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A TRADING SYSTEM BASED ON FUZZY LOGIC

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Summary

Preface

When it comes to technical trading, every operator would love to get the magic technical formula for transforming all trades into profitable ones. There are many ways of doing it. When it comes to improving an already existing profitable system based on technical analysis, the list of viable mechanisms shortens. Many methods have been tested for the improvement of trading systems, including artificial intelligence methods such as genetic algorithms, neural networks and fuzzy logic systems. The aim of this thesis is to propose a fuzzy logic inference system (FIS) based on technical indicators as fuzzy inputs; moreover we test whether it is possible to obtain significant and robust improvements and we describe the type of enhancements obtained along with further system modifications.

Chapter 1

INTRODUCTION

The main aim of financial operators in the financial market is to maximize their profits controlling the risk associated with the operations. It is trivial to state that some proper instruments are required, in order to pursue this aim. Fundamentally, two different (but not mutually exclusive) macro instruments of stock financial analysis have been provided throughout the years: *Fundamental Analysis* and *Technical Analysis*.

The time horizon of the stock analysis and forecasts is the first main difference between this two approaches: the fundamental analysis foresees a medium-long term to buy and hold as well as extended mispricing given a somehow known intrinsic value, while the technical analysis foresees a very short time horizon (minutes, hours, days) to operate as well as price movements which can be brief but, on the other hand, sharp.

While stock fundamental analysis is based on balance sheet and cash flow indicators, technical analysis is based on mathematical indicators that have been built throughout the years by mathematicians and operators and subject to continued empirical tests and improvements. In a few words, the key role of fundamental analysis is to provide support in medium-long term investment decisions, while the one of the technical analysis is to provide instruments for the set-up of a (possibly) profitable trading strategy which could follow a day-trading scheme as well as a scalping scheme (trades are completed within minutes or even seconds).

It is trivial to state that, if one seeks to build a profitable trading strategy, a suitable technical indicator (or a crossover of them) is required to be used or built in order to receive the correct operating signals. The nature of these operating signals is usually a “BUY/SELL” scheme, given some conditions reached by the technical indicator currently in use. Countless profitable strategies have been built, considering the same stock underlying and different technical indicators. We will now raise a question: is it possible to improve some pre-existent trading strategies by introducing a new sort of mechanism, which provides more precise signals?

The aim of this in-depth analysis is to check whether it is possible or not to improve the profitability of the operations by transforming the aforementioned mechanism into a function giving back numbers associated to a quality evaluation scheme of the operating signals. This quality evaluation scheme can be seen as a "recommendation" scale evaluating each of the received signals.

Theoretically, the power of this recommendation scale is, for the operator, to avoid low-profitable operations, since each of the operators would consider, from the simple native "BUY/SELL" scheme, each of the signals appearing on the screen highly reliable.

In other words, this function may be considered as a control function for the operations, similar to the control processes of the industrial factories, which have a mechanism for checking at a certain degree of confidence the quality of the production. It is possible, for the operator, to set a minimum level to observe from the recommendation scale to avoid unprofitable or risky operations unless a certain threshold is reached or even exceeded.

It is needless to state that this mechanism can be also seen as a good risk management method, if each value of the recommendation scale is linked to some values from a riskiness scale or it becomes itself a riskiness indicator.

What is the logic underlying this reasoning and the theoretical function associated with it? It is time to introduce the concept of Fuzzy Logic.

In the next sections we will describe the fuzzy logic reasoning and introduce the most famous and wide used technical indicators.

In addition, we will set up a fuzzy logic evaluation system with technical indicators as inputs for the fuzzy inference and test its power on some stocks or other assets. We will finally draw some conclusions about our research.

Chapter 2

WHAT IS FUZZY LOGIC?

Fuzzy logic is a variable processing method which enables a same variable to process multiple values of other variables. The key point of the use of fuzzy logic is that it allows machines to replicate human semantic constructions, that is linguistic or perception-based (f.e.: True/False), overtaking standard numerical approaches which are not applicable when the values of variables and parameters are not standard or Boolean logical (es: 0=False, 1=True), leading to the desired output which could not be obtained otherwise. In other words, it is a computing approach based on degrees of truth of a variable (or a combination of them) rather than the common Boolean logic.

The inventor of fuzzy logic, Lotfi Aliasker Zadeh (1921-2017), a successful mathematician and computer scientist from Berkeley University who spent his career after the *fuzzy concept*, observed that unlike computers, the human decision making includes a range of possibilities between YES and NO, such as an indefinite YES as well as a non-fully reached NO, and the truth of each statement becomes a **matter of degree**.

The fuzzy logic works on the levels of possibilities of input to achieve the definite output. The rationale for fuzzy logic to be implemented in systems, as highlighted by Professor Zadeh in "Fuzzy Logic Systems: origin, concepts, and trends" (2004), is that there are many classes of problems which cannot be addressed by theories consisting in bivalent logic but need to be addressed by more realistic approaches based on some matter of degree, representing approximation rather than exactness.

The term "fuzzy" refers to the situation in which this type of thinking allows multiple values for the "True" condition, widening the array of possible solution given some initial condition; in other words, it is possible to obtain the desired output from the analysis of a wide imprecise spectrum of data which are not limited to some discrete points, but to a wider continuous range of points (ex.: price data).

Due to its contribution in dealing with uncertainty in engineering, fuzzy logic is used in a wide variety of sectors. As in Zadeh (2004), nowadays the contribution of fuzzy logic-based reasoning is massive in engineering-based economic activities: quality control, consumer products, industrial systems, automotive, decision analysis, medicine, geology, pattern recognition, robotics. Other emerging sectors in which the fuzzy logic already is or will be applied are: computational theory of perceptions, natural language processing, biomedicine, legal reasoning, forecasting, financial engineering. In financial services, fuzzy logic is being used in machine learning and technology AI-based systems providing outputs as a support for investment decisions.

From a practical point of view, the fuzzy logic's basic way of reasoning consists of some basic steps to be followed along with elements that cannot be neglected.

More specifically, the steps to follow according to the fuzzy logic reasoning are the following:

Crisp input values	**Convergence module**	Fuzzy sets	Inference ↓↑ Rule base	Output membership function	Defuzzifier	Crisp output values

Crisp input values are represented by some usual numerical values of something (level of dirt in the water in a washing machine, speed of a car just before the driver brakes) and must be transformed into some other type of values suitable for the fuzzy logic thinking. In our thesis, the crisp values consist of the technical indicators' values obtained from the stock price series.

The **Convergence Module** transforms the system inputs, which are crisp numbers linked to some linguistic perception-based inputs, into auxiliary suitable variables for the construction of the input membership function and consequently the fuzzy sets. There are some rules to follow to create the necessary auxiliary variables: linguistic sets have to be created along with crisp boundaries for the numerical crisp values.

For example, if we had to compute a fuzzy study on the quality of the service in a given restaurant, what would be our analysis' inputs?

From a linguistic perspective, we could determine the service as Poor, Good, Excellent; from a numerical perspective, we could rate the service from 1 to 10, with 1 representing the worst possible mark and 10 the best. We would then have to assign a lower and upper numerical limit to each of the sets, and the choice for each boundary would affect our analysis. For example, the possible lower limit for the Poor set would be of course 0, while the upper may vary: we could assign 3/4/5 as upper limit; the middle set, Good, may obtain 3/4/5/6 as lower limit, and 5/6/7/8 as upper limit, with the sets clearly overlapping. The last set, Excellent may have limits 7/8/9-10.

What we have just obtained are the **Fuzzy Sets**, which are the key element of the fuzzy logic reasoning: their construction influences all further reasoning and results. They are an extension of the classical sets and admit some variable partial membership which will be numerically expressed by a value obtained through a **degree of membership function**. In fact, the term "fuzzy" refers exactly to this: each set's boundaries are not perfectly defined, they overlap, and some values might fall into more than one set, with a high probability in the k_{th} set, with a lower probability in the $k-1_{th}$ and $k+1_{th}$ sets. The degree of membership function will tell us which set this specific crisp value fuzzified has the strongest membership with.

The degree of membership functions are the foundations of fuzzy sets, since fuzziness in a fuzzy set is determined by its membership function, which is not the same one for all fuzzy logic cases, but rather it may take different forms. The form is fundamental, since it affects the degree of membership and hence the whole fuzzy inference system.

There is an infinite number of forms the membership function can take, and there is so far no criterion affecting the choice of the shape, but this may be not needed: in fact, it is well-known that a triangular shape is usually a good starting point. The following is an example of triangular membership function for the fuzzy analysis of a cloth's price:

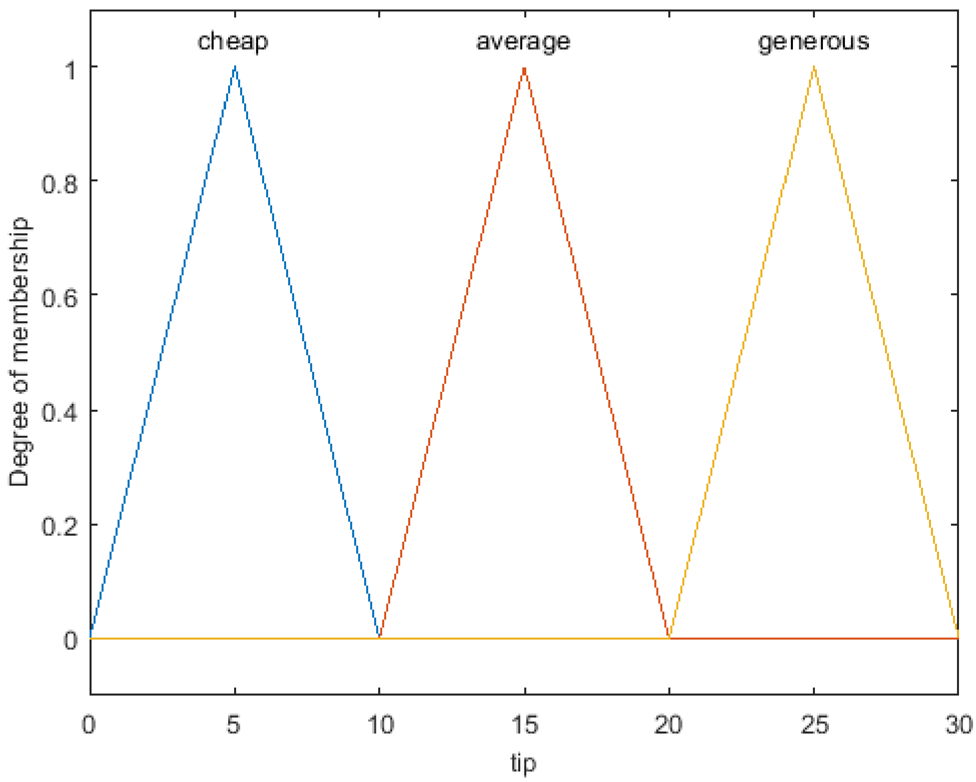


Figure 1, source: Mathworks.com, fuzzy membership functions examples.

The Gaussian-shaped membership function is universally recognized as the most precise to absolve all the tasks imposed by fuzzy logic problems, but its analysis and construction may be tricky and time-consuming. Other fundamental elements of the membership function are the number of intervals of the sets and their boundaries: this lies at the discretion of the problem-solver given the initial problems and the input crisp values obtained from some previous measurements.

The following graph is an example of a possible membership function graphical representation given our restaurant problem:

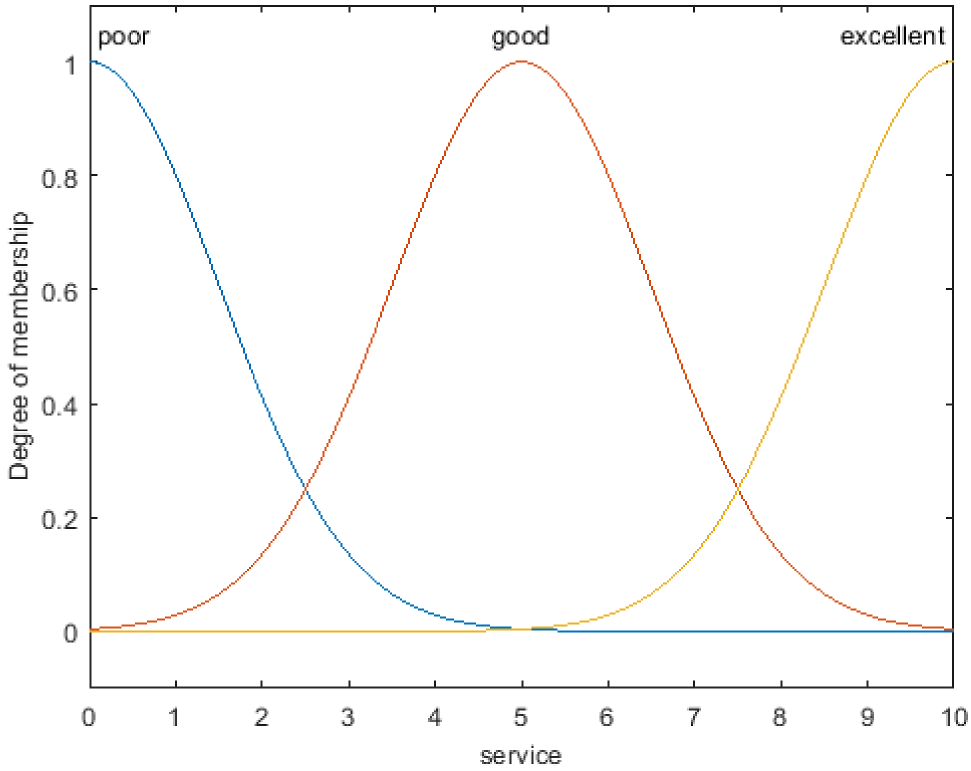


Figure 2, source: Mathworks.com, fuzzy membership functions examples.

As in the example, the shapes of the membership functions are chosen to be Gaussian to better represent the situation. If 0 is considered to be the worst possible mark and 10 the best, Poor is the worst possible linguistic outcome, Excellent the best, we observe then a degree of membership of all possible marks to the linguistic perception-based sets: of course, the lower the mark, the higher the probability for the mark to pertain to the worst linguistic set, and vice versa.

Then, the **Inference Engine** simulates the human reasoning process by making fuzzy inference on the inputs given some rule base, which is usually an IF-THEN rule or a combination of more than one, returning a corresponding to an **output membership function** value, usually ranging from 0 to 100 for simplicity. For example, given our restaurant problem, some possible input rules may be “If the service is Good and/or Excellent, visit it again” or “If the service is Poor, do not come back”. If we collect more than 100 opinions on the restaurant service, we can transform the collection of perception based observations into a value function ranging from 1 to 100: if we stay above 50, the final output will be “Come back one day”, otherwise “Do not come back”.

In other words, the **Defuzzification Module** is deputed to transform the output membership function value into an **output crisp value** which are usually numerical or linguistic instructions to follow, and that can be fed to a software.

Chapter 3

WHAT IS TECHNICAL ANALYSIS?

3.1 The philosophy underlying technical analysis

As already stated in the introduction, technical analysis is a trading discipline which focuses on historical data and repeating patterns of price and volume movements for the setup of some profitable and possibly low-risk trading strategy. Its time horizon is short or very short-term (days, hours, minutes, seconds...). This type of analysis can be applied to all types of security, not only to stocks, and it's prevalent in the *ForEx* market (FOReign EXchange, the platform for foreign currency trading) and in the commodities market (oil, gas, wheat crops…).

The basics of technical analysis as we know it today must be credited to Charles Dow, the founder of the Dow & Jones Company and of the Wall Street Journal, and its Dow Theory in the late 19^{th} century.

Edwards and Magee (1992) offers a summary of the technical analysis' foundations in accordance with the most prominent technical analysts:

- The market value of an asset is determined only by the supply and demand levels;
- Supply and demand are determined by both rational and irrational factors, and none of the individuals is able to catch them and weigh them, only the market will;
- Prices in the long-term follow trends;
- Changes in the balance between supply and demand, whatever caused them, can be spotted by observing market actions and trend inversions.

In other words, it is an attempt to predict future price movements of the stock or else based on the analysis of past sequences of the price series. It is clear, at this point, that technical analysis does not take into account other factors affecting the price, such as political events, trade wars, military wars, the current situation of the economic environment and its future outlook etc.

Technical analysis is also about market participants' expectations: these expectations are strong price and action drivers for all operators, since the price at which one individual is willing to buy, above all in a hectic environment such as a **day trading** set or a **scalping** one, depends on the expectations of future price movements and its will to anticipate it.

Therefore, the success of a trading strategy consists in retrieving as much information about the past as possible, in order to build solid assumptions about all possible trends that might repeat and concluding which technical indicator (or combination of indicators) might be suitable to describe these past and future trends. In other words, this is all about "knowing what you are doing".

Technical analysis displays itself through indicators, which will be described in greater detail later in this dissertation.

3.2 Technical analysis and efficient market hypothesis: a confrontational approach

As we said in the previous paragraph, technical analysis states that past trading activity and price changes of a security can be valuable indicators of the security's future price movements.

The last tenet collides with the so-called *efficient-market hypothesis*, which is, as in Fama (1970), an economic hypothesis that states that the stock price reflects all available information at a certain time T; this would lead to no operational room for technical analysis nor fundamentalism, proving technical analysts to be wrong. In other words, this theory is therefore one of the pillars of all passive investment strategies, namely indexes-buying strategies. For this reason, the market is called "efficient": because all prices fully reflect all available information at that moment. In support of this, Jensen in 1978 famously wrote: "I believe there is no other proposition in economics which has more solid empirical evidence supporting it than the Efficient Market Hypothesis." He defines efficiency thus: "A market is efficient with respect to information set ϑ_t if it is impossible to make economic profits by trading on the basis of information set ϑ_t." On the other hand, in Beja (1977) it is stated and shown that the efficiency of a real market is impossible.

In support of Beja, in Grossman and Stiglitz (1980) it is stated that information is costly, therefore prices cannot fully reflect all information available, as they showed that investors spending money on information and research had excess return on their trading activity. Furthermore, in LeRoy and Porter (1981), the efficiency of the market is rejected as excess volatility is observed and shown.

When it comes back to the Efficient Market Hypothesis, there are 3 forms of it: weak, semi-strong, strong.

The first case, namely the weak form, states that all past information is priced into all securities. As no "patterns" could exist, and all prices move randomly, neither fundamental nor technical analysis would provide long-term returns.

The second case, namely the semi-strong form, states that all securities' prices adjust quickly to newly available information, thus eliminating the use of fundamental and technical analysis to achieve a higher return. That is, the rate of return is given by the random move of the security's price in the appropriate direction.

The third case, namely the strong form, states that the information set reflects only past prices data, not other information, which could provide some operational input to arbitrageurs, technical analysts and fundamentalists.

When it comes to finding an edge in the market through technical analysis and technical operational setup, we are intrinsically accepting the third case and strongly rejecting the weak form of the theory, based on the most recent works and papers, which show operational rooms for technical analysis-based strategies.

In the real world, the evidence of profits obtained by technical traders when technical analysis is correctly deployed violates the weak form of efficient markets and contradicts the work of Fama, leaving enough room to state that technical analysis has its rights to exist and to be employed. This leads us to think of some new methods to improve it, to make it more efficient into recognizing patterns not catchable otherwise and into highlighting market phases' shifts.

As in Matilla-Garcia and Argello (2005) and Kaucic (2010), methods such as genetic algorithms and neural networks have been employed in the past years to better deal with market uncertainty; lesser used has been the fuzzy logic system, which we will instead experiment in the practical part of this thesis. But first, we will enlist the most used technical indicators, which will form our base for the construction of the fuzzy processing.

3.3 THE WIDE USED TECHNICAL MARKET INDICATORS.

A major differentiation among technical indicators is done as follows:

- Sentiment indicators;
- Flow-of-funds indicators;
- Market indicators.

A quick definition of **sentiment** and **flow-of-funds** indicators will be now provided.

Market sentiment indicators are statistics designed to show how whichever type of market operators (investment bankers, retail traders and portfolio managers, for example) feel about the market or economy. In most of the cases, these indicators try to reveal how bullish or bearish a group of people are, which may help forecast this group's future behaviours, often in a contrarian way. For example, when investors are extremely bearish, that is often a contrary signal to sentiment indicator traders that market prices could start heading higher soon.

Market surveys are examples of sentiment indicators, and they are compiled by professionals. They are the most biased indicators, due to the unreliability of professionals' thoughts, forecasts and doubts as well as their willingness to reveal their true expectations. An example of this is the NAAIM SURVEY OF ACTIVE INVESTMENT MANAGERS, which highlights the sentiment and expectations of a group of investment managers from some top-notch active investment banks and firms of the US. The WALL STREET STRATEGIST ASSET ALLOCATION goes along with the previous market sentiment indicator, as it represents the supreme survey among all asset managers, active investors and traders of all major New York City investment firms and investment banks.

Other sentiment indicators are based on other types of statistics, volatility for example: among these, we find the supreme CBOE Volatility Index (VIX) and CSFB Fear Barometer.

The VIX, also known as the "Fear Index", is a real-time market index that represents the market's expectation about the 30-day forward-looking volatility. Derived from the price inputs of the S&P 500 index options, therefore from their implied volatility, it provides a measure of market risk and investors' sentiment. Investors, research analysts and portfolio managers use options or futures derived from the VIX value to hedge their current market position: as the fear rises and VIX increases, the dollar position in the VIX derivatives offsets the eventual loss on their portfolio allocation.

The Credit Suisse Fear Barometer, which is a proprietary indicator developed at Credit Suisse's headquarters, measures the cost of protection against a market crash. It is measured as follows: by selling a 10% OTM [out-of-the-money] call on the S&P and using that premium to purchase downside protection, namely to buy an out-of-the-money put option to hedge a position. The level of the index indicates the %OTM strike of the put that makes the strategy net zero cost. As the market starts to tank, the value of the call on the S&P tanks as well, and we have to go deep down out-of-the-money in the option chain of the S&P to find an equal corresponding put option value, as all at-the or in-the-money put started appreciating fast. Therefore, a high CSFB value, namely how far we have to go out-of-the-money with respect to the actual market strike price, signals a high cost of protection relative to upside calls.

Flow-of-funds indicators are a helpful instrument to quantify current market liquidity and mutual funds and ETFs cash inflows and outflows, which give a qualitative response about current market confidence and ongoing investments. Flow-of-funds indicators can be seen as a type of market sentiment indicators, but their results do not return the same response as the one provided by active ETFs traders' Bull/Bear sentiment index. As the majority of participants to mutual funds, closed-end funds and some other types of ETFs consists of major companies' stakeholders, wealthy investors and corporate sharks, the flow-of funds indicator is supposed to be more reliable, as it highlights the money management of the greatest market players, rather than a vast number of retail investors and traders. Due to this, they have gained their own categorization.

Conversely, only the last type of indicators are the ones on which the technical analysts focus the most when operating daily.

In general, broad types of market indicators technical analysts look at are the following:

- chart patterns, such as trends and support and resistance levels;
- price trends, such as moving averages;

- volume and momentum indicators, built in a more complex way.

Chart patterns belong to the category of the **chart analysis**, which is visual; quantitative (price trends and movements) and qualitative (volume trends and movements) analysis belong to a deeper level of analysis, whereas **trends** and **momentum** technical market indicators are obtained by the manipulation of the time series of financial data (prices or volumes) and can be divided in two categories: those whose purpose is to individuate a *trend*, and those whose purpose is to measure the *strength of that trend*, with the perspective to confirm the trend or to highlight a possible inversion.

3.3.1 Trend Indicators

The most famous and most used Trend Indicators are the following ones:

- Moving Averages;
- Exponential Moving Averages;
- Bollinger Bands.

Their purpose is to discriminate among real trends and false trends caused by abnormal short time-framed price oscillations by smoothing price changes in accordance with some mathematical rules.

The simplest indicator is a simple non-centered Moving Average as an arithmetic average of *k* prices from *t-(k-1) to t:*

$$MA(k)_t = \frac{1}{k}\sum_{i=0}^{k-1} P_{t-i}$$

The bigger *k* is, the greater the smoothing effect against short term abnormal peaks is.

This is just a brief introduction. The widespread category of Moving Averages in use is the Weighted one, where the *Exponential Moving Average (XMA)* plays a fundamental role given his exponentially decreasing weights. In other words, each price observation has a weight: the further is this past price observation from the most recent one, the lesser it is weighted. This procedure provides a first discrimination among all price observations at disposal of the technical analyst for the prediction of future price movements.

The XMA is the so defined:

$$XMA_t = (1 - \alpha)XMA_{t-1} + \alpha P_t$$

Where α is the weight of the most recent observation and should be obtained by estimating the minimizing value of a function objective given a certain number of periods *k,* and P_t is the stock price series at a certain time instant. In technical analysis, the value of α is determined by the formula:

$$\alpha = \frac{2}{k + 2},$$

where k is the number of periods chosen by the analyst. As $k \to \infty$, the smoothing effect on the XMA series will be stronger. We will not provide justification for this latter formula, we will take it as a postulate.

In other words, if $\alpha \to 1$, the XMA series will replicate the original price series.

If $\alpha = 0$, the XMA series will turn into a horizontal line starting from y-axis value of P_0, that is the initial value of the whole price series; if $\alpha \to 0, XMA_t$then will tend to the P_t.

In other words, the Exponential Moving Average is a weighted average of infinite terms with exponentially decreasing weights as a mathematical rule for the weights' assignment. It is proven to be more realistic and more effective in highlighting and describing the existing trend, as well as highlighting its inversion by the comparison between a slower XMA (high number of periods) and faster XMA (low number of periods).

