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Household Finance:
Life-Cycle Strategies

Supervisor
Ch. Prof. Andrea Berardi

Graduand
Riccardo Lambertini
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876247

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Household finance: life-cycle strategies

Riccardo Lambertini

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Abstract

Longevity is increasing and retirees face the risk of outliving their income resources, perceived by the (compulsory) second pillar pension scheme. In this regard the European Commission has launched the pan-European pension program (PEPP) to help workers to deal with this type of risk. In the thesis I address the potential risks and benefits of different life-cycle strategies, which could be adopted in a (voluntary) third pillar pension scheme. I run a simulation and compare the results of eight strategies. The analysis aims at suggesting which one fits best the capital protection characteristics required by a default option in PEPP. The main results of the thesis concern the greater ability of life-cycle strategies in providing a higher wealth realization at retirement with respect to a minimum capital guarantee policy, and the very low probability that these strategies cannot recoup the amount of contributions paid by the worker.

Keywords: household finance, pensions, PEPP, default option, life-cycle strategies

Introduction

Since the beginning of her working life an individual pays compulsory contributions that are deducted from her gross monthly salary. Those payments are accumulated and represent the retirement savings that should guarantee to the worker a stable pension check during retirement years. However, not only has a retiree to meet the basic needs, such as eating, heating, clothing, but she also has to deal with healthcare expenses, for example in case she will be assisted by a caregiver. Moreover, the retiree would like to live a pleased retirement livelihood, for example she might want to travel or to improve education, and then she might want to bequeath her grandchildren. Essentially, since the retiree is a person, she has needs. Anthony (2008) proposed his version of the Maslow's Hierarchy of Needs (1943), named "Maslow Meet Retirement", in terms of the needs that a retired person wants to meet. Moreover, people are afraid of the longevity risk, i.e. the risk of not having enough money to satisfy the basic needs and the healthcare expenses until the death. This last point is very important because, according to Maslow, not only are people's decisions justified by unmet needs, but also by the risks, i.e. the fear of outliving their money. So, a young worker may feel the need for a complementary pension scheme in order to improve her survival chances as well as to meet further needs. These could also be done by voluntarily putting aside parts of the net income, i.e. the income from which compulsory pension contributions are already deducted, and investing them. But then, it arises the problem of how investing the amount of the complementary monthly contributions in an efficient way so that the investor will achieve a satisfying wealth realization at retirement, with an adequate source of risk. It is here that my thesis comes into play, comparing different life-cycle strategies and presenting the results both in terms of wealth realizations and focusing on the most important downside risk, i.e. the probability of not recouping the amount invested during the accumulation period. Essentially, the first purpose of my thesis is to present the benefits that could be achieved in terms of welfare by a worker who sets up a complementary pension scheme.

However, as Richard Thaler said in an interview released for ProfessioneFinanza[1], researchers cannot expect people to save for retirement on their own. : for these reasons there are compulsory pensions that make it easier. Moreover, speaking about complementary pension plans, people cannot save and plan for retirement just because is too hard both cognitively (they have to make hard computations) and behaviorally (they

[1] Evolving behavioral finance: Richard Thaler, event date 23/06/2020.

have to take money every month). Planning and investing require investors to be behaviorally ready to endure the anxiety of the market for a long time. It is for all these reasons that investors need “money doctors”. According to Gennaioli et al. (2015), money doctors should help investors making risky investments by reducing “the investor’s subjective perception of the risk of investments” (Gennaioli, et al., 2015, p. 93). Essentially, financial advisors should both help investors in the choice of the assets, since the bulk of them are financial illiterate, and facilitate them in the investment path[2].

In recent years, the European Parliament and the European Council proposed the so-called pan-European Pension Product (PEPP), i.e. a further effort in the direction of building the Capital Market Union within Europe. Essentially, it is a voluntary personal pension scheme that allows European savers to move across borders without the need to change pension product, i.e. these pension products are portable. PEPP intends to help workers providing a complementary pension check when they retire. It includes up to five saving options, among which there is a default option that is mandatory. This compulsory option is a saving plan for those investors who want at least to recover the initial investment but do not have enough financial knowledge to decide among a wide range of alternative investment options. According to Berardi et al. (2018), even the default option should be tailored with the investor’s age, i.e. it should follow the technique of a life-cycle strategy. So, the second purpose of my thesis is to compare the different life-cycle strategies and to suggest one of them as a default option for the naïve investor.

This thesis is organized as follows. In Chapter I, I briefly present the theory behind the Modern Portfolio Theory and the Intertemporal Portfolio Choice. They represent the theoretical ground upon which is based the maximization of the wealth in the short period and in the long period. Chapter II reports the most important variable that an investor must consider in a defined-contribution plan: the human capital. It has been identified as the labor income that a worker perceives. In Chapter III, I review the different studies on empirical evidence. This section is important because these studies show discrepancies between the theoretical ground and what happens in the real world, i.e. the behavior of investors. In Chapter IV, I present the data and the assumptions made for the simulation. In Chapter V, I present the eight glide paths and I run the simulation. Then, addressing the first purpose of the thesis, I quantify the performance for each life-cycle strategy in

[2] Because the alone-investor feels a higher perception of the risk of an investment than whether she is assisted by a professional advisor. This leads her to invest less in the stock market.

terms of both the capital protection capacity and the risk-return tradeoff. Chapter VI presents the characteristics that a default option should have and concludes with the comparison off the eight life-cycle strategies. At the end, the Appendix reports all statistics computed as well as all the figures that represent the path of wealth realization for each life-cycle strategy.

I. A glance to theory

My purpose in this chapter is to explore, albeit not thoroughly, the theory upon which is based the Modern Portfolio Theory and the Intertemporal Portfolio Choice.

1.1. From the Saint Petersburg Paradox to the risk aversion measures

The Saint Petersburg Paradox was first invented by Nicolas Bernoulli and then, solved by Daniel Bernoulli in 1738. The Paradox is focused on different answers given by different people to the question “How much would you pay to play?”, when they must decide whether to participate in a gamble or not. Bernoulli’s (1954) solution concerned the use of the expected utility function and, most importantly, he reached the conclusion that not every man uses the same rule to evaluate the same gamble. Bernoulli started by arguing that expected value proposition does not account for characteristics of different people. Therefore, what we need to consider is the value of the gamble based not on the price but upon the utility that it could yield; this is because utility is a personal matter, thus it depends on the person making the estimate. Famous is the expression “[…] there is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount […]” (Bernoulli, 1954, p. 12). So, the investor should maximize her utility instead of her value and, since the outcomes in a gamble are not known in advance, Bernoulli talked about maximization of the expected utility[3] rather than expected value. Bernoulli proposed a nonlinear utility function: the logarithmic utility function where the argument could be the gain, the wealth, or other variables. Bernoulli also saw that, any increase of wealth always leads in an increase in utility, but it is inversely proportionate with respect to wealth already possessed. This is explained by Arrow (1974) for the case of a risk-averter investor, where the marginal utility[4] – i.e. the first derivative – of the wealth is strictly decreasing as the wealth increases. That is, the higher the wealth already possessed the lower the utility of an additional unit in wealth.

[3] In Bernoulli’s work expected utility function is called moral expectation.
[4] Marginal utility of wealth is the change in utility from a change in wealth.

Starting from the concept of utility, Von Neumann et al. (1944) assumed that events, as well as utilities, can be combined with probabilities and they derived the four axioms that define the rational decision maker:

- Completeness: an individual with preferences can always express her preferences between two or more alternatives.
- Transitivity: the individual is consistent with her choices.
- Continuity: if there are three lotteries A, B and, C and A is preferred to B, which in turn is preferred to C, then it will exist a combination of A and C such that it is indifferent for the individual to choose between the combination of A and C and the lottery B.
- Independence of irrelevant alternatives: given preferences among alternatives, adding the same quantity to each alternative does not alter the order of the preferences.

Von Neumann et al. (1944) stated that, if these axioms are satisfied – i.e. the decision maker is rational – we can represent preferences through a utility function and, the utility can be expressed as numerical value. Furthermore, and most importantly, utility function can be computed like mathematical expectation[5], using probabilities. Thus, the expected utility has been defined as the sum of the products between the different utilities that are led by different outcomes and the probability that a certain event will occur:

$$E[u(x)] = \sum_{i=1}^{N} p_i \cdot u(x_i) \quad (1.1)$$

where $u(x_i)$ is the utility for the i^{th} outcome with $i = 1,2, \dots, N$, p_i is the probability of the i^{th} outcome, and, N is the number of outcomes. Therefore, the expected utility can be expressed as a linear function of the outcome utilities times the probability of each outcome. According to Bernoulli (1954), the (rational) investor should choose the highest expected utility, instead of the highest expected value. This has been called the expected utility hypothesis. Within this theory I have talked about expectations, meaning that investors are dealing with risky outcomes, since in real life investors face risky situations. So, different people could have different behaviors towards risky outcomes. Arrow (1974) noted that, since Bernoulli solved the Saint Petersburg Paradox, the bulk of

[5] Mathematical expectation is another name for expected value.

researchers started to argue that individuals tend to display risk aversion towards risky events. In his work, Arrow tried to define measures of risk aversion that can be applied also in quantitative terms. Both Arrow (1974) and Pratt (1964) formulated measures of absolute risk aversion

$$R_A(Y) = -\frac{U''(Y)}{U'(Y)} \tag{1.2}$$

and, relative risk aversion,

$$R_R(Y) = -\frac{Y \cdot U''(Y)}{U'(Y)} \tag{1.3}$$

where Y represents the wealth. The difference between absolute risk aversion and relative risk aversion concerns respectively the amount or the fraction that changes if, for example, a person experiences an increase in wealth, according to the slope of the risk aversion measure. These are the coefficients of risk aversion. They allow to measure the degree of curvature of the utility function, which in turn determines the intensity of the investor's risk aversion.

1.2. Modern Portfolio Theory

Markowitz, in (1952) and better explained in (1959), proposed an analysis "[…] to find portfolios which best meet the objectives of the investor." (Markowitz, 1959, p. 3). He also added that, "the proper choice among efficient portfolios depends on the willingness and ability of the investor to assume risk" (Markowitz, 1959, p. 6). It is not the purpose of this chapter to discuss the computational part[6]. However, Markowitz presented what is called the Mean-Variance Optimization Problem, in which he exhibits the powerful tool of diversification and, in which every investor is favorable in increasing the expected returns and unfavorable in increasing the standard deviation of returns. An investor should not choose an asset, or a portfolio, just looking at the highest expected value, because that asset, or portfolio, may have a higher standard deviation than the one the investor would take[7]. This means that when the investor will choose the efficient

[6] Markowitz (1952), (1955) and (1959).

[7] Since Markowitz assumed that asset returns are normally distributed, he was able to find the efficient frontier simply using the mean and the standard deviation of asset returns.

portfolio, her behavior towards risk will make her choose a certain efficient portfolio, rather than another one that a different investor will choose, because of different risk-aversion. So, the efficient will be the one that maximizes investor's expected utility, and, within the utility function enter not just the mean and the standard deviation of the asset returns, but also the risk aversion of the investor. So, the utility functions enter directly in the decision theory. Nevertheless Tobin (1956) showed that the different sequences of weights that constitute the efficient portfolios depend just on the characteristics of the probability distribution of risky assets and not upon the risk aversion of the investor. Thus, it is possible to split the process in two tasks: first, determining the efficient frontier and, then, the optimal risky portfolio[8], through the maximization of the Sharpe ratio. In presence of a riskless asset it is possible to retrieve the Capital Allocation Line (CAL)[9], that is the combination between riskless asset and the optimal risky portfolio. Second, the investor will choose that combination that maximizes her utility function. This result is called the Separation Theorem. Moreover, Hicks (1962), in his presidential address, analyzed the definition of Liquidity Preference and, concluded that "Liquidity Preference then determines the proportions in which capital is divided between money and risky securities; it does not affect the proportions in which the risky securities are themselves combined." (Hicks, 1962, p. 795). However, Sharpe (1963) showed that, in presence of lending and borrowing at the risk-free rate, all efficient portfolios, except for the market portfolio, become inefficient.

Then, Sharpe (1964) focused his attention on the construction of a market equilibrium theory of asset prices under conditions of risk, in order to shed light on the relationship between the price and the components of risk of an asset. Through diversification, part of the risk of an asset can be eliminated, but diversification cannot eliminate all the risk: there exists some part that cannot be avoided, because of the non-zero covariance among assets, and, that part is called systematic risk. Systematic risk is the responsiveness of asset return to changes in the market and, it is usually called and represented by beta (β)[10]. The result achieved is represented by the Security Market Line (SML), that allows to see

[8] Portfolio constituted just by risky assets.

[9] Where the Sharpe ratio represents the slope of the CAL. The CAL is also called Capital Market Line (CML) by assuming that the optimal risky portfolio is the market portfolio, which is a value-weighted portfolio of all assets in the investable universe.

[10] Indeed, by looking at the formula for computing beta (1.5), it includes the covariance between the asset return and the market return and the variance of the market. This is the only part that an investor cannot diversify, no matter the number of assets that she includes in her portfolio.

the linear relationship between the expected return of assets and their systematic risks. Sharpe (1964) stated that the expected return of an asset should depend just on the magnitude of its systematic risk and, that risk is the only one the market must remunerate. The work of Sharpe (1964), Lintner (1965) and, Mossin (1966) leads to the disclosure of the most widely used model for asset pricing: the Capital Asset Pricing Model (CAPM):

$$E(R_i) = R_f + \beta_i \cdot \left[E(R_M) - R_f\right] \quad i = 1, 2, \dots, N \tag{1.4}$$

where $E(R_i)$ represents the expected return for the i^{th} asset, β_i is the systemic risk of the i^{th} asset with respect to the market, R_f is the risk-free rate of return and, $E(R_M)$ the expected return on the market. The systematic risk of the i^{th} asset with respect to the market is given by:

$$\beta_i = \frac{cov(R_i, R_M)}{\sigma_M^2} \tag{1.5}$$

where $cov(R_i, R_M)$ represents the covariance between the returns of the i^{th} asset and the returns of the market and, σ_M^2 is the variance of market returns.

The best-known characteristic and the main problem of the Mean-Variance Portfolio Optimization and the CAPM is the fact that they are meant to hold in one period. However, it may happen that investors do not have short-term horizons but long-term ones, for example 10, 20, 40 years from now. Thus, an investor could not keep the same portfolio for the entire horizon, but he could shift from one asset to another because of new investment opportunities. Tobin (1965) was one of the first to introduce and analyze the problem of multi-period investments. He stated that there is one case where the theory of portfolio choice for one period has a direct application to the choice of multi-period portfolio sequence. In that case, asset returns are statistically independent and stationary over time[11]. Under these assumptions, not just all efficient portfolio sequences are stationary, but also "any portfolio that is efficient for one-period investment is also efficient for multi-period investment […]" (Tobin, 1965, p. 46) .

[11] "Statistical independence of asset returns over time means that the probability distribution of returns over any future time period are unaffected by returns actually realized in any previous period. […]. Stationarity over time means that the same probability distributions apply in every future period of equal length." (Tobin, 1965, p. 42)

1.3. Intertemporal Portfolio Choice

Mossin, Samuelson and Merton, allow to enter the field of Intertemporal Portfolio Choice. That is, generally, the process through which an investor allocates wealth in financial assets, repeatedly over time, in order to optimize some objective function. Intertemporal Portfolio Choice has been translated by researchers with the consumption-investment decision rules in multiperiod context, i.e. the investor must decide how much wealth allocate in consumption and in investment in order to maximize her utility function, where the object is the final wealth. In the subsequent models it is assumed that the income of the investor is derived only from capital gains. Intertemporal Portfolio Choice has been analyzed both in discrete and in continuous time, and the optimal result is achieved through a dynamic programming[12].

Mossin (1968) focused his work on multiperiod context without considering the consumption. He used a backward recursive procedure, because he included in new decisions both the outcomes of past decisions and the information of future probability distributions. Moreover, Mossin discussed myopically decisions. Broadly speaking myopia means making decision disregarding future opportunities, i.e. not considering the horizon. In particular, there are two types of myopia:

- Complete myopia that means behaving as each period were the last one.
- Partial myopia that means behaving as the immediate decision were the last one.

According to Mossin (1968), myopic decisions are not optimal in a multiperiod context, but there exist some class of utility functions, such as the logarithmic and the power utility function[13], that allow myopic decisions to be optimal. Those utility functions allow, in a single-period model, to have constant asset proportions independently on wealth to be optimal. Moreover, Mossin (1968) discussed Tobin's (1965) stationarity optimal portfolio policy, in which "the same proportion is invested in each asset every period" (Mossin, 1968, p. 227). Mossin argued that it is not enough to have independency and stationarity in asset returns, but there is also a need to have optimal complete myopia, i.e. utility functions must be represented by the logarithmic or the power ones. Thus, in order

[12] It is a procedure where the investor starts in last period defining the optimal decision rule and, then she goes backward to retrieve the optimal decision rule for each single period until she reaches the first one.

[13] Both are isoelastic utility functions, that is a special case of HARA utility functions, since isoelastic utility function is the only class of utility functions with constant relative risk aversion.

to have an efficient portfolio in both single-period and multi-period the investor needs to make decision in an optimal complete myopia framework.

Samuelson (1969) proposed a model for lifetime (i.e. multiperiod) planning for consumption and investment decisions. Both Samuelson and Mossin optimized the objective function through the dynamic stochastic programming, but the difference is that the latter optimized an objective function excluding consumption whereas the former formulated a model including both consumption and investment decisions. The more general model by Samuelson led to the conclusions that "for isoelastic marginal utility functions, […], the optimal portfolio decision is independent of wealth at each stage and independent of all consumption-saving decisions, [...]" (Samuelson, 1969, p. 244). If the assumption of isoelastic marginal utility function is released, the above theorem does not longer hold. The main conclusion achieved by Samuelson is that the investor has a constant relative risk tolerance[14], since the main property of isoelastic utility functions, as stated above, is to have constant relative risk aversion[15].

Merton (1969) derived the optimality equation in continuous time, including both consumption and investment decisions, with income given just by the asset returns – which in turn are generated by a Wiener Brownian-motion process – and with isoelastic marginal utility functions (i.e. risk-averse investors). Merton reached the conclusion that, under these assumptions, the optimal proportion invested in risky assets is independent with respect to wealth and time.

Therefore, Mossin (1968), Merton (1969) and Samuleson (1969) found that portfolio that is optimal in a static (one-period) model is also optimal in a dynamic (multiperiod) model. This because of the peculiarity of certain utility function that exhibits constant relative risk aversion, causing (complete) myopia to be optimal.

Moreover, in another paper Merton (1971) retrieved the optimal consumption and portfolio rules, in continuous-time, assuming returns to follow the geometric Brownian motion and furthermore assuming the utility function of the investor to be member of the HARA functions. He concluded that, in presence of a riskless asset, both the optimal consumption and the optimal proportion of wealth invested in risky assets are linear with wealth.

[14] Relative (absolute) risk tolerance is given by the reciprocal of the relative (absolute) risk aversion.

[15] Stated differently: having constant relative risk aversion means that portfolio choice does not depend on wealth (Campbell & Viceira, 2001).

Fama (1970) tried to fulfill the gap between one-period models and multiperiod models, describing the multiperiod problem as a sequence of single-period problems. Furthermore, taking few more assumptions than the general one-period model, Fama showed that an investor's behavior in a multiperiod context, facing her investment-consumption decision for any period, is indistinguishable with respect to the behavior of a risk-averse investor having one-period horizon.

I presented the main results in the field of Modern Portfolio Theory and Intertemporal Portfolio Choice. These results are the starting point of many subsequent studies, which in turn lead to various and different findings in the field of household finance. I will introduce some of these studies in order to give a wider context of the field of household finance, and to see also the difficulties that an investor, who is planning to save money for retirement, faces.

II. Human capital

As shown in the previous chapter Markowitz's findings led to the Tobin's separation theorem, which in turn led to the Sharpe-Lintner-Mossin model, also known as CAPM[16]. This in a single-period context. Then, Merton (1971) assuming asset prices to be stationary and log-normally distributed, in a continuous time frame, found out what has been called the two-fund theorem. That theorem is the analogue of the separation theorem, but in a multi-period framework. Moreover, Merton (1973) formulated an Intertemporal Capital Asset Pricing Model.

In this chapter, I present a possible source of risk that was left out in the above models: human capital[17]. One of the main assumptions made before was that all income was generated just by capital gains. Obviously, in real-life this is not a fair assumption, since it is more likely that an income is generated by work labor only than by capital gains only. Human capital has been identified by labor income[18], but in my opinion labor income is just the result of human capital. That is because, human capital includes the education and the experience that a person has respectively received and lived and, most importantly, what has been called tacit knowledge. Tacit knowledge is what a person has learnt from the environment where she has grown, lived, studied, worked. Computing all those variables and the impact that they have is extremely difficult, or impossible, and that explains why all researchers translate human capital with labor income. It is possible to understand that the greater is the education of a person the higher is the probability to earn a high income. However, there are cases in which labor income of a person does not reflect the true value of her human capital; this concerns the behavioral component of a person as well. For instance, her willingness to play the game in first person.
However, at the end human capital ends up in a job, and so in an income.
Human capital represents an asset and as all assets it is a source of risk. Therefore, it is important to include human capital as present value of future labor income in the computation of optimal portfolio composition between stocks and bonds. However, this

[16] Black (1972) tested the linearity of the CAPM with the assumption that an investor cannot borrow or lend at the risk-free rate (i.e. there is no riskless asset) and then with the assumption that an investor cannot borrow at the risk-free rate but just lend. Black reached the conclusion that in both cases there is a linear relationship between the expected return of a risky asset and its beta.
[17] For more theoretical background about human capital and optimal consumption and portfolio decisions look at the works of (Chan & Viceira, 2000), (Davis & Willen, 2000), (Viceira, 2001).
[18] Present value of all labor incomes earned by a person during her working life.

asset is nontradable, which means that investors cannot sell it. The reason is the moral hazard problem. An investor could sell claims against her future labor income, but she cannot be legally enforceable to work (slavery is illegal) or she has the intention to boost her human capital – to sell it at a higher price – because it is extremely difficult to evaluate and it takes on risk.

All the models described in the previous chapter are formulated in a complete market setting, where there are assumptions – such as no transaction costs, borrowing and lending at the risk-free rate, and others – that do not reflect real-life situations. So, my aim here is to show the literature behind models formulated in incomplete market settings, where labor income risk enters the models and it is not fully insurable using marketable assets. The literature reaches the conclusion that if human capital has been taken into account (even if it is assumed to be riskless) this changes the composition of investors' portfolio.

However, empirical studies observed the different behaviors of investors, with respect to theoretical models. By trying to explain these discrepancies, first studies extended the CAPM model by including human capital risk. Then, researchers moved towards multiperiod contexts, by adding to the consumption-investment decision rule different components of human capital, for instance, education. In this chapter, I will firstly present the literature behind the extension of the CAPM. Secondly, I will present the literature behind the optimal consumption-investment decision in presence of human capital, where researches focused their works on the main determinants of human capital and how they influence investors' portfolio composition. Subsequently, I will introduce other types of risks directly linked with human capital risk. Finally, I will report the more generally accepted investment guidelines. However, it is important to remind that the starting point is the empirical evidence and that theoretical models leave out difficulties that investors must face in real-life situations. In this chapter, most of the time human capital is treated as an investment in riskless assets.

