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Forecasting Value-at-Risk using high frequency data

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Introduction

The uncertainty that characterizes financial markets requires practices and tools necessary to identify, to evaluate and to plan the exposure to risk. This is the task of financial risk management, mitigating the impact of unfortunate events and creating opportunities by an adequate exposure to risk. In this field, the Value at Risk (VaR) is one of the standard statistical tools used by practitioners to measure the occurrence of potential losses. Introduced by J.P. Morgan in July 1993, VaR became one of the most widespread risk management tools, an effective measure to compare the risk through different markets.

This research aims to investigate the field of volatility forecasts that incorporate the VaR measure. Furthermore, this study proposes an empirical effort to its computation with the application of high frequency data (data registered at very short timeframe, such as 1 minute). The volatility gauges the degrees of a price variability. It is important to consider that volatility acts in a predictable way, scholars and practitioner already know that volatility can be predicted, as opposed to returns. Thus, forecasting volatility has important implications for the investors. Good predictions of volatility improve the financial risk management. The importance of the risk management is increased in the last 30 years due to a growing availability of financial derivatives and products, whose prices depend on the volatility of underlying asset and due to the improvements in modeling statistical tools able to forecast volatility. The forecastability of the volatility has to do with the fact that several statistical properties characterize it. Volatility Clustering, Mean Reverting Leverage effects, higher Cross Correlation between volatilities are some important properties that distinguish volatility paths. The challenge is to find a model that could predict volatility in comply with these properties. Volatility prediction has been useful for short-horizon investors, but new econometric models are able to perform in a good way also in the long term. Moreover, the volatility is also barometer for the vulnerability of financial markets and its study can be useful also for policy-makers.[1]

The content of the research moves from the descriptions of VaR as a risk measure that is contained in Chapter 1. It includes the definition, some methodologies for VaR computations, drawbacks and alternatives. In addition, the three methodologies are put into practice. Chapter 2 addresses some practices and models used to predict volatility. It starts

[1]Cf. Poon and Granger, 2003, pp.478-479.

considering the properties of volatility and the importance of intraday data with their features, potentialities and issues. The Chapter keeps on considering different models to depicts and predicts volatility. The models are divided into two parts, models of conditional volatility and models of unconditional volatility. In particular, in the latter group, the attention is drawn to one of the most recent and influencing approach in the field of volatility forecast, the Heterogeneous Autoregressive (HAR) model of Corsi (2008), a model which utilize a simple AR(1) (Auto Regressive) and only few parameter, but at the same time seems to outperform more complex models based on conditional volatility. Chapter 2 includes also some evidence from the literature and a complete literature review of the conditional volatility models. Instead, Chapter 3 consists in a literature review concerning the VaR forecast. In this chapter are analyzed some approaches and practices that may be useful for the empirical analysis in the last chapter. In Chapter 4, I carried out an empirical research that aims to put into practice some remarks given throughout the previous chapters. The final goal of the Chapter is to achieve a one-day-ahead out-of-sample forecast of the VaR for both tails of distribution.

Chapter 1. Value-at-risk as a risk measure

Value-at-Risk is one of the most popular and exploited statistical measure used to evaluate the riskiness of a financial investment, taking into consideration possible portfolio losses. Its wide utilization is due to its simplicity and understandability. For this reason it became a standard in financial industry, and financial software includes functions that make possible to compute it quickly. Furthermore, its utilization is appropriate for different types of portfolios. Limitations and drawbacks will be discussed later in the chapter. This tool combines the effect of assets volatility with exposure to financial risks. Hence, it integrates the price–yield relationship with possibility of negative market movements. In detail, VaR assesses the worst loss which cannot be exceeded, given a specific holding period and a confidence level which measures the risk aversion of a certain investor. In other words, it describes the quantile characterizing the distribution of losses over a specified time horizon:

$$Prob(p_t \leq p_{t-1} - VaR_t(\tilde{\boldsymbol{\alpha}}) \mid \Omega_{t-1}) = 1 - \tilde{\boldsymbol{\alpha}}\,.^{2} \qquad (1)$$

Formula (1) defines the concept of VaR where p_t is the price at time t, p_{t-1} is the price at time t-1, $VaR_t(\tilde{\boldsymbol{\alpha}})$ is the threshold level for $\tilde{\alpha}$ confidence level and Ω_{t-1} is the information set at time t-1. The VaR is based on two quantitative factors: a holding period t and a confidence level $\tilde{\boldsymbol{\alpha}}$. The choices of holding period and confidence level are fundamental determinants for the VaR calculation. In fact, portfolios with comparable characteristic could come up with different VaR estimations. The longer the time horizon, the higher is the borne risk. The higher is the confidence of an investor $\tilde{\boldsymbol{\alpha}}$**,** the higher is the VaR that he is able to tolerate. The time considered by financial firms is usually of one day while the confidence level in most cases varies between 5% to 1%.[3] In this framework, Bank for International Settlements (BIS) established the praxis for the practice of this tool into financial company. During the first Basel committee on banking supervision (Basel I capital accord (1988)), it sets the guidelines and parameters for the assessment of VaR. Banks need to hold capital in order to cover the risk in their portfolio. For trading books, they are suggested to hold a confidence level of 99% and an investment horizon of 10

[2] Cf. Będowska-Sójka, 2015, p. 165.
[3] Cf. Linsmeier and Pearson, 2000, p.49.

days[4]. However, the 99% percent confidence level standard is far from being uniformly adopted by all participants in the market. For example, J.P. Morgan discloses its daily VaR at the 95% percent level[5]. Two different basic approaches can be considered and utilized to compute VaR. The non-parametric approach derives the distribution on returns empirically and look directly at the quantile of interest, while the parametric approach based its validity on statistical assumption of the returns distribution. At the same time semi-parametric methods like the Extreme Value Theory can be implemented. However, a common path for each approach and for each method can be used for the computation[6]:

- Determinate the current value of the portfolio;
- Measure the variance of the risk factor;
- Set the time horizon and the critical level;
- Taking into consideration the previous steps to determine the probability distribution of returns through a parametric or non-parametric approach.

There are different methods that can be applied to achieve a fair assessment of the VaR. The most widespread methods are the Delta-Normal approach, the historical simulation, and the Montecarlo simulation[7]. The Delta-Normal method based its validity on the assumption of normal distribution. It applies mathematical properties of normal distribution to determine the amount of losses for a fixed confidence level. The historical simulation method aims to construct a distribution for possible future profits and losses using historical changes in the current portfolio. Lastly, in the Montecarlo simulation approach, it is chosen a distribution function that can be interpreted as a proxy for possible changes in the market factors. This approach is similar to the historical simulation, however, the distribution assumption does not stem from observed changes. The main features of these methodologies are summed up in Table 1.

[4] Cf. Lopez 1998, p. 119.
[5] Cf. Duffie and Pan, 1997, pp.9-10.
[6] Cf Jorion, 2011 p. 107.
[7] Cf. Linsmeier and Pearson, 2000, pp.50-60.

	Positions		Risk Factors		Risk Engine
	Mapping	*Positions*	*Model*	*Distribution Risk Factors*	
Delta-normal	Delta	Linear	Parametric	Covariance$\sum$	$x' \sum x$
Historical simulation	Full Valuation	Pricing Function	Non Parametric	Historical	Simulation
Montecarlo simulation	Full Valuation	Pricing Function	Parametric	Estimate	Simulation

Table 1: Summary of the main alternatives for the VaR measurement

Source: P. Jorion, Value at Risk, The new benchmark for managing Financial Risk, pp.249-257.

1.1. Calculating VaR , an Empirical example.

The aim of the paragraph is to turn into practice the three methods introduced above. The purpose is evaluating the highest acceptable loss for a hypothetical portfolio and establishing the most reliable methods which do not underestimate risk. The calculation has been implemented on a portfolio composed by four different kind of assets. The assets on which the portfolio is built are S&P 500 stock index, the EUR/USD spot exchange rate, GOLD 1 OZ and the iShares MSCI Japan ET, an Exchange Trade Fund that includes Japanese equity assets belonging to different sectors. The iShares MSCI Japan ET is traded on the London Stock Exchange. Each of those assets composed the 25% of the portfolio, so they are equally weighted. The considered sample starts from 01.01.2015 and ends the 12.31.2016[8]. The frequency of the observations is daily based and only closing prices are selected. Consequently, the number of observations is 504. From Figure (1) to Figure (5) the price paths and the Kernel's frequency distribution of returns for each assets on the portfolio are presented. Kernel's frequency distribution is similar to a histogram, but it defines a smother and continuous shape of the curve. While the histogram produces a

[8] S&P500 and EWJ data are exported from https://it.finance.yahoo.com, while EUR/USD and GOLD 1 OZ are exported from https://it.investing.com.

discrete probability density function using the sample of returns, the kernel distribution is the result of the sum for the individual probability density curves for each observation.[9]

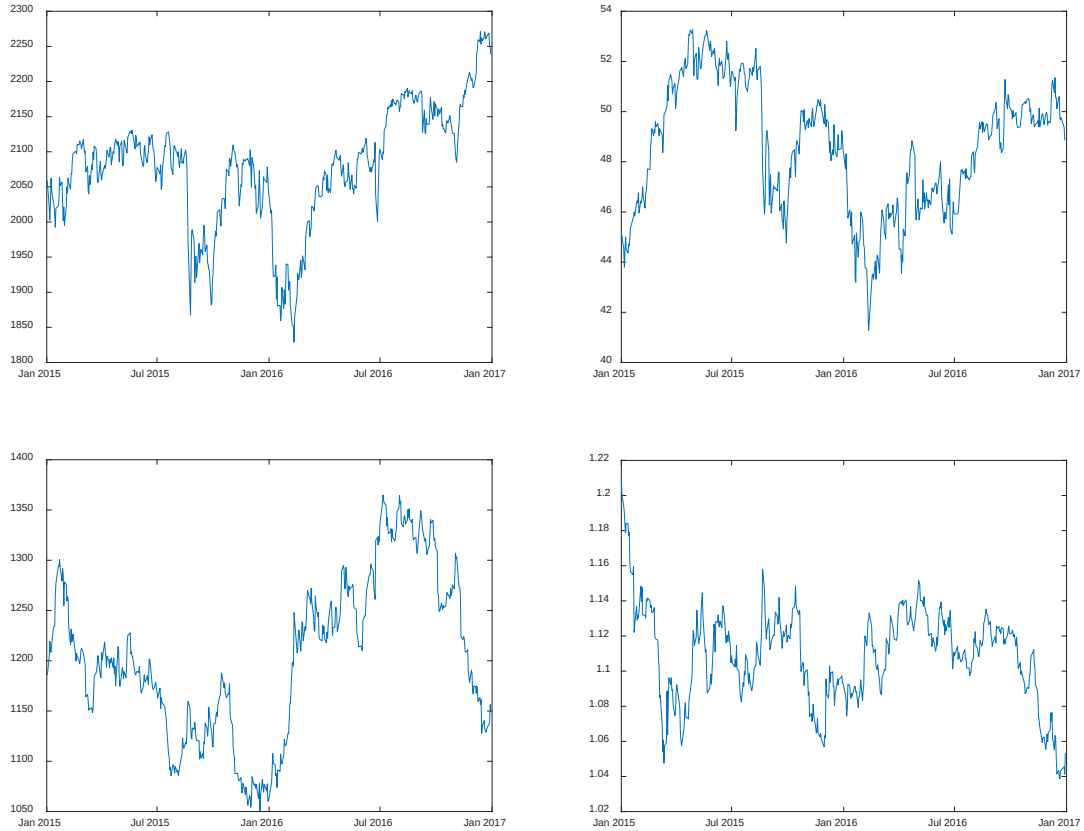

Figure 1: VaR empirical application, representations of historical prices for the assets and timeframe taken into consideration. The considered period lasts from 1 January 2015 to 31 December 2016. The top left corner depicts S&P 500 spot price, the top right corner EWJ spot price, the bottom left corner GOLD spot price, the bottom right corner EUR/DOL spot price.

Source: Author's elaboration

[9]Cf. Matlab documentation, Kernel Distribution.

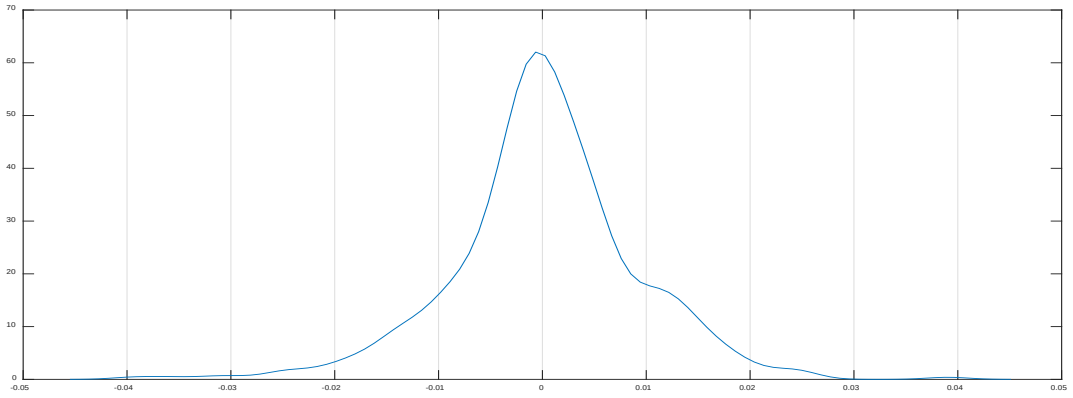

Figure 2: VaR empirical application, S&P500 Kernel's frequency distribution of returns

Source: Author's elaboration

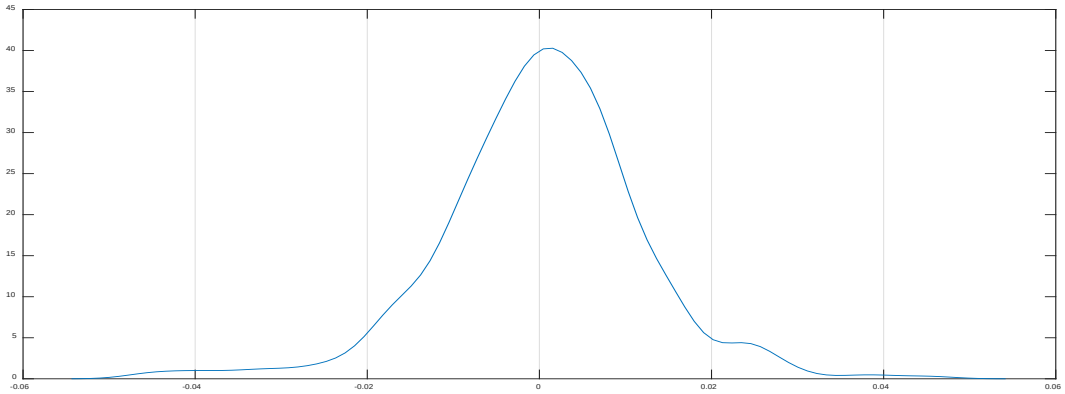

Figure 3: VaR empirical application , EWJ Kernel's frequency distribution of returns

Source: Author's elaboration

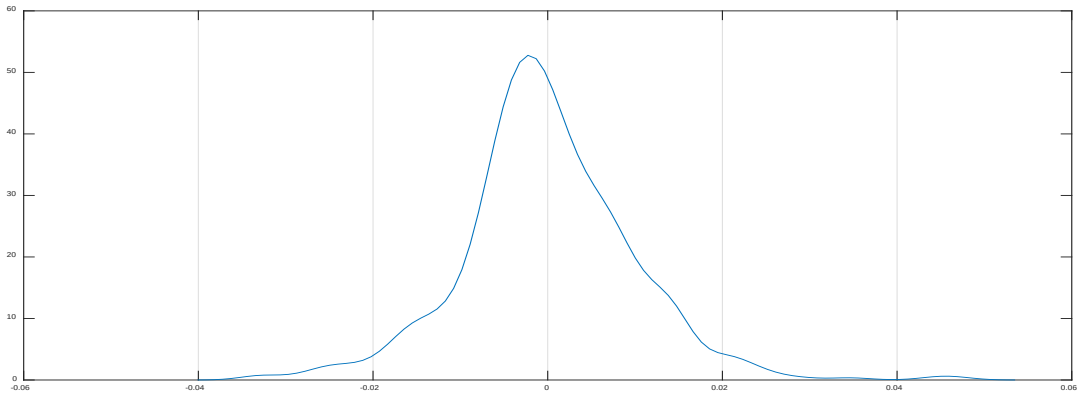

Figure 4: VaR empirical application , Gold Kernel's frequency distribution of returns

Source: Author's elaboration

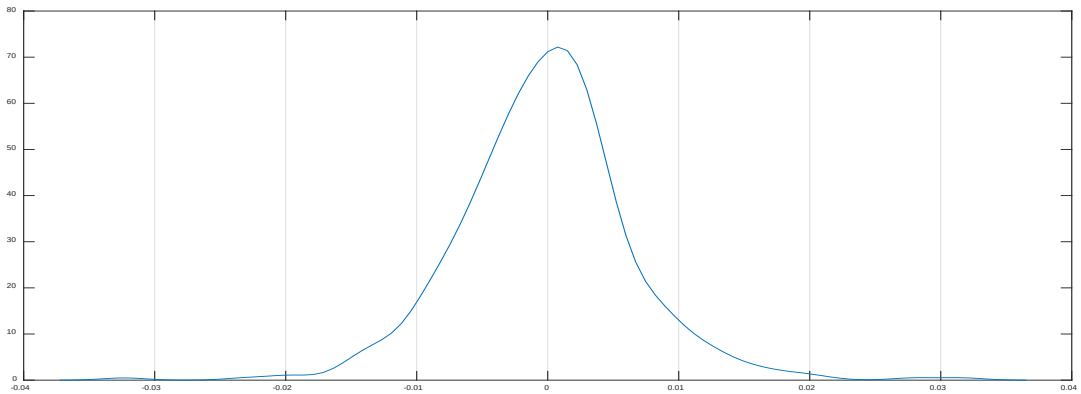

Figure 5: VaR empirical application , EUR/USD Kernel's frequency distribution of returns

Source: Author's elaboration

The aim of the research is to get the VaR using the confidence level and the time horizon that Basel II agreement suggest and J.P. Morgan considers as its standard. Thus, 99% and 95% of confidence intervals and 1-day and 10-day as time horizon are taken into account. The goal is to find out the most precise method to calculate VaR of a portfolio which does not contain option positions. The first step is setting the current value of the investment which is supposed to be 1,000,000 $. The second step is calculating the mean and variance of the portfolio through the variance-covariance matrix. Those estimates are necessary for parametric methods like the Delta-Normal but also for Montecarlo simulation. The portfolios standard deviation is 0.52% while, the mean daily return is 0.004%. The portfolio has a low standard deviation and a daily return close to zero. In fact, EUR/USD exchange rate and gold price are usually negative correlated with S&P500 stocks and EWJ stocks. The main descriptive statistics of the portfolio are summed up in Figure (6) and (7) and Table (2) and (3) below.

