Università
Ca'Foscari
Venezia

Master's degree

in Global Development

and Entrepreneurship

Final Thesis

# The Discount for Lack of Marketability

An Investigation of Privately Held Companies in North Italy

**Supervisor**
Ch. Prof. Guido Massimiliano Mantovani

**Graduand**
Alberto Crisostomo
Matriculation Number 859360

**Academic Year**
2019 / 2020

# Index

# Introduction

Liquidity and marketability are two key concepts in assets' valuation, and they refer to the ease at which an asset can be sold without lowering its price. Due to the increasing uncertainty of markets, investors prefer investing in marketable assets, which are easy to sell. Therefore, in case of adverse market movements, investors can rapidly sell their assets to avoid losses or if the asset 's value has peaked, they can realize the maximum possible. Considering the liquidity spectrum, public stocks included in market indexes, are regarded as very liquid assets. On the contrary, private companies controlling and non-controlling interests are deemed illiquid. Assets exchanged in stock markets can be bought and sold in a matter of minutes, while it can take months or even years to sell private equity unless the investor accepts significant price reductions. The unrealized gains and long holding period must be considered in private equity and private company's valuation. It is broadly accepted among practitioners that the lack of marketability and illiquidity should drive the asset's price down either by applying a discount rate to the overall value of the company or by applying a premium to the investor's return, which lowers the final value of the subject firm.

Although small and medium enterprises account for 99,8% of the total EU companies[1], there is no universally accepted model to value private companies or to precisely compute the DLOM.

There are two main strands of thoughts regarding private companies' valuation. The first computes the value of the subject private company as if it was listed, and then applies a discount rate. The second finds the premium-return required by the investor for investing in a private company and then computes the value of the company based on such return.

The first approach applies a DLOM by looking at the results of analyses on several non-marketable assets. This procedure has been widely used in the past. Even tax courts used to rely on past DLOM analyses. More recently, though, both tax courts and appraisers started believing that DLOMs should reflect the subject firms' characteristics. Therefore, instead of applying a fixed discount rate, now each firm is deeply analyzed.

[1] European Commission. 2019. *Annual Report on European SMEs 2018/2019: Research & Development and Innovation by SMEs.* Karen Hope.

This work is organized as follows. Chapter 1 gives an in-depth look at the DLOM's concept, describing how its computation has changed over time and what factors influence it the most. Chapters 2 and 3 present empirical and theoretical models, respectively, giving an in-depth look at the models and their pros and flaws. Finally, chapter 4 reports the model I applied in a sample of 209 Italian private companies, explaining why I rejected the other several approaches presented in chapters 2 and 3. I will also present the model's hypotheses and test them for my data sample. I find a discount for lack of marketability ranging between 23,96% and 32,44%. My results are consistent with the latest DLOM studies, considering that the Italian market is less liquid than the US market.

# Chapter 1

# The Concept of Discount for Lack of Marketability

## 1.1 Discount for Lack of Marketability (DLOM)

The DLOM is a discount rate used to adjust the value of an asset based on its overall marketability. The basic concept is that private stocks, as well as private companies, do not have a ready market like their public counterparts. This difference has to be taken into account when valuing a private asset, due to investors preferring marketability over non-marketability. Marketability is "the ability to convert the business ownership (at whatever level) to cash quickly, with the minimum transaction and administrative costs in doing so"[2]. An asset does not necessarily have to be either marketable or non-marketable. There are different degrees of marketability, as explained later in paragraph 1.3, that can cause a slight discount even for listed securities.

The main approach for valuing a non-marketable asset is to value it as if it was marketable and then apply the discount rate that accounts for its lack of marketability and lack of liquidity, which will adjust the asset's price downward.

Even today practitioners do not agree on how to compute the DLOM. During the past, many experts have been divided into two main ways of estimating such a discount rate. The first strand of thought is based on empirical studies, which is divided into Restricted Stock Studies, Pre-IPO Studies, and Valuation Multiplier Approach. The first approach compares the price of a listed company's shares with restricted stock prices of the very same firm. The Pre-IPO method compares the price of a company a few months before going public and immediately after being listed. Finally, the Multiplier approach compares a set of financial ratios of a private company to the same ratios of a similar public company.

These models have been developed between the 1960s and mid-1990s, but their results are still relevant today. Towards the early 1990s up to the present day, practitioners have been focusing on theoretical studies, which consist of: Option Pricing Theory,

[2] Pratt, Shannon P. and Niculita, Alina V. 2008. *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*. The McGraw-Hill Companies, Inc.

Quantitative Marketability Discount Model (QMDM), Capital Asset Pricing Model (CAPM) Approach, and AECA Approach.

The first method shows that an investor can use a put option to get protection from future market fluctuations that might drive the price of the restricted asset down. The price of such a put option should represent the marketability value of the subject asset. The DCF model tries to derive the value of a nonmarketable asset as the present value of expected future cash flows. The QMDM, CAPM, and AECA approaches have very similar goals: accounting for the extra return an investor of nonmarketable assets requires at WACC level.

The evolution of the aforementioned methods and the concept of DLOM will be further analyzed in paragraph 1.2, while methods will be detailly analyzed in chapter 2.

Applying a valuation discount to a private stock or a private company's value is extremely important and in line with the Fair Market Value Principle, which states that: "Fair Market Value (FMV) is the price that property would sell for on the open market. It is the price that would be agreed on between a willing buyer and a willing seller, with neither being required to act, and both having reasonable knowledge of the relevant facts"[3]. The concept of both the willing buyer and willing seller is proper even in a non-marketable transaction. On the contrary, the idea of both parties having reasonable knowledge of relevant facts cannot be taken for granted. Bajaj, Denis, Ferris, and Sarin (2001) published a study presenting their method which is an extension of the Restricted Stock Approach (RSE). The Authors presented an RSE based on eighty-eight observations ranging from 1990 to 1995. They noticed how 56% of the total transactions were made by firms operating in hi-tech industries. "Interestingly, these industries tend to be technology driven and hence are plausibly characterized by larger information asymmetries between insiders and potential outside investors"[4]. Information asymmetries exist, especially when restricted stocks are traded on the Over-The-Counter (OTC) market, which oftentimes is where most restricted stocks and private companies' stocks are traded.

[3] IRS Website, Publication 561. https://www.irs.gov/publications/p561#:~:text=Fair%20market%20value%20(FMV)%20is,knowledge%20of%20the%20relevant%20facts.

[4] Bajaj, Mukesh, Denis, David J., Ferris, Stephen P., and Sarin Atulya. 2001. Firm Value and Marketability Discounts. Journal of Corporation Law.

An analyst has to be very careful when computing the DLOM for privately held companies. Empirical studies focus on finding the discount rate on temporary nonmarketable shares to apply on assets that might never be listed. On the other hand, theoretical studies are very difficult to use due to some components being not only difficult to obtain but also highly subjective. Therefore, the Discount for Lack of Marketability divides practitioners even today.

### 1.2 Time Evolution of Literal Review

In the early 20th century, many public companies were offering restricted shares as a way of raising capital. Restricted stocks were easy to place and were less costly than common shares. After the 1929 crisis, the Securities and Exchange Commission (SEC) tried regulating the resell of both common and restricted stocks. Only years after, in 1972, the SEC published its Rule 144, posing restrictions to the resell of letter stocks. In fact, "Under the pre-Rule 144 administrative practice, the "intent" of the person reselling was a crucial element in the determination of whether public resale would be permitted without registration"[5]. Investors had to show no interest in reselling this type of shares. The SEC acknowledged the problem of valuing restricted shares, since in court's appraisers often ended up with conflicting results among them. To tackle this issue, the SEC published the Investment Company Act in 1940. "In respect of securities not registered as to ownership, in lieu of mailing a copy of such advisory report, such registered company shall publish promptly a statement of the existence of such advisory report in a newspaper of general circulation […] and shall make available copies of such advisory report upon request"[6]. From now on, investment companies engaged in valuing and buying restricted shares had to make available the report stating what inputs they considered during the valuation. Despite the publications, many struggled in valuing private company shares due to a lack of principles. Through the Rev Rule 59-60, in 1959, the SEC admitted that no formula was available, but they tried explaining the most common methods and factors that must be considered. The SEC in 1969 expanded even further the Investment Company Act of 1940, through the release number 113. Now

[5] Yale Law Journal. 1972. *Resale of Restricted Securities under SEC Rule 144.* Yale Law Journal.
[6] Securities and Exchange Commission. 1940. Investment Company Act of 1940. Chapter 686 of the 76th Congress.

both investment companies and the board of directors of the firm offering restricted stocks had to disclose the inputs used in the valuation and the method used and the percentage of price discounts compared to the marketable counterpart. This was a turning point. Now more information was easily available for anybody. Many tried developing models to compute the DLOM based on discounts applied by investment companies. It is possible to identify some tendencies in the valuation of DLOMs. The end of a trend corresponds to either a change in the way of thinking the discount rate or to a new method being developed.

"In the 1970s and 1980s, many believed that marketability should be 35%, irrespective of a specific situation's facts"[7]. This first trend was highly influenced by restricted stock studies. Despite many analysts presenting their DLOM based on different data among each other, results were very consistent. Therefore, appraisers and courts tended to apply a 35% discount regardless of the situation.

The earliest RSE study was performed by the Securities and Exchange Commission (SEC) in 1971. Their research considered only transactions of restricted stocks between 1966 and 1969 in four different markets: NYSE, AMEX, OTC (reporting companies) and, OTC (nonreporting companies). "Because most closely held businesses (even substantially close corporations) are much smaller than typical well-known public companies, the smaller nonreporting public companies may have characteristics that are more comparable with the subject closely-held business"[8]. Considering solely the last out of the four markets, the average discount is 33%.

After the SEC study, tax courts used to suggest a discount for lack of marketability of 35% or lower, for tax purposes. Some authors decided to investigate this issue, by comparing some actual investment banks' discounts applied to restricted stocks purchased with courts' results.

Robert E. Moroney, in 1973, reported his study confronting the discounts applied by courts and by ten investment banks investing in 146 letter stocks' blocks. The author pointed out that discount rates in courts were ranging between 10% and 35% as a maximum. Investment banks reported an average discount of 35.6%, with the lowest

[7] Reinemann, Annika M. 2008. Lack of Marketability. Trusts & Estates.

[8] Pratt, Shannon P. and Niculita, Alina V. 2008. Valuing a Business: The Analysis and Appraisal of Closely Held Companies. The McGraw-Hill Companies, Inc. p.422.

reported being 3% and 90% as the highest. Moroney condemns the benchmarking activity suggested by appraisers, stating that: "If, say, a 60 per cent discount is fair and reasonable for a minority block of "letter stock" which enjoys registration rights and will probably become marketable to the public within a year or two or three, then surely a much greater discount would be appropriate for an identical block of unregistered stock enjoying no registration rights, no likelihood of becoming saleable to the public for many years to come, if then"[9].

Another author opposing the benchmarking activity is Michael J. Maher, who in 1976 affirmed that most appraisers underestimate the discount attributed to lack of marketability. The author examined transactions of restricted stocks performed by four different mutual funds, for the years 1969-1973. The average discount rate was 34.73%, but Maher points out how the actual discounts ranged between 2.79% and 75.66%.

So far, the DLOM has been computed by looking at already closed transactions of restricted stocks. Very little importance was attributed to variables affecting the discount. In 1977, Robert R. Trout published his study based on a financial model consisting of a different set of variables: Exchange listing, Number of Shares outstanding, % of Control, Variable indicating purchases less than 1% and, Overall value of the purchase.

The coefficient of variables gave important results. Trout concluded that "the discount may vary according to the level of control"[10]. The data Trout used was based on 60 stock restricted stock purchases by six mutual funds. The average discount using his financial model was 33.45%.

Plenty of other authors gave their contribution to the DLOM matter by presenting their studies. Although some slight changes in the Investment Company Act of 1940, studies reported very similar results.

Toward the end of the 1980s, things were changing rapidly. More hedge funds were being created, more investment companies were buying restricted shares and overall, an asset that once was considered completely nonmarketable was now receiving

[9] Moroney, Robert E. 1973. *Most Courts Overvalue Closely Held Stocks*. Taxes.

[10] Trout, Robert R. 1977. *Estimation of the Discount Associated with the Transfer of Restricted Securities.* Taxes.

importance. The SEC decided to revise Rule 144 in 1997. The most important change concerned the holding period, which was shortened from two years to only one year. SEC Rule 144 will be further analyzed later in paragraph 1.5. Investors now must hold the restricted stock for one year before being able to sell. This was a turning point for Restricted Stock Studies, now the average DLOM of studies decreased significantly, as we can see from Bruce Johnson's study. In 1999, he presented the very first study concerning DLOM from restricted stocks after the holding period reduction. His data consisted of 72 letter stocks' placement with an average discount rate of 20%.

One of the main criticism towards Restricted Stock studies is that "The restricted stock was purchased with an eye toward eventual marketability as rights of registration were usually included"[11]. So that a 35% average discount does not consider the entire lack of marketability of assets. To tackle this issue, a new method was created: the Pre IPO study. The accepted average discount for lack of marketability, now, increased to 45%.

John D. Emory presents the new Pre-IPO method in 1997. He performed a total of 8 studies, considering 310 transactions ranging from 1980 to 1997. The mean discount using this method is 44% and the author pointed out that "The discounts tended to be somewhat lower close to the IPO date and higher four or five months before the IPO date, although the relationship was not clear"[12]. This was probably due to the increasing probability of the company becoming listed. So, after the holding period, the nonmarketable asset would have a market ready for it to be sold.

So far, to compute the value of a privately held company, appraisers used to apply the average discount of one of the previous studies, until the Mandelbaum case, in 1995.

For tax court reasons, Judge David Laro defined a set of nine elements that must be considered when computing the DLOM. The determinants are: 1) Financial Statement Analysis; 2) Company's dividend Policy; 3) Economic Outlook for the Company; 4) Company's Management; 5) Amount of control in transferred shares; 6) Restrictions on transferability; 7) Probable expected holding period; 8) Redemption policy of the firm; 9) Costs related to making a public offer. This case has shown the importance of

[11] Emory, John D. 1997. *The Value of Marketability as Illustrated In Initial Public Offering Of Common Stock*. Business Valuation Review.

[12] Emory, John D. 1997. *The Value of Marketability As Illustrated In Initial Public Offering Of Common Stock*. Business Valuation Review.

identifying an appropriate DLOM based on the subject company rather than applying an average discount.

In this context, new methods have been developed, trying to end up with a universal framework whose inputs are firm-specific. A new concept was developed in 1993, by David B. Chaffe. His option pricing theory needed some firm-specific inputs like volatility, expiration date, and stock price. The model is based on a European option. The author found that "considering that volatility for shares of most smaller, privately held companies fit 'VOL 60%-70%-80%-90%' curves, a range of put prices of approximately 28% to 41% of the marketable price is shown at the two-year intercept"[13].

In 1995 Francis A. Longstaff presented his option pricing study based on two assumptions: the investor has perfect timing, and must hold the nonmarketable asset for the period T. The model is based on a look-back option The author shows that holding period and volatility substantially account for the DLOM, and that "discounts for lack of marketability can potentially be large even when the illiquidity period is very short"[14]. What is more, his model is recognized as an upper bound for the DLOM.

In 2002, John D. Finnerty expands even further the option pricing models. He agrees with Longstaff, stating that "[…] the stock's volatility and the length of the restriction period are key drivers of the discount in their restricted stock model"[15]. Finnerty based his model on an average-strike put option and he did not assume the perfect timing of the investor since usually, no shareholder has a special capability of outperforming the market. This model is more appropriate for standard markets, but Finnerty himself admits that "Longstaff's model may be more appropriate in the presence of asymmetric information"[16]. The author compared the discounts resulted in using his models with discount rates resulting from 205 discounted private stock placements. The results from his models were very consistent with the actual discount rates applied.

The Option Pricing models have received attention and importance in the 1990s and early 2000. At the same time, some appraisers were developing methods based on the

[13] Lance, Travis R. 2007. *The Use of Theoretical Models to Estimate the Discount for Lack of Marketability*. Insights.

[14]Longstaff, Francis A. 1995. *How Much Can Marketability Affect Security Values?*. The Journal of Finance.

[15] Finnerty, John D. 2002. *The Impact of Transfer Restrictions on Stock Prices*.

[16] Finnerty, John D. 2002. The Impact of Transfer Restrictions on Stock Prices.

Discounted Cash Flow Approach (DCF). In 1997, Christopher Z. Mercer presented his newly created method: The Quantitative Marketability Discount Model (QMDM) which computes the DLOM according to the expected growth of the company, expected future cash flows, expected holding period and require holding period return. The main issue with applying this model is the high subjectivity of some inputs and that a slight change in data leads to a critical change in the DLOM. David Tabak in 2002 gave his contribution to DCF approaches through his model based on the Capital Asset Pricing Model. This framework focuses on finding the extra return required by the investor for holding the nonmarketable asset. Tabak explained that "one would use the same cash flows as before and, for each year in which the company is expected to be illiquid, to adjust the WACC by adding the extra return for illiquidity"[17]. The Spanish Accounting and Business Association (AECA) gave its contribution to DCF models in 2005, presented their method of computing the discount rate that represents the extra return at shareholders level. "The idea […] is that an ERI who invests his or her resources in a PHC requires a higher risk than a PFI who operates in the financial market, according to the degree of diversification and liquidity"[18]. Where the Economic Risk Investor (ERI) is the company's owner, and the Purely Financial Investor (PFI) is, for instance, an investment firm.

No universal DLOM method exists yet, and appraisers are still giving their contribution to the matter. Although it is recognized the importance of applying a discount for lack of marketability solely based on the company being valued, many analysts are still using the Restricted Stock Studies as a benchmark to define such a discount.

### 1.3 Factors Affecting the DLOM

The marketability of an asset is not an either/or proposition. In fact, "there is a spectrum of investment marketability, ranging from fully marketable to fully nonmarketable"[19]. Where fully marketable is a publicly-traded asset, that can be converted into cash quickly and without incurring high transaction costs. On the other hand, a fully

[17] Tabak, David. 2002. *A CAPM-Based Approach to Calculating Illiquidity Discounts*. NERA Economic Consulting.

[18] Rojo-Ramirez, Alfonso A. 2014. *Privately Held Company Valuation and Cost of Capital*. Journal of Business Valuation & Economic Loss Analysis.

[19] Reilly, Robert F. 2016. *Measuring the Discount for Lack of Marketability for a Closely Held Taxpayer Company.* Insights.

nonmarketable asset is a privately held company's ownership that has no ready market available and that cannot be turned into cash rapidly.

Many practitioners use the term Discount for Lack of Marketability interchangeably for both controlling and noncontrolling interests. In general, the two ownership level bear differences, the word marketability refers to noncontrolling interests while liquidity is attributed to controlling interests. In the following work, only the word Discount for Lack of Marketability will be used for both levels of ownership.

Several factors affect the degree of marketability and the overall DLOM.

Risk factors faced by a controlling interest's holder are:

1- Uncertainty of the time frame needed for selling the entire block of shares. It may take several months or even years, especially if no investor is interested.
2- Possibility of incurring considerable costs to arrange the company's offering sale and all papers needed for the future transaction.
3- The uncertainty is linked to the actual sale price. The seller might not achieve the desired price due to different factors like adverse market movement.
4- Uncertainty regarding the way the payment occurs. Different ways of paying are available, other than cash. Using restricted public stocks or future contingency payments can drive the overall sale price down and not meet the seller expectations.
5- Inability to hypothecate the privately held company or the subject controlling block of shares as collateral.
6- List of Marketable and Nonmarketable assets: the first can lower the DLOM value, while nonmarketable assets report is crucial since it gives information about values, costs, and description.
7- Valuation and brokerage fees.

Another important factor affecting the DLOM is the presence of several buyers. "The existence of a reasonable number of potential buyers or even one strong potential buyer […] could dampen the discounts for lack of marketability"[20]. This happens because one of the main characteristics of non-marketability is the absence of potential buyers.

[20] Pratt, Shannon P. and Niculita, Alina V. 2008. *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*. The McGraw-Hill Companies, Inc. p 446.

The DLOM impact depends also on the subject company's risk, as a function of private stock's return and size of the business. According to Robert F. Reilly, "Larger earnings typically enable a company to (1) withstand downturns in the economy and the subject industry and (2) capitalize on growth opportunities"[21]. Besides return and size, other factors like market shares, historical earning and financial ratios can have an impact on the DLOM valuation of controlling interest ownership.

In the case of noncontrolling interests, the holder can rely on the methods seen in the previous paragraph to compute the DLOM. The Restricted Stock Studies found a linear relationship between the discount value and the subject company dividend, size, and block size. These risk factors are similar to controlling interest's, but a noncontrolling stock's owner experience additional risk elements. "These factors include contractual restrictions, such as a shareholder agreement, right of first refusal, or buy-sell agreement. Contractual restrictions can severely limit the marketability of the ownership of closely-held company securities"[22]. The shareholder agreement is a document that regulates the way shareholders can operate in the company. Such contract state shareholders' right and obligation, so it can limit share's transfer. The right of first refusal is generally a clause in contracts that gives the right to start a transaction before anyone else can. Finally, the buy-sell agreement is a binding contract that specifies what happens to share in case of the shareholders leaving the company. Typically, the buy-sell agreement forces the shareholders leaving the company to sell its shares to the other shareholders still present.

Other factors affecting the noncontrolling ownership interest are related to

1- Expected dividends: these payments are liquid, so they reduce the DLOM.
2- Bonuses to managers: monetary compensations to managers lower the DLOM.
3- Holding period: the higher the expected holding period the bigger the DLOM.
4- The subject company risk: which is a function of stock price volatility and business size.

[21] Reilly, Robert F. 2016. *Measuring the Discount for Lack of Marketability for a Closely Held Taxpayer Company.* Insights.

[22] Reilly, Robert F. 2016. *Discount for Lack of Marketability Considerations Related to Closely Held Company Securities.* Practical Tax Strategies.

"Most agree that any marketability discount for a controlling interest should be less than the discount for a minority interest in the same entity"[23]. This is true because the owner of a private business, although facing risks, can also pay brokerage and investment firms' fees to make the company more marketable and liquid. On the contrary, a minority interest owner cannot change shareholders' agreement or eventual clauses in contracts reducing the marketability of its block of shares. Therefore, all things equal, a controlling ownership DLOM will be smaller than non-controlling ownership.

### 1.4 DLOM and Private Company Discount

It is generally accepted that a discount for lack of marketability should be applied when valuing minority interest ownership of a private company. On the other hand, applying a DLOM for controlling interests is an issue that divides practitioners. Some believe a discount should not be applied when valuing a private company, while others assume a discount lower than the discount for minority interest, is appropriate.

The theory seems rather general on the topic. The Internal Revenue Service (IRS) in 2009 published a job aid for professionals regarding the discount for lack of marketability, in an attempt of giving some guidelines on best methods and practices. It is stated that "[…] the controlling interest owner may not be able to sell the interest quickly enough or with certainty as to the ultimate sales price. Therefore, it follows that the controlling interest may not be fully marketable"[24]. They do not state clearly whether the discount in case of 100% ownership must be applied or not. This uncertainty regarding the discount is reflected in tax court decisions as well, resulting in a "[…] schizophrenic trail of decisions over the past 20 years regarding DLOMs"[25]. Some tax courts' decisions will be further explained in paragraph 1.4.1.

Authors such as Christopher Z. Mercer, argue that "If enterprise value is determined based on expected cash flows, the expected growth of those cash flows, and the riskiness of those cash flows, then what additional factors would support a discount

[23] Engineering/Valuation Program DLOM Team. 2009. *Discount for Lack of Marketability: Job Aid for IRS Valuation Professionals*. IRS.

[24] Engineering/Valuation Program DLOM Team. 2009. *Discount for Lack of Marketability: Job Aid for IRS Valuation Professionals*. IRS.

[25] Quackenbush, William C. 2016. *DLOMs in N.Y. Statutory Fair Value Cases – A Follow-Up to Matthews.* Business Valuation Update.

from this value?"[26]. There is neither a clear methodology nor evidence accounting for the controlling interest of DLOM. The existing frameworks give good proxy of minority interest discounts and applying those results to the overall value of an enterprise is completely wrong.

