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Testing Standard Technical Analysis Parameters' Efficiency: a Comparison Among PSO, EFWA and DE

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Chapter 1

Introduction

The thesis proposed is aimed to exploit the profitability of Technical Analysis (TA) identifying among the wide set of indicators a sub-set of them, widely used both in literature and in practise, to optimize with metaheuristic algorithms and to test using a trading system. Since its introduction, there has been a wide discussion about the profitability of TA and mostly of the time it has been compared with the fundamental and Buy-And-Hold strategies. A questionnaire conducted by Lui and Mole (1995) [23] highlighted the use of both techniques in the foreign exchange market, in which the TA was mostly used in shorter horizon and more useful in forecasting trends and turning points. Even if TA is widely used in practise for its ability to predict change or constant trend periods it is not always perfect and there are many false signals that have to be correctly interpret. Due to the difficult to evaluate false signals it is not easy to obtain superior performance using the TA and many times trades finish to buy and sell in high and low bottom market moments losing money.

The selection of wrong enter and exit market moments are conditioned by the parameters selection of the trader which can choose the input and the indicator of its trading strategy. The literature and creators of the TA indicators have selected some parameters that considered optimal based on their working experience but they are not perfect for this reason indicators often give false signals. To achieve superior performance a trading system should be run using TA indicators that promptly signal when it is time to buy and sell. However, to find such TA indicators' values it requires the solution of a complex problem that can be expressed as an optimization one. The solution of this branch of optimisation problems has focused the attention of scholars into the usage of computational intelligence methods, as said by Cheng & Wei (2014) [8]

"*Soft computing mechanisms present several advantages when compared with traditional statistical methods. They generally exhibit high tolerance to imprecision and perform well in noisy data environments; they are numeric, data-driven, non-parametric and self adaptive mechanism; they require less historical data than traditional statistical models*".

Recently, many applications of computational intelligence methods have been applied on such problems, Cavalcante et al. (2016) [6] have done a survey of the suitability of such techniques in the finance field looking at the types of problem that scholars want to solve and which kind of computational intelligence algorithms have been proposed. The review shows that mostly of the literature has been focused on instrument such as Genetic Algorithms(GA) and Artificial Neural Network (ANN) leaving almost unexplored the field of metaheuristics and their applications on trading systems. Aguilar-Rivera et al.(2015) [1] have explored a wide set of techniques to solve financial problems: Genetic Algorithm, Genetic programming, Multi-Objective Evolutionary Algorithms, Learning Classifier System, Co-evolutionary Approaches and Estimation of Distribution algorithm. As reported by them, GA is one of the most popular in literature to approach complex problems and results to be a good tool to optimize series of TA indicators proving that optimized indicators perform better than others. L.L. Macedo et al.(2016) [24] still have searched for the superior performance of optimized GA indicators on Forex markets resulting to be true before trading cost were added. A. Kayal (2010) [19], R. Majhi et al.(2009) [25], Ghazali et al.(2009) [16] have applied different Artificial Intelligence techniques on Forex and stock markets showing better performances of optimized indicators respect to the classic value initially proposed by creators and traders.

Even if Artificial Neural Network (ANN) has been widely used it is not the only statistical intelligent learning method applied, Yuan (2013) [40] explained that while an ANN tends to find local optimum solution a Support Vector Machine (SVM) may tend to find a global optimum thanks to the structural risk minimization principle implemented. Chen J. (2010) [7] applied a SVM technique using original times series and empirical technical indicators to predict stock indices with good results. Another paper that follows the same strategy has been proposed by Guo-Qiang (2011) which can precisely predict the third stock closing price day ahead with a combination of particle swam optimization on the parameters optimization and the Support Vector Regressor in the selection.

A further step has been done combining GA and ANN this time to optimize a trading system as showed by Evans, Pappas & Xhafa (2013) [12]. They show that the combination of the two methods applied on an intra-day foreign exchange time series produce outstanding results and a high annualized net return.

Even if the GA has been considered one of the most used metaheuristic techniques to forecast stock market prices, it is not the only one explored. Brasileiro, Souza, Fernandes and Oliveria (2013) [3] propose a intelligent system that use closing stock prices and technical analysis indicators optimised by Artificial Bee Colony Algorithm, for the selection of the best time lags of indicators, and a nearest neighbours classification to decide the best moment to enter or exit the market. Their proposed method shows better result respect to a buy & hold strategy.

Subsequently the survey of Cavalcante et. al, many papers have been published covering evolutionary computational approaches still untested on this specific financial field. O.G. Monakhov et al. (2016) [27] have applied a new version of a Differential Evolution algorithm to improve a set of financial indicators and test them with a trading system showing the efficacy of this approach. Ghasemiyeh et al. (2017) [15] have combined a Hybrid Artificial Neural Network (HANN) with different metaheuristic approaches in the training phase finding that the Particle Swarm Optimization (PSO) algorithm is the one that perform better. In a recent paper, Corazza et al.(2017) [9] implemented a PSO algorithm looking for a solution of a non-linear, non-differentiable and integer problem of a simple trading system based on TA indicators. In line with the previous discovers, the evolutionary approach has improved the predictability of the indicators' set selected both in-the-sample and out-of-sample trading system performances. Following this paper, another one that compare PSO and Fireworks (FWA) has been published by

C.Pizzi et al.(2020). A trading system has been proposed this time composed by the Bollinger Band indicator with the superior and lower bands that do not follow the same standard deviation signalling the best moment for buying and selling once the moving average line touch them. Their work has proved that FWA has a higher ability in the exploitation of the solution space respect to PSO improving the result of the trading strategy. Their results have showed that there is still room for testing new and different evolutionary approaches.

However, even if the literature seems wide and present divergence on the result found, with papers that show good results and other assessing that a profit exist only if the trading costs are not taken into account, a well-established and tested methodology that highlight how to build an autonomous trading system is lacking. Vanstone & Finnie (2010) [39] say that the main reasons of the lack of methodology are due to the investors that try to keep their intellectual capital saved.

The thesis proposed wants to compare three different metaheuristic algorithms: Particle Swarm Optimization (PSO), Differential Evolution (DE) and Fireworks (FWA); searching among them which one perform better. The set of indicators contains a total of 5 instruments: two momentum indicators, MACD and William%R, two trend indicators, Parabolic Stop and Reverse (SAR) and Bollinger Bands (BB), and one volume indicator, Money Flow Index (MFI). The trading system created for this thesis combines the 5 aforementioned TA indicators together and when the combination of buy or sell signals coming from the indicators overcome a specific boundary the algorithm buy or sell from the market. The final performance of the trading system, in other words, the value of the capital invested at the end of the period will tell us the efficiency of the optimized set of indicators. In accordance with Cavalcante et al. the research does not focused only on the already wide tested advance stock market and Forex market but also in the emerging markets.

The remainder of this thesis is organized as follows. Chapter 2 describes the Technical Analysis with the main indicators used in the trading system. Chapter 3 explain what are metaheuristics and describe the algorithm selected for the optimization of the trading system. Chapter 4 shows how the trading system has been implemented and how the metaheuristics' parameters are chosen. Chapter 5 describes the results found. Chapter 6 concludes the thesis.

Chapter 2

Technical Analysis

In finance there has always been a great interest in trying to predict the future behaviour of the stock market. Trying to generate a reasonable and accurate way to anticipate the future has generated a wide amount of studies that involve an large number of techniques. Non-linearity and non-stationary features of the stock markets make them highly complicate to predict. The increase interconnection among different sectors and states have widened even more the complexity of financial markets because political events, market news, quarterly earning reports, international influences and conflicting trading behaviours make a role in shaping them. Among scholars and practitioners two techniques has the majority of consensus: Technical analysis and Fundamental analysis.

Fundamental analysis consist of studying the accountability, health, market strategy, financial statements, competitors and overall state of the economy such as GDP, interest rate, employment rate, production, manufacturing and management to estimate the possible value of the company. The main aim of the strategy is to find the possible companies that in the short period of time are underpriced by the market and buy them. From the identification

moment use the Buy-&-Hold strategy until the security reaches the value initially identify by the fundamental analysis. This technique is usually applied by experts as a long term strategy. It is possible to assess that the fundamental analysis born or it is well studied and spread with the Benjamin Graham's famous book *Intelligent Investor* [17]. Even considering semi-strong form of efficiency market hypothesis, scholars have tested the rules proposed by the strategy achieving superior results respect to the simple Buy-&-Hold as proved by Oppenheimer and Schlarbaum (1981) [32] and Metghalchi, Chang and Marcucci (2008) [26].
Technical analysis consists of studying historical stock data especially prices and volume to predict the future prices' movements. Murphy, John J. (1999) [30] define technical analysis as:

"*Technical analysts look at trends and price levels and believe that trend changes confirm sentiment changes. Recognizable price chart patterns may be found due to investors' emotional responses to price movements. Technical analysts mainly evaluate historical trends and ranges to predict future price movement.*".

This technique is usually applied by experts as a short-medium term strategy. Among scholars, it is not well recognised, in a famous paper of Lo, Mamaysky, and Wang (2000) entitled "Foundations of Technical Analysis: Computational Algorithms, Statistical Inference, and Empirical Implementation" [22], they have quoted:

"It has been argued that the difference between fundamental analysis and technical analysis is not unlike the difference between astronomy and astrology. Among some circles, technical analysis is known as 'voodoo finance'".

On the other hand, many other researches that have been carried out to prove the efficiency of this technique leaving many doubt on its use.
Moving in time, Charles Dow is considered the creator or at least the one which gave the groundwork of the technical analysis at the end of the 19th century. Then one of the first paper that has started to investigate the predictability of TA techniques was Alfred Cowles (1933) with "Can stock market forecasters forecast?" [10]. He has searched among 45 professional agencies if they were able to foresee the future stock prices movements. From that moment many studies have been done. Fama and Blume (1966) [13] have tested the performance of filtered rules showing their inability to beat the buy-&-hold strategy. Few years later their discovery, they came out with the efficient market hypothesis (EMH) theory. Also Jensen and Benington (1970) [18] and Levy (1967) [21] have supported the efficiency of Buy-&-Hold strategy respect to relative strength trading rules.
However, many studies later have started to undermine the theory of the EMH assessing that technical analysis could add value to the investment process. The paper of Brock et al. (1992) [4] was one of the first to show, with two famous trading rules, that returns are not consistent with 4 well known models: random walk, AR(1) , GARCH-M and the exponential GARCH. In their paper, they have highlighted that buy signals generate higher and less volatile returns than sell signals, also, they have reported that sell sig-

nals generate negative returns in contrast with all the existent, at that time, equilibrium models. Ratner and Leal (1999) [36] have examined ten emerging equity markets applying ten Variable Length Moving average. In 3 over 10 markets after the trading costs are taken into account seem to be profitable, at the same time, they have found that the TA indicators are able to predict the future direction in the return series. Another highly cited paper is the one of Lo et al.(2000) [22] with which they have reported the useful of TA indicators such as head-and-shoulder or double-bottoms. They have highlighted how these instruments and others improve the information resulting to have some practical utility for practitioners.

Focusing in the 2000s and 2010s, researchers published results that both support positive and negative outcomes of technical analysis. As it is showed by RTF Nazàrio et al.(2017) with "A literature review of technical analysis on stock markets" [31] over 95 studies, 56 have found pro results, meanwhile, the remaining show negative or mixed results.

2.1 Fundamental of Technical Analysis

The technical analysis is based on three main assumptions:

1. The price reflects every informations;
2. Prices move following trends;
3. Patterns repeated in history and they will do the same in future.

The idea that price reflects all the information available on the market means that the price of a stock today already incorporates fundamental factors, behavioural factors, political exposure and so on. Under this hypothesis, the only thing reaming to study its the price itself because all the other studies will result into discovering something already priced by the market. The starting point of the technical analysis theory comes from demand and supply of securities. If the price is increasing, it means that there is someone that has better information about fundamentals but, also, that these good information are well spread among investors pushing the demand and so raising the price, vice versa when the offer is higher respect to the demand. Based on this reason, graphical analysis does not push up or down the market but simple reflects the psychology. Leaving the fundamental outside the analysis, focusing only on chart and technical indicators, analysts can predict the future of the market without explaining the reason behind that behaviour.

The concept of trend is fundamental for the technical analysis. Analysts have to identify when a trend start, which is its direction and when a trend has changed definitively its path. An important opinion among trend analysts is that a trend has to give clear signal of a behavioural change before to assess

that it is not following any more its previous trend.
The history repeats itself is the third assumption, such idea born observing the cyclicity of markets' movements. Based on the last 100 years observation, it is noticeable that specific patterns reflect the bearish and bullish psychology of the market. If the market repeats itself this means, under the technical analysis hypothesis, that the future might repeat the past.
An advantage of technical analysis respects to fundamental analysis is its adaptability to every type of market. If in the fundamental one an investor has to evaluate many indicators and, at the end, when a result is found that one is applicable only to that specific market, while, such behaviour does not apply for the technical analysis. Another useful feature is that fundamental analysis, sometimes, can not be used in particular market such as futures markets, meanwhile, the technical one does not suffer of such restrictions.
One important distinction that need to be done is between technical analysts and charters. The first group uses technology instruments as an additional help other than their personal graphical interpretation of the stocks' charts. On the other hand, charters focus mostly on their predictions of futures movements of stock markets based on statistical models and indicators. This last group is used to apply automatic trading systems to avoid that emotion influence their decisions.

2.2 Dow Theory

Charles Dow and Edward Jones are widely recognized as the precursors of the modern technical analysis theory and, also, the founders of the Dow Jones & Company. The Down Jones, the first and one of the most famous indicator, its name has been given as a memory for Charles Dow's contribution to the financial market theory. Charles Dow published the indicator for the first time on July 1884. He were used to use it as a benchmark for the national economy status. He believed that the collection of the biggest companies so far were a good measure to evaluate the economy of the state. Such idea became a permanent contribution to today market analysis. His works were mostly based on the indicator created by him, however, the majority of the ideas formulated can be adopted also in other indicators and stock market prices. In the following subsections there will be illustrated the fundamental principles of the Dow Theory.

2.2.1 The indicators reflects everything

"*The sum and tendency of the transactions of the Stock Exchange represent the sum of all Wall Street's knowledge of the past, immediate and remote, applied to the discounting of the future. There is no need to add to the averages, as some statisticians do, elaborate compilations of commodity price index numbers, bank clearings, fluctuations in exchange, volume of domestic and foreign trades or anything else. Wall Street considers all these things*" [1].

[1] Hamilton, pp. 40-41 and also cited by John J. Murphy in Technical Analysis of Financial Markets

Such principle is valid for stock prices and also for indexes. In other words, indicators reflect all expectations at exclusion for something that it is not predictable and unexpected by the market.

2.2.2 The market has three trends

Dow define trend in three categories: primary, secondary and minor (Figure 2.1). Under his hypothesis the primary is a long-term trend that last for

Figure 2.1: Three main trends Source: https://tradeforexsa.co.za/forex-education/dow-theory/

years. Inside the primary trend there are secondary or intermediate trends. They have an average time line of 3 weeks to 3 months and the correction is 1/3 up to 2/3 of the amplitude of the previous trend. The minor, as anticipated by the name, has a lower importance and it last less than 3 weeks .

It is useful to give a definition of trend. It is considered an uptrend a price movement in a long time interval that generates growing maximum and minimum. Vice versa, it is considered a downtrend a price movement in a long time interval that generates decreasing maximum and minimum.

The primary trend is characterized by 3 phases: accumulation, public participation and distribution or excess phase (Figure 2.2). The first one contains the well informed investors together with the most astute buyers, they know information in advance or they have found important result before other investors. The second group is composed by trend followers, when news start to spread out and investors enter in the market just to follow the trend. At the end, when newspapers continue to report extreme positive news, speculators enter in the market but, at the same time, the first group of investors start to go out. A similar theory was proposed by Elliot with the so called 'Elliot waves'. Together with price, also volume has to increase in case of a

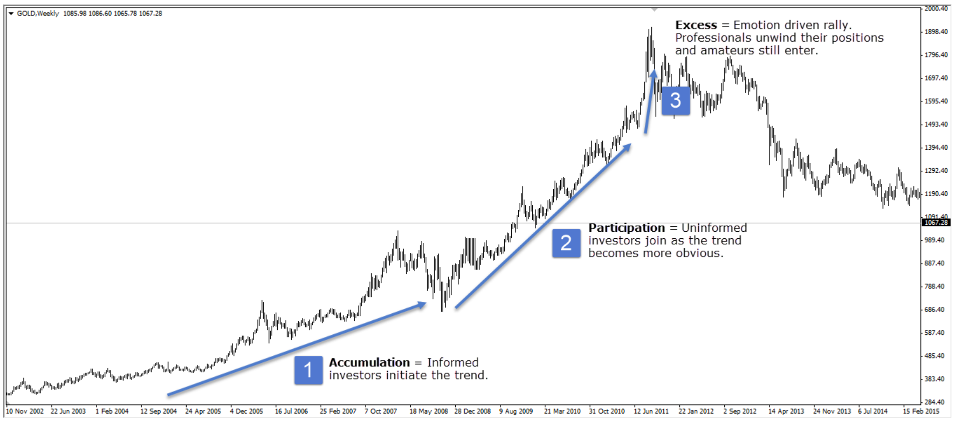


Figure 2.2: Three phases trend Source: https://optimusfutures.com/tradeblog/archives/why-the-last-part-of-a-trend-is-the-most-dangerous

bullish trend and decrease in case of bearish trend. Even if volume is a secondary indicator, it highly sustains the movement of price. When there are divergence among volume and price trend, sooner or later the price change direction.

Dow assessed that only closing price should be considered in technical analysis. In case of an intra-day break of a level, it is not considered important because it is not consolidated.

2.3 Other famous instruments

Outside the Dow theory there are others instruments that time has consolidated and they are widely used. These concepts are considered fundamental for everyone that want to start to study technical analysis and also they are highly used by experts.

2.3.1 Support and Resistance

The market price moves with maximum and minimum, the direction of these maximum and minimum characterize the trend. The support and resistance are the minimum and the maximum, respectively, in case of a bullish trend or bearish trend. In case of raising trend resistance level represent a pause from the main trend. The same in case of decreasing trend, the support stops shortly the price decline.

Only if trend breaks the resistance there is a continuation of the bullish trend, otherwise, there is a lateral trend that continue until a support or a resistance will be broken. When a support is broken, after an increasing trend, this is a signal of inversion, the same in case of break of a resistance during a bearish trend. In case of a uptrend, when a resistance is broken it becomes the new support, on the other hand, in downtrend when the price overcomes a support it becomes the new resistance (Figure 2.3).

If a resistance or a support are touched many times during a long period without being broken, they are considered major support and resistance and they need high buy and sell volume to be overcome (Figure 2.4).

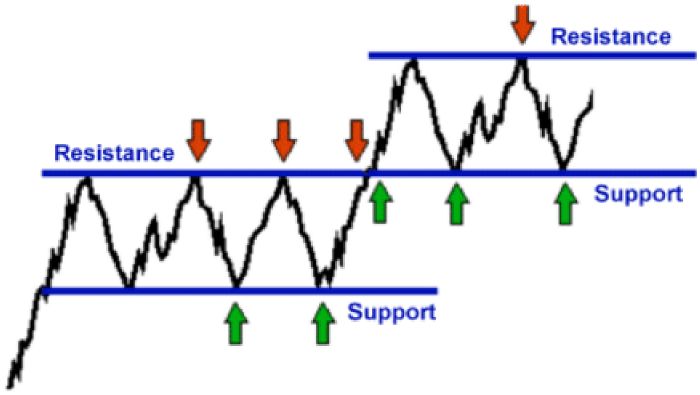


Figure 2.3: Support and resistance interchange Source: https://medium.com/@Rolandexpert/support-and-resistance-indicators-were-reimagined-with-blockchain-datasets-363e3ac5c460

2.3.2 Trendlines

The trendline is simple a straight line that connects all minimum in a bullish trend (positive trendline) or all maximum in a bearish trend (negative trendline, Figure 2.5). Drawing a trendline is difficult and subjective, the same positive trend might generate different trendlines if different analysts observe it. It is considered a trendline when more than 2 minimums or maximums are connected.

There are two characteristics that give importance to a trendline: the timelines and how many times the trendline has been touched.

A long period of time gives importance to the trendline, longer is the period stronger is the importance of that trendline. For example, a 9 months' trendline is stronger than a 6 weeks' trendline. The same principle stands for the number of positive tests that has been done on the line. Every time that price touches the trendline and bounces back without breaking it is a test of resistance. Higher is the number of positive tests stronger is the importance

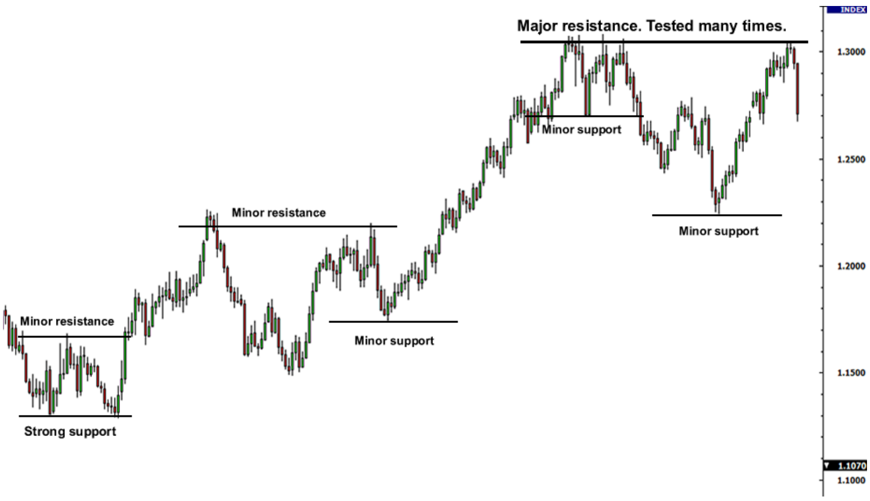


Figure 2.4: Minor and major resistance/support Source: https://zi.media /@yidianzixun/post/Uyc4Mw

of the trendline. The intra-day overcome is not considered important. Sometimes, even an end-day price break is not enough to considered it a change of trend. However, as for the resistance and support, once the line is broken it becomes a resistance in case of previous positive trendline and a support in case of previous negative trendline (bottom right Figure 5).

Figure 2.5: Example of negative trendline Source: https://forextraininggroup.com/how-to-properly-draw-and-trade-trendlines/

2.3.3 Channel trend

The channel trend or return line is the combination of two trendlines where price moves between them (Figure 2.6). Identifying these channels permit traders to profit both with up and down movement founding the possible limit of them. The channel trend is usually composed by two parallel lines but it is not always like that. There are multiple cases where the two lines tend to converge or amplifying. The same principles that apply for the trendline apply also here for the return line, longer in time and frequent are the bounces back stronger is the channel. There are two types of channel

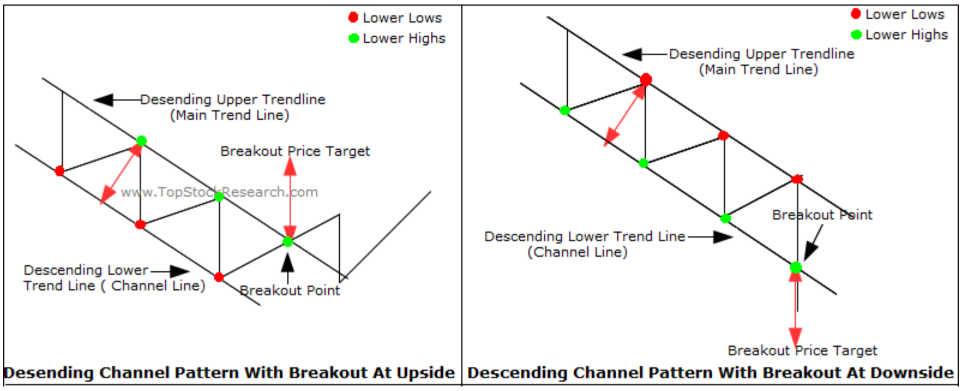


Figure 2.6: Example of channel trend Source: https://smv2 .stockmarketvideo.com/price-channel-know-where-youre-headed/

break in case of positive trend, in other words, when both the trendlines are positive. The first case is represented by a break of the lower trend line where the price tends to follow the same distance of the previous channel. This new movement might be a parallel trend or a complete inversion of the main trend. The second option is a break of the upper line, this comports an even stronger buy signal that consolidate the long position. The two behaviours just explained are the same but with opposite movements when there is a bearish main trend.

2.4 Indicators Comparison

In the previous sections it has been illustrated some famous theory and instrument among the wide possible choice. In this part there will be a comparison of all the technical analysis indicators proposed by two R packages that have been used: 'TTR' and 'QuantMode'. These two packages present a wide set of indicators, mostly of them are very famous and used both in practise and literature, others are less known. Based on the analysis proposed 5 indicators above presented have been selected.
The comparison illustrated is based on 8 characteristics:

1. Indicators: The list of all the packages' indicators
2. Modifiability (Mod): If packages permit to modify the indicator;
3. Only period (On.Per): If packages permit to modify only the selection period. In mostly of the case the intrinsic structure of the indicator permits only the selection of the time interval;
4. Momentum (Mom): If it is a momentum indicator;
5. Trend: If it is a trend indicator;
6. Volatility (S.D.): If it is a volatility indicator;
7. Volume Flow (Vol): If the indicator takes into account also the volume and analyse it together with other data or singularly;
8. Most used (M.Us.): Which are the most famous and used indicators among traders and scholars.

Indicators	Mod	On.Per	Mom	Trend	S.D.	Vol	M.Us.
Average directional index ADX	no	no		x			x
Bollinger bands	yes	no					x
Commodity channel index CCI	yes	no	x				
Simple MA	yes	yes		x			x
Exponential MA	yes	yes		x			x
Weighted MA	yes	yes		x			
Double MA	yes	yes		x			
Elastic volume weight MA	yes	yes		x			
Zero lag EMA	yes	yes		x			
Hull MA	yes	yes		x			
Triple smoothed exp. oscillator	yes	no					
MACD	yes	no	x				x
Rate of change ROC	yes	yes	x				
Relative Strenght index RSI	yes	yes	x				x
Parabolic Stop and Reversal SAR	yes	yes		x			
Stochastic momentum indicator	yes	yes	x				
William%R	yes	yes	x				x
Aroon	yes	yes		x			
Average true range ATR	yes	yes			x		x
Chaikin accumulation/ distribution A/D	no	no				x	
Chaikin volatility	yes	yes			x		
Close location value CLV	no	no			x		
Chaikin money flow	yes	yes				x	
Chande momentum oscillator CMO	yes	yes	x				
De-trending price oscillator DPO	yes	no		x			
Arms'ease of movement value EMV	yes	no	x				

Indicators	**Mod**	**On.Per**	**Mom**	**Trend**	**S.D.**	**Vol**	**M.Us.**
Guppy multi moving average GMMA	yes	no					
Know sure thing KTS	yes	no	x				
Money flow index MFI	yes	yes					
On balance volume	no	no					
Stochastic oscillator	yes	no	x				
Trend detection index TDI	yes	no		x			
The ultimate oscillator	yes	no	x				
Vertical horizontal filter	yes	yes		x			
William A/D	no	no	x				

Based on the flexibility of parameters selection but also for the wide used both in literature and by traders the selected indicators for the trading system are: Bollinger Bands, Moving Average Convergence Divergence (MACD), Money Flow Index (MFI), William%R and Parabolic Stop and Reverse (SAR). These indicators are also selected to involve an instrument for every category above mentioned, in fact, there is one Volume indicator, two Momentum indicators and three Trend indicators. It has been preferred not to include a Volatility indicator because it is not clear how to interpret a buy or sell signal.

2.5 Technical Analysis Indicators

In the previous section it has been illustrated a wide range of indicators and a selection of them that will be used in the analysis illustrated in chapter 4. Here it will be clarified what are technical analysis indicators, why they are used and how some of them works.

Technical indicators are mathematical instruments based on price, volume or open interest data that help the analyst to take buying and selling choice. Indicators have been created to predict the future movements of the market showing signals that are not easy to understand looking simply to the chart or single data. Usually, they are used for short-period market movements and not for long period investment strategies. Analysts use indicators for every type of securities: forex markets, stocks, commodities, futures, fixed-income and so on. Another important characteristic is that fundamental values do not influence indicators' value. It is possible to classify indicators into two main macro category: trend and momentum indicators. There are also Volatility and Volume indicators but they are small categories and not all of its indicators are used in practise.

Trend or break out indicators are the most famous and used, they try to follow the trend or at least identifying it. Among them the most famous are: Moving Average, Exponential Moving Average, Directional Movement Index, Moving Average Converge/Divergence Index (MACD), Parabolic Stop and Reverse (SAR) and Bollinger Bands (BB). These instruments are based on different parameters, some are simple other are more complex, however, they try to find always the same thing, a trend. Once a positive o negative

trend is identify, a change of it due to break of a resistance or a break of a imaginary line generated by an indicator is a buy or a sell signal for the analyst.
Momentum indicators measure price variations respect to its real level, in other words, it measures the speed of price changes. A famous definition is the one given by J.J. Murphy:

"Market momentum is measured by continually taking price differences for a fixed time interval. To construct a 10-day momentum line, simply subtract the closing price 10 days ago from the last closing price. This positive or negative value is then plotted around a zero line. The formula for momentum is:"

$$M = V - V_x \tag{2.1}$$

"where V is the latest price and V_x is the closing price of x days ago.". [2]

The momentum indicators that is part of oscillator category can give 3 important indications:

1. When the value reaches the upper level is a signal of overbought, in the same way, when it reaches the lower level is an oversold signal;

2. If there is a divergence between the oscillator and price it is a warning signal;

3. The overcome of the zero line might be an important buy or sell signal.

[2] John J. Murphy, Technical Analysis of the Financial Markets

Among the momentum indicators category the most famous are: Relative Strength Index (RSI), Stochastic Oscillator and William%R.
It is always suggested by expert not to base a buy or sell action only from one indicator signal but to use a combination of them.
In the following sections there will be illustrated some of the most famous indicators and the 5 ones chosen for the trading system.

2.5.1 Moving Average

The moving average (MA) is the most famous trend-following indicator. It is used to reduce noise of random short-term price movements trying to keep only the real trend behind. It is called trend-following or lagging indicator because it is based on past prices. The moving average line on price generates a support or a resistance line, when this is broken it signals a trend reverse.
There are a large number of MA indicators the most famous are:

Simple Moving Average (SMA)

$$SMA = \frac{P_t + P_{t-1} + \cdots + P_{t-n+1}}{n} \tag{2.2}$$

where $P_t, P_{t-1}, \ldots, P_{t-n+1}$ are closing prices and n is the number of lags selected.
The selected period influence the analysis, a small n value ranging from 10 to 30 is focused on a short-period evaluation. A larger time span between 30 to 50 days is more suitable for medium term analysis (Figure 2.7) and value over 50 days are considered for long-term analysis. However, even if 15, 20, 30, 50, 100 and 200 possible lags of n are widely used in literature and

practise, there is no right time frame to settle a Moving Average indicator. Some time frames might be ideal for a specific security and completely wrong for another one.

One of the main drawback of MA happens when a significant amount of false signals are taken into account in a short-period of time resulting into a line that is less smoothed than it should be.

Figure 2.7: Example of Simple Moving Average. Graph made with R-Studio

Exponential Moving Average (EMA)

$$EMA_t = \begin{cases} P_1 & t = 1 \\ \alpha * P_t + (1 - \alpha) * EMA_{t-1} & t > 1 \end{cases} \tag{2.3}$$

where P_t is the closing price at time t and EMA is the value of the exponential moving average. It is widely used to settle the first EMA_1 equal to P_1. α is a constant smoothing factor between $[0, 1]$. Higher is α faster is the discount of old values. Usually α is settled at:

$$\alpha = \frac{2}{n+1} \tag{2.4}$$

The exponential moving average (Figure 2.8) has been created to give more emphasis to the latest values respect to old values generating a line that reacts faster than the simple moving average. Even if old values weight every time less they are still counted, they never reach zero. The most used time spans are 12 and 26 days, however, as for the SMA there is no a ideal set of parameters. EMA is used together with other indicators to confirm significant market movements. It is more applicable for intra-day and fast-moving markets.

Cumulative Moving Average (CMA)

$$CMA_{n+1} = CMA_n + \frac{P_{n+1} + n * CMA_n}{n+1} \tag{2.5}$$

where

$$CMA_n = \frac{P_t + P_{t-1} + \cdots + P_{t-n+1}}{n} \tag{2.6}$$

$P_t, P_{t-1}, \ldots, P_{t-n+1}$ are closing prices, n is the number of time periods selected and CMA is the cumulative moving average value.

It is simple used when an user wants the average of all data up to the last day. It is also called running average or long running average. Even if it is well known it is not popularly used among traders.

Figure 2.8: Example of Exponential Moving Average, red = SMA, blue = EMA. Graph made with R-Studio

Weighted Moving Average (WMA)

$$WMA_t = \frac{nP_t + (n-1)P_{t-1} + \cdots + P_{t-n+1}}{n + (n-1) + 2 + 1} \tag{2.7}$$

where P_t is the closing price and n is the number of periods selected.

The Weighted Moving Average (Figure 2.9) assigns higher weights to the new values and decrease linearly the weight for the old values. Weights are equally distributed. It is used together with other indicators to confirm significant market movements. It is more applicable for intra-day and fast-moving markets.

The EMA and WMA are very similar indicators, however, which one of them is better it is not clear and highly depends on stock, time frame and parameters selected. Usually traders prefer EMA because they believe it is more responsive to trade change but at the same way it might be highly influence by false signals.

Figure 2.9: Example of Weighted Moving Average, green = WMA, red = SMA. Graph made with R-Studio

2.5.2 MACD

The Moving Average Convergence Divergence (MACD) is a trend-following momentum indicator. It is used to reveal changing in strength, direction, momentum and duration of a trend. It is one of the most famous indicator. MACD is composed by 2 moving average called ”fast” and ”slow” and a third

MA called "signal line". The first one is usually defined by a 12-days Simple Moving Average or Exponential Moving Average, meanwhile, the second is a 26-days. The "signal line" is a 9-days Simple Moving Average or Exponential Moving Average.

The adoption of a SMA or other types of MA as "fast" and "slow" lines is not so common, however, many pre-built software include them and other variations.

The MACD formula is:

$$MACD = EMA_{12} - EMA_{26} \tag{2.8}$$

The indicator is displayed with the MACD line, the Signal line and a histogram that represents the difference among the two lines (Figure 2.10). When the signal line is above the MACD line the histogram's bars are under the Base line and vice versa when lines invert. High and low values of MACD stand for overbought and oversold conditions.

There are many trading interpretations applied to this indicator such as signal-line crossover, zero-crossover and divergence. The most applied method and also the one that has been used in the trading strategy is the "signal-line crossover". When the MACD line crosses up the "signal line" it is suggested to buy and to sell in the opposite case. A "zero-crossover" case happens when the MACD line crosses the Base-line or zero-line, such event occurs when there is no difference between the slow and fast EMAs. When the MACD line becomes negative is considered a bearish signal, on the other hand, when the line moves in the positive area is a bullish signal. In the last case, if the indicator value diverges with respect of the direction of the price trend, it is a synonymous of an end of the current trend, especially in area of overbought and oversold.