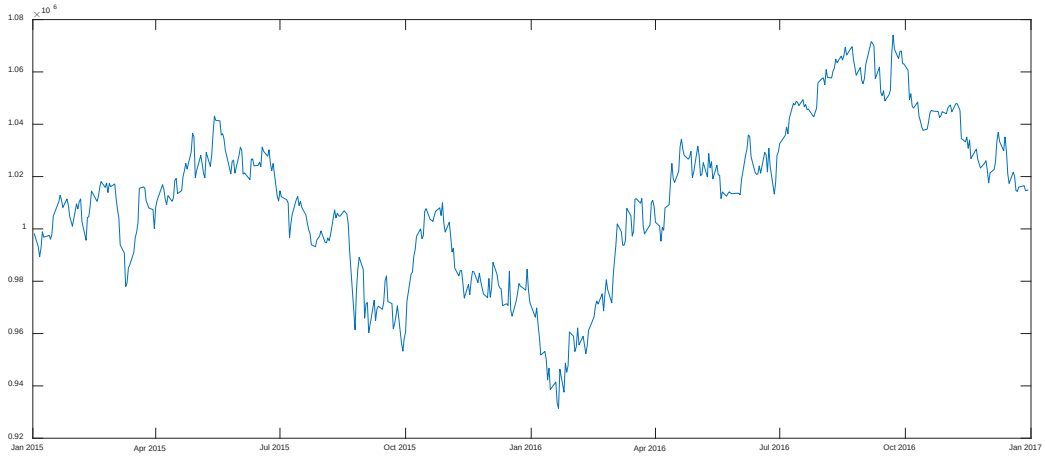

Figure 6: VaR empirical application , Portfolios value over the time horizon.

Source: Author's elaboration

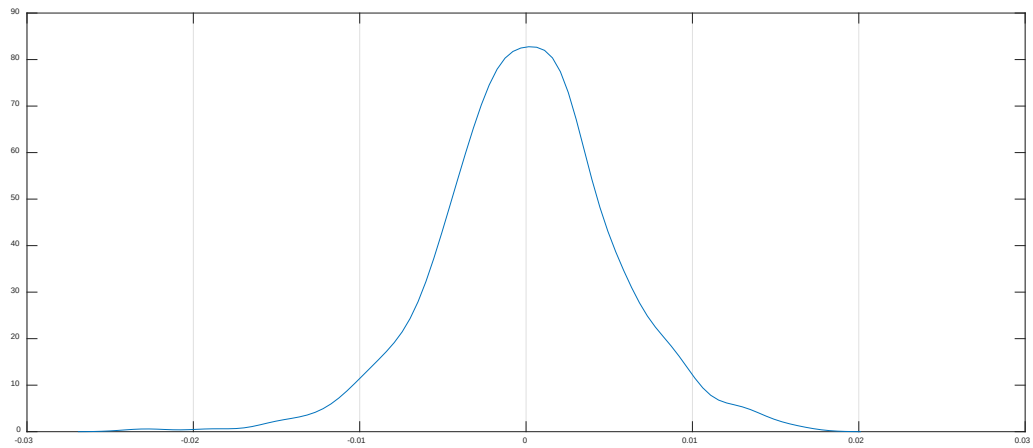

Figure 7: VaR empirical application , Kernel's frequency distribution of portfolio returns

Source: Author's elaboration

	S&P500	EWJ	Gold	EUR/USD
S&P500	1.00	0.74	-0.17	-0.11
EWJ	0.74	1.00	-0.14	-0.10
Gold	-0.17	-0.14	1.00	0.09
EUR/USD	-0.11	-0.10	0.09	1.00
Mean return	0.021%	0.023%	-0.001%	-0.025%
Standard Deviation	0.903%	1.168%	0.978%	0.659%

Table 2: VaR empirical application, Portfolios Correlation matrix

Source: Author's elaboration

Investment	$	1,000,000.00
Mean Return		0.0043%
Standard deviation		0.5204%
Skewness		-0.17
Kurtosis		1.15

Table 3: VaR empirical application, Main descriptive statistics of the portfolio

Source: Author's elaboration

The Delta-Normal method is named in this way because of its linear exposure to risk factors that delta represents. This characteristic makes the method easy to implement but, at the same time, it can be misleading and can overestimate risk if option positions are contained in the portfolio. In this method, portfolios risk is produced by the linear exposure to the risk factors that are normally distributed. The results of covariance matrix are the used for the calculation and they are incorporated in the formula (2):

$$VaR^n = -(\hat{\mu}_n + \sqrt{\hat{\sigma}_n^2}\, z_{\tilde{\alpha}}^{-1}) \tag{2}$$

Where $\hat{\mu}_n$ is the observed mean of the portfolio, $\sqrt{\hat{\sigma}_n^2}$ the observed standard deviation and $z_{\tilde{\alpha}}^{-1}$ is simply the quantile of the normal standard distribution. Essentially, the concept is to sum to the mean a cushion, according to the volatility of the considered portfolio and the confidence level for a specific distribution assumption, in order to obtain a value able to cover a maximal potential loss. In addition, the same approach can be implemented with the assumption of a Student-t distribution with 6 degrees of freedom ($v = 6$). The reason behind this assumption is that Student-t distribution approximates better the distribution of fat tails. In particular, a greater parameter α can achieve more precise tails distribution and for this purpose a student t distribution with a number of degrees of freedom between 4 and 6 is suggested as a good proxy[10]. In details the calculation is carried out as follow:

$$VaR^n = -(\hat{\mu}_n + \sqrt{\hat{\sigma}_n^2}\, t_{v\tilde{\alpha}}^{-1} \sqrt{\frac{v-2}{v}}) \tag{3}$$

[10] Cf. Jorion, 2011, p.269 according to Hendricks, 1996.

Where $\hat{\mu}_n$ and $\hat{\sigma}_n^2$ are respectively the observed mean and variance of the portfolio and t_v^{-1} is the inverted Student-t and v are the degrees of freedom.

The other two methods, instead, can be considered as full valuation techniques. In fact, the linearity of pricing function is no more an assumption and the VaR is generated by looking at quantiles. In the historical method, the portfolio returns can be calculated empirically and quantiles of interest are directly selected. In this example, considering that the number of sampled returns is equal to 504, the observations taken into consideration are the 5th and the 25th that represents respectively the 0.01 and the 0.05 critical value.

However, the most sophisticated method is the Montecarlo simulation. The difference with the Historical method is that the path of returns is created by pseudorandom draws and not by observed returns. Thus, the hypothetical change in price is generated by random draws from a pre-specified stochastic process. For this Purpose, I used the continuous time Geometric Brownian Motion for one random variable as follows:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dz \tag{4}$$

The return $\frac{dS_t}{S_t}$ in Formula (4) is the results of two components: the drift or deterministic component $\mu_t dt$ and the stochastic component $\sigma_t dz$, where σ_t denotes the standard deviation. Given the assumption on the process generating the returns, we can use it to generate future returns through simulation.

The procedure starts with the determination of a vector that includes k=1000 inverted standard normal random components. To achieve the complete stochastic component, the vector is multiplied by the standard deviation of the sample. The addition of the drift completes the 1 day vector of simulated returns. The simulation continues for the next 9 steps following this process:

$$P_{t+1} = P_t + P_t(\hat{\mu}_t d\Delta t + \hat{\sigma}_t z)$$
$$P_{t+2} = P_{t+1} + P_{t+1}(\hat{\mu}_t \Delta t + \hat{\sigma}_t z) \ldots \tag{5}$$

The result of Formula (5) is a vector of 1000 portfolios values on the 10th day. At the end of the computation, exactly as in the historical simulation, I look at the observations of the considered quantiles, so the 10th and the 50th.

The results of the different methodologies are summed up in Table (4) and Table (5).

	VaR 99%		VaR 95%	
	1 Day	10 Days	1 Day	10 Days
Delta-Normal	$ 12,063.99	$ 38,149.67	$ 8,517.26	$ 26,933.93
Delta-student t with 6 d.f.	$ 13,311.10	$ 42,093.40	$ 8,214.08	$ 25,975.22
Historical Simulation	$ 14,669.31	$ 46,388.43	$ 8,611.02	$ 27,230.43
MonteCarlo Simulation	$ 12,190.94	$ 36,187.21	$ 8,772.09	$ 25,175.53

Table 4: VaR empirical application, Results of the research, downside VaR for the four different methodologies.

Source: Author's elaboration

	VaR 99%		VaR 95%	
	1 Day	10 Days	1 Day	10 Days
Delta-Normal	-1.2064%	-3.8150%	-0.8517%	-2.6934%
Delta-student t with 6 d.f.	-1.3311%	-4.2093%	-0.8214%	-2.5975%
Historical Simulation	-1.4669%	-4.6388%	-0.8611%	-2.7230%
MonteCarlo Simulation	-1.2191%	-3.6187%	-0.8772%	-2.5176%

Table 5: VaR empirical application, Results of the research, downside limit returns for the four different methodologies.

Source: Author's elaboration

	VaR 99%		VaR 95%	
	1 Day	10 Day	1 Day	10 days
Delta-Normal	98.30%	98.90%	94.10%	95.70%
Delta-student t with 6 d.	99.20%	99.50%	93.40%	94.80%
Historical Simulation	99.50%	100.00%	94.40%	95.70%
MonteCarlo Simulation	98.50%	98.90%	95.10%	94.60%

Table 6: VaR empirical application, Fraction of covered observations

Source: Author's elaboration

In order to evaluate results, I calculated pseudorandom future returns exactly as in the Montecarlo simulation. On those new K variables, I tested the VaR computed previously calculating the fractions of covered observation (Table 6). The fraction of covered observation is given by 1 –minus the percentage of violations computed considering the

504 returns of the sample. The closer the value to its confidence level, the more precise is the estimation. In this research the historical simulation method overestimates the risk for 99% confidence level. The Delta-Normal method shows, as expected, to underestimate risk for the extreme part of the tail if compared with the Delta Student-t with 6 degrees of freedom. However, the Delta Student-t is less reliable at lower confidence level. The main conclusion is that Delta-Normal method and the Montecarlo simulation shows quite good and very similar results. Hence, for a portfolio like the one considered, without option positions inside, the Delta-Normal method is the best choice to compute value at risk, in fact it gives the same results of Montecarlo simulation but through a faster and more intuitive approach.

1.2. Drawbacks and alternatives

Despite of its simplicity and effectiveness, VaR approach presents various disadvantages. Assuming that returns have a normal distribution, such as in the case of the delta-normal method, can be misleading. In fact, the awareness that the probability density function of returns is leptokurtic and asymmetrically distributed is widely stated among both, scholars and practitioners[11]. An example can be observed in figure (8). Thus, returns are distributed in a higher proportion around the mean than in a Normal distribution function. Another limitation of VaR is that the possible losses that exceed the threshold VaR level are not measurable, hence, we known the probability to drop under a pre-determinate price, but we do not known the amount of those losses. Furthermore, VaR is not sub-additive, thus, $\mathrm{VaR}(X_1 + X_2) \leq \mathrm{VaR}(X_1) + \mathrm{VaR}(X_2)$. It means that the VaR of a portfolio including two assets is not less or equal to the sum of the VaR of those two assets. According to (Artzner et al. 1999), the VaR is not a coherent risk measure. In fact, they define a coherent risk measure as a measure that satisfies the properties of translation invariance, positive homogeneity, monotonicity and subadditivity[12]. Although the first three properties are compliant with VaR, this measure is not sub-additive. Lastly, it can be costly in terms of time required to compute all the correlation between assets for a wide portfolio.

[11] See Sheikh and Qiao, 2009.
[12] Cf. Artzner et al., 1999, pp.215-216.

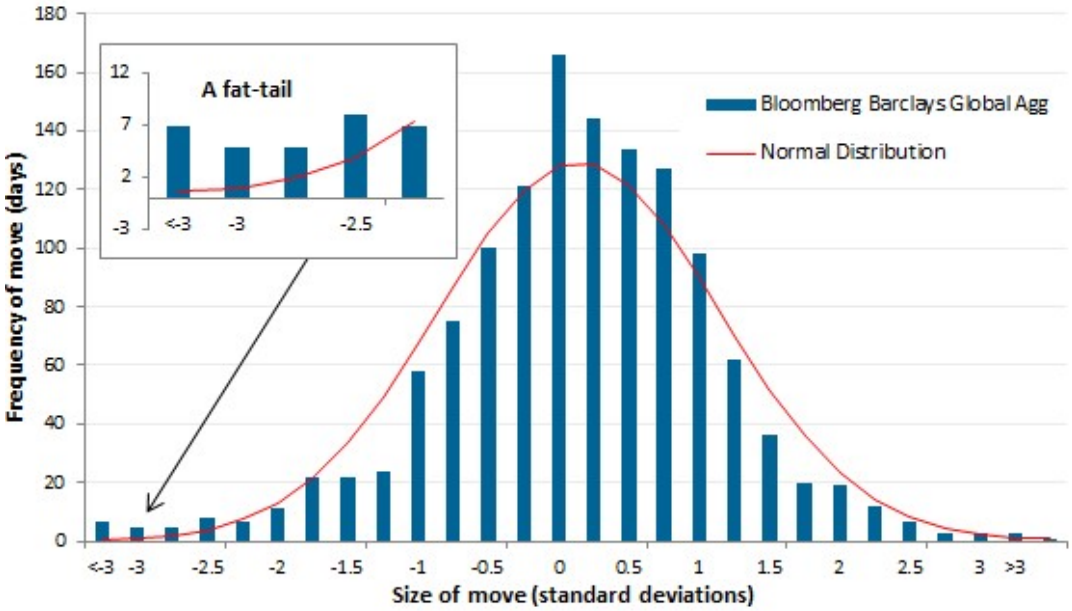

Figure 8: Daily changes in Bloomberg Barclays Global Aggregate Index 2011-2016

Source. Bloomberg, as of 25 October 2016, cited according to https://blog.jpmorganinstitutional.com/2016/10/fatter-tails-and-endogenous-risk

Taking into account those drawbacks, in particular the impossibility to detect a worst case scenario, one alternative to this measure is preferable. The Expected Shortfall (ES) provide a better risk assessment. It has an higher informative power about the amount of possible losses under the threshold level that characterize VaR. If the distribution below maximum accepted drop is far away from normality, ES is more suitable than VaR to estimate the overall risk level. It could be more evident during market stress period, when the fluctuation of asset prices is more evident and the VaR measure could underestimate the risk[13]. Moreover ES is a sub-additive measure. However, this tool suffers from higher estimation error in comparison with VaR[14]. Figure 8 shows the returns density distribution of the Bloomberg Barclays Global aggregated between 2011 and 2016. In this case the distribution is characterized by fat tails and VaR measure could be misleading if used to manage risk in this situation.

[13]Cf. Yamai and Yoshiba, 2005, pp. 998-999.
[14]Cf. Krause, 2003, p.27.

Chapter 2. Modelling Volatility

Assessing portfolios VaR at the current period is a quite simple task if compared to forecasting VaR. In order to forecast VaR we have to consider models that are able to give us a precise estimate of future volatilities.

Forecasting volatility is a critical task in asset evaluation and risk management and it is used by investors and financial intermediaries to price assets in a portfolio. The price of many securities is affected by fluctuations in volatility. Risk management models aim to achieve a fair volatility forecasting, in particular those models should follow the main properties from which the volatilities are characterized.

A first property is that volatility is often dispersed in bunches. This phenomenon is called Volatility Clustering Phenomenon, it means that large volatility changes are usually followed by others large volatility changes and vice versa. This is due to the Investor's inertia encouraged by the incorporation of new information.[15] Another property that characterize volatility is the Leverage effects, according to which the returns and their change in volatility are negatively correlated. In other words, negative returns provoke higher volatility changes than positive returns, hence the distribution of return is asymmetric.[16] Furthermore, volatility's development over time tends to revert to a mean value. This property it is also known as Mean reversion.[17] Making a good forecast of volatility, means also being able to capture its persistence.[18] In fact, while correlation between assets returns is very low, the correlation of volatility between asset classes is clearly observable (Cross Correlation of Volatility).[19] According to Poon and Granger (2003)[20], volatility forecasting models can be split up into four main categories: Historical Volatility models that based their prediction on past standard deviation, Conditional Volatility models that formulate the conditional variance through maximum likelihood approach, Stochastic Volatility models and models based on implied volatility that stems from Black-Scholes model. In this section, will be presented times series models to forecast Volatility, hence the first two categories.

[15] Cf. Marra, 2015, p. 2.
[16] Cf. Marra, 2015, p.3 according to Black, 1976: The pricing of commodity contracts.
[17] Cf. Marra, 2015, p.3.
[18] Cf. Poon and Granger, 2003, pp. 478-479.
[19] Cf. Marra, 2015, p. 4.
[20] Cf. Poon and Granger, 2003, pp.483-486.

A lot of time series models has been developed to provide a good volatility prediction. A large part of empirical works concerns the volatility forecasts through models of conditional variance, the General AutoRegressive Conditional Heteroscedasticity (GARCH) models. However, the increasing availability of high-frequency data enabled the development of alternative approaches based on the evaluation of the in-sample volatility using past data. The Realized Volatility (RV) is the sum of squared returns during a fixed time interval, thus is a non-parametric ex-post estimate. While realized volatility models often show excellent prediction performance[21], there is still much discussion concerning the model for the prediction that can optimize the incorporation of RV. In this context can be useful to take into account the main characteristics of a forecast. In fact, there is not often a model more performant than the other but a model that fit the characteristics of the analysis better than the others. Features like length of the out-of-sample forecast and dataset are determinant in the analysis. In support of this consideration Bentes (2015), carrying out a volatility predictions on gold spot rate, underline the importance of finding a time series model able to catch long-run dependencies typical of gold and metals returns. Furthermore, "large markets seem to be efficient (…)but less developed markets tend to exhibit long-range correlations. A possible explanation for this phenomenon lies on the fact that smaller markets are more likely to experience correlated oscillations and they are therefore more predisposed to being influenced by aggressive investors"[22]

2.1.Daily or Intraday Data

In this context, the increasing availability of intraday data for returns played an important role in making observed volatility be successful. In fact, as it will show in this chapter, the sum of intraday squared returns can give a good performance for the estimation of volatility at long time horizon. For this reason, an increasing number of researchers are trying to exploit these information building less parametrized models that measure volatility directly from past returns. However, it is also possible to incorporate high-frequency data into the conditional volatility or also to add a Realized Volatility (RV) component to more parametrized models like Auto Regressive Conditional

[21] Cf. Andersen et al., 2003, pp. 579-582.
[22] Cf. Bentes, 2014, p.191

Heteroscedasticity (ARCH)[23]. In spite of the higher costs to gather and more time to handle with the data, intraday returns contain a lot of information. However, there are a lot of time horizons in which intraday data can be computed. The data can be collected tick by tick, or can be gathered using different timeframe, such as 1-minute, 5-minute, 15-minutes or 30minutes. The shorter is the timeframe, the wider can be the information set captured by the estimation of RV, but also the time and costs required to handle with the data. However, "while it is theoretically necessary to sum squared returns that are computed over small intervals of time to better identify the volatility over a period, summing different contaminated squared returns entail considerable noise accumulation"[24]. Thus, a very small interval on which RV can be calculated, could be characterized by a huge quantity of microstructure noise. The main drivers of noise are: discreteness of price changes, differences in trade size and bid-ask bounce effects[25]. Thus, the main question that arise is if high frequency intraday data could be useful to improve the forecast and if the timeframe taken into consideration could be affected by microstructure frictions. In addition, the more liquid is the instrument, the higher could be the timeframe in which data can be collected[26].Furthermore, one the of the main issues that stems from collecting this kind of data is the consideration or not of night returns. One of the most widespread approach is taking into account the weight of night returns relatively to daily RV[27]. Another possible approach is to omit night returns.[28] Fuertes and Olmo (2012) find out that night returns do not improve the forecast of VaR.[29] To support the consideration of this paragraph, Figure 9 shows how RV based on very short time interval could contain a large quantity of noise.

