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Static and dynamic setting for prudence and prevention: a health-wealth sight

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To the life that will breath easier because I have lived

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Abstract

This thesis is composed by two parts. In the first section we have examined the former literature of prudence, prevention, risk and precautionary motive in investment and saving, especially in the health-wealth field with a glans on prevention. Then, we have compared the different assumptions and conclusions of the main meaningful and innovative papers. Combined with two of the most latest prevention articles, we suggested a new model to clarify the vaccination thresholds for risk neutral, averse and seeker decision makers, in order to highlight which are the components of the threshold and how they behave.

1 Introduction

If we ask people who a prudent agent is, their answer will most likely involve the behaviour of agents, which can be seen as a mental mechanism activated to answer to risk aversion. The Economics students will say that it needs a $u''' > 0$. . It has many definitions in literature, however, all the Authors wrote in the same direction.

- Kimball (1990) describes prudence as the *sensitivity of the optimal choice of a decision variable to risk*. It is, thus, the measure of the inclination of an agent to get wealthier in order to face a future risk. Intensity is calculated as
$$P(z) = -\frac{u'''(z)}{u''(z)}$$
- Gollier (2001) defines an agent prudent *if adding an uninsurable zero mean risk to his future wealth raises his optimal saving.*
- Eeckhoudt, Schlesinger (2006), moreover, states that an agent is prudent if *the lottery* $B = [-k; \widetilde{\varepsilon}]$ *is preferred to the lottery* $A = [0, \widetilde{\varepsilon} - k]$ *where all outcomes of the lotteries have equal probability, for all initial wealth levels and for all* k *and for all* $\tilde{\epsilon}$.
- Courbage (2006), summing up the previous concepts, stated that *prudence is defined as a type of preference for diversification between a sure loss and a pure risk.*

This risk could involve income or, as we study in this thesis, the future risk of incurring a medical disease. It is important to have a clear idea of what prudence is because it is the central point of the prevention notion. Even the less risk averse agents think that prevention is better than cure, but this old saying assumes that people are risk averse, prudent, with a strictly concave utility function and then doing a preventive activity ex ante instead of a cure ex post.

Prevention is one of those concepts for which, a person, may give one rigorous, or at least personal definition. If a random person is asked to provide the most rigorous definition of prevention they will answer that prevention is acting for the not occurrence of a bad event. In Courbage (2006) we may notice that an agent who sign a Medical Saving Account is prudent because they are saving money to cover some possible future medical expenses. For this reasons prevention is present in many scientific fields.

- In a medical field prevention consists of all those healthy routines, nutrition habits and measures taken in order to avoid a future disease, or to keep under control an already existing disease, which can be of many types, such as genetic or environmental. It is commonly accepted that there are three types of prevention. The primary is composed of all the preventive activities, such as avoiding smoking, having a varied diet and doing physical activity. The secondary prevention aims at the reduction of the impact of an already occurred disease or a disease which will occur with certainty, for example stopping smoking to avoid other heart attacks, going to the nutritionist in order to begin a healthier nutrition program. Tertiary prevention is conceptually similar to a cure, but it is not. If a cure aims at defeating a disease, or an infection, the tertiary prevention consists of all those mechanisms and activities adopted to reduce the impact of permanent effects, for example rehabilitation programs, group therapy for depression, etc. As Nutbeam (1998) said in the WHO forum, "Disease prevention covers measures not only to prevent the occurrence of disease, such as risk factor reduction, but also to arrest its progress and reduce its consequences once established".

- From the point of view of economics, prevention is related to an activity carried out to decrease the probability of the occurrence of a disease or a loss. It can be related to some saving activities in economics topics. One of the most important links is the concept of precautionary saving. In a life cycle, an agent may face some shocks (as an income shock). Assuming that the life plan is done ex ante, if an agent is prudent, they will do some preventive activities in order to reduce the impact of a potential shock. In Consumption Theory, these preventive activities are commonly known as precautionary motive for saving.

This thesis is structured as follows. In chapter two we will study some of the most important and innovative papers in the prevention field with regard to the health issue. Then in the third and fourth chapters we are going to deeply focus on Liu, Menegatti (2019) and Crainich et al. (2019). Finally in the fifth chapter we proposed our model.

2 Literature review

In literature of health prevention the analyses, deeply investigate optimal prevention in one or two period frameworks. All these models assume an increasing concave utility function $u'(\cdot) > 0$, $u''(\cdot) < 0$ for risk aversion, and a positive third derivative for prudence $u'''(\cdot) > 0$. In some models as those of Menegatti (2014) Brianti et al. (2018), Liu, Menegatti (2019) and Courbage, Rey (2006) the utility function is a two argument utility function $U(w, h)$ of respectively wealth[1] and health. In the latter models, $U_2(\cdot) > 0$ represents the non satiation in health, namely the willingness of being even healthier.

Moreover, while some papers, including Menegatti, Rebessi (2011), only focus on a mono variable utility of wealth, others, asCrainich et al. (2019), focus on a mono variable utility of health. Another important assumption made in all the models that include the prevention activity, denoted by e is that the probability of the occurrence of a certain disease is actually a probability function of e so that $p'(e) < 0$ and with $0 < p(e) < 1$. Excluding the boundaries means that the event is neither certain nor certainly unverifiable.

A common procedure in the analyzed models is the optimization problem in relation to a prevention effort, namely e, and, in some cases, to both prevention effort and saving or to prevention effort and cure as described by Brianti et al. (2018). It is important to highlight and take into consideration what seems to be a specific case of prevention activity, as described by Crainich et al. (2019). They wanted to compare the occurrence probability of encountering a medical disease whether the agent is vaccinated or not. We will explore in more detail these two papers.

The main conclusions of the literature in health care prevention show that the more an agent is prudent, the more their marginal cost of effort for self-protection is low and, consequently, an higher prevention activity is carried out in the first period and the probability of encountering the disease in the second period will be lower.

In this thesis we are focusing on the study and potentially the enlargement of the proposals of Liu, Menegatti (2019) and Crainich et al. (2019). These 2 models are respectively the most complete and the one treating an innovative view of health prevention and they will be deeply discussed in the next two chapters. However it is important to summarize some important and fundamental

[1] In some papers as in Eeckhoudt et al. (2012), wealth is described as income x

articles regarding health prevention and its sphere.

2.1 Eeckhoudt, Gollier (2005)'s The impact of prudence on optimal prevention

In this paper the Authors wanted to take into consideration a nonlinear utility function and develop a cost-benefit examination of prevention effort. They strongly rejected the hypotheses of risk neutrality, especially for medical diseases or catastrophic risks. This consideration is well explained and compelling because, taking into account medical diseases, as far as getting ill or not is concerned, no one, or at least a non-statistical significative percentage of the population, is neutral. Generally, there might be some individuals who are reluctant to being ill and a small percentage of self-destructive ones who prefer to incur a disease. Eeckhoudt and Gollier's aim was to analyze the link between prudence and prevention[2].

Given an initial wealth w and consequently a utility function u(w), the consequences in the same period might include:

- A loss $l > 0$ with probability $p(e)$, decreasing in effort e exerted to prevent the loss (i.e. $p'(e) < 0$) so that

  $$u(w - l - e)$$

  This means that prevention and loss both decrease the present wealth, but optimal values of effort reduce the probability of a loss.

- A situation in which the loss does not occur with probability $1 - p(e)$, but the effort for prevention is still exerted. In this case we will have a utility function such as:

  $$u(w - e)$$

It is important to stress that

$$u(w - e) > u(w - l - e)$$

[2]In a previous article, of 1985, the Author, together with Dionne, showed ambiguous links between risk aversion and prudence. Following Dionne, Eeckhoudt (1985) *for the risk-neutral agent not to exert any effort, so that the accident will be incurred with certainty. In such a situation, exerting effort would induce risk, since the probability of accident will be less than unity. This is not desirable for risk averse agents.*

One important result of this article is that, if the optimal probability $p^*(e) = 1/2$, for the risk neutral, only adds risk aversion but not prudence, it will have no effects on the optimal choice of the effort. the Authors focused on an analysis in which there is approximately $\frac{1}{2}$ probability of loss. They conclude that only the sign of the third derivative of the utility function affects preventive effort in case of a non-linear utility function. Furthermore, a prudent decision maker is less willing to undertake effort activities. This is because reducing the effort does not change the marginal level of risk, but, on the other hand, it will increase their precautionary wealth accumulation.

2.2 Courbage (2006)'s Prudence, Savings and Health Care Risks

In this paper, Christophe Courbage wanted to link the concept of prudence with a health care framework, using an innovative (at the time) prevention tool: MSA[3], the Author began expanding the basic mono- dimensional framework from a utility, function of wealth, to a two-periodal and two-dimensional von Neumann-Morgenstern utility[4], function of wealth and health. Moreover, it is assumed that the agent may contract the disease in the second period, which causes their health care expenditure.

1st period: t=0

Courbage developed a utility function $u_0(w_0, h_0)$ where h is the stock of health and w denotes wealth. The agent's prevention activity, so to say, is to decide which amount s to save and insert in their MSA.

2nd period: t=1

In the second period the agent becomes ill with a probability p and their bad health status will become $h_0 - d$, where d is the impact of the disease. Moreover, there might be a financial loss covered by the medical saving account.

[3] MSAs are financial tools which covers possible future medical or health expenses. Similar to an insurance, the agent saves, each period, part of her wealth and cashed out exclusively for health care charge.