Gilbert E. Matthews agrees with Mercer. Good DCF calculations account for most of the DLOM factors. Applying a DLOM to a private company valuation creates the risk of double-counting risk factors, leading to a wrong valuation. He adds that "To the extent that there is a private-company discount, it derives not from the fact that its shares are not traded in a public market but, in fact, from the characteristics of the company"[27]. A private company discount, after a DCF valuation, should be applied only when there are restrictions that might have an impact on the enterprise itself. So, factors that are not common, and that are not considered in the valuation.

Other authors believe that the DLOM for controlling interest exists and it should be applied to business valuations. According to John J. Stockdale, the base theory can answer the dilemma. The definition of DLOM is "an amount or percentage deducted from the value of an ownership interest to reflect the relative absence of marketability"[28]. According to the Encyclopedia of Banking and Finance, marketability is "The relative ease and promptness with which a security or commodity may be sold when desired, at a representative current price, without material concession in price […]"[29]. Therefore, a DLOM must be applied when the subject asset cannot be immediately sold. Considering Bizcomps' and Pratts' databases the average time needed for a private company to close merger and acquisition transactions is around 197 days. Both databases state that the price paid at the closing date is often lower than the price suggested by valuations. Therefore, there is a lot of risk and uncertainty regarding both time and price for selling a business. Stockdale acknowledges that risk and time are taken into account when valuing a business, but he argues that "The risk and timing

[26] Mercer, Christopher Z. and Harms, Travis W. 2007. *Business Valuation: An Integrated Theory.* 2nd ed. John Wiley & Sons, Inc. Hoboken, New Jersey. p 95.

[27] Matthews, Gilbert. 2016. *DLOMs in Fair Value Cases: Lack of Marketability Does Not Cause Private Company Discounts.* Business Valuation Australia.

[28] International Glossary of Business Valuation Terms, https://www.nacva.com/content.asp?contentid=166

[29] Woelfel, Charles J. 1994. *Encyclopedia of Banking & Finance.* p.729. Probus Publishing Company. Chicago, Illinois.

factors that are included are […] of future cash flows, not risk and timing of the receipt of determined value"[30]. The risk and timing considered in the valuation are related to the probability of expected future cash flows to be realized. Risk and timing when computing the DLOM are related to the probability that the business value is realized once the transaction is closed. They refer to two different areas, and no double-counting risk is present.

Shannon P. Pratt claims that DLOM for controlling interests exists since "selling a controlling business ownership interest is a lengthy, expensive, and uncertain undertaking. This assertion is equally true even if the subject investment is a 100 percent (i.e., absolute controlling) ownership interest in a closely held business enterprise"[31]. It may take several months or even years to sell a business. The price itself is a risk factor. First, because the final price is likely to be different from initial valuations; second, the transaction might not be entirely made through cash, but with other currencies. Generally, these factors are not considered in an initial business valuation. So, a DLOM should be applied.

The major criticism moved toward the application of a controlling interest discount is that minority interest has lots of empirical models while controlling interest has none. Ronald D. DiMattia identifies the Merger Arbitrage framework, as a model that can explain the existence of DLOM at controlling interest level. The base concept of the Merger Arbitrage is that investors put their money in companies involved in a probable acquisition or merger. The investor will go long on stocks of the company being acquired and will go short on the firm performing the acquisition. "[…] the target's stock price reacts and tends to rise near to, but generally lower than, the announced acquisition price"[32]. The merger arbitrage method has many similarities with controlling interest's acquisitions: they both are subject to failure; both represent a controlling interest transaction, and both require a lengthy period to complete. According to DiMattia, merger arbitrage used as an empirical model gives a controlling interest DLOM of

[30] Stockdale, John J. 2011. *Lack-of-Marketability Discounts for Controlling Interests.* Valuation Strategies

[31] Pratt, Shannon P. and Niculita, Alina V. 2008. *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*. 5th ed. The McGraw-Hill Companies, Inc.

[32] Matthews, Gilbert. 2016. *DLOMs in Fair Value Cases: Lack of Marketability Does Not Cause Private Company Discounts.* Business Valuation Australia.

between 4% to 9%. This result is consistent with the theory since it is lower than the minority interest DLOM.

DLOMs for controlling interests should be applied given the theoretical findings and the merger arbitrage that can be considered a good empirical method.

### *1.4.1 Evidence in Tax Court cases*

The application of a DLOM for controlling interest is a very controversial topic among practitioners. This chaos is reflected in tax court cases, which is very discontinuous in applying this type of discount rate. It is important to have a clear idea of past judicial decisions to better understand both the factors considered and the circumstances when a DLOM for controlling interest was applied. This analysis should not be used as a means of benchmarking the percentage DLOMs applied.

In Estate of Jephson (87 T.C. 297, 1986), the deceased Lucretia Davis Jephson, owned at 100% level two investment companies: R.B Davis Investment Co. and Davis Jephson Finance Co. Valuation of those companies' stocks was needed. Assets of the two companies were mainly cash and marketable securities held on portfolios. Petitioner performed a valuation assessing a discount for lack of marketability of 28% and 31.3% respectively. Respondent argues that no discount should be applied since any willing buyer would get access to the whole corporation assets, cash, and securities. Petitioner argues that securities and cash held in company form are not readily marketable. The court counters stating that "This argument ignores the fact that complete ownership of each corporation enables petitioner to obtain, at any time, direct ownership of the corporate assets either through a partial or complete liquidation or through a dividend in kind"[33]. Therefore, considering the case of a private company's assets mainly made of cash, it appears that no lack of marketability discount should be applied when transferring 100% controlling shares.

In Estate of Bennett (TCM 1993-34), Mr. Bennett was founder and owner at a 100% level of Fairlawn Plaza Development, Inc., a real estate development company. After the death of Mr. Bennett in 1984, all company's ownership, according to his will, moved to his wife. Two experts valued the company, and considering they used different methods,

[33] Leagle.com: https://www.leagle.com/decision/198638487rtc2971367

results were dissimilar. Both experts failed on proving to the court their method was appropriate and, in this case, the tax court itself must perform a valuation. The tax court ended up with a value of $4.421.631 valuing assets and liabilities and affirmed that a discount for lack of marketability of 15% due to nonliquid assets. The respondent argued that "[…]the 100-percent owner (or a willing buyer of the 100-percent owner's interest) has unconditional control over management and the underlying assets and, thus, can force liquidation to obtain direct ownership over those assets at any time"[34]. He also relied upon the above-mentioned Estate of Jephson case, in which a DLOM was not applied. The tax court claimed that no discount was applied to those corporations since they were investment companies, whose assets are cash.

Another case showing the application of controlling interest DLOM is the Estate of Borgatello (80 T.C.M. 260, 2000). Mr. Borgatello died in 1993, having the participation of 82.76% on Valley Improvement Co., Inc. (VIC), a real estate company. Once again, a DLOM has been applied to the overall value using a build-up method, considering several factors that affect VIC's assets. The appraiser and tax court considered elements like restriction on stock transfer, real estate market, and transaction costs. The final decision was that a willing buyer would purchase that majority interest shares at a discount of 33%.

The application of a DLOM for controlling interest is still a very controversial topic. To tackle this issue, tax courts affirm that every case is different. So there is no one size fits all solution, rather every appraiser has to perform very detailed due diligence on subject companies and present good reasoning to support their ideas on why to apply or why not to apply a DLOM in that case.

### 1.5 SEC Regulations

The Securities Exchange Commission is an authority created in 1934 with a view on regulating trading of shares after the 1929 financial crisis. During the market crash, each state had its laws and practices regarding financial markets and the US government only presented general guidelines. Due to law differences among states, criminals easily took advantage of it. "State laws were inadequate. The common law actions were

[34] Leagle.com: https://www.leagle.com/decision/1993188165ahtcm181611848

impotent"[35]. The SEC had the duty of creating laws having legal effects in all the US territory. The Federal Securities Act of 1933 was written and published to close the legal black holes affecting the trading system. That act focused on common shares of listed companies imposing restrictions in different areas of trading. Under Section 5, for example, the securities act of 1933 prohibits interstate commerce of securities by any means. Section 7 targets transparency, stating that the company offering shares, must file documents disclosing information on the securities, giving investors enough information to decide whether that would be a good investment or not. Section 8 required registration for common shares.

The securities act of 1933 is a first attempt of harmonizing the US trading system and to discourage fraudulent practices. Even in later reviews of the act, "Congress […] specifically exempted from registration certain types of securities and transaction for which it was thought there was no practical need for registration"[36]. One of the exceptions refers to restricted shares, since few people purchase that type of stocks, and such investors generally have the economic stability and knowledge to protect themselves.

SEC Rule 144 published in 1972 targets restricted shares and their resell to the public. Before 1972, restricted stock buyers had to prove their intent was not distribution, rather they purchased restricted shares with investment intent. This required a case by case analysis for every restricted shares investor, leading to a lengthy and intricate process. The 1972 revision introduced an objective set of requirements to allow the reselling of restricted shares. There are 3 conditions, and they all have to be met:

1) The company offering restricted stocks must periodically file reports in compliance with SEC Rule 144 (1934), which specify both the type of reports and what companies must perform the filing; if the subject company does not fall into one of the Sec Rule's companies, then it has to provide to the public its information.
2) The buyer must be the beneficial owner of the subject restricted shares for at least 2 years.

[35] Doerfer, John C. 1934. *The Federal Securities Act of 1933.* Marquette Law Review.
[36] Yale Law Journal. 1972. *Resale of Restricted Securities under SEC Rule 144*. Yale Law Journal.

3) The resell within six months, cannot exceed one percent of the company's total outstanding shares.

The goal of SEC Rule 144 before and after 1972 is the same: the protection of the public investor. At the beginning of restricted shares regulation, such stocks were considered highly risky due to their uncertainty and the SEC tried blocking the resell to the public. During the years restricted shares proved to be less risky than the SEC originally thought. The 1972 review shows that with enough information, a public person can assess the value of restricted stock and decide whether to perform the purchase or not.

Regulating the resale of restricted stocks made them safer. An increasing number of corporations as well as an investor were attracted by the potential of latter stocks. Although they were still considered nonmarketable, the large number of potential buyers lead to a slight increase in the marketability of those stocks. These new trends resulted in a new substantial change in Rule 144.

In 1997 the SEC released a new version of Rule 144 incorporating adjustments to better suit the restricted stocks' market changes that occurred from 1972. The new rule "requires a person to hold restricted stock for a continuous period of one year"[37]. What is more, the holder can now sell a total of one percent of the firm's total outstanding shares in three months, "[…] or the weekly average trading volume during the four calendar weeks preceding the filing of Form 144"[38].

These changes targeted the marketability of restricted stocks. By shortening both the required holding time and the time to sell a large number of stocks, the discount or lack of marketability decreased. There are pieces of evidence of these decrease by confronting RSE studies performed using transactions in the 1970s and transactions that occurred in the late 1990s. The changes in the rule decreased the DLOM from an average of 35% to 20%.

One last considerable change in the rule was published in 2007, having effects from February 2008. "The amendments shorten the holding periods before affiliates and nonaffiliates may sell restricted securities and otherwise loosen restrictions on the

[37] Altstadter, Eric. 1997. *What Exactly is Rule 144 ?.* The CPA Journal.
[38] Altstadter, Eric. 1997. *What Exactly is Rule 144 ?.* The CPA Journal.

public resale of equity and debt securities acquired in private placements"[39]. Once again, these changes affected the liquidity of restricted stocks, increasing it. The result is a decrease in the DLOM. The holding period for non-affiliates, purchasing restricted shares from a reporting company has decreased from a one-year holding period to 6 months. On the other hand, buying restricted sticks from a non-reporting company requires a holding period of one year. Affiliates no longer have to fulfill certain requirements when selling their restricted shares after the required holding period. They are now required to be in line with the current public information obligations.

These changes have increased liquidity of restricted stocks and private companies' shares by making the resell affordable by the vast majority of public investors.

[39] Jacob, Valerie F., Bursky, Daniel J., Gelfond, Stuart H., Levitt, Michael A., Tropp, Paul D. and Tsaganos, Vasiliki B. 2008. *SEC shortens Rule 144 hodling periods and loosens restrictions on resales of privately placed securities.* Journal of Investment Compliance.

# Chapter 2

# Empirical Models to Estimate the DLOM

## 2.1 Empirical Models

The lack of marketability and liquidity is considered a cost that lowers the value of an asset. Such costs should be observable if present. Empirical models study the market considering past transactions of non-marketable assets, comparing them with a marketable counterpart. The average rate represents the discount for lack of marketability, which is often applied during unlisted companies' and investments' valuation. To end up with a more accurate discount, appraisers consider restricted stocks' transactions over several years.

Many authors argue that, despite showing the existence of DLOM, "[…] the study results are unreliable for calculating the DLOM applicable to a particular valuation engagement"[40]. The inaccuracy of empirical studies when applying their results for valuation purposes is due to the small sample size and to the fact that most data do not derive from private companies. Therefore, courts lately prefer a rather firm-specific valuation instead of applying the average of a restricted stock study[41].

The major empirical models are:

- The Restricted Stock Studies
- The Pre-IPO Approach
- The Valuation Multiplier Approach

## 2.2 Restricted Stock Studies (RSE)

Restricted stock studies represent the first attempt at computing the discount for lack of marketability. The Securities and Exchange Commission (SEC) performed the first-ever RS in 1971. Since then, many appraisers performed their analysis publishing similar results. For these reasons, even today RSE results are applied in small and medium enterprise's valuation.

[40] Vianello, Marc. 2019. *Empirical Research Regarding Discounts For Lack of Marketability: Volume 1.1*. VFC DLOM Calculator.

[41] National Association of Certified Valuators and Analysts (NAVCA). 2016. *Discount for Lack of Marketability: Job Aid for IRS Valuation Professionals.* NAVCA. p.33

Restricted stock studies try computing the DLOM by comparing public shares of a listed company with its restricted stocks. The restrictions in transferability derive from SEC rule 144. The difference in price percentage is considered the DLOM.

Restricted stock studies do a fine job of showing that discounts related to illiquidity and non-marketability do exist. They also display that the DLOM must be considered when performing the valuation of a non-marketable asset under the fair market value assumption. Although the importance and attention RSEs have received over time, an appraiser should be very cautious when applying them for companies' valuation purposes. These studies present some major pitfalls that might lead to an erroneous valuation.

One pitfall refers to sample sizes, which tend to be small within studies, and data are generally distributed in different years, resulting in misleading conclusions.

Usually, "[…] most firms do not make restricted stock issues and the firms that do make these issues tend to be smaller, riskier and less healthy than the typical firm"[42]. This evidence shows that most studies might suffer from serious selection bias. Considering that high-risk companies drive up the average DLOM, applying such discounts to healthy firms can cause an unjustified and unfair loss of value.

Another dilemma is how most restricted stock studies do not reveal the standard deviation when reporting the average DLOM results. Standard deviation is a crucial statistical index when presenting the reliability of an average value since it measures dispersion. The larger the standard deviation is, the more the values tend to differ from the mean. On the contrary, a low standard deviation suggests that values in research are generally very close to the average. Through the Monte Carlo Simulation tool in Excel, it is possible to compute a reliable range of values that discount rates can assume having a given mean and standard deviation. Moroney, Maher, and Silber reported both means and standard deviations in their restricted stock studies. Running the simulations on one hundred fifty thousand values, results are the following:

- Moroney reported a mean of 35,6% and a relatively low standard deviation of 18%. The simulation reported values ranging from around negative 40% to positive 120%.

[42] Damodaran, Aswath. 2005. *Marketability and Value: Measuring the Illiquidity Discount*. Stern School of Business

- Maher reported a mean of 35,4% and a standard deviation of 18%. The simulation's results ranged from around negative 45% to positive 115%.
- Finally, Silber reported a mean of 33,8% and a standard deviation of 24%. The simulation reported a probable range of DLOM values ranging from around negative 70% to positive 135%.

*Exhibit 1*: Possible Range of Values of Studies

| | Value | | % of Values | |
|---|---|---|---|---|
| | Min | Max | < 0% | > 100% |
| Moroney | -42,98% | 114,25% | 2,368% | 0,017% |
| Maher | -45,33% | 116,79% | 2,42% | 0,01% |
| Silber | -66,88% | 142,16% | 7,95% | 0,30% |

Exhibit 1 displays the ranges assumed by values based on each study's mean and standard deviation. The table shows that the probability of restricted security to receive a negative discount rate is relatively large. In fact, according to Silber, it is not unlikely for a restricted share to receive a premium, rather than a discount. It may be "because investors are promised representation on the company's board of directors"[43].

Generally, discount rates for lack of marketability should always range between 0% and 100%, since it is reasonable to think a discount rate cannot larger than the asset's value itself. "Although the restricted stock studies demonstrate the appropriateness of a DLOM in determining fair market value, they reflect deficiencies that undermine their reliability for practical application when valuing privately-held businesses"[44].

Restricted stock studies can give a general idea of the DLOM over periods, but their result should be used very cautiously by appraisers.

### *2.2.1 Securities and Exchange Commission (SEC) Study*

The Securities and Exchange Commission, in 1971, published its DLOM analysis in the Institutional Investor Study Report during the 92nd US Congress. This was the first restricted stock study. The SEC collected data from the private placement of shares from 1966 to 1969 spread into four different markets: NYSE, AMEX, Over-The-Counter (OTC),

[43] Silber, William L. 1991. *Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices*. Financial Analysts Journal.

[44] Vianello, Marc. 2014. *Using Restricted Stock and Pre-IPO Studies for Quantifying DLOM: Two Ways of Saying "I Don't Know"?*. Valuation Strategies.

and OTC non-reporting companies. The final data pool consisted of 398 restricted stock purchases.

The study shows that companies whose stocks are traded in either the NYSE or the AMEX tend to issue restricted securities at lower discounts than their OTC's counterparts. Overall, the average discount considering the four markets is 25,8%. It is important to note that the average discount in this analysis is likely to be applied in small and medium enterprises' valuation. Therefore, computing a value in percentage based on listed companies which are usually healthier than SMEs, can result in misleading values.

The SEC considered companies issuing restricted stocks in the OTC and that are non-reporting. According to experts, these types of firms resemble SMEs. "For non-reporting OTC companies (which are more comparative with smaller businesses), the average price discount was 32,6 percent"[45].

### *2.2.2 Gelman Study*

Milton Gelman presented his study in 1972. His analysis is based on the purchase price paid by four investment companies, experts in restricted stock investments. The data consisted of 89 total purchases performed between 1968 and 1970. According to Gelman, the mean discount as well as the median is 33%, although he reported that "almost 60% of the restricted stock sales indicated discounts of 30% or higher"[46].

### *2.2.3 Trout Study*

Robert R. Trout disclosed in 1977 his restricted stock study, performed using data from registered stock purchases by 6 mutual funds. "The author sets up an economic model that can be used to estimate the discount that should be accorded a transfer of restricted securities"[47]. The author removed transactions from the sample for different reasons. For instance, if not enough information regarding the deal was disclosed and if

[45] Pratt, Shannon P. and Niculita, Alina V. 2008. *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*. 5th ed. The McGraw-Hill Companies, Inc.

[46] Reilly, Robert F. and Rotkowski, Aaron. 2007. *The Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies*. The Tax Lawyer

[47] Trout, Robert R. 1977. *Estimation of the Discount Associated with the Transfer of Restricted Securities.* Taxes.

the purchases included warrants, the data were excluded from the analysis. The final sample consisted of a total of 60 purchases executed between 1968 and 1972.

Trout's model is based on relations between different financial variables. Those considered in his analysis are:

- Exchange Listing: this first variable has a value of 1 if shares are listed in important markets such as the New York Stock Exchange. The value equals 0 if otherwise. The variable should be negatively correlated to the discount rate value since being listed in significant markets makes security more marketable.
- The number of Outstanding Shares: the larger the total outstanding shares, the more marketable a security is. This variable is negatively correlated to the discount value.
- The number of shares bought as a percentage of the overall outstanding shares: this has two different types of outcomes. The first one is that the higher this percentage is, the higher the value of the subject block of shares. This happens because owning a high percentage of shares of a company gives you rights of control, which is often viewed as a premium rather than a discount. Therefore, the control effect balances off the discount for lack of marketability. The percentage of shares owned over the total outstanding ones has a negative relationship with the DLOM. On the other hand, owning a high number of securities, especially if restricted stocks, makes it more difficult to sell them. In this case, the percentage increases the DLOM. To tackle this issue, Trout decided to include two more variables in the model:
- The first variable receiving value of 1 if % of the purchase is lower than 1% of the total outstanding shares.
- The second variable added represents the value that the subject shares would have if they were not restricted.

The integration of the above-mentioned variable in the DLOM computation can be expressed through this multiple regression analysis:

$$D_i = B_0 + B_1X_{i1} + B_2X_{i2} + \ldots + B_jX_{ij} + E_i \quad [48] \qquad (2.1)$$

[48] Trout, Robert R. 1977. *Estimation of the Discount Associated with the Transfer of Restricted Securities.* Taxes.

Where:

- $D_i$ = Discount for the i-transaction.
- $B_0$ = Intercept term.
- $B_j$ = Financial variables' coefficient.
- $X_{ij}$ = Financial characteristics of the i-transaction. It refers to the financial variables previously mentioned.
- $E_i$ = random error term.

Regression coefficients are the result of the multiple regression, and they indicate "the percentage point change in the discount resulting from a one-unit change in the variable specified, holding each of the other variables in the equation constant"[49]. Using the 60 purchases as data, the exchange listing variable assumed a value of -8.39. The outstanding shares' coefficient resulted in a value of -4.08, meaning that the discount will be 4.08% lower for each million outstanding common stock the subject firm has. The control variable has a coefficient of -0.87, implying that the final DLOM should be 0.87% lower for every additional 1% of company control derived by the restricted stock's purchase. The financial variable concerning the acquisition of less than 1% of the total outstanding shares of the subject company, has a coefficient of -12.11. Therefore, small purchases of restricted stocks lower the DLOM by around 12%. Finally, the coefficient referring to the value of the total purchase is 4.75%. Hence, the final discount increase by 4.75% for each additional million dollars.

In this study, Trout found and proved that restricted stocks issued by companies listed in national exchange markets have lower average discounts than companies having shares listed in over-the-counter (OTC) markets.

The regression model reported an average discount attributed to the lack of marketability of 33.45%.

### *2.2.4 Moroney Study*

In 1972, Robert E. Moroney published his restricted stock study. The author argues that tax courts are overvaluing private companies' stocks. Moroney performed an analysis of

[49] Trout, Robert R. 1977. Estimation of the Discount Associated with the Transfer of Restricted Securities. Taxes.

DLOMs applied by tax courts, comparing the results with price discounts paid by ten investment companies purchasing restricted stocks.

Generally, courts valued the subject company and applied a discount either related to the non-marketability or related to the cost of creating a market. In both cases, the DLOM was entirely based on courts 'opinions, and they did not have to disclose their assumptions. This situation led to confusion, resulting in tax courts applying DLOMs of 10% as well as 66%.

Judges cannot be blamed though; it is rather legislation's fault. "Until recently the answer was wholly subjective since no objective guidelines were available to the appraisers and the courts"[50]. As stated in the previous chapter, in 1969 the SEC updated the Investment Company Act, requiring all investment companies to disclose criteria considered in the valuation of restricted shares as well as their financial statements. By looking at their prospectus, it is possible to compare the price such companies paid for restricted stock and compare it with the common stocks' price of the same firm. The price reduction is the DLOM and shows the discount at which investment companies purchased subject restricted stocks.

Moroney considered 146 transactions from 10 US investment companies.

The differences among the tax courts' DLOM and cash DLOM derived from the purchases of restricted stocks by investment companies are consistent. The mean discount applied by investment companies is 35,6% while the mean DLOM applied in tax court cases, before 1972, is 23%. The tax court's study is strongly affected by a lack of data. In most cases, courts did not disclose the exact DLOM, or in other cases, a discount for lack of marketability was not even mentioned. Only 14 discounts were available, and they tend to be low, resulting in a lower mean discount if compared to what investment companies applied for the purchase of restricted stocks.

[50] Moroney, Robert E. 1973. Most Courts Overvalue Closely Held Stocks. Taxes.

