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Climate Change Adaptation and the Role of Innovation: A Model of Directed Technological Change

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Abstract

Adaptation is considered a key element in the defense against the negative effects of climate change on human societies. In this thesis, I study adaptation to climate change by focusing on the role of innovation. I develop a three-sector general equilibrium model with endogenous directed technological change and an environmental damage function. The three sectors represent polluting, green, and adaptation technologies and agents are free to choose how much research to allocate to each sector. I describe the decentralized equilibrium of the model and analyse which are the incentives shaping agents' choices on adaptation. Finally, I try to understand whether an interior solution in which innovation happens both in adaptation and green technologies is possible in this model.

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Chapter 1

Introduction

Adaptation is defined as the "process of adjustment to actual or expected climate and its effects" (IPCC, 2014a). This can be referred to ecological, social or economic systems. For human systems, adaptation means seeking to moderate the damages and risks that climate change poses to social and economic structures, or trying to seize the opportunities arising from it (IPCC, 2014a). The latest report from the Intergovernmental Panel on Climate Change of the United Nations (IPCC) shows that the climate has altered significantly in the past decades and it is projected to continue to do so even in the most optimistic scenarios. "Global surface temperature was 1.09 [0.95 to 1.20] °C higher in 2011–2020 than 1850–1900" (IPCC, 2021), with an estimated contribution of +0.19 °C coming from the period 2003-2012. Human-induced climate change is the cause of more frequent and severe climate extremes events, such as heavy precipitation, droughts, heat waves, cyclones, etc. "Global mean sea level increased by 0.20 [0.15 to 0.25] m between 1901 and 2018" (IPCC, 2021), partly due to ice loss from the melting of glaciers and partly due to thermal expansion through ocean warming.

The negative effects of climate change on present human activities are increasing in intensity and scope and the possible consequences of future climate alterations on human societies range from serious to catastrophic. More frequent and increasingly intense extreme climate events are causing disasters that affect human settlements and human activities, resulting in deaths, interruptions to water and food supply, increases in food prices, damage to infrastructures and to human health in general (IPCC, 2014a). Climate change has negatively affected crop production in many areas of the world and represents a threat to future food security. Rising sea levels are endangering the territories of some regions, causing displacement and involuntary migration for people living in those areas.

Moreover, the effects of climate change are unevenly distributed, with poorer regions of the world suffering harsher damages and disadvantaged groups of people being affected the most by food insecurity and destruction of houses and means of living. This poses also a political and international cooperation problem, since those areas of the world that are suffering the most from climate-related damages are the ones that contributed the least to the alteration of the climate itself.

Global surface temperature is projected to continue to rise at least until 2050, under all the emissions scenarios considered by the IPCC, which means that climate change will continue even if the most ambitious plans for cutting CO_2 emissions are implemented. The most optimistic scenario in IPCC (2021) still forecasts global warming to reach a best estimate of 1.6 °C in the mid-term (2041-2060). On the other hand, if timely mitigation actions are not taken, and the green transition is implemented slowly, the alterations to climate in the coming decades would be very severe. "Global warming of 1.5°C and 2°C will be exceeded during the 21st century unless deep reductions in CO_2 and other greenhouse gas emissions occur in the coming decades" (IPCC, 2021). This means that climate-change-related damages to human societies will for sure continue in the next decades, and will intensify if the reduced-emissions scenarios do not materialize.

In this context, adapting to climate change is, and will increasingly be in the future, crucial for the livelihoods and life quality of a great number of people. Finding new tools and strategies to adapt is now recognized as top priority both by scientists and by policy-makers involved in addressing the climate change issue. Innovation has a prominent role in the adaptation effort since new technologies are in many cases the best instruments for coping with a new climate. Examples of adaptation through new technologies are new irrigation systems, more resilient crop varieties, better weather forecasting tools, coastal and river protection infrastructures, new water management systems, etc. (Dechezlepretre et al., 2020). Adaptation was indicated already in the 1997 Kyoto Protocol as the way to cope with the damages of climate change that are already present, especially for less developed countries, but the main focus of climate policy at the time was on mitigation. Mitigating climate change means limiting or preventing greenhouse gases emissions and enhancing activities removing these gases from the atmosphere (IPCC, 2014b). Hence, according to this definition, mitigation encompasses a wide ranges of activities, from carbon capture and storage technologies (CCS), to fighting deforestation to, most importantly, everything that is related to the concept of green energy transition. Over the years and in the successive climate agreements and negotiations (especially the Cancun

Climate Change Conference and the Paris Agreement), the relevance given to adaptation has constantly grown and now the scientific community, but also the policy-making environment, have reached a consensus that climate policy needs necessarily to encompass both adaptation and mitigation. From the policy-making perspective, the most relevant issue, together with selecting the most effective adaptation policies, is how to finance adaptation projects in developing economies. Already back in 2001 the Conference of the Parties to the Kyoto Protocol instituted the Adaptation Fund, whose goal is to provide finance for developing countries parties to the agreement. In the Paris Agreement, which is a legally binding agreement under international law, adaptation is recognised as a global goal that countries should commit to. Funding and technology transfer are recognised as key issues shaping the implementation and success of adaptation policies, especially in developing countries. In the text of the agreement it is clearly stated that developed countries should assist developing partners both through funds and knowledge sharing. In the recent COP26 in Glasgow, adaptation was one of the four pillars of the discussion and countries pledged to double the levels of adaptation finance with respect to 2019 by 2025.

Adaptation is also increasingly incorporated into the economic models studying climate change. Since people are increasingly aware of climate change effects and this is beginning to affect their behavior, it is now possible to study adaptation not just as an exogenous policy instrument in the hands of governments, but also as an endogenous choice variable available to those economic agents that wish to protect themselves from the undesired consequences of climate change. In this thesis, I study adaptation from the perspective of innovation. Dechezlepretre et al. (2020) researched patents data to analyse trends in adaptation research and innovation and found that the global absolute number of adaptation technologies patents has risen steadily between 1995 and 2015, but the share of adaptation research and development over total innovation has been roughly unaltered during the whole period. This evidence is shocking if we consider the relevance that the adaptation problem has gained in recent years and even more so if compared with patent data for the same period for innovation in mitigation. The share of innovation in mitigation over total innovation almost doubled over the same period (Dechezlepretre et al., 2020). This poses the question of which factors are shaping the demand and supply of innovation in adaptation, also vis-à-vis mitigation. Climate change adaptation and innovation is still a quite under-researched topic, hence many different new approaches are possible for the study of this problem.

In this thesis, I decided to use the framework offered by models of directed technical change (DTC), in order to study innovation in adaptation and its interplay with the transition to green energy sources, which according to IPCC (2014b) is a form of mitigation. Models of directed technological change allow to study endogenous innovation dynamics when there are sectors with different characteristics and to analyze which factors will shape the relative allocation of research across sectors. This class of models has already been used to study climate change and the role of innovation in the transition to a low carbon economy, starting from the seminal work of Acemoglu et al. (2012), by modelling an economy in which there are two sectors representing on one hand clean energies and on the other polluting fossil fuels. By introducing adaptation in this kind of framework, it is possible to analyse which forces shape relative demand and supply of research in adaptation vis-à-vis the "green" and "dirty" sectors, and which consequences different innovation possibilities can have on the environment and on the economy as a whole. The purpose of this thesis is to find a reasonable way to incorporate adaptation into a standard environmental model of directed technical change and then to analyse the decentralized equilibrium behaviour of innovation in adaptation, in a setting where adaptation competes with green and dirty technologies in the allocation of research resources. The first research question will therefore be whether in a simple DTC model augmented with an adaptation sector it is possible to have a decentralized equilibrium in which innovation happens in both adaptation and green technologies. If the response to the first question is positive, the second question is what are the factors shaping the relative allocation of research between clean technologies and adaptation in equilibrium.

The rest of the thesis is structured as follows. In the next section an overview is given of the main literature on climate change adaptation and innovation. In Chapter 2 the model is outlined: Section 2.1 illustrates a general and complete specification of the model, while Section 2.2 focuses on how to model the adaptation sector and evaluates different functional forms for adaptation. Chapter 3 presents the decentralized equilibrium of the model. In Chapter 4 a set of parameters is proposed and those are used to conduct a quantitative analysis of the model. Some concluding remarks and the limitations of the model are addressed in the final chapter.

1.1 Previous Literature

Adaptation in the theoretical economics literature has been studied mainly through Integrated Assessment Models (IAMs). IAM models are large scale models containing both an earth system part, modeled according to physical science, and a socio-economic system part, modeled using the techniques of economics and social sciences. These kind of models allow to study the interactions between the physical and socio-economic aspects of the climate change issue and draw policy-relevant conclusions. The IAM literature on adaptation has focused mainly on quantifying the costs and benefits of adaptation, as in Agrawala et al. (2010), and on studying the optimal mix between adaptation and mitigation in designing climate policy (see Bosello et al., 2010). Adaptation indeed does compete with mitigation in the allocation of resources dedicated to climate action, but at the same time, according to many of these models, the benefits of climate policy are the highest when both adaptation and mitigation are implemented (Agrawala et al., 2010). The literature on climate change adaptation however is in continuous evolution and there are still many areas and approaches to be explored and research gaps to be filled.
We can distinguish two ways of incorporating adaptation in this class of macroeconomic models. The first approach is to consider adaptation as an exogenous policy imposed by a social planner: in this case the analysis will be focused on finding the socially optimal mix of adaptation and mitigation. The second approach is considering adaptation as an endogenous variable, determined by the choices of optimizing agents. This second approach makes possible also to study the decentralized equilibrium behaviour of the model, even though in the IAM literature on adaptation the social planner problem is the one studied in many models. In this thesis the approach of adaptation as an endogenous choice variable will be adopted, in order to study which are the drivers of demand for adaptation and how it interacts with mitigation in a decentralized equilibrium setting.

The literature on adaptation and innovation is still quite recent and it comprises some empirical papers trying to estimate how reactive adaptation inventions are to climate extremes manifestations. These kind of works mainly focus on the agriculture sector, where both the damages of climate change and the emergence of adaptation measures are more evident (see Moscona and Karthik, 2021). They generally take a natural disaster caused by climate change and analyse whether it has had any impact on subsequent innovation in adaptation technologies; see Miao and Popp (2014) for an example of this method encompassing different sectors. Moscona (2019) argues that disasters can shape the *direction* of future innovation and he uses a model of directed technical change to describe the change in crop production that happened in the United States after the so-called "Dust Bowl". One relevant topic in this very recent strand of literature is whether innovation in adaptation is reactive to climate change, i.e. it is triggered by past negative effects of climate change, or proactive, i.e. it is carried out in anticipation of future negative consequences of climate change. The majority of these papers focus on analysing reactive adaptation, also because it is easier to test this assumption empirically. The study in this thesis is entirely theoretical, so it does not share much with this literature, but it will focus as well on reactive adaptation only, since I do not incorporate any measure of expectations in the model.

The literature on the environment and directed technical change originates from Acemoglu et al. (2012), who adapted the general framework of directed technological change (Acemoglu, 2002) to describe an economy with a "dirty" sector, producing a negative externality on the whole economy, and a clean sector, having no negative impact. Within this framework, Acemoglu et al. (2012) try to explain why technological progress can be biased towards the more productive "dirty" sectors, and why green technologies are struggling to reach productivity levels comparable to those of more ancient fossil fuels technologies. The main contribution of this work is also to point at a possible way to shape the future direction of technological change towards environment-friendly sectors, through the introduction of research subsidies. In this class of environmental directed technical change models, climate change is modeled in a much simpler way than in IAMs: it is usually represented by a damage function mapping the quantity of dirty output to a decrease in aggregate production or in utility. Much of this literature has focused on comparing two instruments for climate policy: a carbon tax on the production of dirty energy and research subsidies to support innovation in the clean sector, and on finding the optimal mix of the two in order to avoid climate disaster and at the same time do not sacrifice economic growth.