The following is a graphical example of the smoothing effect of the XMA line when setting a higher number of periods in the XMA calculation:

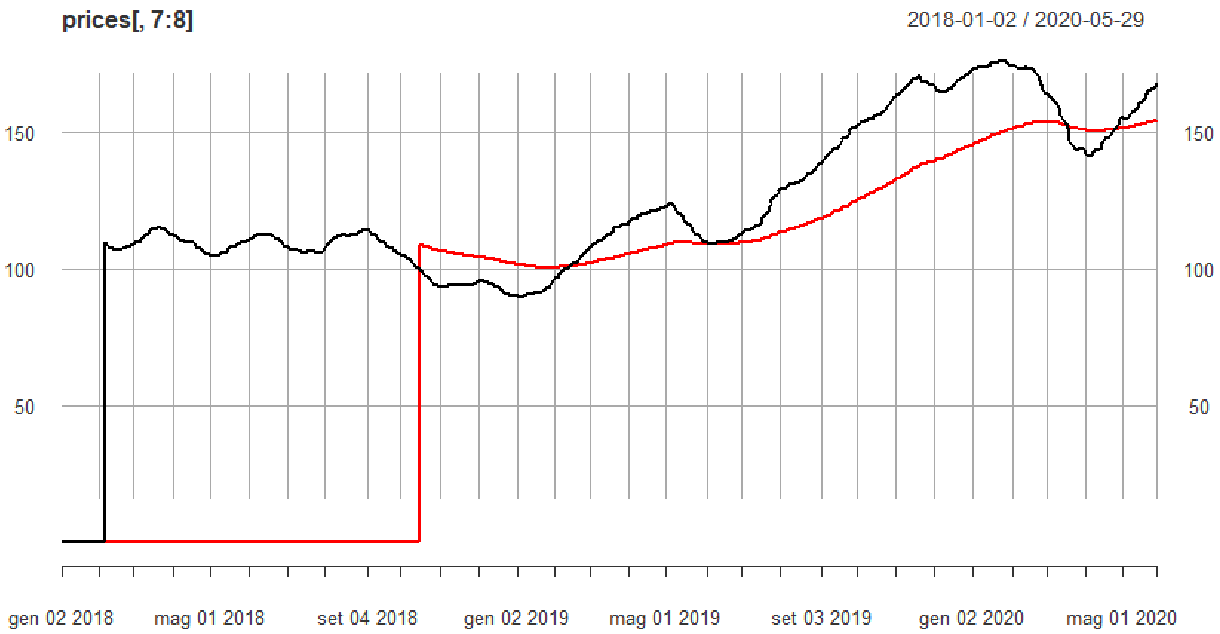


Figure 4, R, the graphical comparison between a 25-period XMA and a 200-period XMA on the ticker KLAC. The price line is omitted to avoid visual confusion. The smoothing effect of a higher k is evident.

Finally, the last trend indicator under observation are the *Bollinger Bands*, developed by the trader and technical analyst John Bollinger in the 1980[1].

This indicator consists of two bands, one upper band and one lower band, which provide an interval for the possible future price swings. In other words, they provide insight in the price volatility of the stock.

Each of the bands is simply defined as the sum between the simple k-periods moving average of the price $\pm$ a constant multiplied by the k-periods moving standard deviation $\boldsymbol{\sigma}$. In formula form:

[1] About this topic, no proper financial publication was made available back then. Bollinger published *"Bollinger on Bollinger Bands"* later in 2001.

$$UPPER\ BAND = MA(k)_t + a * MSD(k)_t$$

$$LOWER\ BAND = MA(k)_t - a * MSD(k)_t$$

where **k** is usually 20, the constant **a** is usually 2 (but these parameters can be adjusted given the trader's preferences) and $MSD(k)_t$ is the moving standard deviation.

The constant multiplying the moving standard deviation acts as an amplifier for the bands, shifting each of the band series upward or downward, enlarging or reducing the bands' interval. The creator of the bands, John Bollinger, usually set the value of the constant equal to 2, so $a = 2$. As someone might think, most price movements ranges between the two bands; on the other hand, each breakout is a major signal, but this latter should not be considered as a trading signal. Conversely, we might expect the opposite price movement as soon as the breakout happens:

- if the price breaks the upper band, given the distance between the actual price and its moving average, one might expect a price pullback or full downward movement, unless some trend continuation signals occur;
- if the price breaks the lower band, someone might think of a reversal to the upside, unless some trend continuation signals occur.

In terms of operational signals, it is common practice not to consider them as a standalone trading system. John Bollinger suggested often using them in cooperation with two or three other non-correlated indicators that provide more direct market signals. He believes it is crucial to use indicators based on different types of data. Some of his favored technical techniques are moving average divergence/convergence (MACD), on-balance volume and relative strength index (RSI).

3.3.2 Momentum indicators

We move onto the next type of indicators, that is, the *momentum* indicators.

The *Rate of Change* is the easiest to interpret indicator: it is a percentage differential of the price of a stock between a certain time *t* and a certain past time *t-k.*

In formula form:

$$\text{ROC} = ((\text{CP}(n) - \text{CP}\,(n-k)) \,/\, \text{CP}(n-k\,) \times 100$$

where CP(n) is the closing price of the n-th period (it might be the last price of the 1-minute candlestick or of a daily price candlestick) and CP(n-k) is the closing price of the previous k-th candlestick (usually 1 candle ahead, either we are looking at the one minute chart or the daily).

Figure 5, source: Investopedia.com, a stock price series and its Rate Of Change over the period.

It measures the strength of a given price movement, whether it is upward (the *ROC* will have a positive value) or downward (the *ROC* will have a negative value). The higher the *ROC* is in absolute value, the stronger the trend is. in case of a zero or approximately zero value ROC, we could find ourselves in a situation of price consolidation, namely a situation in which the stock price hit a support or resistance and buyers and sellers are fighting to prevail: in this case, the ROC provides no further information, and for this reason we should use it in conjunction with other indicators.

The next indicators are much more complex than the *ROC*, as they are more evolved transformations of the price series.

The first one is going to be the *Moving Average Convergence Divergence,* commonly known as MACD, introduced in 1979 by Gerald Appel. The MACD is a trend-following momentum indicator that shows the relationship between two moving averages of a security's price. The MACD is calculated in this way: first, the 26-period Exponential Moving Average (EMA) of the security's price must be subtracted from the 12-period EMA of the same price. The result of that calculation is the MACD line. Secondly, a nine-day EMA of the MACD called the "signal line" is then plotted on top of the MACD line, which can function as a trigger for buy and sell signals. Operators buy the security when the MACD crosses above its signal line and sell - or short - the security when the MACD crosses below the signal line.

Nevertheless, the crossing between the MACD line and its signal line gives birth also to the MACD histogram, that we can see in the graph at the end of the paragraph. The magnitude of each histogram bar derives from the distance between the MACD line and its signal line for the time frame chosen; its positiveness or negativeness depend on whether, respectively, the MACD is above or below the signal line for that timeframe. When the two lines cross, the MACD histogram goes to 0: this value usually represents a reversal in the price trend.

The Moving Average Convergence Divergence (MACD) indicator can be interpreted in several ways, but the more common methods are crossovers, divergences, and rapid rises/falls.

The crossover method represents the case in which the MACD crosses over or under its signal line, therefore generating a possible change-in-the-trend signal. This can be used for trading styles as scalping, day trading, swing trading or position trading, given the number of periods used in the MACD formulation and the time horizon set for the analysis. More than breakout, this crossovers signalling an inversion could be used for reversal trades, in which the operator wants to profit from the classical "buy low, sell high" by finding the best entry point in favour of his plan, either the goes reverses up or down.

The MACD-stock price divergence is another reversal trading method. One of the most common setups is to find chart points at which price makes a new swing high or a new swing low, but the MACD histogram does not, indicating a divergence between price and indicator. In this case, it is said that the indicator is revealing a possible change in the trend before the price action does, and it should signal to the operator what to do and where to position his entry and stop point, given this latter's risk aversion and expected return. This approach is said to fail more times than it wins, but that depends also on where stop losses are set and how long one would want to wait given his trading style: most of the times, we observe a "fakeout", meaning a fake price reversal stopping traders out of the trade before the actual reversal effectively happens.

The third method, that highlights rapid rises or falls in the price, serves to the purpose of trading breakouts as the trend is established and there is no evidence of a reversal: the quicker the MACD crosses above its signal line in conjunction with the primary trend, the more explosive the breakout seems to be. In this case, this MACD signal must be supported by adequate trading volume and compared with another indicator, perhaps the ROC, to gain more insight on the strength of the breakout. Furthermore, it would be helpful to check if the breakout is happening at historical supports or resistances, knowing his certain direction.

The number of periods for each moving average can change due to the behaviours of the screened stock or index, therefore the operator should check past observations to draw conclusions about the best time intervals to use.

Figure 6, source: Investopedia.com, a stock price series and its MACD over the period.

As further examination, another widely used technical indicator measuring the strength of the ongoing trend is the *Relative Strength Index*, also known as *RSI,* originally developed by J. Welles Wilder Jr. in 1978.

The RSI is about the persistence and magnitude of a security's price increases or decreases: the higher the number of price increases throughout the whole analysis period, the higher the demand level is, and therefore the stronger the upward trend will be.

Conversely, the higher the number of negative price differences, the lower the demand level (or the higher the supply level compared with the demand) is, therefore the stronger the downward trend will be.

The RSI is computed as follows:

$$RSI_t = 100 - \frac{100}{1 + \frac{\sum_{t=0}^{t-1} \quad \Delta price_t +}{\sum_{t=0}^{t-1} \quad \Delta price_t -}}$$

Where $\Delta price_t \pm$ counts as a single price difference, either positive or negative, observed from the previous price to the next price given our observation period (either 1m candlesticks or daily price candlesticks).

This indicator can take values ranging from 0 to 100, and usually the areas 80-100 and 0-20, are considered respectively as *overbought* or *oversold* areas. More active thresholds may be set at 30 and 70, increasing the number of operative signals to catch riskier but also more profitable trading opportunities. These thresholds might be adaptive, in the sense that they can be moved up or down based on the current price primary trend: in case of a strong uptrend, the oversold conditions may come already when the RSI enters in the 50 or 40 territory, while overbought conditions may come when the RSI enters the 90 or 95 territory. This could help the operator in smoothing the signals from the RSI. In this dissertation, however, we will hold to the classical RSI *overbought* or *oversold* thresholds as in the beginning of this paragraph.

Figure 7, source: Investopedia.com, a stock price series and its RSI over the period.

The RSI can be used primarily for reversal trades, assuming that after the indicator enters one of the two territories (overbought and oversold) while the price hits a particular level (historical support or resistance, very far from its 9-period moving average) we may expect a price reversal coming soon, and to trade indicator-price divergences similarly to the MACD divergence method. As in the MACD case, a sudden divergence between the price action and the indicator, with the indicator making higher lows (or lower highs) and the price acting not accordingly by making lower lows (or higher highs), may signal a reversal coming soon (of course not with 100% accuracy), actioning reversal trading strategies in the operator's mind. Hence, more than breakouts, the scope of using the RSI (in conjunction with other indicators), is to trade reversals and range trades, in presence of strong supports and resistances which may bring the price to range among them.

The *Stochastic Oscillator* is an indicator developed in the late 1950s by George Lane which signals when the stock is in an overbought and oversold position as well, but it is constructed differently. We identify two lines: the K line (the fast line, capturing finer details), and the D line (the slow line, indicating any major movements of price in the charts), which is a 3-period moving average of %K. The cross between those lines in the proximity of the overbought (usually 80) and the oversold levels (usually 20) triggers operational signals.

The formula for this indicator is as follows:

$$\%K = 100 * \left(\frac{CP_t - L_t(L14)}{H_t(14) - L_t(14)}\right) —$$

$$\%D = 100 * \left(\frac{H_t(3)H3}{L_t(3)L3}\right) —$$

Where CP_t is the closing price of that day (or timeframe), $L_t(14)$ is the lowest price of the previous 14 sessions, and the $H_t(14)$ is the highest of the same previous 14 sessions, while $H_t(3)$ is the highest price of the previous 3 trading sessions, and $L_t(3)$ is the lowest of the same previous 3 sessions.

A BUY signal is usually triggered when the fast line crosses the slow line in the proximity of the oversold level; conversely, a SELL signal is triggered when the slow line crosses the fast one in the proximity of the overbought level.

Figure 8, source: Investopedia.com, a stock price series and its Stochastic Oscillator over the period.

As the RSI, this indicator is useful in reversal and consistent range trading with strong supports and resistances, more than breakouts; moreover, its nature makes it more appropriate for sideways or choppy markets, as it reacts more quickly than the RSI, which is more useful in trending markets whose direction is well defined.

When it comes to the MACD, RSI and SO, their main drawdown is the ability to produce false signals in every environment: it is the task of the technical operator to smooth those false signals improving and adapting each indicator to the historical characteristics of the security or market under observation.

3.3.3 Volume technical indicators

We now move onto volume technical indicators, which are used in common practice as reinforcements to the prediction of future prices given the strength or change of strength in the ongoing trend.

The only indicator hereby exposed is the *On Balance Volume*, also known as OBV, developed by Joseph Granville in 1963, and chosen for its ability to better represent the underlying dynamics of the levels of supply and demand when it comes to market exchange, as volume is considered to be the key force behind major moves in the markets: that is, the imbalance between supply and demand, with the latter taking over the supply levels for some reason pushing the price high, and vice versa when the supply level takes over the demand one.

The OBV, in mathematical terms, is an algebraic cumulative sum: the trading volume at time t must be added to the previous trading volume at time $t-1$, and so on starting from time 0. The time interval remains fixed by a dedicated starting point, meaning the real number value of OBV arbitrarily depends on the start date. The sign of the addition will be determined by the price path: if during that day the asset price registered a positive difference (price increased as final result when the bell rings), the trading volume must be added to the total accumulated OBV value; else, if the price registered a negative difference at the final bell, the same day trading volume must be subtracted to the total accumulated OBV value.

Figure 9, source: Investopedia.com, a stock price series and its OBV over the period.

Despite being plotted on a price chart and measured numerically, the actual individual quantitative value registered on a given day or period of OBV is not relevant. Instead, traders and analysts look to the nature of OBV movements over time; it is the slope of the OBV line that matters, indicating the strength of changes in the price.

The operation signals can be created in different ways: buy/sell signals may be triggered when the changes in the OBV reach a certain threshold, or when the OBV crosses over or under its period moving average, or even more if the changes in the OBV persist for more than 1/2/3 days, therefore if the indicator first spikes and then plummet following a couple of rollercoaster days, false operational signals would be avoided as no clear primary trend is detectable. If the OBV stays level, no operational signals would be triggered.

Chapter 4.

A MIXTURE: TECHNICAL ANALYSIS BASED ON FUZZY LOGIC AND SOME PRACTICAL EXAMPLES

There are two possible scenarios for a technical trader: operating in a scenario of uncertainty or operating in a risky scenario. This latter verifies when all probabilities of all possible outcomes are known; the reality is, probability of each event can be estimated but there will always be some error in the estimate. Therefore, previous case studies exist in which fuzzy logic control has been applied to technical analysis to better deal with uncertainty, improving timing, avoiding wrong market shifts and improving profitability.

More specifically, the fuzzy logic control for technical analysis consists in one or more technical indicators taken as crisp inputs, which will become fuzzy sets that provide an output membership function (the choice of the shape is vital) after some fuzzy inference. After defuzzifying, we obtain an output crisp value which could represent some sort of operational recommendation in both senses (BUY, SELL).

From the previous premises, it can be perceived that fuzzy reasoning blends very well with technical analysis process, although there are some initial issues which should be addressed:

- There is no basic rule that helps to determine whether a training set of a fuzzy system, as well as one of a neural network, is correctly represented and/or computed, although the architecture of a fuzzy system is easier to interpret and therefore less subject to misconstruction;
- The fuzzy technique is purely quantitative, and it excludes qualitative judgements from the human operator;
- The fuzzy modification of the initial analysis set may be too difficult to comprehend and/or too difficult to build or compute.

Our purpose in this chapter is to prove that all addressed issues do not apply to each fuzzy construction, if some basic rules are followed; the back-testing of this fuzzy reasoning then helps us whether to state the adequateness of our fuzzy system or provides us some hints for further modifications.

4.1 TRADING SYSTEM CONSTRUCTION

The architecture of the fuzzy system that will be obtained and presented in this thesis consists in the following scheme:

- The **technical indicators** module;
- The **convergence** module;
- The **fuzzy inference** module;
- The **back-testing** module.

As in Gradojevic and Gencay (2013) and Olufunka et al. (2014), it is possible to think of a fuzzy system based on technical analysis indicators and to build a practical foundation of it. To obtain an augmented system providing highly reliable conclusions, the results are that at least 3 indicators should be involved in the fuzzy architecture, possibly extracted from those describing the direction of the trend as well as those describing the strength of the ongoing trend.

They will be, almost certainly, transformed in inputs for the input membership function through a **convergence module** to fit the fuzzy logic systems' requirements.

From the convergence module we will obtain the **fuzzy sets** on which **fuzzy inference** will be executed: **fuzzy rules** which regulate the process must be specified.

The **defuzzification** of the fuzzy process will eventually return the desired output, which are the fuzzy-derived trading signals constituting the fuzzy trading system to evaluate.

4.1.1 The Technical Indicators’ Choice Module

For what concerns the first module, the most important aspect is the choice of the technical indicators which will be fed to the convergence module for the inference system. As in Ijegwa et al (2014) and Gradojevic and Gençay (2013), very frequently the indicators fed to the system are those describing the strength of the trend, such as ROC, OBV, RSI, Stochastic Oscillator with some reinforcement provided by the MACD or some simple moving average but this latter is very rare. The rationale for this choice is that each of these indicators itself is helpful in drawing conclusions about the possible future path of stock, since their mathematical construction is not trivial nor too basic.

In this thesis, the choice of the indicators to feed will be based on the papers named above and personal working experiences; technical indicators from the **trend-direction** group, namely the *MACD,* and from the **strength-of-the-trend** group: *RSI, SO* and *OBV*.

The choice of the indicators is based on the following reasons:

- The MACD is one of the most famous and widest-used indicators for what concerns the calculations of a trend direction, and it can be considered reliable in most of the cases;
- The RSI is as well one of the most famous and widest-used indicators for what concerns the calculation of a trend strength, and although its construction is simpler than others wide-used indicators, this does not affect reliability;
- The Stochastic Oscillator functions similarly to the RSI, and built in conjunction with the latter, reinforces our forecasts on the direction of the trend or on a possible reversal, returning us the best timing for the anticipation of the reversal;
- The OBV is one the best indicators when it comes to helpful tools for checking the ongoing trading volume during the operations, and this helps us in whether confirming or rejecting our conclusions drawn from the results of the trend indicators: if the trading volume is low, the ongoing trend may not be confirmed as lasting trend, and therefore

we would not have all sufficient elements to confirm the opening of the position. In other words, checking for trading volumes is in no case a disagreeable idea.

4.1.2 The Garch effect and the shift to a semideviation indicator

In this dissertation, a further indicator is added as final control for the triggering of the operational signal: the GARCH volatility model, developed by Robert Engle in 1982. Thia model can assume several forms, while the simplest one is the G(1,1) model, which is exactly the model we will try to fit to our price log-returns.

The formula for the simplest of the GARCH models is the following:

$$\sigma_t^2 = \boldsymbol{\omega} + \boldsymbol{\alpha}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

where σ_t^2 is the conditional volatility, $\varepsilon_{t-1}^2$are the squared unexpected returns of the series for the previous period, while $\boldsymbol{\omega}$ is an always positive constant, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are non-negative coefficients of the unexpected squared returns and of the conditional volatility observed for the previous period, respectively. Stationarity of the GARCH(1,1) requires that ($\boldsymbol{\alpha}$ + $\boldsymbol{\beta}$)<1.

The reason for this choice is the following: besides pure technical indicators, our aim is to add some statistical/econometric indicators to our system, so relying on different sources for our system rules (technical-professional and statistical-mathematical).

We fitted a GARCH model to the log-returns of the stock to find the best estimates for the volatility of returns, as our scope is to determine which trading days had the highest volatility (good for obtaining large returns as the stock moves more) and which days had the lowest volatility (bad as the stock does not move).

The operational signal (BUY, SELL) is triggered on a certain day when the volatility estimated on the previous trading days is above the average GARCH volatility, which means we are in a high volatility regime (GARCH models, as they are built, imply that volatility trends high for a certain period in a certain direction, then it reverses and trends lower); in case of low volatility, we will stop from entering positions, as we do not want to get stuck in something that theoretically will not move as much as we want.

The estimation of a GARCH model provided results which are far from being robust: this is mainly due to the trade-off between using weekly data for the returns or using daily returns tables, as a simple GARCH model would require at least a year of daily data.

The weekly returns frame allows us to scope a wide span of data (even more than 20 years) without overloading our analysis software, while the daily returns, although capturing finer details, would overload our system: we should cut the time span of the data but the robustness of the trading strategy would be somehow heavily affected.

To overcome this problem, we shifted to the **SemiDeviation** as a signal triggering indicator based on the stock volatility.

SemiDeviation is a common metric in finance which captures the downside volatility, namely the below-mean fluctuations: this is outrageously helpful in determining a short entry, and helpful in signalling low downside volatility periods for going also long, as the risk of getting stuck in a position in a fast falling stock is reduced.

The common SemiDeviation formula is the following:

$$SemiDev = \sqrt{\frac{1}{n} * \sum_{r_t < meanrt}^{n} (mean_{rt} - r_t)^2}$$

where $\mathbf{mean_{rt}}$ is the average rate of return of the stock in terms of daily price variations and $\mathbf{r_t}$ is each observed price. This formula is like the common standard deviation one, but it differs as it considers only the price observed values under the mean price, focusing on the downside movements of it.

In conjunction with the technical indicators, we plan to use the Semideviation as follows: given the current technical setup, if the Semideviation indicator signals a low downside risk, we will have then a LONG setup, therefore we will plan to buy the stock; if, on the contrary, a high downside volatility is signalled, given the actual technical setup, we will be shown then a SHORT setup, therefore we will plan to sell short the stock or close the position.

4.2 The Convergence Module

Our fuzzy system requires a preparatory semantic mapping of all our 4 indicators. In addition, the crisp daily input, if the analysis is conducted daily on daily prices, must be specified and, of course, must be numerical; they need to be fuzzified into the fuzzy sets. For the given semantic constructions, numerical thresholds must be specified:

- If MACD>0, Then MACD is high;
- If RSI>70, Then RSI is high;
- If SO>80, Then SO is high;
- If OBV_{t+1}>OBV_t, Then OBV is high;
- If MACD<0, Then MACD is low;
- If RSI<30, Then RSI is low;
- If SO<20, Then SO is low;
- If OBV_{t+1}<OBV_t, Then OBV is low;
- If MACD = 0, Then MACD is med;
- If 30<RSI<70, Then RSI is med;
- If 20<SO<80, Then SO is med;
- If OBV_{t+1}=OBV_t, Then OBV is med.

The trading rules triggering each chosen indicator must be transformed into human linguistic constructions as follows:

- If MACD is high, then buy;
- If RSI is low, then buy;
- If SO is low, then buy;
- If OBV is high, then buy;

- If MACD is low, then sell;
- If RSI is high, then sell;
- If SO is high, then sell;
- If OBV is low, then sell.

These are the schemes to be mapped through the construction of fuzzy sets. In the meantime, it is worth to notice that, by doing so, operational signals will be obtained every day, probably leading to quite high frequency of trading and consequently to a high accumulated amount of transaction costs.

Therefore, some intermediate values of those indicators must be introduced to preserve the goodness of trading signals by highlighting situations in which no operation is required but one: position holding, or no position trading at all if all previous positions were already closed out. The intermediate values are as follows:

- If MACD=0, then hold;
- If 30<RSI<70, then hold;
- If 20<SO<80, then hold;
- If 30<OBV<70, then hold.

The elaboration of the fuzzy sets composed by our fuzzified indicators, after being fed with crisp inputs, must return us a function providing either a set of values or a single value which instructs us on the trading steps to take. If we assume that a single value is returned, and this value ranges from 0 (Sell Immediately) up to 100 (Buy Immediately), representing the strength of the signal in both directions, an additional function must then return us the amount (or portion) of our capital to invest into the trade.

Similarly, the amount of our capital to be deployed is proportional and may follow a linear relationship with the value representing the strength and direction of the trade. For example, we may assume that, if a value of 90 is returned by our fuzzy function, the capital function will return us a value of 50% of capital to invest into the trade; if we obtain 10, the capital function may return us a value of -50%, therefore we have to close out our position or short sell the stock by a half of our capital.

This function is clearly related to the risk preferences and aversion of the investor. We will try to fit some known utility functions to our problem.

The other solution is to set some arbitrary threshold which represents some sort of percentages of the amount of capital to put at work, but that seems too arbitrary and has no foundations.

An extreme analysis to conduct on the goodness of our exploration is to fix a single value for the percentage of capital to put at risk every time a trading signal is produced, and this percentage could be 100% (to test our trading set to the fullest) or a very small amount (for example a 5%) to test the profitability also for the average investor, which is usually risk-averse.

Following the mapping of all indicators, the convergence module requires their transformation into auxiliary variables as inputs for the inference system.

The value returned by this fuzzy module will be valid as an operational signal, and we will compare it with the simple technical indicators-based strategy.

4.3 The Fuzzy Inference Module

The convergence of the crisp inputs into fuzzy ones based on the convergence rules allow us to partition the corresponding fuzzy universe into fuzzy sets for all indicators and statistical measures (semideviation) we are treating; eventually, the necessary fuzzy rules to complete the fuzzy analysis that will return some value to defuzzify need to be added.

The result of the fuzzy sets construction in R for all variables is the following:

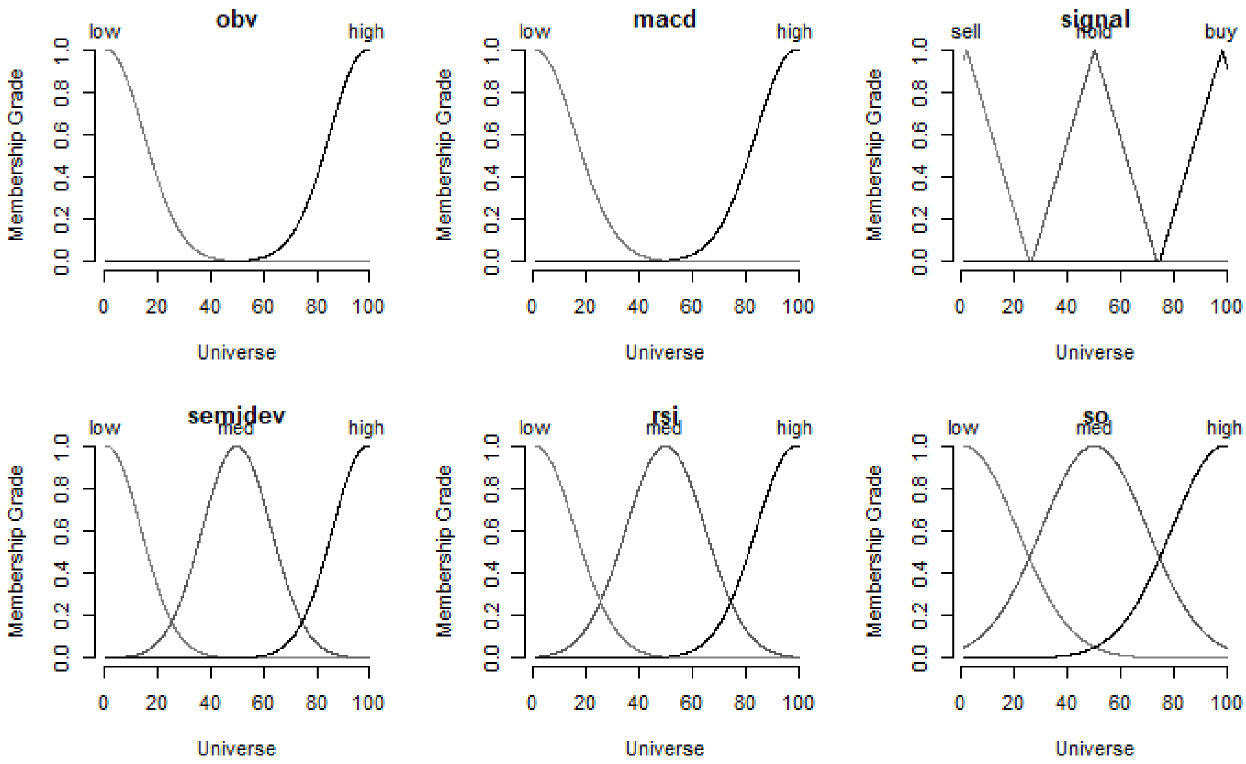


Figure 10, R, fuzzy sets of the WBA stock.