*Exhibit 2*

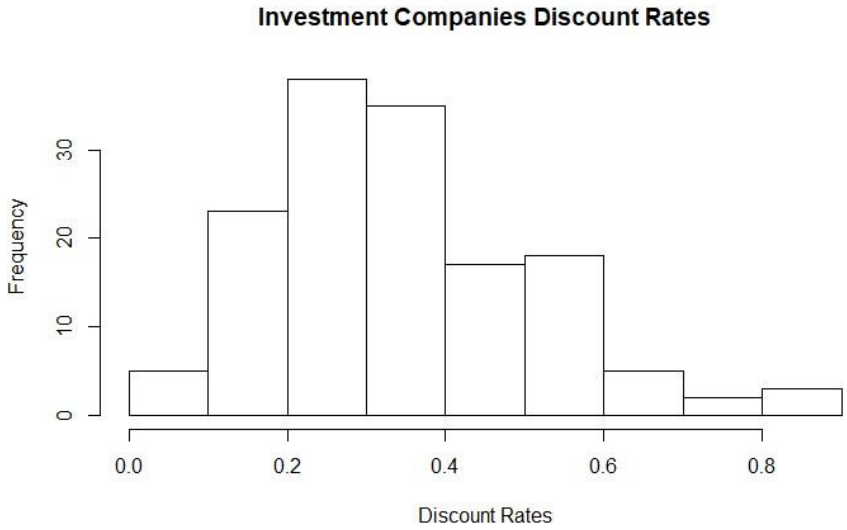
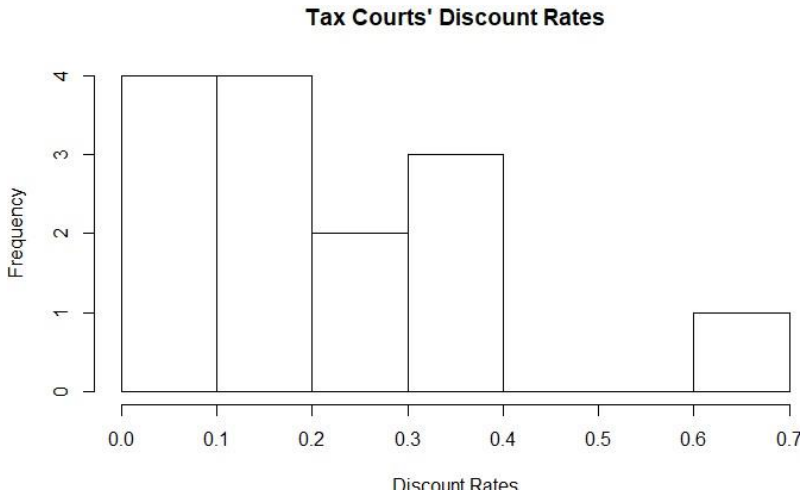



*Exhibit 3*




Histograms related to the density of discount rates distribution for both investment companies and tax courts give a clear idea of the difference. Most of the DLOMs applied by investment companies range between 20% and 40%. On the contrary, most Tax courts' discounts are lower than 20%, with one only case in which a discount rate reached a value of 70%.

The author suggests that now that empirical data are available, tax courts should apply larger discount rates since they are backed by strong empirical evidence.

### *2.2.5 Maher Study*

In his study published in 1976, Michael J. Maher argues that appraisers are underestimating the impact marketability has on fair market value. "Maher's analytical

method was similar to Moroney's in that it compared prices paid for restricted stocks with the market prices of their unrestricted counterparts"[51]. The author considered 34 restricted stock purchases by six mutual funds between 1969 and 1973. He did not include data from 1974-1975 since neither one of the funds performed any restricted share investment. What is more, in 1973 only one purchase was performed. These unusual data are attributed to the crisis that hit worldwide's stock markets.

The average discount was 35,43%. Data ranged from 2,79% to 75.66% leading to possible average bias due to such cases. To end up with a more precise average DLOM, Maher "eliminated the top 10% and bottom 10% of purchases […] to remove an especially high and low-risk situation"[52]. The result was a mean discount of 34,73%, very similar to the previous result without elimination.

Once again, results show that appraisers are underestimating the discount for lack of marketability. Maher agrees with empirical results since investing in restricted stocks prevents an investor from the possibility of rapidly take advantage of market changes. Excluding possible profits.

### *2.2.5 Standard Research Consultants Study*

Standard Research Consultants (SRC), a New York company leader in both business and security valuation, contributed to the restricted stock studies in 1983. SRC experts investigated whether the above-mentioned studies' results were still relevant.

They collected data for 28 restricted stock purchases that occurred from October 1978 to June 1982. The median price discount is 45%, although deductions ranged between 7% to 91%. What is more, no mean of the study was disclosed. The median can be considered a good proxy for the mean, although it is not as precise. The DLOM might have slightly increased from the first studies, probably due to constraints imposed on restricted stocks resell.

[51] Pratt, Shannon P. and Niculita, Alina V. 2008. *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*. 5th ed. The McGraw-Hill Companies, Inc.

[52] Maher, Michael J. 1976. *Discounts for Lack of Marketability for Closely Held Business Interests.* Taxes.

### *2.2.6 Willamette Management Associates Study*

Another consultant company specialized in business valuation performed a restricted stock study. In this case, Willamette Management Associates collected 33 privately placed stocks' transactions between 1981 and May 1984. Part of the study uses the same restricted stocks as the SRC study. Once again, only the median discount was revealed, which is 31,2%.

"The slightly lower average percentage price discounts for private placements during this time may be attributable to the somewhat depressed pricing in the public stock market"[53].

### *2.2.7 Silber Study*

In 1991, William L. Silber presents a statistical model proving the importance of liquidity and marketability in restricted stocks' prices. The author debates that so far, most-restricted stock studies were focusing on bid-ask spreads and different market structures. Hence, he wanted to prove that the price discount affecting restricted shares was also dependant on the firm's characteristics.

The original data sample consisted of a total of 310 private shares purchases between 1981 and 1988. Not all of them could be eligible for the study due to special characteristics some of the purchases presented. "After eliminating issues that had warrants or other special provisions, we identified the precise date of private placements by 69 companies through a computer search"[54].

The statistical model is based on the supply and demand equation. The Demand function for restricted shares is:

$$Q_D = Q_D(D, CR, M, S) \tag{2.2}$$

And supply-demand is:

$$Q_S = Q_S(D, CF) \tag{2.3}$$

[53] Pratt, Shannon P. and Niculita, Alina V. 2008. *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*. 5th ed. The McGraw-Hill Companies, Inc

[54]Silber, William L. 1991. *Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices*. Financial Analysts Journal.

Where:

- D = Discount rate, and it corresponds to 1-p*/p. And p* is restricted stock's price; p is common stock's price.
- CR = Creditworthiness.
- M = marketability of restricted stocks
- S = favorable relationships towards the investor.
- CF = Cash flow of the subject firm.

Solving both equation 2.2 and equation 2.3 for the variable D representing the discount rate, the new equation is:

$$D = D(CR, M, CF, S) \quad (2.4)$$

In equation 2.4, all the variables are negatively related to the value of Discount. An increase in CR lowers the discount rate. Moreover, a high level of marketability, lowers D as well as high cash flows makes a company healthier, resulting in lower required discount. Finally, favorable concessions toward the purchaser of restricted stocks, make the investor willing to accept a lower discount.

The author then performs a logarithmic transformation of equation 2.4, given that "if we take the logarithm of the variable, run the model, make predictions on the log scale, and then transform back (by exponentiating), the resulting predictions are necessarily positive because for any real an exp(a)>0"[55]. The new equation is the following:

$$p*/p = f(CR, M, CF, S) \quad (2.5)$$

In this case, variables are positively related to the price of restricted stocks. For instance, the higher the creditworthiness and the marketability, the higher the price of restricted stocks.

The final step in Silber's regression model is computing a good proxy for creditworthiness, marketability, cash flow, and favorable conditions. To make equation 2.5 more precise the author checked the subject companies' financial statements, balance sheets, and market studies. He also added two dummy variables, assuming a

[55] Gelman, Andrew and Hill, Jennifer. 2007. *Data Analysis Using Regression and Multilevel/Hierarchical Models.* Cambridge University Press. New York. p59.

value of 1 if an event is true, and 0 otherwise. Equation 2.6 is equation 2.5's enhanced version:

$$LN(RPRS) = 4{,}33 + 0{,}036LN(REV) - 0{,}142LN(RBRT) + 0{,}174DERN + 0{,}332\ DCUST \quad (2.6)$$

"The dependent variable, LN (RPRS), is the natural logarithm of the relative price of restricted stock expressed in percentage terms [(p*/p) *100]"[56]. Other variables, defined as explanatory, are LN(REV), which is the natural logarithm of the company's revenues. Then there is LN(RBRT), which represents the natural logarithm of the percentage of restricted stocks over the total common outstanding shares. Then the last two elements are dummy variables. The first one is DERN, which assumes the value of 1 if the company's earnings are positive, 0 if otherwise. This is correct since positive earnings increase the overall value of restricted stocks. The last dummy variable is DCUST, which assumes a value of one if there are favorable relations between the investor and the issuer. Once again, in case of promising future preferential treatment for holding the subject restricted stocks, the value increases.

Equation 2.6 shows that illiquidity and marketability discounts derive not only from market characteristics but also from the company issuing restricted stocks.

The average price discount was 33,75%. In line with previous studies.

### *2.2.8 Hertzel & Smith Study*

In 1993, Michael Hertzel and Richard L. Smith published their study based on the restricted stocks' approach. They believed that many factors in addition to marketability have an impact on the discount applied in private placements. Therefore, the authors "develop and test the hypothesis that […] private placement discounts and stock price reactions also reflect the resolution of asymmetric information about the firms"[57]. What is more, the discounts reflect costs the investor will incur during the monitoring activity. Investors investing in restricted stocks will ask for higher returns the more costly and time-consuming, assessing the value of the firm is. The information hypothesis is crucial

[56] Silber, William L. 1991. *Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices*. Financial Analysts Journal.

[57] Hertzel, Michael and Smith, Richard L. 1993. *Market Discounts and Shareholders Gains for Placing Equity Privately*. The Journal of Finance.

in this study. Therefore, the authors developed an empirical model to prove different concepts. For instance, they proved that retaining information is expensive, but it can be explained as an economy of scale. Meaning that large placements of shares result in smaller discounts. Another important concept is that if a control premium is present in the restricted stock placement, then it can partially counteract the discount rate.

Hertzel and Smith conducted their study by considering private placements that occurred from 1980 to 1987. They retrieved the data in the Dow Jones News Retrieval Service, Standard and Poor's News Reports, and Investment Dealers' Digest. The authors only considered private placements not involving any free trading stocks nor grating warrants to the investor. The final sample consisted of 106 privately placed stocks. Once more, 75% of the total placement was issued by OTC firms.

The average price discount in the study is 13,5%. Although discounts varied significantly among the data sample.

### *2.2.9 Johnson Planning Study*

In 1999, Bruce Johnson presented his study concerning private placements from 1991 to 1995. This is the first study performed after the change in the holding period of Rule 144, so the lower average discount might be a result of that.

His data consisted of 72 private placements of restricted stocks. The reported average discount amounted to 20%. The lower mean value can be attributed to changes applied by Rule 144 rework. For instance, the holding period, which is considered one of the biggest DLOM determinants, was changed from one year to two years[58].

Johnson investigates how factors such as the value of the transaction, sales volume, and net income affected the discount. Generally, the lower the transaction, the higher the discount applied. For high positive net income and high total sales, the average discounts applied are low.

### *2.2.10 Management Planning Study*

Management Planning, Inc. published its DLOM analysis in 2000 in the Handbook of Advanced Business Valuation. Their study considered private placements that took place between 1980 and 1996. They found a total of 231 public companies issuing their stocks

[58] Altstadter, Eric. 1997. *What Exactly is Rule 144 ?.* The CPA Journal

privately. To make the study as significant as possible, the authors developed three criteria the placements must fulfill to be included in the final sample. The tests are: "If the company issuing the restricted shares lost money in the year before the transaction, it was excluded. All start-up companies were excluded. […] Some of the transactions involved restricted shares […] these transactions were excluded."[59] The final sample considered 53 transactions.

The average discount rate attributable to lack of marketability is 27%. The median is around 25%.

The authors performed a further analysis. They tried identifying what factors had the biggest impact on the discount observed. They found a total of 24 possible impactful factors. The determinants that influence the discount the most, according to the analysis, are 7. They are the size of revenues, size earnings, market/price per share ratio, price stability, number of quarters of trading volume, the impact of Rule 144's holding period, the overall value of the block issued.

#### *2.2.11 FMV Study*

FMV Opinions, Inc. presented its study concerning the value of the DLOM. They examined 243 transactions occurring from 1980 to 1997.

They presented possible interactions between the discount observed and the firm's characteristics such as size block and the market it is traded in.

The method they developed is a two-step process. "The first step […] is to analyze the financial characteristics of the subject firm against the parameters in the FMV Study […]. Then, the second and final step of the analysis determines the addition to the discount due to the differential between the liquidity of privately held shares and the restricted shares of public companies"[60].

The average price discount observed using their model is 22,1% with a median of 20,1% and a standard deviation of 16%.

---

[59] Reilly, Robert and Schweihs, Robert P. 2000. *The Handbook of Advanced Business Valuation.* McGraw-Hill Professional. p.102

[60] Robak, Espen. 2001. *FMV Introduces Detailed Restricted Stock Study.* Business Valuation Resources.

### *2.2.12 Bajaj study*

In 2001, Bajaj published a restricted stock study. The authors believed that discounts resulting from previous restricted stock studies cannot be completely considered a DLOM. Part of the discount reflects the lack of marketability, but other factors have an impact on the discount value.

The authors performed a restricted stock study using private placements that occurred between 1990 and 1995. They implemented a cross-sectional analysis, so they "examine how discounts based on this approach […] vary based on various firm and issue characteristics"[61].

Through the Securities Data Corporation (SDC), the authors gathered all restricted stock placements completed between 1990 and 1995. Removing transactions that either did not fit the study, or that were duplicates of the very same transactions. The final study presented a data pool of 88 purchases.

A first look at the data shows that most placements, 82% of the purchases, are performed by corporations on the Over the Counter (OTC) market. This is consistent with the authors' intuition, since "OTC firms have a propensity to be smaller and are typically not followed by many financial analysts"[62]. An investor interested in OTC firms' private placements must bear costs for retrieving information on the industry and for monitoring the firm itself. Therefore, the final discount tends to be higher if compared to private placement whose issuer is a firm listed in huge markets where information is easy to obtain.

The average discount considering both restricted stocks issued by listed companies and privately held companies' shares is 22,21%. The discount is in line with other restricted stock studies performed during the 1990s. The authors argued that the entire discount cannot be exclusively assigned to a lack of marketability. Evidence of this thinking derives from the data itself. 38 out of 88 placements, were the private placement of registered stocks. The discount in this case should only reflect the holding period since

[61] Bajaj, Mukesh, Denis, David J., Ferris, Stephen P., and Sarin Atulya. 2001. *Firm Value and Marketability Discounts*. Journal of Corporation Law.

[62] Bajaj, Mukesh, Denis, David J., Ferris, Stephen P., and Sarin Atulya. 2001. Firm Value and Marketability Discounts. Journal of Corporation Law.

no marketability issue is present in listed corporations' shares[63]. The average discount is 14,04%, which is lower than the average of the study. On the other hand, the remaining 50 private placements were unregistered shares being privately placed. The average discount is 28,13%. Therefore, a 14,04% discount is referred to as unregistered but marketable stocks, and 28,13% is referred to as non-marketable securities. One could argue that the difference, 14,09%, represents the discount only attributable to a lack of marketability. Bajaj claimed that the difference is influenced by economic factors, not only by marketability. Understanding the impact of such determinants on the overall discount can help isolate the actual discount for lack of marketability.

"The discount offered to buyers is compensation for the cost of assessing the quality of the firm and for the anticipated costs of monitoring the future decisions of its managers"[64]. The harder and the more costly it is to compute the firm's value and to track the company performances, the bigger the discount applied during private placements. Factors that authors find important to monitor and assess for the company valuation are the following:

- The Fraction of Total Shares Offered in the Placement: generally, the bigger the fraction of total shares being privately placed, the bigger the discount applied.
- Business Risk: highly risky enterprises are more difficult to evaluate. Leading to more resources needed during the valuation. The result is a bigger discount in the case of risky companies.
- Financial Distress: a company in financial distress struggles to generate profits due to financial duties. The managers need to privately place stocks at a bigger discount to attract investors.
- Total Proceeds from the Placement: Obtaining information regarding private placements can be very expensive. Therefore, the more shares being placed, the easier it is to absorb such costs. So larger proceeds lead to lower discounts.

Through regression analysis, the authors proved that most of their hypothesized variables do account for the discount in private placements. The larger the shares issued,

[63] After the required holding period, restricted stocks issued by listed companies, should not face any marketability issue.

[64] Bajaj, Mukesh, Denis, David J., Ferris, Stephen P., and Sarin Atulya. 2001. Firm Value and Marketability Discounts. Journal of Corporation Law.

the larger the discount observed in their 88 purchases sample. The business risk, measured as volatility of the companies' returns, increases according to the discounts applied. The financial distressed was measured using the Altman Z-Score. Lower discounts presented higher average Z-Score, meaning that companies offering unregistered stocks at a lower discount are financially stable. The authors stated that the variable concerning proceeds does not seem to be consistent among different discount levels.

The 14,09% difference between registered and unregistered shares' private placements does not only reflect the lack of marketability. It also indicates that unregistered companies are not as financially strong as their registered counterparts. The result is a further increase in the discount.

Running a multivariate regression, the authors tried assessing the impact of previously mentioned factors, as well as the marketability factor on the discount applied in private placements. The factors considered in the regression are % of total shares issued; Z-Score of issuing company; Standard deviation of subject company's returns; If the company is registered or not.

The regression model derived is:

$$Discount = \alpha + b_1 * Fraction\ of\ Shares\ Issued + b_2 * Z - Score + b_3 * Standard\ Deviation\ of\ Returns + b_4 * Registration\ Indicator \quad (2.7)$$

$b_i$ values state by how much the discount changes if the factor they refer to changes by one unit.

The coefficient of whether the subject company is registered is -7,23. This means that, if the company issuing private shares is not registered, then the discount should increase by 7,23%. This percentage value represents the value of the discount for lack of marketability.

#### *2.2.13 Comment Study*

In one of the most recent restricted stock studies, published in 2012, Robert Comment presented his analysis. The author strongly criticizes the application of DLOMs up to 40% that are still being used when valuing small companies. Comment believes that large discounts are not appropriate due to redundancy. DLOMs resulted from past restricted

stock studies are flawed and they consider factors that are present in a core company valuation. Applying large DLOMs accounting for factors already considered when valuing a company, creates a repetition. The final value might be strongly undervalued.

In presenting his study, comment indicates that his study focuses only on DLOM from restricted stocks. He does not consider the difference between restricted stock and a free-trading stock to be the entire DLOM. Other factors can alter the discount.

The data of the study consists of 1103 privately placed shares, from 2004 to 2010 issued by 724 firms. Data were collected through Bloomberg's archive of SEC filings.

Taking a first look at the data in the study shows how 80% of firms issuing their shares privately, present negative net income during the year before the deal. Moreover, most private placements, 41%, are performed by OTC companies. "The scarcity of large, well-known companies in this sample is not surprising since such companies have operating cash flow, ready access to debt financing, and correspondingly less need for external equity financing"[65]. This is consistent with Damodaran thinking cited in paragraph 2.2, which affirms that in most cases, private placements are issued by either non-healthy companies, or by companies operating in small markets, such as the OTC.

The study was performed using multiple regression analysis since it can best represent the impact of different factors on the dependent variable, which is the discount in private placements. The author implemented fifteen explanatory variables in the regression. Those variables fall into four major categories identified by the SEC to be the major determinants for the DLOM. Such variables are classified as OTC status, Net Income, Sales, and Registration Rights[66].

By running different regressions, the author finds that the coefficient for the explanatory variable referring to restricted stock without registration rights is 5,238. So around 5%. The result is consistent with the most recent restricted stock studies. The discount rate solely attributable to the security lacking marketability is around 5%.

[65] Comment, Robert. 2012. *Revisiting the Illiquidity Discount for Private Companies: A New (and "Skeptical") Restricted-Stock Study.* Journal of Applied Corporate Finance.

[66] Houlihan Lokey Howard and Zukin. 2004. *Discounts for Lack of Marketability: Discussion Materials.* Investment banking services. p16

The result is much lower than past RS studies probably due to the 2008 Rule 144 rework. Restricted stocks are now required to be held for six-months by the investor[67].

### *2.2.14 Restricted Stock Studies Conclusions*

Several restricted stock analyses have been performed, as shown in Exhibit 4.

*Exhibit 4*: Restricted Stock Studies' Results

| Restricted Stock Studies' Results Overview | | | |
|---|---|---|---|
| **Restricted Stock Study** | **Years Observed** | **# of transactions** | **Average Discount** |
| SEC Entire Study | 1966-1969 | 398 | 25,80% |
| SEC OTC Nonreporting | 1966-1969 | 112 | 32,60% |
| Gelman Study | 1968-1970 | 89 | 33,00% |
| Trout Study | 1968-1972 | 60 | 33,45% |
| Moroney Study | 1969-1972 | 146 | 35,60% |
| Maher Study | 1969-1973 | 34 | 35,43% |
| Standard Research Consultants Study | 1978-1982 | 28 | 45,00% |
| Willamette Management Associates Study | 1981-1984 | 33 | 31,20% |
| Silber Study | 1981-1988 | 69 | 33,75% |
| Hertzel & Smith Study | 1980-1987 | 106 | 13,50% |
| Johnson Planning Study | 1991-1995 | 72 | 20% |
| Management Planning Study | 1980-1996 | 53 | 27% |
| FMV Opinions Study | 1980-1997 | 243 | 22,10% |
| Bajaj, Deni, Ferris and Sarin Study | 1990-1995 | 88 | 22,21% |
| Bajaj & Co. DLOM exclusive | 1990-1995 | 88 | 7,23% |
| Comment Study | 2004-2010 | 742 | 5,24% |

It is possible to note a slight but steady decrease among the first and later studies. This was due to the two major SEC Rule 144 rework, in 1997 and 2008 that decreased the holding period required to sell the restricted shares while increasing the volume of potential selling. Therefore, restricted stocks started slowly becoming more and more marketable within each year.

As it is clear in Exhibit 4, most of the studies analyzed a relatively small number of restricted shares over a long period. This characteristic mined the relevance of studies due to market changes occurring even within the same study. Silber, for instance,

[67] Jacob, Valerie F., Bursky, Daniel J., Gelfond, Stuart H., Levitt, Michael A., Tropp, Paul D. and Tsaganos, Vasiliki B. 2008. *SEC shortens Rule 144 hodling periods and loosens restrictions on resales of privately placed securities.* Journal of Investment Compliance.

considered stocks from 1981 to 1988. The first years of his data are strongly biased by a stock market crisis. Appraisers must use restricted stock studies' results cautiously, considering the market characteristics' in which the subject study was carried out.

Finally, some studies only consider restricted shares issued by listed companies. Those results should be considered as a minimum DLOM since, once the restriction expires, the holder can trade the share on public markets. On the contrary, privately held companies' stocks, might be non-marketable even after the holding period restriction.

### 2.3 Pre-IPO Studies

Around the 1980s most DLOM studies focused on comparing a non-marketable security with a marketable one. In the case of Pre-IPO studies, "the DLOM is quantified by analyzing (with various adjustments) the difference between (1) the public market price at which a stock was issued at the time of the IPO and (2) the private market price at which a stock was sold […] prior the IPO"[68].

According to the authors performing Pre-IPO studies, the method could work extremely well to detect the marketability value. Private security has no market, but once it gets listed through the IPO, it becomes marketable. Therefore, comparing these two security's prices of the very same firm before and after going public, should result in a good proxy for the DLOM value.

The Pre-IPO studies are generally considered to bear several flaws. For instance, it is difficult to set a date before the IPO to value the subject company's securities during their period of non-marketability. If such a moment is near the IPO, then the value might be distorted and biased. On the contrary, if the date is too far from the initial public offering, the valuation might reflect the market and firm characteristics that have already changed.

