Università
Ca’Foscari
Venezia

Master’s Degree

in Economics and Finance

Final Thesis

Modelling electricity price and determination of the futures price

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Academic Year
2018 / 2019

Abstract

We discuss the characteristics of the electricity and its price in order to find suitable models for the specification of the fair futures price. The non-storability feature makes this commodity different from the others bringing several consequences in the determination of its price. Another difference with other commodities is that electricity has regional markets instead of global markets due to the fact that it is not simple to transport energy. It is also linked to other commodities such as natural gas because electricity can be produced from other sources of energy, and of course, this contributes to the formation of power prices. The consumption of electricity is also linked to the weather, so its usage is conditional to the seasons, implying evidence of seasonality in its time series. All these interesting features forced the researchers to find appropriate models to explain the behaviour of electricity spot prices because these models can be used for the determination of the forward prices. In this thesis we analyse the characteristics of electricity and its price, we discuss some of the models used in literature for the power price and the models to determine a fair futures price. The final part of the thesis is dedicated to the application of a model for the electricity spot price using real market data. Then, we want to determine if this model is a good choice for the determination of futures prices using futures contracts traded in the market.

Contents

Introduction

In the last 30 years, the electricity market has changed dramatically because in the 90s it became a competitive market. As a matter of fact, before the deregulation it was monopolistic, the prices were decided by public institutions but now it is very different. Now, the production and distribution of power are governed by the rules typical of the free markets and prices are determined by the competition.

This drastic change is translated into a more volatile market and uncertainty that brought market operators to the necessity of hedging price risk. As it is for assets and commodities, the market developed derivatives regarding electricity to cover the risk of changes in power prices, thus arose also the need for correct pricing of these financial instruments.

The path to the determination of a fair price for an electricity derivative starts from the analysis of the characteristics of this particular commodity, continues with the study of the historical time series of the price and concludes with a correct pricing procedure of the derivative.

In this thesis, we face this topic starting exactly from the analysis of electricity characteristics because it is important to understand the differences with other, simpler commodities. The most important power's property is the non-storability, therefore the impossibility to store it in any way as it is for other commodities, such as oil or natural gas. This physical feature entails the existence of regional markets, sometimes called Pool, where electricity is produced and exchanged through a grid of distribution. Therefore, the power market is not global but there are several small markets that could operate in more than one country. Another consequence of non-storability is the necessity to consume electricity when it is generated, implying a continuous balancing of demand and supply. The impossibility to store this commodity, associated with different ways to produce it and to the correlation with weather changes has several implications on the price movements, as we will see.

This commodity is very volatile, more than the stocks and more than other commodities, as a consequence of the continuous search of equilibrium between demand and supply. Demand can vary according to weather changes that make increase or decrease the request of electricity, modifying the price as well.

Looking at the historical time series of power prices we can notice sudden rises of the prices followed by a likewise decrease back to the previous level. This behaviour is called a spike, and this happens when the producer has to face an unexpected increase of the demand whit the implementation of more expensive producing factors, making the price rises.

The weather influences consumption, not only with unexpected and severe changes, but it does it constantly during the year. Indeed, temperatures change according to the season, so the demand for electricity changes, implying adjustments to the production and to the price.

We are interested in these characteristics because it is important to understand the dynamics of the electricity price, in order to develop a model that is able to explain price movements and replicate them. This aspect is discussed in the second chapter where we schematize several typologies of models, basically divided into two categories, stochastic models and equilibrium models. The former uses stochastic differential equations based on the Geometric Brownian Motion to create the dynamic used to explain price behaviour. The latter tries to find the equilibrium between demand and supply in order to determine the price.

The most important features of the stochastic models we'll see are the mean reversion and the jump-diffusion. The jump-diffusion process is used to simulate indeed the jumps, while the mean reversion process is used to bring back the prices to the long-run mean level. Those two elements are always supported by a Wiener process and sometimes we can find also stochastic volatility instead of a constant one. A similar approach is the regime-switching that divide the process into two parts, one that models the continuous part of the process, while the other models the discontinuous part, that is the part dedicated to the jumps.

A totally different approach is the equilibrium model that uses the information provided by the marker about production and consumption, trying to build the curves of supply and demand. This type of model tries to determine the price by finding the equilibrium of the two curves.

There exist also hybrid models that try to get the best of both, stochastic and equilibrium models. This type of model uses stochastic processes for the drivers of the demand and the supply instead of directly using them for the prices. When the two curves are built,

the research of the equilibrium starts to find the price.
In this thesis, we explain also why is needed a model for electricity and how it could be used to price derivatives. In particular, there will be a review of the theory behind the pricing of forward contracts, with an explanation of why it is not simple in the case of electricity. The main feature of this commodity, the non-storability, forces one to use a different approach than the arbitrage-free pricing theory. This theory implies the possibility to adopt investment strategies that could require to hold the underlying till the end of the contract or to borrow it. This is something impossible if the underlying is electricity. We go further introducing the risk premium approach and we finally explain how electricity models are used for pricing forward contracts. In addition, there is also a section where we explain a totally different approach that aims to model directly the forward curve instead of using electricity spot price models.
The last chapter is dedicated to a practical application that analyses in detail one of the stochastic models introduced in the second chapter. This part is dedicated to a review in depth of this model and to its calibration. We implement this model for Italian's electricity spot prices, giving also our opinion on the results obtained, in addition to the weaknesses of the model.

Chapter I. Physical characteristics of electricity and its spot prices

In the last century, until the ’90s, the electricity industry had a vertical structure and the prices were decided by the regulators in order to set a fair price based on the costs of creation and distribution. At the beginning of the new millennium, we start to see a change in the structure of the market, so now the prices of the electricity are ruled by the supply and the demand of the electricity due to the deregulation occurred after the ’90s. The deregulation of this sector brought the need for protection against the price risk, thus this was the starting point of the creation of new financial instruments (Geman and Roncoroni, 2006). The introduction of the derivatives, as new instruments, bring the market participants to face the interesting characteristics of electricity, such as non-storability~~,~~ for example. This feature forces the investors to treat this commodity in a different manner because there are no reservoirs and what is produced must be consumed immediately. Furthermore, the demand is inelastic and must be balanced constantly with the supply.

An interesting aspect of electricity is that it can be produced from other energy sources such as natural gas and other fuels, so power prices could be correlated to other commodities in certain markets as well as to weather conditions if electricity is produced from renewable energies. Electricity is not traded globally, due to its peculiarities, but it is produced and delivered in regional markets. These markets can diverge for several reasons: they can differ for the weather conditions or the technologies used to produce electricity. For example, Scandinavian countries use mainly hydroelectricity which can be considered as a sort of indirect storage of this energy source and this could influence the prices in a different way if compared to other regions. All the characteristics just listed could be possibly spotted in time series of the power prices: we can observe spikes due to adverse weather conditions, or problems in the transmission of this good, difficulties in the generation of the electricity or all of these in an unfavorable moment (Geman and Roncoroni, 2006).

Besides spikes, a time series gives information regarding the seasonality or the volatility of the prices and we are going through all these elements in this first part of the thesis.

In this chapter, in fact, is highlighted the main features of the electricity as a commodity, such as non-storability (1.1), but also the characteristics of the prices observable analysing the time series: the high volatility as consequence of the non-storability (1.2.1), the exceptional jumps (1.2.2) of the prices and why they occur, the seasonal patterns (1.2.3) due to the connection to the weather and to the schedule of the final users, the mean reversion feature (1.2.4) that has been implemented in the models of several researchers In the last paragraph (1.2.5) is shown the probability distribution of the power prices that resumes some of the characteristics of this commodity such as volatility and spikes and how they influence the shape of the distribution. The study of the time series of electricity prices is a first necessary step to understand what kind of model would fit better. Finding a suitable model is the second step in order to define a fair futures price.

1.1. Physical peculiarities of electricity

Electricity is a commodity that can be used in several ways and for a lot of purposes, it doesn't pollute when it is used but it is also connected to other energy sources because it is converted from coal, natural gas or oil, for example, as well as renewable sources (Burger, Graeber, Schindlmayr, 2014). Regarding these connections, it is important to highlight that electricity prices exhibit very unstable correlations with other fuels as reported in Eydeland and Wolyniec (2003), who provide Figure 1.1 as an example. The figure shows how the correlation between the two elements can change as time goes by. We can see, indeed, values of the correlation close to zero between December 1999 and February 2000, and on the other hand, we can spot values of the correlation close or above 20% in the whole 1999.

Electricity has characteristics that make difficult delivering this good to the final user, more precisely electricity needs a grid infrastructure and for this reason, it has a more regional market if compared to other commodities. In other words, there isn't a global power market but there are several regions in which electricity is produced and delivered. There is also the possibility to exchange power among communicating regions in order to compensate the excess of production or to satisfy an unexpected higher demand (Burger, Graeber, Schindlmayr, 2014).

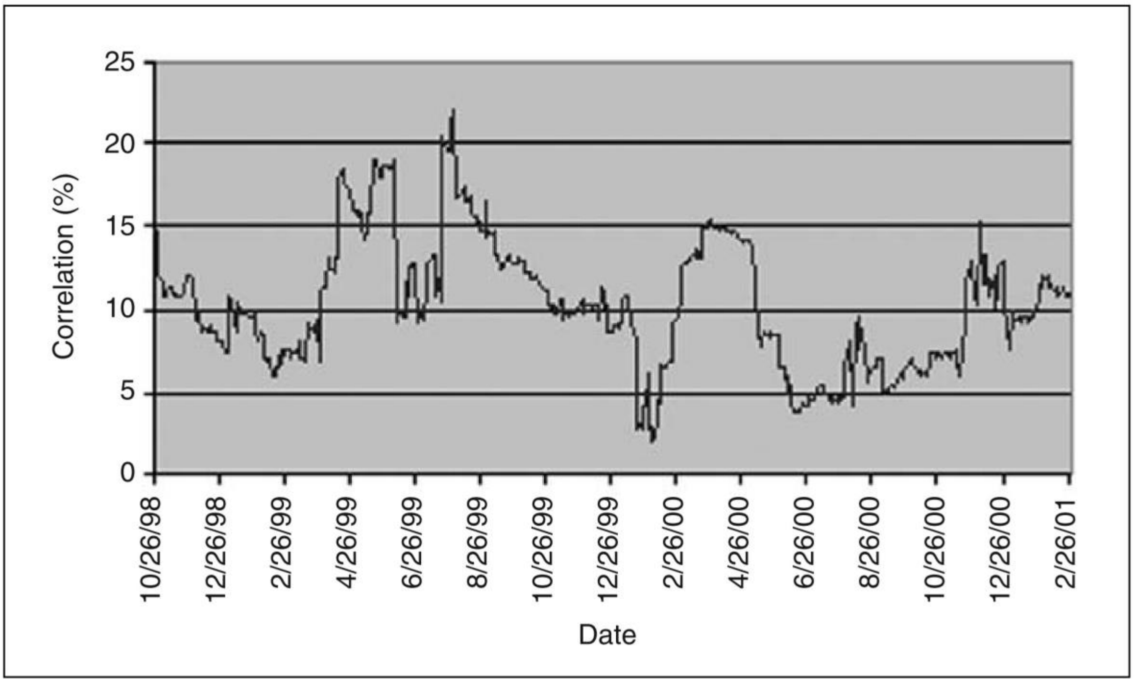

Figure 1.1: the figure shows the correlations between electricity spot prices of the PJM[1] market and natural gas spot prices of the Transco Z6[2] market from 26/10/1998 and 26/02/01 (Eydeland and Wolyniec, 2003).

An important difference with other commodities is that electricity is hardly storable. One can argue that reservoirs exist (for example lakes) from which we can pump water and create electricity. One can see this method as a kind of electricity storage, but it is also true that the actual capacity of the countries to generate hydroelectricity is not able to satisfy the consumptions. This is one of the reasons why researchers in this field prefer to not consider it as storable, neither indirectly (Bettiol, 2013). Another consequence of the non-storability is that it is required to match the supply and the demand constantly. Demand and supply are balanced by the transmission system operator (TSO) that integrate the energy provided by the supplier. A single day is divided into several small periods (30 minutes or 1 hour) in which is provided by the merchant a certain amount of energy based on the average consumption, the TSO integrate and balance the need for electricity according to the continuously changing customer

[1] It is a wholesale electricity market in the north-east of the USA. The acronym stands for Pennsylvania, (New) Jersey and Maryland, three American states.
[2] Spot price of natural gas traded in the north-east of the USA.

requirements (Burger, Graeber, Schindlmayr, 2014).

When one deal with commodity derivatives, we should take into consideration two factors: the convenience yield and the cost of carry. The former is the advantage we have in holding the commodity, the latter is the cost one has in keeping the commodity and bringing it to the future time (Hull, 2012). The convenience yield isn't something one can observe but rather something implicit in the forward curve (Burger, Graeber, Schindlmayr, 2014). This is because the convenience yield represents the expectation the market has for the availability of a certain good in the future, Hull (2012). This expectation is made by the belief of the market of how much of a certain good will be at disposal in the next future based on the inventories, demand, and supply. Since electricity is not storable there aren't inventories affecting the convenience yield or the cost of carry, thus the forward price for the electricity is determined by the expected capacity of production of the power plants and by the cost of production, and not by the inventories available to satisfy an unexpected high demand as it is for other commodities. We don't continue here these topics, the convenience yield and the cost of carry, because they are treated in the third chapter where it is also discussed the relationship between the electricity spot price and its forward price.

We have discussed in this section some of the characteristics of electricity, such as the non-storability, that is the most important feature. It was critical to understand the element we are dealing with before going further and, to summarize, the features just treated are shown in Figure 1.2, where there are also peculiarities of the power spot price that are introduced in the next paragraphs. As we can see in this figure, there is a mention of volatility and seasonality, and in fact, they are treated in the next section among the spikes, the mean reversion and the probability distribution of the prices.

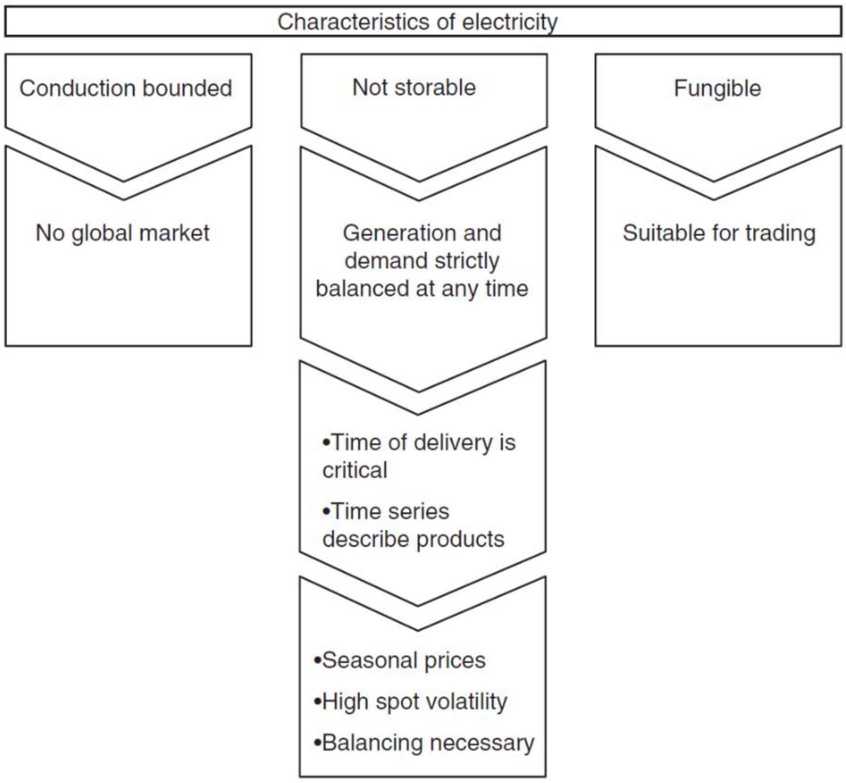

Figure 1.2: Characteristics of electricity (Burger, Graeber, Schindlmayr, 2014).

1.2. Characteristics of electricity spot prices: analysis of power time series

In the last paragraph, we introduced the electricity as a tradable good, with some of the very particular characteristics that identify this commodity, and that distinguish it from the others. It was necessary to understand this commodity before we go through the analysis of its price because the features we have seen are related to the price movements. Indeed, this section treats the characteristics of power prices.

1.2.1. Volatility and stochastic volatility in the power time series

A new more competitive market brought more sources of uncertainty around electricity prices and adding the feature of non-storability of this commodity results in high volatility (Escribano, Peña e Villaplana, 2002). In fact, electricity prices are more volatile than other commodities and even more than the stock market (Renò, 2006). As already said in this chapter, and as stated also by Escribano et al. (2002), the non-storability requires a continuously balancing of supply and demand making the prices move frequently. Moreover, because of the impossibility of storing the power, it is also hard to face unexpected shocks from the side of the demand, as well as the side of the production.

One must take into consideration the rigid demand for electricity that can make large price movements and make the demand change, without forgetting that the consumption of electricity is also very connected to the weather. These two elements are very important to understand why power prices are volatile. Another consideration that has to be made is about the supply: electricity is provided using different plants starting from the ones with lower marginal costs, satisfying a low level of power required. As the electricity demand increases, generators with higher marginal costs enter in action rising electricity prices (we are going to explain better the function later when we will talk about the spikes). Thus, also this characteristic of the supply can make understand that prices could widely move as the need for electricity changes. Due to the fact that the demand curve is inelastic, and taking into consideration the characteristics of the supply, one can imagine there is the possibility to spot high changes in the price as the demand of electricity changes (Escribano at al., 2002).

It is possible to observe not only volatility but also conditional volatility, even though it is not simple to test it due to the presence of seasonality and jumps. Finding the dependence of the volatility is possible by building models that take into consideration the presence of seasonality and taking off the jumps. Renò (2006) tells that implementing this procedure allows us to obtain something similar to what is seen in models for stock prices.

In order to model conditional volatility, one must apply a model of the ARCH[3] type and Escribano, Peña e Villaplana (2002) have shown in their results that it is possible to

[3] The acronym stands for Autoregressive conditional heteroscedasticity. This model was introduced by Engle (1982) for the time series with non-constant variance.

obtain a good model by allowing for jumps in a GARCH[4] model, finding at the end a process with stationary volatility. We describe their model in section 2.1.1 of this thesis.

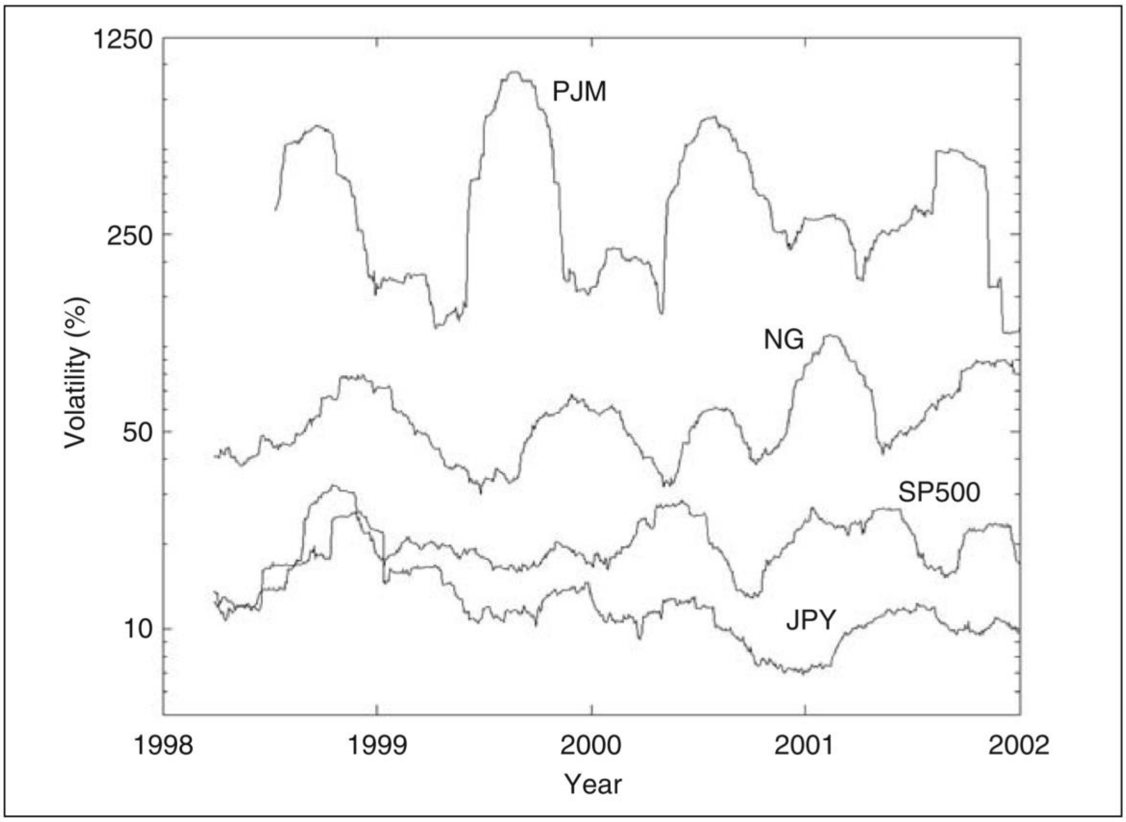

Figure 1.3: The figure shows the volatility of the PJM, the natural gas (NG), the SP500 and the Japanese Yen (JPY) between 1998 and 2002 (Eydeland and Wolyniec, 2003).