Figure 2.10: Example of MACD. Graph made with R-Studio

The MACD should help traders to understand when bullish or bearish price movements are strengthening or weakening.

2.5.3 Parabolic Stop and Reverse

The Parabolic Stop and Reverse (SAR) is a trend-following, break even indicator designed to highlight the direction of the market. It should provide possible enter and exit points and it can also be used to settle stop-losses.

The creator J. Welles Wilder Jr. has created it for commodities, securities and currencies market. The instrument is graphically showed as a series of dots that are under the price line if there is a positive trend and above the market line if the trend is negative (Figure 2.11).
The SAR is constructed in the following way:

$$SAR_{n+1} = SAR_n + \alpha * (EP - SAR_n) \tag{2.9}$$

where SAR_n is the current period SAR.
EP stands for Extreme Point, it is the highest value reached by the price during the current uptrend or the lowest price during a downtrend. Every time that a new maximum or minimum is reached EP is up-date.
α stands for 'Accelerator factor' and it starts with a initial value of 0.02 but it can be selected differently. Every new EP, the accelerator factor increases of 0.02 up to the maximum of 0.2. Also, the maximum is up to traders' decision selecting the one more suitable for their conditions and for the securities. Sometime traders settle the increasing factor at 0.01 to avoid a false price decrease.
SAR has been computed every time in the way just explained with two exceptional cases:

1. If the next period's SAR value is inside, or beyond, the current period or the previous period's price range, the SAR must be set to the closest price bound.

2. If the next period's SAR value is inside, or beyond, the next period's price range, a new trend direction is then signalled. The indicator must switch side.

The old EP is the first EP and the acceleration factor is reset to the initial value.

When the parabola is under the price line it can be considered as a support, meanwhile, when it is above the price line it is considered a resistance. If the increasing SAR is broken a sell signal is generated, in the same way, when a decreasing SAR is crossed by the price line a buy signal is generated. As for other indicators, literature and traders suggests to use SAR in combination with other TA instrument such as MA and Average Directional Index.

Figure 2.11: Example of SAR. Graph made with R-Studio

2.5.4 William%R

The William Percentage R is a momentum indicator and it was created to find enter and exit points from the market. The creator, Larry William, initially has used it to trade commodities, however, it can be used also for currencies and stocks. The indicator range is between 0 and -100, where the area close to zero is an overbought level and oversold on the other border (In Figure 2.12 the limit range between 1 and 0 due to the software's setting). The typical period selected is 14 days. Usually, when the indicator is above -20 is a synonymous of overbought and under -80 of oversold, however, it does not mean that the price will reverse.
The William%R is computed as follows:

$$WPR_t = -100 * \frac{H_n - C_t}{H_n - L_n} \tag{2.10}$$

where H_n is the higher price among the n periods selected, C_t is the closing price at time t and L_n is the lowest price among the n periods selected.
The indicator tells if the current price is close to the highest or lowest price in the time frame selected. When the indicator moves in the overbought or oversold area it does not mean that there will be a reverse action, price might continue to increase or oscillate in that area. However, trades usually interpret as a buy o sell signal when the indicator value move out or move in from the overbought and oversold areas.
The period selection as for the overbought and oversold perimeters are up to the analysts who decide the most suitable for them and for the market analysed.

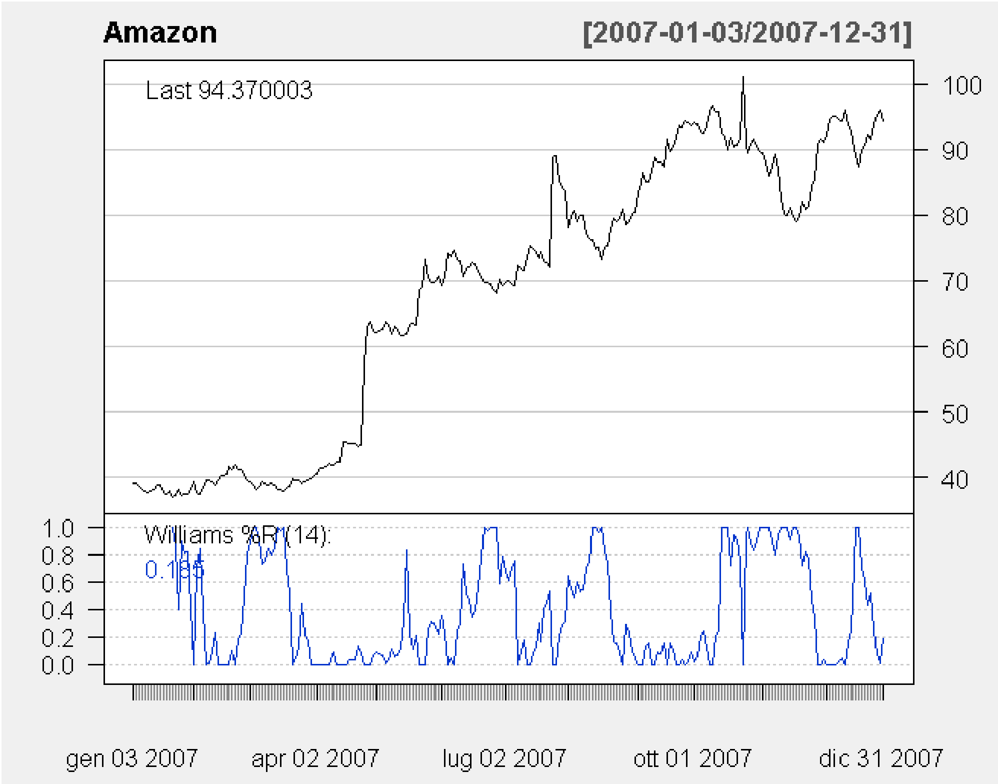


Figure 2.12: Example of William%R. Graph made with R-Studio

2.5.5 Money Flow Index

The Money Flow Index (MFI) is an oscillator designed to compute the ratio of capital flowing into and out of a security. The MFI range between 100 and 0 where values above 80 and below 20 are top and bottom market situations (Figure 2.13). The creator of the index has defined the top and bottom level for overbought and oversold at 90 and 10, respectively. However, it does not mean that if the price reaches these levels there will be a trend reversal, for such reason it is suggested to combine it with other indicators. It is considered a signal when there is divergence between the security price and the indicator. The time period usually selected is 14 days.

The MFI is computed as follows:

$$TypicalPrice = \frac{High + Close + Low}{3} \tag{2.11}$$

$$MoneyFlow = TypicalPrice * Volume \tag{2.12}$$

$$\begin{cases} PMF_t = PMF_{t-1} + MoneyFlow & \text{if } TypicalPrice_t > TypicalPrice_{t-1} \\ NMF_t = NMF_{t-1} + MoneyFlow & otherwise \end{cases} \tag{2.13}$$

$$MoneyRatio_i = \frac{\sum_{i=1}^{n} PositiveMoneyFlow}{\sum_{i=1}^{n} NegativeMoneyFlow} \tag{2.14}$$

$$MoneyFlowIndex = 100 - \frac{100}{1 + MoneyRatio} \tag{2.15}$$

Where PMF and NMF stand for Positive Money Flow and Negative Money Flow, respectively. $High, Close$ and Low refer to the daily price at time t and n is the period selected. One characteristic of this indicator that differ from others is that it takes into account also the volume, for this reason it is also called volume-weight Relative Strength Index.

2.5.6 Bollinger Bands

The Bollinger Bands are a trend-following, break out indicator developed by John Bollinger. The indicator is composed by a Simple Moving Average that follows price and others 2 lines that represent price volatility (Figure 2.14). Usually, the upper and lower bands are 2 standard deviation from the SMA, meanwhile, this one is settled at 20-days period. As for the other indicators these values are up to the trader that can decide other setting. The BB is

Figure 2.13: Example of Money Flow Index. Graph made with R-Studio

computed as follows:

$$\begin{cases} BBup = MA(TP, n) + m * \sigma[TP, n] \\ BBlow = MA(TP, n) - m * \sigma[TP, n] \end{cases} \tag{2.16}$$

where $BBup$ and $BBlow$ stand for Upper Bollinger Band and Lower Bollinger Band, respectively.

TP is the typical price or:

$$TP = \frac{HighPrice + LowPrice + ClosePrice}{3} \tag{2.17}$$

$MA(TP, n)$ is the Simple Moving Average of the typical price of the period selected, n in the period selected, m is the standard deviation multiplier and

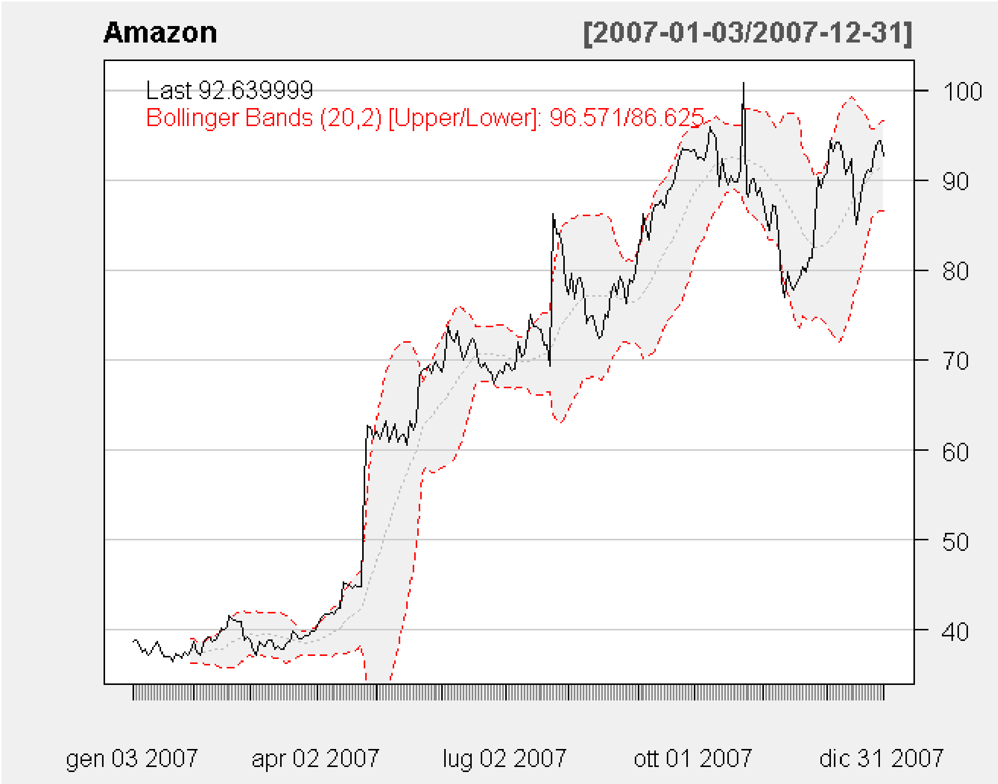


Figure 2.14: Example of Bollinger Bands. Graph made with R-Studio

$\sigma[TP, n]$ is the standard deviation of price in the period selected.

It is called a "squeeze" when the upper and lower bands shrink narrowing the distance. This means that there is low volatility in the period selected and traders interpret it as a quiet period before a potential signal and increase of volatility. However, this does not represent a enter or exit market signal. When the SMA touches the upper band is a selling signal, meanwhile, if the SMA bounce back from the lower band is a buy signal. Even in this case there are divergence among the interpretation of signals, however, the one proposed above is one of the most common and used in practice.

Another widely used strategy is a reverse crossover rule. It states that trades should buy when price touches the upper band and sell when price reaches the lower band.

The indicator effectiveness has been widely studied, Lento et al (2007) [20] and Balsara et al. (2007) [2] have studied it showing that the Buy-&-Hold strategy results to over-perform the single indicator but a contrarian version of the moving-average crossover rules showed some good results. Also Butler at al. (2010) [5] and C. Pizzi et al. (2020) have tested again the BB with the classic crossover rule but applying a metaheuristic approach to improve the parameters' set. Both their results indicate better performances without the default parameters.

Chapter 3

Metaheuristics

Mostly of the real problems related to the financial sector are non-linear, in other cases they are also non continuous, for example thinking about the classic financial problem of a minimum-variance portfolio selection buying and selling stock means to deal with integer quantities. As the portfolio selection problem, also the global optimization one that it has been considered has to deal with a non-linear, non-continuous function. The solution to such problems have been found with algorithms of direct search.
When the optimal solution is difficult or impossible to find with classical methods the only alternative is to trade optimality for efficiency. Heuristics, metaheuristics or approximate algorithms seek to obtain near-optimal solutions at relatively low computational cost. The MIT defines them as

"A heuristic is a technique designed for solving a problem more quickly when classical methods are to slow, or for finding an approximate solution when classic methods fail to find any exact solution. This is achieved by trading optimality, completeness, accuracy, or precision for speed. In a way, it can

be considered a shortcut" [1].

In literature, up to now, it is possible to consult dozens of these approaches that are inspired by nature, physic, chemistry and so on. It is considered a metaheuristic if the procedure proposed have exploitation and exploration capabilities as defined by Osman I.H. et al (1996)

"A metaheuristic is formally defined as an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space, learning strategies are used to structure information in order to find efficiently near-optimal solutions" [2].

All these methods are characterised by testing a function that need to be minimized/maximised with an initial random set of value, then a variation of the parameters is generated following specific rules and the function is recomputed with the new set of parameters. The process just describes is repeated until a condition is fulfilled. Generally, it is widely said that a metaheuristic is not a problem-specific solution but a technique to reach a reasonable and quickly solution. Metaheuristic explores the search space following some stochastic methods and it might embody some kind of memory for improving the search toward the global minimum, however, it might be trapped into some local minimum.
It is possible to classify metaheuristics among two classes: Population based

[1] http://www.mit.edu/~moshref/Heuristics.html
[2] Osman I.H., Laporte, G.:"Metaheuristics: A bibliography", Annals of Operations Research, 1996

and Trajectory based.

Population based is an algorithm that use a so called population during the search activity, each member represents a specific solution in the search space. All candidates move following some specific rules and they are updated when a new optimal point is found.

Trajectory based algorithms rely only in one searcher that starts identifying the first solution. Step by step, the first point in the research space is replaced every time by a new optimal solution that is found moving close to its neighbourhood. Usually such techniques are optimal to find good solutions in the local space for this reason they are called exploitation-oriented.

The algorithms used in the following analysis are all population based, mostly because it is not known if the best solution is close to the starting point and for such reason the exploitation of all the search space is considered the optimal approach to pursue.

3.1 Particle Swarm Optimization

The Particle Swarm Optimization (PSO) [35] algorithm has been developed to copy the behaviour of birds' flock or a swarm of bees during the food research. It is a natural-inspired, iterative, population-based, evolutionary, derivative-free metaheuristic for the solution of global unconstrained optimization problems. PSO makes almost no assumptions regarding the problem to optimize and at the same time searches in a wide space of possible candidates. However, it is not ensured the reach of a global minimum because it is possible that the algorithm sticks into a local minimum without being able to move out. For such reason, many tests are usually applied avoiding that final result is conditioned by a local minimum bias.

Each member of the swarm keeps memory of its best position and the global best, as showed in Figure 3.1, such value is identified computing the fitness function with the parameters founded. This behaviour permits to direct the swarm towards the best global position. The algorithm starts with M particles (the swarm) which are initially random generated respecting bounds fixed by the user. They represent also possible solutions in the search space. Together with the initial position also a random speed is assigned which is used to determine the movement direction. The fitness function is computed for each particle, if the value found is better than the previous fitness function the personal best is update. The global best is update searching among the neighbours together with the personal best. If the fitness function value found is better than the old one among the neighbours than the new global best is update, the number of neighbours considered is settled by the user.

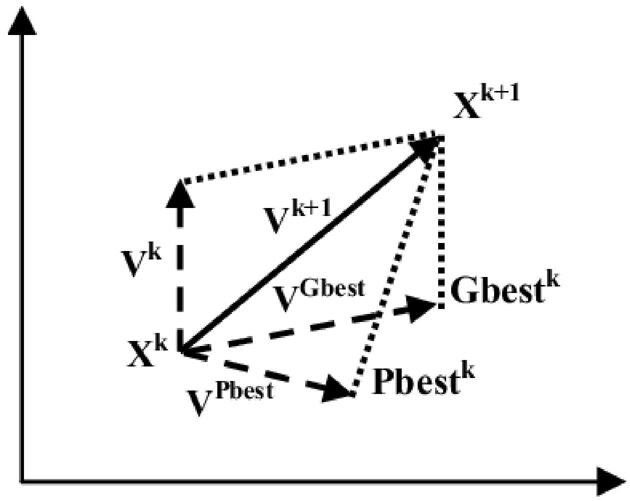


Figure 3.1: PSO searching behaviour Source: https://www.researchgate.net/figure/Concept-of-a-searching-point-by-PSO-This-modification-can-be-represented-by-the-concept_fig2_224091502

Then the position and velocity is update following specific criteria. Such process is repeated until the stop criteria is reached.

In detail, in case of function minimization, with M particles considered:

In the $k-th$ step of the algorithm 4 vectors are associated to the $j-th$ particle:

1. $x_j^k \in \mathrm{R}^d$ position at k' step of the $j-th$ particle;
2. $v_j^k \in \mathrm{R}^d$ velocity at k' step of the $j-th$ particle;
3. $PBestpos_j \in \mathrm{R}^d$ best position visited by the $j-th$ particle ($PBest_j = f(PBestpos_j)$);
4. $GBestpos_j \in \mathrm{R}^d$ global best position visited by the $j-th$ particle and its neighbours ($GBest_j = f(GBestpos_j)$)

then the process starts following these steps:

1. Random generation of the initial value (starting position) x_j^1 and initial velocity v_j^1 for $j = 1, \ldots, M.$;

2. Set $PBest_j = +\infty$ for $j = 1, \ldots, M$ set $GBest = +\infty$ in case of function minimisation;

3. Compute the $f(x_j^k)$ for $j = 1, \ldots, M.$;

4. If $f(x_j^k) < PBest_j$ then set $PBestposition_j = x_j^k$ and $PBest_j = f(x_j^k)$ for $j = 1, \ldots, M$;

5. If $f(x_j^k) < GBest_j$ then set $GBestposition_j = x_j^k$ and $GBest_j = f(x_j^k)$ for $j = 1, \ldots, M$;

6. Update of position and velocity following the criteria:

$$v_j^{k+1} = w^{k+1} v_j^k + \mathrm{U}_{\phi_1} \otimes (PBestpos_j - x_j^k) + \mathrm{U}_{\phi_2} \otimes (GBestpos_g - x_j^k) \quad (3.1)$$

$$x_j^{k+1} = x_j^k + v_j^{k+1} \quad (3.2)$$

   where $\mathrm{U}_{\phi_1}, \mathrm{U}_{\phi_2} \in \mathrm{R}$ are uniform random distribution in $[0, \phi_1]$ and $[0, \phi_2]$ respectively, and $\otimes$ denotes the component-wise product.

7. If the convergent criteria is not match $k = k + 1$ and return to step 3

The parameters choice has a huge influence on the optimization ability of the algorithm. The algorithm to converge into a solution need a careful calibration of the parameters ϕ_1 and ϕ_2 which strongly influence the exploration toward the personal and global best. These parameters need to be selected in accordance with the inertia weight w^k which generally decrease linearly with the increase of the parameter k:

$$w^k = w_{max} + \frac{w_{min} - w_{max}}{K} k \tag{3.3}$$

where K is the maximum number of iterations.
Many studies have been done on parameters convergence ability resulting in guidelines for selecting the correct PSO parameters set. However, there have been also criticisms for the oversimplification of these guidelines limiting the ability of PSO to few parameters.

3.2 Firework Algorithm

The Fireworks Algorithm (FWA) [38] has been created following the behaviour of fireworks and, as the Particle Swarm Optimization, it has been created for global optimization of complex functions. The one used in the analysis proposed by these thesis is not the original version but an improved one titled "Enhanced Firework Algorithm (EFWA)" [41]. It is an optimized version of the previous system that solves the problem of shifting functions and improves the speed. The operation of the algorithm can be explain with few steps (Figure 3.2):

1. The explosion operator that generates sparks around fireworks inside the search space;
2. The creation of Gaussian sparks, these are generated following the Gaussian mutation operator;
3. Checking if the initial conditions are fulfilled, if they are complete terminate;
4. If the condition are not completed than select the population for the next generation by the identification in the searching space of new fireworks starting positions following a mapping rule;

3.2.1 Classic or First FWA

In the first version of the FWA as in the enhance one it has great importance the design of the firework explosion. It is considered a bad explosion (Figure

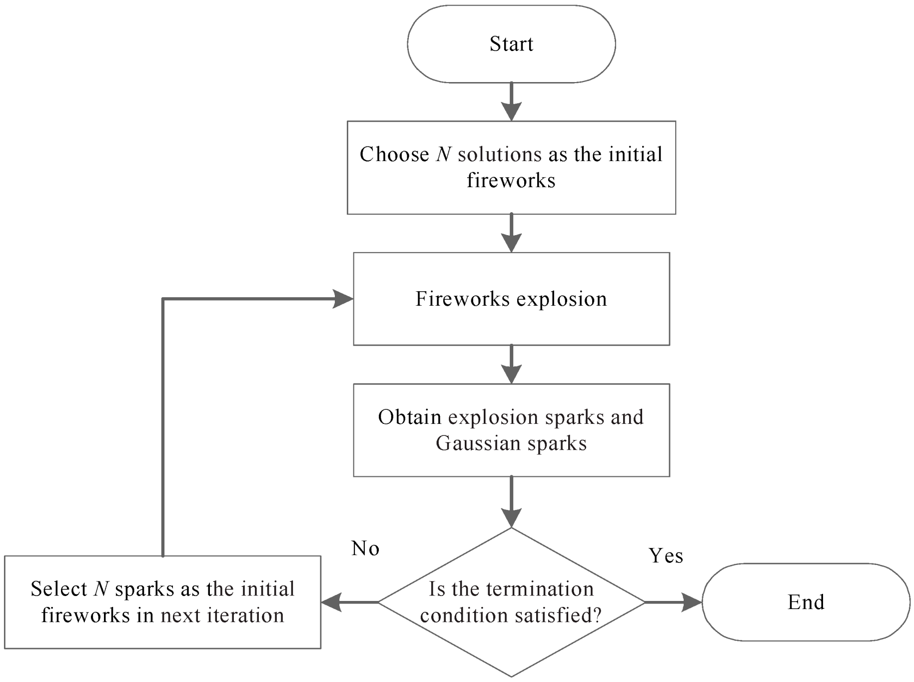


Figure 3.2: Fireworks algorithms steps. Source: https://www.mdpi.com/2076-3417/10/3/1174/htm

3.3.b) if it results to produce few sparks from it and they are far from the centre, meanwhile, a good explosion (Figure 3.3.a) is when numerous sparks are generated and they are close to the centre.

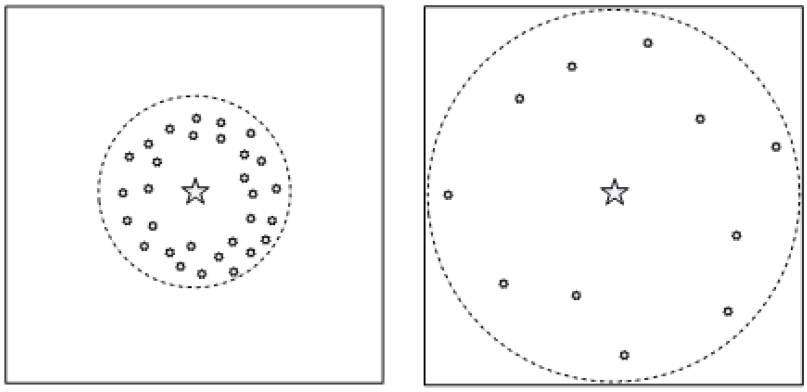
In case of minimization problem:

$$\text{Minimize} f(x) \in \mathrm{R}, x_{min} \leq x \leq x_{max} \tag{3.4}$$

where $x = x_1, x_2, \ldots, x_d$ denotes the possible location in the solution space, $f(x)$ is a generic objective function and x_{min}, x_{max} denote the solution space limits.

The number of sparks generated by each firework (x_i) is:

$$s_i = m * \frac{y_{max} - f(x_i) + \epsilon}{\sum_{i=1}^{n}(y_{max} - f(x_i)) + \epsilon} \tag{3.5}$$



(a) (b)

Figure 3.3: Fireworks Explosions. Source: https://www.mdpi.com/1999-4893/9/2/23/htm

where m controls the total number of sparks generated, $y_{max} = max(f(x_i))(i = 1, 2, \ldots, n)$ is the maximum or worst value of the fitness function among fireworks and ϵ is simple a small constant that avoid a zero denominator value. The number of sparks is bounded to avoid overwhelming effect of spending fireworks.

$$s_i = \begin{cases} round(a * m) & \text{if } s_i < am \\ round(b * m) & \text{if } s_i > bm, a < b < 1, \\ round(s_i) & otherwise \end{cases} \quad (3.6)$$

where a and b are constant parameters.

The amplitude of the explosion is defined by:

$$A_i = \hat{A} * \frac{f(x_i) - y_{min} + \epsilon}{\sum_{i=1}^{n}(f(x_i) - y_{min}) + \epsilon} \quad (3.7)$$

where $\hat{A}$ denotes the maximum explosion amplitude, $y_{min} = min(f(x_i))(i = 1, 2, \ldots, n)$ is the minimum or the best value of the fitness function among the n fireworks.

The FWA, at this point, generates two types of sparks: the first method applies random z directions:

$$z = round(d * rand(0, 1)) \tag{3.8}$$

where d is the dimensionality of the location x and $rand(0, 1)$ is a random uniform distribution bounded at [0,1]. The location of the sparks x_i is computed with the Algorithm in figure 3.4. The spark's location $\tilde{x}_j$ is generated in the same way of the explosion process. The second method called Gaussian

Algorithm 1. Obtain the location of a spark

```
Initialize the location of the spark: x̃_j = x_i;
z=round(d · rand(0,1));
Randomly select z dimensions of x̃_j;
Calculate the displacement: h = A_i · rand(−1,1);
for each dimension x̃_k^j ∈ {pre-selected z dimensions of x̃_j} do
   x̃_k^j = x̃_k^j + h;
   if x̃_k^j < x_k^min or x̃_k^j > x_k^max then
      map x̃_k^j to the potential space: x̃_k^j = x_k^min + | x̃_k^j | %(x_k^max − x_k^min);
   end if
end for
```

Figure 3.4: Source: Fireworks Algorithm for Optimization [38]

Explosion uses a Gaussian function denoted by $Gaussian(1, 1)$, mean 1 and standard deviation 1, to define the explosion coefficient as showed in figure 3.5. The generation of new fireworks follow a specific rule. The previous best location, the one with the best fitness function $f(x_i)$, is always kept for the next explosion. All others locations will be selected based on the previous distance. The distance is defined by:

$$R(x_i) = \sum_{j \in K} d(x_i, x_j) = \sum_{j \in K} \|x_i - x_j\| \tag{3.9}$$

Algorithm 2. Obtain the location of a specific spark

```
Initialize the location of the spark: x̂_j = x_i;
z=round(d · rand(0,1));
Randomly select z dimensions of x̂_j;
Calculate the coefficient of Gaussian explosion: g = Gaussian(1,1);
for each dimension x̂_k^j ∈ {pre-selected z dimensions of x̂_j} do
  x̂_k^j = x̂_k^j · g;
  if x̂_k^j < x_k^min or x̂_k^j > x_k^max then
    map x̂_k^j to the potential space: x̂_k^j = x_k^min + | x̂_k^j | %(x_k^max − x_k^min);
  end if
end for
```

Figure 3.5: Source: Fireworks Algorithm for Optimization [38]

where K defines the current location of all the sparks and fireworks.

At this point, the selection probability of a location x_i is:

$$p(x_i) = \frac{R(x_i)}{\sum_{j \in K} R(x_j)} \tag{3.10}$$

3.2.2 Enhance FWA

The first version of the firework algorithm results to work really well when the best solution is close or nearby the starting point. However, as proved by Shaoqiu et al. (2013) in case of shifted function, where the optimum is far from the starting point, the FWA lacks of convergence due to the structure of the Gaussian operator. Sparks generated by the Gaussian operator deposit close to the origin avoiding to spread all over the solution space. Another drawback of the FWA is the high computational cost required per iteration. The first improvement has been to change the explosion amplitude. With classic FWA, in case of high fitness values the firework algorithm generates sparks really close to the point in which the firework was set off. To improve such behaviour a lower bound A_{min} of explosion amplitude has been added. This value, in the initial exploration phase, is high and then it reduces with time. The amplitude A_i^k is limited as follows:

$$A_i^k = \begin{cases} A_{min}^k & \text{if } A_i^k < A_{min}^k \\ A_i^k & otherwise. \end{cases} \tag{3.11}$$

The lower bound is updated every iteration following two different approaches.
A linear decreasing function:

$$A_{min}^k(t) = A_{init} - \frac{A_{init} - A_{final}}{evals_{max}} * t \tag{3.12}$$

or a non-linear decreasing function (the last one is the function selected in the analysis proposed in the chapter 4/5):

$$A_{min}^k(t) = A_{init} - \frac{A_{init} - A_{final}}{evals_{max}} \sqrt{(2 * evals_{max} - t)t} \tag{3.13}$$

where A_{init} and A_{final} are the initial and the final minimum amplitude of explosion, respectively. t corresponds to the number of function evaluation and $evals_{max}$ is the maximum number of evaluations.
The generating explosion sparks operators has been changed:

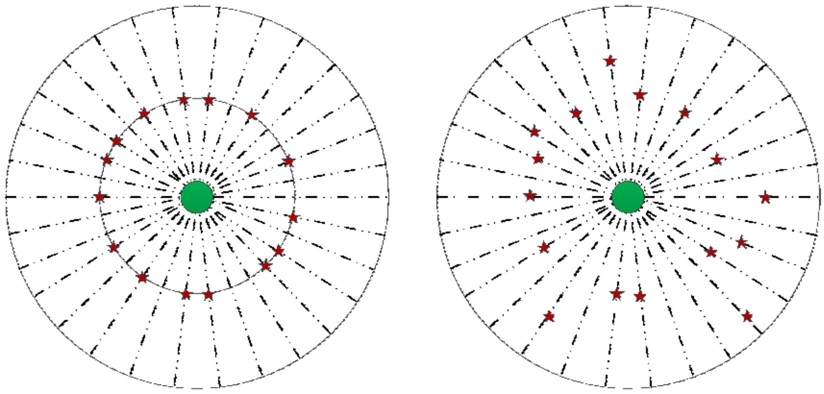
$$z^k = round(rand(0,1)) \tag{3.14}$$

and for each dimension of $\tilde{X}_i^k$ where $z^k == 1$ calculate:

$$\Delta X^k = A_i * rand(-1,1) \tag{3.15}$$

$$\tilde{X}_i^k = \tilde{X}_i^k + \Delta X^k \tag{3.16}$$

now a different offset displacement is computed in every dimension (Figure 3.6).
One of the main drawback of the FWA is the mapping operator. In case of



(a) FWA (b) EFWA

Figure 3.6: Firework new explosion. Source: Enhance Fireworks Algorithm [41]

new sparks that exceed the search range in dimension k they are relocated following the criteria:

$$\tilde{X}^k_{min} = X^k_{min} + |X^k_i| \% (X^k_{max} - X^k_{min}) \tag{3.17}$$

Such behaviour facilitates the deposit of sparks close to the origin avoiding to search in others areas. To deal with such problem Shaoqiu Z. at al (2013) have modified the mapping operator using a uniform random mapping operator:

$$\tilde{X}^k_i = X^k_{min} + rand * (X^k_{max} - X^k_{min}) \tag{3.18}$$

Also the Gaussian spark operator was modified with:

$$X^k_i = X^k_i + (X^k_B - X^k_i) * e \tag{3.19}$$

where X_B is the best firework or spark location up to now discovered and $e = \mathcal{N}(0, 1)$. The new operator proposed permits to explore all the space from the current firework to the best spark/firework founded (Figure 3.7). This approach avoids the focus of Gaussian operator in the centre unless the best optimum found locates there.

The last improvement adopted with the EFWA was a new selection opera-

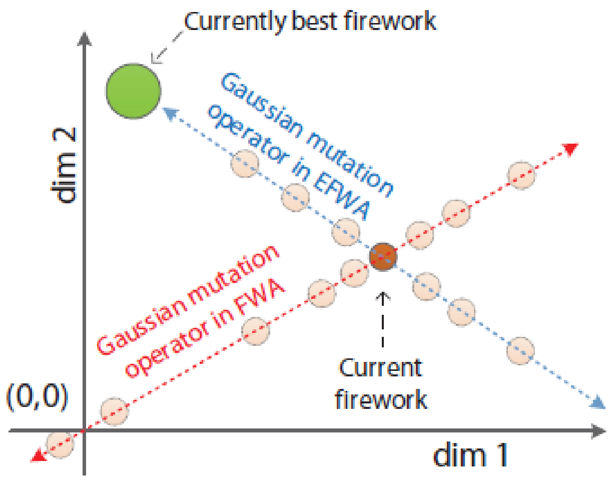


Figure 3.7: Gaussian operator. Source: Enhance Fireworks Algorithm [41]

tor. The one proposed in the FWA version is optimal because it permits to explore less crowded area by new fireworks, however, it results to be computational heavy to sustain. The new one, the Eliotism-Random Selection (ERP) method [11], settles the first firework, as before, in the best optimal point found and than it randomly selects all the other points.

In the paper proposed by S. Zheng et al (2013) the EFWA outperforms the classic FWA in all the benchmark functions selected. Meanwhile, Ying Tan et al. (2010) when they have proposed the firework algorithm they have showed its better performance compared to the standard version of PSO and Clonal PSO.

3.3 Differential Evolution

The Differential Evolution algorithm [37], differently from the other two, is not inspired by any natural behaviour. The creators in the design of the algorithm have tried to fulfil 4 requirements that for them and possible users are necessary. They define them as:

1. *Ability to handle non-differentiable, nonlinear and multimodal cost functions;*

2. *Parallelizability to cope with computation intensive cost functions;*

3. *Ease of use, i.e. few control variables to steer the minimization. These variables should also be robust and easy to choose;*

4. *Good convergence properties, i.e. consistent convergence to the global minimum in consecutive independent trials.*

The algorithm is composed by NP vectors, each contains N elements (number of variables that need to be optimized in our function).

$$x_{i,G} \qquad i = 1, 2, \ldots, NP \tag{3.20}$$

All the elements are initially randomly generated with an uniform distribution.

Than the algorithm generates a new vector called 'mutant' adding the subtraction of other two random vectors selected among the population NP, multiply by a constant to a third vector still selected among the NP vectors. In detail:

$$v_{i,G+1} = x_{r_1,G} + F * (x_{r_2,G} - x_{r_3,G}) \tag{3.21}$$

where $r_1, r_2, r_3 \in (1, 2, \ldots, NP)$ are randomly selected, integer and mutually different. The three variables need to be different from the index value, such constraint requires that $NP \geq 4$. The constant F controls the amplification of the differential variation and it is bounded at [0,2].

The following figure (3.8) shows a two dimensional example of the previous explained operation.