An additional issue that weakens the effectiveness of such studies is the sample size. In the studies summarized below, the data pools include a small selection of IPOs. In the John D. Emory study, for example, only 310 successful IPOs were detected between 1980 and 1997. Around 18 per year, on average. Moreover, the discounts observed were

[68] Reilly, Robert F. and Rotkowski, Aaron. 2007. *The Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies*. The Tax Lawyer.

extremely different among transactions in the same study. It is possible to run Monte Carlo simulations on excel using Pre-IPO studies' average and standard deviation to end up with a probable range of discounts. Performing a total of one hundred fifty thousand simulations, results are the following:

- The last 5-year period, from 1998 to 2002 of the Willamette Management Associates study, reports a standard mean of 23,9% and a standard deviation of 59,9%[69]. Due to such a large mean, the simulation reported results ranging from around negative 240% to positive 285%. A discount cannot be negative. By applying a logarithmic distribution, it is possible to perform the simulations without incurring negative values. By doing so, the minimum value is around 5%, and the largest reach 1600%. Once again, this is due to the large standard deviation of the study.

The most important Pre-IPO studies are:

- The Emory Study
- The Valuation Advisors Study
- The Willamette Management Associates Study

### *2.3.1 Emory Study*

John D. Emory has been a private stocks' appraiser since the early 1960s. Through his work, he observed the importance of marketability in the decisions of investors buying private equity. His major intuition is that a firm before being listed is affected by the lack of marketability, which elapses once the subject company goes public.

The author performed nine studies on companies undergoing an IPO. Each study covers around one and a half years of data and they range from 1980 to 2000.

The author considered in the study only companies that concluded the IPO with a positive outcome. Furthermore, Emory eliminated from the final sample "development stage companies with a history of real operating losses and companies whose IPO was

[69] Garland, Pamela J. and Reilly, Ashley L. 2004. *Update on the Willamette Management Associates Pre-IPO Discount for Lack of Marketability Study for the Period 1998 through 2002.* Insights.

less than $5 per share […]"[70]. The private share valuation was performed five months before the IPO since it generally is the time it takes to conclude the listing procedure[71]. From 1980 to 1997, more than 2200 prospectus were examined and a total of 310 deals were considered in the analysis.

Once the value of the privately held share was estimated, it was compared with the price offered to the public by the very same company. The difference in percentage represented the impact of marketability. The average discount observed in each study is the DLOM. The average discount for lack of marketability on the study covering years 1995-1997 is 43%. The average discount of the entire eight studies is 44%.

Exhibit 5 summarizes the 9 studies' results.

*Exhibit 5*

| Emory Pre-IPO Studies' Results | | | |
|---|---|---|---|
| Years Observed | # of Transactions | Discount | |
| | | Average | Median |
| 1980-198 | 13 | 60,0% | 66,0% |
| 1985-1986 | 21 | 43,0% | 43,0% |
| 1987-1989 | 27 | 45,0% | 45,0% |
| 1989-1990 | 23 | 45,0% | 40,0% |
| 1990-1992 | 35 | 42,0% | 40,0% |
| 1991-1993 | 54 | 45,0% | 44,0% |
| 1994-1995 | 46 | 45,0% | 45,0% |
| 1995-1997 | 91 | 43,0% | 42,0% |
| 1997-2000 | 53 | 54,0% | 54,0% |
| **Total 1980-2000** | **363** | **46,9%** | **46,6%** |

It is noticeable how, besides the first study in 1980, the average DLOM among studies is very similar between each other.

Through an analysis of share prices of the last two studies, the 1995-1997[72] and 1997-2000[73] , it is possible to draw some interesting conclusions, shown through Exhibit 6.

[70] Emory, John D. 1997. *The Value of Marketability As Illustrated In Initial Public Offering Of Common Stock*. Business Valuation Review.
[71] PWC. 2017. *Roadmap for an IPO: A guide to going public.* PWC Deals.
[72] Emory, John D. 1997. *The Value of Marketability As Illustrated In Initial Public Offering Of Common Stock*. Business Valuation Review.
[73] Emory, John D., Dengel III, F. R. and Emory Jr, John D. 2000. *The Value of Marketability as Illustrated in Initial Public Offerings of Dot-Com Companies: May 1997 through March 2000.* Business Valuation Review.

The standard deviation in both studies is between 18% and 20%. Therefore, the statistical reliability of these DLOMs is similar to restricted stock studies. Emory reported the valuation dates when the firm was private and when it concluded the IPO. As stated in exhibit 6, a large percentage of valuations, in both studies, were performed close to the IPO. This can have biased the DLOM value since the closer the company is to be public, the more marketable the share is. In fact, in the case of the 1995-1997 study, the DLOM computed in transactions valued less than 5 months from the IPO date, resulted in a lower discount rate if compared to the average. Considering the 1997-2000 case study for the same type of share, the discount rate is very close to the overall DLOM. This indicates that during 1997-2000, on average, no major discount differences were registered if companies were valued way before the IPO or closer to the listing date. On the contrary, the previous study shows that companies being valued more than five months before the IPO date, register a mean discount higher than the average of the study.

*Exhibit 6: Emory's Study Data Analysis*

| | Total Transactions | Average DLOM | Standard Deviation | # of Transactions | | Average DLOM Considering | |
|---|---|---|---|---|---|---|---|
| | | | | < 5 | ≥ 5 | < 5 | ≥ 5 |
| 1995-1997 Study | 91 | 42,78% | 18,72% | 71 | 20 | 41,16% | 48,50% |
| 1997-2000 Study | 53 | 54,28% | 19,75% | 39 | 14 | 54,67% | 53,18% |

Emory concludes that it is important to apply a DLOM considering the subject firm characteristics, rather than applying an average. Though, his Pre-IPO study shows the general impact of marketability in share prices.

### *2.3.2 Valuation Advisors Study*

Valuation Advisors, LLC. performs small and medium enterprises' valuation for IPOs' purposes. Thanks to their activity, they retained lots of information regarding private firms going public. This persuaded them to perform a Pre-IPO study.

Valuation Advisors conducted its analysis between the years 1999 and 2002. They considered more than 1300 successful IPOs. When estimating the final mean discount, they reduced the sample considering only data between the 10% and 90% percent. Therefore, discarding possible cheap offerings and premiums. The study covering the

year 2000, results in an average discount of 47.07% and an average DLOM after reducing the sample of 52.40%[74].

On Exhibit 7 are reported the four Valuation Advisors' study results. It is possible to notice even further the impact of time on share values. The discount for lack of marketability close to IPO is much lower than private discounts registered on the very same shares' valuation performed one year before the IPO. This clearly shows the importance of marketability in private shares valuation.

*Exhibit 7*

| Valuation Advisors Pre-IPO Studies<br>DLOM Results | | | | | | |
|---|---|---|---|---|---|---|
| IPO Year | Period Before IPO in which shares where valued | | | | | # of Shares in the Study |
| | 0-3 Months | 4-6 Months | 7-9 Months | 10-12 Months | 1-2 Years | |
| 1999 | 30,8% | 54,2% | 75,0% | 76,9% | 82,2% | 695 |
| 2000 | 28,7% | 45,1% | 61,5% | 68,9% | 76,6% | 653 |
| 2001 | 14,7% | 33,2% | 33,4% | 52,1% | 51,6% | 115 |
| 2002 | 6,2% | 17,3% | 21,9% | 39,5% | 55,0% | 61 |

### *2.3.3 Willamette Management Associates Study*

Willamette Management Associates performed its Pre-IPO study. The analysis includes firms undergoing an IPO from 1975 to 2002. WMA included only private stock sales. In the latest study, from 1998 to 2002 "[…] sales of closely held corporation stock in private placements and repurchases of treasury stock by the closely held corporation"[75] were also considered.

The authors retrieved IPO data through Corporate New Issues databases, prepared by Thomson Financial Securities Data. For the sake of the study, stock issued at $1 per share as well as offering including warrants, were excluded from the analysis.

WMA ran a different methodology compared to the Emory study (2.3.1). Instead of valuing a private company's shares before being listed, they used Price/Earnings ratios before and after the IPO is concluded. Therefore, in the case of "companies that had no

[74] Pearson, Brian. 2001. *2000 Marketability Discounts as Reflected in Initial Public Offerings.* Business Valuation Resources.

[75] Garland, Pamela J. and Reilly, Ashley L. 2004. *Update on the Willamette Management Associates Pre-IPO Discount for Lack of Marketability Study For the Period 1998 through 2002.* Insights.

meaningful earnings as of the private transaction date and/or the public offering data were eliminated"[76].

The formula used to perform the studies:

$$\frac{P/E_0 - P/E_p\left(\frac{IP/E_0}{IP/E_p}\right)}{P/E_0} \tag{2.7}$$

Where:

- $P/E_0$ = Price per share of the public offering
- $P/E_P$ = Price Per shares during the private transaction
- $IP/E_0$ = Industry price index during the offering
- $IP/E_p$ = Industry price index during a private transaction

Exhibit 8 reports all WMA studies' results. The DLOM reported is the trimmed discount. Therefore, WMA performed each year's discount for lack of marketability, and to make it more consistent, they eliminated the outliers. They did it to eliminate extremely high and extremely low discounts that could bias the final discount rate.

Private transactions are referred to as private companies selling their stakes at least 36 months before the IPO date. The number of companies undergoing IPO is lower than the number of private transactions, due to firms performing multiple transactions over the above-mentioned time-frame.

The DLOMs are very different among studies, and on average, they are higher than restricted stock studies' results. What is more, standard deviations are also large. This indicates that data among the same study, are very different.

The 2001 study's results are very contradictory. The average DLOM is -195,80%, which indicates a premium rather than a discount. Moreover, although the positive discount presented in the 2002 study, the result might be inaccurate due to the small sample size. Willamette argues that its Pre-IPO studies can be used as a basis for DLOM computation, but they "believe that quantitative conclusions for 2001 and 2002 are not reliable indicators of the current DLOM related to privately held corporation stock"[77].

[76] Reilly, Robert and Schweihs, Robert P. 2000. *The Handbook of Advanced Business Valuation.* McGraw-Hill Professional.

[77] Garland, Pamela J. and Reilly, Ashley L. 2004. *Update on the Willamette Management Associates Pre-IPO Discount for Lack of Marketability Study For the Period 1998 through 2002.* Insights.

*Exhibit 8*

| WMA Pre-IPO Studies (1975-2002) | | | | | |
|---|---|---|---|---|---|
| Year of the Study | # of IPO considered | # of Private Transactions Analysed | Average DLOM | DLOM Median | DLOM Standard Deviation |
| 1975-1978 | 17 | 31 | 43,40% | 52,50% | 58,60% |
| 1979 | 9 | 17 | 56,80% | 62,70% | 30,20% |
| 1980-1982 | 58 | 113 | 51,90% | 56,50% | 29,80% |
| 1983 | 85 | 214 | 55,20% | 60,70% | 34,70% |
| 1984 | 20 | 33 | 52,90% | 73,10% | 63,90% |
| 1985 | 18 | 25 | 47,30% | 42,60% | 43,50% |
| 1986 | 47 | 74 | 44,70% | 47,40% | 44,20% |
| 1987 | 25 | 40 | 44,90% | 43,80% | 49,90% |
| 1988 | 13 | 19 | 42,50% | 51,80% | 29,50% |
| 1989 | 9 | 19 | 46,90% | 50,30% | 18,60% |
| 1990 | 17 | 23 | 33,00% | 48,50% | 42,70% |
| 1991 | 27 | 35 | 28,90% | 31,80% | 37,70% |
| 1992 | 36 | 75 | 47,00% | 51,70% | 42,60% |
| 1993 | 51 | 110 | 49,90% | 53,30% | 33,90% |
| 1994 | 31 | 48 | 38,40% | 42,00% | 49,60% |
| 1995 | 42 | 66 | 47,40% | 58,70% | 76,40% |
| 1996 | 17 | 22 | 34,50% | 44,30% | 45,40% |
| 1997 | 34 | 44 | 30,50% | 35,20% | 46,70% |
| 1998 | 14 | 21 | 39,80% | 49,40% | 43,30% |
| 1999 | 22 | 28 | 27,10% | 27,70% | 45,20% |
| 2000 | 13 | 15 | 22,90% | 31,90% | 58,50% |
| 2001 | 2 | 2 | -195,80% | -195,80% | n/a |
| 2002 | 5 | 7 | 55,80% | 76,20% | 42,80% |

Some criticisms have been moved toward the studies. For instance, the results presented during the analysis are difficult to challenge due to WMA not disclosing the data they used.

### *2.3.4 Pre-IPO Studies Conclusions*

Overall, Pre-IPO studies clearly show that DLOM exists and it affects privately held securities. Pre-IPO studies can be very suited for private companies' valuation if the expected holding period is around two years. In that case, Pre-IPO studies' results are more relevant than Restricted stock studies.

Pre-IPO studies suffer from major flaws, which impact their reliability. The first issue is related to the data selected. All the three above mentioned studies do not consider companies who have failed the IPO. Generally, healthy firms make it through the IPO, while weaker companies tend to flop. Weaker firms are considered less liquid than

sound firms. Therefore, the data selected in each study only contemplate private firm whose DLOM is on average lower than the vast majority of privately held corporations.
The second flaw is that prices tend to be inflated by underwriters when a firm is going public. Evidence shows that tock returns on the first IPO date are strongly influenced by optimist investors.[78] There DLOM resulting from Pre-IPO studies might be upward biased to an increase in prices during the first IPO day.

### 2.4 Multiples-Based Studies

More recent empirical studies focus on firms' multiples to obtain a good proxy for the DLOM. Multiples-Based Studies shows that listed companies are generally sold at a price premium compared to their private counterparts. This happens due to non-public companies being less marketable and liquid than listed firms.
The studies present two variations: The Acquisition studies and Pricing Multiples studies. They present slight differences. The acquisition approach compares a private company purchase with the acquisition of a similar but public company. While the Pricing Multiple studies compare "[…] a publicly-traded company's stock price and a private company's business enterprise value derived from an acquisition transaction"[79].
Multiples-Based approaches prove that public companies are purchased at higher prices. In doing so, they attribute the price difference only to a DLOM. The reliability of each study is related to how similar the two matched firms are. This is crucial since it is impossible to find two identical firms. Therefore, authors must prove they tried to compare two akin-enough companies.
Some authors argue that the DLOM derived using multiples can be the result of systematic differences between public and private companies, rather than an actual discount related to lack of marketability.
The main Acquisition studies are:

- The Koeplin-Sarin-Shapiro Study
- The Kooli-Kortas-L'Her Study

[78] Loughran Tim and Ritter Jay. 2004. *Why has IPO Underpricing Changed Over Time?.* Financial Management. p30-31.
[79] Zanni, Kevin M. 2013. *The Private Company Discount Based on Empirical Data.* Taxation Planning and Compliance Insights.

- The Block Study
- The Officer study

While Harjoto-Paglia published the most relevant Pricing Multiples study.

### *2.4.1 Koeplin-Sarin-Shapiro Study*

In 2000, John Koeplin, Atulya Sarin, and Alan C. Shapiro published their acquisition approach study. The usual framework in private company valuation was using the DCF to value the company as if it was listed and then apply a DLOM. They tried presenting a new method to account for a lack of marketability. They demonstrated that private companies are purchased at price discounts if compared to listed firms.

The authors started by selecting private companies' purchases from 1984 to 1998. They searched for the companies in the SDC merger and acquisition database. They considered only transactions that disclosed enough data to run the analysis.

From the initial data pool, the authors eliminated purchases of both financial companies and regulated utilities, as well as non-controlling ownership transactions. For each private transaction, a public company's purchases had to be detected. The similar-but-public purchase had to occur during the same year, in the very same country and the two firms had to operate in identical industries. The 4 digit SIC code was used to check whether the two companies were active in the same industry.

The final sample consisted of 84 transactions in the US and 104 in foreign markets.

The authors identified 4 different multiples that can efficiently measure value in both private and public companies:

- EBIT
- EBITDA
- Sales Multiple
- Book Multiple

Both EBIT and EBITDA are a good proxy for a company's cash flow, and they do not consider the capital structure. EBITDA is like EBIT, but it is computing adding back the depreciation and subtracting capital expenditures, and it represents a better proxy for representing the free cash flow. Assuming capital expenditures equals depreciation, makes it so that EBIT can be equally good. "The rationale for using sales multiples is that companies seeking to expand their operations are often interested in the price paid per

additional dollar of sales. […] Similarly, the market/book multiple tells how much was paid for every dollar of capital invested in the business"[80].

To end up with ratios that can allow the analysis between private and public companies, the multiples are compared to the enterprise value (ENTVAL).

The authors made some assumptions for multiples computation. They assumed the margin of free cash flows (FCFF) on sale to be constant, called "*m*". Moreover, they assumed a steady growth rate of sales, "*g*". Considering that sales refer to sales revenues, it is now possible to compute the FCFF. Using the following formula, one can compute the sales multiples:

$$ENTVAL/Sales = \frac{m}{(k_0 - g)} \tag{2.8}$$

Where $K_0$ is the weighted average cost of capital.

Assuming that both depreciation and capital expenditure are equal, the company's worth is $EBIT(1-t)/K_0-g)$. The subject company's EBIT multiple is:

$$\frac{ENTVAL}{EBIT} = \frac{EBIT(1-t)(k_0 - g)}{EBIT} = (1-t)(k_0 - g) \tag{2.9}$$

Where t is the corporate tax rate.

Concerning Book multiples, the authors developed some further assumptions. For instance, assets generate a fixed return "*R*" and a constant sum of such return "*b*" is reinvested. Considering these premises, the FCFF produced can be computed as:

$$(1-b)\ X\ Book\ Value \tag{2.10}$$

Expanding equation 2.10, the overall value of the company is:

$$ENTVAL = (1-b)R\ X\ Book\ Value\ /\ (k_0 - bR) \tag{2.11}$$

Finally, considering equation 2.11 it is possible to compute the Book multiple using the following formula:

[80] Koeplin, John. Sarin, Atulya and Shapiro, Alan C. 2000. *The Private Company Discount*. Journal of Applied Corporate Finance.

$$\frac{ENTVAL}{Book\ Value} = \frac{(1-b)R\ X\ Book\ Value\ /\ (k_0 - bR)}{Book\ Value} = \frac{(1-b)R}{(k_0 - bR)} \qquad (2.12)$$

The discount for lack of marketability was computed by comparing the privately held companies' discount with their similar public counterparts.

Results show that in the US market the average discount in cases of EBIT and EBITDA multiples is 28,26% and 20,39% respectively. In foreign markets, discounts are larger. In fact, EBIT multiple discount is 43,87% while EBITDA discount is 53,85%. "It appears from these results that the sales of both the private and public companies are valued similarly"[81]. Resulting in non-statistically significant results. The Book multiple is 17,81% for US markets and 34,86% for foreign markets. Once again, the analysis shows that foreign private companies are valued at a higher discount rate than US companies.

The authors tried to adjust the Sales' low significance level by running a regression analysis, due to sales being very dependent on growth rates which tend to differ a lot among private and public firms. The regression proves even further that EBIT, EBITDA, and Book multiple largely influence the DLOM, while sales are not statistically significant.

The study shows that US private companies are generally purchased at a DLOM of around 20% to 30% while foreign private companies receive discount rates up to 50%.

### *2.4.2 Kooli-Kortas-L'Her Study*

In this analysis published in 2003, the authors compute the DLOM through the acquisition of several US companies. In contrast to the previous study, Kooli performed a reference portfolio framework, rather than using multiples.

The data were collected through the DoneDeals and SDC databases. The authors selected both private and public companies' acquisitions from 1995 to 2002. The sample was then limited to only controlling interest transactions and US companies' purchases, due to possible differences in accounting standards. The final data pool includes 331 private deals.

[81] Koeplin, John. Sarin, Atulya and Shapiro, Alan C. 2000. *The Private Company Discount*. Journal of Applied Corporate Finance.

The discount applied in private transactions was computed by "using the available multiples provided by the DoneDeals database, namely, the sales, the earnings, and the cash flow multiples, which appear to be the most relevant for valuation purposes"[82]. Instead of picking a similar but public company to measure the discount in multiples, the Kooli sets up a public acquisition reference portfolio. They first classify private transactions based on industry, year, and size, forming size-quartile portfolios, and computing their breakpoints. Next, they classify public companies according to their size, year, and industry forming "industry-period-size portfolios based on the breakpoint of each private size-quartile, previously formed"[83]. They created portfolios of public companies based on one characteristic to compare it with private companies.

The private company discount is computed for each multiple as follows:

$$Discount = 1 - \left(\frac{private\ company\ multiple}{median\ public\ company\ multiple}\right) \tag{2.13}$$

The authors prefer comparing private and public companies according to their median to not account for possible outliers that can severely impact the mean.

Results show that public companies' medians are higher than private companies multiples.

Public companies' portfolios have higher multiples than that of private companies. The discount based on sales multiples is 17%, earnings discount is 34%, and discount according to cash flows multiple is 20%.

To better describe the discount for lack of marketability affecting private transactions, the authors performed a multivariate regression analysis on two factors. The regression model considers 219 private companies' transactions, and it can be described as follows:

$$DLM_j = \alpha + \beta_1 * Asset_1 + \beta_2 * (1 - Asset_1) + \gamma_1 * P/SE_1 + \gamma_2 * (1 - P/SE_1) + \sum_{i=1}^{7} \delta_1\, Industry_1 + \sum_{t=1995}^{2002} \Phi_t\, Year_t + \varepsilon_j \tag{2.14}$$

[82] Kooli, Maher., Kortas, Mohamed and L'Her, Jean-François. 2003. *A New Examination of the Private Company Discount: The Acquisition Approach.* The Journal of Private Equity.

[83] Kooli, Maher., Kortas, Mohamed and L'Her, Jean-François. 2003. *A New Examination of the Private Company Discount: The Acquisition Approach.* The Journal of Private Equity.

$Asset_1$ is a dummy variable, therefore it has a value of either 1 if the subject private company is larger in size than the median of the assets, or it assumes a value of 0 otherwise. $P/SE_1$ is another dummy variable assuming a value of 1 when the private company has P/SE larger than measured P/SE's medians, the value is 0 otherwise. Both Industry and Year are dummy variables having values of 1.

The regression analysis shows that DLOM depends largely on the subject firm characteristics and the industry it is operating in.

### *2.4.3 Block Study*

In 2007, Stanley Block published his analysis extending the Koeplin study, mentioned in paragraph 2.4.1. The methodology is the same, although Block applied some changes in the sample selection.

The author examined private company's transactions from 1999 to 2006 using the Thompson Financial Transaction Database, SDC and, Wall Street Journal. He included financial firms as well as regulated utilities, which were excluded in the Koeplin study. Moreover, Block limited his study to US deals. The final data sample consisted of 91 purchases.

Just like in the Koeplin study, the multiples considered are four: EBIT, EBITDA, Book value, and sales multiples. The average discount rate at which private companies were sold compared to listed firms is around 25%.

Block investigated the impact of the industry in the DLOM. He divided his data into eight industries to compute the average multiples for each industry. The results are shown in Exhibit 9.

*Exhibit 9*: Block's Study Multiples by Industry

| | Average Multiples Discount | | | | |
|---|---|---|---|---|---|
| **Industry** | **Price/Earning per Share** | **ENTVAL/EBIT** | **ENTVAL/ EBITDA** | **ENTAVL/Book Value** | **ENTVAL/ Revenue** |
| Energy | 28,33% | 30,08% | 29,29% | 20,08% | 23,91% |
| Manufacturing | 37,17% | 40,08% | 36,08% | 25,19% | 31,08% |
| Finance | 9,97% | 12,08% | 13,03% | 7,61% | 8,42% |
| Utilities | 21,01% | 23,18% | 25,05% | 15,42% | 26,36% |
| Technology | 26,17% | 24,72% | 28,07% | 14,87% | 25,19% |
| Retail | 24,18% | 28,68% | 26,26% | 17,92% | 23,08% |
| Healthcare | 30,17% | 32,16% | 27,06% | 19,04% | 26,06% |
| Transportation | 26,29% | 27,07% | 26,34% | 18,26% | 27,17% |

Block points out that "the liquidity discount highest for manufacturing firms (generally 30%-40%) and smallest for financial firms (8%-10%). Statistical tests indicate a significant relationship between the industry of origin and the size discount"[84]. The difference in the discount rate between Block's study and Koeplin's study is not only due to time differences but also to data considered. Block analyzed financial firms, reporting that they were operating in the industry with the lowest discount compared to public companies. While Koeplin did not include such firms in its analysis.