Moreover, also Eydeland and Wolyniec (2003) talk about the presence of volatility like the authors we've mentioned in this part. They noticed a "consistently and significantly higher" volatility if it is compared with other assets and commodities. They continue saying that the prices suddenly rise in a more intensive manner, showing also a "nonstationary behaviour". They provide some plots to better understand what are referring to: in Figure 1.3 we can see a comparison of the volatilities of the PJM market with the natural gas (NG), with the stock exchange SP500 and with the Japanese Yen. As we can see the two commodities show similar volatility, with the electricity that has stronger spiky behaviour. The SP500 and the Yen are placed below the commodities to

[4] The acronym stands for Generalized Autoregressive conditional heteroscedasticity and it is an extension of the ARCH model introduced by Bollerslev (1986) and Taylor (1986). The difference with the ARCH is that the GARCH includes a moving average component in addition to the autoregressive one.

make understand how much different the volatility between the commodities and other assets is, and especially to highlight how relevant volatility is for the study of electricity prices. Figure 1.4 reveals that not only the PJM market is characterized by a peculiar behaviour of the volatility but also other power markets as a demonstration that electricity prices are very volatile, much more than other commodities and more than any other asset.

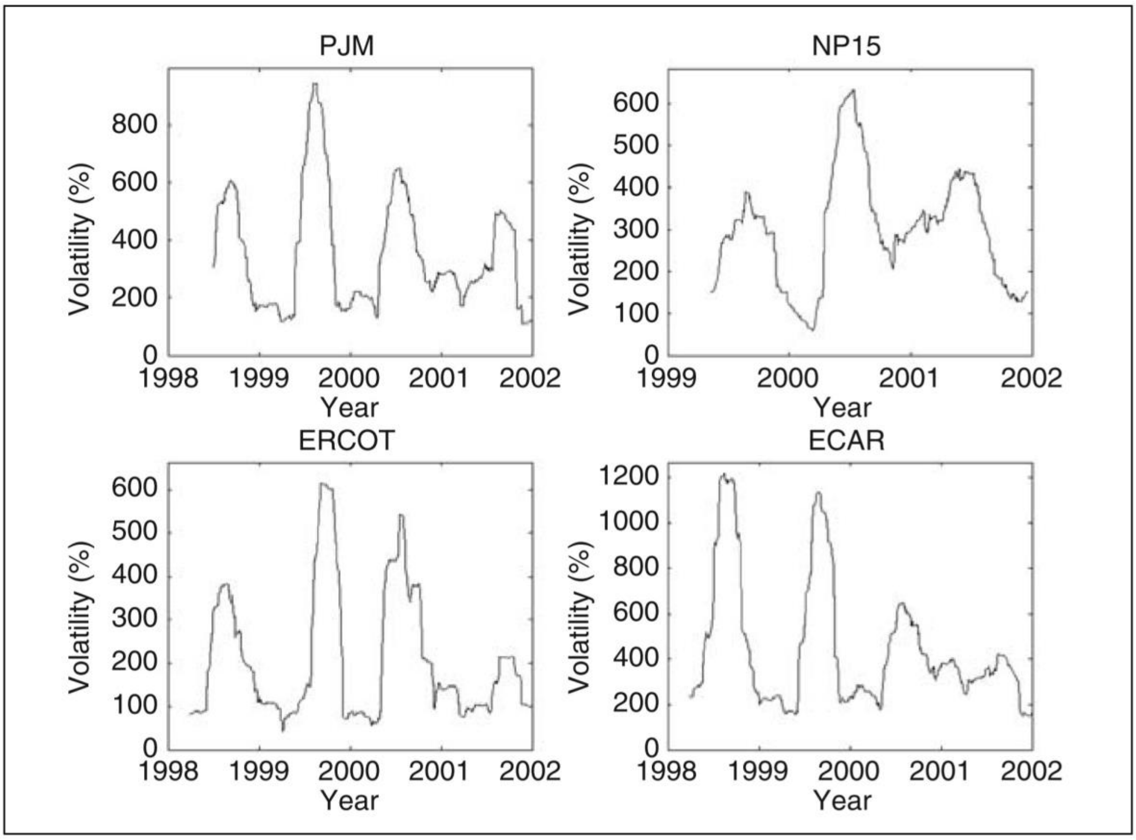

Figure 1.4: Volatility of power prices in different electricity markets (NP15, ERCOT, ECAR)[5] between the 1998 and the 2002 (Eydeland and Wolyniec, 2003).

1.2.2. Evidence of spikes in electricity prices

Geman and Roncoroni (2006) suggest us one more feature, the presence of spikes in the process which are a sudden rise in the prices followed by a deep decrease, and vice

[5] NP15 is the electricity market of the California region. ERCOT is the Texas electricity market. ECAR is the east-central electricity market of the USA.

versa. An example of spikes has been observed in the midwest of the U.S. during June 1998 when the electricity prices went from 25 dollars per megawatt-hour to several thousand, decreasing to 50 in a few days. This behaviour can be explained by the happening of several events, the unfavourable weather and the bottleneck created in the transmission of the electricity produced in Canada due to the high demand at that moment. Also, in Europe has been observed something similar in December 2001, even though with a "smaller" increase due to the lower magnitude of the event and to the higher reserves (hydroelectricity). This type of events can be explained by the power stack function (Eydeland and Geman, 1998), that is the "marginal cost of the electricity supply". It is possible to graph the supply function if one knows the properties of the several plants in a certain region. In practice it is enough to put in order the production sources of energy from the cheapest to the most expensive (per unit), resulting in a function that is almost flat for the part of the low-cost energy and at a certain point increases exponentially due to the use of very expensive plants. Figure 1.5, the power stack, shows the pattern just described for the electricity process and in other words is the graph of the supply and demand functions.

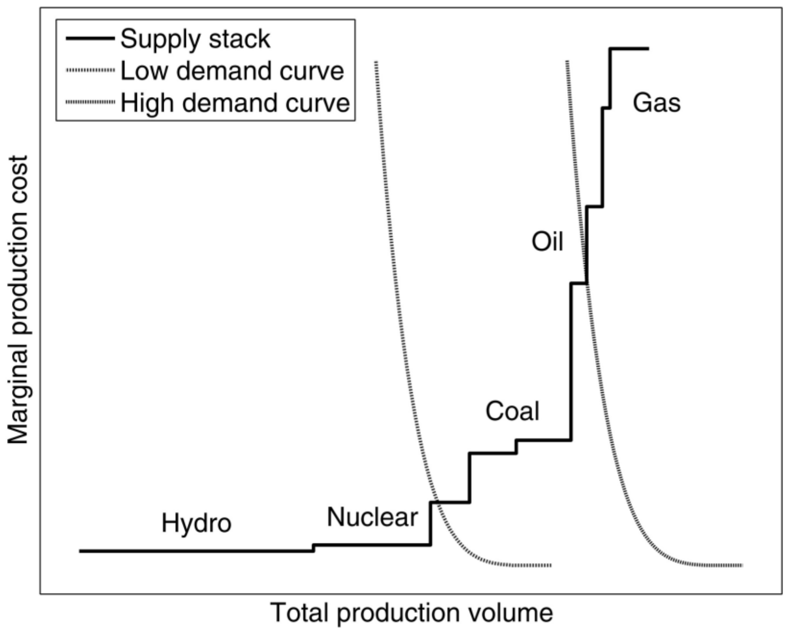

Figure 1.5: this is a simplified model that explain very well how the marginal cost to produce electricity increases using different sources. The two demand curves make us understand how the price increases as the demand shift to the right, that is when more electricity is needed (Weron, 2006).

Also, Weron (2006) defines the spikes in the electricity prices as the most evident characteristic of this commodity, finding the cause in the non-storability feature of course. The author says that spikes are more evident especially during on-peak hours, on working days and during the seasons in which the demand is higher depending on the region one is referring to; it could be winter for the regions in the north of Europe or summer for a region with very hot weather in that season. Just to give an example, Figure 1.6, provided by Weron (2006), shows very well the spiky nature of electricity prices in the Nord Pool[6] market.

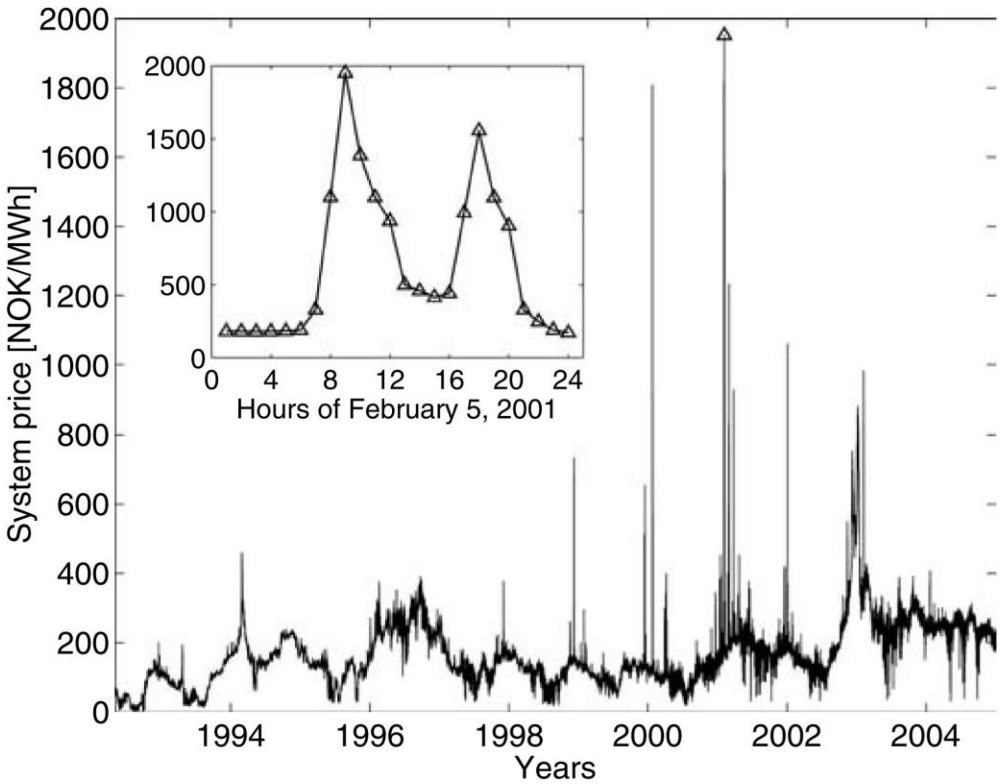

Figure 1.6: Hourly electricity prices of the Nord Pool market during the period 1992-2004. In the small square inside the plot is reported the hourly price series of February 5, 2001, where the increase in the prices reached 10 times the initial level. The prices are measured in units of Norway Krones per megawatt of electricity distributed in one hour (Weron, 2006).

In the figure, the jumps are very clear of the prices during the whole time series, followed suddenly by a decrease to the initial level there was before the jump taken into consideration.

[6] Nord Pool market is the electricity market of several north European countries.

In Figure 1.6 there is also an example, in the square inside the plot, of the magnitude of a jump during a single day; in fact, prices can rise 10 times in a single day.

The same author, Weron (2006), provides another interesting plot in three dimensions reporting, also in this case, the hourly spot prices of the Nord Pool market. The difference of Figure 1.7 with Figure 1.6 is that in this case, one can see better the sudden rise of the prices on the peak hours and during the high demand season. The peak hours are in the morning around 9:00 and in the evening, around 18:00, whereas the period of the year in which the demand increase is of course when it is colder, that in winter.

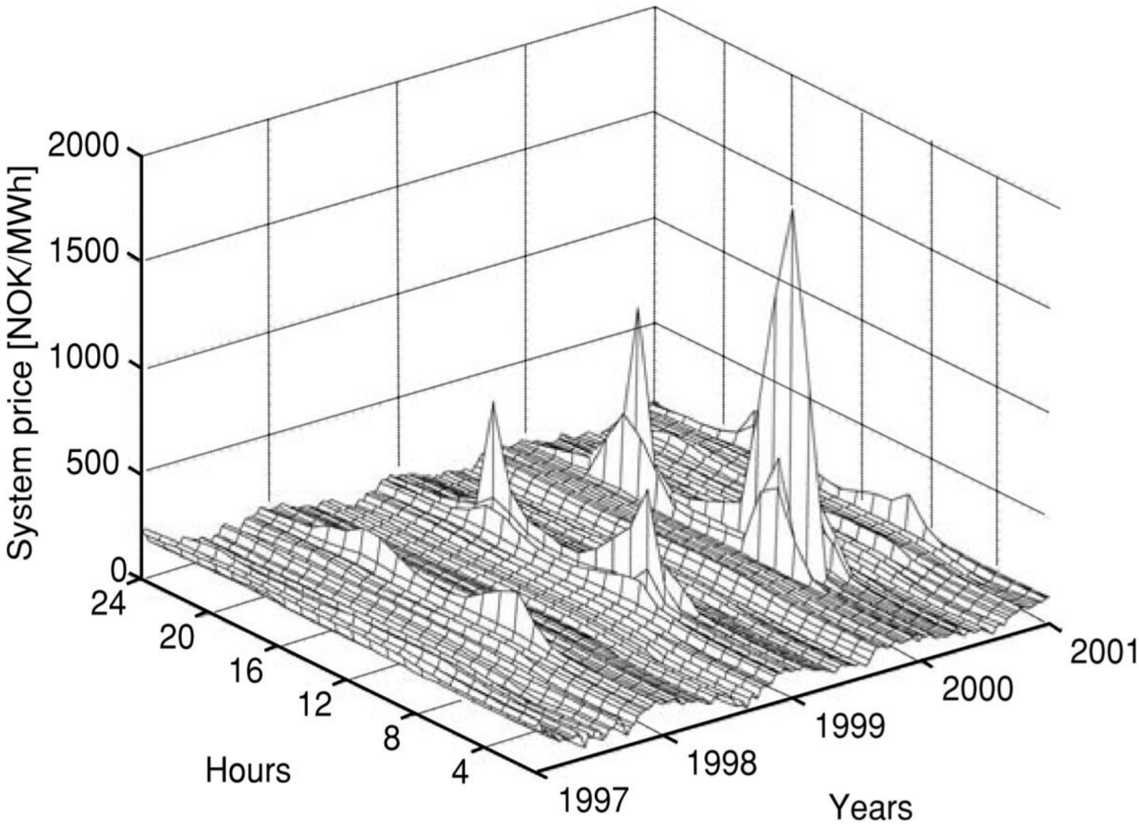

Figure 1.7: the figure shows the hourly price of electricity per megawatt-hour in Norway Krones. The period at issue goes from 1997 to 2001 and the data are provided also on an hourly basis. In fact, it is possible to see the peaks during the days and during the years (Weron, 2006).

Geman and Roncoroni (2006) say that several researchers had to add a Poisson[7] element

[7] A certain discrete random variable X that have the following density

$$P[X = k] = \frac{e^{-\mu}\mu^{k}}{k!}$$

to their models, inspired by the jump-diffusion model introduced by Merton (1976), in order to explain the sudden rising of electricity prices, but it came up also the necessity to explain how to bring down the prices in the model created to explain them. A regime-switching[8] representation could be a good solution to this problem, as was thought by Deng (1999), but the spot prices obtained by this representation were pretty different from the one observed in the market. The intuition that adding jumps in the model could help to represent well the electricity spot prices is supported also by Lucia and Shwartz (2002), who found that even in the regions with apparently enough electricity reserves (hydro), power prices don't have a continuous path.

In contrast to the spikes, there is another characteristic, the white noise[9] factor that makes the prices move around the average trend. It is not predictable and represents the temporary daily inequalities between demand and supply in the market. This is the first element of uncertainty in a model, as explained by Geman and Roncoroni (2006) who added a wiener process[10] in their model to explain the small unpredictable fluctuations of the power prices.

It is important to be aware of the presence of spikes in the time series taken into consideration if we want to create a good model for electricity prices. Besides the spike feature, there is also the seasonality component that is very important, considering that the demand for electricity varies depending on the seasons. Indeed, this is the argument treated in the next subsection.

1.2.3. Different seasonalities for electricity prices

is a Poisson variable. Where μ is the expected value of X, and k is the random variable representing the number of events occurring in the time interval taken into consideration.

[8] We will treat regime switching models in the second chapter.

[9] A white noise is an independent identically distributed random variable and can be discrete or continuous. It has zero mean a variance equal to σ^2.

[10] A Wiener process $W = (W_t)_{t\geq 0}$, or Brownian Motion, is a stochastic process with the following properties:

- $W_0 = 0$ or in other words, the process starts at zero;
- It has stationary, independent increments ;
- For $t > 0$, (W_t) has a Gaussian $\mathcal{N}(0, t)$ distribution;
- It has continuous sample path.

It is possible to observe seasonality generated by the demand side, which affects heavily electricity prices, but also by the supply side. As regards to the demand, we can imagine that the request for electricity could be very different depending on the temperature or how many hours of light we have in a specific day (Renò, 2006). Thus, it is possible to imagine that during the summer period in which we have higher temperature we have also a higher demand for electricity which is used for air-conditioning. On the other hand, it is likely to use more electricity in that period of the year in which the hours of sun are less, and it is needed to use artificial light for much more time in a day. Looking at the supply of hydroelectricity, it is possible to understand that seasonality could be generated by the quantity of rain that falls in a certain period of the year (Renò, 2006). All these characteristics that contribute to the presence of seasonality are also strongly connected to the region in which the plants and the final users are. Imagine how much different is the situation in a Scandinavian country compared to the Nevada state in the U.S. for example. The first one has more hydroelectric plants, colder and darker winters, whereas the second one has hot summers and probably lower reservoirs at its disposal. In addition to yearly seasonality, one can also observe weakly and daily seasonality. This happens because during the weekends many workplaces are closed, so it is consumed less electricity. This is valid also for the night when less power is consumed with respect to the day when the prices are higher because the demand is higher. In addition, Renò (2006) tells that we can find evidence of seasonality also for the variance of the prices.
Lucia and Schwartz (2002) have studied the Scandinavian electricity prices and, as we said, they noticed the influence of business activities in spot prices detecting hourly, working hours against non-working hours, and weekly patterns, working days against non-working days. Furthermore, they noticed also seasonal patterns based on the period of the year, distinguishing between the cold period, with lower temperatures and shorter days, and warm period with opposite conditions.
Figure 1.8 shows some of the findings of Lucia and Schwartz (2002), the average intraday changes in electricity prices, whereas the difference among prices during the working days, weekends and holidays is shown in Figure 1.9.

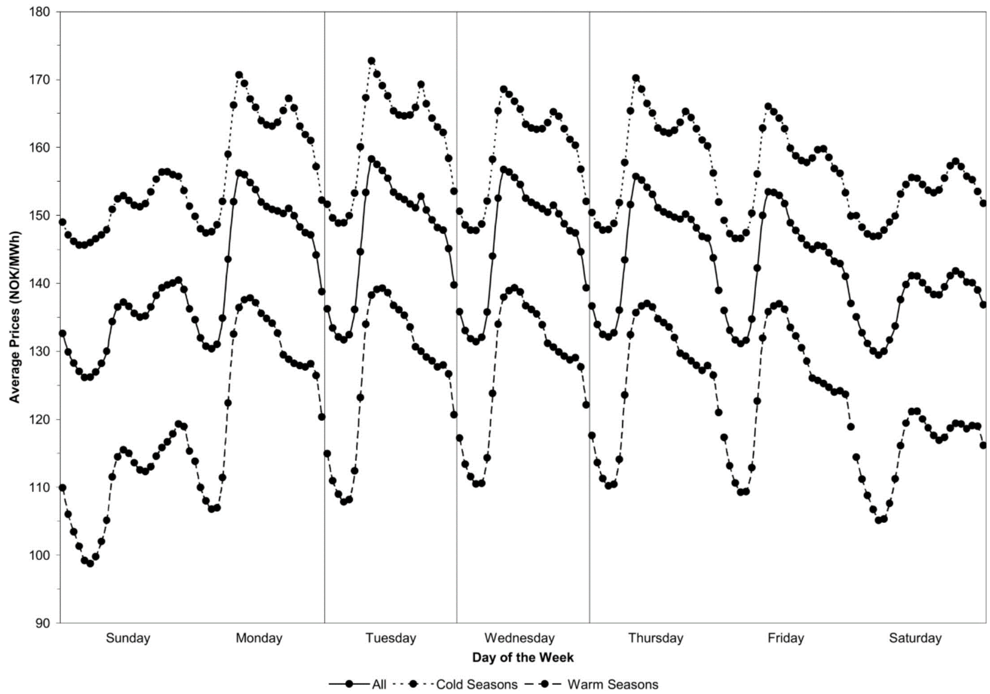

Figure 1.8: the figure shows the hourly mean prices throughout the week across seasons in the period 1993-1999 (Lucia and Schwartz, 2002).