[4] Utility functions explaining well risk averse, risk neutral and risk lovers

This paper investigated savings and prudence from a health care point of view. It proves that marginal utility of wealth is a decreasing function of health and that convexity of marginal utility is required. Despite the latter results, the Author indirectly explained that, in countries where tools as MSA are mandatory, observing data on the application of such tools may give us important results on the prudent behaviour of a person. In aggregate terms, the average prudence level of a social group. All this information may be useful to policy makers in order to 'produce' social benefits.

2.3 Menegatti (2009)'s Optimal prevention and prudence in a 2 period model

In this paper, Menegatti extended the work of Eeckhoudt, Gollier (2005) from a mono-periodal framework to a two period one, even though Menegatti maintained all the assumptions and the aim of the paper. In other words, the Author wanted to analyze the behaviour of optimal prevention in relation to the concept of prudence. In Eeckhoudt, Gollier (2005)'s model, starting from a level of effort chosen by a risk neutral agent, the prudent agent exerts a smaller effort compared to that of the starting point. This result means that prudence tends to decrease prevention.

Conversely, Menegatti (2009) explained this result and provided a less counter intuitive result. He states that we can have two 'timing' situations. In the first one, the prevention activity is carried out in the same period as the possible occurrence of the risk. In the second situation, the preventive effort is made in one period, and the risk occurs in the following one.

It is important to remember that a prudent agent wants to maximize their wealth in the period of the occurrence of the risk, hence, if the risk is in the same period of the preventive effort, they will exert less prevention activity than a risk neutral agent. On the contrary, in presence of two periods, the prudent agent will do more preventive activities in the first period (compared to the risk neutral one) in order to reduce the potential loss of wealth in the second period.

1st period: t=0

In the first period, the given initial wealth w_0 is reduced by the prevention activity, namely the effort, $e \in [0, w_0]$ implying a utility, for the first period so that $u(w_0 - e)$.

2nd period: t=1

In the second period, as usual, we can encounter a bad period with probability $p(e)$ implying a utility $u(w_1 - l)$. With probability $1 - p(e)$ we will have a good period represented by $u(w_1)$. Note that w_1 represents the wealth endowment in the second period, l is the potential loss and p is a probability function so that $p'(e) < 0$, implying that an increase in the prevention effort decreases the probability of facing the loss.

The agent decides her optimal level of prevention aiming to maximize her total utility which is given by

$$V(e) = u\left(w_0 - e\right) + p(e)u(w - l) + [1 - p(e)]u(w)$$

Assuming that $V''(e) < 0$ implies that there will be a solution to the maximization problem and also that the agent is risk averse. Starting from these assumptions, one of the most important results is: assume $p(e_n) = 1/2$, where e_n denotes the effort exerted by the risk neutral, a risk averse prudent agent (i.e. $u'''(.) > 0$) will exert higher effort compared to the risk neutral agent.

This paper is important in literature because it proves that the optimal preventive activity in a one-period framework is antipodal to the one in the two-period framework. In conclusion, both frameworks are plausible, but the affection of prudence in terms of prevention e depends on the timing of such an activity.

2.4 Menegatti, Rebessi (2011)'s On the substitution between saving and prevention

The prevention topic has begun to be considered as a two-period framework by Menegatti (2009), instead of only a single period. On the other hand, saving is a concept related to multi-period frameworks[5] by definition. For this purpose, this work by Menegatti and Rebassi aims to fill the gap in literature about the effects of analyze saving and prevention together in a two-period context. Hereafter, in the prevention and prudence literature, more and more articles and works

[5] An agent, consumer, decision maker, decides to save today in order to face a possible future negative shock or, simplier, to smooth consumption through time.

abandon the monoperiodical environment taking for granted the bi-periodical one.

1st period: t=0

In the first period, the given initial wealth w_0 is reduced by the prevention activity, namely the effort, $e \in [0, w_0]$ implying a utility, for the first period so that $u(w_0 - e)$. In this period, the agent also chooses the level of saving $s \in [0, w_0]$. In other words the initial wealth w_0 can be divided in saving s and prevention e.

2nd period: t=1

In the second period, as usual, we can encounter a bad period with probability $p(e)$ implying a utility equal to $u(w_1 - l)$. With probability $1 - p(e)$ we will have a good period so that $u(w_1)$. Note that w_1 represents the wealth endowment in the second period, l is the potential loss and p is a probability function so that $p'(e) < 0$, implying that an increase in the prevention effort decreases the probability of facing the loss.

The model introduces a time preference discount rate ρ representing the fact that usually, agents are myopic, i.e. they care more about the present than the future. Moreover, as far as saving $s \in [0, w_0]$ is concerned, an interest rate $r \in [0, 1)$ is introduced as well. After all these considerations and assumptions the optimization problem becomes[6]:

$$\max_{s,e} V(s, e)$$

where

$$V(s, e) = u(w_0 - s - e) + \frac{p(e)u(w_1 + s(1 + r) - l) + [1 - p(e)]u(w_1 + s(1 + r))}{(1 + \rho)}$$

We know that an equilibrium, even if not unique, exists[7] because the domain $0 \leq s \leq e \leq w_0$ is considered closed and bounded and, moreover, the objective function is supposed to be continuous. the Authors highlighted two main results in this model. One in which the more the agent 'spend' in effort, the less She will save (if $e \uparrow$ then $s \downarrow$). The other shows that, since $\frac{\partial s}{\partial r} > 0$ and $\frac{\partial e}{\partial r} < 0$, a

[6] With respect to Menegatti, Rebessi (2011).
[7] The Weierstrass theorem can be applied.

growth in the interest rate increases saving and lowers the prevention activity effort (if $r \uparrow$ then $s \uparrow$ and $e \downarrow$). Finally, this paper aims to point out the effect of a change in endowments on prevention and saving. The results are ambiguous until we make some strong assumptions on the marginal intertemporal utility function. the Authors conclude by saying that the assumption with which the effort is measurable in terms of wealth (this assumption is commonly accepted in the literature, and in future works) is quite fragile. They rightly claim that effort is measurable in terms of wealth if it is physical. If it is mental it is only a perception and is not fully measurable in terms of wealth.

2.5 Eeckhoudt et al. (2012)'s Precautionary effort: a new look

Eeckhoudt et al. (2012) work wanted to extend these frameworks, studying what happens when a background risk[8] is introduced in the prevention analysis. They argued that (precautionary) saving is needed to increase wealth status in both good and bad periods, effort increases the probability of the good period and insurance covers wealth losses. Assuming a non-zero mean background risk $\tilde{\epsilon}$ we will observe an increase in the optimal level of effort. This phenomenon is called 'precautionary effort' by the Authors. After adjusting the notation according to this literature review chapter, we will have a two-period framework.

1st period: t=0

In the first period, the given initial income w_0[9] can be divided into consumption and effort e. The effort reduces the probability, in the second period, to face a loss $l > 0$. Implying, as it is commonly accepted, that $p'(e) < 0$.

2nd period: t=1

In the second period we can encounter two states of income, a good one w_1^g and a bad one w_1^b:

$$u(w_1^b - l) \text{ with probability } p(e)$$

[8] A background risk is a non fully insurable risk because the decision marker cannot take it in consideration into her analysis.

[9] In the original paper, income is denoted by x_0. We preferred w_0 for an standardization purpose.

$$u(w_1^g) \text{ with probability } 1 - p(e)$$

where g represents the good period and b the bad one. Moreover, by assuming a time discount rate δ for the utility function, the decision maker will consider the following optimization problem:

$$\max_e V = u\left(w_0 - e\right) + \frac{1}{1+\delta}[(1-p(e))u(w_1) + p(e)u(w_1 - l)]$$

Furthermore, assuming a zero mean risk $\tilde{\epsilon}$, the maximization problem becomes

$$\max_e \hat{V} = u\left(w_0 - e\right) + \frac{1}{1+\delta}[(1-p(e))Eu(w_1 + \tilde{\epsilon}) + p(e)Eu(w_1 - l + \tilde{\epsilon})]$$

The Main result for this paper by Eeckhoudt, Huang and Tzeng is that, the introduction of a generic background risk, hence unknown in its nature, causes a sudden increase in the level of effort exerted for a preventive motive.

2.6 Menegatti (2014)'s Optimal choice on prevention and cure: a new economic analysis

The utility function, equal in both periods has the following assumptions on the derivatives in order to ensure risk aversion and non satiation.

$$\frac{\partial u}{\partial w} > 0, \quad \frac{\partial u}{\partial^2 w} < 0, \quad \frac{\partial u}{\partial h} > 0, \quad \frac{\partial u}{\partial^2 h} < 0$$

After these assumptions, it is convenient to divide the case in the relative periods.

