Ca' Foscari University of Venice

Single Cycle Degree programme

in Economics and Finance

Final Thesis

GDP-linked bonds

A smart financing instrument for few countries

Supervisor
Prof. Monica Billio

Graduand
Michele Bado
871265

Academic Year
2017 / 2018

Introduction

One of the concerns of present times is given by the substantial levels of public debt accumulated since the financial crisis, whether they were already high or have risen subsequently, both in advanced economies and, to a somewhat less degree, emerging markets. This rise was due from governments responding with countercyclical fiscal policies in order to contain the economic damage, parallel to expansionary monetary policies from central banks from all over the world. Together with low inflation rates, a decade of extremely low interest rates across the globe has gone by; this has immensely helped in keeping high debt levels sustainable.

As monetary policies fall short in bringing an additional stimulus. to the economy past a certain threshold and are not supposed to be a long term tool, since they are effective only for a (not so) short period of time, highly indebted countries are likely to face higher costs in facilitating their gross financing needs. Moreover, if the long term intertemporal budget constraint assumption is to be hold true, we should see interest rates on sovereign debt rise at least at the growth rate of the economy, which brings also to the stagnant growth of some of the advanced countries and a general relative slowdown thereof, especially in Europe and the so-called PIIGS countries.

A lot of countries have now to deal with the need to bring down debt to safer level and move away the possible ignition of explosive debt dynamics, an issue that is constantly being monitored by financial markets, as media reminds us. At the same time low growth rates demand a continuation of expansionary fiscal policies by governments, which have then the highly delicate duty to weight the two opposing needs; the degree of freedom for governments adding further debt without causing worries about their solvency and the maximum public debt level that a country can carry without defaulting are known as fiscal space and debt limit.

A question that should be asked is whether is there any financing tools available to public administrations to answer their need for a more flexible debt mix. In the wake of the global financial crisis shock to growth, the case for more growth sensible debt instruments has been made, namely debt securities that temporarily lower payments, and so the government budget associated with debt service, as negative shocks happen to the economy. The answer could be GDP-linked bonds, whose payout depend on the level of economic output of a country, its growth rate, or both, depending of the specific design. In fact, if we assume that there is a strong link between growth and tax revenues, indexing a portion of public debt service to growth seems to be a match made in heaven between assets and liabilities. The goal here is to both lower default fears and increasing the fiscal space available to governments.

In addition to that, a new kind of an indexed public debt security depending on the economic output of a whole country, and thus an intrinsically diversified investment vehicle, has been in the wishlists of some economists for decades. In “Macro markets” (Shiller, 1993), the author calls for the need of a security linked to GDP as a step towards completeness in financial markets, letting sovereigns share their risk associated with growth uncertainty and allowing financial agents to both invest and hedge their economic exposure to a whole country; such an instrument would offer potentially more upside to investors was the economy to rise more than expected, while allowing sovereigns for an automatic cut in their debt service expense during times of economic distress. As a consequence of the issuance of such an instrument, a market estimation of future growth rates for an economy would be possible to derive.

The practicalities of issuing a new kind of public debt instrument are multiples, from a lack of objective definitive standardized design to the coordination effort that would be needed for it to be issued by a number of countries in parallel, thus mitigating the risks perceived by investors towards a specific country through diversification.

Having made the case for the need of investigating the possibility of a new more flexible financing tool for sovereigns, the present work tackles the problems of design and standardization, dwelling upon the simpler design that replicates the logic of inflation linked bonds, the major case of a successful lunch and spread of a new indexed public debt security, whose need was born in the wake of the large inflation rates of the ‘70s and ‘80s. Maybe, by the same token, GDP-linked bonds will be a response to the negative growth shocks that we have seen in the last decades. The specific question that the thesis seeks to answer is whether a specific chosen indexation design is worth it, namely if a sovereign would be better off in terms of fiscal space gains and lower defaults spreads - which go hand in hand, of such a security would prove to be to costly in terms of risk premia assigned to it by investors. the problem is faced by following the experiment proposed in a Bank of England research paper (Benford, 2016) and expanding it by accounting for the presence of a market implied default risk (and so, a default risk that is part of interest rates on public debt) inside sovereign CDS. By extrapolating a pseudo debt limit from this risk and running one hundred thousands simulations of future debt path per country, an estimation of a new default risk under the allocation of 25% indexed debt and 75% conventional debt is given.

There is certainly need for further work about the design of indexation analyzed here and other possible designs, as a consensus towards an optimal is desirable before large scale issuance in order to facilitate investors and issuers alike, while keeping faith to the debt service flexibility and the higher asset liability matching premises of this underused kind of debt indexation.

Chapter I – Why index sovereign debt to GDP

1 Sovereign debt

Since the time of Babylon almost 4000 years ago, government (or sovereign) debt has been issued by state entities, which may be central governments or even municipalities, in order to finance themselves in their continuous public expenditures. Debt can be view as a time shift of future money to the present, as a consequence of an immediate need of money for a number of purposes. Historically, for public debt, those may have been war funding, from the ancient and modern times, like the long standoff for world power between England and France during the “second hundred years’ war” (18th century) to the episodes of the world wars and the decade following the burst of the financial crisis, rather than investing in technological development, thus productivity, or temporarily boosting the economy by increasing spending in times of economic recession, known as “stimulus”. Worse, if the debt stock gets large enough, a lot of debt might be needed just to pay down to make good on current debt obligations, which raises concerns about the possibility of the kickoff of a self feeding vicious circle, in which current debt service leads to the issuance of more debt at an higher interest rate, which subsequently raises the debt service. This loop continues until the country either manages to cut off the debt spiral, maybe through raising its primary balance or by growing faster than its debt, or leaves current bond claimholders worse off either by defaulting on part or all of its debt (stopping payments), or by monetizing its debt, meaning servicing it by minting of new currency, thereby risking an heavy currency depreciation. I’ll talk about the various case later on.

In the historic evolution of public debt two major breakthrough happened: in the 13th century in the Republic of Venice at Rialto Market the Europe’s first bond market emerged; but all came together in 1694 with the creation of the first central bank, the Bank of England, an institution originally design to lend to the government (Goetzmann, 2016). Nowadays almost every country in the world is capable of and does issue public debt, aided by a central bank institution as "lender of last resort", while previously issued bonds keep being exchanged in a secondary market, where their prices give governments (treasury departments in particular) a pretty accurate estimate of the discount they will have to given up to investors at the following debt auctions.

Present day debt mix of a government varies a lot across countries. The first difference can be seen in the currency denomination of the outstanding debt: whereas advanced economies issue almost always bonds denominated in their local currency which they control, emerging markets tend to issue a sizeable portion of their debt in an international currency, primary in dollars, but also in euros, yens and pounds, in order to raise the appetite of foreign investors. It may be argued that euro denominated bonds of Eurozone countries are not denominated in a sovereign currency at all, given the inability of single members of minting euros and monetize their debt, or to even default without risking their very membership. We have grown accustomed to the strange situation of having a monetary union while lacking a fiscal union, which is close to a paradox. For the scope of this thesis I'll consider Eurozone countries as having their debt stock denominated in sovereign currency, an assumption that could be proven wrong in the future. A more clear distinction can be made by talking about local and foreign currency. Figure 1 reports the average amount of foreign denominated sovereign debt at the global level and by the subgroups of Advanced Economies (AE) and Emerging Markets (EM); the average is displayed both equally weighting each country and using the total dollar amount of debt at market value. Arguing that denominating bonds in the local currency is preferable to the issuer, two observations can be made:

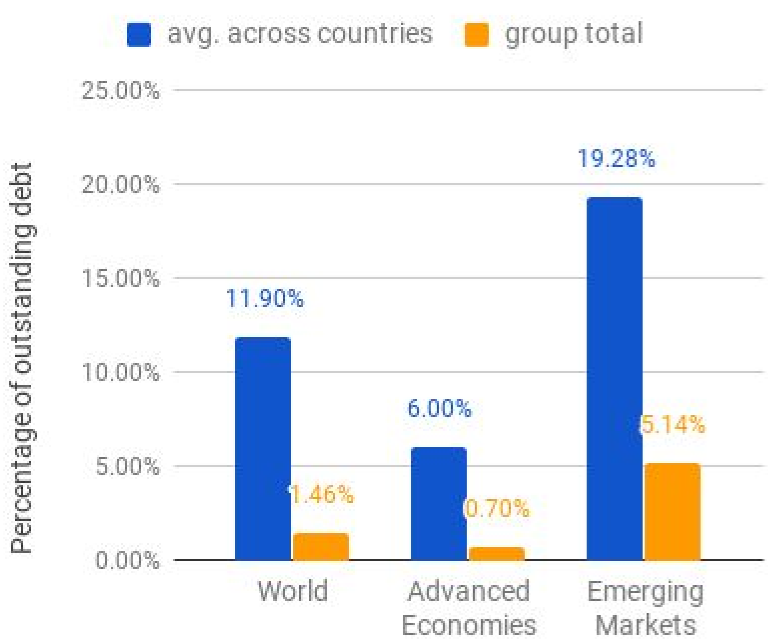
- EMs exhibit a stronger need for foreign currency denomination, probably because they lack a stable local currency or feel that bonds denominated in a stable currency are an easier sell
- averages weighted by market value of debt are consistently lower that their equal weighted counterpart, helping to make the case that countries issuing prevalently local

currency debt are able to issue more debt than the ones who don't.

Beyond currency denomination, bonds can also differ in their payment structure. Most of government securities are fixed rate bond, but there are also floating rate ones, whose interest payment is recalculated on a variable, like an updated interbank lending rate. In the last two decades the inflation-linked bonds market has grown in absolute and relative terms, now comprising 11.6% of the global sovereign debt stock; their peculiarity is that both interest payments and principal reimbursement move with an index of goods' prices. It has to be noted that the overwhelming majority of the latter kind of security comes from very few countries, namely United States, United Kingdom, France, Italy, Brazil (Krämer, 2017).

In spite of thousands of years of financial innovation, fixed rate bonds are still the norm. The world debt stock composition is reported in Table 1.

Figure 1. Sovereign debt denominated in foreign currency, world and selected groups of countries



Sources: BIS, author's computations

Table 1. Global sovereign debt mix structure, interest rate structure

Fixed rate	Floating rate	Inflation-linked	Other
83.67%	4.20%	11.62%	0.51%

Sources: BIS, author's computations

1.1 Debt levels

The global public debt stock is estimated to be around $69.6t, or roughly 82% of world GDP ($84.8t). Although most of it is issued by developed countries, namely us, Japan and the European Union, until the last decade most of the problems had come out from the lower debt carrying capabilities of economically smaller countries. From an historical point of view it was war funding that demanded high level of debt, but, since the 1950's, it has been the case that excessive loose fiscal policy (too much spending), hyperinflation, and currency depreciations caused severe sovereign debt crises. Referring to a median level of public debt among the G7 countries (Canada, France, Germany, Italy, United Kingdom, United States), for which there is availability of long term historical data, one can visualized the two biggest increases in debt levels during the 20th century, namely from 30% to 110% during the First World War and from 70% to 130% during the Second one. Figure 2 gives context around the increased happened in last 10 years, from roughly 68% to 97% of debt over GDP.

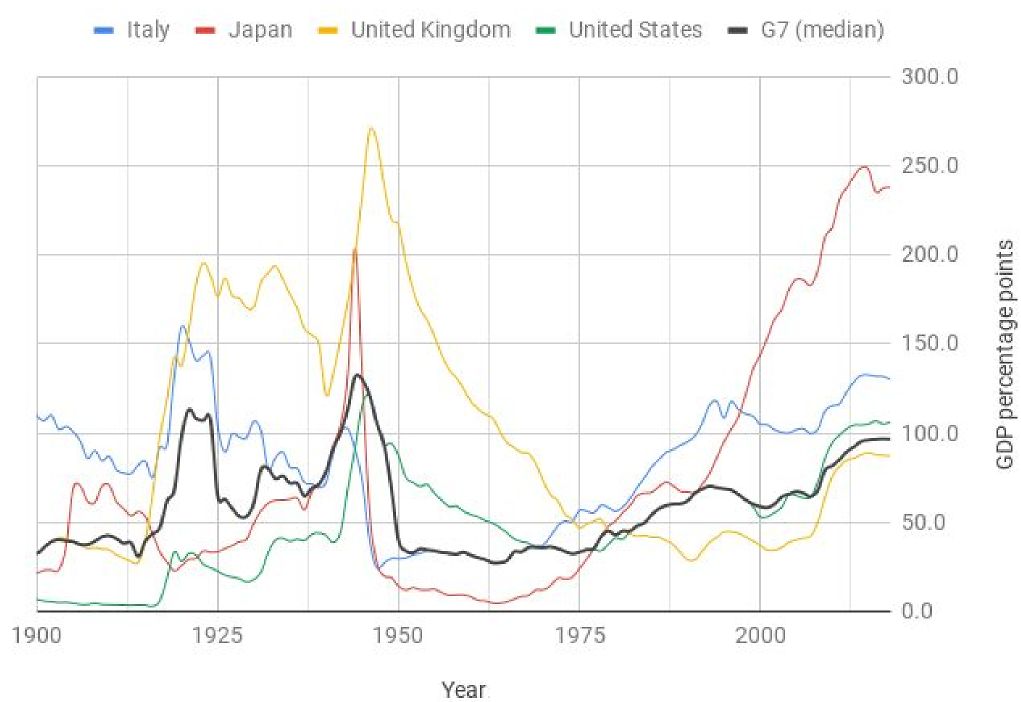
As a consequence of the commitment of governments in not allowing the great recession (2008-2009) to become a sequel to the great depression (1929-1939), which ended only with the Second World War, the excessive debt carried by homebuyers and insolvency of major financial institutions across the world was absorbed by the various central banks of the world, thus making sovereign debts skyrocket both in nominal terms and relative to the GDP level. Considering that a full decade has gone by, one might wonder how we came to still hold such an level of total world public debt, with the most extreme high values coming from major developed economies. In particular just 11 countries make up for $57.3t, or 82.4%, of world debt; they are reported in

Figure 3.

China's marvelous growth hasn't translated in an excessive public debt level, meaning that it is in effect lowering the world public debt to GDP ratio, yet it faces another problem: a credit bubble at the corporate level. China's credit to the non-financial corporations over GDP stands at 155.1%, an increase of 66 percentage points in the last decade. From a global point of view, the so-called deleverage since 2008 only happened at the households level, while corporate and government total credit have increased, respectively 19% and 28%, according to BIS data. The present credit to GDP statistics are displayed in Figure 4.

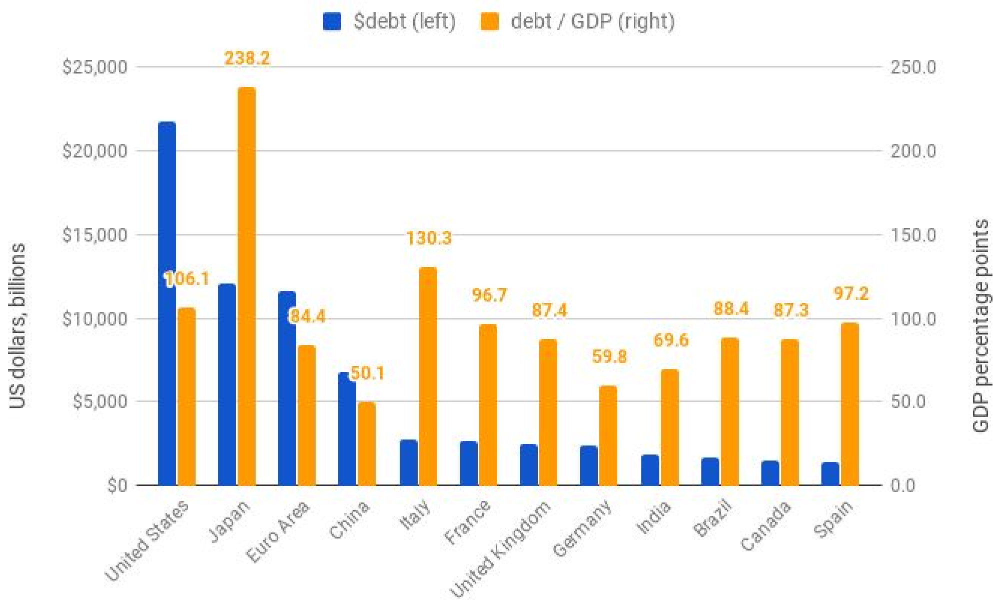
To sum it up today the entire world faces the problem of extremely dangerous levels of indebtedness from the state to the corporate level (Buttiglione, 2014); of course this thesis will focus only on the first issue.