2.1. CAPM and Human Capital

One of the main assumptions of the CAPM is that each investor wants to hold the market portfolio. However, this is not reflected by empirical evidence, where investors tend to hold different portfolios. Mayers (1972) tried to explain it, by including – in the CAPM – non-marketable assets. Mayers examined the returns of human capital as a proxy for non-marketable assets. He reached the conclusion that human capital risk – computed as the covariance between the market portfolio and the payoffs to human capital – is different for each investor and this leads different investors to hold different portfolios of risky assets[19], i.e. "each investor holds a portfolio of marketable assets that solves his personal and possibly unique portfolio problem, and, hence the model allows for unique portfolios to be held by investors" (Mayers, 1973, pp. 259-260). It is obvious that an investor holds a marketable portfolio, but now in the computation of her optimal portfolio the investor must take into account not just asset return distributions, but also human capital risk. This could change the composition of her optimal portfolio computed only using asset return distributions. Moreover, Mayers' (1972) extended model confirmed a similar linear relationship between the expected return and the risk of an asset; similar because it is still linear, but now the systematic risk includes the human capital risk. The Mayers' extended model is

$$E(\tilde{R}_{it}) = R_{ft} + [E(\tilde{R}_{Mt}) - R_{ft}]\beta_i^* \quad i = 1, 2, \dots, N \tag{2.1}$$

and the systematic risk is represented by

$$\beta_i^* = \frac{V_{M,t-1} \cdot \text{cov}(\tilde{R}_{it}, \tilde{R}_{Mt}) + \text{cov}(\tilde{R}_{it}, \widetilde{H}_t)}{V_{M,t-1} \cdot \sigma^2(\tilde{R}_{Mt}) + \text{cov}(\tilde{R}_{Mt}, \widetilde{H}_t)} \tag{2.2}$$

where, in (2.1) all the variables are simply the analogue of (1.4) just expressed in a different way, but the market portfolio and the systematic risk. The M here is the market portfolio of marketable assets. The systematic risk β_i^*, of the i^{th} marketable asset, includes $V_{M,t-1}$as the total market value of all marketable assets, $\widetilde{H}_t$ as the payoff (labor income) on all non-marketable assets. In addition, the standard systematic risk also depends on the covariance between the i^{th} asset and the payoff of all non-marketable assets and the covariance between the marketable portfolio and the payoff of all non-marketable assets.

Furthermore, Mayers (1973) extended the previous model allowing for the presence of risky assets only and further assuming that an investor must determine not only her allocation among risky assets, but also the wealth allocation between consumption and investment. Again, Mayers observed both a different composition of portfolios of risky marketable assets held by investor and a preservation of the general linear relationship

[19] Even with the assumption of homogeneous expectations.

between risk and expected return on individual assets. Finally, Mayers (1974) derived the equilibrium structure of wages under uncertainty. He reached the conclusion that under uncertainty with a perfect labor and a perfect capital market, the equilibrium structure of expected wages is linear, and its measure of risk is represented by the covariance between the wage income and the total wealth.

Essentially, the general assumption of zero-covariance (i.e. zero-correlation) between the market and the human capital implies zero-covariance between every single asset in the market and labor income. Therefore, human capital is riskless. However, it still changes the composition of the investor's portfolio because it is possible to consider human capital as an investment in a riskless asset (reminding that is a non-marketable asset). Obviously, labor income does not always come without risk and its riskiness depends on its covariance with the market. Hence, each investor is exposed to a different magnitude of risk because of the different jobs.

Fama and Schwert (1977) tried to estimate the magnitude of the difference between the beta from Mayers' extended model and the beta from the CAPM. They compared estimates of (1.5) and (2.2) for different classes of marketable assets, i.e. they estimated $(\beta_i^* - \beta_i)$. They found that this difference is very small, resulting from a weak relationship between human capital and the returns on bonds and stocks. So, it is possible to see that if the correlation between human capital and other marketable assets is low, then CAPM could be used to measure the expected return-risk relationship. However, the Fama and Schwert's (1977) study focused just on NYSE stocks and governments bonds and, as stated by themselves, there are subgroups for which the relationship is not weak. Today, there exist larger financial markets in the world with widely spread correlations among each other and considerations about human capital risk cannot be omitted in a portfolio composition analysis for long-time horizon. Furthermore, the beta from Mayers' extended model could be different from the beta from the CAPM, if there is covariance between labor income and the market. This means that the SML of Mayers' extended model could be different from the general one. Both are still linear, but one could be above or below the other.

2.2. Consumption-investment decisions and human capital determinants

In multiperiod context we need to consider many aspects of human capital. That is because there are various components that affect human capital. The main determinants are education, labor flexibility, length of remaining working lifetime and riskiness of human capital. These four determinants allow to see why one of the most important financial advice in a life-cycle strategy is to take greater risks when individuals are young and gradually reduce those risks as individuals become older and older.

Williams (1978) introduced an educational-work trade-off. An individual can choose her level of education[20] at the expense of working time. Hence, time spent for education instead of working is an investment for a future higher human capital, where the opportunity cost is represented by the current labor income that could be earned working. Williams developed an educational-consumption-investment model in order to also find the optimal investment in education, as well. Then, Williams (1979) included the ability for the individual, through education, to learn more about her ability to acquire additional skills.

Bodie, Merton and Samuelson (1992) examined the effect of labor-leisure trade-off on consumption-investment model[21]. That trade-off is called labor flexibility. They included it in their model and, through different levels of flexibility individuals determine their personal optimal levels of consumption, labor effort and financial investments. Bodie et al. (1992) stated that investment decisions of individuals must be seen in terms of total (financial and human capital) wealth. They showed that two individuals, one with flexibility labor supply and the other with fixed labor supply, hold different explicit portfolios[22]. Moreover, the numerical value of human capital is given by the present value of labor income, i.e. workers assume to know all future wages until retirement age, and they discount them depending on their riskiness. Obviously, the present value function depends also on the time, so human capital will also depend on the length of individual remaining working lifetime.

Firstly, education is important because it allows individuals to earn a higher income in the future. If an individual earns a high income then human capital will be greater than, *ceteris paribus*, another individual's human capital with lower income. Secondly, depending on the type of occupation, people can enjoy a certain degree of labor flexibility,

[20] Where the outcome in term of human capital is uncertain because uncertain is education process. Since that, the investment in education also depends on risk aversion of the individual.

[21] Labor-leisure trade-off was also considered in (Mayers, 1974) and (Williams, 1979).

[22] In Bodie et al. (1992) explicit portfolio is the portfolio made only using financial assets, i.e. it is the one that individuals hold in the market; while, implicit portfolio is the optimal portfolio because it includes also human capital. Bodie et al. showed that starting from the implicit portfolio and subtracting to it human capital it is possible to achieve the explicit portfolio.

but generally speaking the younger the individual, *ceteris paribus*, the higher the labor flexibility she will be able to provide. According to Bodie et al. (1992), labor flexibility means the ability to decide how much time spent either in working or in leisure but it also means the decision to retire before the conventional age. Thirdly, the length of the remaining working lifetime, *ceteris paribus*, changes the value of human capital: the lower the remaining working lifetime the lower the number of wages to be discounted, thus, the lower the human capital. Therefore, it possible to see that a younger individual has a greater (riskless) present value of income, a greater labor flexibility and a greater length of remaining working lifetime than an older one. Therefore, the younger should invest greater proportion in risky assets in order to offset her more-than-safe position.

Obviously, I have talked about three of the four determinants as separate components, but they are linked together. For example, higher education is more likely to allow the individual to achieve greater labor flexibility. In such a way, if the investment does not turn out as planned the worker can recoup the losses by working more hours, i.e. the individual has a greater margin of safety, meant as magnitude of labor flexibility that she can provide. As the age increases, or the remaining working lifetime decreases, riskless human capital decreases and so decreases the proportion invested in risk-free assets, hence, the individual should decrease the investment in risky assets. Indeed, Jagannathan and Kocherlakota (1996) explained the shifting investments away from stocks and toward bonds, as investors age, as a way in which individuals can offset the declining of their human capital values.

All in all, a younger individual should invest more in risky assets than an older one because of the greater amount of remaining working lifetime, the greater amount of labor flexibility and the greater probability of high labor income (depending on level of education). Obviously, there exist many combinations between education, human capital risk, labor flexibility and remaining working lifetime that allow the individual to split human capital in parts of investment in risky assets and in risk-free assets. However, the main idea – assuming riskless human capital – is that better educated individuals, as well as younger workers with greater labor flexibility and a plenty of remaining lifetime, have greater human capital and they should be willing to accept more risks in financial markets.

Finally, heterogeneity in jobs implies heterogeneity in the riskiness of human capital. Up to now it has been assumed that human capital is riskless, but it has been seen above that it depends on its covariance with the market. Essentially, a high-risk human capital that derives from a high covariance with market (due to the investor's job), leads to treating human capital as a risky asset and it will reduce the investment proportion in risky assets.

2.3. Background risks

Background risks include labor income (human capital), proprietary income and housing. The latter two are marketable assets, meaning that it is possible to sell or exchange them for something else. Proprietary income is also called entrepreneurial income, that is the fact that an individual is the owner of a company whereas housing is the fact that every individual holds at least one house where she can live or rent it. All these three risks associated with the income sources are implicitly held by the individual. This means that they can alter the portfolio composition of the investor. The income of the owner of a company is subject to the market, i.e. it is highly correlated with market conditions and add some risk to the portfolio of the owner. Obviously, a company could be sold but for housing is more difficult: if the individual has just one house that she uses as a consumption good it can be considered as a risk-free asset (Bodie & Crane, 1997), while if the worker has other houses that she rents then housing will represent an asset that is correlated with the housing-market, i.e. it will add some risk to the portfolio. Therefore, background risks must be considered in the composition of the optimal portfolio. Heaton and Lucas studied the three sources of background risk once at a time. Firstly, Heaton and Lucas (1997) calibrated a portfolio-choice model in which agents face labor income uncertainty that cannot be insured directly. Individuals self-insure labor income shocks by accumulating a buffer stock of savings (stocks, bonds, risk-free assets), in order to smooth consumption over time. When labor income is assumed to be uncorrelated to asset returns, i.e. there is no systematic risk, the model predicts that all savings are held in risky stocks. Consistent with above, individuals see human capital as a risk-free asset and so, substitutes for risk-free bond holding. Furthermore, they tend to offset this asset by allocating their wealth in stocks. Then, Heaton and Lucas (1999) included entrepreneurial income[23] in their analysis. They reached the point that if an individual is the owner of a business activity, her income is more highly correlated with market returns (business cycle) than labor income earned as an employee. So, this leads to the conclusion that

[23] Both entrepreneurial income and proprietary income represent the income earned by an individual because she is the owner of a business activity.

investors with proprietary income are less inclined to hold stocks, or at least tend to hold less wealth in stocks, due to the higher background income risk that they face. Heaton and Lucas also noticed that, contrary to conventional wisdom, entrepreneurs at retirement age tend to increase the proportion of wealth in stocks, but this perhaps because older property-investors tend to substitute riskier proprietary business ownership, by selling it, with safer assets such as stocks, bonds and cash. Hence, it has been possible to see how the presence of entrepreneurial income risk alters the portfolio composition of investors who face that risk. Finally, Heaton and Lucas (2000) explored the effects of background risks – labor income, proprietary income, and real estate – on portfolio allocations, and their results are consistent with the previous studies. However, they also noticed that "[…] changes in the general economic environment […] will result in differences in household exposure to background risk over time." (Heaton & Lucas, 2000, p. 23).

It is possible to see why background risks are so important: because they link individual's survival with economy. Without a labor income an individual cannot invest, she will be worried to consume and then if something is left she may think to invest. The same argument for housing, without a house to live in the investor will not think about investing. So, when an individual has to choose the allocation of her portfolio she also has to take into account the risks associated with her background and try at least to insure them.

2.4. Accepted financial advices

Bodie and Crane (1997) listed generally accepted financial advices, that represent the recap of what I have exposed in this chapter. Financial advices are:

- Funds saved for retirement should be invested in primarily in equities and log-term fixed income securities.
- The fraction of assets invested in equities should decline as the investor's age advances, also known as age effect.
- The fraction of assets invested in equities should increase with wealth because a wealthier individual should be able to handle more risk, also known as wealth effect.
- All investors should diversify their total portfolios across asset classes, and the equity portfolio should be well-diversified across industries and companies.

- Young investors, with long investment horizon, should take more risk than older investors.
- Conservative investors are typically encouraged to hold more bonds, relative to stocks, than aggressive investors. However, this is the opposite to the constant bond-stock ratio of Markowitz and Tobin. Indeed, it has been called by Canner et al. (1994) asset allocation puzzle.

2.5. Conclusions

Campbell and Viceira (2001) wisely summed up by saying "household adjusts explicit asset holdings to compensate for the implicit holding of human capital and reach the desired allocation of total wealth" (Campbell & Viceira, 2001, p. 170). This adjustment should continue during the lifetime of the investor, because changes in human capital value and riskiness lead to changes in household's portfolio composition.

III. Different studies on empirical evidence

In this chapter, I show what are the main differences between theoretical models and empirical evidence and what are the possible explanations given by researchers to justify these discrepancies.

In the previous chapters I have shown an extremely important type of risk, human capital, which must be taken into account as it alters the theoretical portfolio allocation. Moreover, I have also briefly exposed which rules the investors should follow according to financial advices. Nevertheless, empirical evidence shows significant discrepancies between the investors' portfolios and recommended ones, and it shows some heterogeneity among investor's portfolios. Researchers suggest that investors who are better educated, wealthier and show specific sociodemographic characteristics are more inclined to be closer to financial advices rather than less educated and poorer ones[24]. Furthermore, researchers try to explain these discrepancies by claiming that there are other aspects to consider in the portfolio allocation which could lead different investors to alter their portfolio in a personal portfolio allocation.

3.1. Empirical evidence

Empirical evidence has shown that the main discrepancies concern:

- Lack of participation in the stock market and low share of wealth invested in stocks.
- Lack of asset diversification.
- Lack of stock diversification, including the problem of employee stock ownership.
- Lack of international diversification.
- Inertia, in the sense of doing nothing when economic or financial conditions changes lead to a better or a worse scenario.

[24] It is important to understand that wealthier and better educated groups tend to follow closer financial advices, but this does not mean that they exactly follow recommended portfolios (the same is true when I will talk about diversification). Even within these groups a lot of households present some of the aspects showed by poorer and less educated households. For example, if the financial advice is to invest 40% of investor's wealth in stocks, empirical evidence observes that poorer and less educated household tend to hold a small percentage (say no more than 5%) in stocks or not at all. On the other hand, some wealthier and better educated households will tend to hold a greater percentage of stock (say 20%) but it is still well below the recommended one.

3.1.1. *Lack of participation and low share in risky assets*

Since the discover of the equity premium puzzle by Mehra and Prescott (1985), people should be encouraged in holding stocks. The fact that empirical evidence has shown that the bulk of investors tends to hold a little percentage of stocks[25] or not at all leads to the so called “stockholding puzzle” (Haliassos & Bertaut, 1995) and “participation puzzle” (Guiso, et al., 2002).

Mankiw and Zeldes (1991) estimated that three-fourths of US families do not hold stocks. They tried to explain it by liquidity constraints, but this is not the only reason. That is, less than half of households with more liquid assets own equity. This view is confirmed by Haliassos and Bertaut (1995). Moreover, Haliassos and Bertaut consider business cycle risk (the positive correlation between labor income and the stock market) as a reason for lack of participation. Furthermore, they also included the informational costs as a reason for the stockholding puzzle. Generally speaking, Mankiw and Zeldes (1991) showed that wealthier and better educated households are more likely to be stockholders. However, King and Leape (1987) showed that this was not always the case in the past.

Gomes and Michaelides (2005) updated to one half the number of US households that hold stocks, in 2001. However, this improvement is still far away from the idea that every household should participate in the stock market because of the equity premium puzzle. They set up a model in which the stock market is inhabited mainly by risk-averse households, who accumulate more wealth and are less reluctant to pay the fixed entry cost to participate with respect to less risk-averse households who have a weaker incentive to pay it. Henceforth, since there are risk-averse investors they end up by investing just a small part of their wealth in stocks.

Furthermore, Gomes and Michaelides (2005) summarized Ameriks and Zeldes’ (2004) empirical findings and noted that the participation rates increase during working life and then decrease during retirement. Essentially, the participation rate follows a hump-shaped profile as well as the risky asset share held by investors over lifetime (Poterba & Samwick, 2001). This could be justified by the fact that as investors accumulate knowledge and experience, they tend to participate and invest more in risky assets (Ameriks and Zeldes (2004), King and Leape (1987), Guiso et al. (2002)). However, the problem is that financial planners (and equity premium) suggest that households should all participate in the stock market and then decrease their participation rate as their age

[25] In the sense of either directly or indirectly through mutual funds, retirement accounts, etc.

increases; and they should all invest in risky assets when they are young and then decrease the proportion as they age. However, empirical evidence shows a different story: households are reluctant, when they are young, not just to participate in the stock market, but even invest in risky assets (Guiso, et al., 2002).

3.1.2. Lack of asset diversification

There are two important meanings of lack of asset diversification: first, a household could hold within her portfolio a very low number of assets; second, a household could hold within her portfolio a very low number of a specific type of asset, for example stocks.

Regarding the first part, King and Leape (1987) showed that also among wealthy households there is a very low number of assets held, and this amount increases with age. It is, thus, hump shaped. Focusing on the second part, Canner and Mankiw (1994) called it the "asset allocation puzzle". That is, the discrepancy between the mutual fund theorem[26] and the financial advice which argues that for more risk-averse investors, portfolios should be rebalanced towards a higher amount of bonds rather than stocks[27]. This advice, if followed, for example, by an extreme risk-averse investor leads either to a lack of participation in the stock market or in a low level of stocks held in households' portfolios.

However, as stated above, empirical evidence has shown that wealthier and better educated households tend to have a better diversified portfolio than poorer and less educated ones. Finally, one solid finding reported by Guiso et al. (2002) is that most households tend to concentrate their portfolios on those assets that are the safest and most liquid.

3.1.3. Lack of stock diversification

Empirical evidence presented by Blume and Friend (1975) showed highly undiversified portfolios of stocks among households. It is possible to observe even here that wealthier

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[26] Recall that the mutual fund theorem says that an investor has the market portfolio (where there is the optimal allocation between bonds, stocks and other assets) and a risk-free asset. The optimal allocation, depending on investor's risk aversion, is between the market portfolio and the risk-free asset.

[27] For more details about the puzzle and the discussion look at (Canner, et al., 1994) and (Campbell & Viceira, 2001, pp. 40-72).

households tend to have more diversified stock portfolios. However, at all levels of wealth there is substantial low stock diversification.

Stock diversification has two meanings: first, having an adequate number of stocks; second, stocks must have low correlation among each other, this means that stocks should be spread among different sectors and countries. An exception to the first part is, for example, self-employed households that tend to have a lower amount of stocks because of their highly correlated job with the market. Nevertheless, they seem to have a more diversified stock portfolio (Blume & Friend, 1975). More generally, Kelly (1995) showed that households tend to hold a very low number of stocks. This number tends to increase with wealth, even though it is still far away from the theoretical finance textbooks. As mentioned above, the stock holdings curve is hump shaped since it increases with the age of households.

Another problem is the employee stock holdings (Benartzi & Thaler, 2001). Employees who are stockholders may work harder because their wealth is linked to the performance of the firm for which they work. Usually, a firm offers to its employees a certain number of stock (call) options as part of payment. However, the call option has five price determinants: the price of the stock, the time to maturity, the exercise price, the interest rate and the volatility of the stock. The volatility of the stock has a positive relationship with the value of the call option. This means that an employee who wants to increase the value of her call option could do so by working harder and/or by engaging in riskier activities than normal that could boost up the stock price, but can also take down the firm. Despite this, it is obvious that for an employee having part of her wealth invested in her working company represents a lack of diversification. This is because if the firm performs badly, she does not just face a sharp decrease in portfolio value because the stock price has declined but she could also lose the job. Essentially, a household portfolio should not take the same risks that its major source of income faces (Malkiel, 1996).

3.1.4. Lack of international diversification

French and Poterba (1991) presented evidence that, despite the great benefit that could be achieved by households in investing in different countries, they tend to invest in their own country assets. However, the argument concerns real estate investments as well. Quan and Titman (1997) concluded that the relation between stock returns and housing in 17 countries is a strong and positive one. This could be a reason to diversify not just in international stock markets but also in international real estate markets.

3.1.5. Inertia

Inertia is the no-action done by an investor that instead should adjust her portfolio because age increases or simply because economic conditions evolved. An example could be the refinancing of a mortgage when interest rates decrease (Campbell, 2006).
Ameriks and Zeldes (2004) demonstrated that households tend to make few or no changes to their portfolio allocations over time.

3.2. Possible explanations given by researchers and other challenges

Researchers consider not just human capital, but also other aspects that affect the investor’s decisions and lead her to a determined portfolio allocation. These possible explanations and challenges that an investor faces, are:

- The presence of human capital in different features.
- The presence of housing
- Behavioral aspects that lead to biases.
- The presence of transaction costs, that include informational costs.
- Problems in understanding the inflation.
- Difficulties in optimizing the tax choice.
- Literacy, i.e. financial knowledge that affects directly all the six points before.

3.2.1. Human capital

Cocco et al. (2005) argued that it is the shape of the labor income over life that affects investors to increase or decrease the stock proportion in their portfolio. If labor income has a smooth decreasing function over age (meaning a bond-like feature) the household will have a smooth decreasing function of equity shares over age. However, Benzoni et al. (2007) modeled the case in which labor income is cointegrated with dividends. They argued that labor income is more a stock-like asset when the household is younger. As time passes, the (positive) correlation with stock market decreases as age increases transforming labor income in a bond-like asset. Labor income has a hump-shaped profile that results: firstly in a zero participation (or even a short sale position) in stock markets at the early stage of investor's working life; secondly in an increase in equity shares with a peak at middle-later age (40-60 years old) and finally it decreases when investor retires. Essentially, it causes a hump shape in the stock holdings because of the riskiness change on labor income during the life pattern.

In addition, Baxter and Jermann (1997) showed that human capital is highly correlated to returns of domestic stock market and an investor should diversify internationally. Moreover, they suggest that a well-diversified portfolio will establish a short position in domestic marketable assets and a long position in foreign markets.

3.2.2. Housing

King and Leape (1987) showed that the most important asset held by households is housing (confirmed then by Heaton and Lucas (2000)). This is also reported by Tracy et al. (1999), and they assumed that households who want to live in a house are constrained to buy it entirely and end up having little (liquid) capital for other kind of investments, like equity. Therefore, households will result with a poorly diversified portfolio where the bulk of wealth is invested in housing. Not affected by this problem are wealthy households, whose weight burden of housing results to be much lower in the portfolio and thus have greater wealth available to diversify.

The housing effect could explain the fact that young households tend to abstain from participating in stock markets. Because they have a very little wealth amount, housing – that is an illiquid and an undiversifiable consumption good – represents the major asset in their portfolios, i.e. they are highly leveraged, liquidity constrained and poorly diversified. This view is confirmed by Cocco (2005), and he adds that this situation is faced by poor households[28] as well. Flavin and Yamashita (2002) suggested that young households have incentive to reduce risk and one way to do so is to pay down the mortgage. The other way is to invest in bonds rather than stocks. Then, the investment in stocks will increase as wealth is accumulated and so its profile will be hump shaped. It is therefore possible to understand that there is another reason for which the percentage of

[28] Cocco (2005) studied the different portfolio allocation decisions between owning and renting a house.

stock holding increases with age, not for mere human capital reasons, but because of the proportion of housing value with wealth value, the so-called housing constraint. The higher the ratio, the higher the leverage of the young household and the higher the incentive to balance the portfolio by investing in bonds. Over time the ratio will decline, and the household will tend to invest more in stocks rather than bonds.

3.2.3. Borrowing constraints

Guiso et al. (2002) suggested as a possible explanation the fact that young households do not participate in the stock market because they have a small amount of cash and because they are borrowing constrained, i.e. they cannot borrow money and invest in risky asset taking advantage of the equity premium puzzle. They cannot borrow for different reasons: one could be that they are young and with little wealth on hand as collateral and the bank will not lend to them or, more likely, they have purchased a house through mortgage and they are already leveraged and it will be difficult to borrow again at a rate that they can afford it.

3.2.4. Behavioral aspects

French and Poterba (1991) explained the lack of international diversification by showing that investors who live in a certain country tend to expect their own country market to outperform (with a huge difference) other countries markets. So, this leads to invest all their wealth in securities within that country, leading to a lack of international diversification. French and Poterba tried to give two explanations. The first and less likely one is that there are institutional constraints, for example institutional barriers, tax purposes, transaction costs, however, all these alone are unable to explain this type of lack of diversification. The second one concerns the behavior of investors. Investors tend to think that expected returns are higher in domestic markets and that foreign countries bear extra "risk" because they know less about foreign markets. Behaviorally speaking, this is known as home bias. Home bias is an implication of the heuristic known as familiarity. This bias could be explained by the ambiguity aversion showed by investors as well as by the fact that people are comfort-seeking (so-called status quo bias). The former is simply the fear of something that a person does not know, while the latter is the fear to feel regret because a person has changed something. Zhu (2002) documented this local bias as well. However, he excluded the advantageous information that people think to have as a possible explanation.