1st period: t=0

The agent has a utility function $u(w, h)$. The health status is equal to h. This paper introduces a peculiar difference compared to the first period formalization. Wealth is represented by $w - qe$ where effort e is chosen by the agent and q is the financial cost for effort

2nd period: t=1

In the second period the health status is equal to $h - d$ where $d \in [0, h)$[10] is the disease effect. The bad state, as usual, may occurs with probability $p(e) \in (0, 1)$

[10] The disease magnitude is assumed to never reach the level of the health status, i.e. $d \neq h$. Otherwise the disease would have been lethal. We will see a case with lethal disease in section 3 thanks to Crainich et al. (2019)

so that $p'(e) < 0$ (if $e \uparrow$ then $p \downarrow$). In the case in which the agent did prevention, but the disease occurred anyway, She can try to exert a cure with a cost kc and, moreover, reducing the disease effect by αc.
After all those assumptions we have the optimization problem as:

$$\max_{e,c} V(e,c) = \max_{e,c} u(w - qe, h) + p(e)u(w - kc, h - d + \alpha c) + [1 - p(e)]u(w, h)$$

Along with this maximization problem, Menegatti found some interesting results. The first one shows that the optimal levels of cure and effort are independent. This seems to be trivial because a generic agent chooses the effort level to lower the probability of getting ill and, if the illness occurs anyway, they will choose the level of cure. A second result points out that an increase in the cost of prevention q will reduce the level of prevention e. Moreover, any change in the cost of prevention q will not affect the optimal level of cure c. The third conclusion is, conversely, that an increase in the cost of the cure k will increase the optimal level of prevention e as well (if the cure costs more, the agent prefers to spend more for prevention instead of expensive cures). The Authors finallyshowed that the cross derivative $u_{12} \geq 0$ then an increase in the cost of a cure k, decreases optimal prevention e. The results above are valid only for two-period frameworks because with more periods, the possibility of reinfection has to be inserted and the results may be different. The paper continues with an analysis that observes what will happen when the magnitude of the disease increases ($d \uparrow$) and when when the existence of a better cure will occurs ($c \uparrow$). Menegatti showed that increasing (decreasing) the magnitude of the disease causes an increase (decrease) of the optimal level of prevention. Moreover, if there is a better medical cure, the optimal level of prevention will be higher. As far as cross derivatives are concerned, if $u_{12} \geq 0$ then an higher disease effect causes more prevention, however, if $u_{12} < 0$ then an increase in disease effect leads to ambiguous results.

2.7 Liu, Menegatti (2019) Optimal saving and health prevention

The main objective of this paper is to study a model with a bivariate utility, a function of wealth and health, by inserting costs on wealth and health when exerting a preventive activity. A peculiarity of this work is the analysis of optimal saving and preventive effort together, for the first time in the context of health prevention. The bracketing of saving and effort is natural if we consider

a maximization problem of wealth in a two- period framework. In this paper, we are going to observe that saving and preventive effort may be both complements or substitutes[11] depending on the correlation aversion or love of the decision maker. If they are correlation lovers, they will choose optimal saving and effort levels together, otherwise they will decide as if they were substitutes. The following analysis by Liu and Menegatti represents the effect of changes in interest rate giving financial returns over saving. The results may be different, depending on the influence of substitution effect and income effect, the correlation-type of the decision maker, and if the decision maker is a borrower or a saver.

Assume a decision maker with a utility function such as $u(w, h)$, where w is wealth and h is health. Assume that

$$\frac{\partial u}{\partial w} = u_1 > 0 \quad \frac{\partial u}{\partial^2 w} = u_{11} < 0 \quad \frac{\partial u}{\partial h} = u_2 > 0 \quad \frac{\partial u}{\partial^2 h} = u_{22} < 0$$

so that the agents are risk averse and the non satiation condition is satisfied. There are no assumptions on the cross derivative of the utility function, but we can simply state that if $u_{12} > 0$, the agent is correlation lover and if $u_{12} < 0$ is correlation averse. Liu, Menegatti (2019) considered, as we have already seen, a two-period framework, with a constant utility function over time.

1st period: t=0

In the first period, the Authors assumed a strictly positive initial endowment of wealth w_0 which can be consumed or saved. Saving gives a return $R \geq 1$. The initial health status h_0 is given. Moreover, in this period, the (effort level) preventive activity e is decided and made. According to this assumption, the initial health level has to be reduced by the effort level, $h_0 - e$ and the initial wealth level has to be reduced by saving and the financial cost of prevention, hence $w_0 - s - q(e)$. Thus, the utility function is:

$$u(w_0 - s - q(e), h_0 - e)$$

[11] The difference between substitute and complementary goods is quite trivial. Substitutes can be intended as two goods for which a change in the demand of the first triggers an opposite fluctuation in the market of the second. An example may be mango and papaya: if the price of mango rises, the agent will choose papaya and vice versa. Two complementary goods, such as sugar and coffee, are two goods for which an increase of the price of one causes a reduction in the demand of the other.

2nd period: t=1

In the second period, it is still assumed a $w_1 > 0$ certain and divisible in saving and consumption. Moreover, health status h_1 can be different considering whether the agent is facing a good or a bad period. It is commonly accepted to divide the second period in good and bad situations. h_1^g is the health's good period occurred with probability $1 - p(e)$. For the same reason h_1^b is health's bad period occurred with a probability $p(e)$. It is necessary to be warned that the Authors assumed $h_1^g > h_1^b > 0$. It is, furthermore, appropriate to remember that $p'(e) < 0$ meaning that a higher prevention activity exerted, will reduce the probability of facing a bad period. To summarize, in the second period we have:

$$u(sR + w_1, h_1^b) \text{ with probability } p(e)$$

$$u(sR + w_1, h_1^g) \text{ with probability } 1 - p(e)$$

After these assumptions the decision maker will solve the following optimization problem:

$$\begin{aligned}\max_{s,e} U(s,e) =& u\left(w_0 - s - c(e), h_0 - e\right) \\ &+ p(e)u\left(sR + w_1, h_1^b\right) + (1 - p(e))u\left(sR + w_1, h_1^g\right)\end{aligned}$$

The main results for the optimal choices of saving and health prevention may be summarized as follows. Starting from an equilibrium situation, in case of a correlation lover decision maker, increasing (decreasing) one instrument level (e or s) will increase (decrease) the marginal utility of the other. Conversely, in the case of correlation aversion, increasing (decreasing) one instrument level (e or s) will decrease (increase) the marginal utility of the other, implying that the agent will choose oppositely saving and preventive activities. The presence of costs in both health and wealth pushes the decision maker to be more correlation averse, confirming the literature relative to this issue.

Liu and Menegatti then wanted to show the consequences of an increment in the interest rate level to saving and the preventive effort. In the literature of Consumption and Saving Theory, when the interest rate is increasing, there are two opposite possibilities. An increase in saving, known as substitution effect, and a reduction in saving known as income effect. The results of this problem may be interpreted as follows:

- When the decision maker is correlation averse, increasing the interest rate causes a reduction in health care prevention. This result implies, again, that saving and health prevention can be seen as substitutes.
- When the decision maker is correlation lover, increasing the return on saving implies an increase in the optimal level of prevention exerted. This result implies, again, that saving and health prevention can be seen as substitutes.

3 Brianti et al. (2018)'s Optimal choice of prevention and cure under uncertainty on disease effect and cure effectiveness

In the literature seen so far, we observed agents or decision makers dealing with uncertainty in the future level of wealth or health. As far as the health sphere is concerned, the agent has and wants to react to an uncertain future state of health. They may be sick or not and can reduce this probability exerting a prevention activity. In the unlucky case in which they face a bad period, they may reduce the magnitude of the disease by finding a cure. One of the papers about this topic is due to Menegatti (2014). It is important to say that uncertainty on the possibility of facing a disease is not the only risk the agent has to deal with. Moreover, if a patient has an immune system disorder, the same sickness will have different consequences compared to the ones who do not have it. This was the reason such uncertainties and risks on disease effect were introduced. Along the same reasoning procedure, the disease may be different by nature such as heart attacks and flu. Respectively, the first can have lethal or almost null consequences, the second may have many more implications with higher variability of side effects. In these lines we have just spoken colloquially about the variability of side effects and disease strength. It is natural to formalize it mathematically inserting another risk to the framework. Introducing uncertainty in the side of cure is important for multiple reasons. One can be that the effect of a cure or a treatment depends on the 'body resilience' of the patient, it may depend on the clinic where the patient receives the treatment (in some countries expensive private clinics provide better services than public ones, in others public clinics provide a really good service). This goes by the name of 'cure effectiveness' and it is commonly accepted to be treated as the disease effect, but with a positive effect on the health status. This paper by Brianti et al. (2018) aims at introducing uncertainty on effects of disease and cure, and verifying the implications on optimal prevention and cure choices.

3.1 Assumptions and notification

Here, as it is commonly accepted in the health prevention literature, the agent has a utility function with two arguments $u(w, h)$ where W represents wealth

and H the health status. It is assumed non-satiation in both wealth and health, hence $u_1 > 0$ and $u_2 > 0$. Moreover, conveniently, is assumed risk aversion in both variables, i.e. $u_{11} < 0$ and $u_{22} < 0$. Consequently, since it does not change the framework's results, it is assumed that the utility function does not change through periods and the time discount rate is negligible.

1st period: t=0

In the first period wealth and health are given as the initial endowment. Moreover, the choices about optimal prevention and cure have to be made in this period, hence the individual exerts a prevention activity e to lower the probability of getting ill in the second period. For this purpose the bivariate utility function can be rewritten as $u(w - e, h)$.

2nd period: t=1

Health status h_1 can be different considering a good or a bad period. We may have a good period with probability $1 - p(e)$ where health is represented by h_1^g and we may have a bad period with probability $p(e)$ where, despite of the effort exerted doing prevention activities, the illness occurs and health is h_1^b so that $h_1^b < h_1^g$. The health status in the 2nd period is

$$h - d + \alpha c$$

where d represents the disease effect, $c \geq 0$ the cure level and $\alpha > 0$ the effectiveness of the cure. In the case of a good period, both the disease effect, the cure level and, consequently the cure effectiveness are null.

In terms of occurrence, the agent, the probability of the bad event (i.e. $p(e)$) may be reduced by exerting more preventive effort e. It is assumed, indeed, $p'(e) < 0$. Wealth, also in the second period is positive, but it has to be reduced by the eventual cost of the cure level. That is

$$w - kc$$

where k is the cost of the cure.