The contributions to this strand of literature mainly differ in the modelling choices they do for the innovation sector, which lead to different theoretical and simulation results. Greaker et al. (2018) assume decreasing returns to research and the possibility of longer-lasting profits for scientists from successful innovation compared to Acemoglu et al. (2012) and this leads them to conclude that subsidizing green research can even substitute carbon pricing if the dirty and clean sectors have a high degree of substitability. Hart (2019) incorporates technology spillovers across the two sectors, allowing technological progress to take place in both sectors in the long run and he finds that a large carbon tax is still needed to avoid climate disaster. Some of these models include in the analysis also exhaustible resources, like Lemoine (2020) and the same Acemoglu et al. (2012). Other models are built to produce more accurate quantitative analysis, like the micro-data founded model of Acemoglu et al. (2016) and the model of Fried (2018), which does not study optimal policy, but focuses on quantifying the dynamic effects of a carbon tax in a model built to match accurately US data on energy production. The model of Fried (2018) is probably the most relevant three-sector model in the directed technological change literature and this thesis work draws many features from it. The third sector in Fried (2018) is the non-energy sector, which is an essential component of final output. In this thesis model, the third sector is adaptation, which is not an essential component of final output, but rather a factor reducing the negative impact of climate change, hence the way the third sector is modeled in this work is quite different from Fried (2018). Durmaz and Schroyen (2020) also build a three-sector directed technological change model, where the third sector is represented by carbon capture and storage (CCS) technologies. However, they analyse just the Pareto optimal equilibrium of the model, while this thesis focuses on the decentralized equilibrium of the model economy. To my knowledge, there are no other models incorporating adaptation to climate change into a directed technological change framework.

Chapter 2

The Model

2.1 General specification

The model extends the standard model of directed technological change and environmental damage from Acemoglu et al. (2012). It is therefore a general equilibrium model with endogenous technological progress. Technological change is an outcome of investment in research activity and agents are given the possibility to choose in which sector or sectors they want to carry out productivity-enhancing research, hence also the *direction* of technological change is an outcome of agents choices.

The main novelty of the model is the introduction of a sector modelling adaptation activities and research in adaptation. Hence the three sectors of this model are: a **dirty** sector, representing all those energy production processes that are emission-intensive (i.e. fossil fuels sectors); a **green** sector, representing those energy sectors that are low emissions intensity (e.g. solar, wind, but also nuclear); and an **adaptation** sector, whose function is to shield final output from the damages of climate change. The model draws many features from Fried (2018), especially in the modelling of the intermediate goods and the innovation sectors. The main difference between this model and the one of Fried (2018) is the way in which the third sector enters the final production function. The environmental part is modeled through a damage function, mapping the production of dirty goods to the degradation of the environment and then to a negative impact on total production, as in much of the environmental DTC models. The model economy is an infinite horizon one in discrete time, populated by a continuum of households comprising a fixed mass of workers L, a constant mass of scientists S, intermediate goods producers, final goods producers and machines producers.

2.1.1 Households

Households derive utility just from consuming the final good and their preferences can be summarized by the inter-temporal utility function of a representative household,

$$U(C) = \sum_{t=0}^{\infty} \frac{1}{(1+\sigma)^t} u(C_t), \tag{1}$$

where σ represents the inter-temporal discount rate, C_t is the household's consumption at time t and $u(C_t)$ is an instantaneous utility function.

2.1.2 Final good

There is a unique final good, produced competitively using two types of inputs as in Acemoglu et al. (2012): one **clean** input, whose production does not have a negative effect on the quality of the environment, and one **dirty** input, whose use in the production of the final good implies a negative externality on the environment. As already mentioned above, the climate is modeled through a damage function, that in this case is a function of environmental quality and of adaptation effort. The third intermediate input, **adaptation**, therefore enters final good production not directly, but through the damage function. The production of aggregate final good Y_t is given by

$$Y_t = (1 - D_t(S_t, Y_{at}))(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)}, \tag{2}$$

where Y_{ct} is the quantity of the clean intermediate input used in production, Y_{dt} is the dirty intermediate input and Y_{at} is the quantity of adaptation good that is produced and used.

I will now analyse Equation (2) in its parts, in order to gain further insights on this functional form. First, $(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)}$ is the constant elasticity of substitution (CES) production function used by Acemoglu et al. (2012), and $\epsilon \in (0, \infty)$ is the elasticity of substitution between the two inputs, dirty and clean. During the whole analysis, I will make the following assumption:

Assumption 1. *The elasticity of substitution between clean and dirty intermediate goods is above unity, $\epsilon > 1$, which means that the two goods are gross substitutes.*

With this kind of functional form and with this assumption, we can clearly see that a complete switch form dirty to clean inputs is theoretically possible, since one of the two inputs can be zero and total output can still be positive.

Second, $(1 - D_t(S_t, Y_{at}))$ is the environmental part of the model and $D_t(S_t, Y_{at})$ is the damage function, that determines which share of the final good is foregone due to the negative effects of environmental degradation. S_t represents the stock of greenhouse gases accumulated in the atmosphere up to time t and it is our measure of environmental degradation. Y_{at} is the quantity of adaptation produced at time t. In this model adaptation is a flow variable, not a stock one, so with this modelling choice we are assuming, and it is a not so innocent simplification, that past adaptation efforts do not protect from present environmental damages. Hence, if agents want to protect themselves from the damages of climate change, they should demand adaptation in every period, until the stock of emissions reaches a level where it does no longer harm. The damage function $D_t(S_t, Y_{at})$ must necessarily be increasing in S_t and decreasing in Y_{at}. Moreover, the damage function must be well defined for zero adaptation output, since damage should be positive if humans do not adapt to climate change. The case of $S_t = 0$ is not so relevant since the stock of accumulated greenhouse gases is very unlikely to go back to zero (or better to pre-industrial levels, as I will explain right below). A more specific functional form for the damage function and the adaptation contribution to it will be analysed in Section 2.2.

The way that the use of dirty good Y_{dt} contributes to build the stock of emissions S_t is described by the following formulation, introduced by Golosov et al. (2014),

$$S_t - \bar{S} = \sum_{s=0}^{t+T} (1 - d_s) \kappa Y_{dt-s}, \tag{3}$$

where $\bar{S}$ is the stock of greenhouse gases present in the atmosphere before industrial times and it is the standard benchmark against which human-induced climate change is measured. Equation (3) is a stylized representation of the carbon cycle that happens in nature (Golosov et al., 2014). Here, $1 - d_s$ represents the share of carbon emissions that are left in the atmosphere for s periods in the future and κ is the share of dirty output that translates into emissions. In order to give a more realistic description of the carbon cycle, Golosov et al. (2014) decomposed $(1 - d_s)$ into

$$1 - d_s = \theta_L + (1 - \theta_L)\theta_0(1 - \theta)^s, \tag{4}$$

where θ_L is the share of carbon that enters the atmosphere and stays there forever. Of the part that exits the atmosphere in one way or another, a share $1 - \theta_0$ is absorbed by the biosphere or by oceans surfaces, while the rest slowly decays at a geometric rate θ.

2.1.3 Intermediate goods

The intermediate goods Y_{dt}, Y_{ct}, and Y_{at} are produced competitively using labor L_{jt} and capital, which is represented by a continuum of sector specific machines x_{jit}. Here, the index $j \in (d, c, a)$ indicates one of the three sectors, i.e. dirty, clean or adaptation and i is the subscript indicating one specific machine type. The production functions of the three goods are symmetric, as in Fried (2018), and are modeled as Cobb-Douglas, i.e.

$$Y_{dt} = L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha} x_{dit}^{\alpha} di \tag{5}$$

$$Y_{ct} = L_{ct}^{1-\alpha} \int_0^1 A_{cit}^{1-\alpha} x_{cit}^{\alpha} di \tag{6}$$

$$Y_{at} = L_{at}^{1-\alpha} \int_0^1 A_{ait}^{1-\alpha} x_{ait}^{\alpha} di. \tag{7}$$

α is the machines share in production and it is equal across the three sectors. A_{jit} is the quality of machine i specific for sector j, which represents the technology embodied in that machine. Workers can move freely across sectors and the decision in which sector to work is based solely on relative wages. The sum of labor in the three sectors must be equal to or lower than total labor L,

$$L_{dt} + L_{ct} + L_{at} \leq L. \tag{8}$$

2.1.4 Machines and innovation sector

Every sector has a unit mass of machines producers, which produce sector-specific machines at the fixed unit cost, equal across sectors, of ψ units of the final good and sell them to intermediate good producers. The machines sector is monopolistically competitive, so machine producers are also price setters and earn positive profits from selling the machines, as in Acemoglu et al. (2012) and Fried (2018). Every period machine producers hire scientists to increase the productivity of machines in their sector, A_{jit}, through innovation. Endogenous technological progress is therefore modeled as productivity-enhancing innovation in the technology embodied in the machines used for the production of intermediate goods. Following Fried (2018), the evolution of machines quality for machine type i in sector j is modeled in the following way

$$A_{jit} = A_{jt-1} \left(1 + \lambda \ s_{jit}^{\eta} \left(\frac{A_{t-1}}{A_{jt-1}} \right)^{\phi} \right). \tag{9}$$

The parameter λ describes the efficiency of scientists in producing innovation and it is therefore always positive. η is a parameter describing how returns to research evolve when the number of scientists increase. $\eta \in (0, 1)$ means that scientific research has diminishing returns within one period, which models what in the endogenous innovation literature is called *stepping on toes* effect (Greaker et al., 2018). This refers to the fact that, increasing the number of scientists working in the same sector increases the probability of duplicating a discovery, and innovation is productive only insofar it discovers something new. η equal to one means that there are constant returns to research and η greater than one means increasing returns. Throughout this analysis diminishing returns to research will be assumed, hence $\eta \in (0, 1)$. A_{jt-1} and A_{t-1} are respectively the average machines quality in sector j at time $t-1$ and the general level of productivity at time $t-1$, i.e.

$$A_{jt} = \int_0^1 A_{jit} di \tag{10}$$

$$A_t = A_{dt} + A_{ct} + A_{at}. \tag{11}$$

The parameter ϕ measures the extent to which there are innovation spillovers across sectors. ϕ can range from zero to one, with $\phi = 0$ corresponding to the case when there are no spillovers across sectors, as in the original model of Acemoglu et al. (2012). The ratio $\frac{A_{t-1}}{A_{jt-1}}^{\phi}$ is called the *TFP catch-up ratio* and it describes the contribution that productivity in a sector receives from the outside through cross-sectors spillovers. This ratio expresses the natural concept that sectors that are relatively backward compared to others are those that gain the most from spillovers, i.e. this ratio is higher the lower is past productivity of a sector compared to the average.

Finally, s_{jit} is the number of scientists innovating on machine i in sector j. Since machines are sector specific, scientists will have to choose in which sector they want to innovate and, as the market for scientists is perfectly competitive and there is free mobility across sectors, they will decide solely on the base of relative wages across sectors. The sum of the scientists in all the sector must be at most equal to the total number of scientists available.

$$s_{dt} + s_{ct} + s_{at} \leq S \tag{12}$$

Looking at equation (9), it is clearly visible that the forces shaping innovation dynamics in this model are: i) the level of past productivity in the same sector, implying that there is path dependence in innovation; ii) the number of scientists that are hired to innovate, hence the degree of investment in a given direction; iii) the contribution of cross-sectors spillovers.

2.2 A functional form for Adaptation

In this section, I will analyse different possibilities to find an appropriate functional form for adaptation. In order to have a damage function in line with previous literature, we could split the general form $(1 - D_t(S_t, Y_{at}))$ in Equation (2) into two multiplicative and separate parts, a *pure damage* part and an *adaptation compensation* part, i.e.

$$1 - D_t(S_t, Y_{at}) = (1 - D_t(S_t))f(Y_{at}). \tag{13}$$

The first part $(1 - D_t(S_t))$ is the pure environmental damage function and it can be modeled similarly to the damage function of the literature on the environment and directed technological change without adaptation. For example, we could use the exponential functional form proposed by Golosov et al. (2014),

$$(1 - D_t(S_t)) = exp(-\gamma_t(S_t - \bar{S})), \tag{14}$$

where γ_t is a (possibly) time-varying scaling parameter. This function is always between zero and one. It is equal to one when the stock of emission S_t is equal to the pre-industrial level: in this situation the damage on output resulting from climate change is null. As the stock of carbon emissions above pre-industrial levels grows bigger, the function gets closer to zero, making the foregone share of output closer to one as $S_t - \bar{S}$ increases towards infinity.

The second part, $f(Y_{at})$, is a function of the adaptation good that should model how adaptation activity translates into a reduced impact of emissions onto the final output. Since in the formulation (13) the pure damage part is already a number between zero and one, representing the share of output that is unaffected by climate change, the adaptation part should act as a sort of compensation, increasing the total function (13) to be closer to unity as Y_{at} increases. Hence, $f(Y_{at})$ must be increasing in Y_{at} and must be equal to one when adaptation output is zero, meaning that with no adaptation the negative effect of environmental damage will fall entirely on the production of final output. The function should also have an upper bound at the inverse of (14), so that adaptation effort can at most compensate for all the damages caused by environmental degradation, but it does not add to final output beyond that. This means that the following boundary condition[1] must always hold

$$f(Y_{at}) \leq \frac{1}{exp(-\gamma_t(S_t - \bar{S}))}. \tag{15}$$

Now I will analyse two possible more specific forms for the adaptation compensation function $f(Y_{at})$.

