library(dplyr)
library(ggplot2)
library(corrplot)

library("dint")
library(date)
library(lubridate)
library(varhandle)

library(DateMatch)

require(riskParityPortfolio)
```

B Some Functions Used

Here below we download the FAANG Dataset and we create the risk parity function and the max sharpe ratio function.

```r
faang_data <- stockDataDownload(c("GOOG", "NFLX", "AAPL", "AMZN", "FB"),
                                from = "2014-01-01", to = "2019-06-25")
```

```r
risk_parity <- function(dataset) {
  prices <- dataset #$adjusted
  log_returns <- diff(log(prices))[-1]
  return(riskParityPortfolio(cov(log_returns))$w)
}

library(quadprog)

max_sharpe_ratio <- function(dataset) {
  prices <- dataset #$adjusted
  log_returns <- diff(log(prices))[-1]
  N <- ncol(prices)
  Sigma <- cov(log_returns)
  mu <- colMeans(log_returns)
  if (all(mu <= 1e-8))
    return(rep(0, N))
  Dmat <- 2 * Sigma
  Amat <- diag(N)
  Amat <- cbind(mu, Amat)
  bvec <- c(1, rep(0, N))
  dvec <- rep(0, N)
  res <- solve.QP(Dmat = Dmat, dvec = dvec, Amat = Amat, bvec = bvec, meq = 1)
  w <- res$solution
  return(w/sum(w))
}
```

In the peace of code below, we test the two portfolios against each other. The code gives a summary of the results, then it prints the cumulative returns as well as the max drawdown. See Figure 3 and Figure 4.

```r
bt <- portfolioBacktest(list("risk parity portfolio" = risk_parity,
                             "tangency portfolio"    = max_sharpe_ratio),
                        list(faang_data),
                        T_rolling_window = 12*20,
                        optimize_every = 3*20, rebalance_every = 3*20)

index(bt$tangency$data1$w_designed)

backtestSummary(bt)$performance

backtestChartCumReturns(bt)
backtestChartDrawdown(bt)


backtestChartStackedBar(bt, portfolio = "risk parity portfolio", legend = TRUE)
backtestChartStackedBar(bt, portfolio = "tangency portfolio" , legend = TRUE)
```

Even though we cannot include all the code since it will become too cumbersome, here below we can see a technique used, when we dealt with the Dynamic allocation, to assign the Portfolio State according to the CPI value and the GDP value. See Figure 25.

```r
# checkign the conditions ans assigning the portfolio allocation state
for(i in 1:dim(dataPrices)[1]){
  if(dataPrices$Inflation[i] == 1 & dataPrices$gdp_growth[i] ==1){
    dataPrices$port_state[i] =1
  }

  if(dataPrices$Inflation[i] == 1 & dataPrices$gdp_growth[i] == -1){
    dataPrices$port_state[i] =2
  }

  if(dataPrices$Inflation[i] == -1 & dataPrices$gdp_growth[i] == 1){
    dataPrices$port_state[i] =3
  }

  if(dataPrices$Inflation[i] == -1 & dataPrices$gdp_growth[i] == -1){
    dataPrices$port_state[i] =4
  }

  if(dataPrices$Inflation[i] == 0 & dataPrices$gdp_growth[i] == 1){
    dataPrices$port_state[i] =5
  }else{
    if(dataPrices$Inflation[i] == 0 & dataPrices$gdp_growth[i] == -1){
      dataPrices$port_state[i] =5
    }
  }
}
```

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