[23] Cf. So and Xu, 2013, pp.93-95.
[24] Cf Bandi and Russell, 2008, pp 361-363.
[25] Cf. Aït-Sahalia and Yu, 2009, p.2.
[26] Cf. Będowska-Sójka, 2015, p.162, according to Corsi(2008).
[27] Cf. Będowska-Sójka, 2015, p.162, according to Hansen and Lunde (2005)
[28] Cf. Będowska-Sójka, 2015, p.162, according to Fuertes and Olmo (2012)
[29] Cf. Będowska-Sójka, 2015, pp. 158-159.

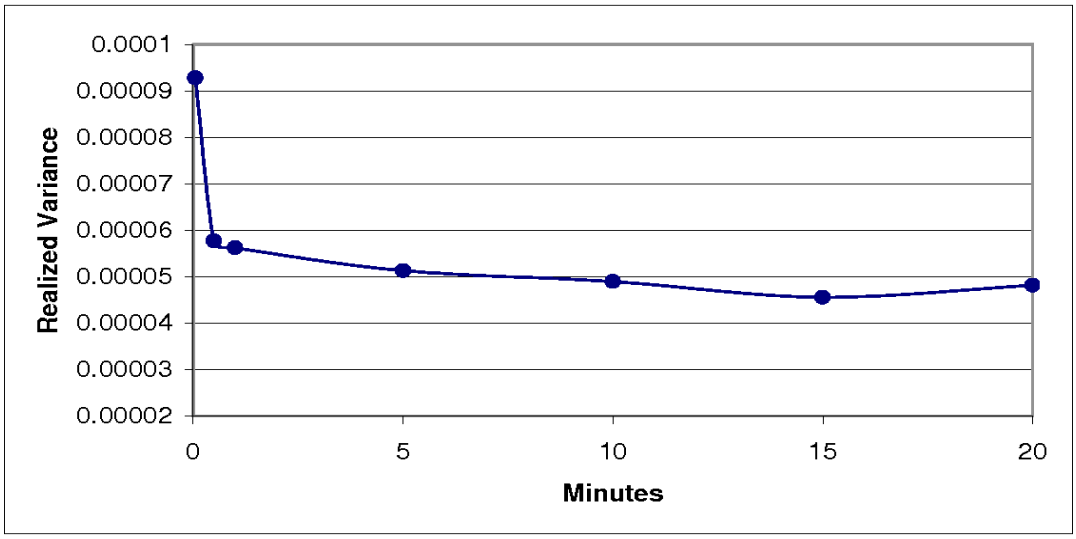

Figure 9: Intraday Realized measure, an example of an average signature plot.

Source: Bandi and Russell (2008), pp. 340.

2.2. Modelling Conditional volatility, GARCH Class Models

2.2.1. Basic GARCH models

In finance, most relationships do not behave linearly, such as option payoff considered in Chapter 1. Particularly, in the majority of financial series, the variance of the innovation terms is not constant over time. This property is known as heteroscedasticity. For this reason, in order to forecast the volatility, it is necessary to implement a model able to shape heteroscedasticity. GARCH models are created to describe this phenomenon, since they are non-linear in variance at odds with ARMA models, linear in mean and variance.

In GARCH models the volatility is conditional, it means that cannot be observed directly but, it is a dependent random variable tied to the progress of another variable. They are able to catch many financial relationship that linear model cannot gather, such as leptokurtosis and the tendency for volatility trends to gather in bunches (Volatility Clustering). Moreover, linear functions are not suited to explain leverage effects. The forerunner models for volatility forecast are the ARCH model introduced by Engle (1982) and GARCH introduced by Bollerslev (1986). In both models the variance is conditional, thus, it depends on the temporal parameter:

$$\sigma_t^2 = var(\ \mu_t \mid \mu_{t-1}, \mu_{t-2}, \dots\ ) = E[(\mu_t - E(\mu_t))^2 \mid \mu_{t-1}, \mu_{t-2}, \dots],$$

With $E(\mu_t) = 0 \Rightarrow \sigma_t^2 = E[\mu_t^2 \mid (\ \mu_{t-1}, \mu_{t-2}, \dots)]$.

The basic representation of a process including the conditional variance is the ARCH(q).

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 = \alpha_0 + \alpha_q(L)\varepsilon_t^2 \qquad (6)$$

Where $\varepsilon_t = \sigma_t Z_t$ with $Z_t \sim i.i.d.(0;1)$ and $\alpha_0 > 0$, $\alpha_i > 0$. In formula (6) the conditional variance (σ_t^2) is linked to the development of the q lags of the squared error (ε_t^2). L is the lag operator with $\alpha_q(L) = 1 - \sum_{i=1}^{q} \alpha_i L^i$ and α_0 and $\alpha_i$are the parameters to estimate. In order to guarantee a positive conditional variance, parameters α_0 and $\alpha_i$should be positive.

The ARCH(q) moves like a Moving Average filter over a noise sequence power which is unobservable. In detail, in the ARCH(1), the conditional variance of the error term depends

on the immediately previous value of the squared error. However, in ARCH(1) model a huge jump which could cause a large error would not have the pattern of persistence. Moreover, in the extended ARCH(q), where the disturbance variance is based on q lags of the squared error, the lags to choose in order to catch all the dependencies could be very large. Therefore, it is not a parsimonious model. Another limitation of ARCH(q) is that the more are the parameter included in the model, the higher is the likelihood to violate a non-negativity constraint

In order to overcome these drawbacks, Bollerslev (1986) formulated the generalized ARCH. As opposed to the ARCH, the generalized model allows the conditional variance to be also tied to its other previous lags. Hence, GARCH (p,q) model lets an infinite quantity of past squared errors to affect the current conditional variance. This model moves like an ARMA filter on the noise sequence power which is unobserved. GARCH model is more parsimonious than ARCH because avoids overfitting. Thus, a good prediction is achievable only with few parameters. The formula of GARCH is given by:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 = \alpha_0 + \alpha_q(L)\varepsilon_t^2 + \beta_p(L)\sigma_t^2, \qquad (7)$$

Where $\varepsilon_t = \sigma_t Z_t$ with $Z_t \sim i.i.d.(0;1)$, $\alpha_0 > 0$, $\alpha_i > 0$, $\beta_j > 0$. In addition, L is the Lag operator, $\alpha_q(L) = 1 - \sum_{i=1}^{q} \alpha_i L^i$ and $\beta_p(L) = 1 - \sum_{i=1}^{p} \beta_i L^i$. GARCH (p,q) models the conditional variance considering a long-term average value (α_0), the information about volatility of the previous period ($\alpha_i \varepsilon_{t-i}^2$) and the conditional variance of the previous period ($\beta_j \sigma_{t-j}^2$). So that, by the addition of the time-lagged conditional variances, it can be reduced the number of parameters in many cases. In many cases, as it will be explained in the literary review in this chapter, the simple model GARCH(1,1) in formula (8) could be sufficient to capture volatility clusters:

$$\sigma_t^2 = \alpha_0 + \alpha_i \varepsilon_{t-i}^2 + \beta_j \sigma_{t-j}^2 . \qquad (8)$$

2.2.2. GARCH extensions

Subsequently to GARCH model introduction, more sophisticated and highly parametrized implementations of this model were developed. The aims of these models is to characterize some properties of financial series that the generalized version cannot catch. First and foremost the leverage effect, i.e. the presence of asymmetries in the returns' distribution of the financial series. Furthermore, some of those models address the issue about how the volatility persistence after a shock should be quantified.

IGARCH (p,q)

The Integrated GARCH introduced by Nelson (1990) is a specific case of GARCH characterized by the presence of an unit root. The model can be written in this way:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} (1-\alpha_i)\sigma_{t-j}^2 =$$

$$= \alpha_0 + \alpha_q(L)\varepsilon_t^2 + \left(1-\alpha_q\right)(L)\sigma_t^2 =$$

$$= \alpha_0 + (L)\sigma_t^2 + \alpha_q(L)(\varepsilon_t^2 - \sigma_t^2) \qquad (9)$$

Where $\alpha_0 > 0$, $0 < \alpha_i > 1$, and $1 - \alpha_i(x) - \beta_j(x) = 0$.

The process is similar to a Random Walk with a drift $\alpha_0 > 0$. In this specification, the unconditional variance does not exist. The IGARCH (p,q) complies with the property of persistent variance. It means that the current variance keeps its informative power for more time. In other words, the current information set has an important weight on the forecast of the conditional variances for all horizons. In the IGARCH, shocks have the highest pattern of persistence.

FIGARCH (p,d,q)

The Fractionally Integrated GARCH, introduced by Baillie et al. (1996), aims to model volatility persistence through the utilization of a decay parameter (*d*). The model is presented as follows:

$$(1-L)^d\left[1-\alpha_q(L)-\beta_p(L)\right]\varepsilon_t^2=\alpha_0+\left[1-\beta_p(L)\right]V_t$$

$$(1-L)^d\Phi(L)\varepsilon_t^2=\alpha_0+\left[1-\beta_p(L)\right]V_t \qquad (10)$$

With $V_t=\varepsilon_t^2-\sigma_t^2$ as the disturbance variable, $\alpha_0>0$, $\alpha_i>0$, $\beta_j>0$, $\Phi(L)=1-\alpha_q(L)-\beta_p(L)$ and d the decay parameter. The FIGARCH is a long memory model where the persistence of the variance lies between the exponential decaying of the GARCH and the infinite persistence which characterizes the IGARCH. In opposition with the ARFIMA(p,d,q), the higher the parameter d the lower is the persistence of the volatility. The reason is due to the fact that decay parameter acts on the squared error.

EGARCH (p,q)

The Exponential GARCH introduced by Nelson (1991) is a widespread approach for modelling asymmetries in financial series:

$$ln\sigma_t^2=\alpha_0+\sum_{i=1}^{q}\alpha_i\left[\phi z_{t-i}+\gamma_j(|Z_{t-i}|-E|Z_{t-i}|)\right]+\sum_{i=1}^{p}\beta_j ln\left(\sigma_{t-j}^2\right) \qquad (11)$$

Where $Z_{t-i}=\frac{\varepsilon_{t-i}}{\sigma_{t-i}}$, with $Z_t\sim n.i.d.(0,1)$, and $g(z_t)=\phi z_t+\gamma(|z_t|-E|z_t|)$, where $\gamma(|z_t|-E|z_t|)$ detects the magnitude effect and ϕz_t the sign effect. EGARCH can catch the volatility persistence and is able to detect mean reversion effects, such as in the case of GARCH. Moreover, it is able to accommodate asymmetrical effects. The conditional variance is expressed as an asymmetric function of ε_{t-j}. The EGARCH is a model where the conditional variance depends on the size and the sign of the lagged residuals. The feature that differentiate this model from the other GARCHs is that parameters have no positive restrictions. Moreover, the behavior is not affected by the square of the data like in the GARCH. Thus, shocks can have a lower persistence than in GARCH. Looking at the formula (11), it is possible to notice that negative shocks occur when $\phi<0$ and $\alpha_i>0$.

GJR-GARCH (p,q) and TGARCH (p,q)

The Glosten, Jagannathan and Runkle GARCH of Glosten et al. (1993) and the Threshold GARCH of Zakoian (1994) are other different approaches able to represent the skewed Generalized Error Distribution which characterize the leverage effect of a financial series. The two models are very similar. The GJR-GARCH can be written as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \gamma_i \mathrm{I}_{t-i} \varepsilon_{t-i}^2 \qquad (12)$$

With $I_{t-1} = \begin{cases} 1 \;\; if \;\; \varepsilon_{t-i} < 0 \\ 0 \;\; if \;\; \varepsilon_{t-i} \geq 0 \end{cases}$

Where parameters have non negative restrictions, γ_i is the asymmetry parameter and I_{t-i} is a dummy variable which considers the propensity for the volatility to increase more after negative shocks. The properties of this model are very similar to the EGARCH. Therefore, it is able to detect asymmetric effects of positive and negative returns, where negative shocks influence more the conditional volatility. However the GJR-GARCH has non-negative restrictions of the parameters.

APARCH (p,q)

Introduced by Ding et al. (1993), the Asymmetric power ARCH is presented as follows

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^{q} \alpha_i (|\varepsilon_{t-i}| - \gamma \varepsilon_{t-i})^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta \;, \qquad (13)$$

Where parameters have to be estimated taking into account non-negative restrictions. Hence, $\delta > 0$ $\alpha_0 > 0$, $\alpha_i > 0$ and $\beta_j > 0$. Formula (13) models asymmetric innovation effects and parameter γ capture the asymmetry, with $-1 < \gamma < 1$. The model is able to capture asymmetries, such as in the case of EGARCH and GJR-GARCH.

GARCH-M (p,q)

The GARCH-in-mean model was introduced by Engle et al. (1987). This model considers innovations of mean and variance at the same time. Particularly, it explores the relation between risk premium and its conditional variance. A positive risk premium is present when data series is positively correlated with its volatility. The model is represented by system (14):

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 = \alpha_0 + \alpha_q(L)\varepsilon_t^2 + \beta_p(L)\sigma_t^2,$$

$$y_t = \mu + \vartheta g(\sigma_t^2) + \varepsilon_t$$

$$\varepsilon_t = \sigma_t Z_t \text{ with } Z_t \sim i.i.d.(0;1) \qquad (14)$$

The conditional expected return on the financial system (y_t) equals the risk free rate μ when $\sigma_t^2 = 0$. When a risky asset is included in the portfolio, to this the risk premium $g(\sigma_t^2)$ is added with a proportional coefficient ϑ.

2.2.3. GARCH Literature Review

The aim of this section is investigating with regard to volatility forecasting in the field of GARCH universe models. Seven different papers have been selected to find out which models of the GARCH family are recommended to improve the accuracy of the forecast. The analysis of the models is carried out in relation to the features of the sample and the specifications that could improve the significance and the predictive power for the analysis of the conditional variance. These seven researches have been chosen regarding their importance and different approaches that could be considered complementary for this analysis.

The papers considered are:

1) T. G. Andersen and T. Bollerslev (1998), “Answering the Skeptics: Yes, Standard Volatility Models do Provide Accurate Forecasts”.

2) S. R. Bentes (2015), “Forecasting volatility in gold returns under the GARCH, IGARCH and FIGARCH frameworks: New evidence”.

3) C. Brownlees, R. Engle and B. Kelly (2011), “A practical guide to volatility forecasting through calm and storm”.

4) P.H. Franses and D. van Dijk (1996), “Forecasting Stock Market Volatility Using (Non-Linear) GARCH Models”.

5) R.F. Engle and V. K. Ng (1993),” Measuring and testing the impact of news on volatility”.

6) P. R. Hansen and A. Lunde (2005), “A forecast comparison of volatility models: does anything beat a GARCH(1,1)?”.

7) S. H. Kang , S.M. Kang , S.M. Yoon (2008), “Forecasting volatility of crude oil markets”.

The choice of the financial time series, the timeframe and the sampling frequency are important determinants for the forecast of volatility (see first paragraph Chapter 2). The accuracy of the forecast for many financial series could be higher with more sophisticated models, when these series present enhanced features such as leverage effect and persistence in the volatility. However, in some cases, as it will be explained in this chapter, even simpler models can track volatility in an accurate and efficient way. The information about the dataset of the seven researches considered are summed up below in table (7). Three of these papers carry out a forecast for exchange spot rates, four paper consider stocks spot rates in the analysis and two other papers forecast volatility for commodity series. Different sampling frequencies are selected for the estimation of the models parameters, nevertheless it seems to be enough to estimate them on daily data. Differently, intra-daily data, through the estimation of realized variance, helps to increase the performance power of the prediction. In fact, the squared returns can approximate the conditional variance, but they contain more noise[30]. Some of the papers consider in the analysis periods characterized by financial distress, such as the crises of the 1987 and 2008. In these situations, the level of volatility increases and it could be interesting to consider the statistical properties of the asymmetrical models.

[30]Cf. Andersen e Bollerslev, 1998, pp. 885-888.

	Market	Estimation timeframe	Out-of sample timeframe	Sampling frequency for estimation	Sampling frequency for forecast
T. G. Andersen and T. Bollerslev (1998)	DM/USD and JPY/USD spot rates	Oct. 1, 1987 - Sept. 30, 1992	Oct. 1, 1992- Sept. 30, 1993	Daily returns	5 minute Returns
S. R. Bentes (2015)	Gold spot rates	Aug. 2, 1976 - Oct. 24, 2008	Oct. 27, 2008 – Feb. 6, 2015	Daily returns	Daily returns
C. Brownlees, R. Engle and B. Kelly (2011)	S&P 500 spot rates	from 1990 to 2008	from 2001 to 2008	Daily returns	5 minute returns
	10 exchange rates	Jan. 5, 1999 - Dec. 30, 2000	from 2001 to 2008	Daily returns	5 minute returns
	9 US equity indixes	Dec. 23,1998 - Dec. 30, 2000	from 2001 to 2008	Daily returns	5 minute returns
	18 international equity indexes	Mar. 19, 1996 - Dec. 30, 2000	from 2001 to 2008	Daily returns	5 minute returns
R. F. Engle V. K. Ng (1993)	Japan TOPIN index spot rates	Jan. 1, 1980 - Dec, 31, 1988		Daily returns	
P. H. Frances and D. van Dijk (1996)	DAX (ger), OEO(ned), MAD (spa), MIL (ita), VEC (swe) spot rates	from 1986 to 1990	from 1991 - 1994	Weekly returns	Weekly Returns
P. R. Hansen and A. Lunde (2005)	DM/USD spot rates	Oct. 1, 1987 - Sept. 30, 1992	Oct. 1, 1992 - Sept. 30, 1993	288 intraday returns	288 intraday returns
	IBM spot rates	Jan. 2, 1990 - May 28, 1999	June 1, 1999 - May 31, 2000	Artificial 3 minutes returns	Artificial 3 minutes returns
S. H. Kang , S.M. Kang , S.M. Yoon (2008)	Brent, Dubai and West Texas Intermediate (WTI) crude oil spot rates	Jan. 6, 1992 - Dec. 29, 2005	Jan. 6, 2006 - Dec 29, 2006	Daily closing prices	Daily closing prices

Table 7: GARCH literature review, dataset

Source: Author's elaboration

The chosen papers differ from each other in relation to the research methodologies used by the authors. First question that stems from the analysis is whether more parametrized models are always, or at least in many cases, more performant than the progenitors, including the GARCH(1,1), the most used and widespread base model. This is the research demand of Hansen and Lunde (2005). Making a large comparison for the model part of the GARCH universe, their aim is to figure out which rank has the most parsimonious model of this type, precisely the GARCH(1,1). This analysis comprehends all existing GARCH models and the comparison is done for two different kinds of assets (exchange rates and stocks) via two tests: Superior Predictive Ability (SPA), Reality Check (RC) (Hansen (2005)). These tests use as baseline the benchmark models, ARCH(1) and GARCH(1,1), and test if these two model are the most performant of the sample.

The same goal characterizes the research of Franses and van Dijk (1996), however they focus the content of the paper on tracking the performance and features of asymmetric GARCH models, such as Quadratic GARCH (QGARCH) and GJR-GARCH.