The function *SETS* has been applied to each indicator and statistical measure for the WBA stock case, in addition to the final signal function, in which we distinguish the three main cases: when to sell, when to hold, when to buy.

In general, the "base" of each bell (if we can call it this way for a Gaussian curve) and therefore its height is determined by the fuzzy convergence rules which are input of the convergence module. The function *SETS* is responsible for the fuzzy sets creation based on the input given in the convergence module, for which the operator is responsible.

Each of the fuzzy sets needs to be discussed, as each of them presents some differences from the other, due to its nature and the scope of our analysis.

The MACD fuzzy set and the OBV fuzzy set are similar: they consist just in two membership functions, namely the “Low” and the “High” ones, as they’re the only ones triggering trading signals. Their mean value 0 has taken value 50; the values of 50 or around 50 are not considered, as they do not trigger any trade. This mean value, according to the fuzzy logic reasoning, should belong to both membership functions equally, with a certain degree of participation; here, the system is built in order for this value not to belong to any membership function, to process a lower amount of data.

In other words, the rationale of the fuzzy logic reasoning is all in the last paragraph: each of the converged values does not belong to a single membership function, as it may belong to at least two membership functions at the same time, with different degrees of participation, overcoming the ambivalent ***true-false*** reasoning which is typical for hypothesis testing (this does belong to the null and does not belong at all to the alternative) and embracing different fuzzy level of “truth” (this does belong with probability 0.80 to the first case or set and probability 0.20 to the second case, and 0 elsewhere, or even better: this is true with 80% probability in this linguistically perceivable case or set, and true with 20% probability in the other linguistically perceivable case, and true with 0% probability in all other cases).

For the MACD case, the values around 50 are not considered, as a value of the MACD around zero does not trigger any trade.

The fuzzy architecture of the fuzzy sets for the Semideviation, the RSI and the SO follow the common scheme of overlapping membership functions, where the starting points of the beginning and the final functions are, respectively, the lower and the highest value of the universe of converged values. The value of the universe with the highest degree of membership for the median membership function is always 50.

The signal rule fuzzy set is set triangular, as no overlapping of membership function is intended, in order to send a single clear instruction to the machine. As we can see, the more the signal function value approaches one of the extremes, the higher the degree of membership to one of the two triggering signals functions, hence the stronger the quality of the signal is. When it comes to the HOLD signal, we can see that the closer we get to the median value of this function, the stronger the HOLD signal will be; while, as we move away from it, we can reach certain values whose degree of membership is to the HOLD and to one of the other two functions is equal. To avoid confusion and misunderstandings, we created triangular membership functions that will not overlap.

The next step, as we approach the end of the process, is to present the fuzzy rules which will determine the final shape of this model.

The fuzzy system consists of 6 variables (the 4 technical indicators, the Semideviation indicator and the signal function), whose 2 of them can assume medium values, which has been specified in the convergence module: hence, we have 36 possible fuzzy combinations. We identified 8 rules which try to combine all previous indicators in different ways, even in ways that may seem completely the opposite of each other. The rationale for it is that, by combining the indicators in these particular ways, we want to see if we can profit from breakout movements along with reversals. Moreover, the amount of time needed for the cycle and computational capacity of the consolle have imposed to exclude some other rules, reducing the total number of rules to an amount of less or equal to 10.

The fuzzy rules are the following:

BUY SIGNAL
fuzzy_rule 1: IF (macd is high && rsi is low && so is low && obv is high, signal is buy) fuzzy_rule 2: IF (macd is low && rsi is high && so is high && obv is low, signal is buy) fuzzy_rule 3: IF (macd is high && rsi is med && so is med && obv is high, signal is buy)
SELL SIGNAL
fuzzy_rule 5: IF (macd is low && rsi is med && so is high && obv is low, signal is sell) fuzzy_rule 6: IF (rsi is high && so is low && obv is low, signal is sell) fuzzy_rule 7: IF (macd is low && rsi is high && so is high, signal is sell)
HOLD SIGNAL
fuzzy_rule 8: IF (macd is low && rsi is high && so is med, signal is hold) fuzzy_rule 9: IF (macd is high && rsi is med && so is med && obv is low, signal is hold)

In addition to this, there is the fuzzy rule of the Semideviation indicator to consider: as a measure of downside risk, we want to go long when this downside risk is reduced, therefore when the volatility measured through this indicator is low; on the other hand, the system will trigger a SELL order when the downside risk increases.

The combination and recombination of the indicators is crucial: they can be combined in several ways. The inclusion or exclusion of one or more indicators plays also a fundamental role in the system. As in this model, stating that this model is overleveraged and could be made more parsimonious does not seem so erroneous, but the opposite could be right as well: the inclusion of further indicators could increase the effectiveness of the system. The answer to this doubt is to search into different systems' comparisons, where a higher or lower number of indicators and a change into the fuzzy rules' combination could lead to extremely different results.

But the comparison between fuzzy models is not the scope of this analysis: our aim is to compare pure technical systems with fuzzy models.

4.4 The realization of the fuzzy strategy: the Back-testing Module

To perform the construction of the indicators and the assessment of the strategy, the following R packages were needed:

- *tidyverse*, to clean, process, model, and visualize data;
- *lubridate*, to work with dates;
- *TTR*, to construct technical trading rules from a set of technical indicators;
- *PerformanceAnalytics*, to perform an overall analysis on system performance and risk (in our case, it is necessary to compute the Semideviation indicator);
- *Quantmod*, to add numerous technical indicators to the system (es: MACD, ATR et cetera) after extracting prices;
- *htmltab*, to collect structured information from HTML tables;
- *Sets,* to build the fuzzy sets for our fuzzy analysis.

Furthermore, a random stock data-downloading function has been added to the code through the latter *htmltab* package, to create a code chunk that generates a table of prices of a casual Nasdaq-listed stock to analyse, each time we run the code: we want to check if, for each stock prices data, the fuzzy logic algorithm is better-performing than the simple pure technical indicators-based strategy.

As the time span increases, the computational power of our console decreases, therefore we reduced the time span of data to a period slightly longer than 2 years, precisely a period starting from January 1st, 2018 lasting to May 31st, 2020. We can consider a time span of 2 years as the minimum requirement for a trading system to be properly backtested.

In this dissertation, commission costs and others (slippage, orders unexecuted, technical issues) are not considered.

As a random stock prices table is called from the *Nasdaq slickcharts*, all OHLC prices and trading volume values need to be extracted along with the Nasdaq index performance in that period, for control.

Subsequently, all the necessary technical indicators are computed through appropriate functions, while the Semideviation indicator requires a *for* cycle, as it requires a minimum number of observations and there is no built-in function which serves our purpose.

After obtaining our first overview of the pure technical strategy performance, we then move onto building the fuzzy system. To build the necessary fuzzy sets, the convergence module containing the fuzzy rules must be specified starting from crisp values, similarly to the technical indicators mapping section of this dissertation. Fuzzy sets are then briefly built using the *fuzzy_partition* R-function.

The last step is to apply the fuzzy setup to real data, creating a numerical output which represents one of the possible operational outcomes: BUY, HOLD, SELL.

An object representing the evolution of our starting capital is computed: eventually, a graphical comparison between the two systems performance in terms of cumulative returns and drawdowns is returned by calling the *charts.PerformanceSummary* R-function, making us able to draw some conclusion on the effectiveness of the fuzzy processing. Further numerical statistics are obtained calling numerous *table* functions, such as *table.Stats* and *table.AnnualizedReturns*.

We will now discuss 5 cases where both systems have been applied to random real Nasdaq-listed stocks. The choice of the following stocks is purely random and is to ascribe to the random ticker generator chunk of code: in fact, the first 5 random tickers who have been called are the following five in the order in which they are presented.

For none of the following tickers we managed to obtain the price series and its indicators on a single graph due to the fallacy of all possible codes we deployed; nevertheless, a useful graphical comparison can still be provided.

4.4.1 Case 1: “WBA”

The first random stock the code has provided us is **Walgreens Boots Alliance Inc,** ticker “WBA”. Just for curiosity, this company is one of the global leaders in retail and wholesale pharmacy. In the following tables, a graphical representation of the series and its indicators is provided:

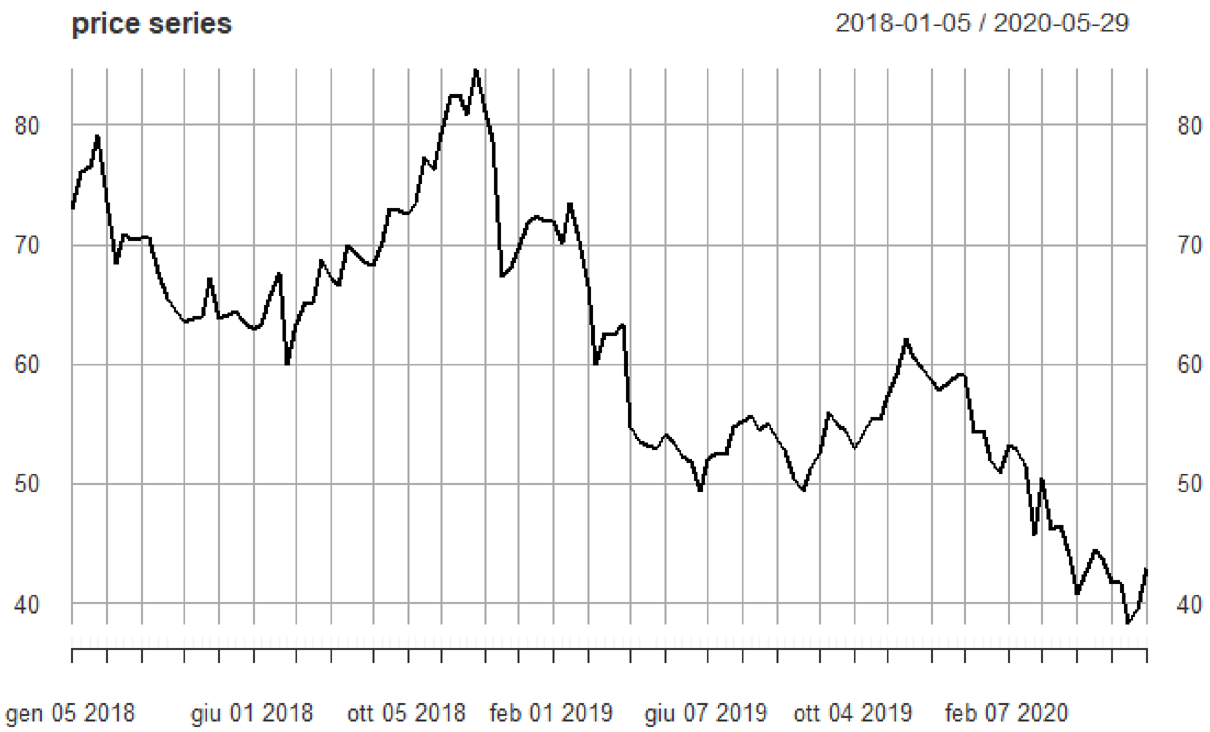


Figure 11, R, WBA stock price series.

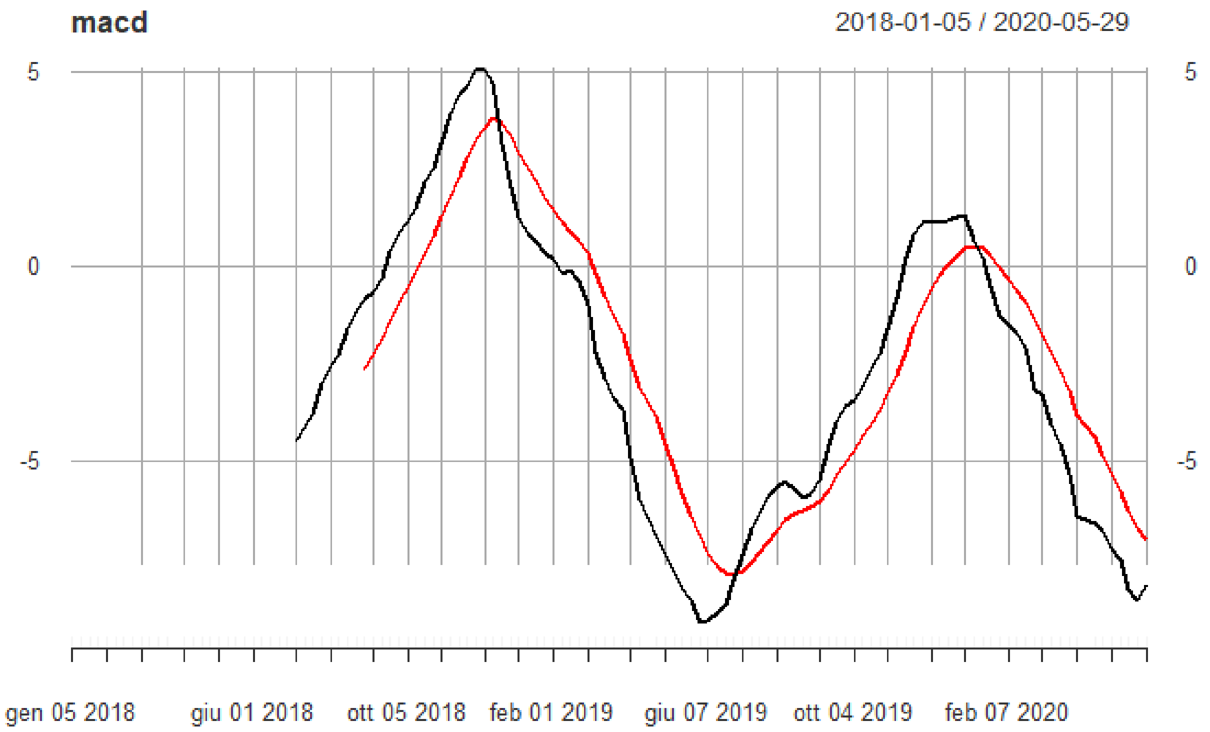


Figure 12, R, MACD of the price series of the WBA stock.

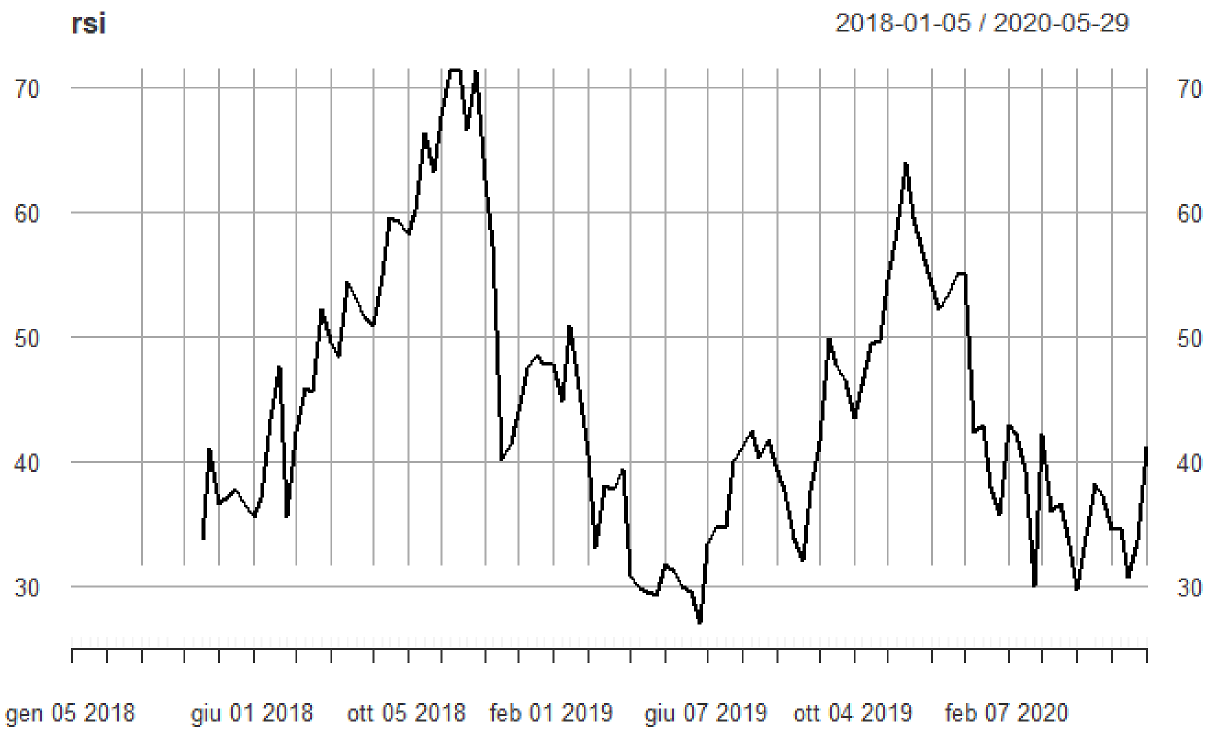


Figure 13, R, RSI of the price series of the WBA stock.

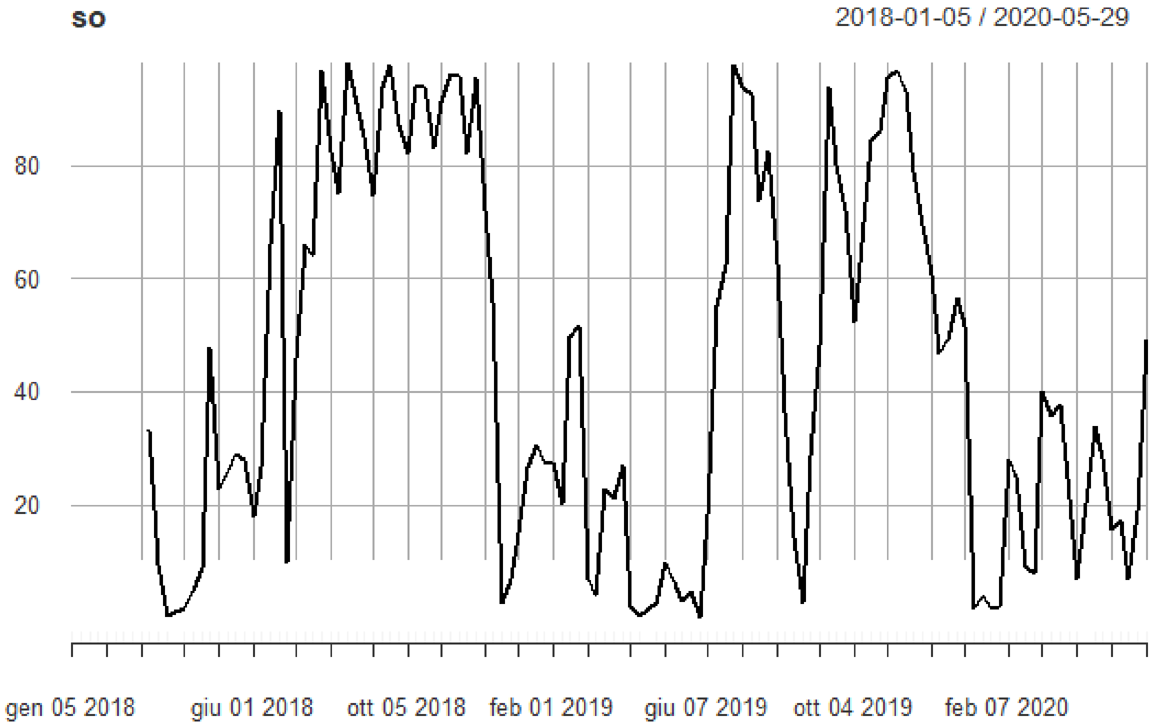


Figure 14, R, SO of the price series of the WBA stock.

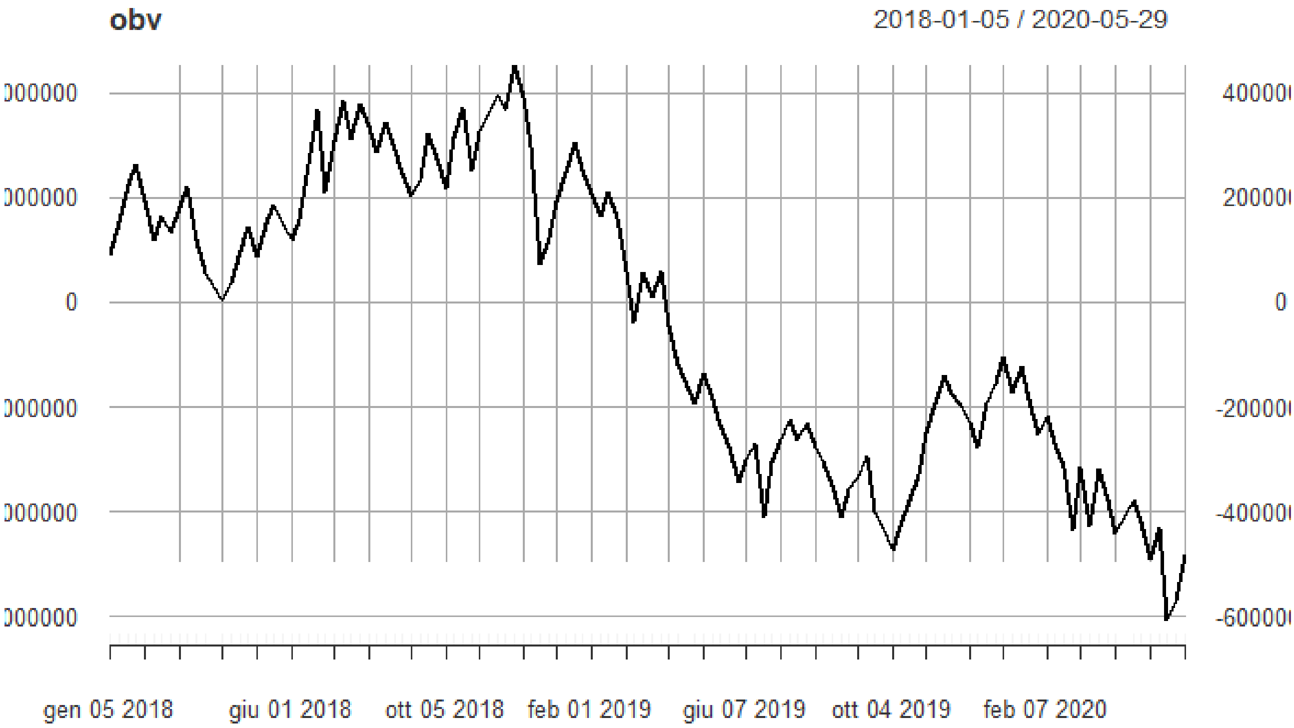


Figure 15, R, OBV of the price series of the WBA stock.

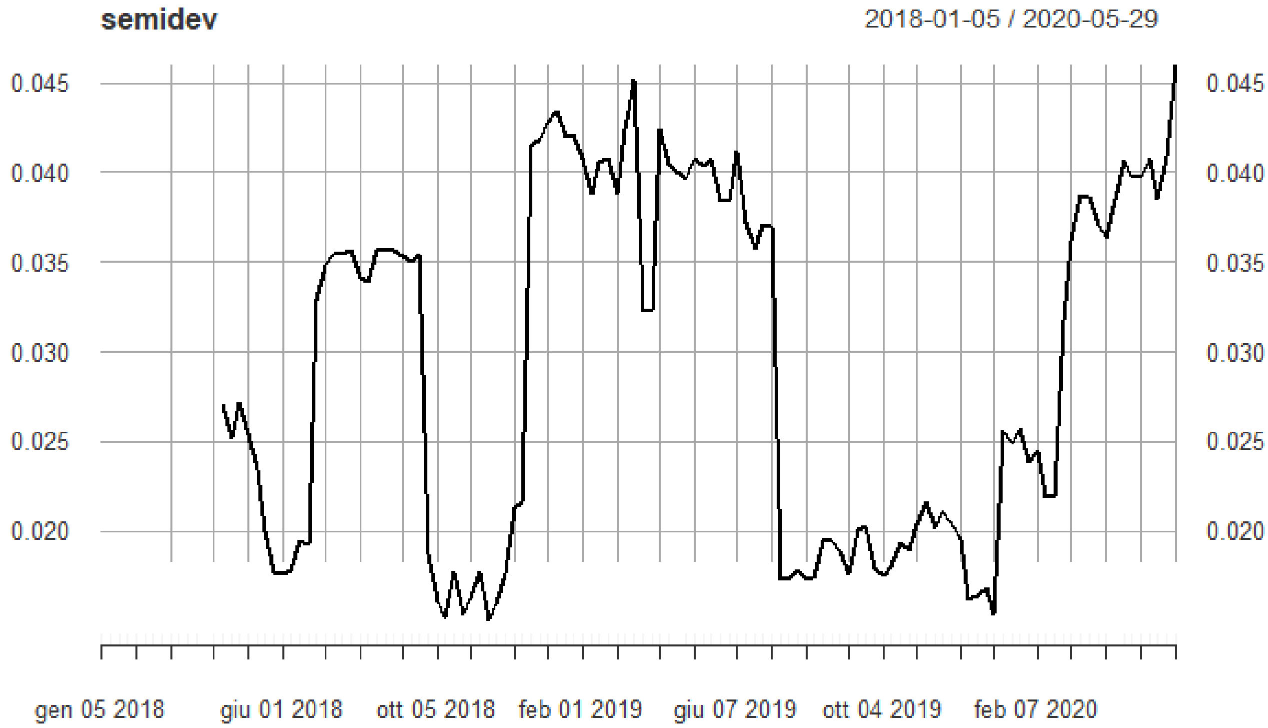


Figure 16, R, Semidev indicator of the price series of the WBA stock.