Figure 2. Historical debt to GDP ratios of selected G7 countries and median of all G7 countries



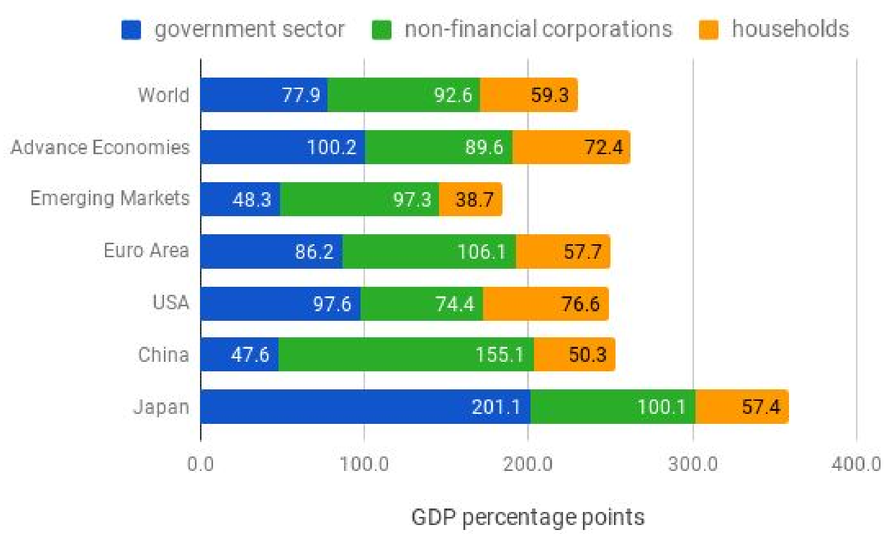
Source: IMF

Figure 3. Public debt amounts, first 11 countries by nominal debt



Source: IMF

Figure 4. Total credit to non-financial sector; world total, selected groups of countries and selected individual countries



Source: BIS

1.2 Interest rates level

It has to be noted that the huge increase in public debt of central governments in the current century has been matched with ever lower real interest rates (particularly with lower nominal rates *and* lower inflation), thereby annual debt payments haven't increased as much. This is of course sensible to the level of interest rates, which are certainly at an historical extreme, as shown in Figure 5. Are negative interest rates irrational? They are at least a mind-boggling concept. When real rates, and for a certain span of time even nominal rates, are negative, something is not *natural*. Who would ever surely lose money on purpose? An heavy downward pressure is due to central banks asset buying programs, asset requirements for financial institutions and pension funds usual purchases. The Federal Reserve and the Bank of England have stopped buying assets only in 2014 (indeed the first one has been selling assets since last year), while the European Central Bank is ending its own program just now and the Bank of Japan basically owns half of the Japanese government debt and has been keeping an easing monetary policy since the 1990's, which has costed Japan an increase of 130 GDP points in public debt (Figure 2); the BoJ went as far as massively buying local stocks, ending up as one of the 10 largest shareholders in 40% of listed japanese companies (Tomita, 2018).

To contextualize sovereign debt interest rates levels one can look at historical rates for most developed countries and, in particular, to the very long time series reconstructed for the U.K.. Thanks to this data, the computed median long term nominal interest rate for the latter country since 1800 is 3.3%, while the (geometric) average inflation across the same period is 2%; then a sort of natural long term real interest rate can be roughly tough to be 1.3%. In comparison inflation-linked Gilts with a maturity of 20 years are yielding -1.7%. Historical nominal rates are low across the globe; Figure 5 displays data from two countries with very long history available, U.K. and us, and two countries with currently near 0% nominal long term rates, Japan and Germany.

The concern here is that the normalization process will bring higher nominal interest rates globally, as in the long term the intertemporal budget constraint should mean that they are higher than nominal growth. A process that necessarily will need a reduction in debt levels to allow countries to service their newly debt issues. This has of course already begun in the us, ever since 10 years nominal interest rates have hit rock bottom July 8th, 2016, at a yield of 1.37%, and now both short and long term nominal rates are above the inflation level. For US dollar interest rates maybe the process has already happen and there is less a concern about further increases; also futures on LIBOR imply that rates won't rise in the next 4 years (Table 2).

By comparison, in the next few years it would be logical to expect for nominal rates in the EU and Japan to rise at least to the expected inflation level, and even more for non default free countries. That being said, futures markets are implying a continuation of low interbank rates and, as a proxy, short term public debt rates. For interest rates are forecasted to rise ¾ of 1% for the euro and 0.5 percentage point for the British pound. Data is summarized in the table below.

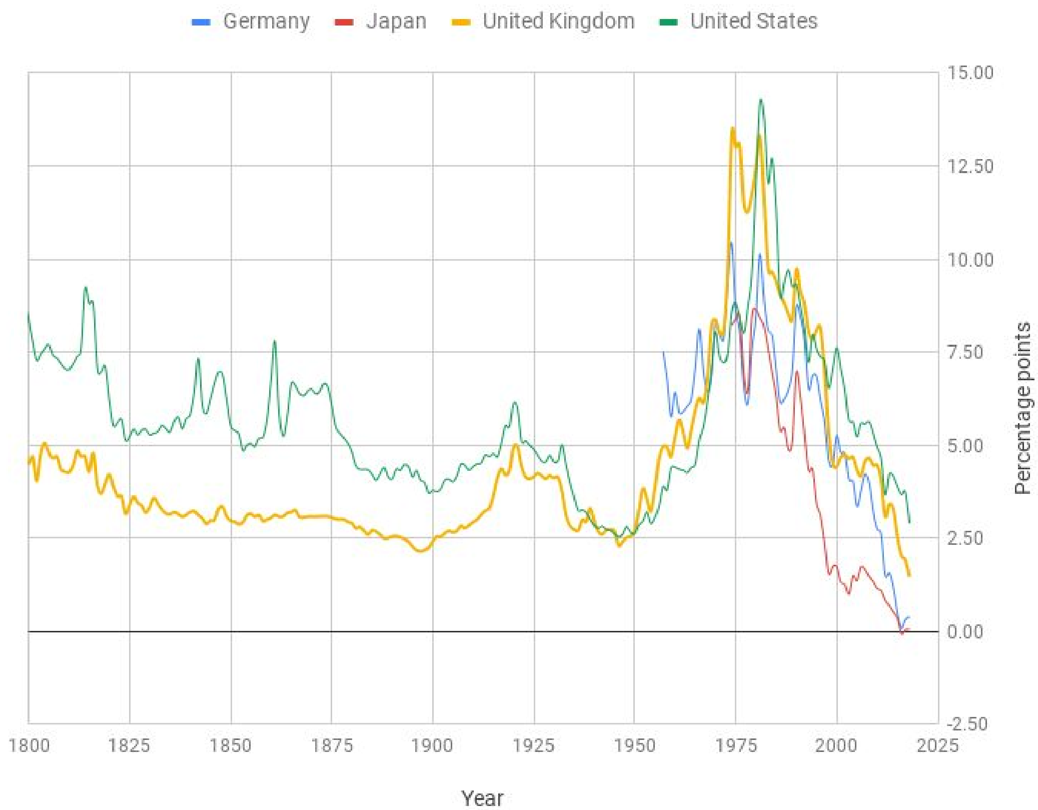
For Japan most of public debt is held inside the country, so that outside investors are not a concern, with the BoJ plus japanese insurances and pension funds holding a total of 69% of central government debt; if the government won't be able to make good on payments, it may not face particularly harsh consequences.

Table 2. Futures prices on 3 months interbank lending rates and implied future change in basis points, 4 years out

	March 2019	March 2023	implied change in short term rates, basis points
LIBOR (USD) eurodollar	97.360	97.405	-4.5
EURIBOR (EUR)	100.300	99.555	74.5
LIBOR (GBP)	99.145	98.655	49.0
TIBOR (JPY) euroyen	99.970	99.885	8.5

Sources: CME, ICE, TFX (as of 16/02/2019), author's computations

Figure 5. Nominal long term interest rates on sovereign debt, selected countries

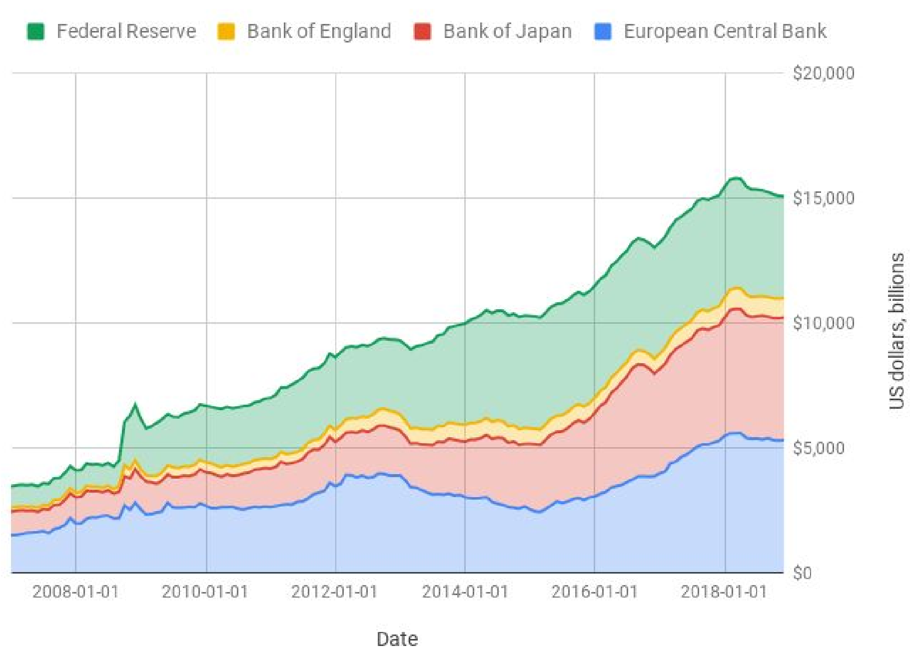


Sources: BoE, MeasuringWorth, OECD, Ministry of Finance (Japan)

Moreover Japan owns its currency and its debt is denominated entirely in yen, so the worst case is a debt monetization rather than a proper default; this would of course cause a currency depreciation, but I argue that it wouldn't be a severe one, considering it holds the first place in the world when it comes to the Economic Complexity Index, meaning that it is too much interconnected in global trade to let the yen fade out as a currency. This is how the third country by economic output, the second by public debt, at a level of 238% of its GDP can still manage to issue debt a near 0% interest (Figure 5).

In Europe there are supposedly default free countries with debt trading at negative real rates, in particular for the U.K. the market implied inflation, obtained by computing the interest rates in non indexed debt versus inflation linked securities, is high, ranging about 3.24% (Table 3), which might be due to concerns about the British pound and the U.K. economy after Brexit takes place. When talking about normalization of interest rates, one cannot ignore the activity of central banks, which are been key buyers of assets, especially local sovereign debt securities. The sum of the increase in the combined balance sheet of the Federal Reserve, the European Central Bank, the Bank of Japan and the Bank of England since 2008 exceeds the $10 trillion mark (Figure 6). The process of normalization, in this case, would be the progressive selling of their assets, a process already started only by the FED. The amount of assets already bought by central banks, together with the general high levels of public debt in advanced countries and resulting need for reducing it, should have the effect of limiting fiscal space for said countries in the next decade, exposing themselves more to potential macroeconomic shocks, in addition to a likely general increase in debt service costs, deriving from higher interest rates.

Figure 6. Total assets of selected central banks



Sources: FRED, BoE

2 State contingent debt instruments

Central governments are supposed to operate a counter-cyclical fiscal policy, allowing for a reactive and temporary increase in public spending during period of economic recessions, while keeping a surplus in their budget in "normal" periods. Sadly empirically this is not always the case, mainly because of political (and behavioural) reasons. Politicians seek approval for re-elections, so they keep spending too much in periods of growth, letting their country face an harmful lack of fiscal space during downturns in the economic cycle; therefore a spending cut will be deemed necessary in order to keep the debt expense (interest payments) from running out of control, since the very fear of a sovereign insolvency makes the interest rates of government debt rise.

If the very declaration of default takes place when a government is simply unable to find the capital to fulfill its immediately due payments obligations, the reason usually comes from holding too much debt relative to its revenues, which triggers the deadly debt spiral in which interest rates and debt payments rise and feed upon each other, thereby making debt refinancing more and more costly. Adding to that, governments may be even contractually obliged to not run "too much" of a budget deficit, as it theoretically is for the eurozone members – although during the years of the great recession an exception to the rule was made for most of them and other exceptions take place from time to time.

There is evidence for government running a pro-cyclical fiscal policy, thereby adding to the economic cycle making it more volatile, not less, as one would be inclined to expect. The takeaway is that a government counter-cyclical fiscal policy is desirable, as Keynesian economics say, and it can be so only by designing debt obligations to be flexible to the current payment capacity of the debtor.

Since a government cannot issue shares in its country, the instrument needed has to be an hybrid security, in particular a debt instrument able to reflect the country effective ability to pay in its contractual obligations to pay. To do so, the general kind of securities that has to be explored is called "state-contingent debt instruments" (SCDIs), which are debt instruments whose payoff is tied to the outcome of state variables or trigger events (Novikova, 2017). The variables can range from commodity prices, as in certain Latin America bonds which are linked to crude oil, to price indexes, as in inflation linked bonds. There can even be trigger events to temporarily either activate or shut off payments, such as contractually defined natural disasters.

The main feature of this kind of securities is the intrinsic additional risk-sharing capabilities that they provide: an oil producer country whose exports (and so currency value) and economic performance are tied to crude oil sees its tax revenues move with oil prices and might want to hedge that risk away, whereas a country with high expected inflation might find easier to auction debt whose obligation are denominated in real terms, i.e. grow with the level of prices of goods and services. What diversifying away from just straight conventional debt brings to the table is additional risk transfers; in particular indexing to commodity prices exported by a country, or enabling trigger events to lower payments transfers risk from the state to bondholders, while indexing to inflation or enabling trigger events to increase payments transfers risk from bondholders to the state. While the first kind of trade-off enhances the debt carrying capabilities of the state, the purpose of the latter is to find a market for debt securities at issuance by overcoming some of investors' fears. It is not a coincidence that the first strong issuance of sovereign inflation-linked securities happened in 1981 (HM Treasury, 1981), with the United Kingdom auctioning index-linked Gilts after seeing 8 years of annual inflation in the double digits, in the U.K. and across the globe.

2.1 Equity component

Corporate history that there are two basic categories of financing tools: debt and equity. Of course, then we have to add hybrid securities. The distinction can be summarized as this: a security granting a fixed claim on the issuer assets is debt, while every other claim is equity. (Damodaran, 2014)

A firm ideally chooses the correct mix of debt and equity based on the stability of its future income and its position in the life cycle of a company. The more certain future positive cash flows are, the less the risk of failing to meet debt obligations, thus defaulting; the early stages of the life cycle require more equity than debt because growth, and in particular high growth, brings along uncertainty about the future almost by definition, since future growth is, well, an estimate, thus it can be wrong.

This is sort of reflected also by sovereigns' financing history, where it is blatant that more developed economies have the means – a more developed financial market – and follow a more stable growth path, while emerging economies encounters, usually, more difficulties in raising funds. The difference between corporations and sovereigns is, of course, the lack - and impossibility - of equity instruments in raising capital for the latters. What an equity or equity-like instrument provides is the possibility of making contractual obligations more flexible, by having the option of delay *ad libitum* or lower payments without incurring in legal consequences. For corporations equity is the mean though which investors gain control over corporate decisions, at the cost of having to delay personal compensations, in terms of shares buybacks and dividends, for the sake of the company and its fixed debt obligations. Of course this is not a possibility when talking about a country: it is not owned by investors, but by its population. The discussion then has to focus more on hybrid instruments, or, better, on debt designed as featuring equity characteristics. Still referring to corporate debt in the case of high growth companies the most common of these kind of securities is given by convertible debt, or by attaching a call option or warrant to straight debt; but we've seen that an equity stake to call doesn't really exist.

There are at least three dimensions to explore in order to enhance straight conventional debt: coupons, principal, duration. The possibility of contractually giving the issuer the option of lowering or stopping coupon repayments as specific events happen can be achieved by using floating coupons linked to an external variable, as it happens with floating rate bonds; by adjusting the bond principal the overall bond characteristics can be altered forever; finally, as duration moves further and further away in time, debt looks more and more like equity. On the latter point a word has to be said about perpetuals, or bonds that never expire. What you get is an annuity, a coupon stream that (should) last forever; this structure reminds very much of preferred stock in equities, which features a fixed dividend to be paid until the company ceases to exist, without giving voting powers to the holders. Perpetuals don't seem to offer good conditions to the issuer, in fact the most important sovereign securities of this kind were auctioned in time of high distress and extreme need of funding, namely by the United Kingdom during the Second Hundred Years' War against France during the 18th century, known as Consols, and by the United States during the Civil War (Goetzmann, 2016). Most of these securities have been fully redeemed - meaning bought back - by the respective governments. A thing to consider is that perpetuals wisely provide a nominal cash flow that gets annihilated in time by inflation, meaning that their payments in real terms get to 0 as decades go by. The concept of perpetual bonds is not dead, as they are actually gaining in popularity among asian corporations. (Weinland, 2017) (Zhang and Shen, 2019)

2.2 Indexing to inflation

There is a not so ancient example of introduction of a sovereign debt instrument linked to an economic index published by the public statistics organization. Linking the principal amount of debt to the change in the consumer price level is an idea that has been along for a long time. For example, even in 1780's Massachusetts we can find an example of indexation to a basket of core agricultural goods, while during the last century has taken place the idea of linking the principal to the price of the most important export commodity (usually crude oil), which also functions as a proxy for economic performance of a country relying heavily on the production and sale of that good. The hyperinflation that hitted even developed countries in the '70s and early '80s, caused by skyrocketing oil prices due to the establishment of the OPEC cartel, highlighted the need for debt instruments immune to changes in the level of prices. As previously said, the first country to issue inflation-linked bonds (ILBs) on a large scale was the United Kingdom in 1981, followed by Australia, Canada, Sweden; the United States started issuing the famous TIPS (Treasury Inflation Protected Securities) only in 1997. Eurozone countries arrived later, but France and Italy are now big players in the inflation-linked sovereign debt market. Surprisingly even low to null inflation countries like Germany and Japan have a portion of their debt indexed, but, not surprisingly, its very small.