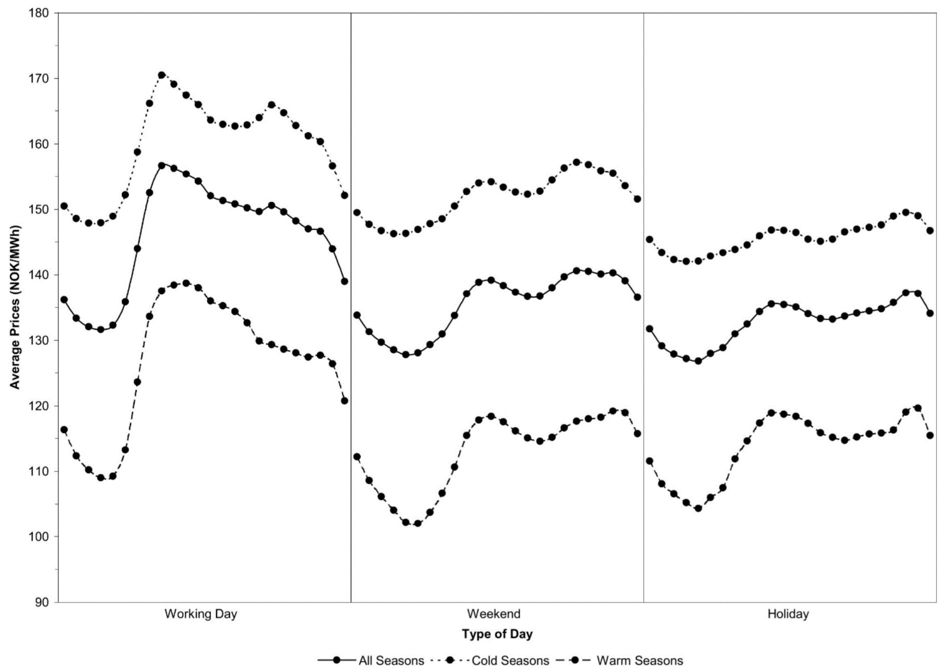

Figure 1.9: The figure shows the hourly mean prices separately for working days, weekends, and holidays (only official holidays in Norway have been considered) across seasons in the period 1993-1999. A warm season includes the months from May through September. Lucia and Schwartz (2002).

Weron (2006), as well as Lucia and Schwartz (2002), studied the Nord Pool system and, also this author can provide a useful plot representing very well the seasonality. What appears in Figure 1.10 is the graph of the electricity spot prices of the Nord Pool market. The figure gathers more than three years of observations and illustrates clearly the seasonal behaviour of the prices which are higher during the winter and lower during the summer. Notice that the annual seasonality (dotted line) is sinusoidal and that this factor captured the attention Pilipovic (1998) who proposed, indeed, a sinusoidal function to model power prices.

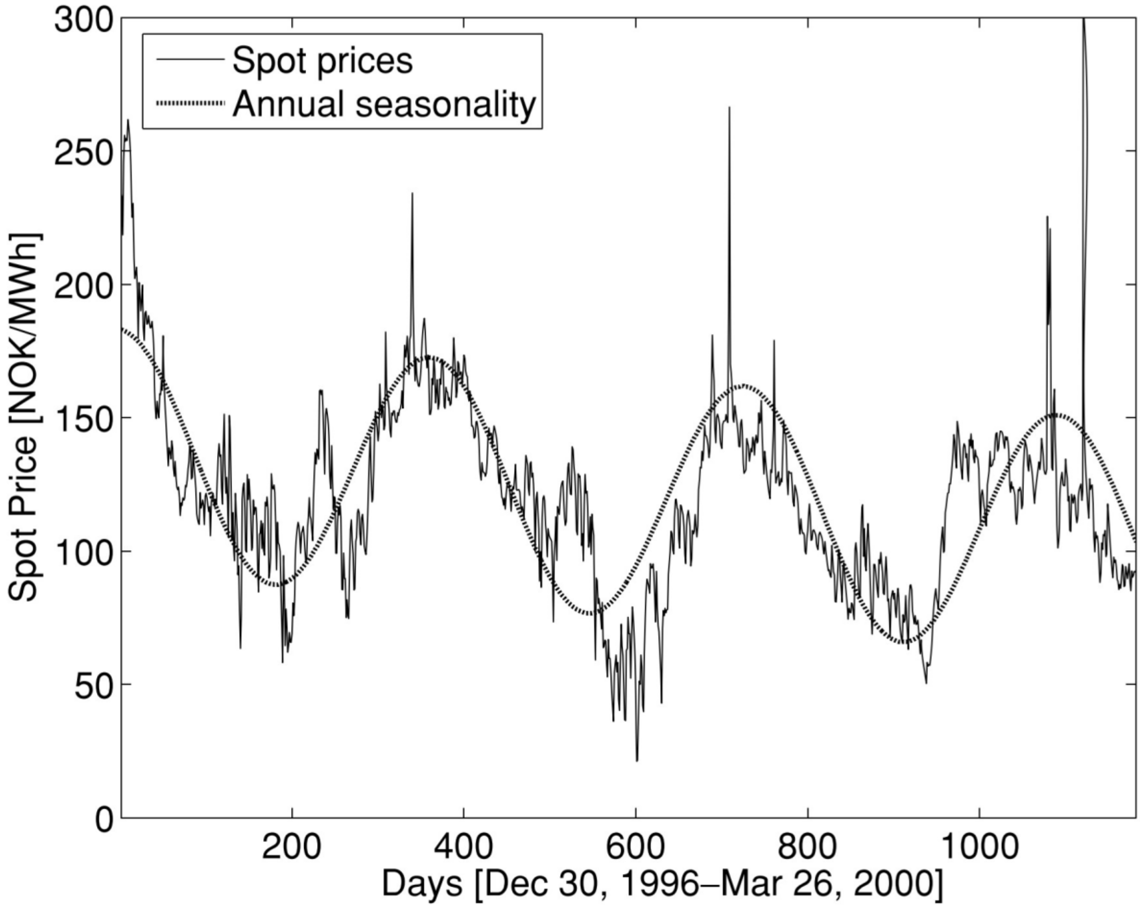

Figure 1.10: Daily average spot price of the Nord Pool market. The period taken into consideration foes from December 30 to March 26, 2000. The price in Norway Krones per megawatt-hour. The dotted line is the annual seasonality (Weron, 2006).

1.2.4. Indication of mean reversion in electricity's time series

Among the characteristics of electricity prices, there is the mean reversion, implying that prices revert to a certain level that is assumed to be the marginal cost and it is possibly "*constant, periodic, or periodic with a trend*" Geman and Roncoroni (2006). Escribano et al. (2002) suggest that the presence of mean reversion could be assumed because of the nature of electricity consumption, strictly connected to the weather. They also give another intuition, that is, they defined as natural an expectation of a certain "degree of mean reversion" since the supply follows the demand after an increase in the consumption of power, bringing down the price that raised at the moment in which more electricity was needed. In Escribano et al. (2002), we find several authors that applied mean reversion to their models such as Bhanot (2000), Karesen and Husby (2000), Lucia and Schwartz (2002) and Knittel and Roberts (2001), but also a few researchers who tried models without mean reversion; among them, there are De Vany and Walls (1999) and Leon and Rubia (2001). Johnson and Barz (1999) tried to compare different models including or excluding mean reversion and jump processes. They combined these two elements in different ways discovering that the best solution was a model with mean reversion and jumps (Escribano et al. (2002).

Eydeland and Wolyniec (2003) argued that the presence of mean reversion is strictly connected to the "spiky nature" of the electricity time series, so it depends on the model that is used. If the model takes into consideration the spikes and the non-constant volatility, we will find strong evidence of mean reversion. They found that if we take off the spikes or analyse a section of the series where there aren't spikes or just a few, we'll not find any presence of mean reversion.

In Weron (2006) one can even find an analysis of four time series, regarding different markets, which aim is to demonstrate the presence of long memory, previously named in this paper as mean reversion. Author's work examines the Nord Pool market, the European Energy Exchange, the California Power Exchange and the over the counter on-peak spot prices of some firms in three regions in the south of the USA. Weron (2006) uses four different techniques to conduct the analysis and what the author found is that all the examined series show evidence of mean reversion. Weron (2006) continues the analysis with a deseasonalised dataset of the same series to test if the seasonality component is responsible for the significant mean reversion. The result is that in only one case there is no more evidence of mean reversion, thus he concludes saying that the presence of seasonality doesn't seem to interfere with the mean reversion

component of the power prices. Notice that this is exactly the contrary of what Eydeland and Wolyniec (2003) say.
With this paragraph about the mean reversion, we are almost at the end of our analysis regarding the characteristics of power spot prices. What remains is to see how these prices are represented by the probability distribution function. What we expect to see is a non-normal distribution, skewness, and fat tails and these are the main features of the pdf of power prices we are going to talk about next.

1.2.5. The probability distribution of power prices

We have seen several peculiarities of electricity prices such as the seasonality and the spikes, in addition to other and to the last one we have treated, the mean reversion. Now, is the time to talk about one of the most useful tools in statistics, the probability distribution function and how it looks like in the case of power prices.
We start telling part of the research of Lucia and Schwartz (2002) who uses the changes in the prices of the Nord Pool market to understand better the distribution of its returns and properties it shows. They found that electricity returns show non-normal distribution, skewness, and relatively fat tails. The time series taken into consideration by the two authors goes from 1993 to 1999 and shows an asymmetric distribution, giving a large probability to low and high returns. The skewness they found was positive, indicating that extremely high returns were more likely than low ones. The two authors associate the high level of skewness and kurtosis to the spikes typical of the return of power price time series, or better, to the jumps caused by higher demand which calls to action more expensive units of the supply stack (see Figure 1.5).
Eydeland and Wolyniec (2003) reached the same conclusions analysing the daily log price returns of the PJM market, finding that the spikes of the series give as result the fat tails in a distribution which is a non-normal distribution. Their result is shown in Figure 1.11 which is the histogram of the distribution of the PJM market's returns during the period 1998-2001 (notice how different is the figure from the canonical bell-shaped

normal distribution).

The outcome of the tests carried out by the two authors gave a unique positive answer to the presence of non-normality of the distribution, showing a stronger divergence from the normal distribution then other commodities.

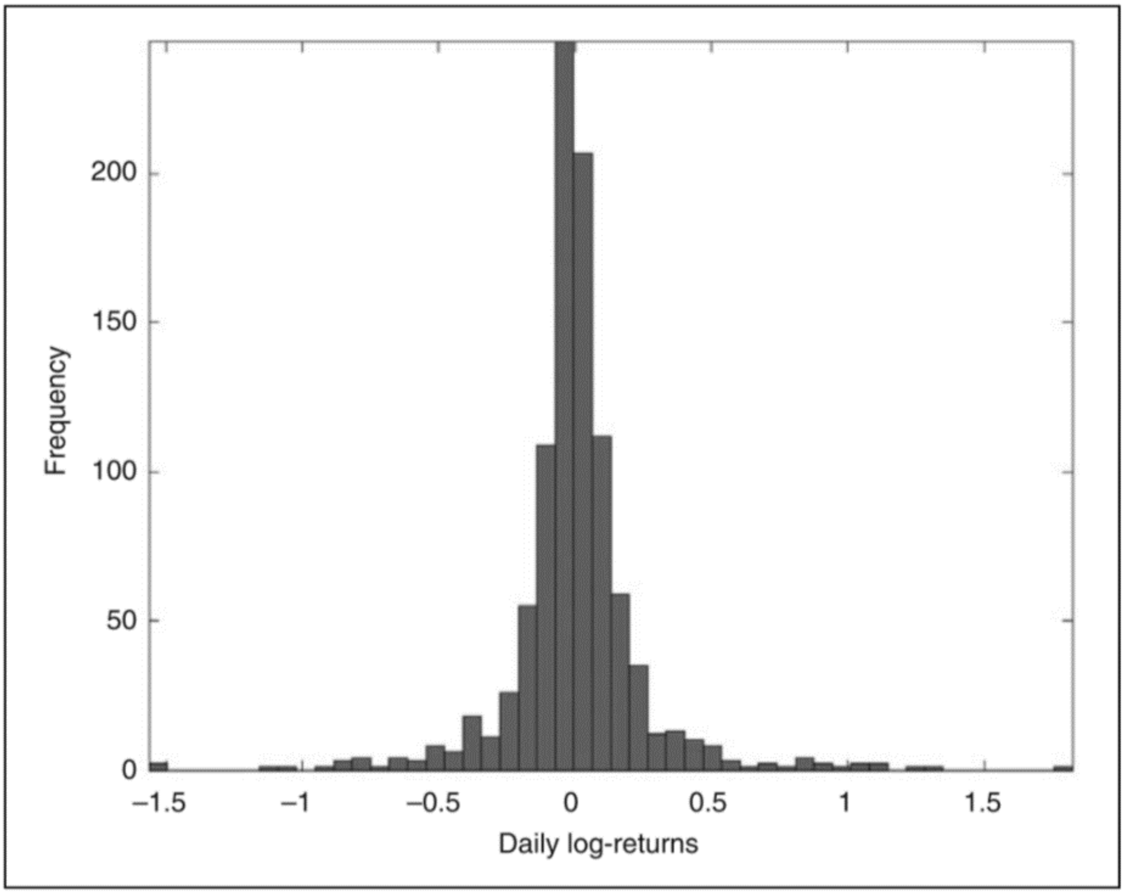

Figure 1.11: Histogram of Daily Log-returns for PJM Prices of the period 1993-1999 (Eydeland and Wolyniec,2003).

This chapter started with the most important characteristic of electricity, that is non-storability. This feature influences the prices and most of the times is the reason explaining certain movements of the same prices. We learned that jumps of the price can be caused by an increase in the demand which can be related to weather conditions and thus, to a seasonal component. For this commodity, there could be daily, weekly or yearly seasonality depending on the hour of the day if that day is a working day or a holiday, and depending on the season if it is winter or summer. It is possible to find long term memory in the time series of electricity prices and that power's probability distribution function is far from being normal.

All the information provided in this chapter are useful to build models for electricity spot prices and these models must represent very well the prices. A good model for spot prices could help to find a fair price for forward contracts that are essential for hedging price risk, especially with this commodity which price can suddenly jump causing enormous losses to the buyers. On the other hand, also the producers of electricity might want to fix a fair price, hedging against the risk of having a low price in the future, thus having lower earnings. This is the reason why in the next chapter we will introduce some of the models studied in literature which aim is to represent as well as possible power spot prices, fixing the second step to find a fair price for a forward contract.

Chapter II. Modelling electricity price

In the previous chapter, we talked about the characteristics of the electricity as a tradable good and resumed the peculiarities of power prices. The main electricity's feature is non-storability, in fact, this commodity cannot be stored, and this changes how to relate with this type of asset. Another concept introduced in the last chapter was the definition of spikes and how they are generated. It easy, indeed, to see in the time series of the electricity prices sudden rises followed by a likewise sudden drop. This behaviour can be called spike and it is due to an increase in the demand that forced to use more expensive production sources with a direct influence on the final price. Following the spike feature, there is also the mean reversion characteristic that is strictly linked to the way in which the prices suddenly go back to their previous level after a jump is verified. As well as spikes and mean reversion, seasonality is also a very important characteristic of power prices because the consumption of electricity depends on the weather and therefore on the seasons. Yearly seasonal movements aren't the only type of seasonality we can find analysing power prices but, also weakly and daily seasonality are spotted in the time series. Furthermore, this commodity is very volatile, more than other energy sources and more than any other asset. The information provided in the first chapter showed also the presence of stochastic volatility, thus the necessity to deal with a non-stationary behaviour of this feature when analysing power prices.

It is important now to remember all the characteristics just listed because in this chapter the argument treated is "modelling electricity spot prices". To model a certain behaviour, one has to know what a model has to replicate, that is why the first chapter was absolutely a first necessary step. This chapter continues, though, introducing the ways to model these peculiarities in order to obtain feasible models that represent electricity power prices as well as possible. The aim of this part of the thesis is not to create a complete taxonomy of the models in this field, but rather giving an example of the most important types. It could be considered very interesting the way in which Eydeland and Wolyniec (2003) divide the several models studied in literature, indeed we

use the chapters 4 and 7 of their book to create a better and concise subdivision, where it is possible to understand better the possibilities one has for modelling electricity spot prices. In any case, we suggest also a different way to create a summary of the models we are going to study, more precisely, it is possible to find an interesting work of this type in Aïd (2015): the author tells that is not his intention to create a complete list of all the models for spot prices but, he wants to give an intuition about the relation between the most popular models and the forward price curve. In doing so, he creates a resume similar to the one we are going to present.
Thus, the chapter is divided into three parts: the first one is dedicated to the reduce form processes (2.1) that start with the most simple stochastic processes used in finance, which are actually not very interesting in this case but useful to understand the successive and more relevant model. Hence, in this section, are offered examples of jump-diffusion models and regime-switching models with their more complex variations also. Section (2.2) introduces production cost and fundamental equilibrium models, in contrast to the stochastic processes because these models aim to find the right price through the equilibrium of microeconomic elements. Lastly, section (2.3) explains what Eydeland and Wolyniec (2003) call hybrid models whose intention is to use the best of the stochastic and structural models. Also, hybrid models can be divided in the stochastic models of elements connected to electricity prices, and models that aim to find an equilibrium of demand and supply through the drivers of the electricity prices. Stochastic models are used for the drivers of the demand and the supply combined to find an equilibrium.

2.1. Reduced form processes

This chapter can start by introducing a stochastic[11] model that in finance has been widely used, the Geometric Brownian Motion (see Samuelson, 1965), which differs from the

[11] A stochastic process is a collection $\{X(t) \;\; with\; t \geq 0\}$ of random variables. For each point $\omega \in \Omega$, the mapping $t \to X(t, \omega)$ is the corresponding sample path, a realization or trajectory.

standard Brownian Motion[12] because it gives positive values, to model the evolution of the prices (Eydeland and Wolyniec, 2003). The GBM satisfies the following SDE

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \qquad S_0 = 0, \tag{2.1}$$

where μ is the constant drift, $\sigma > 0$ is the constant volatility and W_t is the Wiener process.

This is a very simple model, and therefore not very useful for our purposes because it doesn't show the characteristics listed in the first chapter and resumed at the beginning of this one. Remember that mean reversion is one of the key aspects of power prices and as you can see there isn't any mean reversion element in equation (2.1), that is why they have been modified to add this feature. Recall the behaviour of mean reversion: defined a certain level, named long-term mean, higher the distance between the price and this level, higher the probability that the price will go back to the long-term mean. The corrections introduced in the GMB we are talking about are of course the long-term mean (S_∞), and the element (k) that defines the intensity of reversion, changing the GMB as follows (Eydeland and Wolyniec, 2003)

$$\frac{dS_t}{S_t} = k(S_\infty - S_t)dt + \sigma dW_t. \tag{2.2}$$

Equations of type (2.2) are taken from the Vasicek model for the instantaneous interest rate[13], introduced in 1977, and Vasicek defined this very model from the Ornstein-Uhlenbeck[14] stochastic process.

Another version of equation (2.2) is obtained by using the logarithm of the prices and

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[12] A Brownian motion can be defined as follows

$$W(t) = d \lim_{n \to +\infty} W^{(n)}(t)$$

where $W^{(n)}(t)$ is a scaled random walk evaluated at time t. In words, once $t \geq 0$ is fixed, as $n \to \infty$, the distribution of $W^{(n)}(t)$ evaluated at time t converges to a random variable of type N(0,t), a normal distribution with mean zero and variance t.

[13] Vasicek used this model to explain the changes of the interest rates.

[14] The Ornstein-Uhlenbeck is the first mean reverting process from which come the Vasicek model and the Cox-Igersoll-Ross model. This process takes the form of equation 2.3 but sometimes can be written also as

$$dX_t = -\theta X_t dt + \sigma dW_t$$

where θ>0.

using a long-term mean of logarithms of spot prices, with the only advantage of obtaining a normal distribution of the process.

As already explained, power prices show seasonality and for this reason, it is important to build a model that considers this element. Eydeland and Wolyniec (2003) report briefly the work of Pilipovic (1997) who uses a GBM as a base of his model, including seasonality. It was included also a mean reversion element through the long-term mean of the logarithms of the prices, giving as result a lognormal distribution. The main criticism is that the distribution of power prices, actually, is not normal as we already said in the first chapter, but it shows skewness[15] and kurtosis[16], therefore this type of distribution, the lognormal, shouldn't be sought when modelling power prices.

2.1.1. Jump diffusion models

Due to the inability of the GBM to explain the fat tails and nonzero skewness, bring us to consider other variations in order to implement other elements such as the jumps and the stochastic volatility.

