Modeling Exchange Rate Volatility in Central Africa (CEMAC) Using GARCH Models

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Abstract

Modeling and forecasting exchange rate volatility has important implications in a range of areas in macroeconomics and finance. A number of models have been developed in empirical finance literature to investigate this volatility across different regions and countries. This research work considers the autoregressive conditional heteroscedastic and the generalized autoregressive conditional heteroscedastic approaches in modeling daily exchange volatility between CEMAC CFA Franc and United States Dollar (XAF/USD) from 1st January 2010 to 4th January 2018. Both the symmetric (ARCH and GARCH) and the asymmetric (APARCH, GJR-GARCH and EGARCH) GARCH families of models have been taken into consideration to capture some stylized facts about exchange rate returns such as volatility clustering and leverage effect. All models are estimated using the maximum likelihood method under the assumption of several distributions of the innovation terms such as: Normal (Gaussian), Student-t and skew student-t distributions. Evaluating the models using standard information criteria (AIC and BIC) showed that the conditional volatility models are best estimated under the student-t distribution with EGARCH (1, 1) being the best fitted model. In accordance with the estimated models there is empirical evidence at some point that negative and positive shocks imply a different next period volatility of the daily XAF/USD exchange rate return. Finally, the research work concludes that the exchange rates volatility can be adequately modeled by the class of GARCH models.

Keywords: Exchange rate volatility, Heteroscedasticity, GARCH models, Volatility clustering, Leverage effect, CEMAC, Value-at-Risk

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Chapter 1

1 Introduction

After the collapse of the Bretton wood system of fixed exchange rate among major industrial countries in 1973, exchange rate movement and fluctuation (floating exchange rate) over the last decades has become an interesting macroeconomics topic of discussion, that requires analysis and have received great deal of interest from academics, financial analyst/economist, stakeholders and policy makers as they seek to design polices to mitigate the adverse effect of exchange volatility on important economic indicators. Volatile exchange rates are likely to affect countries' international trade flow, capital flow, inflation, risk management and overall economic welfare (***Hakkio, 1984; De Grauwe, 1988; Asseery & Peel, 1991***). It is also crucially important to understand the behavior of exchange rate in order to design proper monetary policy (***Longmore & Robinson, 2004***). Consequently, a number of models have been developed in empirical financial literature to investigate this volatility across different regions and countries. A well-known and frequently applied model to estimate exchange rate volatility are the Autoregressive Conditional Heteroscedastic (ARCH) model proposed by ***Engle*** (***1982***) and Generalized Autoregressive Conditional Heteroscedastic (GARCH) model developed by ***Bollerslev*** (***1986***) ***and Taylor*** (***1986***). In many cases, the basic GARCH model provides a reasonably good model for analyzing financial time series and estimating conditional volatility. However, GARCH models have been criticized in that they do not provide a theoretical explanation of volatility or what information flows, are in the volatility generating process according to ***Tsay*** (***2010***). The symmetric GARCH models (ARCH and GARCH) assume that both positive and negative shocks are the same and therefore cannot cope with significantly skewed time series which results in biased estimates. Another significant problem encountered when using GARCH models is that they do not always fully embrace the heavy tails property of high frequency financial time series. To overcome this drawback ***Bollerslev et al.*** (***1987***) used the Student-t distribution. Also, there has been some extension in the GARCH model to capture the aspect of asymmetry (new in the form of leverage effect) using Exponential GARCH, Threshold GARCH, GJR-GARCH and the Asymmetric Power GARCH (APARCH), which nest several GARCH type models. Nowadays this issue of exchange rate modeling has gained considerable importance as many countries now desire to shift from a fixed exchange rate to a floating exchange rate regime which is the case with the CFA Franc Zone. Although there have been an extensive empirical studies focusing on modeling and estimating exchange rate volatility using different currency pairs in developed countries and by applying different specifications, little attention has been paid in Central African countries, and to the best of my knowledge, empirical research on this topic of XAF/USD exchange rate volatility is fragmented as there is little or no work that compares the ability of different volatility models in Central African countries.