2.2.1 First possibility

A functional form for $f(Y_{at})$ could be the following one, which has all the characteristics mentioned above

$$f(Y_{at}) = 1 + \delta Y_{at}^{\beta}, \tag{16}$$

where δ is a parameter describing the *effectiveness* of adaptation, i.e. which share of the adaptation output translates into an effective protection against climate change damages, while β describes the returns to scale of adaptation production within one period. $\beta \in (0,1)$ means that adaptation has decreasing returns to scale, which means that one unit of adaptation is very productive when adaptation is scarce in period t, but it gets marginally less productive when adaptation is abundant.

Decreasing returns to scale seems a reasonable assumption for adaptation in one period, hence for the rest of the thesis the assumption will be that $\beta \in [0,1]$ with the linear case $\beta = 1$ also included in the range of possible values. With this functional form, the upper bound on adaptation (15) is given by

$$\delta Y_{at}^{\beta} \leq \frac{1}{exp(-\gamma(S_t - \bar{S}))} - 1. \tag{17}$$

By inserting this adaptation function (16) and the pure damage function of Golosov et al. (2014) into (2), we get the following expression for the final output

$$Y_t = exp(-\gamma_t(S_t - \bar{S}))(1 + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)}. \tag{18}$$

As current emissions S_t increase relative to the pre-industrial level $\bar{S}$, so the greater is the environmental damage $S_t - \bar{S}$, the closer the function $exp(-\gamma_t(S_t - \bar{S}))$ gets to zero, and hence the the greater is the share of output that is foregone. On the contrary when

[1] Another technique that can be used to limit the quantity of adaptation output to the one necessary to compensate for climate change damages is to use a step parameter. This step parameter will turn marginal adaptation to zero when the quantity of adaptation reaches the amount needed to compensate fully for cumulative emissions.

$S_t - \bar{S}$ is zero, the pure damage part $(1 - D_t)$ is equal to one, so no output is lost due to climate change. On the other hand, when the adaptation output increases, for the same level of emissions, the damage on output decreases. When Y_{at} tends to its maximum, the expression $exp(-\gamma_t(S_t - \bar{S}))(1 + \delta Y_{at}^{\beta})$ tends to one, that means that the damage from environmental degradation is fully compensated. If on the other hand $Y_{at} = 0$ the emissions will translate fully into damage on final output.

2.2.2 Second possibility

Another possible functional form for $f(Y_{at})$ could be

$$f(Y_{at}) = exp(\delta Y_{at}^{\beta}), \tag{19}$$

where δ is again the efficiency of adaptation and β describes returns to adaptation within one period. Also in this case, $f(Y_{at})$ is increasing in Y_{at} and it is equal to one when adaptation is zero. Combining (19) with (14), the complete damage function hence appears as

$$1 - D_t(S_t, Y_{at}) = (exp(-\gamma_t(S_t - \bar{S}))(exp(\delta Y_{at}^{\beta})) = exp(-\gamma_t(S_t - \bar{S}) + \delta Y_{at}^{\beta}). \tag{20}$$

Equation (20) is bounded between zero and unity only insofar as $-\gamma_t(S_t - \bar{S}) + \delta Y_{at}^{\beta}$ is negative. This means that for this function to have sense, i.e. to describe the share of final output that survives environmental damages, the following boundary condition, which is a special case of (15), must apply

$$\delta Y_{at}^{\beta} \leq \gamma_t(S_t - \bar{S}), \tag{21}$$

which essentially means that the amount of effective adaptation in one period should not exceed the one needed to compensate for the stock of greenhouse gases present in the atmosphere in the same period.

Provided that condition (21) is satisfied, function (20) has all the desired properties mentioned before: it is between zero and one; it is well defined when adaptation is null; it is decreasing in Y_{at} and increasing in S_t; it approaches one as (21) gets closer to hold with equality, which means that the amount of effective adaptation in period t, δY_{at}^{β}, is compensating fully for the damages created by the stock of carbon present in that period, $-\gamma_t(S_t - \bar{S})$.

With this functional form for the damage function, it is possible to rewrite expression

(2) for final output as

$$Y_t = exp(-\gamma_t(S_t - \bar{S}) + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon)} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)}. \quad (22)$$

Chapter 3

The Equilibrium

In this chapter the decentralized equilibrium of the model is derived. The maximization problems of all the agents in the economy are analysed and solved and market clearing conditions are considered to derive equilibrium expressions for all the variables of interest of the model. In particular, special attention is given to the question of whether it is possible to have an equilibrium path with positive innovation in all the three intermediate input sectors. In this work, I will concentrate on the case in which the economy is on a balanced growth path, hence on a situation where the growth rate of technological change is constant across time and all the main aggregate variables grow at the same rate of technological progress.

A decentralized equilibrium is a sequence of wages $[w_{dt}, w_{ct}, w_{at}, w^s_{dt}, w^s_{ct}, w^s_{at}]^{\infty}_{t=0}$, prices for intermediate inputs $[P_{dt}, P_{ct}, P_{at}]^{\infty}_{t=0}$, prices of machines $[p_{dit}, p_{cit}, p_{ait}]^{\infty}_{t=0}$, quantities of intermediate inputs $[Y_{dt}, Y_{ct}, Y_{at}]^{\infty}_{t=0}$, quantities of machines $[x_{dit}, x_{cit}, x_{ait}]^{\infty}_{t=0}$, labor allocations $[L_{dt}, L_{ct}, L_{at}]^{\infty}_{t=0}$, allocations for scientists $[s_{dt}, s_{ct}, s_{at}]^{\infty}_{t=0}$, atmospheric carbon concentration $[S_t]^{\infty}_{t=0}$, such that in each period t:

i. (Y_{dt}, Y_{ct}, Y_{at}) maximize the profits of final good producers, (L_{dt}, L_{ct}, L_{at}) and demand for machines $(x^d_{dit}, x^d_{cit}, x^d_{ait})$ maximize the profits of intermediate goods producers, $(s_{dit}, s_{cit}, s_{ait})$, $(p_{dit}, p_{cit}, p_{ait})$ and supply of machines $(x^s_{dit}, x^s_{cit}, x^s_{ait})$ maximize machines producers' profits;

ii. prices (P_{dt}, P_{ct}, P_{at}) clear the market for intermediate goods, $(p_{dit}, p_{cit}, p_{ait})$ clear the markets for machines and wages $(w_{dt}, w_{ct}, w_{at}, w^s_{dt}, w^s_{ct}, w^s_{at})$ clear the two labor markets.

3.1 Final good producer problem

Final good producers maximize profits, taking prices of intermediate inputs and the environmental damage as given. Final good producers are infinitesimal and they do not consider the aggregate environmental impact of their production choices. They do not internalize the environmental cost of dirty energy use, i.e. they ignore (3) and act as if the environmental damage $S_t - \bar{S}$ were exogenous and not a function of current and past dirty input production. Nevertheless, they see how environmental damage affects their output and they can demand adaptation to protect themselves from the consequences of current environmental damages, i.e. they are aware of the damage function (13).

The maximization problem for final good producers can be summarized as the problem of a single representative final good producer:

$$\max_{Y_{dt},Y_{ct},Y_{at}} Y_t - P_{dt}Y_{dt} - P_{ct}Y_{ct} - P_{at}Y_{at}. \tag{23}$$

subject to the definition of final output (2). Since the problem of the final good depends on the shape of the environmental damage function and of the adaptation function, I will now distinguish two cases, one for each of the two functional forms explored in Section 2.2.

3.1.1 Polynomial functional form

Using (16) the as adaptation function, if we substitute Y_t with (18) the final producer problem becomes

$$\max_{Y_{dt},Y_{ct},Y_{at}} (exp(-\gamma(S_t - \bar{S}))(1 + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)} - P_{dt}Y_{dt} - P_{ct}Y_{ct} - P_{at}Y_{at}). \tag{24}$$

The first order conditions for the demands of the three intermediate goods Y_{dt}, Y_{ct}, Y_{at} lead to the following expression for the three equilibrium prices P_{dt}, P_{ct}, P_{at}:

$$P_{dt} = exp(-\gamma(S_t - \bar{S}))(1 + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{1/(\epsilon-1)} Y_{dt}^{-1/\epsilon} \tag{25}$$

$$P_{ct} = exp(-\gamma(S_t - \bar{S}))(1 + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{1/(\epsilon-1)} Y_{ct}^{-1/\epsilon} \tag{26}$$

$$P_{at} = exp(-\gamma(S_t - \bar{S}))(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)} \delta\beta Y_{at}^{\beta-1}. \tag{27}$$

The ratio between the equilibrium prices of clean and dirty intermediate goods will therefore be equal to

$$\frac{P_{ct}}{P_{dt}} = \frac{Y_{ct}^{-1/\epsilon}}{Y_{dt}^{-1/\epsilon}}, \tag{28}$$

which means that relative prices are inversely related to relative demands. Relative prices of clean and dirty goods depend just on their own relative demands and are not affected by the adaptation good or by the level of environmental damage. This is because in the choice of production the two goods are just substitutes (because of Assumption 1) and agents are blind with respect to their diversity in how the use of two goods impacts the environment.

The ratio of clean to adaptation intermediate goods prices is

$$\frac{P_{ct}}{P_{at}} = \frac{(1+\delta Y_{at}^{\beta})Y_{ct}^{-1/\epsilon}}{(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})\delta\beta Y_{at}^{\beta-1}}. \tag{29}$$

Substituting Y_{dt} with the expression for it derived from (28), the ratio of clean to adaptation price becomes (full derivation in Appendix A.1)

$$\frac{P_{ct}}{P_{at}} = \frac{1}{Y_{ct}}\left(\frac{1+\delta Y_{at}^{\beta}}{\delta\beta Y_{at}^{\beta-1}}\right)\left(\frac{P_{ct}^{1-\epsilon}}{P_{ct}^{1-\epsilon}+P_{dt}^{1-\epsilon}}\right). \tag{30}$$

Since the dirty and the clean sector are symmetric, the same kind of ratio can be derived also for the dirty sector vis-à-vis adaptation

$$\frac{P_{dt}}{P_{at}} = \frac{(1+\delta Y_{at}^{\beta})Y_{dt}^{-1/\epsilon}}{(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})\delta\beta Y_{at}^{\beta-1}}. \tag{31}$$

Substituting Y_{ct} from (28)

$$\frac{P_{dt}}{P_{at}} = \frac{1}{Y_{dt}}\left(\frac{1+\delta Y_{at}^{\beta}}{\delta\beta Y_{at}^{\beta-1}}\right)\left(\frac{P_{dt}^{1-\epsilon}}{P_{ct}^{1-\epsilon}+P_{dt}^{1-\epsilon}}\right). \tag{32}$$

It is worth noting that the relative prices of clean to adaptation good and of dirty to adaptation good do not depend on the level of environmental damage. This is because in the perspective of agents, adaptation is just another way of increasing final output. In their perspective, final good producers can increase their profits in two ways: by producing more output, i.e. by increasing their demand for Y_{dt} or Y_{ct}, or by adapting more. They will pursue the strategy that gives them more output, not considering that after a certain level of emissions the environmental damages might become too high.