Jump diffusion processes come from the mind of Merton (1976) and became widely used in the financial markets to explain how their prices evolve. Its ability to capture "big market movement correctly" (Eydeland and Wolyniec, 2003), is the reason why it has become so used to explain electricity prices. As already explained in the first chapter, one characteristic of energy prices is the presence of spikes, that is why it is needed a model, like the jump-diffusion, that takes into consideration this feature and that can also capture fairly well the distribution of power prices.

A JDP can be defined as a mixture of a diffusion process like the Gaussian Brownian Motion (GBM), which has been shown already in this chapter, and a jump process, typically a Poisson process with jumps having a random magnitude instead of a constant

[15] Skewness is a measure of the asymmetry of a certain probability distribution. It can be positive or negative indicating that the distribution has long right tail or long left tail respectively. Having a long right or left tail says that the distribution is not centred on the zero, but it is shifted to the right or to the left.

[16] Kurtosis is a measure that says where the distribution is more concentrated. In other words, it gives a measure of the probability that is on the tails of the probability distribution. Kurtosis is useful because says if extreme events have a relatively high probability.

one as the Poisson is supposed to have. Eydeland and Wolyniec (2003) give an example of the typical JDP:

$$\frac{dS_t}{S_t} = (\mu - \lambda k)dt + \sigma dW_t + (Y_t - 1)dJ_t\,, \tag{2.3}$$

where it is possible to see a structure similar to the GBM but with more elements defining the jumps such as J_t. The jumps are modelled with a Poisson process and in equation (2.3) there is indeed the intensity of the Poisson process λ. Moreover, $Y_t - 1$, with $t > 0$ is the random variable representing the magnitude of the jumps and k is the expected jump magnitude.

The JDP just presented is still too simple for our purpose because it can explain the jumps in series but, once the jump is verified, the prices are inclined to stay at the level just reached. As already discussed, mean reversion is a characteristic of power prices that after a jump they quickly go back to the previous level. That is why a model that explains not only jumps but rather spikes is required, thus a new element has to be added to bring down the price after a jump. In this chapter, one can find a few modifications to the GBM, including the additional element of mean reversion, and now we are going to show a modification to the JDP, introducing a jump-diffusion process with mean reversion. As reported by Eydeland and Wolyniec (2003), a process of this type, that have both jumps and mean reversion, has been proposed by Deng (1998) and it presents itself as

$$dS_t = k(\theta - \lambda k - logS_t)dt + \sigma dW_t + (Y_t - 1)dJ_t\,, \tag{2.4}$$

where k is the strength of the mean reversion and all the other elements assume the same role they have in equation (2.3).

This model can represent and show all the important elements of the electricity spot prices, but it is now very complex and requires to estimate all the parameters we can see in the model.

The main positive point of the JDP is that it can represent at the best the most important characteristics of power prices. For example, it works better than the GBM that cannot express the spikes and fat tails. It has also some drawbacks such as the parameter estimation and calibration, or the inability to consider information about the future.

One can find another interesting work in Geman and Roncoroni (2006) who present their model for the spot prices as the following stochastic differential equation

$$dS_t = \mu'_t dt + \theta_1(\mu_t - S_{t^-})dt + \sigma dW_t + h_{t^-} dJ_t \,, \qquad (2.5)$$

where μ' is the standard first order derivative, $f(t^-)$ is the left limit of the function at time t, μ is the seasonal trend, $\theta_1[\mu(t) - S(t^-)]$ is the drift factor that guaranties the mean reversion towards the long-run value $\mu(t)$, θ_1 is the speed of reversion, W is the standard Brownian motion and σ is the volatility of the Brownian motion.

It is difficult to find a clear definition of spikes but Geman and Roncoroni (2006) offer their interpretation: according to them "a spike is a cluster of upward shocks of relatively large size with respect to normal fluctuations, followed by a sharp return to normal price levels". They say that their model can reproduce this behaviour first by defining a threshold that divides the series into two regimes in which the power price could be: a normal regime and regime where demand and supply aren't in equilibrium, so where the price is high. Second, they define an intensity process for the jumps through the following deterministic function

$$\iota(t) = \theta_2 \times s(t) \,, \qquad (2.6)$$

where $s(t)$ is the normalized jump intensity shape and θ_2 is the maximum expected number of jumps per time unit

Third, they define a process for the jump's size

$$J(t) = \sum_{i=1}^{N(t)} J_t \,, \qquad (2.7)$$

Where J_i's are independent, identically distributed random variables and $N(t)$ is a counting process that specifies the number of jumps experienced up to time t.

The density of the jump is

$$p(x;\ \theta_3, \psi) = c(\theta_3) \times exp[\theta_3 f(x)]\,, \tag{2.8}$$

where x is a value between zero and ψ that is the maximum jump size, while $c(\theta_3)$ is a constant ensuring that p is a probability distribution density.

The last step they take is to define a function that decides whether the jump should be upward or downward, according to the position of the spot price with regard to the threshold. This is translated as follows

$$h\big(h(S_t)\big) = \begin{cases} +1, & E(t) < \mathcal{T}(t) \\ -1, & E(t) \geq \mathcal{T}(t) \end{cases} \tag{2.9}$$

where $\mathcal{T}(t)$ is the threshold. In words: if the spot price is below the threshold, the function h assumes value +1, and if it is above it assumes value -1.

The authors call this type of JDM the "level-dependent signed-jump model with time-varying intensity" and it is the response to other types of jump-diffusion models with some drawbacks the authors found in the work of Deng (1999) and Escribano at al. (2002). Geman and Roncoroni (2006) criticize also the regime-switching models, giving as example Huisman and Mahieu (2001) and Barone-Adesi and Gigli (2002), because the Markov property is not respected. It seems that they found the solution to the problems observed in other jump-diffusion models maintaining also the Markov property.

In Eydeland and Wolyniec (2003) one can find also a type of jump-diffusion model that accounts for stochastic volatility[17]. The two authors justify this decision because there is the possibility of having biased parameters due to the volatility and in this case, it could be a good idea to implement a stochastic component for the volatility in the model. They give in addition an example of this kind of model

$$\frac{dS_t}{S_t} = \ \theta(\mu - lnP)dt + \ \sqrt{\sigma_t}dW_t^1 + (Y_t - 1)dN_t\,, \tag{2.10}$$

[17] When the volatility is not constant but varies with time is called stochastic volatility.

$$\frac{d\sigma_t}{\sigma_t} = \theta(\omega - ln\sigma_t)dt + \eta\sqrt{\sigma_t}dW_t^2 , \tag{2.11}$$

where the expected value of $W_t^1 \times W_t^2$ is equal to ρdt.

This model is a mean-reverting jump-diffusion model similar to the one in equation (2.3) presented at the beginning of this section with a substantial difference: the volatility is not constant and is modelled with a stochastic process (2.11) inspired by the Cox-Igersoll-Ross model. The CIR model is a modification of the Vasicek model introduced earlier in this chapter and its main improvement is that it doesn't allow negative interest rates. In fact, one of the most important criticisms of the Vasicek model was that it could give negative values for the interest rates while the CIR model avoids this possibility. It isn't probably a big problem nowadays having negative interest rates since we are living in a period in which they are actually negative, while they weren't in the past when these models were created.

This type of model that includes stochastic volatility is very interesting and the literature can offer several papers examining this feature. We decided to propose the work of Escribano, Peña and Villaplana (2002): the authors proposed a model that tried to include all the characteristics showed by electricity prices such as seasonality, mean reversion, jumps and lastly stochastic volatility explained by a GARCH model. The model presents itself as follow:

$$S_t = f(t) + X_t , \tag{2.12}$$

$$dX_t = -kX_t dt + v_t^{\frac{1}{2}}dW + J(\mu_J, \sigma_J)dq , \tag{2.13}$$

$$dv_t = k_v(\theta_v - v_t)dt + v_t^{\frac{1}{2}}\sigma dW_v , \tag{2.14}$$

where S_t is the electricity spot price, $f(t)$ is the seasonal component, W is the Wiener process, q is the Poisson process with intensity λ_t that is conditional to the time, (μ_J, σ_J) is the jump process with mean μ_J and standard deviation σ_J and v_t is the stochastic volatility process.

The model above is in continuous time and the authors decided to show it because, in finance, continuous time models are used for hedging and valuation. However, they continued their study using a discrete time model because it's more adaptable and the estimation bias created by the discretization is small and can be ignored if daily data are used. Thus, below is presented the general discrete model:

$$S_t = f(t) + X_t \,, \tag{2.15}$$

$$X_t = \begin{cases} \phi X_{t-1} + h_t^{1/2}\varepsilon_{1t};\ prob.1 - \lambda_t \\ \phi X_{t-1} + h_t^{1/2}\varepsilon_{1t} + \mu_J + \sigma_J\varepsilon_{2t};\ prob.\lambda_t \end{cases}, \qquad 2.(16)$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} \,, \tag{2.17}$$

$$\lambda_t = L1 \cdot winter_t + L2 \cdot fall_t + L3 \cdot spring_t + L4 \cdot summer_t \,,$$

where $\varepsilon_{1t} \sim i.i.d. N(0,1)$, $\varepsilon_{2t} \sim i.i.d. N(0,1)$, ϕ is the degree of mean reversion, α and β are the parameters of the GARCH(1,1) model for the volatility and λ_t is the time dependent intensity process for jumps.

In $\lambda_t$there are 4 dummy variables that recognize the seasons: for example, if one takes a certain observation of a certain day in July, the dummy variable $summer_t$ takes value 1, while the remaining three dummies take value 0. The function $f(t)$ is the deterministic seasonality:

$$\begin{aligned} f(t) = B0 + B2 \cdot t + C1 \cdot sin(t + C2) \cdot \left(\frac{2\pi}{365}\right) + C3 \cdot sin\left((t + C4) \cdot \left(\frac{4\pi}{365}\right)\right) \\ + D1 \cdot wkd_t \,. \end{aligned} \tag{2.18}$$

This is a sinusoidal function with a dummy variable (wkd_t) for identifying weekends and weekdays.

The model just seen has been compared with five different models, from the simplest to the most complex (the model above), to check the relevance of the stochastic volatility and the other elements. The models are:

- Pure-Gaussian model with constant variance (no jumps);
- GARCH(1,1,)-Gaussian model (no jumps);
- Poisson-Gaussian model with constant variance;
- Poisson-Gaussian model with time-varying intensity for jumps;
- GARCH(1,1,)-Poisson-Gaussian model with constant intensity;
- GARCH(1,1,)-Poisson-Gaussian model with time-varying intensity for jumps.

Escribano et al. (2002) introduced the GARCH(1,1) in the second model, noticing immediately an improvement in the fit. The last model, the one they consider the best and the one we reported above, is what they define as “GARCH(1,1,)-Poisson-Gaussian model with time-varying intensity for jumps”. The last thing to say about this model is that, even though the authors didn’t find significance in the addition of the intensity of jumps that varies with time, they still consider it important for the comprehension of other characteristics of the electricity market.

2.1.2. Regime switching models

The jump-diffusion model can interpret very well the jumps occurring in the nature of the electricity prices but, this could have a negative aspect according to Eydeland and Wolyniec (2003). More precisely, the two authors say that this approach requires the introduction of a strong mean reversion component to bring down the price and to transform that jump in a spike, and this could affect the diffusive part of the model, giving biased parameters. For this reason, regime-switching models are introduced, bringing an element that distinguishes the state in which the system is and apply different mean reversions according to it. In a regime-switching model, there are several regimes that the price series goes through and, there is a certain probability for the process to cross from one regime (or state) to the other one. Eydeland and Wolyniec (2003) suggest, as an example, the following process with two states for the return of the prices

$$dS = \begin{cases} \mu_L dt + \sigma_L dW_L \longrightarrow P_L = 1 - \lambda_{LU} dt \\ \mu_0 + \mu_U dt + \sigma_U dW_U \longrightarrow P_U = 1 - \lambda_{UL} dt \end{cases}. \tag{2.19}$$

The transition matrix shows the probabilities of switching from one regime to the other one:

$$P = \begin{pmatrix} 1 - \lambda_{LU} dt & \lambda_{LU} dt \\ \lambda_{UL} dt & 1 - \lambda_{UL} dt \end{pmatrix}. \tag{2.20}$$

The probability of remaining in a given state is $P_L = 1 - \lambda_{LU} dt$, or $P_U = 1 - \lambda_{UL} dt$. Instead, the probability of changing the given state and switching to the other is λ_{UL} or λ_{LU}, according to the state in which the process is. Summing up, the model is divided into two states, upper and lower, and the process stays or changes the state according to a certain probability. The two states have different drift and volatility and are based on the GBM. The advantage of this model is that we can choose the rate at which the upper state change to the lower one in a way that simulates the mean reversion, or in other words we can assign to λ_{UL} a higher value than λ_{LU}. With this method, we can obtain a model that can revert rapidly toward the lower state, in which the prices are lower, right after a sudden jump. One of the criticisms of the jump-diffusion model is that the element of the mean reversion is always present without discerning between a situation in which there is a spike or there isn't one. The regime-switching model allows solving this problem creating a model that can distinguish the different situations in which the process is.

Regime switching models can be different depending on weather one chooses 2 or 3 regimes, and on the stochastic process chosen for each regime. For example, in Bierbrauer, Trück and Weron (2005) is used a mean reverting process for the base regime and is suggested to use a Pareto process for the pike regime. The authors concluded their analysis saying that the Pareto distribution contributes to overestimate the number of extreme events, while normal and lognormal distribution underestimate them.

Since we have mentioned the possibility of having 3 regimes, it seems fair introducing the model of Janczura, Trück, Weron and Wolff (2013) who used a regime-switching model for deseasonalized spot prices with 3 states: the first regime is governed by a mean reversion process which account for heteroskedasticity and it is called base regime. The process, in continuous time, can be described by means of the following SDE

$$dX_{t,1} = \left(\alpha_1 - \beta_1 X_{t,1}\right)dt + \sigma_1 W_t \,, \tag{2.21}$$

while in discrete time is

$$X_{t,1} = \alpha_1 + (1 - \beta_1)X_{t-1,1} + \sigma_1 \varepsilon_t \,, \tag{2.22}$$

where β_1 is the speed of mean reversion, $W_t$is the Wiener process and $\varepsilon_t$is a white noise. The second regime is what is called the spike regime, that is the regime that is responsible for representing the typical spikes in the behaviour of the electricity spot prices. This regime can be expressed in this model with the shifted log-normal distribution

$$log\left(X_{t,2} - q_2\right) \sim N(\mu_2, \sigma_2^2), \qquad X_{t,2} > q_2 \,. \tag{2.23}$$

The third regime is used to bring down the prices after a sudden rise with a shifted inverse log-normal distribution

$$log\left(-X_{t,3} - q_3\right) \sim N(\mu_3, \sigma_3^2), \qquad X_{t,3} > q_3 \,. \tag{2.24}$$

The q_n's are arbitrary, become constants once estimated and assume the role of filters for the spikes in the dataset. Janczura et al. (2013) give a hint regarding the values one should choose: they estimated the third and first quartile for q_2 and q_3 respectively. This means that the values below the third quartile in the first case will not be treated as spikes, as well as the values above the first quartile for the first case.

It's concluded here the general review of some of the interesting stochastic models for electricity spot prices because there are other types of models that take into consideration more the environment of this commodity rather than only historical time series. In fact, the next section is dedicated to the production cost and fundamental equilibrium models which aim is to use demand and supply to find equilibrium prices.

2.2. Production cost and fundamental equilibrium models

In the last section, we reviewed some of the stochastic processes one can find in the literature and that are used to model electricity spot prices. Recall that a good model is useful to explain the behaviour of this commodity and therefore to find a fair value for the forward price. That is why we continue in this section to show other types of models that differ from the stochastic processes we have seen in the previous section. The first topic regards the production cost models, continuing then with the more interesting fundamental equilibrium models and reporting the contribution of Barlow that offered a first good alternative to the stochastic models studied earlier in this chapter.
Production cost model's objective is to minimize the production cost and satisfy the demand for power at the same time. In order to create such a model, one must have some information like the capacity of production and the demand for electricity. In other words, this kind of model doesn’t use market data but instead, tries to find the price drivers that influence electricity, starting from the demand and production capacity, as we have already said, adding to the model the cost of every single plant and finding the correlation with other commodities or the weather (Eydeland and Wolyniec, 2003). The authors continue saying that fundamental equilibrium models, on the other hand, aim to find and explain the connections between demand and supply, optimizing a certain function in order to find the right price. The difference with the production cost model is that this model now takes into consideration the market data. In any case, both models can’t capture the characteristics of power prices. It is possible to find a few works of this type in Bessembinder and Lemmon (2001), and in Supatgiat, Zhang and Birge (2001).
Aïd (2015) reports the main contribution to the side of the structural models for electricity spot prices, that is the work of Barlow (2002) who captured the spiky nature of the prices with a model including demand and supply. This is, of course, an equilibrium model where the spot price is determined by the balance between demand and supply.

Barlow's model (2002) is explained in Aïd (2015) as follows: a supply function $u_t(x)$ has to be defined as well as the demand function $d_t(x)$. In this case, the unknown variable x is actually the spot price S_t, and since this is an equilibrium model, it is necessary to set the supply equal to the demand as follows

$$u_t(S_t) = d_t(S_t)\,. \tag{2.25}$$

Supply and demand curves are obviously increasing and decreasing respectively as usual for these curves. In this case, a few assumptions are needed: the first one is that the supply function is a certain constant g and the second one is that the demand curve is rigid. Actually, with regards to the second assumption, it is known from the first chapter that electricity demand is inelastic because it is a necessary good and the quantity asked doesn't change too much when the price changes. Therefore, summing up the assumptions we obtain

$$\begin{cases} u_t = g \\ d_t(x) = D_t \end{cases}, \tag{2.26}$$

where D_t stands for the consumption of electricity. Now, it is possible to derive a first equation for the spot price

$$S_t = g^{-1}(D_t)\,, \tag{2.27}$$

where $g(x) = a_0 - b_0 x^{\alpha}$ with $\alpha < 0$.

If the demand is higher than a certain level a_0 corresponding to the maximum capacity, the price is upward limited. The situations that we can face are

$$S_t = \begin{cases} \left(\dfrac{a_0 - D_t}{b_0}\right)^{1/\alpha}, & D_t \le a_0 - \varepsilon_0 b_0 \\ \varepsilon_0^{1/\alpha}, & D_t \ge a_0 - \varepsilon_0 b_0 \end{cases}, \tag{2.28}$$

where ε_0 is defined according to a certain maximum price.

The explanation of the model continues with a simplification, obtaining, in the end, a non-linear Ornstein-Uhlenbeck process:

$$S_t = \begin{cases} (1+\alpha X_t)^{1/\alpha}, & 1+\alpha X_t \leq \varepsilon_0 \\ \varepsilon_0^{1/\alpha}, & 1+\alpha X_t \geq \varepsilon_0 \end{cases}. \tag{2.29}$$

$$dX_t = -\lambda(X_t - a)dt + \sigma dW_t\,. \tag{2.30}$$

This model was a first good alternative to stochastic models, but it showed a relevant drawback found by Carmona and Coulon (2013): the two authors noticed that Barlow's model tends to simulate a lot of spikes at the same level of the price cap, as resumed in Aïd (2015). Still, Aïd (2015) suggests a few successive works aiming to improve Barlow's model: Kanamura and Ohashi (2007) and Kanamura (2009).

So far, we have seen stochastic models in contraposition to equilibrium models, but it is also true that someone tried to combine these two models to obtain different better results. This combination gave the so-called hybrid models which we are going to treat in the next and last section of this chapter, completing the review of the models for power spot prices.

2.3. Hybrid models

The last part of this chapter is focused on the hybrid models for the energy spot prices and it concludes this brief summary of the models studied in the literature. In the previous sections of this chapter are reported stochastic and equilibrium models, but now it's time to tell how the researchers tried to combine the best of the two techniques. This final part is divided into two sections where firstly we'll talk about reduced form hybrid models (2.3.1) and lastly fundamental hybrid models are explained. Hybrid models, as the name suggests, combine fundamental models and stochastic models in order to obtain the positive aspects of both the techniques examined before. In other words, the capacity to explain the relationship between supply and demand is unified to the evolution of those variables captured by the market data. This combination gives more information to use but it also increases the number of parameter estimations (Eydeland and Wolyniec, 2003).

2.3.1. Reduced form hybrid models

This type of model simply takes stochastic models introduced in this chapter and adds an element in order to include information not related to the prices. As an example, Eydeland and Wolyniec (2003) propose a jump-diffusion model where the volatility is explained by a stochastic model. In this model, the jumps are a function of the volatility or, in other words, the likelihood of having spikes is linked to market circumstances. It is well known that if there is hot weather, one can see the activation of more expensive plants to generate energy (remember the stack function in chapter 1). This can be viewed as a direct connection between spikes and temperature, a connection that can be used to correct the jump-diffusion model and to obtain a model in which the jump rate depends on the temperature. With the introduction of the temperature, one obtains a hybrid model with more parameters to estimate but if one looks closer, he/she can find the advantage of this type of model. It is true that the number of parameters that have to be estimated now is increased but, there isn't anymore the necessity to estimate the volatility process of the price data.