The main objective of this research work is to model exchange rates return volatility in Central African countries (CEMAC) using XAF/USD, by applying different univariate specifications of GARCH type models for daily

observations of exchange rate returns series for the period 1st January 2010 to 4th January 2018. The volatility models applied in this paper include ARCH (1), ARCH (2), GARCH (1,1), GARCH(2,1), GARCH(1,2), EGARCH(1,1) and GJR-GARCH(1,1) and APARCH (1,1). The rest of this paper is organized as follows; Section 2 provides a brief review of past studies, exchange rate regimes, stylized facts about exchange rate volatility and describes exchange rate system in Central Africa (CEMAC). Section 3 presents the methodology, while section 4 presents the empirical results, and finally section 5 concludes the research work.

Chapter 2

2 Literature Review

Exchange rate volatility modeling has remained crucially important because of its diverse implications. More specifically, the GARCH family of models is widely used in analyzing and forecasting of volatility in financial time series. The number of GARCH models is extremely large but the most influential were the first. ***Engle*** (***1982***) introduced the Autoregressive Conditional Heteroscedasticity (ARCH), with the main to estimate the conditional variance of a time series. Engle described the conditional variance by a simple quadratic function of its own lagged values. ***Bollerslev*** (***1986***) introduced the Generalized ARCH model (GARCH), by extending the basic ARCH model and described the conditional variance by its own lagged values and the square of the lagged values of the disturbance or shocks. ***Taylor*** (***1986***) and ***Schwert*** (***1989***) introduced the Power GARCH in order to capture the power effect. ***Nelson*** (***1991***) formulated the Exponential GARCH (EGARCH) model by extending the GARCH model to capture news in the form of leverage effects. Thereafter, the GARCH model extension was developed to test for this asymmetric news impact (***Glosten et al., 1993; Zakoian, 1994***). ***Ding et al***. (***1993***) combined the leverage and power effects in the same model, to produce the so-called asymmetric power GARCH or APARCH model. ***Olowe*** (***2009***) modeled volatility of Naira/US Dollar exchange rates on a sample of monthly data from 1970 to 2007 using six different GARCH models. The paper concluded that the best fitted models are the Asymmetric Power ARCH and the Threshold Asymmetric GARCH. ***Cyprian et. Al,*** (***2017***) modeled exchange rate volatility in Kenya, using daily observations over the period starting 3rd January 2003 to 31st December 2015, between USD/KES. The used both symmetric and asymmetric models that capture most of the stylized facts about exchange rate returns such as volatility clustering and leverage effect. They concluded that the most adequate models for estimating volatility of the exchange rates are the asymmetric APARCH model, GJR-GARCH model and EGARCH model with Student's t-distribution. ***Ivan Novak et al*** (***2016***) modeled exchange rate volatility in Croatia using GARCH family of models. Specifically, they used EUR and USD against the HRK on daily data sets from 1997 to 2015. They concluded that GARCH (2 1) and GARCH (1 1) are the best fitted models for EUR/HRK and USD/HRK respectively. ***Andreea C*** (***2016***) modeled exchange rate volatility of daily returns for EUR/RON , from 1999 to 2016 using both symmetric and asymmetric GARCH models and concluded that, the best fitted model for estimating daily returns of EUR/RON exchange rate is EGARCH(2,1) with Asymmetric order 2 under the assumption of Student's t distributed innovation terms**.** ***Ştefan Cristian*** (***2015***) examines daily exchange rate volatility for RON/EURO time series from 2005 to 2015 time period using GARCH models and concluded that ARCH (5) is the best fitted model. ***Kane G. and Jean .J*** (***2014***) analyzed the determinants of food price volatility in Cameroon using GARCH model. ***Clement and Samuel*** (***2011***) also aimed to model Nigerian exchange rate volatility. They used the monthly exchange rate of the naira against the US dollar and British pound for the period from 2007 to 2010. They found that the exchange rate return series was non-stationary and that the series residuals were asymmetric. Since return volatility was found to be