3.1.2 Exponential functional form

Using (19) as the functional form for adaptation and substituting Y_t with (22) in the final good producer problem, we have the following version of the final producers maximization problem

$$\max_{Y_{dt},Y_{ct},Y_{at}} exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)} - P_{dt}Y_{dt} - P_{ct}Y_{ct} - P_{at}Y_{at}. \tag{33}$$

The first order conditions for the demands of the three intermediate goods Y_{dt}, Y_{ct}, Y_{at} lead to the following expression for the three equilibrium prices P_{dt}, P_{ct}, P_{at}:

$$P_{dt} = exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{1/(\epsilon-1)} Y_{dt}^{-1/\epsilon} \tag{34}$$

$$P_{ct} = exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{1/(\epsilon-1)} Y_{ct}^{-1/\epsilon} \tag{35}$$

$$P_{at} = exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)} \delta\beta Y_{at}^{\beta-1}. \tag{36}$$

Also in this case the ratio between clean and dirty input prices depends solely on relative demands and it is equal to (28). What changes are the relative prices including the adaptation good. The ratio of clean to adaptation prices is

$$\frac{P_{ct}}{P_{at}} = \frac{Y_{ct}^{-1/\epsilon}}{(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})\delta\beta Y_{at}^{\beta-1}}. \tag{37}$$

Also in this case the relative price of clean to adaptation good is a function of Y_{dt}. We can remove Y_{dt} from the expression by substituting it from (28)

$$\frac{P_{ct}}{P_{at}} = \frac{Y_{at}^{1-\beta}}{Y_{ct}\delta\beta} \left(\frac{P_{ct}^{1-\epsilon}}{P_{ct}^{1-\epsilon} + P_{dt}^{1-\epsilon}} \right). \tag{38}$$

From this expression we can see that also the relative prices of clean and adaptation goods are inversely related to their demands. Since we made the assumption that β is comprised between zero and one, the exponent of Y_{at} in the above equation cannot be smaller than zero. From (38) it is also possible to notice that, everything else equal, a greater effectiveness of adaptation δ makes the adaptation good relatively more expensive with respect to the clean good. In the same way, recalling that $\beta \in [0, 1]$, a smaller parameter for returns to scale, makes adaptation relatively cheaper with respect to the clean good. Symmetrically, we can derive the same expressions also for the relative price of the dirty good with respect to adaptation.

$$\frac{P_{dt}}{P_{at}} = \frac{Y_{dt}^{-1/\epsilon}}{(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})\delta\beta Y_{at}^{\beta-1}}. \tag{39}$$

We can remove Y_{ct} from the expression by substituting it from (28)

$$\frac{P_{dt}}{P_{at}} = \frac{Y_{at}^{1-\beta}}{Y_{dt}\delta\beta}\left(\frac{P_{dt}^{1-\epsilon}}{P_{ct}^{1-\epsilon} + P_{dt}^{1-\epsilon}}\right). \tag{40}$$

3.2 Intermediate goods producers problem

The problem for intermediate goods producers is symmetric across the three sectors, hence it can be written for a generic sector $j \in d, c, a$. Intermediate good producers choose the quantity of sector-specific machines to buy and of labor to employ in order to maximize their profits. Since the intermediate goods sector is perfectly competitive, these profits should be zero in equilibrium. Their maximization problem can be expressed as the problem of a representative agent for sector j

$$\max_{L_{jt}, x_{jit}} P_{jt} L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di - w_t L_{jt} - \int_0^1 p_{jit} x_{jit}. \tag{41}$$

The first order conditions with respect to the quantity of machines lead to the following demand for machines:

$$x_{jit} = \left(\frac{\alpha P_{jt}}{p_{jit}}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit}. \tag{42}$$

The first order condition for the demand for labor lead to the following equilibrium wage

$$w_{jt} = (1-\alpha) L_{jt}^{-\alpha} P_{jt} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di, \tag{43}$$

and demand for labor

$$L_{jt} = \left(\frac{(1-\alpha)P_{jt}}{w_{jt}} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di\right)^{1/\alpha}. \tag{44}$$

3.3 Machines producers problem

The market for machines is monopolistically competitive, hence machine producers are able to choose both the quantity and price of machines to sell. Machines producers also decide how much to invest in the innovation of machines, hence how many scientists to hire for carrying out research in a specific sector. With unit costs of machines equal to ψ units of the final good, the maximization problem of machine producers, here written for a generic sector j and machine i, is given by:

$$\max_{p_{jit}, x_{jit}, s_{jit}} p_{jit}x_{jit} - \psi x_{jit} - w^s_{jit}s_{jit} \tag{45}$$

subject to the demand for machines from intermediate inputs producers (42) and the evolution of the innovation frontier (9).

From the first order conditions for this problem, we get the following expression for the equilibrium price of machines (see full derivation in Appendix A.1)

$$p_{jit} = \frac{\psi}{\alpha}. \tag{46}$$

The equilibrium price for machines is hence the same across sectors and constant over time. It is indeed a constant markup over marginal cost of machines, ψ, which is also constant. The fact that the equilibrium price is different from marginal cost is a consequence of the monopolistic power of machine producers. The equality across sectors is on the other hand due to the fact that the three sectors have the same machines-intensity, i.e. the share of machines in production is the same. We can assume without loss of generality, as in Acemoglu et al. (2012), that $\psi \equiv \alpha^2$. Then the equilibrium price of machines in the three sectors is just equal to

$$p_{jit} = \alpha. \tag{47}$$

Substituting (47) into (42) we have the equilibrium quantity for machines:

$$x_{jit} = P_{jt}^{1/(1-\alpha)} L_{jt} A_{jit}. \tag{48}$$

The machines producers problem relative to the choice of the number of scientists can be rewritten in the following way. Substituting x_{jit} with the demand for machines (48), p_{jit} with the equilibrium price for machines (47) and A_{jt} with the definition of technological change (9), the machines producers problem appears as

$$\max_{s_{jit}} \left(\alpha(1-\alpha)P_{jt}^{1/1-\alpha} L_{jt} A_{jt-1} (1 + \lambda\ s_{jit}^{\eta} \left(\frac{A_{t-1}}{A_{jit}} \right)^{\phi} - w_{jit}^{s} s_{jit} \right). \tag{49}$$

By substituting (48) to have back in the equation x_{jit}, we obtain the following equilibrium wage for scientists w_{jit}^{s}:

$$w_{jit}^{s} = \frac{\eta\lambda A_{jt-1} \left(\frac{A_{t-1}}{A_{jt-1}} \right)^{\phi} \alpha x_{jit}}{(\frac{1}{1-\alpha}) s_{jit}^{1-\eta} A_{jit}}. \tag{50}$$

Since the equilibrium in the machines sector is symmetric across all machines in the same sector, we can remove the machine-specific index i and write the equilibrium wage for scientists in sector j (which means $s_{jit} = s_{jt}$, $x_{jt} = \int_0^1 x_{jit}$, $A_{jt} = \int_0^1 A_{jit}$)

$$w_{jt}^{s} = \frac{\eta\lambda\alpha(1-\alpha) A_{jt-1}^{1-\phi} (A_{t-1})^{\phi} x_{jt}}{s_{jt}^{1-\eta} A_{jt}}. \tag{51}$$

3.4 Scientists problem

The scientists face a discrete optimization problem since they have to choose in which sector to work. Since there is free movement across sectors, they will choose the sector with the highest wage w_{jt}^{s}. This means that if one sector offers a higher wage than the other two, innovation will happen in that sector only. This is the case in the model of Acemoglu et al. (2012), where there are no spillovers across sectors, i.e. $\phi = 0$. Without cross-sector spillovers, there is full path-dependence in innovation and the sector that is technologically more advanced will always be the one offering a higher wage and hence securing all the scientists. The case in which innovation happens in one sector only is not very interesting for two reasons: first, it is counterfactual, since in reality innovation takes place also in relatively backward sectors; second, it is not interesting from an environmental economics point of view, since in this case innovation will only take place in the dirty sector, not allowing to analyse the equilibrium dynamics of innovation in clean and adaptation technologies. In this work I will focus on finding an equilibrium in which there is innovation in all the three sectors and in understanding the conditions under which this kind of equilibrium can exist in the first place. For innovation to happen in all the three sectors there must exist an equilibrium in which the three wages $w_{dt}^{s}, w_{ct}^{s}, w_{at}^{s}$ are equal. This is a three-sector model, hence there is not a unique ratio governing the whole innovation sector as in Acemoglu et al. (2012) and this makes the theoretical analysis on the existence of an equilibrium with innovation in all the three sectors more complex.

As mentioned in the beginning of the chapter, I will focus here on the case of a *balanced growth path* equilibrium, hence on a situation in which the growth rate of technological progress is constant. In this model, constant growth rate of aggregate technology can imply three scenarios: i) there is a balanced growth path with innovation in all the three sectors and hence all the three sector must grow *at the same constant rate*; ii) technological change happens just in one sector, whose productivity grows at a constant rate, while the productivity of the other two sectors is fixed across time in the balanced growth path; iii) two sectors have positive innovation in the balanced growth path, and experience the same rate of technological change, while the third sector will stay forever at its initial technological level. In the model of Fried (2018), the existence of an interior balanced growth path with innovation in more than one sector depends on the strength of cross-sectors spillovers, hence on the parameter ϕ. If ϕ is high enough there exists an interior solution where innovation happens in all the three sectors. Otherwise the path dependence effect leads innovation to happen just in the sector that is already more advanced. This consideration should be valid also for the model of this thesis.

3.5 Utility maximization problem of households

The representative household maximizes utility given its budget constraint. The household utility maximization problem can be written as

$$\max_{C_t} \left(\sum_{t=0}^{\infty} \frac{1}{(1+\sigma)^t} u(C_t) \right) \tag{52}$$

subject to the budget constraint

$$C_t = w_{dt}L_{dt} + w_{ct}L_{ct} + w_{at}L_{at} + w^s_{dt}s_{dt} + w^s_{ct}s_{ct} + w^s_{at}s_{at} + \int_0^1 (\pi_{fit} + \pi_{cit} + \pi_{ait})di, \tag{53}$$

where $(\pi_{fit}, \pi_{cit}, \pi_{ait})$ are the profits of machine producers in the three sectors. Since in this model the household derives utility just from consumption, it will consume all its income. The representative household earns income from the wages of scientists and workers and from the profits of machine producers. In equilibrium intermediate and final good producers do not earn positive profits since their markets are perfectly competitive. The representative household will supply labor and scientists for whatever positive wage, choosing the sector with the highest wage. The final good cannot be stored, but it must be consumed or used to produce machines, hence market clearing for the final good implies

$$C_t = Y_t - \psi\left(\int_0^1 x_{dit} + \int_0^1 x_{cit} + \int_0^1 x_{ait}\right). \tag{54}$$

3.6 Equilibrium allocation of workers and scientists

By putting together the five maximization problems considered above and exploiting market clearing conditions, it is possible to rewrite some equilibrium expression in a more meaningful way and to derive the equilibrium allocation of workers and scientists.

Substituting the equilibrium machines demand (48) and price (47) into the production function for intermediate outputs, we obtain the following equilibrium supply of intermediate goods, here expressed for a generic good $j \in (d, c, a)$

$$Y_{jt} = L_{jt} A_{jt} P_{jt}^{\alpha/1-\alpha}. \tag{55}$$

In the same way, substituting equilibrium machine demand (48) and price (47) into the equation for equilibrium wage for labor (43), the equilibrium wage becomes

$$w_{jt} = (1-\alpha) P_{jt}^{1/(1-\alpha)} A_{jt}. \tag{56}$$

The ratio between labor wages in two different sectors $j, z \in (d, c, a)$, with $j \neq z$, is therefore equal to

$$\frac{w_{jt}}{w_{zt,z\neq j}} = \left(\frac{P_{jt}}{P_{zt}}\right)^{1/1-\alpha} \frac{A_{jt}}{A_{zt}}. \tag{57}$$

Since the market for labor is perfectly competitive and there is free movement across sectors, wages will be equalized across sectors in equilibrium, i.e. $w_{dt} = w_{ct} = w_{at}$. Hence, from (57), labor market clearing implies the following relations between relative prices and relative productivities in the three sectors

$$\frac{P_{dt}}{P_{ct}} = \left(\frac{A_{dt}}{A_{ct}}\right)^{-(1-\alpha)}, \tag{58}$$

$$\frac{P_{dt}}{P_{at}} = \left(\frac{A_{dt}}{A_{at}}\right)^{-(1-\alpha)}, \tag{59}$$

$$\frac{P_{ct}}{P_{at}} = \left(\frac{A_{ct}}{A_{at}}\right)^{-(1-\alpha)}. \tag{60}$$

Relative prices of intermediate inputs are inversely proportional to relative productivities, meaning that the sector that is relatively backward compared to another, has also a higher price for the intermediate good produced in that sector. This reflects the natural idea that technological progress makes production processes more efficient and increases productivity, which, in the context of a perfectly competitive market for intermediate inputs, drives down prices.