3.2 The basic model

In the previous section we have seen the assumptions for the frameworks and the models we are going to study. Moreover, the Authors want to neglect

saving in the analysis because it is focused on health and cure. Given all the above statements, the decision maker wants to solve the following optimization problem

$$\max_{e,c} V(e,c,d,\alpha) = u(w-e,h) + (1-p(e))u(w,h) + p(e)u(w-\kappa c, h-d+\alpha c)$$

where k is the financial cost of the cure. Taking into consideration the first order conditions with respect to both e and c we have

$$V_e\left(e^*,c^*,d,\alpha\right) = p'\left(e^*\right)\left[u\left(w-\kappa c^*, h-d+\alpha c^*\right) - u(w,h)\right] - u_1\left(w-e^*,h\right) = 0$$

and

$$V_c\left(c^*,d,\alpha\right) = \alpha u_2\left(w-kc^*, h-d+\alpha c^*\right) - \kappa u_1\left(w-\kappa c^*, h-d+\alpha c^*\right) = 0$$

it is easily provable that

$$\left[u\left(w-\kappa c^*, h-d+\alpha c^*\right) - u(w,h)\right]$$

is negative, implying that the agent's utility is higher when She is healthy than when She is not.

3.3 The model with uncertainty on disease effect

In this section, Brianti et al. (2018) assumed the disease effect as a generally distributed random variable $\tilde{d}$ with $E[\tilde{d}] = d$. This problem is represented by the following

$$\max_{e,c} E[V(e,c,\tilde{d},\alpha)] = u(w-e,h) + (1-p(e))u(w,h) + p(e)E[u(w-\kappa c, h-\tilde{d}+\alpha c)]$$

Solving this maximization problem, the Authors, stated that uncertainty on disease effect increase the cure level if and only if

$$\alpha u_{222}\left(w-kc^*, h-d+\alpha c^*\right) > \kappa u_{122}\left(w-kc^*, h-d+\alpha c^*\right)$$

u_{222} measures the prudence in health and u_{122} measures the cross imprudence in wealth. A result for Brianti et al. (2018) is that *under prudence in health and cross-imprudence in wealth, the presence of uncertainty raises marginal benefit and reduces marginal cost of cure, determining, in turn, an increase in its optimal level.* The latter result is the sum of two concepts. One for which the marginal benefit provided by the cure increases when uncertainty is introduced (prudence in health). For the other, there is a lower marginal cost of cure when there is uncertainty. Hence, in case of uncertainty, prevention is always increased, despite of the cure level depending on prudence/imprudence.

3.4 The model with uncertainty on cure effectiveness

In this section, Brianti et al. (2018) assumed the cure effectiveness as a generally distributed random variable $\tilde{\alpha}$ with $E[\tilde{\alpha}] = \alpha$. This problem is represented by the following

$$\max_{e,c} E[V(e,c,d,\tilde{\alpha})] = u(w-e,h)+(1-p(e))u(w,h)+p(e)E[u(w-\kappa c,h-d+\tilde{\alpha}c)]$$

One main result for this section is the one for which the cure effectiveness level is twofold. An uncertain effect of the cure increases the optimal cure level when $\eta - \zeta > 2$ where

$$\eta = -c^* \alpha \frac{u_{22}\left(w - kc^*, h - d + \alpha c^* + \gamma e^*\right)}{u_{22}\left(w - kc^*, h - d + \alpha c^+ + \gamma e^*\right)}$$

and

$$\zeta = -c^* K \frac{u_{122}\left(w - kc^*, h - d + \alpha c^* + \gamma e^*\right)}{u_{22}\left(w - kc^*, h - d + \alpha c^* + \gamma e^*\right)}$$

Introducing uncertain effects of cure as well, we observe an increase of optimal prevention prompting us that uncertainty in general implies more prevention for a precautionary motive. Another important result in this section is that, since uncertain effects of cure is a multiplicative risk[12], a higher cure level increases the expectation of health in the second period but also increases its randomness.

If we want to compare the result of uncertainty of both cure effectiveness and disease effects analyzed by this paper, we may have observed that, regarding prevention, it raises in both cases, and regarding the cure level it depends on the case examined. *The proposition n.5 of Brianti et al. (2018) states: if an agent raises cure in the presence of uncertainty on cure effectiveness then she will also raise cure in the presence of uncertainty on disease effect. if an agent reduces cure in the presence of uncertainty on disease effect then she will also reduce cure in the presence of uncertainty on cure effectiveness.*

3.5 The model with the interactions between prevention and cure

Brianti et al. (2018) wanted to enlarge their study to a more complex but realistic situation in which the prevention activity and the cure effectiveness interact.

[12] According to Wong, Chang (2005) *An individual is multiplicative risk prudent, if multiplying a pure risk to her future wealth raises her optimal savings.*

The term γe is introduced as the proof of the fact that increasing prevention increases health status level. the Authors begin with the basic model (with the new γ) and assume that the problem admits solutions. The agent may wants to solve the following optimization problem:

$$\max_{e,c} V(e,c,d,\alpha) = u(w-e,h)+(1-p(e))u(w,h)+p(e)u(w-\kappa c, h-d+\alpha c+\gamma e)$$

Implying first order conditions, with respect to the cure level and the effort exerted

$$\begin{aligned} V_c\left(c^+, e^+ d, \alpha\right) =& \alpha u_2\left(w-kc^+, h-d+\alpha c^+ + \gamma e^+\right) + \\ & - \kappa u_1\left(w-\kappa c^+, h-d+\alpha c^+ + \gamma e^+\right) = 0 \end{aligned}$$

and

$$\begin{aligned} V_e\left(e^+, c^+, d, \alpha\right) =& p'\left(e^+\right)\left[u\left(w-\kappa c^+, h-d+\alpha c^+ + \gamma e^+\right) - u(w,h)\right] + \\ & - u_1\left(w-e^+, h\right) + \gamma p\left(e^+\right) u_2\left(w-\kappa c^+, h-d+\alpha c^+ + \gamma e^+\right) = 0 \end{aligned}$$

3.5.1 Uncertainty on Disease Effect

Introducing the γ term in the health part, meaning interactions through cure and prevention, and uncertainty on the effect of the disease, the agent wants to maximize

$$\max_{e,c} V(e,c,\tilde{d},\alpha) = u(w-e,h)+(1-p(e))u(w,h)+p(e)E[u(w-\kappa c, h-\tilde{d}+\alpha c+\gamma e)]$$

In order to have an increasing optimal level of cure and prevention, differently from the framework with no interactions, the following three sufficient conditions have to be satisfied.

- Prudence in health, $u_{222} > 0$
- Cross imprudence in wealth, $u_{122} < 0$
- $V_{c,e}(e,c,d,\alpha) > 0$

The first two conditions need to be studied together. These two conditions entail, in presence of uncertainty, a higher marginal benefit of the cure level and, at the same time, its lower marginal cost. Even though this result is similar to the non-interaction model, it has to be completed by a further step. Here, prevention and cure interact, indeed prevention affects both the possibility of occurrence of the bad period and the health status when the disease occurs.

As far as the third condition is concerned, stating $V_{c,e}(e, c, d, \alpha) > 0$, hence a positive cross derivative, in this case, points a complementarity between cure and prevention. The latter notion implies that, since there is complementarity between the two parameters, with uncertainty, the decision maker, will move her decisions on cure and prevention levels in the same direction. That is, if $e \uparrow$ then also $c \uparrow$.

3.5.2 Uncertainty on cure effectiveness

Introducing the γ term in the health part, meaning interactions through cure and prevention, and uncertainty on the effectiveness of the cure, the agent wants to maximize

$$\max_{e,c} V(e, c, d, \tilde{\alpha}) = u(w-e, h)+(1-p(e))u(w, h)+p(e)E[u(w-\kappa c, h-d+\tilde{\alpha}c+\gamma e)]$$

The main result, showed by Brianti et al. (2018), for this problem is that, assuming Edgeworth-Pareto complementarity of cure and prevention, i.e. $V_{c,e}(e, c, d, \alpha) > 0$, then prudence in health, $u_{222} > 0$, and $\eta' - \zeta' > 2$ are *sufficient conditions for optimal prevention and cure to increase.*

$$\eta = -c'^{+}\alpha \frac{u_{22}\left(w - kc^{+}, h - d + \alpha c^{+} + \gamma e^{+}\right)}{u_{22}\left(w - kc^{+}, h - d + \alpha c^{+} + \gamma e^{+}\right)}$$

and

$$\zeta = -c^{+}K \frac{u_{122}\left(w - kc^{+}, h - d + \alpha c^{+} + \gamma e^{+}\right)}{u_{22}\left(w - kc^{+}, h - d + \alpha c^{+} + \gamma e^{+}\right)}$$

where c^{+} and e^{+} the optimal levels of cure and prevention.

3.6 Implications

With no interactions, in the case with uncertainty on the disease effectiveness or in the case with uncertainty on effect of cure, the optimal prevention effort increases. This means that more uncertainty presses the agent to put more effort in order to decrease the probability of encountering a disease. This is because agents are usually prudent and risk averse. On the other hand, with uncertainty on both factors[13], the agent prefers to increase both prevention and cure effort in order to minimize the probability of encountering a disease. Moreover it is sufficient to assume $V_{c,e}(e, c, d, \alpha) > 0$ and prudence in health to state that, in presence of uncertainty, both prevention and cure are raised if $\eta' - \zeta' > 0$.