persistent, the study recommended further investigation of the impact of government policies on foreign exchange rates. ***Spulbar et. al.,*** (***2012***) analyzed the impact of political news and economic news from euro area on the exchange rate between Romanian currency and Euro using a GARCH models. ***Hsieh*** (***1989***) proved on the daily data sample during a 10-year period (1974 – 1983) for five countries in comparison to the US dollar, that these two models, the ARCH and GARCH models were capable to remove all heteroscedasticity in price changes. It was also proved that the standardized residuals from each of the ARCH and GARCH models using the standard normal density were highly leptokurtic, and the standard GARCH (1, 1) and EGACH (1, 1) were found to be efficient for removing conditional heteroscedasticity from daily exchange rate movements. ***Dahiru A. and Joseph O.*** (***2013***) examines exchange–rate volatility with GARCH models using monthly exchange–rate return series from 1985 to 2011 for Naira/US dollar return and from 2004 to 2011 for Naira/British Pounds and Naira/Euro returns. They compared estimates of variants of GARCH models with break in respect of the US dollar rates with exogenously determined break points. Their results reveal presence of volatility in the three currencies and equally indicate that most of the asymmetric models rejected the existence of a leverage effect except for models with volatility break. ***Marreh et al***. (***2014***) modeled the Euro/GMD and USD/GMD daily returns on a sample period from 2003 to 2013. Based on Akaike information criteria authors found the ARMA (1, 1) –GARCH (1, 1) and the ARMA (2, 1) – GARCH (1, 1) the best fitting models

2.1 Exchange Rate Volatility

An exchange rate (also known as a foreign-exchange rate) between two currencies is the rate at which one currency (based currency) will be exchanged for another (counter currency). It allows you to determine how much of base currency you can exchange for the counter. Therefore Exchange rate volatility refers to the tendency for foreign currencies to appreciate or depreciate in value over time, thus affecting the profitability of foreign exchange trades. Hence, volatility is the measurement of these amounts of changes and the frequency of those changes. It is also known as a measure of risk, whether in asset pricing, portfolio optimization, option pricing, or risk management, and presents a careful example of risk measurement, which could be the input to a variety of economic decisions. It can be measured on an hourly, daily, weekly, monthly or annual basis. Exchange rate volatility, like any other financial asset with volatile movement in price is measured using the standard deviation (square root of the variance) and are usually assumed to follow a normal distribution. Understandably, it is unobservable variable and thus its measure is a matter of serious contention. Recent literature, however, seems to be increasingly adopting the use of the GARCH model, proposed by ***Bollerslev*** (***1986***) which is an extension of the ARCH proposed by ***Engle 1982.*** The phenomenon of leptokurtosis in exchange rates changes that have been documented by a number of studies and ARCH effects are consistent with the phenomenon of leptokurtosis (***McFarland, 1982***).

Usually, two measures of volatility are commonly employed in financial calculations; historical and implied volatility. The former is calculated from past values of an exchange rate (backward looking). Thus given a