Huberman (2001) showed how people tend to be affected by this geographical bias and end up not only by concentrating portfolios in the country in which they live, but also by holding employer's stocks in their retirement account. Essentially, home bias and more generally familiarity could also explain the lack of stock diversification. Huberman further suggested that familiarity could be linked to availability of information by the local investor. "Familiarity may represent information available to the investor, but not yet to the market. It may represent the investor's illusion that he has superior information." (Huberman, 2001, p. 675).

3.2.5. Transaction costs

Ameriks and Zeldes (2004) included in transactions costs: minimum balance requirements, pre-trade fees and information costs. All these monetary and mental efforts could represent reasons for which households not only tend to stay away from stock markets but also have inertia behaviors. The inertia costly equivalent[29] amount depends, for example, on the risk aversion of the household, on her wealth, and other factors, even psychological ones. If an investor is highly risk-averse, the entry fee for the stock market could be even very low, but for her it could be high enough to decide not to participate (Campbell, 2006). The same is true for information costs (Haliassos & Bertaut, 1995). Empirical evidence showed that "higher-income families are more likely to choose to pay the fixed cost [to enter in the stock market] because they have larger portfolios, and the fixed cost is lower for the more educated because information acquisition and processing are less costly" (Mankiw & Zeldes, 1991, p. 101).

Transaction costs could be a possible explanation, according to King and Leape (1987), for the lack of asset and stock diversification. When a household decides to invest in a certain number of assets, not only she has to pay the cost for the transaction, but also the cost for selecting the adequate assets to insert in her portfolio, and this implies an effort that is not monetary. Even here the wealthier and better educated households tend to have

[29] The cost for which a household decides to not adjust her position or to not enter in the stock market.

a higher number of assets (meaning a better diversification as suggested by financial planners). Thus, the solution could be to simply give every information to a household, and she will be able to process and make the right decision. Despite this scenario being very appealing it is very unlikely that the household will get hold of complete information. That is, even assuming that the household has the knowledge to process the information, she in reality has to deal with incomplete information (King & Leape, 1987). Furthermore, transaction costs could also be an explanation for low stock diversification, even if it is not sufficient to explain the entire lack of diversification (Blume & Friend, 1975). Transaction costs is less likely to be a possible explanation for lack of international diversification (French & Poterba, 1991) even if it is not sufficient, because now transaction costs are almost smoothly among countries.

3.2.6. Inflation

Inflation is the rate at which the average price level in an economy increases over some period of time. Inflation causes through time the erosion of the purchasing power of consumers. However, Lusardi and Mitchell (2011a) found out that there is still about 25% of people surveyed that do not know what inflation is. One consequence for this lack of knowledge is what has been called "money illusion" by Shafir et al. (1997). Shafir et al. explained that people tend to think in terms of nominal monetary values rather real ones. Moreover, they showed that people also tend to think in both nominal and real terms in different representations, and then they mix these representations. Eventually, the result is a bias in the assessment of the real value induced by a nominal evaluation, i.e. people evaluate in nominal terms to take decisions because it is easier to grasp. They explained this difficulty in valuation of alternatives because of inflation as framing, loss aversion, risk attitudes and fairness concerns, and there are still other behavioral reasons. Moreover, Canner et al. (1994) tried to use money illusion to explain the asset allocation puzzle.
Inflation also concerns the decision between nominal bonds and inflation-indexed bonds in the portfolio allocation. This argument is treated by Campbell and Viceira (2001) and they concluded that in the long-term "an inflation-indexed long-term bond is actually less risky than cash" (Campbell & Viceira, 2001, p. 72) because it delivers real income giving the possibility to the investor to keep her living standards.

3.2.7. Tax

Poterba (2002) pointed out the importance of taxes in the determination of the household portfolio structure. Particularly, taxes rise the problem in which asset invest and how much to borrow and when to trade assets. The last problem is of concern among investors because it seems that they are unable to behave tax-efficiently. Dyl (1977) provided the traditional view for which the capital gains tax gives incentive to investor to realize capital losses, as a tax shield, and to defer the realization of capital gains. Following this hypothesis, investors follow year-end tax strategy in which investors, toward the end of the year, decide to realize capital losses for tax purposes. This behavior suggests, according to Dyl, that there will be a high trading volume in stocks that represents unrealized capital losses at the end of the year, and a low trading volume for the unrealized capital gains. According to Constantinides (1984) this could predict a seasonal pattern in trading volume. This could explain the so called "January effect" which suggests the hypothesis that stock prices tend to increase in the month of January more than in any other month. This could be explained by the fact that investors and traders realize (sell) capital losses in December causing an abnormally high trading volume and buy another security with similar characteristics of the one sold in December to rebalance their portfolios.

The one just described is the rational behavior for an investor who wants to take advantage in a tax system. However, Shefrin and Statman (1985) observed financial markets and reached the conclusion that investors are reluctant to realize losses[30], moreover, they tend to keep the stocks following the so called "break-even effect". Shefrin and Statman explained this behavior starting from the prospect theory (Kahneman & Tversky, 1979), in which people tend to be more risk averse in the domain of gains and more risk seeking in the domain of losses. They also included as a possible explanation the mental accounting theory (Thaler, 1985), in which people's loss aversion and endowment effect lead to the reluctance to sell losers. That is because people want to avoid the pain of closing an account, i.e. sell a stock, in which they have lost money. Moreover, people try to avoid feelings of regret that are caused by their personal (investment) choices. In the end, investors who succeed in the selling of losers and in the holding of winners are the ones who employed the self-control (Thaler & Shefrin, 1981)[31].

[30] (Odean, 1998).

[31] See also Tversky and Kahneman (1974) and Thaler (1980).

3.2.8. Literacy

Financial illiteracy is the first reason for which investors fail to design and carry out retirement saving plans (Lusardi & Mitchell, 2011a). Lusardi and Mitchell have submitted a simple questionnaire about three main arguments needed to be understood by investors to design an effective saving plan. At the end, they conclude that (Americans) investors have a low level of financial literacy. The result is extremely discouraging since in most countries of the world people have defined contribution pensions. That means that households have the responsibility to choose where to allocate their pension wealth (Lusardi, 2002). Moreover, empirical evidence reported by King and Leape (1987) supports the importance of information, directly linked with literacy, since one reason because households do not hold stocks is that they do not have enough knowledge of the stock market.

Most importantly, people without financial knowledge make mistakes in the portfolio allocation and this potentially leads to lose part of their pension wealth, and as a result they have to adjust to a different retirement lifestyle from the one they had planned for.

People who have no financial knowledge will rely on heuristics, for example, in Benartzi and Thaler (2001) people tend to follow the “1/n heuristic” where “someone using this rule simply divides her contributions evenly among the *n* options offered in her retirement saving plan” (Benartzi & Thaler, 2001, p. 79). Essentially, they can adopt strategies that do not follow any life-cycle pattern, i.e. not considering the different risks that they face during different time in life. In addition, Haliassos and Michealides (2002) suggested that the lack of knowledge leads the investor to rely just on her impressions about the market. In particular, “misperceptions, ignorance, and even prejudice can contribute to inertia” (Haliassos & Michaelides, 2002, p. 63). The message is that without knowledge investors’ decisions depend on their impressions, on their standpoints and on their past experiences. They represent the only source of knowledge for the investor.

Another relevant problem is that people talk with their friends, family and so they are affected by family and friends’ opinions (Lusardi & Mitchell, 2011a). They talk to them instead of talking to experts. A friend, instead, could be seen as a person who gives advice based on self-experience and personally recommends a certain expert rather than another, i.e. she acts as a referral.

Finally, an investor who wants to improve her financial decisions, or at least understand what her financial advisor is doing with her pension wealth, should take retirement seminars (Lusardi, 2002). Moreover, the government should raise awareness about the financial choices that students will have to face when they become adults (Bernheim, et al., 1997). That is because, there are encouraging results showing the improving of decisions about retirement plans by individuals. In addition, financial literacy programs in school allow all students to learn, as documented by Lusardi et al. (2010) who state that financial literacy is strongly related to sociodemographic characteristics and family financial sophistication.

3.3. Conclusions

Guiso et al. (2002) showed that these puzzles are present in all the main developed countries from the United States to Europe to Asia. Moreover, financial illiteracy is even widespread around the world (Lusardi & Mitchell, 2011b). It is possible to see that almost every researcher showed that better educated (and wealthier) household tend to be closer to financial advices. So, how can households improve their decisions? Through education. However, regulation can also mitigate the mistakes made by investors by, for example, setting default option or by limiting retirement income risk (Antolin, et al., 2009).

IV. Data

In this chapter I present the assumptions made and the asset classes used in the simulation. Since the estimation window starts from July 1999 and ends in July 2020, I present the most significant events happened during that period.

4.1. History from 1999 to 2020

The period 1999-2020 has been characterized by the introduction of the single currency called EURO within the European Union. That was one of the major steps conceived in order to create the European Market Union. After that, the World suffered from the dot.com bubble (2000-2001) and from the debt crisis in Argentina (2000-2002). In response, the Federal Reserve reduced interest rates in order to encourage investments and boost the economy. However, few years later in 2008 a new bubble bursted: the Subprime bubble or the American Housing Market bubble. It began in the US and then spread across the world, particularly in Europe. That led to the famous speech given by the president of the ECB, Mario Draghi, "Whatever it takes" and then to the Quantitative easing, the program through which the ECB started to inject liquidity in the European banking system. In 2016, the European Union faced another issue: Brexit, when the UK referendum to exit from the EU passed. Meanwhile, Donald Trump has been elected as president of the US and started an economic war against China and the European Union to protect the American domestic market. More recently the spread of Covid-19 (Sars-CoV-2) has brought down the economy of the world, including the economy of the EU. The virus produced a situation of lockdowns that stemmed the whole economy. Meanwhile, the markets have experienced another oil crisis and, for the first time ever in history, negative oil prices due to the inactivity of the economy and the inability to stock oil barrels within the industries' warehouses. Finally, the twenty-seven countries within the EU has reached an agreement about the Recovery Fund (NextGeneration EU).

4.2. Assumptions

In order to set up the model I need to make some assumptions. They are the following:

- An annual initial wage of €19,200, which corresponds to the Euro Area median net income[32].
- An annual wage growth rate of 2.4%. Every month the wage increase by 2.4% on annual basis[33], i.e. every month the salary grows at 0.2% compounded rate.
- A monthly wage contribution of 10% to the defined contribution pension plan.
- The worker starts paying contributions at 25 years old, i.e. she joins the voluntary defined contribution pension plan.
- The worker retires when she is 65 years old. That is a 40 years contribution phase or wealth accumulation period.

At the beginning the worker starts contributing €160, i.e. the 10% of her monthly wage. At the retirement, the worker will have a final annual salary of about €49,000 and a total contributions paid during the accumulation phase of about €127,970 for the defined contribution pension plan.

Most of these assumptions, such as the contribution rate and the length of the contribution period, are crucial as well as the investment policy chosen for the final wealth that the individual will benefit. However, these assumptions were made before the spread of Covid-19, which caused a decrease of the amount of contributions, a down in the markets and a decrease of the annual growth rate of wages. Hence, the impact on the result of the different pension schemes will be seen in the future.

4.3. Asset classes

The worker's portfolio could be composed by five asset classes.

4.3.1. Equity asset class

It is represented by FTSEurofirst 300 Total Return Index (ETOP300 Index). It includes the 300 largest companies ranked by market capitalization within the FTSE Developed Europe Index, which in turn represents the benchmark for European investments.

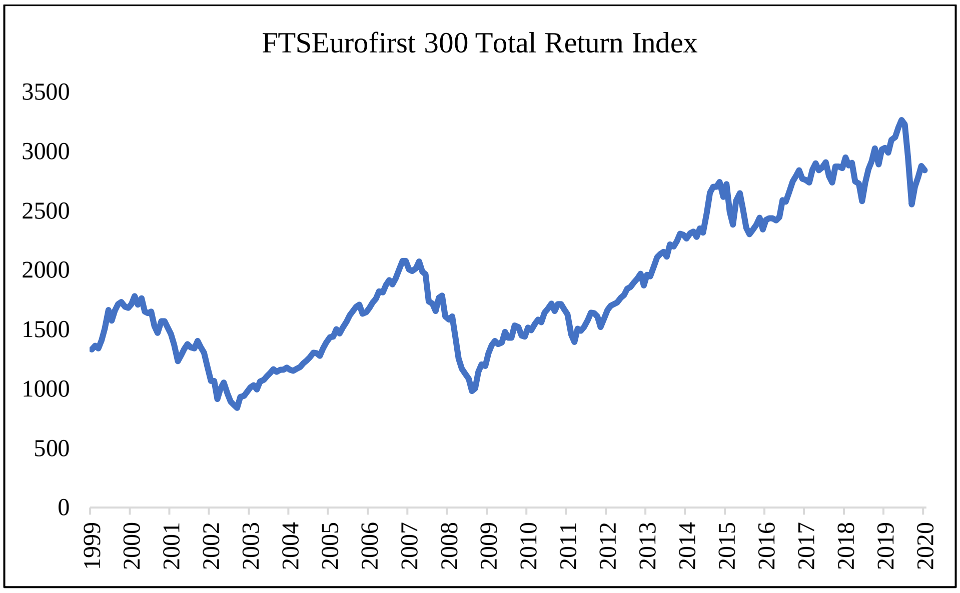
In Figure 1 it is possible to see that the market went sharply down in 2002, 2008 and 2020 due to the Argentina crisis, subprime crisis and pandemic crisis, respectively. However,

[32] *Source*: EuroStat

[33] *Source*: Trading economics

has the graph showed the market recovered well and overtake the levels before crisis. That is exactly what is happening with the NASDAQ and the S&P500 nowadays.

Figure 1 – FTSEurofirst 300 Total Return Index chart

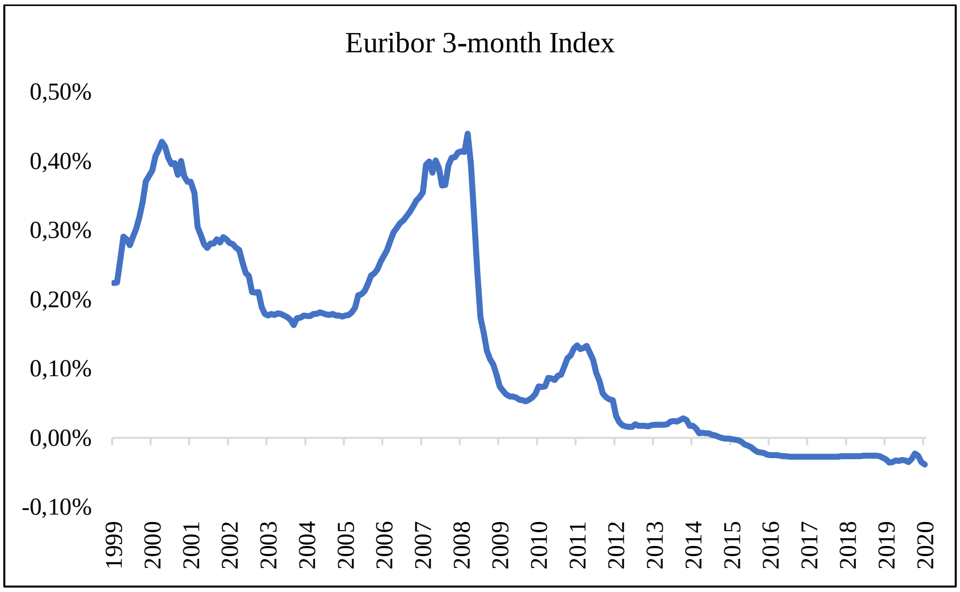


Source: Bloomberg

4.3.2. Cash asset class

It is represented by Euribor 3-month Index in Figure 2. Since 2015, i.e. the starting years of the Quantitative Easing program, it has a negative interest rates due to the surplus of money in the market. That was caused by the injection of liquidity by ECB to relaunch the investments. That also explains the negative trend. I choose it as a proxy for cash because it is highly liquid, it decreases the risk of the reinvestment rate, since there are not intermediate cash flows, and it has no default risk.

Figure 2 – Euribor 3-month Index chart

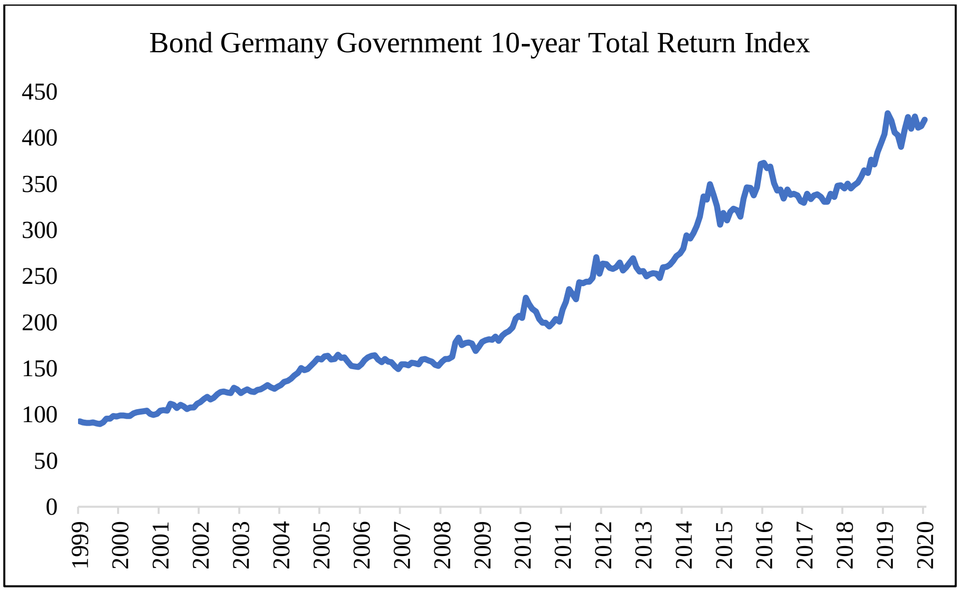


Source: Bloomberg

4.3.3. Risk-free asset class

It is represented by Bond Germany Government over 10-year Total Return Index (BCEG4T Index). It proxies for the risk-free asset within the Eurozone. In Figure 3 it is possible to notice the upside trend and ability to recovery from the crisis.

Figure 3 – Bond Germany Government 10-year Total Return Index chart

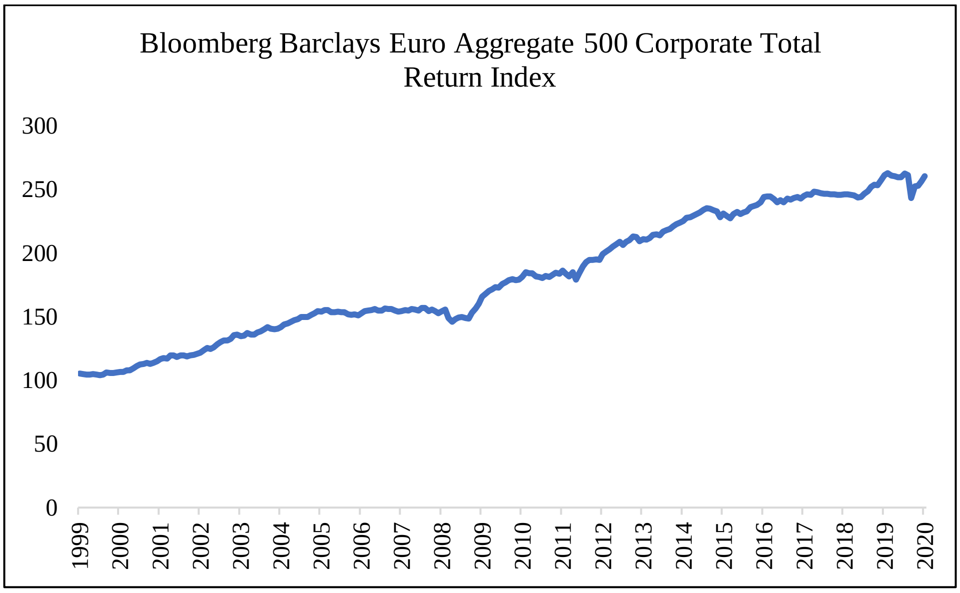


Source: Bloomberg

4.3.4. *Corporate bond asset class*

It is represented by Bloomberg Barclays Euro Aggregate 500 Corporate Total Return Index (LE5CTREU Index). It is a benchmark that measures the corporate component of the Euro Aggregate Index, which in turn includes fixed-rate, investment-grade Euro denominated bonds. It is a proxy for the investment in defaultable bonds issued by corporates, that allow investors to take more risk, but also a higher possibility of remuneration. Figure 4 shows a quite flat upside trend with some downfall in 2008 and 2020 due to the subprime crisis and the pandemic, respectively.

Figure 4 – Bloomberg Barclays Euro Aggregate 500 Corporate Total Return Index chart

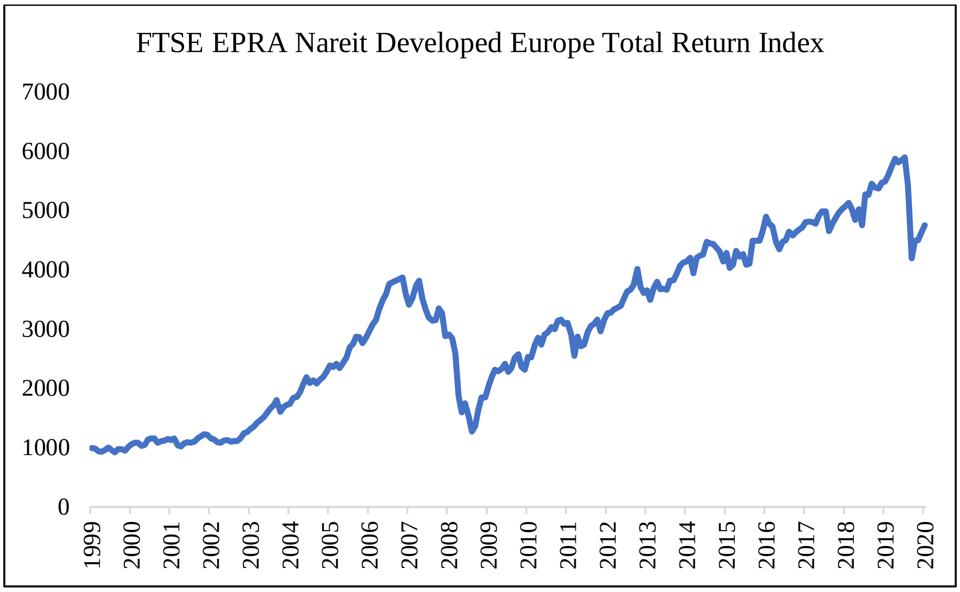


Source: Bloomberg

4.3.5. *Real estate asset class*

It is represented by FTSE EPRA Nareit Developed Europe Total Return Index (RUGL Index). It is a subset of the FTSE EPRA Nareit Developed Index and it is designed to measure the performance of real estate companies and REITS within the developed countries in Europe. I choose to include real estate in my simulation since it allows to benefit from diversification and because it represents a real term asset class. Figure 5 shows an upside and a very volatile trend with a sharply downfall in 2008, where the very housing market collapsed, and then in 2020, due to the spread of the pandemic.

Figure 5 – FTSE EPRA Nareit Developed Europe Total Return Index chart



Source: Bloomberg

4.4. Statistics

I used European Euro-denominated assets, but households could and should benefit from international diversification and hedge the currency risk. Moreover, I used total return because I assumed that all the coupons/dividends paid to the households are reinvested according to the glide paths that I will expose in the next chapter.

For each asset class I have estimated monthly returns using a 21-year period of monthly observations from July 1999 to July 2020 and then historical statistics based on returns, such as:

- Monthly and annual average, variance and volatility.
- Variance-covariance matrix of monthly returns.
- Correlation matrix of monthly returns.