When looking at global public debt, nowadays the inflation-linked portion accounts for more than a tenth of the total debt composition (Table 1). This give the possibility to hold sovereign debt even to investors fearing future inflation, where previously precious metals where the only instruments available as an hedge to inflation and, most of all, currency crises. The common design for indexing to inflation is to let the principal amount of the outstanding security float with an index of prices of goods and services, typically the Consumer Price Index, published by the relevant national statistics institute, effectively holding the cash flows of the bond inflation invariant, or stable in real terms.

Let's consider sovereign bonds paying a single annual coupon. For conventional debt the total cash flow, its present value and its standard approximation, considering a unique interest rate or yield to maturity, are the usual

$$L(cT+1)$$
$$L\Big(c\sum_{t=1}^{T} exp(-i_t t) + exp(-i_T T)\Big)$$
$$L\Big(c\sum_{t=1}^{T} exp(-yt) + exp(-yT)\Big) \ (1)$$

using the simplification of a flat interest rate curve

$$y = i_t \forall t$$

Considering the indexation of the principal to goods and services prices

$$L_t = L(CPI_t/CPI_0) = L\prod_{n=1}^{t}(1+\pi_n)$$

For inflation-indexed bonds the previous set of equations becomes

$$c\sum_{t=1}^{T} L_t + L_T$$

$$L\Big(c\sum_{t=1}^{T}\prod_{n=1}^{t}(1+\pi_n)exp(-i_t t) + \prod_{n=1}^{T}(1+\pi_n)exp(-i_T T)\Big)$$

$$L\Big(c\sum_{t=1}^{T} exp\big(-(y-\pi)t\big) + exp\big(-(y-\pi)T\big)\Big) \ (2)$$

using the simplification that

$$exp(\pi t) \approx (1+\pi)^t = (1+\pi_n)^t$$

Where L is the principal amount, c is the coupon rate, or the coupon as a percentage of the principal, *CPI* is the consumer price index, i_t is the nominal interest rate at year t, y the yield to maturity, π_n the inflation rate in year n; a single inflation rate, π, comes from the same simplification adopted when considering a single interest rate in determining a yield to maturity.
Then, we can define the average real interest rate until bond maturity, which also represents the yield to maturity of an inflation-linked bond, as

$$r := y - \pi$$

Equation (2) becomes

$$L\Big(c\sum_{t=1}^{T} exp(-rt) + exp(-rT)\Big) \ (2.1)$$

The takeaway is that, by comparing yields derived from market prices of straight and inflation-linked bonds with the same maturity, we're able to extrapolate a market implied expected inflation rate. Table 3 considers the largest issuers of inflation-linked bonds, namely us, U.K. and Eurozone, here represented by french OATs indexed to eurozone inflation (OAT€i). Japan is added because of the importance of the Yen and for continuity with previous comparison between major central banks.

Table 3. Nominal and real 10 years yields on sovereign debt and market implied inflation expectations, selected currencies

	Nominal yield	Real yield	inflation breakeven
United States ($)	2.70%	0.82%	1.88%
France (€)	0.68%	-0.48%	1.16%
United Kingdom (£)	1.27%	-2.00%	3.27%
Japan (¥)	-0.03%	-0.26%	0.24%

Sources: FED, Agence France Trésor, BoE, Japan Bond Trading Co. (as of 01/02/2019)

2.3 Indexing to GDP

Gross domestic product is a common standardized and generally accepted measure of a country economic output. It can be defined in the expenditure approach as the sum of spending from individuals (consumer purchases of durable goods, nondurable goods and services), the government, corporations (in the form of investments) and the net balance of payments.

Whereas the inflation-linked instrument keep the principal amount inflation invariant by multiplying it by the change in the CPI (or similar index), the main feature of a GDP-linked bonds (GLB onwards) is to link either or both the bond coupon rate and principal to the level of

GDP or its growth. If one is to make the debt service float with its income in order to let it be more flexible to ups and downs in the economy, linking to a measure of debt repayment capability is key; it can be done deploying GLBs.

The basic intuition of this kind of linkage is the following: the lower the country income becomes, the lower the government ability to service conventional debt (lower tax revenue and higher political pressures to spend), the lower the expenditure for GLB securities; the opposite applies. Of course this line of thinking resembles the classical problem of asset liabilities matching in identifying and achieving the optimal financial mix for a corporation, which also argues in favour of floating rate debt for firms whose earnings move with inflation, among other things. Assuming the GDP is a good enough proxy of government tax revenues, thus government ability to pay, then contractually matching debt service with GDP will certainly enhance said country ability to carry debt, raising its debt limit, or the maximum level of debt over GDP that a country can make good on without either monetize the debt or having to default on payments. Direct consequences of lowering the probability of an adverse event such as those mentioned should have the consequence of an overall fall in interest rates for public debt, which are usually followed by lower rates for companies operating in that particular country, and an increase in the potential for a government boost of the economy via fiscal policies, should the conditions arise.

An indirect consequence stems from the relative stabilization of the level of the spread between interest rates on public debt and growth in GDP. The basic debt dynamics equation is as follows and will be exploited throughout this work:

$$\Delta d_t = \frac{i_t - \gamma_t}{1 + \gamma_t} d_{t-1} - pb_t + oadj_t \ (3)$$

where d is the debt to GDP ratio, pb a government primary balance, or its budget balance prior to interest expenses, denominated in GDP points, i_t represents the nominal average interest rates on debt issued in year t, γ_t represents the nominal growth rate for the same year and *oadj* stands for other adjustments to debt stock. The debt stabilizing properties of indexing to GDP, conversely linking i to γ and stabilizing their spread towards a constant, can be clearly seen after having computed the variance of the change in the debt ratio.

$$var(\Delta d_t) = var(pb_t) + d_{t-1}^2 var(i_t - \gamma_t) - 2d_{t-1}cov(pb_t, i_t - \gamma_t)$$

So that for GLBs the following holds true:

$$var(\Delta d_t) \approx var(pb_t)$$

By assuming the covariance between primary balance and the spread between nominal interest rates and nominal growth to be negative, indexing debt to GDP surely decreases the variance of changes in the debt stock. The assumption seem to be empirically true, since the median of the correlation computed on 20 years of data for the 38(+1) countries selected is -.349. More on this data in Chapter 3.

The very idea of indexing sovereign debt has been developed since Shiller's bonds (Shiller, 1993), which, similarly to how inflation-linked debt instruments work, links the bond principal to nominal GDP, thereby behaving like an extension of the latter, providing both an hedge to inflation and a positive exposure to the growth in the economy of the issuing country. The second school of thought is given by the design proposed by Borensztein and Mauro in 2004, where the coupon payments are linked to real growth, building a debt instrument which features an "equity-kicker", akin to a conversion option into equity, but also providing a contractually automated temporary partial default on coupon payments when real growth is depressed.
So, what kind of security is a GLB? It is debt, given it is issued on the promise to repay, but payments are dependant on the economic output of a country, as a proxy to the tax revenues and ability to pay of the government, tying the fortunes of investors to the country economic performance, similar to what happens with equity investors - VC firms, private equity, shareholders - and corporations. This kind of bond is therefore an hybrid instrument, a debt

vehicle with an equity kicker.

In summary, the two main benefits of GLBs discussed in the literature are the debt stabilizing effects, as the ratio of debt over GDP will fall in volatility, and the boost in fiscal space (Shiller, 2018). I argue that a lower debt over GDP volatility also means a lower real interest rate for all the sovereign debt of the issuing country, not only for the indexation portion, as the default risk premium charged should be lower, given the introduction of frictions in the potential debt spiral.

3 Fiscal space

A key issue for conscientious governments is to let there be room for a possible expansionary fiscal maneuver which could be needed in the future, if the conditions arise, in order to be able to face possible future slowdowns in the economy. By fiscal space I therefore mean how much more can a government push down its fiscal balance, either by lowering its taxes, raising its non interest spending or both, without undermining public debt sustainability; but it can be more properly defined as the difference between the debt limit and current debt. Fiscal space is very much valuable, especially considering that its lack thereof ignites the explosive debt dynamics where higher debt and higher interest rates, caused at least by higher default risk, feed unto each other, therefore causing debt to spiral out of control, until an inevitable default - or debt monetization by the central bank at best.

Considering the lower marginal benefits of the global ongoing monetary expansion - or lack of subsequent tightening - financial markets have become increasingly worried by fiscal concerns, given the lack of substantial growth in advanced economies and their demographic pressures. One might wonder at what yield would the sovereign bonds of Japan, Italy, France and others trade and be issued at if the respective central banks where not there to buy huge amounts; likely neither nominal nor real rates would be negative. The result is that most governments in the advanced countries currently do not feel to have or do not have any fiscal space - or room for more government stimulus of the economy - left, while the variability of emerging countries economic performance always demands some kind of stricter fiscal responsibility from their governments.

In practice fiscal space translates in how much public debt can a country carry without raising founded default concerns: this limit is empirically higher for advanced economies than for the others, and their current debt levels reflect this. Anyway the point is that some kind of fiscal consolidation is needed in order to reduce overall debt levels, but can it be done without hurting growth to much? Conversely, are there ways to design public debt such that heavily positive primary balances won't be needed, even assuming that they could ever be feasible, but rather enable a growth friendly fiscal consolidation?

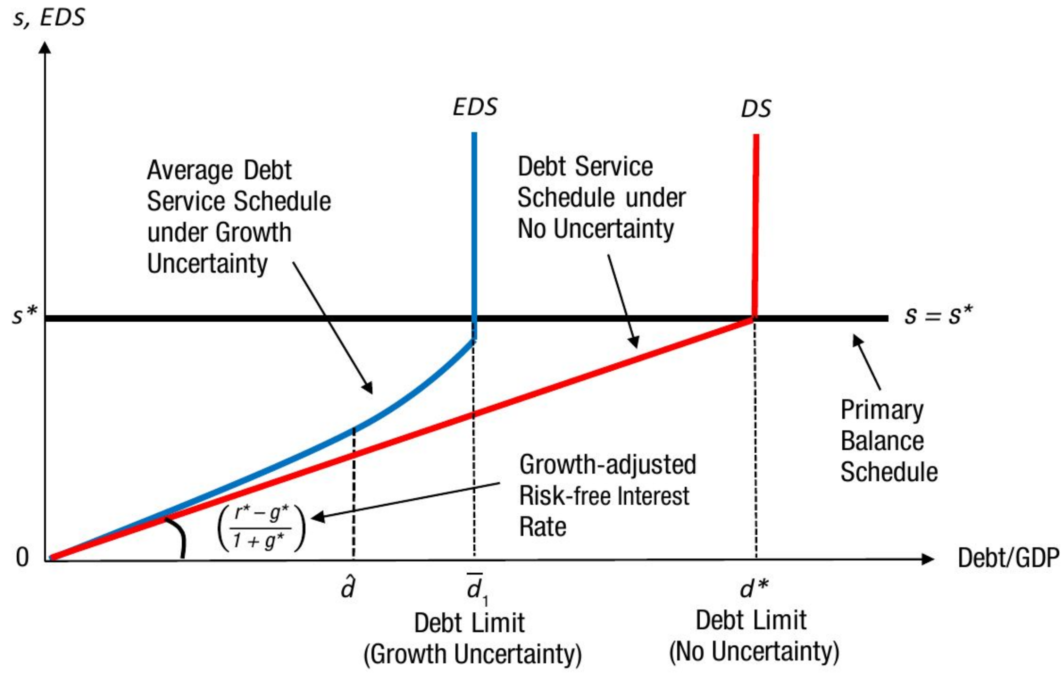
The two main possible solutions are indexing to GDP and issuing bonds with longer duration (Kim and Ostry, 2018).

The dynamics through which indexing to GDP are able to enhance fiscal space were explored in the last paragraph, but can be described slightly differently as transferring the uncertainty about future growth from the issuer to bondholders.

The second way, the lengthening of maturities, works as well. In the extreme case of perpetual bonds, debt would be almost equivalent to preferred stock; without going that far, longer dated debt still looks more like equity, or at least hybrid instruments, and enables a risk-sharing benefit to the issuer.

If by indexing to GDP it was uncertainty about growth that was being reduced, in this case it's rollover risk that is being lowered: in the case of high debt levels a good portion of new debt has the sole purpose of repaying the current one, or just making good on fixed contractual obligations. Of course, that alone increases the importance of future nominal interest rates at debt auctions, or how much money and how cheaply the sovereign issuing entity will be able to raise funds in the futures, since explosive debt dynamics could then be triggered by even a short period of time during which interest rates are high.

Figure 7. Effect of growth uncertainty on debt limit and illustration of explosive debt dynamics.



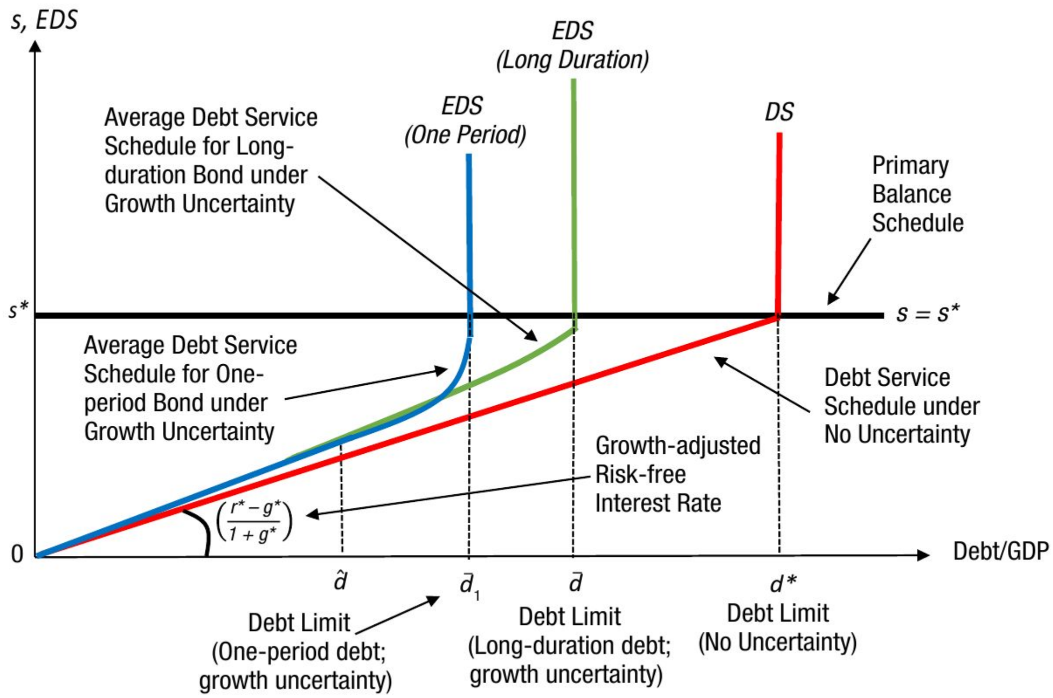
Source: Kim and Ostry, 2018

The uncertainty about the future ability to service current debt, let alone new one, that comes with an high rotation of debt, negatively impacts fiscal space by increasing default risk, thus interest rates. The gross financing need would be sensibly lower with a debt stock exhibiting a lower rotation, higher maturities, and, thus, reducing the vulnerability of debt dynamics to to adverse shocks either to growth, primary balances, interest rates.

With shorter duration bonds, there are many more expectations about future conditions; every single expectation comes with uncertainty and, thus, requires a risk premium to be paid by the issuer to the bond investor in the form of an higher discount on new debt securities. Longer dated securities are overall less vulnerable to short term shocks simply because their cash flows are spread out in time, so a smaller chunk of payments is likely to be affected by the same shock than as it were to happen to a shorter dated security; furthermore, if there are not only negative shocks out there, then longer dated bonds are likely to reflect that. It's true that moving to longer dated maturities is not an easy task: if done by converting existing debt to longer dated bonds,that would imply a debt consolidation, which won't be welcomed by most bondholders, even if rationally that could increase their probability of ever getting their money back. The other way, auctioning debt with a maturity very distant in time, seems more feasible: a country that has done that, also taking advantage of its currently abismal nominal interest rates, has been Japan, which has been issuing in the last few years Japanese Government Bonds (JGBs) with a maturity of 40 years.

Gains in the debt limit have been estimated to be in a range between 21 and 56 GDP percentage points for a full indexation to GDP and to be around 35 GDP percentage points for increasing the average duration from 10 to 20 years (Kim and Ostry, 2018).