As we can see, the MACD, the RSI, the SO (although it oscillates far more than the other two indicators) and the OBV replicate with fidelity the price series of the stock, and that is valid for each of the tickers we analyzed. The OBV replicates with fidelity the stock price path due to the way it is calculated, as it involves trading volume, yet its variation is based on the closing price difference between two periods.

The Semidev indicator presents the most interesting features: it does not replicate the price series but represents its volatility to the downside. In fact, we notice spikes in the volatility when the stock price starts to tank, and rapid falls in the volatility when the stock price starts to grow quickly; moreover, this indicator seems to be proportionally reactive to the magnitude of the changes in the stock price.

Without further ado, here is a graphical performance comparison of the systems' results provided by R:

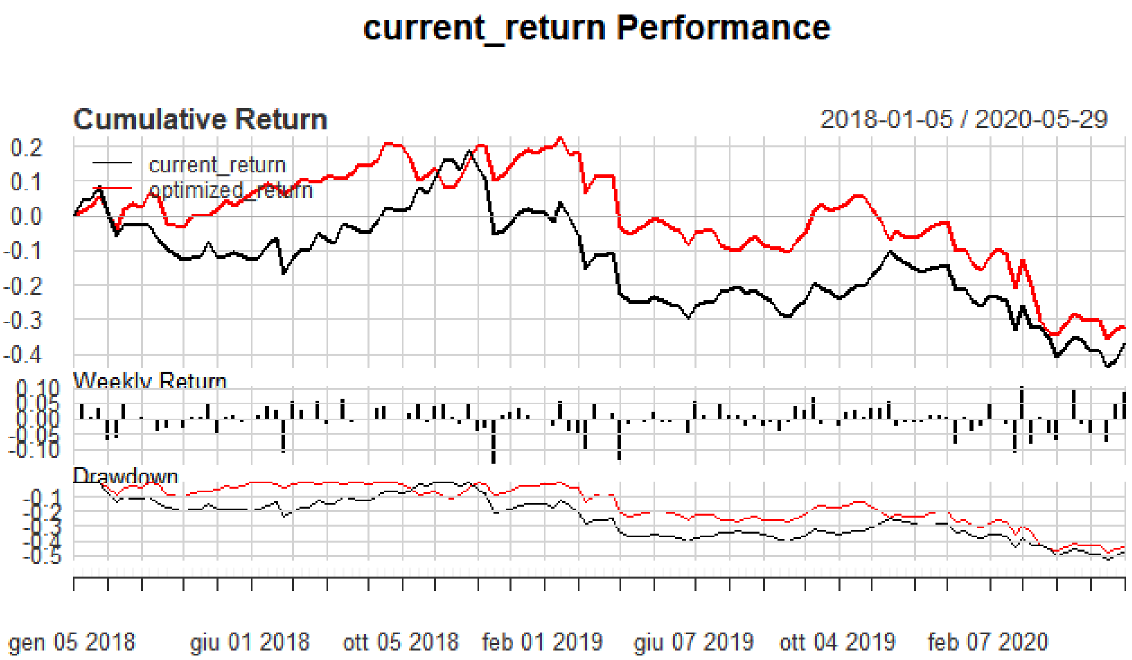


Figure 17, R, cumulative return comparison for the WBA stock.

The red line represents the cumulative return for the fuzzy system, the black one represents the cumulative return for the pure technical system. As we can see, the fuzzy system performed outstandingly for the first half of our analysis period (beginning 2018-beginning 2019), with an impressive cumulative return of 20%, against a poor swinging performance by the pure technical system, which eventually reached an 18% cumulative return by the end of 2018, but it was not able to hold it thoroughly. The maximum drawdown provided by the pure technical system during the same period is also strong, higher than the one of the fuzzy system.

The path of the cumulative returns for both systems eventually reversed, turning negative and closing negative: that may be due to the entry of new players into the playing field for this stock, who ensured more liquidity to the game or even drew liquidity away to the overall market for this stock, or to changes in the company fundamental which pushed human operators and *Algos* to respond differently to price moves, market moves and company announcements.

In this case too, the fuzzy system performed better than the pure technical system, as it limited the cumulative losses to a -30%, 10% less than the technical system (given that the trader/investor would hold onto these losses without cutting them earlier).

As the fuzzy system is projected to consistently increase the profitability of our operations, so it can limit the false operations, and therefore the losses.

The following is a statistics table with numerical values in details, which confirms our system results' assessment for WBA:

	current_return <dbl>	optimized_return <dbl>
Observations	123.0000	123.0000
NAs	0.0000	0.0000
Minimum	-0.1458	-0.1356
Quartile 1	-0.0217	-0.0136
Median	-0.0001	0.0014
Arithmetic Mean	-0.0027	-0.0024
Geometric Mean	-0.0037	-0.0031
Quartile 3	0.0271	0.0206

Figure 18, R, systems' return distribution comparison for the WBA stock.

Those statistics depict a much more favourable environment for the fuzzy system, in terms of maximum drawdown (minimum return), median return and geometric mean return.

In terms of annualized return and risk, we have another confirmation for the goodness of our fuzzy system on this stock, as its Sharpe ratio[2] is less negative than the one from the pure technical system.

[2] The Sharpe ratio is a measure of expected return calculated as follows: $\mathrm{ER\text{-}R_fVar(R)}$, where R_f a constant risk-free return.

	current_return <dbl>	optimized_return <dbl>
Annualized Return	-0.1749	-0.1493
Annualized Std Dev	0.3133	0.2759
Annualized Sharpe (Rf=0%)	-0.5584	-0.5412

Figure 19, R, systems' annualized return, Sharpe ratio and st.dev. comparison for the WBA stock.

4.4.2 Case 2: “MDLZ”

The second ticker we are provided is “MDLZ”. It refers to the company **Mondelez International Inc**, which is a huge snacks company serving food markets worldwide.

The following is the graphical representation of all indicators:

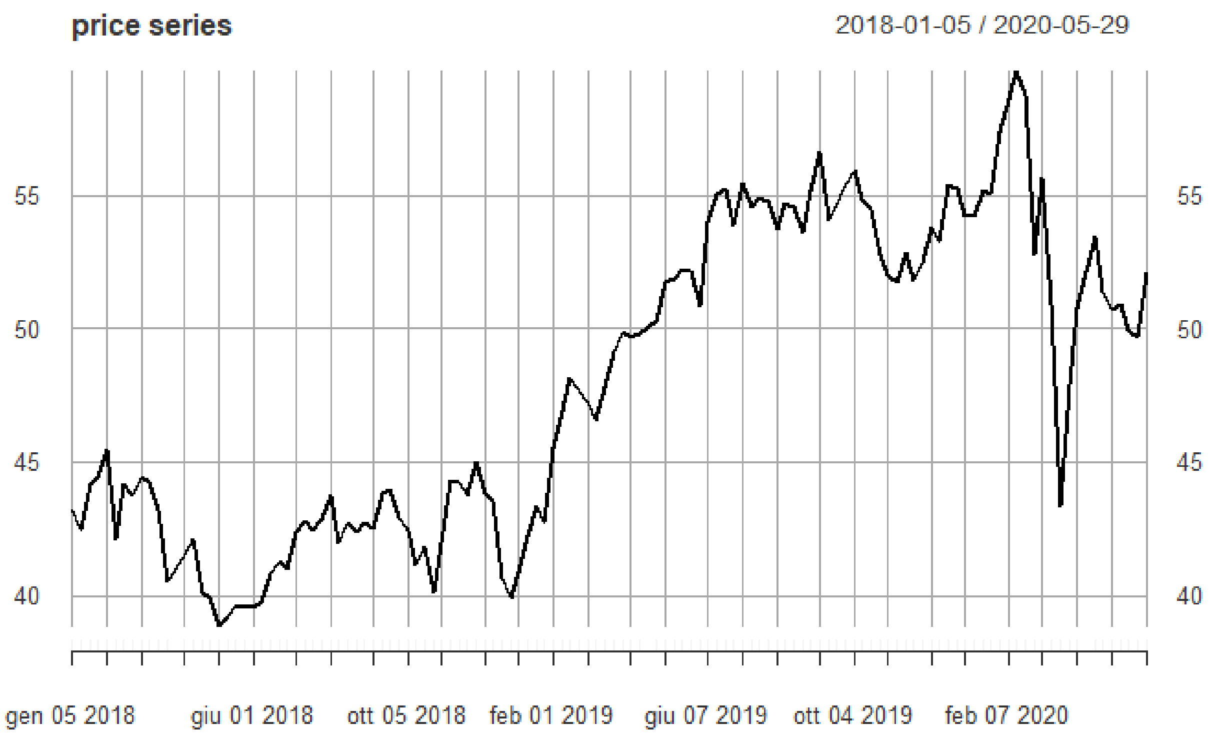


Figure 20, R, MDLZ stock price series.

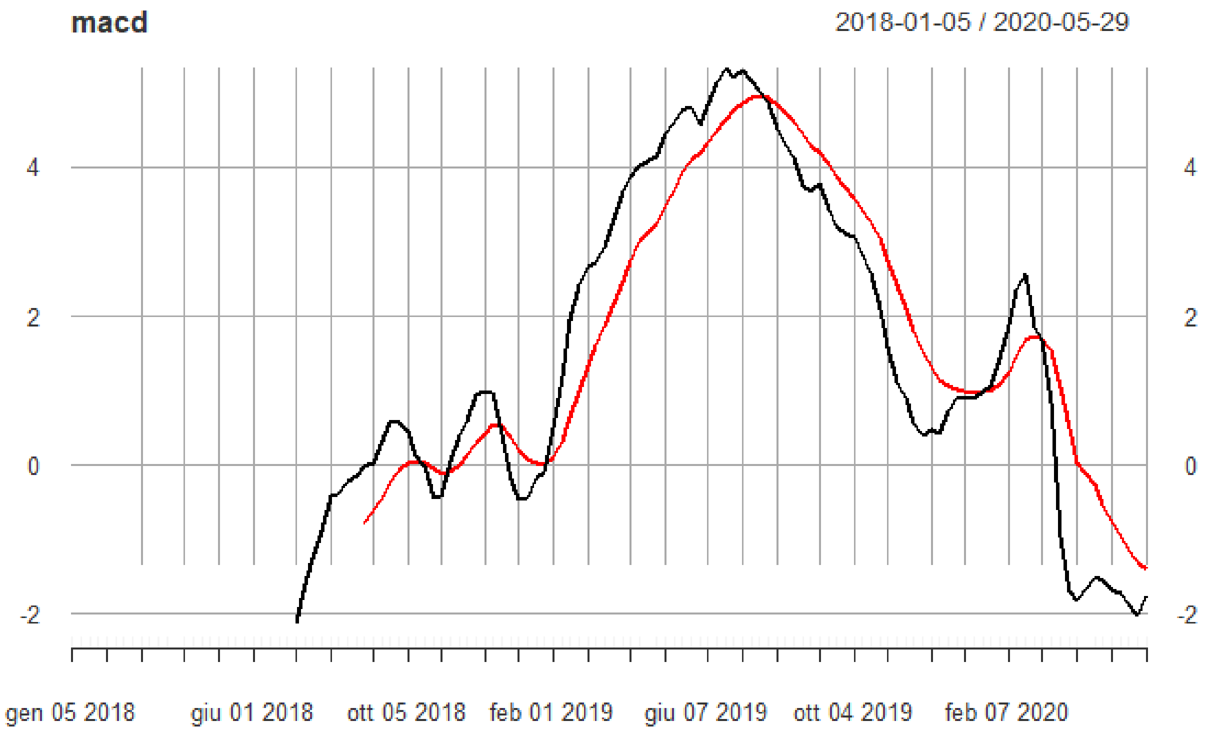


Figure 21, R, MACD of the price series of the MDLZ stock.

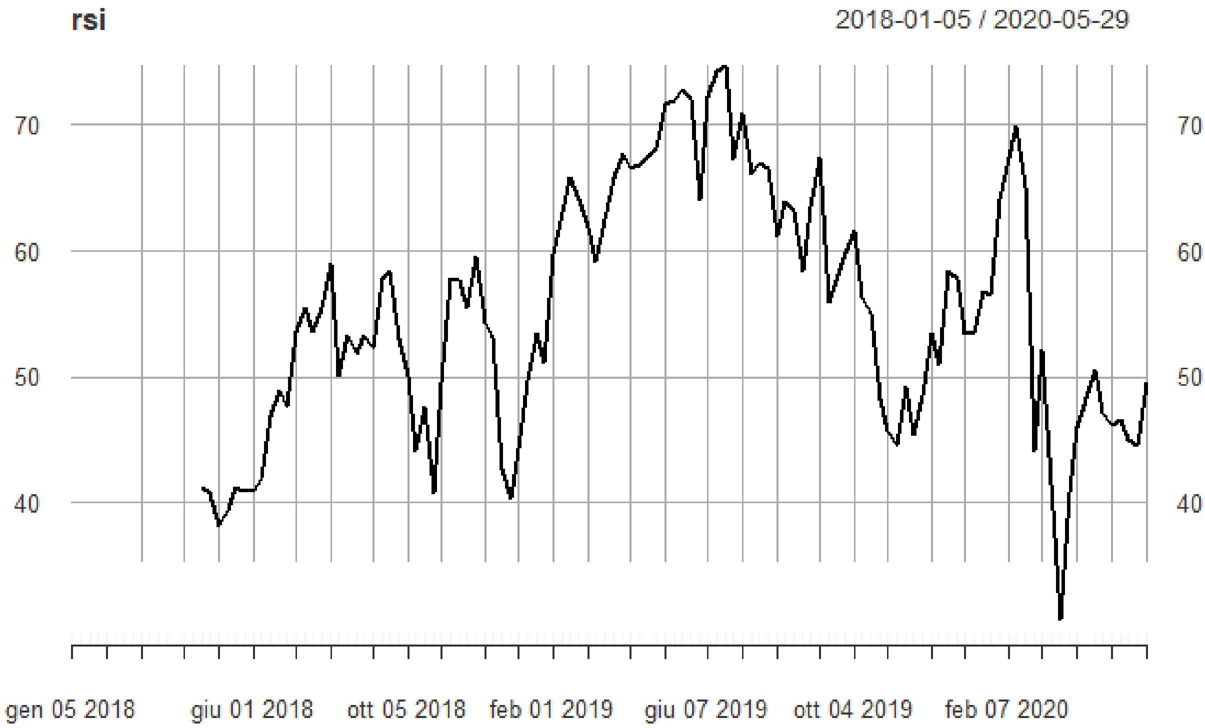


Figure 22, R, RSI of the price series of the MDLZ stock.

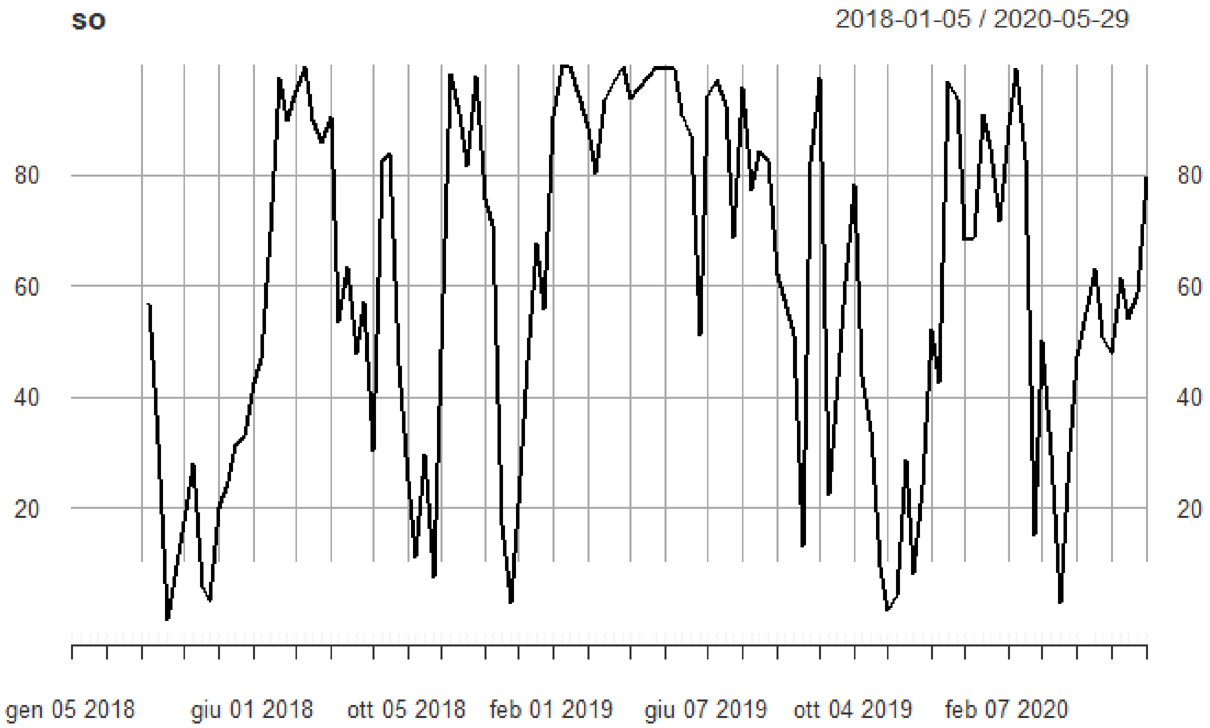


Figure 23, R, SO of the price series of the MDLZ stock.

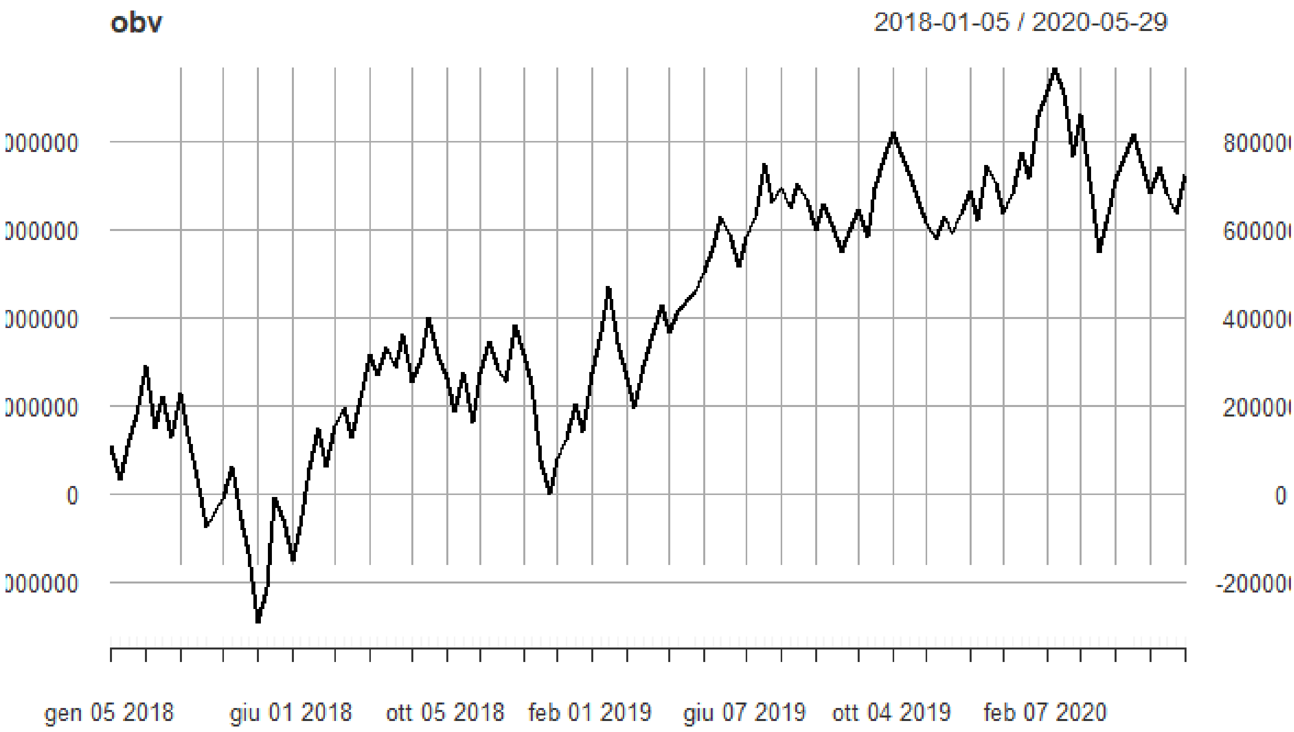


Figure 24, R, OBV of the price series of the MDLZ stock.

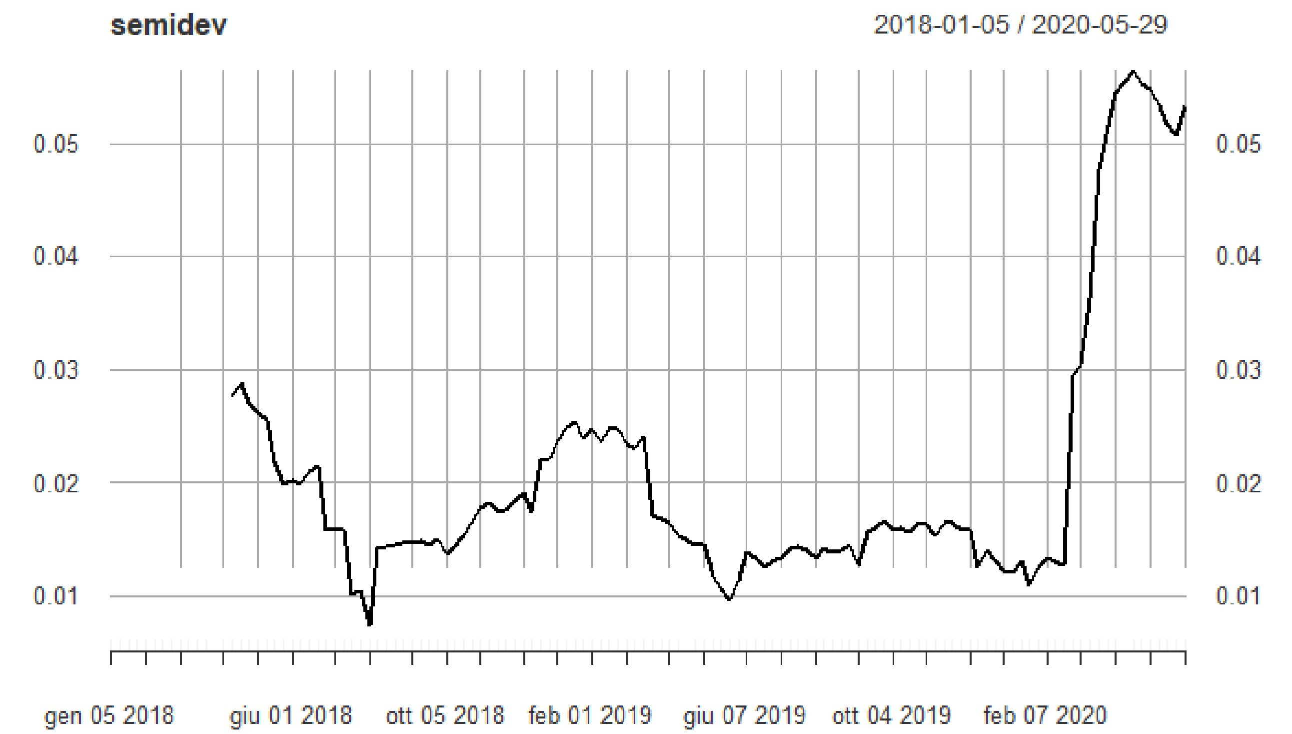


Figure 25, R, Semidev indicator of the price series of the MDLZ stock.

In this case, the spike in the volatility is caused by strong swings in the stock price, which lead to important drawdowns during both uptrends and downtrends. The other indicators replicate with quite fidelity the price series, although the MACD highlighted a reversal which did not happen in the price action of the stock.

The performance of our systems is the following, as provided by R:

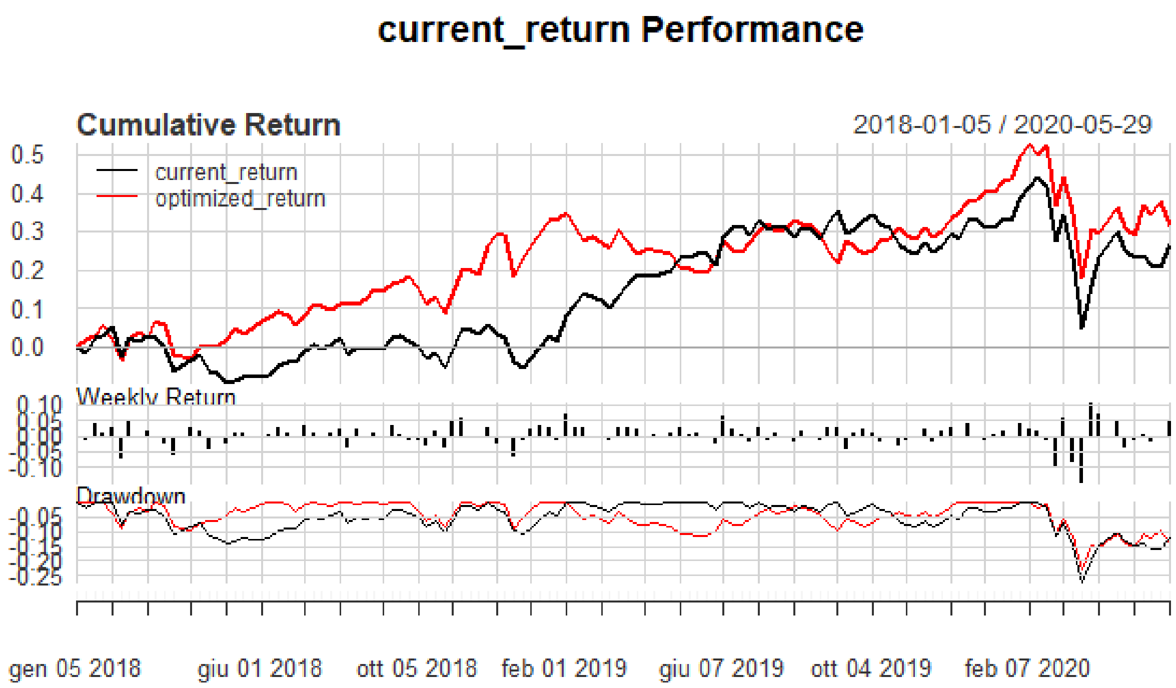


Figure 26, R, cumulative return comparison for the MDLZ stock.