Equilibrium supply of dirty input Y_{dt} relative to clean input Y_{ct}, from (55), can be written as

$$\frac{Y_{dt}}{Y_{ct}} = \frac{L_{dt}}{L_{ct}}\frac{A_{dt}}{A_{ct}}\left(\frac{P_{dt}}{P_{ct}}\right)^{\alpha/(1-\alpha)}. \tag{61}$$

Substituting (58), the expression above becomes

$$\frac{Y_{dt}}{Y_{ct}} = \frac{L_{dt}}{L_{ct}}\left(\frac{A_{dt}}{A_{ct}}\right)^{1-\alpha}. \tag{62}$$

This is also valid for the other two relative supplies of intermediate inputs

$$\frac{Y_{dt}}{Y_{at}} = \frac{L_{dt}}{L_{at}}\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tag{63}$$

$$\frac{Y_{at}}{Y_{ct}} = \frac{L_{at}}{L_{ct}}\left(\frac{A_{at}}{A_{ct}}\right)^{1-\alpha}. \tag{64}$$

In equilibrium, relative supplies of intermediate goods should match relative demands, hence the ratios in equations (62-64) should be equal to relative demands as derived from the final producer problem in Section (3.1). Without loss of generality, we can define the parameter φ, following Acemoglu et al. (2012), as

$$\varphi \equiv (1-\alpha)(1-\epsilon). \tag{65}$$

By substituting (28) and (58) into (62) and using the definition of φ (65), the relative labor allocation between the dirty and clean sector can be expressed as (see Appendix A.1 for full derivation)

$$\frac{L_{dt}}{L_{ct}} = \left(\frac{A_{dt}}{A_{ct}}\right)^{-\varphi}. \tag{66}$$

The other two relative labor allocations are given by

$$\frac{L_{ct}}{L_{at}} = \frac{Y_{ct}}{Y_{at}} \left(\frac{A_{ct}}{A_{at}}\right)^{\alpha-1} \tag{67}$$

$$\frac{L_{dt}}{L_{at}} = \frac{Y_{dt}}{Y_{at}} \left(\frac{A_{dt}}{A_{at}}\right)^{\alpha-1}. \tag{68}$$

With the two functional forms for adaptation explored in Section 2.2 it is not possible to work out relative demands $\frac{Y_{dt}}{Y_{at}}, \frac{Y_{ct}}{Y_{at}}$ as a function of relative prices only. Hence expressions (63) and (64) cannot be simplified to be functions of relative productivities only.

Turning now to the innovation sector and to the choice of scientists, which is based solely on the wage, we can rewrite the expression for the equilibrium scientists wage in sector j as

$$w^s_{jt} = \frac{\eta\lambda\alpha(1-\alpha)A^{1-\phi}_{jt-1}(A_{t-1})^{\phi}Y_{jt}P_{jt}}{s^{1-\eta}_{jt}A_{jt}}. \tag{69}$$

This is obtained by noting from (5)-(7) and (42) that $x_{jit} = Y_{jt}P_{jt}$ and substituting this expression into (51) (see Appendix A.1). Since the market for scientists is perfectly competitive, the wage of a scientist will be equal to marginal return from innovation in that sector. From (69) we can see that returns on innovation in a sector are proportional to the value of the intermediate good produced in that sector, $Y_{jt}P_{jt}$. Hence if a sector has too small value of output compared to the other two, there will not be innovation in that sector.

By rewriting (51) substituting demand for machines (48), relative wage for scientists in clean and dirty sector is equal to

$$\frac{w^s_{ct}}{w^s_{dt}} = \frac{A^{1-\phi}_{ct-1}P^{1/(1-\alpha)}_{ct}L_{ct}s^{-(1-\eta)}_{ct}}{A^{1-\phi}_{dt-1}P^{1/(1-\alpha)}_{dt}L_{dt}s^{-(1-\eta)}_{dt}}. \tag{70}$$

The market for scientists is perfectly competitive, so in order to have innovation in all the three sectors the wage for scientists must be equal across sectors in equilibrium. If we assume that a balanced growth path with innovation in all the three sectors exists, the relative allocation of scientists in the green and dirty sector along the interior balanced growth path can be written as

$$\frac{s_{ct}}{s_{dt}} = \left(\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{1-\phi}\left(\frac{P_{ct}}{P_{dt}}\right)^{1/(1-\alpha)}\frac{L_{ct}}{L_{dt}}\right)^{1/(1-\eta)}. \tag{71}$$

Within the same assumption, we can write the equivalent expression for relative scientists allocation in the clean versus adaptation sector and in the dirty versus adaptation sector

$$\frac{s_{ct}}{s_{at}} = \left(\left(\frac{A_{ct-1}}{A_{at-1}} \right)^{1-\phi} \left(\frac{P_{ct}}{P_{at}} \right)^{1/(1-\alpha)} \frac{L_{ct}}{L_{at}} \right)^{1/(1-\eta)}, \tag{72}$$

$$\frac{s_{at}}{s_{dt}} = \left(\left(\frac{A_{at-1}}{A_{dt-1}} \right)^{1-\phi} \left(\frac{P_{at}}{P_{dt}} \right)^{1/(1-\alpha)} \frac{L_{at}}{L_{dt}} \right)^{1/(1-\eta)}. \tag{73}$$

By looking at these equations it is possible to analyse the forces governing the relative allocation of research in equilibrium: (i) there is a **past productivity effect** given by the ratio of past productivities elevated at $1-\phi$; (ii) a **price effect**, represented by $\left(\frac{P_{ct}}{P_{dt}}\right)^{1/(1-\alpha)}$, that directs innovation towards the sector with higher prices, and hence, according to (58), the relatively backward sector; and (iii) a **market size effect**, represented by the ratio of labor, which attracts scientists towards the sector with higher employment and consequentially also a higher number of machines (Fried, 2018). The **past productivity effect** can be decomposed into two sub-components. There is a **direct productivity effect** (Acemoglu et al., 2012) given by the ratio of past productivities $\frac{A_{ct-1}}{A_{dt-1}}$, which will attract scientists towards the more advanced sector and which models path dependence in research and cross-periods spillovers within the same sector. And then there is a **spillover effect**, given by $\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\phi}$, which attracts scientists towards the more backward sector, and this effect is stronger the higher is the spillover parameter ϕ. This effect embodies the idea that the relatively backward sector is the one that can benefit the most from spillovers from other sectors. Since ϕ is between zero and one, $1-\phi$ will always be positive, hence the direct productivity effect will always dominate, but it will be smaller the greater is the spillover parameter, capturing the idea that innovation spillovers across sectors smooth down the effect of path dependence in scientific research.

By substituting (66) and (58) into (71), the ratio of scientists in the clean versus dirty sector can be expressed as a function of productivity and parameters only

$$\frac{s_{ct}}{s_{dt}} = \left(\left(\frac{A_{ct-1}}{A_{dt-1}} \right)^{1-\phi} \left(\frac{A_{ct}}{A_{dt}} \right)^{-\varphi-1} \right)^{1/(1-\eta)}. \tag{74}$$

Since in the balanced growth path productivity growth is constant and equal across sectors, relative productivities across sectors are fixed in time. Hence $\frac{A_{ct-1}}{A_{dt-1}} = \frac{A_{ct}}{A_{dt}}$, so the above equation becomes

$$\frac{s_{ct}}{s_{dt}} = \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{-\phi-\varphi}{1-\eta}}. \tag{75}$$

By substituting (59) and (63) into (72) and (60) and (64) into (73), and considering the fact mentioned above, that along the balanced growth path relative productivities are fixed over time, relative scientists allocations with respect to adaptation appear as

$$\frac{s_{ct}}{s_{at}} = \left(\left(\frac{A_{ct-1}}{A_{at-1}}\right)^{-\phi+\alpha-1} \frac{Y_{ct}}{Y_{at}}\right)^{1/(1-\eta)}, \tag{76}$$

$$\frac{s_{dt}}{s_{at}} = \left(\left(\frac{A_{dt-1}}{A_{at-1}}\right)^{-\phi+\alpha-1} \frac{Y_{dt}}{Y_{at}}\right)^{1/(1-\eta)}. \tag{77}$$

These expressions are still functions of relative demands for inputs, since we do not have an expression linking relative demands to relative prices for the ratios involving the adaptation sector.

Relative allocation of scientists in a balanced growth path with innovation in all the three sectors can be analysed also starting from the expression for the evolution of the technology frontier (9). The number of scientists in a sector j in period t can be rewritten as

$$s_{jt} = \left(\left(\frac{A_{jt}}{A_{jt-1}} - 1\right)\left(\frac{A_{jt-1}}{A_{t-1}}\right)^{\phi} \frac{1}{\lambda}\right)^{1/\eta}. \tag{78}$$

The expression $\frac{A_{jt}}{A_{jt-1}} - 1$ is the growth rate of A_j from time $t-1$ to time t. If the economy is on a balanced growth path, the growth rate of the quality of machines is constant and equal across sectors, which means

$$n = \frac{A_{ct}}{A_{ct-1}} - 1 = \frac{A_{dt}}{A_{dt-1}} - 1 = \frac{A_{at}}{A_{at-1}} - 1, \tag{79}$$

where n is the common growth rate of technological progress along the balanced growth path. From (78), considering (79), the ratio between the number of scientists in two sectors is therefore equal to

$$\frac{s_{ct}}{s_{dt}} = \frac{(A_{ct-1})^{\phi/\eta}}{(A_{dt-1})^{\phi/\eta}}, \tag{80}$$

which here is expressed as ratio of clean to dirty sector scientists, but the same relation is valid for all sectors.

Equations (75) and (80) are both describing the allocation of scientists needed to support constant technology growth equal to rate n in both the clean and dirty sector. Putting them together can hence help to understand under which circumstances could a balanced growth path with innovation in both clean and dirty sector can exist.

$$\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\phi/\eta} = \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{-\phi-\varphi}{1-\eta}}. \tag{81}$$

By taking logs of expression (81), I can derive a threshold value for the spillover parameter, $\bar{\phi}$, above which there exists a balanced growth path with innovation in both dirty and clean sectors (see Appendix A.1 for full derivation)

$$\bar{\phi} = -\varphi\eta. \tag{82}$$

Note that $-\varphi$ is a positive quantity, since $\alpha < 1$ and $\epsilon > 1$, as in Assumption 1, means that $\varphi = (1-\alpha)(1-\epsilon)$ is negative.

By applying the same reasoning as above to the allocation of clean scientists relative to adaptation scientists needed to support a balanced growth path with innovation in both these sectors, we can analyse together equation (76) and the equivalent expression of (80) for clean-versus-adaptation scientists' ratio. In this way, I can get some insights on the conditions governing the existence of a balanced growth path with innovation in both clean and adaptation sectors.

$$\frac{(A_{ct-1})^{\phi/\eta}}{(A_{at-1})^{\phi/\eta}} = \left(\left(\frac{A_{ct-1}}{A_{at-1}}\right)^{-\phi+\alpha-1}\frac{Y_{ct}}{Y_{at}}\right)^{1/(1-\eta)}, \tag{83}$$

which can be simplified to

$$\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{\frac{\phi}{\eta}-\frac{-\phi+\alpha-1}{1-\eta}} = \left(\frac{Y_{ct}}{Y_{at}}\right)^{1/(1-\eta)}. \tag{84}$$

By taking logs of the expression above, it is possible also in this case to derive a threshold condition for the spillovers parameter, $\bar{\phi}$, above which a balanced growth path with innovation in both clean and adaptation sector is possible (see Appendix A.1 for full derivation)

$$\bar{\phi} = \eta\frac{\ln\left(\frac{Y_{ct}}{Y_{at}}\right)}{\ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right)} - \eta(1-\alpha). \tag{85}$$

Since both the conditions on $\bar{\phi}$, Equation (82) and Equation (85), must hold for innovation to take place in all the three sectors, I can combine the two expressions in order to analyse under which conditions the spillover parameter is exactly at the threshold value $\bar{\phi}$. When $\phi = \bar{\phi}$ and the value of $\bar{\phi}$ is greater or equal to both (82) and (85), an interior equilibrium with innovation in all the three sectors should theoretically exist. Combining (82) and (85)

$$\eta \frac{\ln\left(\frac{Y_{ct}}{Y_{at}}\right)}{\ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right)} - \eta(1-\alpha) = -\varphi\eta, \tag{86}$$

which simplifies to (see Appendix A.1)

$$\frac{\ln\left(\frac{Y_{ct}}{Y_{at}}\right)}{\ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right)} = (1-\alpha)\epsilon. \tag{87}$$

If condition (87) is satisfied, an interior balanced growth path with innovation in all the three sector should theoretically exist. The problem is that Equation (87) is not a function of parameters only, but depends also on relative outputs in each period and relative technological levels. This means that ensuring that this condition holds in equilibrium is more complicated.