Table 1 reports the annualized average returns, variance and volatility for each asset class. As it is possible to notice real estate is the asset class with the highest expected return and with the highest volatility. Surprisingly, it is followed by the risk-free asset class that presents the second largest expected return, but the third largest volatility. Essentially, it dominates the equity asset class[34]. This could be due to the challenging 21-year period, where stocks have faced a high volatile pattern and low returns due to the many crises. After equity, there is the corporate bond asset class with a similar expected return and a much lower standard deviation. However, the standard deviation of the corporate bond asset class is lower than the risk-free one. In the last seed, there is the cash asset class with an almost-zero standard deviation. Essentially, the last 20 years have changed the general view of asset classes, where equity has been experiencing a high volatility which in turn has been compensated with high expected returns. However, Table 1 reports a different story, where equity asset class is dominated by the risk-free one and is riskier than the corporate bond one because it presents a relative five times higher volatility. This could lead to changes in the way in which financial institutions allocate the contributions of the workers to reach a satisfactory wealth realization.

Table 1 – Annualized average returns, variance and volatility

	Average Returns	Variance	Standard Deviation
Equity	4.782%	2.301%	15.170%
Cash	1.667%	0.003%	0.511%
Risk-Free	7.638%	0.851%	9.224%
Corporate Bond	4.390%	0.145%	3.804%
Real Estate	9.190%	3.285%	18.124%

Table 2 reports the variance-covariance matrix among the monthly returns of the asset classes. As it is possible to notice there is a negative monthly covariance between equity and cash as well as between equity and the risk-free asset classes. On the contrary, the covariance is positive with respect to corporate bond and real estate asset classes. Risk-free asset class presents some negative covariance with real estate asset class and positive covariance with corporate bond one. Cash asset class presents almost a null covariance with every asset class. Corporate bond asset class presents a positive covariance with each asset class. Finally, real estate asset class presents a negative covariance with cash and risk-free asset classes and positive covariance with the remaining asset classes.

[34] The risk-free asset class has a higher expected return and a lower standard deviation than the equity one.

Table 2 – Variance-covariance matrix of monthly returns

	Equity	Cash	Risk-Free	Corporate Bond	Real Estate
Equity	0.192%				
Cash	-0.001%	0.000%			
Risk-Free	-0.024%	0.000%	0.071%		
Corporate Bond	0.014%	0.000%	0.014%	0.012%	
Real Estate	0.146%	-0.001%	-0.004%	0.029%	0.274%

In the end, Table 3 reports the monthly correlation between asset classes. It is this table that allows to understand the strength of the relationship among the asset classes. Equity asset class has a discrete negative correlation with cash and risk-free asset classes. However, it has a quite large positive correlation with corporate bond and even larger one, above 50%, with real estate asset classes. Cash asset class is very low positive correlated with risk-free asset class and it exhibits a low but negative correlation with the remaining asset classes. Risk-free asset class shows a quite large positive correlation with the corporate bond asset class, probably because both deal with Eurozone bonds, and a discrete negative correlation with equity asset class. Corporate bond asset class presents consistent positive correlations with equity, risk-free and real estate asset classes, in ascending order. Finally, real estate asset class shows large positive correlation with equity and corporate bond asset classes and low negative correlation with cash and risk-free asset classes.

Table 3 – Correlation matrix of monthly returns

	Equity	Cash	Risk-Free	Corporate Bond	Real Estate
Equity	100.00%				
Cash	-16.75%	100.00%			
Risk-Free	-20.34%	0.92%	100.00%		
Corporate Bond	30.03%	-2.70%	47.55%	100.00%	
Real Estate	63.58%	-9.88%	-2.78%	50.65%	100.00%

4.5. Caveats of the model

The main omissions in the thesis concern:

- Not considering taxation.

- Focusing only on the Eurozone, resulting in Euro-denominated assets, i.e. not taking into account the beneficial role of international diversification.
- Considering as risk-free an asset class that does not hedge against the inflation risk, i.e. the result is not in real terms.
- There is no possibility for the investors to keep the investment after retirement, therefore forcing them to disinvest when they retire.
- When the investor retires she will receive the wealth realization as a lump sum, i.e. I simply assume that the sum will be deposited in a current account and withdrawn every month.

V. Life-cycle strategies

In this chapter I present, test and show the comparison between different life-cycle strategies. Some of them are suggested in the literature by Poterba and Malkiel, whereas others are used in practical contexts by Vanguard, T. Rowe Price, PIMCO, Fidelity. Moreover, I compute a monthly optimal portfolio allocation à la Markowitz and then I test it following the allocation suggested. At the end, I will run a sensitivity analysis in order to see how the results vary when the accumulation period, i.e. the investment period, is halved.

I start with a brief introduction about the World Bank Pension Conceptual Framework to provide for a proper context to this study.

5.1. The World Bank Pension Conceptual Framework

The World Bank suggests a multi-pillar model in order to "better address the needs of diverse populations to manage the risks in old age" (World Bank's Pension Reform Primer, 2008). The five pillars are:

- A non-contributory "zero pillar": it has to provide all the retirees with basic protection in old age giving them a minimum pension amount even if they had not participated in the pension scheme.
- A mandatory "first pillar": it is a public pension scheme that has to provide for basic needs. It takes the pay-as-you-go form.
- A mandatory "second pillar": it is an individual occupational pension scheme financed by employers to support the first pillar. It could be defined-benefit or defined-contribution[35].
- A voluntary "third pillar": it is discretionary and consists of individual savings (that could be invested) to provide further support to the individual when retired.
- A non-financial "fourth pillar": it includes access to family support and other social programs (such as health care and housing).

The focus of my thesis concerns the third pillar, where workers voluntary put aside part of their salary and invest it in an efficient way. They should do that in order to get a better

[35] A defined-benefit pension scheme is based on the employee's salary and years of service. Instead, a defined-contribution pension scheme is based on the employee's amount and frequency of contributions paid.

pension which allow them to smooth consumption as well as helping to meet healthcare expenses due to their older age. The complementary pension scheme has to be a monetary support for different types of retirees' needs.

5.2. Life-cycle strategies

Target-date funds are funds that seek to reach a determined target through asset growth over a specified time. For example, an investor would like to live retirement without changing her living standards. Meanwhile, glide paths are the formula that define the asset allocation of a target-date fund through years, and they are based on the investor's age and on the number of years left before she retires. Essentially, an investor invests her monthly contributions following a glide path that has been designed to reach a predetermined goal.

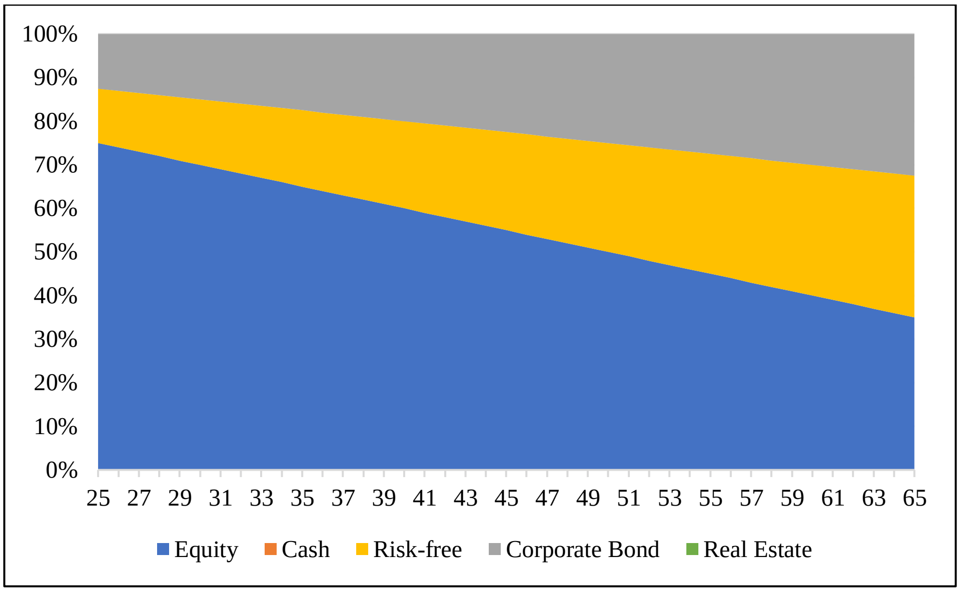
Each glide path (or life-cycle strategy) represents a different defined-contribution pension plan, and I test the wealth allocation according to the different life-cycle strategies that will be presented below. The various life-cycle investment strategies are set for a wealth accumulation phase of 40 years. These strategies follow the main advices given by financial planners I have discussed in the Chapter II. While some of these strategies are taken out from the literature, other stem from practical contexts and are usually applied in real life planning. It is important to note that the glide paths proposed in the literature are considered "heuristics" by the authors themselves. That is because researchers try to simply give the general idea of how the rebalancing process should work as the age increases.

5.2.1. 100-minus-age rule

Figure 6 shows the glide path of the so-called "100-minus-age rule". In this first strategy the allocation in equity consists in the 100% minus the age of the investor. Obviously, as the age increases the percentage allocated in equity decreases. The opposite happens for safer assets such as corporate bond and risk-free asset classes. The percentage that is not allocated in equity is equally split between such safer asset classes. The investor starts the accumulation with 75% of her wealth invested in equity at the age of 25. Every year the worker decreases by 1% in equity. At retirement, the percentage has decreased to 35%.

Meanwhile, corporate bond and risk-free asset classes start with 12.50% each at 25 years old and increase by 0.5% each year. They finish both with 32.50% at retirement.

Figure 6 – 100-minus-age rule glide path for a 40-year accumulation period

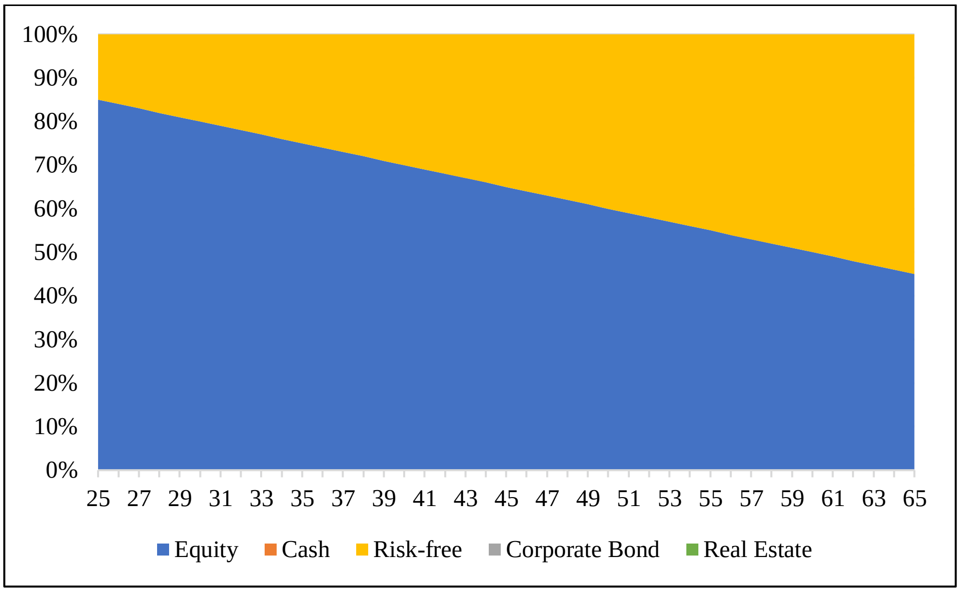


5.2.2. Poterba age-based scheme

Figure 7 shows the glide path called "Poterba age-based scheme"; see Poterba et al. (2006). The idea is very similar to the 100-minus-age rule, but here the investor starts with an allocation in equity equal to 110% and then she subtracts her age. Therefore, the initial allocation in equity is equal to 85% when the worker is 25 years old and ends with 45% at age of 65. The remaining part is allocated in risk-free asset class. Risk-free asset class allocation begins with 15% in the first year of accumulation period and ends up with 55% at retirement.

In case of two asset classes only, it is possible to see that the risk-free allocation exposure increases by 1% per year as the equity exposure decreases by the same rate per year.

Figure 7 – Poterba age-based scheme glide path for a 40-year accumulation period

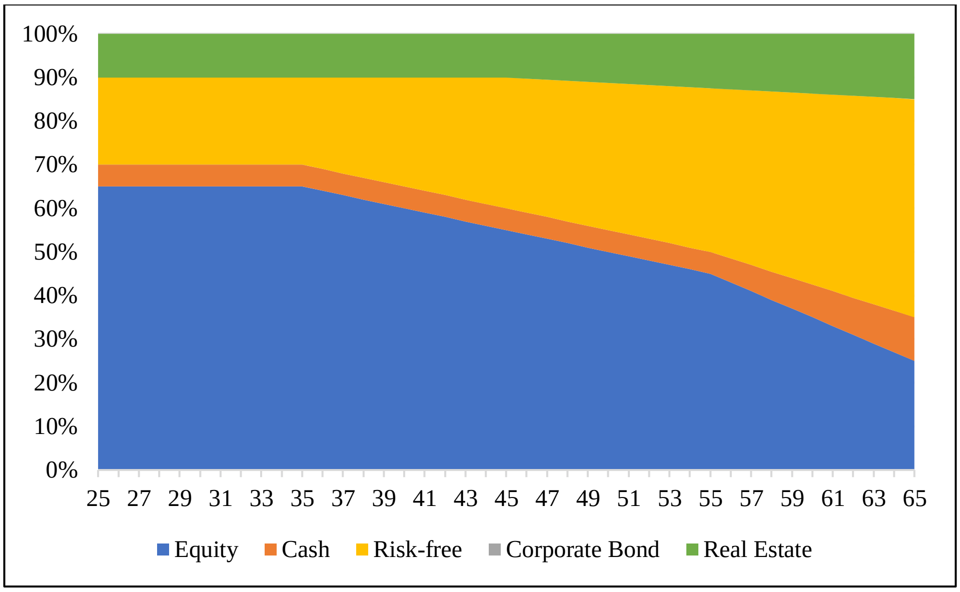


These first two life-cycle strategies do not include any other type of asset classes, such as cash or real estate. However, they follow the main idea of diminishing the equity exposure as age increases, even if they lack in terms of asset class diversification.

5.2.3. Malkiel age-based scheme

A more complete and diversified strategy is presented in Figure 8. Malkiel (1996) age-based scheme starts when the investor is young (in her mid-twenties), where she could hold a very aggressive portfolio. As the investor ages, she should decrease the proportion invested in risky assets and starts investing in those assets that pay generous dividends such as bonds and REITs. When the investor is around 55 years old, she starts thinking about a more stable and safer income, i.e. "thinking about the transition to retirement and moving portfolio toward income production" (Malkiel, 1996, p. 368). Essentially, the worker switches toward a more conservative portfolio as age increases. The investor starts with a 65% allocation in equity when she is 25 years old. Then, after 10 years the equity exposure decreases about 1% per year for the next 20 years, when she is 55 years old. Then, the equity allocation starts decreasing at 2% per year to reach 25% when the investor retires. For the first time the cash asset class is present since the beginning of the glide path. It remains constant at 5% until 10 years before retirement, when it starts increasing by 0.5% per year and finishes at 10% when the worker retires. Risk-free asset class starts with an allocation of 20% in the mid-twenties and after 10 years it increases until reaching 50% at retirement. Real estate allocation begins with an initial allocation of 10% that remains constant for the first 20 years. Then, it increases by 0.25% each year to reach 15% when the investor retires.

Figure 8 – Malkiel age-based scheme glide path for a 40-year accumulation period



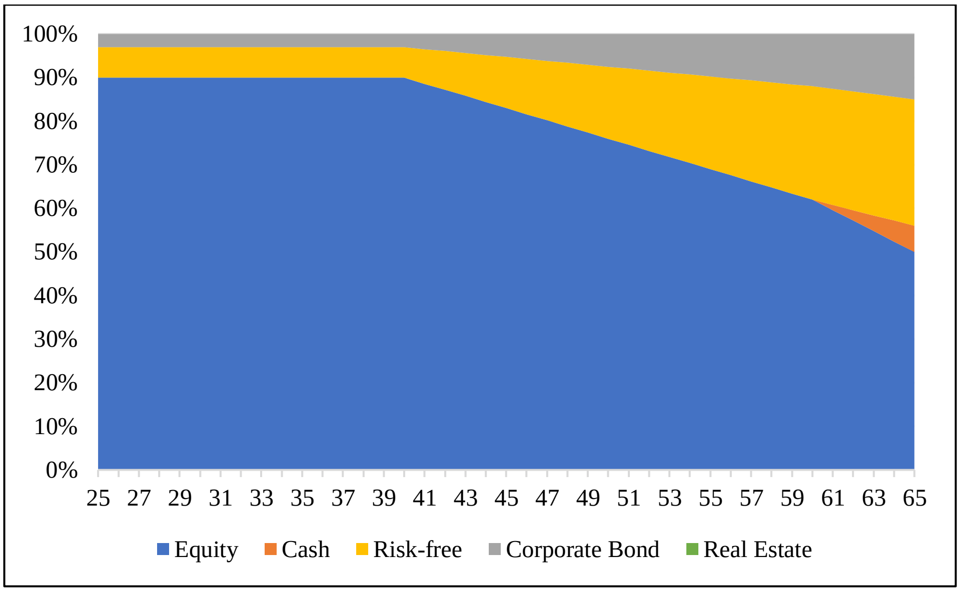
These three strategies represent glide path suggestions presented in the literature. They follow the financial advices examined in Chapter II. However, I would also like to present what are the main players' proposes.

5.2.4. Vanguard Target Date Fund

Figure 9 shows the glide paths proposed by Vanguard Group Inc. (Donaldson, et al., 2015). Vanguard's report recognizes the potential rewards for taking market risks and the importance of human capital in different stages of an investor's life. The 25-year-old worker starts with an equity asset class allocation of about 90% that remains constant until she turns 40 years old. According to Vanguard, this is justified by the dominant role that human capital plays in the early stages of the accumulation period. Then, the equity allocation decreases steadily until the worker retires, ending up at 50%. The corporate bond and the risk-free asset classes remain constant for the first 20 years at 3% and 7%, respectively. Then, they both start to increase. However, the risk-free allocation grows more rapidly than the corporate bond one. Indeed, at retirement the investor has 15%

allocation in corporate bond and 29% in risk-free asset classes. Finally, there is no cash component until the last 5 years before retirement when it increases to reach 6%.

Figure 9 – Vanguard Target Date Fund glide path for a 40-year accumulation period

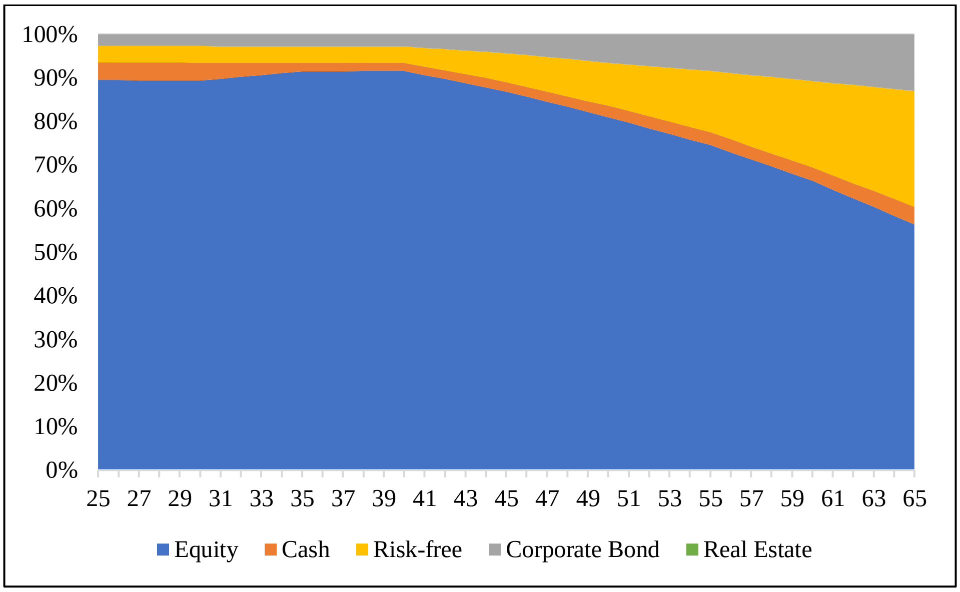


5.2.5. T. Rowe Price Retirement Fund

Figure 10 shows the combination of the different T. Rowe Price Retirement Funds[36] that have different target dates. It is possible to see a slightly hump-shaped profile in the equity allocation. The investor starts with an equity asset class allocation of about 89.50%, that increases with a peak of about 91.60% when the investor is 40 years old. After the middle age, the allocation decreases to reach 56.40% at retirement. Both corporate bond and risk-free asset classes increase with age, starting from 2.60% and 3.80% and ending with 13% and 26.6%, respectively. Instead, cash asset class follows a U-shaped profile: it starts at 4.10% when the investor is 25 years old; it reaches the bottom at 1.80% when she is 40 years old, i.e. when there is the peak in equity allocation, and it ends again at 4% at retirement.

[36] https://www.troweprice.com/personal-investing/tools/fund-research/target-date-funds

Figure 10 – T. Rowe Price Retirement Fund glide path for a 40-year accumulation period

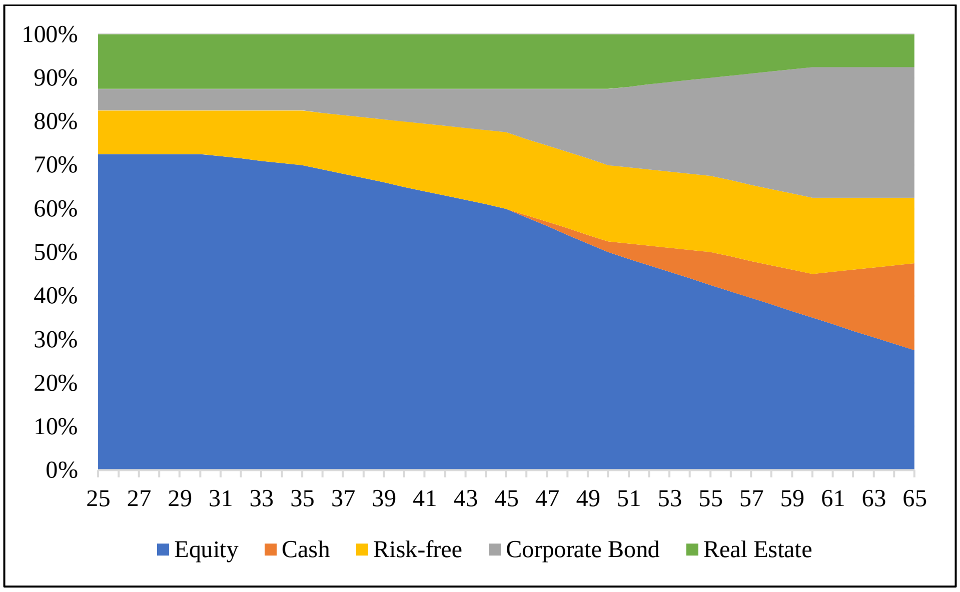


There is no presence of real estate investment. The equity hump-shaped profile at middle age is something that has been vastly examined in the literature. So, it is not wrong to increase equity exposure up to 40 years old since everything depends on the initial assumption about human capital.

5.2.6. PIMCO Glide Path

Figure 11 shows the PIMCO (Whitton & Thuerbach, 2015) glide path. The equity exposure starts with a percentage of about 72.50% that remains constant for the first five years and then decreases until reaching 27.50% when the investor retires. The risk-free asset class begins with a 10% allocation that increases with a peak at 45 years old that remains constant until 60 years old when it slightly decreases. The corporate bond asset class starts at 5% to reach 30% at retirement. A similar pattern is followed by the cash asset class that starts being part of the portfolio when the investor is 46 years old and increases until reaching 20% at retirement. Real estate is present with a 12.50% initial allocation than remains constant for the first 25 accumulation periods and then decreases reaching 7.50% when the investor retires.