In this case, we assess that the fuzzy system performed better than the pure technical one by a 5% (stopping on May, 31st) in terms of cumulative returns for the period considered, which steadily increased through time, besides a period of uncertainty in the second half of 2019 where the pure technical system performed equally or slightly better.

In terms of drawdown, we are satisfied with the behaviour of our fuzzy system, although its drawdowns have been stronger than those of the technical system for nearly the whole 2019, so about a 30/35% of the time, giving back a 10% of the overall cumulative returns accumulated until then. Those returns have been eventually gap-filled and increased until a new drawdown occurred.

Overall, as we obtained a final higher return on our capital and more stable drawdowns, we can state that the fuzzy system worked greatly for the MDLZ case as well.

The following is a statistics table with numerical values in details, which confirms our system results' assessment for MDLZ:

	current_return <dbl>	optimized_return <dbl>
Observations	123.0000	123.0000
NAs	0.0000	0.0000
Minimum	-0.1493	-0.1252
Quartile 1	-0.0137	-0.0124
Median	0.0022	0.0022
Arithmetic Mean	0.0025	0.0027
Geometric Mean	0.0019	0.0022
Quartile 3	0.0231	0.0216
Maximum	0.1055	0.1055
SE Mean	0.0030	0.0029

Figure 27, R, systems' return distribution comparison for the MDLZ stock.

Those statistics confirm what is said in the chart analysis paragraph for this stock. In this case the maximum drawdown for the fuzzy system is smaller than the pure technical but they exhibit the same maximum win and median return as well. The gap between the systems' geometric mean returns is very small.

Further annualized statistics are provided:

	current_return <dbl>	optimized_return <dbl>
Annualized Return	0.1052	0.1221
Annualized Std Dev	0.2410	0.2296
Annualized Sharpe (Rf=0%)	0.4366	0.5318

Figure 28, R, systems' annualized return, Sharpe ratio and st.dev. comparison for the MDLZ stock.

In terms of annualized return and risk, the fuzzy system performed slightly (but still satisfactory) better. The difference of goodness in terms of Sharpe Ratio is impressive.

4.4.3 Case 3: “MU”

The third case involves an analysis around the stock **Micron Technology**, ticker “MU”. Micron Technology is a producer of computer memory and computer data storage.

The following tables highlight the stock price series and its indicators:

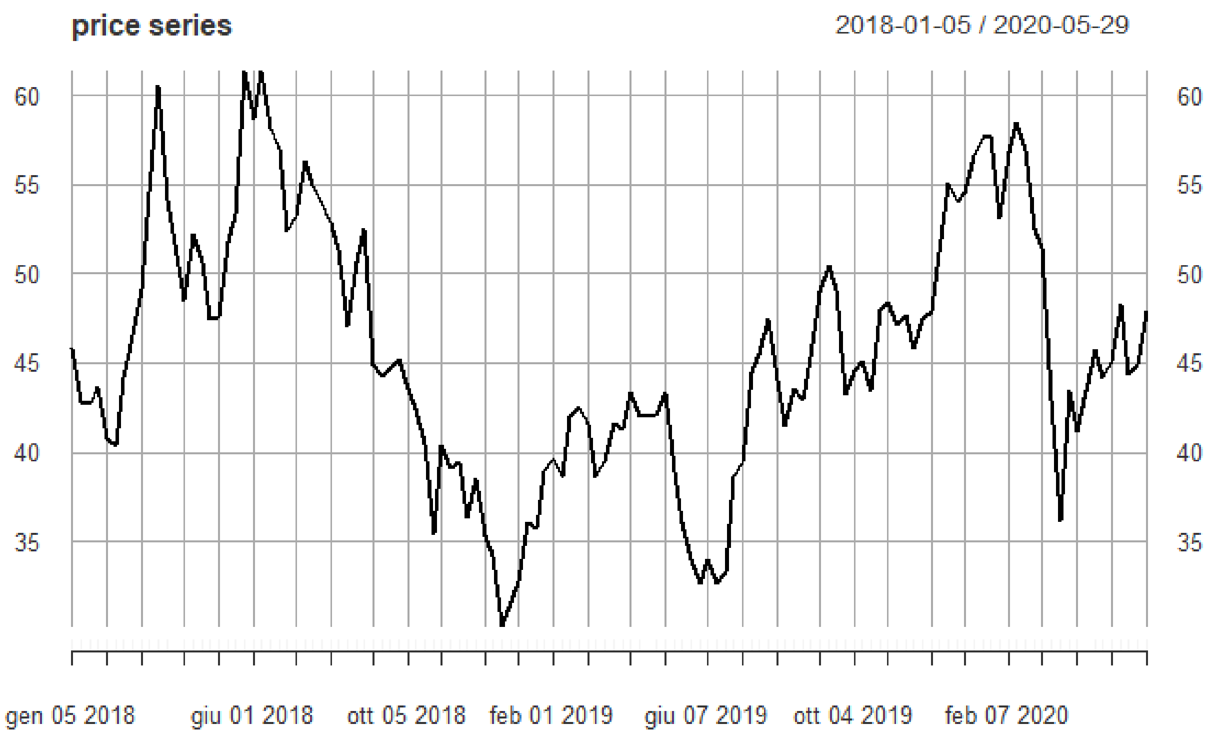


Figure 29, R, MU stock price series.

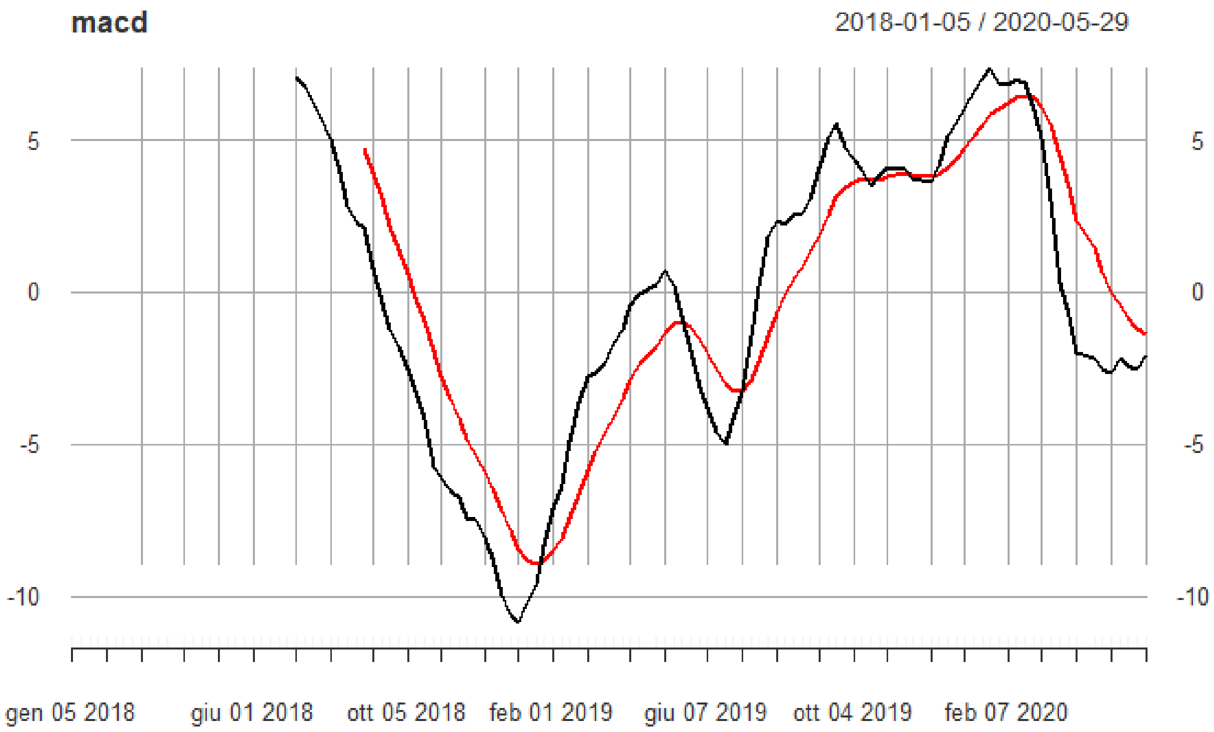


Figure 30, R, MACD of the price series of the MU stock.

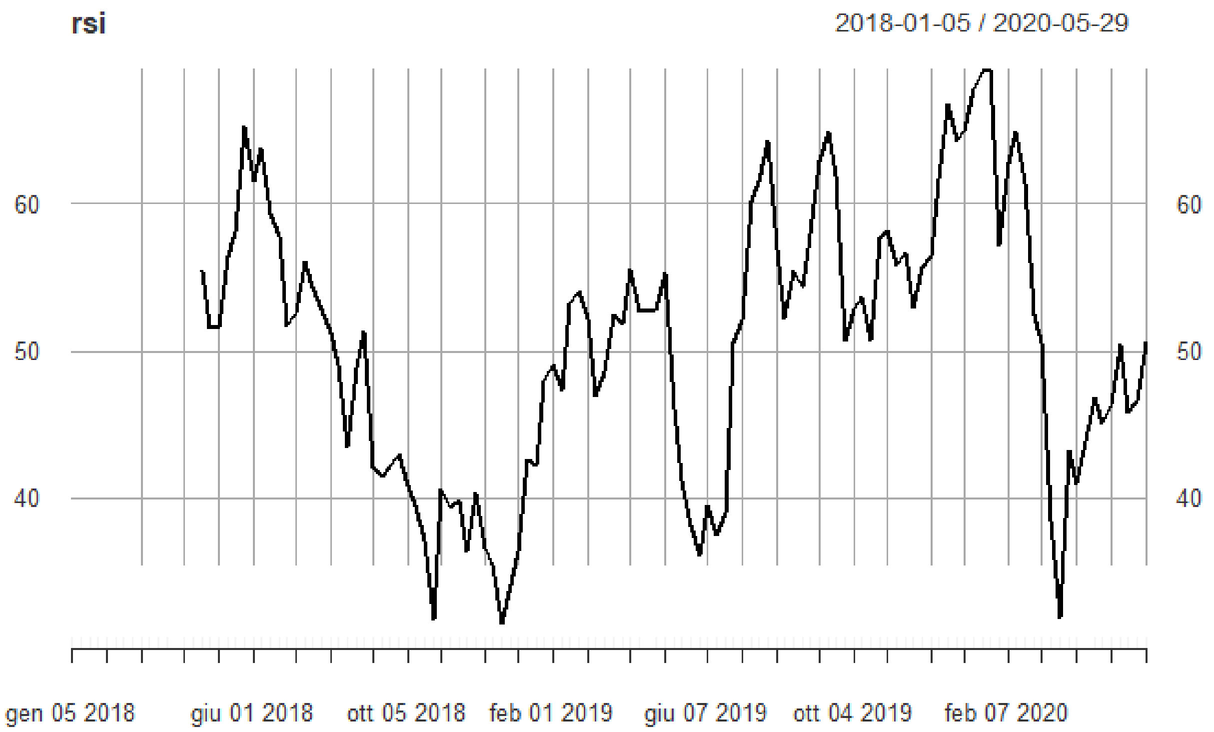


Figure 31, R, RSI of the price series of the MU stock.

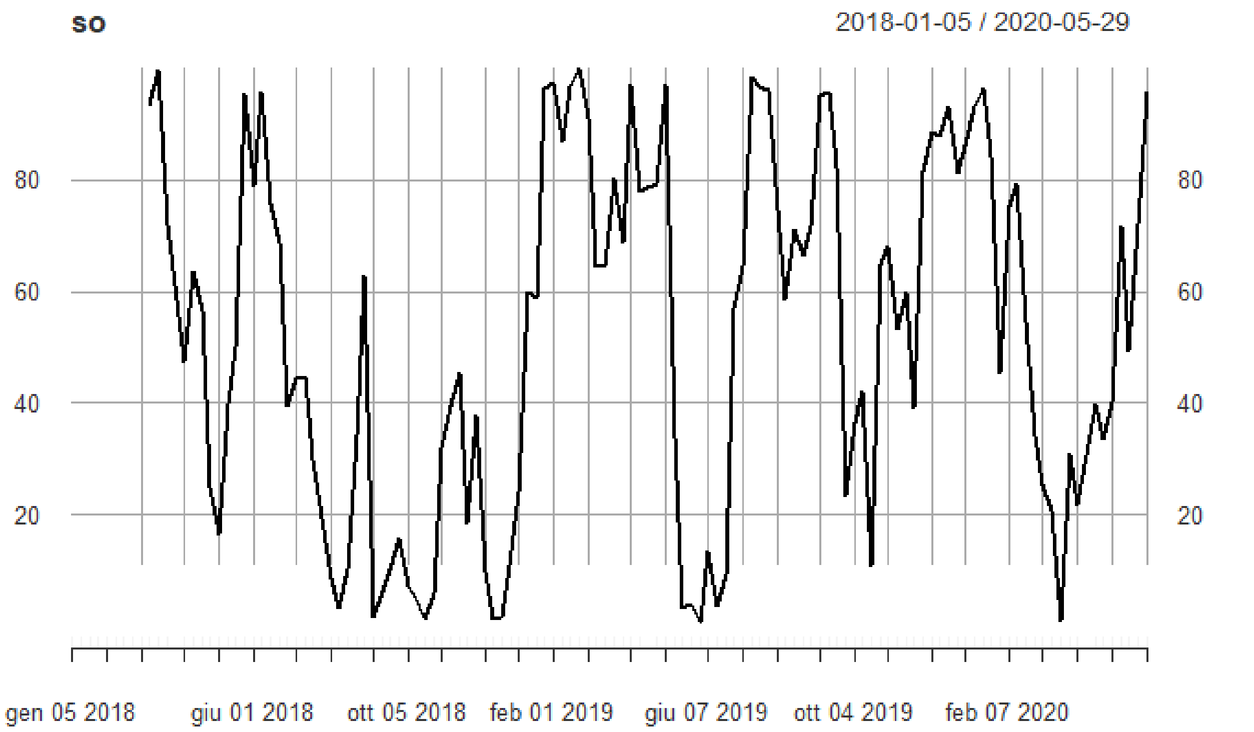


Figure 32, R, SO of the price series of the MU stock.

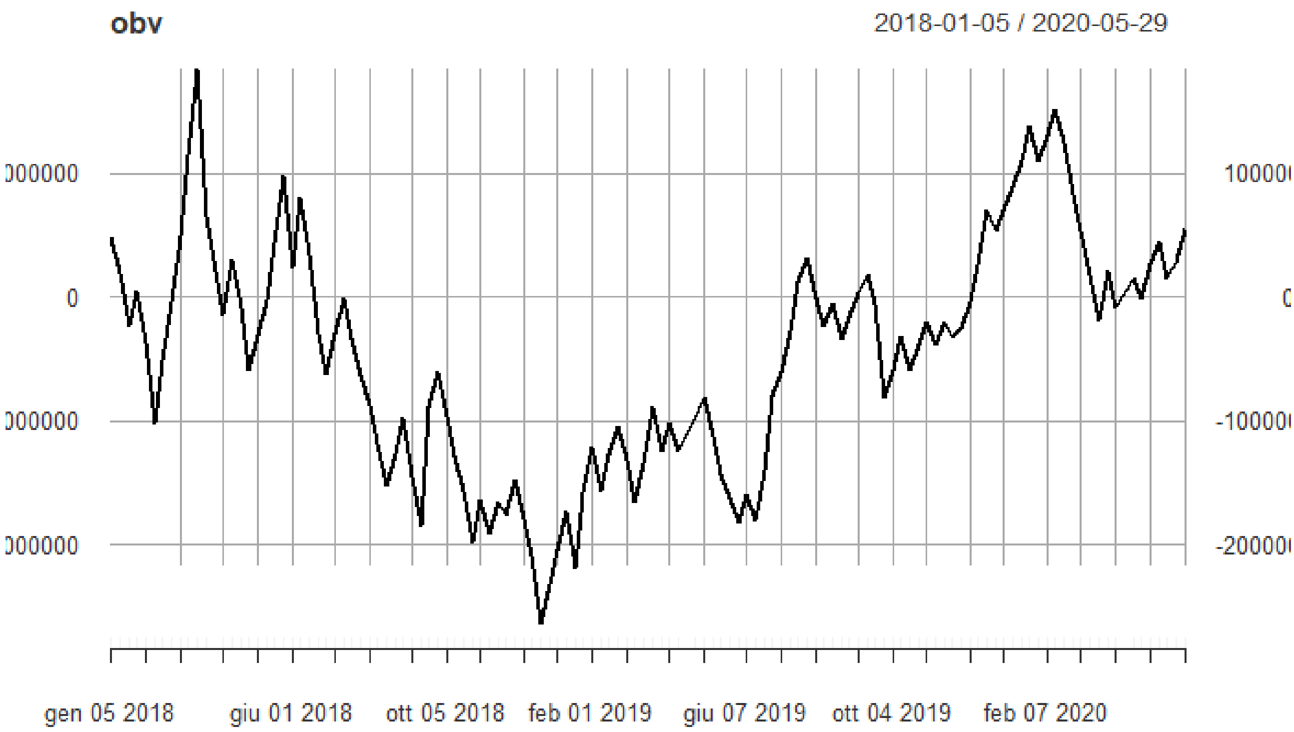


Figure 33, R, OBV of the price series of the MU stock.

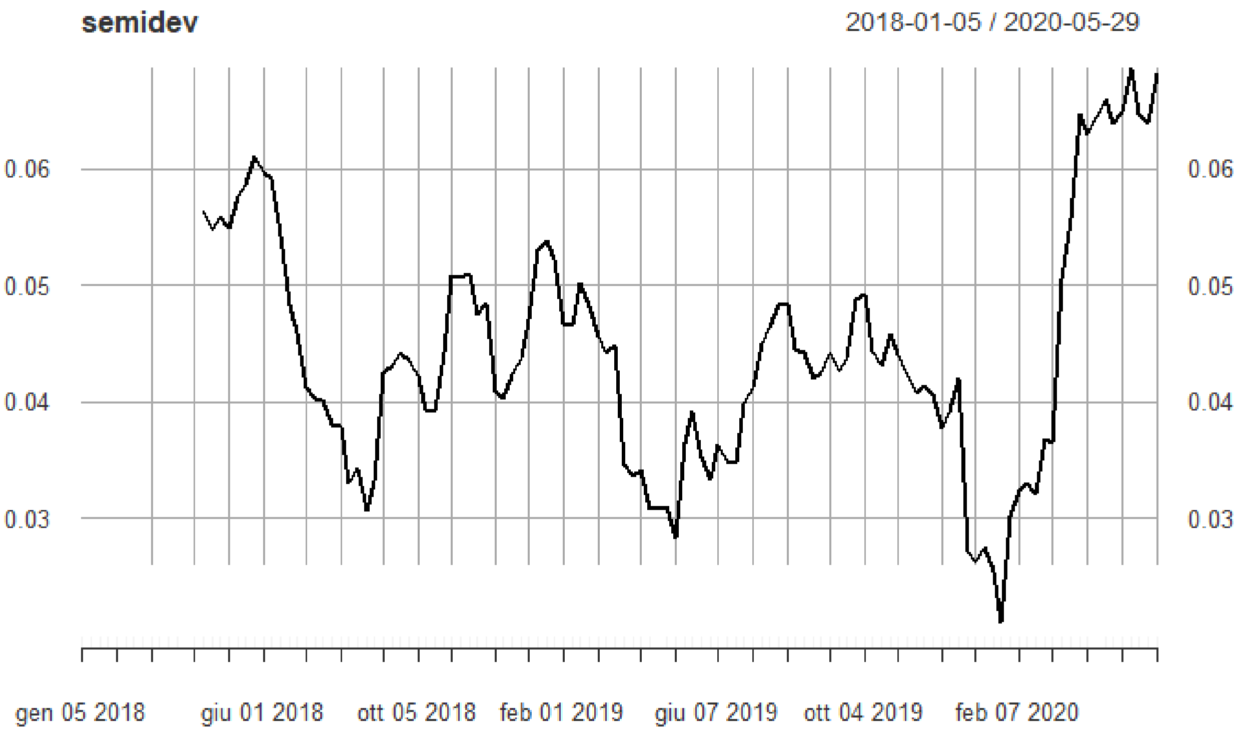


Figure 34, R, Semidev indicator of the price series of the MU stock.

In the MU case, all indicators replicate with fidelity the stock price path, with the Semidev spiking when the stock price tanks, and tanking when the stock price soars.

The performance of both our systems is the following, as provided by R:

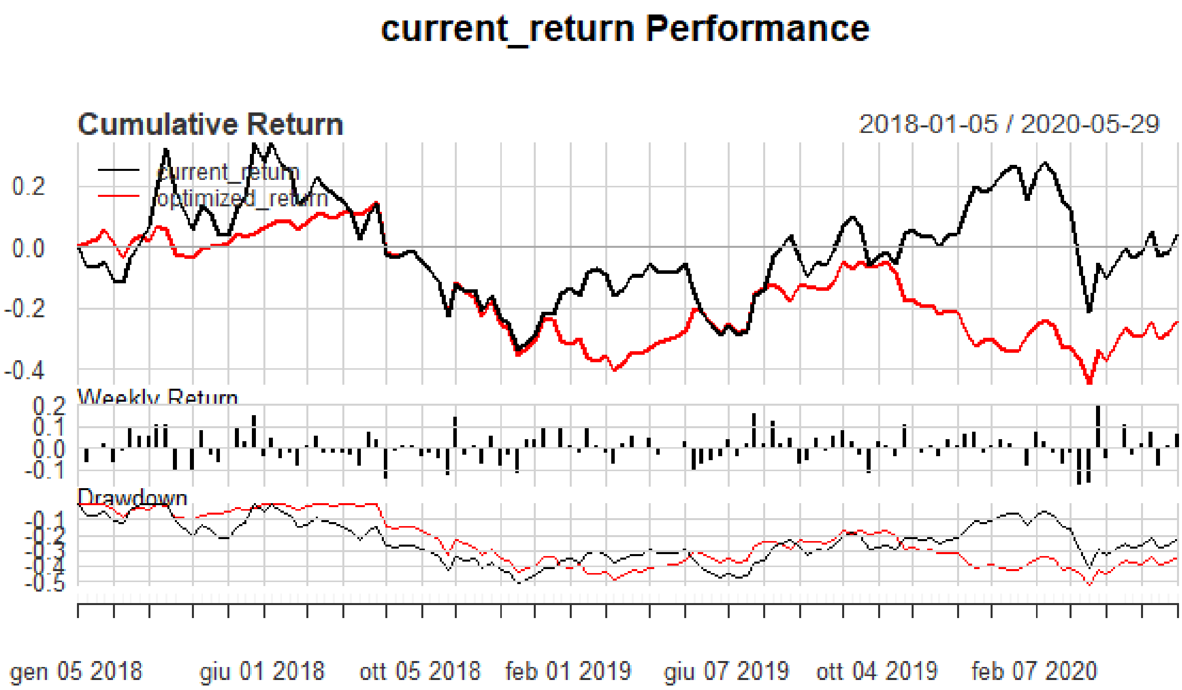


Figure 35, R, cumulative return comparison for the MU stock.

As we can see, in terms of cumulative returns throughout the analysis period, in this case the fuzzy system performed horribly, as in no case it offered a cumulative return so far greater than the pure technical system.

Although it stabilized the drawdowns and consequently the average losses, this case finds us unsatisfied with the overall fuzzy system performance.

These results may be due to some peculiar characteristics of the stock, and perhaps to some wrong inputs fed to the two systems, as it appears that plenty of false trading signals have been generated throughout the period. If someone is interested in trading this stock, he should review thoroughly all inputs and back-test the system almost from scratch.

The statistics table reinforces our assessment:

	current_return <dbl>	optimized_return <dbl>
Observations	123.0000	123.0000
NAs	0.0000	0.0000
Minimum	-0.1648	-0.1458
Quartile 1	-0.0375	-0.0306
Median	0.0073	0.0000
Arithmetic Mean	0.0027	-0.0006
Geometric Mean	0.0004	-0.0022
Quartile 3	0.0442	0.0259
Maximum	0.2041	0.2041
SE Mean	0.0061	0.0050

Figure 36, R, systems' return distribution comparison for the MU stock.

As the maximum win is the same and the drawdown for the fuzzy system is the lower between the two, the average return in geometrical terms does not reward the fuzzy system, whose return is negative, while the one of the pure technical setup finishes positive, although not significantly different from zero.

Further annualized statistics are provided:

	current_return <dbl>	optimized_return <dbl>
Annualized Return	0.0192	-0.1070
Annualized Std Dev	0.4896	0.4034
Annualized Sharpe (Rf=0%)	0.0393	-0.2654

Figure 37, R, systems' annualized return, Sharpe ratio and st.dev. comparison for the MU stock.

In terms of annualized return, the difference of performance in favour of the pure technical setup is massive, while the fuzzy setup still works outstandingly in terms of risk and loss control. Due to a massively higher annualized return, the Sharpe ratio comparison ends up in favour of the pure technical system.

4.4.4 Case 4: "NVDA"

The fourth case revolves around **Nvidia** (ticker: "NVDA"), which is a famous company active in the Artificial Intelligence field.