In Aïd (2015) one finds that Barlow's model has been improved with the addition of several parameters, like the fuel prices. Therefore, it is obtained a model based on statistical elements, such as the relation between fuel prices and energy prices that are put in relation to the microeconomic elements. We already know that electricity can be obtained from gas or coal, for example, and it seems reasonable considering this element in such a model. Aïd (2015) mentions the work of Pirrong and Jermakyan (2001) who developed a model with two state variables, demand and fuel prices, obtaining the following equation for the electricity prices

$$dS_t = \varphi(q_t, f_t, t)dt + S_q\sigma_q q_t du_t + S_f\sigma_f f_t dz_t \,, \qquad (2.31)$$

with

$$\varphi(q_t, f_t, t) = S_q\alpha_q(q_t, t)q_t + S_f\alpha_f(f_t, t)f_t + \frac{1}{2}S_{qq}\sigma_q^2 q_t^2 + \frac{1}{2}S_{ff}\sigma_f^2 f_t^2 + S_{qf}q_t f_t \sigma_q \sigma_f \rho_{qf}\ , \tag{2.32}$$

where ρ_{qf} is the correlation between electricity demand and fuel prices.

Aïd (2015) shows another example of a model that isn't based on microeconomic principle but rather on statistical relations that links energy prices with other elements. The author briefly explains the model proposed in Cartea and Villaplana (2008) which is another equilibrium model using demand and capacity: thus, the power prices, in this context, are formed according to the following equation:

$$S_t = \beta e^{(\gamma C_t + \alpha D_t)}\ , \tag{2.33}$$

where D is the power demand, C is the capacity, α and β > 0 and γ < 0.

Demand and capacity dynamics are the followings

$$\begin{cases} D_t = g_t^D + X_t^D \\ C_t = g_t^C + X_t^C \end{cases}, \tag{2.34}$$

Where g_t^D and g_t^C are deterministic seasonal components, while X_t^D and X_t^C are mean reverting processes with the following dynamics

$$\begin{cases} dX_t^D = -k^D X_t^D + \sigma_t^D dW_t^D \\ dX_t^C = -k^C X_t^C + \sigma_t^C dW_t^C \end{cases}, \tag{2.35}$$

where W^D and W^C are independent Wiener processes, while k^D and k^C are speed of mean reversion.

Other works of this kind can be found in Benmenzer, Gobet, Jérusalem (2007), De Jong and Schneider (2009), Frikha and Lemaire (2013), Grine and Diko (2010), Lyle and Helliot

(2009).

2.3.2. Fundamental hybrid models

One of the critics made by Eydeland and Wolyniec (2003) to the reduce-hybrid models is that the low amount of historical data gives unstable results. For this reason, they introduce a fundamental hybrid model that relies on the underlying drivers of the prices, giving a natural representation of the real market. As shown in Figure 2.1 the two authors define the cost of the fuel and influence of the outages as determinants of the supply of energy.

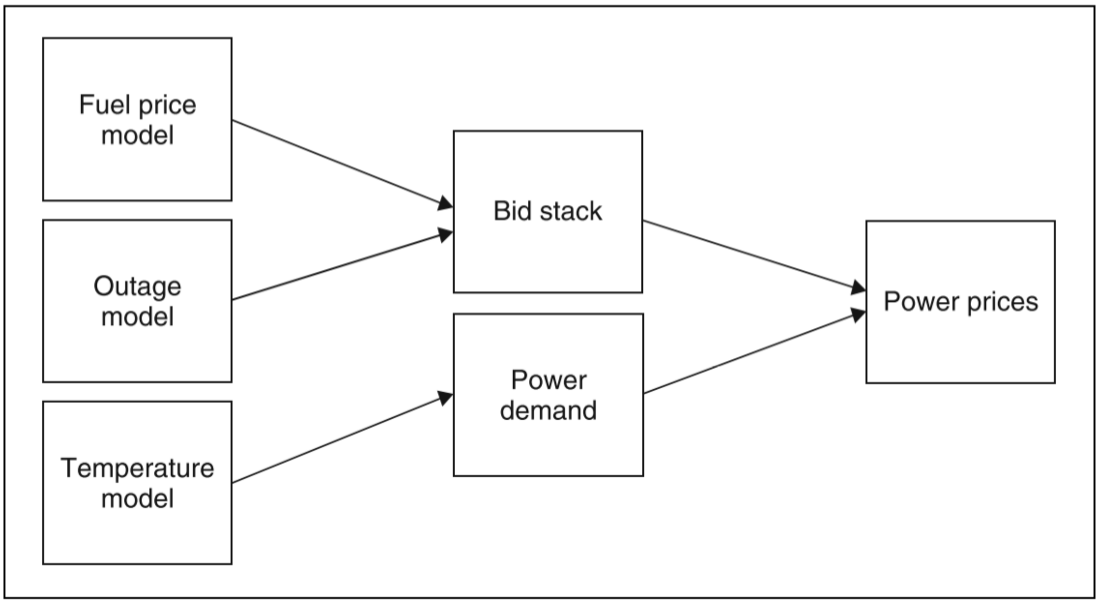

Figure 2.1: Hybrid model for power prices (Eydeland and Wolyniec, 2003).

The temperature is used as a driver for the demand, but it is suggested to use every parameter linked to meteorological elements, such as the humidity. Once obtained a suitable model explaining the historical data of each driver, it is possible to merge the results in order to find the final model for the power prices. The equation for the price provided by Eydeland and Wolyniec (2003) seems simple and is presented by the authors as follows

$$S_t = s^{bid}(D_t) \tag{2.36}$$

where S_t is the electricity price in a certain instant, D_t is the demand for electricity and $s^{bid}(D)$ is the bid stack function, or the supply function.

With the help of Figure 2.1, it is possible to understand that equation (2.36) is not so simple because it is necessary to obtain an estimation for every driver of the demand and supply. We preferred to do not write every variable of the extended equation (2.36) because the reading would become tedious and impossible to remember. Anyway, the peculiarity of this model is that historical price data aren't used in this case to calibrate the model, but only to test it, while market data are used for the calibration. The fundamental hybrid model presented by the two authors is based on structural features of the market, the supply and demand, easily allowing for projections of different changes in the supply framework or from the demand side (Eydeland and Wolyniec, 2003).

In this chapter, it's shown how it is possible to model electricity spot prices with different methods. We explained that simple models such as the Geometric Brownian Motion are only the basis from which all the stochastic models studied in this field started. In fact, first attempts have been made with models based on Ornstein-Uhlenbeck mean reverting processes that contain a GBM component. These models were probably too simple and for this reason, some researchers took a look to past also in this case and decided to use the jump processes introduced by Merton. As a consequence, the mean reverting processes have been modified by adding a jump component to model in a better way the typical spikes observable in the time series of power prices. Since someone noticed the presence of dependence in the volatility structure of the power prices, more complex models took place with the modelling of the stochastic volatility that was placed to the side of mean reverting jump diffusion processes. Some researchers tried a different approach, the regime switching model, that is a model built to divide into 2 or more regimes the evolution of the prices, assigning different processes to each regime. On the other side of the stochastic models, there are the production cost and fundamental models which try to study the spot price from the point of view of the microeconomics variables affecting electricity price movements. These models are simply based on the research of the equilibrium between demand and supply, requiring a lot of information. First, it has to be known the supply curve and the costs to generate electricity in a certain region at each level of demand. Secondly, one needs to know the consumptions of electricity of that region, that vary depending on the part the world that region is, on the seasons on the day of the week or on the hour of the day we are looking at. Someone tried also to combine stochastic models and fundamental models obtaining a new concept that uses stochastic modelling on the drivers of the electricity drivers, rather than using only time series of the prices. These drivers could be the price of natural gas, the temperature or the amount of rain that falls every year in the region we are analysing, for example.

In the end, we have seen though that none of these models prevail the others and that all of them have their strengths and their weaknesses. Anyway, all the models showed have one common goal, that is creating a tool to find a fair price for a forward contract on electricity. This chapter is indeed the second step of our path that continues in the next chapter which aims is to explain the relationship between spot and forward prices and to study the ways to obtain the forward price of a contract with electricity as the underlying asset.

Chapter III. Forward prices for electricity

In the second chapter of this thesis, we resumed the variegated world of the models for the electricity spot price and we went from the simplest stochastic processes to the most complicated, like the regime-switching models with three regimes. We have seen not only pure stochastic approaches but also equilibrium models based on the characteristics of a market with its own peculiarities. All of those models have their strengths and their drawbacks and none of them prevails on the others but, all of them can be useful to the final purpose of giving information about what will possibly be the spot price at a certain time in the future.

In this chapter, we aim to explain how the models we studied before can be used for the calculation of a fair forward price of a certain contract. It may be not clear the connection between the spot price and the futures price and this is why this chapter starts talking about the relation between these two elements (3.1). First of all, it is necessary to review the theory regarding the process to obtain a forward price for a certain derivative. After this first part, we introduce an alternative to the classical no-arbitrage theory implemented for the assets and storable commodities for the calculation of the futures price. Indeed, we will explain the risk premium approach (3.2) providing an example of its application. In the last part (3.3) we conclude with a different way to calculate the forward price through the direct modelling of the forward curve instead of the use of stochastic processes for the spot price.

3.1 Relation between forward and spot price: preliminary remarks

We start this chapter with some preliminary remarks to explain how a general forward (futures)[18] price is determined, in order to reveal the difficulties in doing so if the

[18] In this chapter we treat forward and futures prices as they are the same thing, even though these two types of contract are slightly different.

underlying asset is the electricity.

In Hull (2012) one find a very clear way to treat a likely troublesome topic such this. First of all the authors starts saying that, for simplicity, we have to state several assumptions regarding the actors who operate in the market, in particular: there aren't transaction costs for the trades that take place in the market, there is the same tax rate for everyone, all the investors can borrow money at a certain risk-free rate that is equal for everyone and last, If an arbitrage opportunity occurs, the investors immediately seize the opportunity. Remember that these statements are made to simplify the problem of deriving a forward price of a certain contract, with a certain underlying asset while the reality is much more complex. It isn't necessary though that all these assumptions are verified for all the market participants but, it is enough that they are true for the most important investors of the market. The actions they operate in the market are sufficient to explain the relationship between forward and spot prices.

We consider now a very simple forward contract for the delivery of an asset that doesn't give any income. In a situation with absence of arbitrage opportunities, the relation between the forward price and the spot price is expressed as follows

$$F_{0,T} = S_0 e^{rT} , \tag{3.1}$$

where $F_{0,T}$ is the forward price at time $t = 0$, S_0 is the spot price of the underlying asset at time $t = 0$, r is the annual zero-coupon risk-free interest rate and T is the time until the expiration date. In words the forward price is the spot price capitalized till the expiring date at the interest risk-free rate.

In practice, equation (3.1) tells us that if the market offers a different price for the forward contract there is an arbitrage opportunity: if the forward price is higher than $S_0 e^{rT}$ an investor can buy the asset and short the forward contract related to the same asset. On the contrary, if it is lower, the investor should short the asset and assume a long position on the forward contract.

It is very important for every investor to check the value of the assets in its portfolio, as well as the value of the derivatives the same market participant subscribed. For this reason, we introduce now the equation to find the value of a long forward contract in $t = 0$:

$$f_{0,T} = (F_0 - K)e^{-rT} \, , \tag{3.2}$$

The value of a forward contract should be zero at the moment in which is written, meaning that the value K, the delivery price that will be paid at the maturity, is equal to F_0, that is calculated with equation (3.1). The delivery price K is constant because it is set at the beginning of the contract, while the value of the contract f changes according to the changes of the forward price that happens between $t = 0$ and the expiration date T. In words, the value of the contract in $t = 0$ is the payoff $F_0 - K$ actualised at the continuously compounded risk-free rate. If the contract is priced correctly $F_0 - K$ is equal to zero, thus the value of the contract is zero.

Since we already know how to obtain the forward price from equation (3.1), it is possible then to rewrite the value of the forward contract substituting equation (3.1) in equation (3.2), obtaining

$$f_{0,T} = S_0 - Ke^{-rT} \, , \tag{3.3}$$

It' been obtained again the value of the forward contract today, that is in $t = 0$, written in a different way from equation (3.2).

A more general way to write equation (3.3) in order to evaluate the value of a forward contract at any time of the contract's life is

$$f_{t,T} = S_t - Ke^{-r(T-t)} \, , \tag{3.37}$$

where t is the time in which we want to calculate the value of the forward contract, T is the expiration date, and $(T - t)$ is the time to maturity.

Now that the procedure to calculate the forward price for an asset is known, it is possible to move on and introduce the case in which we have to deal with a commodity. As we have seen, when one can take advantage of an arbitrage opportunity, it would be required by the strategy involved to buy and hold the asset underlying the derivative contract. In the case of a commodity (gold, silver, copper, etc.), this strategy makes arise storage costs that can be considered as a negative income. Thus, we rewrite the equation for the forward price, taking into consideration the present value of the storage costs U:

$$F_{0,T} = (S_0 + U)e^{rT} \, , \tag{3.5}$$

In some cases, a commodity can give an income, so U has to be considered as storage costs, already netted from any possible income. In the case of proportionality of the storage costs in relation to the commodity price, the equation above becomes

$$F_{0,T} = S_0 e^{(r+u)T} \, , \tag{3.6}$$

Now, storage costs aren't anymore treated as a negative income but are considered as a negative yield.

When we deal instead with consumption commodities, we have to take into consideration that this type of asset can be useful to the owner by using the commodity in some way. A producer of a certain good, feels to have a benefit in keeping that good instead of a forward contract because the commodity may help the continuity of the production and facing unexpected shortfalls of the raw materials. To be clearer, Hull (2012) makes an example: for an oil refiner, writing a forward contract on crude oil is not the same as having real crude oil available for the production and to keep the inventory at a safe level. Thus, holding an asset is perceived as an advantage or a benefit that we can call convenience yield:

$$F_{0,T} = S_0 e^{(r+u-y)T} \, , \tag{3.7}$$

Under the no-arbitrage condition, the convenience yield y must be zero if the investment is not a consumption commodity, but rather an investment asset. Hull (2002) explains very well in words what is the convenience yield: "*The convenience yield reflects the market's expectations concerning the future availability of the commodity. The greater the possibility that shortages will occur, the higher the convenience yield*". Therefore, now we know that the forward price is linked also to the expectations of the investors with regards to the quantity of the good that will be available in the future.

The author continues saying that If one sum the risk-free interest rate and the storage costs, he/she obtains a new element that is called cost of carry (we've talked about the cost of carry in the first chapter where we defined it as the cost the investor bears to bring the asset into the future)

$$c = r + u \,, \tag{3.838}$$

therefore, equation (3.7) becomes

$$F_{0,T} = S_0 e^{(c-y)T} \,. \tag{3.9}$$

One important aspect that has to be taken into consideration is the time to delivery of the asset. With commodities is not unusual to have more than one delivery date or a continuous delivering in a period (for example a week or a month) specified by the contract (Burger at al., 2007). In the former case, we can consider every delivery date as it is linked to a single forward contract, then we sum the values of each contract,

$$f_{t,T_1,T_n} = \sum_{i=1}^{n} (F_{t,T_i} - K) e^{-r(T_i - t)} \,. \tag{3.10}$$

Equation 3.10 simply divide a futures contract with multiple delivery dates between T_1 and T_n in a sequence of futures contracts with different delivery dates going from T_1 to T_n. The sum of these contracts gives the value of the whole single contract. Now, one can obtaine the fair price for this contract, that is the value of K that makes f_{t,T_1,T_n} equal to zero:

$$K = \frac{\sum_{i=1}^{n} F(t, T_i) e^{-r(T_i - t)}}{\sum_{i=1}^{n} e^{-r(T_i - t)}} \,. \tag{3.11}$$

One can do the same for the second case where there is a continuous time delivering, that is the case regarding usually electricity contracts. The equation (3.10) and (3.11) are transformed now in integrals because we have a function which is continuous in time and defined in the interval (T_1, T_2), where T_1 is the first delivery date and T_2 is the last delivery date, assuming a constant delivery of electricity during this period:

$$f_{t,T_1,T_2} = \int_{T_2}^{T_2} (F_{t,T} - K)e^{-r(T-t)} dT \, , \tag{3.12}$$

$$K = \frac{\int_{T_1}^{T_2} (F_{t,T} - K)e^{-r(T-t)} dT}{\int_{T_1}^{T_2} (F_{t,T} - K)e^{-r(T-t)} dT} . \tag{3.13}$$

With the information provided in the first chapter, one can recognize that this relation between forward and spot price doesn’t hold in the case of electricity. We have seen that the cost of carry is the cost of carrying on into the future the underlying asset, but electricity is not storable, thus none of the strategies implied by the no-arbitrage theory can be applied. Moreover, the convenience yield that represents the expectation about the future availability of the underlying asset, cannot be directly observed, so it is difficult also for a storable commodity to define exactly the convenience yield. In the case of electricity, this is even more difficult due to the non-storability of power (Aïd, 2015).

3.1.1 Relation between forward and expected spot price

Once derived the forward price, the chapter can go on explaining the relation between the forward price and the expected spot price, but before doing this, some explanations about the risk we face when we take futures position are required. Hence, assume now a situation in which an investor takes the following actions: in time $t = 0$ the investor takes a long position in a forward contract with delivery date in T; at the same time the same investor invests at a risk-free interest rate an amount equal to the present value of the forward price F_0. At the delivery date T the earnings of the investment are used to buy the asset underlying the forward contract and finally, the investor sells the asset at the market price S_T. Now, is likely this market player wants to evaluate this investment, therefore, the cashflows of this operation can be summarized as follows: today the investor has a negative cash flow equal to $-F_0 e^{-rT}$; whereas on the delivery day the investor will have a positive cash flow equal to $+S_T$ (Hull, 2012).

If the securities used in this process are priced correctly, we can now proceed to obtain the present value of the cash flows we have just seen:

$$-F_0 e^{-rT} + E_t(S_T) e^{-kT} = 0 \,, \tag{3.14}$$

where $E_t(S_T)$ is the expected spot price (since we don't know now what will be exactly the spot price) on the delivery date T given the information provided in time t; k is the return required by the investor and depends on the systematic risk[19] of the investment. We rewrite the equation above in a more useful form

$$F_0 = E(S_T) e^{(r-k)T} \,, \tag{3.15}$$

Now, it is clear the connection between the futures price and the expected spot price. What can be noticed is also that if the investment's return is uncorrelated with the market's return, the return required by the investor is equal to risk-free interest rate, giving that the forward price is equal to the expected spot price:

$$F_0 = E(S_T) \,. \tag{3.16}$$

This is the result we would obtain in a risk-neutral world.

If the return has instead a positive correlation, we'll have $k > r$ and therefore the forward price we'll be below the expected spot price. The last case is the one in which the return has negative correlation obtaining $k < r$ and F_0 higher than $E(S_T)$. Keynes (1930) and Hicks (1939) observed that futures prices and expected spot prices could be

[19] The systematic risk is the risk that cannot be diversified away coming from the correlation of a certain asset's return and the market's return. If the correlation of this asset with the market is positive, an investor asks for a higher return than the risk-free interest rate on the investment. On the opposite case, the investor requires a lower return if the correlation is negative.

respectively in different points according to the positions held by the different market participants, speculators and hedgers. It is possible to see a situation in which the expected spot price stays above the forward price when hedgers are holding a short position and speculators are holding a long position. The forward price is below the expected spot price because speculators require a premium for the risk they are accepting. The inverse situation is also possible, and in this case, the positions of the two type of investors will be inverted and the expected spot price will be below the future price (Hull, 2012).

It is clear now why we had to introduce stochastic models for electricity price in the second chapter: they can be used to understand what the spot price will be at a certain moment in the future allowing us to determine the forward price. The next section analyses in depth the relation between the forward price, the risk premium, the market price of risk and the market power.

3.2 Risk premium, market price of risk and market power

Equity forwards are very simple to price because under no-arbitrage argument it is possible to borrow the amount necessary to buy the underlying asset and keeping it until the delivery date. In the case of the commodities is not that simple because the cost of carry is hardly measurable as well as the convenience yield, assuming that we are talking about storable commodities (Benth, Cartea and Kiesel, 2007).

Giving the non-feasibility of the arbitrage-free pricing theory, when one has to deal with non-storable commodities, something else has been tried to understand how marker players' behaviour changes when they face risks in these markets. In fact, the shape of the forward curves has been studied, and from the observations, it's known that when there is a situation of backwardation it is likely that the market is facing a supply deficiency with respect to the demand. The opposite happens, instead, when a situation of contango is observed, that is when the spot price is above the forward price. If this is true, it is easy to understand that forward contracts with long expiring date are usually in backwardation, whilst they are in contango when the expiring date is close (Benth, Cartea and Kiesel, 2007).

According to Benth, Cartea and Kiesel (2007), there is a relevant measure that links the forward price of a contract to the expected spot price of the underlying: the market risk premium

$$\pi(t,T) = F(t,T) - E^P[S(T)|\, \mathcal{F}_t]\,, \tag{3.17}$$

where $E^P[]$ is the expected value under the historical measure P, giving the information provided till time t. They studied connections between the market risk premium, the market price of risk and the risk preference of the market actors in order to explain the relation showed in equation (3.17).