series of past daily exchange rates, the standard deviation can be calculated. Historical volatility provides a good assessment of possible future changes when the financial markets and economies have not gone through structural changes. On the other hand, the latter (implied volatilities) are used to monitor the market's opinion about the volatility of a particular stock or currencies and it is a forward looking measure of volatility. It is calculated from the market participant's estimates of what is likely to happen in the future. More precisely, implied volatility is estimated from the quoted price of a currency option when the values of all other determinants of the price of an option are known. The basis for this calculation is the Black Scholes option pricing model, according to which the price of an option is determined by the following: the current price of the asset (the exchange rate or a stock or a commodity), the strike price at which the option can be exercised, the time to maturity of the option, the risk free interest rate, and the volatility of the asset or the exchange rate (***John C Hull 2009***). Exchange rate volatility, like the volatility of any other financial asset, changes in response to information. Currency traders are sensitive to information that might influence the value of one currency in terms of another. The most important information is that related to the macroeconomic performance of the economies behind the two currencies. Changes in the levels of uncertainty about the future of either economy will cause traders to become restless and less willing to hold a particular currency. Uncertainty about the future is the most important reason for the change in the volatility in the currency markets. Changes in the proportions of hedgers versus speculators can also change the volatility of a currency. Central banks can also influence the volatility of their currencies with their announcements of their intentions to either intervene or otherwise in the markets for their currencies. While it is commonly believed that central banks can influence the value of their currency at most in the short run, they can certainly cause a change in the volatility. The nature of exchange rate fluctuations (volatility) in the FOREX market strongly depends on the exchange rate regimes the currency pairs are operating upon.

2.2 Exchange Rate Regimes

Exchange rate regimes are classified in to two broad regimes by monetary authorities, based on their flexibility. These regimes are the fluctuating or floating exchange regime and the fixed or fixed peg or hard peg exchange rate regime. In the former, exchange rates (prices) are determined by the market forces of demand and supply while in the latter, exchange rates are determined by political decisions. Usually, within these two broad regimes of exchange rate, there exist some intermediate ranges of different systems with limited flexibility, known as the soft pegs. Some of these exchange rate systems are discussed below;

2.2.1 Fluctuating or Floating Exchange Rate Regime

This exchange rate regime can take the form of independent floating or managed floating systems. In the independent floating exchange rate, there's existence of a free or competitive foreign exchange market where the

prices of one currency in terms of another are determine by the free forces of demand and supply operating without any official interference. Within the context of this study, the value of XAF (CFA franc) in terms of USD would depend upon the demand of XAF from holders of USD and the supply of XAF from holders of XAF seeking to buy USD. Here, Foreign exchange interventions are rare and there's no prevention of undue fluctuation. Hence the monetary policies usually function without exchange rate consideration. Hence the XAF/USD exchange rate is a typical example of an independent floating exchange rate. On the other hand, with managed floating exchange system, the prices of one currency in terms of another are determine by the free forces of demand and supply operating but the central banks (monetary authorities) usually intervenes in the market to "manage" the fluctuations if need arises, to prevent high volatility and stimulate growth. These adjustments can be done using interest rates to influence the flow of funds in and out of a country[1]. The IMF calls this practice a "Managed Floating with No Predetermined Path for the Exchange Rate".

2.2.2 The Fixed or Fixed Peg or Hard Peg

This regime exists when the exchange rates of the home currency is fixed or peg to a foreign currency in terms of some common standards. In order to maintain the currency at a fixed value, the monetary authorities of the home country must be ready to buy and sell currencies at a fixed price. This means that they must have large supplies of their own currency, gold and convertible foreign currencies in order to remove any excess demand or supply at fixed price. This is the case with economies having currency boards or with no separate national currency of their own. Countries do not have a separate national currency, either when they have formally dollarized, or when the country is a member of a currency union, for example Euro. The IMF categorizes these two processes as "Exchange Arrangement with No Separate Legal Tender". The XAF/EUR exchange rate is a typical example of a fixed exchange rate.

2.2.3 Intermediate Regimes

Intermediate exchange rate regimes consist of an array of differing systems allowing a varying degree of flexibility, such as conventional fixed exchange rate pegs, crawling pegs, currency board and exchange rate bands. A Currency Board Arrangement is a monetary regime based on an explicit legislative commitment to exchange domestic currency for a specified foreign currency at a fixed exchange rate, combined with restrictions on the issuing authority to ensure the fulfillment of its legal obligation. This implies that domestic currency will be issued only against foreign exchange and that it remains fully backed by foreign assets, eliminating traditional central bank functions, such as monetary control and lender-of-last-resort, and leaving little scope for discretionary monetary policy. Some flexibility may still be afforded, depending on how strict the banking rules of the currency board arrangement are. A typical example is the CFA Franc zone. On the other hand, Conventional Fixed Peg