Figure 8. Effect of longer durations on debt limit and illustration of explosive debt dynamics.



Source: Kim and Ostry, 2018

3.1 Local currency

The difference in debt limits across governments is exacerbated in the case in which a country holds a significant portion of its public debt in bonds denominated in foreign currency, which hinders the option of debt monetization - a procedure that if done in size can be compared to actually defaulting, with the exception that there would be less difficulties than in a default negotiations / debt restructuring. Furthermore, by issuing a lot of debt in foreign currency a country exposes itself to possibile currency depreciation - just look at the usDARS exchange rate, which, until 2001, was fixed 1:1, where now it trades at about 38 Argentinian pesos for 1 US dollar. This exposure requires the issuing government to hold significant reserves of the currency the bonds issued are denominated in, or to having to buy it at the market at the time of payment. Of course knowing the demand of foreign currency worsens the local currency depreciation in times when there are default concerns regarding the issuing entity, making debt payments pricier and pricier until the loop feeds itself and the government defaults. All of this translates in a tremendously lower debt limit and fiscal space.

But why it is common for emerging countries to issue debt denominated in foreign currencies, typically in those with “reserve status” like usD, EUR, JPY? They are clearly thinking that doing so their debt issues will be more appetible to foreign investors, who would then feel reassured that they don’t have to hedge away their currency exposure: their payment is in a fixed amount of - presumably - their own currency, so they need to to worry about potential currency depreciation of the issuer currency. The consequence is that, during the auction of new debt issues, the sale of those will be easy (or at least easier), enabling the issuer to access to a larger market and get a more competitive bid, i.e. issue at a lower discount or lower implied interest rate.

This line of thinking might be flawed in my opinion. Investors are still exposed to currency depreciation in the measure that it makes less likely for the issuer to make ever growing debt payments in he vicious circle of progressively lower local currency value and higher probability of default on foreign denominated bonds and default or monetization of local currency denominated bonds. Of course, knowing this, investors in the foreign denominated ones will still purchase them only at a discount reflecting the problem: the issuer might not even get better conditions by denominating in foreign currency.

As far as making it easier for global investors to purchase their bonds, emerging markets can anchor the coupon and principal reimbursement to local currency, but make the actual payment in foreign currency, converting the amount at the prevailing market exchange rate at the time of payment, thus making it easier for investors to receive payments. This, obviously, still transfers the exchange risk to the global investor, but it makes the issuer appeal to a wider audience in issuing debt. Receiving payments directly in a preferred currency would really be a marginal benefit for financial institutions, but it would be more appealing for smaller investors. Conversely, it doesn’t matter so much in which currency the payments are in, but if the amount to be paid out is calculated in local currency or not.

The gap in credit risk between local and foreign public debt securities in emerging economies has been decreasing over the last two decades, in particular thanks to the accumulations of higher forex reserves by the issuing country (Amstad et al., 2018). Similarly to the previous explanations for identifying growth uncertainty and future refinancing costs uncertainty as drivers of lower debt limits and fiscal space, I argue that the evidence of FX reserves lowering the gap previously mentioned translates directly in uncertainty about the future relative value of the local currency being a key driver of fiscal space. The gains in terms of debt limit are estimated to be of about 40 GDP percentage points for emerging markets (Pienkowski, 2017).

Chapter II – Designing GLB

1 Historical precedents

The idea of linking public debt to GDP is not new and their benefits as a debt stabilizer appear to be obvious. Indeed there have been GDP indexed debt securities issued by sovereigns in the past, featuring a variety of complex designs and featuring different degrees of success of the product, measured both in terms of risk-sharing properties for the issuer and presence of a liquid secondary market. In particular past issuance always happened during time of distress, usually during debt restructurings following governments defaulting on public debt. But, as the saying goes, "necessity is the mother of invention".

1.1 GDP-linked warrants

There have been a few cases of clauses or warrants included in the Brady bonds context, following the Mexican default in 1982, which triggered a series of other defaults, that provided extra payments to bondholders if certain favorable economic conditions were to happen. They were called Value Recovery Rights; the ones adopted by Costa Rica, Bulgaria and Bosnia and Herzegovina in early the 90's where liked to growth or the GDP level (Borensztein, 2016). Despite that, the focus will be on the largest issuance of growth indexed instruments in terms of size, namely the examples provided by Argentina in 2005 and 2010, after its default on $93b of external debt in 2001, and the most recent example of Greece in 2012, with the restructuring a €172b of its outstanding debt (Zettelmeyer et al., 2013).
What the two countries did was to issue detachable GDP-linked warrants (GLWs) together with new long term bonds and swapped them with some of the defaulted ones. In the case of

Argentina swapped bonds gave bondholders an haircut on notional value of about ⅓ , but the attached GLWs could then reward them with a maximum payout of 48% of the original bonds' notional value. Of course they acted as sweeteners during a very complicated debt restructuring, which took years to be settled after the original default. Bonds were swapped in 2004, but the GLWs were detached in 2005 and become a security with a secondary market of their own. A second round happened in 2010 in a smaller size. To give an hint about the complexity of these issues, which, nevertheless, managed to develop decent activity in a secondary market, just consider that there were five kinds of them, classified by currency of payment and legal enforcement: ARS under Argentinian law, usD under Argentinian law, usD under New York law, EUR under British law, JPY under Japanese law. The payments was triggered by real GDP being above a baseline trajectory and proportional to real growth, with a minimum and maximum cap. Conditions on the greek GLW were similar with some additional complexities, such as calculating the level of GDP in nominal terms, while the growth in real terms. The link to the prospectus of these securities is reported in the the Data Sources.

The value of the argentinian and greek GLWs experienced an opposite fate, determined in the first place by the change in GDP that happened in the first years following the default and restructuring of debt: while Argentinian economy quickly recovered, triggering initially an high payment almost every year, the Greek one didn't, making its GLW akin to a far out-of-the-money call option on its GDP.

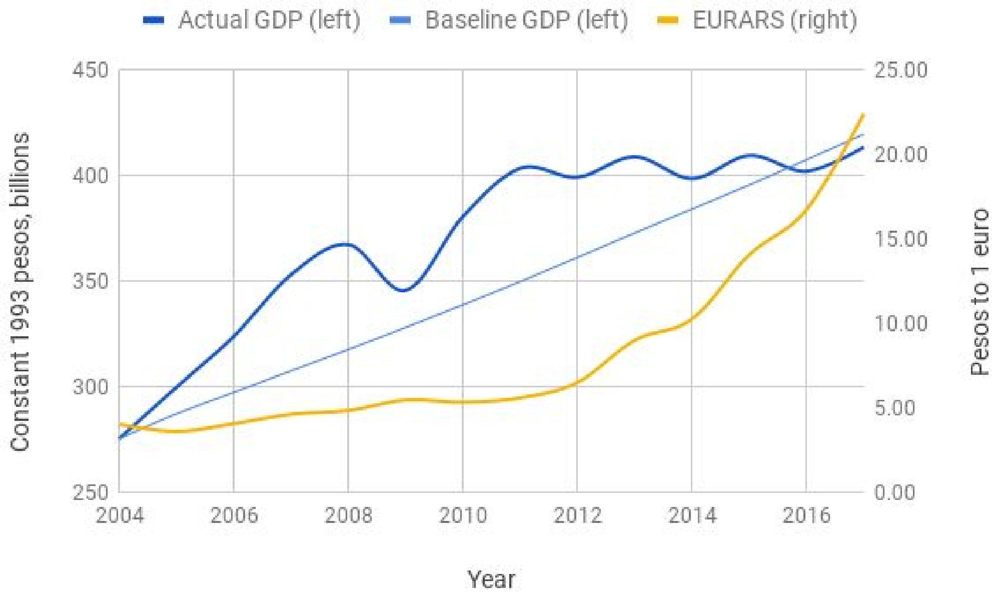
Figure 9. Argentina's euro denominated GLW - ISIN:XS0209139244

Source: Bloomberg

Figure 10. Greece GLW - ISIN:GRR000000010

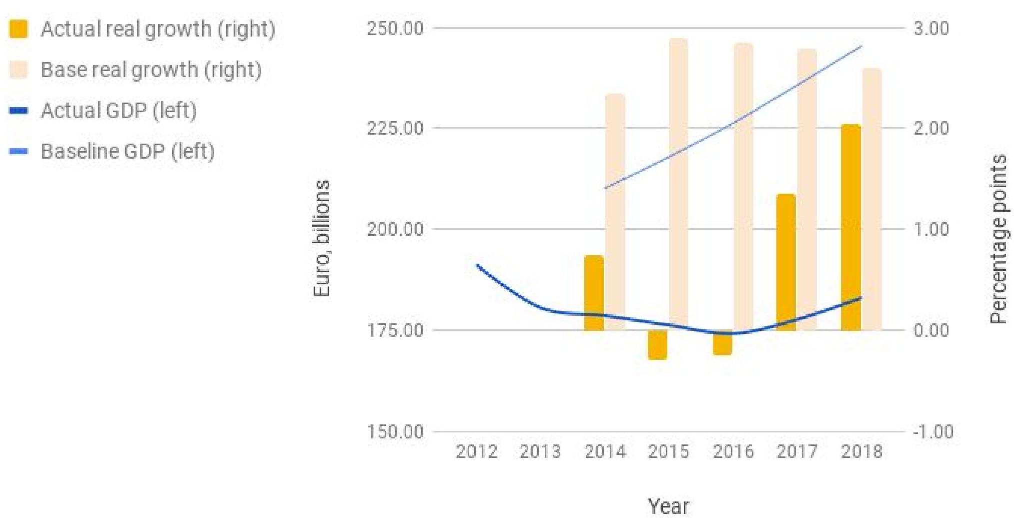
Source: Bloomberg

Figure 11. Argentina's euro denominated GLW - underlying variables



Sources: IMF, SEC

Figure 12. Greece GLW - underlying variables



Sources: IMF, Public Debt Management Agency (Greece)

Argentinian GLWs were to pay a positive function of excess of real GDP over a baseline, with a trigger condition of real growth being above the one implied by the baseline trajectory; thus it payed coupons from 2005 to 2011, with the exception of year 2009 (Figure 11) and it didn't pay further, as the condition wasn't triggered anymore.

Viceversa, the Greek GLW was to pay a coupon in a range from 0 to 1% of notional on a principal amount decaying after 2023, setting the coupon rate equal to real growth rate in excess of a baseline of about 2.3%. As excess growth has been negative (Figure 12), no payment has been made; the trigger here was the nominal GDP being above a baseline, which, for 2018, is 25% still above the actual level.

From the investor point of view, the Argentina GLW was a tremendous investment, holding to a 12.35% annualized total return over more than a decade, with the country growing economically in the middle '00s. conversely the Greek GLW would have been a 71% loss, as Greece faced recession years until 2017 and now it seems almost impossible that it will grow to the point of triggering coupon payments.

A few words on why payments on the Argentinian GLWs were so high and why the indexation to growth didn't provide as much of a relief for the issuer. Following the default, the exchange rate with the US dollar shifted from a fixed to a floating regime, jumping from a 1:1 ratio to 3 ARS per usD (now it's 37.2, as of February 6, 2019). Currency depreciation and hyperinflation happened, with CPI rising at a geometric average of 22% a year in the 2002-2018 period. If the main problem of the GLW was the the high payments as the real GDP followed a path quite above the baseline case, the second one comes from the indexation lag, with the period between the end of the reference period for the GDP number and the coupon payment lasting 1 year. This fact vastly diminishes the counter-cyclical properties of the indexation, with payments that can be high even is the economy is slowing and vice versa; moreover, hyperinflation affected the value real value of payments for ARS denominated GLWs, as an inflation of, let's say, 10% in that year would imply a cut of the same amount in the real dividend. From the issuer point of view, foreign denominated GLWs were the most dear ones, as the payments was determined in nominal ARS from a formula base on excess real growth with respect with a base real growth and then converted at the current exchange rate, thus effectively rising the dividend amount by a year of depreciation in the Argentinian peso.

GLWs might have been an instrument too complex to find a large acceptance and have been used only as sweeteners for debt issuance in distress countries. Nonetheless these experiment are important in the path to a viable standalone GLB instrument issued in large sizes and key lessons can be appreciated. Payment structure has to be simple in order to find acceptance by investors and not create obstacles to the creation of a liquid secondary market. Most of all, the indexation should hold true to its premise of providing fiscal space by make debt service pro-cyclical, conversely, the premise to contractually specifying a temporarily interest payment relief in times of economic distress. There is also the problem of data continuity, with the example of Argentina changing the base year for GDP calculation in 2013: the bond documentation was far from comprehensive and gave rise to different interpretations on which GDP methodology to adopt for the coupon calculation.

On the bright side, Argentinian GLWs managed to find some liquidity, despite their complexity, thus indicating that novelty premiums won't hinder the effectiveness of future attempts at indexation (Costa, 2008). In addition to that, fears about data misreporting to the advantage of the issuer seem to be misplaced: there is a natural incentive for governments not to underestimate growth, as politicians are heavily judged by the population on that basis. In fact, private economists estimated was that official statistics in fact overstated real economic growth during 2008-2014 (Borensztein, 2016).

When considering what the design of an indexed debt security should be, there should be room for the consideration that countries until now have chosen different forms of indexation, plus a number of additional trigger points.

2 Risk premia

For a new sovereign debt instrument to be successful it should be simple to comprehend its cash flows and it shouldn't be too complex to price.

When in comes to the benefits of standardization, we are mainly talking about reducing or, possibly, eliminating risk premiums other than the default risk of the issuer. Their direct consequence is that the issuer has to sold the debt security at an excessessive discount in the form of risk premia, thus raising less money than it should. They can be clearly identified as:

- GDP or growth risk premium, which seems to be inherent with a GDP indexed instrument
- novelty premium, or the learning process needed for a new product
- liquidity premium, as the fear that there won't be a secondary market where to potentially sell the bonds later feeds itself
- agency costs, as the institution publishing the economic data has to be trusted

On a bright note, default risk should decrease for all debt, if the proportion of it that is indexed to GDP really provides for some debt stabilization. The lunch of such a product needs be successful from the get-go, namely there shouldn't be much of a novelty premium, which might be caused by initial skepticism, the security was to be easily sold in particular to long term funds and buy and hold investors and, in general, if the interest in the new security, as measured by its ability to find a place in the secondary market, was enough to bring to life a liquid marketplace in which the liquidity and novelty premiums peters out (Benford et al., 2016). That being the case, further issuance would be much more straightforward, with indexed securities being auctioned near a price estimated from the market price of then trading bonds.

Firstly, the national institution that publishes economic data has to be trustworthy, as it already happens with inflation-linked bonds, where the indicator is the price level of goods. To be sure of the data quality there could be a constraint on bonds that requires the institution to qualify for the IMF standards in data reporting (Joy, 2017).

The novelty and illiquidity premia should decay over time, as investors and financial institutions understand the product and its pricing more and more. Of course the work required to address a new debt instrument can be stimulated only at product launch. As in the case of the United Kingdom first issuance of sovereign inflation-linked bonds in 1981, the first GLB issuer should schedule an heavy issuance of the same instrument over time, both in terms of size and continuity. It has to be clear that from that point onwards, a proper percentage, say in a range between 10% and 30%, of total new sovereign debt issues will be made of GLB having that specific design. This way financial institution won't be allowed (by competition and the cost of lost opportunities) to forego the pricing of that instrument.

The problems and following risk premia caused by a novelty factor and uncertain future liquidity should be tackle together and immediately, to help the first issuer. In order to do so, the first issuance of GLB should come from multiple sovereigns at once, each scheduling an heavy further issuance of them over time. Moreover, the debt instrument issued should share the same design, helping the market in pricing these bonds. The parallel issuance would facilitate investor diversification among different countries, thereby pricing in only the risk associated with that specific country and letting a portfolio of indexed product coming from a number of issuers bear only market risk, in this case a global growth risk. This is of course a big coordination problem; it's one of the reasons why GLB are not a thing yet, even when they appear an obviously helpful debt instrument in letting governments hedge growth uncertainty.

Another consideration is that if the instrument was to be designed with the buy and hold investor in mind as primary buyer, much of the illiquidity premium would be gone, since there are interested in buying the security and holding it to maturity, receiving a stream of variable payments in the meantime. Moreover, GDP indexed product should be an easy sell to pension funds, as they commonly offer benefits linked with the rage rate. Real wages, at least in the long run, are related to the productivity of labour, which is in fact the biggest component of GDP growth (Shiller and Ostry, 2018); a GDP-linked security would be an asset properly matching pension funds' liabilities.

As far as the GDP risk premium goes, that will always be there for a good reason: there is uncertainty regarding the future growth of a country. If this premium was to be too much, the issuance of GLBs would be too costly, offsetting its debt stabilizing benefits. Which means that this premium has to be estimated *ex ante* in order to determine whether the issue this new kind of debt instrument is worth it or not.

This premium has been estimated to be around 150 basis points (bps) for United States (Kamstra and Shiller, 2009), while the average for advanced economies might be 140 bps and 80 bps for emerging markets, reflecting their lower exposure to systematic GDP risk (Benford et al., 2016). While this premium is definitely a cost, it may still be worth the trouble. The estimation process makes use of the infamous CAPM, either using a global stock index or the world GDP as the market portfolio.