Comparing the performance of different kind of GARCH is also the goal of Bentes (2015) and Kang et al. (2009), but in these cases the intention is to explore the persistence and long term volatility properties of commodities financial series. The former carries out a volatility forecasting taking into consideration gold spot rates, the latter considers as reference samples the crude oil spot rates of the three main regional markets Brent (North Sea-Europe), Dubai (Persian Gulf), and West Texas Intermediate (WTI) Cushing (US). Both papers include forecast models, widely known for their long-term memory properties, such as IGARCH(1,1) and FIGARCH (1,1). In addition, Kang et al. (2009) include a Component-GARCH (CGARCH(1,1)), a model where each component allows the variance innovations to decay at a different rate. Some components captures the long-run movements in volatility, while some other components model the short-run movements.

Forecasting volatility is not only an issue about the choice of the right model, but also about incorporating the correct specifications into the models. In particular, Brownlees et al. (2011) explore how the predictive ability of a volatility forecast can be affected by the decisions of the estimation window length, of the innovations distribution, and the frequency of the parameter re-estimation. For this purpose GARCH(1,1), TARCH, EGARCH, nonlinear GARCH and APARCH performance are compared in the research.

In this context, Andersen and Bollerslev (1998) tried to provide a support to the predictive power of the GARCH family models blamed on providing poor volatility forecast. In order to do that, they reconfigured the ex-post inter-daily volatility measurement. In detail, they incorporated high-frequency data to latent volatility factor, in order to achieve a less noisy estimation of the ex-post volatility measure.

A different purpose characterizes the work of Engle and Ng (1993), in fact they do not aim to provide a comparative study for volatility forecasting, but to provide some tools to analyze how new information are incorporated in some GARCH model, such as ARCH, GARCH(1,1), EGARCH (1,1), Non-linear GARCH (NGARCH(1,1)), multivariate GARCH (VGARCH(1,1)), GJR-GARCH (1,1) and Asymmetric GARCH (AGARCH(1,1)). The main tool provided by them is the News Impact Curve. It shows the relationship between the current shock and the one-period-ahead conditional variance, holding constant all other past and current information. An example taken from Engle and Ng (1993) is shown in figure (10).

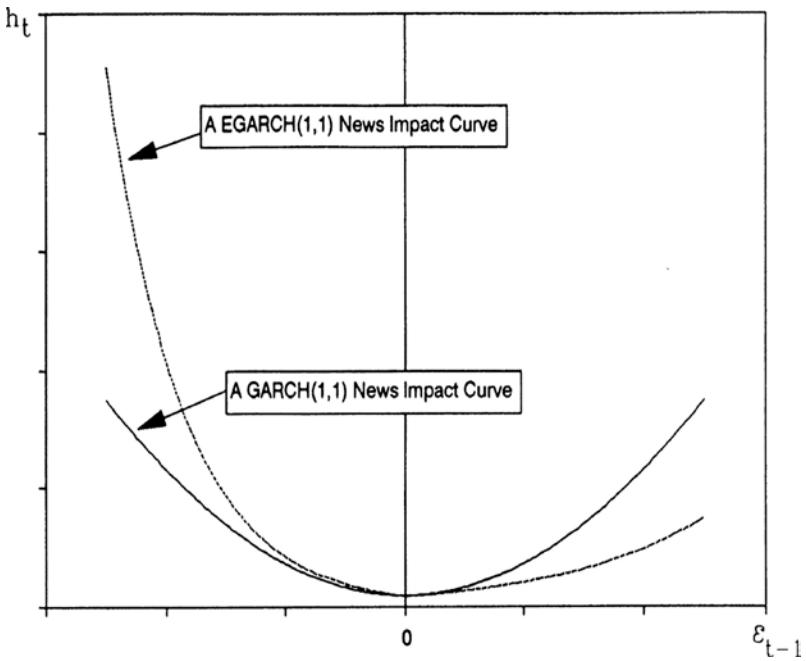

Figure 10: The news impact curves of the GARCH(1,1) and the EGARCH(1, 1) models

Source: Engle and Ng (1993), p. 1754

Most of the researches of the sample consider the one-step-ahead as forecasting horizon. Brownlees et al. (2011) extend the analysis to one week, two weeks, three weeks and one-

month-ahead. This multi-step approach is due to the particular attention paid for extremely volatile period such as the 2008. In fact, the work provides a test for this class of model on the long term horizon. In order to make the long term ahead period forecast more accurate, it is necessary to consider a long window for the estimation of the parameter and monthly or even daily estimation updates, in order to mitigate the effects of parameter drift[31].

Similar reasons lead Kang et al. (2009) to take into account a longer time ahead forecast horizon but, in this case, the goal is using models suited catch volatility persistence.

A fundamental specification involves the determination of the ex-post volatility measure as supported by Andersen and Bollerslev (1998). As already introduced in section 2.1., the utilization of high-frequency data is becoming essential for the forecast of conditional volatility and also for the implied volatility factor volatility of GARCH unconditional models. Andersen and Bollerslev (1998) propose a RV measure based on 5 minute returns that became a reference point for many other works. Two of the researches considered in the literary review, implement a realized volatility measure as ex-post volatility measure. Hansen and Lunde (2005) construct the RV based on intraday returns obtained artificially by fitting a cubic spline to all mid-quotes. They extract 3 minutes artificial returns.[32]They also compare their RV measure with other RV measures in order to demonstrate that their final result is not sensitive to the choice of the realized measure. Also Brownlees et al. (2011) use a RV measure to characterize ex-post volatility. In detail, they sample every mid-quote in order to have an average sampling duration of five minutes[33].

The loss functions used for the valuation of the forecasts are widespread tools and are presented in table(8). Particular remarkable tests introduced in the analyzed papers are the Superior Predictive Test (SPA) by Hansen e Lunde (2005) and three diagnostic tests introduced by Engle and Ng (1993): sign-bias, negative size-bias and the positive size-bias. The last three tests emphasize the model response to shocks.

[31]Cf. Brownlees et al., 2011,p.5.
[32]Cf. Hansen and Lunde, 2005, p.881.
[33]Cf. Brownlees et al., 2011, p. 9.

	Content and goal	Models included	Evaluation and R squared	Forecasting horizon
T. G. Andersen and T. Bollerslev (1998)	Proving that traditional GARCH models produced powerfull volatility forecast. New specifications for the ex-post interdaily volatility mesurements.	GARCH(1,1)	R squared	One day ahead
S. R. Bentes (2015)	Forecasting gold volatility to examine long memory property.	GARCH(1,1), IGARCH(1,1) and FIGARCH(1,d,1)	Information criteria (Ln(L), BIC and AIC) to select the best model. Loss functions : MAE, RMSE, TIC.	One step ahead
C. Brownlees, R. Engle and B. Kelly (2011)	Volatility forecasting comparative study using ARCH class of models. Identifying successful predictive models given some specification: estimation window length, innovation distribution and frequency of parameter reestimation.	GARCH(1,1),TARCH, EGARCH, nonlinear GARCH, APARCH	QLIKE	One day, one week, two weeks, three weeks, One month ahead forecast
R. F. Engle V. K. Ng (1993)	Existing ARCH models are compared to determine the shape of the News Impact Curve. New diagnostic tests are presented which emphasize the asymmetry of the volatility response to news.	ARCH, GARCH(1,1), EGARCH (1,1), NGARCH(1,1), VGARCH(1,1), GJR-GARCH(1,1), AGARCH(1,1)	The estimations are performed using the QML. Tests for adequacy: sign-bias, negative size-bias and the positive size-bias	No forecast
P. H. Frances and D. van Dijk (1996)	Tracking the performance of the GARCH model and two its asymmetric specifications to forecast weekly stock market volatility.	QGARCH, GJR-GARCH, GARCH (1,1), RW	The diagnostics are AIC, Ln(L), Q. Loss function: MedSE	One week ahead estimated from rolling 4 years
P. R. Hansen and A. Lunde (2005)	Volatility forecasting study that aims to understand if more sophisticated models can clearly outperform the simple GARCH (1,1).	330 GARCH type models	Loss functions: two types of MSE, QLIKE, R^2LOG, two types of MAE1. Tests for performance: Superior predictive test (SPA), Reality Check (RC)	One day ahead
S. H. Kang , S.M. Kang , S.M. Yoon (2008)	Testing the efficacy of a volatility models for crude oil markets. Proving GARCH ability to forecast and identify volatility stylized facts, such as volatility persistence or long memory.	GARCH(1,1), IGARCH(1,1), FIGARCH(1,1), component-GARCH (CGARCH)(1,1)	Evaluation of estimates: Ln(L), ARCH (5). Loss functions: MSE, MAE.	One, five and 20 trading days ahead

Key: Akaike Information Criterion (AIC), Bayesian Infromation Criterion (BIC), Log-likelihood (Ln(L)), Mean Absolute Error (MAE), Median Squared Error (MedSE), Mean Squared Error (MSE),Box-Pierce test statistics (Q), Quasi-Likelihood (QLIKE), Quasi-Maximum-Likelihood (QML), Root-Mean Squared Error (RMSE),Theil's inequality coefficient (TIC)

Table 8: GARCH literature review, methodologies

Source: Author's elaboration

Looking at the results, I derived some of the most relevant findings. The papers that analyze the persistence of the volatility for commodity assets agree on the fact that long term specification are useful to detect the persistence of volatility. In Bentes (2015) and Kang et al. (2009), the sum of estimated GARCH values for the parameter α and β is close to unity in both cases. These values demonstrate the presence of autocorrelation in the volatility process. In particular, both models agree on the fact that FIGARCH (1,d,1) is the model which gives the most accurate forecast, also in longer than one-day-ahead horizons and at the same time GARCH(1,1) is outperformed by all the other models.

Models suited to represent better the leverage effect phenomenon seem to be accurate too and they are particularly effective during tumultuous periods, such as the financial crises of the 2008. Brownlees et al. (2011), in this economic context, find out T-GARCH as the best forecaster. In general, the asymmetric models outperforms the others. They support that a frequent re-estimation of the parameters on weekly basis is necessary to maintain accurate the forecast and that a long estimation window is always more informative. Different conclusion for Student-t likelihood specification, in this case do not improve the forecast ability. Similar economic circumstances characterize the research of Franses and van Dijk (1996). In the context of financial turbulence of the 1987, QGARCH is the best model, however, Random Walk models are the best with the crash included. GJR-GARCH in this case is not recommended.

Although the volatility prediction is not the goal of their investigation, Engle and Ng (1993) find out important features about models consistent with leverage effect property during a period of financial distress. EGARCH(1,1) and GJR-GARCH(1,1) have the highest variation over time and have the curve impact news close to the one suggested by a non-parametric benchmark. The EGARCH(1,1) is the most extreme and asymmetrical model, instead GJR-GARCH(1,1) is more reasonable and highly significant.

The most extensive investigation in the sample is carried out by Hansen and Lunde (2005). In fact, their empirical research includes 330 ARCH class models and two different datasets with different characteristics. The analysis of the exchange rates series shows the GARCH (1,1) as one of the most accurate forecaster of volatility. In this case, also the Student-t specifications seem to improve the forecast. As opposed, in the prediction of IBM volatility the models with asymmetric specification outperforms the GARCH (1,1), which have, in this a case, a low rank in the benchmark. For this series, a very highly parametrized model, the APARCH (2,2) is ranked as the best forecaster for IBM one-day

ahead volatility prediction. In this circumstances, Student-t specifications do not seem to improve the accuracy of the forecast. On the other hand, Andersen and Bollerslev (1998) do not compare time series models and, utilizing the simple GARCH(1,1), demonstrate that the traditional models do not have a low informative power. The new specification, obtained from daily cumulative five minute squared returns, gives a R^2 close to 0.5 for both samples. This value is approximately ten times higher than the R^2 of daily squared returns measure.

The importance of the result in Andersen and Bollerslev (1998) allows to derive a main conclusion. Although it is proved that more sophisticated models could give in some case a more appropriate description and forecast of volatility, the role and the choices of the specifications seems to be more important that the choice of the models itself. Particularly, the ex-post volatility measure can be more accurate if it includes intraday returns measure, sampled with a reasonable timeframe. In addition, long estimation window and frequent re-estimations of the parameters helps to improve the prediction.

2.3. Modelling unconditional Volatility

In econometrics, two factors determine the convenience of the model rather than others. These factors are the quality of the estimation and forecast and the parsimony of the model in use. Information criteria, such as the AIC (Akaike Information Criteria) and the BIC (Bayesian Information Criteria), are yardsticks utilized to assess the quality of the estimate, taking into consideration the degrees of parametrization of a model. The weight of the parametrization, that has a different impact on the two different criteria, penalizes the complexity of the model. For this reason the concept of parsimony is critical for the choice of a model. The way to increase parsimony is simply achieving, as far as possible, the qualitative information from historical high-frequency data rather than from a huge parameterization. In this context, a first evidence has been shown in the previous chapter. Building the proxy of latent volatility factor with the sum of intraday returns explains a higher proportion of changes in volatility rather than classical daily squared returns measure. Starting with these considerations, this chapter examines the most common approaches to model directly volatility, exploiting the potentiality of intraday returns. The models taken into consideration are those that nested the ARMA process inside and some recent remarkable approaches such as the Heterogeneous AutoRegressive (HAR), the High-frEquency-bAsed VolatilitY (HEAVY) and the Realized GARCH.

2.3.1. ARIMA (p,d,q)

The AutoRegressive Integrated Moving Average model, ARIMA, is probably the most general type of model for forecasting a time series. Through differencing, this approach is able to "make stationary" a series which is not stationary in its original form. The number of difference transformations is given by *d*, the integer order of differencing. The model is a combination of AR(p) and MA(q) components that represents, respectively, the weighted sum of the values of *y* and of the forecast errors. The model is described as follows:

$$\Phi(L)\Delta^d(y_t) = \alpha_0\Theta(L)\varepsilon_t, \tag{15}$$

With $\varepsilon_t \sim WN(0, \sigma^2)$. Formula (15) represents the formal statement of the model, where L is the lag operator and respectively $\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$ and $\Theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$, where p are the lags of AR type and q the lags of the MA type. In this model the autocorrelation remains stable over time.

2.3.2. ARFIMA(p,d,q)

The Autoregressive Fractional Integrated Moving Average (ARFIMA) model, conceived by Granger and Joyeux (1980), is a process that makes part of the ARMA class models' framework. Such as in the GARCH family model, the Fractional integrated models are suited to catch long time dependences. It has a wide capability of application, since it could capture both short-time dependence and long-time dependence. ARFIMA(*p,d,q*) processes are very utilized tools in modeling long memory time series, especially for the high frequency trading data. The model, that presents the same linear filter of formula (15), can be described in this way:

$$\Phi(L)(1 - L)^d (y_t - \mu) = \Theta(L)\epsilon_t, \tag{16}$$

Where $(1 - L)^d = \frac{\Gamma(K-d)L^k}{\Gamma(-\mathrm{d})\Gamma(\mathrm{K}+1)}$ and $\epsilon_t \sim i.i.d.(0, \sigma_\epsilon^2)$

In formula (16), $(1 - L)^d$ is the fractional differencing operator, where $\Gamma(.)$ is the generalized factorial function. The autocorrelation function of the ARFIMA(p,d,q) decays hyperbolically to zero. Many features of the model rely on the value of the differencing operator *d*. The process becomes stationary when $|d|$<0.5. Particularly, in the case that $-0.5 < d < 0$, we have an anti-persistence model, while, when we have $0 < d < 0.5$, the model becomes more effective in catching long range positive dependence. Lastly, when $0.5 < d < 1$ the model is not covariance stationary and it depicts a mean reverting process. Koopman et al. (2005) test the ARFIMA(p,d,q) for the prediction of S&P100. An out-of-sample volatility prediction from one day to one week is carried out, comparing the ARFIMA model based on intraday measures, GARCH class model based on RV and stochastic volatility models. They find out that ARFIMA (1,d,0) gives the most

parsimonious description of the estimates with an achieved *d* between 0.4 and 0.48 and at the same time, makes the most accurate prediction. Furthermore, they employ the Superior Predictive Ability test that confirm the ARFIMA –RV as the most accurate predictor.

In another study concerning ARFIMA, Pong et al. (2004) compare four kinds of different models: short memory ARMA, long memory ARFIMA, GARCH models and implied volatility models. The analysis shall consist on a one-day and one-week-ahead forecast for three different exchange rates (Pound/$, mark/$, yen/$). The estimated ARFIMA is (1,d,1) in one case and (0,d,0) in the other two cases. The Realized measures are obtained through the processing of 5 minute and 30 minute returns. ARFIMA and ARMA models has a lower mean squared error than GARCH and implied volatility models for the short horizon. Instead, implied volatility models are the best in the long window. The same situation characterize the quantity of volatility explained by the models. ARFIMA models R squared is the best in the short run while the implied volatility R squared is the highest in the long run. It seems that historical forecast accuracy ups more to the usage of intraday data than the implementation of a long memory model like the ARFIMA.

Oomen (2001), implements a 25-minutes RV for the forecast of the FTSE 100 variance. In this case the ARFIMA slightly outperforms the GARCH (1,1), but the flexibility and the parsimony of the latter makes the simple GARCH preferable.

2.3.3. HEAVY model

High-frEquency-bAsed VolatilitY (HEAVY) forecast approaches are based on realized measures of the conditional covariance matrix of daily returns. Those models exploit the increasing availability of high-frequency financial data and is a recent approach proposed by Shephard and Sheppard (2010). The succeeding model proposed by Noureldin et al. (2012) includes the multivariate version of the model but, since the purpose of the research is to investigate the univariated approaches, we analyze only the univariate model. The model includes a Realized Volatility component, high frequency based and non-parametric estimator of the variation of returns. The realized volatility estimator included in the model is the Realized Kernel introduced by Barndorff-Nielsen et al. (2008), that is robust with market microstructure effect. The first model of the HEAVY class, the univariate HEAVY is presented as follows:

$\begin{cases} var(r_t|f_{t-1}^{HF}) \\ E(RK_t|f_{t-1}^{HF}) \end{cases}$ with $t = 2,3, \dots, T$,

$$var(r_t|f_{t-1}^{HF}) = \sigma_t = \alpha_0 + \alpha RK_{t-1} + \beta\sigma_{t-1}, \tag{17}$$

with $\alpha_0, \alpha \geq 0$ and $\beta \epsilon [0,1)$.

$$E(RV_t|f_{t-1}^{HF}) = \varepsilon_t = \alpha_0 + \alpha_R RK_{t-1} + \beta_R \varepsilon_{t-1}, \tag{18}$$

with $\alpha_0, \alpha_R, \beta_R \geq 0$ and $\alpha_R + \beta_R \epsilon [0,1)$.