[1] G. F Standlake and S J Grant, 1995. Introductory Economics 6th edition,pp 470-477

Arrangements occurs when the country (formally or de facto) pegs its currency at a fixed rate to another currency or a basket of currencies, where the basket is formed from the currencies of major trading or financial partners and weights reflect the geographical distribution of trade, services, or capital flows. The currency composites can also be standardized, as in the case of the SDR (Special Drawing Rights) and there is no commitment to keep the parity irrevocably .The exchange rate may fluctuate within narrow margins of less than ±1 percent around a central rate-or the maximum and minimum value of the exchange rate may remain within a narrow margin of 2 percent for at least three months. The monetary authority stands ready to maintain the fixed parity through direct intervention[2].

2.3 Stylized Facts about Exchange Rates Volatility

Extensive research on the properties of financial returns such as exchange rate and stock returns has demonstrated that returns usually exhibit three main statistical properties that are present in most, if not all, financial returns. These are often called the stylized facts of financial returns and they are usually considered crucial for correct model specification and forecasting. They include volatility clustering, fat tails, nonlinear independence. Other properties may include; leverage effect, Regular Events, long memory and Co-movements in Volatility.

2.3.1 Volatility clustering

It is the tendency of large changes in prices of financial assets to cluster together, which results in the persistence of these magnitudes of price changes. Another way to describe the phenomenon of volatility clustering is to quote the definition of famous scientist-mathematician ***Benoit Mandelbrot*** (***1963***), who defines it as the observation where large and small values of log-returns turns to occur in a cluster. That is "large changes tend to be followed by large changes, of either signs and small changes tend to be followed by small changes". This phenomenon is observed when there are extended periods of high market volatility or the relative rate at which the price of a financial asset changes, followed by a period of "calm" or low volatility. Usually when volatility is high it is likely to remain high and when it is low it is likely to remain low. Volatility clustering is nothing but accumulation or clustering of information. This feature reflects on the fact that news is clustered over time (***Engle, 2004***). Almost all financial returns exhibit volatility clusters and this feature of financial time series gained widespread recognition with the publication of ***Engle*** (***1982***) and is now one of the accepted stylized facts about asset returns. If we can capture predictability in volatility, it may be possible to improve portfolio decisions, risk management and option pricing, among other applications. This concept of volatility clusters can be illustrated in Figure 2.1 below which shows exaggerated simulated volatility clusters. Panel (a) shows returns and Panel (b) volatility. In the beginning,

[2] See IMF – classification of Exchange Rate Arrangements and Monetary Policy Frameworks, June 2004 for more insight on exchange rate regimes

volatility increases and we observe periods of high volatility followed by period of high volatility (high volatility clusters). Around day 180, volatility decreases (periods of low volatility followed by periods of low volatility) only to increase again after a while and so on. This sort of movement is usually driven by FOREX market sentiments.