Figure 11 – PIMCO glide path for a 40-year accumulation period

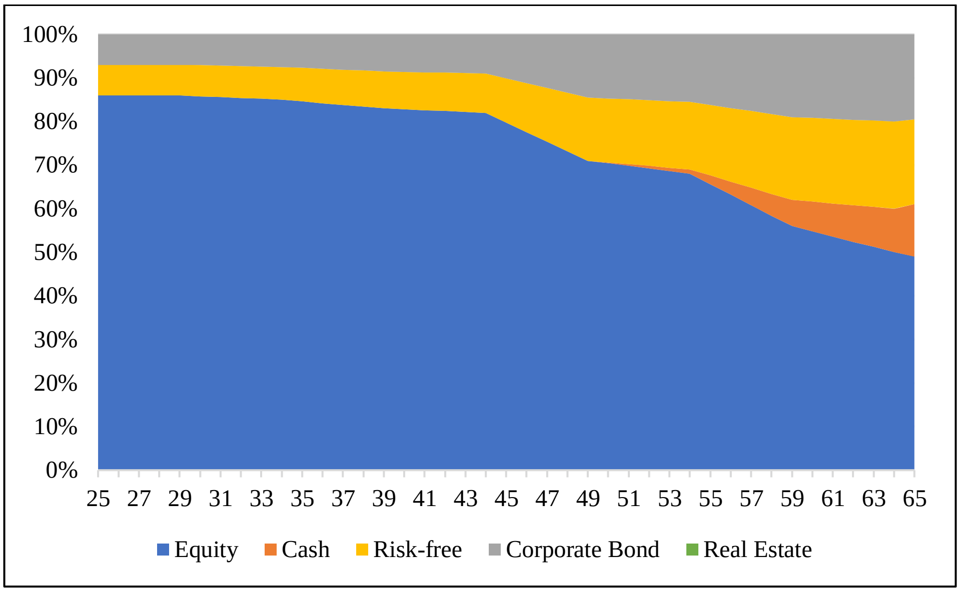


5.2.7. Fidelity Freedom K Funds

Figure 12 shows the glide path proposed by Fidelity[37], called Fidelity Freedom K Funds. The investor at the beginning starts with a portfolio composed by 86% in equity, 7% in corporate bond and 7% in risk-free asset classes. The equity exposure decreases with age ending at 49% at retirement. Corporate bond and risk-free asset classes both increase ending with a 19.50% when the investor retires. Cash asset class is included in the portfolio after 25 accumulation periods and increases up to 12% at retirement.

[37] https://www.bowdoin.edu/hr/pdf/retirement-plan-freedom-fundk-brochure.pdf

Figure 12 – Fidelity Freedom K Funds glide path for a 40-year accumulation period



The players' proposals include investments in foreign equities and foreign bonds, such as investments in emerging markets, taking advantage from the international diversification. Thus, main players set up portfolios with many different types of assets. To make things simpler I have gathered different types of assets in a unique asset class.

All the life-cycle strategies presented so far follow the main advice found in the literature and given by financial planners: the equity exposure should decrease as the investor ages. Hence, the investor's portfolio moves towards a safer allocation due mainly by her decreasing value of human capital.

5.2.8. Mean-Variance Glide Path

I allocate wealth to the five asset classes according to the mean-variance optimization approach, in which the portfolio return volatility is minimized for any target level of portfolio return. I impose no short-selling constraint on each asset class and I bound the investment to cash asset class to be no more than 5%. Formally, there is a quadratic optimization problem of this type:

$$\min_{\boldsymbol{w}} \ \boldsymbol{w}'\Sigma\boldsymbol{w} \tag{5.1}$$

$$\text{s.t.:} \quad \boldsymbol{w}'\mathbf{1} = 1 \tag{5.2}$$

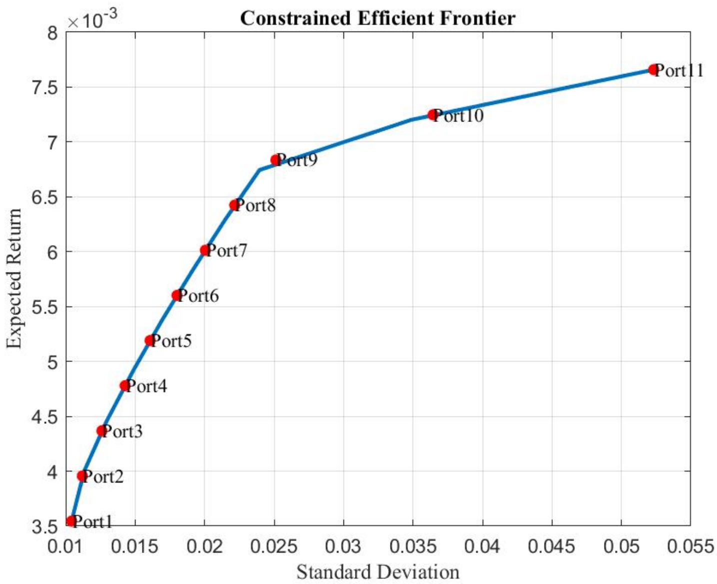
$$\boldsymbol{w}'\mu = \bar{\mu} \tag{5.3}$$

$$\boldsymbol{w} \geq \mathbf{0} \tag{5.4}$$

Where Σ is the variance-covariance matrix of monthly asset returns, $\boldsymbol{w} = (w_1, w_2, w_3, w_4, w_5)'$ is the vector of portfolio weights of equity, cash, risk-free, corporate bond and real estate, respectively, $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5)'$ is the vector of asset average returns, $\mathbf{1} = (1,1,1,1,1)'$ is a vector of ones and $\mathbf{0} = (0,0,0,0,0)'$ is a vector of zeros.

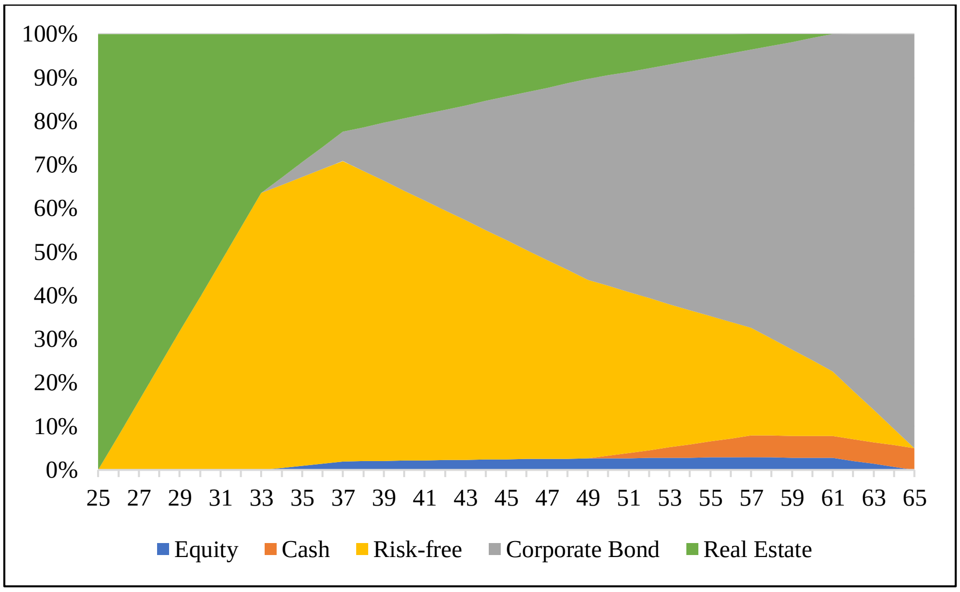
The mean-variance efficient frontier obtained is shown in Figure 13, where I have computed 11 efficient portfolios. The 11th portfolio represents the initial wealth allocation among each asset class, then in the next four years (or 48 months) the allocation shifts toward the 10th portfolio, that presents a lower standard deviation. Essentially, the allocation shifts toward a more defensive portfolio every four years, until the investor reaches the wealth allocation of the 1st portfolio at retirement (Antolin, et al., 2009).

Figure 13 – Constrained efficient frontier



The efficient frontier and the subsequent proposed allocations are mean-variance efficient in one-period (month) context given the monthly estimates of expected returns and variance-covariance matrix. Figure 14 shows the glide path of the mean-variance optimization. The wealth allocation starts with a fully investment in the real estate asset class, which does not help diversification, and then it starts decreasing while the risk-free asset class increase. At the age of 33 begins the allocation in the corporate bond and in the equity asset classes. The corporate bond allocation increases until reaching the 95% at retirement. The equity exposure increases until the middle-fifties when it reaches a peak of about 3% after which it decreases down to 0% at retirement. Risk-free asset class increases up to about 69% in the late thirties and then decreases with real estate until reaching 0% at retirement. The 15 years before retirement cash asset class starts increasing, reaching the upper bound of 5% in the late fifties. The year before retirement the investor will have a portfolio with 95% of wealth invested in corporate bond asset class and 5% in the cash asset class. This last portfolio is extremely defensive since it is in proximity of the minimum variance portfolio on the efficient frontier.

Figure 14 – Mean-Variance glide path for a 40-year accumulation period



Surprisingly, the equity exposure is very little with respect to other life-cycle strategies. As reported in Table 1, this is probably due to the fact that the corporate bond asset class shows an average return that is only slightly lower than the equity one, while the standard deviation of the corporate bond is one-fifth of the equity volatility. Thus the corporate bond asset class tends to substitute in part the equity exposure. Moreover, as shown in Table 1, the risk-free asset class dominates the equity asset class in terms of both expected returns and standard deviation: the risk-free asset class presents higher average returns but lower volatility. For this reason, the risk-free asset class behaves as a theoretical hump-shaped equity exposure[38] and essentially it substitutes the equity asset class.

Table 4 reports the different asset allocation of the different portfolios. As it is possible to notice Portfolio 11 starts with a full investment in real estate that decreases to 0% in

[38] It increases until investor's middle-age and then it decreases as investor ages.

Portfolio 1. Then the risk-free asset class and then the corporate bond asset class increase, even if the corporate bond reaches its peak at retirement and risk-free asset class presents a hump-shaped profile. The same happens for the equity exposure, while cash asset class reaches its upper bound in Portfolio 3 and it keeps it constant until retirement.

Table 4 – Monthly efficient portfolio weights

	Equity	Cash	Risk-Free	Corporate Bond	Real Estate
Portfolio 1	0.00%	5.00%	0.00%	95.00%	0.00%
Portfolio 2	2.75%	5.00%	14.87%	77.39%	0.00%
Portfolio 3	2.91%	5.00%	24.69%	63.78%	3.63%
Portfolio 4	2.78%	2.39%	32.80%	55.07%	6.95%
Portfolio 5	2.64%	0.00%	41.02%	46.01%	10.33%
Portfolio 6	2.41%	0.00%	50.31%	32.94%	14.35%
Portfolio 7	2.17%	0.00%	59.60%	19.86%	18.37%
Portfolio 8	1.94%	0.00%	68.88%	6.79%	22.38%
Portfolio 9	0.00%	0.00%	63.59%	0.00%	36.41%
Portfolio 10	0.00%	0.00%	31.80%	0.00%	68.20%
Portfolio 11	0.00%	0.00%	0.00%	0.00%	100.00%

Table 5 shows the comparison between annualized expected returns and standard deviations for each monthly efficient portfolio. Obviously, the riskiest portfolio is the one with the real estate asset class only. However, as age increases the allocation shifts toward a more defensive portfolio and a better diversified one. It is possible to notice than while the volatility ends up being only one-sixth of the volatility of Portfolio 11, the expected return only halved.

Table 5 – Annualized expected return and standard deviation of the monthly efficient portfolios

	Expected Return	Standard Deviation
Portfolio 1	4.25%	3.61%
Portfolio 2	4.75%	3.88%
Portfolio 3	5.24%	4.37%
Portfolio 4	5.73%	4.95%
Portfolio 5	6.23%	5.57%
Portfolio 6	6.72%	6.24%
Portfolio 7	7.22%	6.95%
Portfolio 8	7.71%	7.68%
Portfolio 9	8.20%	8.71%
Portfolio 10	8.70%	12.62%
Portfolio 11	9.19%	18.12%

5.3. Simulation

I follow an historical Monte Carlo simulation approach to simulate the distribution of retirement wealth under the eight different life-cycle strategies. Using the MATLAB function for the Monte Carlo simulation of correlated asset returns, I simulate series of 40 years of monthly returns for each asset class for 5000 times. Each simulation for each asset class consists of 480 time series observations. Essentially, each asset class has more than 2 million of simulated monthly returns; given the five asset classes the total simulated monthly returns is about 12 million. At the end, what I obtain for each life-cycle strategy is a distribution of 5000 simulated portfolio wealth realizations at the retirement age. Furthermore, to make the model more realistic I consider an annual management fee of 1%.

5.4. Results

Having obtained 5000 paths of wealth realization for each glide path[39], here I compute the main statistics in order to understand the distribution of the results. The main statistics are:

- Mean.
- Median. It is important since it is not influenced by outliers as it is the mean.

[39] Figures available in the Appendix.

- Variance: the level of dispersion.
- Standard deviation: the volatility.
- Skewness: it measures the asymmetry in the distribution. If the skewness is higher than zero there will be a positive asymmetry, i.e. a longer right tail with respect to the left one, while if the skewness is less than zero there will be a negative asymmetry, i.e. a longer left tail with respect to the right one. If it is zero there will be a normally distributed function. A right-skewed distribution means a longer right tail in the distribution, i.e. a higher probability of having extreme positive events. On the contrary, in case of a left-skewed distribution the opposite occurs. However, that probability also depends on the kurtosis measure.
- Kurtosis: it measures the magnitude of fat tails in the distribution. The kurtosis of a normally distributed function is equal to 3. A kurtosis greater than 3 means that the distribution has fatter tails than the normal one, i.e. there is higher probability for both positive and negative extreme events. On the other hand, a kurtosis lower than 3 means that the distribution has slighter tails than the normal one, i.e. there is lower probability for both positive and negative extreme events.
- The 5^{th} lowest percentile (5^{th} percentile): it represents the minimum amount of wealth realization that an investor could reach with a possibility of 95%. The higher this amount the lower the distribution probability of the downside risk.
- The 1^{st} lowest percentile (1^{st} percentile): it represents the minimum amount of wealth realization that an investor could reach with a possibility of 99%. As before, the higher this amount the lower the distribution probability of the downside risk.
- The median replacement ratio: it is the median of the ratio between the annual income perceived by a person after retirement and the income perceived the year before retirement (Aon Consulting, 2008). I compute the annual income after retirement as the sum perceived after the disinvestment divided by the difference between the life expectancy (80 years) and the age at which the investor retires (65 years old). This ratio shows how much of the investor's last working-year income has been replaced by her annual complementary pension. Obviously, much depends on the life expectancy assumption.
- Internal Rate of Return (IRR): it represents the discount rate that equals the net present value of a discounted cash flows to zero. Essentially, it summarizes in a

single number the merits of an investment. The higher is the IRR, the higher is the discount rate that an investor could bear before the project does not create value for her. As for the replacement ratio, I also compute the median and the 5th percentile.

- Level of capital protection: the probability that the wealth at retirement is less than the amount of contributions paid by the investor during the accumulation period. Essentially, the lower the probability of having a wealth-over-contributions ratio smaller than 1 the higher the probability of having at least the full amount of contributions paid at retirement.

Statistics in terms of wealth realizations are important, but they do not allow a comparison with what an investor had to give up during the accumulation period, i.e. the contributions paid. For this reason, I have also computed the same statistics in terms of wealth realization over the total contributions paid by the investor[40]. The wealth-over-contributions ratio shows, disregarding time, how much retirement wealth an investor could get based on the total amount of contributions paid during the accumulation period. This is crucial, since it allows to compare how much wealth an investor get relative to the contribution paid. However, it is important to understand that relative measures, such as the replacement ratio, IRR and wealth-over-contributions statistics are scale-free, i.e. they are simply a ratio between two variables and its value equally depends on both of them. For this reason, it is also important to take a look at the tables that report the absolute amounts of the simulation.

Before focusing on the wealth-over-contributions ratio and its related statistics I need to discuss the median replacement ratio and the IRR statistics. Table 6 reports the median replacement ratio for different life-cycle strategies. It is well known that the annual income after retirement covers only a part of the last annual income before retirement. Obviously, an investor could not expect to perfectly replace the income perceived the year before retirement, that is earned through hard work and experience. However, the wealth perceived at retirement is the result of a complementary pension scheme. That means that the investor received her pension amount from her compulsory pension scheme to which she adds the retirement wealth generated from her voluntary pension scheme. For example, investor's last year salary amount is about €49,000 and the median

[40] For all the statistics in terms of wealth realization and in terms of the ratio between wealth accumulation and contributions look to the Appendix.

replacement ratio following the 100-minus-age rule is 39.02%. It means that the complementary pension is able to fulfill the investor's basic pension with about €19,120, annually. It means that every month the investor will have additionally €1,593 on her pension check[41].

Table 6 – Median replacement ratio for the different life-cycle strategies

	Median Replacement Ratio
100-minus-age	39.02%
Poterba	43.00%
Malkiel	45.06%
Vanguard	35.56%
T. Rowe Price	33.62%
PIMCO	38.96%
Fidelity	34.53%
Mean-Variance	47.97%

This ratio is very important because it provides the nominal support that the investor could benefit by setting up a complementary pension scheme. This additional amount, which is not inflation-indexed, could be able to better sustain possible healthcare expenses that an old person would face because of the age. Moreover, it corresponds to a median statistic, that means that in the 50% of the cases an investor could reach a ratio greater or equal.

Table 7 reports the median and the 5th percentile IRR for each glide path for a 40-year accumulation period. It is possible to see that literature-strategies tend to outperform practical-strategies with a greater median IRR and a lower risk. The exception is represented by the Mean-Variance strategy that greatly outperform literature-strategies both in terms of median IRR and 5th percentile IRR.

[41] Recalling the assumption made about life expectancy.

Table 7 – Median and 5th percentile of the IRR for a 40-year accumulation period for each glide path

	Median IRR	5th per. IRR
100-minus-age	4.11%	1.92%
Poterba	4.55%	2.04%
Malkiel	4.75%	2.31%
Vanguard	3.69%	0.66%
T. Rowe Price	3.43%	0.18%
PIMCO	4.11%	1.65%
Fidelity	3.56%	0.70%
Mean-Variance	5.02%	3.43%

Table 8 reports the wealth-over-contributions ratio statistics computed after the simulation. The lower median ratio is represented by the T. Rowe Price life-cycle strategy and it is 1.929, that means that in 50% of the cases the investors can expect to accumulate a retirement wealth slightly less than twice the level of the contributions they paid. I have also calculated the downside risk using the 5th percentile and the 1st percentile of the ratio distribution, i.e. the threshold value of the ratio such that a lower outcome occurs with a probability not higher than 5% and 1%, respectively. In a 50% of the cases, if an investor sets up the T. Rowe Price Retirement Fund she should expect final wealth at least doubled with respect to the contributions paid. Moreover in the 95% of the cases the worker will fully recoup her amount of contributions paid during the accumulation period. That life-cycle strategy represents the worst one with respect to the other life-cycle strategies. Table 8 also shows that as the median increases the two measures of downside risk also increase, leading to a higher degree of capital protection. The most efficient strategy in terms of median and 5th percentile is represented by the Mean-Variance one, and it also shows a great result with respect to of 1st percentile, since it allows the investor at retirement to fully recoup the amount of contributions paid.. Meanwhile, practical life-cycle strategies such as Vanguard, T. Rowe Price and Fidelity bear the risk of not recouping the full amount of contributions paid in 1% of the cases.

Table 8 – Wealth-over-contributions ratio statistics for a 40-year accumulation period

	Mean	Median	5^{th} per.	1^{st} per.
100-minus-age	2.372	2.239	1.417	1.185
Poterba	2.674	2.467	1.449	1.184
Malkiel	2.781	2.585	1.529	1.262
Vanguard	2.278	2.040	1.122	0.897
T. Rowe Price	2.171	1.929	1.031	0.821
PIMCO	2.401	2.235	1.347	1.125
Fidelity	2.186	1.981	1.130	0.907
Mean-Variance	2.843	2.752	1.927	1.685

Table 9 reports the degree of capital protection, i.e. the probability of not recouping the full amount invested. The Mean-Variance glide path seems to assure the investor a fully reimbursement of the investment. It is followed by the three theoretical life-cycle strategies that exhibit a lower probability. Then, there is the PIMCO glide path that shows a similar probability. However, there are strategies that are riskier, such as Vanguard, T. Rowe Price and Fidelity, arguably due to the higher exposition in the equity asset class during the first 20 years of accumulation period. The investor who follows one of these strategies bears the risk of not recouping the full amount invested with a probability that ranges between 2.08% and 3.96%.