The following are the indicators' graphical tables:

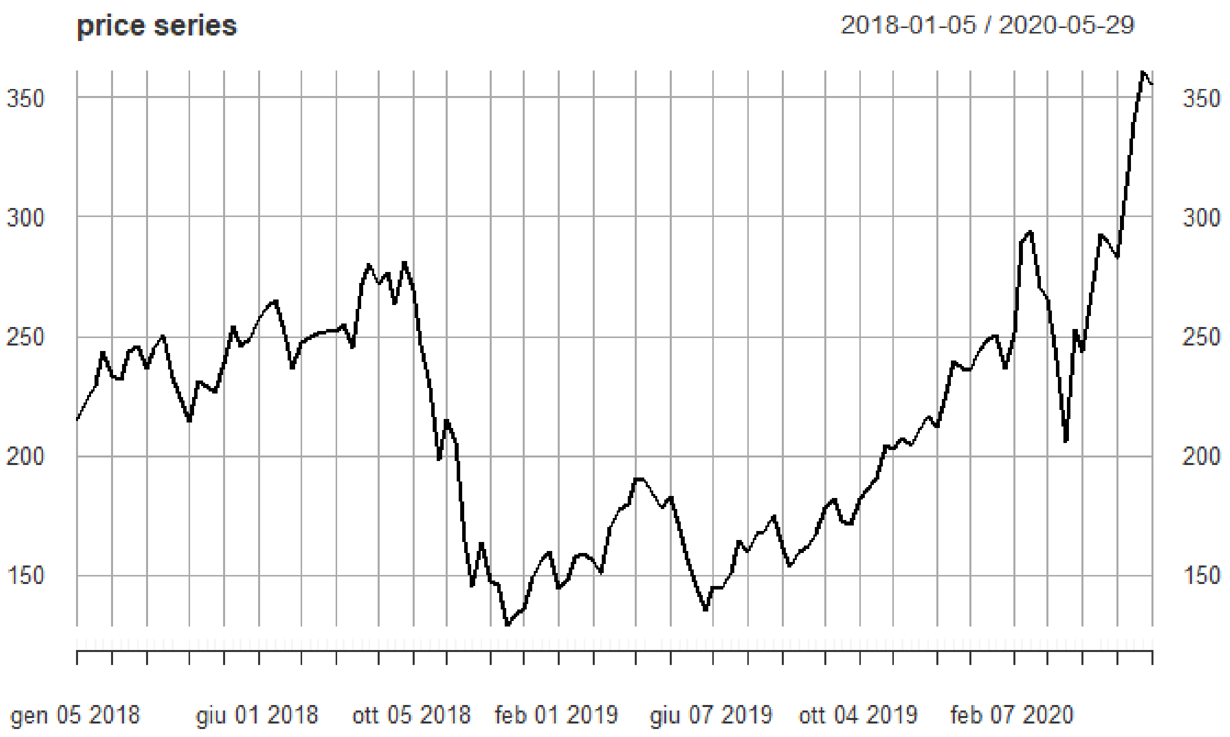


Figure 38, R, NVDA stock price series.

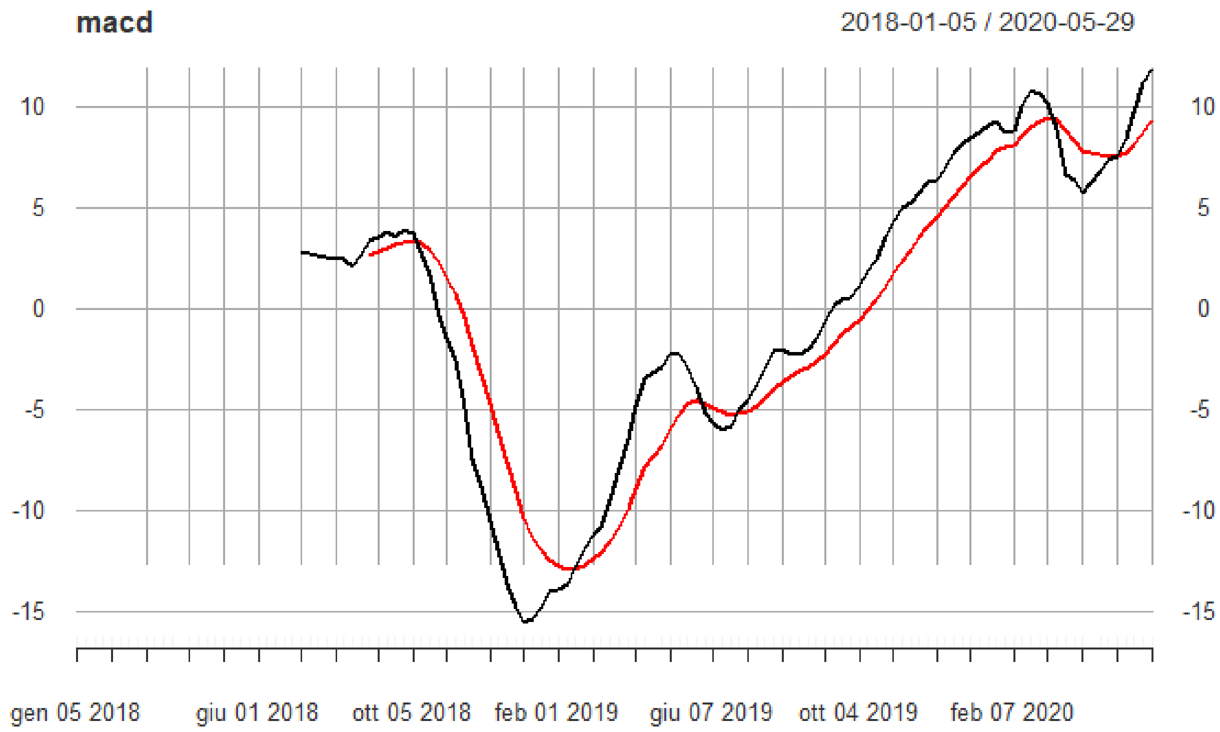


Figure 39, R, MACD of the price series of the NVDA stock.

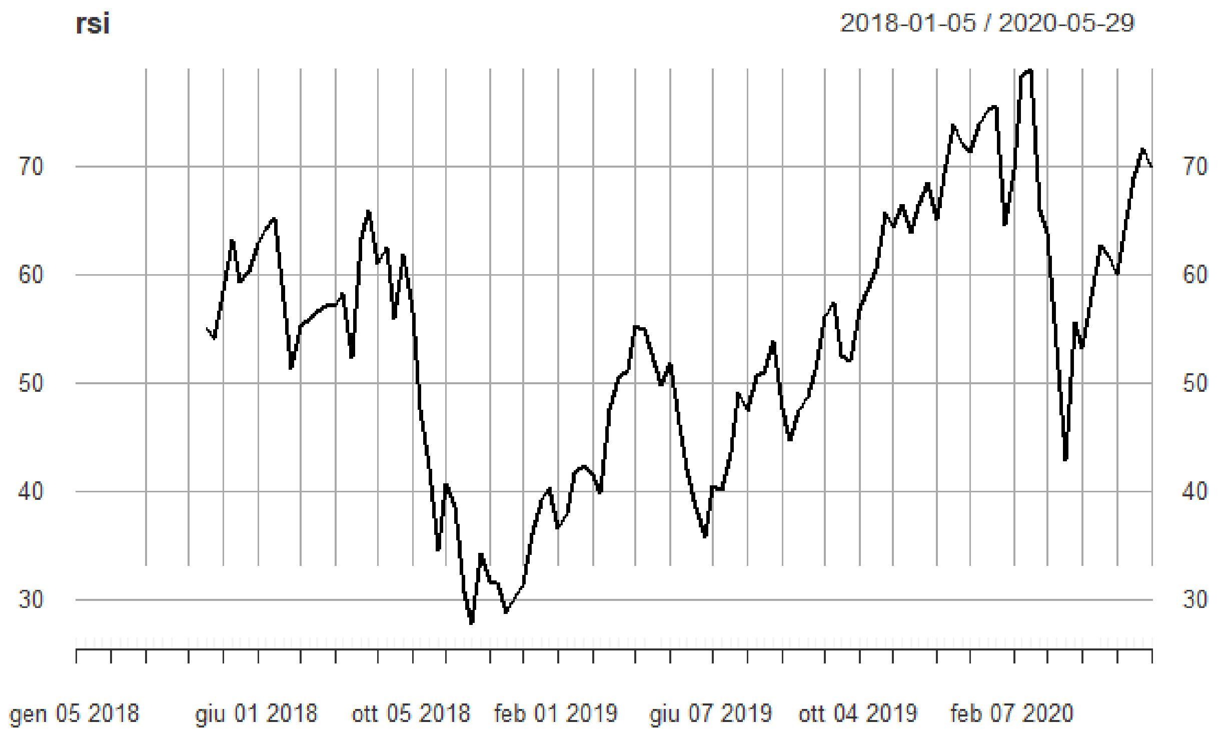


Figure 40, R, RSI of the price series of the NVDA stock.

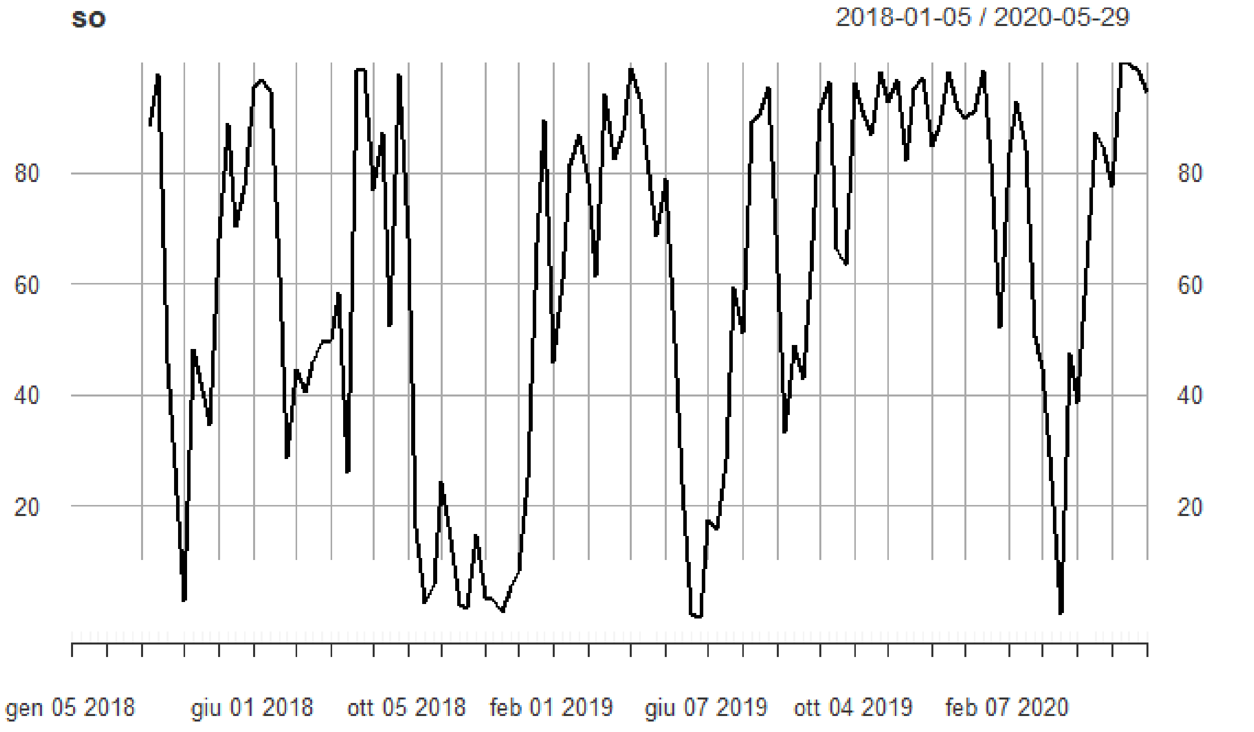


Figure 41, R, SO of the price series of the NVDA stock.

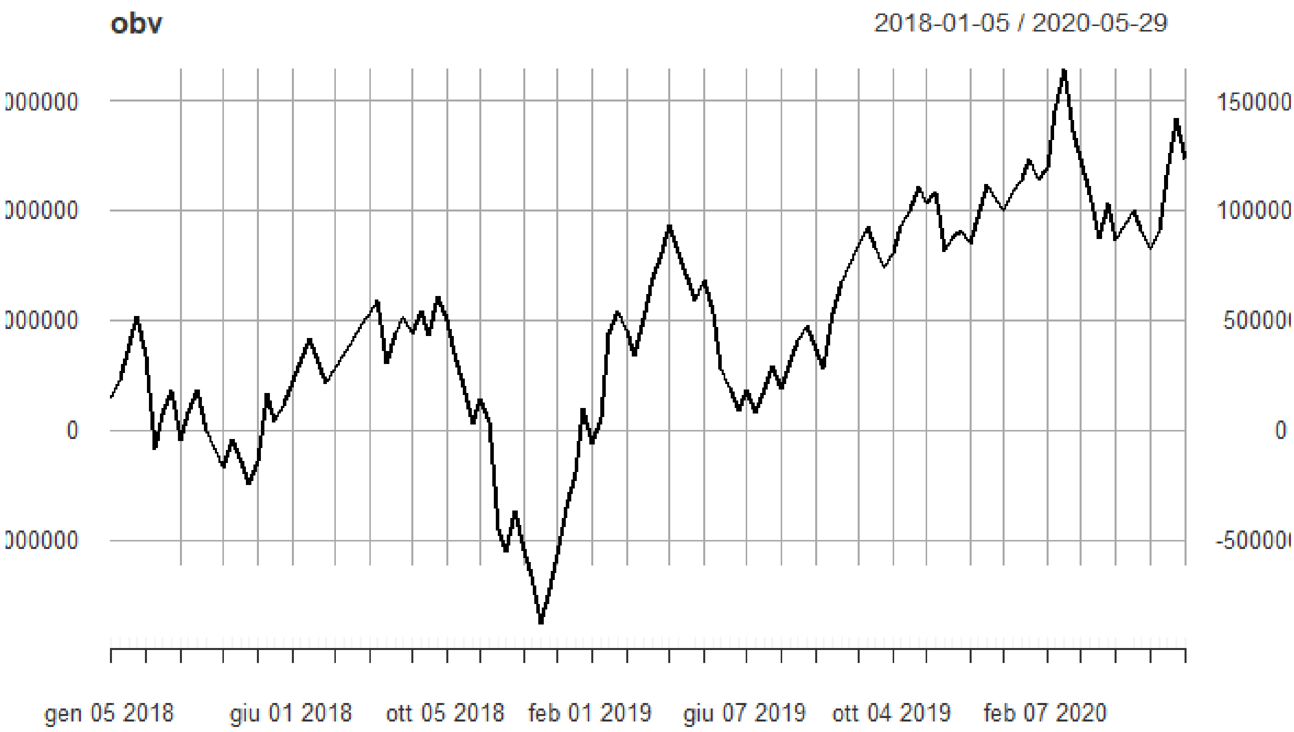


Figure 42, R, OBV of the price series of the NVDA stock.

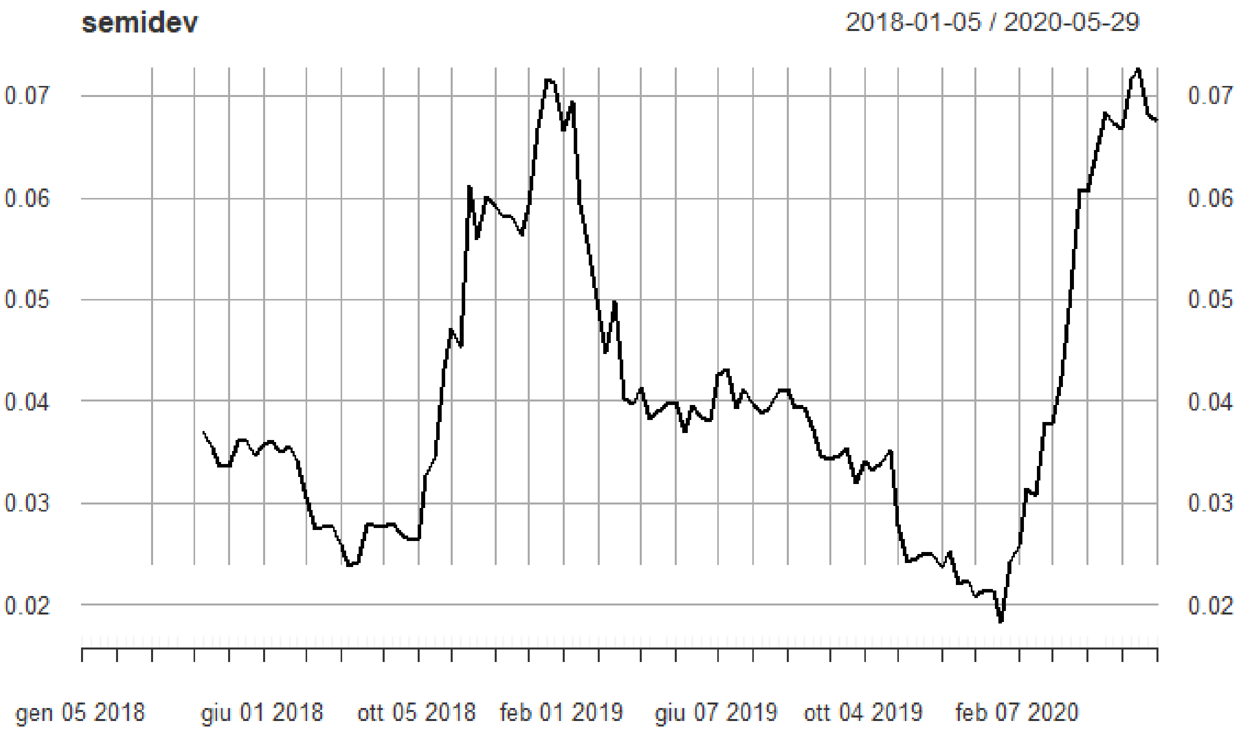


Figure 43, R, Semidev indicator of the price series of the NVDA stock.

In this case as well, we notice a spike in the downside volatility during a price uptrend, mainly due to the strong drawdowns during this trend, which caused the volatility to increase, instead of stabilizing. The MACD has been very effective in signalling a bottom and its consequent price reversal.

The performance of our system is the following, as provided by R:

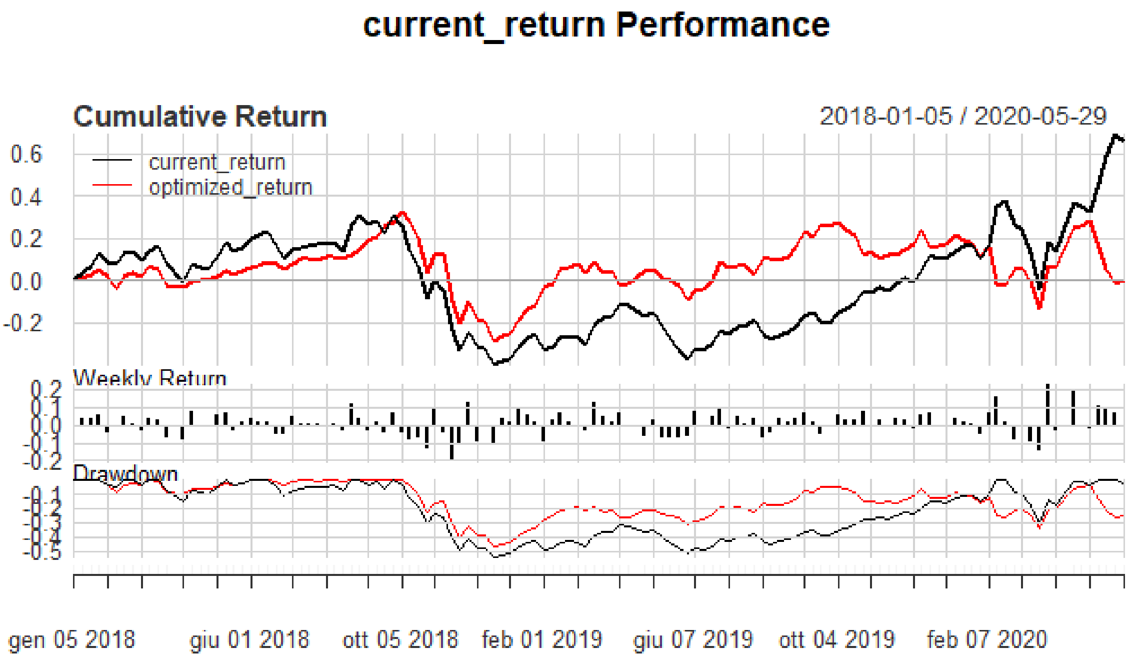


Figure 44, R, cumulative return comparison for the NVDA stock.

In this last case, the performance is quite controversial: in terms of loss control and drawdowns, the fuzzy system performs much better than the pure technical one for more than half of the period, stabilizing and reducing the average loss over the period.

But when it comes to the overall cumulative returns, those of the fuzzy system are not as stable as expected, and for half of the analysis period they are even lower than those provided by the technical setup. Those fuzzy system's returns are higher than those from the technical system only during the 2019 tech stocks rally, suggesting a stronger influence more by this sector rally in the returns rather than a significant improvement provided by the fuzzy setup.

Eventually, the gap between the cumulative returns of the fuzzy system and those of the pure technical surges: the fuzzy system closes even, while the pure technical closes at an impressive +65%. That represents an instant but still, the gap is huge.

In this case too, the inputs to the system should be reviewed and changed, perhaps fully from scratch.

The statistics table does not show a significant difference between the two systems in terms of average performance.

	current_return <dbl>	optimized_return <dbl>
Observations	123.0000	123.0000
NAs	0.0000	0.0000
Minimum	-0.2005	-0.2005
Quartile 1	-0.0360	-0.0210
Median	0.0114	0.0094
Arithmetic Mean	0.0063	0.0017
Geometric Mean	0.0041	0.0000
Quartile 3	0.0447	0.0254
Maximum	0.2283	0.2283
SE Mean	0.0059	0.0052

Figure 45, R, systems' return distribution comparison for the NVDA stock.

The annualized return table exhibits statistics in line with our first results' assessment:

	current_return <dbl>	optimized_return <dbl>
Annualized Return	0.2390	0.0009
Annualized Std Dev	0.4721	0.4157
Annualized Sharpe (Rf=0%)	0.5062	0.0021

Figure 46, R, systems' annualized return, Sharpe ratio and st.dev. comparison for the NVDA stock.

As the annualized performance in terms of returns turns in favour of the pure technical setup, the fuzzy system reveals itself once again as a powerful loss and risk control tool.

As the gap between the two systems' annualized return is heavy, the Sharpe ratio revolves massively in favour of the pure technical setup.

4.4.5 Case 5: “MAR”

Lastly, we are randomly assigned the stock **Marriott International Inc** (ticker: “MAR”), which is a very famous luxury hotels chain operating worldwide.

The following tables highlight the stock price series and its indicators:

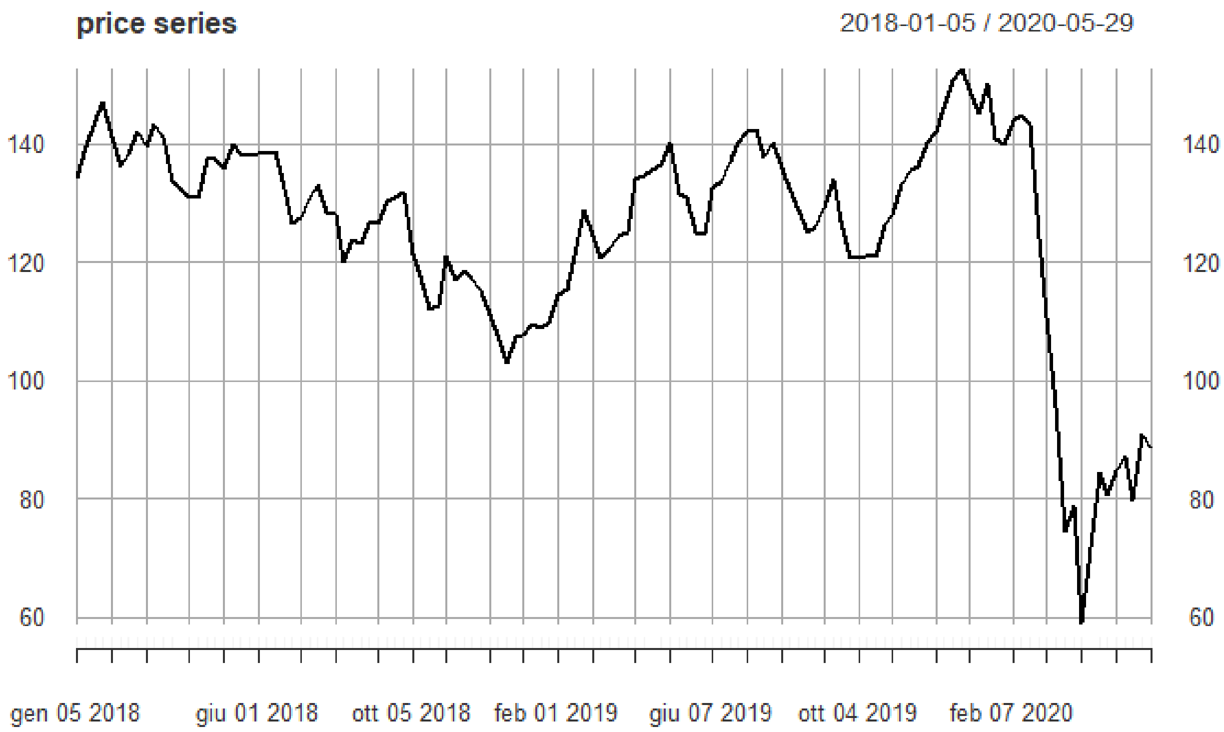


Figure 47, R, MAR stock price series.