Chapter 4

Quantitative Exercise

With the two functional forms for adaptation chosen in Section 2.2 there is not an analytical solution of the model equilibrium in terms of relative equilibrium demands for intermediate goods ($\frac{Y_{ct}}{Y_{dt}}, \frac{Y_{ct}}{Y_{at}}, \frac{Y_{dt}}{Y_{at}}$). In this chapter, I propose a possible parametrization for the model and I look for a numerical solution to the equilibrium problem outlined in Chapter 3. The parameters are mainly taken from the literature, except those for the adaptation function, for which a new calibration is proposed in Section 4.2. In this quantitative exercise the equilibrium problem is analysed under the assumption that a balanced growth path with innovation in all the three sectors exists. Hence the parameter ϕ is set equal to (82) and it is assumed that in equilibrium ϕ will be greater or equal than expression (85). Condition (86) for the existence of an interior balanced growth path with innovation in all the three sectors is indeed not a function of parameters only, so a correct parameter choice cannot ensure that this condition holds in equilibrium.
The equilibrium of the model is rewritten as a system of equations depending on intermediate input prices (P_{ct}, P_{dt}, P_{at}) and on parameters only. I then use a numerical solver that tries to find a combination of initial prices satisfying general equilibrium conditions. I repeat this exercise for both the functional forms presented in Section 2.2.

4.1 Parameters

The parameters used for this numerical exercise are taken from the literature and are summarized in Table 1. In this numeric analysis I will assume that a period of the model is equal to five years. Fried (2018) argues that five years are a reasonable period of time for within sector spillover to take place, i.e. for technological progress in period $t-1$, A_{jt-1}, to give a contribution to innovation in period t, A_{jt}.

The annual growth rate of technology in the balanced growth path is set to 2%, hence the per-period growth rate of the model, n, is equal to 10%. The inter-temporal discount rate σ is set to 1.5% per annuum as in Acemoglu et al. (2012). The share of machines in the production of intermediate goods, α, is fixed at 1/3, in order to match the value for the share of capital in production that is consolidated in the literature. There exists a quite wide range of values in the literature for the elasticity of substitution between green and dirty technologies, and this range goes from unity to 10 (see Acemoglu et al. (2012), Greaker et al. (2018), Hart (2019)). In this work I set it to 3, as in the benchmark case of Greaker et al. (2018). Scientists efficiency is calibrated in order to match an annual growth rate of technology of 2% in the balanced growth path. Returns to research η is set to 0.7, as in Greaker et al. (2018) and close to Fried (2018), 0.79. The cross-sectors spillover parameter is calibrated to be equal to the value for which it exists a balanced growth path with innovation in both green and dirty sectors, $\phi = \bar{\phi}$, from equation (82). With the set of parameters mentioned above this means $\bar{\phi} = 0.9333$.

Table 1: Parameters

Description	Parameter	Value
Per-period growth rate of productivity in the BGP	n	10%
Inter-temporal discount rate	σ	0.15
Elasticity of substitution	ϵ	3
Machines share	α	1/3
Returns to research	η	0.7
Cross-sector spillovers	ϕ	0.933
Number of scientists	S	0.01
Number of workers	L	1
Permanent carbon share	θ_L	0.2
Carbon exit share	$1 - \theta_0$	0.5981
Carbon decay rate	θ	0.0115
Pre-industrial carbon concentration	$\bar{S}$	581
Initial carbon concentration	S_0	802
Damage parameter	γ	$3.65e^{-05}$
Share of dirty output that contributes to emissions	κ	1
Efficiency of adaptation	δ	0.1
Returns to adaptation	β	0.9

The parameters of the environmental damage function follow entirely the calibration done by Golosov et al. (2014). The share of emissions that stays forever in the atmosphere θ_L is set to 0.2, following IPCC (2007), which states that 20% of total carbon emissions will stay in the atmosphere for thousands of years. The part of carbon that does not stay in the atmosphere forever has, according to Archer (2005), a mean lifetime of 300 years, hence we can derive the geometric rate of decay θ from $(1-\theta)^{300/5} = 0.5$. IPCC (2007) also estimates that half of the CO_2 pulse is removed from the atmosphere after 30 years, hence we can calibrate the carbon cycle expression (4) as $0.5 = 0.2 + 0.8\theta_0(1-\theta)^{30/5}$. This leads to $\theta = 0.0115$ and $\theta_0 = 0.4019$. Finally for the calibration of γ, I follow Nordhaus (2017), who assumes a logarithmic function mapping greenhouse gases concentration to temperature

$$T_t = \frac{3}{\ln 2} \ln \frac{S_t}{\bar{S}}. \tag{88}$$

This functional form implies that a 3°C increase above pre-industrial temperature will be reached when emissions S_t are the double of pre-industrial emissions. This consideration, together with the assumption, again from Nordhaus (2017), that a 3°C increase of temperature above pre-industrial average will translate into a damage of 2.1% on output, means that γ can be calibrated from 2.1% damage, i.e. $(1-0.21) = exp(-\gamma(2(\bar{S}) - \bar{S}))$. The starting values for the stock of carbon in the atmosphere at period zero are also taken from Golosov et al. (2014).

4.2 Calibration of adaptation parameters

Adaptation parameters δ and β do not have references in the literature, hence here I will outline some considerations in order to identify a suitable range of values for their calibration. The actual values that will be chosen for the quantitative exercise are of course a rather imprecise guess, but the considerations made on the possible range for these values could be a starting point for any future research interested in finding a calibration for adaptation using this type of functional form.

For finding the relevant subset of the parameter space for δ and β, I consider boundary condition (15) on the maximum quantity of adaptation. Under functional form for adaptation (16), substituting S_t with the initial value for the stock of greenhouse gases emissions in Table 1, S_0, and using the values for $\bar{S}$ and γ from the set of chosen parameters, we can write the following condition for adaptation in time zero

$$\delta Y_{a0}^{\beta} \leq 0.0081 \tag{89}$$

This implies that i) adaptation output in time zero must be quite low: we will here assume that it should be below one (so that it won't be necessary for δ and β to be excessively low for the condition to be met); ii) since β is below or equal to one, a β below one, but close to it will be the best choice, if we assume that Y_{a0} is below one, iii) δ should be smaller than one and also quite low, if the boundary condition is to be met for the equilibrium value of Y_{a0}.

Considering functional form (19) instead, and substituting the chosen parameters into (21) we get the following boundary condition for adaptation

$$\delta Y_{a0}^{\beta} \leq 0.0081 \tag{90}$$

The two boundary conditions are equal, hence calibration for the two adaptation parameters can be the same regardless of the functional form that is chosen. I choose parameters $\beta = 0.9$ and $\delta = 0.1$ according to the reasoning above, but recognizing the high degree of arbitrariness of this choice. One important aspect that it is worth noting is that this calibration choice does not ensure that boundary conditions (17) and (21) are met. The only thing that this calibration does is making *possible* that these conditions are met, but if the equilibrium quantity of adaptation good Y_{at} is too high, this will result in the boundary condition to be exceeded in period t.

4.3 Result of the quantitative exercise

I run the quantitative exercise with both the functional forms for adaptation considered in Section 2.2 and, for both functional forms, the algorithm did not converge to a real solution for none of the several initial guesses for (P_{ct}, P_{dt}, P_{at}) that were chosen as starting points. It might be that the initial guesses were not close enough to the equilibrium prices for the algorithm to find the solution of the problem, but given the amount of trials, with different random guesses, I believe it is more likely that the problem, in both functional forms, does not have real solutions. This means that for the set of parameters in Table 1 a balanced growth path equilibrium with innovation in all the three sectors does not exist. The numerical exercise was indeed based on the assumption that an interior solution to the problem existed, so the failure of the exercise might be a signal that this assumption was wrong.

Chapter 5

Conclusions

The objective of this thesis was to develop a model of directed technical change and the environment incorporating adaptation to climate change. The purpose of developing this kind of model was studying what determines the equilibrium dynamics of innovation in adaptation in a setting where it is possible to innovate also in the development of clean technologies, hence in mitigation. The two research questions presented in the introduction were: first, whether the model developed in this thesis has an interior equilibrium in which innovation takes place in both adaptation and green technologies; and second, if such an equilibrium exists, which are the forces shaping the relative allocation of research in adaptation vis-à-vis clean technologies.

Given these objectives, I will now list the conclusions that this thesis has reached on the above points. The first contribution of this work to the existing literature is the development of a model of directed technical change with an adaptation sector, something that, to my knowledge, was still unexplored. In the second Chapter I analysed how to best insert adaptation into the environmental damage function of a standard DTC model and I discussed the desirable properties that a good modelling choice for adaptation should have. In Section 2.2 I proposed two specific functional forms for adaptation and the results illustrated here depend on the choice of these two functions.

As a second contribution, in Chapter 3 I analysed the equilibrium of the model and I found that the relative allocation of research in adaptation vis-à-vis clean and dirty technologies is shaped by the following forces, which are well known in the DTC literature and are expressed by equations (71), (72) and (73). One of the main forces driving innovation is past technological level, which directs innovation towards relatively more advanced sectors. This effect is moderated by the influence of technology cross-sectors spillovers and by the price effect, that are both favoring innovation in the relatively backwards sectors. The last factor shaping the equilibrium allocation of scientists is the size of the labor market, which drives research towards the bigger sector. Which of these effects will prevail in equilibrium depends on the initial level of technological development in the three sectors, on the strength of cross-sectors spillovers and on the size of the three sectors in terms of employment and of output, hence also on the demands for the three intermediate goods.

In this work I did not manage to find a positive reply to the first research question stated in the introduction. In Chapter 4 I provided a set of parameters suitable for a quantitative evaluation of the model. However, for the set of chosen parameters, the model does not seem to have an interior balanced growth path in which there is positive innovation in all the three sectors. Or at least it is unlikely that such a solution exists in the region where the values for the three equilibrium prices P_{ct}, P_{dt}, P_{at} have some economic sense. With this I am referring to the region of possible values for P_{ct}, P_{dt}, P_{at} where: first, the three prices are all positive real quantities; second, the initial dirty sector price is lower than the other two (if this is not the case there is still economic sense to the model, but this is no longer suitable for describing the climate change problem). Hence this work provides, and this can be considered as the third contribution of the thesis, an indication on a set of parameters for which the model does not converge to an interior equilibrium. This failure to find an interior solution could be a starting point for reasoning on a different parametrization, for which an equilibrium with innovation in all the three sectors might exist. Another choice of parameters could indeed result in an interior solution for this model, but, also in this case, these alternative choices for parameters need to make economic sense and to be suitable to describe the problem of climate change adaptation and innovation. For example, increasing the value of elasticity of substitution ϵ would be a valid alternative parameter choice, or also changing the returns to research parameter η. I would not change for example the share of machines parameter α or the γ of the damage function, since those are well-established in the literature. For sure one aspect that can be improved in the choice of parameters is the calibration of β and δ. These two adaptation parameters are not present in the literature, hence, and this can be considered as a fourth contribution of this thesis, the considerations illustrated in Section 4.2 could be a good starting point for a more accurate calibration using the functional forms proposed in this model.

The other possibility is that a balanced growth path solution for this model does not exist with any economically-meaningful parametrization. This would mean that the functional form chosen to represent the adaptation sector is probably not well suited for the study of adaptation in a DTC type of model. Also in this case, this thesis could be a starting point for modifying the adaptation function into a more suitable alternative.

5.1 Limitations of the study and further research directions

This model and the approach adopted in this thesis have clearly many limitations. One of the main limitations is that, as stated in Section 4.2, the two functional forms proposed for adaptation, together with the calibration choice for the two adaptation parameters, do not ensure that the boundary conditions (17) and (21) are met. This means that in both the current versions of the model there is the possibility that the equilibrium quantity of adaptation exceeds the one needed to compensate for the negative effects of climate change on output. A way to improve the model would therefore be to find a functional form and a respective calibration for which the boundary condition of the maximum amount of adaptation is met with certainty.

Another issue related to the modelling choice for adaptation is that in this work adaptation is modeled as a flow variable, while including at least a part of adaptation that is a stock variable would describe better the dynamics of investing in adaptation. This however would increase the number of state variables to consider, raising the complexity of the analysis. Another issue is that in this model agents are only backward looking and expectations of future climate damages are not taken into account. Incorporating in the model forward looking expectations and a stock part of the adaptation good would probably lead to higher equilibrium demand for adaptation for two reasons: i) current adaptation preserves also future output since it is a stock variable, ii) agents are able to anticipate this future benefit and embody it into their present decisions.