Another way to compute the premium of a single country would be to start from a "market" or global GDP risk premium (GRP) and use beta coefficients calculated for a single country growth against world growth. In particular, considering *g* as the real growth rate, a specific country GDP risk premium can be estimated starting from:

$$\beta = cov(g_{country}, g_{world})/var(g_{world})$$

To compute a base GRP I used data from the U.S.A., as the largest world economic superpower in the last century (1919-2018). I started from the construction of four fictional "portfolios", the first representing the total return of the equity market, the second growing at the rate of return of a 10 years Treasury Note, the third growing at the nominal growth rate of the economy. The fourth and last portfolio grows at the combined return of the nominal growth rate and the rate of return on bonds, or what a GLB characterized by a fixed coupon rate and principal indexation to GDP would yield if issued at the same price of a simple conventional bond. The assumption here is absurd, since a GLB offers plenty more upside, but I shall use the latter portfolio just to compute its volatility, relatively to the volatility of others.

Table 4. Annualized statistics of fictional portfolios, 1919-2018

Portfolio	Total return	Volatility
US Equities	10.05%	18.60%
US Treasury Notes	4.91%	7.09%
US GDP	5.77%	7.05%
GLB, principal indexation	10.97%	10.07%

Sources: Shiller, author's computations

Historically, using the last 100 years of data, the us equity risk premium ERP has been 4.90%, compounding returns once a year. By exploiting the consideration that in a perfectly efficient market assets with the same return should be similar in risk, as measured by the volatility of their returns, I obtained a GRP by scaling down the ERP to the excess volatility of the equities and GLB portfolios relatively to the bonds portfolio.

$$GRP = ERP \frac{\sigma_{GLB} - \sigma_{bonds}}{\sigma_{equities} - \sigma_{bonds}} = 4.90\% \frac{10.07 - 7.09}{18.60 - 7.09} = 1.27\%$$

Although computed with a subjective methodology, results are in line with previous estimates; I shall use this new one as the world GRP in computations for each country own GDP risk premium.

3 Standardization

Standardization would help the creation of a new healthy market, where investors and traders would easily understand and be interested in this new kind of bonds and where the difference among GLB issued by multiple countries would only be the different default risk, future growth estimations and, possibly, currency risk, but not in their payment structure.
Considering the importance of issuing and refinancing debt for a whole country, the path towards standardization even prior to the first sizable issuance matters, as it could be costly to let the market decide itself a consensus about a preferred design, meaning that countries that issued GLB in a non standard one would than have to probably switch later, possibly paying the price of having to convert existing securities.

There have been several proposal of how a GLB should be structured. Recently, the London Term Sheet has been draft by the ICMA (International Capital Market Association) as a consequence of a collaboration between Allen & Overy, an international law firm, the IIF (International Institute of finance), a global association of 500+ financial institutions and the EMTA, an association of traders and investors in the emerging markets. The document deals with the practicality of a GLB, allowing for a variety of designs and letting the actual contract be valid under English law (Allen & Overy, 2017). This work prepares the ground for an hypothetical GLB which will have already passed a legal scrutiny.
Furthermore, the process of generating this term sheet, which is now at its third version, involved a survey of potential investors regarding their actual preferences in the debt instrument design. Of course these options should be accounted for, but the original purposes for issuing such an instrument should come first.

No. 1

There is consensus about using GDP as the economic variable as a proxy for both a performance indicator, from the investor perspective, and for a country's payment capacity, determined by its tax revenues, which is the issuer perspective. Almost every country in the world has a good number of historical GDP data, giving the foundation for a trustworthy standard of data and context around the dynamics of its growth. There have been cases of indexation to other relevant economically relevant variables or events, like commodity prices for commodity-exporting countries, making debt service oscillates with them, catastrophe bonds that are able to automatically cut coupon payout if certain negative events happen, providing relief for the issuer, and inflation linked bonds, of course. Economic output is a comprehensive measure that takes everything into account, particularly if denominated in nominal terms. It serves as the most universal risk-sharing vehicle when it comes to economic conditions and would be an ideal indexation from the point of view of governments, if their payment capacity is truly proxied by them.

No. 2

It is common for countries going through debt restructurings to issue debt denominated in foreign currencies at the request of investors in order to regain market access. Even the Argentinian GLWs were issued mostly in euros and dollars. Theoretically, payments were to be computed in nominal pesos and them converted, without creating a substantial difference between pesos and foreign currency denominated securities; in practice, however, there was a lag between the reference period for GDP and the reference period for the forex conversion, and the depreciation of the peso compounded the amount to be paid on foreign denominated GLWs. The practice of denominating in international currency, which forces the bearing of currency exchange rates uncertainty on the issuer, is dangerous to say the least, as the issuing country gives up the possibility of monetize its debts, if needed, in additional to potentially face a shortage of foreign currency reserves.

The investor preference is for a foreign currency denomination in case of debt restructurings, but

I argue that this position should be rejected, as it nullifies the GDP indexation purpose, as the switch between foreign and local debt is bigger in terms of fiscal space gains.

The steps for a country towards a fiscal space increase then are: in the first place to issue only debt in the local currency, which mainly involves non advanced economies, then try to increase the average duration of public debt stock, in order to decrease rolling risk due to future interest rates uncertainty, which heavily affects countries exhibiting high public debt levels. Only as a third step, if certain conditions are favourable, namely low GDP risk premium and believable national statistical institutions respecting the IMF requirements in data publishing, issue a slice of total debt indexed to GDP.

What is implied here is that, rather than going through the trouble of GLB denominated in foreign currency, a country has a bigger problem that has to be faced firstly: being able to control its debt.

No. 3

Which kind of indexation to apply is a more debatable subject and, as such, will be dealt with later, as the possible points of view are rather debatable and no clear solution is available.

No. 4

The maturity of debt linked to economic performance should reflect the fact that investors are more willingly to bet on long term growth of a country than short term fluctuation. From the issuer point of view it is even more obvious that this kind of debt should be issued with a long maturity, say 10 to 30 years: keep in mind that long term growth is less volatility than short term fluctuations, as longer maturities span across multiple economic cycles, with one lasting from 6 to 8 years historically, so that short term shocks to the economy are smoothed out. The takeaway is that the GDP risk premium should be lower with long term maturities. Of course, if the GLB issuance happens at a time of distress, the market short term forecasts may be catastrophic, bidding a low price for the bond, while it would be incredibly pessimistic not to incorporate some kind of a positive long term growth pattern in the price of a GLB at its issuance.

There has been even the proposal of the extreme design of a perpetual GLB paying only annual coupons (for obvious reasons), calling said bonds *"trills"*, as one contract could give its owner the right to 1 trillionth of a country GDP (Kamstra and Shiller, 2009). This instrument would be the ultimate dream towards satisfying the purpose of accounting only for long term growth patterns and letting the issuer be the most flexible with each year payments, as a few bad years for its economy would cut its interest expense and not negatively impact the price of its GLB at the same time. At the same time there will be a practical problem in issuing annuities as fractions of GDP: the central government can issue just as much of them, totalling at best a few GDP percentage points. The trick with known perpetual bonds precedents is that they were denominated in nominal currency, thus real payments fell nearly to 0 over time - in the long term the average inflation has always been positive. Being the trills also an hedge to inflation, as the latter increases nominal GDP, they constitute a tremendous long term committed that is fixed in terms of GDP points. There appear to be little appetite towards perpetuals – I certainly don't blame sovereigns for that. There seem to be little appetite from investors too, even as a perpetually growing annuity should please them.

No. 5

As experience with the GLWs tough us, it is important to define how to deal with publications delays, data revisions and which indexation lag length to choose.

In practice GDP is a lagged measure, as it isn't readily available right after the end of a period; the calculation of this metric requires time to collect and verify all the data needed. In many developed countries there is a first flash estimate of GDP two months after the end of the reference period, but the exact number is adjusted multiple times and even revised years after. Given that the GLB actual payment has to be based on a specific number, there is the problem of selecting how much to wait between the end of the reference period for the GDP calculation and the coupon payment. Paying early would give more sensibility in reflecting in the coupon size the payment capability of the issuer, which is the whole reason for GLBs, while waiting for a more refined GDP estimate, less subject to future revisions, would be closer to the truth, helping in dealing with GDP revisions. History tells us that a full year between the end of the reference period and the actual payment is far too long: it was one of the pitfalls of Argentina's GLWs. The fact it that in a year economic conditions may have dramatically changed, resulting in a high dividend during times of economic distress or vice-versa, letting either the issuer of investors worse off. Referring to the lag tradeoff, inflation-linked bonds uses a three months lag, while there is consensus for a six months lag for GLBs.

There are both frequent and small routine adjustments to GDP and non-routine revisions. The latter ones tends to be difficult to predict in terms of magnitude and also include periodic GDP methodology changes, which should happen every 5 years. It should be noted that such methodologies changes have little impact in the year to year growth, at least usually.

There are two approaches in dealing with revisions:

1. index each payment to the last available GDP data, letting the market deal with the changes in the methodology;
2. index the payments to a notional, chain-linked series that is constructed by cumulating together lagged estimates of recorded GDP-growth, thereby stripping out the effect of revisions, adding work duties to the institution publishing the economic data.

The second options seems to be preferred from investors, which is similar to what happens generally with inflation-linked, whose principal is linked to an index that is not revised. On the one hand, considering that a GDP measure lagged ½ a year is already at its first or second revision, routine adjustments are not worrisome; on the other hand GDP methodology changes - rebasements - seem to make an even bigger case for the second option, as a chained series built with growth rates of individual years won't be revised in the future. It argues for computing each year growth with the most updated methodology, as it already happens.

No. 6

Other data problems may arise. If there is a technical failure at the statistical office of a country and so the data dissemination process is delayed, the issuer should be allowed a grace period before any put event is triggered.

Investor protection against data manipulation should trigger put events – the possibility of redeeming the bond at full value now – if the issuers fails to satisfy the quality of data required by the IMF, including its data dissemination standards. It should be noted that a government has little incentives in under reporting its GDP due to political pressures, while sizeable later GDP revisions usually happen to the upside.

No. 7

With respect to the reference period for the calculation, investors would favor more the indexation to quarterly data, which would also give a more responsive link between economic performance and following payment. It should be argued that indexing to annual data smooths out some additional volatility and seasonal patterns. For the sake of design simplicity, consider only annual data should be investigated first.

No. 8

Debt seniority has to be properly defined in order to avoid possible litigations later. Investors indicates preference towards a *pari passu* clause, meaning that GDP indexed debt should be ranked on the same legal footing of all other unsecured debt.

Cross-default clauses allows investors to require an early payment on their debt if the sovereign issuer fails to pay some other debt. All the remaining creditors can sit at the same discussion table in order to propose a solution on how to make the most of their holdings, i.e. should the issuer restructure its debt and should the remaining creditors take a haircut. If the GDP linkage works as intended, the GLB holders won't face an haircut, rather it will be the bond that, by design, doesn't pay that year o pays a very little amount; therefore the sovereign should be able to service the GDP-linked debt well even under times of distress.

3.1 Shiller macro bonds and trills

There are two main ways to link debt payments to GDP.
The older one was proposed by Shiller in 1993 and is very similar to the way inflation-linked bonds are structured: the coupon is still a fixed fraction of the principal, as in straight bonds, but the principal amount grows over time with nominal GDP.
Considering the last GDP prior to GLB issuance as the GDP at time 0 and the GDP of the year *t*, the bond principal over which the coupon rate will be applied in computing coupon payments will change year by year is given by

$$L_t = GDP_t / GDP_0$$

After all the coupon payments, at maturity T, L_T will be returned to bondholders as the principal reimbursement.
This way the indexation methodology is akin to inflation linked bonds. Moreover, this type of GLB even offers inflation protection to the investor, as the principal is linked to nominal GDP. The Shiller design might be more familiar to investors and governments alike; its benefits stems from tweaking a very well know debt structure by working on the principal alone. Of course, there is a downside to it: although debt becomes more stable, debt payments become less sensible to recent economic performance over time, as the most recent growth rate won't change the principal that much. If a country enjoys years of economic growth, coupons on GLBs will grow as well; then, yearly payments will remain relatively high for quite some time, even in the wake of negative growth shocks: the indexation will take time to adjust downward payments in the case of a recession.

3.2 Borensztein and Mauro

Borensztein and Mauro in 2004, in their famous paper "The case for GDP-linked bonds", proposed to link each coupon payment to the real GDP growth, calculated from the GDP denominated in constant currency, meaning:

$$g = \frac{GDP_t^C}{GDP_0^C} - 1$$

They proposed to index the coupon rate, letting the principal to stay fixed, deploying a fixed starting coupon or base coupon, c_0, and a minimum growth, g^*. The actual coupon rate shall be:

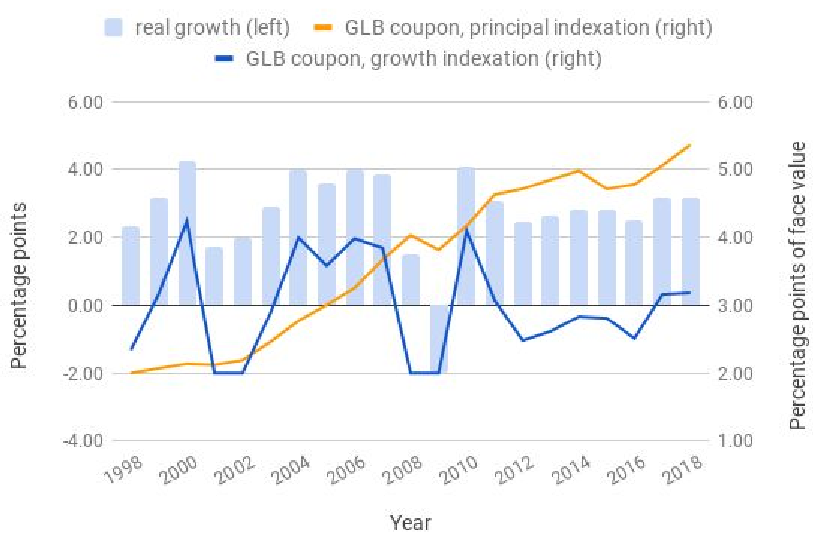
$$c = c_0 + max(g - g^*, 0)$$

so that what is actually added to the base coupon is the excess growth with respect to a minimum. The coupon indexation approach allows for a more strong linkage between recent economic performance and debt expense; the annual GLB payments fluctuate more than they do with the Shiller version. The coupon calculations closely resemble the Greek GLW, without bringing in further complications as trigger points; in fact, linking each yearly payment to the then most recent growth rate can be complicated enough in terms of pricing.

The cash flows added to the base payment should be valued separately from the straight bond component given by the base coupon rate. They are akin call options on an underlying that changes each and every year, namely that year real growth rate. It can be argued that they can be priced as a function of the country real growth distribution, realizing that, in the long term, the mean payment should reflect the mean growth rate for that country. Problems arise at least on two fronts: it is true that the growth rate oscillates around its mean, but in which year it will be below or above matters very much, as payments further out in time are subjected to a higher discount factor. Moreover, growth rates are not independent and identically distributed: there are autocorrelations between a year and another that also change in time and across countries, adding to the complexity in pricing.

It has to be noted that, in spite of this, the Greek GLW has found a market, even if a not very liquid one, and that it is the most growth sensible form of indexation: it contractually provides for a temporary halt on GLBs' payments is the base coupon is set to 0. Conversely, they come with an automatic temporary default clause. Figure 13 provides for a comparison of different payouts in the two broad GLBs design; the underlying data refers to nominal world GDP denominated in US dollars and world real growth rate in the 1998-2018 period. The starting principal is set at \$100, the coupon rate for the security under the Shiller design and the base coupon rate under Borensztein and Mauro is set 2% and so is the average real growth rate, which is subtracted from the coupon rate in the latter design. It has to be beared in mind that actual payments would happen with a half year delay from the reference period for GDP calculation. Both the inflation-hedged and growth sensibility properties of the two different designs are blatant.

Figure 13. World real growth and coupon rates of GLBs linked to nominal world \$GDP and world real growth rate respectively, setting: $c_0 = 2\%; \quad g^* = 2\%$



Sources: IMF, author's computations

Investor feedback has be more positive towards linking the principal to GDP, rather than coupons to growth. The original GLB version is less responsive to recent growth shocks, but it allows for a simpler instrument similar to others already in the market, such as inflation linked bonds.
Being this the case, this thesis will now focus on the simpler design and ask it allows for the GDP indexation premises to hold true.