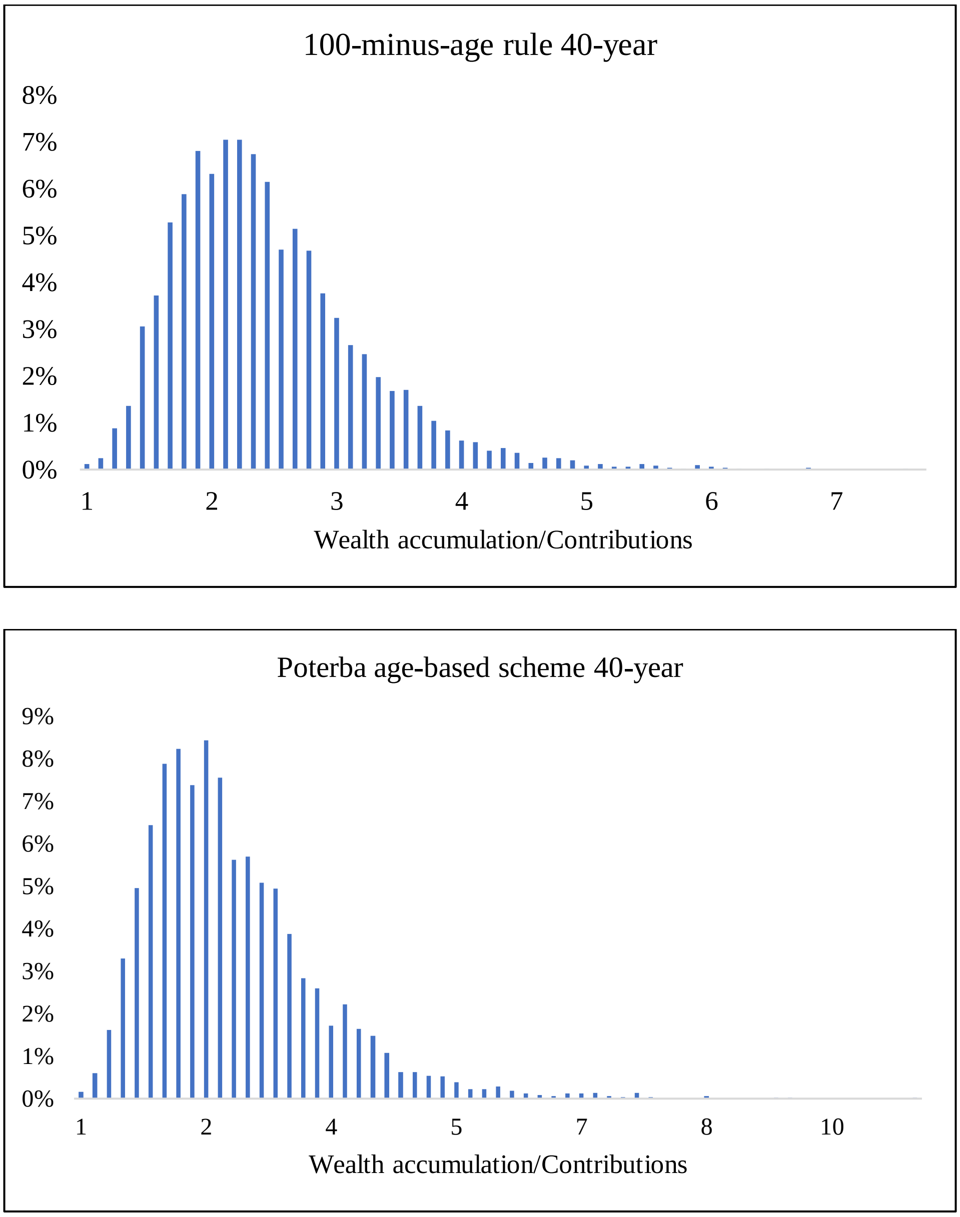
Table 9 – Probability of having wealth accumulation below total contributions paid

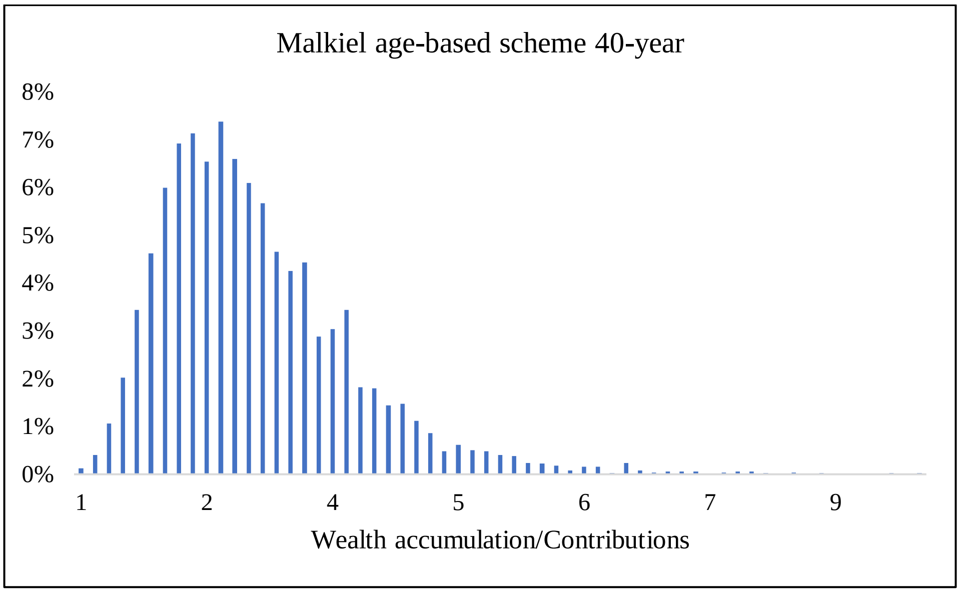
	Percentage
100-minus-age	0.12%
Poterba	0.20%
Malkiel	0.10%
Vanguard	2.16%
T. Rowe Price	3.96%
PIMCO	0.28%
Fidelity	2.08%
Mean-Variance	0.00%

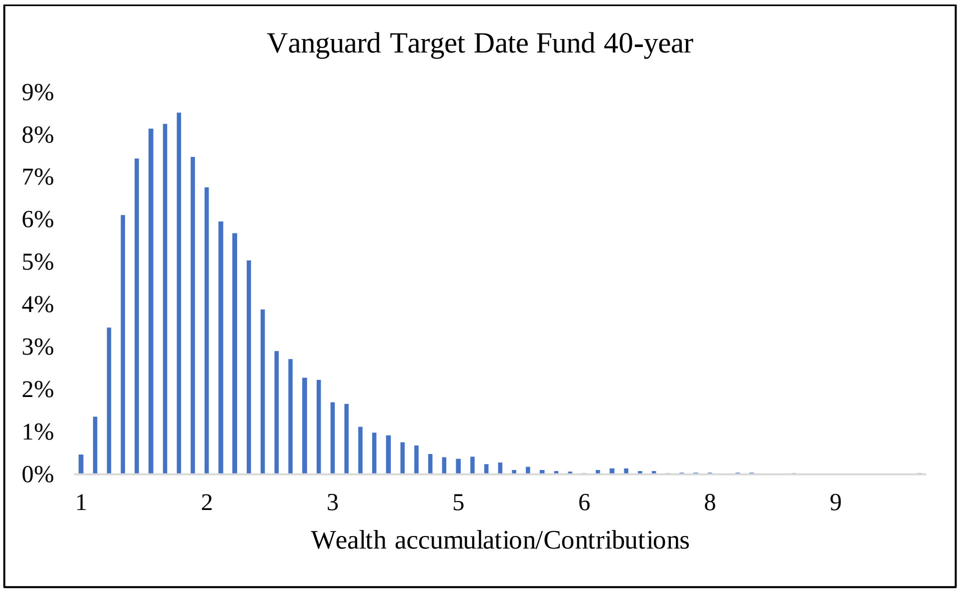
So far, the Mean-Variance life-cycle strategy seems to be the best one since it shows the highest median replacement ratio associated with the highest median, 5^{th} percentile and 1^{st} percentile of the wealth-over-contributions ratio. Moreover, it has the highest level of capital protection.

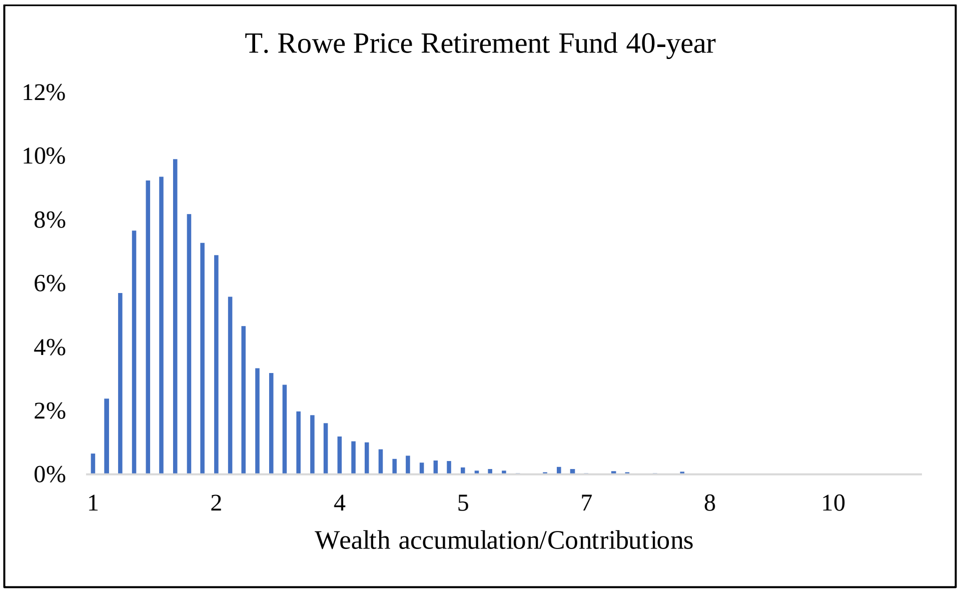
Figure 15 shows the simulated distributions of the ratio between accumulated wealth at retirement and total contributions paid during the accumulation period. As it is possible to see, all the distributions are positively skewed, that means that the probability of achieving a ratio not lower than the mean ratio is less than 50%. By looking at Table 16 in the Appendix, the skewness measure is greater than 0 proving a positive skewed distribution. Moreover, the kurtosis measure is greater than 3 showing a fatter right tail. Then, putting together these last two features the investor gets a higher probability of reaching positive extreme events.

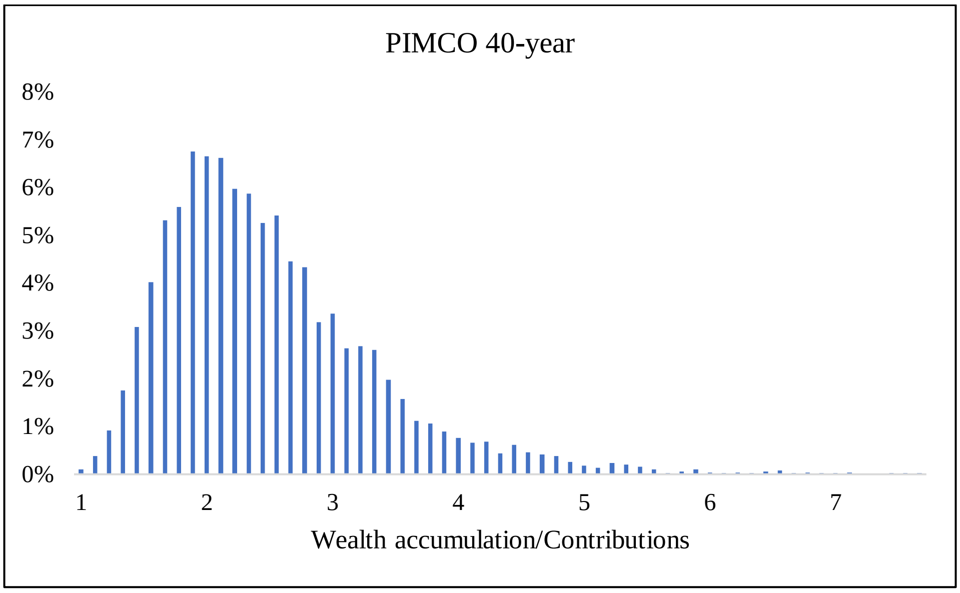
Figure 15 – Distribution functions of the ratio wealth accumulation/Contributions for a 40-year accumulation period

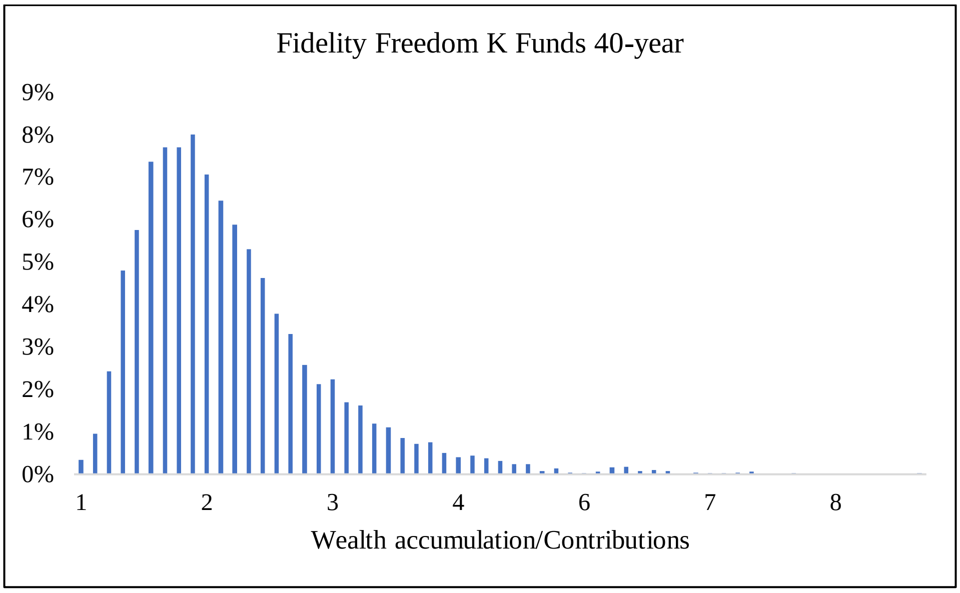



Malkiel age-based scheme 40-year
8%
7%
6%
5%
4%
3%
2%
1%
0%
1
2
4
5
6
7
9
Wealth accumulation/Contributions


Vanguard Target Date Fund 40-year
9%
8%
7%
6%
5%
4%
3%
2%
1%
0%
1
2
3
5
6
8
9
Wealth accumulation/Contributions


T. Rowe Price Retirement Fund 40-year
12%
10%
8%
6%
4%
2%
0%
1
2
4
5
7
8
10
Wealth accumulation/Contributions


PIMCO 40-year
8%
7%
6%
5%
4%
3%
2%
1%
0%
1
2
3
4
5
6
7
Wealth accumulation/Contributions


Fidelity Freedom K Funds 40-year
9%
8%
7%
6%
5%
4%
3%
2%
1%
0%
1
2
3
4
6
7
8
Wealth accumulation/Contributions

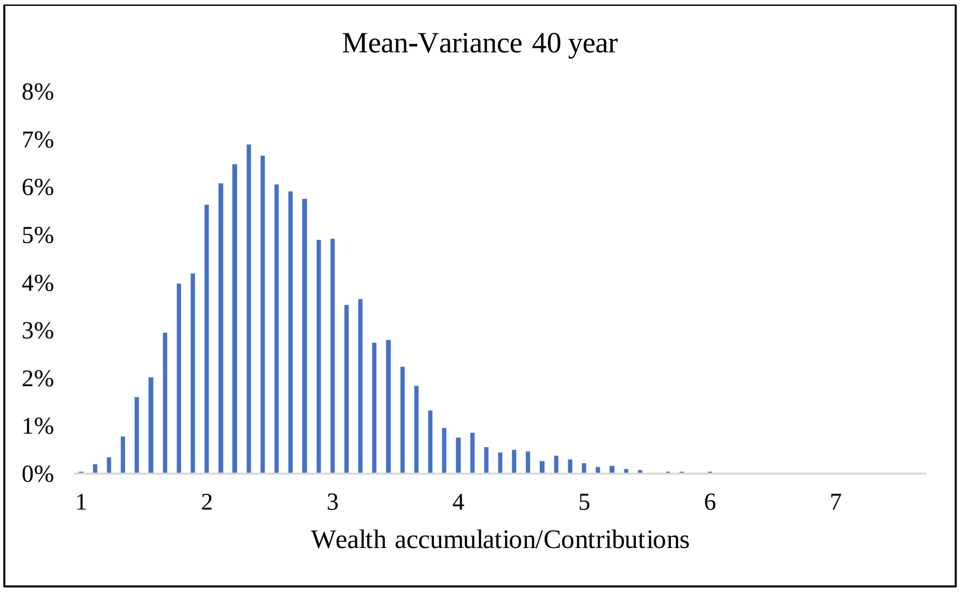

Mean-Variance 40 year
8%
7%
6%
5%
4%
3%
2%
1%
0%
1
2
3
4
5
6
7
Wealth accumulation/Contributions

Table 10 reports the annualized average of returns obtained in the simulation for each asset class for a 40-year accumulation period. As it is possible to notice, real estate asset class is the one that performed better, followed by the risk-free asset class and then by the equity and corporate bond asset classes. However, the difference between the expected return of real estate and the expected return of other asset classes is astonishing.

Surprisingly, the worst life-cycle strategies are represented by the main players in the market, such as Fidelity, Vanguard and T. Rowe Price. Arguably, the reason could be that in their glide paths there is no evidence of real estate asset class allocation. As it is possible to see in Chapter IV when I described the glide paths, these three strategies present a high exposure to equity asset class. This in turn presents a halved expected return with respect to the real estate asset class and a quite similar volatility (Table 1). In addition, all these three strategies present a consistent allocation in the risk-free asset class that shows a quite high volatility (Table 1).

I would also like to shed light on the reason why the Poterba life-cycle strategy is one of the strategies that did better despite the fact that it has only equity and risk-free asset classes in its glide path. Since the first year, the decrease of the equity asset class allocation and the increase of the risk-free allocation start. Meanwhile, Vanguard, T. Rowe Price and Fidelity keep an allocation higher than 80% of equity, at least until the investor turns 40 years old. This exposes the investors to a quite higher volatility and a lower expected return with respect to the risk-free asset class.

The two best strategies are the Mean-Variance and the Malkiel ones. These include real estate asset classes. However, the Malkiel glide path limits the investment in no more than 15%, while the Mean-Variance strategy allocates all the wealth in real estate and then it decreases as age increases. Perhaps, it is for this reason that the results of the Mean-Variance strategy and the Malkiel one are so different, despite they are the best ones. Essentially, the gap could be due to the higher exposure in real estate.

Table 10 – Simulated annualized mean for each asset class

	Annualized Mean
Equity	4.73%
Cash	1.67%
Risk-Free	7.64%
Corporate Bond	4.37%
Real Estate	9.11%

100-minus-age and PIMCO life-cycle strategies are the middle-position strategies. 100-minus-age does not present allocation in the real estate asset class. Instead its wealth is allocated in the corporate bond component, that has a lower but similar expected return than equity and a volatility that is much lower. Even the simulated annualized mean reports the lower but quite similar return of the corporate bond asset class with respect to the equity one. Thus, by limiting the allocation in risk-free asset class and investing in the corporate bond one, 100-minus-age rule has limited in part, the higher volatility of the risk-free giving up some potential return. PIMCO includes all asset classes in its glide path. In particular, it keeps real estate allocation from the beginning and it allocates more wealth to the corporate bond component as age increases, while decreasing equity. Essentially, PIMCO life-cycle strategy did not overperformed because of the real estate exposure decreases with age and did not underperformed because of the corporate bond asset class allocation.

5.5. Sensitivity analysis

Lastly, I study the retirement wealth that a worker could benefit if she starts the complementary pension scheme when she is 45 years old, i.e. how much the retirement wealth would be when the accumulation period is only 20 years. The investor perceives an initial annual salary amount of about €31,191 when she starts the plan. The total amount of contributions paid during the 20-year accumulation period is about €78,880. I run the sensitivity analysis for each glide path, where obviously the initial allocation will be the one that corresponds to the age of the investor, i.e. starting from 45 years old. Then, I compute statistics both in terms of wealth realizations and in terms of wealth-over-contributions ratio[42], as I did before. The goal is to compare the statistics presented above for a 40-year accumulation period with the ones for a 20-year accumulation period.

Table 11 reports the comparison between the median replacement ratio of the two different accumulation periods. As it is possible to notice, the ratio of the 20-year period is more than halved with respect to the one of the 40-year period. That means that an investor who starts the plan at 45 years old should expect, *ceteris paribus*, a complementary pension check that is able to replace less than a half of what would replace a 40-year complementary pension check. As it is possible to see, the Mean-Variance

[42] Appendix.

strategy is the most affected one, both because it is no longer the best strategy in terms of median replacement ratio and because the median of the ratio for a 20-year period represents just the 34% of the ratio for a 40-year period.

Table 11 – Comparison between median 20-year replacement ratio and median 40-year replacement ratio

	20-year	40-year
	Median Replacement Ratio	Median Replacement Ratio
100-minus-age	16.59%	39.02%
Poterba	17.68%	43.00%
Malkiel	18.11%	45.06%
Vanguard	16.18%	35.56%
T. Rowe Price	15.74%	33.62%
PIMCO	16.19%	38.96%
Fidelity	15.72%	34.53%
Mean-Variance	16.31%	47.97%

Table 12 reports the comparison between mean, median, 5th percentile and 1st percentile of wealth-over-contributions for the two different accumulation periods. As it is possible to see, the median for a 20-year accumulation period ranges between 0.464 and 1.234 lower relative to the 40-year accumulation period. In the 20-year period no strategy comes near to double the contributions paid, while in the 40-year period six-out-of-eight strategies reach and exceed that threshold. In particular, the Mean-Variance strategy is once again the is most affected by the halved of the investment period. This is because the median of the ratio for a 20-year period represents only the 55% of the median ratio for a 40-year period. However, in the 5th and in the 1st percentile for a 20-year period that strategy has the highest ratio. Essentially, the Mean-Variance glide path could be no more the one that gives the maximum median wealth-over-contributions ratio, but it keeps the highest level of capital protection.

In the 5th percentile column for the 20-year period there is one strategy that shows the possibility of not recovering the full amount invested in at least the 95% of the cases. This possibility has not shown up in the 40-year period. Moreover, if I diminish the percentile to the 1st percentile there are half of the strategies that do not allow the investor to recoup the initial investment for a 20-year period. However, this last case is similar to the 40-year period, where three-out-of-eight glide paths bear that risk.

Table 12 – Comparison between wealth-over-contributions for a 20-year period and for a 40-year period

	20-year				40-year			
	Mean	Median	5th per.	1st per.	Mean	Median	5th per.	1st per.
100-minus-age	1.578	1.544	1.144	1.025	2.372	2.239	1.417	1.185
Poterba	1.695	1.646	1.155	1.004	2.674	2.467	1.449	1.184
Malkiel	1.728	1.686	1.188	1.049	2.781	2.585	1.529	1.262
Vanguard	1.564	1.506	1.013	0.873	2.278	2.040	1.122	0.897
T. Rowe Price	1.529	1.465	0.957	0.812	2.171	1.929	1.031	0.821
PIMCO	1.543	1.507	1.107	0.982	2.401	2.235	1.347	1.125
Fidelity	1.513	1.463	1.008	0.877	2.186	1.981	1.130	0.907
Mean-Variance	1.531	1.518	1.265	1.170	2.843	2.752	1.927	1.685

Table 13 reports the comparison between the probability of not recouping the full amount investment between the 20-year accumulation period and the 40-year accumulation period. As it is possible to notice, the percentage from 40-year to 20-year period for each glide path increased substantially, and in four-out-of-seven cases it has more than tripled. Essentially, the shorter the investment period the lower the degree of capital protection. Finally, as I said before, the Mean-Variance strategy keeps the highest degree of capital protection, because the percentage of not recouping the amount invested, for both the 20-year and the 40-year periods, remains equal to zero.

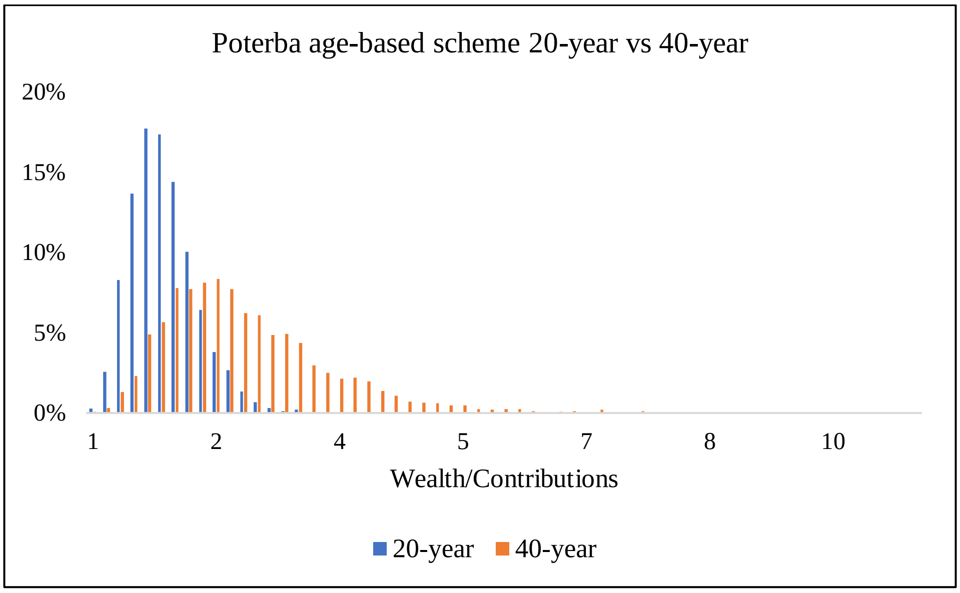
Table 13 – Comparison between the wealth accumulation below total contributions paid for a 20-year period and for a 40-year period

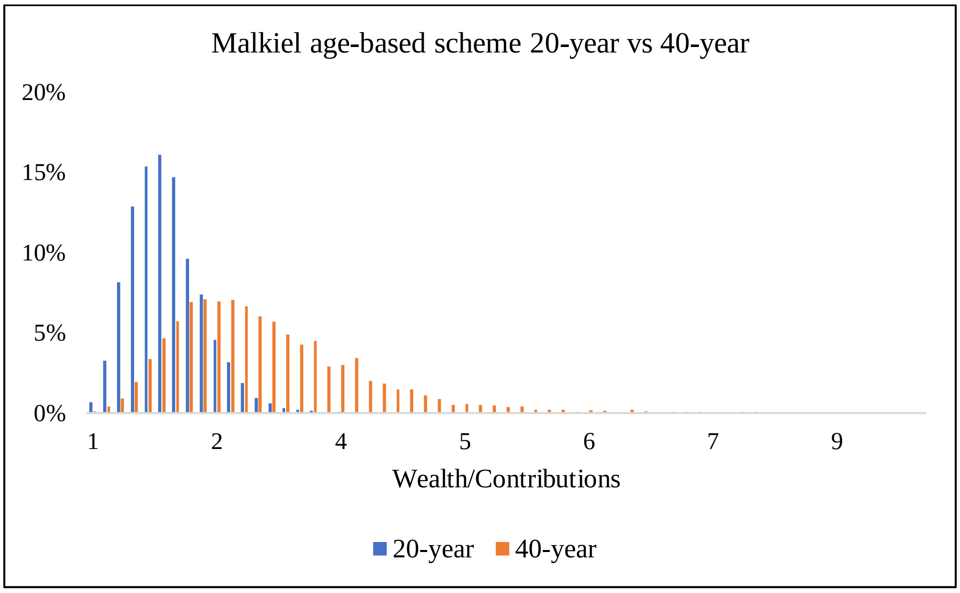
	20-year	40-year
100-minus-age	0.78%	0.12%
Poterba	0.96%	0.20%
Malkiel	0.52%	0.10%
Vanguard	4.62%	2.16%
T. Rowe Price	7.18%	3.96%
PIMCO	1.34%	0.28%
Fidelity	4.64%	2.08%
Mean-Variance	0.00%	0.00%

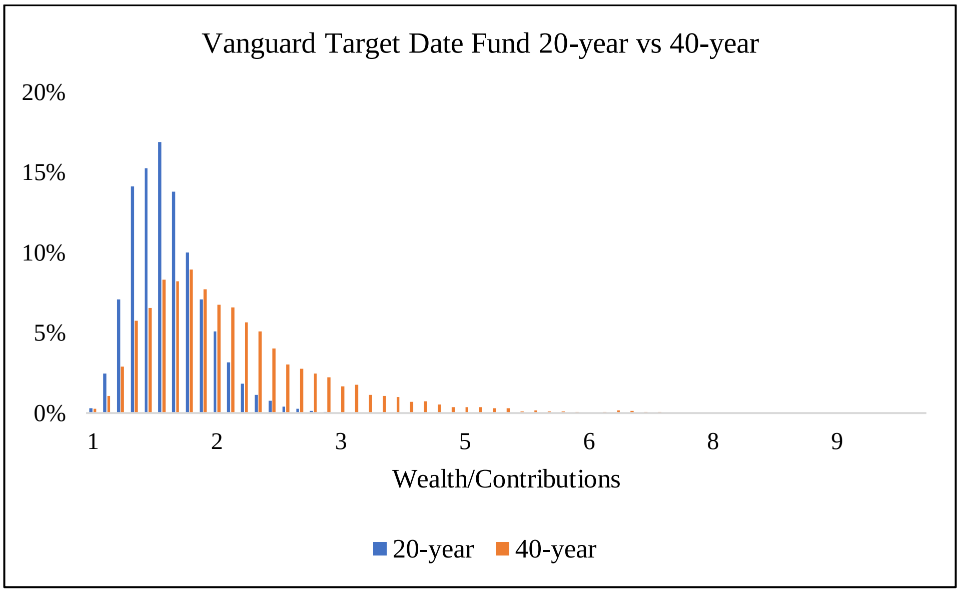
Even in the 20-year accumulation period the Mean-Variance life-cycle strategy remains the best one. However, it is no more the best strategy in terms of the median replacement ratio result. In fact, the literature-strategies overcome it, even if it is the best one in the level of protection of the capital.