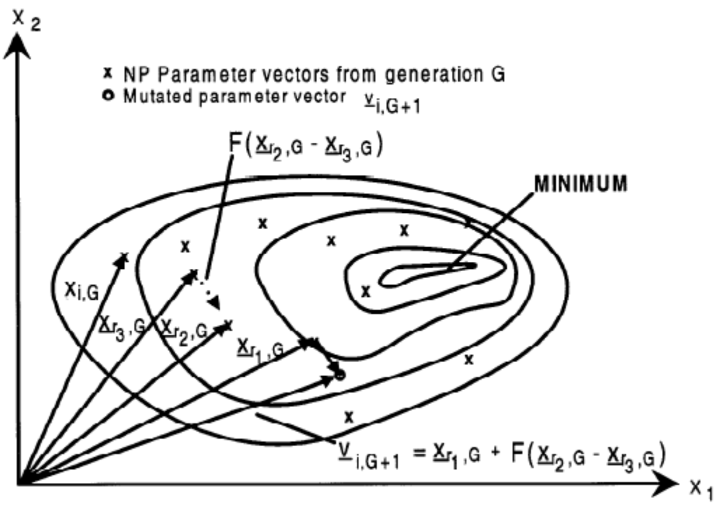


Figure 3.8: Two dimensional cost function showing its counter lines and the process for generating $v_{i,G+1}$. Source: Differential evolution- a simple and efficient heuristic for global optimization over continuous spaces [37]

Rainer et al. (1996) show that the usage of two different vectors in the subtraction passage seems to improve the efficiency only if NP is enough large.

At this point the crossover is introduced in the mutant vector. This step permits to increase the diversity of perturbed parameters vectors:

$$u_{i,G+1} = (v_{1i,G+1}, v_{2i,G+1}, \ldots, v_{Ni,G+1}) \tag{3.22}$$

where

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if } (rand(j)) \leq CR) \text{ or } j = rand(i), \\ x_{ji,G} & \text{if } (rand(j)) > CR) \text{ and } j \neq rand(i) \end{cases} \quad j = 1, 2, \ldots, N \tag{3.23}$$

$Rand(j)$ is a random number $\in [0, 1]$, meanwhile, CR $\in [0, 1]$ is the crossover constant selected by the user. $Rand(i)$ is a random number selected among the total number of variables N, this passage ensures that at least one parameter comes from $v_{i,G+1}$. One example in imagine 3.9.

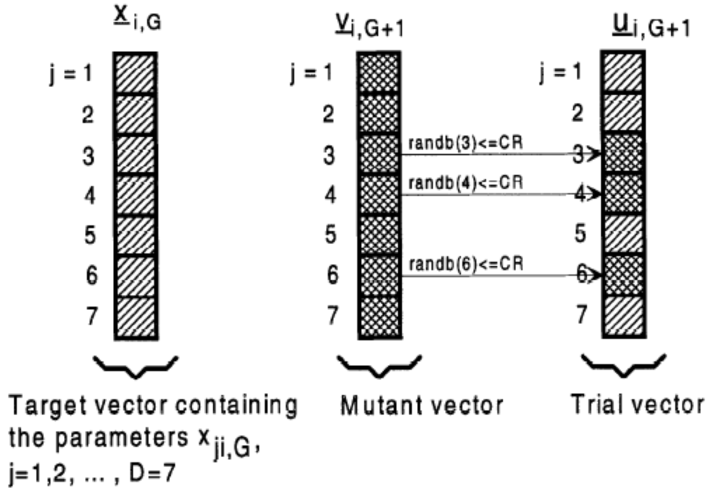


Figure 3.9: Example of crossover process. Source: Differential evolution- a simple and efficient heuristic for global optimization over continuous spaces [37]

Once the trial vector is created, the fitness function is computed with values from $v_{ji,G+1}$ and $x_{i,G}$ selecting the one that minimizes the function. This process is computed for all NP vectors and for all iterations settled by the user. The algorithm runs until the maximum number of iterations is reached or another stopping criteria is settled.

Chapter 4

Trading System

The trading system has been developed with R language code [1]. A total of 5 functions have been created:

1. Fitt: The trading system function, where data are analysed and it returns the result of the trading;

2. diff.evol: The Differential Evolution function;

3. pso.cf: The Particle Swarm Optimization function;

4. EFWA: The Enhance Firework Algorithm function;

5. tempo: An additional function that counts the time spent for each iteration.

The second, third and fourth function are explained in the Metaheuristic section. The code implemented with these functions have been adapted with little change to be suitable for the tasks required in the trading computation. The fifth function is a support function created for showing how much time each iteration takes to complete a cycle, with this function it is possible to

[1] https://www.r-project.org/about.html

compare not only the final result obtained by algorithms but also the speed to reach it.
Focusing in the first function, the trading system, it has been created to be highly personalized and to be used with different combination of indicators, parameters and weight of each signal. It requires 7 inputs:

1. dati: A matrix containing all the data required for the indicators selected. If all 5 indicators mentioned in chapter 2 are selected, 5 times series need to be given: Open, Close, High and Low prices and Volume;

2. param: A vector containing all parameters necessary to run indicators functions;

3. from: If a different starting point is selected respect to the starting date of the given time series;

4. to: If a different ending point is selected respect to the ending date of the given time series;

5. indi: The list of indicators that the user wants to test;

6. soglia: With this input the user can personalize how many indicators need to be agree before a buy or a sell signal is converted into an action;

7. weight: It is possible to give a different weight for each buy or sell signal of every indicator. If the user considers a buy or sell signal of a specific instrument less or more important respects to others a value different from one will be settled. However, the weight needs to be settled in comparison with the input 'soglia'.

The performance of the trading system is given by the total gain at the end of the period settled. A buy/sell signal is given when the level is equal or bigger/lower to the bound indicated by the parameter 'soglia'. Once the signal is received, the system buys at the closing price of the next day and keeps the stock until a sell signal is received. The system does not take short position.

Bollinger Bands The indicator gives the following signals:

$$signal(t) = \begin{cases} +1 & \text{if } SMA > UppenBand \\ -1 & \text{if } SMA < LowerBand \end{cases} \tag{4.1}$$

The parameters suggest in literature are: 20 days period for the Simple Moving Average, 2 standard deviation for the upper band and 2 standard deviation for the lower band. Both upper and lower bands follow the same standard deviation in the classic approach. Meanwhile, in the case proposed, a higher freedom has been given permitting the metaheuristic to find, if it gives better results, different amplitudes for the two lines. As showed in the system above a reverse crossover rule has been used. The optimization rule is settled as follows:

$$max\ f(t) \tag{4.2}$$

with the following constraints:

$$\begin{cases} 0.5 \leq UpperBand \leq 5 \\ 0.5 \leq LowerBand \leq 5 \\ 1 \leq SMA \leq 100 \end{cases} \tag{4.3}$$

where SMA period is an integer value, while, others are not. The value of the $UpperBand$ and $LowerBand$ refer to the parameter m that multiply the

σ of the Bollinger Band function.

MACD It has been adopted the cross-over strategy, in other words, buy and sell signals are generated when the MACD line overcomes the signal line. The rule can explained as follows:

$$signal(t) = \begin{cases} +1 & \text{if } MACD_t > SMA_t \ \& \ MACD_{t-1} < SMA_{t-1} \\ -1 & \text{if } MACD_t < SMA_t \ \& \ MACD_{t-1} > SMA_{t-1} \\ signal_{t-1} & \text{otherwise} \end{cases} \tag{4.4}$$

The buy signal is kept until a sell signal is given and vice versa.
Initially the literature suggests an EMA_{12} as "fast line", EMA_{26} as "slow line" and a SMA_9 as "signal line". However, the aim of these research is to search among a wide set of possible choices parameters which best optimize our problem. Then, the optimization problem becomes:

$$max \ f(t) \tag{4.5}$$

with the following constraints:

$$\begin{cases} 1 \leq EMA_{fast} \leq 26 \\ 1 \leq EMA_{slow} \leq 78 \\ 1 \leq SMA \leq 27 \end{cases} \tag{4.6}$$

regarding only this indicator. All the time periods selected require integer values.

Parabolic Stop and Reverse The SAR as all others trend-following, break out indicators generates a buy signal when the upper line is broken by the price line and vice versa a sell signal once the price overcomes the lower line. The rule is defined as follows:

$$signal(t) = \begin{cases} +1 & \text{if } SAR_t < Price_t \ \& \ SAR_{t-1} > Price_{t-1} \\ -1 & \text{if } SAR_t > Price_t \ \& \ SAR_{t-1} < Price_{t-1} \\ signal_{t-1} & \text{otherwise} \end{cases} \tag{4.7}$$

The literature propose an acceleration factor $\alpha = 0.02$ that increase each time up to the maximum of 0.2. The optimization problem is settled as follows:

$$max f(t) \tag{4.8}$$

with the following constraints:

$$\begin{cases} 0.005 \leq \alpha \leq 0.05 \\ 0.05 \leq ExtremePoint \leq 0.5 \end{cases} \tag{4.9}$$

regarding only SAR. Both parameters are not integers values

William%R The literature suggests to pay particular attention when the William%R indicator's value reaches the overbought and oversold areas settled at -20 and -80, respectively. In the trading system, this suggestion has been converted into a buy and sell rule. When one threshold is reached, buy and sell signals are given. It has been used the function from the package TTR which uses a different set of values and identifying the overbought condition for figures below 20 and oversold condition for figures above 80. [2]

[2]The package takes the formula from https://www.fmlabs.com/reference/default.htm?url=WilliamsR.htm

Such action can be summarized as follows:

$$signal(t) = \begin{cases} +1 & \text{if } WPR_t > 80 \text{ \& } WPR_{t-1} < 80 \\ -1 & \text{if } WPR_t < 20 \text{ \& } WPR_{t-1} > 20 \\ signal_{t-1} & \text{otherwise} \end{cases} \tag{4.10}$$

Larry William suggests to select a period of 14 days with -20 and -80 as overbought and oversold bounds area. In the optimization problem proposed, buy and sell areas are the one settled by the TTR's function that it is just a positive expression of the Larry William's boundaries, however, a wide selection period range has been proposed. The optimization problem becomes:

$$max\ f(t) \tag{4.11}$$

with the following constraints:

$$1 \leq n \leq 45 \tag{4.12}$$

regarding only this indicator. The time period selected is an integer value.

Money Flow Index The MFI, as explained in chapter 2, does not have a precise level to consider the indicator figure inside an overbought or oversold area. The creator has defined them as 90 and 10, respectively, however, many traders and scholars prefer a more reasonable 80 and 20 because they are easier to reach respect to the creator's parameters set. Based on such opinion, the trading rule is summarized in the following way:

$$signal(t) = \begin{cases} +1 & \text{if } MFI_t < 20 \text{ \& } MFI_{t-1} > 20 \\ -1 & \text{if } MFI_t > 80 \text{ \& } MFI_{t-1} < 80 \\ signal_{t-1} & \text{otherwise} \end{cases} \tag{4.13}$$

As it has been done with the William%R indicator, boundary areas are left untouched but the selection period has been subject of metaheuristic optimization. The optimization problem is settled as follows:

$$max f(t) \tag{4.14}$$

with the following constraints:

$$1 \leq n \leq 45 \tag{4.15}$$

regarding only MFI. The time period selected is an integer value.

Combining all indicators together the profit/loss constrained function $f(t)$ that need to be optimized has the following boundaries:

$$\begin{cases} 0.5 \leq \textit{BB-UpperBand} \leq 5 \\ 0.5 \leq \textit{BB-LowerBand} \leq 5 \\ 1 \leq \textit{BB-SMA} \leq 100 \\ 1 \leq \textit{MACD-EMA}_{fast} \leq 26 \\ 1 \leq \textit{MACD-EMA}_{slow} \leq 78 \\ 1 \leq \textit{MACD-SMA} \leq 27 \\ 0.005 \leq \textit{SAR-}\alpha \leq 0.05 \\ 0.05 \leq \textit{SAR-ExtremePoint} \leq 0.5 \\ 1 \leq \textit{WPR-n} \leq 45 \\ 1 \leq \textit{MFI-n} \leq 45 \end{cases} \tag{4.16}$$

Mostly of these parameters require integer values, this means that the function is not continuous and non-derivable.

To recap, a buy o sell signal is converted into a real action when the sum of signals is above the level settled at the beginning, if the trading system uses more than one indicator. Initially, signals can take 3 value: +1, 0, -1 but one the first +1, -1 signal is returned all future signals will be a +1 or -1. Testing many times the algorithms, the best 'soglia' level found is +3 and -3, in other words, when the sum of all indicators signals reaches +3 or higher values the function buys the stock and keeps it until a sum of signals equal or lower than -3.

4.1 Stocks Selected

The stocks selected for the following analysis are Amazon (AMZN), Brf S.A (BRFS) and EUR/USD.

The idea to test the trading system on these securities is based on the paper "Computation intelligence and Financial Markets: A Survey and Future Directions" R.C. Cavalcante et al.(2016) [6] and "A literature review of technical analysis on stock markets" R.T.F. Nazàrio (2017) [31] which suggest to explore also emerging markets because they are not highly tested by literature. Based on this conclusion a stock from an advance market a stock from an emerging market and a currency will be tested.

The period selected range from 1st January 2007 to 1st April 2020, divided in sub-periods of 5 years for the in-sample test (Training period), 2 years for the out-of-sample test (Validation period 1), other 2 years the second out-of-sample test (Validation period 2) and the whole time series after the in-sample test for the third out-of-sample test (Validation period 3) of the parameters found.

Training period	Validation Period 1	Validation Period 2	Validation Period 3
1/1/2007-1/1/2012	1/1/2012-1/1/2014	1/1/2014-1/1/2016	1/1/2012-1/1/2020
1/1/2009-1/1/2014	1/1/2014-1/1/2016	1/1/2016-1/1/2018	1/1/2014-1/1/2020
1/1/2011-1/1/2016	1/1/2016-1/1/2018	1/1/2018-1/1/2020	1/1/2016-1/1/2020
1/1/2013-1/1/2018	1/1/2018-1/1/2020	1/1/2020-1/4/2020	1/1/2018-1/4/2020
1/1/2015-1/1/2020	1/1/2020-1/4/2020		

Only the last validation period is shorter than others to test parameters founded during a highly volatile situation as the Coronavirus Pandemic.

More than one validation period has been proposed to evaluate if the initial

set of parameter identified might be profitable only close to the training period, first two year a head, or it might result to be profitable in the following, second years after and whole time series.
The multiperiod test has been applied as suggest in the paper "Predictability of the simple technical trading rules: an out of sample test" J. Fang et al. (2014) [14] to avoid data snooping and statistical bias. They have showed that the technical analysis well-performed in their historical sample but out-of-sample performances did not perform well. They assess that technical analysis does not give good results and the only profit previous obtained are due to statistical bias because markets have become more efficient. To test if their conclusions are still valid with the optimized parameters indicators founded by metaheuristics approaches a similar time frame testing has been applied, however, stocks selected are different.

4.1.1 Amazon

Amazon, an American multinational company, is one of the biggest company in the world and is considered one of the four big tech companies. Its industry is Cloud-computing, E-commerce, Artificial Intelligence, Consumer electronics, Digital distribution and Grocery stores. It were founded by Jeff Bezos in July 1994 as an online book store market and than it has started to sell electronic, software, videogames and so on. Now it is the biggest online market place, Artificial Intelligence provider and cloud computing platform. As financial point of view, the company has had a steady increase of revenue since 2007 as for total assets and employees, meanwhile, the net income has fluctuated during the period taken into account due to the continue re-investment of the company liquidity for its expansion. [3].

The stock price, figure 4.1, in the time period selected shows a steady increase with some lateral fluctuation from 2007 to 2009 and between 2014 and 2015. There were some major price correction of 10 to 20 percent but in the end Amazon stocks had a considerably increase of its value.

It has been selected because it is a major company in a high industrialise country. It is highly liquid and does not present large difference of price between bid and ask price.

Figure 4.1: Amazon closing price time series. Graph made with R-Studio

[3]https://en.wikipedia.org/wiki/Amazon_(company)

4.1.2 BRF S.A

BRF S.A. is a Brazilian society and one of the biggest food company in the world with branches in 150 countries over all 5 continents. It is the result of the merge of two major Brazilian companies, Sadia and Perdigao, in 2009. The main sector is food processing with meats, processed food, margarines, pastas, pizzas and frozen vegetables as main products. It is listed in two stock market, the Brazilian stock market Novo Mercado of BM&FBovespa (BRFS3) and New York Stock Exchange.

As a financial point of view, the company has a rapid increase of revenue from 2007 with a peak in 2014 and then a rapid decline in 2016 when, from that year, the revenue has started to fluctuate [4]. In the period selected the net income started with a decline until 2009 when a rapid increased took place with a peak in 2011. The following year the revenue halved and then doubled in 2014 staying stable only for 1 year. From 2015 up to 2018 the company had losses and concluded with a small positive income only in 2019 [5].

In this analysis it is considered the BRFS quoted in New York Stock Exchange due to the higher liquidity and to keep the confront with the dollar as the main currency. The stock price, figure 4.2, increased in the first year and then plummeted in the middle of 2008 due to the world crisis. From April 2009, BRF S.A.'s price raised and declined following the movement of the revenue with a peak in 2014 over 26 dollar and a bottom price around 5 in 2018. With the Corona Virus crisis the price has declined fast reaching

[4] https://www.macrotrends.net/stocks/charts/BRFS/brf-sa/revenue

[5] https://www.macrotrends.net/stocks/charts/BRFS/brf-sa/net-income

the lowest price ever under 3 dollar per stock.
It has been selected because it is a major company but in an emerging market. Being a big company permits to trade the stock without suffering of liquidity problem and at the same time false and miss-price error due to the liquidity.

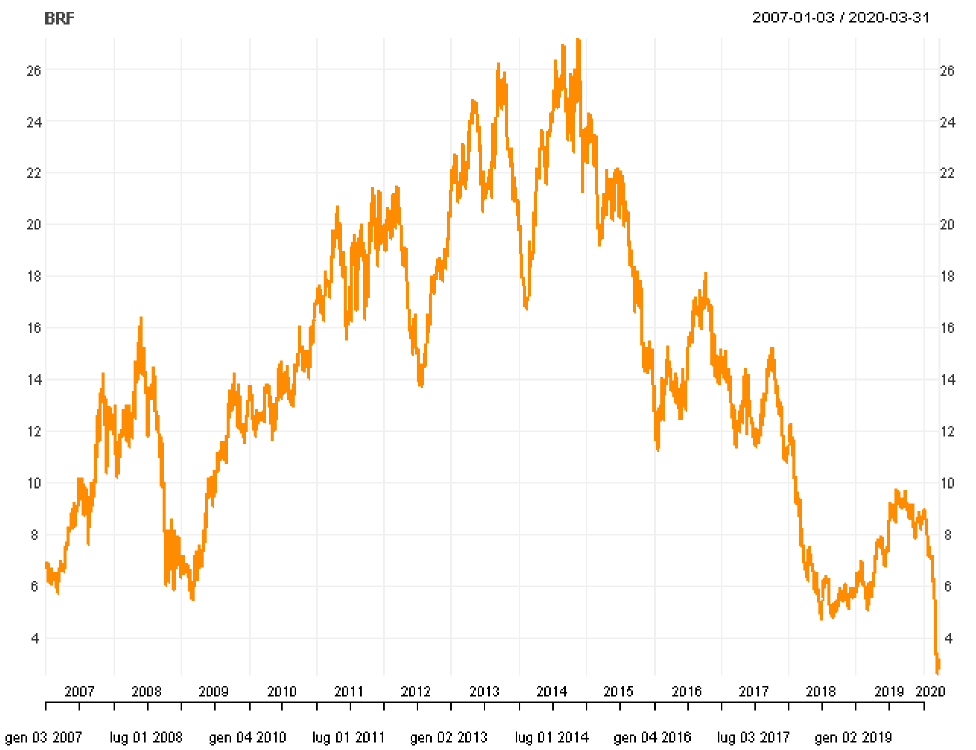


Figure 4.2: BRF S.A. closing price time series. Graph made with R-Studio

4.1.3 Euro/Dollar

The exchange rate is the rate at which one currency is exchanged for another currency, in other words, how much you have to pay one currency with another currency. The price is decided in the foreign exchange markets (Forex) that is open 24 hours on 24 from Monday to Friday.

Euro and Dollar are the two major currencies in the world. Being the biggest currencies mean also that the EUR/USD is the most liquid market in the world. This market, however, it is not free because it is managed by two central banks, the European Central Bank and the FED with which their operations influence the price among the two currencies.

In the analysis proposed the EUR/USD has been considered because of its liquidity and to continue to test the effectiveness of the metaheuristic algorithms in market different from the stock market.

The price, figure 4.3, presents an high volatility, it had its peak in the middle of 2008 before the crisis at almost 1.6 Euro for Dollar and than it collapsed under 1.3 in 3 months. From that moment the exchange rate highly fluctuated between 1.2 and 1.5 for 6 years. Since 2015 the price level moved in the area from 1.05 and 1.2 with the bottom price of 1.042 reached at the end of 2016.

Figure 4.3: EURUSD closing price time series. Graph made with R-Studio

4.2 Selection of metaheuristics' parameters

Among the metaheuristics selected to improve indicators previous mentioned a wide choice of metaheuristics' parameters should be tested. As introduce in the third chapter, each metaheuristic has a set of parameters that is not fixed but they should be modify to find the best optimum in the area in which they are applied. Testing all parameters would require an amount of time unsustainable with the computational power at disposition. However, there are many papers that give some guidelines of the most performing sets of parameters, based on them and with additional tests the results illustrated in the following subchapter have been found.
All tests have been carried out with Amazon stock from 1/1/2015 to 1/1/2020.

4.2.1 PSO parameters

The Particle Swarm Optimization algorithm has 5 parameters that need to be explored:

1. Number of iterations: how many times the algorithm need to run. The limit is settled by the user or if another condition is reached. In the analysis proposed the following additional condition is imposed: when the algorithm complete 3 cycles without having an improve above 0.000001 the algorithm stops to work;

2. Swarm population: there is no limit of the dimension of the population;

3. Initial w: the velocity of the swarm. There are no bounds specification;

4. ϕ_1: the velocity of the best personal position. There are no bounds specification;

5. ϕ_2: the velocity of the best global position. There are no bounds specification.

Parameters tested are proposed by the paper "Good parameters for Particle Swarm Optimization" of M.E.H. Pederson (2010) [34] and "Exploration and exploitation in global optimization problems: a comparison between PSO and Fireworks" of C. Pizzi et al.(...).
See Table A.1 in the appendix.
The comparison has been carried out selecting all indicators for a total of 10 indicators' parameters analysed. Each test has been computed 5 times and then average, maximum and minimum results are showed and have been compared with the benchmark, a Buy-&-Hold strategy. The results show the final value of 1 dollar of investment at the end of the period.
Looking at the data the use of a population of 100 with $w = 0.279, \phi_1 = 1.49168$ and $\phi_2 = 1.49168$ give the best results. This set of parameters has already given good results with a smaller population but with the increase of the swarm the average returns increase even more. It is also noticeable to say that the algorithm almost has never reached 1000 iterations but ended before with the reach of the second break condition. In most of the cases it converged before 400 iterations and only in few cases it went further without improving considerably the final result but with a very high time cost.

4.2.2 EFWA parameters

The Enhanced Fireworks algorithm has 3 parameters that need to be explored:

1. Number of iterations: how many times the algorithm need to run. The limit is settled by the user or if another condition is reached. In the analysis proposed the following additional condition is imposed: when the algorithm complete 3 cycles without having an improve above 0.000001 the algorithm stops to work;

2. Number of fireworks: there is no limit of the dimension of the population;

3. m: it is used to control the number of sparks for each firework. Higher is the number of fireworks larger need to be 'm' to generate the same amount of sparks.

The parameters that have been tested are proposed by the paper "Exploration and exploitation in global optimization problems: a comparison between PSO and Fireworks" of C. Pizzi et al.(...) together with additional parameters set.
See Table A.2 in the appendix.
The comparison has been carried out selecting all indicators for a total of 10 indicators' parameters analysed. Each test has been computed 5 times and then average, maximum and minimum results are showed as comparison with the benchmark, a Buy-&-Hold strategy. The results show the final value of 1 dollar of investment at the end of the period.

It appears that 5 fireworks are not enough to explore all the research space in contrast with the parameters set proposed initially by C. Pizzi et. al., this might be due to the increase and difference of the solution space this time proposed. However, with the increase of number of fireworks better results are achieved. Looking at the results seem that 50 fireworks and $m = 200$ perform as good as 100 fireworks with $m = 400$ but when they were retested increasing the number of test from 5 to 100 the average results show a different view. The case with lower fireworks does not perform as good as the other one on average. The false initial result might be due to a good random fireworks initialization that when the number of repetition increase did not last. In the end, 100 fireworks with $m = 400$ is the best set of parameters even if the computational time necessary to complete a cycle is higher that others. It is also important to say that the algorithm almost never reached 100 iterations but it ended before with the reach of the second break condition.

4.2.3 Differential Evolution parameters

The Differential Evolution algorithm has 4 parameters that need to be explored:

1. Number of iterations: how many times the algorithm need to run. In this case there is not a second condition that might interrupt prematurely;

2. np: it is the number of vectors generated, there is no upper limit on the number of vectors but the minimum is 4;

3. F: a constant that controls the amplification of the differential variation with a minimum of 0 and a maximum of 2;

4. CR: crossover constant ranging between 0 and 1.

Parameters tested are proposed by the paper "An Analysis of the Operation of Differential Evolution at High and Low Crossover Rates" J.Montgomery et al. (2010) [29], "Good parameters for differential evolution" M.E.H Pedersen et al.(2010) [33] and "Differential evolution: Difference vectors and movement in solution space" J.Montgomery (2009) [28].
See Table A.3 in the appendix.
The comparison has been carried out selecting all indicators for a total of 10 indicators' parameters analysed. Each test has been computed 5 times and then average, maximum and minimum results are showed as comparison with the benchmark, a Buy-&-Hold strategy. The results show the final value of 1 dollar of investment at the end of the period.
Tests have been carried out with all parameters suggested in papers previous mentioned. Looking at the results taking as first comparison the standard 100 iterations, the algorithm seems to perform better with high value of CR, meanwhile, the F parameter does not influence results except for the outstanding results obtained with $F = 0.2$ and $F = 0.5$. Focusing on the same parameters' results but with an higher number of iterations the average of returns increase considerably among all the set of parameters and contemporarily the computation time required. The average returns are still higher with large value of CR and the best mean is still obtained by the combination $CR = 0.9, F = 0.2$.

Increasing the number of iterations up to 1000 on the best set of parameters and observing the convergence it results that convergence has been reached, on average, before 100 iterations. In conclusion, in line with the previous considerations the set $CR = 0.9, F = 0.2$ with 100 iterations is the fastest and most performing combination of parameters.

Chapter 5

Results

In this chapter there will be showed the results of the application of the algorithms illustrating their performance in the in-sample and out-of-sample analysis. To recap, the in-sample analysis has been computed with 5 period of 5 years, meanwhile, the out-of-sample analysis has been calculated, where it is possible, first in the next two years than the following two years and the last in the full period from the day after the in-sample-analysis to the last day available.

The parameters that have been applied in the metaheuristics algorithms are:

Differential Evolution	parameters	NP	iterations	F	CR	Indicators
Amazon	10	100	100	0.2	0.9	ALL
Brf S.A	10	100	100	0.2	0.9	ALL
EUR/USD	9	100	100	0.2	0.9	No MFI

Table 5.1: Differential Evolution parameters

PSO	Param.	Iter.	Swam	W	B. Pers.	B. Glob.	Indicators
Amazon	10	400	100	0.279	1.49618	1.49618	ALL
BRF S.A	10	400	100	0.279	1.49618	1.49618	ALL
EUR/USD	9	400	100	0.279	1.49618	1.49618	No MFI

Table 5.2: Particle Swarm Optimization parameters

Enhance Fireworks	Parameters	Fireworks	Iterations	M	Indicators
Amazon	10	100	100	400	ALL
Brf S.A	10	100	100	400	ALL
EUR/USD	9	100	100	400	No MFI

Table 5.3: Enhance Firework parameters

The results that will be presented in the tables in the following sections are presented in the following way:

- Time: Range of time requires for each iteration in minutes;
- Average: Arithmetic average of the 100 iterations, result in percentage;
- Best: It is the best result in the 100 iterations, result in percentage;
- Worst: It is the worst result in the 100 iterations, result in percentage;
- B&H: It is the result if a Buy and Hold strategy is applied, result in percentage;
- Standard: It is the result with the standard set of parameters, result in percentage;
- Rank: Refers to the ten best in-sample set of parameters that are tested in the out-of-sample analysis;
- P1-P3: Bollinger Bands parameters;
- P4-P5: Parabolic Stop and Reverse parameters;
- P6-P8: MACD parameters;
- P9: William%R parameter;

- P10: Money Flow Index parameter.

The iterations just mentions this times refer to the number of times that the algorithm has been repeated. This methods is pursued to obtain a possible distribution of the solutions reached by the algorithms.

The results obtained in the in-sample analysis are computed reinvesting all the capital without fulfilling the unitary condition of the stock acquisition. In the out-of-sample analysis it has been used an initial portfolio with a starting capital of 10000 dollar. In this case no fractions of stocks are allowed. A similar analysis but with a capital of 1 dollar and with not unitary stock acquisition are showed in the Appendixes B,C and D. The values presented in all the tables are the net income/loss from the initial capital invested.

5.1 Amazon Algorithms results

5.1.1 PSO Amazon In-sample Results

Period	Time	Average	Best	Worst	B&H	Standard
2015-2020	6/50	7,099	10,481	4,877	4,98	0.422
2013-2018	6/50	5,836	7,914	4,479	3,544	0.621
2011-2016	6/50	5,852	7,699	4,118	2,668	0.223
2009-2014	6/50	8,356	11,512	5,492	6,336	0.463
2007-2012	6/50	8,848	16,972	2,850	3,472	0.958

Table 5.4: PSO In-sample Result

The application of the PSO algorithm, results showed in table 5.4, as indicated in the column 'Time' reaching a solution require a time that oscillate from 6 to 50 minutes. The algorithm, depending on the initial random allocation or for the difficult to overcome a local minimum, might take a short period of time to find a solution or it could continue running until the maximum number of iteration has been reached taking a considerably high amount of time. The reaching of the stopping criteria of the maximum number of iterations does not mean that the solution found is optimal, similarly, leaving the algorithm free without a maximum limit does not mean that it will converge into the global minimum. However, mostly of the time the second stopping criteria, in other words, when the algorithm does not have any progress on the solution, has been fulfilled. On average to compute 100 solutions the algorithm took 36 hours.

Focusing of the solution, on average the algorithm always beat the Buy and Holding strategy and the standard set of parameters. The best results show outstanding returns mostly of the time more than double of the Buy and Hold strategy, meanwhile, only in two worst case period results the algorithm do not beat the market.

5.1.2 PSO Amazon Out-of-sample Results

The application of the best ten set of parameters found in the period 2007-2012 seems not to obtained outstanding results, table 5.6, as in the in-sample analysis. In the first two years only in 1 case we obtained a result better than Buy and Hold strategy, however, respect to the standard set of parameter the gain is always better. Better results are obtained in the second period from 1/1/2014 up to 1/1/2016 where 6 over 4 set of parameters beat the B&H strategy and as in the previous period the standard set almost does not generate a gain. The last three columns results which take into account a very long period are in line with the first observation, a worst outcome respect to B&H and a substantial gain compared to the standard set of parameters. It seems that a set of parameters with values that are lower respect to the standard set (second rank in table 5.5) is able to promptly follow the high volatility of Amazon time series entering and leaving the market in the correct time. However, a faster set of parameters is not the best choice for all the time series as it could be notice in the values of the second biennium.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	18	1,046	1,481	0,034	0,306	18	12	25	21	7
2°	21	1,426	2,978	0,038	0,500	4	12	5	9	3
3°	19	1,214	2,846	0,010	0,351	10	14	13	41	43
4°	20	1,145	3,077	0,010	0,500	11	12	15	41	28
5°	19	1,233	3,018	0,010	0,327	12	21	8	41	39
6°	44	1,457	1,697	0,050	0,368	26	24	13	14	21
7°	40	1,419	1,670	0,047	0,105	23	12	12	16	43
8°	42	1,485	1,728	0,050	0,247	26	12	12	16	29
9°	37	1,000	1,522	0,050	0,180	26	12	3	14	12
10°	37	1,000	1,605	0,050	0,105	25	19	3	16	14

Table 5.5: PSO parameters'set 2007-2012

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
8526,6	12086,8	714,8	6275,9	6948,0	177,5	42234,3	91784,5	10006,6
12399,6	12086,8	714,8	6973,1	6948,0	177,5	79086,9	91784,5	10006,6
10217,9	12086,8	714,8	4992,6	6948,0	177,5	67825,8	91784,5	10006,6
10031,9	12086,8	714,8	3660,6	6948,0	177,5	60018,3	91784,5	10006,6
9485,9	12086,8	714,8	4992,6	6948,0	177,5	65078,5	91784,5	10006,6
4493,4	12086,8	714,8	7674,7	6948,0	177,5	40475,3	91784,5	10006,6
5831,0	12086,8	714,8	10567,8	6948,0	177,5	59338,5	91784,5	10006,6
3858,5	12086,8	714,8	10567,8	6948,0	177,5	40726,8	91784,5	10006,6
5610,6	12086,8	714,8	9209,5	6948,0	177,5	57408,7	91784,5	10006,6
4587,3	12086,8	714,8	8575,2	6948,0	177,5	50146,8	91784,5	10006,6

Table 5.6: PSO Out-of-sample Results

The time series in the second in-sample period analysed ranging from 1/1/2009 to 1/1/2014 has improved on average the set of parameters, table 5.7. They beat both standard and B&H strategy in the first out-of-sample period, table 5.8. However, this improvement do not last for long, in fact, in the out-of-sample period from 1/1/2016 to 1/1/2018 the optimised parameters result to beat the market only in 3 cases. On the other hand, the new set of parameter still beat the standard 8 times over 10. Focusing in the longer period, the last 3 columns, the new parameters obtain good returns close to the B&H in 4 cases and beat it in one situation. There is still an outstanding improvement respect to the classic set.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	16	1,000	3,560	0,038	0,050	22	12	22	20	45
2°	16	1,000	4,074	0,038	0,444	26	15	18	20	38
3°	5	1,000	3,428	0,034	0,500	8	12	4	20	24
4°	5	1,000	3,640	0,034	0,300	7	12	4	20	45
5°	5	1,000	3,446	0,034	0,053	4	12	7	19	45
6°	29	1,000	5,000	0,049	0,266	4	12	3	15	26
7°	30	1,021	3,848	0,049	0,353	4	12	3	15	26
8°	24	1,021	3,620	0,035	0,260	25	17	15	22	18
9°	24	1,003	4,186	0,035	0,495	24	19	14	22	18
10°	16	1,000	1,775	0,038	0,319	26	12	18	22	45

Table 5.7: PSO parameters'set 2009-2014

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
8785,2	6948,0	177,5	5225,6	7987,2	3915,6	26157,0	36246,7	8557,7
8785,2	6948,0	177,5	5225,6	7987,2	3915,6	26157,0	36246,7	8557,7
9093,0	6948,0	177,5	5020,7	7987,2	3915,6	34143,2	36246,7	8557,7
9093,0	6948,0	177,5	5020,7	7987,2	3915,6	34143,2	36246,7	8557,7
9093,0	6948,0	177,5	6662,3	7987,2	3915,6	40091,2	36246,7	8557,7
7605,4	6948,0	177,5	9609,5	7987,2	3915,6	34228,5	36246,7	8557,7
7460,9	6948,0	177,5	9609,5	7987,2	3915,6	34376,5	36246,7	8557,7
6532,6	6948,0	177,5	3629,5	7987,2	3915,6	16890,8	36246,7	8557,7
6563,1	6948,0	177,5	4749,8	7987,2	3915,6	18867,9	36246,7	8557,7
6428,3	6948,0	177,5	397,1	7987,2	3915,6	13183,4	36246,7	8557,7

Table 5.8: PSO Out-of-sample Results

Applying again the PSO in the period from 2011 to 2016 seems to penalize the predictability of the new set of parameters, table 5.9, in the next two years, column 1-3 table 5.10, respect to the one proposed in table 5.7. The PSO has found in the training period a set of parameters that reflect a slow reaction to the market movements with mostly of the parameters considerably bigger than the standard ones. However, the returns obtained still beat the one of the standard set 8 times over 10. It might be possible that there has been a sudden change of the time series behaviour with high volatility that could have been caught with a faster combination of parameters. Such conduct it can not be taken into consideration if the training period behave to much differently from the out of sample period.