4 Cash flows and pricing

In the Shiller design coupon payments and principal reimbursement are referred to a modified principal

$$L_t = L\frac{GDP_t}{GDP_0} = L\prod_{n=1}^{t}(1+\gamma_n)$$

where L is the principal, GDP_t the nominal GDP at year t and γ_t its growth rate.
Which gives rise to the serie

$$\{L_0, L_1, ..., L_t, L_{t+1}, ..., L_T\}$$

which is equivalent to

$$\frac{L}{GDP_0}\{GDP_0, GDP_1, ..., GDP_t, GDP_{t+1}, ..., GDP_T\}$$

With conventional bonds we would discount the cash flows by multiplying by nominal interest rates, being i the nominal interest rate that incorporates both the real interest rate r, the inflation expectation π. The risk premium associated with the default risk resides inside the real interest rate, such that

$$r = r_f + \text{default spread}$$

Considering the additional risk involved in GLBs, I argue that we are able to discount the cash flows in similar fashion by multiplying by an higher discount rate, where the country GDP risk premium is given by

$$\theta = \beta GRP_{world} = GRP_{country} \ (4)$$

then the discount rate should be

$$i_{glb,t} = i_t + \theta$$

By indexing the principal to nominal GDP the bond cash flows and present value become, respectively,

$$c \sum_{t=1}^{T} L_t + L_T$$

$$L\Big(c \sum_{t=1}^{T} \prod_{n=1}^{t} (1+\gamma_n) exp(-i_{glb,t} t) + \prod_{n=1}^{T} exp(-i_{glb,T} T) \ (5)$$

which exploits the approximation of a yield to maturity and an average long term growth rate γ

$$exp(\gamma t) \approx (1+\gamma)^t = (1+\gamma_n)^t$$

Equation (5) can be simplified to

$$L\Big(c\sum_{t=1}^{T} exp\big(-(y+\theta-\gamma)t\big) + exp\big(-(y+\theta-\gamma)T\big)\Big) \quad (5.1)$$

which represents the present value of a GLB with the principal indexed to GDP.

Consider now the ratio between a GLB and a conventional bond.

Multiplying the conventional bond present value of equation (1) by

$$exp(\gamma - i_{glb} + i) = exp(\gamma - i - \theta + i) = exp(\gamma - \theta)$$

yields a GLB with the principal indexed to GDP. Which means that, if the forecasted nominal growth is higher than the risk premium to be paid in the GLB, the latter prices more than a conventional bond. Moreover

$$\gamma - y = (g+\pi) - (r+\pi)$$

so that the present value of a GLB is inflation invariant and equation (5.1) can be rewritten as

$$L\Big(c\sum_{t=1}^{T} exp\big(-(r+\theta-g)t\big) + exp\big(-(r+\theta-g)T\big)\Big) \quad (5.2)$$

meaning that a GLB indexed to nominal GDP also provides inflation protection.

Chapter III – Simulations

1 Set-up

Following the example of James Benford et al. in the Bank of England's Financial Stability Report No. 39, September 2016, titled "Sovereign GDP-linked bonds", the general lower volatility of debt issued with a link to GDP rather than is demonstrated. But lower debt volatility is not enough, as high debt is an issue that has to be resolved in order to push away the possibility of it spinning out of control and cause a sovereign default, with the economic damage that comes with it. Across a selection of 13 defaults happened in the last 40 years (Latin American debt crisis in the 80s, Asian debt crisis in '97, Russia '98, Argentina '01, Greece '12) economies of defaulted countries lost a median value of 9% in real GDP and 38% in GDP denominated in current US dollars in a 2 years period, with unemployment rising 4 percentage points; from the discrepancy in changes in local currency units and US dollar denominated GDP, it is clear that this kind of crises tend to happen together with a local currency devaluation.

To ensure against default, debt has to still be headed down, not just stabilize. One of the premise of GLBs is that, by lowering debt change volatility, the possibility of default weaken; the consequence is that default risk should lessen for all debt, including the non indexed portion, letting the issuer gain fiscal space in the form of lower interest rates, thus a lower debt service expenditure.

The first measure to overcome the premium that comes from the agency costs of having to trust national statistics is to restrict the issuance to believable countries. The consensus found by the London Term Sheet is to require the sovereign issuer to adhere to the IMF dissemination data standards; of course, there is also the precedent of inflation linked bonds. Most of them have been issued by United States and other advanced economies. Among emerging markets Brazil has been indexing a large portion of its debt to IPCA, an index of consumer prices, though *Notas do Tesouro Nacional – Série B*, totalling 27.5% of outstanding federal debt as of December 2018. To solve the problem at the root, for the scope of the simulation, I decided to start by using data from 34 OECD countries, as this organization publishes accurate and complete data of its members.

The 38(+1) countries included in the simulation are: every OECD country but three, with the addition of the Euro Area as a separate *pseudo-country*, China, India, Brazil, Russia and South Africa - meaning that the so-called BRICS countries are all included. The data used here was taken from the IMF World Economic Outlook published in October 2018, with the exception of average annual nominal interest rates for long term debt, which are taken from the OECD database, and other sources when lacking data. The considered countries combined account for 85% of global GDP and 92% of global public debt.

Following the IMF categories of Advanced Economies on one side and Emerging Markets and Developing Economies, the countries selected for the simulation are:

- AE (28): Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Korea, Lithuania, Netherlands, New Zealand, Norway, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, United Kingdom, United States
- EM (10): Brazil, Chile, China, Colombia, Hungary, India, Mexico, Poland, Russia, South Africa

The 100,000 simulations for each individual country are based on real and nominal growth rates, primary balance and outstanding gross public debt in GDP percentage points, nominal interest rates on long term debt. Historical data cover the 1998-2018 window, with data for 2018 being an IMF estimate, and IMF forecasts cover the 2019-2023 window.

The experiment is intended to exhibit the debt path and its estimated range 20 years out, both with existing conventional debt and with the indexation of bonds' principal to GDP: the obtained 98% confidence interval should be much less volatile and featuring lower upper confidence intervals, if the indexation was to be of any help.

From the resulting conventional debt paths, together with default probabilities and default spreads implied in by the CDS spread market, a back-of-the-envelope debt limit is estimated, such that the limit is hit in the first 10 forecasted years (2019-2028) almost exactly with a frequency equivalent to the probability of the country defaulting on its public debt in the same time interval; then said limit is used in order to extrapolate a new default risk for a mixed debt stock of 75% conventional bonds and 25% GLBs. In order for the latter to prove themselves worth it, an overall reduction in default risk should happen. In spite of simulating 20 years in the future, only debt variability and probabilities of default in the next 10 years are accounted for deriving a possible benefit in default spreads by indexing to GDP a portion of the debt. The reason is twofold: on the one hand using 20 years to forecast yet another 20 years is a bit of a stretch statistically speaking and debt ranges started to skyrocket in the second half of the forecast period, on the other hand average outstanding debt maturities are hardly above 10 years even in developed countries (Slok, 2018), meaning that longer forecast horizons are a bit outside the scope of estimating the perceived risk of the possibility of default.

1.1 Debt dynamics

In order to make a forecast for the next 20 years of a debt / GDP range for a given country, first start from building the pieces needed in the debt dynamics equation. Recalling equation (3) and denoting that it describes conventional debt issued in local currency:

$$\Delta d_t^C = \frac{i_t - \gamma_t}{1 + \gamma_t} d_{t-1}^C - pb_t + oadj_t \ (6)$$

which can be further expanded by accounting for the errors in forecasting interest rates, growth rates and primary balances. The errors in these three components forecasts can be modeled by a multivariate normal distribution.

$$\Delta d_t^C = \frac{(i_t + \epsilon_{i,t}) - (\gamma_t + \epsilon_{\gamma,t})}{1 + (\gamma_t + \epsilon_{\gamma,t})} d_{t-1}^C - (pb_t + \epsilon_{pb,t}) + oadj_t \quad (6.1)$$

where

$$\epsilon_t = (\epsilon_{i,t}, \epsilon_{\gamma,t}, \epsilon_{pb,t}) \sim N(0, \Sigma)$$

Although assuming a normal distribution is an approximation, but it has been chosen in order to facilitate the estimation process: only two decades of yearly data have been used as inputs - training data - to the model, mainly because the fundamentals of the economy of a country, and of the entire world economy for that matter, evolve over time.

Correlation tends to be positive for interest rates and growth, which is to be expected considering they are being taken at their nominal values, meaning that they share an inflation component and inflation expectations are based on today's inflation; in spite of that, the only correlation that is positive for every country but two is the one between primary balance and nominal growth (Table 5), which seems to be in line with the argument that governments of countries in economic slowdown have an hard time in raising the primary balance – either cutting public expenditures or raising taxes. Primary balance tends to be a key driver of changes in the debt level; putting the two things together explains the practical near impossibility of indebted slow-growth countries in putting an effort in setting healthy fiscal budgets.

There is also the issue of autocorrelations in the three input variables; in particular the first lag autocorrelation of interest rates comes up at a median value of 0.92 across the considered countries. Modelling the change in interest rates would add an order of magnitude higher in the complexity of the simulation, with changes that have to account for the level of the underlying variable, as interest rates are somewhat bounded in an interval: a soft rational low cap of 0% and debt quickly igniting explosive dynamics and causing a default after a certain threshold in interest rates.

To avoid unneeded additional complexities and assumptions, the multivariate normal model is preferred, as it still provides some benefits: in estimating 6 model variables for each country, 20 data points are enough to let the algorithm converge, plus the different variability of the variables is accounted for, as well as the basic dynamics that bound these variables together.

Table 5. Median correlations for the whole dataset

$\rho(pb,i)$	$\rho(pb,\gamma)$	$\rho(\gamma,i)$	$\rho(pb_t,pb_{t-1})$	$\rho(\gamma_t,\gamma_{t-1})$	$\rho(i_t,i_{t-1})$
0.20	0.51	0.23	0.73	0.35	0.92

Sources: IMF, OECD, author's computations

From the debt dynamics future debt levels can then be forecasted, providing both a baseline or point estimate, which assumes the variables to be deterministic, and a series of debt paths obtained by a large number of simulations, that allow for an estimate of a possible future range.
For the 2019-2023 period the point forecast of the three variables comes from the World Economic Outlook database of the IMF, updated in October 2018, while the point forecast for the 2024-2038 period are simply repetitions of the 2023 point forecasts, allowing for adjustments given by the assumptions discussed below. Future other adjustments to debt stock are set to 0.

Turning to GLBs, there is the problem of estimating a different interest rate for this new kind of instruments. The hypothesis about an *ex-post* return on GLB, which can be translated to its cost for the issuer, is this: supposing both no novelty and no liquidity premiums, which should decay over time anyway, the return of a bond whose principal is indexed to nominal GDP (Shiller bonds) should be given by the growth rate in the economy and a premium as a reward for withstanding growth uncertainty, namely a risk premium that can be though as a predetermined coupon rate, plus a constant accounting a default risk premium, an average risk free rate and whatever adjustment needed in the constans estimation in the ensuing section.
The yield should then be the sum of the nominal growth rate in the economy g, which is also the growth rate of the bond principal, a constant k and a GDP risk premium θ.

$$i_t^{glb} = \gamma + k + \theta$$

Substituting i_t^{glb} in place of i_t in the debt dynamics equation (6) yields

$$\Delta d_t^{glb} = \frac{k+\theta}{1+\gamma_t} d_{t-1}^{glb} - pb_t + oadj_t \ (7)$$

$$\Delta d_t^{glb} = \frac{k+\theta}{1+(\gamma_t+\epsilon_{\gamma,t})} d_{t-1}^{glb} - (pb_t + \epsilon_{pb,t}) + oadj_t \ (7.1)$$

where interest rates uncertainty doesn't play a role.

1.2 Constants estimation

The GRP for each country has to be estimated from raw data. In order to avoid both distortions from the currency market, lessening the believability of growth rates in US dollar denominated GDP, and the different inflation ratios in local currencies and and US dollars, statistics have been computed using real growth rates, the rate of change in GDP denominated in constant local currency units. Table 6 shows the correlation, beta coefficients and GDP risk-premia of selected groups of countries' real growth rates and the real growth rate in the world economy, represented by the world GDP in constant US dollars. The degree of correlation ranges quite a bit, but its median value for the selected countries is 68%. The typical value of beta is nearly 1, as one would expect from a group of countries accounting for the vast majority of the world economy; the variability of the coefficients, however, translates in a large range of GRP, from the 0.23% of Australia to the 3.53% of Lithuania. As this value will be a positive component of the cost of GDP-linked debt for the issuer, it quantifies how much a sovereign should be willing to introduce this kind of securities: were the GRP too high, it may not be worth the trouble. Moreover, I'm assuming a coordinated effort in making a reality of GLBs, meaning these estimates assume a potential for investment diversification across countries. You can also notice that the correlation for the Euro Area as a whole is higher than the average correlation of the single 13 out of 19 Euro Area countries considered, while its GRP is lower: its an example of portfolio diversification, meaning that the closer you get to the market portfolio, the higher the correlation and the lower the specific risk.

Table 6. Averages of GDP risk premium (GRP) estimates for groups of countries

Country Group	Correlation	Beta	GRP
Euro Area	0.835	1.121	1.42%
Euro Area, individual countries	0.670	1.325	1.68%
Advanced Economies ex Euro Area	0.696	1.071	1.36%
Advanced Countries	0.684	1.189	1.51%
Emerging Markets	0.520	0.990	1.25%
Dataset	0.646	1.136	1.44%

Sources: IMF, author's computations

The other component of the debt dynamic equation (7) that has to be estimated country by country is k, namely the average spread between the *ex-post* return and the sum of nominal interest rates, country GRP and nominal growth rates. The referenced simulation requires to set the constant k such that, in the base case where there are no shocks, the debt ratio after 20 years is the same for GDP-linked bonds as it is for conventional debt. Conversely, the baseline for debt is calculated for both conventional debt and GLBs assuming no variability in the underlying variables; then, k is algorithmically chosen such that the 2038 point debt forecast doesn't change with the debt composition. The algorithm is a simple binary search algorithm, but calculations are easy enough to let it converge rather quickly. In order not to favour indexed debt, the algorithm works with the country GRP θ set to 0; meaning that, once the risk premium is added, the baseline for indexed debt will end up higher than for the conventional one.

1.3 Long Term assumptions

In forecasting debt with the two debt dynamics equations (6) (7), one for the conventional side of debt and one for the GDP index side, two adjustments have been made in order to make a long term forecast more realistic.

The first one regards the notion that, in the long run, nominal interest rates tends to be above nominal growth, which is known in literature as the modified golden rule (Escolano, 2010).

$$\lambda = \frac{i-\gamma}{1+\gamma} = \frac{r-g}{1+g} > 0$$

The theoretical significance of this rule comes from efficiency considerations of the growth path and the preference of economic agents for current versus future consumption, under the assumption of looking at an economy to be around its long term dynamic steady state. Simply, an intertemporal budget constraint for sovereign should make the case for interest rates being higher than growth rates, even if this a notion that is being questioned (Olivier, 2019). In any way, I acknowledge the empirical significance of this rule, which holds true for most developed countries over sufficiently long periods of time. As an example, the spread between average interest rates and average growth rates for U.S.A. and U.K. over the last half century are about 0.5%, while it tends to be higher for other european countries. Simulations will account for a long term equilibrium of this spread at 0.5% - a subjective decision, but there is not a good theoretical base for how much it should be, literature only agrees on its non-negativity. Every path will tend to reach this level in 2038, the farthest simulated year. As Table 7 illustrates, the adjustment will be done in the interest rates, rather than the rate of growth, setting a yearly constant increase or decrease in the first proportional to the distance between the spread in 2018 and 0.5%. Conversely, this yearly adjustment to long term nominal interest rates will be equal to

$$\frac{0.5\% - (i_{2018} - \gamma_{2018})}{20}$$

I deem this adjustment necessary and sensible since, on the one hand, in the last decade we have seen interest rates hitting negative levels in advanced economies, either in real terms of both in nominal and real terms; this is mainly due to central bank interventions and it would be foolish to let it hold in a long term forecast. On the other hand, the modified golden rule should also holds true for now high growth emerging countries, as they are expected to reach steady-state in the long run. Of course this will, in turn, increase forecasted public debt levels of low interest rates countries overall.

Table 7. Yearly adjustment to nominal interest rates in simulated debt paths, examples

Country	Interest - growth spread	Yearly adjustment
China	-5.43%	+29.7 bps
India	-4.35%	+24.3 bps
Germany	-3.28%	+18.8 bps
United States	-2.36%	+14.3 bps
Japan	-1.84%	+11.7 bps
Italy	0.13%	+1.9 bps

Sources: IMF, OECD, author's computations

The second assumption an adjustment stems as a consequence of the first one: by substituting a strictly positive λ in the debt dynamics (6)(7), the level of debt will raise exponentially long term, all else equal. The only way for a government to to default on debt in the long run is to sustain a positive primary balance and target at least

$$pb_{LT} = d_0\lambda_{LT}$$

where primary balance and the ratio λ are long term values and debt refers to the current level.
Primary balances range quite a lot across countries and across time, but the median IMF forecast for the selected countries is slightly above 0.5% of GDP. This will be the long term target for the selected countries and they will tend to reach it in 2038; primary balance adjustments will work in a similar fashion to the interest rates adjustments previously mentioned, which implies that countries continuously running negative primary balances, like the United States, will slowly stop to do it in the future.
By arbitrarily choosing to set both the interest rates growth rate spread (in percentage points) and primary balance (in points of GDP) to a long term value of 0.5%, I'm implying a sort of turning point level in debt (in points of GDP): if it is slightly below 100%, it will tend to decay. The opposite is true, such that countries which higher debt levels will require to target higher levels of primary balance or, conversely, to grow more that what is expected.
I deemed these two adjustments necessary, as today's global interest rates level are still at an extraordinary low point from an historical point of view, as discussed in Chapter 1.