When investors enter in a forward contract to diversify their risk, and also in the case of electricity, one can observe the willing of the producers to decrease the uncertainty of their earnings, as though consumers want to protect themselves from price risk. Benth et al. (2007) make notice that these two players, producers and consumers, have different time horizons when arises the necessity to diversify their risk, as well as different risk aversions. In fact, the producer plans to stay in the business for a long time with a longer exposition to the profit's variability, while the consumer has a really short time horizon. The authors continue saying that is exactly this difference that justifies the risk premium and if it is positive or negative. Thus, when the result of equation (3.17) is greater than zero, consumers are more willing to hedge their risk then the producers are. On the other hand, when it is negative, is verified the opposite situation and the producers want to cover their exposure more than the consumers.

Benth, Cartea and Kiesel (2007) start their study about the formation of the forward prices through a utility curve for producers and consumers: this utility should describe the preference of these two actors regarding how much of their production or consumption sell or buy respectively in the forward market or in the spot market. The first step is to find the forward price for both actors that makes the choice of forward price and spot price absolutely indifferent for them. The next step is to study how the creation of the forward price is affected by the different desires of protection of the two actors. Thus, we introduce now the utility function proposed by the authors:

$$U(x) = 1 - e^{-\gamma x}\,, \tag{3.18}$$

In equation (3.18) γ is the risk aversion of the market participants and it will become γ_p and γ_c when it will refer to producers and consumers, respectively. Using this equation, the authors aim to find two bounds: an upper bound defining the highest price consumers are willing to pay in the forward market before becoming indifferent between spot and forward market; the lower bound will be the lowest price producers are willing to pay in the forward market before becoming indifferent between the two markets. Between the bounds defined above, according to the authors, there is a set of possible forward prices for the market's equilibrium. The final objective is to find out what is the market forward price that lies in this set.

For their study, Benth, Cartea and Kiesel (2007), use a mean-reverting multi-factor process as a model for the electricity spot price, as the one shown in Lucia and Schwartz (2002) that we mentioned in the second chapter of this thesis:

$$S_t = \Lambda(t) + \sum_{i=1}^{m} X_i(t) + \sum_{i=1}^{n} Y_i(t)\,, \tag{3.19}$$

where $\Lambda(t)$ is the seasonal trend function and $X_i(t)$, $Y_i(t)$ are the solutions of the following SDE

$$dX_i(t) = -\alpha_i X_i(t)dt + \sigma_i(t)dW_i(t)\,, \tag{3.20}$$

$$dY_j(t) = -\beta_j Y_j(t)dt + dL_i(t)\,, \tag{3.21}$$

where $W(t)$ is a standard Brownian motion and $L(t)$ is a Lévy process.[20] It's not our goal discussing here the stochastic model above, since we are more interested now in the process to obtain the forward price, thus for more information see Benth, Cartea and Kiesel (2007). The very same authors derive the forward price for the producers, or

[20] A Lévy process is a stochastic process with independent, identically distributed random increments. An example of this type is the Wiener process.

better, the price that makes them indifferent:

$$F_{pr}(t, T_1, T_2) = -\frac{1}{\gamma_p}\frac{1}{T_2 - T_1} lnE^P\left[\exp\left(-\gamma_p \int_{T_1}^{T_2} S(u)du\right)|\mathcal{F}_t\right], \qquad (3.22)$$

where the risk-free interest rate is assumed to be zero aiming to make all the derivation simpler. In equation (3.22), $\int_{T_1}^{T_2} S(u)du$ is the earning of the producer coming from the sale of the commodity in the spot market between T_1 and T_2. On the other hand, the producer would receive $(T_2 - T_1)F_{pr}(t, T_1, T_2)$ if the commodity was sold on the forward market.

A similar result is obtained for the indifference price of the consumer:

$$F_c(t, T_1, T_2) = -\frac{1}{\gamma_c}\frac{1}{T_2 - T_1} lnE^P\left[\exp\left(-\gamma_c \int_{T_1}^{T_2} S(u)du\right)| \mathcal{F}_t\right]. \qquad (3.23)$$

The only difference is that in this case, we have the risk aversion of the consumer (γ_c) instead of the risk aversion of the producer (γ_p).

Now there are the elements to define the upper and lower bounds for the market forward price as previously stated:

$$F_{pr}(t, T_1, T_2) \leq F(t, T_1, T_2) \leq F_c(t, T_1, T_2)\,. \qquad (3.24)$$

In this context, the consumer wants to buy in the spot market if the forward price is higher than the indifference price $F_c(t, T_1, T_2)$, while the producer wants to sell the commodity in the forward market if the forward price is higher than the indifference forward price $F_{pr}(t, T_1, T_2)$.

Suppose now that the risk aversion decreases to zero for both the market's participants, obtaining in this way the following equation

$$\lim_{\gamma_{p,c}\to 0} F_{pr,c}(t, T_1, T_2) = E^P\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(u)du \mid \mathcal{F}_t\right]. \qquad (3.25)$$

This tells that the now the forward price for the producer is the expected value of the proceeds obtained from the sale of the commodity in the spot market in the period $[T_1, T_2]$.

The same result is obviously obtained for the consumer when the risk aversion goes to zero, as shown in equation (3.25). We rewrite now equation (3.24) with the information provided in equation (3.25):

$$F_{pr}(t, T_1, T_2) \leq E^P\left[\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(u)du \mid S(t)\right] \leq F_c(t, T_1, T_2)\,. \qquad (3.26)$$

All the proofs of the inequalities shown so far are provided by Benth et al. (2007).

Inequality (3.24) shows the range in which lies the forward market price that producers and consumers can and are willing to pay. The determination of the fair price depends on the demand and supply equilibrium like every other product. In order to ease this last part of the reasoning, recall the utility function we presented previously and that the actors in this market have different time horizons. Imagine that our market's producers want to cover themselves from price risks but, there just a few consumers willing to cover their risks in the long run. Now, we say that the consumers have "*market power*" because they can push down the price of the forward market price.

We formalize this new concept defining the "*market power of the representative producer*" as $p(t, T_1, T_2) \in [0,1]$: it is a number between zero and one, that explains how much power the producer has in the market, with regards to a forward price of a contract with delivery date between T_1 and T_2, at the time of evaluation t. If the market power is equal to 1, then the producer has an absolute power to impose a certain price, while it is equal to zero if the power is inexistent (Benth, Cartea and Kiesel, 2007).

This is the last missing piece to obtain the market forward price, defined as follows

$$\begin{aligned} F^p(t, T_1, T_2) = {} & p(t, T_1, T_2)F_c(t, T_1, T_2) \\ & + \big(1 - p(t, T_1, T_2)\big)F_{pr}(t, T_1, T_2)\,, \end{aligned} \qquad (3.27)$$

where $(1 - p)$ is the market power of the consumers.

In words, the price defined in equation (3.27) is the price the market's participants will pay based on the equilibrium reached in the market between demand and supply, and moreover, it is one of the prices in the range of the possible prices bounded by the indifference prices of the market's actors. The authors conclude this part saying that in this case, they used a constant value for the market power to make the explanation easier but, it could be stochastic and, the authors found evidence of a term structure for this element.

With the help again of Benth et al. (2007) it is highlighted below the relation between the market risk premium, the market power and the risk aversion of the participants with a simple example: suppose one wants to find the forward price of a contract with a single delivery date and the spot price is represented by the following a mean-reverting process, like the one we introduced in the second chapter

$$dS(t) = \big(\mu - \alpha S(t)\big)dt + \sigma dW(t)\ , \tag{3.28}$$

where μ is the drift, α is positive, σ is the volatility that assumes values above or equal to zero, and last $W(t)$ is the Wiener process. The SDE in equation (.328) can be solved and the result is

$$\begin{aligned} S(T) = S(t)e^{-\alpha(T-t)} + \frac{\mu}{\alpha}(1 - e^{-\alpha(T-t)} \\ + \sigma \int_t^T e^{-\alpha(T-t)}\, dW(s)\ . \end{aligned} \tag{3.28}$$

We explained before how to find the indifference forward price for the consumer and for the producer by using their utility curves and now the process is the same. Therefore, the indifference price for the consumer is

$$F_c^{\gamma_c}(t,T) = \frac{1}{\gamma_c} E^P\big[\exp\big(\gamma_c S(T)\big) \mid \mathcal{F}_t\big]\ , \tag{3.29}$$

and the price for the producer is

$$F_p^{\gamma_p}(t,T) = -\frac{1}{\gamma_p} E^P\left[\exp\left(-\gamma_p S(T)\right) \mid \mathcal{F}_t\right]. \tag{3.30}$$

Simple operations bring the following explicit solutions for the forward prices:

$$F_c^{\gamma_c}(t,T) = S(t)e^{-\alpha(T-t)} + \frac{\mu}{\alpha}\left(1 - e^{-\alpha(T-t)}\right) \\ + \gamma_c \frac{\sigma^2}{4\alpha}\left(1 - e^{-2\alpha(T-t)}\right), \tag{3.31}$$

for the consumer, while the price for the producer is

$$F_{pr}^{\gamma_p}(t,T) = S(t)e^{-\alpha(T-t)} + \frac{\mu}{\alpha}\left(1 - e^{-\alpha(T-t)}\right) \\ + \gamma_p \frac{\sigma^2}{4\alpha}\left(1 - e^{-2\alpha(T-t)}\right). \tag{3.32}$$

These equations represent the indifference prices for the market's actors and now we have to find the market price as we showed earlier in this chapter. Recall the idea behind the power market p and applying what learnt in this chapter, one can find the forward market price

$$F^p(t,T) = p(t,T)F_c^{\gamma_c}(t,T) + \left(1 - p(t,T)\right)F_{pr}^{\gamma_p}(t,T)\,. \tag{3.33}$$

Substituting the forward prices for the consumer and the producer in equation (3.33) one obtains

$$F^p(t,T) = S(t)e^{-\alpha(T-t)} + \frac{\mu}{\alpha}\left(1 - e^{-\alpha(T-t)}\right) \\ - p(t,T)\gamma_p \frac{\sigma^2}{4\alpha}\left(1 - e^{-2\alpha(T-t)}\right) \\ + \left(1 - p(t,T)\right)\gamma_c \frac{\sigma^2}{4\alpha}\left(1 - e^{-2\alpha(T-t)}\right). \tag{3.34}$$

The market risk premium, in this case, presents itself as follows

$$\pi(t,T) = \left(\gamma_c - p(t,T)\left(\gamma_p + \gamma_c\right)\right)\frac{\sigma^2}{4\alpha}\left(1 - e^{-2\alpha(T-t)}\right). \tag{3.35}$$

Notice that the sign of the risk premium depends on the sign of $\left(\gamma_c - p(t,T)\left(\gamma_p + \gamma_c\right)\right)$ obtaining a positive risk premium when $p(t,T)$ is less than $\gamma_c/\left(\gamma_p + \gamma_c\right)$ for example. Equation (3.35) can be written in a different way to highlight something relevant:

$$\begin{aligned} F^p(t,T) = \frac{\mu}{\alpha} + \frac{\sigma^2}{4\alpha}\left(p(t,T)\left(\gamma_p + \gamma_c\right) - \gamma_p\right) + \left(S(t) - \frac{\mu}{\alpha}\right) e^{-\alpha(T-t)} \\ - \left(p(t,T)\left(\gamma_p + \gamma_c\right) - \gamma_p\right) e^{-2\alpha(T-t)}\,. \end{aligned} \tag{3.36}$$

Notice that $F^p(t,T)$ is made of a constant, a slow mean reversion level $e^{-\alpha(T-t)}$ and a fast mean reversion level $e^{-2\alpha(T-t)}$. Furthermore, suppose that market power is constant and less than $\gamma_p/\left(\gamma_p + \gamma_c\right)$, then the last element of equation (3.36) is exponentially increasing towards zero creating a hump in the final part of the forward curve (Benth et al., 2007). The authors provide also a more complicated example with the stochastic processes introduced at the beginning of this section. In this case, they used a measure of risk for the jump component of the process and one for the Brownian motion in the evaluation of continuous time delivery. Anyway, the results are similar to the ones showed above, thus the relations between market power, risk aversion and risk premium holds in the same way.

Due to the incapability of the storage theory to price forward contracts on non-storable commodities, researchers had to rely on the risk premium approach. In this section, is explained the idea behind this technique with the help of Benth et al., 2007 but we can find other studies with the same aim. Recall that in the second chapter of this thesis we explained the different typologies of processes for electricity spot price; among them, there are also equilibrium models based on the characteristics of a market rather than be based on an analysis of historical data. Bessembinder and Lemmon (2002) used their equilibrium model to find the forward price of a contract under the risk premium approach obtaining results not so different from the conclusions shown in this section. Moreover, Longstaff and Wang (2004) continued on the same path of Bessembinder and Lemmon (2002) giving one more proof to their results. Other works of this type are Botterud et al. (2002) on the Nordic market, Huisman and Kilic (2012) as well, or Redl and Bunn (2011) for the German market.[21]

3.3 Forward curve modelling

Eydeland and Wolyniec (2003) assert that the Black model is one of the most used to price certain derivatives and this is built as a Geometric Brownian Motion where the drift is null[22]:

$$\frac{dF_{t,T}}{F_{t,T}} = \sigma dW_t \, , \qquad t_0 \leq t \leq T \, , \tag{3.37}$$

where t is the time of evaluation and T is the delivery date.

Equation (3.38) is only the starting point due to the fact that it has some limitations. However, this model doesn't take into consideration how the particular contract is related to other contracts (with different maturities on the same underlying). An investor should be interested in the evolution of the forward curve because in most of the cases, the cash flow generated is related to this curve. The studies that followed the implementation of equation (3.37) ended up in the field of the forward curve models and in particular with the model implemented by Heath, Jarrow-Morton (1992) (HJM). This procedure allows for the direct study of the term structure of a forward contract, instead of giving a model for a single particular contract (Eydeland and Wolyniec, 2003). Eydeland and Wolyniec (2003) suggest an example of a typical model based on the HJM methodology and is presented as follows

$$dF_{t,T} = \mu\left(t, T, F_{t,T}\right)dt + \sum_j \sigma_j\left(t, T, F_{t,T}\right)dW_t^j \, , \tag{3.38}$$

[21] For more information regarding the literature on risk premium approach and the pricing of forward contracts in general see Kännö (2014).

[22] At the beginning of chapter 2 you can find the GBM (equation 2.1). In the case me mentioned in this page $\mu = 0$.

with t between 0 and T. Equation (3.38) is the dynamic that describes the evolution of the forward curve. As can be seen, the difference with equation (3.37) is that now there is a drift term μ in addition to the oscillations created with the Wiener process and the volatility function that can be defined also as the deterministic shape function. We point the presence of the drift and recall that in the electricity spot time series we found evidence of the presence of a seasonal component. Well, there is the same characteristic for the forward curve. In the second chapter, we told that several authors introduced a seasonal trend in their models, and this is possible also when we model forward curves, but this requires the estimation of more elements. This wouldn't be a problem usually, but in this case, there isn't the huge amount of historical data required for the estimation and this is an issue that some researcher tried to solve (Aïd, 2015). One idea, for example, is to take advantage of the dependence of the forward prices with the consumption of electricity and the marginal cost to generate it. Thus, it is possible to use the seasonality of the spot prices for forward prices (Fleten and Lemming, 2003).

Equation (3.38) is just a general case of HJM-style forward curve model and now we provide another version of that equation that fits better to the topic we are studying, the electricity market:

$$dF_{t,T} = \sum_j \sigma_j\big(t, T, F_{t,T}\big) dW_t^j \;, \tag{3.39}$$

In Eydeland and Wolyniec (2003) one can also find an example that applies equation (3.39) that is reported here: the following dynamic aims to explain how the forward curve changes and it is presented as follows

$$\log\big(F_{t,T}\big) = e^{-k(T-t)}\chi_t + \xi_t + A(T-t) \;, \tag{3.40}$$

where χ_t and ξ_t are random variables related to the spot price:

$$\log(S_t) = \chi_t + \xi_t \;, \tag{3.41}$$

The same variables χ_t and ξ_t have the following dynamics

$$d\chi_t = \left(-k\chi_t - \lambda_\chi\right)dt + \sigma_\chi dW_t^\chi \,, \tag{3.42}$$

and

$$d\xi_t = \left(\mu_\xi - \lambda_\xi\right)dt + \sigma_\xi dW_t^\xi \,, \tag{3.43}$$

where the expected value of the multiplication of the two Wiener processes is equal to $\rho_{\chi,\xi}dt$, while the function A can be expressed as

$$\begin{aligned} A(\tau) = \mu_\xi\tau - (1 - e^{-kT})\frac{\lambda_\chi}{k} \\ + \frac{1}{2}\bigg[(1 - e^{-2kT})\frac{\sigma_\chi^2}{2k} + \sigma_\chi^2\tau \\ + 2(1 - e^{-kT})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{k}\bigg]. \end{aligned} \tag{3.44}$$

Parameters $k, \sigma_\chi, \sigma_\xi, \mu_\xi$ and $\rho_{\chi,\xi}$ are respectively the decay, the volatility of χ, the volatility of ξ, the drift of ξ and the correlation between ξ and χ. Whereas, λ_χ and λ_ξ are "*risk premiums introduced for proper risk adjustment*" (Eydeland and Wolyniec, 2003).

The model proposed above can be found in Schwartz and Smith (2000) and one can call it "*Two-factor forward curve dynamics model*". According to Eydeland and Wolyniec (2003), ξ_t should represent the behaviour of the spot price in the long run, and χ_t should represent the movements in the short-term horizon. One criticism coming from the authors is that the lognormal distribution obtained from this model doesn't belong to the world of electricity prices as we already know from the first chapter. Moreover, this model was built for oil prices and calibrated with the high number of observations provided by the time series of this commodity, therefore it's not sure it would be that simple to obtain a good fit in the case of electricity.

Anyway, it is more likely to use in this context equation (3.39), or a similar form:

$$\frac{dF_{t,T}}{F_{t,T}} = \sum_j \sigma_j(t,T) dW_t^j \; . \tag{3.45}$$

Here one can see that now the returns of the forward prices are modelled with a GBM with zero drift instead of having a dynamic for the forward price. The main issue of this representation is that the goodness of this model depends on the number of random shocks. Litterman and Scheinkman (1991)[23] showed that just a few factors are enough to explain most of the variance of the futures returns in the stock market, reaching higher percentages in the storable commodities markets (Aïd, 2015).

Eydeland and Wolyniec (2003) recall that there are three kinds of random movements when one models interest rates term structure: the first one is the most important because it can explicate most of the variations of the shocks and it is called parallel shift. Whereas, the second one is the twist and the last one is the one that explains the variations of the twist. Of course, in this case, are mentioned interest rates but, in the case of the power market it is not that easy to obtain the right number of shocks and furthermore, we have to rely on a smaller amount of data. Even though we specified that just a few factors are enough to explain the volatility nature of the forward curve, it has to be taken into consideration that this happens especially for storable commodities. With electricity, there is evidence that a higher number of factors is needed due to the high volatility of this commodity.[24]

The beauty of equation (3.46) is that it leads to an explicit solution for the forward price

$$F_{t,T} = F_{0,T} exp\left\{ \sum_j \left[-\frac{1}{2} \int_0^t \sigma_j^2(u,T) du + \int_0^t \sigma_j(u,T) dW_t^j \right] \right\}. \tag{3.46}$$

As said earlier, the first shock is the parallel shift which is a decaying function like the following one

$$\sigma_1(t,T) = \sigma_1 - \rho\sigma_2 \frac{1 - e^{-k(T-t)}}{k}, \tag{3.47}$$

[23] See also Knez et al. (1994).

[24] For more insights see Koekebakker and Ollmar (2005).

where k is the decay parameter. The second shock is the twist and can be expressed as

$$\sigma_2(t,T) = -\sigma_2\sqrt{1-\rho^2}\,\frac{1-e^{-k(T-t)}}{k}\,. \tag{3.48}$$

Notice that when one defines the perturbation functions, he/she creates automatically the dependence structure between contracts with distinct expiring dates (Eydeland and Wolyniec, 2003).

One more example regarding forward curve modelling is given from Aïd (2015) who talks about the possibility of modelling electricity futures jointly with fuel prices using the typical correlation that exists between these two commodities. The author proposes the following dynamic for the electricity

$$\frac{dF_{t,T}^{e}}{F_{t,T}^{e}} = \sigma^{e}(t,T)dW_t^{e}\ , \tag{3.49}$$

and the following one for the coal

$$\frac{dF_{t,T}^{c}}{F_{t,T}^{c}}(t,T) = \sigma^{c}(t,T)dW_t^{c}\ , \tag{3.50}$$

The correlation of the two Wiener processes assures the dependence between the two futures. As we have seen in the second chapter, electricity is correlated with other energy commodities, thus Aïd (2015) thought reasonable to find a way to reflect this correlation also in the dynamic of the forward price.