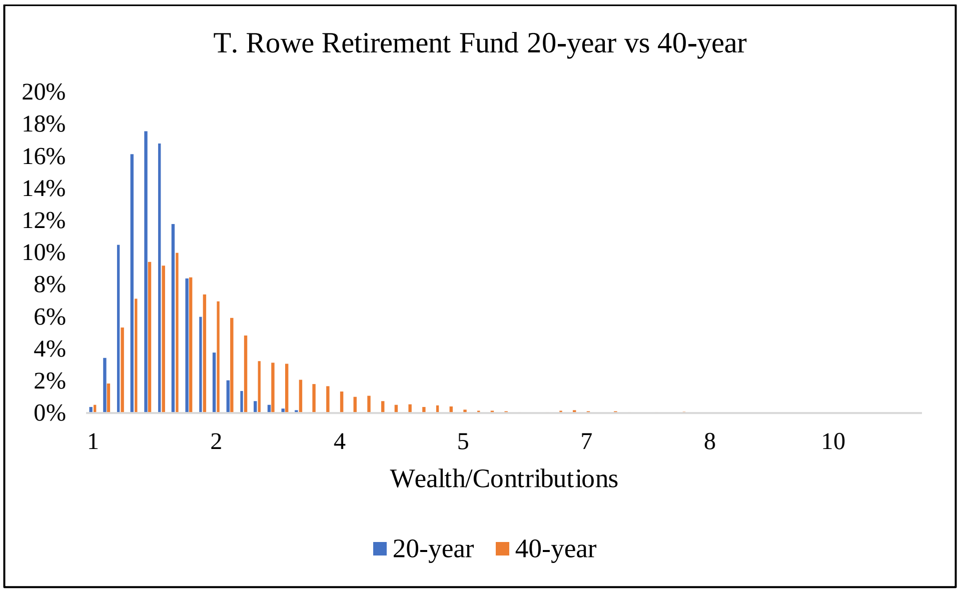
Figure 16 shows the comparison between the distribution function for the 20-year and the 40-year accumulation periods. It is possible to see that the distribution for a 20-year period is less positively skewed and the kurtosis is closer to the normal distribution than for a 40-year period distribution. Most importantly, the distribution function is shifted toward left, proving the huge gap between the median (and the mean) of the 20-year period distribution and the medina (and the mean) of the 40-year one. The most evident case is represented by the Mean-Variance strategy, where, as I said before, the difference between the median results is higher. It is also possible to notice that those life-cycle strategies that performed worse in both the periods, such as T. Rowe Price, Vanguard and Fidelity, are the ones with a lower difference between the distribution functions.

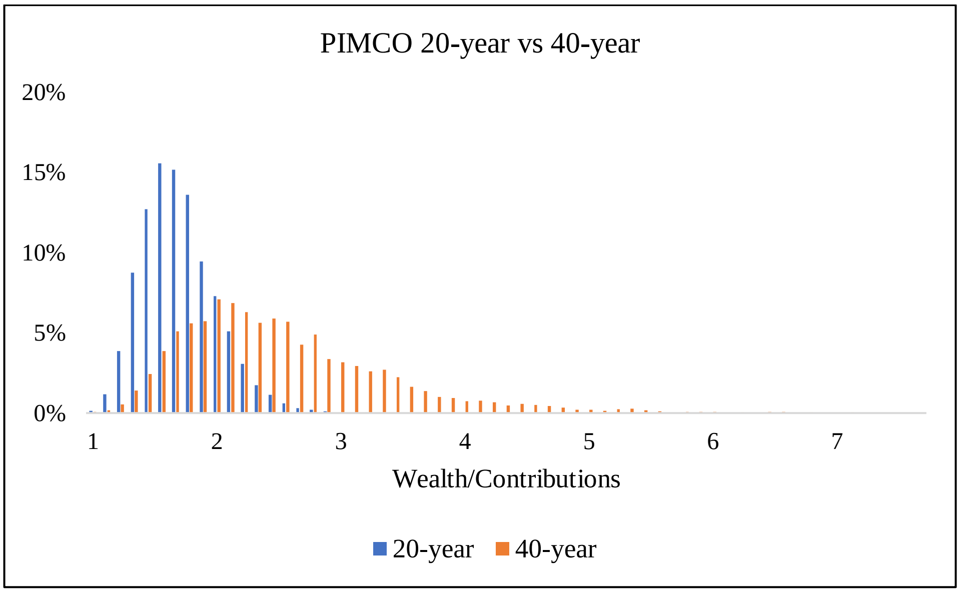
Figure 16 – Comparison between distribution functions for a 20-year accumulation period and for a 40-year accumulation period

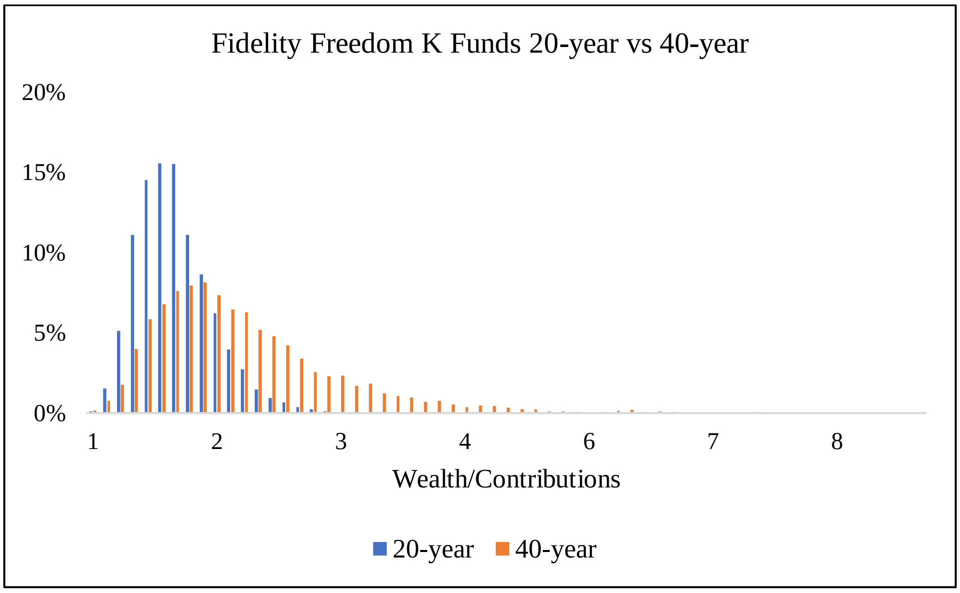

Poterba age-based scheme 20-year vs 40-year
20%
15%
10%
5%
0%
1
2
4
5
7
8
10
Wealth/Contributions
20-year
40-year


Malkiel age-based scheme 20-year vs 40-year
20%
15%
10%
5%
0%
1
2
4
5
6
7
9
Wealth/Contributions
20-year
40-year


Vanguard Target Date Fund 20-year vs 40-year
20%
15%
10%
5%
0%
1
2
3
5
6
8
9
Wealth/Contributions
20-year
40-year


T. Rowe Retirement Fund 20-year vs 40-year
20%
18%
16%
14%
12%
10%
8%
6%
4%
2%
0%
1
2
4
5
7
8
10
Wealth/Contributions
20-year
40-year


PIMCO 20-year vs 40-year
20%
15%
10%
5%
0%
1
2
3
4
5
6
7
Wealth/Contributions
20-year
40-year


Fidelity Freedom K Funds 20-year vs 40-year
20%
15%
10%
5%
0%
1
2
3
4
6
7
8
Wealth/Contributions
20-year
40-year

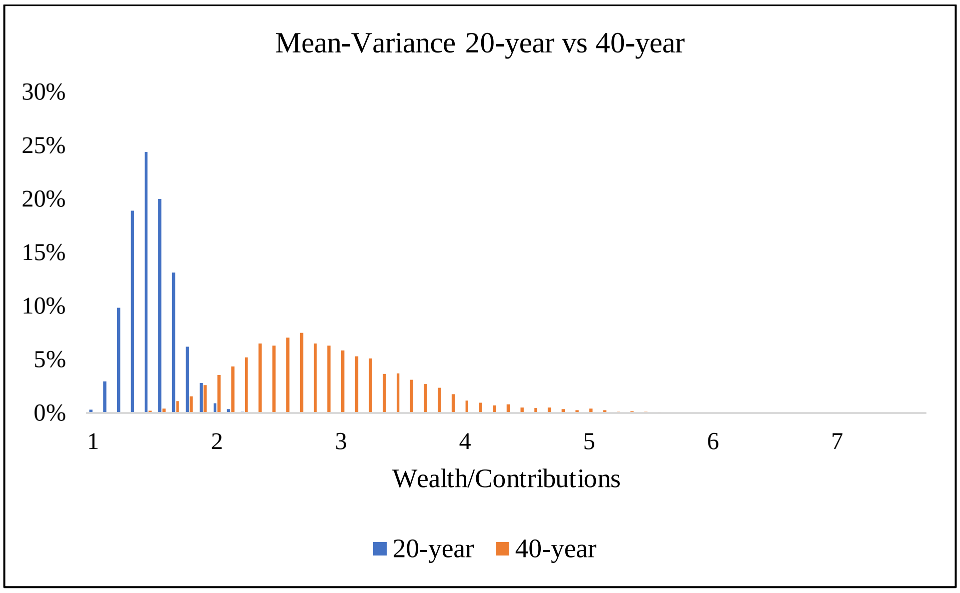


Table 14 reports the comparison between the simulated annualized means for each asset for 20-year period and the 40-year period. As it is possible to see the average returns for the risk-free asset class and the corporate bond one are exactly the same. Instead, for the other asset classes the 20-year simulated annualized mean of returns are greater than the 40-year ones. Equity and corporate bond asset classes tend to better perform in a shorter horizon and henceforth improving those strategies that rely more on them. This could be the reason why there is a reduction of the gap between the Mean-Variance strategies and the literature strategies concerning the 40-year accumulation period. However, even real estate asset class tend to better perform in the shorter period.

Table 14 – Comparison between the simulated annualized mean of returns for each asset class for a 20-year accumulation period and for a 40-year accumulation period

	20-year	40-year
Equity	4.95%	4.73%
Cash	1.67%	1.67%
Risk-Free	7.64%	7.64%
Corporate Bond	4.40%	4.37%
Real Estate	9.31%	9.11%

VI. Conclusions

Since Berardi et al. (2018) have already considered the advantages of a life-cycle strategy with respect to a life insurance minimum guarantee rate policy, my conclusions regard the choice among the glide paths simulated in Chapter V as a possible guide for the default option in a pan-European Pension Product. An efficient default option should at least guarantee the nominal capital protection, and this concerns the protection of investors' savings. That is the only obligation that the default investment option must offer. In Table 13 of Chapter V, it is possible to see that a 40-year period results much way better than a 20-year accumulation period in capital protection. Thus, in order to benefit from a more complete capital protection a long-term commitment is suggested. The best life-cycle strategies are represented by the Mean-Variance and the literature ones, such as the 100-minus-age rule, the Poterba and the Malkiel age-based schemes. However, the practical ones, represented by Vanguard, T. Rowe Price, PIMCO and Fidelity, are also quite good in terms of capital protection. It all depends on the threshold that the European Commission will decide to place, but quantitative regulations in a defined contribution plan should be carefully considered, as suggested by Antolin et al. (2009). However, since the worker's effort of giving up part of her monthly salary, she would expect to be at least inflation protected by the default option. Then, an investor should also consider inflation-protected bonds in addition to other real assets, such as real estate and equity. For these reasons, even if the practical strategies performed worse they are none the less suggested. That is because, as I said before, the main players include inflation-indexed assets, foreign bonds and foreign equities improving diversification and protection against inflation. Moreover, the overall result is quite good, since every glide path shows a high median replacement ratio, and a high median wealth-over-contributions ratio with very low downside risks, in a 40-year accumulation period. From Table 11 in Chapter V it is possible to see that the median replacement ratio is never lower than 33% and the median wealth is likely to double the amount of the contributions paid (Table 12). Then, the probability of not recouping the initial investment is very low for the bulk of the life-cycle strategies, and the riskier one does not have a probability higher than 4%. In particular, from Table 13 it is possible to see that five-out-of-eight life-cycle strategies has a probability lower than 0.20%.

Therefore, default options that rely on life-cycle strategies help investors to bear the longevity risk by providing a complementary retirement wealth. An efficient default option should aim mainly at capital protection in order to face the longevity risk. That means that it should have very low downside risk and should allow the investor to add a substantial welfare to the national compulsory pension scheme in order to face the longevity risk. In addition, the default option should aim at the protection of the capital from inflation. Life-cycle strategies are able to fulfill all these requirements by the diversification between the equity exposure and the bond exposure with the supplement of real assets in the portfolio.

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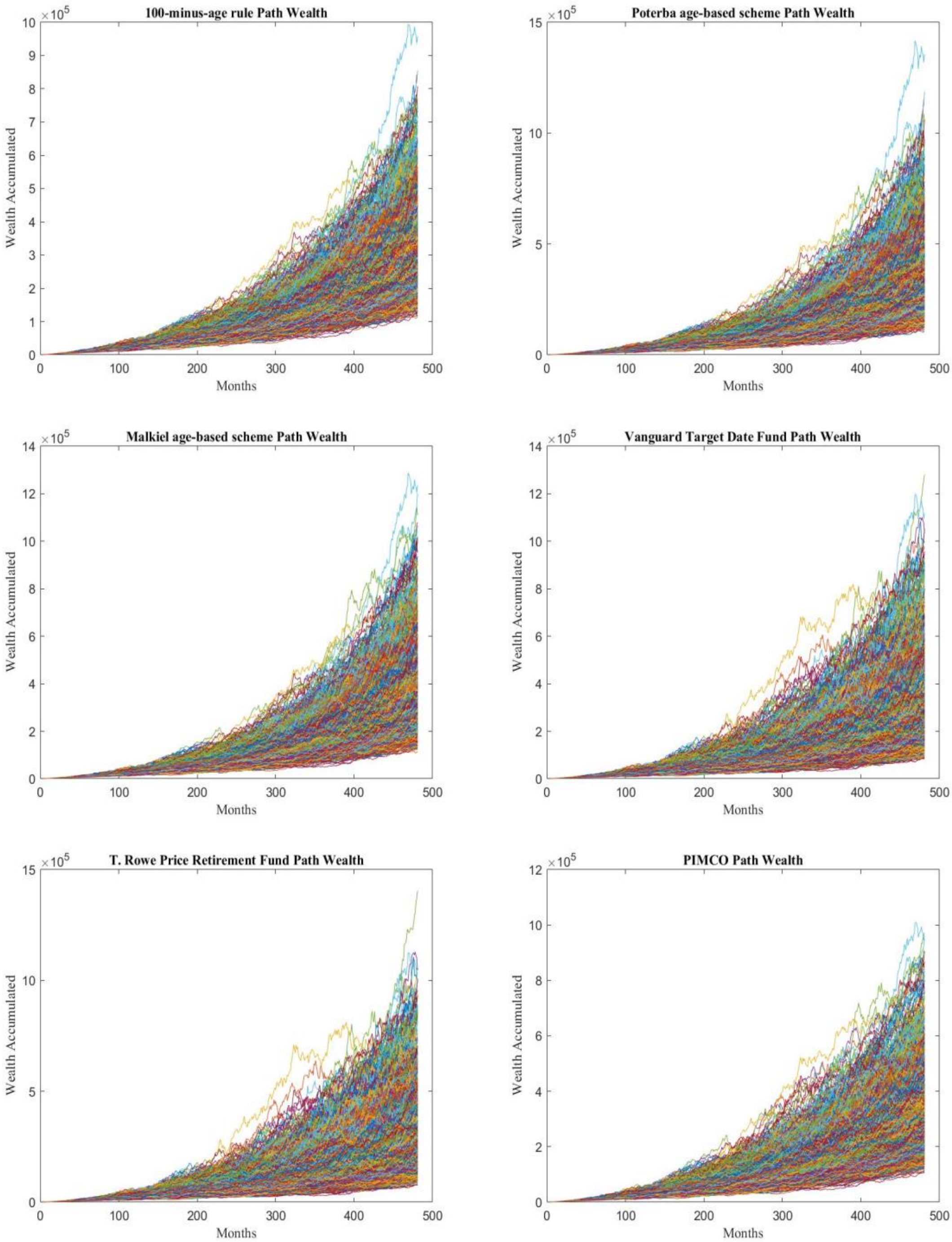
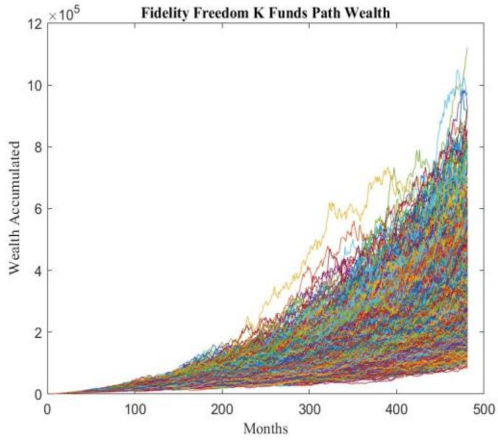
Appendix

Figure 17 represents the paths wealth for each life-cycle strategy for 40-year accumulation period simulation.

Figure 17 – Paths wealth for a 40-year accumulation period of each glide path (in €)





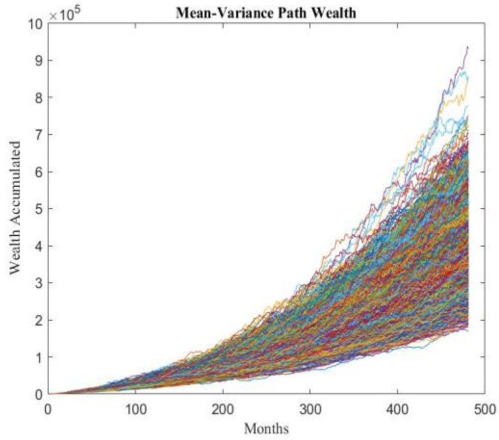


Table 15 reports all the wealth realizations statistics computed for a 40-year accumulation period for each glide path considered.

Table 15 – Wealth realization statistics computed for a 40-year period

	Mean (€)	Median (€)	Variance	Standard Deviation
100-minus-age	303,489.053 €	286,501.636 €	9188717870.088	95857.800
Poterba	342,226.616 €	315,742.899 €	16747380554.820	129411.671
Malkiel	355,912.774 €	330,855.592 €	17048415768.778	130569.582
Vanguard	291,454.547 €	261,064.829 €	17267449476.998	131405.668
T. Rowe Price	277,838.294 €	246,813.061 €	17457726650.760	132127.691
PIMCO	307,309.568 €	286,040.208 €	12399088084.110	111351.193
Fidelity	279,683.291 €	253,505.194 €	13717567127.975	117122.018
Mean-Variance	363,870.036 €	352,165.746 €	7291293905.482	85389.074

	Skewness	Kurtosis	5th per. (€)	1st per. (€)
100-minus-age	1.239	5.721	181,388.13 €	151,660.90 €
Poterba	1.483	6.969	185,462.30 €	151,526.07 €
Malkiel	1.430	6.580	195,715.91 €	161,537.15 €
Vanguard	1.709	7.677	143,606.96 €	114,753.63 €
T. Rowe Price	1.791	8.221	131,932.07 €	105,023.57 €
PIMCO	1.403	6.174	172,329.61 €	143,953.51 €
Fidelity	1.581	6.970	144,540.66 €	116,105.16 €
Mean-Variance	0.965	4.825	246,588.20 €	215,607.59 €

	Median IRR	5^{th} per. IRR	Median Replacement Ratio
100-minus-age	4.11%	1.92%	39.02%
Poterba	4.55%	2.04%	43.00%
Malkiel	4.75%	2.31%	45.06%
Vanguard	3.69%	0.66%	35.56%
T. Rowe Price	3.43%	0.18%	33.62%
PIMCO	4.11%	1.65%	38.96%
Fidelity	3.56%	0.70%	34.53%
Mean-Variance	5.02%	3.43%	47.97%

Table 16 reports all the wealth accumulation-over-contributions paid statistics for a 40-year accumulation period for each glide path considered.