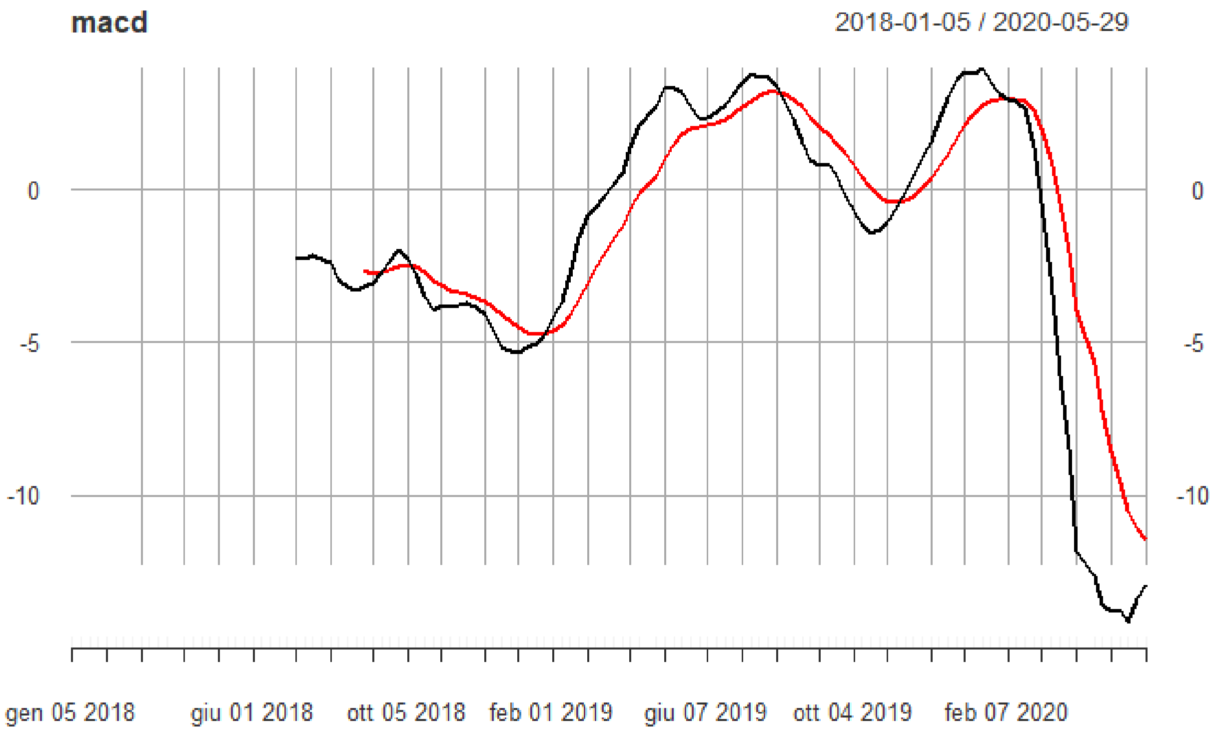


Figure 48, R, MACD of the price series of the MAR stock.

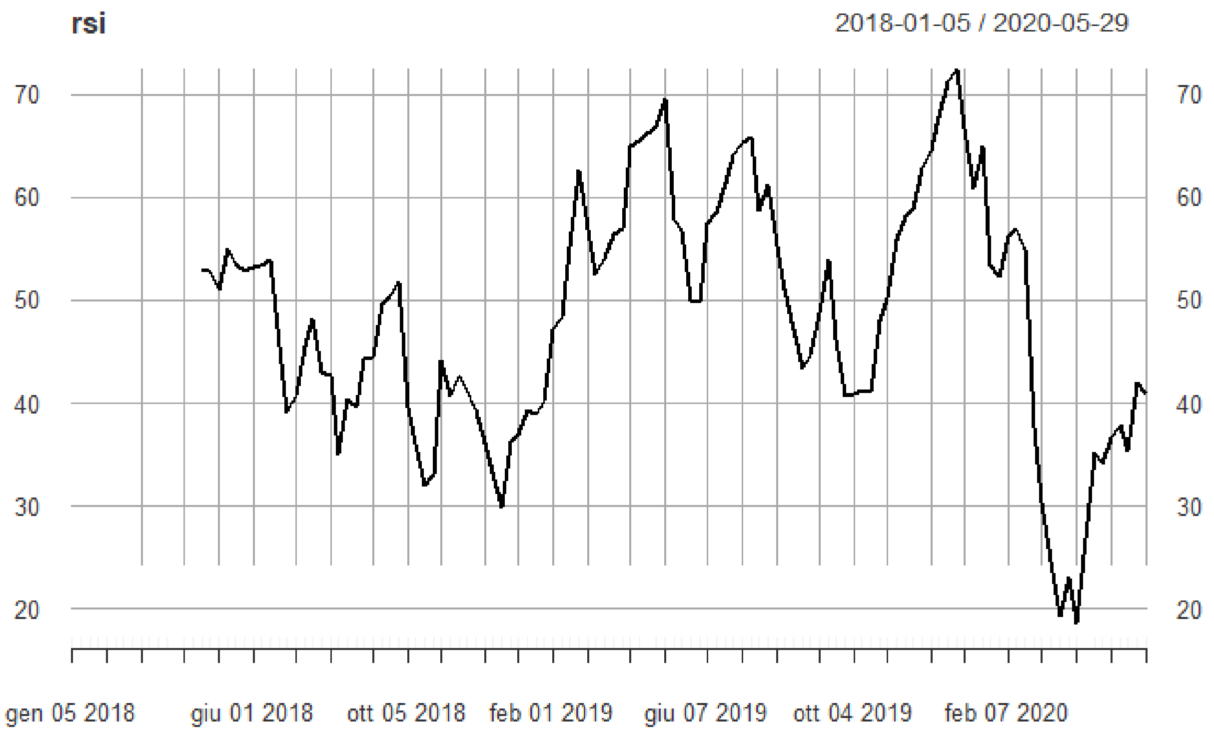


Figure 49, R, RSI of the price series of the MAR stock.

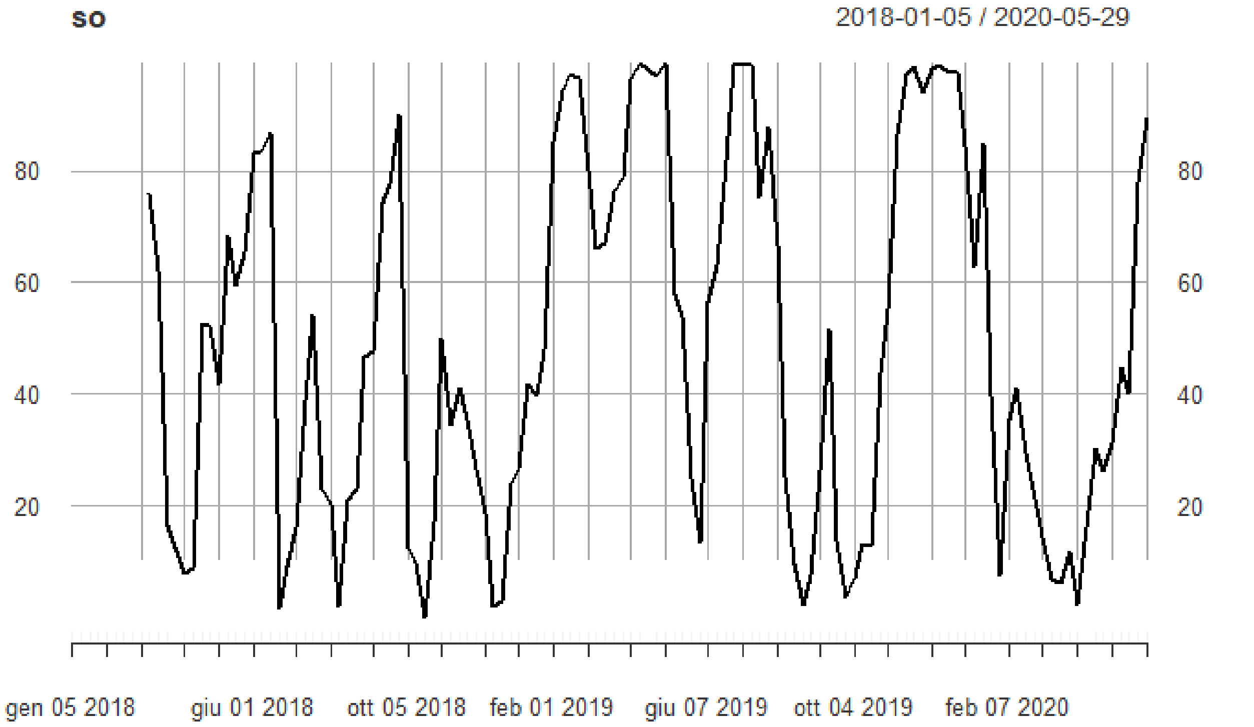


Figure 50, R, SO of the price series of the MAR stock.

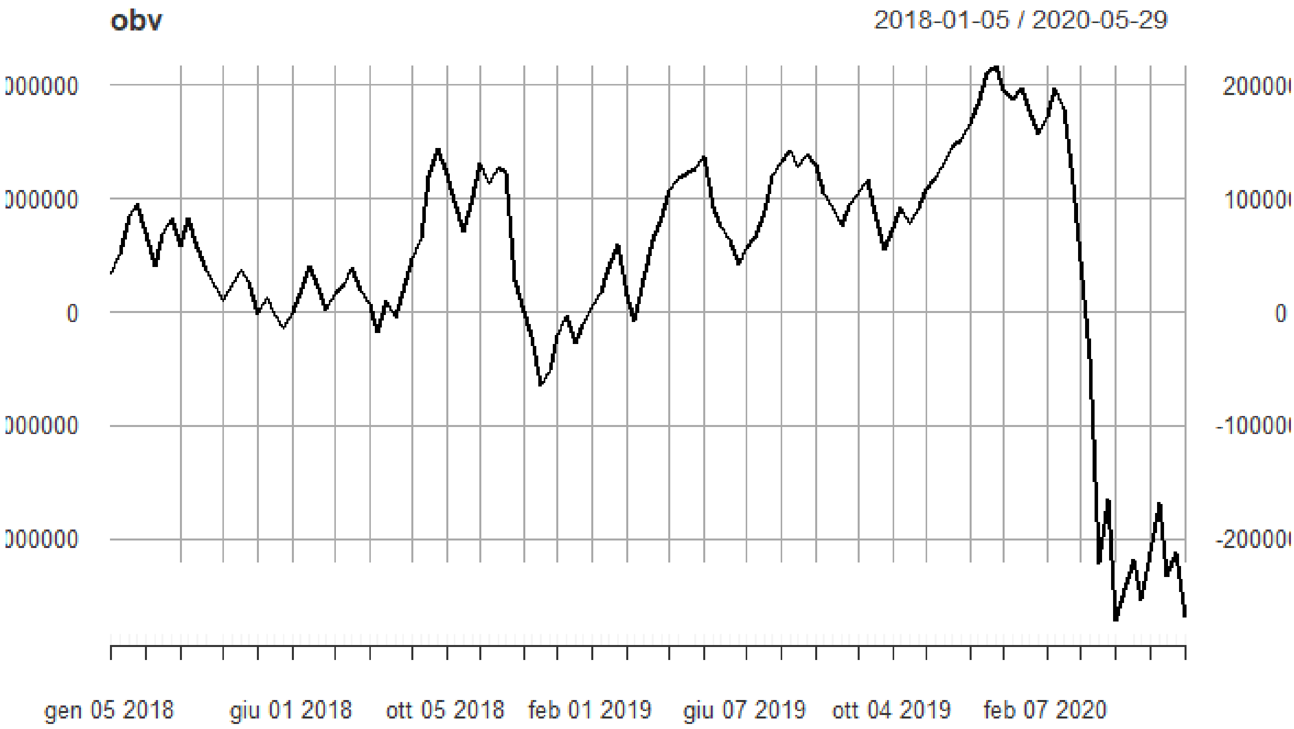


Figure 51, R, OBV of the price series of the MAR stock.

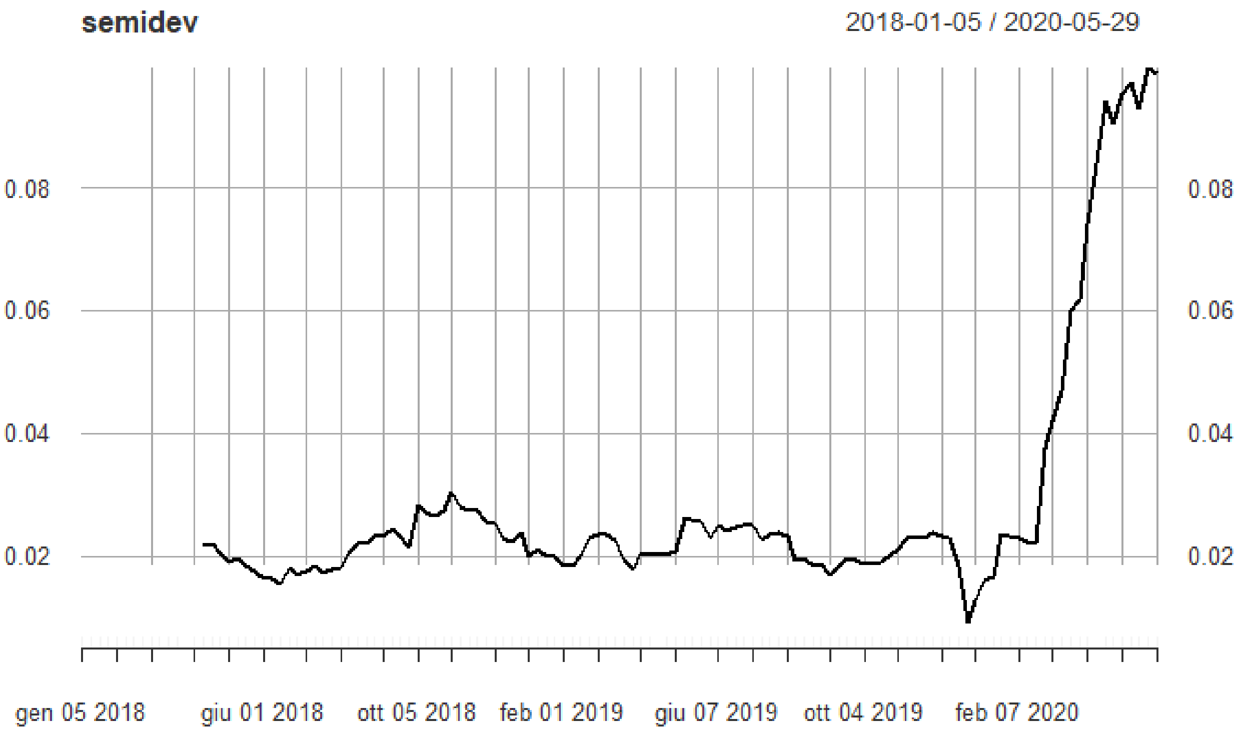


Figure 52, R, Semidev indicator of the price series of the MAR stock.

In this case, the Semidev spiked incredibly as the stock price tanked abruptly, while the RSI and SO signalled a strong Oversold situation and bounced back up.

The performance of our system is the following, as provided by R:

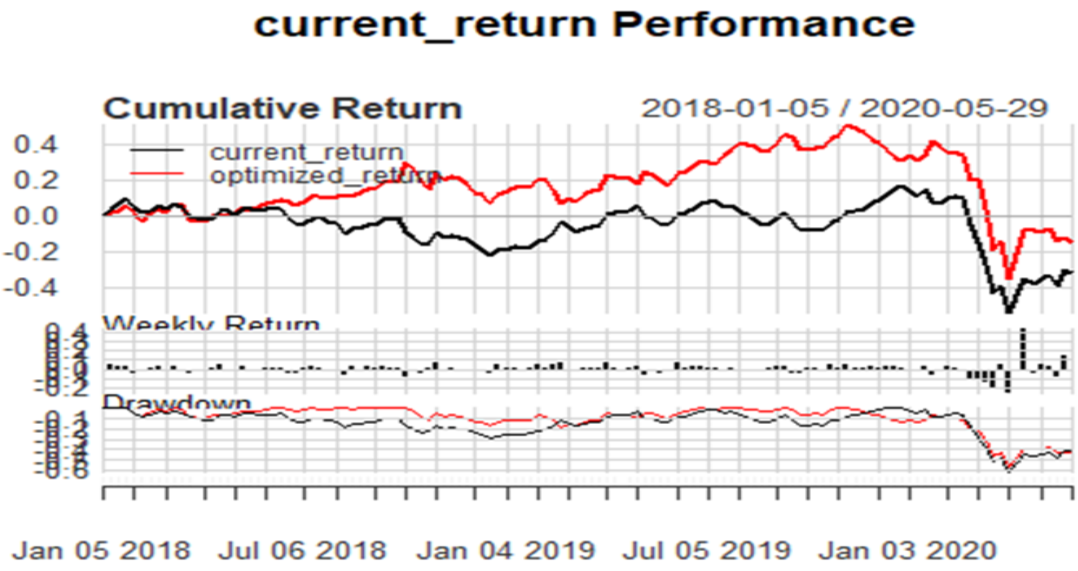


Figure 53, R, cumulative return comparison for the NVDA stock.

As we can see, the fuzzy system performed outstandingly compared to the pure technical in terms of cumulative return, as it offered thoroughly a better return than the pure technical for the whole period.

It also provided smaller drawdowns, besides a couple of months in 2020, where the difference with those provided by the pure technical setup started being less significant, when a black swan event occurred and the overall market tanked.

The statistics table and the annualized return table exhibit results in line with our initial overview: the fuzzy system performed incredibly better on this stock than the pure technical system, according to all necessary statistics:

	current_return <dbl>	optimized_return <dbl>
Observations	123.0000	123.0000
NAs	0.0000	0.0000
Minimum	-0.2507	-0.2507
Quartile 1	-0.0277	-0.0179
Median	0.0036	0.0004
Arithmetic Mean	-0.0013	0.0004
Geometric Mean	-0.0031	-0.0013
Quartile 3	0.0264	0.0232
Maximum	0.4262	0.4262
SE Mean	0.0057	0.0054

Figure 54, R, systems' return distribution comparison for the MAR stock.

	current_return <dbl>	optimized_return <dbl>
Annualized Return	-0.1511	-0.0648
Annualized Std Dev	0.4523	0.4294
Annualized Sharpe (Rf=0%)	-0.3341	-0.1509

Figure 55, R, systems' annualized return, Sharpe ratio and st.dev. comparison for the MAR stock.

4.5 Results' Assessment

We assume that a random sample of 5 stocks of different market cap and sector is sufficient to assess the first results for our fuzzy system; clearly, a larger sample would be advantageous for our analysis, but we run the risk of being redundant. The indicators to be used in the system may vary considerably: for example, a high number of real world technical setups use moving average as foundation for the system, while here they have not being practically considered, but we decided to feed more complex indicators to the system.

In retrieving the necessary graphical and numerical results, the function *"table.PerformanceSummary"* had to be substituted by single statistics computing functions as it was not possible, under this dissertation's terms, to retrieve a statistics table embracing all necessary statistics like the Profit Factor and Average Win/Average Loss of both systems; furthermore, the computation of those statistics outside a built-in function would have required an extremely long process, as the R developers know.

Considering the previous results, the performance of our fuzzy system in terms of overall cumulative returns is quite variable: over a time span of 2 and a half years, for some stocks we were able to generate higher returns than those of the pure technical system by even 10 percentage points, while for others we obtained lower final returns, in the last case lower even by 60 percentage points!

Clearly, overall different results would be obtained if the time span is changed, possibly enlarged, but a stronger computational capacity by the console is required. For each stock analysis, the consolle took 30 minutes to complete the whole cycle, and not all functions for system performance assessment were available to run, perhaps due to the complexity of the code.

Furthermore, final results were heavily influenced by the market downturn in early 2020 due to the Coronavirus outbreak and subsequent crisis, as the overall market tanked, dragging almost all stocks to the downside: in this case, we can not state whether a system would have prevailed on the other, but surely it is not wrong to assess that both of them would have performed better, as the stock itself. We decided not to exclude the early 2020 from our time span as we wanted to verify how both systems behaved in presence of a *black swan* event, in terms of loss and risk control at first stance.

What we can be more satisfied about is the loss control in terms of average and maximum drawdown: in no cases we experienced an average drawdown truly larger than the one generated by the pure technical system. We also experienced more stable drawdowns, protecting our capital more effectively: in a field where emotions play a key role, and mental and behavioural bias can be limited but not eliminated, this is truly fundamental for building a consistently profitable automated trading system.

If we enlarged the random sample to 10, 20, 50 random stocks, our opinion is that the final results would be levelled, meaning that the gap between both systems' performance in terms of overall cumulative returns would still be variable but we would collect evidence in favour of the fuzzy system when it comes to risk management. In other words, as the performance to the upside may be variable and, in given cases, unsatisfying and worse than the pure technical system, the performance to the downside would be assessed through a risk measurement indicator; then controlled, limited and stabilized.

No further analysis on the scrutinized companies' fundamentals (market cap, business sector, dividends announcements or stock split) is provided, as this is not the field of interest of a technical trader.

Conversely, we can discriminate the performance of the fuzzy system based on the single stock's average volatility expressed by the Semideviation indicator. In fact, by checking the indicators' statistics calculated by R at the end of the pure technical chunk of code for each stock, we can assess that the better overall fuzzy system performance has been obtained from stocks which showed to have the lowest average volatility of the sample, or a volatility belonging to the low-end of the sample volatility range.

The reason for this may be due to the ability of the fuzzy system to catch finer details in the stock price action, combining many inputs at the same time in a finer way with respect to the pure technical system, which, on the other hand, proved itself to be more efficient for higher volatility stocks, which probably do not require a finer further elaboration but instead need a simpler and more parsimonious system.

When assessing the performance of the systems, the eventual statistical distribution of the returns and its first four moments have been assessed but not considered in this dissertation.

Chapter 5

Conclusions and final remarks

The aim of this dissertation was simply to compare a pure technical indicators-based trading strategy with one involving a fuzzy transformation of those indicators; hence, our aim was not to analyse the performance of a more complex strategy compared to a more parsimonious one, or to analyse the goodness of a single input or a group of inputs with respect to another inputs. The results, as shown, proved us that the fuzzy logic algorithm can be a truly interesting way to improve the precision of our operations, whether they are trading signals or estimates, as it takes into account multiple "worlds" or sets at the same time, extracting the higher probability for a transformed crisp value to pertain to a certain set, accounting for multiple evolutions all in once.

Clearly, the improvements of our trading strategy obtained through the fuzzy logic, although only in terms of risk assessment and control and not necessarily in terms of overall higher cumulative returns, show a "new world" of possibilities for the trader, as this algorithm could be applied thoroughly to the whole universe of tradable stocks and even to their derivatives. For example, next developments based on fuzzy logic algorithm strategies could lead to edges in the Stock Futures trading, perhaps even in Stock Options trading.

Another branch based on some fuzzy logic algorithm could be the one trading Indexes Futures and Options, as indexes behave slightly differently than single stocks.

Hedge funds, pension funds and Asset managers might want to create fuzzy rules-based portfolio optimization algorithms, starting from simple fuzzy rules-based algorithmic strategies similar to ours; professional and institutional traders (banks, aggressive funds, proprietary trading desks) might want to join the stream and base some of their day- and swing-trading strategies on this simple yet effective fuzzy rules-based trading strategy and even build a more complex one.

A further branch of developments could be the final implementation of a GARCH model-based indicator, overcoming our initial difficulties in matching robust GARCH estimates with our data and strategy. Perhaps some different volatility measures could be added to the trading system: a bold move could be implementing the CBOE VIX index value into our system. The CBOE VIX index is known as the fear index, as it measures the volatility in the overall market given the implied volatility of the stocks being part of the market.

This approach may sound interesting for systems which will trade indexes and its derivatives, but it may sound not so effective when it comes to single stock. In this case, the single stock options' implied volatility may sound more effective when it comes to catch the single stock's periods of high volatility, increasing the odds of finer price moves and higher returns.

From a purely statistical point of view, a simple volatility measure which could be added might be the Mean Absolute Deviation, defined as: simple value – mean, where the mean is a common sample average (a moving average perhaps exponential could be used on the spot instead of a common sample average). As we fix a threshold, we will then be able to recognize shifts between different volatility regimes.

From a technical point of view, an interesting measure of a stock volatility could be the ATR, that is, the Average True Range, which is the average possible range of oscillation of a stock price given a period of analysis (9 days, 14 days, intraday): this is an effective metric widely used by Wall Street professionals, given that the right span of time is used for operational signals.

A further point for the operator trader is linked to the effectiveness of the fuzzy system based on the average volatility regime of the stock: further research should be conducted on whether the fuzzy system is more effective for stocks exhibiting lower average volatility or for stocks showing a high average volatility (the choice of the measure of the volatility is left to the researcher), in order to confirm or disprove our statement in the results' assessment section, as the system in this dissertation is either subject to improvements by other researcher or abandoned.

In terms of forward-thinking, further development could consist of creating a trading system which takes into account price forecasts obtained through a proper model, which could be based both on statistical forecasts based on past price behaviours and econometric forecasts.

The road is now open: it is up to the operator to find new solutions and improvements to reach his financial goals, whatever they are.

Appendix

R code Markdown

The following is an example of the whole code used to create the fuzzy system from the pure technical setup and to examine its final performance.

Predictive Analysis using Fuzzy Logic

Packages necessary to load data and compute financial indicators.