In this chapter, we reviewed the theory for pricing futures contract on assets and commodities and pointed out that is not easy to apply the free-arbitrage theory to non-storable commodities. Exploiting an arbitrage opportunity involves borrowing or bringing in the future a certain asset and this is not possible if the underlying is electricity. We showed the relationship between forward and futures prices and explained why models for electricity spot price are needed. Indeed, stochastic or equilibrium models are useful tools that help in the research of a fair forward price, since the futures price could be seen as an expectation of the spot price at the end of the contract.

We have said though, with the introduction of the risk premium approach, that the forward price is actually a biased estimator of the spot price at the delivery date. This happens because the actors in the market are willing to hedge their risks, therefore they are ready to pay a premium or concede discount to protect themselves. It was proposed a simple example with a market divided into two categories, producers and consumers. The former has a long vision, thus are willing to hedge their positions for long term horizons, whilst the latter have can focus mainly in the short period, so they cover their risks in the short term. These differences give a different power in making decisions to the two actors, depending on the type of contract we are taking into consideration. There is also another factor to recall, the risk aversion of the market participants that is used by Benth et al. (2007) together with the market power to create a link with the risk premium.

At the same time of the risk premium approach, researchers tried to model the forward curve with stochastic processes as it is for the spot prices (see second chapter). Heath, Jarrow-Morton (1992) (HJM) had the idea to model the term structure of interest rates with stochastic models an alternative to the researchers for pricing electricity forward contracts.

In conclusion, our opinion is that none of the methodologies introduced in this chapter prevails on the others, like in the case of electricity spot models. This is still a new field for the research, thus counting the difficulty in treating this particular commodity results that it will take time for the development of a concrete technique in pricing derivatives which is in absolute better than the others.

In the next chapter, it is possible to find a detailed explanation of one of the stochastic processes encountered in the second chapter, in order to highlight the difficulties in the calibration of such a model.

Chapter IV. Practical application: study of a stochastic spot price model for electricity

In the second chapter, we talked about the models used for power prices and summarized the different typologies. These models try to represent at best the characteristics of electricity prices and there are several approaches to do this. Stochastic models use historical data to understand the behaviour of the price to replicate it. Equilibrium models use the information provided by demand and supply to determine the price. In the third chapter, is also explained the purpose of price modelling and why obtaining a good model is important for derivatives pricing.

In this chapter, we explain in detail one of the stochastic models already introduced. This is a model with mean reversion plus a jump diffusion component that Geman and Roncoroni introduced in 2006. Part of the model is explained in the second chapter and now we go into details (4.1) and discuss its calibration (4.2). In the last part of this chapter, are summarized the results obtained by the two authors and at the same time we use this model for more recent data from the Italian market. Finally, We will give also our impressions about the procedures adopted and the results obtained.

4.1. The electricity spot price model

The unique peculiarities of electricity brought the researchers to develop several models to explain its movements and to reproduce them in order to provide a suitable tool for pricing derivatives on the very same commodity. Starting from the end of the 90s, the simplest models based on the Geometric Brownian motion aren't enough to reproduce power spot prices, thus more elements are added to improve those models. The mean reversion element is one of the first we see in this field, and after that, the jump diffusion is considered to explain the spiky nature of these prices. Someone noticed also the presence of stochastic volatility in the time series, bringing to more complicated models with several components to estimate.

One of the models we want to analyse in detail is the one proposed by Geman and Roncoroni (2006) that is built with a mean reversion and a jump diffusion processes: the authors call it *"level-dependent signed-jump model with time-varying intensity"*. Is presented now the model that is based on the following stochastic differential equation

$$dS_t = \mu'_t dt + \theta_1(\mu_t - S_{t^-})dt + \sigma dW_t + h_{t^-} dJ_t \,. \tag{4.1}$$

In the dynamic of the spot price we can see that the mean reversion $\theta_1(\mu_t - S_{t^-})$ makes the price coming back to the mean price level represented by a seasonal component μ_t, at a speed of mean reversion equal to θ_1. The element S_{t^-} indicates the value of the price a moment before the time t, whilst σ is the volatility and W_t is the Wiener process. The combination of these two elements under the form equal to σdW_t makes possible the random oscillations around the mean trend. The last element of equation (4.1), $h_{t^-} dJ_t$, is composed of a direction function and a jump process, representing the discontinuous part of the process. h_{t^-} gives the direction to the jump that could go up or down according to the value of the price a moment before time t. J_t gives the size of the jump, once the jump occurred. The jumps occur only when the spot price goes beyond a certain level, defining a certain instant that tells us we are not anymore in a normal regime but, instead, in a jump regime. This could be similar to a regime switching model but in this case we have only one dynamic describing the whole process.

More in detail, the function that gives the direction to the jump can be written as follows

$$h(S_t) = \begin{cases} +1, & S_t < \mathcal{T}_t \\ -1, & S_t \geq \mathcal{T}_t \,, \end{cases} \tag{4.2}$$

where $\mathcal{T}_t$ is a certain threshold. Basically, when the spot price S_t at time t is below the threshold, the function h is equal to 1 and when the spot price is above that threshold, the function h assumes a value equal to -1. Geman and Roncoroni (2006) make notice the relevance of this function because it allows the creation of a cluster of a subsequent sudden rise of the prices followed by a rapid decrease. The authors criticise other jump diffusion models because they show a highly positive skewness in the return of the prices when the model is used to simulate the prices. Moreover, the two researchers don't agree with the use of certain regime switching models because they don't represent the jump clustering as it should be on their opinion.

If the function h gives the direction of the jump, there are other elements in the model that signal when the jump happens and how big is this jump. To ensure the right timing of the jumps, Geman and Roncoroni (2006) defined a deterministic intensity function that describes the mean amount of jumps we can observe in a unit of time:

$$\iota(t) = \theta_2 \times s_t \,, \tag{4.3}$$

where s_t is the jump intensity shape, while the parameter θ_2 is the highest number of jumps we can expect in a unit of time. This function is related to a counting process $N(t)$ that count the jumps happened till time t.

When there is a jump, it has to be defined also the size of the jump, therefore the authors provided us with a compound jump process $J(t)$:

$$J(t) = \sum_{i=1}^{N(t)} J_i \,, \tag{4.4}$$

where the J_i's are random variables, uncorrelated with each other and, the same distribution and the same density function defined as follows

$$p(x;\ \theta_3, \psi) = c(\theta_3) \times \exp[\theta_3, f(x)] \,, \qquad 0 \leq x \leq \psi \,, \tag{4.5}$$

where $c(\theta_3)$ is a constant allowing p to be a probability distribution density, while ψ is the highest size the jump could have.

In the first part of this chapter, by Geman and Roncoroni (2006) the model developed is explained in detail while in the next section is reported the calibration composed of two steps that define the structural elements and estimate the parameters of the model.

4.2 Model calibration

The authors calibrate their model on the COB (California Oregon Border) market, the PJM (Pennsylvania-New Jersey-Maryland) and the ECAR (East Center Area Reliability coordination agreement) market. The time series taken into consideration consists of 750 prices[25] taken from the late 90s and the authors decided to use these markets because they represent extremely different areas of the United States and because they differ also for the type of production of electricity. There are huge differences in the prices of these three markets because we can go from a region of high pressure (ECAR), where it is possible to observe spikes between 1750 and 2950 dollars[26], to a low pressure market (COB), where the prices are lower and the observed spikes lie between 90 and 115 dollars. In the middle, there is the PJM that experienced spikes between 263 and 412 dollars in the three years of analysis.

Once the model has been defined, it has to be calibrated. Geman and Roncoroni (2006) propose a procedure made of two phases. In the first one, it is necessary to give a detailed form of the structural elements of the model. More precisely, one has to define the mean trend, the jump intensity shape, the threshold and the distribution of the jump size.

After the definition of such elements, one has to estimate the parameters. In the dynamics there are θ_1 and σ, mean reversion force and volatility, respectively. Whilst in the jump intensity function there is the maximum expected number of jumps θ_2, and finally the reciprocal expected jump size θ_3.

4.2.1 Structural elements

The authors decided to select the structural elements according to the analysed data set. Therefore, they noticed a seasonal pattern in their time series but with spikes

[25] The prices are the average of each day.

[26] The prices are expressed in megawatt-hour.

grouped in different periods. In some markets, higher prices are observed during a certain period of the year, while in others there are spikes grouped in different seasons. They built a trend function that was able to fit in both cases, thus mean trend function they thought to be appropriate is a sinusoidal function with two different periodicities:

$$\mu(t;\ \alpha,\beta,\gamma,\delta,\varepsilon,\zeta) = \alpha + \beta t + \gamma\cos(\varepsilon + 2\pi t) + \delta\cos(\zeta + 4\pi t)\,. \qquad (4.6)$$

α is the average log price level, β is the average log price slope and they represent respectively a sort of fixed cost of production and the trend of the price in the long run. γ is the yearly trend, δ is the six-month trend, ε is the yearly shift and ζ is the six-month shift. Indeed, the last 2 elements of equation (4.6) define the two seasonalities observed during a year and the respective amplitudes (Geman and Roncoroni, 2006).

The second structural element to define is the intensity shape of the jumps which is built by the authors as follows

$$s(t) = \left\{\frac{2}{1 + \left|sin\left[\frac{\pi(t-\tau)}{k}\right]\right|}\right\}^{d}. \qquad (4.7)$$

This equation is developed in order to create convexity in the jumps shape with yearly periodicity. The authors aim to replicate the properties of the power stack function showed in the first chapter, thus a sudden increase in the price as soon as more expensive production sources are used to serve the demand. In equation (4.7), k is the frequency of the jumps we observe in a year, so if one notices that jumps are concentrated in two different times of the period $k = 0.5$. If one observes a concentration of the spikes only one time in a year then $k = 1$. τ represents the time in which we observe most of the spikes, thus for example, if we observe the spikes with a maximum on May 1, then $\tau = 4/12$. The last parameter in this equation is d which is the dispersion of the jumps in the period they occur.

The third structural element is the threshold $\mathcal{T}$ that tells if the jump has to go upward or downward. Geman and Roncoroni (2006) defined the threshold as follows

$$\mathcal{T}(t) = \mu_t + \Delta\,, \tag{4.8}$$

where Δ is chosen by the authors.

The last structural element is the distribution of the jump size which is

$$p(x;\ \theta_3, \psi) = \frac{\theta_3 e^{-\theta_3 x}}{1 - e^{-\theta_3 \psi}}\,. \qquad 0 \leq x \leq \psi\,, \tag{4.9}$$

As we can see, the authors chose a truncated exponential density where θ_3 is the reciprocal average jump and ψ is the maximum jump size. Equation (4.9) comes from the type of distributions described in equation (4.5) with $c(\theta_3) = \theta_3/[1 - \exp(-\theta_3\psi)]$ and $f(x) = -x$.

The first phase, where the structural elements are defined is now concluded and the next step is to estimate the parameters.

4.2.2 Parameter estimation

In the previous subsection, we introduced the structural elements of the model and estimated or calculated the single parameters belonging to these elements. This was the first important step before estimating the parameters of the model, a procedure explained in this subsection.

Geman and Roncoroni (2006) maximize the following log likelihood in order to find the set of parameters θ that are needed to complete the model

$$
\begin{aligned}
L(\theta|\theta^0,S) = &\sum_{i=0}^{n-1} \frac{[\mu_{t_i} - S_i]\theta_1}{\sigma^2}\left[\Delta S_i^c - \mu'_{t_i}\Delta t\right] \\
&- \frac{\Delta t}{2}\sum_{i=0}^{n-1}\left\{\frac{[\mu_{t_i} - S_i]\theta_1}{\sigma}\right\}^2 \\
&- (\theta_2 - 1)\sum_{i=0}^{n-1} s_{t_i}\Delta t + log\theta_2 N_t \\
&+ \sum_{i=0}^{n-1}\left[-(\theta_3 - 1)\frac{\Delta S_i^d}{h(S_i)}\right] + N_t log\left[\frac{1 - e^{-\theta_3\psi}}{\theta_3(1 - e^{-\psi})}\right].
\end{aligned}
\tag{4.10}
$$

The maximization starts from an initial set θ^0 of parameters. The authors established $\theta_1 = 0$, $\theta_2 = 1$ and $\theta_3 = 1$ as starting parameters. There are also a few elements that we didn't introduce before: ΔS_i^c and ΔS_i^d. The authors divided the original dataset and obtained the sampled jumps ΔS_i^d and the continuous part ΔS_i^c.

In order to complete the computation, one has to estimate also the volatility:

$$
\sigma = \sqrt{\sum_{i=0}^{n-1} \Delta\bar{S}\,(t_i)^2}\,, \tag{4.11}
$$

where $\Delta\bar{S}(t_i)^2$ is the square of the continuous part ΔS_i^c.

The calibration procedure in now completed and in the next subsection it is shown how in practice the authors obtained their results.

4.2.3 Empirical results: Geman and Roncoroni (2006)

The first step now is to obtain the parameters of the structural elements: the mean trend, the intensity jump shape, the threshold and the probability density function of the jump's size.

The mean trend has six parameters as we explained before and the authors estimated them through a sequential OLS. First of all, they took the average of every daily log price of the 3-year data set and then they excluded the values above the 0.7 quantile of the empirical distribution. After that, they fitted the trend function in equation (4.6) to this new filtered dataset.

The second element contains parameters that are actually chosen arbitrarily by the authors. Indeed, Gemand and Roncoroni (2006) noticed that the best values for the parameters in equation (4.7) could be $k = 1$, $\tau = 0.5$ and $d = 2$. The choice of the first two parameters relies on the fact that they observed a group of jumps at the centre of the year in the markets they studied. While they tried several values for d and decided that 2 was the best for their data.

The choice of Δ is even more arbitrarily because the authors set it equal to 50% of the range spanned by the observed log prices. In other words, they took the difference between the maximum log price and the minimum log price, then they divided it by two. The results obtained by Geman and Roncoroni (2006) are reported in Table 4.1

Parameter	Interpretation	ECAR	PJM	COB
α	Average log price level	3.0923	3.2002	2.8928
β	Average log price slope	0.0049	0.0036	0.1382
γ	Yearly trend	-0.013	0.0952	0.1979
δ	6-month trend	0.0292	0.217	0.0618
ε	Yearly shift	0.3325	2.4383	1.7303
ζ	6-month shift	0.7417	0.2907	1.7926
Δ	Jump regime level	2.5	1.5	1
ψ	Maximum jump size	3.3835	1.6864	1.0169

Table 4.1: Prameters estimation of the structural elements (Geman and Roncoroni, 2006)

The results in the second line of Table 1 tell us that there is no evidence of a linear trend for the first two markets, whereas there is a little positive trend for the third market. With regards to γ, the yearly trend, one can notice that in the COB market is higher, while the same market has a lower value of δ, the 6-month trend. In seems though, that for the first two markets prevail a relevance of the semi-annual periodicity, while the annual periodicity is more important in the third case. We can also see a great difference in the value of ψ, the maximum jump size, that is particularly high for the ECAR market if compared with the others because this region has very warm summers and very cold winters. Gemand and Roncoroni (2006) explain that the low ψ of the COB market is probably due to the high hydroelectric capacity, allowing this market to face in a better way unexpected increases of the demand.

With the estimation of the parameters of the structural elements, the first step is complete, so now it is possible to proceed with the second step. It consists in the estimation of the model parameters through the maximization of the log likelihood reported in equation (4.10). The authors decided to use the Levenberg-Marquardt nonlinear maximum search algorithm[27] for the estimation of θ_1, θ_2 and θ_3. The results are summarized in Table 4.2:

Parameter	Interpretation	ECAR	PJM	COB
θ_1	Smooth mean reversion	38.8938	42.8844	13.3815
θ_2	Maximum expected number of jumps	59.5210	63.9301	13.2269
θ_3	Reciprocal average jump size	0.3129	0.5016	1.0038
σ	Brownian local volatility	1.8355	1.4453	1.3631
Γ	Jump threshold	0.92	0.6	0.62
$N(1)$	Average number of jumps	9	9.6667	2

Table 4.2: Estimated model parameters (Geman and Roncoroni, 2006)

The volatility is obtained as shown in equation (4.11), while the average number of jumps is obtained by calculating the integral of the intensity function shown in equation (4.3). The jump threshold required much more efforts because it is chosen so that the model obtained matches the fourth moment of the empirical distribution of the return of the log prices.

[27] It's an algorithm applied to nonlinear problems to find a local minimum. In this case we have to find a local maximum of a function.

The results obtained by Geman and Roncoroni (2006) seem to reflect very well the characteristics of the markets taken into consideration even though the skewness is the result that less fit the empirical dataset. The explanation is that probably the mean reverting element should be more vivid. The results are resumed in is Table 4.3:

Statistic	ECAR		PJM		COB	
	Empirical	Simulated	Empirical	Simulated	Empirical	Simulated
Average	-0.0002	-0.0001	-0.0006	0	0.0009	0.0006
Standard deviation	0.3531	0.3382	0.2364	0.2305	0.1586	0.1121
Skewness	-0.5575	2.1686	0.3949	1.6536	0.1587	0.9610
Kurtosis	21.6833	22.5825	13.1507	14.8429	6.7706	6.5402

Table 4.3: Moments of the empirical and simulated daily price variations (Geman and Roncoroni, 2006)

The authors point out the good results of the Kurtosis that has been represented very well by the model. They explain also that it is the main advantage of the signed-jump model developed by the two researchers.

Geman and Roncoroni (2006) continue with a simulation, replicated 1000 times, of the model in order to re-estimate the parameters and obtaining better results. Thus, the model in equation (4.1) has been discretized, then the following equation is built

$$S_{t_k+1} = S_{t_k} + \mu'_{t_k}\Delta t + \theta_1\left(\mu_{t_k} - S_{t_k}\right)\Delta t + \sigma\sqrt{\Delta t}\mathcal{Z}_{k+1} + h_{t_k}\mathbf{1}_i J_{k+1}\,, \tag{4.12}$$

where $\mathcal{Z}$ is a sample from a Gaussian distribution with zero mean a variance equal to one while $t \in [0, T]$. Then, J is a random sample generated from a probability distribution $p(\cdot, \theta_3)$. The function $\mathbf{1}_i$ is a dummy variable that assumes a value equal to one or zero according to the presence or not of a jump. The authors simulate the times of the jumps with a random exponential distribution, with parameter equal to maximum value of the intensity function. Then the simulated jumps are accepted or rejected with the aim of obtaining a set of sample jumps with the statistical properties needed. In

other words, the times of the jumps are simulated, then the size of the jump is given by a sample from a random Poisson distribution with parameter equal to the θ_3.
The re-estimated parameters are shown in Table 4.4

Parameter	ECAR		PJM		COB	
	Original	Re-estimation	Original	Re-estimation	Original	Re-estimation
θ_1	38.8938	37.7559	42.884	40.0285	13.381	11.7956
θ_2	59.5210	57.9367	4.1578	4.0188	2.5822	2.4001
θ_3	0.3129	0.2957	.5016	0.48	1.0038	1.1897

Table 4.4: Results of the simulation and re-estimation of the model parameters (Geman and Roncoroni, 2006).

The more important information in Table 4 is the difference between original and re-estimated θ_3 of the COB market. This is a higher difference if compared with the other two markets, but the authors confirm the lack of relevance of this change since the jump component is not very important for the COB market. Indeed, this change could be due to the low number of jumps observed in the dataset of this market, according to what the authors say.
It is concluded here the resume of the study conducted by Geman and Roncoroni (2006) and in the next section, their model and their procedures are applied to more recent data regarding taken from the Italian market.

4.3 Practical application

This last section is dedicated to a practical application of the model developed by Geman and Roncoroni (2006) to represent the electricity prices of the PUN (Prezzo Unico Nazionale) market, in Italy. (For this practical application we used two softawares: Microsoft Excel and Matlab).
The historical time series are reported in Figure 4.1 that shows the daily price between 01/01/2017 and 28/12/2019. The observations are in total 1092.

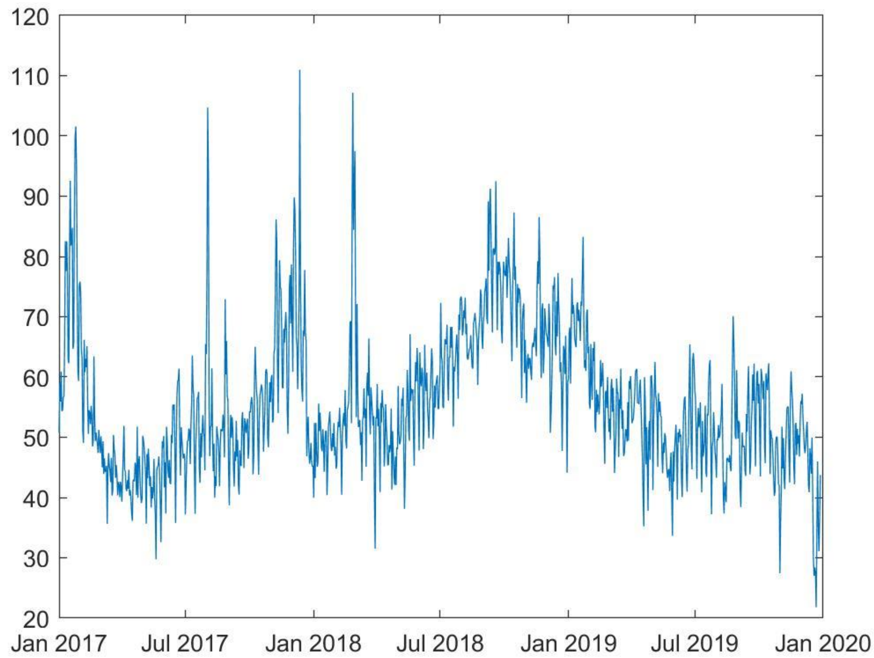

Figure 4.112: PUN dataset. Average daily prices between 01/01/2017 and 28/12/2019.