### *2.4.4 Officer Study*

Micah Officer in 2007 published his multiples-based study. His goal is "explaining the cross-sectional variation in […] acquisition discounts, to demonstrate that acquisition discount varies with unlisted firms' owners need for liquidity and/or the availability of alternate sources of liquidity"[85].

The author collected data using the SDC database, including both successful and failed transactions from 1979 to 2003. The transactions contain purchases of private companies as well as unlisted subsidiaries owned by public companies.

The sample consisted of 12716 transactions which then decreased due to the removal of purchases disclosing insufficient data.

The methodology applied by Officer is like Kooli's analysis in paragraph 2.4.2. It consists of identifying private companies' transactions and creating public companies' portfolios having similar attributes. In this case, "comparable acquisitions are those for which the publicly traded target is in the same two-digit SIC code as the unlisted target, has deal value excluding assumed liabilities within 20% of the deal value excluding assumed liabilities for the unlisted target […] and is announced within the three-calendar-year window centered on the announcement of the unlisted acquisition"[86]. After that, the author computed acquisition multiples for both listed and unlisted transactions. Multiples considered are price to earnings, deal value to EBITDA, deal value to sales, or

[84] Block, Stanley. 2007. *The liquidity Discount in Valuing Privately Owned Companies.* Journal of Applied Finance.

[85] Officer, Micah S. 2007. *The Price of Corporate liquidity: Acquisition discounts for unlisted targets.* Journal of Financial Economics.

[86] Officer, Micah S. 2007. *The Price of Corporate liquidity: Acquisition discounts for unlisted targets.* Journal of Financial Economics.

price to book value of equity. The discount emerges from the comparison of an unlisted company's multiple with its public companies' portfolio multiple.

The average discount observed ranges between 15% to 30%.

The author also proves that selling a private company or a subsidiary is the last possible remedy to get cash financing when borrowing money is unattractive for the company. The analysis shows that many parental companies reported negative Z-Scores. Meaning that the company had severe debt issues. Additionally, parental companies presented very low stock returns among the year before the selling of their subsidiaries. Finally, most of the subsidiaries' transactions were executed through cash payments.

### *2.4.5 Paglia-Harjoto Study*

In 2010 Paglia and Harjoto published their study on DLOM using a multiples approach. They argue that similar studies based on acquisition multiples suffer from strong bias due to strategic determinants. Therefore, the authors introduce their analysis based on comparing only multiples of a private company and a set of similar listed counterparts.

The authors retrieve data concerning privately held companies' sales through Pratt's Stats Database. The data collected was limited to private companies having sales larger than $50 million since "these companies are potential candidates for being publicly traded. Therefore, we can better isolate the "public" versus "private" valuation discount"[87]. Utilities and services companies were excluded from the final sample.

Public firms' information was collected using the Compustat database. The authors paired private and public firms according to the year of the transaction, annual net sales, and the six digits of the North America Industry Classification System.

The data used in the study consisted of 431 companies from the year 1994 to 2008.

The analysis computes the DLOM using two market multiples. The market value of invested capital (MVIC) over sales and EBITDA. They used the following formulas:

$$DLOMSALE(\%) = [1 - (MVIC/Sale\ for\ private\ firm)/(MVIC/ Sale\ for\ public\ firms)]X100 \quad (2.15)$$

[87] Paglia, John and Harjoto, Maretno A. 2010. *The Discount for Lack of Marketability in Privately Owned Companies: A Multiples Approach.* Journal of business Valuation and Economic Loss Analysis.

$$DLOMEBITDA(\%) = [1 - (MVIC/EBITDA\ for\ private\ firms)/ (MVICEBITDA\ for\ public\ firms)]X100 \quad (2.16)$$

DLOM resulting from equations 2.15 and 2.16 should normally assume values ranging from 0% to 100%. Therefore, the authors discarded all the pairs of companies whose discount for lack of marketability fell outside that range. The result is 431 pairs for DLOMSALE and 283 pairs for DLOMEBITDA. The authors also examine how DLOM changes according to industry. Results are presented in Exhibit 10. It is possible to note that, on average, the Sale multiple has a higher DLOM, if compared to DLOMEBITDA. Sales seem to be more impactful on determining the private discount than EBITDA. Considering DLOMSALE multiple, the industry registering the highest discounts is Retail Trade. On the contrary, healthcare registered the lowest discount rate.

*Exhibit 10*: Paglia-Harjoto's Study Results by Industry

| DLOM's Multiples By Industry | | | | |
|---|---|---|---|---|
| **Industry** | **Companies** | **DLOMSALE** | **Companies** | **DLOMEBITDA** |
| Mining | 20 | 55,24% | 14 | 47,96% |
| Construction | 14 | 65,46% | 12 | 53,14% |
| Manufacturing | 163 | 55,40% | 91 | 49,63% |
| Wholesale Trade | 22 | 60,93% | 20 | 42,44% |
| Retail Trade | 17 | 70,82% | 16 | 44,45% |
| Transportation | 9 | 57,54% | 9 | 28,99% |
| Information | 63 | 57,19% | 47 | 60,15% |
| Fianance and Insurance | 22 | 70,02% | 13 | 48,08% |
| Real Estate | 5 | 65,51% | 6 | 53,13% |
| Professional Services | 63 | 66,31% | 36 | 67,66% |
| Staff Support and Waste Management | 13 | 66,91% | 6 | 34,21 |
| Healthcare | 9 | 40,68% | 6 | 29,25% |
| Art and Entertainement | 2 | 60,25% | 2 | 17,93% |
| Accomodation and Food Services | 9 | 58,89% | 5 | 68,47% |
| Total | 431 | | 283 | |

As displayed in Exhibit 10, DLOM is significantly higher than the average studies from 1995 onward.

Paglia and Harjoto examined the determinants that influence the DLOM using the following multivariate regression:

$$DLOM(\%) = a + b_0(LOGSALE) + b_1(EBITDA/SALE) + b_2(POSTIVE\ INCOME) + b_3(PUBLIC\ BUYER) + b_4(ASSET\ PURCHASE) + b_5(CCORP) + Sector\ Dummy + Year\ Dummy\ + Error \quad (2.17)$$

The regression model 2.17 is based on the following hypothesis: Privately held companies reporting positive profits show lower discount and the more profitable the firm the lower the discount applied; private companies acquired by public companies will exhibit lower discount due to potential synergies deriving from the transaction; high level of potential financial distress will result in higher discounts; lower liquidity leads to higher discounts and DLOM increases in case of a C corp.

The regression shows that the more profitable a company is, the lower the DLOM. on the other hand, no evidence that the type of firm influences the discount rate.

### *2.4.6 Multiples-Based Approaches Conclusions*

Multiples-based approaches show that private companies are purchased at lower multiples than their public counterparts. Some authors have addressed the whole multiples difference to lack of marketability. The DLOMs in these studies are significantly higher than the discount rate generated from other analyses. Some argue that DLOM in the Multiples-based approach accounts for other factors in addition to marketability.

# Chapter 3

# Theoretical Models to Estimate the DLOM

## 3.1 Theoretical Models

The DLOM computations have raised great interest among practitioners and appraisers. Some focused on further extending empirical models. Others explored new ways of addressing the right discount for non-controlling ownerships' valuation in private companies. The second strand of models is referred to as Theoretical models.

While empirical models focus on the pricing of transactions involving either restricted securities or private companies, theoretical models "are based on fundamental microeconomic relationships"[88]. They consider firm-specific parameters.

There are two types of Theoretical models:

- Option Pricing Models
- Discount Cash Flow Models

## 3.2 Option Pricing Models

Put options and securities affected by Rule 144 share some similarities. For instance, restricted stocks have reselling constraints, which can be associated with the loss of value in holding a put option until you can sell the underlying asset.

Appraisers started investigating the possibility of using put options[89] to assess the DLOM's impact on the pricing of non-controlling interest shares in private companies. The frameworks are based on assuming that purchasing stock options to sell restricted stocks at their free-market value, means that the investor has bought marketability for those shares.

In the case of a European put option, the investor can sell the underlying assets at a predetermined value at the expiration date. Therefore, if the put options' strike price is the market value at which restricted stocks can be sold once the limitations on their

[88] Lance, Travis R. 2007. *The Use of Theoretical Models to Estimate the Discount for Lack of Marketability*. Insights.

[89] A put option gives the owner the right, but not the obligation, to sell the underlying asset at a planned price within a period.

resell are relaxed, the put option's price is the value of marketability for the subject shares.

Option pricing studies all reach similar conclusions. However, they present several pitfalls. The first, and major flaw, is referred to as inputs considered in the models. Option prices are mostly influenced by both the holding period and the underlying asset's volatility. "Option pricing studies may understate the DLOM because they ignore other factors that may reduce the marketability for a privately held company"[90]. Moreover, experts question the suitability of using option-based models to portray the discount related to marketability. According to Aswath Damodaran, the value of a put option is not related to its liquidity, instead, it is the amount an investor is willing to offer as insurance on the underlying security[91]. Finally, put options are often issued on listed companies. It is extremely rare to witness options on private companies' equity. Therefore, option-based studies compute the DLOM using liquid assets and their results can be considered slightly downward biased.

The major option pricing models are:

- The Chaffe Study
- The Longstaff Study
- The Finnerty Study

### *3.2.1 The Chaffe Study*

In 1993, David B.H. Chaffe published an analysis of DLOM computations. He argues that the discount for lack of marketability in minority interests' of privately held companies, can be linked to the price of a European put option.

The intuition behind this study is that, according to Chaffe, "if one holds restricted or non-marketable stock and purchases an option to sell those shares at the free market price, the holder has, in effect purchased marketability for those shares"[92]. The price of the subject put option represents the discount for lack of marketability. Chaffe used the

[90] Reilly, Robert F. and Rotkowski, Aaron. 2007. *The Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies*. The Tax Lawyer.

[91] Damodaran, Aswath. 2005. *Marketability and Value: Measuring the Illiquidity Discount*. Stern School of Business.

[92] Chaffe, David B.H. 1993. *Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations. Business Valuation Review.*

Black-Scholes formula to price put options in his analysis. The Black-Scholes' model values the option depending on stock prices assuming "ideal conditions in the market for the stock price and the option"[93]. For instance, stock prices follow a log-normal distribution; no dividends are paid; the option considered must be European and no transaction costs are present when buying or selling either the option or the stock. The following formula is the Black-Scholes model used by Chaffe:

$$w(x,t) = xN(d_1) - ce^{r(t-t^*)}N(d_2) \tag{3.1}$$

$$d_1 = \frac{ln\, x/c + (r + \frac{1}{2}\delta^2)(t^* - t)}{\delta\sqrt{t^* - t}} \tag{3.2}$$

$$d_2 = \frac{ln\, x/c + (r - \frac{1}{2}\delta^2)(t^* - t)}{\delta\sqrt{t^* - t}} \tag{3.3}$$

The main inputs in equation 3.1 are stock price (x), strike price in the put option (c), time to the expiration date ($t\text{-}t^*$), volatility (δ), and finally the interest rate (r) which is constant through time by assumption.

For his study, Chaffe considered both stock price and strike price to be equal to the price of the subject company's shares as if it were marketable. Moreover, the time to maturity is set equal to the time needed for the restricted stock to become marketable. The constant interest rate is changed into the cost of capital. Finally, the volatility "is a judgmental factor based on the volatility of guideline publicly traded stocks"[94].

By looking at OTC companies, Chaffe argued that volatility for private companies could range from 60% to 90%. Some authors claim that terminal volatility can easily exceed 100% making log-normal distribution less accurate[95].

DLOM ranged from 28% to 41% depending on the volatility level from 60% to 90% considering marketability restrictions set at 2 years.

[93] Black, Fischer and Scholes, Myron. 1973. *The Pricing of Options and Corporate Liabilities.* The Journal of Political Economy.

[94] Reilly, Robert F. and Rotkowski, Aaron. 2007. *The Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies*. The Tax Lawyer.

[95] Brooks, Robert. 2013. *A General Option Valuation Approach to Discount for Lack of Marketability.* p 6-7

According to the author, results are a lower bound and a minimum applicable to discount for lack of marketability in case of non-controlling ownership.

### *3.2.2 Longstaff Study*

In 1995, Francis A. Longstaff published his study, presenting further conclusions on option studies for DLOM computation. He derives the maximum discount rate applicable, using look-back options. These types of options are substantially different from put options used in the Chaffe study. An investor, holding a look-back option, can check all the prices the underlying asset has reached during the option life after the purchase, and exercise the option choosing the price that brings the maximum benefits to the investor. This leads to the major assumption in Longstaff's study. The author assumes that investors have perfect timing and the market has no frictions. Although that hypothesis might sound idealistic, "strategic investors and insiders might have valuable private information that would enable them to time the market were it not for the transferability restrictions"[96].

The framework developed by Longstaff considers the investor holding a stock in which resells is limited until period T. The value of the stock is $V_T$ depends on the cash flows it generates up to time T when its restrictions are relaxed. Considering the investor having perfect timing, he can "sell the security and reinvest the proceeds in the riskless asset at the time $\tau$ that maximizes the value of his portfolio"[97]. $M_T$ is the payoff the investor would achieve if he could perfectly sell the underlying asset. The perfect timing's value is represented by the incremental cash flow $M_T - V_T$.

An investor generally cannot have perfect timing. Therefore, the Longstaff model presents an upper bound. He portrays the maximum trade-off an investor can achieve. The present value of the cash flows derived from $M_T - V_T$ "can be determined using standard risk-neutral valuation techniques familiar from option-pricing theory"[98]. Considering F(V, T) the above mentioned present value, then it is true that:

[96] Finnerty, John D. 2002. *The Impact of Transfer Restrictions on Stock Prices*.
[97] Longstaff, Francis A. 1995. *How Much Can Marketability Affect Security Values?*. The Journal of Finance.
[98] Longstaff, Francis A. 1995. *How Much Can Marketability Affect Security Values?*. The Journal of Finance.

$$F(V,T) = e^{-rT}E[M_T] \; - \; e^{-rT}E[V_T] \qquad (3.4)$$

Considering E as expected values in a normal density function, equation 3.4 can be expressed as follows:

$$F(V,T) = V\left(2 + \frac{\vartheta^2 T}{2}\right) N\left(\frac{\sqrt{\vartheta^2 T}}{2}\right) + V\sqrt{\frac{\vartheta^2 T}{2\pi}}\, exp\left(-\frac{\vartheta^2 T}{8}\right) - V \qquad (3.5)$$

Equation 3.5 shows that the discount for lack of marketability can be expressed as a percentage of the value of the stock. Moreover, the discount increases the more the restrictions on the stock last. The variance strongly influences the DLOM. This is consistent since high volatility in returns results in high potential losses if the investor had perfect timing.

Applying equation 3.5 having V set equal to $100 and variances at 10%, 20%, and 30% as applied by Longstaff, the DLOM are the following:

*Exhibit 11*: Longstaff's DLOMs by Variance

| Marketability Restriction (T) | σ = 10% | σ = 20% | σ = 30% |
|---|---|---|---|
| 1 Day | 0,421% | 0,844% | 1,268% |
| 5 Days | 0,944% | 1,895% | 2,852% |
| 10 Days | 1,337% | 2,688% | 4,052% |
| 20 Days | 1,895% | 3,817% | 5,768% |
| 30 Days | 2,324% | 4,691% | 7,100% |
| 60 Days | 3,299% | 6,683% | 10,153% |
| 90 Days | 4,052% | 8,232% | 12,542% |
| 180 Days | 5,768% | 11,793% | 18,082% |
| 1 Year | 8,232% | 16,984% | 26,276% |
| 2 Years | 11,793% | 24,643% | 38,605% |
| 5 Years | 19,128% | 40,979% | 65,772% |

One major finding is that depending on the volatility of the stock, the DLOM can substantially change even in a matter of days. Moreover, as the author stated, "the upper bound can also be viewed as the maximum amount that any investor would be willing to pay to obtain immediacy in liquidating a security position"[99].

[99] Longstaff, Francis A. 1995. *How Much Can Marketability Affect Security Values?*. The Journal of Finance.

According to Longstaff, volatilities ranging from 10% to 30% are consistent with the average returns' volatilities of stocks. On the other hand, Chaffe reported that most companies having their shares restricted under Rule 144 are OTC firms, which can produce return volatilities up to 90%. Extending the Longstaff formula using a range of volatility from 60% to 90% leads to the following results:

*Exhibit 12: Longstaff's Model DLOMs Applying High Variances*

| Marketability Restriction (T) | σ = 60% | σ = 70% | σ = 80% | σ = 90% |
|---|---|---|---|---|
| 1 Day | 2,548% | 2,978% | 3,409% | 3,841% |
| 5 Days | 5,768% | 6,754% | 7,748% | 8,748% |
| 10 Days | 8,232% | 9,654% | 11,091% | 12,542% |
| 20 Days | 11,793% | 13,860% | 15,956% | 18,082% |
| 30 Days | 14,587% | 17,171% | 19,801% | 22,475% |
| 60 Days | 21,093% | 24,921% | 28,841% | 32,856% |
| 90 Days | 26,276% | 31,131% | 36,128% | 41,269% |
| 180 Days | 38,605% | 46,020% | 53,735% | 61,755% |
| 1 Year | 57,588% | 69,235% | 81,519% | 94,459% |
| 2 Years | 87,716% | 106,673% | 127,009% | 148,774% |
| 5 Years | 159,901% | 198,514% | 241,041% | 287,638% |

As predicted, the higher the volatility, the higher the difference in discount rate among close dates. In the case of high volatilities and the discount rate can exceed 100%, which is not possible. It is important to note that this is not a DLOM, it is just the upper bound, so the maximum DLOM possible.

### *3.2.3 Finnerty Study*

John D. Finnerty published his study on discounts on restricted stocks in 2007. He conducted many analyses: the first is similar to restricted stock studies, then he ran three regression analyses, and finally, he developed his option-based model.

Finnerty tested his intuition that "a 'private placement discount' may reflect both a 'marketability discount' as well as information and equity ownership concentration effects, although the relative importance of the two sets is unresolved"[100].

Finnerty first conducted the restricted stock analysis. He selected privately placed stocks from 1991 to 2005 using the 10kwizards database. The preliminary sample consisted of 348 transactions. The author removed offerings including non-US companies' stocks;

[100] Finnerty, John D. 2007. *The Impact of Transfer Restrictions on Stock Prices*.

restricted stocks performed by utilities and depository institutions; shares that were announced less than five months than the date of execution and shares whose companies revealed noteworthy information five to ten days before the private placement announcement, due to potential price bias. The final sample amounted to 244 deals.

Finnerty computed two discounts. One is related to the closing price registered the day before the private placement has been priced. The other discount is computed considering the closing price ten days after the announcement has been disclosed.

The reason for the second type of discount is to test how share price changes since generally "market react to the information contained in the private placement announcement"[101]. Moreover, considering that in 1997 the SEC relaxed restriction time on privately placed shares, Finnerty divided data before and after 1997. Results are the following:

*Exhibit 13: DLOMs Before and After the 1997 Change*

| | February 1997 | |
|---|---|---|
| | Before | After |
| Day Prior | 21,00% | 15,94% |
| 10 Days After | 21,61% | 18,70% |

Comparing the same class of discount before and after 1997 shows how discount decreased. This is due to lower restrictions to resell of privately placed stocks. Results are not significant, probably due to information and ownership affecting discount rates. By considering discount rates in cases of the large market value of equity and gross proceeds, Finnerty concludes that smaller companies are exposed to larger information asymmetries. Additionally, a larger block of shares requires higher discount rates due to larger costs and time required by the investor to estimate the value.

Finnerty then ran three regressions to investigate what variables affect the discount for lack of marketability the most. He included several determinants: a dummy variable with a value of 1 if he firm issued stocks privately before 1997, 0 otherwise; the boot value

[101] Finnerty, John D. 2002. *The Impact of Transfer Restrictions on Stock Prices*.

to market value ratio, to consider possible market reactions after private issue's announcement; the rights' dummy variable, having a value equal to 1 if the holder has registration rights, 0 if not; registered is another dummy variable, assuming the value of 1 if the company issued stocks after 45 days from the private issue. Finally, both time and volatility are considered in the regression.

Regression's results display that volatility and time account for the largest part of discount rates applied to privately placed stocks. The conclusion complies with Longstaff's thinking in paragraph 3.1.2.

Considering the importance of volatility and time, option-based models appear to be accurate for DLOM computation. Therefore, Finnerty develops his Average-Strike put option[102] model.

The model is based on several assumptions. For instance, restricted stocks have reselling limitations over the period T; for simplicity, the market is considered frictionless; dividends (q) are paid at a fixed rate; the risk-free interest rate (r) is constant over the period T. Finally, the investor does not have perfect timing ability. Although Finnerty himself admits that "Longstaff's model may be more appropriate in the presence of asymmetric information"[103].

Considering the assumptions and regressions' results, the value of the discount using an average-strike option is the following:

$$D(T) = V_0 \left[ e^{(r-q)T} N\left(\frac{r-q}{v}\sqrt{T} + 1/2\ v\sqrt{T}\right) - N\left(\frac{r-q}{v}\sqrt{T} - 1/2\ v\sqrt{T}\right)\right] \quad (3.6)$$

And

$$v^2 = \sigma^2 T + ln\left[2\left\{e^{\sigma^2 T} - \sigma^2 T - 1\right\}\right] - 2ln\left[e^{\sigma^2 T} - 1\right] \quad (3.7)$$

Applying both equation 3.6 and 3.7, assuming that if dividends are paid, thy equal to q=0,02 and the risk-free rate is 5% the DLOMs are:

[102] It is an Option having the strike price equal to the average price assumed by the underlying asset over a specified period.

[103] Finnerty, John D. 2002. *The Impact of Transfer Restrictions on Stock Prices*.

*Exhibit 14: Finnerty's Study DLOMs with No Dividends*

| | DLOM Considering No Dividends (q = 0) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| T | σ = 0,1 | σ = 0,2 | σ = 0,3 | σ = 0,4 | σ = 0,5 | σ = 0,6 | σ = 0,7 | σ = 0,8 |
| 1 Day | 0,02% | 0,02% | 0,03% | 0,03% | 0,04% | 0,05% | 0,05% | 0,06% |
| 5 Days | 0,08% | 0,10% | 0,13% | 0,17% | 0,20% | 0,23% | 0,26% | 0,29% |
| 10 Days | 0,16% | 0,21% | 0,27% | 0,33% | 0,39% | 0,46% | 0,52% | 0,58% |
| 20 Days | 0,31% | 0,42% | 0,54% | 0,66% | 0,79% | 0,91% | 1,04% | 1,17% |
| 30 Days | 0,47% | 0,63% | 0,81% | 1,00% | 1,18% | 1,37% | 1,56% | 1,75% |
| 60 Days | 0,94% | 1,26% | 1,62% | 1,99% | 2,37% | 2,74% | 3,12% | 3,49% |
| 90 Days | 1,41% | 1,89% | 2,44% | 2,99% | 3,55% | 4,11% | 4,67% | 5,23% |
| 180 Days | 2,84% | 3,81% | 4,90% | 6,01% | 7,12% | 8,22% | 9,31% | 10,38% |
| 1 Year | 5,76% | 7,71% | 9,88% | 12,09% | 14,27% | 16,40% | 18,45% | 20,42% |
| 2 Years | 11,81% | 15,76% | 20,12% | 24,43% | 28,56% | 32,42% | 35,97% | 39,15% |
| 5 Years | 31,84% | 42,08% | 52,64% | 62,13% | 70,09% | 76,39% | 81,07% | 84,32% |

*Exhibit 15: Finnerty's Study Results with Dividends*

| | DLOM Considering Dividends (q = 0,02) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| T | σ = 0,1 | σ = 0,2 | σ = 0,3 | σ = 0,4 | σ = 0,5 | σ = 0,6 | σ = 0,7 | σ = 0,8 |
| 1 Day | 0,01% | 0,02% | 0,02% | 0,03% | 0,04% | 0,04% | 0,05% | 0,06% |
| 5 Days | 0,06% | 0,09% | 0,12% | 0,15% | 0,18% | 0,21% | 0,25% | 0,28% |
| 10 Days | 0,11% | 0,17% | 0,24% | 0,30% | 0,36% | 0,43% | 0,49% | 0,55% |
| 20 Days | 0,23% | 0,35% | 0,47% | 0,60% | 0,73% | 0,85% | 0,98% | 1,11% |
| 30 Days | 0,34% | 0,52% | 0,71% | 0,90% | 1,09% | 1,28% | 1,47% | 1,66% |
| 60 Days | 0,69% | 1,05% | 1,42% | 1,80% | 2,18% | 2,56% | 2,93% | 3,31% |
| 90 Days | 1,03% | 1,57% | 2,13% | 2,70% | 3,27% | 3,83% | 4,39% | 4,95% |
| 180 Days | 2,07% | 3,15% | 4,27% | 5,40% | 6,52% | 7,63% | 8,72% | 9,79% |
| 1 Year | 4,17% | 6,34% | 8,58% | 10,81% | 13,00% | 15,12% | 17,17% | 19,12% |
| 2 Years | 8,46% | 12,82% | 17,27% | 21,58% | 25,68% | 29,50% | 33,00% | 36,14% |
| 5 Years | 22,08% | 33,06% | 43,54% | 52,77% | 60,45% | 66,51% | 71,00% | 74,11% |

The tables show that dividend-paying stocks are issued at a lower discount than their non-dividend counterpart. What is more, results are within Longstaff's discounts shown in Exhibit 11, giving further proof of Longstaff's model being an upper bound.