Another issue is that in this model adaptation is an aggregate variable, while the adaptation problem has often a very local nature. It would be interesting to build a similar framework in a two regions model, with the two regions having different climate damage functions. The region having a more *severe* climate damage function will be also relatively backward with respect to the aggregate level of technology and will have lower regional output with respect to the more technologically advanced region. This kind of setting will allow to model the different dynamics of adaptation between the rich and poor regions of the world. A model like this would describe the drivers for adaptation in a context in which those that have higher benefits from adapting to climate change are also those in the worse position to carry out productivity-enhancing innovation in adaptation. In this context it would be possible for example to assess the relative weight of need for adaptation vis-à-vis technological advancement in driving the amount of research in the adaptation sector.

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Appendix A

Appendix

A.1 Derivation of equations in main part

A.1.1 Final good producer problem

Derivation of equation (30)

$$
\begin{aligned}
\frac{P_{ct}}{P_{at}} = \frac{(1+\delta Y_{at}^{\beta})Y_{ct}^{-1/\epsilon}}{(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})\delta\beta Y_{at}^{\beta-1}} \\
&= \frac{(1+\delta Y_{at}^{\beta})Y_{ct}^{-1/\epsilon}}{(Y_{ct}^{(\epsilon-1)/\epsilon} + \left(Y_{ct}\frac{P_{dt}^{-\epsilon}}{P_{ct}^{-\epsilon}}\right)^{(\epsilon-1)/\epsilon})\delta\beta Y_{at}^{\beta-1}} \\
&= \frac{(1+\delta Y_{at}^{\beta})Y_{ct}^{-1/\epsilon}}{Y_{ct}^{(\epsilon-1)/\epsilon}\left(1+\frac{P_{dt}^{1-\epsilon}}{P_{ct}^{1-\epsilon}}\right)\delta\beta Y_{at}^{\beta-1}} \\
&= \frac{1+\delta Y_{at}^{\beta}}{Y_{ct}\left(\frac{P_{ct}^{1-\epsilon}+P_{dt}^{1-\epsilon}}{P_{ct}^{1-\epsilon}}\right)\delta\beta Y_{at}^{\beta-1}} \\
&= \frac{1}{Y_{ct}}\left(\frac{1+\delta Y_{at}^{\beta}}{\delta\beta Y_{at}^{\beta-1}}\right)\left(\frac{P_{ct}^{1-\epsilon}}{P_{ct}^{1-\epsilon}+P_{dt}^{1-\epsilon}}\right)
\end{aligned}
\tag{A.1.1}
$$

Derivation of equation (38)

$$
\begin{aligned}
\frac{P_{ct}}{P_{at}} &= \frac{Y_{ct}^{-1/\epsilon}}{(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})\delta\beta Y_{at}^{\beta-1}} \\
&= \frac{Y_{ct}^{-1/\epsilon}}{(Y_{ct}^{(\epsilon-1)/\epsilon} + \left(Y_{ct}\frac{P_{dt}^{-\epsilon}}{P_{ct}^{-\epsilon}}\right)^{(\epsilon-1)/\epsilon})\delta\beta Y_{at}^{\beta-1}} \\
&= \frac{Y_{ct}^{-1/\epsilon}}{Y_{ct}^{(\epsilon-1)/\epsilon}\left(1 + \frac{P_{dt}^{1-\epsilon}}{P_{ct}^{1-\epsilon}}\right)\delta\beta Y_{at}^{\beta-1}} \\
&= \frac{Y_{at}^{1-\beta}}{Y_{ct}\delta\beta}\left(\frac{P_{ct}^{1-\epsilon}}{P_{ct}^{1-\epsilon} + P_{dt}^{1-\epsilon}}\right)
\end{aligned}
\tag{A.1.2}
$$

A.1.2 Machines producers problem

This is the Lagrangian function for the machine producers problem:

$$
\begin{aligned}
L : p_{jit}x_{jit} - \psi x_{jit} - w_{sjt}s_{jit} - \mu_1 \left(\left(\frac{\alpha P_{jt}}{p_{jit}}\right)^{\frac{1}{1-\alpha}} L_{jt}A_{jit} - x_{jit} \right) \\
- \mu_2 \left(A_{jt-1}(1 + \lambda\ s_{jit}^{\eta} \left(\frac{A_{t-1}}{A_{jt-1}}\right)^{\phi}) - A_{jit} \right)
\end{aligned}
\tag{A.1.3}
$$

where μ_1 and μ_2 are the Lagrange multipliers for the two constraints.
The first order conditions are:

$$\frac{\partial L}{\partial p_{jit}} = x_{jit} - \mu_1(\alpha P_{jt})^{1/(1-\alpha)} L_{jt}A_{jit}(-\frac{1}{1-\alpha})p_{jit}^{(\alpha-2)/(1-\alpha)} \tag{A.1.4}$$

$$\frac{\partial L}{\partial x_{jit}} = p_{jit} - \psi + \mu_1 \tag{A.1.5}$$

$$\frac{\partial L}{\partial s_{jit}} = -w_{sjt} - \mu_2(A_{jt-1}\lambda \left(\frac{A_{t-1}}{A_{jt-1}}\right)^{\phi} s_{jit}^{\eta-1}) \tag{A.1.6}$$

By substituting x_{jit} with (42) into (A.1.4) we can get a value for μ_1. By replacing it into (A.1.11) we get the following expression for the equilibrium price of machines

$$p_{jit} = \frac{\psi}{\alpha} \tag{A.1.7}$$

A.1.3 Equilibrium allocation of workers and scientists

Derivation of equation (66)

Substituting (28), (62) becomes

$$\left(\frac{P_{dt}}{P_{ct}}\right)^{-\epsilon} = \frac{L_{dt}}{L_{ct}}\left(\frac{A_{dt}}{A_{ct}}\right)^{1-\alpha} \tag{A.1.8}$$

Substituting also (58) into it, the expression becomes

$$\left(\frac{A_{dt}}{A_{ct}}\right)^{\epsilon(1-\alpha)} = \frac{L_{dt}}{L_{ct}}\left(\frac{A_{dt}}{A_{ct}}\right)^{1-\alpha}$$
$$\frac{L_{dt}}{L_{ct}} = \left(\frac{A_{dt}}{A_{ct}}\right)^{\epsilon(1-\alpha)-(1-\alpha)} = \left(\frac{A_{dt}}{A_{ct}}\right)^{(\epsilon-1)(1-\alpha)} \tag{A.1.9}$$

Considering (65) this can be written as

$$\frac{L_{dt}}{L_{ct}} = \left(\frac{A_{dt}}{A_{ct}}\right)^{-\varphi} \tag{A.1.10}$$

Derivation of equation (69)

From (42) we can derive the following expression for labor

$$L_{jt} = \frac{x_{jit}}{A_{jit}}\left(\frac{p_{jit}}{\alpha P_{jt}}\right)^{1/(1-\alpha)} \tag{A.1.11}$$

Substituting (A.1.11) into the production function for intermediate inputs (5)

$$Y_{jt} = \frac{x_{jit}^{1-\alpha}}{A_{jt}^{1-\alpha}}\frac{p_{jit}}{\alpha P_{jt}}\int_0^1 A_{jit}^{1-\alpha}x_{jit}^{\alpha}di$$
$$Y_{jt} = x_{jit}\frac{p_{jit}}{\alpha P_{jt}}$$
$$p_{jit}x_{jit} = \alpha P_{jt}Y_{jt} \tag{A.1.12}$$

Since $p_{jit} = \alpha$

$$x_{jit} = P_{jt} Y_{jt} \tag{A.1.13}$$

Substituting (A.1.13) into (51)

$$w_{sjt} = \frac{\eta \lambda \alpha (1-\alpha) A_{jt-1}^{1-\phi} (A_{t-1})^{\phi} P_{jt} Y_{jt}}{s_{jt}^{1-\eta} A_{jt}} \tag{A.1.14}$$

Derivation of equations (76) **and** (77)

By substituting (59) and (63) into (72) and (60) and (64) into (73), relative scientists allocation with respect to adaptation appears as

$$\frac{s_{ct}}{s_{at}} = \left(\left(\frac{A_{ct-1}}{A_{at-1}} \right)^{1-\phi} \left(\frac{A_{ct}}{A_{at}} \right)^{-1} \frac{Y_{ct}}{Y_{at}} \left(\frac{A_{ct}}{A_{at}} \right)^{\alpha-1} \right)^{1/(1-\eta)} \tag{A.1.15}$$

$$\frac{s_{dt}}{s_{at}} = \left(\left(\frac{A_{dt-1}}{A_{at-1}} \right)^{1-\phi} \left(\frac{A_{dt}}{A_{at}} \right)^{-1} \frac{Y_{dt}}{Y_{at}} \left(\frac{A_{dt}}{A_{at}} \right)^{\alpha-1} \right)^{1/(1-\eta)} \tag{A.1.16}$$

which simplifies to

$$\frac{s_{ct}}{s_{at}} = \left(\left(\frac{A_{ct-1}}{A_{at-1}} \right)^{-\phi+\alpha-1} \frac{Y_{ct}}{Y_{at}} \right)^{1/(1-\eta)} \tag{A.1.17}$$

$$\frac{s_{dt}}{s_{at}} = \left(\left(\frac{A_{dt-1}}{A_{at-1}} \right)^{-\phi+\alpha-1} \frac{Y_{dt}}{Y_{at}} \right)^{1/(1-\eta)} \tag{A.1.18}$$

Derivation of equation (82)

Taking logs of equation (81) we get to

$$\frac{\phi}{\eta} = \frac{-\phi - \varphi}{1-\eta} \tag{A.1.19}$$

which simplifies to

$$\bar{\phi} = -\varphi \eta \tag{A.1.20}$$

Derivation of equation (85)

Taking logs of (84) we get

$$\frac{\phi(1-\eta) + \phi\eta - \alpha\eta + \eta}{\eta(1-\eta)} \ln \left(\frac{A_{ct-1}}{A_{dt-1}} \right) = \frac{1}{1-\eta} \ln \left(\frac{Y_{ct}}{Y_{at}} \right) \tag{A.1.21}$$

that is equal to

$$\frac{\phi - \alpha\eta + \eta}{\eta} \ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right) = \ln\left(\frac{Y_{ct}}{Y_{at}}\right)$$

$$\frac{\phi}{\eta} - \alpha + 1 = \frac{\ln\left(\frac{Y_{ct}}{Y_{at}}\right)}{\ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right)}$$

$$\bar{\phi} = \eta\frac{\ln\left(\frac{Y_{ct}}{Y_{at}}\right)}{\ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right)} - \eta(1-\alpha) \tag{A.1.22}$$

Derivation of expression (87)

$$\eta\frac{\ln\left(\frac{Y_{ct}}{Y_{at}}\right)}{\ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right)} - \eta(1-\alpha) = -\varphi\eta$$

$$\eta\frac{\ln\left(\frac{Y_{ct}}{Y_{at}}\right)}{\ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right)} - \eta(1-\alpha) = -(1-\alpha)(1-\epsilon)\eta$$

$$\eta\frac{\ln\left(\frac{Y_{ct}}{Y_{at}}\right)}{\ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right)} = \eta(1-\alpha)(1-1+\epsilon)$$

$$\eta\frac{\ln\left(\frac{Y_{ct}}{Y_{at}}\right)}{\ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right)} = \eta(1-\alpha)\epsilon$$

$$\frac{\ln\left(\frac{Y_{ct}}{Y_{at}}\right)}{\ln\left(\frac{A_{ct-1}}{A_{dt-1}}\right)} = (1-\alpha)\epsilon \tag{A.1.23}$$

A.2 Equations for quantitative exercise

Here are derived the equilibrium demands for intermediate goods that are used in the quantitative exercise, for both functional forms. These equation will be part of the equilibrium system that is passed as an input to the numerical solver in the quantitative analysis.