[13] Topic treated for the first time in the literature

4 Crainich et al. (2019)'s Vaccination as a trade-off between risks

When discussing about prevention with others, one of the first thought are vaccines. Vaccination should counter some illnesses, or at least drastically reduce the impact of the disease. Even though in first world countries it defeated certain diseases (thanks to the so-called herd immunity) and it is beneficial for the society, groups of people criticized and wanted to boycott this system[14]. Today, the members of a socio-political movement, known as anti-vax, claim that they have not been vaccinated both because they do not want to give money to pharmaceutical multinational firms and because they are scared of 'becoming' autistic as a consequence of the vaccination. The latter common statement and excuse comes from a fake study by a former doctor, Andrew Wakefield. Vaccines usually cause side effects, but sometimes people overestimate and misperceive them. In lower income countries vaccination is not a matter of an individual's or a decision maker's choice. Agents vaccine themselves only if the vaccine is available.

This paper is important because it introduces, in literature, the risks of side effects deriving from prevention activity which, in this case, is vaccination. Deciding whether to get vaccinated or not is similar to deciding whether doing prevention activities or not. The Decision Makers' problem is similar because it aims at reducing the probability of contracting a future disease. However, it differs in the structure of the problem. While the standard 'health prevention problem' is related to a maximization problem, this vaccination problem is similar to the vaccination and non-vaccination scenarios and because it is a dichotomous problem (the agent vaccinates or does not), it has more of a game theory structure. In this paper Crainich et al. (2019), will cover three cases. First, a mono-period framework. Secondly, a mono periodical framework, but with a lethal disease preventable with vaccines. Finally, a two-period framework in which side effects occur before the main disease. In the last part of the present work uncertainty is discussed in the severity of the diseases.

[14] In early history people thought that vaccines were harmful, since they came from cowpox bacteria, ignoring that was precisely the purpose of doctor Edward Jenner's discovery.

4.1 Assumptions and notation

This paper has a framework structure that is slightly different from the previous paper of Brianti et al. (2018). The agent may incur two types of diseases: a primary disease and side effects. The agent choose whether to get a voluntary vaccine with no financial costs, or not.

- The primary disease, or disease 1 is the disease preventable with the vaccine which may occur with probability $0 < p < 1$ and an intensity measured with M_1. In those terms, the vaccination drastically reduces the probability of contracting the primary disease. This effect is denoted by Δ, which can be seen as a parameter of the effectiveness of the vaccination. We can say, therefore, that $0 < \Delta \leq p < 1$[15].

- The secondary disease, or disease 2 are all the side effects deriving from the vaccination. The secondary disease may occurs with probability $0 < q < 1$ and its intensity measured with M_2. It is relevant to clarify that this disease will surely not occur in case of no vaccination. Moreover, another important assumption is that $M_2 < M_1$

4.2 One-period framework

In this framework, the agent decides to get vaccinated or not and both the primary and side effects may occur simultaneously in the same period. For this purpose, Crainich et al. (2019) wanted to find the risk averse threshold Δ for which is indifferent being vaccinated or not. Both for the risk averse, indicated with a and for the risk neutral n. To do this it is convenient to equalize the case of no-vaccination and the case with vaccination as it follows

$$\begin{aligned}(1-p)u\left(H_0\right)+pu\left(H_0-M_1\right)&=\left(1-p+\Delta^a\right)\left[(1-q)u\left(H_0\right)+qu\left(H_0-M_2\right)\right]\\&+\left(p-\Delta^a\right)\left[(1-q)u\left(H_0-M_1\right)+qu\left(H_0-M_2-M_1\right)\right]\end{aligned}$$

This equality represents, on the left hand side a weighted average between the benefit of vaccination without and in presence of sides effects. On the right hand side of the equality, is represented a weighted average as well, but between the presence and absence of disease 1. Isolating Δ^a

[15] The case of $\Delta = p$ represents the complete effectiveness of the vaccine.

$$\Delta^a \left[(1-q)\left[u\left(H_0\right)-u\left(H_0-M_1\right)\right]+q\left[u\left(H_0-M_2\right)-u\left(H_0-M_1-M_2\right)\right]\right]$$
$$= q\left((1-p)\left[u\left(H_0\right)-u\left(H_0-M_2\right)\right]+p\left[u\left(H_0-M_1\right)-u\left(H_0-M_1-M_2\right)\right]\right)$$

$$\Delta^a = \frac{q\left((1-p)\left[u\left(H_0\right)-u\left(M_2\right)\right]+p\left[u\left(H_0-M_1\right)-u\left(H_0-M_1-M_2\right)\right]\right)}{(1-q)\left[u\left(H_0\right)-u\left(H_0-M_1\right)\right]+q\left[u\left(H_0-M_2\right)-u\left(H_0-M_1-M_2\right)\right]}$$

Moreover, once applied the mean value theorem, the Authors wrote the risk averse threshold Δ^a as

$$\Delta^a = \frac{qM_2\left((1-p)u'\left(\alpha_2\right)+pu'\left(\alpha_3\right)\right)}{M_1\left((1-q)u'\left(\alpha_1\right)+qu'\left(\alpha_4\right)\right)}$$

For a risk neutral agent, which has a linear marginal utility function, the affinity condition for the utility function applies. Affinity holds if and only if $f(x_1, ..., x_n) = ax_1 + ... + ax_n + b$. Applying this condition to the utility functions, leads us to the following threshold for a risk neutral agent:

$$\Delta^n = \frac{qM_2}{M_1}$$

Comparing the two thresholds, it is shown that risk averse agents are more willing to get vaccinated, indeed $\Delta^a < \Delta^n$, if the following inequality holds. Otherwise, we will have that a risk averse agent less prone to vaccination than the risk neutral one.

$$(1-p)u'\left(\alpha_2\right)+pu'\left(\alpha_3\right) < (1-q)u'\left(\alpha_1\right)+qu'\left(\alpha_4\right)$$

This may holds for sufficiently low values of p and sufficiently high values of q.

4.3 1t framework lethal disease

In this section the Author presented a one-period model in which when disease 1 occurs, it is lethal. This condition implies that the secondary disease, i.e. the side effects of the vaccination, occurs only if the primary disease have not occurred. The model will be almost identical to the no-lethal first disease model.

It is slightly different because it will not consider the side effects in the case of occurrence of the primary disease. That is:

$$\begin{aligned}&(1-p)u\left(H_0\right)+pu\left(H_0-M_1\right)\\&\quad=\left(1-p+\Delta^a\right)\left[(1-q)u\left(H_0\right)+qu\left(H_0-M_2\right)\right]\\&\quad+\left(p-\Delta^a\right)u\left(H_0-M_1\right)\end{aligned}$$

Isolating Δ^a we reach

$$\Delta^a=\frac{(1-p)q\left[u\left(H_0\right)-u\left(H_0-M_2\right)\right]}{u\left(H_0\right)-u\left(H_0-M_1\right)-q\left[u\left(H_0\right)-u\left(H_0-M_2\right)\right]}$$

And, as usual, applying the mean value theorem, a more precise result is found.

$$\Delta^a=\frac{(1-p)qM_2u'\left(\alpha_2\right)}{M_1u'\left(\alpha_1\right)-qM_2u'\left(\alpha_2\right)}=\frac{(1-p)qM_2}{M_1\frac{u'(\alpha_1)}{u'(\alpha_2)}-qM_2}$$

With a linear utility function, hence when the agent is risk neutral, the threshold is represented by:

$$\Delta^n=\frac{(1-p)qM_2}{M_1-qM_2}$$

Given the fact that $M_1>M_2$ holds $u'(\alpha_1)>u'(\alpha_2)$ as well, implying that the risk averse agent is more vaccination prone.

4.4 Two-period framework

In this section Crainich et al. (2019) wanted to stretch some assumptions introducing a two-period model and the secondary disease or side effects. These effects are assumed to occur in the first period, just vaccinated. According to these assumptions and the studied framework we have:

1st **period: t=0**

The agent decides to get vaccinated or not and she can remain healthy with probability $1-q$ or face side effects with probability q.

2nd **period: t=1**

In the second period the agent may face the primary disease with probability p if She is not vaccinated or with probability $p-\Delta$ later on vaccination.

Now, as in the one period framework, the vax-NoVax situations have to be equalized to find both the risk averse and risk neutral thresholds.

$$\begin{aligned}&u(H_0)+(1-p)u(H_0)+pu(H_0-M_1)\\&=(1-q)u(H_0)+qu(H_0-M_2)+(1-p+\Delta)u(H_0)\\&+(p-\Delta)u(H_0-M_1)\end{aligned}$$

The risk averse threshold, is calculated as

$$\Delta^a = q\frac{u(H_0)-u(H_0-M_2)}{u(H_0)-u(H_0-M_1)}$$

And, through the mean value theorem we obtain

$$\Delta^a = q\frac{M_2u'(\alpha_2)}{M_1u'(\alpha_1)}$$

Meanwhile, the risk neutral's threshold is

$$\Delta^n = q\frac{M_2}{M_1}$$

Given these results, the Authors showed that risk neutral decision makers are less vaccination prone than the risk averse, i.e. $\Delta^a < \Delta^n$.

4.5 Random disease effects

We have seen two frameworks in which the occurrences of the diseases are uncertain, but their magnitude are certain. Now the paper continues analyzing the same frameworks with uncertainty also in the disease's effects. To formalize this, it is better to set, first M_1 as a random variable $\tilde{M}_1$ with its expectation

$E[\tilde{M}_1]$. The findings of this section are equivalent to those in previous ones, namely without randomness in the effect of primary disease. Hence, the risk averse agents are more vaccination prone compared to risk neutral agents, but in a stronger way. Conversely, having a random effect on the secondary disease, $\tilde{M}_2$ with its expected value $E[\tilde{M}_2]$, gave us ambiguous results in all three cases.