2 Simulations

2.1 Debt limit

In order to find possible advantages in terms of lower default spreads (and so lower borrowing costs), due to a reduction in debt volatility under indexation to GDP, first it has to be set same kind of debt limit over which a country can be counted as having defaulted. The probability of hitting the debt limit can be estimated from a large number of simulated paths for debt itself; this probability is taken as the probability of actually defaulting and corresponding to a default spread, namely and annualized probability of default. Conversely, a lower probability of hitting the debt limit will be considered a lower probability of defaulting and be associated with a lower default spread and, thus, a lower interest rate.
Given the complexity of the matter at hands and searching for a solution that would depend only on the debt level, I decided to reverse engineer a fictional debt limit, useful only for the scope of this analysis and not a real forecast of a country's debt carrying capabilities.

1. A CDS spread s is obtained for each country where available, while an average of spreads of countries with the same credit rating is taken for countries without one of their own readily available; I assigned to the Euro Area the same CDS spread of Germany. Using a recovery rate of 30%, which has been the weighted average recovery rate of defaulted debt in the past few decades and measures how much value a defaulted bond still holds (Liu, 2017), an annualized default spread or probability of default is calculated as

$$PD = 1 - exp\Big(- \frac{s}{1 - 30\%}\Big) \ (8)$$

and the probability a country defaulting in the next 10 years or, equivalently, hitting the debt limit in the same time window is assumed to be

$$PD_{10} = 1 - exp\big(- 10PD\big) \ (8.1)$$

2. Debt limit for the scope of these simulations is then a specific level of the debt over GDP ratio, algorithmically chosen such that it is hit across different conventional debt paths with a frequency equal to PD_{10}. Doing so I constructed a procedure through which a sovereign default is simply triggered at a condition "*if, then*" that takes only one variable into account, namely the level of debt itself, which is, in turn, a function of growth rates, interest rates, primary balance paths and their related simulated shocks. Returning to the notion of a debt limit, the indexation to GDP should increase it. What I'm going to do instead is to count how much less the same debt limit it is hit under debt indexation, which is just looking at the same problem from another point of view.

2.2 Default spread adjustment

Parallel to the construction of paths for conventional debt, the very same simulated triplets of shocks from the multivariate model used will serve for the construction of the same number of both 100% indexed debt and 25% indexed debt paths. The mixed debt case follows the debt dynamics

$$\begin{aligned} x_1 =& (i_t + \epsilon_{i,t}) + (\gamma_t + \epsilon_{\gamma,t}) \\ x_2 =& k + \theta \end{aligned}$$

$$\Delta d_t^{mix} = d_{t-1}^{mix} \frac{0.75x_1 + 0.25x_2}{1 + (\gamma + \epsilon_{\gamma,t})} - (pb_t + \epsilon_{pb,t}) + oadj_t \ (9)$$

The number of path for the mixed debt that hit the previously found debt limit any time between in the next 10 years, divided by the number of simulations, is taken as an estimate of a first probability of defaulting under a 25% indexation. From these defaulting frequencies a new default spread can be worked back with the inverse equation (8.1), namely:

$$PD_{new} = 0.10ln\left(\frac{1}{1 - PD_{10}^{mix}}\right) \ (10)$$

The Matlab script then investigates whether there is a gain in terms of a lower annualized probability of default, which I'm taking as default spreads incorporated in the long term interest rates of sovereign debt. If there is, the simulation process is iterated as long as there is at least a 10 basis points reduction in the default spread.

The reduction in interest rates for all debt, conventional and indexed, is applied to starting interest rates, so that the long term equilibrium of a spread of 0.5% between 2038 interest rate and growth rate still holds. At each iteration

$$\begin{aligned} i_{2018,new} =& i_{2018,old} - (PD_{old} - PD_{new}) \\ k_{new} =& k_{old} - (PD_{old} - PD_{new}) \end{aligned}$$

This procedure is able to provide a debt limit level for conventional debt such that it is hit with a frequency equivalent to default probability implied by the CDS market, then associate the change in the hitting frequency under partial indexation to a change in default probability, thus default premium in interest rates, assuming no effect in the growth rates and primary balance paths.

3 Results

The comparison between debt level forecasts under current debt stock and using a fictional debt stock composed of 75% conventional debt and 25% GLBs can be explored by the change in three variables, namely the mean debt level, its the standard deviation as a measure of its variability and the new default spread extrapolated with the procedure described in the last paragraph. The changes are measured as values under partial indexation versus values under conventional debt, both taken at the end of year 2028, or 10 years out in the simulation.

Results for groups of countries are reported as averages in Table 8, while the ones for every single country are reported in their totality in Table 11, as they are the main output of the simulations run. As an introduction to results, median changes for the three variables can be of some use. The increase in mean debt levels has been shallow, at a median value of 2.2 points of GDP. The promise of a lower volatility in debt levels seems to have hold true, but it came in small, as the median decrease in standard deviation of debt levels is just 2 points of GDP; on a brighter note, mixing conventional debt and GLBs didn’t increase volatility in future debt ranges for any country, when measured 10 years out in time. As far as adjustments in the default spread, partial indexing lowered it by a median value of 12 basis points, resulting in a lower default spread for 31 countries out of 38 (~79%), but varying a lot across countries. If the improvement in default spread is to be considered significant only across a certain threshold, say 25 basis points, or ¼ of 1%, then indexing a portion of the debt seems to make sense only for selected country groups, namely Emerging Markets and, to a lesser degree, Euro Area.

Of course the significance of the improvement in default spreads has to be further investigated. As the groups most interested by a change are those with an higher average starting default spread, one might think that the reduction works in percentage terms as a proportional cut to default risk. Other variables that can be seen as explanatory variables are the starting debt level and starting interest rate.

Table 8. Changes in key variables by indexing 25% of the debt to GDP and improvement consistency, averages for groups of countries

Country Group	mean, GDP p.p.	volatility, GDP p.p.	default spread, bps	improvement >= 25 bps
Euro Area	3.8	-2.2	-10.0	0%
Euro Area, individual countries	1.5	-6.4	-48.3	54%
Advanced Economies ex Euro Area	2.8	-1.4	-5.6	13%
Advanced Countries	2.2	-3.7	-25.4	32%
Emerging Markets	0.2	-3.9	-68.4	80%
Dataset	1.7	-3.7	-36.1	44%

Sources: IMF, OECD, DataGrapple, WorldGovernmentBonds, author's computations

On a second though the starting debt level doesn't matter in absolute terms, as some countries are more stable and can sustain an higher one, but it is the distance to debt limit that matters, which is something that should be reflected in the CDSs market and, thus, in the default spreads used. Tables 9 and 10 report the linear regressions run, from which results that the starting level of the default spread, namely the probability of default of conventional debt, is key in determining the improvement obtained by indexing to GDP 25% of debt. Its p-value in both regressions, nearly 0, highlights its statistical significance. The relative coefficient of -0.4643 means that the partial indexation has the effect of almost cutting in half, on average, the default spread. The actual change in default spreads can be visualized in Figure 14.

Table 9. Regression of change in default spreads to LT growth, interest rates and default spread under 100% conventional debt

Regression Statistics	
Multiple R	94.31%
R Square	88.94%
Adjusted R Squa	87.99%
Standard Error	18.66
Observations	39

ANOVA	*df*	*SS*	*MS*	*F*
Regression	3	97,982.80	32,660.93	93.81
Residual	35	12,185.10	348.15	
Total	38	110,167.90		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	3.9697	6.6002	0.6015	0.5514
LT growth	0.0328	0.0270	1.2141	0.2328
interest rate	-0.0164	0.0165	-0.9915	0.3283
PD_old	-0.4421	0.0354	-12.4835	0.0000

Sources: IMF, OECD, DataGrapple, WorldGovernmentBonds, author's computations

Table 10. Regression of change in default spreads to LT growth, interest rates and default spread under 75% conventional debt and 25% indexed to nominal GDP

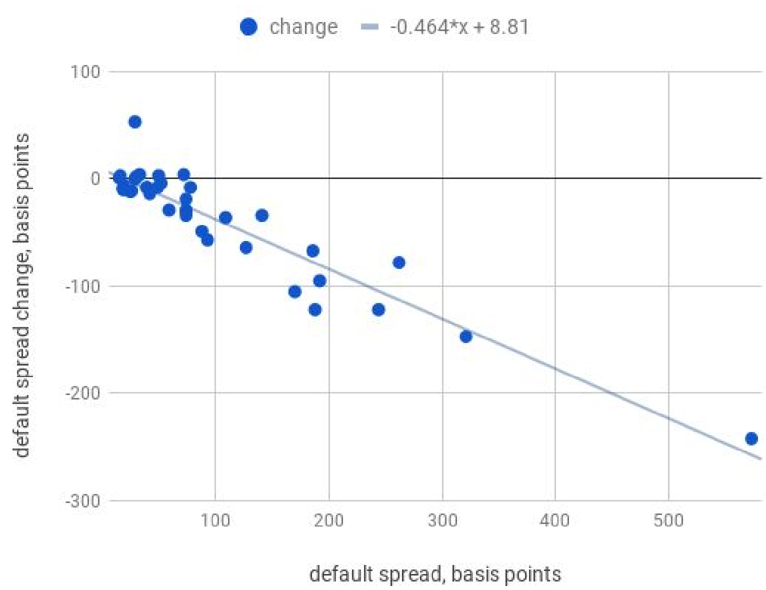
Regression Statistics	
Multiple R	94.03%
R Square	88.41%
Adjusted R Squa	88.10%
Standard Error	18.58
Observations	39

ANOVA	*df*	*SS*	*MS*	*F*
Regression	1	97,401.39	97,401.39	282.29
Residual	37	12,766.51	345.04	
Total	38	110,167.90		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	8.8056	3.9969	2.2031	0.0339
PD_old	-0.4643	0.0276	-16.8015	0.0000

Sources: IMF, OECD, DataGrapple, WorldGovernmentBonds, author's computations

Figure 14. Change in default spreads vs starting levels



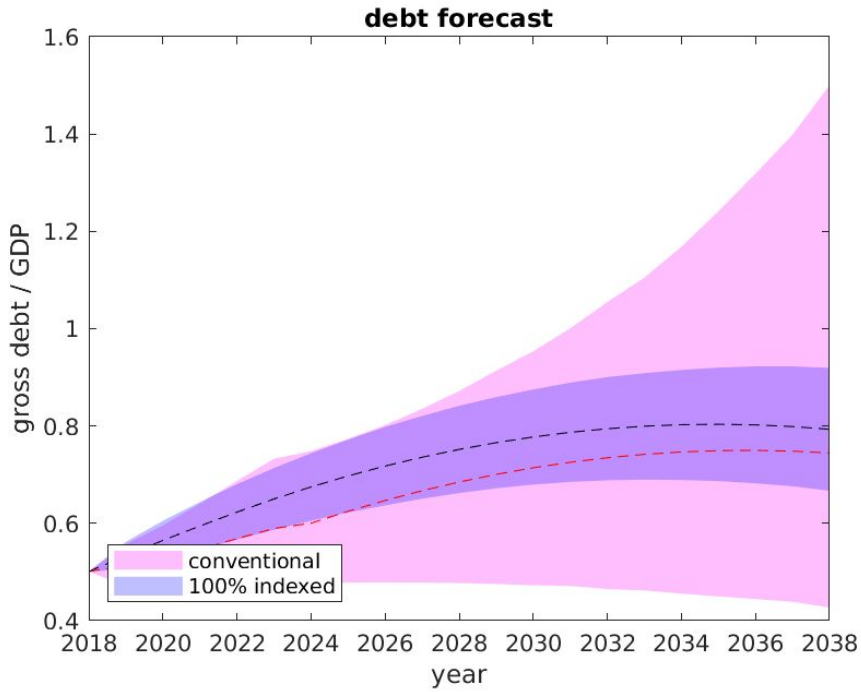
Sources: IMF, OECD, DataGrapple, WorldGovernmentBonds, author's computations

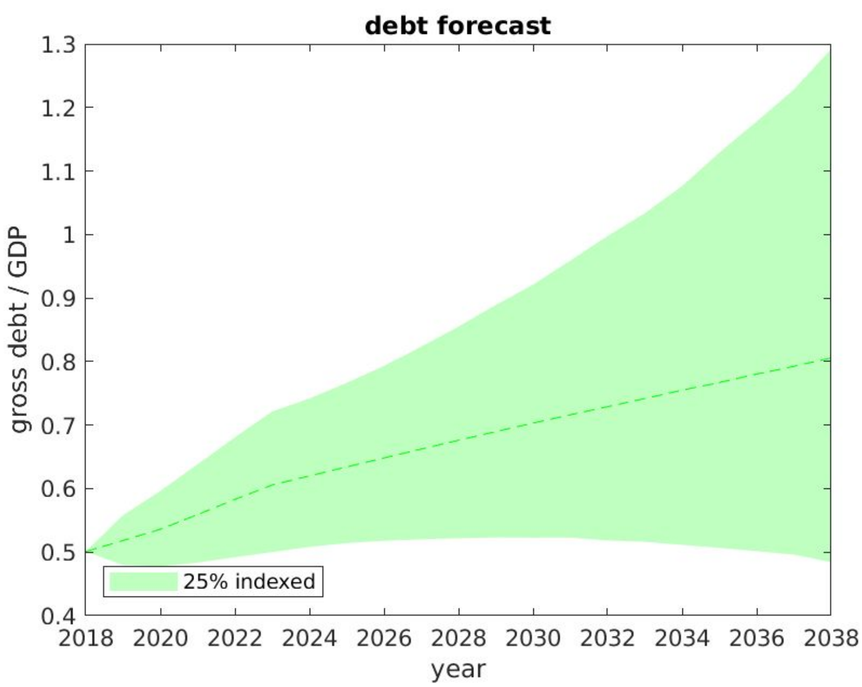
Table 11. Changes in key variables by indexing 25% of the debt to GDP and improvement consistency, individual countries

Country	mean, GDP p.p.	volatility, GDP p.p.	default spread, bps
Australia	1.0	-0.2	4
Austria	3.3	-1.9	-9
Belgium	4.4	-1.9	-8
Brazil	-0.3	-7.5	-122
Canada	3.1	-1.5	-8
Switzerland	1.6	-0.8	-11
Chile	1.2	-0.2	4
China	2.2	-1.3	-8
Colombia	0.1	-2.1	-105
Czech Republic	1.5	-0.8	-14
Germany	3.1	-1.6	-10
Denmark	2.0	-0.3	1
Euro area	3.8	-2.2	-10
Spain	3.1	-3.8	-36
Finland	4.0	-1.7	-6
France	4.5	-2.0	-4
United Kingdom	3.9	-1.7	-8
Greece	-17.1	-45.7	-242
Hungary	2.3	-3.9	-67
India	2.2	-2.0	-34
Ireland	2.2	-6.1	-29
Iceland	0.9	-1.3	-19
Israel	0.9	-3.5	-49
Italy	0.2	-5.9	-147
Japan	15.7	-4.9	2
Korea	1.6	-0.5	3
Lithuania	2.2	-3.1	-34
Mexico	-2.0	-3.7	-122
Netherlands	1.9	-1.3	-9
Norway	1.0	0.0	3
New Zealand	0.7	-0.2	0
Poland	-0.2	-2.3	-57
Portugal	3.9	-5.3	-64
Russia	-4.0	-14.5	-95
Slovak Republic	2.3	-2.0	-29
Slovenia	3.6	-3.5	-30
Sweden	3.3	-0.1	53
United States	2.4	-2.6	-12
South Africa	0.1	-1.8	-78

Sources: IMF, OECD, DataGrapple, WorldGovernmentBonds, author's computations

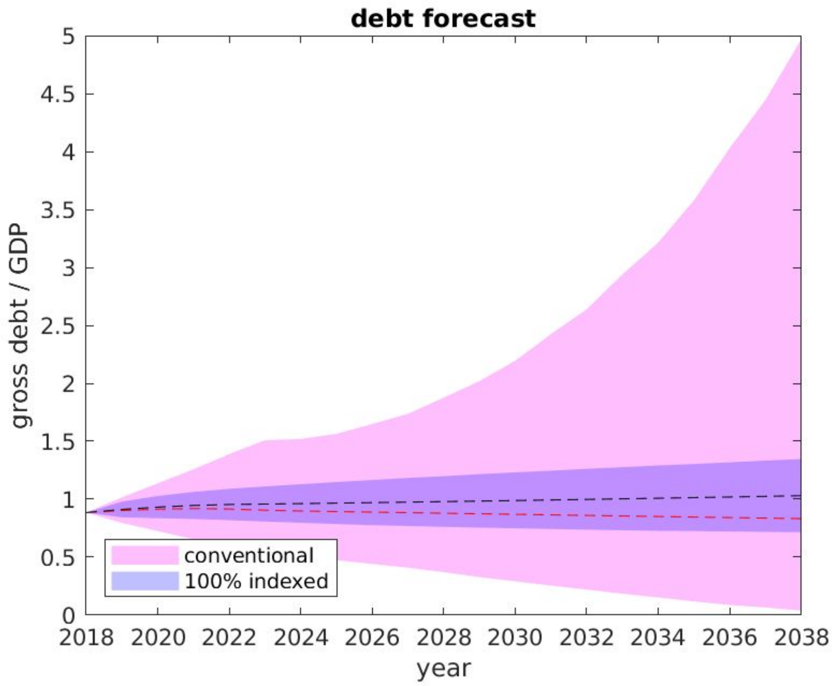
Figure 15. Debt path simulations for China