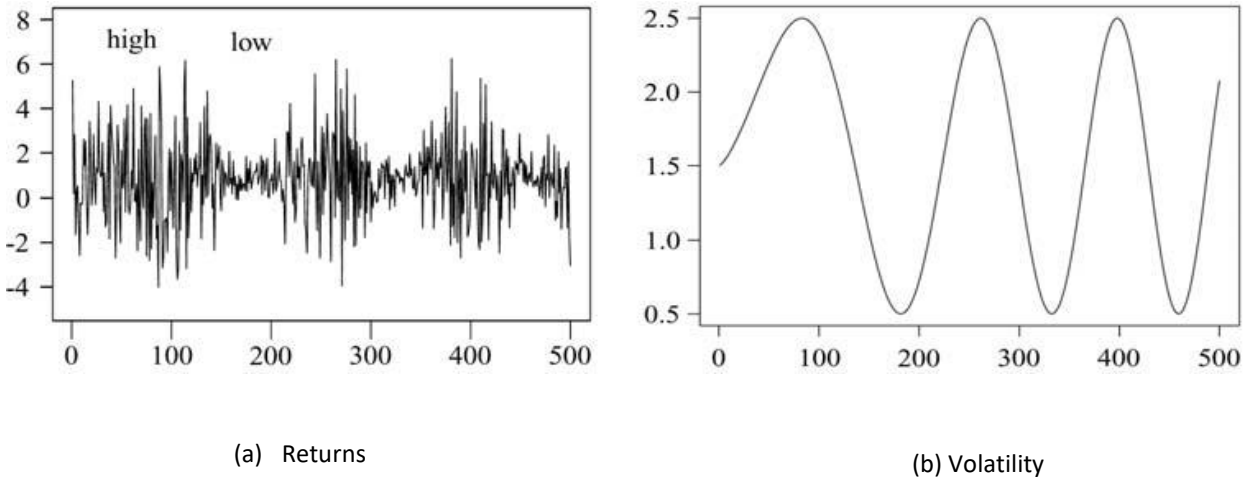

Figure 2. 1. Exaggerated simulated Volatility Clustering

The standard graphical method for exploring predictability in statistical data is the autocorrelation function (ACF), which measures how returns on one day are correlated with returns of the previous days. If such correlations are statistically significant, then we have strong evidence of predictability. The Ljung-Box test is usually used to test for joint significance of autocorrelation coefficients of several lags.

2.3.2 Non-normality and Fat Tail

When the distribution of financial time series such as exchange rate returns is compared with the normal distribution, fatter tails are observed (leptokurtosis). Most often, there are two main approaches for identifying and analyzing tails of financial returns: statistically test and graphical methods. The former compare observed financial returns with some based distribution, typically but not always the normal distribution. The latter relate observed returns with the values predicted from some distribution, often the normal.

Statistical test for fat tail analysis

Financial time series usually, are said to have fat tail if they exhibits more extreme outcomes than that of a normal distribution with the same mean and variance. This means that the market has more relatively large and small outcomes than one would expect under the normal distribution, and conversely fewer returns of an intermediate magnitude. In particular, the probability of large outcomes is much higher than the normal would predict. The fat-tailed property of returns has been known since ***Mandelbrot*** (***1963***) ***and Fama*** (***1963, 1965***). Usually, a basic property of normal distributions is that, they are completely described statistically by the first (mean) and second (variance) moments. This means that both skewness and kurtosis are the same for normally distributed variable (that is 0 and 3 respectively). Skewness is a measure of asymmetry of the probability distribution and kurtosis

measures the degree of peakedness of a distribution relative to the tails. Positive skewness indicates a distribution with an asymmetric tail extending towards more positive values (right), while Negative skewness indicates a distribution with an asymmetric tail extending towards more negative values (left). The formulae for computing skewedness and kurtosis are given below respectively

$$S_k = \sum_{t=1}^{T} \left(\frac{X_t - \bar{X}}{\hat{\sigma}_X} \right)^3 \qquad and \quad kur = \sum_{t=1}^{T} \left(\frac{X_t - \bar{X}}{\hat{\sigma}_X} \right)^4 - 3$$

High kurtosis generally means that more of the variance is due to infrequent extreme deviations than predicted by the normal, and is a strong, but not perfect, signal that a return series has fat tails. Excess kurtosis is defined as kurtosis above 3. It is important to note that using the *Jarque-Bera (JB) test;* we actually test for normality in our time series. That is testing if skewedness and kurtosis are significantly different from zero. The formula for computing JB test is given as

$$JB\ test = T\left(\frac{S_k^2}{6} + \frac{Kur^2}{24} \right) \xrightarrow{d} \chi_2^2$$

Under the $H_0: X_t \sim N(0, \sigma^2)$

Graphical method for fat tail analysis

Even though a good number of graphical techniques do exist to detect the presence of fat tail, they are not efficient because they do not provide a precise statistical description of the data. Nevertheless, these methods indicate the nature of the tails (fat or thin) and can reveal information of how the data deviate from a normal distribution. Within the context of this work, the quantile-quantile plot or QQ plot will be used to illustrate fat tail. The QQ plot compares the quantiles of the sample data against the quantiles of a reference distribution (normal or student t distribution). It is used to assess whether a set of observations have a particular distribution, or whether two datasets have the same distribution.