In the following two years the strategy poorly perform, it still generates positive returns but only in 3 cases they are better of the standard set and no one beat the Buy and Hold strategy. Taking into consideration the whole period, 1/1/2016-1/1/2020, it shows improvement for the strategy propose that generate mostly a higher return respect to the standard set but not enough to beat the B&H.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	30	1,000	3,907	0,039	0,339	24	13	22	25	16
2°	31	1,000	2,938	0,044	0,493	18	15	27	26	9
3°	37	1,183	4,233	0,038	0,493	6	12	6	28	18
4°	60	1,750	1,932	0,044	0,078	26	12	3	25	16
5°	30	1,000	2,248	0,041	0,050	25	13	27	31	3
6°	45	1,002	2,243	0,048	0,339	4	38	19	45	10
7°	44	1,000	2,603	0,049	0,175	4	36	19	45	10
8°	44	1,000	5,000	0,048	0,141	4	33	22	45	10
9°	61	1,772	5,000	0,044	0,500	26	12	3	24	18
10°	16	1,007	3,013	0,039	0,147	26	19	16	20	24

Table 5.9: PSO parameters'set 2011-2016

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
3510,8	7987,2	3915,6	682,3	5270,6	3123,1	8113,1	18162,7	8299,7
4169,3	7987,2	3915,6	3080,0	5270,6	3123,1	9243,6	18162,7	8299,7
4729,2	7987,2	3915,6	2627,7	5270,6	3123,1	10721,7	18162,7	8299,7
6608,3	7987,2	3915,6	1161,9	5270,6	3123,1	12244,6	18162,7	8299,7
4126,3	7987,2	3915,6	268,6	5270,6	3123,1	7076,6	18162,7	8299,7
6393,9	7987,2	3915,6	4072,6	5270,6	3123,1	13178,8	18162,7	8299,7
6393,9	7987,2	3915,6	4072,6	5270,6	3123,1	13178,8	18162,7	8299,7
6393,9	7987,2	3915,6	3434,9	5270,6	3123,1	12078,5	18162,7	8299,7
5768,1	7987,2	3915,6	1266,2	5270,6	3123,1	10995,4	18162,7	8299,7
3066,8	7987,2	3915,6	1593,8	5270,6	3123,1	9082,5	18162,7	8299,7

Table 5.10: PSO Out-of-sample Results

Focusing on the set of optimized parameters obtained from the in-sample analysis between 1/1/2013 and 1/1/2018, table 5.11, they still do not gain as much as the other strategies and the results, table 5.12, are similar with that showed in table 5.10 4th column. However, in a period of high volatility as the first quarter of the 2020 the optimise sets do not lose as much as the standard set and in two cases it gains more than the Buy and Hold strategy. Merging the two out-of-sample periods and retesting the new parameters shows that the strategy propose can deal with the high volatility generating better results respect to the standard set but not enough to beat the Buy and Hold strategy.

These results might be due to a combination of parameters too specific to be adapted outside the training period. In fact, value of the parameters' indicators closer to the standard ones such as the set ranked in 9th position in the table 5.11 generate higher returns.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	14	1,000	2,253	0,048	0,452	4	12	3	15	14
2°	12	1,368	4,025	0,027	0,134	4	12	9	16	23
3°	9	1,796	4,411	0,022	0,179	10	21	3	17	23
4°	15	1,271	5,000	0,027	0,485	9	52	5	17	23
5°	12	1,387	3,342	0,027	0,224	18	68	3	17	23
6°	12	1,375	4,134	0,026	0,415	19	64	3	17	23
7°	14	1,362	3,851	0,026	0,380	4	78	3	19	29
8°	12	1,408	2,559	0,019	0,284	25	26	12	18	45
9°	5	1,178	2,653	0,025	0,307	8	12	12	16	23
10°	12	1,407	4,995	0,026	0,280	8	29	8	19	45

Table 5.11: PSO parameters'sets 2013-2018

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
1630,7	5270,6	3123,1	482,2	258,5	-957,8	2363,0	6085,7	1540,3
2336,9	5270,6	3123,1	-487,6	258,5	-957,8	2948,2	6085,7	1540,3
2568,2	5270,6	3123,1	0,0	258,5	-957,8	3179,5	6085,7	1540,3
4714,7	5270,6	3123,1	-487,6	258,5	-957,8	5427,9	6085,7	1540,3
1745,6	5270,6	3123,1	-487,6	258,5	-957,8	2356,9	6085,7	1540,3
1745,6	5270,6	3123,1	-487,6	258,5	-957,8	2356,9	6085,7	1540,3
1737,3	5270,6	3123,1	-487,6	258,5	-957,8	2348,6	6085,7	1540,3
2313,5	5270,6	3123,1	-487,6	258,5	-957,8	2924,8	6085,7	1540,3
4477,0	5270,6	3123,1	561,3	258,5	-957,8	5190,1	6085,7	1540,3
1916,0	5270,6	3123,1	-399,8	258,5	-957,8	1654,0	6085,7	1540,3

Table 5.12: PSO Out-of-sample Results

The last training period proposed has been computed only to see if it can produce better results respect to the previous set and if these last two years are enough to prepare the strategy to deal with the high volatility of the coronavirus pandemic.

The new set of parameters, table 5.13, generates better results, table 5.14, respect to the one in table 5.12, in fact, half of them are able to obtain a profit and the other half lose less respect to before. It is noticeable to say that the optimized parameters always perform better than the standard set but still not enough to beat the Buy and Hold strategy.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	23	1,150	3,931	0,008	0,387	14	12	6	14	32
2°	24	1,152	4,015	0,008	0,192	18	12	5	14	15
3°	10	1,044	2,818	0,030	0,053	7	12	3	13	13
4°	13	1,548	4,295	0,008	0,395	26	12	7	4	3
5°	25	1,086	2,558	0,008	0,311	18	12	3	14	45
6°	39	1,462	4,118	0,008	0,314	15	12	6	14	9
7°	22	1,000	2,542	0,007	0,341	14	12	3	15	23
8°	36	4,512	1,247	0,008	0,175	24	15	6	11	6
9°	11	1,568	4,999	0,007	0,050	13	12	3	17	42
10°	93	1,000	1,614	0,008	0,464	25	12	5	3	3

Table 5.13: PSO parameters'sets 2015-2020

1/2020 4/2020	B&H	Stand.
162,9	258,5	-957,8
162,9	258,5	-957,8
-487,6	258,5	-957,8
471,0	258,5	-957,8
-27,3	258,5	-957,8
-27,3	258,5	-957,8
-27,3	258,5	-957,8
240,5	258,5	-957,8
520,5	258,5	-957,8
-159,2	258,5	-957,8

Table 5.14: PSO Out-of-sample Results

5.1.3 EFWA Amazon In-sample Results

Period	Time	Average	Best	Worst	B&H	Standard
2015-2020	20/45	8,040	10,481	5,312	4,98	0.422
2013-2018	20/45	6,508	7,809	5,067	3,544	0.621
2011-2016	20/45	5,959	7,785	5,010	2,668	0.223
2009-2014	20/45	8,767	12,241	6,125	6,336	0.463
2007-2012	20/45	10,563	14,295	3,875	3,472	0.958

Table 5.15: EFWA In-sample Result

The application of the Enhance Fireworks Algorithm, results showed in table 5.15, takes a substantial amount of time for each test in line with the PSO algorithm. Differently from the PSO the maximum require almost 45 minutes, meanwhile, the minimum takes at least 20 minutes to reach one of the two stopping criteria. To compute 100 tests in average the algorithm took 42 hours. This heavier computational effort, however, produce in-sample results better than the PSO algorithm results. The column 'best' shows a gain at least 2 times bigger than the Buy and Holding strategy and even the worst results produce a performance higher than the B&H one.
Comparing the best, worst and average in-sample trading gains all of them are higher than the ones obtain by the PSO resulting into an higher efficiency of the algorithm to explore the research space, on the other hand, this higher profitability is penalised by the higher computation effort required.

5.1.4 EFWA Amazon Out-of-sample Results

The optimised sets of parameters perform well in the first period out of sample as showed in the first three columns in table 5.17. It is able to generate a profit considerably higher respects to the standard set of parameters, however, such gain is still behind the Buy and Hold strategy. In the second period from 1/1/2014 to 1/1/2016 the strategy 6 times over 10 obtained an higher gain respect to the B&H and always beat with a huge discrepancy the standard parameters' set. Testing the optimised set in the full time series, in other words, in the period between 1/1/1/2012 and 1/1/2020 our strategy perform quite well generating higher return in all the period but only one time is able to overcome the gain generated by the Buy and Hold strategy. Also in this case as showed in the table 5.17 and in line with the result obtained with the PSO in table 5.6 the strategy proposed always generate an higher return respect to the standard composition of the set of parameters. As it has been seen in the previous in-sample comparison between EFWA and PSO' s results, even in this out-of-sample test the Enhance Fireworks algorithm on average found better results respect to the Particle Swarm Optimization algorithm. Such outcome might be due to a composition of low values of the sets of parameters which were the ones that permit to perform better also for the PSO. However, not all the sets ,even if they are very similar, are good as it is noticeable in the last two rows in table 5.16 and their respective results in table 5.17

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	39	1,326	1,725	0,046	0,486	24	18	19	15	6
2°	20	1,093	2,911	0,010	0,198	14	15	7	41	10
3°	45	1,306	1,702	0,047	0,230	18	14	22	16	6
4°	39	1,475	1,644	0,047	0,399	19	14	3	16	6
5°	41	1,301	1,783	0,049	0,407	23	17	12	16	6
6°	40	1,188	1,674	0,047	0,438	22	19	19	16	6
7°	43	1,314	1,721	0,049	0,280	19	16	16	16	6
8°	20	1,095	2,879	0,010	0,321	10	19	8	41	10
9°	19	1,201	3,133	0,010	0,336	13	15	7	41	25
10°	19	1,202	3,038	0,010	0,367	12	13	9	41	24

Table 5.16: EFWA paramters'sets 2007-2012

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
5658,6	12086,8	714,8	10567,8	6948,0	177,5	39952,7	91784,5	10006,6
10439,8	12086,8	714,8	3775,6	6948,0	177,5	64153,1	91784,5	10006,6
10916,1	12086,8	714,8	10567,8	6948,0	177,5	97120,3	91784,5	10006,6
11548,2	12086,8	714,8	10567,8	6948,0	177,5	68481,5	91784,5	10006,6
7405,6	12086,8	714,8	10567,8	6948,0	177,5	47943,2	91784,5	10006,6
10891,1	12086,8	714,8	10567,8	6948,0	177,5	87856,4	91784,5	10006,6
7405,6	12086,8	714,8	10567,8	6948,0	177,5	79490,3	91784,5	10006,6
10439,8	12086,8	714,8	3775,6	6948,0	177,5	64153,1	91784,5	10006,6
5742,4	12086,8	714,8	4992,6	6948,0	177,5	50282,5	91784,5	10006,6
10217,9	12086,8	714,8	4992,6	6948,0	177,5	67825,8	91784,5	10006,6

Table 5.17: EFWA Out-of-sample Results

The parameters set generated from the in-sample training period from 2009 to 2014, table 5.18, do not help to find better results. In the out-of-sample analysis, in fact, the returns generated in the first two years are not enough to beat on average the Buy and Hold strategy, however, the gains produced result to be very close. On the other hand, the gains generated are still considerably higher respect to the standard set. It is possible that a more general combination of parameters generated by a lower number of iterations in the algorithm might have produced a combination of parameters more suitable for the first out-of-sample period. This point of view born observing the last three rows in table 5.18 which returns are the highest but the values are not to far from the ones ranked in higher positions. However, such combination it would have not last for long. In fact such results do not last in the period ranging from 1/1/2016 to 1/1/2018 where the strategy proposed is not able to beat the standard set 6 times over 10. Focusing on the six years ahead from the in-sample time series the optimised set of parameters obtained always a positive gain, almost the double of the standard set, but half respect to the Buy and Hold strategy. In this time frame the EFWA is not able to continue to produce better performances respect to the PSO algorithm even if its in-sample gains resulted to be better.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	24	1,026	1,567	0,035	0,115	24	19	14	22	18
2°	24	1,026	1,573	0,036	0,090	21	20	15	22	17
3°	24	1,006	1,631	0,035	0,126	23	21	13	22	16
4°	24	1,001	1,541	0,036	0,424	19	15	20	22	17
5°	24	1,008	1,512	0,037	0,383	22	16	16	23	15
6°	18	1,082	1,333	0,038	0,248	26	21	11	23	16
7°	22	1,115	1,445	0,037	0,180	21	19	16	22	15
8°	16	1,020	2,248	0,038	0,326	23	21	15	20	28
9°	21	1,047	2,865	0,037	0,395	22	15	22	20	37
10°	22	1,089	3,005	0,036	0,470	24	16	20	20	29

Table 5.18: EFWA paramters'sets 2009-2014

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
6532,6	6948,0	177,5	3629,5	7987,2	3915,6	16890,8	36246,7	8557,7
6532,6	6948,0	177,5	4501,2	7987,2	3915,6	17644,8	36246,7	8557,7
6563,1	6948,0	177,5	4749,8	7987,2	3915,6	18867,9	36246,7	8557,7
6563,1	6948,0	177,5	4749,8	7987,2	3915,6	18867,9	36246,7	8557,7
6499,7	6948,0	177,5	3640,1	7987,2	3915,6	22378,9	36246,7	8557,7
3672,3	6948,0	177,5	194,6	7987,2	3915,6	7509,3	36246,7	8557,7
6018,6	6948,0	177,5	387,1	7987,2	3915,6	12571,2	36246,7	8557,7
8785,2	6948,0	177,5	3066,8	7987,2	3915,6	21263,9	36246,7	8557,7
10884,7	6948,0	177,5	590,5	7987,2	3915,6	17382,9	36246,7	8557,7
10635,3	6948,0	177,5	992,0	7987,2	3915,6	17450,4	36246,7	8557,7

Table 5.19: EFWA Out-of-sample Results

Following the observation done in the previous out-of sample analysis, also in this case the out-of-sample results point out that the EFWA perform poorly. In the period from 1/1/2016 to 1/1/2018, as shown in table 5.21, the optimised set of parameters, table 5.20, still obtaining positive returns it performs worst than the previous set, table 5.18, in fact only two times the strategy beats the standard set of parameters and it never generates a gain higher than the Buy and Hold strategy. The same poor performances can be seen in the next two years, columns 3 to 6 table 5.21. Focusing in the longer period that analyse the two just mentioned bienniums together the strategy still is not able to beat both the other two strategies. These results are in contrast with what has been found by the PSO algorithm which is considerably able to beat the standard set 8 times over 10.

Even if the EFWA performed better than the PSO algorithm in the in-sample period the combination of parameters identified is not as good as the other algorithm. The problem might be re-conducted to a combination of parameters too specific for the in-sample time series that is not adapt for the out-of-sample period.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	43	1,208	4,442	0,044	0,439	26	23	25	34	9
2°	46	1,275	2,596	0,034	0,191	24	21	27	27	18
3°	15	1,073	4,074	0,039	0,137	24	23	16	20	28
4°	32	1,010	2,183	0,039	0,120	23	19	18	25	6
5°	46	1,260	3,573	0,040	0,065	6	12	6	37	16
6°	43	1,191	4,821	0,049	0,449	25	24	27	38	23
7°	31	1,033	2,713	0,043	0,162	19	16	22	25	7
8°	30	1,002	4,337	0,042	0,245	23	15	23	35	3
9°	16	1,025	2,533	0,035	0,319	21	18	17	21	21
10°	16	1,104	2,214	0,038	0,242	12	56	4	22	10

Table 5.20: EFWA paramters'sets 2011-2016

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
2456,6	7987,2	3915,6	1133,1	5270,6	3123,1	4025,3	18162,7	8299,7
1798,1	7987,2	3915,6	437,1	5270,6	3123,1	2254,6	18162,7	8299,7
4304,0	7987,2	3915,6	1654,4	5270,6	3123,1	11184,8	18162,7	8299,7
3546,0	7987,2	3915,6	-96,1	5270,6	3123,1	6758,1	18162,7	8299,7
3653,4	7987,2	3915,6	1520,2	5270,6	3123,1	6089,2	18162,7	8299,7
2601,9	7987,2	3915,6	2907,2	5270,6	3123,1	6574,0	18162,7	8299,7
3822,6	7987,2	3915,6	2335,7	5270,6	3123,1	7326,8	18162,7	8299,7
2124,2	7987,2	3915,6	3381,9	5270,6	3123,1	6707,0	18162,7	8299,7
2402,4	7987,2	3915,6	1593,8	5270,6	3123,1	7909,1	18162,7	8299,7
4158,7	7987,2	3915,6	2281,7	5270,6	3123,1	9516,8	18162,7	8299,7

Table 5.21: EFWA Out-of-sample Results

Moving to the next out-of-sample test, the new set of optimised parameters found running the algorithm in the in-sample period between 2013 to 2018, table 5.22, shows some improvement. In the first biennium ahead of 2018 the new set of parameters perform better than the standard one and sometime the returns are very close to the Buy and Hold strategy but not enough to beat it. Testing the strategy in the first quarter of 2020 generates 8 times over 10 a loss and the other two no trading signals, however, the losses are lower than the one of the standard set. Combining the two time series and testing them together appears to be a better option respect to consider them singularly, in fact, the strategy proposed performs highly better than the standard one and close to the Buy and Hold, as shown in table 5.23. In contrary with what has been found in the previous two out-of-sample analysis aforementioned the Enhance Firework algorithm allocates more efficiently the parameters in the space showing better results both in the in-sample and out-of-sample analysis than the PSO.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	16	1,230	4,112	0,027	0,381	13	14	5	17	23
2°	15	1,262	3,345	0,026	0,167	9	12	8	16	23
3°	15	1,266	3,859	0,027	0,281	7	20	6	17	23
4°	15	1,252	3,297	0,026	0,249	6	19	8	16	23
5°	15	1,258	4,229	0,023	0,312	5	23	6	16	23
6°	9	1,913	4,732	0,024	0,399	9	22	4	17	23
7°	12	1,361	3,009	0,027	0,112	7	30	3	17	23
8°	12	1,389	4,222	0,027	0,080	5	14	6	16	23
9°	12	1,346	3,998	0,026	0,324	6	23	3	16	23
10°	9	1,839	3,143	0,023	0,074	8	14	9	16	19

Table 5.22: EFWA paramters'sets 2013-2018

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
3989,3	5270,6	3123,1	-487,6	258,5	-957,8	4702,5	6085,7	1540,3
4714,7	5270,6	3123,1	-487,6	258,5	-957,8	5427,9	6085,7	1540,3
4714,7	5270,6	3123,1	-487,6	258,5	-957,8	5427,9	6085,7	1540,3
4714,7	5270,6	3123,1	-487,6	258,5	-957,8	5427,9	6085,7	1540,3
4714,7	5270,6	3123,1	-487,6	258,5	-957,8	5427,9	6085,7	1540,3
2853,7	5270,6	3123,1	0,0	258,5	-957,8	3465,0	6085,7	1540,3
2336,9	5270,6	3123,1	-487,6	258,5	-957,8	2948,2	6085,7	1540,3
3366,6	5270,6	3123,1	-487,6	258,5	-957,8	4079,7	6085,7	1540,3
3504,3	5270,6	3123,1	-487,6	258,5	-957,8	4217,5	6085,7	1540,3
3156,3	5270,6	3123,1	0,0	258,5	-957,8	3869,4	6085,7	1540,3

Table 5.23: EFWA Out-of-sample Results

The last training period from 1/1/2015 to 1/1/2020 has been proposed to see if the algorithm is able to predict somehow the high volatility that emerged due to the coronavirus pandemic. As report by the table 5.25 the last two years of training period permit to adapt the set of optimised parameter with more efficiency respect to the out-of-sample result showed in table 5.23. The strategy proposed 8 times over 10 generates positive returns respect to the negative one of the standard set of parameters. The PSO even in this case is behind the return of the EFWA as shown in table 5.14, only in 3 cases the PSO over perform the EFWA but on average the results are not as good as the one in table 5.23.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	23	1,179	2,561	0,008	0,165	15	13	5	14	14
2°	25	1,257	3,306	0,008	0,326	19	15	3	14	21
3°	26	1,188	3,696	0,008	0,285	21	14	3	14	21
4°	25	1,252	2,959	0,008	0,147	16	14	4	14	19
5°	25	1,115	2,813	0,008	0,242	18	13	4	14	18
6°	23	1,170	2,786	0,008	0,099	16	12	6	14	16
7°	23	1,197	2,733	0,008	0,338	17	14	4	14	14
8°	10	1,107	3,604	0,031	0,415	9	14	3	13	15
9°	26	1,172	2,543	0,008	0,362	15	14	4	17	7
10°	15	1,510	3,179	0,008	0,391	19	13	4	14	21

Table 5.24: EFWA paramters'sets 2015-2020

1/2020 4/2020	B&H	Stand.
162,9	258,5	-957,8
162,9	258,5	-957,8
162,9	258,5	-957,8
162,9	258,5	-957,8
-27,3	258,5	-957,8
162,9	258,5	-957,8
162,9	258,5	-957,8
-487,6	258,5	-957,8
162,9	258,5	-957,8
162,9	258,5	-957,8

Table 5.25: EFWA Out-of-sample Results

5.1.5 DE Amazon In-sample Results

Period	Time	Average	Best	Worst	B&H	Standard
2015-2020	11/12	9,677	12,203	5,345	4,98	0.422
2013-2018	11/12	6,534	7,256	5,123	3,544	0.621
2011-2016	11/12	6,117	7,451	4,352	2,668	0.223
2009-2014	11/12	9,020	11,530	6,872	6,336	0.463
2007-2012	11/12	10,739	14,908	9,041	3,472	0.958

Table 5.26: DE In-sample Results

The application of the Differential Evolution algorithm, results showed in table 5.26, as indicated in column 'Time' requires an amount of time considerably lower than PSO and EFWA. The algorithm on average is able to compute 100 iteration in 11/12 minutes, such results permit to calculate 100 cycle in almost 20 hours that is half of the time required by both PSO and EFWA. With half of the time necessary to converge into a solution the DE algorithm is the fastest among the metaheuristics proposed and the returns found are not far from the other two algorithms. Comparing the average return the DE rank first, finding higher profit in all 5 in-sample periods. Its primacy do not remain regarding its best results obtained because in 3 over 5 periods the EFWA and PSO is able to find a maximum higher than the one of DE. This means that the DE algorithm converges more times into local maximum and few times can reach the best global maximum, if the ones found by the other two algorithm is really the global maximum. However, the best results found are in 4 cases more than double of the Buy and Hold strategy such characteristic is in line with what has already been discovered with the other two algorithms. Focusing on the worst results, the Differential Evolution algorithm rank first with considerable difference from the other two algorithms. Such results explain why the algorithm on average obtain a higher return than the other even if it does not have the maximum return.

5.1.6 DE Amazon Out-of-sample Results

Analysing the results discovered, the optimised parameters sets, table 5.27, perform quite well in the first out-of-sample biennium obtaining 6 times over 10 results close to the Buy and Hold strategy and returns substantially higher respect to the standard set of parameters, table 5.28. Moving to the next two years the performances are not as good as the ones obtained by the other algorithms. DE still finds positive results larger than the standard set but only in two cases it can beat the Buy and Hold strategy. Observing the long times series, the last three columns of the table 5.28, the sets of parameters proposed has been able to record high returns respect to the standard set but not enough to beat the Buy and Hold strategy. In the whole period the differential evolution algorithm rank second, the EFWA seems to capture better the return in this first phase, meanwhile, the PSO is no to far from the DE.

The differential evolution as the EFWA has found an higher compositions of low parameters' values this permit as already discussed before a higher return in the first biennium.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	26	1,357	3,466	0,010	0,415	26	25	3	39	28
2°	25	1,297	3,354	0,010	0,074	5	2	11	35	28
3°	28	1,252	1,784	0,009	0,201	9	1	14	39	13
4°	20	1,092	3,266	0,010	0,340	11	13	9	41	10
5°	35	1,314	2,115	0,047	0,135	23	20	2	14	6
6°	21	1,193	3,483	0,010	0,256	12	14	8	41	10
7°	20	1,098	2,776	0,010	0,204	9	12	15	41	10
8°	20	1,103	3,193	0,010	0,453	11	18	9	41	10
9°	20	1,102	2,931	0,010	0,387	17	18	9	41	10
10°	20	1,096	3,186	0,010	0,295	9	16	11	41	10

Table 5.27: DE paramters'sets 2007-2012

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
0,0	12089,8	714,8	12559,8	6948,0	177,5	38711,0	91784,5	10006,6
7572,6	12086,8	714,8	6764,4	6948,0	177,5	61046,5	91784,5	10006,6
8045,9	12086,8	714,8	6344,3	6948,0	177,5	65064,8	91784,5	10006,6
11198,7	12086,8	714,8	3775,6	6948,0	177,5	66891,1	91784,5	10006,6
6217,9	12086,8	714,8	10567,8	6948,0	177,5	36211,1	91784,5	10006,6
10031,9	12086,8	714,8	3660,6	6948,0	177,5	63793,2	91784,5	10006,6
11198,7	12086,8	714,8	3775,6	6948,0	177,5	66891,1	91784,5	10006,6
11198,7	12086,8	714,8	3775,6	6948,0	177,5	66891,1	91784,5	10006,6
10439,8	12086,8	714,8	3797,8	6948,0	177,5	64352,8	91784,5	10006,6
11198,7	12086,8	714,8	3775,6	6948,0	177,5	66891,1	91784,5	10006,6

Table 5.28: DE Out-of-sample Results

Moving to the next time slot and analysing the results obtained applying the set of parameters, table 5.29, estimated by the in-sample training pe-

riod from 2009 to 2014, the first two out-of-sample years taken into account show higher returns respect to the ones found in table 5.28. The optimised set of parameters generate higher returns that Buy and Hold strategy and the standard set of parameters 6 times over 10, the remaining 4 results still show substantially better returns than the standard set, table 5.30. The next two years between 1/1/2016 and 1/1/2018 are not as good as the previous two, the strategy proposed is more profitable than the standard one but not enough to be compared with the Buy and Hold strategy. The same opinion can be assessed testing the optimised parameters in the longer period from 1/1/2014 up to 1/1/2020.
Studying all three algorithms' observations the PSO this time rank first having on average best returns, followed by the DE and at the end the EFWA.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	24	1,026	1,661	0,035	0,245	23	20	14	22	18
2°	24	0,967	1,552	0,035	0,134	26	19	13	22	15
3°	24	1,020	1,585	0,036	0,337	22	18	16	22	16
4°	16	0,956	1,394	0,038	0,076	24	20	12	22	17
5°	5	0,608	4,143	0,034	0,376	5	9	9	20	25
6°	5	0,577	3,772	0,036	0,292	8	11	4	20	24
7°	5	0,578	4,759	0,036	0,320	8	14	3	20	24
8°	16	0,936	3,004	0,038	0,265	24	13	20	20	31
9°	16	0,919	4,640	0,038	0,160	25	12	19	20	42
10°	16	0,982	3,732	0,038	0,167	17	14	27	20	29

Table 5.29: DE parameters'sets Results2009-2014

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
6532,6	6948,0	177,5	2425,8	7987,2	3915,6	14054,1	36246,7	8557,7
6499,7	6948,0	177,5	4749,8	7987,2	3915,6	25352,2	36246,7	8557,7
6532,6	6948,0	177,5	3203,3	7987,2	3915,6	15461,1	36246,7	8557,7
5490,8	6948,0	177,5	818,6	7987,2	3915,6	13723,3	36246,7	8557,7
9022,0	6948,0	177,5	5641,1	7987,2	3915,6	30542,7	36246,7	8557,7
7623,2	6948,0	177,5	5641,1	7987,2	3915,6	32041,9	36246,7	8557,7
7623,2	6948,0	177,5	5641,1	7987,2	3915,6	32041,9	36246,7	8557,7
8785,2	6948,0	177,5	3462,6	7987,2	3915,6	25348,5	36246,7	8557,7
8785,2	6948,0	177,5	5379,3	7987,2	3915,6	29580,8	36246,7	8557,7
8785,2	6948,0	177,5	5225,6	7987,2	3915,6	26157,0	36246,7	8557,7

Table 5.30: DE Out-of-sample Results

Observing the values illustrated in table 5.32 obtained by the set of parameters in table 5.31 resulting from the in-sample algorithm training from 2011 to 2016, they are not as good as before. The algorithm seems to struggle to find a set of parameters that outperform the standard one in the first biennium, in fact, only 5 over 10 are better than the others. This proportion reduces even more in the second biennium showing poorly returns but still positive. In all the period analysed the optimised parameters are not able to beat the Buy and Hold strategy. Better results are observed when the all period is taken into account, the new set of parameters overcome the returns generated by the standard one.

Comparing the algorithms observations even in this case the EFWA rank third, meanwhile, the Differential evolution and Particle Swarm Optimization algorithm perform better the in the whole period and in the first part, respectively.

As for the other two algorithms the DE struggles to find a set that beat the Buy and Hold and Standard set probability due to a combination of parameters too specific for the in-sample time series instead of adapting also for the out-of-sample period.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	58	1,420	2,420	0,039	0,295	3	1	2	26	20
2°	38	1,197	2,412	0,039	0,181	3	1	2	26	23
3°	17	0,742	1,815	0,039	0,253	24	17	17	23	12
4°	16	1,009	1,627	0,039	0,273	24	13	20	20	36
5°	17	0,740	1,438	0,039	0,152	26	11	24	23	13
6°	36	1,145	3,096	0,039	0,387	7	11	6	30	16
7°	41	1,297	2,961	0,034	0,159	5	7	9	28	25
8°	37	1,162	3,103	0,050	0,386	6	11	4	31	23
9°	36	1,146	3,762	0,050	0,305	6	8	6	31	23
10°	14	1,268	4,335	0,034	0,417	5	1	4	20	11

Table 5.31: DE parameters'sets Results2011-2016

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
6571,9	7987,2	3915,6	1689,6	5270,6	3123,1	11554,9	18162,7	8299,7
6712,6	7987,2	3915,6	3326,6	5270,6	3123,1	14872,6	18162,7	8299,7
1697,6	7987,2	3915,6	2886,7	5270,6	3123,1	9431,8	18162,7	8299,7
397,1	7987,2	3915,6	900,6	5270,6	3123,1	3960,4	18162,7	8299,7
3001,7	7987,2	3915,6	1706,6	5270,6	3123,1	9150,9	18162,7	8299,7
5405,7	7987,2	3915,6	2627,7	5270,6	3123,1	11884,9	18162,7	8299,7
3598,5	7987,2	3915,6	4304,1	5270,6	3123,1	11724,5	18162,7	8299,7
7792,4	7987,2	3915,6	2817,5	5270,6	3123,1	14089,5	18162,7	8299,7
7792,4	7987,2	3915,6	1100,7	5270,6	3123,1	10608,8	18162,7	8299,7
1533,3	7987,2	3915,6	1639,3	5270,6	3123,1	3369,6	18162,7	8299,7

Table 5.32: DE Out-of-sample Results

Focusing in the next period, the returns computed with the new sets of parameters, table 5.33, are more contained respect to the one obtained with the previous set, in the periods between 1/1/2018 and 1/1/2020. The optimised sets still generate some gains but far from both Buy and Hold and standard set, table 5.34. The same observations can be done taking into account the first quarter of 2020, the application of the parameters in the whole period produce a gain slightly lower than the standard set and far from the one of the Buy and Hold strategy. Better values are visible only if the first quarter of 2020 is taken into account, in fact, even if the strategy produce a loss it is less significant than the one produced by the standard strategy.

In comparison with the other two algorithms the DE achieves the worst results ranking third. The one that has been able to deal with the highly volatility period has been the EFWA. Even the PSO obtained good observations but not as good as the Fireworks algorithm.

In this case, mostly of the sets obtained by the DE algorithm are localised in the same area, in fact, from the 3rd to the 10th set they obtain the same returns. The maximum found by the DE in this case it is not optimal in the out of sample analysis but better results are obtained with the combination of parameters in top rank. Probability increasing the number of iteration leaving the algorithm explore more in the search area it would have permit to obtain returns.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	9	1,832	3,971	0,023	0,390	7	22	6	16	23
2°	12	1,397	4,379	0,025	0,347	3	26	7	17	23
3°	14	1,362	4,083	0,026	0,301	2	23	9	19	29
4°	14	1,366	3,528	0,026	0,252	2	13	14	19	26
5°	14	1,368	3,703	0,026	0,352	2	16	12	19	27
6°	14	1,358	3,781	0,026	0,246	10	21	2	19	35
7°	14	1,373	4,132	0,026	0,280	5	31	3	19	43
8°	14	1,375	3,416	0,026	0,398	11	19	2	19	24
9°	14	1,365	4,470	0,026	0,336	3	35	5	19	37
10°	14	1,361	3,218	0,026	0,209	2	18	12	19	25

Table 5.33: DE parameters'sets Results2013-2018

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
3156,3	5270,6	3123,1	0,0	258,5	-957,8	3869,4	6085,7	1540,3
2336,9	5270,6	3123,1	-487,6	258,5	-957,8	2948,2	6085,7	1540,3
1737,3	5270,6	3123,1	-399,8	258,5	-957,8	1475,2	6085,7	1540,3
1737,3	5270,6	3123,1	-399,8	258,5	-957,8	1475,2	6085,7	1540,3
1737,3	5270,6	3123,1	-399,8	258,5	-957,8	1475,2	6085,7	1540,3
1737,3	5270,6	3123,1	-399,8	258,5	-957,8	1475,2	6085,7	1540,3
1737,3	5270,6	3123,1	-399,8	258,5	-957,8	1475,2	6085,7	1540,3
1737,3	5270,6	3123,1	-399,8	258,5	-957,8	1475,2	6085,7	1540,3
1737,3	5270,6	3123,1	-399,8	258,5	-957,8	1475,2	6085,7	1540,3
1737,3	5270,6	3123,1	-399,8	258,5	-957,8	1475,2	6085,7	1540,3

Table 5.34: DE Out-of-sample Results

Testing the Differential Evolution in the last in-sample period has been optimal because the optimised set of parameters found, table 5.35, obtained a profit in the first quarter of the 2020 changing completely the values generated with the previous data. As can be observed in table 5.36, the new sets signal the same enter and exit points as in the majority of the sets of parameters found by the other two algorithms having the same final returns. This strategy permits to beat the standard set that shows a consistent loss but do not surpass the gain of the Buy and Hold strategy.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	15	1,475	4,754	0,008	0,409	15	2	2	14	31
2°	20	1,218	3,080	0,008	0,127	21	1	25	14	33
3°	20	1,217	3,331	0,008	0,450	18	1	27	14	36
4°	21	1,278	3,295	0,008	0,047	20	1	24	14	23
5°	20	1,285	3,592	0,008	0,103	20	2	18	14	15
6°	23	1,355	4,457	0,008	0,228	17	2	22	14	22
7°	23	1,384	3,763	0,008	0,378	20	1	25	14	42
8°	25	1,153	4,249	0,008	0,368	15	8	8	14	44
9°	23	1,101	2,873	0,008	0,392	15	8	9	14	21
10°	25	1,208	2,994	0,008	0,120	13	3	21	14	22

Table 5.35: DE parameters'sets Results 2015-2020

1/2020 4/2020	B&H	Stand.
162,9	509,4	-957,8
162,9	509,4	-957,8
162,9	509,4	-957,8
162,9	509,4	-957,8
162,9	509,4	-957,8
162,9	509,4	-957,8
162,9	509,4	-957,8
162,9	509,4	-957,8
162,9	509,4	-957,8
162,9	509,4	-957,8

Table 5.36: DE Out-of-sample Results

5.2 BRF S.A Algorithms results

5.2.1 PSO BRF S.A In-sample Results

Period	Time	Average	Best	Worst	B&H	Standard
2015-2020	6/50	1,737	3,168	0,937	-0,621	-0.47
2013-2018	6/50	1,989	3,072	1,209	-0,471	-0.279
2011-2016	6/50	1,851	3,366	0,982	-0,187	-0.156
2009-2014	6/50	4,699	7,463	2,871	2,055	0.837
2007-2012	6/50	5,719	10,401	3,651	1,869	0.514

Table 5.37: PSO In-sample Results

The application of the PSO algorithm, results showed in table 5.37, requires the same time in minutes estimated with the Amazon time series. The returns are always positive differently from what it is observable with the Buy and Holding strategy. In line with the in-sample results obtained with the Amazon time series the PSO is able to find a sets of parameters that perform substantially better than the Buy and Hold strategy and the standard set. Even the worst results generate a returns higher than the other strategies.