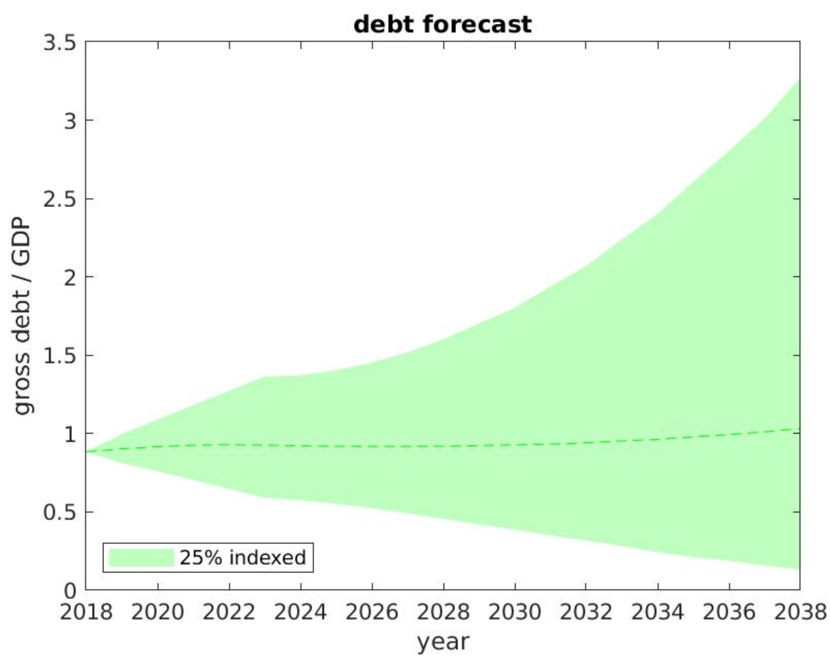




Sources: IMF, OECD, DataGrapple, WorldGovernmentBonds, author's computations

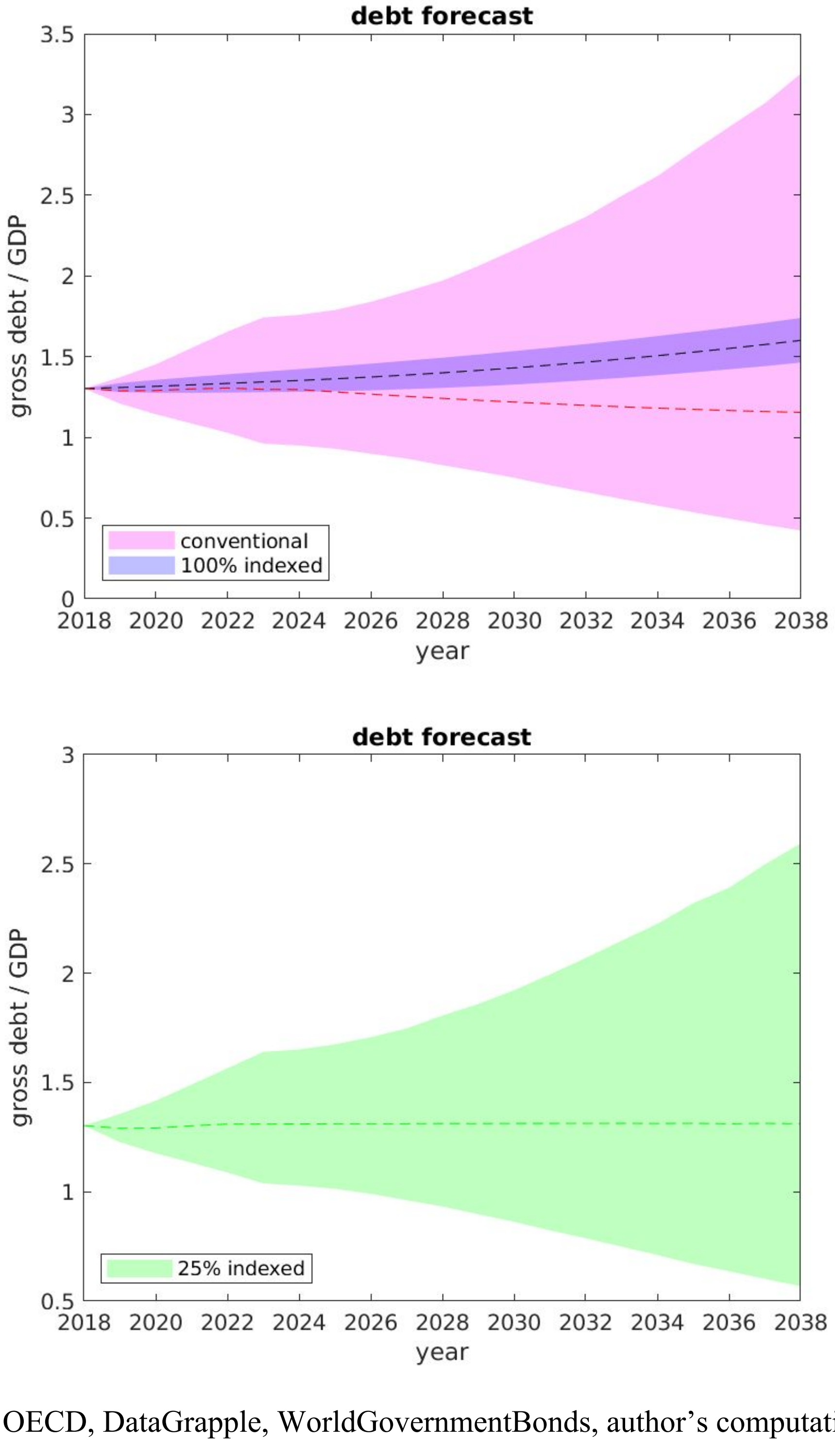
Figure 16. Debt path simulations for Brazil





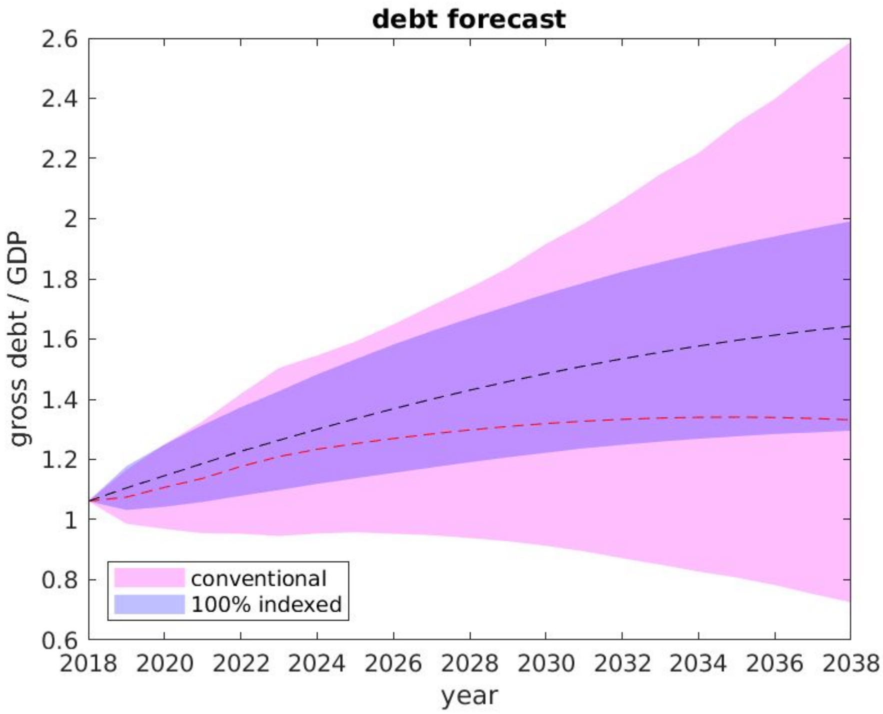
Sources: IMF, OECD, DataGrapple, WorldGovernmentBonds, author’s computations

Figure 17 Debt path simulations for Italy



Sources: IMF, OECD, DataGrapple, WorldGovernmentBonds, author's computations

Figure 18 Debt path simulations for the United States



Sources: IMF, OECD, DataGrapple, WorldGovernmentBonds, author's computations

Conclusions

Is it realistic?

Be looking at Figure 14 and Table 11, it can be noticed that a relevant reduction in default spread, say ¼ of 1% - 25 basis points - happens only for countries with default spreads of at least roughly 0.75%, which, considered a recovery rate of 30% for defaulted debt, corresponds to a sovereign CDS pricing at 97 basis points or more. As this is true only for 14 countries out of 38 in the sample (~36.8%), which themselves combined account for 92% of world public debt, it give rise to an issue, which is twofold: firstly, too few suitable countries annihilates the diversification assumption, then there would be the difficulty of countries accounting for a much smaller slice of global public debt, 24.4%, with a not so crystalline credit rating and, sometime, holding a large portion of sovereign debt denominated in foreign currency. It would require them to make the leap forward to local currency denominated debt indexed to nominal GDP.

Against the latter point it has to be noted again that necessity is truly the mother of innovation, as the heavy inflation rates of the '70s and 80s made investors demand an insurance o hedge against it, or as Argentina and Greece issued detachables GLWs in debt restructuring processes as sweeteners to regain investors' confidence. Academics have been making the case for GDP indexed debt for a quarter of a century, with the most relevant work being the call for "Macro markets" (Shiller, 1993), the opportunity for sovereigns in hedging growth uncertainty (Borensztein and Mauro, 2004) and the case for indexation as a mean to let sovereign regain control on skyrocketing debt levels (Benford et al., 2016). In spite of that, the previously said GLWs remain the only strong piece of evidence about an actual issuance: whereas the Argentinian one failed in the risk-sharing properties by bad design, letting the government worse off, the Greek one might have been too complicated and was only issued once, even as there have been further proposals of Greek debt indexation by deutsche economists (Fratzscher et al., 2014). The bottom line is that GDP indexed securities appear to be only one-off measures sporadically used during times of distress. It can be argued that this is due to them being seen only as sweeteners do debt claimholders' losses and far too costly to be issued as a standalone instrument during normal times. Thus, past issuance arises the problem of a correct design of these instruments: their actual benefits and perceived costs have to be estimated with confidence, if GLBs are not to stay one-off measures.

So, what happens if only for a few countries indexing to GDP a slice of the debt stock would be beneficial overall, thereby making them less likely to default? Sadly allowing for a parallel issuance among a number of different countries, such that specific risk could be nullified, this work is probably underestimating the cost of issuing GLBs. Accounting for the lack of investor diversification can be done by dividing each country GDP risk premium by its correlation to world growth. Doing so the average premium increases from 1.44% to 2.30%, or a 86 basis points increase in the demanded reward for accepting the sharing of growth risk, which has to be paid by the issuer to investors - a high cost that could potentially make GLBs not worth the trouble.

Risk Sharing

Looking at the difficulties of making a reality of GLBs asks for more general questions about the risk sharing premises of different kinds of debt. To take a broader view, we have to reason in terms of uncertainty and ask who will bear it - after all, financial markets are markets for risk.

Referring to public debt, a number of uncertainties and their associated risks, in addition to pure default risk, can be identified.

Future interest rates risk is clearly beared by sovereigns for all debt securities, and the only way for them to decrease their exposure to this is to aim at longer bond maturities. As a matter of fact, the usual positive slope of the interest rate curve might also be due to investors demanding an higher reward for sharing the rollover risk of sovereign debt.

Making a list of sovereign debt securities other than straight local currency denominated bonds, it can be notice a common theme across most of them. Foreign currency denominated ones, floating rate bonds with a rate linked to some updated reference lending rate, inflation linked bonds, growth or GDP-linked bonds. The first three do exist and are being issued in size, while the latter doesn't. Why is that? A possible answer is that the other flavours of debt securities listed above are a response to investors' needs, allowing them to reduce the risk beared by them or, equivalently, the risks being hedged away by the issuer, whereas in the latter they are assuming more risk than straight bonds.

By denominating bonds in foreign currency, the sovereign expose itself to international currency reserve shortages, letting foreign investor feel safer. Floating rate bonds display a variety of designs, but all of them offer some kind of protection; with respect to future interest rates they are a double edged sword, as they will bring benefit the investors were interest rates to rise, the vice versa applies. The inflation risk is generally beared by bondholders, as payments promises are made in nominal terms. Inflation rates higher than 10% bought governments to the issuance of inflation-linked bonds, which basically promise a stream of payments and principal repayment in constant currency. By issuing them sovereigns relieve investors from inflation risk, thus bearing it themselves.

The common theme of successful sovereign debt securities but straight bonds is that they relieve investors of some risk in addition to pre default risk at the expense of the issuer, or at least they offer some other kind of a possible greater upside to investors. The problem with GLBs is that it's a demand from the sovereigns to hedge away growth risk by sharing it with creditors. The two main GLBs designs offer different risk sharing properties: the Shiller design, indexing the principal to nominal GDP, lets governments hedge away long term growth risk, a the expense of paying a GDP risk premium and relieving investors of inflation risk. On the other side the Borensztein and Mauro design, indexing the coupon rate to the real growth rate, performs a lot better in eliminating mismatches between a sovereign ability to pay and its debt service duties, while still letting investors bear the inflation risk. The question on GLBs reason to be than becomes a question about their ability to offer a greater upside to investors as a reward for sharing additional risks..

The way forward

I have shown that, under principal indexation to nominal GDP, debt ratios are likely to stabilize for every country issuing them, but debt ratios might than just get stuck at high levels! Debt stabilization will be useful only to the degree that it lowers the probability of default, thereby decreasing the default risk or default spread attached to every debt security as a component of interest rates, which should, thus, fall and help in reducing debt levels.

As simulations have shown, the Shiller design of indexation to GDP under the stated assumptions might not be worth it for most of the countries, thereby prohibiting a coordinated large scaled issuance across the globe and, thus, the very possibility of investors' diversification. At a first look it would seem that it is high expected growth rates in the economy to determine the worthiness of GLB issuance, as the reduction in default spreads happened mostly in the Emerging markets group, but, at a second look, it seems that the only relevant explanatory variable is current default spreads themselves. This could make the case that GLBs seem to have a beneficial role only in cutting excessive fears of default, a topic that will need further research. If this is true, then GDP indexation is bound to remain a one-off tool to be used under times of high public debt distress, may it be debt restructurings and after sovereign defaults, with GLBs being used as sweeteners to regain investor confidence by giving them the upside of a future potential economic recovery, making this kind of security akin to an out-of-the-money call option on a country future economic output.

A lot of work has been put in estimating a GDP risk premium by previous research, whether exploiting the CAPM model or not. Accounting for the impracticality of investor diversification across the globe, this premium will be mostly a function of uncertainty of growth rates and can be estimated to range about a global average of 2.3%, as stated early, a price that issuers might or might not be inclined to pay. Is there a way to make this uncertainty a positive component of GLBs' present value, rather than a negative one, as it is when adding a growth premium to the discount rate?

The first kind of financial instruments that come to mind as being a positive function of uncertainty are, naturally, options. For this reason GLBs of the Borensztein and Mauro design should be further explored. Under the latter design each coupon payment is, at least for the variable component, akin to a call option on the the real growth rate in the relevant period, making the whole bond the sum of a conventional bond and a set of call options all referring to a different growth rate, acting as underlying. In addition the median value of the first lag autocorrelation in the real growth rates for all the selected countries is a low value of 0.29, making a weak case for indexed coupon payments being sticky. If this line of thinking has some merit, than the price of a GLB with indexed coupon rate gains from growth uncertainty indefinitely, as long as there is not a payment cap; the exact opposite of what happens under the design analyzed in the previous chapter.

Of course this kind of instrument reminds of the Greek GLW, a detachable instrument featuring a coupon rate indexed to excess real growth with a very low cap of 1% and a trigger in the nominal GDP level, with a principal amount decaying linearly over time after 2023. The underlying idea had a lot of merit, but partially failed in finding a liquid secondary market, with bid-ask spreads slightly above 10% of mid-price, as of October, 2018. The price drop made interest towards this kind of instrument fade out over time. It has to be said that, as a sweetener, it helped the Greek government in raising €62b worth of bonds, the underlying debt to which the GLW had been attached, and the security itself firstly traded at 7.5% of maximum redemption value.

On a final note, a word has to be spent regarding standardization. Before calling for a large scale issuance of any GLB, there should be consensus about a simple design that doesn't leave sovereigns worse-off on average, as this thesis has shown for the principal indexation design. The optimal design should hold true to the premises of linking to economic performance, namely higher stability in debt changes, a lower default probability, higher fiscal space for governments. Since the London Term Sheet for GDP linked instruments allows for both designs, a legal structure is already in place.

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