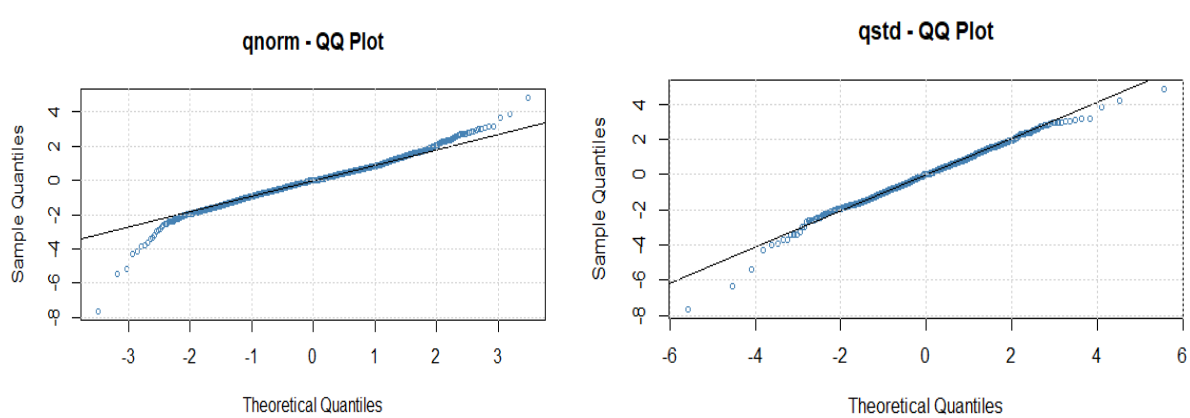

Figure 2. 2. QQ plots for daily XAF/USD Exchange Rate returns, January 2010 to January 2018

Figure 2.2 above shows the QQ plots for daily XAF/USD Exchange Rate returns. The x-axis shows the standard normal while the y-axis measures outcomes from the data. The straight line (blue) is the normal prediction. It can be observed that many observations seem to deviate from normality, both on the downside and on the upside. This is evident that fat tail is present and that data violate normality. Introducing a fat-tailed distribution (student-t distribution) the returns still seems to have fat tails at both ends but the student-t distribution provides a much more better fit (lesser tails) that the normal. Perhaps increasing the degrees of freedom, we might achieve normality.

2.3.3 Non Linear Dependence

It is the observation that the dependence between different financial returns series changes according to different market conditions. That is most of the time, prices of financial assets move relatively independently of each other, but during crisis, the all drop together. In practice, joint extreme outcomes are likely to occur than predicted by multivariate normality and linear correlations. Usually, most statistical model assumes that the relationship between different asset returns is linear. But in recent times research has shown that this assumption of linear dependence does not necessary holds for financial returns where correlations are usually lower in bull markets than in bear markets. More so, if financial data were jointly normally distributed, correlations would decrease for extreme events whereas empirically we see that correlations tend to increase during periods of crisis. To capture such phenomena, models of nonlinear dependence allow the dependence structure to change according to market conditions. In this case, linear correlations overestimate dependence in non-crisis periods and underestimate correlations during crises. Research such as ***Ang et al.*** (***2001***) ***and Patton*** (***2002***) has found that these nonlinear dependence structures command a premium in the market as investors require higher expected returns for portfolios where assets are highly correlated under bad market conditions. Aside from asset allocation, applications in risk analysis, economic capital and financial stability also focus on large outcomes. In such applications it is essential to address nonlinear dependence[3].