### *3.2.4 LEAPS Study*

In 2003, Robert Trout presented his Long-Term Equity Anticipation Securities (LEAPS) study. It is a long-term put option. A LEAP grants price protection, against possible contractions in the underlying security value. The insurance given by such a put option lasts for 2 years, which is very close to the restriction period affecting privately issued stocks. Trout "examined the costs of buying LEAPS puts and determined the relative

insurance cost by dividing the put cost by the underlying stock price"[104]. He reported the LEAPS puts costs for eight large, listed companies. The average LEAPS' prices presented a premium of 24% on the underlying stock's market value. Trout concluded that the average discount for lack of marketability is 24,6%, and a median of 22,9%.

Ronald Seaman extended Trout's findings in 2005. His goal was to show how impactful time and risk are in drawing discounts using LEAPS. Computing the costs of issuing LEAPS for 100 casually selected listed firms, he concluded the following results:

*Exhibit 16: Impact of Time and Risk on DLOM*

| | | Safety Ranks | | |
|---|---|---|---|---|
| | | 1 | 3 | 5 |
| β | Average | 0,82 | 1,10 | 1,87 |
| | Median | 0,8 | 1,10 | 1,88 |
| 1 Year Discount | Average | 7,0% | 10,6% | 20,0% |
| | Median | 7,3% | 9,2% | 17,0% |
| 2 Year Discount | Average | 9,4% | 14,9% | 30,6% |
| | Median | 9,3% | 13,8% | 31,0% |

The safety ranks indicate how risky the data are. A safety rank of 1, means the volatilities are low. While a safety rank of 5 considers risky companies.

As it is possible to see, the lower the safety rank is, the lower the discounts. And as the holding period increases, the DLOM increases as well.

Therefore, the riskier the company is, the bigger the discount rate, and a longer time frame results in an increasing DLOM.

Finally, Seaman presented his second analysis, again published in 2005. He analyzed the impact of size on the discount rate. The sample increased to 261 companies. Seaman concludes that large-sized companies, in terms of revenues and assets, face lower discount rates than otherwise smaller companies.

Arguments have been moved toward the reliability of LEAPS' results for different reasons. First, the underlying asset of a LEAPS is traded stocks, not facing any resell constraints. This is very far from restricted stocks or private companies' equity. Moreover, companies considered in the study are larger than companies facing liquidity issues. Finally, LEAPS can be sold at any time.

[104] Dorrell, Darrell D., Gadawski, Gregory A. and Brown, Thomas S. 2008. *2008 Update: Marketability Discounts, A comprehensive Analysis.* The Value Examiner.

For these reasons "the authors of the three LEAPS studies concluded that the observed DLOMs are appropriately viewed as benchmark minimum price discount when applied to privately held companies"[105].

### *3.2.5 Option-Based Studies Conclusions*

Results derived from the four mentioned option-based studies are similar to empirical models DLOMs. Nonetheless, such studies give new insights on the discount for lack of marketability. For instance, option pricing established the importance of volatility in DLOM. According to Chaffe's study, privately held companies' volatility amounts to at least 60%. On the contrary, Longstaff argues that volatility should be not higher than 30%. An appraiser using option-based models need to pay attention to correctly estimate the subject private company's volatility,

Finally, the second conclusion is referred to as the importance of holding period. The studies covered up to 5 years but holding non-marketable securities can lead to longer holding periods.

## 3.3 Discounted Cash Flow Models

The second type of theoretical approach is discounted cash, flow models. They use quantitative inputs to derive the value of the company's share.

Computing the value of illiquid shares, based on factors such as dividends, holding period, and level of sales can establish the discount attributable to lack of marketability. The most important contributions on quantitative models are given by:

- QMDM model
- Tabak Model
- AECA Model

### *3.3.1 The QMDM Model*

In 1997, Christopher Z. Mercer published his study concerning a quantitative model that, depending on inputs used, resulted in the appropriate discount for lack of marketability. According to Mercer, the Quantitative Marketability Discount Model (QMDM) "provides

[105] Reilly, Robert F. and Rotkowski, Aaron. 2007. *The Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies*. The Tax Lawyer.

a standardized format for analyzing, projecting, and discounting relevant shareholder cash flows that apply to almost any subject nonmarketable minority interest"[106].

The framework designed by Mercer is made of different steps an appraiser has to follow to end up with the subject company's DLOM.

The first step is to value the company using the DCF method. Mercer used the two-stage DCF approach to compute the stocks' value as if they were marketable. The following formula is made of two segments: the first one referred to the company's value based on the present value of future cash flows for *f* years. The second part of the formula is referred to as the terminal value.

$$V_e = \sum_{i=1}^{f} \left[ \frac{CF(1+g_e)^i}{(1+r)^i} \right] + \left[ \frac{CF(1+g_e)^{(f+1)}/(r\ -\ g)}{(1+r)^f} \right] \qquad (3.8)$$

Where: CF is yearly cash flows, g is the expected growth and r is the risk.

The next step is computing the shareholders' value. The formula used is the same as 3.8 with changes in inputs to have a result that better represent the value shareholders get by owning shares of the subject company. In this variation of equation 3.8, *f* represents the holding period an investor expects to own the shares. $CF_{sh}$ refers to cash flow generated by dividends to which is applied the expected dividends yearly growth rate $g_d$. On the side of terminal value, $g_v$ "defines the terminal enterprise value in terms of an anticipated annual capital appreciation from the current enterprise value"[107]. Sometimes, shareholders will be rewarded for keeping non-marketable participation. Or in other cases, they might face discounts. Either way, the model has the P/D% input that refers to the premium/reduction on the firm's value. Finally, investors are likely to have a minimum threshold of returns they expect from the risk they incur by holding illiquid shares. This last input is represented by $R_{hp}$.

Considering all the shareholders' level inputs, the DCF formula at the shareholder level is:

[106] Mercer, Christopher Z. and Harms, Travis W. 2007. *Business Valuation: An Integrated Theory.* 2nd ed. John Wiley & Sons, Inc. Hoboken, New Jersey. p172

[107] Mercer, Christopher Z. and Harms, Travis W. 2007. *Business Valuation: An Integrated Theory.* 2nd ed. John Wiley & Sons, Inc. Hoboken, New Jersey. p174

$$V_{sh} = \sum_{i=1}^{f} \left[ \frac{CF_{sh}(1+g_d)^i}{(1+R_{hp})^i} \right] + \left[ \frac{V_e(1+g_v)^f(1+P/D\%)}{(1+R_{hp})^f} \right] \quad (3.9)$$

Once the appraiser has identified both the company's value at shares' value (3.8) and shareholders value (3.9) the DLOM is obtained as follows:

$$MD = 1 - \frac{V_{sh}}{V_e} \quad (3.10)$$

Mercer developed a first attempt at creating a framework, based entirely on firms' characteristics and where no benchmarking was needed. The main issue is that results given by this model dramatically change when a slight change is applied to one of its inputs. This strongly mined its reliability, especially in tax courts. For instance, Judge Whalen in Estate of Weinberg v. Commissioner[108], turned down the valuation produced by the QMDM due to its large difference in the DLOM if the holding period was changed to 10 to 15 years.

### *3.3.2 Tabak Model*

In 2002 David Tabak presented his discounted cash flow model. It is an extension of the Capital Assets Pricing Model (CAPM). The CAPM is generally used to estimate returns of listed companies' stocks. Tabak then added the risk an investor faces when holding restricted stocks and non-marketable assets.

In the case of private company valuation, Tabak argues that to account for its non-marketability, the equity risk premium needs to be added to the WACC used in the discounted cash flow. The formula is:

$$WACC_{private} = WACC + \left( \frac{\sigma_i^2}{\sigma_m^2} - \beta \right) \theta \quad (3.11)$$

Where $\sigma_i^2$ is the standard deviation of the return of non-marketable asset i. $\sigma_m^2$ is referred to as the standard deviation of return of the market portfolio. β is the covariance and θ is the equity risk premium.

[108] https://www.leagle.com/decision/2000158679aytcm150711536

Differently from Mercer's model, Tabak's only has the estimated time the asset i is expected to be restricted from reselling, as a subjective variable.

### *3.3.3 Rojo-Ramirez Study*

In 2013 Alfonso Rojo-Ramirez provided further information on the use of the CAPM for estimating privately held companies. The standard CAPM formula is not accurate for non-listed companies since they are generally riskier than their traded matches. Moreover, no beta can be estimated for private companies. The author proposes his framework based on the behaviorism of investors.

Rojo-Ramirez first distinguishes between two types of investors. The Purely Financial Investor (PFI) and the Economic Risk Investor (ERI). The PFI invests in the market. He has a well-diversified portfolio that minimizes the risk and maximizes the profits. The PFI rate of return is:

$$k_e = r_f + (r_m - r_f) = R_f + P_m \tag{3.12}$$

Equation 3.12 states that a PFI requires a return of at least the return of a riskless asset (Rf) plus the market risk premium (Pm).

The second category of investors is the ERI which is usually the owner of a small and medium enterprise. He cannot diversify his investments. He does not have a portfolio of investments; he just has one. Therefore, an ERI faces a higher risk than a PFI. He will require his returns to be at least equal to PFI's plus a premium that reflects the return volatilities of the non-marketable asset and the market portfolio. The equity return required by an investor, investing in privately held companies in the following 3 component CAPM variation:

$$k_e = R_f + P_m + P_m \frac{\sigma_e}{\sigma_m} \tag{3.13}$$

Rojo-Ramirez then applied his model on 712 Spanish SMEs (683 were non listed companies and the remaining 29 were public companies). The data was collected through the SABI database, from 2002 to 2007.

Results show that applying the ERI model to the valuation of SMEs "corrects the overestimated value of a company by applying the discount rate of the CAPM, which is consistent with professional practice"[109].

The results portray a discount rate that averaged between 29% and 49%, depending on the industry.

A limitation affecting the model is that it is entirely based on Spanish companies, which might bias the result due to firms' characteristics.

### *3.3.4 Discounted Cash Flow Approaches Conclusions*

The Discounted Cash Flow methods are correct at the theoretical level. The main issue that leads both the QMDM and the Tabak's model to be very little used is the high subjectivity of some inputs. On top of that, a slight change in the input can create contrasting results. Therefore, an appraiser should apply the DCF models only when most, if not all, the inputs are objectives and well defined.

[109] Rojo-Ramirez, Alfonso A. 2013. *Privately Held Company Valuation and Cost of Capital*. Journal of Business Valuation & Economic Loss Analysis.

# Chapter 4

# A North Italy Investigation

## 4.1 Model Selection

Framework selection is crucial for collecting the right data and drawing results as reliable as possible. Depending on the model, one might need restricted stocks, IPO prospectuses, or balance sheet data.

For my study, I decided not to adopt the Restricted Stock Study approach[110] for several reasons. According to such analysis, the DLOM should be attributed entirely to the observable price-percentage difference between a company's common and restricted stocks. Some authors argue that this leads to a rather shallow discount representation. The value difference indeed demonstrates the existence of a price reduction for non-marketable assets, but it also incorporates other factors, such as the subject company's risk and financial distress, the cost of obtaining information, and the percentage of total shares offered in the deal[111]. Moreover, restricted stocks only have transfer impairment for a limited time. Meaning that they are non-marketable for a fixed period. Before 1997, the restriction was two years of holding period for the investors, between 1997 and 2008 it was reduced to one year, and now it is six-months. Therefore, restricted stock studies base their DLOM on temporarily restricted assets, which are very likely to become marketable once the mandatory holding period expires. On the contrary, a private company's shares might never become marketable and can be hard to sell. Finally, it is difficult to retrieve information on restricted stocks and the final sample could have been rather small, resulting in statistically non-significant conclusions.

I also rejected the Pre-IPO approach[112]. Although the method is straightforward: the DLOM is computed as the difference in shares' value before and after the IPO, finding data is somewhat difficult. The first firm valuation needs to be made around five months before the IPO. If the estimation date is closer to the IPO, then the price might be inflated. On the contrary, if the valuation is performed too early, there might have been

[110] Chapter 2, Paragraph 1

[111] Bajaj, Mukesh, Denis, David J., Ferris, Stephen P., and Sarin Atulya. 2001. *Firm Value and Marketability Discounts*. Journal of Corporation Law.

[112] Chapter 2, Paragraph 2

market changes, affecting shares' prices. So, despite prospectuses giving detailed information about the firm going public, valuing its shares within the five months is still very challenging. Moreover, the yearly successful IPOs' average in the Italian stock exchange market is 30. According to the Borsa Italiana website: in 2019, 35 IPOs took place. In 2018 and 2017 a total of 32 successful IPOs took place each year, and in 2016 only 14 companies went public[113]. Therefore, several years need to be examined to have a statistically significant sample. Incorporating companies being valued in different market dynamics can result in misleading DLOM results. Finally, many argue that IPOs' prices are inflated depending on investors' demand[114]. Comparing the value of a company's shares before and at the IPO date might not be an effective way of researching the discount for lack of marketability. It might be more precise to consider companies' prices sometime after the IPO date when the shares' prices better represent their actual value.

I decided not to use the Multiples-based models[115] mainly due to data problems. These models are based on finding a private company similar and comparable enough to a public counterpart. The reliability of the study depends on how similar the companies being matched are. Finding two comparable companies while having access to limited data can be complex. Additionally, the differences in multiples are entirely attributed to a discount related to the difference in marketability the two companies have. Due to high DLOMs reported by multiple-based studies, appraisers argued that these models incorporate systematic differences among private and public firms, instead of the discount for lack of marketability itself.

I did not use an Option-Based Model[116] since stock options only exist for listed companies. There is no option for a privately held company's shares. It is impossible to purchase liquidity for a non-marketable asset. Most Option-Based models price the option, and the subsequent marketability, using the Black-Scholes Model (BSM). This framework value an option according to different variables[117] and some are extremely

[113] https://www.borsaitaliana.it/azioni/ipoematricole/paginaipo2019.htm
[114] Derrien, Francois. 2005. *IPO Pricing in "Hot" Market Conditions: Who Leaves Money on the Table ?.* The Journal of Finance.
[115] Chapter 2, Paragraph 3
[116] Chapter 3, Paragraph 1
[117] Chapter 3, p.58

difficult to compute when they are referred to private companies. For instance, the volatility and the holding period cannot be computed precisely, and they account for the most value using the BSM. In Chaffe's model, the volatility is a judgmental factor, while the holding period equals the restricted stocks' holding period. The main issue is that today, privately-held stocks are likely to be non-marketable for more than six months. Some appraisers critique Longstaff results. According to them "such an estimate would be speculative and unreliable for several reasons"[118]. First, the investor with perfect marketability is an impracticable hypothesis. Moreover, Longstaff himself consider his model to be an upper-bound for the DLOM. Finally, the Finnerty model is the most reliable and used option-based model[119]. It works well with relatively low volatilities, but as shown in Exhibit 14 and Exhibit 15, once both the firm's volatility and holding period are increased, the model might result in high discounts for lack of marketability. Considering that the volatility for non-marketable assets paired with the long-expected holding period can be extremely high[120], the Finnerty model's results must be used cautiously.

Considering that I am investigating the discount related to lack of marketability, I decided to reject the above-mentioned models which strongly rely on liquid markets' data and dynamics. I opted for Discount Cash Flow (DCF) Approaches[121] which are not based on marketable securities, rather they consider the lack of marketability's premium at shareholders' level.

I rejected Mercer's QMDM model due to some of its inputs being subjective. What is more, slight changes in the inputs produced dramatically different results. I wanted to adopt a highly objective model.

I did not use the Tabak Model either. One of the major inputs is the expected holding period which cannot be estimated, and it is highly subjective.

---

[118] Hitchner, James R. 2011. *Financial Valuation: Application and Models.* John Wiley & Sons, Inc. Hoboken, New Jersey. Third Ed. Chapter 9 p.24.

[119] Duffy, Robert E. 2011. *Why Finnerty's Put Option Model is the DLOM Model of Choice.* Financial Valuation Litigation Expert

[120] Brooks, Robert. 2013. *A General Option Valuation Approach to Discount for Lack of Marketability.* p 6-7

[121] Chapter 3, Paragraph 3

I decided to use the Rojo-Ramirez model which computes the DLOM as a difference in the company's value using the cost of equity (Ke) of an Owner-Investor and the Ke of an investor with the possibility of creating a well-diversified portfolio. The premium for the owner-investor is computed as the volatility of the subject private company's return over the overall market return, as shown in equation 3.13. The only market data considered in the model are the risk-free rate and the market return, which are objective.

The model gives a great contribution to DLOM computations. For instance, it shows and proves that privately-held companies grant a higher return on equity than that of public companies. What is more, the premium calculated using this model is computed directly by comparing the private company's performance with the market. Therefore, no subjective or mean data are used. Each company in the study shows the premium an investor requires for investing in that specific firm. The results display the average, but each company has its own DLOM, premiums, and ROEs.

Nonetheless, the model displays some flaws. For instance, there is no focus on a company's assets. A private company owning several non-marketable properties might be subject to a higher DLOM than a similar company with more marketable assets[122]. What is more, the number of shareholders in the subject companies has been ignored. This can strongly impact the marketability value of private companies' shares, since the more the stakeholders the higher the DLOM.

### 4.2 The Model's Hypotheses

The Rojo-Ramirez model is based on one main concept: the owner-investor will require a higher rate of return than an investor having a well-diversified portfolio. An investor faces two types of risk: the systematic risk and the idiosyncratic risk. The first one is market risk. It cannot be lowered since it does not depend on the assets owned by the investor, rather it affects the whole market. The idiosyncratic risk is at shares level. It indicates the possibility that an adverse market movement may lower the value of the asset itself. Diversification can lower the idiosyncratic risk. An owner-investor that invests most of his wealth in a company cannot achieve diversification, and therefore

[122] Fazzini, Marco. 2018. *Valutazione D'azienda. Premio di Maggioranza e Sconto di Minoranza.* Fazzini & Partners

will require higher returns compared to a portfolio's owner[123]. What is more, Elisabeth Mueller, investigating the impact of risk for entrepreneurial investors, discovered that "the degree of risk exposure has a statistically and economically significant positive influence on return on equity"[124].

An investor exploiting the market to create a well-diversified portfolio is called a "purely financial investor". On the other hand, the owner-investor putting all his resources in one company is defined as "economic risk investors". The two, hereafter, will be referred to as PFI and ERI, respectively.

The rate of return required by a PFI is the capital assets pricing model (CAPM).

The CAPM equation is the following:

$$k_e = R_f + \beta_i(R_M - R_f) \quad (4.1)$$

Where $R_f$ is the riskless asset; β indicates the correlation between the share I and the market; $R_M$ is the return expected by investing in the market. Equation 4.1 demonstrates that an investor, investing in listed and marketable shares will require a return at least higher than a riskless asset, plus the expected return that the shares are likely to generate. The second part of the equation represents a premium that the investor requires investing in risky, yet marketable, assets. A PFI having the possibility of constructing a diversified portfolio will try to achieve a portfolio's beta (β) of 1. Equation 4.1 can be rewritten as follows:

$$k_e = R_f + (R_M - R_f) = R_f + P_M \quad (4.2)$$

Where $P_M$ is the difference between the expected return of the market and the risk-free rate. It is the premium return one expects to gain for investing in risky securities.

This leads to the first hypothesis of the model:

[123] Kerins, Frank. Smith, Janet K. and Smith Richard. 2004. *Opportunity Cost of Capital for Venutre Capital Investors and Entrepreneurs.* The Journal of Financial and Quantitative Analysis.

[124] Mueller Elisabeth. 2010. *Returns to Private Equity – Idiosyncratic Risk Does Matter!.* Review of Finance.

H1: "Investors' financial profitability in QCs ($ROE_M$), […] should be greater than the minimum rate of return $R_M$ to safeguard their wealth.[125]

In privately held businesses, the owner-investor has put most of his wealth in his company and he possesses most of the shares. According to Rojo-Ramirez "practitioners use the capital assets pricing model […] to estimate the cost of equity capital (Ke) […] of the owner investor"[126]. The CAPM works for PFIs, but it defective when it comes to ERIs' rate of return. A rational investor interested in financing private companies will only accept a rate of return higher than that of PFIs. If this was not the case, it would be unreasonable to invest in private companies, since less-risky assets would generate higher returns. Thus, an ERI's rate of return will be equal to that of the PFI plus a premium ($P_e$) to offset the higher idiosyncratic risk he faces.

The minimum rate of return required by an ERI can be expressed as an extension of equation 4.2:

$$k_e = R_f + P_M + P_e \tag{4.3}$$

Considering this rational investor's behavior, the second assumption is:

H2. The minimum rate of return required by an investor in private companies (ERI) must be larger than that of an investor having a diversified portfolio (PFI).

Rojo-Ramirez, RR from now on, starts by considering an ERI that invests all of his wealth in just one asset that by characteristics is similar to a market index. The profitability of the subject asset is $r_M$, the standard deviation is $\sigma_M$. Right after the investment, the ERI issue debt at a risk-free rate, denoted $r_f$, investing the debt amount into an activity which grants him profitability of r, having a standard deviation of σ. The ERI will obtain profitability of:

$$r_T = r_M - r_f + r \tag{4.3}$$

[125] Rojo-Ramirez, Alfonso A. 2013. *Privately Held Company Valuation and Cost of Capital*. Journal of Business Valuation & Economic Loss Analysis.
[126] Rojo-Ramirez, Alfonso A. 2013. *Privately Held Company Valuation and Cost of Capital*. Journal of Business Valuation & Economic Loss Analysis.

Assuming for simplicity that the market risk premium: $r_M - r_f = P_M$, equation 4.3 becomes:

$$r_T = P_M + r \tag{4.4}$$

The premium return required by an ERI, besides the market risk premium ($P_M$) is r, and it can be computed and proved through a set of due assumptions.

Modigliani and Miller (1958), MM from now on, proved that the market value of a company is not influenced by its leverage[127], which is the debt to equity ratio.

$$L = \frac{D}{E} \tag{4.5}$$

Considering the market value (V) of a company being the market value of equity (E) plus the market value of debt (D):

$$V = E + D \tag{4.6}$$

Now consider Company *e*, operating in a market environment that incorporates both levered and unlevered companies. According to MM the cost of capital of Company *e*, Ke, depends on neither the market value of equity nor debt, since the cost of capital for a company is "the expected value of the before-tax income stream generated by its assets"[128] divided by the market value of the company:

$$k = \frac{\bar{X}}{V} \tag{4.7}$$

Where X refers to the expected value of the pre-tax stream of cash produced by the assets owned by Company *e*. According to MM, X "does not stand for the earnings generated by a firm during a given period […] it represents, rather, the return that accrues to those who own the firm's securities"[129]. Therefore, RR assumes X as the return on equity (ROE), hence the return generated by one single share of Company *e*.

[127] Modigliani, Franco and Miller, Merton H. 1958. *The Cost of Capital, Corporation Finance and The Theory of Investment.* The American Economic Review.

[128] Rojo-Ramirez, Alfonso A., Cruz-Rambaud, Salvador, and Alonso-Cañadas, Juana. 2011. *A note on The Operating Return of a Company Under Modigliani-Miller Assumptions.* Working Paper

[129] Modigliani, Franco and Miller, Merton H. 1969. *Reply to Heins and Sprenkle.* The American Economic Review.