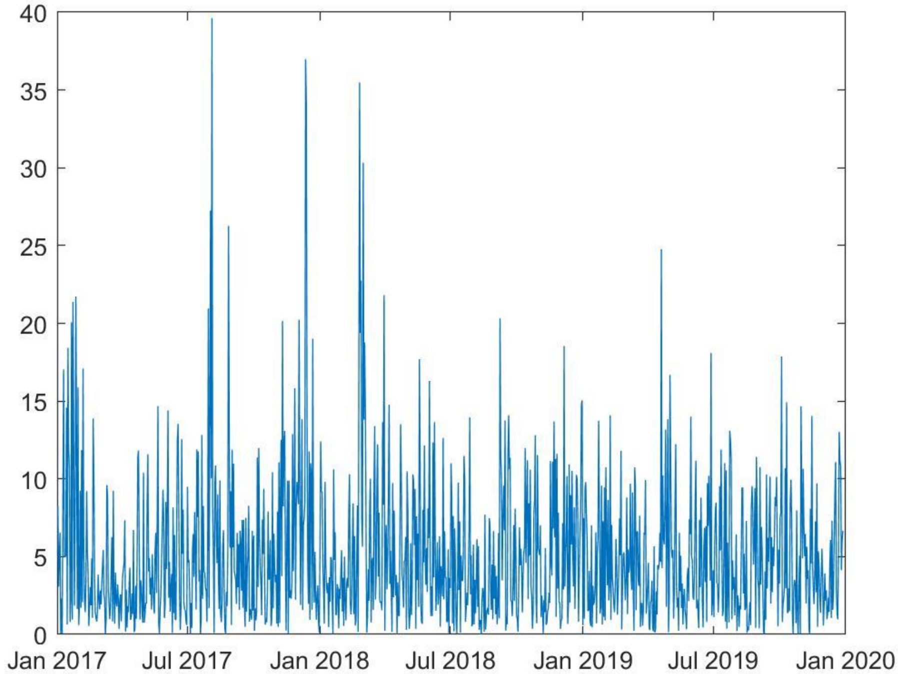

Figure 4.2: Price variation of the PUN market between 01/01/2017 and 28/12/2019.

The average price during this period is 55.81 euros per Mw/h, the maximum value reached is 110.98 euros while the minimum is 21.78. The biggest spikes are observed in the summer of 2017, at the end of the same year and during the spring of 2018. On the contrary, 2019 seems to be slightly stable if compared with the other two years. In Figure 4.2 we report the differences in the prices in absolute value, and it is possible to see better the dramatic changes in the prices.

The first step is to take the mean of each day of the 3 years data set in logarithmic scale, then we excluded the values above the quantile 0.7 as the authors suggest. The vector obtained is used for the OLS regression for the mean trend.

The objective is to find the set of parameters $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta$ that minimize the following equation

$$\sum_{t=0}^{n} \left(y_t - \hat{\mu}_t\right)^2 , \tag{4.13}$$

where y_t is the dataset averaged in a one-year period and bounded from above by the 0.7 quantile of the empirical distribution. $\hat{\mu}_t$ is instead equation (4.6) that has to be substituted inside our minimization problem. This is a non-linear optimization problem and required the implementation of an algorithm in Matlab for the research of a local minimum. One possible way to estimate the parameters of the seasonal trend is to assign arbitrarily one value to the six-month shift and to the yearly shift, allowing for a transformation of t:

$$\mu(x;\ \alpha, \beta, \gamma, \delta) = \alpha + \beta X_1(t) + \gamma X_2(t) + \delta X_3(t) \tag{4.14}$$

where the observation t is a positive integer that goes from zero to the maximum number of observations. $X_1(t) = t$, $X_2(t) = \cos(\varepsilon + 2\pi t)$ and $X_3(t) = \cos(\zeta + 4\pi t)$. With these transformations the minimization problem is now linear and simpler but, this procedure requires to assign a value to two parameters: ε, ζ.

With the algorithm for the nonlinear optimization, we obtained the results shown in Table 4.5

Parameter	Interpretation	PUN
α	Average log price level	-17.58
β	Average log price slope	9.89e-05
γ	Yearly trend	8.985
δ	6-month trend	45.0913
ε	Yearly shift	0.4514
ζ	6-month shift	1.3200

Table 4.5: Estimation of the seasonal trend's parameter

In Figure 4.3 we report the estimated seasonal trend.

With regards to the intensity shape function we decided to use the same parameters decided by Geman and Roncoroni (2006) for their model, thus $\tau = 1/2$, $k = 1$ and $d = 2$.

$$s(t) = \left\{ \frac{2}{1 + \left| sin\left[\frac{\pi(t - 0.5)}{1}\right] \right|} \right\}^2. \qquad (4.15)$$

In words, this function should group the jumps around the middle of the year with a dispersion coefficient equal to two.

On other structural elements is the threshold seen in equation (4.8) where there is a parameter Δ we must estimate. Actually, is not required an estimation but rather a simple calculation. Indeed, the authors obtained this parameter by subtracting the minimum log price from the maximum log price and divided by two. We did the same, obtaining Δ= 0.149865.

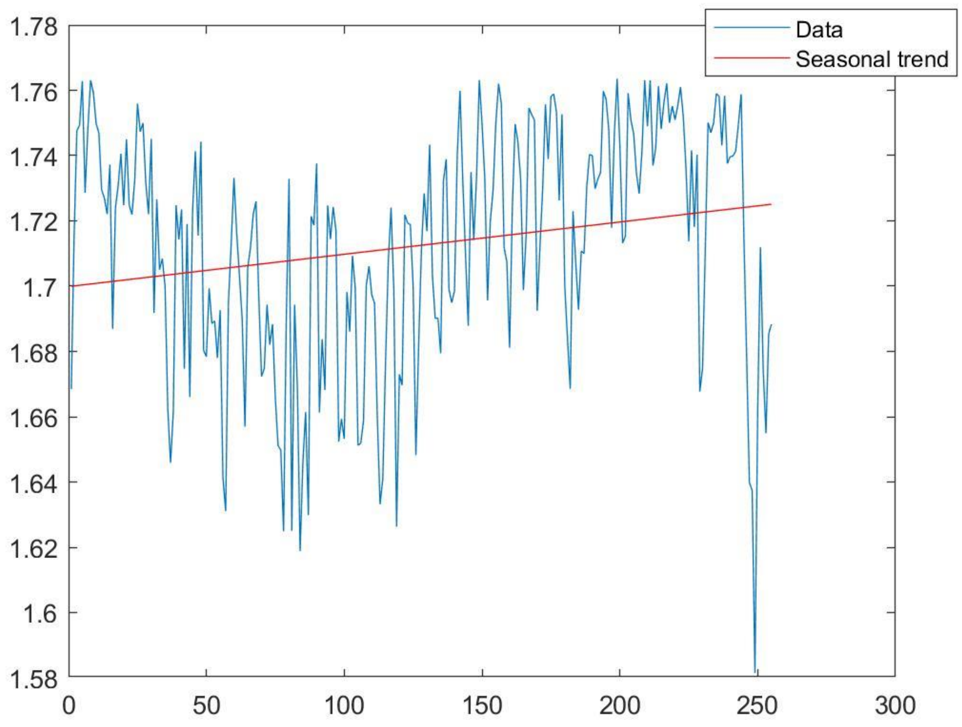

Figure 4.3: Estimated seasonal trend for the PUN market.

The last parameter to calculate in order to complete the structural elements of the model is ψ, that we find in the probability density function of the jump size, equation (4.9). We took the variation of the log price of the entire dataset and found the maximum, thus we found that $\psi = 0.235366$.

The next step is the estimation of the model's parameters, but first, it is required to divide the dataset to obtain the continuous part and the discontinuous part where there are the jumps. In doing so we decided to use the threshold obtained by the authors for the ECAR market because the procedure to obtain the threshold is particularly complicated. We should find the threshold that allows the simulated model to match the fourth moment of the empirical distribution, but this requires to run the log likelihood maximization for several times with different discriminations of the original dataset.

Once obtained the three datasets (the entire dataset, the dataset containing the simple fluctuations around the mean trend and the discontinuous part) we can proceed to obtain the volatility because it is required in the equation of the log likelihood. Applying equation (4.11) we obtain $\sigma = 0.55$.

With regards to the counting function $N(t)$ that is present in the log likelihood, it has been decided to use the dataset obtained from the discontinuous part and to count every time we find a jump remaining above the threshold.

We maximized the log likelihood function the Levenberg-Marquardt algorithm as the authors did with their data. The results of the estimation are reported in Table 4.6.

Parameter	Interpretation	PUN
θ_1	Smooth mean reversion	0.7331
θ_2	Maximum expected number of jumps	1.0014
θ_3	Reciprocal average jump size	0.5753

Table 4.6: model's parameters estimates for the PUN market.

The results obtained for the model's parameters aren't what we expected. The smooth of mean reversion should be stronger due to the presence of the jumps and the maximum expected number of jumps is obviously a higher number.

We can suggest several reasons why the estimates obtained are so far from reasonable results. First, recall that for the calculation of the seasonal trend we averaged the entire dataset in a one-year period and then, bounding from above that vector of average prices calculating the 0.7 percentile of the empirical distribution. The choice of the percentile is totally arbitrary, thus this parameter could be changed, obtaining a different dataset for the regression of the seasonal trend function. Speaking of which, the seasonal trend function is another element that could be changed, due to the fact that the authors decided to include 2 types of seasonalities, instead of any other amount. The number of seasonalities can be changed according to the market we are analysing.

The parameters used for the intensity shape function are the same adopted by Geman and Roncoroni (2006). The authors noticed a cluster of spikes in the middle of the year, thus they decided certain parameters that could be adequate for their dataset. In the case of the PUN market, one can notice a group of jumps during the spring instead of the summer, thus one could change one of the parameters to fit better the data. We have to say though, that this spikes clustering is visible only in the first two years of the dataset, but not in the last one. Also, the parameter for the dispersion of the spikes is something that could be analysed separately because in some markets maybe it is possible to observe longer periods of clustered spikes.

One arbitrary procedure, we took from the authors, is the way in which is obtained the parameter Δ that we find in the threshold function $\mathcal{T}$ that tells of the jump should be upward or downward. The result obtained is very high and we noticed the corresponding high value of the threshold. As far as we are concerned, we should decrease such value or find another way to estimate in order to obtain a reasonable threshold.

Γ is the last parameter to discuss in this section: recall that Γ defines the filtration of the original dataset with the aim to divide it into two subsets, the continuous part and the discontinuous part, or in other words the normal regime and the jump regime. We used the one estimated for the ECAR market that is a very high value and we don't know if it is optimal in this case. We should use different Γ's in order to find the one that makes the estimated model matches the fourth moment of the empirical distribution.

In any case, it is proposed below the simulation of the electricity prices for the Italian market over a one-year period. The parameters of the model are the same used by Geman and Roncoroni (2006) given that the calibration procedure wasn't successful with the data of the PUN market. The prices obtained are shown in Figure 4.3

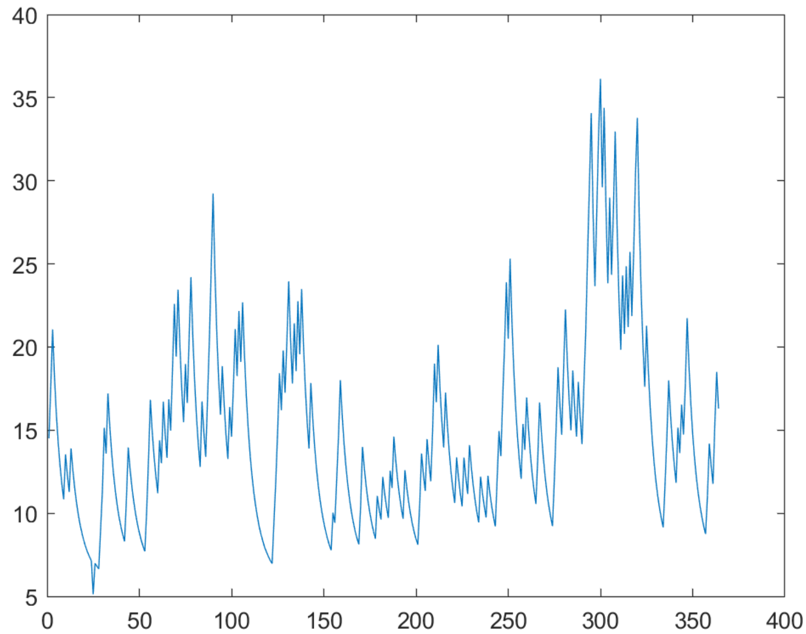

Figure 4.4: simulation of the PUN prices.

The simulation of the trajectories described in equation (4.1) is based on the Monte Carlo simulation, following the procedure adopted by Geman and Roncoroni (2006) as described in equation (4.12).
The statistical properties of the returns of the log prices of the empirical and simulated time series are reported in Table 4.7

Statistic	PUN	
	Empirical	Simulated
Average	1.46e-04	3.30e04
Standard deviation	0.0722	0.1512
Skewness	-0.0526	0.7045
Kurtosis	3.2671	1.8863

Table 4.713: Moments of the empirical and simulated daily price variations *of the PUN market.*

As expected, the simulation using the parameters estimated by the two authors doesn't give good results.

In conclusion, the most important part of the study offered by Geman and Roncoroni (2006) is the calibration procedure. With this procedure, one has to define a form for the structural elements that could be slightly different from the ones proposed by the authors. Moreover, we noticed that several arbitrary parameters have to be used depending on the dataset and on the market analysed. Lastly, the estimation of the parameters has to be conducted multiple times before finding the optimal set, and this has to be done with different filtrations of the dataset.

Conclusion

After the deregulation of the electricity market, a new interesting field has been opened in the world of derivatives. It is appealing because it is impossible to use the same theories applied to derivatives on stocks or other commodities. Thus, the peculiarities of electricity forced the operators to find alternative approaches to price derivatives with electricity as underlying.

In the first chapter, we summarized the characteristics of electricity, such as the non-storability that is the main feature influencing the behaviour of the price. It is indeed impossible to store and when it is produced, it has to be consumed immediately. This is translated in a very volatile asset with a price that can dramatically change when the demand changes. The consumption of electricity is strictly linked to the temperatures and therefore to the seasons. In fact, one of the characteristics it is possible to observe looking at the historical time series of the power price is the evidence of seasonal behaviour. A typical pattern we can see is also a sudden rise in the price, followed by a rapid decline, forming what is called a spike.

Knowing the peculiarities of this commodity is essential for the creation of a model that tries to replicate the dynamics of the electricity price. In the second chapter, are listed several types of models created by the researchers who aim to explain price behaviours and to create a useful tool for pricing derivatives. The chapter starts with the reduced form models like the Geometric Brownian Motion used in finance for decades but, too simple to model electricity price. Already at the end of the 90s, someone tried to use mean reverting processes based on the researches of Ornstein-Uhlenbeck with some good results. Anyway, this model alone can only explain how to bring the price to the long-run level and doesn't explain very well the jumps, typical of electricity. In fact, in literature, one can find models with a jump diffusion process, based for example on a Poisson distribution of the jumps, in combination with mean reverting processes. These two elements together can explain well the behaviour of power price because the jump diffusion process succeeds to replicate the unexpected rise of the price, while the mean reverting component helps to bring down the price to the level the price was before the jump. It is then considered the possibility of the introduction of a stochastic volatility component instead of using constant volatility. Some researchers found evidence of dependence in the volatility of the historical time series, suggesting to model this component with some heteroskedastic process. On the contrary, some authors recognized the possibility of having stochastic volatility but preferred to don't model it, being able to obtain good results without it.

The second approach in price modelling is the regime-switching model. In this case, exist two or more regimes that can be modelled in different ways, and there is a certain probability to go from one state to another. Usually, there are two regimes, but they could be three, with different processes depending on the period one has to model. The first one typically models the movements of the price around the trend mean, while the second one models the jumps. This type of model has been criticized because, once the transition from the first regime to the second one that models the jumps has been verified, the price tends to go back to the normal state immediately, avoiding the possibility of having multiple jumps.

On the other side, there are the so-called equilibrium models that aim to define the supply and demand curves, then by balancing them, it is possible to find an equilibrium price. Instead of using historical prices, this kind of model aims to explain how supply and demand are linked. It is necessary to build the supply curve, taking into consideration the production costs for example, and the demand curve, observing the average consumption. This requires a lot of information that is not available and easy to find in all the markets.

In addition, we talked about the combination of stochastic models and equilibrium models as the last argument of the second chapter. In this case, the drivers of the demand and supply are modelled with stochastic processes, then, once the curves are built, it is possible to find the equilibrium price. The drivers for the supply could be historical prices of other commodities if the electricity in that market is produced from different energy sources like natural gas or coal. With regard to the demand, one possible driver could be the historical data of the temperatures, considering that consumption of energy is related to the weather.

What we learned from the analysis of spot price models is that, in the last twenty years of studies, none of the models introduced prevailed on the others. In literature, one can find different opinions about the models used in this field and it is not clear which one is better. It will take more time to define a model that is efficient and that is not cumbersome to estimate.

In the third chapter, is explained why it is important to obtain a good spot price model, given that it is necessary for pricing forward contracts. The connection between the forward contract and the spot price couldn't be very clear immediately, that is why we took a little bit of space to review the theory for pricing futures. What is sure is that the free-arbitrage theory cannot be used for pricing electricity derivatives due to the impossibility to take advantage of certain investment strategies. As a matter of fact, electricity cannot be carried on in the feature and cannot be borrowed as it is possible with stocks and other commodities. Therefore, is needed a different approach to price derivatives with electricity as underlying.

It is reasonable to assume that investors use derivatives to hedge risks, in particular price risk. Producers of a certain commodity are willing to concede a discount to fix the price of the product that has to be sold on a specific day in the future. As well as the buyers of the same commodity are willing to pay a premium in order to fix the price and protect themselves from price variations. This approach based on the risk premium has been used to price forward contracts. More precisely, utility curves and the risk aversion of two market players, producers, and consumers, are used to find a range of prices the two actors are willing to pay for the contract that allows them to hedge the price risk. At this point, stochastic models for the spot price are used to find the equilibrium price, making understand how important is to have a good model.

There is also a different approach that aims to model the forward curve directly with stochastic models, instead of creating a model for the spot price and then use it to price the derivative. The idea for this approach comes from the technique used by Heath, Jarrow-Morton (1992) for the modelling of the term structure of the interest rates. Some authors decided to do the same with the term structure of the electricity forward curve, finding a different way to price forward contracts. The main difficulty, in this case, is to determine the right number of disturbance factors that model the shocks of the price. Many authors noticed that for electricity are needed more elements if compared with other commodities if one wants to explain most of the price variations. One more thing to take into consideration is the possibility of adding a seasonal trend function since there is some evidence verifying the presence of seasonality also in the forward price.

Anyway, as it is for electricity spot price models, it is not clear which one is the best procedure and none of the one reported in this thesis prevails the others.
The last chapter is dedicated to a detailed explanation of the stochastic spot price model developed by Geman and Roncoroni (2006). This model explains electricity price movements through several components: a seasonal trend, a mean reversion, and jump-diffusion processes. One interesting characteristic of this model is that the function dedicated to the jumps is associated with another function that gives the direction to the price. If the price is above a certain threshold, the direction given to the jump is downward, otherwise, it is upward.
The authors propose a calibration of the model in two phases: the first one is dedicated to the definition of the structural elements, such as the form of the seasonal trend, the intensity function, and the threshold. This phase is also necessary to estimate the parameters of these elements. In the second phase, the authors proceed with the estimation of the model's parameters with the maximization of an approximated log-likelihood. The results obtained in their case are pretty good and their model was able to replicate very well the prices of the markets analysed.
We used the prices of the Italian market to replicate this model, but the results obtained aren't satisfactory and the parameters estimated couldn't simulate efficiently the prices. There are several arbitrarily parameters that could be changed to improve the estimations. These parameters are for example the ones of the intensity shape function or the parameter that influence the threshold that gives the sign to the jump. One more idea to obtain better results is to change the form of the seasonal trend function for instance.
In conclusion, some of the spot price models studied in this thesis could be very effective for the simulation of the prices but are probably still complicated to calibrate, making difficult their usage in reality.

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