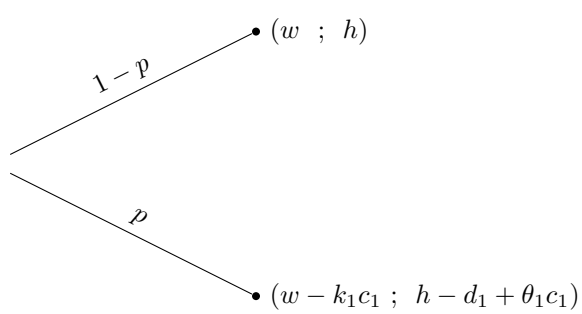
5 Thresholds for vaccination in a wider model including costs of cure on both side and primary effects

In our model the determinants of vaccination are no longer only the probability and severity of the primary disease, the efficiency of vaccination and its side effects. We have broadened the framework for the economic expenses of vaccination and cure for both the main and secondary impacts of the disease and the vaccine itself, and the cure for both illnesses. This chapter is intended to confirm or deny the prevalent understanding that a risk lover and a risk averse agent are acting against to preserve a higher standard of health and wealth. By the manner, the question is, instead of the risk seeker the risk lover results to be more willing to vaccinate.

5.1 Assumptions and notification

Case without vaccination

In this simple case of no vaccination the agent may incurs, with a probability $p \in (0,1)$[16], in a primary disease, denoted as d_1. The effect of the primary disease is countered with a cure c_1 which has a positive effect $\theta_1 c_1$ on health and also has a cost $k_1 c_1$ imputable to the the wealth side. On the other hand, the agent may remain healthy with probability $1-p$. The framework is more understandable looking at the following diagram:

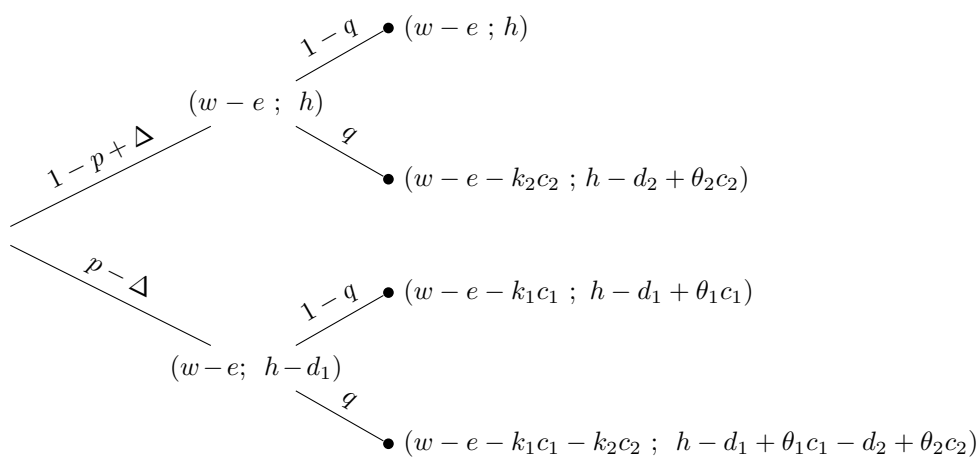


[16] It is important to recall that the choice of exclude the certain event and the null event create a more risky and realistic situation.

Case with vaccination

In the case of vaccination the latter framework has to be slightly complicated. We are assuming that exists a vaccination with effect Δ aimed to reducing the probability of occurrence of the primary disease. In other words it can be considered the preventive activity. For sufficiently high values of Δ, the individual will be more vaccination prone. Even though the vaccination is done, the agent may incurs in the primary disease anyway. In this case She may incur in the disease 1 with probability $p - \Delta$ and She will counteract the illness with a cure c_1 having an effect $\theta_1 c_1$, positive on the health side, and a cost $k_1 c_1$, negative on the wealth side. Then, with probability $1 - p + \Delta$ remains healthy.

Vaccination may provoke side effects, which we have summarized with the variable d_2. Those effects may happen with a probability q, which, although remote, is still positive such as $q \in (0, 1)$. The effect d_2 can be mitigated or, in some cases eliminated, with a cure c_2 whose effect is $\theta_2 c_2$ and whose cost is $k_2 c_2$. The framework is more understandable looking at the following diagram:



We have equalized all the possible situations in the following form, both with and without vaccination.

$$
\begin{aligned}
&(1-p)u(w,h) + p(u(w-k_1c_1, h-d_1-\theta_1c_1)) = \\
&= (1-p+\Delta)[(1-q)u(w-e,h) + q(u(w-e-k_2c_2, h-d_2+\theta_2c_2))] + \\
&+(p-\Delta)[(1-q)u(w-e-k_1c_1, h-d_1+\theta_1c_1) + \\
&+q(u(w-e-k_1c_1-k_2c_2, h-d_1+\theta_1c_1-d_2+\theta_2c_2))]
\end{aligned}
\tag{1}
$$

In the following chapters, we would like to evaluate and address the three primary agent behaviors, i.e. risk-neutral, risk-averse and risk-seeking. To do this, we started assuming that the generic utility function U(w,hH) is additive separable[17], which is

$$U(w,h) = u(w) + u(h)$$

In addition, we suggested a sum of logarithmic functions for the risk averse agent, i.e. concave utility functions, and a paraboloid to please the risk seeker.

To find an expression for the threshold in order to satisfy an agent we can now, rename, for simplicity:

- $A = u(w,h)$
- $B = u(w - k_1c_1, h - d_1 - \theta_1c_1)$
- $C = u(w - e, h)$
- $D = u(w - e - k_2c_2, h - d_2 + \theta_2c_2)$
- $E = u(w - e - k_1c_1, h - d_1 + \theta_1c_1)$
- $F = u(w - e - k_1c_1 - k_2c_2, h - d_1 + \theta_1c_1 - d_2 + \theta_2c_2)$

Substituting in the equality (1) we have

$$\begin{aligned}(1-p)A + p(B) = (1 - p + \Delta[(1-q)C+ \\ +q(D)] + (p-\Delta)[(1-q)E + q(F)]\end{aligned} \tag{2}$$

$$\begin{aligned}(1-p)A + pB = (1-p)[(1-q)C + qD] + \Delta[(1-q)C + qD] \\ +p[(1-q)E + qF] - \Delta[(1-q)E + qF]\end{aligned}$$

$$\Delta[(1-q)C+qD-(1-q)E-qF] = (1-p)A+pB-(1-p)[(1-q)C+qD]-p[(1-q)E+qF]$$

$$\Delta = \frac{(1-p)[A-(1-q)C-qD] - p[B-(1-q)E-qF]}{(1-q)(C-E)+q(D-F)}$$

The latter expression for Δ may be rearranged as

$$\Delta = \frac{[(1-p)(A-C)+p(B-E)] + q[(1-p)(C-D)+p(E-F)]}{(1-q)(C-E)+q(D-F)} \tag{3}$$

[17] This topic is covered in Gorman (1968).

5.2 A threshold for a risk averse agent

Knowing that a risk averse agent has a concave utility function, we we consider the case of the sum of logarithms. That is, given the parameters $a, b > 0$, assuming separability and additivity on a concave utility function so that

$$u(w,h) = a \ln w + b \ln h$$

Since the first and second derivatives are, respectively

$$u_1'(w,h) = \frac{a}{w},\ u_2'(w,h) = \frac{a}{h}$$

and

$$u_1''(w,h) = -\frac{a}{w^2},\ u_2''(w,h) = -\frac{a}{h^2}$$

recalling that the absolute risk aversion is $A = -\frac{u''(\bullet)}{u'(\bullet)}$, quod erat demostrandum, we have a positive aversion.

The generalized Mean Value Theorem as in Carter (2001) says that *Let $f : S \to \mathbb{R}$ be a differentiable functional on a convex neighborhood S of $\mathbf{x_0}$. For every $\mathbf{x} \in S - \mathbf{x_0}$, there exists some $\overline{\mathbf{x}} \in (\mathbf{x_0}, \mathbf{x_0} + \mathbf{x})$ such that*

$$f(\mathbf{x_0} + \mathbf{x}) = f(\mathbf{x_0}) + Df[\overline{\mathbf{x}}](\mathbf{x}) \tag{4}$$

Given the expression (3) we apply the Mean Value Theorem six times. By stating the existence of six points, let us say (w_j, h_j) where $j = 1, ..., 6$ such that:

- $A - C = u(w,h) - u(w-e,h) = \frac{a}{w_1} e$
  where $w - e < w_1 < w$ and $h < h_1 < h$[18]
- $C - D = u(w-e,h) - u(w-e-k_2c_2, h-d_2+\theta_2c_2)$
  $= \frac{a}{w_2} k_2c_2 + \frac{b}{h_2}(d_2 - \theta_2c_2)$
  where $w - e - k_2c_2 < w_2 < w - e$ and $h - d_2 + \theta_2c_2 < h_2 < h$
- $B - E = u(w-k_1c_1, h-d_1-\theta_1c_1) - u(w-e-k_1c_1, h-d_1+\theta_1c_1) = \frac{a}{w_3} e$
  where $w-e-k_1c_1 < w_3 < w-k_1c_1$ and $h-d_1+\theta_1c_1 < h_3 < h-d_1+\theta_1c_1$[19]
- $E - F = u(w-e-k_1c_1, h-d_1+\theta_1c_1) - u(w-e-k_1c_1-k_2c_2, h-d_1+\theta_1c_1-d_2+\theta_2c_2) = \frac{a}{w_4} k_2c_2 + \frac{b}{h_4}(d_2 - \theta_2c_2)$
  where $w - e - k_1c_1 - k_2c_2 < w_4 < w - e - k_1c_1$ and
  $h - d_1 + \theta_1c_1 - d_2 + \theta_2c_2 < h_4 < h - d_1 + \theta_1c_1$

[18] Hence we can write $h_1 = h$.
[19] Hence we can write $h_3 = h - d_1 + \theta_1c_1$.