A.2.1 Functional form one

The equilibrium prices and demands for inputs for functional form one are the following. For Y_{dt}:

$$P_{dt} = exp(-\gamma(S_t - \bar{S}))(1 + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{1/(\epsilon-1)} Y_{dt}^{-1/\epsilon} \tag{A.2.24}$$

By expressing this for Y_{dt} we can derive equilibrium demand

$$\begin{aligned} Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon} &= \frac{P_{dt}^{\epsilon-1} Y_{dt}^{(\epsilon-1)/\epsilon}}{(exp(-\gamma(S_t - \bar{S}))(1 + \delta Y_{at}^{\beta}))^{\epsilon-1}} \\ Y_{ct}^{(\epsilon-1)/\epsilon} &= \left(\left(\frac{P_{dt}}{exp(-\gamma(S_t - \bar{S}))(1 + \delta Y_{at}^{\beta})} \right)^{\epsilon-1} - 1 \right) Y_{dt}^{(\epsilon-1)/\epsilon} \\ Y_{dt}^{d} &= \frac{Y_{ct}}{\left(\left(\frac{P_{dt}}{exp(-\gamma(S_t - \bar{S}))(1+\delta Y_{at}^{\beta})} \right)^{\epsilon-1} - 1 \right)^{\epsilon/(\epsilon-1)}} \end{aligned} \tag{A.2.25}$$

For the clean good Y_{ct} the same considerations apply

$$P_{ct} = exp(-\gamma(S_t - \bar{S}))(1 + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{1/(\epsilon-1)} Y_{ct}^{-1/\epsilon} \tag{A.2.26}$$

$$\begin{aligned} Y_{dt}^{(\epsilon-1)/\epsilon} &= \left(\left(\frac{P_{ct}}{exp(-\gamma(S_t - \bar{S}))(1 + \delta Y_{at}^{\beta})} \right)^{\epsilon-1} - 1 \right) Y_{ct}^{(\epsilon-1)/\epsilon} \\ Y_{ct}^{d} &= \frac{Y_{dt}}{\left(\left(\frac{P_{ct}}{exp(-\gamma(S_t - \bar{S}))(1+\delta Y_{at}^{\beta})} \right)^{\epsilon-1} - 1 \right)^{\epsilon/(\epsilon-1)}} \end{aligned} \tag{A.2.27}$$

And finally for the adaptation good Y_{at}:

$$P_{at} = exp(-\gamma(S_t - \bar{S}))(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)} \delta\beta Y_{at}^{\beta-1} \tag{A.2.28}$$

$$Y_{at}^{d} = \left(\frac{P_{at}}{exp(-\gamma(S_t - \bar{S}))(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)} \delta\beta} \right)^{1/(\beta-1)} \tag{A.2.29}$$

By substituting (A.2.25) into (A.2.27) we get

$$Y_{ct} = \frac{Y_{ct}}{\left(\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1\right)^{\epsilon/(\epsilon-1)}} \left(\left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1\right)^{\epsilon/(1-\epsilon)}$$

$$Y_{ct} = Y_{ct}\left(\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1\right)^{\epsilon/(1-\epsilon)}$$
$$\left(\left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1\right)^{\epsilon/(1-\epsilon)}$$

$$1 = \left(\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1\right)^{\epsilon/(1-\epsilon)} \left(\left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1\right)^{\epsilon/(1-\epsilon)}$$

$$1 = \left(\frac{P_{dt}P_{ct}}{(exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta}))^2}\right)^{\epsilon-1} - \left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}$$
$$- \left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} + 1$$

$$\left(\frac{P_{dt}P_{ct}}{(exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta}))^2}\right)^{\epsilon-1} = \left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}$$
$$+ \left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}$$

$$\left(\frac{P_{ct}P_{dt}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} = P_{dt}^{\epsilon-1} + P_{ct}^{\epsilon-1}$$

$$(exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta}))^{1-\epsilon} = \frac{P_{dt}^{\epsilon-1} + P_{ct}^{\epsilon-1}}{(P_{ct}P_{dt})^{\epsilon-1}}$$

$$(exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta}))^{1-\epsilon} = \frac{1}{P_{dt}^{\epsilon-1}} + \frac{1}{P_{ct}^{\epsilon-1}}$$

$$exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})) = (P_{dt}^{1-\epsilon} + P_{ct}^{1-\epsilon})^{1/(1-\epsilon)}$$

$$(1+\delta Y_{at}^{\beta}) = \frac{(P_{dt}^{1-\epsilon} + P_{ct}^{1-\epsilon})^{1/(1-\epsilon)}}{exp(-\gamma(S_t-\bar{S}))}$$

$$Y_{at} = \left(\frac{(P_{dt}^{1-\epsilon} + P_{ct}^{1-\epsilon})^{1/(1-\epsilon)}}{exp(-\gamma(S_t-\bar{S}))\delta} - \frac{1}{\delta}\right)^{1/\beta} \tag{A.2.30}$$

Substituting (A.2.25) into (A.2.29) we have

$$Y_{at} = \left(\frac{P_{at}}{exp(-\gamma(S_t-\bar{S}))\left(Y_{ct}^{(\epsilon-1)/\epsilon} + \frac{Y_{ct}^{(\epsilon-1)/\epsilon}}{\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1}\right)^{\epsilon/(\epsilon-1)} \delta\beta} \right)^{1/(\beta-1)}$$

$$Y_{at} = \left(\frac{P_{at}}{exp(-\gamma(S_t-\bar{S}))\left(Y_{ct}^{(\epsilon-1)/\epsilon}\left(1 + \frac{1}{\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1}\right)\right)^{\epsilon/(\epsilon-1)} \delta\beta} \right)^{1/(\beta-1)}$$

$$Y_{at} = \left(\frac{P_{at}}{exp(-\gamma(S_t-\bar{S}))Y_{ct}\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})}\right)^{-\epsilon} \delta\beta} \right)^{1/(\beta-1)}$$

$$Y_{at} = \left(\frac{P_{at}}{exp(-\gamma(S_t-\bar{S}))^{1+\epsilon} P_{dt}^{-\epsilon}(1+\delta Y_{at}^{\beta})^{\epsilon} Y_{ct}\delta\beta} \right)^{1/(\beta-1)}$$

$$Y_{ct} = \frac{P_{at}Y_{at}^{1-\beta}(1+\delta Y_{at}^{\beta})^{-\epsilon}}{exp(-\gamma(S_t-\bar{S}))^{1+\epsilon}P_{dt}^{-\epsilon}\delta\beta} \tag{A.2.31}$$

The demands for the three intermediate inputs to be used in the quantitative exercise are:

$$Y_{at}^{d} = \left(\frac{(P_{dt}^{1-\epsilon} + P_{ct}^{1-\epsilon})^{1/(1-\epsilon)}}{exp(-\gamma(S_t-\bar{S}))\delta} - \frac{1}{\delta} \right)^{1/\beta} \tag{A.2.32}$$

$$Y_{ct}^{d} = \frac{P_{at}Y_{at}^{1-\beta}(1+\delta Y_{at}^{\beta})^{\epsilon}}{exp(-\gamma(S_t-\bar{S}))^{1-\epsilon}P_{ct}^{\epsilon}\delta\beta} \tag{A.2.33}$$

$$Y_{dt}^{d} = \left(\left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S}))(1+\delta Y_{at}^{\beta})} \right)^{\epsilon-1} - 1 \right)^{\epsilon/(\epsilon-1)} Y_{ct} \tag{A.2.34}$$

A.2.2 Functional form two

For the second functional form, equilibrium demands for intermediate inputs are derived here. For Y_{dt}:

$$P_{dt} = exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{1/(\epsilon-1)} Y_{dt}^{-1/\epsilon}, \tag{A.2.35}$$

Which leads to the following equilibrium demand for Y_{dt}

$$\begin{aligned} Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon} &= \frac{P_{dt}^{\epsilon-1} Y_{dt}^{(\epsilon-1)/\epsilon}}{(exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta}))^{\epsilon-1}} \\ Y_{ct}^{(\epsilon-1)/\epsilon} &= \left(\left(\frac{P_{dt}}{exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1\right) Y_{dt}^{(\epsilon-1)/\epsilon} \\ Y_{dt}^{d} &= \frac{Y_{ct}}{\left(\left(\frac{P_{dt}}{exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1\right)^{\epsilon/(\epsilon-1)}}. \end{aligned} \tag{A.2.36}$$

For the clean good Y_{ct} the same considerations apply

$$P_{ct} = exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{1/(\epsilon-1)} Y_{ct}^{-1/\epsilon}. \tag{A.2.37}$$

$$\begin{aligned} Y_{dt}^{(\epsilon-1)/\epsilon} &= \left(\left(\frac{P_{ct}}{exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1\right) Y_{ct}^{(\epsilon-1)/\epsilon} \\ Y_{ct}^{d} &= \frac{Y_{dt}}{\left(\left(\frac{P_{ct}}{exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})}\right)^{\epsilon-1} - 1\right)^{\epsilon/(\epsilon-1)}}. \end{aligned} \tag{A.2.38}$$

For the adaptation good instead we have

$$P_{at} = exp(-\gamma(S_t - \bar{S}) + \delta Y_{at}^{\beta})(Y_{ct}^{(\epsilon-1)/\epsilon} + Y_{dt}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)} \delta\beta Y_{at}^{\beta-1}. \tag{A.2.39}$$

Deriving equilibrium demand for Y_{at} from this expression is not possible since Y_{at} enters this expression both as exponential and as $Y_{at}^{\beta-1}$.

Substituting (A.2.36) into (A.2.38)

$$
\begin{aligned}
Y_{ct} &= \frac{Y_{ct}}{\left(\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}-1\right)^{\epsilon/(\epsilon-1)}\left(\left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}-1\right)^{\epsilon/(\epsilon-1)}} \\
1 &= \frac{1}{\left(\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}-1\right)^{\epsilon/(\epsilon-1)}\left(\left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}-1\right)^{\epsilon/(\epsilon-1)}} \\
1 &= \left(\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}-1\right)^{\epsilon/(\epsilon-1)}\left(\left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}-1\right)^{\epsilon/(\epsilon-1)} \\
1 &= \left(\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}-1\right)\left(\left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}-1\right) \\
1 &= \left(\frac{P_{dt}P_{ct}}{(exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta}))^2}\right)^{\epsilon-1} \\
&\quad -\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}-\left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}+1 \\
&\left(\frac{P_{dt}P_{ct}}{(exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta}))^2}\right)^{\epsilon-1} = \\
&\left(\frac{P_{dt}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1}+\left(\frac{P_{ct}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} \\
\left(\frac{P_{dt}P_{ct}}{exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta})}\right)^{\epsilon-1} &= P_{dt}^{\epsilon-1}+P_{ct}^{\epsilon-1} \\
(exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta}))^{1-\epsilon} &= \frac{P_{dt}^{\epsilon-1}+P_{ct}^{\epsilon-1}}{(P_{ct}P_{dt})^{\epsilon-1}} \\
(exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta}))^{1-\epsilon} &= P_{ct}^{1-\epsilon}+P_{dt}^{1-\epsilon} \\
exp(-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta}) &= (P_{ct}^{1-\epsilon}+P_{dt}^{1-\epsilon})^{1/(1-\epsilon)} \\
-\gamma(S_t-\bar{S})+\delta Y_{at}^{\beta} &= \ln{(P_{ct}^{1-\epsilon}+P_{dt}^{1-\epsilon})^{1/(1-\epsilon)}} \\
\delta Y_{at}^{\beta} &= \ln{(P_{ct}^{1-\epsilon}+P_{dt}^{1-\epsilon})^{1/(1-\epsilon)}}+\gamma(S_t-\bar{S}) \\
Y_{at} &= \left(\frac{\frac{1}{1-\epsilon}\ln{(P_{ct}^{1-\epsilon}+P_{dt}^{1-\epsilon})}+\gamma(S_t-\bar{S})}{\delta}\right)^{1/\beta}.
\end{aligned}
\tag{A.2.40}
$$

By rewriting Equation (38) in the main part, I can obtain an expression of Y_{ct} just as a function of Y_{at} and of prices

$$Y_{ct} = \frac{Y_{at}^{1-\beta} P_{at}}{\delta\beta} \frac{P_{ct}^{-\epsilon}}{P_{ct}^{1-\epsilon} + P_{dt}^{1-\epsilon}}. \tag{A.2.41}$$

In the quantitative exercise we can hence use the three following equations linking equilibrium demands for intermediate goods

$$Y_{at} = \left(\frac{\frac{1}{1-\epsilon} \ln \left(P_{ct}^{1-\epsilon} + P_{dt}^{1-\epsilon} \right) + \gamma (S_t - \bar{S})}{\delta} \right)^{1/\beta}, \tag{A.2.42}$$

$$Y_{ct} = \frac{Y_{at}^{1-\beta} P_{at}}{\delta\beta} \frac{P_{ct}^{-\epsilon}}{P_{ct}^{1-\epsilon} + P_{dt}^{1-\epsilon}}, \tag{A.2.43}$$

$$Y_{dt} = \frac{P_{ct}^{-\epsilon}}{P_{dt}^{-\epsilon}} Y_{ct}. \tag{A.2.44}$$