According to another assumption proposed by MM, "firms can be divided into "equivalent return" classes such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with ) the return on the shares issued by any other firm in the same class"[130]. Therefore, companies operating in the identical industry display perfectly correlated returns with their own securities' returns. Hence, the return for Company *e* can be formulated as:

$$r = r_f + (r_s - r_f)\frac{\sigma_e}{\sigma_s} \tag{4.8}$$

Where $\sigma_e$ represents the standard deviation of returns generated by Company *e*, while $r_s$ is referred to the return of market "s" in which the operates in. And finally, $\sigma_s$ is the standard deviation of returns of market "s".

The mathematical proof for equation 4.8 is the following:

The goal is finding the minimum premium return required by an investor, investing most, if not all his budget in risky assets, leaving no room for diversification. The standard deviation of the risky assets the ERI has invested in is:

$$\hat{\sigma}_P = \sum_{i=1}^{n} X_i \sigma_i \tag{4.9}$$

While the return generated by the very same risky assets is:

$$r_P = \left(1 - \sum_{i=1}^{n} X_i\right) r_f + \sum_{i=1}^{n} X_i r_i \tag{4.10}$$

Where: $R_f$ is the riskless rate; $X_i$ represents the weight of the total budget put in the i-th asset; $r_i$ indicates the return generated by the i-th asset, and $\sigma_i$ is the standard deviation of the i-th asset.

It is important to note that, equation 4.9 is referred to as the average mean of all i-assets' standard deviations that compose the risky portfolio. If:

[130] Modigliani, Franco and Miller, Merton H. 1958. *The Cost of Capital, Corporation Finance and The Theory of Investment.* The American Economic Review.

$$\sigma_{ij} = \sigma_i \cdot \sigma_j \tag{4.11}$$

Then, equation 4.9 is the abbreviation of the following:

$$\sigma_P = \sqrt{\sum_{i=1}^{n} X_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{j>i}^{n} X_i X_j \sigma_{ij}} \tag{4.12}$$

Considering that securities operating in the same industry are assumed perfectly correlated, then:

$$\rho_{ij} = 1 \tag{4.13}$$

To compute the minimum return required by the ERI for investing in a risky portfolio, RR used the Lagrange multipliers method which is a mathematical approach to detect both the minimum and the maximum for a given formula. To perform the Lagrange method (L), RR introduced the Lagrange multiplier variable (λ):

$$L = \sum_{i=1}^{n} X_i \sigma_i + \lambda \left[ r_P - \sum_{i=1}^{n} X_i r_i - \left( 1 - \sum_{i=1}^{n} X_i \right) r_f \right] \tag{4.14}$$

By computing the derivates for each $X_i$ and λ and making them equal zero, the new set of equations fulfil the conditions for representing a minimum.

$$\frac{\partial L}{\partial X_i} = \sigma_i + \lambda(-r_i + r_f) = 0 \tag{4.15}$$

$$\frac{\partial L}{\partial \lambda} = r_P - \sum_{i=1}^{n} X_i r_i - \left( 1 - \sum_{i=1}^{n} X_i \right) r_f = 0 \tag{4.16}$$

Hence, the equation representing the standard deviation of the risky portfolio is

$$\sigma_P = \lambda \left( \sum_{i=1}^{n} X_i r_i - r_f \sum_{i=1}^{n} X_i \right) \tag{4.17}$$

In this case, the ERI is expected to invest his entire budget in only one risky asset. Hence, $\sum_{i=1}^{n} X_i = 1$. Equation 4.15 can be further written as:

$$\sigma_M = \lambda(r_M - r_f) \tag{4.18}$$

$\sigma_M$ and $r_M$ represent the standard deviation and the expected return of a market portfolio made of risky assets. Equation 4.18 can be turned into:

$$\lambda = \frac{\sigma_M}{(r_M - r_f)} \tag{4.19}$$

Considering $\sum_{i=1}^{n} X_i = 1$, equation 4.15 can be rewritten as follows:

$$r_i = r_f + \frac{\sigma_i}{\lambda} \tag{4.20}$$

Replacing equation 4.19 into equation 4.20, lead to the following formula:

$$r_i = r_f + (r_M - r_f)\frac{\sigma_i}{\sigma_M} \tag{4.21}$$

This last equation is the same as formula 4.8. where $\sigma_i$ is the standard deviation of returns from Company *e* and $\sigma_M$ represents the standard deviation of returns of the market in which the company operates. Sadly, most of the time the return of one specific industry is not known. RR extends the formula to overcome this issue and come up with a function that works for the market, rather than just one industry.

RR starts by looking at the cost of capital derived from the return generated by the $X_i$ assets the company *e* owns.

The firm is now assumed to be made of a number n of assets: $A_1$, $A_2$, …, $A_n$. The acquisition of asset $A_i$ assumes a monetary payment by Company $e_i$ equal to $M_i$. RR continues stating that "generalizing the restrictions of a market economy to the sector in which the firm […] is included, we can state that the profitability obtained by the investment on $A_i$ must be $k_i$"[131] which corresponds to Company $e_i$ cost of capital since no arbitrage is present and any speculative trait in the market is eliminated.

Therefore, the profitability of Company *e* is equal to the weighted average of all profitabilities gained by investing in each asset. Hence:

[131] Rojo-Ramirez, Alfonso A., Cruz-Rambaud, Salvador, and Alonso-Cañadas, Juana. 2011. *A note on The Operating Return of a Company Under Modigliani-Miller Assumptions.* Working Paper

$$k = \frac{\sum_{i=1}^{n} M_i k_i}{\sum_{i=1}^{n} M_i} \quad (4.22)$$

If the subject company has issued debt at the risk-free rate, then some $k_i$ in equation 4.22 will be equal to the riskless rate of return of that market.

It is now possible to presume that the entire $e_i$ companies are quoted in the market they operate in, assuming that "in the stock market all productive sectors in the economy are represented, that is, there exists at least a company of each equivalent return class"[132]. So, Company *e* is now viewed as a portfolio made of two resources: a riskless asset and a risky portfolio:

$$r = X_1 r_f + X_2 r_P \quad (4.23)$$

$r_f$ is related to the return of the risk-free asset, while $r_p$ represents the return generated by the risky portfolio. $X_1$ and $X_2$ represent the percentage of the total budget invested in the risk-free asset and risky portfolio, respectively. According to RR, it makes sense choosing $r_p$ to represent the return of the market portfolio M[133]. The following formula represents the profitability generated by the assets owned by the company, assuming it has both riskless and risky assets (equation 4.23):

$$r = r_f + \frac{r_M - r_f}{\sigma_M}\sigma \quad (4.24)$$

Considering both formulas 4.3 and 4.4, the equation can be turned into the following:

$$r_T = r_f + P_M + P_M \frac{\sigma}{\sigma_M} \quad (4.25)$$

Where σ is the standard deviation of profitability of Company *e*; $\sigma_M$ is the standard deviation of returns of the market, and, finally, $P_M$ is the market risk premium.

RR explains that a company can be seen as a portfolio of two assets: a riskless one and a risky one. What is more, "the operating return of a (quoted or unquoted) company can be measured as the aggregate of the risk-free rate and the market risk premium

[132] Rojo-Ramirez, Alfonso A., Cruz-Rambaud, Salvador, and Alonso-Cañadas, Juana. 2011. *A note on The Operating Return of a Company Under Modigliani-Miller Assumptions.* Working Paper

[133] Rojo-Ramirez, Alfonso A., Cruz-Rambaud, Salvador, and Alonso-Cañadas, Juana. 2011. *A note on The Operating Return of a Company Under Modigliani-Miller Assumptions.* Working Paper

adjusted by the variability of the risk company concerning the market"[134]. This adjustment is seen in equation 4.25. The third term: $P_M \frac{\sigma}{\sigma_M}$, is the idiosyncratic risk faced by the investor investing all of his wealth in just one risky asset. Therefore, the cost of equity ($K_e$) required by an ERI investor is computed through formula 4.25.

The equation is built upon MM propositions: the cost of capital is independent of the subject firm leverage; firms can be grouped in classes where securities are homogeneous; and in every economy, there is at least one firm per sector. The propositions are linked to the capital markets' theory, stating that the profitability generated by one company's assets is derived from the risk-free rate and market risk premia adjusted by a determinant.

Most appraisers tend to use the CAPM even though private companies cannot sell their shares easily nor they allow the owner-investor to have a well-diversified portfolio. Therefore, comparing companies' value using the standard CAPM and equation 4.25 for an investor's return, shows the average overestimation of private companies. That overestimation rate is a DLOM that can be applied when an appraiser value a company using standard valuation tools, as if the private company were, instead, public.

This finding leads to the third and final hypothesis of the model:

H3. "Investors' financial profitability in NQC ($ROE_e$) […] should be greater than the minimum rate of return $k_e$ to safeguard their wealth"[135].

Once the cost of equity for both ERI and PFI has been established, it must be applied in the weighted average cost of capital, which enables the valuation of companies through the Discounted Cash Flow approach. After randomly selecting several private firms and listed companies, they are valued using the WACC computed with the $K_{eri}$ and the $K_{capm}$. Therefore, they are valued both as private and as public firms. The percentage difference between the two values is the Discount for Lack of Marketability.

[134] Rojo-Ramirez, Alfonso A., Cruz-Rambaud, Salvador, and Alonso-Cañadas, Juana. 2011. *A note on The Operating Return of a Company Under Modigliani-Miller Assumptions.* Working Paper

[135] Rojo-Ramirez, Alfonso A. 2013. *Privately Held Company Valuation and Cost of Capital*. Journal of Business Valuation & Economic Loss Analysis

### 4.3 Sample Selection

The data were collected through the Cerved database, which is an Italian leader in the balance sheet's analysis and companies' creditworthiness and the AIDA (Analisi Informatizzata Delle Aziende Italiane).

The initial sample consisted of 388 privately held companies. Since the Rojo-Ramirez model compares private and public companies for the DLOM valuation, it was reasonable to keep only medium-large enterprises. Comparing the profitability of a small private company against that of a public firm is unreasonable. According to the European Commission, medium companies must have annual revenues of 10 million euros or more[136]. The number of private medium-large companies eligible for my study is 209.

To have statistically significant results, I decided to consider mostly companies operating in the manufacture and retail industry in north Italy. Exhibit 17 shows how companies were distributed per industry.

*Exhibit 17: Total Sample by Industry*

| Industry | # of PHC | % of Total Sample |
|---|---|---|
| Agriculture | 3 | 1,44% |
| Manifacture | 174 | 83,25% |
| Utilities | 1 | 0,48% |
| Water Supply | 3 | 1,44% |
| Constructions | 1 | 0,48% |
| Retail | 19 | 9,09% |
| Logistics | 4 | 1,91% |
| Communication Services | 1 | 0,48% |
| Rent Services | 3 | 1,44% |
| Total | 209 | 100,00% |

Then I selected 21 listed companies whose shares are exchanged in the Italian stock exchange. The data concerning these corporations were retrieved in the Borsa Italiana website. I mainly selected companies operating in the manufacturing industry, to match PHC companies and quoted companies operating in similar market dynamics.

[136] Commissione Europea. 2019. *Guida dell'Utente alla Definizione di PMI.* Ufficio delle Pubblicazioni dell'Unione Europea. Lussemburgo.

To end up with a result as statistically significant as possible, I considered companies' data for 7 years, from 2013 to 2019.

### 4.4 Empirical Results

Once the data were collected, I computed the accounting profitability for both private and listed companies using the formula suggested by Rojo-Ramirez:

$$ROE_{AT} = \frac{EBITDA - FE - Taxes}{Equity} \quad (4.14)$$

Where: EBITDA is the earnings before interests, taxes, depreciation, and amortization and FE corresponds to financial expenses.

Some companies can earn significant financial income. So, as suggested by the author of the model, I computed the financial profitability for both types of companies considering their financial income (FI):

$$ROE_{FI} = \frac{EBITDA - FE + FI - Taxes}{Equity} \quad (4.15)$$

As expected, due to the large PHC sample, some outliers compromise the validity of results. Considering the entire 209 companies' sample, results are the following:

*Exhibit 18: PHC Accounting Profitability w/ Outliers*

| | PHC Sample | | | |
|---|---|---|---|---|
| | **MIN** | **MAX** | **MEAN** | **SD** |
| ROEAT | -428,79% | 186,82% | 22,79% | 40,60% |
| ROEIF | -428,72% | 187,16% | 24,37% | 41,38% |

Due to the high standard deviation and the large difference among the mean and both the minimum and maximum value, I decided to eliminate the first and last percentile of the sample. I removed a total of five data from ROEat and seven from ROE if. The consequence is a more statistically significant result, that can be compared to listed companies' profitability ratios.

*Exhibit 19: $ROE_{AT}$ and $ROE_{IF}$ Descriptive Statistics (w/o Outliers)*

| Type of Firms | Profitability Ratio | # of data | Min | Max | Mean Value | Standard Deviation | Standard error |
|---|---|---|---|---|---|---|---|
| PHC | $ROE_{AT}$ | 203 | -8,98% | 135,15% | 24,08% | 21,49% | 1,51% |
| | $ROE_{IF}$ | 200 | -5,81% | 135,18% | 25,60% | 20,59% | 1,48% |
| Listed | $ROE_{AT}$ | 21 | 3,16% | 63,39% | 23,96% | 16,52% | 3,61% |
| | $ROE_{IF}$ | 21 | 8,32% | 63,43% | 26,56% | 15,59% | 3,40% |

Exhibit 19 shows that when it comes to financial profitability ($ROE_{AT}$), private companies perform slightly better than their public counterparts. The average $ROE_{AT}$ is 24.08% for privately held companies against 23.96% of listed firms. When the financial income is considered as well ($ROE_{IF}$), the results are the opposite. Private companies' profitability becomes 25.60% while that of public companies is 26.56%. According to Rojo-Ramirez, very akin results among PHC and public companies is due to the sample presenting firms with similar size[137]. This is interesting since roughly half of the companies I analyzed are considered large and very large companies. I decided to include such companies to try and isolate the DLOM even more, since the difference in the value of two large companies, one being listed and the other being private, can be attributed to the lack of marketability.

Therefore, broadly speaking, the companies considered in the study are a good private comparison to public companies in the analysis.

The slightly larger $ROE_{IF}$ for public companies was predictable for several reasons. For instance, public companies generally own shares of other firms or simply and they tend to be better financially managed to meet the shareholders' expectations on dividends.

The second step was computing the $K_e$ for both ERI and the PFI, using the three-components formula (4.12) and the capital asset pricing model (4.1) respectively. To better represent the Italian market return on the years of the analysis, I computed the yearly ex-post market return[138] for years 2013-2019. The average market returns for the period considered is 6,40%. The risk-free rate is 1.42%[139], and it is the yield rate of a ten-

[137] Rojo-Ramirez, Alfonso A. 2013. *Privately Held Company Valuation and Cost of Capital*. Journal of Business Valuation & Economic Loss Analysis

[138] I checked FTSE MIB Index returns.

[139] The data is displayed on Il Sole 24 Ore website. https://mercati.ilsole24ore.com/obbligazioni/spread/GBITL10J.MTS?refresh_ce

years Italian treasury bond (BTP) on 31/12/19. The beta for listed companies was taken from the Yahoo Finance website.

Considering these inputs, the average cost of equity capital for public enterprises ($K_{CAPM}$) is 7,44%%.

For what concerns PHC, I first computed the total beta as volatility of profitability over the volatility of market returns. The first being the average standard deviation of the three-year accounting profitability ($ROE_{AT}$) for each company. The cost of equity capital for privately-held enterprises ($K_{ERI}$) is 10,32%.

*Exhibit 20: Costs of Equity Capital*

| | Mean | Standard Deviation |
|---|---|---|
| KERI | 10,32% | 4,38% |
| KCAPM | 7,44% | 2,45% |

The 2,88% excess return required by the owner-investor represents, on average, the return linked to the idiosyncratic risk that he cannot lower due to lack of diversification.

I ran a few t-tests to investigate whether the hypotheses expressed in paragraph 4.2 are valid or not according to my sample results.

Hypothesis 2 states that the cost of equity capital should be higher for non-quoted companies. By running a two-tailed t-test to confirm that $K_{ERI} > K_{CAPM}$, the outcome is the following:

*Exhibit 21: Testing H2*

| Hypothesis | t-value | Mean Difference | Significance Level | 95% Confidence Level | |
|---|---|---|---|---|---|
| | | | | Lower | Higher |
| KERI > KCAPM | 4,623825772 | 2,88% | 0,000049 | 1,690 | 2,030 |

Considering both the low significance level and t-value being larger than 2,030, it is safe to assume that, normally, ERI's equity rate of return will be larger than a PFI's equity return. Therefore, the second hypothesis is well-founded.

Hypotheses 1 and 3 states that the accounting profitability ($ROE_{AT}$) is larger than the cost of equity for private and listed companies. I ran two more two-tailed t-tests and the results are shown in Exhibit 22:

*Exhibit 22: Testing H1 and H3*

| Hypothesis | t-value | Significance Level | Mean Difference | 95% Confidence Level | |
|---|---|---|---|---|---|
| | | | | Lower | Upper |
| ROEAT > KERI | 8,93522 | 0,0000 | 13,76% | 1,652 | 1,971 |
| ROEAT > KCAPM | 4,53358 | 0,00018 | 16,52% | 1,721 | 2,080 |

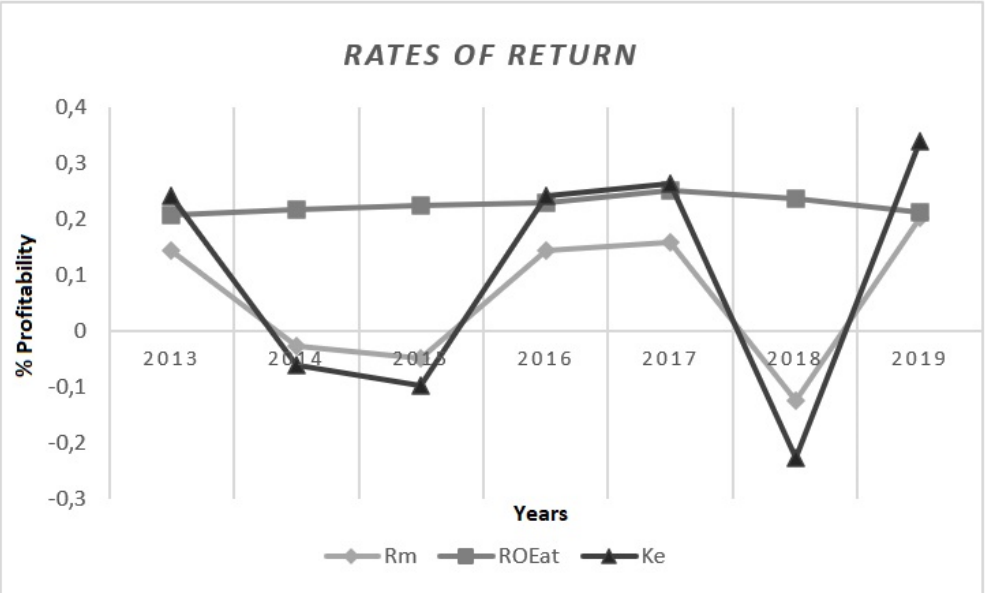
Considering the tests' outcomes, the accounting profitability is always larger than the cost of equity capital for both private ad public companies, with a very low significance level. In the ERI case, the level of significance is roughly zero, meaning that typically the ROE is larger than Ke. Once again, the two assumptions are proved to be valid.

*Exhibit 23: Data*

| | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | SD |
|---|---|---|---|---|---|---|---|---|
| Rm | 14,4% | -2,7% | -4,9% | 14,4% | 15,8% | -12,5% | 20,3% | 12,76% |
| ROEat | 20,7% | 21,8% | 22,6% | 23,0% | 25,1% | 23,7% | 21,2% | 1,52% |
| Ke | 24,2% | -6,1% | -9,8% | 24,2% | 26,3% | -22,6% | 34,0% | 22,20% |

Taking a look at the seven-years data, it is possible to see how the accounting profitability of a company is not as volatile as the market and how Keri depends on the market return. The correlation coefficient for Rm and Ke is 0.99. Data presented in exhibit 23 are presented graphically in the following table, to give a clearer representation of the seven years analyzed.

*Exhibit 24*



Since results are in line with the hypotheses of the model, I computed the value of 120 private companies and 21 public companies using both the cost of equity of an ERI and PFI. The difference in the price displayed by each company is the discount for lack of marketability.

To value my companies, I used the discounted cash flow approach:

$$DCF = \sum_{i=1}^{n} \frac{FCFF(1+g)^n}{(1+r)^n} + \frac{FCFF(1+g)^{n+1}}{(WACC - g)} \qquad (4.16)$$

Where: FCFF is the free cash to the firm, computed for the year 2019; g = is the expected growth rate of the firm. Appraisers usually base g on past FCFF growth rate. Due to a lack of data, I applied the expected growth rate of the manufacturing industry published by a Prometea[140] report performed at the end of 2019. Both r and WACC are referred to as the weighted average cost of capital. This input is computed using the Ke. So, I created two WACC to reflect both the ERIs and PFIs' cost of equity capital. The Ke for an owner-investor reflects the higher risk he faces when investing in a non-marketable asset. The high risk will be reflected in the WACC as well, increasing it. As suggested by the DCF formula, the higher the discount rate, the lower the present value of future free cash flows. The higher risks faced by the ERI will be shifted to the overall value of the company, decreasing it. After computing the value of each company both as if it was private and then public, the DLOM is derived as follows:

$$DLOM = \frac{FIRM\ VALUE_{CAPM} - FIRM\ VALUE_{ERI}}{FIRM\ VALUE_{CAPM}} \qquad (4.17)$$

The final results of my study, are the following:

*Exhibit 25: DLOM Results*

| | Average DLOM | Median | Standard Deviation |
|---|---|---|---|
| PHC | 32,44% | 31,18% | 10,85% |
| Listed companies | 23,96% | 27,37% | 7,491% |

[140] https://www.prometeia.it/news/rapporto-analisi-settori-industriali-ottobre-2019-highlights

In the case of PHC companies, I rejected three results due to results showing either a DLOM larger than 100% or a large premium.

**Result's Discussion**

Results displayed in Exhibit 25 are in line with recent studies and tax court decisions.

Rojo-Ramirez analyzed a sample of 683 privately held Spanish companies and 29 listed firms, collecting data from 2002 to 2007. His results portray a discount rate ranging between 28% and 49% with a reported standard deviation of 29,13%[141]. My DLOM range is lower. My standard deviation is lower as well, indicating that my DLOMs are less volatile than that of Rojo-Ramirez, but I believe this is mainly due to him computing the DLOM on 286 private companies, while I considered 120 private firms, and 96 listed firms when I considered 21. Therefore, the difference in standard deviation might be due to Rojo-Ramirez having a larger sample than me.

Considering recent DLOMs using different approaches, my results still appear to be in line with the average results. For instance, the Valuation Advisors Pre-IPO study, performed between 2007 and 2012, are displayed in Exhibit 26, and it shows large DLOMs

*Exhibit 26: Valuation Advisors Pre-IPO Study*

| | Period Before the IPO | | | | |
|---|---|---|---|---|---|
| | 0-3 Months | 4-6 Months | 7-9 Months | 10-12 Months | 1-2 Years |
| Average 2008-2012 | 16,82% | 28,93% | 37,85% | 45,39% | 46,96% |

For Pre-IPO studies, valuing a company five months before the IPO date is considered a good proxy. Considering the discount rate on the 4 months before the IPO, the average DLOM is 28,93%, having a standard deviation unknown. What is more, a slight increase in the months, lead to even larger results.

The Trugman Valuation Associates, Inc. (TVA) published its restricted stock studies in 2009 considering a total of 80 transactions occurred between 2007 and 2008. They reported an average discount of 18,1% and a relatively low standard deviation of

[141] Rojo-Ramirez, Alfonso A. 2013. *Privately Held Company Valuation and Cost of Capital*. Journal of Business Valuation & Economic Loss Analysis,

15,6%[142]. The average DLOM is lower than that of my study. It is important to keep in mind that, as explained in paragraph 4.1, restricted stocks becomes marketable after a six-month holding period. Therefore, a lower DLOM might be due to restricted stocks having the high probability of being as marketable as their listed counterpart in a short period.