5.2.2 PSO BRF S.A Out-of-sample Results

The outstanding returns obtained in the in-sample period from 2007 to 2012 do not reflect in the out-of-sample periods. The optimized parameters in the first biennium generates 9 times over 10 a loss in contrast with the gain obtained by the other two strategies, table 5.39. Such behaviour could be reconducted to a wrong parameters selection. Mostly of the sets of parameters found with good local maximum by the algorithm, in the training period, are composed by very slow SMA of the BB and a fast reactivity of the other indicators. Such combination might be a good one in the training period but perform really badly in the first out of sample analysis. A less extreme combination of parameters as the one that rank third in table 5.38 perform better in the first period.

In the following two years both Buy and Hold and the standard set of parameters sustain a huge decline of the capital as opposite of the optimised parameters sets. The strategy proposed is able to gain 3 times over 10 and in the remaining cases the losses are less that the other strategies. Focusing in the whole time series from 1/1/2012 to 1/1/2020 the B&H shows a substantial loss, also the TA is not able to generate a profit both with classic parameters and with the optimised. However, the losses sustained with the new sets of parameters are worst than with the standard set in 7 cases over 10 but less than the Buy and Hold with all the sets presented.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	76	5,000	1,575	0,026	0,117	26	24	3	3	13
2°	43	1,000	1,652	0,021	0,321	15	12	3	21	3
3°	5	4,678	2,888	0,019	0,383	26	12	21	7	6
4°	78	1,000	4,119	0,023	0,137	15	12	3	21	3
5°	100	1,328	5,000	0,026	0,346	13	12	3	3	32
6°	98	3,834	1,085	0,026	0,387	13	12	3	3	3
7°	44	1,036	2,259	0,032	0,278	13	12	3	5	21
8°	59	1,000	4,887	0,020	0,500	23	12	4	20	3
9°	85	2,685	1,588	0,026	0,500	21	12	13	3	16
10°	100	1,000	2,309	0,025	0,500	15	12	3	12	3

Table 5.38: PSO parameters'set 2007-2012

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
-3296,0	513,1	1482,0	894,0	-3161,6	-3060,9	-5368,8	-5608,5	-3307,0
-2443,4	513,1	1482,0	-2473,9	-3161,6	-3060,9	-6127,3	-5608,5	-3307,0
2671,5	513,1	1482,0	-2449,6	-3161,6	-3060,9	-3284,2	-5608,5	-3307,0
-2794,7	513,1	1482,0	-2909,3	-3161,6	-3060,9	-5697,8	-5608,5	-3307,0
-2853,1	513,1	1482,0	473,3	-3161,6	-3060,9	-4301,9	-5608,5	-3307,0
-1229,4	513,1	1482,0	-354,2	-3161,6	-3060,9	-4219,6	-5608,5	-3307,0
-1132,3	513,1	1482,0	307,1	-3161,6	-3060,9	-528,7	-5608,5	-3307,0
-2783,7	513,1	1482,0	-2681,1	-3161,6	-3060,9	-5444,7	-5608,5	-3307,0
-472,4	513,1	1482,0	-184,1	-3161,6	-3060,9	-3390,2	-5608,5	-3307,0
-2255,5	513,1	1482,0	-1402,0	-3161,6	-3060,9	-2400,6	-5608,5	-3307,0

Table 5.39: PSO Out-of-sample Returns

Shifting the trading time series with the period between 2009 and 2014 it has produced better out-of-sample observations as shown in table 5.41. The new sets of parameters, table 5.40, perform better than the previous one in the first biennium losing overall a small amount of capital in contrary with the average losses of 30% of the other two strategies. The same observations are visible both in the next biennium and considering the whole period from 1/1/2014 to 1/1/2020. It is possible to assess that in this case the new sets of parameters are the ones that contained the most losses and sometime they have even generated some gains.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	22	1,918	3,417	0,008	0,277	20	16	3	10	13
2°	25	1,786	3,595	0,008	0,480	21	16	3	10	34
3°	40	2,481	5,000	0,008	0,054	26	15	3	10	42
4°	86	4,201	2,991	0,008	0,103	26	15	3	10	29
5°	52	2,915	4,916	0,008	0,290	26	15	3	10	21
6°	38	3,391	2,284	0,008	0,253	26	15	3	10	32
7°	40	2,327	3,312	0,008	0,161	21	16	3	10	38
8°	60	2,064	4,530	0,008	0,476	26	12	3	10	26
9°	49	2,899	2,401	0,008	0,377	19	12	5	10	9
10°	89	2,958	2,188	0,008	0,173	21	12	5	10	9

Table 5.40: PSO parameters'set 2009-2014

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-316,7	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3066,9	-5690,9	-4168,2
-1034,7	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3581,7	-5690,9	-4168,2
-848,5	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3241,3	-5690,9	-4168,2
-848,5	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3241,3	-5690,9	-4168,2
-848,5	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3241,3	-5690,9	-4168,2
-495,6	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-2983,9	-5690,9	-4168,2
-688,2	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3331,7	-5690,9	-4168,2
85,0	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-2216,7	-5690,9	-4168,2
-288,9	-3161,6	-3060,9	-115,5	-1670,1	-594,0	-2191,6	-5690,9	-4168,2
57,0	-3161,6	-3060,9	-51,3	-1670,1	-594,0	-1857,8	-5690,9	-4168,2

Table 5.41: PSO Out-of-sample Returns

Focusing on the next sets of optimised parameters, table 5.42, they do not show as good values as the previous one. It is possible to observe, in table 5.43, that the strategy can generate a profit 3 times over 10 and other two times it loses less than the standard set. Overall the strategy proposed beat the Buy and Hold strategy in the first biennium but it is not possible to say which one is better between the standard and the one proposed. The algorithm has found sets of parameters with high discrepancy among the values that compose them. It might be possible that leaving the PSO to search better toward the global maximum it resulted to find a more stable an optimal combination of parameters.

In the following two years it is possible to do the same observation of the first biennium among Buy and Hold strategy and the one with the optimised parameters but the standard seems to perform better even if it presents a slight loss. The same conclusion just mentioned can be done combining the whole period, from 1/1/2016 to 1/1/2020.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	8	1,859	1,791	0,009	0,233	24	45	14	9	35
2°	10	4,952	1,724	0,014	0,385	14	40	9	13	22
3°	10	3,661	1,759	0,014	0,051	9	32	19	13	28
4°	7	2,638	1,553	0,014	0,171	10	29	18	13	27
5°	7	1,938	2,049	0,009	0,237	15	47	12	9	40
6°	8	1,741	1,742	0,011	0,066	12	40	21	12	19
7°	7	4,998	2,049	0,015	0,091	5	55	16	32	4
8°	85	4,142	2,485	0,008	0,341	15	13	4	8	13
9°	6	2,324	1,836	0,014	0,050	14	26	27	12	18
10°	6	4,089	2,073	0,016	0,452	4	74	10	8	9

Table 5.42: PSO parameters'set 2011-2016

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
-1813,9	-1670,1	-594,0	-3935,2	-2386,0	-81,3	-5030,6	-3562,0	-528,5
-117,5	-1670,1	-594,0	1084,1	-2386,0	-81,3	954,2	-3562,0	-528,5
-2213,0	-1670,1	-594,0	-2138,4	-2386,0	-81,3	-6336,5	-3562,0	-528,5
-110,6	-1670,1	-594,0	-2138,4	-2386,0	-81,3	-2225,9	-3562,0	-528,5
-2936,2	-1670,1	-594,0	-2536,5	-2386,0	-81,3	-4601,8	-3562,0	-528,5
-1616,9	-1670,1	-594,0	-1824,3	-2386,0	-81,3	-3146,3	-3562,0	-528,5
5238,9	-1670,1	-594,0	813,6	-2386,0	-81,3	6478,1	-3562,0	-528,5
95,0	-1670,1	-594,0	-2677,1	-2386,0	-81,3	-2259,5	-3562,0	-528,5
-1616,9	-1670,1	-594,0	-1222,2	-2386,0	-81,3	-2641,0	-3562,0	-528,5
1290,6	-1670,1	-594,0	1235,0	-2386,0	-81,3	2685,5	-3562,0	-528,5

Table 5.43: PSO Out-of-sample Returns

Updating the training period moving the in-sample time series from 1/1/2013 to 1/1/2018 for helping the algorithm to test new sets of parameters that could have found a better performance in the following two years, did not work. In fact, the first biennium figures, table 5.45, show a loss less huge than the previous, table 5.43, but not enough to beat the standard set. The standard set of parameters obtain few losses because it does not trade to much in the market , meanwhile, the strategy proposed try to enter and exit more times but mostly without success. There is not to much difference among the parameters' values found by the PSO to focus more on a specific characteristic of them, however, even if with slight differences some sets perform really well in the first period. It might be possible to implement some stop losses strategy to improve the return or if the market condition permits the possibility to have short market position.

The same observations done in the first biennium can be assessed observing the results of the first quarter of 2020. There, the strategy proposed 7 times over 10 trades producing a loss, meanwhile, the standard set do not operate in the market. Combining the two periods together is favourable for the strategy proposed. The values computed depict a gloomy result for the standard set with huge losses, while, the losses in our strategy are restrained. Overall the Buy and Hold strategy always lose against the set of parameters proposed.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	5	3,602	1,085	0,007	0,402	8	37	19	9	4
2°	6	5,000	1,332	0,007	0,500	16	22	9	9	4
3°	6	2,884	1,245	0,007	0,475	18	24	8	9	4
4°	6	2,231	1,295	0,007	0,258	9	46	15	9	4
5°	5	4,020	1,186	0,007	0,449	18	38	8	5	4
6°	5	4,324	1,179	0,006	0,318	16	22	16	5	4
7°	7	1,949	1,640	0,006	0,318	5	29	16	8	4
8°	11	3,228	1,927	0,015	0,112	7	44	17	31	4
9°	18	4,798	2,112	0,013	0,246	7	29	22	6	11
10°	26	3,911	1,627	0,011	0,164	11	30	19	39	4

Table 5.44: PSO parameters'set 2013-2018

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
-1539,7	-2386,0	-81,3	-1630,5	-6722,9	0,0	-3065,8	-7455,2	-6693,3
-1023,9	-2386,0	-81,3	0,0	-6722,9	0,0	-7055,2	-7455,2	-6693,3
-1023,9	-2386,0	-81,3	0,0	-6722,9	0,0	-7055,2	-7455,2	-6693,3
-2847,8	-2386,0	-81,3	-6650,9	-6722,9	0,0	-7656,5	-7455,2	-6693,3
1972,5	-2386,0	-81,3	-1649,7	-6722,9	0,0	-187,8	-7455,2	-6693,3
-626,1	-2386,0	-81,3	-1649,7	-6722,9	0,0	-2317,0	-7455,2	-6693,3
1049,8	-2386,0	-81,3	0,0	-6722,9	0,0	-6379,7	-7455,2	-6693,3
-666,2	-2386,0	-81,3	-6604,2	-6722,9	0,0	-6825,1	-7455,2	-6693,3
-3246,0	-2386,0	-81,3	-6756,2	-6722,9	0,0	-7806,2	-7455,2	-6693,3
98,4	-2386,0	-81,3	0,0	-6722,9	0,0	-6629,6	-7455,2	-6693,3

Table 5.45: PSO Out-of-sample Returns

Testing the algorithm in the last time period trying to contain the collapse of the price due to the coronavirus pandemic results into picking the wrong set of parameters. The optimised sets, table 5.46, take positions in the market concluding the period with a huge loss, table 5.47. The capitals reduction are not as large as with the Buy and Hold strategy but they are not comparable with the non operation of the standard set.

The wrong parameters selection of the algorithm could be re-conduct to a suddenly change of the market conditions in which the parameters found do not work any more.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	66	1,911	1,380	0,047	0,338	9	12	3	39	20
2°	77	2,369	1,502	0,047	0,374	25	19	21	39	20
3°	77	2,379	1,485	0,047	0,201	24	18	22	39	17
4°	79	4,506	1,494	0,047	0,131	24	19	19	39	20
5°	79	4,940	1,519	0,047	0,272	26	19	19	39	17
6°	84	4,352	1,504	0,050	0,248	26	20	23	39	20
7°	78	2,376	1,478	0,046	0,288	26	17	13	39	11
8°	79	4,983	1,521	0,046	0,128	26	20	10	39	20
9°	46	4,475	1,237	0,016	0,345	4	12	17	25	3
10°	78	2,103	1,521	0,032	0,220	18	13	13	43	20

Table 5.46: PSO parameters'set 2015-2020

1/2020 4/2020	B&H	Stand.
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
0,0	-6722,9	0,0
-6021,9	-6722,9	0,0

Table 5.47: PSO Out-of-sample Returns

5.2.3 EFWA BRF S.A In-sample Results

Period	Time	Average	Best	Worst	B&H	Standard
2015-2020	20/45	1,749	3,622	1,007	-0,620	-0.47
2013-2018	20/45	2,232	3,208	1,660	-0,471	-0.279
2011-2016	20/45	2,007	2,995	1,418	-0,187	-0.156
2009-2014	20/45	5,339	7,529	2,920	2,055	0.837
2007-2012	20/45	6,304	8,373	3,983	1,869	0.514

Table 5.48: EFWA In-sample Results

The application of the Enhance Firework algorithm, results showed in table 5.48, requires the same time on average estimated with the Amazon time series. The returns are always positive respect to the values obtained by the Buy and Hold strategy and the standard set of parameters. In fact, where the stock shows a sharp decline of its value only the optimised set can still produce a gain. In line with what has been commented with Amazon time series the EFWA is able to explore better the space finding 4 times over 5 a best return higher than PSO and on average always beat the gain of the other algorithm. Even the worst results are better than the ones obtained by the other two strategies. Also in the comparison of the lower returns with what has been found by the PSO the Enhance Firework algorithm always win.

5.2.4 EFWA BRF S.A Out-of-sample Results

Observing the figure obtained testing the parameters found in the period between 2007 and 2012, table 5.49, the returns are good but not outstanding as the ones showed in the in-sample analysis. In the first biennium out-of-sample the optimised parameters sets are generally able to generate a gain substantially higher respect to the Buy and Hold strategy and the standard set of parameters. However, in the next two years no one of the strategies have obtained a positive results, on the other hand, the optimised sets of parameters are the ones that lose less. Analysing the results of the whole period from 1/1/2012 to 1/1/2020 they depict a different view, the strategy proposed struggles to beat the standard set in fact only 3 times is able to generate a profit and 1 times it loses less. Comparing it with the Buy and Hold still do not show a clear advantage because in half of the figures are better for the B&H and the other half for the other strategy.

It seems that the EFWA is able to find, with the top sets of parameters in the table 5.49, good combinations that permit to predict the market movements in the first period a head. Such initial good parameters picking do not last up to the 2020 probability due to a change in the behaviour of the time series that require new analyses.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	6	2,614	3,140	0,020	0,120	18	15	17	7	23
2°	5	4,018	2,691	0,019	0,097	19	14	16	7	33
3°	40	1,120	3,351	0,023	0,375	8	14	10	6	17
4°	37	1,099	4,916	0,022	0,443	8	14	10	6	16
5°	39	1,089	4,254	0,022	0,289	8	14	10	6	17
6°	28	1,223	4,167	0,023	0,274	9	19	11	9	15
7°	29	1,226	3,312	0,023	0,437	10	12	14	9	15
8°	29	1,241	4,373	0,023	0,207	11	16	10	9	16
9°	6	1,131	4,352	0,018	0,266	24	20	3	18	26
10°	29	1,243	4,058	0,025	0,219	10	18	10	9	13

Table 5.49: EFWA parameters' sets 2007-2012

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
2014,3	513,1	1482,0	-1327,4	-3161,6	-3060,9	-4348,7	-5608,5	-3307,0
2117,8	513,1	1482,0	-1327,4	-3161,6	-3060,9	-3049,5	-5608,5	-3307,0
2857,3	513,1	1482,0	-632,5	-3161,6	-3060,9	110,5	-5608,5	-3307,0
3072,2	513,1	1482,0	-632,5	-3161,6	-3060,9	277,2	-5608,5	-3307,0
2857,3	513,1	1482,0	-632,5	-3161,6	-3060,9	110,5	-5608,5	-3307,0
-1440,3	513,1	1482,0	-1450,4	-3161,6	-3060,9	-7115,4	-5608,5	-3307,0
-1123,6	513,1	1482,0	-1242,5	-3161,6	-3060,9	-6956,5	-5608,5	-3307,0
-1110,4	513,1	1482,0	-867,1	-3161,6	-3060,9	-6796,8	-5608,5	-3307,0
3404,0	513,1	1482,0	-4104,1	-3161,6	-3060,9	-6139,9	-5608,5	-3307,0
-1104,3	513,1	1482,0	-2835,8	-3161,6	-3060,9	-7488,3	-5608,5	-3307,0

Table 5.50: EFWA Out-of-sample Returns

Moving to the following period, the analysis of the next two years respect to the training period proposed before improved the sets of parameters, table 5.51, in the period from 1/2014 to 1/2016 producing on average a loss more contained than before, table 5.52. Also the next two years figures are all negative with exceptions of two positive returns obtained by the optimised set of parameters. However, the strategy proposed is mostly able to reduce the decline of the portfolio better than the Buy and Hold strategy and the standard set of parameters. This loss reduction is highlighted even analysing the longer period from 1/1/2014 to 1/1/2020 where the other two strategies perform worst. Comparing the returns obtained by the EFWA algorithm respect to the PSO, table 5.41, do not shows substantially difference in contrary the values are not far from each others.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	10	4,020	2,623	0,008	0,219	22	15	3	10	10
2°	21	1,579	3,827	0,008	0,067	22	16	3	10	13
3°	48	3,763	2,647	0,008	0,319	19	18	3	10	13
4°	22	3,596	2,297	0,008	0,216	19	17	3	10	13
5°	22	1,632	3,468	0,009	0,299	21	17	3	10	31
6°	20	1,883	3,175	0,008	0,261	20	16	3	10	31
7°	22	2,192	3,515	0,008	0,354	21	17	3	10	27
8°	40	2,793	4,456	0,008	0,217	26	15	3	10	20
9°	73	4,324	2,915	0,008	0,470	21	17	3	10	8
10°	45	2,860	3,234	0,008	0,430	24	14	3	10	8

Table 5.51: EFWA parameters' sets 2009-2014

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-1518,1	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3847,0	-5690,9	-4168,2
-447,2	-3161,6	-3060,9	5,4	-1670,1	-594,0	-3549,9	-5690,9	-4168,2
-688,2	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3331,7	-5690,9	-4168,2
57,0	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-2799,3	-5690,9	-4168,2
-764,1	-3161,6	-3060,9	-1651,3	-1670,1	-594,0	-4841,1	-5690,9	-4168,2
-316,7	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3066,9	-5690,9	-4168,2
-1034,7	-3161,6	-3060,9	67,0	-1670,1	-594,0	-3692,3	-5690,9	-4168,2
-495,6	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-2983,9	-5690,9	-4168,2
-1034,7	-3161,6	-3060,9	-74,4	-1670,1	-594,0	-2659,7	-5690,9	-4168,2
-1034,7	-3161,6	-3060,9	-74,4	-1670,1	-594,0	-2798,7	-5690,9	-4168,2

Table 5.52: EFWA Out-of-sample Returns

Focusing on the sets of parameters produced by training the algorithm in the in-sample period from 2011 to 2016, table 5.53, their test on the first biennium of the out-of-sample time series do not improve the results in contrary the outputs now are 6 times over 10 worst than the standard set but still always better than the Buy and Hold strategy results. Probably the metaheuristic algorithm adapt to much the parameters in the training period resulting to be highly profitable in the in-sample but not in the out-of-sample analysis. It is also possible that during the period from 2014 to 2016 the time series present some outliers that modified the behaviour of the algorithm in the tempt to pick them. Such option can be sustain observing the good return obtained by the previous sets in table 5.41 and 5.51.

The initial conclusion assessed in the first biennium can be repeated evaluating the figures in the next two years as showed in table 5.54 column 4 to 6 and in the whole period, columns 7 to 9. These results are similar but slightly better to the ones obtained by the PSO.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	6	1,606	1,456	0,009	0,378	14	40	15	9	19
2°	6	1,649	1,456	0,009	0,235	14	58	12	9	28
3°	13	4,255	1,754	0,014	0,282	7	31	21	13	13
4°	10	3,889	1,678	0,014	0,342	10	23	25	13	24
5°	10	4,438	1,686	0,014	0,107	16	38	8	13	31
6°	13	3,474	1,756	0,014	0,288	18	22	14	13	18
7°	11	3,887	1,606	0,014	0,263	16	21	16	13	35
8°	13	4,377	1,734	0,014	0,296	16	21	18	13	35
9°	13	3,172	1,747	0,014	0,309	18	32	9	13	26
10°	10	4,536	1,665	0,014	0,296	13	19	23	13	27

Table 5.53: EFWA parameters' sets 2011-2016

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
-1697,4	-1670,1	-594,0	-2341,5	-2386,0	-81,3	-3642,6	-3562,0	-528,5
-986,4	-1670,1	-594,0	-2341,5	-2386,0	-81,3	-3093,1	-3562,0	-528,5
-2121,4	-1670,1	-594,0	3389,8	-2386,0	-81,3	549,7	-3562,0	-528,5
-2121,4	-1670,1	-594,0	-2582,8	-2386,0	-81,3	-4156,0	-3562,0	-528,5
-2121,4	-1670,1	-594,0	1084,1	-2386,0	-81,3	-1267,3	-3562,0	-528,5
-200,4	-1670,1	-594,0	-941,6	-2386,0	-81,3	-1123,3	-3562,0	-528,5
-200,4	-1670,1	-594,0	-941,6	-2386,0	-81,3	-1123,3	-3562,0	-528,5
-200,4	-1670,1	-594,0	-941,6	-2386,0	-81,3	-1123,3	-3562,0	-528,5
-2121,4	-1670,1	-594,0	-941,6	-2386,0	-81,3	-2863,3	-3562,0	-528,5
-200,4	-1670,1	-594,0	-2582,8	-2386,0	-81,3	-2731,9	-3562,0	-528,5

Table 5.54: EFWA Out-of-sample Returns

The new in-sample training from 2013 to 2018 with the sets of parameters identified in table 5.55, has improved a little the portfolio results in the first biennium a head, table 5.56, from the out-of-sample training period respect to the results found in table 5.54 but still not enough to beat the standard set. In fact, only 4 times over 10 sets generate a positive return or a loss lower than the standard set. However, all the figures with the exception of 1 case are better than the Buy and Hold strategy. Testing the parameters in the first quarter of 2020 with the crash of the corona virus pandemic shows values close but more contained respect to the loss of the Buy and Hold strategy but not enough respect to the non trading activity of the standard set. Analysing both periods together there are almost equivalent results between the optimised sets of parameters and the standard one, however, both of them are better than the Buy and Hold strategy. Similar results are obtained by the PSO algorithm.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	7	3,815	1,400	0,007	0,394	9	37	15	9	4
2°	6	2,277	1,594	0,006	0,149	8	25	24	8	4
3°	9	4,454	1,685	0,006	0,201	14	21	16	8	4
4°	11	4,133	1,976	0,014	0,118	9	45	14	31	4
5°	11	4,568	1,971	0,015	0,378	21	40	6	31	4
6°	21	1,970	1,659	0,014	0,329	12	53	9	31	4
7°	5	3,233	1,097	0,013	0,474	18	23	8	9	4
8°	22	3,900	1,411	0,014	0,364	15	46	8	31	4
9°	10	4,192	1,983	0,014	0,386	21	37	7	35	4
10°	10	2,020	1,436	0,014	0,146	16	23	14	9	33

Table 5.55: EFWA parameters' sets 2013-2018

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
-234,2	-2386,0	-81,3	-6657,3	-6722,9	0,0	-6797,9	-7455,2	-6693,3
-3300,0	-2386,0	-81,3	0,0	-6722,9	0,0	-7804,5	-7455,2	-6693,3
-614,9	-2386,0	-81,3	-6657,3	-6722,9	0,0	-6921,2	-7455,2	-6693,3
-66,8	-2386,0	-81,3	-6604,2	-6722,9	0,0	-6626,1	-7455,2	-6693,3
-666,2	-2386,0	-81,3	-6604,2	-6722,9	0,0	-6825,1	-7455,2	-6693,3
87,6	-2386,0	-81,3	-6604,2	-6722,9	0,0	-6573,2	-7455,2	-6693,3
-361,6	-2386,0	-81,3	0,0	-6722,9	0,0	-361,6	-7455,2	-6693,3
212,8	-2386,0	-81,3	-6604,2	-6722,9	0,0	-6527,0	-7455,2	-6693,3
1365,5	-2386,0	-81,3	-6604,2	-6722,9	0,0	-6135,7	-7455,2	-6693,3
-1417,2	-2386,0	-81,3	0,0	-6722,9	0,0	-7136,0	-7455,2	-6693,3

Table 5.56: EFWA Out-of-sample Returns

Moving to the next sets of optimised parameters and permitting them to adapt up to the end of 2019 for trying to prevent the crash of the first quarter of 2020 have not worked. The results highlighted in the first three columns in table 5.58 show that the decline of the capital has been contained more than in table 5.56 and better than the Buy and Hold strategy but it can not be compare with the no trading activity exercised by the standard set of parameters.

The wrong parameters selection of the algorithm is re-conduct to a suddenly change of the market condition that could not be predicted by the analysis of the previous time series period.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	55	3,860	1,468	0,046	0,261	6	16	4	39	20
2°	75	2,364	1,596	0,047	0,202	24	19	21	39	20
3°	55	3,383	1,461	0,047	0,203	24	22	23	39	20
4°	83	4,345	1,507	0,047	0,478	25	21	17	39	20
5°	84	4,016	1,489	0,046	0,493	25	17	25	39	20
6°	81	2,711	1,507	0,047	0,231	25	23	25	39	20
7°	48	1,606	1,317	0,047	0,297	4	30	4	39	14
8°	29	4,845	1,020	0,025	0,393	5	49	10	5	8
9°	58	4,276	1,484	0,046	0,199	15	14	14	33	16
10°	90	2,052	1,420	0,035	0,313	22	15	16	42	12

Table 5.57: EFWA parameters' sets 2015-2020

1/2020 4/2020	B&H	Stand.
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-6021,9	-6722,9	0,0
-1638,4	-6722,9	0,0
-6021,9	-6722,9	0,0
-6148,7	-6722,9	0,0

Table 5.58: EFWA Out-of-sample Returns

5.2.5 DE BRF S.A In-sample Results

Period	Time	Average	Best	Worst	B&H	Standard
2015-2020	11/12	1,983	3,622	1,362	-0,621	-0.47
2013-2018	11/12	2,245	2,566	1,661	-0,471	-0.279
2011-2016	11/12	2,044	3,366	1,178	-0,187	-0.156
2009-2014	11/12	6,813	7,748	3,935	2,055	0.837
2007-2012	11/12	7,397	13,406	4,995	1,869	0.514

Table 5.59: DE In-sample Results

The application of the Differential Evolution algorithm, results showed in table 5.59, requires the same time on average estimated with the Amazon time series. The returns are always positive respect to the values obtained by the Buy and Hold strategy and the standard set of parameters. In fact, where the stock shows a drop reduction of its value only the optimised sets can produce a gain. With the BRF S.A the Differential Evolution rank first in the in-sample results obtained in all 4 categories: time, average, best and worst. It takes considerably less time and it has found mostly higher returns in all the categories and times series analysed.

5.2.6 DE BRF S.A Out-of-sample Results

Testing the sets of parameters found in the in sample analysis in the period from 2007 to 2012, table 5.60, perform well with 8 sets that generate a return higher than the standard set and the Buy and Hold strategy in the first biennium, table 5.61, however, the values are not outstanding as the ones that have been reported in the in-sample analysis. The second biennium a head from the end of the in-sample time series still show that the optimised set of parameters are better than the other strategies with a better reduction of the losses. Analysing the whole period the strategy proposed produce a gain 5 times over 10 and, on the other results, 4 times the losses are lower than the one of the standard set. The optimised sets are always better than the Buy and Hold strategy.
In comparison with the other two algorithm discovers the DE evolution parameters perform better also in the out-of-sample analysis ranking first.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	7	2,351	3,872	0,033	0,100	15	7	3	5	20
2°	12	0,918	3,644	0,033	0,137	20	3	6	5	22
3°	60	1,392	2,421	0,032	0,243	15	7	3	5	25
4°	29	1,874	4,492	0,033	0,178	15	7	3	5	15
5°	18	1,829	3,926	0,033	0,326	15	7	3	5	15
6°	62	1,313	3,947	0,034	0,318	15	7	3	5	24
7°	8	2,374	3,878	0,033	0,393	14	3	7	5	7
8°	45	1,523	4,040	0,033	0,332	6	3	18	5	25
9°	61	1,444	4,367	0,032	0,203	8	3	13	5	26
10°	62	1,425	3,773	0,033	0,458	5	4	16	5	10

Table 5.60: DE Parameters' sets 2007-2012

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
439,1	513,1	1482,0	-2896,7	-3161,6	-3060,9	-3684,1	-5608,5	-3307,0
2655,4	513,1	1482,0	-1458,3	-3161,6	-3060,9	1872,4	-5608,5	-3307,0
3179,9	513,1	1482,0	-81,0	-3161,6	-3060,9	1171,4	-5608,5	-3307,0
439,1	513,1	1482,0	-1547,4	-3161,6	-3060,9	-2485,4	-5608,5	-3307,0
1923,6	513,1	1482,0	-1547,4	-3161,6	-3060,9	-1447,7	-5608,5	-3307,0
3179,9	513,1	1482,0	-211,2	-3161,6	-3060,9	7131,6	-5608,5	-3307,0
2959,3	513,1	1482,0	-2896,7	-3161,6	-3060,9	-2282,5	-5608,5	-3307,0
3037,0	513,1	1482,0	-822,8	-3161,6	-3060,9	4222,7	-5608,5	-3307,0
1654,8	513,1	1482,0	-720,0	-3161,6	-3060,9	-1294,3	-5608,5	-3307,0
4514,2	513,1	1482,0	-745,1	-3161,6	-3060,9	4117,5	-5608,5	-3307,0

Table 5.61: DE Out-of-sample Returns

Moving to the next sets of parameters optimised in the training period between 2009 and 2014, table 5.62, the figures in the next two year of the out-of-sample period are better than the ones in table 5.61 showing an improvement. The optimised sets of parameters are still better in the next biennium containing the losses definitely better than the other two strategies. Such conclusions remain analysing the results obtained in the longer period from 1/1/2014 to 1/1/2020 where the standard set rank second and the buy and hold third.

Comparing the values obtained with the other two algorithms show that the results are almost the same, there is not an algorithm that prevail over the others.