Table 16 – Wealth accumulation/contributions statistics computed for a 40-year period

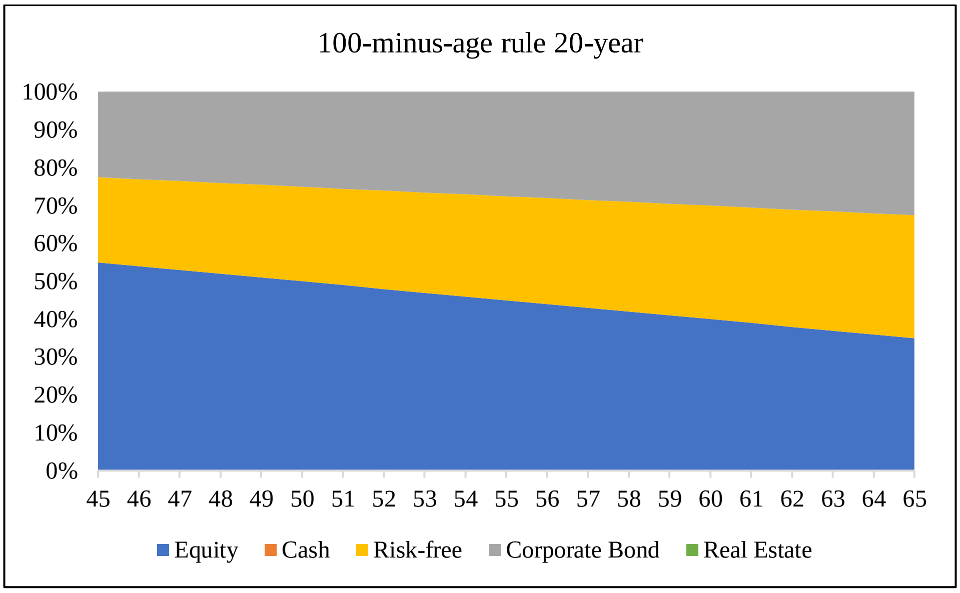
	Mean	Median	Variance	Standard Deviation
100-minus-age	2.372	2.239	0.561	0.749
Poterba	2.674	2.467	1.023	1.011
Malkiel	2.781	2.585	1.041	1.020
Vanguard	2.278	2.040	1.054	1.027
T. Rowe Price	2.171	1.929	1.066	1.033
PIMCO	2.401	2.235	0.757	0.870
Fidelity	2.186	1.981	0.838	0.915
Mean-Variance	2.843	2.752	0.445	0.667

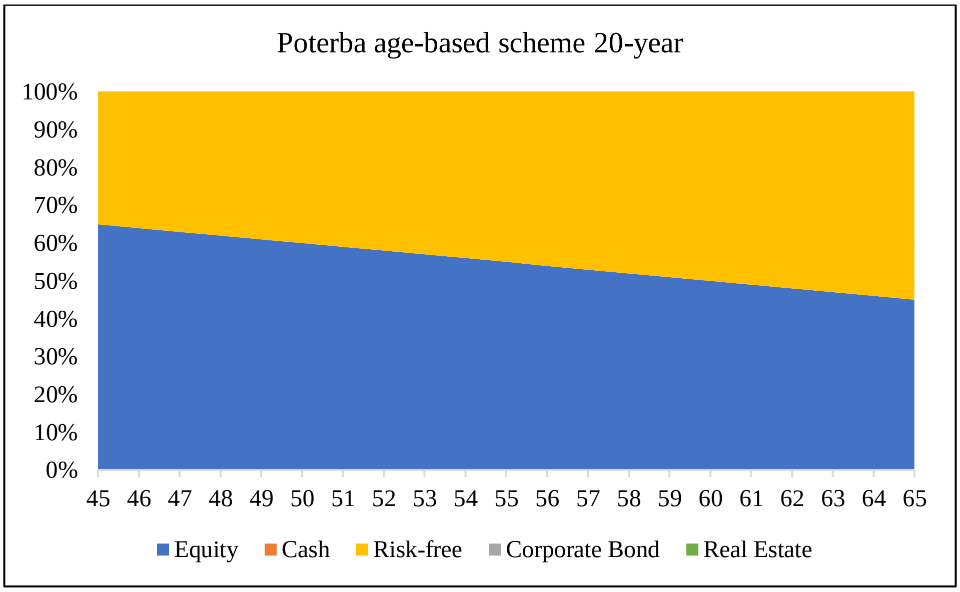
	Skewness	Kurtosis	5^{th} per.	1^{st} per.
100-minus-age	1.239	5.721	1.417	1.185
Poterba	1.483	6.969	1.449	1.184
Malkiel	1.430	6.580	1.529	1.262
Vanguard	1.709	7.677	1.122	0.897
T. Rowe Price	1.791	8.221	1.031	0.821
PIMCO	1.403	6.174	1.347	1.125
Fidelity	1.581	6.970	1.130	0.907
Mean-Variance	0.965	4.825	1.927	1.685

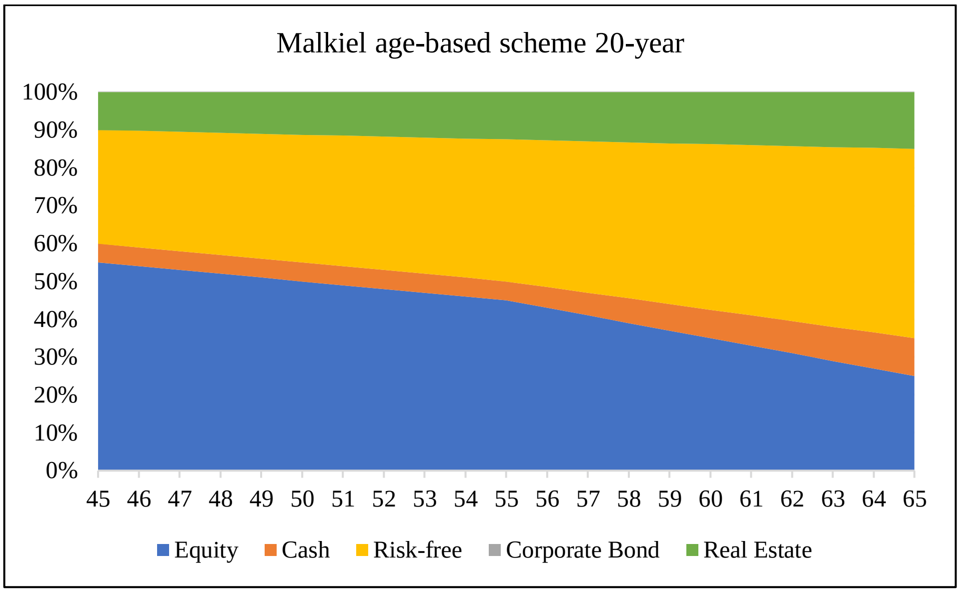
Figure 18 reports the glide paths for a 20- year accumulation period. It is possible to notice that the strategies start when the worker is 45 years old and so the wealth accumulation must be appropriate for the age of the investor. for example, in the 100-minus-age rule a

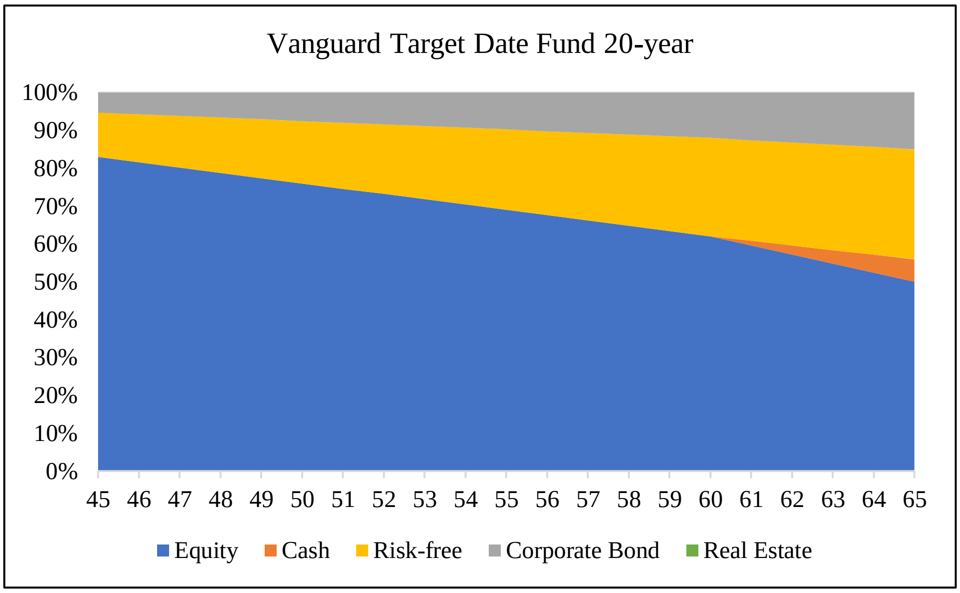
25-year-old worker starts with an allocation in the equity asset class equals to 75%, while a 45-year-old must start with 55%. This happens for each life-cycle strategy.

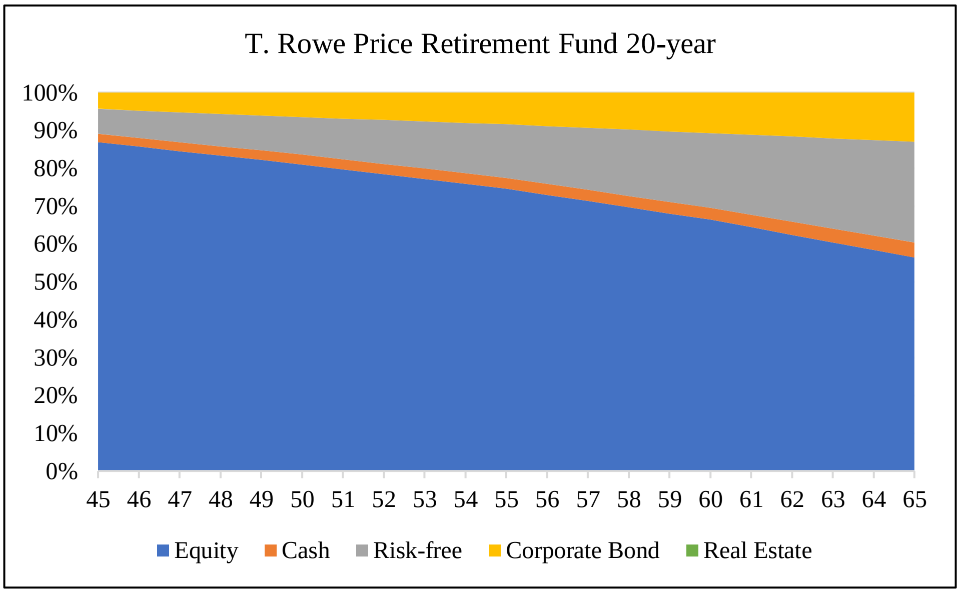
Figure 18 – Glide paths for a 20-year accumulation period

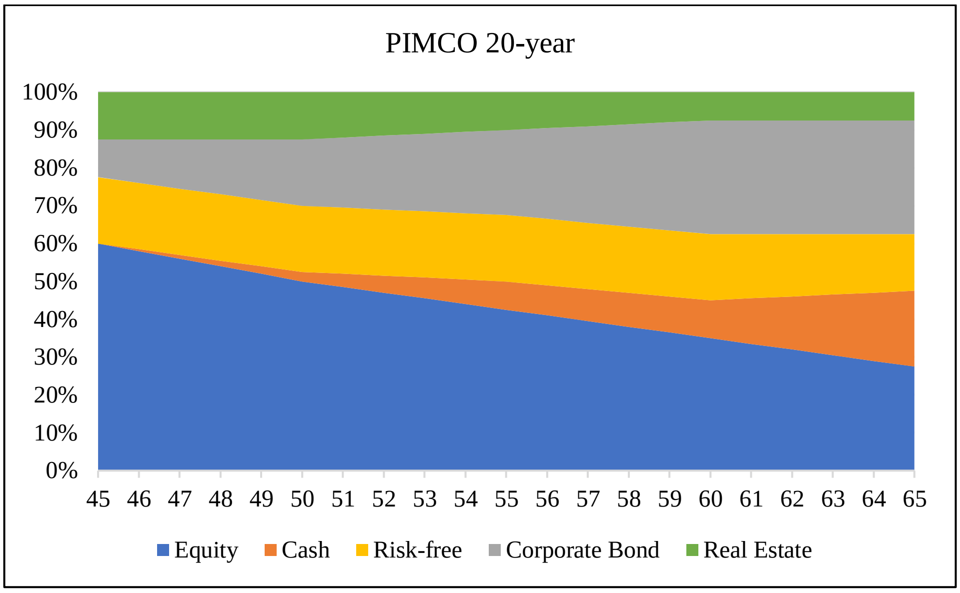


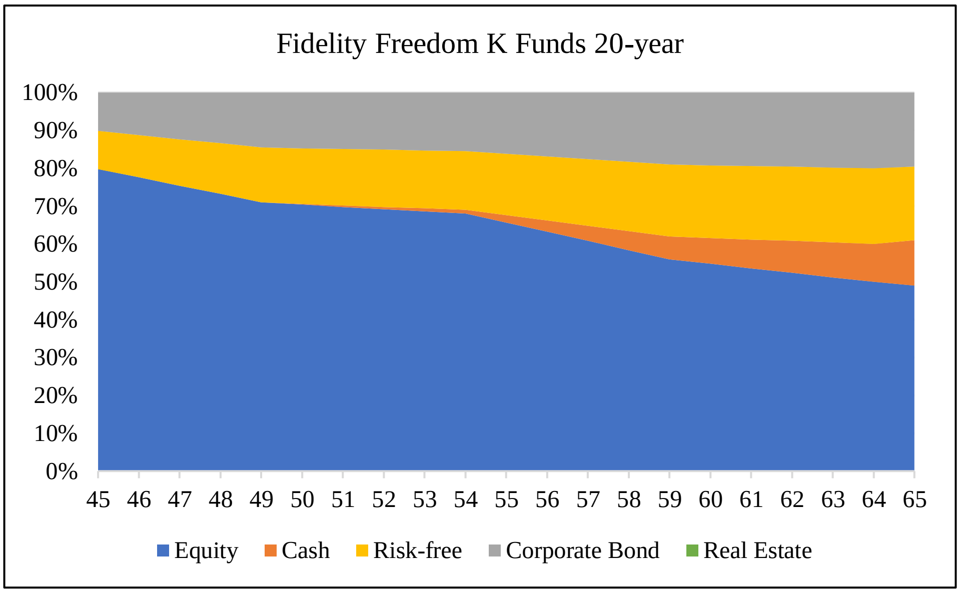



Malkiel age-based scheme 20-year
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65
Equity
Cash
Risk-free
Corporate Bond
Real Estate


Vanguard Target Date Fund 20-year
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65
Equity
Cash
Risk-free
Corporate Bond
Real Estate


T. Rowe Price Retirement Fund 20-year
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65
Equity
Cash
Risk-free
Corporate Bond
Real Estate


PIMCO 20-year
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65
Equity
Cash
Risk-free
Corporate Bond
Real Estate


Fidelity Freedom K Funds 20-year
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65
Equity
Cash
Risk-free
Corporate Bond
Real Estate

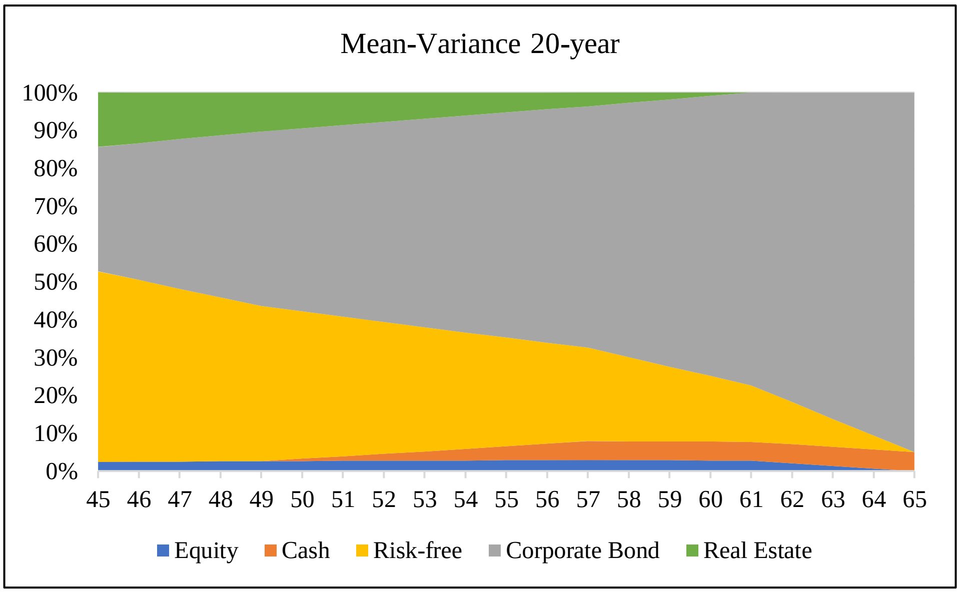
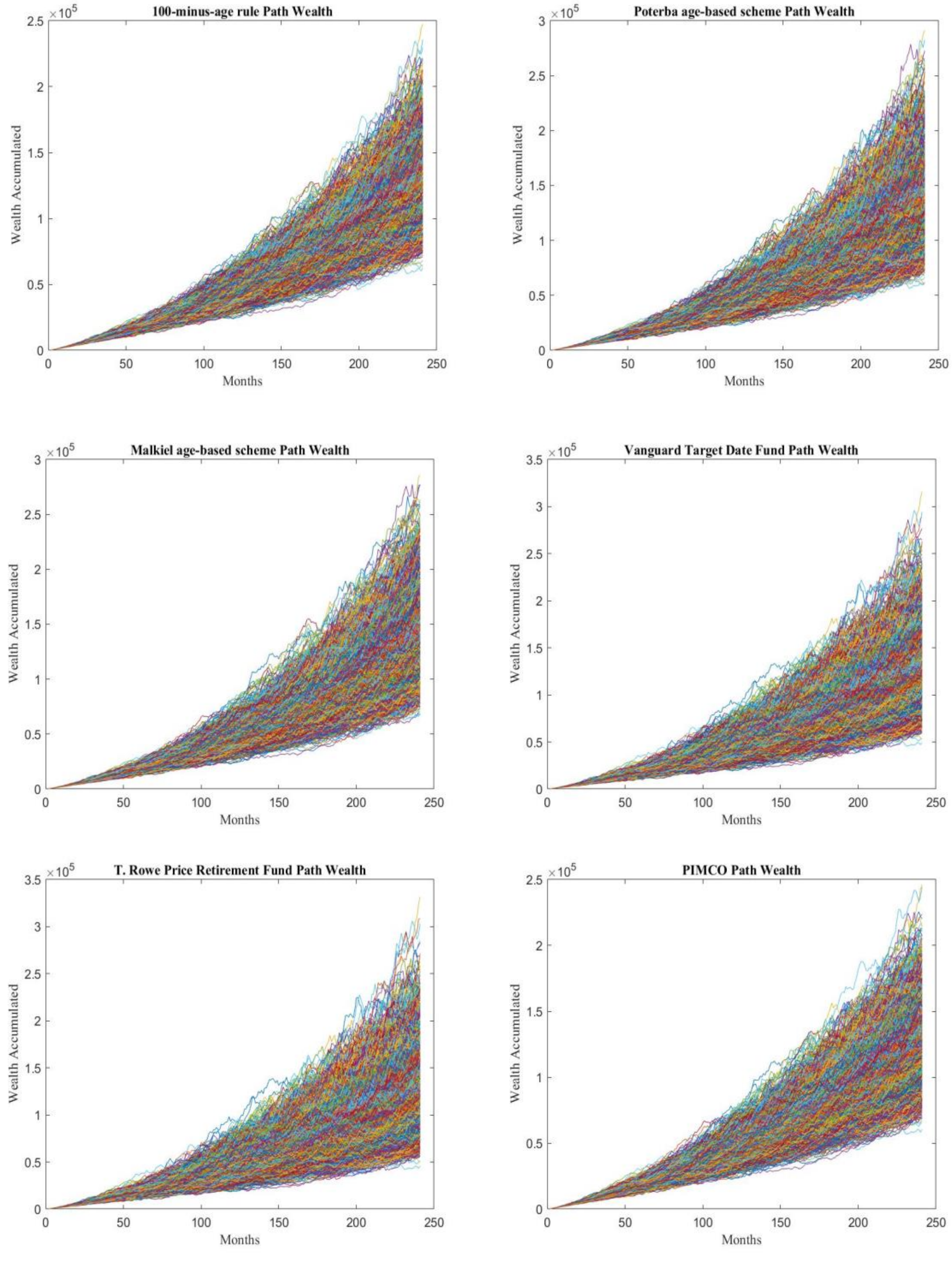
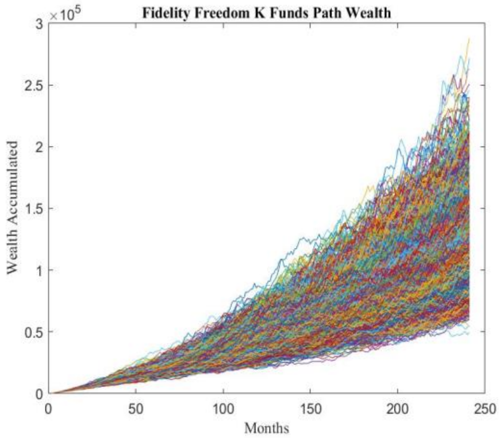

Mean-Variance 20-year
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65
Equity
Cash
Risk-free
Corporate Bond
Real Estate

Figure 19 represents the paths wealth for each life-cycle strategy for 20-year accumulation period simulation.

Figure 19 – Paths wealth for a 20-year accumulation period of each glide path (in €)





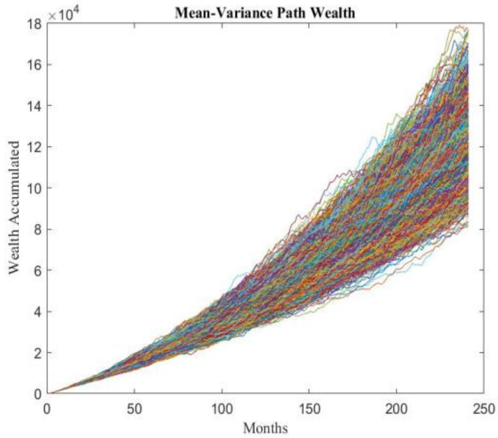


Table 17 reports all the wealth realizations statistics computed for a 20-year accumulation period for each glide path considered.

Table 17 – Wealth realization statistics computed for a 20-year period

	Mean (€)	Median (€)	Variance	Standard Deviation
100-minus-age	124,508.95 €	121,818.79 €	570220789.610	23879.296
Poterba	133,677.92 €	129,816.98 €	939757513.395	30655.465
Malkiel	136,334.59 €	132,970.06 €	907278808.080	30121.069
Vanguard	123,402.77 €	118,805.08 €	1061946816.840	32587.525
T. Rowe Price	120,598.96 €	115,570.93 €	1166139216.839	34148.781
PIMCO	121,681.13 €	118,879.51 €	581834798.980	24121.252
Fidelity	119,306.02 €	115,397.55 €	851101420.631	29173.643
Mean-Variance	120,790.46 €	119,739.92 €	193340281.735	13904.686

	Skewness	Kurtosis	5th per. (€)	1st per. (€)	Median Replacement Ratio
100-minus-age	0.649	3.657	90,277.11 €	80,871.84 €	16.59%
Poterba	0.771	3.924	91,089.78 €	79,219.25 €	17.68%
Malkiel	0.752	3.874	93,674.21 €	82,768.11 €	18.11%
Vanguard	0.919	4.339	79,934.67 €	68,822.67 €	16.18%
T. Rowe Price	0.997	4.625	75,514.04 €	64,048.41 €	15.74%
PIMCO	0.683	3.683	87,287.50 €	77,494.90 €	16.19%
Fidelity	0.854	4.153	79,502.66 €	69,197.84 €	15.72%
Mean-Variance	0.425	3.319	99,807.86 €	92,316.69 €	16.31%

Table 18 reports all the wealth accumulation-over-contributions paid statistics for a 20-year accumulation period for each glide path considered.

Table 18 – Wealth accumulation/contributions statistics computed for a 20-year period

	Mean	Median	Variance	Standard Deviation
100-minus-age	1.578	1.544	0.092	0.303
Poterba	1.695	1.646	0.151	0.389
Malkiel	1.728	1.686	0.146	0.382
Vanguard	1.564	1.506	0.171	0.413
T. Rowe Price	1.529	1.465	0.187	0.433
PIMCO	1.543	1.507	0.094	0.306
Fidelity	1.513	1.463	0.137	0.370
Mean-Variance	1.531	1.518	0.031	0.176

	Skewness	Kurtosis	5^{th} per.	1^{st} per.
100-minus-age	0.649	3.657	1.144	1.025
Poterba	0.771	3.924	1.155	1.004
Malkiel	0.752	3.874	1.188	1.049
Vanguard	0.919	4.339	1.013	0.873
T. Rowe Price	0.997	4.625	0.957	0.812
PIMCO	0.683	3.683	1.107	0.982
Fidelity	0.854	4.153	1.008	0.877
Mean-Variance	0.425	3.319	1.265	1.170