- $C - E = u(w - e, h) - u(w - e - k_1c_1, h - d_1 + \theta_1c_1)$
  $= \frac{a}{w_5}k_1c_1 + \frac{b}{h_5}(d_1 - \theta_1c_1)$
  where $w - e - k_1c_1 < w_5 < w - e$ and $h - d_1 + \theta_1c_1 < h_5 < h$

- $D - F = u(w - e - k_2c_2, h - d_2 + \theta_2c_2) - u(w - e - k_1c_1 - k_2c_2, h - d_1 + \theta_1c_1 - d_2 + \theta_2c_2) = \frac{a}{w_6}k_1c_1 + \frac{b}{h_6}(d_1 - \theta_1c_1)$
  where $w - e - k_1c_1 - k_2c_2 < w_6 < w - e - k_2c_2$ and
  $h - d_1 + \theta_1c_1 - d_2 + \theta_2c_2 < h_6 < h - d_2 + \theta_2c_2$

Now, substituting those values in (3) we have a threshold for the risk averse agent, namely Δ^{RA} so that

$$\Delta^{RA} = \frac{(1-p)\frac{a}{w_1}e + p\frac{a}{w_3}e}{(1-q)[\frac{a}{w_5}k_1c_1 + \frac{b}{h_5}(d_1 - \theta_1c_1)] + q[\frac{a}{w_6}k_1c_1 + \frac{b}{h_6}(d_1 - \theta_1c_1)]} \tag{5}$$

$$+ \frac{q[(1-p)(\frac{a}{w_2}k_2c_2 + \frac{b}{h_2}(d_2 - \theta_2c_2)) + p(\frac{a}{w_4}k_2c_2 + \frac{b}{h_4}(d_2 - \theta_2c_2)]}{(1-q)[\frac{a}{w_5}k_1c_1 + \frac{b}{h_5}(d_1 - \theta_1c_1)] + q[\frac{a}{w_6}k_1c_1 + \frac{b}{h_6}(d_1 - \theta_1c_1)]}$$

The latter expression of the vaccination threshold for a risk averse agent implies that, in order to maintain a certain level of wealth and health, the risk averse decision maker is more vaccination prone. That is, increasing w and h will increase the values of the interval in which w_j and h_j lie (where $j = 1, ..., 6$). Since w_j and h_j increase, Δ^{RA} decreases.

Moreover, as a further result, we may observe that, the cost of the cure, the disease impact on health and, even if negatively, the cure effectiveness, together represent the 'magnitude' of the disease, either primary or secondary. Made those niceties, the latter expression for Δ^{RA}, shows that there may be a common pattern with the expression in the article by Crainich et al. (2019)

$$\Delta^a = \frac{qM_2[(1-p)u'(\alpha_2) + pu'(\alpha_3)]}{M_1[(1-p)u'(\alpha_1) + qu'(\alpha_4)]}$$

where M_1 and M_2 represent respectively the impact of the primary and secondary disease (and, in the latter case, α does not represent the effect of the cure).

5.3 A threshold level for a risk neutral agent

We know that, a risk neutral agent has a linear utility function. Assuming $a, b, g > 0$, we can state the following linear combination

$$u(w, h) := a + bw + gh$$

where w is the total wealth and h the total health. Following the linear expression and denoting the threshold for a risk neutral agent as Δ^{RN} we may rewrite the expression (1) as

$$\begin{aligned}
&(1-p)(a+bw+gh) + p(a+bw-bk_1c_1+gh-gd_1-g\theta_1c_1) = \\
&= (1-p+\Delta^{RN})[(1-q)(a+bw-be+gh) + q(a+bw-be-bk_2c_2+gh-gd_2+g\theta_2c_2)]+ \\
&+(p-\Delta^{RN})[(1-q)(a+bw-be-bk_1c_1+gh-gd_1+g\theta_1c_1)+ \\
&+q(a+bw-be-bk_1c_1-bk_2c_2+gh-gd_1+g\theta_1c_1-gd_2+\theta_2c_2)]
\end{aligned} \tag{6}$$

Semplifying

$$\begin{aligned}
&-pbk_1c_1 - pgd_1 + pg\theta_1c_1 = -be - qbk_2c_2 - qgd_2 + q_g\theta_2c_2+ \\
&-p(bk_1c_1 + gd_1 - g\theta_1c_1) + \Delta^{RN}(bk_1c_1 + gd_1 - g\theta_1c_1)
\end{aligned}$$

$$\Delta^{RN}(bk_1c_1 + gd_1 - g\theta_1c_1) = be + qbk_2c_2 + qgd_2 - qg\theta_2c_2$$

$$\Delta^{RN} = \frac{be + q[bk_2c_2 + g(d_2 - \theta_2c_2)]}{bk_1c_1 + gd_1 - g\theta_1c_1} \tag{7}$$

For several reasons, the outcome in equation (7) is essential. It informs us which element affects the limit and in which direction. Regarding the numerator part, when we increase (decrease) the cost of the vaccine e, the cost of the cure for the side effects k_2c_2 and the side effects impact d_2, the threshold increases (decreases) as well, but when the effectiveness of the cure for side effects θ_2c_2 raises (decreases), the threshold decreases (increase). On the other hand, the denominator, regarding the cost of the cure for the primary disease k_1c_1 and the effect of the primary disease d_1, when decreased (increased), they increase the threshold and, oppositely the effect of the cure for the primary disease θ_1c_1 increases the Δ^{RN} level when it increases as well. This really intuitive dynamics may be more clear with the following statement.

The threshold level for the risk neutral agent increases $\Delta^{n\uparrow}$ *when* $be^{\uparrow} + q[bk_2^{\uparrow}c_2 + g(d_2^{\uparrow} - \theta_2 c_2^{\downarrow})]$ *and* $bk_1c_1^{\downarrow} + gd_1^{\downarrow} - g\theta_1 c_1^{\uparrow}$

A further conclusion discusses the similarity of this result with the risk neutral threshold found by Crainich et al. (2019). They found, assuming no financial costs of the vaccine[20], that

$$\Delta^n = \frac{qM_2}{M_1}$$

In our model, we have introduced costs for the vaccine e, and costs and effects of the cure in the eventuality a disease occurrence, either primary or secondary. In this sense, if we intend $bk_2c_2 + g(d_2 - \theta_2 c_2)$ as total effect of the side effects, namely m_2 and $bk_1c_1 + gd_1 - g\theta_1 c_1$ as total effect of the primary effects, namely m_1 (where g and b constants and negligible for an analysis purpose), we can rewrite our threshold as

$$\Delta^{RN} = \frac{be + qm_2}{m_1} = \frac{be}{m_1} + \frac{qm_2}{m_1}$$

The more the magnitude of the impact of the primary diseases is increased, i.e. increasing m_1, the less the cost of the vaccine has impacts on the willingness of vaccinating.

5.4 A threshold level for a risk seeker agent

Knowing that a risk seeker agent has a convex utility function, we availed ourselves of a paraboiloid. That is, given the parameters $a, b, g, l > 0$, assuming separability and additivity on a convex utility function we consider

$$u(w, h) = bw^2 + aw + gh^2 + lh$$

Since the first and second derivatives are, respectively

$$u_1'(w, h) = 2bw + a, \;\; u_2'(w, h) = 2gh + l$$

and

$$u_1''(w, h) = 2b, \;\; u_2''(w, h) = 2g$$

recalling that the absolute risk aversion is $A = -\frac{u''(\bullet)}{u'(\bullet)}$, quod erat demostrandum, we have a negative aversion. Given the expression (3) we have to apply the Mean Value Theorem six times:

[20] the Authors assumed new risks, such as side effects, as costs.

- $A - C = u(w, h) - u(w - e, h) = (2bw_1 + a)e$
  where $w - e < w_1 < w$ and $h < h_1 < h$[21]

- $C - D = u(w - e, h) - u(w - e - k_2c_2, h - d_2 + \theta_2c_2)$
  $= (2bw_2 + a)e + (2gh_2 + l)(d_2 - \theta_2c_2)$
  where $w - e - k_2c_2 < w_2 < w - e$ and $h - d_2 + \theta_2c_2 < h_2 < h$

- $B - E = u(w - k_1c_1, h - d_1 - \theta_1c_1) - u(w - e - k_1c_1, h - d_1 + \theta_1c_1) = (2bw_3 + a)e$
  where $w - e - k_1c_1 < w_3 < w - k_1c_1$ and $h - d_1 + \theta_1c_1 < h_3 < h - d_1 + \theta_1c_1$[22]

- $E - F = u(w - e - k_1c_1, h - d_1 + \theta_1c_1) - u(w - e - k_1c_1 - k_2c_2, h - d_1 + \theta_1c_1 - d_2 + \theta_2c_2) = (2bw_4 + a)k_2c_2 + (2gh_4 + l)(d_2 - \theta_2c_2)$
  where $w - e - k_1c_1 - k_2c_2 < w_4 < w - e - k_1c_1$ and
  $h - d_1 + \theta_1c_1 - d_2 + \theta_2c_2 < h_4 < h - d_1 + \theta_1c_1$