These results as the ones in the previous table demonstrate how the DE is able to adapt to the data in the training period finding combinations of parameters that results to be optimal also in the initial following period.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	8	3,220	2,884	0,008	0,203	15	3	25	10	34
2°	9	3,266	2,956	0,008	0,388	18	3	21	10	18
3°	5	4,953	2,354	0,008	0,405	22	3	17	10	13
4°	9	3,280	2,932	0,008	0,216	25	3	16	10	39
5°	5	3,862	2,814	0,008	0,490	17	3	21	10	13
6°	8	3,178	2,873	0,008	0,197	25	21	2	10	26
7°	52	1,665	3,944	0,008	0,218	22	2	27	10	13
8°	8	3,196	2,885	0,008	0,247	20	17	3	10	38
9°	5	4,050	2,866	0,008	0,141	24	2	25	10	26
10°	4	4,457	2,411	0,008	0,122	26	15	3	10	38

Table 5.62: DE Parameters' sets 2009-2014

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-848,5	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3152,9	-5690,9	-4168,2
-848,5	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3241,3	-5690,9	-4168,2
-848,5	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3152,9	-5690,9	-4168,2
-848,5	-3161,6	-3060,9	-115,5	-1670,1	-594,0	-3280,2	-5690,9	-4168,2
-848,5	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3318,9	-5690,9	-4168,2
-848,5	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3041,6	-5690,9	-4168,2
280,2	-3161,6	-3060,9	-1391,2	-1670,1	-594,0	-3951,4	-5690,9	-4168,2
-688,2	-3161,6	-3060,9	-61,0	-1670,1	-594,0	-3244,6	-5690,9	-4168,2
280,2	-3161,6	-3060,9	-1391,2	-1670,1	-594,0	-3417,5	-5690,9	-4168,2
-848,5	-3161,6	-3060,9	363,0	-1670,1	-594,0	-2956,1	-5690,9	-4168,2

Table 5.63: DE Out-of-sample Returns

Focusing the attention on the next sets of parameters, table 5.64, it is noticeable to say that the new in-sample period as it has been seen also with the

other two algorithms have not improved the returns in the first biennium, table 5.65. In fact, the losses generated by the optimised parameters are bigger than all the other strategies and the same observations can be done in the following biennium and analysing the two time series together.
This time the DE evolution respects to the other algorithms rank third showing its incapability to contain the losses as good as the other.
Observing the similarity of the behaviour of the algorithms in this period it is possible to assess what was previous mentioned describing the result of the EFWA algorithm. It could be that algorithm adapt to much the parameters in the training period resulting into finding a solution that is not any more suitable for the following time series.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	8	1,835	1,792	0,009	0,293	20	37	20	9	34
2°	8	1,854	1,790	0,009	0,319	17	44	20	9	23
3°	7	1,823	2,024	0,009	0,173	19	20	25	9	39
4°	8	1,934	1,742	0,009	0,134	25	35	10	9	26
5°	8	1,753	1,708	0,008	0,384	22	27	22	9	41
6°	8	1,914	1,800	0,011	0,153	23	44	15	9	18
7°	13	3,605	1,728	0,014	0,349	8	26	26	13	34
8°	13	4,198	1,771	0,014	0,222	8	32	18	13	19
9°	5	1,531	1,827	0,009	0,123	23	24	18	9	22
10°	23	3,704	2,881	0,008	0,117	15	6	9	8	7

Table 5.64: DE Parameters' sets 2011-2016

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
-1813,9	-1670,1	-594,0	-3935,2	-2386,0	-81,3	-5030,6	-3562,0	-528,5
-1813,9	-1670,1	-594,0	-3935,2	-2386,0	-81,3	-5030,6	-3562,0	-528,5
-1813,9	-1670,1	-594,0	-3178,1	-2386,0	-81,3	-4411,6	-3562,0	-528,5
-1813,9	-1670,1	-594,0	-3432,8	-2386,0	-81,3	-4619,0	-3562,0	-528,5
-1218,9	-1670,1	-594,0	-3935,2	-2386,0	-81,3	-4672,1	-3562,0	-528,5
-1188,8	-1670,1	-594,0	-2643,9	-2386,0	-81,3	-3517,6	-3562,0	-528,5
-2121,4	-1670,1	-594,0	-941,6	-2386,0	-81,3	-2863,3	-3562,0	-528,5
-2121,4	-1670,1	-594,0	-10000,0	-3562,0	-81,3	-2121,4	-3562,0	-528,5
-1728,6	-1670,1	-594,0	-2952,7	-2386,0	-81,3	-4168,4	-3562,0	-528,5
-115,8	-1670,1	-594,0	-1986,9	-2386,0	-81,3	-1705,0	-3562,0	-528,5

Table 5.65: DE Out-of-sample Returns

Continuing moving the attention on the next sets of parameters, table 5.66, it is important to say that the new in-sample period as it has been seen also with the previous period it has not improved the return in the first biennium, table 5.67. In fact, the losses generated by the optimised parameters are bigger than all the other strategies and the same observations can be done in the first quarter of 2020 and analysing the two time series together.

The results obtained by the optimised set of parameters are definitively worst that the ones obtained with the standard set and the Buy and hold strategy. Respect to the other two algorithms the differential evolution is the one that could not contain the losses.

The DE algorithm also in the in-sample analysis result to be the one that have generated the lowest 'best result'. It could be that the algorithm was stopped before to reach a better maximum or that it has been blocked inside a local maximum without being able to overcome it. Probably increasing the number of iterations of the algorithm it would have found a better combination of parameters.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	4	4,694	1,816	0,014	0,296	17	24	12	5	9
2°	14	3,118	1,687	0,013	0,288	8	33	18	6	11
3°	13	3,622	1,734	0,013	0,254	8	33	18	6	10
4°	14	3,476	1,738	0,013	0,168	8	33	18	6	10
5°	14	3,643	1,715	0,013	0,222	8	33	18	6	11
6°	13	3,352	1,734	0,013	0,304	8	33	18	6	11
7°	13	4,322	1,709	0,013	0,279	7	27	24	6	10
8°	14	2,748	1,759	0,013	0,283	8	32	19	6	10
9°	14	4,085	1,695	0,013	0,123	8	32	19	6	11
10°	14	4,418	1,744	0,013	0,155	7	28	23	6	10

Table 5.66: DE Parameters' sets 2013-2018

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
2400,4	-2386,0	-81,3	-1522,2	-6722,9	0	391,1	-7455,2	-6693,3
-3246,0	-2386,0	-81,3	-6756,2	-6722,9	0	-7806,2	-7455,2	-6693,3
-3246,0	-2386,0	-81,3	-6756,2	-6722,9	0	-7806,2	-7455,2	-6693,3
-3246,0	-2386,0	-81,3	-6756,2	-6722,9	0	-7806,2	-7455,2	-6693,3
-3246,0	-2386,0	-81,3	-6756,2	-6722,9	0	-7806,2	-7455,2	-6693,3
-3246,0	-2386,0	-81,3	-6756,2	-6722,9	0	-7806,2	-7455,2	-6693,3
-3246,0	-2386,0	-81,3	-6756,2	-6722,9	0	-7806,2	-7455,2	-6693,3
-3827,6	-2386,0	-81,3	-6756,2	-6722,9	0	-7995,2	-7455,2	-6693,3
-3246,0	-2386,0	-81,3	-6756,2	-6722,9	0	-7806,2	-7455,2	-6693,3
-3246,0	-2386,0	-81,3	-6756,2	-6722,9	0	-7806,2	-7455,2	-6693,3

Table 5.67: DE Out-of-sample Returns

Retraining the algorithm in the last period from 2015 to 2020 has been carried out to try to find if the algorithm is able to contained the sharp decline fo the first quarter of 2020. The sets of parameters found, table 5.68, tested in the highly volatile environment of the coronavirus pandemic show an improvement respect to the figure in table 5.67 such that the decline is sightly contained in comparison with the Buy and Hold strategy but far from the no trading activity of the standard set. Almost the same returns are obtained by the others two algorithms.

The wrong parameters selection of the algorithm is re-conduct to a suddenly change of the market condition that could not be predicted by the analysis of the previous time series period.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1°	55	3,131	1,479	0,047	0,149	3	17	5	39	20
2°	63	2,371	1,486	0,048	0,137	3	24	5	39	13
3°	67	1,868	1,368	0,049	0,156	2	21	9	39	6
4°	83	3,198	1,540	0,047	0,202	3	20	5	39	13
5°	63	1,807	1,399	0,047	0,358	4	41	2	39	11
6°	59	1,678	1,423	0,046	0,349	1	30	6	39	11
7°	68	1,853	1,357	0,046	0,203	3	7	17	39	11
8°	76	2,378	1,503	0,047	0,025	24	20	19	39	20
9°	75	2,344	1,539	0,047	0,063	21	18	24	39	17
10°	76	2,346	1,509	0,047	0,109	21	17	27	39	11

Table 5.68: DE Parameters' sets 2015-2020

1/2020 4/2020	B&H	Stand.
-6021,9	-6722,9	0
-6021,9	-6722,9	0
0,0	-6722,9	0
-6021,9	-6722,9	0
-6021,9	-6722,9	0
-6021,9	-6722,9	0
-6021,9	-6722,9	0
-6021,9	-6722,9	0
-6021,9	-6722,9	0
-6021,9	-6722,9	0

Table 5.69: DE Out-of-sample Returns

5.3 EUR/USD Algorithms results

5.3.1 PSO EUR/USD In-sample Results

Period	Time	Average	Best	Worst	B&H	Standard
2015-2020	4/30	0,305	0,362	0,224	-0,073	-0.076
2013-2018	4/30	0,267	0,379	0,137	-0,090	-0.169
2011-2016	4/30	0,131	0,314	0,063	-0,186	-0.222
2009-2014	4/30	0,476	0,611	0,327	-0,016	0.056
2007-2012	4/30	0,553	0,731	0,383	-0,019	-0.014

Table 5.70: PSO In-sample Returns

The application of the PSO algorithm requires a considerably less amount of time due to the reduction of the number of parameters analysed. With the time series considered it has been impossible to find data that have contained the volume and for this reason the MFI indicator has been removed from the analysis. The reduction of only one parameter influences considerably on the time required to the algorithm to converge as it is possible to notice in the column 'time' in table 5.70.

The returns generated by the optimised sets of parameters in the in-sample analysis are superior respect to the other two strategies, in line with what has already been observed in all the other previous in sample analysis. The total gains in all the three categories, even in the worst case, are higher than the ones produced by the Buy and Hold strategy and the standard set of parameters.

5.3.2 PSO EUR/USD Out-of-sample Results

Testing the in-sample parameters sets, table 5.71, in the out-of-sample time series data show that in the first biennium the optimised parameters are not able to find better returns, with exception of 1 case, respect to the other two strategies. On the other hand, in the following two years the strategy propose performs better in all the 10 sets analysed. Observing the values obtained investing in the time frame from 1/2012 to 1/2020 the optimised sets of parameters always beat the standard one but only 4 times over 10 with the Buy and Hold strategy.

Some combination of parameters as the one than rank first shows a higher gain in initial biennium that it might be generated by a continuation of the behaviour of the time series forecast by the set picked by the algorithm. Following this option it could be suggested to increase the searching power of the algorithm to identify a better maximum in the training period. On the other hand, a more general solution as the ones produced by the sets that rank in the 7th and 8th position results to be better in the overall future time series losing less and gaining more in the whole period ahead.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	8	4,706	2,851	0,009	0,378	10	12	10	25
2°	62	1,451	1,857	0,014	0,250	17	23	4	9
3°	87	2,724	3,747	0,013	0,050	13	12	3	29
4°	50	5,000	5,000	0,013	0,208	13	12	3	28
5°	5	2,221	2,441	0,024	0,469	4	65	8	18
6°	40	1,705	1,700	0,009	0,483	11	16	15	25
7°	57	1,381	1,758	0,009	0,209	4	14	26	9
8°	57	1,368	1,761	0,014	0,396	15	24	4	9
9°	39	4,625	2,470	0,015	0,274	13	12	3	24
10°	54	2,022	4,254	0,015	0,391	23	15	4	24

Table 5.71: PSO Parameters' sets 2007-2012

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
864,9	638,1	688,6	-1361,9	-2056,7	-1971,1	-527,6	-1327,9	-1930,2
371,0	638,1	688,6	-1403,8	-2056,7	-1971,1	537,1	-1327,9	-1930,2
0,0	638,1	688,6	-1947,8	-2056,7	-1971,1	-1520,7	-1327,9	-1930,2
0,0	638,1	688,6	-1947,8	-2056,7	-1971,1	-1520,7	-1327,9	-1930,2
122,3	638,1	688,6	-1874,5	-2056,7	-1971,1	-1835,6	-1327,9	-1930,2
360,7	638,1	688,6	-1841,2	-2056,7	-1971,1	-1620,9	-1327,9	-1930,2
437,9	638,1	688,6	-663,3	-2056,7	-1971,1	1297,6	-1327,9	-1930,2
321,3	638,1	688,6	-663,3	-2056,7	-1971,1	1007,9	-1327,9	-1930,2
0,0	638,1	688,6	-1947,8	-2056,7	-1971,1	-1436,0	-1327,9	-1930,2
0,0	638,1	688,6	-1947,8	-2056,7	-1971,1	-1558,2	-1327,9	-1930,2

Table 5.72: PSO Out-of-sample Returns

Moving to the next in-sample trading period, the sets of parameters computed, table 5.73, contain better the losses in the biennium of 1/2014-1/2016, table 5.74, in comparison with both the other two strategies but also with respect of the previous sets parameters proposed in table 5.71. These improvements do not last enough to beat the Buy and Hold strategy in the following biennium but the gains are still higher than the standard set. Observing the whole period together after the end of the in-sample analysis, the optimised sets of parameters values are 6 times over 10 positives in contrast

with the losses of the other two strategies and even the remaining 4 sets that produce a negative returns they are more contained that the others.
The PSO algorithm has been able to correctly adapt in the training period such that its sets of parameters produced perform well in the first biennium. Also in the following period even if the optimised sets do not always beat the Buy and Hold strategy. The figures shows that the standard set values are completely unsuitable for this time series.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	7	3,303	3,471	0,009	0,206	6	12	15	26
2°	77	3,628	1,938	0,009	0,082	10	12	11	10
3°	78	4,155	1,971	0,009	0,166	11	18	7	10
4°	66	1,492	3,513	0,005	0,172	11	12	12	8
5°	56	4,076	2,072	0,009	0,351	9	15	10	10
6°	79	2,337	1,895	0,009	0,227	6	17	14	10
7°	56	3,239	1,852	0,009	0,497	10	18	8	10
8°	41	1,345	1,800	0,008	0,407	12	28	4	9
9°	62	3,161	1,672	0,005	0,237	9	12	16	10
10°	48	1,237	1,372	0,008	0,500	10	19	7	9

Table 5.73: PSO Parameters' sets 2009-2014

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-1620,4	-2056,7	-1971,1	619,1	1088,5	291,5	-1440,3	-1799,2	-2331,4
-778,1	-2056,7	-1971,1	1059,6	1088,5	291,5	345,4	-1799,2	-2331,4
-1116,1	-2056,7	-1971,1	1059,6	1088,5	291,5	-179,7	-1799,2	-2331,4
-2100,3	-2056,7	-1971,1	515,4	1088,5	291,5	-1338,7	-1799,2	-2331,4
-694,5	-2056,7	-1971,1	797,7	1088,5	291,5	267,6	-1799,2	-2331,4
-1425,4	-2056,7	-1971,1	972,4	1088,5	291,5	-612,7	-1799,2	-2331,4
13,3	-2056,7	-1971,1	1059,6	1088,5	291,5	1191,6	-1799,2	-2331,4
-604,5	-2056,7	-1971,1	840,4	1088,5	291,5	476,5	-1799,2	-2331,4
-284,8	-2056,7	-1971,1	822,6	1088,5	291,5	539,1	-1799,2	-2331,4
-571,7	-2056,7	-1971,1	726,4	1088,5	291,5	630,4	-1799,2	-2331,4

Table 5.74: PSO Out-of-sample Returns

Analysing the returns obtained by investing with the sets of parameters at-

tained from the in-sample training in the period between 2011 and 2016, table 5.75, they have improved the results respect to the previous sets of parameters, table 5.73. In fact, in the first biennium a head from the in-sample period 5 times over 10 the gains are higher respect to the Buy and Hold strategy and 8 times over 10 in comparison with the standard set, table 5.76. The following biennium shows losses with all the strategies but always the strategy propose contains better the losses than the other two. Combining the two period together, last three columns table 5.76, depicts a good outlook for the strategy propose because almost with each set the results are greater than the other two strategies.
In line with the previous observations, the PSO algorithm efficiently enter and exit from the market in the first biennium ahead after having trained in the in-sample-period. As described before, not always the top ranking sets of parameters are the one that perform the best in the out-of-sample period.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	35	4,174	1,320	0,006	0,171	20	12	5	17
2°	50	2,441	1,690	0,006	0,492	22	12	7	17
3°	37	1,514	1,511	0,009	0,050	24	18	3	17
4°	12	4,253	2,334	0,006	0,401	22	14	27	4
5°	100	2,952	1,354	0,006	0,050	23	22	8	17
6°	87	1,811	1,029	0,006	0,261	23	14	7	17
7°	63	1,966	1,881	0,006	0,050	21	15	26	3
8°	84	2,429	2,909	0,006	0,415	26	16	20	3
9°	100	3,307	1,000	0,006	0,203	21	20	11	17
10°	79	4,971	2,288	0,019	0,357	23	17	3	14

Table 5.75: PSO Parameters' sets 2011-2016

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
852,5	1088,5	291,5	-579,2	-702,3	-772,8	561,0	351,7	-486,4
931,5	1088,5	291,5	-579,2	-702,3	-772,8	604,5	351,7	-486,4
-403,8	1088,5	291,5	-552,1	-702,3	-772,8	-800,1	351,7	-486,4
1068,2	1088,5	291,5	-259,0	-702,3	-772,8	781,6	351,7	-486,4
1174,2	1088,5	291,5	-467,2	-702,3	-772,8	889,8	351,7	-486,4
1474,3	1088,5	291,5	-554,2	-702,3	-772,8	1169,5	351,7	-486,4
1232,4	1088,5	291,5	-478,0	-702,3	-772,8	939,4	351,7	-486,4
1232,4	1088,5	291,5	-514,9	-702,3	-772,8	897,0	351,7	-486,4
1174,2	1088,5	291,5	-557,8	-702,3	-772,8	786,3	351,7	-486,4
0,0	1088,5	291,5	-202,8	-702,3	-772,8	-202,8	351,7	-486,4

Table 5.76: PSO Out-of-sample Returns

The following period of training from 2013 to 2018 that produced the sets of parameters in table 5.77 still perform well in the first out-of-sample period, columns 1 to 3 table 5.78, in comparison to the the Buy and Hold strategy and the standard set of parameters. Trying to testing these sets in the first quarter of 2020 results to be apparently good because the algorithms do not return a buy signal to enter in the market avoiding losses that both the other two strategies have taken. Merging the two out-of-sample periods still returns a positive conclusion due to the higher reduction of losses of the strategy proposed in all the sets tested.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	25	3,538	1,582	0,005	0,348	13	12	25	17
2°	73	1,414	3,270	0,005	0,500	17	12	21	16
3°	25	5,000	1,646	0,005	0,068	17	12	21	17
4°	54	2,819	4,525	0,005	0,500	13	12	25	17
5°	100	4,323	1,985	0,005	0,499	13	12	25	17
6°	42	3,065	2,731	0,005	0,347	13	12	25	17
7°	80	2,036	4,509	0,005	0,189	25	12	13	17
8°	70	5,000	2,532	0,005	0,214	25	12	13	17
9°	45	2,425	5,000	0,005	0,189	26	14	11	17
10°	100	3,771	4,120	0,005	0,053	25	12	13	17

Table 5.77: PSO Parameters' sets 2013-2018

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 1/2020	B&H	Stand.
-668,9	-702,3	-772,8	0,0	-186,2	-174,6	-779,0	-908,0	-933,9
-368,9	-702,3	-772,8	0,0	-186,2	-174,6	-482,6	-908,0	-933,9
-530,2	-702,3	-772,8	0,0	-186,2	-174,6	-642,0	-908,0	-933,9
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9
-460,5	-702,3	-772,8	0,0	-186,2	-174,6	-573,1	-908,0	-933,9
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9

Table 5.78: PSO Out-of-sample Returns

Training the technical analysis indicators in the last period ranging from 2015 to 2020 and testing the new sets, table 5.79, in the first quarter of 2020 do not change the output, table 5.80, respect to the previous table 5.78. The optimised sets of parameters do not enter in the market, meanwhile, the standard set does with a negative income as the one of the Buy and Hold strategy.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	5	3,222	2,543	0,009	0,413	17	23	10	10
2°	5	3,767	2,379	0,009	0,230	14	20	14	10
3°	5	2,837	4,471	0,009	0,053	8	21	21	10
4°	100	4,031	1,135	0,010	0,277	14	22	16	10
5°	5	3,770	3,755	0,009	0,285	11	16	16	10
6°	5	5,000	3,671	0,009	0,431	9	18	22	10
7°	5	3,241	3,596	0,009	0,086	15	19	19	10
8°	6	4,946	2,509	0,009	0,163	26	39	11	9
9°	6	4,531	2,513	0,009	0,403	18	35	17	9
10°	6	4,675	2,505	0,009	0,294	26	33	13	9

Table 5.79: PSO Parameters' sets 2015-2020

1/2020 4/2020	B&H	Stand.
0,0	-186,2	-174,6
0,0	-186,2	-174,6
0,0	-186,2	-174,6
0,0	-186,2	-174,6
0,0	-186,2	-174,6
0,0	-186,2	-174,6
0,0	-186,2	-174,6
0,0	-186,2	-174,6
0,0	-186,2	-174,6
0,0	-186,2	-174,6

Table 5.80: PSO Out-of-sample Returns

5.3.3 EFWA EUR/USD In-sample Results

Period	Time	Average	Best	Worst	B&H	Standard
2015-2020	6/35	0,327	0,363	0,287	-0,073	-0.076
2013-2018	6/35	0,281	0,356	0,164	-0,090	-0.169
2011-2016	6/35	0,158	0,240	0,077	-0,186	-0.222
2009-2014	6/35	0,506	0,608	0,409	-0,016	0.056
2007-2012	6/35	0,626	0,808	0,388	-0,019	-0.014

Table 5.81: EFWA In-sample Returns

The application of the Enhance Firework algorithm, in line with what has been found with the PSO algorithm, needs a lower amount of times in minutes to compute a full cycle of calculation until a stopping criteria is reached respect to the previous analysis. This reduction as it has been explained previously it is due to the reduction of the number of parameters require to analysis the EUR/USD time series. In the case of the EFWA the maximum time required reduce of 10 minutes and sometime an output has been given after 6 minutes. On average it still takes more time than PSO algorithm.

Moving to the performances the EFWA generates always better return in comparison with the other two strategies, table 5.81, that in case of Buy and Hold always return a negative value and the standard set does not go to far from the negative area only in one case. In contrast with the PSO algorithm the EFWA always perform better on average due to a higher worst case scenario values but the PSO is able to find 3 time over 5 a best maximum that EFWA.

5.3.4 EFWA EUR/USD Out-of-sample Results

Testing the optimised parameters sets, table 5.82, in the out-of-sample time series results to be optimal in all the period analysed. Observing the first biennium a head from the first in-sample time series, table 5.83, the strategy propose gain 8 times over 10 more than the Buy and Hold strategy and the standard set of parameters. In the second biennium the optimised sets of parameters perform even better, in fact, all the sets propose beat the other two strategy containing the losses. The figures that come from test of the period from 1/1/2012 to 1/1/2020 still show a good better reduction of losses in contrast with the deeper reduction of the capital of the other strategies.
The EFWA in comparison with the PSO algorithm seems to perform better with better results in all the period just mentioned.
Differently from the PSO algorithm the EFWA have reached higher maximums in the in-sample. The combination of parameters of these sets have permit to attain a higher return even in the out-of-sample analyses. Probably the PSO could have reached the same results if it had continued to search in the parameters space without being blocked by the stopping criteria.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	9	3,011	2,733	0,009	0,127	11	12	9	25
2°	9	3,019	2,790	0,009	0,270	9	18	7	25
3°	9	3,032	2,702	0,009	0,384	11	14	13	25
4°	9	2,996	2,721	0,009	0,322	14	15	10	25
5°	8	4,157	2,848	0,009	0,356	10	13	14	25
6°	8	4,346	2,850	0,009	0,383	12	14	10	25
7°	8	3,877	2,850	0,009	0,240	10	15	12	25
8°	25	1,805	4,324	0,013	0,342	13	20	13	12
9°	26	1,318	1,379	0,009	0,141	16	17	10	26
10°	7	4,142	2,680	0,009	0,223	11	13	13	25

Table 5.82: EFWA Parameters' sets 2007-2012

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
864,9	638,1	688,6	-1398,3	-2056,7	-1971,1	-57,3	-1327,9	-1930,2
864,9	638,1	688,6	-1382,5	-2056,7	-1971,1	-39,0	-1327,9	-1930,2
864,9	638,1	688,6	-1559,1	-2056,7	-1971,1	-509,0	-1327,9	-1930,2
864,9	638,1	688,6	-1559,1	-2056,7	-1971,1	-243,1	-1327,9	-1930,2
864,9	638,1	688,6	-1559,1	-2056,7	-1971,1	-1063,2	-1327,9	-1930,2
864,9	638,1	688,6	-1559,1	-2056,7	-1971,1	-1063,2	-1327,9	-1930,2
864,9	638,1	688,6	-1559,1	-2056,7	-1971,1	-1063,2	-1327,9	-1930,2
324,9	638,1	688,6	-1169,5	-2056,7	-1971,1	-157,4	-1327,9	-1930,2
115,9	638,1	688,6	-1051,3	-2056,7	-1971,1	-1409,3	-1327,9	-1930,2
864,9	638,1	688,6	-1421,3	-2056,7	-1971,1	-192,8	-1327,9	-1930,2

Table 5.83: EFWA Out-of-sample Returns

The second sets of parameters, table 5.84, coming from the in-sample period between 2009 and 2014 once tested in the first biennium it results to perform better than the previous sets with a higher reduction of losses respect to the other two strategies. The following two years from 1/2016 to 1/2018 results are not as good as before because the optimised set of parameters still do better than the standard set but not enough to beat the Buy and Hold strategy. However, taking into consideration a broad period as showed in the last three column in table 5.85, the strategy proposed perform extremely better than the other two generating 5 times over 10 positive returns, meanwhile, Buy and Hold and standard set strategies show huge losses.

The values obtained with the Enhance Firework algorithm are almost the same of the PSO one.

In line with the previous observations, the PSO and the EFWA algorithms efficiently enter and exit from the market in the first biennium ahead after having trained in the in-sample-period. As described before, not always the top ranking sets of parameters are the one that perform the best in the out-of-sample period.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	38	1,292	1,742	0,009	0,272	7	17	10	9
2°	62	1,459	1,927	0,007	0,288	14	22	4	9
3°	62	1,457	1,927	0,007	0,386	13	19	5	9
4°	60	1,409	2,033	0,006	0,223	5	19	11	9
5°	40	4,051	1,678	0,009	0,264	6	17	14	10
6°	72	3,432	2,037	0,009	0,238	9	18	9	10
7°	48	2,222	1,834	0,009	0,448	9	20	8	10
8°	43	4,113	1,741	0,009	0,239	7	19	11	10
9°	56	2,640	1,889	0,009	0,270	8	18	10	10
10°	76	2,120	1,967	0,009	0,259	9	20	8	10

Table 5.84: EFWA Parameters' sets 2009-2014

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-604,5	-2056,7	-1971,1	547,6	1088,5	291,5	584,9	-1799,2	-2331,4
-1412,2	-2056,7	-1971,1	844,9	1088,5	291,5	-560,0	-1799,2	-2331,4
-1412,2	-2056,7	-1971,1	835,6	1088,5	291,5	-522,6	-1799,2	-2331,4
-1114,8	-2056,7	-1971,1	1000,4	1088,5	291,5	-228,7	-1799,2	-2331,4
-147,6	-2056,7	-1971,1	777,4	1088,5	291,5	895,0	-1799,2	-2331,4
-778,1	-2056,7	-1971,1	1059,6	1088,5	291,5	422,2	-1799,2	-2331,4
-837,6	-2056,7	-1971,1	692,4	1088,5	291,5	-156,3	-1799,2	-2331,4
126,1	-2056,7	-1971,1	777,4	1088,5	291,5	1197,7	-1799,2	-2331,4
-653,4	-2056,7	-1971,1	777,4	1088,5	291,5	179,9	-1799,2	-2331,4
-1377,6	-2056,7	-1971,1	972,4	1088,5	291,5	-318,3	-1799,2	-2331,4

Table 5.85: EFWA Out-of-sample Returns

Focusing in the next out-of-sample tests using the parameters sets obtained from the in-sample period ranging from 2011 to 2016, table 5.86, The figures in the first biennium are not good, table 5.87. The values obtained are worst than the ones with the previous sets because 6 times over 10 there are a negative returns in contrast with the always positive outcome of the other two strategies. However, the remaining 4 figures are higher than the standard set of parameters. The following in-sample test shows better results with losses mostly of the time more contained that the Buy and Hold strategy and standard set of parameters and 2 times they finish with a gain.

The combination of the two just mention periods do not produce positive results for the trading strategy propose resulting to be worst of both the two comparing strategies.

This time the EFWA rank second behind the PSO algorithm that in the same period has been able to achieve better performances.

Differently from the PSO algorithm the EFWA have reached lower maximums in the in-sample. The combination of parameters attain do not always beat the other strategies but as it is noticeable the sets that rank first is able to obtain higher return in the first biennium. Probably the EFWA could have reached the same results if it had continued to search in the parameters space without being blocked by the stopping criteria.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	21	2,450	2,258	0,005	0,472	20	16	12	17
2°	7	1,719	1,866	0,012	0,087	23	13	15	15
3°	37	3,680	1,891	0,009	0,077	11	13	12	10
4°	40	2,417	1,447	0,009	0,099	18	14	6	17
5°	25	3,343	4,902	0,006	0,253	19	18	25	3
6°	37	1,989	1,895	0,040	0,364	17	14	21	5
7°	33	2,165	2,035	0,040	0,217	16	15	19	5
8°	25	1,089	3,117	0,034	0,298	24	22	22	4
9°	26	1,142	3,015	0,034	0,398	25	21	22	4
10°	98	1,323	3,807	0,040	0,251	25	20	25	4

Table 5.86: EFWA Parameters' sets 2011-2016

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
1825,1	1088,5	291,5	-699,0	-702,3	-772,8	1243,8	351,7	-486,4
292,9	1088,5	291,5	-48,3	-702,3	-772,8	243,2	351,7	-486,4
512,6	1088,5	291,5	27,6	-702,3	-772,8	541,6	351,7	-486,4
-403,8	1088,5	291,5	-738,7	-702,3	-772,8	-848,5	351,7	-486,4
1003,7	1088,5	291,5	-510,7	-702,3	-772,8	679,9	351,7	-486,4
-429,4	1088,5	291,5	-382,8	-702,3	-772,8	-795,7	351,7	-486,4
-429,4	1088,5	291,5	108,6	-702,3	-772,8	-325,5	351,7	-486,4
-716,9	1088,5	291,5	-535,5	-702,3	-772,8	-1213,9	351,7	-486,4
-716,9	1088,5	291,5	-535,5	-702,3	-772,8	-1213,9	351,7	-486,4
-164,8	1088,5	291,5	-630,0	-702,3	-772,8	-784,5	351,7	-486,4

Table 5.87: EFWA Out-of-sample Returns

The following period of training from 2013 to 2018 that produced the sets of parameters in table 5.88 still perform well in the first out-of-sample period, columns 1 to 3 table 5.89, in comparison to the the Buy and Hold strategy and the standard set of parameters. Trying to testing these sets in the first quarter of 2020 results to be apparently good because the algorithms do not enter in the market avoiding losses that both the other two strategies have taken. Merging the two out-of-sample periods still returns a positive conclusion due to the higher reduction of losses of the strategy proposed in all the sets tested.

Comparing the results obtained with the PSO algorithm the EFWA shows almost the same result in the first and last part and the same behaviour in the second.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	14	3,503	3,926	0,005	0,140	25	12	12	17
2°	23	2,261	1,931	0,005	0,181	26	22	14	16
3°	69	3,747	4,412	0,005	0,326	25	12	13	16
4°	24	1,730	2,008	0,005	0,157	19	17	25	16
5°	38	2,114	2,025	0,005	0,420	24	12	14	16
6°	75	2,290	2,813	0,005	0,371	21	13	16	17
7°	14	4,718	2,604	0,005	0,350	18	12	21	17
8°	49	3,095	2,650	0,005	0,139	22	12	15	17
9°	18	2,815	4,435	0,005	0,238	23	13	14	17
10°	63	3,556	4,118	0,005	0,084	24	13	14	17

Table 5.88: EFWA Parameters' sets 2013-2018

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9
-442,7	-702,3	-772,8	0,0	-186,2	-174,6	-555,5	-908,0	-933,9
-379,9	-702,3	-772,8	0,0	-186,2	-174,6	-493,5	-908,0	-933,9
-442,7	-702,3	-772,8	0,0	-186,2	-174,6	-555,5	-908,0	-933,9
-433,6	-702,3	-772,8	0,0	-186,2	-174,6	-546,6	-908,0	-933,9
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9
-599,2	-702,3	-772,8	0,0	-186,2	-174,6	-710,2	-908,0	-933,9
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9
-609,8	-702,3	-772,8	0,0	-186,2	-174,6	-720,6	-908,0	-933,9

Table 5.89: EFWA Out-of-sample Returns

Training the technical analysis indicators in the last period ranging from 2015 to 2020 and testing the new sets, table 5.90, in the first quarter of 2020 it do not change the output, table 5.91, respect to the previous table 5.89. The optimised sets of parameters do not enter in the market, meanwhile, the standard set does with a negative income as the one of the Buy and Hold strategy.

Both PSO and EFWA produce the same results.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	9	2,272	2,531	0,009	0,236	8	21	21	10
2°	9	2,300	2,516	0,009	0,223	15	23	10	10
3°	5	4,491	2,622	0,009	0,106	18	19	12	10
4°	5	4,812	2,575	0,009	0,202	13	18	17	10
5°	5	4,836	2,615	0,009	0,285	11	18	20	10
6°	87	4,731	1,959	0,009	0,057	20	21	9	10
7°	5	2,870	2,698	0,009	0,219	8	18	22	10
8°	5	2,860	2,720	0,009	0,422	18	22	9	10
9°	5	2,835	2,587	0,009	0,299	9	18	20	9
10°	5	2,886	3,387	0,009	0,139	13	18	12	10

Table 5.90: EFWA Parameters' sets 2015-2020

1/2020 4/2020	B&H	Stand.
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6

Table 5.91: EFWA Out-of-sample Returns,unitary investment

5.3.5 DE EUR/USD In-sample Results

Period	Time	Average	Best	Worst	B&H	Standard
2015-2020	7/8	0,331	0,364	0,297	-0,073	-0.076
2013-2018	7/8	0,337	0,405	0,205	-0,091	-0.169
2011-2016	7/8	0,177	0,272	0,101	-0,186	-0.222
2009-2014	7/8	0,547	0,608	0,331	-0,016	0.056
2007-2012	7/8	0,634	0,808	0,375	-0,019	-0.014

Table 5.92: DE In-sample Returns

The application of the Differential evolution algorithm follows the same trend showed by the Enhance Firework and PSO algorithm with a substantial reduction of the time. With the EUR/USD time series the algorithm takes an average of 7/8 minutes to compute 100 iterations.
Analysing the figures of the in-sample training periods the DE rank first both in speed and efficiency in the data found. The algorithm is able to beat, mostly of the times, both the other two algorithms in all the categories obtaining higher worst, average and best values, 5.92.

5.3.6 DE EUR/USD Out-of-sample Results

The outstanding performances that have been seen in table 5.92 are not maintained in the out-of-sample test. The sets of parameters of the first in-sample period, table 5.93, show a positives return higher than Buy and Hold and standard set 7 times over 10, table 5.94. The same good results are obtained in the following biennium in which the optimised sets always beat the other two strategies. Observing the returns of the period from 1/1/2012 to 1/1/2020, last three columns of the table 5.94, they highlight a better loss reduction of the strategy proposed in 7 cases respect to the Buy and Hold strategy and 9 times in comparison with the standard set.