If a package is missing, installation is done via the command: install.packages('name')

```
library(tidyverse)
library(lubridate)
library(TTR)
library(PerformanceAnalytics)
library(quantmod)
library(htmltab)
options(scipen = 999)
```

Load data for a list of stocks

We can get a list of symbols from the web. The following website contains a list of stocks in NASDAQ: https://www.slickcharts.com/nasdaq100 We can parse the list and use it in our code.

```
table = htmltab('https://www.slickcharts.com/nasdaq100', 1)



table = select(.data = table, Company, Symbol)

print(head(table))

##          Company Symbo
## 2      Apple Inc   AAPL
## 3 Microsoft Corp   MSFT
## 4 Amazon.com Inc   AMZN
## 5   Facebook Inc     FB
## 6   Alphabet Inc  GOOGL
## 7   Alphabet Inc   GOOG
print(tail(table))

##            Company Symbol
```

```
## 99     Ulta Beauty Inc   ULTA
## 100        NetApp Inc   NTAP
# 101 Liberty Global PLC  LBTYK
## 102          Fox Corp   FOXA
## 103          Fox Corp    FOX
## 104 Liberty Global PLC  LBTYA
```

We have now a list of 100 symbols we can analyse. We can start with choosing randomly a stock:

```
 stock=table$Symbol[sample(1:nrow(table),1)]
#stock=table$Symbol[3]
print(stock)

## [1] "MAR"
```

Prices will be extracted using quantmod's getSymbols function and yahoo as a source.

```
##We need market's performance for the hold period

start='2018-01-01'
end='2020-05-31'
market = getSymbols('^NDX',src='yahoo', from= start,  to=end, env=NULL)

##'getSymbols' currently uses auto.assign=TRUE by default, but will
##use auto.assign=FALSE in 0.5-0. You will still be able to use
##'loadSymbols' to automatically load data. getOption("getSymbols.env")
##and getOption("getSymbols.auto.assign") will still be checked for
##alternate defaults.
##
##This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.

#converting daily data to weekly: fridays only
market=market[wday(time(market),
week_start=getOption("lubridate.week.start",1))==5,]



## Our investment stock

start = '2018-01-01'

end = '2020-05-31'

prices=getSymbols(stock,src='yahoo', from= start, to=end, env=NULL)
#converting daily data to weekly: fridays only
prices=prices[wday(time(prices),
```

```
week_start=getOption("lubridate.week.start",1))==5,]
#check the class
print(class(prices))
```

```
## [1] "xts" "zoo"
```

```
#this is directly output as a xts object which is very helpful for time series analysis

print(head(prices))
```

```
##             MAR.Open  MAR.High  MAR.Low  MAR.Close  MAR.Volume  MAR.Adjusted
##  2018-01-05   136.45    136.45   134.20     134.40     3404400      130.3903
##  2018-01-12   140.11    140.26   138.53     139.78     1806800      135.6098
##  2018-01-19   141.62    144.07   141.44     144.07     3267100      139.7719
##  2018-01-26   145.71    147.15   144.92     147.14     1113300      142.7503
##  2018-02-02   146.69    147.01   140.97     141.17     2672300      136.9584
## 2018-02-09  134.54  137.47  131.10  136.29  2885100  132.2240
```

```
#renaming necessary for calculations
names(prices) = c('Open','High','Low','Close','Volume','Adjusted')
```

Compute factors

For fuzzy logic we need 4 indicators: MACD, RSI, SO, OBV

MACD

We will use the **MACD** function from the TTR package.

```
macd=MACD(prices[,4],nFast=12,nSlow=26,nSig=9)

class(macd)
```

```
## [1] "xts" "zoo"
```

```
tail(macd)
```

```
##                    macd          signal
## 2020-04-24    -13.55614       -7.285536
## 2020-05-01    -13.82941       -8.594310
##2020-05-08     -13.73411       -9.622270
##2020-05-15     -14.14539      -10.526894
##2020-05-22     -13.40745      -11.103005
##2020-05-29     -12.89944      -11.462293
```

The macd output is an xts object which contains two columns: macd and signal. These will be used to compute the trading indicators.

```
names(macd) = c('macd_value','macd_signal') #naming convention such as later the word signal is not confusing
```

```
prices = merge.xts(prices, macd) #merge all into one single table
```

RSI

We will use the **RSI** function from the TTR package.

```
rsi=RSI(prices[,4],n=14)

class(rsi)

## [1] "xts" "zoo"

tail(rsi)

##                    rsi
##2020-04-24     34.22014
##2020-05-01     36.55094
##2020-05-08     37.96883
##2020-05-15     35.36296
##2020-05-22     41.90515
##2020-05-29     40.89827
```

The rsi output is an xts object which contains one column: rsi. These will be used to compute the trading indicators.

```
prices = merge.xts(prices, rsi) #merge all into one single table
```

SO

We will use the **stoch** function from the TTR package.

```
so = stoch(prices[,2:4], nFastK = 10, nFastD = 3, nSlowD = 3) * 100

class(so)

## [1] "xts" "zoo"

tail(so)

##               fastK      fastD      slowD
##2020-04-24  26.23949   19.56987   13.74359
##2020-05-01  31.34885   29.25218   21.21360
##2020-05-08  44.80914   34.13249   27.65151
##2020-05-15  40.19781   38.78527   34.05665
##2020-05-22  77.77525   54.26073   42.39283
## 2020-05-29 89.36170   69.11159   54.05253
```

The SO output is an xts object which contains 3 columns: fastK, fastD, slowD. We will use fastK as the SO.

```
names(so)[1]='so'

prices = merge.xts(prices, so) #merge all into one single table
```

OBV

We will use the **OBV** function from the TTR package.

```
obv=OBV(prices[,4],prices[,5])

##we need to compute the exponential moving average with n = 3

obv$ema=EMA(obv$obv,n=3)
obv$ema[is.na(obv$ema)] = 0 # necessary later for lag calculatios

class(obv)

## [1] "xts" "zoo"

tail(obv)

##                 obv       ema
##2020-04-24 -25313300 -23116087
##2020-05-01 -21079000 -22097543
##2020-05-08 -16809400 -19453472
##2020-05-15 -23336300 -21394886
##2020-05-22 -21076900 -21235893
## 2020-05-29 -26951300 -24093596

prices = merge.xts(prices, obv) #merge all into one single table
```

Additional factor: Semidev Calculations

We will use the **PerformanceAnalytics** package to compute the semi-deviation. We will use a rolling window of 14 weeks for observation. The semi-deviation will indicate the volatility for negative returns only, and therefore allow us to be long on the market even when the volatility is very high. Often markets have very strong uptrend and very high volatilty.

```
library(PerformanceAnalytics)

semidev=rep(NA,13) #we want at least 14 obversations

for(I in 14:length(prices$Adjusted)){
  semidev=c(semidev, SemiDeviation(R = ROC(prices$Adjusted[(i-13):i], n = 1, type = "discrete"),
MAR=0)

          )
}
```

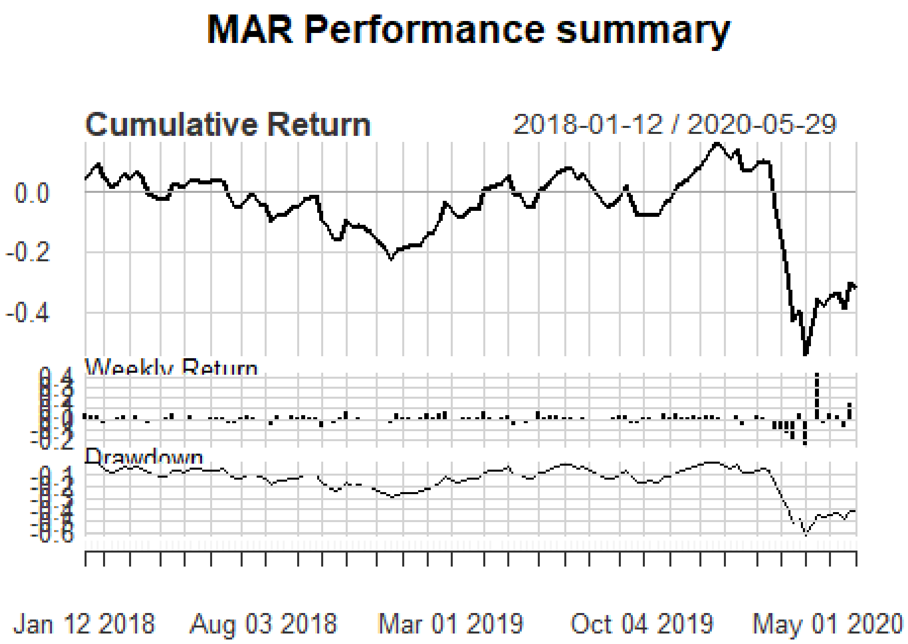
```
prices$semidev = semidev

#par(mfrow=c(5,1)) it does not work


plot(prices[, 4], main = "price series")

plot(prices[, 8:9], main = "macd")

plot(prices[, 10], main = "rsi")

plot(prices[, 11], main = "so")

plot(obv[, 1], main = "obv")

plot(prices$semidev, main = "semidev")
```

Data overview

```
head(prices)

##               Open   High    Low  Close  Volume Adjusted macd_value macd_signal
## 2018-01-05 136.45 136.45 134.20 134.40 3404400 130.3903         NA          NA
## 2018-01-12 140.11 140.26 138.53 139.78 1806800 135.6098         NA          NA
## 2018-01-19 141.62 144.07 141.44 144.07 3267100 139.7719         NA          NA
## 2018-01-26 145.71 147.15 144.92 147.14 1113300 142.7503         NA          NA
## 2018-02-02 146.69 147.01 140.97 141.17 2672300 136.9584         NA          NA
## 2018-02-09 134.54 137.47 131.10 136.29 2885100 132.2240         NA          NA
##               rsi   so  fastD  slowD      obv          ema semidev
##2018-01-05      NA   NA     NA     NA   3404400   0            NA
##2018-01-12    NA   NA     NA        NA   5211200        0       NA
##2018-01-19    NA   NA     NA        NA   8478300   5697967     NA
##2018-01-26      NA   NA     NA       NA   9591600   7644783     NA
##2018-02-02      NA   NA     NA       NA   6919300   7282042     NA
## 2018-02-09      NA  NA    NA    NA 4034200 5658121      NA

summary(prices)

##        Index                    Open              High               Low
##    Min.:    2018-01-05      Min.:   64.36      Min.:   64.71      Min.:   57.0
##    1st  Qu.:  2018-08-13   1st  Qu.:  119.77  1st  Qu.:  121.47  1st  Qu.:  118.9
##    Median:  2019-03-15     Median:  129.82    Median:  131.16    Median:  128.3
##   Mean    :2019-03-17      Mean:   125.94     Mean:   127.38     Mean:   124.4
##    3rd  Qu.:  2019-10-21   3rd  Qu.:  137.80  3rd  Qu.:  139.33  3rd  Qu.:  137.2
##   Max.:    2020-05-29      Max.:   153.38     Max.:   153.39     Max.:   152.5
##
```

```
##     Close                 Volume              Adjusted            Macd_value
##      Min.:    59.08        Min.:    445200      Min.:    59.08       Min.:   -14.1454
##    1st  Qu.:  120.73     1st  Qu.:  1380900   1st  Qu.:  118.58    1st  Qu.:  -3.7105
##     Median  :130.43      Median:   2022600    Median:   127.45     Median:   -0.9713
##      Mean:   125.99       Mean:   2559360      Mean:   124.04       Mean:   -1.5093
##    3rd  Qu.:  138.36     3rd  Qu.:  2823450   3rd  Qu.:135.25     3rd  Qu.:    2.3568
##      Max.:   152.73      Max.:   18154700     Max.:   152.12       Max.:     3.9915
##                                                                     NA's      :25
##      Macd_signal              Rsi                 So                   fastD
##    Min.:    -11.4623        Min.:    18.74      Min.:    0.1405        Min.:    6.576
##  1st  Qu.:   -3.3419      1st  Qu.:  40.32    1st  Qu.:  19.0903     1st  Qu.:23.162
##  Median:   -0.2797        Median:   49.70     Median:  45.7495       Median  :38.947
##    Mean:    -1.0163        Mean:    48.52      Mean:    48.9760       Mean:48.450
##  3rd  Qu.:   2.0584       3rd  Qu.:  56.81    3rd  Qu.:  79.8468     3rd  Qu.:76.054
##    Max.:     3.2206        Max.:     72.43       Max.:99.3134          Max.:99.069
##  NA's:   33                  NA's:   14          NA's:   9              NA's:   11
##        slowD                 Obv                 Obv_ma               Semidev
##    Min.:    8.118        Min.:-27067800       Min.:-24093596       Min.:0.009298
##  1st  Qu.:23.750     1st  Qu.:    1217000   1st  Qu.:    1314217   1st  Qu.:0.019147
##  Median:39.226       Median:    6267700   Median:    6540140     Median  :0.022504
##   Mean:48.348       Mean:     4947021      Mean:     5096579      Mean       :0.027885
##  3rd  Qu.:73.891     3rd  Qu.:  11778950   3rd  Qu.:  10873606   3rd  Qu.:0.024901
##    Max.:97.810          Max.:    21712100      Max.:    20334748        Max.:0.099534
## NA's :13                      NA's   :13
```

```
charts.PerformanceSummary(
  ROC(prices$Adjusted,n=1,type="discrete"),
  main=paste(stock,"Performancesummary")
)
```



Convergence module

```r
fis_variables=prices[,c('Adjusted')]

##MACD SIGNAL calculation

#High = 1, Low = -1, easier to manipulate later
fis_variables$macd_signal = ifelse(prices$macd_value > prices$macd_signal, 1, -1 )
print('Macd signals')

## [1] "Macd signals"

table(fis_variables$macd_signal)

##
##-1 1
## 47 43

#for fuzzy, scaling between 0 and 100
#Since this is binary, we will assume 33 low and 66 high
fis_variables$macd_signal_fuzzy = fis_variables$macd_signal
fis_variables$macd_signal_fuzzy[fis_variables$macd_signal_fuzzy == 1] = 66
fis_variables$macd_signal_fuzzy[fis_variables$macd_signal_fuzzy == -1] = 33


##RSI SIGNAL calculation
#low -1, medium 0, high = 1
```

```
fis_variables$rsi_signal=ifelse(prices$rsi>70, 1, 0)
fis_variables$rsi_signal=ifelse(prices$rsi<30, -1, fis_variables$rsi_signal)
print('RSI signals')

## [1] "RSI signals"

table(fis_variables$rsi_signal)

##
##-1 0 1
## 5  102  2

#values are already scaled between 0 and 100, so no adjustment
fis_variables$rsi_signal_fuzzy = prices$rsi

##SOSIGNAL calculation
#low -1, medium 0, high = 1

fis_variables$so_signal=ifelse(prices$so>80, 1, 0)
fis_variables$so_signal=ifelse(prices$so<20, -1, fis_variables$so_signal)
print('SO signals')

## [1] "SO signals"

table(fis_variables$so_signal)

##
##-1 0 1
## 29 56 29

#values are already scaled between 0 and 100, so no adjustment
fis_variables$so_signal_fuzzy = prices$so


##OBV SIGNAL calculation
# High = 1, Low = -1, easier to manipulate later

fis_variables$obv_signal = ifelse(prices$ema > stats::lag(prices$ema), 1, -1)

print('OBV signals')

## [1] "OBV signals"

table(fis_variables$obv_signal)

##
##-1 1
## 58 64
```

```
#Since this is binary, we will assume 33 low and 66 high
fis_variables$obv_signal_fuzzy = fis_variables$obv_signal
fis_variables$obv_signal_fuzzy[fis_variables$obv_signal_fuzzy == 1] = 66
fis_variables$obv_signal_fuzzy[fis_variables$obv_signal_fuzzy == -1] = 33


##SEMIDEV SIGNAL CALCULATION
# long when below the 33th percentile, short when above the 66th
fis_variables$semidev_signal_fuzzy = 50
fis_variables$semidev_signal_fuzzy[prices$semidev > quantile(prices$semidev, 0.7, na.rm = T)
] = 70
fis_variables$semidev_signal_fuzzy[prices$semidev < quantile(prices$semidev, 0.3, na.rm = T)
] = 30

table(fis_variables$semidev_signal_fuzzy)

##
##30 50 70
## 33 57 33

##
tail(fis_variables)

## Adjusted macd_signal macd_signal_fuzzy rsi_signal rsi_signal_fuzzy
##2020-04-24 80.71 -1 33 0 34.22014
##2020-05-01 84.75 -1 33 0 36.55094
##2020-05-08 87.17 -1 33 0 37.96883
##2020-05-15 79.76 -1 33 0 35.36296
##2020-05-22 91.05 -1 33 0 41.90515
##2020-05-29 88.50 -1 33 0 40.89827
## so_signal so_signal_fuzzy obv_signal obv_signal_fuzzy
##2020-04-24 0 26.23949 -1 33
##2020-05-01 0 31.34885 1 66
##2020-05-08 0 44.80914 1 66
##2020-05-15 0 40.19781 -1 33
##2020-05-22 0 77.77525 1 66
##2020-05-29 1 89.36170 -1 33
## semidev_signal_fuzzy
##2020-04-24 70
##2020-05-01 70
##2020-05-08 70
##2020-05-15 70
##2020-05-22 70
## 2020-05-29 70
```

Fuzzy model

Implementation of the fuzzy model.

```
library(sets)

## Warning: package 'sets' was built under R version 3.6.3

## Registered S3 method overwritten by 'sets':
## method from
## print.element ggplot2

##
## Attaching package: 'sets'

##The following objects are masked from 'package:lubridate':
##
## as.interval, interval, is.interval

##The following object is masked from 'package:forcats':
##
## %>%

##The following object is masked from 'package:stringr':
##
## %>%

##The following object is masked from 'package:dplyr':
##
## %>%

##The following object is masked from 'package:purrr':
##
## %>%

##The following object is masked from 'package:tidyr':
##
## %>%

##assuming all values are between 1 and 100
sets_options("universe", seq(1, 100, 0.1))


variables <- set(
 #assuming if 33, is low, if 66 is high
 macd = fuzzy_partition(varnames = c(low = 33, high = 66),
 sd = 4.99),

 #rsi is defined as low if under 30, to assume a bell curve(see plot below), setting low at 15 is
```

```
correct
 #same logic for high = 80 if we need values from 70 to 100
 rsi = fuzzy_partition(varnames = c(low = 15, med = 50, high = 85),
              sd = 5.0),
 #same logic, low is between 0 and 20, so 10, high is between 80 and 100, so 90
 so = fuzzy_partition(varnames = c(low = 10, med = 50, high = 90),
          sd = 5.0),

 obv = fuzzy_partition(varnames = c(low = 33, high = 66),
          sd = 4.98),

 semidev = fuzzy_partition(varnames = c(low = 30, med = 50, high = 70), sd = 5),

 signal = fuzzy_partition(varnames = c(sell = 10, hold = 50, buy = 90),
            FUN=fuzzy_cone, radius = 10)
)
```

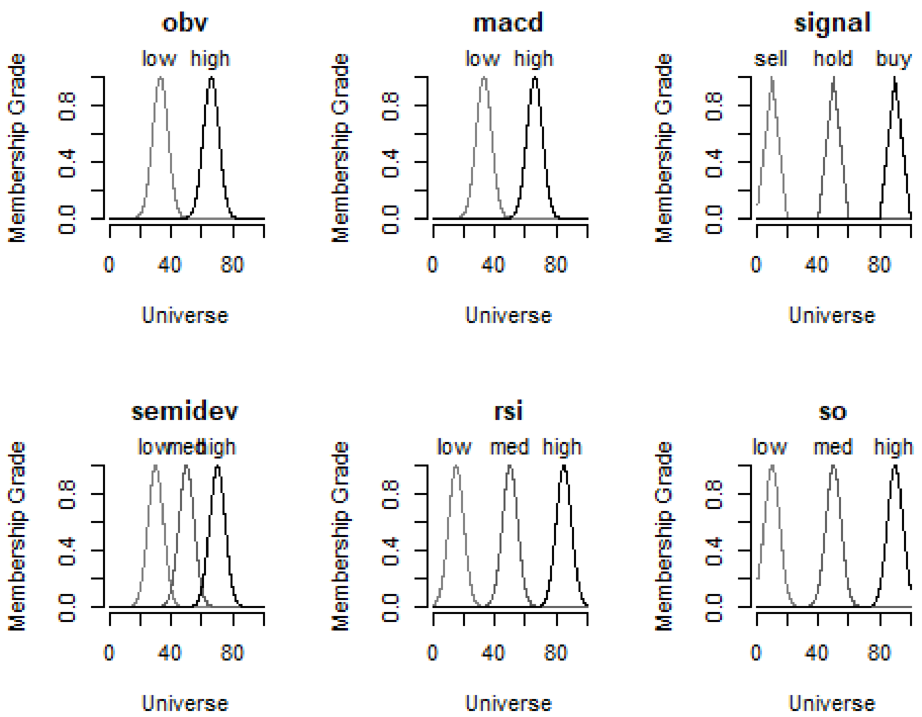
fuzzy rules

```
rules<-set(
 fuzzy_rule(macd %is% high && rsi %is% low && so %is% low && obv %is% high, signal %is%
buy),

 fuzzy_rule(macd %is% low && rsi %is% high && so %is% high && obv %is% low, signal %is%
buy),

 fuzzy_rule(macd %is% high && rsi %is% med && so %is% med && obv %is% high , signal %is%
buy),

 fuzzy_rule(macd %is% low && rsi %is% med && so %is% high && obv %is% low , signal %is%
sell),

 fuzzy_rule(rsi %is% high && so %is% low && obv %is% low , signal %is% sell),

 fuzzy_rule(macd %is% low && rsi %is% high && so %is% high , signal %is% sell),
 fuzzy_rule(macd %is% low && rsi %is% high && so %is% med , signal %is% hold),

 fuzzy_rule(macd %is% high && rsi %is% med && so %is% med && obv %is% low , signal %is%
hold)

)
```

fuzzy model

```
model<-fuzzy_system(variables,rules)
print(model)
```

```
## A fuzzy system consisting of 6 variables and 9 rules.
##
## Variables:
##
##obv(low,high)
##macd(low,high)
##signal(sell,hold,buy)
##semidev(low,med,high)
##rsi(low,med,high)
##so(low,med,high)
##
##Rules:
##
## rsi %is% low && so %is% low && obv %is% high => signal %is% buy
## rsi %is% high && so %is% low && obv %is% low => signal %is% sell
## macd %is% low && rsi %is% high && so %is% med => signal %is% hold
## macd %is% low && rsi %is% high && so %is% high => signal %is% sell
## macd %is% low && rsi %is% med && so %is% high && obv %is% low => signal %is% sell
## macd %is% low && rsi %is% high && so %is% high && obv %is% low => signal %is% buy
## macd %is% high && rsi %is% low && so %is% low && obv %is% high => signal %is% buy
## macd %is% high && rsi %is% med && so %is% med && obv %is% low => signal %is% hold
## macd %is% high && rsi %is% med && so %is% med && obv %is% high => signal %is% buy

plot(model)
```



applying model to current data

```
start_time<-Sys.time()

final_signal=c()
#Buy=1, Hold = 0, Sell -1
#Will be very handy to compute returns as well

for(i in 1:nrow(fis_variables)){

 # print(paste(i,'out of', nrow(fis_variables)))

 row_i=fis_variables[i,]
 if(any(is.na(row_i))){
  final_signal[i]=50 #assume hold for the beginning
 }else{

  v1=as.numeric(row_i$macd_signal_fuzzy)
  v2=round(as.numeric(row_i$rsi_signal_fuzzy))
  v3=round(as.numeric(row_i$so_signal_fuzzy ))
  v4=round(as.numeric(row_i$obv_signal_fuzzy))

  res <- fuzzy_inference(model, list(macd = v1, rsi = v2, so = v3, obv = v4))
  final_signal[i]=gset_defuzzify(res,"largestofmax")
  #print(final_signal[i])
 }

}

## Warning in max(unlist(.get_memberships(x)), na.rm = na.rm): no non-missing
## arguments to max; returning -Inf

## Warning in max(na.rm = FALSE): no non-missing arguments to max; returning -Inf

end_time <- Sys.time()
print(end_time - start_time)

## Time difference of 3.216166 mins

prices$final_signal = final_signal

##correction for -Inf
 prices$final_signal[which(is.infinite(prices$final_signal))] = 50

summary(prices$final_signal)

## Index final_signal
## Min. :2018-01-05 Min. :18.00
```

```
##  1st Qu.:2018-08-13  1st Qu.:50.00
##  Median :2019-03-15  Median :58.60
##  Mean :2019-03-17  Mean :63.23
##  3rd Qu.:2019-10-21  3rd Qu.:99.90
## Max. :2020-05-29 Max. :99.90
```

```
table(prices$final_signal)
```

```
##
##  18 19.9 50 56.2 57.2 58.6 59.6 59.9 99.6 99.8 99.9
## 1 23 34 2 1 1 1 17 1 1 41
```

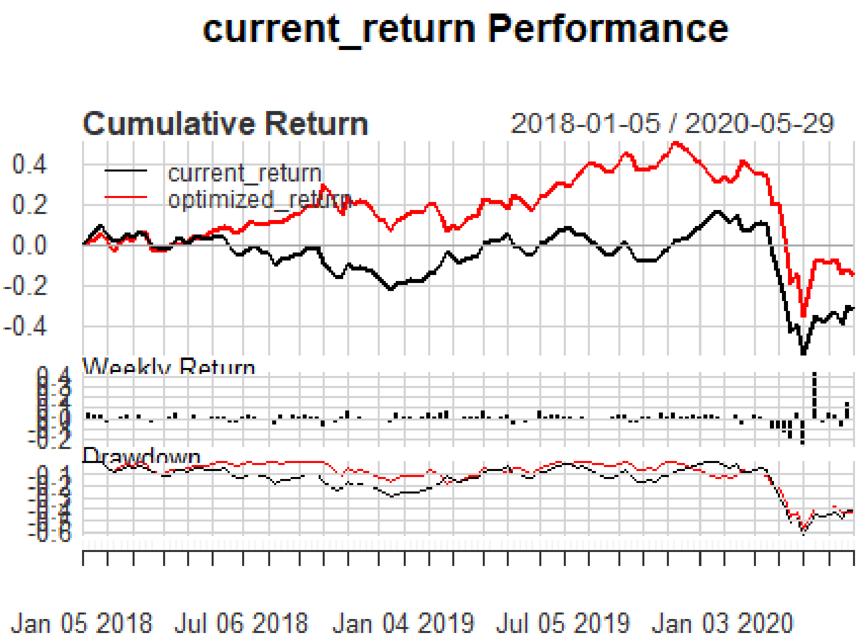
#Conversion to buy/hold/sell

```
prices=merge.xts(prices, market) ##adding market performance


prices$final_signal_adj = ifelse(prices$final_signal <=20, -1, prices$final_signal )
prices$final_signal_adj = ifelse(prices$final_signal_adj>61, 1, prices$final_signal_adj)
prices$final_signal_adj[prices$final_signal_adj !=1 & prices$final_signal_adj !=-1] = 0
table(prices$final_signal_adj)
```

```
##
##-1 0 1
## 24 56 43
```

```
##when it is -1 we sell, when 0 invert in NDX, when 1, we buy current stock

optimized_return=c(0)

for(I in 2:(nrow(prices))){

 #optimized_return[i]=0
 if(prices$final_signal_adj[i-1]==0){
  optimized_return[i] =
   as.numeric(prices$NDX.Adjusted[i])/as.numeric(prices$NDX.Adjusted[i-1]) - 1
 }

  if(prices$final_signal_adj[i-1] == 1){
  optimized_return[i] = 1 * (as.numeric( prices$Adjusted[i])/as.numeric(prices$Adjusted[i-1]) -
1)
  }
 #
  if(prices$final_signal_adj[i-1] == -1){
  optimized_return[i] = -1 * (as.numeric(prices$Adjusted[i]) /as.numeric(prices$Adjusted[i-1])
- 1)
 }
```

```
}

#optimized_return=c(optimized_return,0)

prices$optimized_return=optimized_return
prices$current_return=prices$Adjusted/stats::lag(prices$Adjusted)-1
prices$current_return[1]=0

charts.PerformanceSummary(prices[,c("current_return",'optimized_return')])

table.Stats(prices[,c("current_return",'optimized_return')])

table.AnnualizedReturns(prices[,c("current_return",'optimized_return')])
```



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