Equation (17) uses the lagged realized measure, RK_{t-1}, to drive to dynamics of σ_t, The second equation (18) of the HEAVY model is necessary for multi-step forecasts of h_t. As it is possible to notice, the two formulas are not nested. This approach is built on the literature GARCH framework with the aim to strengthen its predictive power with the inclusion of a realized measure. The main capability of HEAVY model is the short response time, that is the peculiarity to perform well in periods where the level of volatility or correlation is affected by sharp changes. The model has short-run momentum effect, so that volatility forecasts may show a continuation of a trend before mean reverting[34]. The results of the work shows that HEAVY models outperforms GARCH in the short-run forecast horizon and catch better the persistence of a trend.[35]

2.3.4. Realized GARCH

Introduced by Hansen et al. (2012), the Realized GARCH is a joint model of returns and realized measures that exploit the features of the GARCH models framework. The model utilized the realized measure as an exogenous variable. The Realized GARCH can be represented in this way:

[34] Cf. Sheppard and Xu, 2014, pp. 2-4.
[35] Cf. Shephard and Sheppard, 2010, p.228.

$$r_t = \sqrt{h_t} z_t$$
$$h_t = w + \beta h_{t-1} + \gamma x_{t-1}$$
$$x_t = \xi + \varphi h_t + \tau(z_t) + \mu_t \tag{19}$$

With $z_t \sim i.i.d.(0,1)$ $\mu_t \sim i.i.d.\left(0, \sigma_\mu^2\right)$

In formula (19) some components can be specified as follow, $h_t = \mu + \phi h_{t-1} w_{t-1}$, $\mu = u + \gamma\xi$ and $\phi = \beta + \varphi\gamma$, $w_t = \gamma\tau(z_t) + \gamma\mu_t$. w_t is an autoregressive model of order one, while x_t is the measurement equation with an ARMA representation. Furthermore, $\tau(z_t)$ constitutes the specification for leverage function. Since the normal GARCH incorporates only a weak signal of the current volatility, the joint usage of a realized volatility measure helps to increase the responsiveness of the model. In fact, the main feature of the model is the short responsiveness in time, a statistical property that models, such as the HEAVY, one model with a GARCH-X (GARCH with exogenous variables) structure, can detect. The valuation of the model can be carried out with Quasi-Maximum-Likelihood. The model specifies the dynamic properties of returns and Realized measure considering a 6.5 hours of realized measure component based on high frequency data and a close to close return component based on daily data.

Hansen et al. (2012) found out that the Realized GARCH improves the fit of the standard GARCH based only on daily returns.

The model raises interest in the field and after its introduction, some further studies look into its properties and capabilities. Huang et al. (2016) demonstrate that Realized GARCH is not suited to catch long memory properties of the underlying volatility. Through a HAR approach, which uses three different components for the determination of the realized volatility, they are able to describe the autocorrelation behavior in the long run. This approach will be widely shown in the next section. The improvement they have achieved are significant in most samples they considered. The new HAR specification of the Realized GARCH give a more accurate out of sample forecast.

The results obtained by Sharma and Vipul (2016) are not unanimous concordant on the superior forecasting performance of the Realized GARCH in comparison with classical GARCH methods based on daily returns and realized measure models. Their results change with regard to the loss function considered when the comparative study takes into account Realized GARCH and EGARCH, the most performant standard GARCH in the analysis. Instead, when the Realized GARCH is compared with EWMA models based on

Realized Kernel or RV, the latter outperforms the former in most cases. In detail, the results are explained with the following remarks: the necessary estimation of nine parameters induces to a large error that may make difficult the forecast, while the inclusion of daily returns is likely to introduce noise in the model[36].

Jiang et al. (2018) follow a different approach. They figure out that, when future volatility is handled as belonging to a risk measure, such as VaR and Expected Shortfall, Realized GARCH provides a more accurate forecast.

2.3.5. HAR-RV model

The HAR-RV (Heteregeneous autoregressive model of realized volatility) model proposed by Corsi (2008), is one of the most influencing approaches stated in the last years in order to forecast volatility. The model based on the AR family model, a simple linear structure without long term memory seems to overcome the widely used volatility forecasts based on GARCH models based on daily return. This has been made possible through the utilization of the RV and an approach based on different time horizons.

The reason behind the consideration of different time horizons is given by the presence of heterogeneity among traders. The statement stems from the Heterogeneous Market Hypothesis (HMH) proposed by Müller et al. (1997). Trading behavior differs across trader: Short-term traders evaluate the market at a higher frequency and have a shorter memory than long-term traders. Thus, volatilities measured with different time resolutions reflects the expectations and actions of different market components. Consequently, the interval length on which volatility is estimated is an essential parameter. Volatility estimations on short time interval gather different information rather than volatility estimations taken from long time intervals.[37] Furthermore, Müller et al.(1997), studying the interrelations of volatilities of different time interval, reveals some dynamics of market components. In detail, they found out that the information flow between short-term and long-term traders is asymmetric: short-term traders react to volatility peaks by increasing their trading activity, thus increasing the final volatility. At the same time long-term traders

[36]Cf. Sharma and Vipul, 2016 p. 229.
[37]Cf. Corsi, 2008, pp.178-179.

mostly ignore the level of short time volatility.[38] Referring to those findings, Corsi realized a cascade model of volatility from low frequency to high frequency. Therefore, he separated the overall volatility measure into three different components: a short term volatility component which affects traders with high trading frequency, such as market makers and intraday speculators, a medium-term volatility component which influences traders that rebalance weekly their position and a long-term volatility component which is taken into account by institutional investors and insurance companies.

The HAR-RV incorporates the estimate of RV to improve its predictive power. As already introduced in Chapter 2, models based on RV can exploit the informative power of high-frequency data so this tool has a wide potential however, there is a lack of convergence in the literature about which of them could outperform clearly the others in volatility prediction.

RV is defined in terms of the sum of intraday squared returns, as follows:

$$RV_t^{(d)} = \sqrt{\sum_{j=0}^{M-1} r_{t-j\cdot\Delta}^2}. \tag{20}$$

In Formula (20), $\Delta = 1d/M$ is the time interval and $r_{t-j\cdot\Delta} = p(t - j\cdot\Delta) - p(t - (j+1)\cdot\Delta)$ is the continuous compounded frequency return. The three component of volatility stated for different agents profile are embodied into RV calculate on daily, weekly and monthly basis. In addition, weekly and monthly RV are taken into account as the simple average of the daily measure. Considering that weeks and months have respectively 5 and 22 trading days, the calculation results as below:

$$RV_t^{(n)} = \frac{1}{n}\left(RV_t^{(d)} + RV_{t-1d}^{(d)} + RV_{t-2d}^{(d)} + \cdots + RV_{t-(n-1)d}^{(d)}\right). \tag{21}$$

In Formula (21) *n* is the number of days on which RV is calculated. The model takes into consideration three different latent partial volatility $\tilde{\sigma}_t^{(n)}$, one for each timeframe component. Each volatility component has approximately an AR(1) structure. However, only the monthly latent volatility $\tilde{\sigma}_t^{(m)}$is structured as a simple AR(1). Since the HAR-RV is a cascade model, it is designed to incorporate the correlation of a latent volatility

[38]Cf. Corsi, 2008, pp.178-181.

component and the longer time scale component. Thus, the weekly market component has expectation for the monthly period and the daily component has expectation for the weekly period. This cascade approach refers to the findings of HMH and in particular it refers to the asymmetry dynamics between market component. In details:

$$\tilde{\sigma}_{t+1m}^{(m)} = c^{(m)} + \phi^{(m)} RV_t^{(m)} + \tilde{\varepsilon}_{t+1m}^{(m)}, \tag{22}$$

$$\tilde{\sigma}_{t+1w}^{(w)} = c^{(w)} + \phi^{(w)} RV_t^{(w)} + \gamma^{(w)} E_t\left[\tilde{\sigma}_{t+1m}^{(m)}\right] + \tilde{\varepsilon}_{t+1w}^{(w)}, \tag{23}$$

$$\tilde{\sigma}_{t+1d}^{(d)} = c^{(d)} + \phi^{(d)} RV_t^{(d)} + \gamma^{(d)} E_t\left[\tilde{\sigma}_{t+1w}^{(w)}\right] + \tilde{\varepsilon}_{t+1d}^{(d)}. \tag{24}$$

In Formulas (22), (23) and (24) $\tilde{\varepsilon}$ is the volatility innovation, independent noise measure with truncated left tail in order to ensure positivity of partial volatilities. In Formula (24), daily latent volatility is affected by the weekly latent component while in Formula (23) the weekly component includes the effect of the longest volatility component in the model.

$$\tilde{\sigma}_{t+1d}^{(d)} = \sigma_{t+1d}^{(d)} = RV_{t+1d}^{(d)} + \varepsilon_{t+1d}^{(d)}. \tag{25}$$

In Formula (25) daily latent volatility $\tilde{\sigma}_{t+1d}^{(d)}$ is assumed to be equal to daily integrated measure $\sigma_{t+1d}^{(d)}$ intended as the sum of realized volatility at time t+1d and the $\varepsilon_t^{(d)}$component that embed latent daily volatility assessment and estimation errors. Substituting the other volatility components and equation (12) into equation (11) the HAR-RV is obtained:

$$RV_{t+1d}^{(d)} = c + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{w} + \beta^{(m)} RV_t^{m} + \varepsilon_{t+1d}. \tag{26}$$

In Formula (26) $\varepsilon_{t+1d} = \tilde{\varepsilon}_{t+1d}^{(d)} - \varepsilon_{t+1d}^{(d)}$. Because of the construction over three levels the model can be called HAR(3)-RV. The same model can be used for different number of volatility component.[39]

Furthermore, I derived some about the performance given by the HAR evidence from the literature.

[39] Cf. Corsi, 2008, pp.178-181.

Corsi in "A simple approximated Long-memory model of realized volatility", compares the performance of HAR model with simple AR models and an ARFIMA (5,d,0). Using dataset composes by tick-by-tick logarithmic mid prices for USD/CHF exchange rate, S&P500 Futures and 30-year US treasury bond futures. However, the tick-by-tick RV takes into account the microstructure noise's issue and use a two scale estimator introduced by Zhang et al. (2005)[40]. HAR model clearly outperforms simple AR(1) and AR(3) models in the one-week and two-week ahead forecasting and it shows a prediction's power comparable with ARFIMA (5,d,0). Nevertheless, the HAR model is preferable because of his higher parsimony. In fact, it obtains similar results using only few-parameters. Instead, Będowska-Sójka (2015), using GARCH class model and models based on RV, among which HAR models, developed a daily forecast of VaR. Daily data have been used to structure GARCH model and Intraday data with sampling of 5-minute returns to compute RV. The dataset is represented by the WIG20 index prices (Poland most capitalized firms), from 4th April 2007 to 21st April 2011, a period characterized by high volatility. Through the utilization of three different loss function, each penalizing losses in different ways, it has been tried to find the model that predict VaR minimizing loss functions. The main result that stems from the analysis is that there is not clear approach that outperforms the others. However, HAR class models are often rejected by the Proportion of failure, the Kupiec test and ARFIMA seems to be the most performant model based on RV, minimizing the loss function that take into account opportunity cost of capital. In conclusion, FIGARCH, GJR-GARCH and ARFIMA seems to be the best approaches to forecast VaR but there is no a model that is clearly preferable to the others.

2.3.6. HAR-RV Extensions

Corsi and Renò (2009) extended the model taking into consideration leverage effects and possible jumps in volatility. The LHAR-CJ (Leverage Heterogeneous Auto Regressive with Continuous volatility and Jumps) is the extension of HAR-RV model on which is included the principle that the increasing of volatility due to a negative shock is higher than the increasing of volatility after a positive shock (Leverage Effect). Moreover, discontinuous variation of asset prices or Jumps are taken into account.

[40] Cf. Corsi, 2008, pp.186.

$$
\begin{aligned}
logRV_{t+h}^{(h)} = c + \beta^{(d)} logC_t^{(d)} + \beta^{(w)} logC_t^{(w)} + \beta^{(m)} logC_t^{(m)} \\
+ \alpha^{(d)} \log\left(1 + J_t^{(d)}\right) + \alpha^{(w)} log\left(1 + J_t^{(w)}\right) \\
+ \alpha^{(m)} log\left(1 + J_t^{(m)}\right) + \gamma^{(d)-} r_t^{(d)-} + \gamma^{(w)-} r_t^{(w)-} \\
+ \gamma^{(m)-} r_t^{(m)-} + \varepsilon_t^h.
\end{aligned} \tag{27}
$$

In Formula (27) leverage affects that influence each market component separately ($\gamma^{(n)-} r_t^{(n)-}$) and Jumps ($J_t^{(n)}$) are taken into account.

Chapter 3. Use of intraday data in VaR forecast. A comparison with models based on daily returns

Forecasting volatility and forecasting VaR are different tasks. The same model able to make an accurate forecast of volatility could be not as accurate as in forecasting VaR. For this reason, while various researches agree on the fact that high-frequency data can improve the informative power of the dataset to make an accurate volatility prediction, it should be verified the same for VaR forecast. In this framework, this chapter analyzes some papers that aims to answer to this issue. The papers taken into account are heterogeneous with regard to financial markets considered in the study, with regard to results on the ground that there is not yet an unique approach that is clearly preferable to incorporate intraday data in the forecast. As indicated in chapter 1, VaR is a widespread tool among practitioners. Considering this fact, most of the works focus their attention on one-day-ahead out of sample forecast or even a shorter timeframe. In fact, for many active agents and market makers, the risk should be assessed more times during a trading day and their investment horizon is generally shorter than one day[41]

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3.1. Datasets and methodologies used in the literature

Through the five researches chosen for the analysis, the authors analyze different financial markets in terms of traded indexes and market's location. The works jointly included six stock indexes (CAC40, SHCI, SZCI, S&P 500, HIS, WIG20), two exchange rate indexes (YEN-USD, DEM-USD) and one future index (S&P 500 future index). Furthermore, those indexes are globally geographically distributed and characterize developed markets but also emerging markets. In order to address different goals and market's features, distinct sampling intervals have been selected for the computation of high frequency VaR forecast. Giot and Laurent (2004) use a 15-minute frequency for stock returns and one hour frequency for exchange rate returns. The other studies in the analysis use five minutes as sampling interval, as for example suggested by Andersen and Bollerslev (1998)[42].

[41]Cf. Dionne et al., 2009, pp.777-778.
[42]Cf. Andersen and Bollerslev, 1998, pp.886-887.

The researches differ in terms of out-of-sample time interval of the forecast. Giot and Laurent (2004), Będowska-Sójka (2015) and Shao et al. (2009) carry out a one-day-ahead VaR forecast. Będowska-Sójka (2015) makes a comparison between GARCH class models and non-parametric models. The Kupiec test and the Angle-Manganelli test are used for the evaluation of the forecast accuracy, whereas for the assessment of the performances, three different loss function are taken into account, for each of these the possible loss is penalized in a different way. She considers a Binomial loss function, a dummy variable that counts the number of failures, the Regulatory loss function and Firm loss function[43] that penalized failures drawing attention respectively to the magnitude of the failures and the opportunity cost of capital. A similar approach has been undertaken previously by Giot and Laurent (2004). The paper demonstrates that daily returns of the one-day-ahead forecast of the RV are not normally distributed, it compares the performances an of an ARCH model built on daily measures, a Skewed Student-t APARCH (Asymmetric Power ARCH) and a skewed Student-t ARFIMAX (*0,d,1*),[44] a model based on RV. The APARCH model from Ding et al. (1993) is an ARCH class model suited to capture the asymmetry effect and to provide an accurate VaR forecast for both tails of distribution and it includes a power or heteroscedasticity parameter. The models are tested with the failure rate test of Kupiec and the Dynamic Quantile Test.

Another one-day-ahead forecast is applied for two of the most capitalized Chinese markets with the aim to compare the performance of different models. The analysis of Shao et al. (2009) differs from the other previous approaches because of the utilization of the concept of volatility range in the estimation of the VaR forecast. The CARR (Conditional AutoRegressive Range) model introduced by Chou (2005) combines the idea of a volatility range, the difference between the lowest and the highest price, with the structure of the GARCH framework. The conditional volatility of the GARCH can be interpreted as a conditional range. In this case the conditional range is built with the Realized Range, intended as the sum of squared intraday high–low ranges. In order to evaluate the possible utility of RV measure in the forecast, the authors use the CARR model and an ARFIMA (*0, d, 0*) to incorporate the high frequency information. Instead, for the low-frequency volatility measure are used Skewed Student-t GARCH, the Skewed Student-t APARCH, the same models that Giot and Laurent (2004) include in their research, and the IGARCH

[43] Sarma et al., 2003.
[44] ARFIMA with exogenous variables

(*1,1*)[45]. For the evaluation of the results the Kupiec Test and the Engle-Manganelli Test are considered.

A distinct approach is pursued by Huang and Lee (2013). The work focuses on the forecast of a quantile regression model of S&P 500 index daily returns, thus, an extension that looks at all the quantiles, where the VaR is simply the negative part of them. In fact, quantile forecasts are very important for risk management purposes. This approach can be worth also for the estimation of the lower tail of daily VaR. In opposition with the previously presented author approaches, the work does not include a goodness evaluation of the volatility forecast. The analysis combines two different approaches by which high-frequency data can be incorporated for the prediction of VaR: a direct approach and an indirect approach that combine forecasts obtained from high-frequency information into daily low-frequency models. The idea behind is to consider high-frequency return observations as multiple replication of the distribution of daily returns. Hence, the intraday data are used to achieve subsamples of the daily series at different time of a day (subsampling approach)[46]. The 5 minutes intraday block information is the subsample considered to build the models. For the indirect incorporation of high frequency data a subsample averaging, a bootstrap averaging and forecast averaging methods are taken into account. These methods are compared with daily closed models and models that incorporate directly intraday data.

Underlying the needs that characterize a growing number of practitioners that focus their attention on very short investment horizon, So and Xu (2013) conduct a less than one-day-ahead VaR forecast. For this purpose, it is important to consider the distribution of intraday returns. Indeed, the distribution of intraday return is strongly affected by a seasonal component due to "market participants praxis". Thus, for this reason the intraday volatility pattern appears as W-shaped distributed.[47] The main feature that makes the intraday returns W-shaped is the higher volatility that characterized the first minutes after the market re-open, also after lunch break, and the minutes before the closing when some trader pre-empt their position. So and Xu (2013), pursue a research which for some aspects follows the Giot and Laurent (2004) examination. They make a 30 minutes ahead VaR forecast for eight different working time intervals and, acting in the GARCH class model context, they focus on the study of three parameters that could improve the VaR forecast of the

[45]Model used by Riskmetrics to forecast VaR
[46]Cf. Huang and Lee, 2013, pp. 128-129.
[47]Cf. So and Xu, 2013, pp. 84-85.

GARCH(1,1). They consider the asymmetry of returns distribution using Student-t GARCH, implement a measure of volatility that includes a seasonal component and take into consideration the inclusion of a measure of RV, as a replacement of the conditional variance component. Alternatively, the RV has been included as an additional term in the GARCH models. Therefore, they used two different way to incorporate RV in the model. In particular the RV volatility is computed with five minutes returns with the aim to achieve a 30 minutes RV component. Moreover, the VaR composition is characterized by the seasonality. In detail, the IVaR (Intraday VaR) is presented as follows:

$$IVaR_{t,i}(\alpha) = -\sqrt{S(i)}\tau_{t,i}t_{\tilde{\alpha},v}^{-1} \tag{28}$$

In Formula (28) the volatility component is $\sigma_{t,i}^2 = S(i) \times \tau_{t,i}^2$, with $S(i)$ seasonal component independent of the market movement and $\tau_{t,i}^2$ variance component attributed to previous return movement.[48] $t_{\tilde{\alpha},v}^{-1}$ is the inverted Student-t cumulative distribution function with $\tilde{\alpha}$ confidence level and *v* degrees of freedom.

3.2. Results and conclusions

The papers analyzed in this Chapter do not clearly agree on the usefulness of intraday data for a more accurate VaR forecast. However, different approaches are taken into consideration and the topic leads itself to further researches. Although there is not a clear shared conclusion to this question, results can be interpreted and it can be found out that in the majority of the paper high frequency data are not useful to improve the accuracy of VaR forecast.