Among the algorithm compared the DE seems to rank first with a better losses reduction than EFWA and in the end there is the PSO algorithm that have not produced as good results as the others.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	9	3,027	2,838	0,009	0,309	11	12	13	25
2°	9	3,005	2,743	0,009	0,279	12	16	7	25
3°	8	4,037	2,848	0,009	0,137	14	16	8	25
4°	7	3,852	2,670	0,009	0,185	8	14	12	25
5°	7	3,665	2,647	0,009	0,374	8	13	13	25
6°	7	3,715	2,657	0,009	0,491	12	13	8	25
7°	8	3,628	2,933	0,009	0,354	10	15	8	25
8°	16	4,804	2,350	0,009	0,300	10	14	11	25
9°	16	4,103	2,335	0,009	0,186	12	18	6	25
10°	16	4,008	2,345	0,009	0,136	7	17	10	25

Table 5.93: DE Parameters' sets 2007-2012

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
864,9	638,1	688,6	-1559,1	-2056,7	-1971,1	-718,4	-1327,9	-1930,2
864,9	638,1	688,6	-1382,5	-2056,7	-1971,1	-39,0	-1327,9	-1930,2
864,9	638,1	688,6	-1559,1	-2056,7	-1971,1	-1063,2	-1327,9	-1930,2
864,9	638,1	688,6	-1382,5	-2056,7	-1971,1	-35,1	-1327,9	-1930,2
1450,1	638,1	688,6	-1398,3	-2056,7	-1971,1	363,3	-1327,9	-1930,2
864,9	638,1	688,6	-1398,3	-2056,7	-1971,1	-53,4	-1327,9	-1930,2
864,9	638,1	688,6	-1382,5	-2056,7	-1971,1	-809,5	-1327,9	-1930,2
360,7	638,1	688,6	-1406,0	-2056,7	-1971,1	-1972,8	-1327,9	-1930,2
360,7	638,1	688,6	-1361,9	-2056,7	-1971,1	-1872,5	-1327,9	-1930,2
360,7	638,1	688,6	-1361,9	-2056,7	-1971,1	-1872,5	-1327,9	-1930,2

Table 5.94: DE Out-of-sample Returns

Moving to the next in-sample trading period, the sets of parameters computed, table 5.95, contain better the losses in the biennium of 1/2014-1/2016 in comparison with both the other two strategies but also with the previous sets of parameters proposed in table 5.93. These improvements do not last enough to beat the Buy and Hold strategy in the following biennium but the gains are still higher than the standard set. Observing the whole period together after the end of the in-sample analysis, the optimised sets of parameters values are 6 times over 10 positives in contrast with the losses of the other two strategies and even the remaining 4 sets that produce a negative returns they are more contained that the others.

This time the PSO and the DE algorithms are able to find sets of parameters that in the out-of-sample periods invest better than EFWA.

Even if the DE and the EFWA have found the same local maximums in the in-sample period the sets compositions are different. However, leaving the algorithms running more time without being stopped prematurely by the stopping criteria it would have not found better sets to test in the out of sample period as observable by the values found by the PSO algorithm.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	37	1,270	1,726	0,008	0,079	11	13	8	9
2°	37	1,205	1,718	0,009	0,236	11	16	6	9
3°	38	1,224	1,737	0,009	0,229	6	17	11	9
4°	37	1,269	1,683	0,008	0,005	8	18	8	9
5°	38	1,296	1,703	0,009	0,189	8	18	8	9
6°	42	1,292	1,722	0,009	0,283	10	19	6	9
7°	65	2,310	1,571	0,005	0,220	9	38	5	7
8°	62	1,439	1,958	0,007	0,152	5	15	16	9
9°	61	1,439	1,875	0,008	0,441	12	30	4	9
10°	62	1,455	2,007	0,007	0,147	10	26	5	9

Table 5.95: DE Parameters' sets 2009-2014

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-563,8	-2056,7	-1971,1	491,0	1088,5	291,5	545,1	-1799,2	-2331,4
-538,4	-2056,7	-1971,1	491,0	1088,5	291,5	411,8	-1799,2	-2331,4
-546,2	-2056,7	-1971,1	547,6	1088,5	291,5	351,9	-1799,2	-2331,4
-563,8	-2056,7	-1971,1	438,6	1088,5	291,5	419,9	-1799,2	-2331,4
-675,1	-2056,7	-1971,1	373,9	1088,5	291,5	208,7	-1799,2	-2331,4
-415,0	-2056,7	-1971,1	373,9	1088,5	291,5	176,7	-1799,2	-2331,4
-1423,4	-2056,7	-1971,1	599,7	1088,5	291,5	-499,5	-1799,2	-2331,4
-1514,8	-2056,7	-1971,1	835,6	1088,5	291,5	-667,0	-1799,2	-2331,4
-1374,3	-2056,7	-1971,1	979,6	1088,5	291,5	-400,7	-1799,2	-2331,4
-1486,3	-2056,7	-1971,1	844,9	1088,5	291,5	-641,4	-1799,2	-2331,4

Table 5.96: DE Out-of-sample Returns

Observing the returns obtained using the sets of parameters, table 5.97, coming from the training period of 2011 - 2016, they reduce the efficiency before obtained resulting to get 7 times over 10 a negative return while the Buy and Hold strategy and the standard set perform better. A substantial change is observable in the following biennium where the optimised sets of parameters are always able to contain the losses more than the other strategies, table 5.98. However, merging the two time series and analysing the data together do not result to be optimal for the strategy proposed because it underperforms in comparison with the others.

The values that have been showed are in line with what has been found with EFWA in all the three periods considered, meanwhile, the PSO algorithm dissociate showing higher returns and better losses reduction in all the periods.

Differently from the PSO algorithm the EFWA and the DE have reached lower maximums in the in-sample. The combination of parameters of these sets have obtain lower returns even in the out-of-sample analyses. Probably the DE as the EFWA could have reached the same results of the PSO if they had continued to search in the parameters space without being blocked by the stopping criteria.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	71	3,947	1,649	0,006	0,071	23	8	8	17
2°	2	4,030	2,757	0,045	0,133	17	11	23	5
3°	2	4,053	2,695	0,046	0,431	19	11	21	5
4°	30	3,755	1,303	0,006	0,355	22	16	23	4
5°	8	3,980	2,501	0,040	0,304	26	23	25	4
6°	8	3,742	2,377	0,040	0,072	25	23	27	4
7°	8	3,389	2,567	0,040	0,379	26	22	23	4
8°	8	4,287	2,538	0,040	0,323	25	21	24	4
9°	8	3,347	2,618	0,040	0,296	26	20	25	4
10°	8	4,010	2,539	0,040	0,262	26	25	27	4

Table 5.97: DE Parameters' sets 2011-2016

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
931,5	1088,5	291,5	-502,6	-702,3	-772,8	724,4	351,7	-486,4
-150,2	1088,5	291,5	49,2	-702,3	-772,8	-101,7	351,7	-486,4
572,4	1088,5	291,5	52,9	-702,3	-772,8	628,3	351,7	-486,4
1019,1	1088,5	291,5	-465,4	-702,3	-772,8	506,2	351,7	-486,4
-520,3	1088,5	291,5	-535,5	-702,3	-772,8	-1028,0	351,7	-486,4
-520,3	1088,5	291,5	-535,5	-702,3	-772,8	-1028,0	351,7	-486,4
-428,6	1088,5	291,5	-535,5	-702,3	-772,8	-941,1	351,7	-486,4
-744,9	1088,5	291,5	-535,5	-702,3	-772,8	-1240,5	351,7	-486,4
-744,9	1088,5	291,5	-535,5	-702,3	-772,8	-1240,5	351,7	-486,4
-502,5	1088,5	291,5	-535,5	-702,3	-772,8	-1011,0	351,7	-486,4

Table 5.98: DE Out-of-sample Returns

The following period of training from 2013 to 2018 that produced the sets of parameters in table 5.99 lose its ability to contain the losses, columns 1 to 3 table 5.100, resulting to be worst than the Buy and Hold strategy and the standard set of parameters. Trying to testing these sets in the first quarter of 2020 results to be apparently good because the algorithm lose slightly less than the other strategies. The combination of the two time series seems to produce better results from the others strategies but the values produced do not dissociate to much.

Comparing the results obtained with the PSO algorithm the EFWA show almost the same results in the first, last part and the same behaviour in the second, while, the DE always operates in the market concluding the period with huger losses than the other algorithms.

This time the DE evolution have found a combination of parameters that highly perform in the training period, in fact, it obtained the top best profit. However, the test on the first biennium out of sample do not produce as good results as the other algorithms' sets. It might be possible that the combination of parameters found it is too specific for the training period to be adapted outside of it.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	18	1,942	2,269	0,003	0,101	26	8	19	22
2°	18	1,943	2,259	0,003	0,433	23	9	19	22
3°	20	1,975	2,188	0,003	0,357	22	9	17	22
4°	20	2,007	2,178	0,003	0,211	18	9	22	22
5°	20	1,952	2,152	0,003	0,359	26	12	13	22
6°	20	1,930	2,167	0,003	0,470	24	12	16	22
7°	20	1,983	2,219	0,003	0,309	22	9	18	22
8°	20	1,949	2,213	0,003	0,235	26	9	17	22
9°	18	1,953	2,279	0,003	0,101	24	8	21	22
10°	20	2,019	2,149	0,003	0,147	23	11	17	22

Table 5.99: DE Parameters' sets 2013-2018

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	4/2018 1/2020	B&H	Stand.
-848,0	-702,3	-772,8	-118,1	-186,2	-174,6	-956,1	-908,0	-933,9
-848,0	-702,3	-772,8	-118,1	-186,2	-174,6	-956,1	-908,0	-933,9
-789,3	-702,3	-772,8	-118,1	-186,2	-174,6	-898,0	-908,0	-933,9
-330,2	-702,3	-772,8	-118,1	-186,2	-174,6	-444,3	-908,0	-933,9
-330,2	-702,3	-772,8	-118,1	-186,2	-174,6	-444,3	-908,0	-933,9
-789,3	-702,3	-772,8	-118,1	-186,2	-174,6	-898,0	-908,0	-933,9
-330,2	-702,3	-772,8	-118,1	-186,2	-174,6	-444,3	-908,0	-933,9
-789,3	-702,3	-772,8	-118,1	-186,2	-174,6	-898,0	-908,0	-933,9
-848,0	-702,3	-772,8	-118,1	-186,2	-174,6	-956,1	-908,0	-933,9
-789,3	-702,3	-772,8	-118,1	-186,2	-174,6	-898,0	-908,0	-933,9

Table 5.100: DE Out-of-sample Returns

Training the technical analysis indicators in the last period ranging from 2015 to 2020 and testing the new sets, table 5.101, in the first quarter of 2020 the outputs change with the avoidance to trade in the market , table 5.102. The optimised sets of parameters do not enter in the market, meanwhile, the standard set does with a negative income as the one of the Buy and Hold strategy.

DE, PSO and EFWA algorithms produce the same results.

Rank	P1	P2	P3	P4	P5	P6	P7	P8	P9
1°	4	2,060	2,882	0,009	0,076	9	26	13	9
2°	5	4,277	2,612	0,009	0,132	13	16	20	10
3°	5	3,367	2,579	0,009	0,280	15	22	12	10
4°	5	3,755	2,341	0,009	0,140	14	19	15	10
5°	5	4,030	2,448	0,009	0,301	14	23	12	10
6°	5	2,822	2,720	0,009	0,257	13	29	10	10
7°	5	2,865	2,687	0,009	0,207	10	24	13	10
8°	5	2,880	2,391	0,009	0,349	18	21	9	10
9°	5	2,823	2,683	0,009	0,468	11	30	11	10
10°	5	2,858	2,457	0,009	0,381	9	16	22	10

Table 5.101: DE Parameters' sets 2015-2020

1/2020 4/2020	B&H	Stand.
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6
0	-186,2	-174,6

Table 5.102: DE Out-of-sample Returns

5.4 Overview results

In the previous sections have been showed in detail the returns of all the analysis executed in the thesis highlighting how the profit/losses of the optimised sets of parameters might be good at the beginning and worse in the second biennium or vice versa. In this section, it will be highlight the total returns of the best ten in sample sets of parameters at the end of the period considered. In other words, the returns of the first two years out-of-sample period are compounded until the first quarter of 2020. In the columns from 2 to 6 there are the annual returns of the first biennium out-of-sample and in the last three column there are the capitalised returns of the optimised sets of parameters compared with the Buy and Hold strategy and the standard set.

Amazon algorithms' results comparison

The returns generated by the Buy and Hold strategy regarding the Amazon time series seems to be the best options among the three proposed. However, successfully with the aim of the thesis the algorithms used are able to find some parameters that perform better than the standard set as it appears from the figures in table 5.103, 5.104 and 5.105. In fact, with all the algorithms the optimised set of parameters have always generated a higher return. On the other, it is difficult to evaluate which one among the algorithms have produced the best returns because the results are highly volatile and present huge difference among them.

Rank	Year 1/2	Year 3/4	Year 5/6	Year 7/8	1-4/2020	Optimised	B&H	Standard
1°	0,000	0,294	0,292	0,160	0,018	2,829	9,603	0,818
2°	0,329	0,294	0,300	0,122	0,018	5,401	9,603	0,818
3°	0,347	0,294	0,087	0,086	0,018	3,308	9,603	0,818
4°	0,460	0,251	0,019	0,086	0,018	3,158	9,603	0,818
5°	0,280	0,383	0,140	0,086	0,018	3,890	9,603	0,818
6°	0,419	0,329	0,246	0,086	0,018	5,625	9,603	0,818
7°	0,460	0,329	0,178	0,086	0,018	5,263	9,603	0,818
8°	0,460	0,371	0,345	0,086	0,018	7,683	9,603	0,818
9°	0,433	0,371	0,345	0,086	0,018	7,366	9,603	0,818
10°	0,460	0,371	0,077	0,086	0,018	4,573	9,603	0,818

Table 5.103: Differential Evolution algorithm results

Rank	Year 1/2	Year 3/4	Year 5/6	Year 7/8	1-4/2020	Optimised	B&H	Standard
1°	0,367	0,371	0,172	0,093	0,018	4,860	9,603	0,818
2°	0,502	0,371	0,197	0,122	0,018	6,770	9,603	0,818
3°	0,423	0,388	0,225	0,133	-0,049	6,153	9,603	0,818
4°	0,419	0,388	0,295	0,234	0,047	9,369	9,603	0,818
5°	0,397	0,388	0,196	0,089	-0,003	5,364	9,603	0,818
6°	0,207	0,334	0,288	0,089	-0,003	4,082	9,603	0,818
7°	0,263	0,321	0,288	0,086	-0,003	4,428	9,603	0,818
8°	0,180	0,294	0,288	0,119	0,025	3,972	9,603	0,818
9°	0,252	0,296	0,270	0,227	0,052	5,725	9,603	0,818
10°	0,210	0,286	0,145	0,100	-0,017	2,779	9,603	0,818

Table 5.104: PSO algorithm results

Rank	Year 1/2	Year 3/4	Year 5/6	Year 7/8	1-4/2020	Optimised	B&H	Standard
1°	0,254	0,294	0,124	0,204	0,018	3,911	9,603	0,818
2°	0,433	0,294	0,087	0,234	0,018	5,297	9,603	0,818
3°	0,451	0,296	0,201	0,234	0,018	6,908	9,603	0,818
4°	0,472	0,296	0,174	0,234	0,018	6,779	9,603	0,818
5°	0,325	0,294	0,177	0,234	-0,003	5,180	9,603	0,818
6°	0,450	0,170	0,132	0,148	0,018	3,941	9,603	0,818
7°	0,325	0,273	0,181	0,122	0,018	4,080	9,603	0,818
8°	0,433	0,371	0,107	0,172	-0,049	5,176	9,603	0,818
9°	0,256	0,445	0,123	0,178	0,018	4,873	9,603	0,818
10°	0,423	0,449	0,191	0,160	0,018	7,276	9,603	0,818

Table 5.105: Enhance Firework algorithm results

BRF S.A. algorithms' results comparison

The figures illustrated analysing the BRF S.A. shows an imagine completely different from the one of Amazon (tables 5.106, 5.107 and 5.108), because in this case instead of a skyrocket of the value of the company there is a plummet of its price. The Buy and Hold strategy results to be the worst sustaining the biggest losses, meanwhile, the standard set seems to be the best option. Among the algorithms, it is possible to say that the PSO is the one that rank third while the EFWA seems to rank first.

An observation that is noticeable to be highlighted is that the standard set rank first only because it does not enter in the market during the Pandemic crisis avoiding a huge loss. Without the losses of that event that could be avoided adding some stop-loss criteria the final performance would have been completely different with the optimised sets ranking first.

Rank	Year 1/2	Year 3/4	Year 5/6	Year 7/8	1-4/2020	Optimised	B&H	Standard
1°	0,022	-0,043	-0,095	0,114	-0,602	-0,614	-0,857	-0,331
2°	0,125	-0,043	-0,095	-0,178	-0,602	-0,745	-0,857	-0,331
3°	0,148	-0,043	-0,095	-0,178	0,000	-0,333	-0,857	-0,331
4°	0,022	-0,043	-0,095	-0,178	-0,602	-0,790	-0,857	-0,331
5°	0,092	-0,043	-0,063	-0,178	-0,602	-0,743	-0,857	-0,331
6°	0,148	-0,043	-0,061	-0,178	-0,602	-0,714	-0,857	-0,331
7°	0,139	0,014	-0,112	-0,178	-0,602	-0,718	-0,857	-0,331
8°	0,142	-0,035	-0,112	-0,214	-0,602	-0,765	-0,857	-0,331
9°	0,080	0,014	-0,091	-0,178	-0,602	-0,734	-0,857	-0,331
10°	0,205	-0,043	-0,006	-0,178	-0,602	-0,647	-0,857	-0,331

Table 5.106: Differential Evolution algorithm results

Rank	Year 1/2	Year 3/4	Year 5/6	Year 7/8	1-4/2020	Optimised	B&H	Standard
1°	-0,181	-0,016	-0,095	-0,080	-0,602	-0,821	-0,857	-0,331
2°	-0,131	-0,053	-0,006	-0,053	-0,602	-0,761	-0,857	-0,331
3°	0,126	-0,043	-0,118	-0,053	-0,602	-0,678	-0,857	-0,331
4°	-0,151	-0,043	-0,006	-0,154	-0,602	-0,815	-0,857	-0,331
5°	-0,155	-0,043	-0,160	0,094	-0,602	-0,780	-0,857	-0,331
6°	-0,064	-0,025	-0,084	-0,032	-0,602	-0,740	-0,857	-0,331
7°	-0,058	-0,035	0,235	0,051	-0,602	-0,447	-0,857	-0,331
8°	-0,151	0,004	0,005	-0,034	-0,602	-0,727	-0,857	-0,331
9°	-0,024	-0,015	-0,084	-0,178	0,000	-0,476	-0,857	-0,331
10°	-0,120	0,003	0,063	0,005	-0,602	-0,647	-0,857	-0,331

Table 5.107: PSO algorithm results

Rank	Year 1/2	Year 3/4	Year 5/6	Year 7/8	1-4/2020	Optimised	B&H	Standard
1°	0,096	-0,079	-0,089	-0,012	-0,602	-0,671	-0,857	-0,331
2°	0,101	-0,023	-0,051	-0,182	-0,602	-0,722	-0,857	-0,331
3°	0,134	-0,035	-0,112	-0,031	-0,602	-0,648	-0,857	-0,331
4°	0,143	0,003	-0,112	-0,003	-0,602	-0,591	-0,857	-0,331
5°	0,134	-0,039	-0,112	-0,034	-0,602	-0,653	-0,857	-0,331
6°	-0,075	-0,016	-0,010	0,004	-0,602	-0,674	-0,857	-0,331
7°	-0,058	-0,053	-0,010	-0,018	-0,602	-0,701	-0,857	-0,331
8°	-0,057	-0,025	-0,010	0,011	-0,164	-0,293	-0,857	-0,331
9°	0,158	-0,053	-0,112	0,066	-0,602	-0,572	-0,857	-0,331
10°	-0,057	-0,053	-0,010	-0,074	-0,615	-0,742	-0,857	-0,331

Table 5.108: Enhance Firework algorithm results

EUR/USD algorithms' results comparison

The EUR/USD differently from the other two times series do not present a higher increase or decrease of the price but in the period considered it shows high volatility. In this scenario the strategy proposed rank first beating both the standard set of parameters and the Buy and Hold strategies. Where the other strategies resulted into a loss of capital, many times the optimised sets of parameters returned a positive gains and where they showed losses they were more contained.

Comparing the algorithms the one that present the majority of the positive results is the PSO algorithm, while EFWA and DE almost have the same results, tables 5.109, 5.110 and 5.111.

Rank	Year 1/2	Year 3/4	Year 5/6	Year 7/8	1-4/2020	Optimised	B&H	Standard
1°	0,042	-0,029	0,046	-0,043	0,000	0,026	-0,149	-0,207
2°	0,042	-0,027	-0,008	-0,043	0,000	-0,073	-0,149	-0,207
3°	0,042	-0,028	0,028	-0,040	0,000	0,000	-0,149	-0,207
4°	0,042	-0,029	0,050	-0,017	0,000	0,092	-0,149	-0,207
5°	0,070	-0,034	-0,026	-0,017	0,000	-0,021	-0,149	-0,207
6°	0,042	-0,021	-0,026	-0,040	0,000	-0,091	-0,149	-0,207
7°	0,042	-0,074	-0,022	-0,017	0,000	-0,138	-0,149	-0,207
8°	0,018	-0,079	-0,038	-0,040	0,000	-0,251	-0,149	-0,207
9°	0,018	-0,071	-0,038	-0,043	0,000	-0,243	-0,149	-0,207
10°	0,018	-0,077	-0,025	-0,040	0,000	-0,228	-0,149	-0,207

Table 5.109: Differential Evolution algorithm results

Rank	Year 1/2	Year 3/4	Year 5/6	Year 7/8	1-4/2020	Optimised	B&H	Standard
1°	0,042	-0,085	0,042	-0,034	0,000	-0,078	-0,149	-0,207
2°	0,018	-0,040	0,046	-0,019	0,000	0,007	-0,149	-0,207
3°	0,000	-0,057	-0,020	-0,027	0,000	-0,193	-0,149	-0,207
4°	0,000	-0,111	0,052	-0,031	0,000	-0,179	-0,149	-0,207
5°	0,006	-0,035	0,057	-0,023	0,000	0,004	-0,149	-0,207
6°	0,018	-0,074	0,071	-0,031	0,000	-0,043	-0,149	-0,207
7°	0,022	0,001	0,060	-0,031	0,000	0,102	-0,149	-0,207
8°	0,016	-0,031	0,060	-0,031	0,000	0,023	-0,149	-0,207
9°	0,000	-0,014	0,057	-0,031	0,000	0,019	-0,149	-0,207
10°	0,000	-0,029	0,000	-0,031	0,000	-0,115	-0,149	-0,207

Table 5.110: PSO algorithm results

Rank	Year 1/2	Year 3/4	Year 5/6	Year 7/8	1-4/2020	Optimised	B&H	Standard
1°	0,042	-0,031	0,087	-0,031	0,000	0,134	-0,149	-0,207
2°	0,042	-0,073	0,015	-0,022	0,000	-0,082	-0,149	-0,207
3°	0,042	-0,073	0,025	-0,019	0,000	-0,056	-0,149	-0,207
4°	0,042	-0,057	-0,020	-0,022	0,000	-0,115	-0,149	-0,207
5°	0,042	-0,007	0,049	-0,022	0,000	0,127	-0,149	-0,207
6°	0,042	-0,040	-0,022	-0,031	0,000	-0,100	-0,149	-0,207
7°	0,042	-0,043	-0,022	-0,030	0,000	-0,104	-0,149	-0,207
8°	0,016	0,006	-0,037	-0,031	0,000	-0,089	-0,149	-0,207
9°	0,006	-0,033	-0,037	-0,031	0,000	-0,176	-0,149	-0,207
10°	0,042	-0,071	-0,008	-0,031	0,000	-0,135	-0,149	-0,207

Table 5.111: Enhance Firework algorithm results

Chapter 6

Conclusion

The aim of this thesis is to exploit the profitability of the Technical Analysis respect to other techniques of investment decision but also testing the profitability of the standard parameters of the indicators used to see if the literature has correctly chosen them or there is margin of improvement. To answer these questions three metaheuristic approaches have been proposed with which they permit to exploit the research space of the possible solutions identifying the combination of parameters that perform the best. Some of these algorithms have been already used but they have never been compared together for such reason testing them solving the same problems give the possibility to highlight the most efficient among them.

After the careful analysis presented in the previous chapters the Technical Analysis with the optimised set of parameters performs better than the Buy and Hold strategy 2 times over three, precisely in the EUR/USD time series and the BRF S.A. time series. Both time series show a decline of the price but in the emerging market this is deeper respects to the foreign one. Within this market characteristics the optimised sets proposed are able to contain more the losses and sometimes even generates gains. However, when the

price is characterised by a sharp increase of its value as the case of Amazon time series the strategy proposed is not able to enter in the market promptly after small correction resulting to gain less than the Buy and Hold strategy.
The outputs showed in the results chapter highlight two important things in the comparison between optimised sets of parameters and standard set. First, the algorithms have found parameters that different from the classic values propose in literature and these values always perform better in the in-sample period and still do the same in the out-of-sample analysis with Amazon and EUR/USD time series. Second, the sharp decline of the price visible in the first quarter of 2020 gives important to include a stop-loss strategy that in case of similar situation block the trading system avoiding to take all the losses. Such action would have resulted to be advantageous within the strategy proposed making it more profitable of the standard set in the case of BRF S.A. time series.
The metaheuristic algorithm, in line with the literature up to now written, are well suitable to solve problems of trading efficiency showing extremely good results in the in-sample analysis and good out-of-sample application. Among the algorithms analysed the Enhance Firework algorithm is able to find the best in-sample results but with the double of the time required by the Differential Evolution that has obtained values close to the other algorithms.
In the out-of-sample analysis the best in-sample set is not always the one that generates the higher return. The returns generated in the out-of-sample analysis are almost similar among the algorithms, however, if it is not possible to find the best looking at the results because they are very similar it

is easy to find the fastest. In fact, the algorithm that requires less time to find a solution is the Differential Evolution followed by the Particle Swarm Optimization and in the end the Enhance Firework algorithm.

In conclusion, it is possible to say that the metaheuristic algorithms are a good approach to find solutions for trading efficiency problems and with these instruments it is possible to gain or contain the losses better respects to classical Technical Analysis approach and sometimes respect to the Buy and Hold strategy.

Appendix A

Selection of metaheuristics' parameters

Here it will be presented the result obtained by testing the metaheuristics' parameters introduced in chapter 4 with the time series Amazon ranging from 1st January 2015 to 31st December 2019. The data shows the total return of a capital of 1 dollar invested in the stock with the possibility to trade fraction quantity of the security. A total of 3 tables are showed: PSO, EFWA and DE results.

param.	iterat.	swarm	w	Best personal	Best global	Indicators	average	best position	worst position	B&H
10	2000	63	0,6571	1,6319	0,6239	ALL	6,82929	9,66021	4,69685	4,98
10	2000	204	-0,2134	-0,3344	2,3259	ALL	5,42057	7,48217	4,26082	4,98
10	2000	53	-0,3488	-0,2746	4,8976	ALL	6,51445	7,02285	5,95058	4,98
10	100	10	0,279	1,49618	1,49618	ALL	7,35385	7,44084	7,31852	4,98
10	1000	10	0,279	1,49618	1,49618	ALL	6,08045	6,43784	6,01473	4,98
10	1000	20	0,279	1,49618	1,49618	ALL	6,98673	6,98673	6,98673	4,98
10	1000	30	0,279	1,49618	1,49618	ALL	7,62045	7,62045	7,62045	4,98
10	1000	40	0,279	1,49618	1,49618	ALL	7,31502	7,31502	7,31502	4,98
10	1000	50	0,279	1,49618	1,49618	ALL	7,21644	7,21644	7,21644	4,98
10	1000	100	0,279	1,49618	1,49618	ALL	8,50466	8,50466	8,50466	4,98

Table A.1: PSO parameters' tests results

Param.	fireworks	Iter.	m	time min.	Indicat.	Average	Best position	Worst position	B&H
10	20	100	200	3/20	ALL	7,29876	11,42620	3,85012	4,98
10	10	100	100	3/10	ALL	7,65003	10,87030	3,67690	4,98
10	5	100	100	3/10	ALL	4,47339	7,57744	3,41800	4,98
10	50	100	200	4/14	ALL	9,68441	12,96301	3,85012	4,98
10	50	100	400	6/19	ALL	7,01026	8,34069	6,34041	4,98
10	100	100	200	8/30	ALL	8,97028	10,23136	8,06523	4,98
10	100	100	400	22/35	ALL	9,57066	11,00993	8,10330	4,98
10	50	100	200	3/40	ALL	7,45508	10,43884	4,46021	4,98
10	100	100	200	22/35	ALL	8,29138	11,48169	6,13923	4,98
10	100	100	400	20/45	ALL	9,13760	11,48169	6,31279	4,98

Table A.2: EFWA parameters' tests results

Param.	NP	Iter.	F	CR	Time min.	Indicat.	Average	Best position	Worst position	B&H
10	45	100	0,7153	0,9148	4/5	ALL	6,14882	7,83552	5,09271	4,98
10	28	100	0,6607	0,9426	2/3	ALL	6,70914	8,25632	5,69432	4,98
10	12	100	0,6702	0,2368	1/2	ALL	6,02759	7,62865	5,11217	4,98
10	12	500	0,6702	0,2368	4/6	ALL	9,31549	10,51132	7,56533	4,98
10	45	500	0,7153	0,9148	18/20	ALL	11,56384	11,62756	11,47583	4,98
10	28	500	0,6607	0,9426	11/12	ALL	10,67503	10,71641	10,41795	4,98
10	100	100	0,2	0,1	8/9	ALL	6,41655	10,11526	4,69956	4,98
10	100	100	0,5	0,1	8/9	ALL	6,28972	10,30884	4,38794	4,98
10	100	100	0,8	0,1	8/9	ALL	6,09223	9,74523	4,45743	4,98
10	100	100	0,2	0,9	7/8	ALL	10,40541	10,42063	10,39994	4,98
10	100	100	0,5	0,9	7/8	ALL	8,57773	10,82653	7,34658	4,98
10	100	100	0,8	0,9	8/9	ALL	6,34501	9,06523	5,24060	4,98
10	100	1000	0,2	0,9	11/12	ALL	11,72828	11,72828	11,72828	4,98

Table A.3: Differential Evolution parameters' tests results

Appendix B

Amazon Algorithms unitary Results

In this chapter it will be presented the results of the application of the sets of parameters obtained by the in-sample analyses. The trading returns in the following tables consider a initial capital of 1 dollar and an investment of non integer quantity of Amazon stock differently from what has been presented in chapter 5. The same conclusions that has been reached in chapter 5 can be done also with the following values, the only differences is that the returns are slightly different.

B.1 PSO Amazon Out-of-sample Results

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
0,868	1,228	0,072	0,629	0,698	0,019	4,307	9,321	1,060
1,256	1,228	0,072	0,713	0,698	0,019	8,097	9,321	1,060
1,026	1,228	0,072	0,511	0,698	0,019	6,902	9,321	1,060
1,013	1,228	0,072	0,374	0,698	0,019	6,136	9,321	1,060
0,952	1,228	0,072	0,511	0,698	0,019	6,614	9,321	1,060
0,456	1,228	0,072	0,780	0,698	0,019	4,137	9,321	1,060
0,595	1,228	0,072	1,088	0,698	0,019	6,097	9,321	1,060
0,393	1,228	0,072	1,088	0,698	0,019	4,200	9,321	1,060
0,568	1,228	0,072	0,951	0,698	0,019	5,919	9,321	1,060
0,465	1,228	0,072	0,889	0,698	0,019	5,112	9,321	1,060

Table B.1: PSO Out-of-sample Results, unitary investment

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
0,879	0,698	0,019	0,522	0,836	0,403	2,695	3,643	0,920
0,879	0,698	0,019	0,522	0,836	0,403	2,695	3,643	0,920
0,926	0,698	0,019	0,532	0,836	0,403	3,560	3,643	0,920
0,926	0,698	0,019	0,532	0,836	0,403	3,560	3,643	0,920
0,926	0,698	0,019	0,701	0,836	0,403	4,110	3,643	0,920
0,779	0,698	0,019	0,961	0,836	0,403	3,518	3,643	0,920
0,745	0,698	0,019	0,961	0,836	0,403	3,456	3,643	0,920
0,675	0,698	0,019	0,389	0,836	0,403	1,725	3,643	0,920
0,680	0,698	0,019	0,504	0,836	0,403	1,958	3,643	0,920
0,654	0,698	0,019	0,039	0,836	0,403	1,359	3,643	0,920

Table B.2: PSO Out-of-sample Results, unitary investment

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
0,373	0,836	0,403	0,081	0,554	0,343	0,865	1,901	0,884
0,433	0,836	0,403	0,319	0,554	0,343	0,945	1,901	0,884
0,501	0,836	0,403	0,289	0,554	0,343	1,140	1,901	0,884
0,677	0,836	0,403	0,117	0,554	0,343	1,251	1,901	0,884
0,430	0,836	0,403	0,006	0,554	0,343	0,729	1,901	0,884
0,658	0,836	0,403	0,440	0,554	0,343	1,387	1,901	0,884
0,658	0,836	0,403	0,440	0,554	0,343	1,387	1,901	0,884
0,658	0,836	0,403	0,369	0,554	0,343	1,269	1,901	0,884
0,612	0,836	0,403	0,129	0,554	0,343	1,163	1,901	0,884
0,312	0,836	0,403	0,177	0,554	0,343	0,930	1,901	0,884

Table B.3: PSO Out-of-sample Results, unitary investment

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
0,195	0,554	0,343	0,059	0,027	-0,117	0,280	0,640	0,173
0,258	0,554	0,343	-0,049	0,027	-0,117	0,327	0,640	0,173
0,284	0,554	0,343	0,000	0,027	-0,117	0,355	0,640	0,173
0,523	0,554	0,343	-0,049	0,027	-0,117	0,607	0,640	0,173
0,187	0,554	0,343	-0,049	0,027	-0,117	0,252	0,640	0,173
0,187	0,554	0,343	-0,049	0,027	-0,117	0,252	0,640	0,173
0,179	0,554	0,343	-0,049	0,027	-0,117	0,245	0,640	0,173
0,253	0,554	0,343	-0,049	0,027	-0,117	0,322	0,640	0,173
0,505	0,554	0,343	0,056	0,027	-0,117	0,588	0,640	0,173
0,209	0,554	0,343	-0,049	0,027	-0,117	0,175	0,640	0,173

Table B.4: PSO Out-of-sample Results, unitary investment

1/2020 4/2020	B&H	Stand.
0,018	0,027	-0,117
0,018	0,027	-0,117
-0,049	0,027	-0,117
0,047	0,027	-0,117
-0,003	0,027	-0,117
-0,003	0,027	-0,117
-0,003	0,027	-0,117
0,025	0,027	-0,117
0,052	0,027	-0,117
-0,017	0,027	-0,117

Table B.5: PSO Out-of-sample Results, unitary investment

B.2 EFWA Amazon Out-sample Results

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
0,573	1,228	0,072	1,088	0,698	0,019	4,029	9,321	1,060
1,053	1,228	0,072	0,379	0,698	0,019	6,540	9,321	1,060
1,106	1,228	0,072	1,088	0,698	0,019	9,847	9,321	1,060
1,166	1,228	0,072	1,088	0,698	0,019	6,918	9,321	1,060
0,755	1,228	0,072	1,088	0,698	0,019	4,871	9,321	1,060
1,102	1,228	0,072	1,088	0,698	0,019	8,886	9,321	1,060
0,755	1,228	0,072	1,088	0,698	0,019	8,039	9,321	1,060
1,053	1,228	0,072	0,379	0,698	0,019	6,540	9,321	1,060
0,577	1,228	0,072	0,511	0,698	0,019	5,151	9,321	1,060
1,026	1,228	0,072	0,511	0,698	0,019	6,902	9,321	1,060