The Pluris DLOM Database is an updated source of DLOM under the restricted stock approach. Considering 3538 transactions between 2001 and 2013, the average DLOM and standard deviation reported are 22,47% and 29,9% respectively[143]. The database collected restricted stocks being issued at discount ranging from around 84% to -53%, hence the large standard deviation.

Finally, the BIZCOMPS database provides real private companies transaction prices. They compute the DLOM considering both the asking price and sale price and the subject firm's financial information. Considering 141 transactions, the BIZCOMPS database reported an average discount for lack of marketability of 17,44%[144]. Results show that the highest DLOM reported is 92,00% and the lowest is -46,70%. No standard deviation is reported.

Considering the last case, my results seem to be overestimating the DLOM, especially since the BIZCOMPS database considers actual privately-held companies' transactions. Exhibit 27 report this paragraph's results for a clearer idea.

*Exhibit 27: Recent DLOM Studies' Results*

| | Years Observed | # of Transactions | Average DLOM | Standard Deviation |
|---|---|---|---|---|
| This Work's Study | 2013-2019 | 141 | 23,96% - 32,44% | 7,49%-10,45% |
| Rojo-Ramirez | 2002-2007 | 382 | 27,63% - 49,26% | 1,22%-29,13% |
| Valuation Advisors | 2008-2012 | 1050 | 28,93% | N/A |
| PLURIS Database | 2001-2013 | 3538 | 22,47% | 29,90% |
| BIZCOMPS Database | 2008 | 141 | 17,44% | N/A |

[142]Harris, William. 2009. *Trugman Valuation Associates, Inc. (TVA) Restricted Stock Study.* Business Valuation Review.

[143] ValuSource. 2014. *Pluris DLOM Database.* The 2014 UserGuide available at www.valusource.com

[144] Rudich, Ronald D. 2020. *Introducing the Rudich/Jaroudi BIZCOMPS DLOM Study.* Business Valuation Update

## Conclusions

The discount for lack of marketability is one of the biggest issues in company valuation. As shown in this work, there is a multitude of studies and approaches to computing the DLOM, but a universally accepted method is yet to be found. Appraisers used to rely on past studies' results, applying them as discount rates. More recently, tax courts suggest using analyses' results as a starting point to the model according to the subject firm's characteristics. Recent surveys show that most appraisers count on past studies as a basis for DLOM computation. According to a Business Valuation Resources' 2018 survey on 96 practitioners, 75% of the respondents admitted using the restricted stock studies to derive DLOMs. 38% of the appraisers use Pre-IPO results. Option-based models are used as well: 18% of the respondents use Finnerty's average-strike option, while 13% and 12% use Longstaff's and Chaffe's models, respectively. What is more, 11% used Pluris Database and another 11% used the QMDM[145]. Another survey conducted in 2008 by Eva Lang reported that on average, 67% of appraisers spend less than 5 hours in DLOM computations. 22% spend between 5 and 10 hours, 8% consume 10 to 20 hours and finally, 3% employ more than 20 hours into DLOM computation[146].

Appraisers should not carelessly apply DLOM depending on the discount value they prefer applying. For instance, valuing a private company operating in an expanding market might lead the company to be easier to sell in just a couple of years. If an analyst expects this to happen in two years, then one should apply the pre-1990s restricted stock studies, when the holding period imposed by the SEC was two years. On the contrary, if the investor is expected to hold the private firm's interests for more than two years than the Pre-IPO results might be more relevant. Dividends are important as well. In case an investor is concerned about receiving a dividend payment over the expected holding period, the Finnerty model is more suitable.

[145] Business Valuation Wire: https://www.bvresources.com/articles/bvwire/how-valuation-experts-estimate-dlom

[146] Hitchner, James R. 2011. *Financial Valuation: Application and Models.* John Wiley & Sons, Inc. Hoboken, New Jersey.

The value of a private company operating in a mature market having many listed participants can be computed using multiples. Although results depend on how similar the companies are.

Theoretical models are not used often. This is due to the QMDM and Tabak model being highly subjective. A slight change in their inputs leads to extremely difficult results. So, one should carefully use such models to estimate the DLOM, showing that their subjectivity is as objective and grounded as possible.

I used Rojo-Ramirez's suggestion, which is a theoretical model since it did not rely on subjective estimations. All the inputs can either be observed in the market or computed using objective formulas. What is more, the subject model gives the cost of equity for each company. This can result in a clearer DLOM once the company's value is computed.

On the contrary, using past restricted stock studies as a starting point on today's DLOM computations, can be misleading. Past studies are influenced by past market dynamics, giving results valuable for firms operating in that specific market.

Therefore, appraisers should be careful when using past DLOM averages, even as a starting point. Each discount round has to be estimated depending on the subject company characteristics to end up with a fair estimation. For instance, a firm owning assets that cannot be sold separately will face larger DLOMs. What is more the smaller the company, the larger the DLOM and finally, the shareholder's agreement can imply larger discounts.

## References


- 73rd United State Congress. 1933. *Securities Act of 1933.*
- Adams III, Frank A., Manners Jr., George E., Astrachan, Joseph H., and Mazzola Pietro. 2004*. The Importance of Integrated Goal Setting: The application of Cost-of-Capital Concepts to Private Firms*. Family Business Review.
- Alonso-Cañadas, Juana A., and Rojo-Ramirez, Alfonso A. 2011. *The Discount Rate in Valuing Privately Held Companies*. Business Valuation Review.
- Alonso-Cañadas, Juana and Rojo-Ramirez, Alfonso A. 2020. *Trends in the Discount for Lack of Marketability*. Valuation Strategies.
- Altstadter, Eric. 1997. *What Exactly is Rule 144 ?.* The CPA Journal.
- Ang, James S. 1992*. On the Theory of Finance for Privately Held Firms*. The Journal of Entrepreneurial Finance.
- Artemenkov, A., I., Mikerin, G.,I., and Artemenkov I., L,. 2008. *Professional Valuation and Investment-Financial Valuation: Distinctions in Valuations for Private and Public Markets.* The Appraisal Journal
- Ashok, Abbott B. 2012. *Estimating the Discount For Lack of Marketability: A Best Fit Model*. Valuation Strategies.
- Bajaj, Mukesh, Denis, David J., Ferris, Stephen P., and Sarin Atulya. 2001. *Firm Value and Marketability Discounts*. Journal of Corporation Law.
- Bini, Mauro. 2011. *La valutazione degli intangibili. Business Combinations e Purchase Price Allocation*. Egea.
- Black, Fischer and Scholes, Myron. 1973. *The Pricing of Options and Corporate Liabilities.* The Journal of Political Economy.
- Block, Stanley. 2007. *The liquidity Discount in Valuing Privately Owned Companies.* Journal of Applied Finance.
- Bortnick, Edward A. 2014. *Do Not Be Too Quick To Discount A Discount For Controlling Interests.* Valuation Strategies.

- Brooks, Robert. 2013. *A General Option Valuation Approach to Discount for Lack of Marketability.*

- Butler, Peter and Pinkerton, Keith. 2006. *Company-Specific Risk – A Different Paradigm: A New Benchmark*. Business Valuation Review.

- Chaffe, David B.H. 1993. *Option Pricing as a Proxy for Discount for Lack of Marketbility in Private Company Valuations. Business Valuation Review.*

- Comment, Robert. 2012. *Revisiting the Illiquidity Discount for Private Companies: A New (and "Skeptical") Restricted-Stock Study.* Journal of Applied Corporate Finance.

- Commissione Europea. 2019. *Guida dell'Utente alla Definizione di PMI.* Ufficio delle Pubblicazioni dell'Unione Europea. Lussemburgo.

- Committee on Interstate and Foreign Commerce. 1971. *Institutional Investor Study Report of the Securities and Exchange Commission: Volume 1.* 92nd Congress, First Session.

- Covrig, Vicentiu and McConaughy, Daniel L. 2015. *Public versus Private Market Participants and the Prices Paid for Private Companies*. Journal of Business Valuation & Economic Loss Analysis.

- Damodaran, Aswath. 2002. *Investment Valuation.* 2nd Edition. John Wiley and Sons. New York, New York.

- Damodaran, Aswath. 2005. *Marketability and Value: Measuring the Illiquidity Discount*. Stern School of Business.

- Dawson, Peter C. 2016. *An Independent Evaluation of the Reliability of the Implied Private Company Pricing Line Model in Appraisal Practice*. Journal of Business Valuation & Economic Loss Analysis.

- Derrien, Francois. 2005. *IPO Pricing in "Hot" Market Conditions: Who Leaves Money on the Table ?.* The Journal of Finance.

- DiMattia, Ronald D. 2008. *Controlling Interests – Discount For Lack of Marketability: The Empirical Evidence.* CPA Expert.

- Doerfer, John C. 1934. *The Federal Securities Act of 1933.* Marquette Law Review.
- Dorrell, Darrell D., Gadawski, Gregory A. and Brown, Thomas S. 2008. *2008 Update: Marketability Discounts, A comprehensive Analysis.* The Value Examiner.
- Douglas, William O. and Bates, George E. 1933. *The Federal Securities Act of 1933.* Yale Law Journal.
- Duffy, Robert E. 2011. *Why Finnerty's Put Option Model is the DLOM Model of Choice.* Financial Valuation Litigation Expert.
- Dukes, William P. 1 Spring 2001. *Where Do We Stand on Closely-Held Firm Valuation?.* The Journal of Entrereneurial Finance.
- Elkounovitch, Ron. 2018. *Marketability Discounts, Fair Value, and the Forgotten Market Participant: When Do Discounts Represent Distortions ?.* The CPA Journal.
- Emory, John D. 1997. *The Value of Marketability As Illustrated In Initial Public Offering Of Common Stock*. Business Valuation Review.
- Emory, John D., Dengel III, F. R. and Emory Jr, John D. 2000. *The Value of Marketability as Illustrated in Initial Public Offerings of Dot-Com Companies: May 1997 through March 2000.* Business Valuation Review.
- Engineering/Valuation Program DLOM Team. 2009. *Discount for Lack of Marketability: Job Aid for IRS Valuation Professionals*. IRS.
- European Commission. 2019. *Annual Report on European SMEs 2018/2019: Research & Development and Innovation by SMEs.* Karen Hope.
- European Private Equity and Venture Capital Association. January 2013. *Risk Measurement Guidelines*. Brussels.
- Fazzini, Marco. 2018. *Valutazione D'azienda. Premio di Maggioranza e Sconto di Minoranza.* Fazzini & Partners.
- Fédération des Experts Comptables Européens. July 2001. *Business Valuation: a guide for Small and Medium sized Enterprises*.
- Finnerty, John D. 2002. *The Impact of Transfer Restrictions on Stock Prices*.

- Fogelson, James H. 1982. *Rule 144 – A Summary Review.* Business Lawyer.
- Garland, Pamela J. and Reilly Ashley L. 2003. *Update on The Willamette Management Associates Pre-IPO Discount for Lack of Marketability Study.* Insights.
- Garland, Pamela J. and Reilly, Ashley L. 2004. *Update on the Willamette Management Associates Pre-IPO Discount for Lack of Marketability Study For the Period 1998 through 2002.* Insights.
- Garvey, Gerald T. 2001. *What is an Acceptable Rate of Return for an Undiversified Investor?*. Clermont Graduate School of Management.
- Gelman, Andrew and Hill, Jennifer. 2007. *Data Analysis Using Regression and Multilevel/Hierarchical Models.* Cambridge University Press. New York.
- Gelman, Milton. 1972. *An Economist-Financial Analyst's Approach to Valuing Stock in s Closely Held Company.* Journal of Taxation.
- Glazer, Russel T. 2005. *Understanding the Valuation Discount for Lack of Marketability*. The CPA Journal.
- Grandis, Fabio Giulio and Palazzi Federica. 2015. *The valuation methods for small and medium sized enterprises*.
- Hall, Lance S. 2004*. Counteracting the New and Winning IRS Approach to Determine Discounts for Lack of Marketability*. Valuation Strategies.
- Hall, Lance S. 2013. *IRS Revisits Method for Determining DLOM*. Valuation Strategies.
- Harris, William. 2009. *Trugman Valuation Associates, Inc. (TVA) Restricted Stock Study.* Business Valuation Review.
- Hawkins, George B. and Paschall, Michael A. 1955. *Marketability Discounts – The Mandelbaum Case Raises Key Issues.* Banister Financial, Inc.
- Heaton, H, B. 1998. *Valuing Small Businesses: The Cost of Capital*. The Appraisal Journal.

- Hertzel, Michael and Smith, Richard L. 1993. *Market Discounts and Shareholders Gains for Placing Equity Privately*. The Journal of Finance.
- Hitchner, James R. 2011. *Financial Valuation: Application and Models.* John Wiley & Sons, Inc. Hoboken, New Jersey.
- Houlihan Lokey Howard and Zukin. 2004. *Discounts for Lack of Marketability: Discussion Materials.* Investment banking services. Found at: https://www.sec.gov/Archives/edgar/data/48272/000104746904029419/a2143709zex-99_c2.htm
- Hull, Robert H. and Price, David P. 1 spring 2015. *Pass-Through Valuation*. The Journal of Entrepreneurial Finance.
- International Private Equity and Venture Capital. December 2018. *Valuation Guidelines*.
- Invest Europe. 2018. *Professional Standards Handbook*. Invest Europe, Brussels.
- Jacob, Valerie F., Bursky, Daniel J., Gelfond, Stuart H., Levitt, Michael A., Tropp, Paul D. and Tsaganos, Vasiliki B. 2008. *SEC shortens Rule 144 hodling periods and loosens restrictions on resales of privately placed securities.* Journal of Investment Compliance.
- Johnson, Bruce. 1999. *Restricted Stock Discounts, 1991-95.* Business Valuation Update.
- Jones, Lawrence D. 1972. *Some Contributions of the Institutional Investor Study.* The Journal of Finance.
- Kam, Steven D. and Wan, Darren. 2010. *Measuring the Incremental Discount for Lack of Marketability*. Valuation Strategies.
- Kasper, Larry J. 2013. *The Effect of Willing Sellers on Valuation of Privately Held Businesses.* Valuation Strategies.
- Kerins, Frank. Smith, Janet K. and Smith Richard. 2004. *Opportunity Cost of Capital for Venutre Capital Investors and Entrepreneurs.* The Journal of Financial and Quantitative Analysis.

- Klein, Christian and Scheibel, Marcus. 2012. *The Private Company Discount from a European Prospective: An Analysis Based on the Acquisition Approach for Comparable Transactions of European Target Companies*. The Journal of Private Equity.

- Koeplin, John. Sarin, Atulya and Shapiro, Alan C. 2000. *The Private Company Discount*. Journal of Applied Corporate Finance.

- Kooli, Maher., Kortas, Mohamed and L'Her, Jean-François. 2003. *A New Examination of the Private Company Discount: The Acquisition Approach.* The Journal of Private Equity.

- Lance, Travis R. 2007. *The Use of Theoretical Models to Estimate the Discount for Lack of Marketability*. Insights.

- Lerch, Mary A. 2008. *Quantification of marketability discount using regression*. Valuation Strategies.

- Longstaff, Francis A. 1995. *How Much Can Marketability Affect Security Values?*. The Journal of Finance.

- Longstaff, Francis A. 2001. *Optimal Portfolio Choice and the Valuation of Illiquid Securities*. The Review of Financial Studies.

- Loughran Tim and Ritter Jay. 2004. *Why has IPO Underpricing Changed Over Time?.* Financial Management.

- Maher, Michael J. 1976. *Discounts for Lack of Marketability for Closely Held Business Interests.* Taxes.

- Matthews, Gilbert. 2016. *DLOMs in Fair Value Cases: Lack of Marketability Does Not Cause Private Company Discounts.* Business Valuation Australia.

- Mellen, Chris M. and Evans, Frank C. 2010. *Valuation for M&A: Building Value in Private Companies*. John Wiley & Sons, Inc., Hoboken, New Jersey.

- Mercer, Christopher Z. 2005. *Valuing Shareholders Cash Flows: Quantifying Marketability Discounts*. Peabody Publishing, LP.

- Mercer, Christopher Z. and Harms, Travis W. 2007. *Business Valuation: An Integrated Theory.* 2nd ed. John Wiley & Sons, Inc. Hoboken, New Jersey.
- Mork, Randall., Shleifer, Andrei and Vishny, Robert W. 1987. *Management Ownership and Market Valuation: An Empirical Analysis*.
- Moro, Andrea, Nolte, Sandra and Diaz Alexandro. 2014. *Entrepreneur's Wealth Firm Performance and Cost of Capital: A Bayesian Approach to the Capital Structure of Entrepreneurial Ventures*.
- Moro, Andrea, Lucas, Mike R. and Kodwani Devendra. 2012. *Trust and the Demand for Personal Collateral in SME-Bank Relationships*. The Journal of Entrepreneurial Finance.
- Moroney, Robert E. 1973. *Most Courts Overvalue Closely Held Stocks.* Taxes.
- Mueller Elisabeth. 2010. *Returns to Private Equity – Idiosyncratic Risk Does Matter!.* Review of Finance.
- National Association of Certified Valuators and Analysts (NAVCA). 2016. *Discount for Lack of Marketability: Job Aid for IRS Valuation Professionals.* NAVCA.
- Officer, Micah S. 2007. *The Price of Corporate liquidity: Acquisition discounts for unlisted targets.* Journal of Financial Economics.
- Oliver, Robert P. and Meyers, Roy H. 2000. *Discounts Seen in Private Placements of Restricted Stock: The Management Planning; Inc. Long-Term Study (1980-1996).* Published in Handbook of Advanced Business Valuation.
- Paglia, John and Harjoto, Maretno A. 2010. *The Discount for Lack of Marketability in Privately Owned Companies: A Multiples Approach.* Journal of business Valuation and Economic Loss Analysis.
- Pearson, Brian. 2001. *2000 Marketability Discounts as Reflected in Initial Public Offerings.* Business Valuation Resources.
- Pratt, Shannon P. 2004. *Small Business Transactions Databases Can Be Used Reliably.* Valuation Strategies.

- Pratt, Shannon P. and Niculita, Alina V. 2008. *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*. 5th ed. The McGraw-Hill Companies, Inc.
- PWC. 2017. *Roadmap for an IPO: A guide to going public.* PWC Deals.
- Quackenbush, William C. 2016. *DLOMs in N.Y. Statutory Fair Value Cases – A Follow-Up to Matthews.* Business Valuation Update.
- Rawley, Thomas and Gup, Benton E. 2010. *The Valuation Handbook: Valuation Techniques from Today's Top Practitioners*. John Wiley & Sons, Inc. Hoboken, New Jersey.
- Reilly, Robert F. 2016. *Discount for Lack of Marketability for the Closely Held Company.* The Practical Tax Lawyer.
- Reilly, Robert F. 2016. *Discount for Lack of Marketability Considerations Related to Closely Held Company Securities.* Practical Tax Strategies.
- Reilly, Robert F. 2016. *Measuring the Discount for Lack of Marketability for a Closely Held Taxpayer Company.* Insights.
- Reilly, Robert F. and Rotkowski, Aaron. 2007. *The Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies*. The Tax Lawyer.
- Reilly, Robert F. 2005. *Valuation Adjustments in Business and Securities Valuations.* Valuation Strategies.
- Reilly, Robert and Schweihs, Robert P. 2000. *The Handbook of Advanced Business Valuation.* McGraw-Hill Professional.
- Reinemann, Annika M. 2008*. Lack of Marketability*. Trusts & Estates.
- Ritter, Jay R. and Welch, Ivo. 2002. *A Review of IPO Activity, Pricing, and Allocations.* The Journal of Finance.
- Robak, Espen. 2001. *FMV Introduces Detailed Restricted Stock Study.* Business Valuation Resources.

- Robak, Espen. 2007. *Lemons or Lemonade ?: A Fresh Look at Restricted Stock Discounts.*

- Robak, Espen. 2010. *An Updated Approach to Determine Lack of Marketability Discounts*. Valuation Strategies. Valuation Strategies.

- Rojo-Ramirez, Alfonso A. 2013. *Privately Held Company Valuation and Cost of Capital*. Journal of Business Valuation & Economic Loss Analysis.

- Rojo-Ramirez, Alfonso A., Cruz-Rambaud, Salvador and Alonso-Cañadas, Juana. 2012. *Discount Rate and Cost of Capital: Some More About the Puzzle.* Working Paper.

- Rotkowski, Aaron M. and Harter, Michael A. 2013. *Current Controversies Regarding Option Pricing Models.* Taxation Planning and Compliance Insights.

- Rudich, Ronald D. 2020. *Introducing the Rudich/Jaroudi BIZCOMPS DLOM Study.* Business Valuation Update.

- Schroeder, Hans P. 2009. *The Holy Grail of a Valid DLOM*. Trusts and Estates.

- Securities and Exchange Commission. 1940. *Investment Company Act of 1940*. Chapter 686 of the 76th Congress.

- Securities and Exchange Commission. 1959. *Irs Revenue Ruling 59-60.*

- Securities and Exchange Commission. 1969. *Investment Company Act of 1940: Release No. 113.*

- Securities and Exchange Commission. 1995. *The Investment Company Act Amendaments of 1995.* House of Representatives, Committee on Commerce, Subcommitee on Telecommunications and Finance. Washington, DC.

- Sheeler, Carl L. 2004. *Assessing the Emprical Support for Illiquidity Discount in Closely Held Business Transactions.*

- Sheeler, Carl L. 2004. *Empirical Support for Illiquidity Discount Levels*. Valuation Strategies.

- Silber, William L. 1991. *Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices*. Financial Analysts Journal.
- Stockdale, John J. 2011. *Lack-of-Marketability Discounts for Controlling Interests.* Valuation Strategies.
- Tabak, David. 2002. *A CAPM-Based Approach to Calculating Illiquidity Discounts*. NERA Economic Consulting.
- Tatum, Toby. 2014. *Using the Bizcomps Database to Value Small Businesses.* Valuation Strategies.
- Tiest, Roger. June 2013. *Business Valuation of small and medium-sized companies: Due Diligence and Valuation Techniques*.
- Trout, Robert R. 1977. *Estimation of the Discount Associated with the Transfer of Restricted Securities.* Taxes.
- ValuSource. 2014. *Pluris DLOM Database.* The 2014 UserGuide available at www.valusource.com
- Vianello, Mark. 2014. *Calculating Probability-Based Discounts for Lack of Marketability.* Valuation Strategies.
- Vianello, Marc. 2014. *Using Restricted Stock and Pre-IPO Studies for Quantifying DLOM: Two Ways of Saying "I Don't Know"?*. Valuation Strategies.
- Vianello, Marc. 2019. *Empirical Research Regarding Discounts For Lack of Marketability: Volume 1.1*. VFC DLOM Calculator.
- Vidal-Garcia, Raül. Ribal, Javier. 2019. *Terminal Value in SMEs: Testing Multiple EV/EBITDA Approach*. Journal of Business Valuation and Economic Loss Analysis.
- Whittington, Vara P. 2013. *Risk-Based New Venture Valuation Technique: Win-Win for Entrepreneur and Investor*. Journal of Business Valuation & Economic Loss Analysis.
- Woelfel, Charles J. 1994. Encyclopedia of Banking & Finance. 10th Edition. Probus Publishing Company. Chicago, Illinois.

- Wruck, Karen H. 1989. *Equity Ownership Concentration and Firm Value: Evidence from Private Equity Findings*. Journal of Financial Economics.
- Yale Law Journal. 1972. *Resale of Restricted Securities under SEC Rule 144*. Yale Law Journal.
- Yazdipour, Rassoul and Constand, Richard. 3 Fall 2010. *Predicting Firm Failure: A Behavioural Finance Perspective*. The Journal of Entrepreneurial Finance.
- Zanni, Kevin M. 2013. *The Private Company Discount Based on Empirical Data.* Taxation Planning and Compliance Insights.
- Zanni, Kevin M. 2015. *Quantifying the Private Company Discount: Multiples Approach and Acquistion Approach.* Wolters Kluwer.

## Websites

- Borsa Italiana: https://www.borsaitaliana.it/homepage/homepage.htm
- International Glossary of Business Valuation Terms: https://www.nacva.com/content.asp?contentid=166
- Leagle.com: https://www.leagle.com/
- Securities and Exchange Commission (SEC): https://www.sec.gov/