Giot and Laurent (2004) find out that ARFIMAX (0,d,1) outperforms the APARCH low-frequency model in the volatility forecast. However, the performances of the two models are similar in forecasting VaR and the main conclusion of their work is that the assumption of skewed Student-t distribution of returns implemented in the models makes those models more performant in comparison with the same models based on normal distribution assumptions. This finding will influence other authors such as So and Xu (2013). Analyzing the Hong Kong stock index, they compare the GARCH model taking into

[48]Cf. So and Xu, 2013, pp. 98-99.

consideration asymmetry, seasonality characteristic of intraday 30 minutes time intervals and in addition, the aggregation of RV, in order to understand which of those three variables could add worth to the VaR forecasting framework. Their work, which aims to be applied for markets participants that trade frequently, shows that the Student-t GARCH with seasonal index and additional term of RV is the best model for the volatility forecast. Hence, all the three component added to the classical GARCH helps to make a more precise volatility prediction. Nevertheless, the simple student t-GARCH with seasonal index model is the most performant in the 30 minutes-ahead VaR forecast. Therefore, RV does not provide an improvement for the achievement of this task. The results obtained by Będowska-Sójka (2015) lead to a similar conclusion: there is no evidence that RV models should be used for this goal. In particular, the three loss functions represented are individually optimized by different kinds of models, some built with daily measures and some built with intraday measures.

Shao et al. (2009) come to a different conclusion. Indeed, their paper demonstrate that CARR model and ARFIMA (*0, d, 0*) reach a similar improvement of the accuracy for VaR prediction in comparison with low frequency models. Thus, in this case the informative power of intraday data can improve the forecast. A Similar result is achieved by Huang and Lee (2013). Averaging methods based on intraday information clearly provide useful information for the calculation of the quantile and, above all, for the estimation of the downside risk. So the high frequency information are beneficial for this methodology. In Table (9) are summed up some information about the 5 papers analyzed.

Research	dataset	Methodology	Results
Giot eand Laurent (2004)	CAC40 stock index (from 1995 to1999) S&P500 futures index (from January 1989 to December 2000) YEN-USD and DEM-USD (from January 1989 to February 2001)	Comparison between model of RV (ARFIMAX(0,d,1)) and APARCH, model with daily data. Volatility and VaR prediction. The sampling frequency 15-minute for stock frequency and a 1-hour exchange rates. In both cases Student-t distribution of returns is taken into	While RV model outperforms the APARCH model in volatility forecasting, the VaR specification based on ARFIMAX (0,d,1) does not improve one-day-ahead VaR forecasting.
Shao et al. (2009)	Shanghai Composite Index (SHCI) and Shenzhen Component Index (SZCI) 5-min frequency data from June 2th 2005 to November 21st 2007. 598 trading days	One day-ahead-VaR forecast in the CARR (Conditional Autoregressive Range) framework and ARFIMA (0,d,0) in which high frequency data are used. Daily IGARCH, tGARCH, APARCH are utilized for the comparison.	VaR forecasting performances of the intraday models are better than the traditional volatility models based on the daily data. High frequency data can improve VaR forecast.
Huang e Lee (2013)	S&P 500 stock index (from 9 June, 1997 to 30 May, 2003) 5 minutes sampling frequency	Incorporating intraday data to forecast daily quantiles. Direct and indirect approach (Subsample averaging, bootstrap averaging, forecast averaging meth ods).	Using High-frequency information to forecast VaR downside risk is beneficial. Averaging methods which use high-frequency data have better performances than models that incorporate only low-frequency information.
So and Xu (2013)	HIS (Hang Seng stock Index's) (from March 25th 2008 to May 31st 2009), volatile period of the subprime financial crisis	Forecasting VaR and volatility with a shorter than one day investment horizon. GARCH models incorporate RV based on 5 minutes returns Daily seasonality and student –t distributions are implemented in the models.	Student t GARCH model with seasonal index and an additional RV improve volatility forecast but not IVaR forecast. Student t GARCH models with seasonal index and time-varying degrees of freedom performs better in IVaR forecast
Będowska-Sójka (2015)	WIG20 (20 blue chips Warsaw Stock exchange) (from April 4th 2007 to April 21st 2011) 5 minute sampling frequency, Crisis period	Calculation of One-day-ahead forecast of VaR using dailyGARCH class models and models based on RV (HAR, HAR-J and ARFIMA). The considered loss function are : the Binomial loss function, Regulatory loss function and Firm loss function	VaR forecast from daily data performs better than those relied on higher frequency for binomial and regulatory loss function . For the firms loss function ARFIMA is the best model. There is not an approach that clearly outperforms the other.

Table 9: Summary of High-Frequency VaR Forecast Literature Review
Source: Author's elaboration

Chapter 4. Forecasting Value at Risk: an empirical analysis

The goal of this chapter is turning into practice the main evidence brought to the attention during this work, with the focus on the exploitation of the informative power of high frequency data. In particular, the attention is drawn to different lengths of the estimation window and to the comparison of the estimates achieved from data diffent in terms of sampling frequency. These features are applied for a one-day-ahead forecast using the HAR-RV model. Some further widespread literature practices are reported and enforced for the prediction of VaR. The time series used in the study is a currency spot rate series, the EUR/UDS spot rate.

4.1. Stylized facts of high-frequency returns and Realized Variance measures

Before starting with the analysis of the data and with the empirical application, it could be useful recaping some properties and characteristics of returns and some measures of volatility characterizing high frequency time series:

a) Working with intraday returns modifies the results of the descriptive analysis: the mean becomes approximately equal to zero. ACF declines very slowly, while returns are more independent in time looking at PACF. Kurtosis of returns distribution is more marked.

b) A period of the estimation relatively short can contain enough information for a significant estimation. For example, an exchange rate dataset of one week five minute returns counts 1440 observations. The same number of daily observations can be gathered taking into consideration more than five years.

c) Due to microstructure effects, a volatility measure could contain noise when it is built on very short timeframes. In this context, the higher the liquidity of an asset, the shorter can be the timeframe on which volatility measure can be derived.

d) The RV achieved from intraday data is often lower than daily variance calculated with daily squared returns.

e) It is possible to detect seasonality components for both, low inter-daily and high intra-daily frequencies. Some findings in the literature proved the presence of time-of-the-day patterns and day-of-the-week patterns in the volatility. A U shape pattern can be detected for both timeframe. Moreover, a macroeconimic announcements can affect volatility in the short run[49]

f) The ex-post standardized returns have an almost normal distribution (see Andersen et al. (2003) or Andersen et al. (2001)) while the ex-ante standardized returns, i.e. returns standardized by the square root of the one-day-ahead forecast of the daily realized, are not normally distributed [50].

4.2. Description of the time series and dataset

The EUR/USD exchange rate is the most traded currency in the FOREX market with a market share of 23.1%[51] during the 2016. The FOREX market was registering in that period around $5.1 trillion per day. These numbers made the FOREX the most liquid market in the world. The currencies can be traded continuously, 24 hours per 5 days for week and they do not a have a central location where they are traded. For these reason the EUR/USD exchange rate is considered one of the most efficient assets. The exchange rate series, as the other financial series, shows some stylized facts. Non-stationarity, leptokurtosis, fat tails and skewness of the returns' distribution are characteristics of exchange rate series and of most financial series. In particular, currencies affected by different economic policies tend to have a more skewed distribution of returns.[52] Volatility clustering and heteroscedasticity are features widely observed with regard to variability of financial series. A largely utilized trading strategy in the FOREX market, the momentum strategy, consist in keeping a long position when the asset is well performing and keeping a short position when the asset is bad performing. In fact, relatively long trend in the

[49]Cf Bollerslev et al., 2000, pp. 40-48
[50]Cf. Laurent and Giot, 2004, pp.389-390.
[51]https://www.stofs.com/en/newsroom/entry/GENERAL/the-most-traded-currency-pairs-in-the-forex
[52]Cf. Vries and Leuven, 1994., p. 15.

FOREX market has been observed[53]. Furthermore, the EUR/USD is considered a high volatility asset due to its high liquidity.

The dataset of the analysis includes four years five minutes returns of the EUR/USD spot rate from January 2, 2013 to December 31, 2017. The first four years are considered for the estimation of the models' parameters, while the last year is taken into account as out-of-sample period for the forecast. The five minute returns are extrapolated from hisdata.com one minute returns files[54]. The choice of selecting five minute returns is suggested, inter alia, by Andersen and Bollerslev (1998) and it is taken as a good proxy to combine the aim of exploiting higher informative power of high frequency data with a reasonable non-noisy estimate. Moreover, since the asset is very liquid, this sampling frequency should not contain a huge amount of noise (see stylized fact at point c). The estimation sample from 2st January 2013 to 31st December 2016 contains 297,659 observations. Looking at Figure (11) and Table (10) it is possible to notice some main features of the series during the estimation period. The two windows differ with respect to some important aspects. First of all, the second period is more tumultuous than the first due to a drop of the price at the confluence of the end of the 2014 and the beginning of the 2015. This becomes evident looking at Figure (12), where it is possible to see the increasing fluctuation starting from January 2015 and reaching a peak in July 2015. The main drivers of this drop were the decision of European Central Bank to keep on with quantitative easing policy and the perspective of a Federal Reserve rate hike. As a consequence, these opposite policies probably increases the skewness of the period, which was close to zero during the biennium 2013-2014. The series is evidently non-stationary as confirmed by the Dickey-Fuller Test with the acceptance of the null hypothesis, hence, the presence of a unit root. Furthermore, by means of the Engle ARCH test, the presence of heteroscedasticity effects in the series is ascertained. Lastly, the Autocorrelation function (ACF) and the Partial Autocorrelation function (PACF) graphs (figure 13) show a persistence in the price development and the non-correlation of returns. Furthermore, those descriptive statistics, with regard to intraday returns, confirms the fairness of the stylized fact at point a).

[53] Cf. Bayas, 2018.,p.19.
[54] http://www.histdata.com/download-free-forex-data/

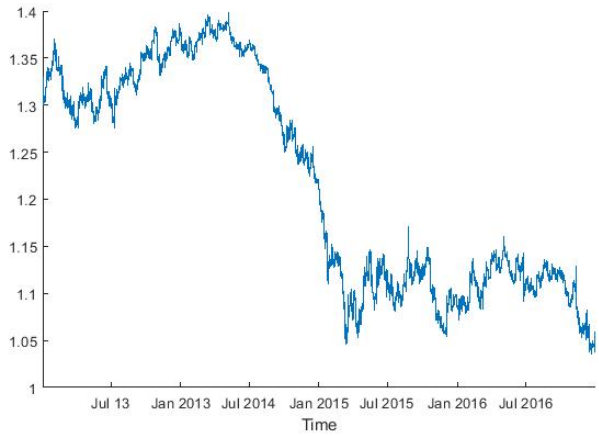

Figure 11: Daily closing price of EUR/USD spot rate during the estimation period

Source: Author's elaboration

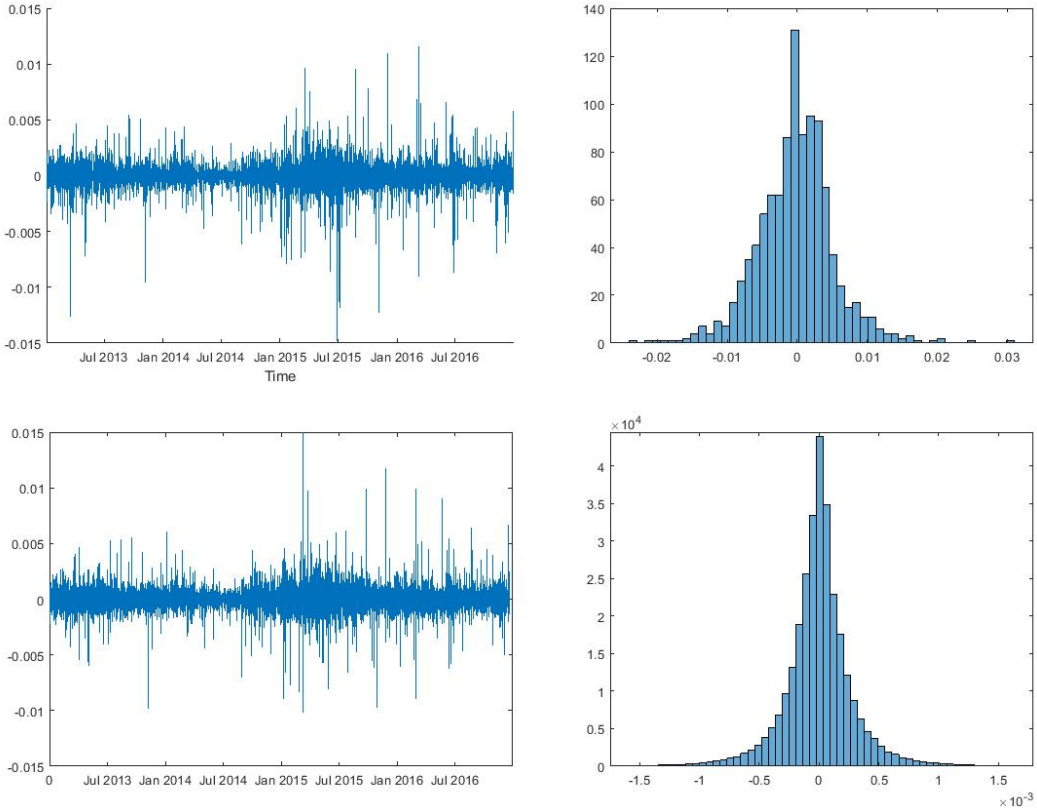

Figure 12: Plots of daily returns and returns distributions during the whole estimation period. On the top part are represented the plots based on daily returns. On the bottom part are represented the plots based on 5 minute returns

Source: Author's elaboration

	From 2013 to 2014	From 2015 to 2016	From 2013 to 2016
Mean	-0.0002 (0.0000)	-0.0002 (0.0000)	-0.0002 (0.0000)
Standard Dev.	0.0042 (0.0003)	0.0065 (0.0004)	0.0055 (0.0003)
Min	-0.0157 (-0.0098)	-0.0238 (-0.0102)	- 0.0238 (-0.0102)
Max	0.0158 (0.0061)	0.0307 (0.0185)	0.0307 (0.0185)
Skewness	0.0262 (-0.4408)	0.2188 (0.8635)	0.1402 (0.5981)
Kurtosis	4.1907 (51.2787)	4.9550 (80.6896)	5.5697 (87.0629)

Table 10: Main descriptive statistics of the EUR/USD spot returns for the three estimation windows. Daily returns statistics are presented above while 5 minute returns statistics are in brackets.

Source: Author's elaboration

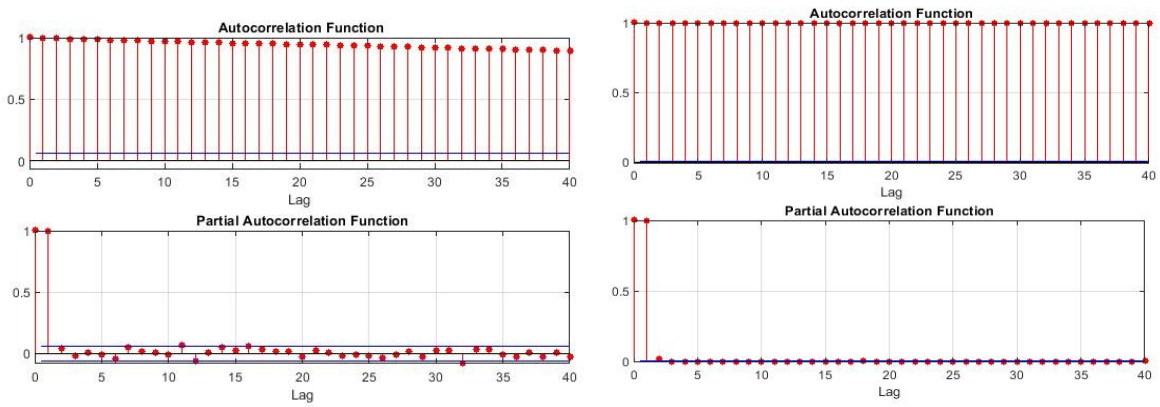

Figure 13:Price autocorrelation and Partial Autocorrelation functions for the whole estimation window. On the left are described the processes based on daily Returns.On the right are described the processes based on 5 minute returns

Source: Author's elaboration

4.3. Estimation of the model

One of the purpose of the research is to try to shorten as much as possible the estimation window using high-frequency data, achieving at the same time significant parameters and an accurate VaR prediction. For this reason, the estimation of HAR-RV has been implemented for three different windows: for the first two years (from 2nd January 2013 to 31st December 2014) , for the last two years (from 2nd January 2015 to 31st December 2016) and for the whole period. Given the features of the examined windows, it could be interesting to investigate if a more tumultuous period affects the goodness of the estimation. Previously, due to a comparative purpose, a preliminary esteem of the model is carried out through the implementation of daily data. Only the estimation windows that gives significant result will be applied for the one-day-ahead VaR forecast.

The methodology used to estimate the model differs from the work of Corsi (2008) in regard to the step length of the model components. Corsi (2008) uses a two-hour time step for the calculation of the volatility measures, in order to achieve a more accurate evaluation. Due to the final purpose of the analysis, the one day ahead forecast of VaR, I consider the choice of the one day step length for the model valuation as the simplest and enough accurate way of proceeding. The estimate model appears in this way:

$$\sigma_{t+1day}^{d} = c + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{w} + \beta^{(m)} RV_t^{m} + \omega_{t+1day}^{d} \qquad (29)$$

Formula (29) The three components of volatility are updated every day. For the model based on daily data, the daily volatility measure is given by the standard deviation between the observad prices at day t and day t-1 :

$$\sigma_d = \sqrt{\frac{(x_t - x_{t-1})^2}{2}}$$

For high frequency models, the daily realized volatility is simple the squared root of the sum of the five minute squared log returns, as indicated previously in formula (20):

$$RV_t^{(d)} = \sqrt{\sum_{j=0}^{M-1} r_{t-j\cdot\Delta}^2}.$$

The weekly and monthly RV observations are compounded through a moving average of, respectively, 5 and 22 days of the daily RV.

The 2013-2014 window contains 497 RV observations, while the 2015-2016 window contains 519 RV observations. The first window is shorter because the first month has been used to calculate the first monthly RV component. In Figure (14), it is possible to notice the different persistence of the components calculated for the model. It is also interesting to observe figure (15), where it is possible to discern a day-of-the-week seasonality pattern in ACF and PACF of the RV, supporting stylized fact at point e).