- $C - E = u(w - e, h) - u(w - e - k_1c_1, h - d_1 + \theta_1c_1)$
  $= (2bw_5 + a)k_1c_1 + (2gh_5 + l)(d_1 - \theta_1c_1)$
  where $w - e - k_1c_1 < w_5 < w - e$ and $h - d_1 + \theta_1c_1 < h_5 < h$

- $D - F = u(w - e - k_2c_2, h - d_2 + \theta_2c_2) - u(w - e - k_1c_1 - k_2c_2, h - d_1 + \theta_1c_1 - d_2 + \theta_2c_2) = (2bw_6 + a)k_1c_1 + (2gh_6 + l)(d_1 - \theta_1c_1)$
  where $w - e - k_1c_1 - k_2c_2 < w_6 < w - e - k_2c_2$ and
  $h - d_1 + \theta_1c_1 - d_2 + \theta_2c_2 < h_6 < h - d_2 + \theta_2c_2$

Now, substituting those values in (3) we may express the threshold for a risk lover agent (i.e. Δ^{RL}) as

$$\Delta^{RL} = \frac{(1-p)(2bw_1+a)e+p(2bw_3+a)e}{(1-q)[(2bw_5+a)k_1c_1+(2gh_5+l)(d_1-\theta_1c_1)]+q[(2bw_6+a)k_1c_1+(2gh_6+l)(d_1-\theta_{c1})]} +$$

$$+\frac{q[(1-p)((2bw_2+a)k_2c_2+(2gh_2+l)(d_2-\theta_2c_2))+p((2bw_4+a)k_2c_2+(2gh_4+l)(d_2-\theta_2c_2))]}{(1-q)[(2bw_5+a)k_1c_1+(2gh_5+l)(d_1-\theta_1c_1)]+q[(2bw_6+a)k_1c_1+(2gh_6+l)(d_1-\theta_{c1})]} \tag{8}$$

The latter expression of the vaccination threshold for a risk lover agent implies that, in order to maintain a certain level of wealth and health, the risk seeker decision maker is less vaccination prone. That is, increasing w and h will increase the values of the interval in which w_j and h_j lie (where $j = 1, 2, 3, 4, 5, 6$). Since w_j and h_j increase, Δ^{RL} increases as well.

[21] Hence we can write $h_1 = h$.
[22] Hence we can write $h_3 = h - d_1 + \theta_1c_1$.

5.5 Implications

As a result, if we compare the risk neutral with the risk averse agent we may conclude that risk averse is more vaccination prone (i.e. $\Delta^{RA} < \Delta^{RN}$) when the following inequality is satisfied.

$$(1-p)\frac{a}{w_1}e + p\frac{a}{w_3}e + q[(1-p)(\frac{a}{w_2}k_2c_2 + \frac{b}{h_2}(d_2 - \theta_2c_2)) + p(\frac{a}{w_4}k_2c_2 + \frac{b}{h_4}(d_2 - \theta_2c_2)] < (1-q)[\frac{a}{w_5}k_1c_1 + \frac{b}{h_5}(d_1 - \theta_1c_1)] + q[\frac{a}{w_6}k_1c_1 + \frac{b}{h_6}(d_1 - \theta_1c_1)] \tag{9}$$

The logic behind this latter outcome is simple. If we compare Δ^{RA} and Δ^{RN}, we must keep the effects of disease and vaccination constant to observe what happens by changing the shape of the utility function, i.e. the degree of aversion of the agent. In other words, neglecting e, d_1, d_2, k_1c_1, θ_1c_1, k_2c_2 and θ_2c_2, not relevant for the comparison, we need a numerator smaller then the denominator. In addition, we have no strong conclusion on this comparative issue.

As far as Δ^{RN} is concerned, elements affecting the threshold, as seen in section 5.3, are only those representing the disease's expenses and effects. On the other hand, regarding Δ^{RA} and Δ^{RL} there is also wealth and health levels in the threshold explicit expression. To observe which are the differences in Δ^{RA} and Δ^{RL} we can assume e, d_1, d_2, k_1c_1, θ_1c_1, k_2c_2 and θ_2c_2 constant.[23] Reducing Δ implies that the agent, whatever her risk attitude is the inclination to vaccinate grows. The terms we discovered for risk-neutrals and risk-seekers say that greater values of wealth and health reduce the threshold for a risk-averse decision-maker and increases the risk seeker one. The latter outcome is quite intuitive, because a wealthy and healthy agent prefers to keep these concentrations as high as possible, but to do so, they will act oppositely.

5.6 Different disease effects when the vaccination is done

In this chapter we want to broaden our structure to observe what happens when vaccination impacts not only the likelihood of the event, but also the effect of the disease. For this purpose we rename the main disease impact with vaccination d_{1e} and the main disease impact without vaccination $d_{1\overline{e}}$. In addition, we slightly change the framework assuming that the main disease after vaccination (i.e. d_{1e})

[23] This is justifiable because in our real assessment they impact both thresholds in the same manner and this is why they are not so crucial.

has less impact than the primary disease without vaccination (i.e. $d_{1\overline{e}}$)

$$d_{1e} < d_{1\overline{e}}$$

We can define the no vaccination and vaccination, respectively as follows

$$(1-p)u(w,h) + pu(w - k_1c_1, h - d_{1\overline{e}} - \theta_1 c_1)$$

and

$$\begin{aligned}&(1-p+\Delta)[(1-q)u(w-e,h) + q(u(w-e-k_2c_2, h-d_2+\theta_2c_2))]+\\&+(p-\Delta)[(1-q)u(w-e-k_1c_1, h-d_1+\theta_1c_1)+\\&+q(u(w-e-k_1c_1-k_2c_2, h-d_1+\theta_1c_1-d_2+\theta_2c_2))]\end{aligned}$$

Therefore, since $\frac{\partial}{\partial d_{1\overline{e}}} u(w - k_1c_1, h - d_{1\overline{e}} - \theta_1 c_1) < 0$

if $d_{1\overline{e}} \uparrow$, as seen in page 34 the utility $B = u(w - k_1c_1, h - d_{1\overline{e}} - \theta_1 c_1) \downarrow$

Recalling, by (3) that the threshold for a generic agent may be written as

$$\Delta = \frac{[(1-p)(A-C) + p(B-E)] + q[(1-p)(C-D) + p(E-F)]}{(1-q)(C-E) + q(D-F)}$$

reducing B implies a reduction of the whole threshold.

In other words, *when we introduce the realistic hypothesis of a reduced impact of the main disease after vaccination compared to the selection of no vaccination in which the effect of the disease remains constant, the risk averse agent is more vaccination prone.*

6 Conclusions

In this dissertation, first we selected the most important and innovative papers in the field of prevention of health and prudence in economics. Initially, we saw that prevention was seen precisely as a precautionary motive for *saving health or wealth.* At the beginning, the analyses concerned a one period maximization problem in which the agent selects the amount of effort for prevention to reduce the likelihood of a loss, either physical or financial. Then the papers began to embrace more realistic two period models in which today's effort impacts the likelihood that a loss will occur tomorrow. To this purpose, as a risk-averse agent wants to maximize her wealth in the period in which the loss may occur, the effort is much lower in a one-period framework than in a two-period model. We saw the turning point in Brianti et al. (2018) where the preventive effort is selected thanks to a maximization of the two-period model, but there is uncertainty about the effect of the disease and the effectiveness of the cure. In this way, the officer is worried not only with maximizing her wealth and health in terms of avoidance, but also with cure costs and impacts. In addition, prevention is regarded in a particular manner in Crainich et al. (2019): vaccination. That is, the agent may decide whether or not to vaccinate. It was not essential to establish a issue of maximization for this decision, but what is the value for which the agent decides to vaccinate or also does not consider the possible occurrence of vaccination side effects. The Authors discovered this risk-averse threshold and risk-neutral decision-makers concluded that the risk averse is more prone to vaccination with regard to the risk neutral when the magnitude of a secondary disease's effect is lower than the main disease's effect.

Our model enlarge the vaccination framework presented by Crainich et al. (2019). We assumed that the considered vaccine are only voluntary vaccines because, otherwise, there was not a decision problem. Then we considered that the threshold for the vaccine depends not only on the magnitude of the primary disease and side effects. We thus introduced in the threshold analysis the cost of vaccination, the cost and the effectiveness of the cure, both for side effects and primary disease (as in Brianti et al. (2018)). We concluded that, even if the threshold is higher than the simpler model, a risk averse decision maker is more vaccination prone than a risk neutral decision maker, again, if the magnitude of the impact of a secondary disease is smaller than the impact of the primary disease, comprehensive of the cure effectiveness and costs. Then we found a explicit formulation for risk neutral, seeker and averse decision makers;

showing that wealth and health component are present in the risk averse and lover formulation and not in the risk neutral, it is possible to deduce that greater values of wealth and health reduce the threshold for a risk-averse decision-maker and increases the risk seeker one. This latter result is quite intuitive because a risk averse agent is more prone to preserve a certain level of wealth or health and a risk seeker will be less prone. Moreover a risk neutral decision maker will make her choice not considering wealth and health. Finally we showed that, assuming that the vaccine reduces also the impact and not only the probability of the preventable disease, for a risk averse agent the threshold will decrease.

Some other interesting aspects that can be studied, for example, a maximization problem with respect to the effort exerted (it can be seen as the cost of vaccination) and the cost of both primary disease and side effects. Another interesting aspect, in our opinion, the future literature has to focus on is the analysis of models with a multiperiod framework where the time discount is not constant over time. Introducing a discount of time preferences may be surprisingly innovative and may induces different results.

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