Table B.6: EFWA Out-of-sample Results, unitary investment

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
0,675	0,698	0,019	0,389	0,836	0,403	1,725	3,643	0,920
0,675	0,698	0,019	0,467	0,836	0,403	1,878	3,643	0,920
0,680	0,698	0,019	0,504	0,836	0,403	1,958	3,643	0,920
0,680	0,698	0,019	0,504	0,836	0,403	1,958	3,643	0,920
0,674	0,698	0,019	0,392	0,836	0,403	2,339	3,643	0,920
0,368	0,698	0,019	0,019	0,836	0,403	0,779	3,643	0,920
0,620	0,698	0,019	0,038	0,836	0,403	1,289	3,643	0,920
0,879	0,698	0,019	0,312	0,836	0,403	2,200	3,643	0,920
1,088	0,698	0,019	0,063	0,836	0,403	1,808	3,643	0,920
1,101	0,698	0,019	0,106	0,836	0,403	1,824	3,643	0,920

Table B.7: EFWA Out-of-sample Results, unitary investment

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
0,263	0,836	0,403	0,102	0,554	0,343	0,433	1,901	0,884
0,181	0,836	0,403	0,042	0,554	0,343	0,230	1,901	0,884
0,441	0,836	0,403	0,191	0,554	0,343	1,146	1,901	0,884
0,379	0,836	0,403	-0,006	0,554	0,343	0,721	1,901	0,884
0,385	0,836	0,403	0,170	0,554	0,343	0,620	1,901	0,884
0,282	0,836	0,403	0,302	0,554	0,343	0,719	1,901	0,884
0,396	0,836	0,403	0,233	0,554	0,343	0,772	1,901	0,884
0,226	0,836	0,403	0,352	0,554	0,343	0,706	1,901	0,884
0,262	0,836	0,403	0,177	0,554	0,343	0,857	1,901	0,884
0,420	0,836	0,403	0,255	0,554	0,343	0,973	1,901	0,884

Table B.8: EFWA Out-of-sample Results, unitary investment

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
0,449	0,554	0,343	-0,049	0,027	-0,117	0,529	0,640	0,173
0,523	0,554	0,343	-0,049	0,027	-0,117	0,607	0,640	0,173
0,523	0,554	0,343	-0,049	0,027	-0,117	0,607	0,640	0,173
0,523	0,554	0,343	-0,049	0,027	-0,117	0,607	0,640	0,173
0,523	0,554	0,343	-0,049	0,027	-0,117	0,607	0,640	0,173
0,318	0,554	0,343	0,000	0,027	-0,117	0,390	0,640	0,173
0,258	0,554	0,343	-0,049	0,027	-0,117	0,327	0,640	0,173
0,374	0,554	0,343	-0,049	0,027	-0,117	0,449	0,640	0,173
0,388	0,554	0,343	-0,049	0,027	-0,117	0,465	0,640	0,173
0,346	0,554	0,343	0,000	0,027	-0,117	0,420	0,640	0,173

Table B.9: EFWA Out-of-sample Results, unitary investment

1/2020 4/2020	B&H	Stand.
0,018	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117
-0,003	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117
-0,049	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117

Table B.10: EFWA Out-of-sample Results, unitary investment

B.3 DE Amazon Out-of-sample Results

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
0	1,228	0,072	1,289	0,698	0,019	3,973	9,321	1,060
0,767	1,228	0,072	0,696	0,698	0,019	6,252	9,321	1,060
0,814	1,228	0,072	0,638	0,698	0,019	6,626	9,321	1,060
1,130	1,228	0,072	0,379	0,698	0,019	6,825	9,321	1,060
0,638	1,228	0,072	1,088	0,698	0,019	3,710	9,321	1,060
1,013	1,228	0,072	0,374	0,698	0,019	6,505	9,321	1,060
1,130	1,228	0,072	0,379	0,698	0,019	6,825	9,321	1,060
1,130	1,228	0,072	0,379	0,698	0,019	6,825	9,321	1,060
1,053	1,228	0,072	0,382	0,698	0,019	6,557	9,321	1,060
1,130	1,228	0,072	0,379	0,698	0,019	6,825	9,321	1,060

Table B.11: DE Out-of-sample Results, unitary investment

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
0,675	0,698	0,019	0,259	0,836	0,403	1,468	3,643	0,920
0,674	0,698	0,019	0,504	0,836	0,403	2,607	3,643	0,920
0,675	0,698	0,019	0,329	0,836	0,403	1,607	3,643	0,920
0,566	0,698	0,019	0,085	0,836	0,403	1,431	3,643	0,920
0,913	0,698	0,019	0,598	0,836	0,403	3,195	3,643	0,920
0,766	0,698	0,019	0,598	0,836	0,403	3,290	3,643	0,920
0,766	0,698	0,019	0,598	0,836	0,403	3,290	3,643	0,920
0,879	0,698	0,019	0,369	0,836	0,403	2,607	3,643	0,920
0,879	0,698	0,019	0,548	0,836	0,403	3,079	3,643	0,920
0,879	0,698	0,019	0,522	0,836	0,403	2,695	3,643	0,920

Table B.12: DE Out-of-sample Results, unitary investment

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
0,668	0,836	0,403	0,180	0,554	0,343	1,178	1,901	0,884
0,691	0,836	0,403	0,360	0,554	0,343	1,543	1,901	0,884
0,181	0,836	0,403	0,332	0,554	0,343	0,966	1,901	0,884
0,039	0,836	0,403	0,103	0,554	0,343	0,432	1,901	0,884
0,300	0,836	0,403	0,185	0,554	0,343	0,926	1,901	0,884
0,553	0,836	0,403	0,289	0,554	0,343	1,214	1,901	0,884
0,387	0,836	0,403	0,472	0,554	0,343	1,258	1,901	0,884
0,808	0,836	0,403	0,321	0,554	0,343	1,460	1,901	0,884
0,808	0,836	0,403	0,129	0,554	0,343	1,101	1,901	0,884
0,160	0,836	0,403	0,146	0,554	0,343	0,330	1,901	0,884

Table B.13: DE Out-of-sample Results, unitary investment

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
0,346	0,554	0,343	0	0,027	-0,117	0,420	0,640	0,173
0,258	0,554	0,343	-0,049	0,027	-0,117	0,327	0,640	0,173
0,179	0,554	0,343	-0,049	0,027	-0,117	0,146	0,640	0,173
0,179	0,554	0,343	-0,049	0,027	-0,117	0,146	0,640	0,173
0,179	0,554	0,343	-0,049	0,027	-0,117	0,146	0,640	0,173
0,179	0,554	0,343	-0,049	0,027	-0,117	0,146	0,640	0,173
0,179	0,554	0,343	-0,049	0,027	-0,117	0,146	0,640	0,173
0,179	0,554	0,343	-0,049	0,027	-0,117	0,146	0,640	0,173
0,179	0,554	0,343	-0,049	0,027	-0,117	0,146	0,640	0,173
0,179	0,554	0,343	-0,049	0,027	-0,117	0,146	0,640	0,173

Table B.14: DE Out-of-sample Results, unitary investment

1/2020 4/2020	B&H	Stand.
0,018	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117
0,018	0,027	-0,117

Table B.15: DE Out-of-sample Results, unitary investment

Appendix C

BRF S.A Algorithms unitary results

In this chapter it will be presented the results of the application of the sets of parameters obtained by the in-sample analyses. The trading returns in the following tables consider a initial capital of 1 dollar and an investment of non integer quantity of BRF S.A stock differently from what has been presented in chapter 5. The same conclusions that has been reached in chapter 5 can be done also with the following values, the only differences is that the returns are slightly different.

C.1 PSO BRF S.A Out-sample Results

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
-0,330	0,051	0,148	0,090	-0,317	-0,307	-0,537	-0,562	-0,331
-0,245	0,051	0,148	-0,248	-0,317	-0,307	-0,613	-0,562	-0,331
0,268	0,051	0,148	-0,245	-0,317	-0,307	-0,329	-0,562	-0,331
-0,280	0,051	0,148	-0,291	-0,317	-0,307	-0,570	-0,562	-0,331
-0,286	0,051	0,148	0,047	-0,317	-0,307	-0,430	-0,562	-0,331
-0,123	0,051	0,148	-0,035	-0,317	-0,307	-0,422	-0,562	-0,331
-0,113	0,051	0,148	0,031	-0,317	-0,307	-0,053	-0,562	-0,331
-0,279	0,051	0,148	-0,269	-0,317	-0,307	-0,545	-0,562	-0,331
-0,047	0,051	0,148	-0,018	-0,317	-0,307	-0,339	-0,562	-0,331
-0,226	0,051	0,148	-0,140	-0,317	-0,307	-0,240	-0,562	-0,331

Table C.1: PSO Out-of-sample Returns, unitary investment

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-0,032	-0,317	-0,307	-0,006	-0,167	-0,059	-0,307	-0,570	-0,417
-0,104	-0,317	-0,307	-0,006	-0,167	-0,059	-0,358	-0,570	-0,417
-0,085	-0,317	-0,307	-0,006	-0,167	-0,059	-0,325	-0,570	-0,417
-0,085	-0,317	-0,307	-0,006	-0,167	-0,059	-0,325	-0,570	-0,417
-0,085	-0,317	-0,307	-0,006	-0,167	-0,059	-0,325	-0,570	-0,417
-0,050	-0,317	-0,307	-0,006	-0,167	-0,059	-0,299	-0,570	-0,417
-0,069	-0,317	-0,307	-0,006	-0,167	-0,059	-0,334	-0,570	-0,417
0,009	-0,317	-0,307	-0,006	-0,167	-0,059	-0,222	-0,570	-0,417
-0,029	-0,317	-0,307	-0,012	-0,167	-0,059	-0,219	-0,570	-0,417
0,006	-0,317	-0,307	-0,005	-0,167	-0,059	-0,186	-0,570	-0,417

Table C.2: PSO Out-of-sample Returns, unitary investment

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
-0,182	-0,167	-0,059	-0,394	-0,239	-0,008	-0,504	-0,357	-0,053
-0,012	-0,167	-0,059	0,108	-0,239	-0,008	0,095	-0,357	-0,053
-0,221	-0,167	-0,059	-0,214	-0,239	-0,008	-0,634	-0,357	-0,053
-0,011	-0,167	-0,059	-0,214	-0,239	-0,008	-0,223	-0,357	-0,053
-0,294	-0,167	-0,059	-0,254	-0,239	-0,008	-0,460	-0,357	-0,053
-0,162	-0,167	-0,059	-0,183	-0,239	-0,008	-0,315	-0,357	-0,053
0,524	-0,167	-0,059	0,081	-0,239	-0,008	0,648	-0,357	-0,053
0,010	-0,167	-0,059	-0,268	-0,239	-0,008	-0,226	-0,357	-0,053
-0,162	-0,167	-0,059	-0,122	-0,239	-0,008	-0,264	-0,357	-0,053
0,129	-0,167	-0,059	0,124	-0,239	-0,008	0,269	-0,357	-0,053

Table C.3: PSO Out-of-sample Returns, unitary investment

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
-0,154	-0,239	-0,008	-0,163	-0,673	0,000	-0,307	-0,746	-0,669
-0,103	-0,239	-0,008	0,000	-0,673	0,000	-0,706	-0,746	-0,669
-0,103	-0,239	-0,008	0,000	-0,673	0,000	-0,706	-0,746	-0,669
-0,285	-0,239	-0,008	-0,665	-0,673	0,000	-0,766	-0,746	-0,669
0,197	-0,239	-0,008	-0,165	-0,673	0,000	-0,019	-0,746	-0,669
-0,063	-0,239	-0,008	-0,165	-0,673	0,000	-0,232	-0,746	-0,669
0,105	-0,239	-0,008	0,000	-0,673	0,000	-0,638	-0,746	-0,669
-0,067	-0,239	-0,008	-0,660	-0,673	0,000	-0,683	-0,746	-0,669
-0,325	-0,239	-0,008	-0,676	-0,673	0,000	-0,781	-0,746	-0,669
0,010	-0,239	-0,008	0	-0,673	0,000	-0,663	-0,746	-0,669

Table C.4: PSO Out-of-sample Returns, unitary investment

1/2020 4/2020	B&H	Stand.
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
0,000	-0,673	0,000
-0,602	-0,673	0,000

Table C.5: PSO Out-of-sample Returns, unitary investment

C.2 EFWA BRF S.A Out-sample Results

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
0,202	0,051	0,148	-0,133	-0,317	-0,307	-0,435	-0,562	-0,331
0,212	0,051	0,148	-0,133	-0,317	-0,307	-0,305	-0,562	-0,331
0,286	0,051	0,148	-0,064	-0,317	-0,307	0,011	-0,562	-0,331
0,307	0,051	0,148	-0,064	-0,317	-0,307	0,028	-0,562	-0,331
0,286	0,051	0,148	-0,064	-0,317	-0,307	0,011	-0,562	-0,331
-0,144	0,051	0,148	-0,145	-0,317	-0,307	-0,712	-0,562	-0,331
-0,113	0,051	0,148	-0,125	-0,317	-0,307	-0,696	-0,562	-0,331
-0,111	0,051	0,148	-0,087	-0,317	-0,307	-0,681	-0,562	-0,331
0,340	0,051	0,148	-0,410	-0,317	-0,307	-0,614	-0,562	-0,331
-0,111	0,051	0,148	-0,284	-0,317	-0,307	-0,749	-0,562	-0,331

Table C.6: EFWA Out-of-sample Returns, unitary investment

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-0,152	-0,317	-0,307	-0,006	-0,167	-0,059	-0,385	-0,570	-0,417
-0,045	-0,317	-0,307	0,001	-0,167	-0,059	-0,355	-0,570	-0,417
-0,069	-0,317	-0,307	-0,006	-0,167	-0,059	-0,334	-0,570	-0,417
0,006	-0,317	-0,307	-0,006	-0,167	-0,059	-0,280	-0,570	-0,417
-0,076	-0,317	-0,307	-0,165	-0,167	-0,059	-0,484	-0,570	-0,417
-0,032	-0,317	-0,307	-0,006	-0,167	-0,059	-0,307	-0,570	-0,417
-0,104	-0,317	-0,307	0,007	-0,167	-0,059	-0,370	-0,570	-0,417
-0,050	-0,317	-0,307	-0,006	-0,167	-0,059	-0,299	-0,570	-0,417
-0,104	-0,317	-0,307	-0,007	-0,167	-0,059	-0,266	-0,570	-0,417
-0,104	-0,317	-0,307	-0,007	-0,167	-0,059	-0,280	-0,570	-0,417

Table C.7: EFWA Out-of-sample Returns, unitary investment

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
-0,170	-0,167	-0,059	-0,234	-0,239	-0,008	-0,364	-0,357	-0,053
-0,099	-0,167	-0,059	-0,234	-0,239	-0,008	-0,310	-0,357	-0,053
-0,212	-0,167	-0,059	0,339	-0,239	-0,008	0,055	-0,357	-0,053
-0,212	-0,167	-0,059	-0,258	-0,239	-0,008	-0,416	-0,357	-0,053
-0,212	-0,167	-0,059	0,108	-0,239	-0,008	-0,127	-0,357	-0,053
-0,020	-0,167	-0,059	-0,094	-0,239	-0,008	-0,112	-0,357	-0,053
-0,020	-0,167	-0,059	-0,094	-0,239	-0,008	-0,112	-0,357	-0,053
-0,020	-0,167	-0,059	-0,094	-0,239	-0,008	-0,112	-0,357	-0,053
-0,212	-0,167	-0,059	-0,094	-0,239	-0,008	-0,286	-0,357	-0,053
-0,020	-0,167	-0,059	-0,258	-0,239	-0,008	-0,273	-0,357	-0,053

Table C.8: EFWA Out-of-sample Returns, unitary investment

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
-0,024	-0,239	-0,008	-0,666	-0,673	0,000	-0,680	-0,746	-0,669
-0,330	-0,239	-0,008	0,000	-0,673	0,000	-0,781	-0,746	-0,669
-0,061	-0,239	-0,008	-0,666	-0,673	0,000	-0,693	-0,746	-0,669
-0,007	-0,239	-0,008	-0,660	-0,673	0,000	-0,663	-0,746	-0,669
-0,067	-0,239	-0,008	-0,660	-0,673	0,000	-0,683	-0,746	-0,669
0,009	-0,239	-0,008	-0,660	-0,673	0,000	-0,657	-0,746	-0,669
-0,036	-0,239	-0,008	0,000	-0,673	0,000	-0,036	-0,746	-0,669
0,021	-0,239	-0,008	-0,660	-0,673	0,000	-0,653	-0,746	-0,669
0,137	-0,239	-0,008	-0,660	-0,673	0,000	-0,614	-0,746	-0,669
-0,142	-0,239	-0,008	0,000	-0,673	0,000	-0,714	-0,746	-0,669

Table C.9: EFWA Out-of-sample Returns, unitary investment

1/2020 4/2020	B&H	Stand.
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,602	-0,673	0,000
-0,164	-0,673	0,000
-0,602	-0,673	0,000
-0,615	-0,673	0,000

Table C.10: EFWA Out-of-sample Returns, unitary investment

C.3 DE BRF S.A Out-of-sample Results

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
0,044	0,051	0,148	-0,290	-0,317	-0,307	-0,369	-0,562	-0,331
0,266	0,051	0,148	-0,146	-0,317	-0,307	0,187	-0,562	-0,331
0,318	0,051	0,148	-0,008	-0,317	-0,307	0,117	-0,562	-0,331
0,044	0,051	0,148	-0,155	-0,317	-0,307	-0,249	-0,562	-0,331
0,192	0,051	0,148	-0,155	-0,317	-0,307	-0,145	-0,562	-0,331
0,318	0,051	0,148	-0,021	-0,317	-0,307	0,714	-0,562	-0,331
0,296	0,051	0,148	-0,290	-0,317	-0,307	-0,228	-0,562	-0,331
0,304	0,051	0,148	-0,082	-0,317	-0,307	0,422	-0,562	-0,331
0,165	0,051	0,148	-0,072	-0,317	-0,307	-0,129	-0,562	-0,331
0,452	0,051	0,148	-0,075	-0,317	-0,307	0,413	-0,562	-0,331

Table C.11: DE Out-of-sample Returns, unitary investment

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-0,085	-0,317	-0,307	-0,006	-0,167	-0,059	-0,316	-0,570	-0,417
-0,085	-0,317	-0,307	-0,006	-0,167	-0,059	-0,325	-0,570	-0,417
-0,085	-0,317	-0,307	-0,006	-0,167	-0,059	-0,316	-0,570	-0,417
-0,085	-0,317	-0,307	-0,012	-0,167	-0,059	-0,328	-0,570	-0,417
-0,085	-0,317	-0,307	-0,006	-0,167	-0,059	-0,332	-0,570	-0,417
-0,085	-0,317	-0,307	-0,006	-0,167	-0,059	-0,305	-0,570	-0,417
0,028	-0,317	-0,307	-0,139	-0,167	-0,059	-0,395	-0,570	-0,417
-0,069	-0,317	-0,307	-0,006	-0,167	-0,059	-0,325	-0,570	-0,417
0,028	-0,317	-0,307	-0,139	-0,167	-0,059	-0,342	-0,570	-0,417
-0,085	-0,317	-0,307	0,036	-0,167	-0,059	-0,296	-0,570	-0,417

Table C.12: DE Out-of-sample Returns, unitary investment

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
-0,182	-0,167	-0,059	-0,394	-0,239	-0,008	-0,504	-0,357	-0,053
-0,182	-0,167	-0,059	-0,394	-0,239	-0,008	-0,504	-0,357	-0,053
-0,182	-0,167	-0,059	-0,318	-0,239	-0,008	-0,442	-0,357	-0,053
-0,182	-0,167	-0,059	-0,343	-0,239	-0,008	-0,462	-0,357	-0,053
-0,122	-0,167	-0,059	-0,394	-0,239	-0,008	-0,468	-0,357	-0,053
-0,119	-0,167	-0,059	-0,264	-0,239	-0,008	-0,352	-0,357	-0,053
-0,212	-0,167	-0,059	-0,094	-0,239	-0,008	-0,286	-0,357	-0,053
-0,212	-0,167	-0,059	-1,000	-0,356	-0,008	-0,212	-0,357	-0,053
-0,173	-0,167	-0,059	-0,295	-0,239	-0,008	-0,417	-0,357	-0,053
-0,012	-0,167	-0,059	-0,199	-0,239	-0,008	-0,171	-0,357	-0,053

Table C.13: DE Out-of-sample Returns, unitary investment

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
0,240	-0,239	-0,008	-0,152	-0,673	0	0,039	-0,746	-0,669
-0,325	-0,239	-0,008	-0,676	-0,673	0	-0,781	-0,746	-0,669
-0,325	-0,239	-0,008	-0,676	-0,673	0	-0,781	-0,746	-0,669
-0,325	-0,239	-0,008	-0,676	-0,673	0	-0,781	-0,746	-0,669
-0,325	-0,239	-0,008	-0,676	-0,673	0	-0,781	-0,746	-0,669
-0,325	-0,239	-0,008	-0,676	-0,673	0	-0,781	-0,746	-0,669
-0,325	-0,239	-0,008	-0,676	-0,673	0	-0,781	-0,746	-0,669
-0,383	-0,239	-0,008	-0,676	-0,673	0	-0,800	-0,746	-0,669
-0,325	-0,239	-0,008	-0,676	-0,673	0	-0,781	-0,746	-0,669
-0,325	-0,239	-0,008	-0,676	-0,673	0	-0,781	-0,746	-0,669

Table C.14: DE Out-of-sample Returns, unitary investment

1/2020 4/2020	B&H	Stand.
-0,602	-0,673	0
-0,602	-0,673	0
0,000	-0,673	0
-0,602	-0,673	0
-0,602	-0,673	0
-0,602	-0,673	0
-0,602	-0,673	0
-0,602	-0,673	0
-0,602	-0,673	0
-0,602	-0,673	0

Table C.15: DE Out-of-sample Returns, unitary investment

Appendix D

EUR/USD Algorithms unitary results

In this chapter it will be presented the results of the application of the sets of parameters obtained by the in-sample analyses. The trading returns in the following tables consider a initial capital of 1 dollar and an investment of non integer quantity of EUR/USD differently from what has been presented in chapter 5. The same conclusions that has been reached in chapter 5 can be done also with the following values, the only differences is that the returns are slightly different.

D.1 PSO EUR/USD Out-of-sample Results

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
0,087	0,064	0,069	-0,136	-0,206	-0,197	-0,053	-0,133	-0,193
0,037	0,064	0,069	-0,140	-0,206	-0,197	0,054	-0,133	-0,193
0,000	0,064	0,069	-0,195	-0,206	-0,197	-0,152	-0,133	-0,193
0,000	0,064	0,069	-0,195	-0,206	-0,197	-0,152	-0,133	-0,193
0,012	0,064	0,069	-0,187	-0,206	-0,197	-0,184	-0,133	-0,193
0,036	0,064	0,069	-0,184	-0,206	-0,197	-0,162	-0,133	-0,193
0,044	0,064	0,069	-0,066	-0,206	-0,197	0,130	-0,133	-0,193
0,032	0,064	0,069	-0,066	-0,206	-0,197	0,101	-0,133	-0,193
0,000	0,064	0,069	-0,195	-0,206	-0,197	-0,144	-0,133	-0,193
0,000	0,064	0,069	-0,195	-0,206	-0,197	-0,156	-0,133	-0,193

Table D.1: PSO Out-of-sample Returns, unitary investment

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-0,162	-0,206	-0,197	0,062	0,109	0,029	-0,144	-0,180	-0,233
-0,078	-0,206	-0,197	0,106	0,109	0,029	0,035	-0,180	-0,233
-0,112	-0,206	-0,197	0,106	0,109	0,029	-0,018	-0,180	-0,233
-0,210	-0,206	-0,197	0,052	0,109	0,029	-0,134	-0,180	-0,233
-0,069	-0,206	-0,197	0,080	0,109	0,029	0,027	-0,180	-0,233
-0,143	-0,206	-0,197	0,097	0,109	0,029	-0,061	-0,180	-0,233
0,001	-0,206	-0,197	0,106	0,109	0,029	0,119	-0,180	-0,233
-0,060	-0,206	-0,197	0,084	0,109	0,029	0,048	-0,180	-0,233
-0,028	-0,206	-0,197	0,082	0,109	0,029	0,054	-0,180	-0,233
-0,057	-0,206	-0,197	0,073	0,109	0,029	0,063	-0,180	-0,233

Table D.2: PSO Out-of-sample Returns, unitary investment

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
0,085	0,109	0,029	-0,058	-0,070	-0,077	0,056	0,035	-0,049
0,093	0,109	0,029	-0,058	-0,070	-0,077	0,060	0,035	-0,049
-0,040	0,109	0,029	-0,055	-0,070	-0,077	-0,080	0,035	-0,049
0,107	0,109	0,029	-0,026	-0,070	-0,077	0,078	0,035	-0,049
0,117	0,109	0,029	-0,047	-0,070	-0,077	0,089	0,035	-0,049
0,147	0,109	0,029	-0,055	-0,070	-0,077	0,117	0,035	-0,049
0,123	0,109	0,029	-0,048	-0,070	-0,077	0,094	0,035	-0,049
0,123	0,109	0,029	-0,051	-0,070	-0,077	0,090	0,035	-0,049
0,117	0,109	0,029	-0,056	-0,070	-0,077	0,079	0,035	-0,049
0,000	0,109	0,029	-0,020	-0,070	-0,077	-0,020	0,035	-0,049

Table D.3: PSO Out-of-sample Returns, unitary investment

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 1/2020	B&H	Stand.
-0,067	-0,070	-0,077	0,000	-0,019	-0,017	-0,078	-0,091	-0,093
-0,037	-0,070	-0,077	0,000	-0,019	-0,017	-0,048	-0,091	-0,093
-0,053	-0,070	-0,077	0,000	-0,019	-0,017	-0,064	-0,091	-0,093
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093
-0,046	-0,070	-0,077	0,000	-0,019	-0,017	-0,057	-0,091	-0,093
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093

Table D.4: PSO Out-of-sample Returns, unitary investment

1/2020 4/2020	B&H	Stand.
0,000	-0,019	-0,017
0,000	-0,019	-0,017
0,000	-0,019	-0,017
0,000	-0,019	-0,017
0,000	-0,019	-0,017
0,000	-0,019	-0,017
0,000	-0,019	-0,017
0,000	-0,019	-0,017
0,000	-0,019	-0,017
0,000	-0,019	-0,017

Table D.5: PSO Out-of-sample Returns, unitary investment

D.2 EFWA EUR/USD Out-of-sample Results

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
0,087	0,064	0,069	-0,140	-0,206	-0,197	-0,006	-0,133	-0,193
0,087	0,064	0,069	-0,138	-0,206	-0,197	-0,004	-0,133	-0,193
0,087	0,064	0,069	-0,156	-0,206	-0,197	-0,051	-0,133	-0,193
0,087	0,064	0,069	-0,156	-0,206	-0,197	-0,024	-0,133	-0,193
0,087	0,064	0,069	-0,156	-0,206	-0,197	-0,106	-0,133	-0,193
0,087	0,064	0,069	-0,156	-0,206	-0,197	-0,106	-0,133	-0,193
0,087	0,064	0,069	-0,156	-0,206	-0,197	-0,106	-0,133	-0,193
0,032	0,064	0,069	-0,117	-0,206	-0,197	-0,016	-0,133	-0,193
0,012	0,064	0,069	-0,105	-0,206	-0,197	-0,141	-0,133	-0,193
0,087	0,064	0,069	-0,142	-0,206	-0,197	-0,019	-0,133	-0,193

Table D.6: EFWA Out-of-sample Returns, unitary investment

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-0,060	-0,206	-0,197	0,055	0,109	0,029	0,059	-0,180	-0,233
-0,141	-0,206	-0,197	0,085	0,109	0,029	-0,056	-0,180	-0,233
-0,141	-0,206	-0,197	0,084	0,109	0,029	-0,052	-0,180	-0,233
-0,111	-0,206	-0,197	0,100	0,109	0,029	-0,023	-0,180	-0,233
-0,015	-0,206	-0,197	0,078	0,109	0,029	0,090	-0,180	-0,233
-0,078	-0,206	-0,197	0,106	0,109	0,029	0,042	-0,180	-0,233
-0,084	-0,206	-0,197	0,069	0,109	0,029	-0,016	-0,180	-0,233
0,013	-0,206	-0,197	0,078	0,109	0,029	0,120	-0,180	-0,233
-0,065	-0,206	-0,197	0,078	0,109	0,029	0,018	-0,180	-0,233
-0,138	-0,206	-0,197	0,097	0,109	0,029	-0,032	-0,180	-0,233

Table D.7: EFWA Out-of-sample Returns, unitary investment

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
0,183	0,109	0,029	-0,070	-0,070	-0,077	0,124	0,035	-0,049
0,029	0,109	0,029	-0,005	-0,070	-0,077	0,024	0,035	-0,049
0,051	0,109	0,029	0,003	-0,070	-0,077	0,054	0,035	-0,049
-0,040	0,109	0,029	-0,074	-0,070	-0,077	-0,085	0,035	-0,049
0,100	0,109	0,029	-0,051	-0,070	-0,077	0,068	0,035	-0,049
-0,043	0,109	0,029	-0,038	-0,070	-0,077	-0,080	0,035	-0,049
-0,043	0,109	0,029	0,011	-0,070	-0,077	-0,033	0,035	-0,049
-0,072	0,109	0,029	-0,054	-0,070	-0,077	-0,121	0,035	-0,049
-0,072	0,109	0,029	-0,054	-0,070	-0,077	-0,121	0,035	-0,049
-0,016	0,109	0,029	-0,063	-0,070	-0,077	-0,078	0,035	-0,049

Table D.8: EFWA Out-of-sample Returns, unitary investment

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093
-0,044	-0,070	-0,077	0,000	-0,019	-0,017	-0,056	-0,091	-0,093
-0,038	-0,070	-0,077	0,000	-0,019	-0,017	-0,049	-0,091	-0,093
-0,044	-0,070	-0,077	0,000	-0,019	-0,017	-0,056	-0,091	-0,093
-0,043	-0,070	-0,077	0,000	-0,019	-0,017	-0,055	-0,091	-0,093
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093
-0,060	-0,070	-0,077	0,000	-0,019	-0,017	-0,071	-0,091	-0,093
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093
-0,061	-0,070	-0,077	0,000	-0,019	-0,017	-0,072	-0,091	-0,093

Table D.9: EFWA Out-of-sample Returns,unitary investment

1/2020 4/2020	B&H	Stand.
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017

Table D.10: EFWA Out-of-sample Returns, unitary investment

D.3 DE EUR/USD Out-of-sample Results

1/2012 1/2014	B&H	Stand.	1/2014 1/2016	B&H	Stand.	1/2012 1/2020	B&H	Stand.
0,087	0,064	0,069	-0,156	-0,206	-0,197	-0,072	-0,133	-0,193
0,087	0,064	0,069	-0,138	-0,206	-0,197	-0,004	-0,133	-0,193
0,087	0,064	0,069	-0,156	-0,206	-0,197	-0,106	-0,133	-0,193
0,087	0,064	0,069	-0,138	-0,206	-0,197	-0,004	-0,133	-0,193
0,145	0,064	0,069	-0,140	-0,206	-0,197	0,036	-0,133	-0,193
0,087	0,064	0,069	-0,140	-0,206	-0,197	-0,005	-0,133	-0,193
0,087	0,064	0,069	-0,138	-0,206	-0,197	-0,081	-0,133	-0,193
0,036	0,064	0,069	-0,141	-0,206	-0,197	-0,197	-0,133	-0,193
0,036	0,064	0,069	-0,136	-0,206	-0,197	-0,187	-0,133	-0,193
0,036	0,064	0,069	-0,136	-0,206	-0,197	-0,187	-0,133	-0,193

Table D.11: DE Out-of-sample Returns, unitary investment

1/2014 1/2016	B&H	Stand.	1/2016 1/2018	B&H	Stand.	1/2014 1/2020	B&H	Stand.
-0,056	-0,206	-0,197	0,049	0,109	0,029	0,055	-0,180	-0,233
-0,054	-0,206	-0,197	0,049	0,109	0,029	0,041	-0,180	-0,233
-0,055	-0,206	-0,197	0,055	0,109	0,029	0,035	-0,180	-0,233
-0,056	-0,206	-0,197	0,044	0,109	0,029	0,042	-0,180	-0,233
-0,068	-0,206	-0,197	0,037	0,109	0,029	0,021	-0,180	-0,233
-0,042	-0,206	-0,197	0,037	0,109	0,029	0,018	-0,180	-0,233
-0,142	-0,206	-0,197	0,060	0,109	0,029	-0,050	-0,180	-0,233
-0,151	-0,206	-0,197	0,084	0,109	0,029	-0,067	-0,180	-0,233
-0,137	-0,206	-0,197	0,098	0,109	0,029	-0,040	-0,180	-0,233
-0,149	-0,206	-0,197	0,085	0,109	0,029	-0,064	-0,180	-0,233

Table D.12: DE Out-of-sample Returns, unitary investment

1/2016 1/2018	B&H	Stand.	1/2018 1/2020	B&H	Stand.	1/2016 1/2020	B&H	Stand.
0,093	0,109	0,029	-0,050	-0,070	-0,077	0,072	0,035	-0,049
-0,015	0,109	0,029	0,005	-0,070	-0,077	-0,010	0,035	-0,049
0,057	0,109	0,029	0,005	-0,070	-0,077	0,063	0,035	-0,049
0,102	0,109	0,029	-0,047	-0,070	-0,077	0,051	0,035	-0,049
-0,052	0,109	0,029	-0,054	-0,070	-0,077	-0,103	0,035	-0,049
-0,052	0,109	0,029	-0,054	-0,070	-0,077	-0,103	0,035	-0,049
-0,043	0,109	0,029	-0,054	-0,070	-0,077	-0,094	0,035	-0,049
-0,074	0,109	0,029	-0,054	-0,070	-0,077	-0,124	0,035	-0,049
-0,074	0,109	0,029	-0,054	-0,070	-0,077	-0,124	0,035	-0,049
-0,050	0,109	0,029	-0,054	-0,070	-0,077	-0,101	0,035	-0,049

Table D.13: DE Out-of-sample Returns, unitary investment

1/2018 1/2020	B&H	Stand.	1/2020 4/2020	B&H	Stand.	1/2018 4/2020	B&H	Stand.
-0,085	-0,070	-0,077	-0,012	-0,019	-0,017	-0,096	-0,091	-0,093
-0,085	-0,070	-0,077	-0,012	-0,019	-0,017	-0,096	-0,091	-0,093
-0,079	-0,070	-0,077	-0,012	-0,019	-0,017	-0,090	-0,091	-0,093
-0,033	-0,070	-0,077	-0,012	-0,019	-0,017	-0,044	-0,091	-0,093
-0,033	-0,070	-0,077	-0,012	-0,019	-0,017	-0,044	-0,091	-0,093
-0,079	-0,070	-0,077	-0,012	-0,019	-0,017	-0,090	-0,091	-0,093
-0,033	-0,070	-0,077	-0,012	-0,019	-0,017	-0,044	-0,091	-0,093
-0,079	-0,070	-0,077	-0,012	-0,019	-0,017	-0,090	-0,091	-0,093
-0,085	-0,070	-0,077	-0,012	-0,019	-0,017	-0,096	-0,091	-0,093
-0,079	-0,070	-0,077	-0,012	-0,019	-0,017	-0,090	-0,091	-0,093

Table D.14: DE Out-of-sample Returns, unitary investment

1/2020 4/2020	B&H	Stand.
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017
0	-0,019	-0,017

Table D.15: DE Out-of-sample Returns, unitary investment

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