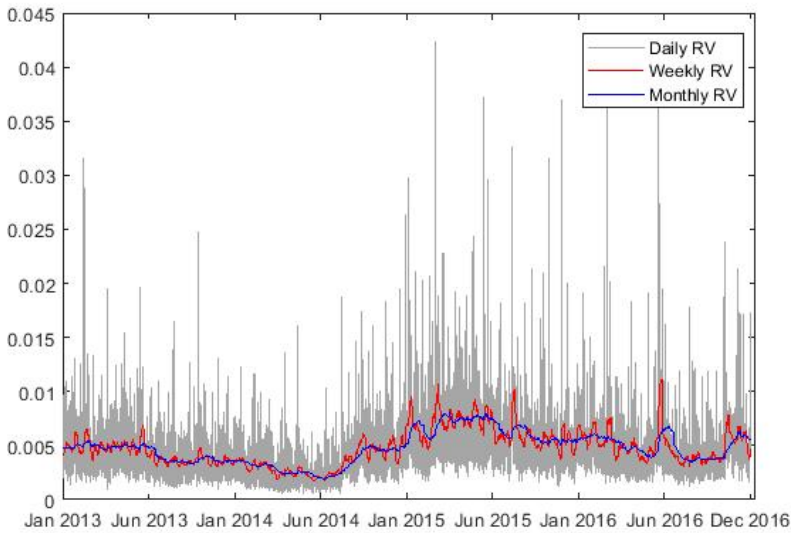

Figure 14: Development of the three different observed RV components during the whole estimation period

Source: Author's elaboration

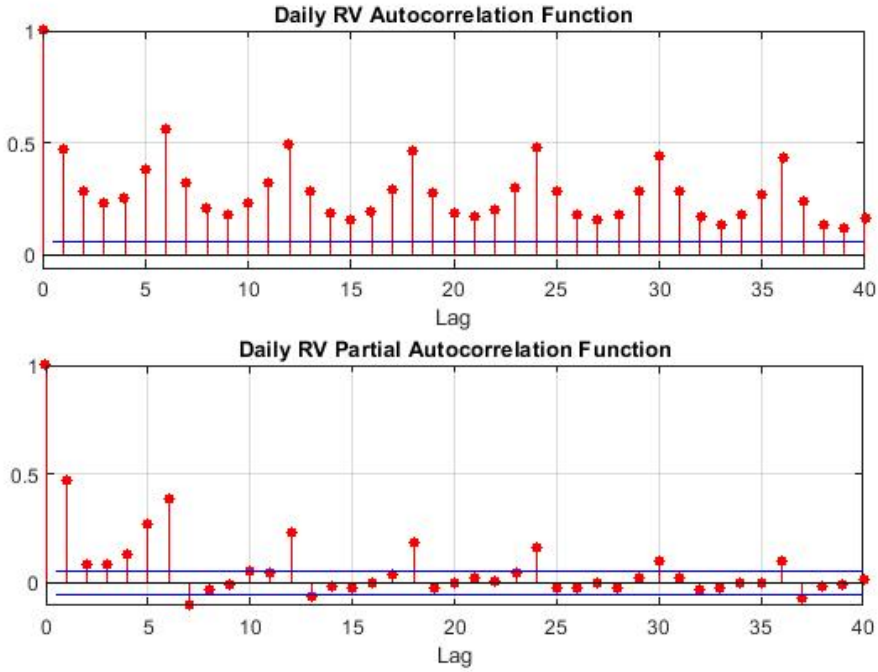

Figure 15: Autocorrelation and Partial Autocorrelation functions on daily RV during the whole estimation window

Source: Author's elaboration

4.3.1.Daily model estimation

The estimation on daily data is carried out only for the whole estimation sample. A shorter period will not certainly provide a good esteem. The results of the model estimation based on daily data is summed up in Table(11) below. As expected, the results are not significant, in detail, the daily component suffers from significance issue. The model is not suited to incorporate low frequency data, in fact, there are not so many evidence in literature about daily volatility HAR model. For this kind of data, it could be reasonable to apply a GARCH family model. Moreover, the model would explain only a minimal part of the volatility movement, in this case around the 6%.

```
Estimated Coefficients:
                   Estimate        SE         tStat       pValue
                   ________    __________    ______    __________

    (Intercept)    0.0012395    0.00029725    4.1699    3.3071e-05
    x1              0.050971      0.036027    1.4148       0.15744
    x2               0.19824       0.08095    2.4489      0.014495
    x3               0.38339       0.10925    3.5091    0.00046921


Number of observations: 1022, Error degrees of freedom: 1018
Root Mean Squared Error: 0.00296
R-squared: 0.0635,  Adjusted R-Squared 0.0607
F-statistic vs. constant model: 23, p-value = 2.07e-14
```

Table 11: Results of HAR estimation on daily data. Period of the estimation from 2013 to 2016.

Source: Author's elaboration

4.3.2. High Frequency data models estimation

In this section the estimation results of the three windows considered for the high frequency HAR model are presented. In order, the evaluation of 2013-2014 window, of 2015-2016 window and of the whole estimation window are explained. Table (12) below sums up the statistics of the estimation for the first window, the smoothest period of the sample. The esteem suffers from significance issues as regards the weekly component and the constant term. In this case, a relatively short estimation window with one day frequency step does not give an acceptable result. I proceed with the estimation of the 2015-2016 window to evaluate if it is possible to keep on working with short estimation timeframes.

```
Estimated Coefficients:
                   Estimate        SE         tStat       pValue
                   _________    __________    ______    __________

    (Intercept)    0.0003613    0.00022431    1.6107       0.10789
    x1               0.29483      0.050546    5.8328    9.8731e-09
    x2              0.063632       0.10418    0.6108       0.54161
    x3               0.55397       0.10602    5.2251     2.574e-07


Number of observations: 497, Error degrees of freedom: 493
Root Mean Squared Error: 0.00119
R-squared: 0.415,  Adjusted R-Squared 0.412
F-statistic vs. constant model: 117, p-value = 4.01e-57
```

Table 12: Results of HAR estimation on high-frequency data. Period of the estimation from 2013 to 2014

Source: Author's elaboration

Table(13) below summarizes the results of the 2015-2016 window, the most tumultuous timeframe. As opposed to the previous table, the parameters derived by the model are all significant. The weekly component is the most influencing parameter in the estimation. Instead, the daily component seems to produce the weakest effect. The model is able to explain around a quarter of the volatility process. Given these considerations, the model will be applied for the VaR forecast. From here on out, this model will be mentioned as "Window 1" model.

```
Estimated Coefficients:
                   Estimate        SE         tStat       pValue
                   _________    __________    ______    __________

    (Intercept)    0.0013174    0.00047462    2.7758     0.0057075
    x1               0.19675      0.051504    3.8201     0.0001497
    x2               0.33997      0.095504    3.5598    0.00040552
    x3                0.2532       0.10592    2.3906      0.017179


Number of observations: 519, Error degrees of freedom: 515
Root Mean Squared Error: 0.00228
R-squared: 0.237,  Adjusted R-Squared 0.232
F-statistic vs. constant model: 53.2, p-value = 5.45e-30
```

Table 13: Results of HAR estimation on high frequency data. Period of the estimation from 2015 to 2016

Source: Author's elaboration

Lastly, Table(14) presents the results of the estimation during the whole sample. Parameters are all significant but, the values are very different from those of the Window 1 model. In this case, the highest parameters is the daily one, while in the Window 1 estimation it was the lowest. The influence of the parameters is decreasing in terms of length of the component. The wider is the volatility component, the lower is its weight on the model. It seems that a more volatile period could be more affected by long term trading purpose. The R-squared is enhanced, probably due to the longer sample. It reaches one-third of explanatory power. The RMSE indicates that this model is slightly less accurate than Window 1 model. I consider the model suited for the VaR forecast. From here on out, the model will be mentioned as "Window 2". Finally, in Figure (16) are depicted the in sample volatility forecast behaviors for Window 1 and Window 2 models. The models are compared with the actual volatility during the years 2015 and 2016. It is difficult to discern differences between the two models, they look very similar.

Estimated Coefficients:

	Estimate	SE	tStat	pValue
(Intercept)	0.00019697	9.0098e-05	2.1861	0.028825
x1	0.43838	0.0082493	53.141	0
x2	0.35135	0.034377	10.22	2.0156e-24
x3	0.16832	0.037884	4.4431	8.9453e-06

Number of observations: 12151, Error degrees of freedom: 12147
Root Mean Squared Error: 0.00289
R-squared: 0.322, Adjusted R-Squared 0.321
F-statistic vs. constant model: 1.92e+03, p-value = 0

Table 14: Results of HAR estimation on high-frequency data. Period of the estimation from 2013 to 2016

Source: Author's elaboration

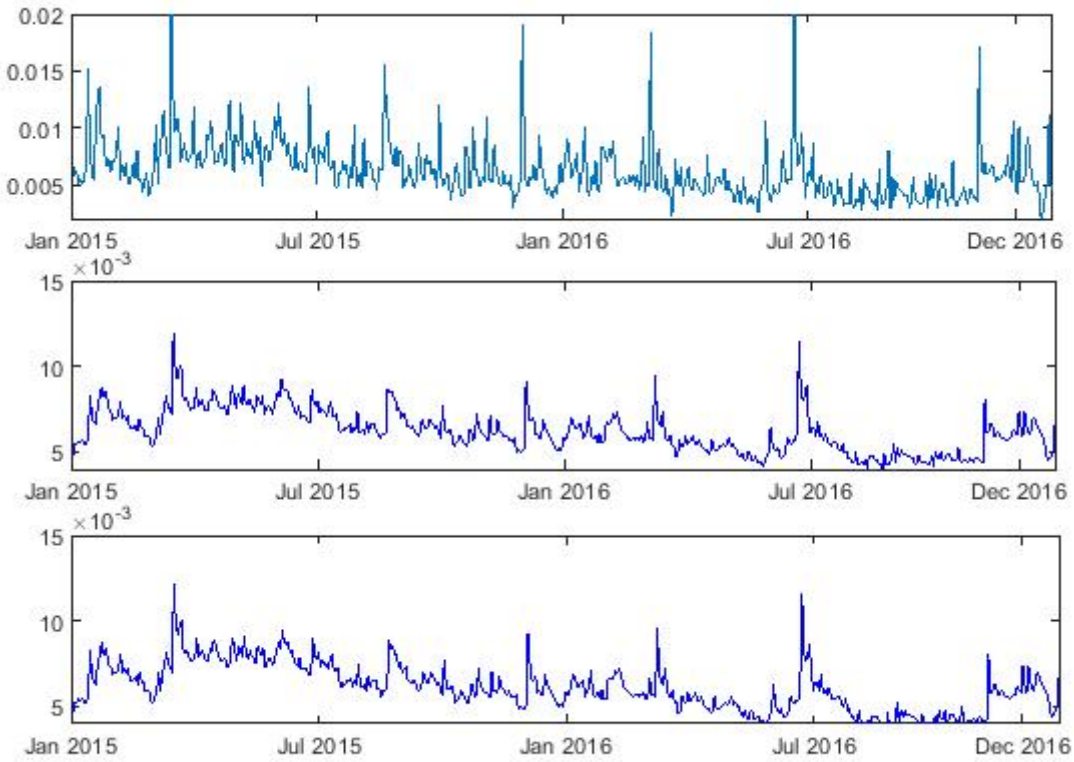

Figure 16: Representation of the in sample volatility prediction from 2015 to 2016.

Top to bottom: Actual RV, One day ahead volatility prediction with Window 1 model, One day ahead volatility prediction with Window 2 model.

Source: Author's elaboration

4.4. Out-of-sample forecast

To recap, in the previous section, I try to estimate the model utilizing different kind of data and different windows. As expected the daily model is not significant, such as the high frequency model estimated for biennium 2013-2014. The significant high frequency estimates achieved from biennium 2015-2016 ("Window 1") and the whole sample ("Window 2") are used to carry out the one-day-ahead volatility forecast during the year 2017. Figure (17) represents the ex-post price development and the returns' path. The period seems to be less volatile than the previous years. Moreover, the period is clearly non-stationary and it shows a positive trend.

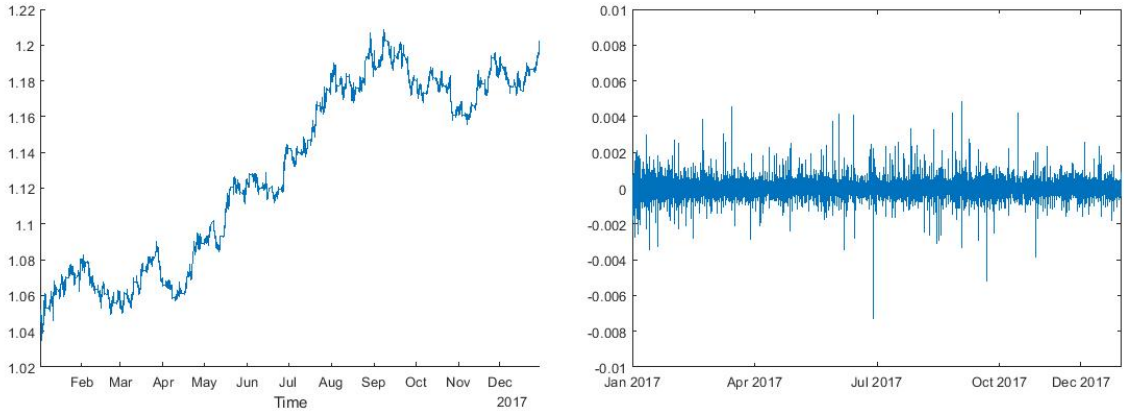

Figure 17: Out-of-sample price (left) and returns development (right)

Source: Author's elaboration

Table(15) below shows the results of the forecast. The results comprehend 9 different measures of loss, the quasi-likelihood estimate, the R-squared and the success ratio[55]. The Window 2 model minimize 7 of the 9 loss function implemented, while for HMSE loss function Window 1 model has a lower error. Window 2 model performs better also for the Quasi-Likelihood, but only slightly. However, the R-squared is higher for Window 1 model. In Figures (18) and (19) the results of the two forecast and their residuals are, respectively, depicted. Window 1 model seems to be slightly smoother than Window 2 model.

```
-------------------------------------------------
Volatility Loss Functions
-------------------------------------------------

 MSE:   1: 0.0000,   2: 0.0027
 MAD:   1: 0.0010,   2: 0.0071
 MLAE:  1: -7.2889,  2: -5.2842
 HMSE:  0.0606
 R2LOG: 0.0662
 QLIKE: -4.4215
 SR:    1.0000
```

RMSE	0.2573
R-squared	0.1470

```
-------------------------------------------------
Volatility Loss Functions
-------------------------------------------------

 MSE:   1: 0.0000,   2: 0.0002
 MAD:   1: 0.0009,   2: 0.0066
 MLAE:  1: -7.4583,  2: -5.4351
 HMSE:  0.0714
 R2LOG: 0.0624
 QLIKE: -4.4212
 SR:    1.0000
```

RMSE	0.2498
R-squared	0.1381

Table 15:Volatility loss functions for the one-day-ahead out-of-sample forecast. On the left side the Window 1 model. On the right side the Window 2 model

Source: Author's elaboration

[55] Volatility loss functions are performed through the utilization of "Volatility Loss Functions and VaR Conditional, Independence and Regulatory BackTests" developed by Alexandros Gabrielsen (2011)

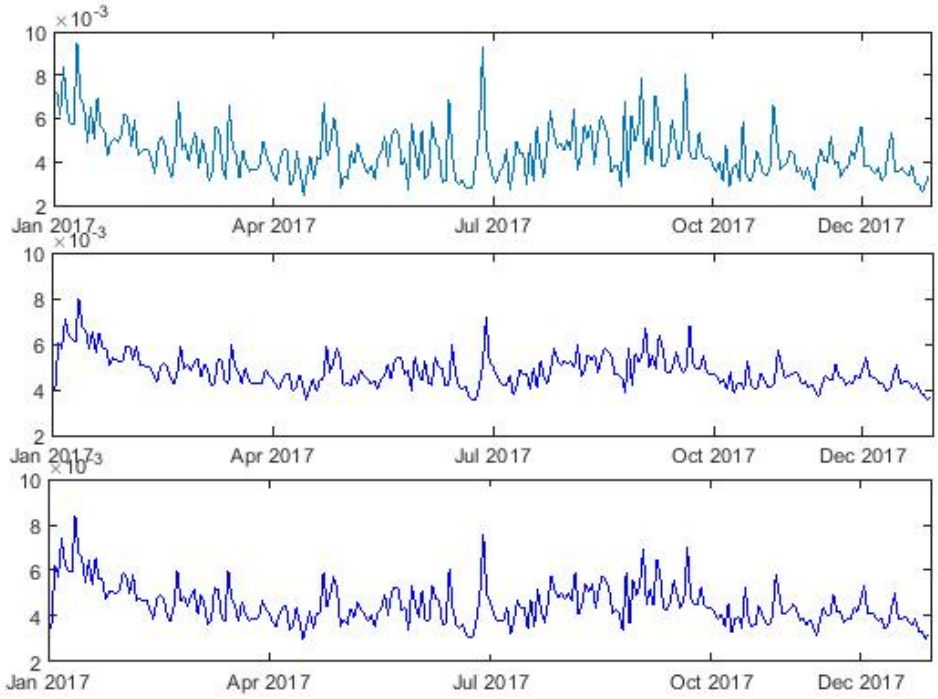

Figure 18: Representation of the out-of-sample volatility forecast from 2015 to 2016.

Top to bottom: Actual future RV, One day ahead volatility prediction through Window 1 model, One day ahead volatility prediction through Window 2 model.

Source: Author's elaboration

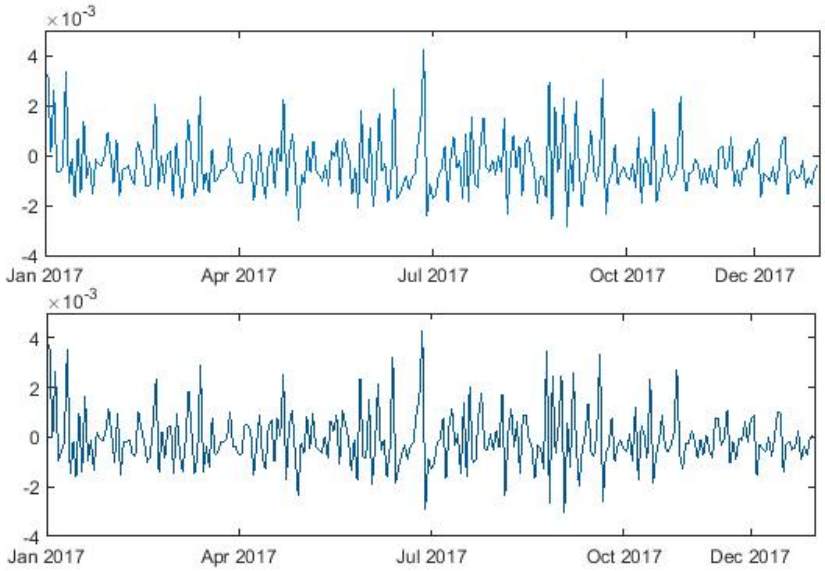

Figure 19: Residuals of Window 1 forecast (top) and Window 2 forecast (bottom)

Source: Author's elaboration

4.5. Forecasting Value at Risk

Although, the model estimated on the whole period seems to give the most accurate forecast, it should be evaluated the model that describes better the risk behavior of volatility. In order to carry out a VaR forecast using the HAR model, a last step should be applied. In particular, it is sufficient to incorporate the one-step-ahead RV measure of Formula (20) into the VaR function of Formula (30):

$$\mathrm{VaR}^{d}_{t+1d} = \mu_d + \sqrt{\mathrm{RV}^{(d)}_{t+1d}}\, z^{-1}_{\tilde{\alpha}} \tag{30}$$

In formula (30), the distribution of one-day-ahead RV is assumed to be Normal with z_α left quantile of normal distribution. However, realized volatility measures must be combined with reliable density distributions. In details, the normal distribution assumption, for a volatility measure computed ex-ante, could give an inaccurate representation. In fact, while the distribution of ex-post standardize returns are nearly Gaussian Andersen et all. (2003), the distribution of the returns of the observed one-day-ahead RV is uncertain[56] . For this reason it can be taken into account the Kurtosis and skewness of returns of the ex-post RV measure, considering respectively Student-t and Student-t with skewness as a proper proxy for the distribution's assumption.[57]

	Long position	Short Position
VaR with normal Distribution	$\mu_d + \sqrt{\mathrm{RV}^{(d)}_{t+1d}}\, z^{-1}_{\tilde{\alpha}}$	$\mu_d + \sqrt{\mathrm{RV}^{(d)}_{t+1d}}\, z^{-1}_{1-\tilde{\alpha}}$
VaR with t-student distribution	$\mu_d + \sqrt{\mathrm{RV}^{(d)}_{t+1d}}\, t^{-1}_{\tilde{\alpha},v}$	$\mu_d + \sqrt{\mathrm{RV}^{(d)}_{t+1d}}\, t^{-1}_{1-\tilde{\alpha},v}$
VaR with skewed t-student distribution	$\mu_d + \sqrt{\mathrm{RV}^{(d)}_{t+1d}}\, t^{-1}_{\tilde{\alpha},v,\xi}$	$\mu_d + \sqrt{\mathrm{RV}^{(d)}_{t+1d}}\, t^{-1}_{1-\tilde{\alpha},v,\xi}$

Table 16: VaR parametric methodologies for long and short positions with different distributions.

Source: Author's elaboration according to Giot and Laurent (2004)

[56] Cf. Giot and Laurent, 2004, pp. 389-390.
[57] Cf. Giot and Laurent, 2004, p. 384.

In table (16) v are the degrees of freedom and ξ the asymmetry coefficient. In the table are considered a long and a short position. Having a long position means that you have bought and own those assets. Conversely, having a short position means that you owe those assets to someone, but you do not actually own them yet. If the asymmetry coefficient is higher than one, the VaR for the trading long position is higher than the value of the VaR for the short position and vice versa[58]. Regulatory frameworks practice makes the VaR assessment over one day length period. I compute the VaR maintaining this periodicity utilizing the widespread parametric approach of Laurent and Giot (2004) previously considered. The procedure does not take into consideration day-of-the-week seasonality effects. The results can be evaluated considering this issue.

The loss function considered in the assessment are the percentage of failure, the Time Until First Failure (TUFF), the likelihood ratio of unconditional coverage (or Kupiec test), likelihood ratio of independence coverage, the likelihood ratio of conditional coverage (Christoffersen's Interval Forecast Test) and the traffic light coverage test (Basel II accord test). The Kupiec test measures the percentage of failure, but ignores the time dependent dynamics of the exceptions, it measures if the number of exceptions is reasonable or not. The likelihood of independence coverage gauges if the VaR exceptions observed at two different dates are independently distributed. The presence of a cluster of violations compromises the validity of the assessment. Christoffersen's Interval test[59] connects the validity of the VaR forecast on both unconditional coverage hypothesis and independence hypothesis. These last three backtest are the most important to consider in the analysis. In addition, the percentage of failure loss function considers if models in the analysis underestimate or overestimate the risk.

Both tails of distribution are assessed through the parametric approach indicated above. The first evaluation concerns normal distribution assumption of the ex-post volatility measure. The examination is divided into long position and short position assessment.

In Table (17), the results of left tail are listed. It is possible to notice that the two models gives the same results, considering also the accuracy values of the tests for the 1% confidence level with only one violation over 258 returns. The 2 models are violated at the same point, after 213 days in November 2017. The Kupiec test demonstrates that both models give an accurate description of the violations, in particular for the 1% confidence

[58]Cf. Giot and Laurent, 2004, pp. 389-390.
[59] Christoffersen, 1998.

level. A positive results is achieved also for the independence coverage test which proves that there is no dependency between violations. Hence, in this case, day-of-the-week seasonality patterns do not influence the goodness of the assessment. Accordingly, the conditional coverage test confirms the accuracy of the forecast. The percentage of failure indicates that both models overestimate risk. The overestimation of the Window 1 model is higher for the 5% Long position.

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 0.39
The first violation is observed after 213 periods
According to BASEL II the model is in the Green zone
-------------------------------------------------
LRTest      Value      chi-square(0.95)      p-value
-------------------------------------------------
 TUFF       0.7538          3.841            0.385
 Uncond.    1.2742          3.841            0.259
 Indep.     0.0014          3.841            0.970
 Cond.      1.2756          5.991            0.528
-------------------------------------------------
```

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 0.39
The first violation is observed after 213 periods
According to BASEL II the model is in the Green zone
-------------------------------------------------
LRTest      Value      chi-square(0.95)      p-value
-------------------------------------------------
 TUFF       0.7538          3.841            0.385
 Uncond.    1.2742          3.841            0.259
 Indep.     0.0014          3.841            0.970
 Cond.      1.2756          5.991            0.528
-------------------------------------------------
```

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 2.71
The first violation is observed after 36 periods
According to BASEL II the model is in the Yellow zone
-------------------------------------------------
LRTest      Value      chi-square(0.95)      p-value
-------------------------------------------------
 TUFF       0.4430          3.841            0.506
 Uncond.    3.3825          3.841            0.066
 Indep.     0.0000          3.841            1.000
 Cond.      3.3825          5.991            0.184
-------------------------------------------------
```

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 3.49
The first violation is observed after 36 periods
According to BASEL II the model is in the Red zone
-------------------------------------------------
LRTest      Value      chi-square(0.95)      p-value
-------------------------------------------------
 TUFF       0.4430          3.841            0.506
 Uncond.    1.3817          3.841            0.240
 Indep.     0.0000          3.841            1.000
 Cond.      1.3817          5.991            0.501
-------------------------------------------------
```

Table 17: Long position results of the one-step-ahead VaR forecast using normal specifications.

On the left side are described VaR backtests for Window 1 model, while on the right side the Window 2 model . On the top are represented VaR backtests for 1% confidence level , while on the bottom the VaR backtests with 5% confidence level.

Source: Author's elaboration [60]

The results of the short position assessment, instead, are presented in Table (18) below. Given the positive trend in Figure (17), we expect to have more issues on the evaluation of the right tail of distribution. The Unconditional coverage test fails only under one circumstance, the 1% confidence for Window 2 model. However, the independence coverage test says that there are no clusters of violations. The model that fails in the unconditional test is also inaccurate for the conditional coverage test. The results of the

[60] VaR backtest statistics are performed through the utilization of "Volatility Loss Functions and VaR Conditional, Independence and Regulatory BackTests" developed by Alexandros Gabrielsen (2011)

percentage of failures shows that the two models underestimate the risk for both slices of the tail, but the Window 1 model seems to evaluate better the risk. Looking at Figure (20) below, it is possible to observe the points on which the exception came up. The main remark that can be drawn from the graph is that in all the cases for the two different positions and for the two confidence levels, the model estimated on the longer period tends to give a less conservative assessment of risk.

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 2.33
The first violation is observed after 52 periods
According to BASEL II the model is in the Yellow zone
-------------------------------------------------
LRTest      Value      chi-square(0.95)      p-value
-------------------------------------------------
 TUFF       0.3523          3.841            0.553
 Uncond.    3.3336          3.841            0.068
 Indep.     0.0000          3.841            1.000
 Cond.      3.3336          5.991            0.189
-------------------------------------------------
```

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 3.49
The first violation is observed after 44 periods
According to BASEL II the model is in the Red zone
-------------------------------------------------
LRTest      Value      chi-square(0.95)      p-value
-------------------------------------------------
 TUFF       0.5292          3.841            0.467
 Uncond.    9.813           3.841            0.002
 Indep.     0.0000          3.841            1.000
 Cond.      9.813           5.991            0.007
-------------------------------------------------
```

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 6.20
The first violation is observed after  3 periods
According to BASEL II the model is in the Red zone
-------------------------------------------------
LRTest      Value      chi-square(0.95)      p-value
-------------------------------------------------
 TUFF       2.3776          3.841            0.123
 Uncond.    0.7309          3.841            0.393
 Indep.     0.0000          3.841            1.000
 Cond.      0.7309          5.991            0.694
-------------------------------------------------
```

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 7.75
The first violation is observed after  3 periods
According to BASEL II the model is in the Red zone
-------------------------------------------------
LRTest      Value      chi-square(0.95)      p-value
-------------------------------------------------
 TUFF       2.3776          3.841            0.123
 Uncond.    3.5479          3.841            0.060
 Indep.     0.0000          3.841            1.000
 Cond.      3.5479          5.991            0.170
-------------------------------------------------
```

Table 18: Short position results of the one-step-ahead VaR forecast using normal specifications.

On the left side are described VaR backtests for Window 1 model while on the right side the window 2 model . On the top side are represented VaR backtests for 1% confidence level while on the bottom side the VaR backtest with 5% confidence level.

Source: Author's elaboration[61]

[61] VaR backtest statistics are performed through the utilization of "Volatility Loss Functions and VaR Conditional, Independence and Regulatory BackTests" developed by Alexandros Gabrielsen (2011)

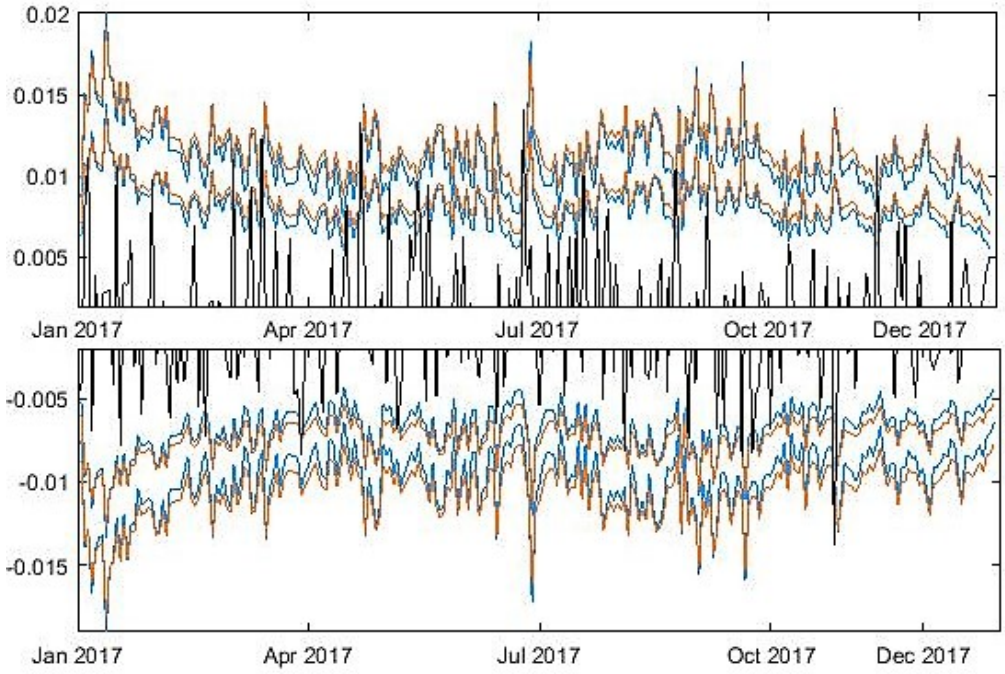

Figure 20: Graphic description of VaR violations for short position (top) and long position (bottom) considering normal assumption distribution. Blue lines represent the VaR forecast of Window 2 model while orange lines the VaR forecast of Window 1 model.

Source: Author's elaboration

I look further to test if Student-t parametric assumptions are able to improve the forecast argued by Laurent and Giot (2004). Through the Kolgomorov-Smirnov test, I tried to find the Student-t probability distribution curve that fits at best the probability distribution of the returns on the whole estimation period. Under the null Hypothesis, the test says that the probability distribution of the sample is drawn from the baseline distribution. The rejection of the null hypothesis was found for the Student-t curves considered (from 2 to 12 degrees of freedom). The slight positive skewness of returns in Table (10) at the beginning of the chapter may make this task more difficult to accomplish. By mean of the Distribution filter on Matlab, it is obtainable a specification of the Student-t probability function that reproduce in the closest way the actual distribution of the sample. I obtained a value close to 5 degrees of freedom. I keep this value as representative of the most accurate Student-t curve for this sample.

As previously, the analysis is divided into estimation of left and right tails of distribution. On the right tail in Table (19), for both models, serious problems about the accuracy assessment has been found for the Kupiec test under the 5% confidence level. Consequently, also the Christoffersen test fails in these circumstances. Nevertheless, violations keep on being uncorrelated. In general, we can say that the two models strongly overestimate the risk.

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 0.00
The first violation is observed after 500 periods
According to BASEL II the model is in the Green zone
-------------------------------------------------
LRTest     Value     chi-square(0.95)     p-value
-------------------------------------------------
 TUFF      0.0000          3.841           1.000
 Uncond.   0.0000          3.841           1.000
 Indep.    0.0000          3.841           1.000
 Cond.     0.0000          5.991           1.000
-------------------------------------------------
```

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 0.39
The first violation is observed after 213 periods
According to BASEL II the model is in the Green zone
-------------------------------------------------
LRTest     Value     chi-square(0.95)     p-value
-------------------------------------------------
 TUFF      0.7538          3.841           0.385
 Uncond.   1.2742          3.841           0.259
 Indep.    0.0014          3.841           0.970
 Cond.     1.2756          5.991           0.528
-------------------------------------------------
```

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 0.39
The first violation is observed after 213 periods
According to BASEL II the model is in the Green zone
-------------------------------------------------
LRTest     Value     chi-square(0.95)     p-value
-------------------------------------------------
 TUFF      15.022          3.841           0.000
 Uncond.   19.254          3.841           0.000
 Indep.    0.0014          3.841           0.970
 Cond.     19.256          5.991           0.000
-------------------------------------------------
```

```
-------------------------------------------------
Back Testing Value-at-Risk
The percentage of failures is 0.78
The first violation is observed after 36 periods
According to BASEL II the model is in the Green zone
-------------------------------------------------
LRTest     Value     chi-square(0.95)     p-value
-------------------------------------------------
 TUFF      0.4430          3.841           0.506
 Uncond.   14.821          3.841           0.000
 Indep.    0.0000          3.841           0.998
 Cond.     14.821          5.991           0.001
-------------------------------------------------
```

Table 19: Long position results of the one-step-ahead VaR forecast using Student-t specifications.

On the left side are described VaR backtests for Window 1 model, while on the right side the Window 2 model . On the top are represented VaR backtests for 1% confidence level , while on the bottom the VaR backtests with 5% confidence level.

Source: Author's elaboration

For the shorts position, Table (20) depicts a different situation. Through the four evaluations, all the tests are widely accurate and the percentage of failures is close to the wanted level. Especially, Window 2 model makes a correct estimation of the risk.

In general, we can derive from these assessments built on different parametric assumption that, in this case, the Student-t specification with 5 degrees of freedom do not improve the

accuracy of the forecast: under normality condition the models underestimate considerably the risk on the right tail of distribution while, under the Student-t specification, the left tails of distribution suffer from inaccuracy issues. Given these condition, a further addition of an asymmetry component may help to improve the valuation.

Back Testing Value-at-Risk
The percentage of failures is 0.00
The first violation is observed after 500 periods
According to BASEL II the model is in the Green zone

LRTest	Value	chi-square(0.95)	p-value
TUFF	0.0000	3.841	1.000
Uncond.	0.0000	3.841	1.000
Indep.	0.0000	3.841	1.000
Cond.	0.0000	5.991	1.000

Back Testing Value-at-Risk
The percentage of failures is 0.78
The first violation is observed after 126 periods
According to BASEL II the model is in the Green zone

LRTest	Value	chi-square(0.95)	p-value
TUFF	0.0583	3.841	0.809
Uncond.	0.1427	3.841	0.706
Indep.	0.0000	3.841	0.998
Cond.	0.1428	5.991	0.931

Back Testing Value-at-Risk
The percentage of failures is 3.49
The first violation is observed after 44 periods
According to BASEL II the model is in the Red zone

LRTest	Value	chi-square(0.95)	p-value
TUFF	0.8572	3.841	0.355
Uncond.	1.3817	3.841	0.240
Indep.	0.0000	3.841	1.000
Cond.	1.3817	5.991	0.501

Back Testing Value-at-Risk
The percentage of failures is 3.88
The first violation is observed after 44 periods
According to BASEL II the model is in the Red zone

LRTest	Value	chi-square(0.95)	p-value
TUFF	0.8572	3.841	0.355
Uncond.	0.7413	3.841	0.389
Indep.	0.0000	3.841	1.000
Cond.	0.7413	5.991	0.690

Table 20: Short position results of the one-step-ahead VaR forecast using Student-t specifications.

On the left side are described VaR backtests for Window 1 model, while on the right side the Window 2 model . On the top are represented VaR backtests for 1% confidence level , while on the bottom the VaR backtests with 5% confidence level.

Source: Source: Author’s elaboration

4.6. Final Remarks of the empirical analysis

I try to employ the use of high frequency data developing around a model suited for this purpose, the HAR model of Corsi (2009), a comparison with daily volatility measures and different samples for the model estimation. One of the two estimates obtained from the shorter windows gives significant parameters and it has been compared with the model estimated on the whole sample. Although, the values of the three components are clearly different between the two estimation, the results of the goodness of the forecast are not so

distant. The model that includes more data minimizes most the loss functions that evaluate the one-step-ahead out of sample forecast. Despite this first results, the capability of the shorter model to catch risk behavior of the volatility is not clearly outperformed by the model estimated on longer dataset. Hence, in the case of this study, a 2 years of intraday 5 minute observations for an exchange rate asset like EUR/USD spot rate, a very liquid market, can be sufficient to achieve a good result for a VaR forecast. In addition, the normal parametric assumption on returns distribution gives an accurate description of the VaR assessment, even if the considered models underestimate the risk on the right tail of distribution. In this context, the Student-t specifications do not enhance the evaluation, however, adding a skewness adjustment to this parametrization could be useful to further improve the assessment. Daily-of-the-week seasonality effects do not influence the goodness in predicting VaR.

5.Conclusions

Forecasting volatility is become a task of primary importance in Risk Management. One of its possible applications is to incorporate it into a measure of risk. Throughout this work, I considered the features and potentialities of VaR. We have seen that VaR is not a coherent measure of risk but it is widely adopted and it can give an approximate evaluation of risk. The mainly advantage of the VaR is its understandability and that is easy to apply. We investigated the properties of volatility and high-frequency data features. Handling with this kind of data is difficult and it is essential to take into account the presence of microstructure effects, that could make the realized measure noisy when sampled on very short timeframe. Cleaning issues and the consideration of overnight returns are also important when we are working with intraday data. I considered some GARCH family models, including the models able to accommodate the leverage effect and long memory GARCH. The literature review indicates that the more sophisticated models in some cases are able to represents these properties especially in the tumultuous periods. However, in some cases the simple GARCH(1,1) remains a performant model. Specifications of the latent volatility factors through the usage of high-frequency data can improve the informative power of conditional volatility models. Later, I considered the most interesting approaches to model conditional volatility. I drew a particular attention on the HAR-RV of Corsi (2009), a model with a simple AR structure based on the Heterogeneous Market Hypothesis framework. Thereafter, I developed a literature review over the VaR forecast in order to find out which approaches and models are suited to represent the risk behavior of predicted volatility. There is no clear approach that performs better than the others. Moreover, Realized measures do not always improved the performance of the VaR prediction. However, when the step of the forecast is shorter than one day, it should be taken into consideration models than can represent intraday seasonality patterns. In the last Chapter, I carried out a One-day-ahead out-of-sample VaR forecast by the HAR-RV model over a relatively short dataset. The model estimated over two years gives significant parameters and it is only slightly outperformed by the model estimated over 4 years. The results of the forecast of the VaR shows that the model estimated on the larger window does not give a more accurate and performant prediction than the model estimated on a shorter timeframe. In this case, the parametric VaR with Normal distribution assumptions gives accurate predictions.

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