2.3.4 Leverage Effect

The leverage effect was first identified by ***Black (1976).*** It refers to the tendency for most measures of returns (for example the sample variance of returns over a given period, or the size of squared returns or absolute returns) to increase as the prices of stocks decreases (***R Cont, 2001***). This suggests that positive and negative returns have asymmetric effect on most measures of volatility (***Taylor, 2005***); with negative returns correspond to reductions in stock prices and hence tend to correlate with increases in volatility measures, whereas positive returns correspond to increases in stock prices and hence tend to correlate with reductions in volatility measures. With such a situation, one might expect negative returns today to lead to higher volatility tomorrow and vice versa for positive

[3] Jon Danielsson, 2011. Financial Risk Forecasting: Theory and Practice of forecasting Market Risk, pp 9-17

returns. In financial markets, it usually evident that downward movement (depression) is always followed by high volatility and any shock which increases volatility of the market, increases the risk of holding the currency. Thus negative shocks increase predictability of financial returns than positive shocks *(**Longmore and Robinson, 2004**)*. Empirical evidence of leverage effect can be found in ***Taylor (2005), Cambell and Kyle (1993), Engle and Ng** (1993/94) **and Nelson (1991)**.*

2.3.5 Long Memory

Usually common especially for high-frequency data like exchange rates, volatility is highly persistent and there exists evidence of near unit root behavior of the conditional variance process. This observation led to two propositions for modeling persistence: unit root or long memory process *(**Longmore and Robinson, 2004**).*

2.3.6 Co-movements in Volatility

In viewing different financial markets, such as exchange rate markets, it is usually observed that large movement in a particular currency is often matched by large movement in another. This brings forth the importance of multivariate models in modeling cross-correlation in different markets.

2.3.7 Regular Events

Holidays and weekends which are regular event that keep occurring, has a significant effect in the exchange rate markets and thus affects the volatility of exchange rates. Recent studies have highlighted that during these periods, percentage changes in currencies or exchange returns are usually lower than trading days. This is usually attributed to accumulation effect of information during non-trading days (holidays and weekends)[4].

2.4 Exchange Rate in Central Africa (CEMAC Zone)

In recent years, exchange rate system in central Africa has become an increasingly discussed topic as researchers and policy makers seeks to know which regime best suits the zone in order to increase trade flow and economic development. *CEMAC*[5] *zone* unites six nations under a common currency, which are mostly French colonies. The currency is popularly known as *Central Africa CFA Franc*, with currency code *XAF*, issued by a single central bank called *BEAC* (Bank of Central African States)[6] located in Yaoundé Cameroon. This monetary zone (*CEMAC*) include; Cameroon, the Central African Republic, Congo-Brazzaville, Gabon, Equatorial Guinea and Chad). The *CFA*[7] *Franc* (or *Franc CFA*) was created in 1945 by France, apparently as a noble gesture to protect its African colonies from a

[4] Scott Thompson 2011; The Stylised Facts of Stock Price Movements

[5] CEMAC stands for Economic and Monetary community of Central Africa (Communauté Économique et Monétaire de l'Afrique Centrale)

[6] Carine Foe, 2009. The Central African CFA Franc: Catalyst For Cross Border Business

[7] CFA stands for Coopération financière en Afrique centrale ("Financial Cooperation in Central Africa").

devaluation of the French franc. Under this carefully-considered control mechanism, participating countries were required to deposit most (65% as of 1973) of their foreign currency reserves with the French Treasury, which in turn dictates monetary policies, and mandate when and how governments within the CEMAC zone could access the money. The reason for this deposition of reserves with the French Treasury was/is to enable the buying and selling of the excess and shortages in the currency exchange to always maintain the fixed rate. To an extent, this decision has helped stabilized the national currency of Central African Franc Zone and greatly facilitates the flow of exports and imports between France and the member-countries[8].

At its creation, it was pegged to the French franc at 50 CFA francs to 1 French franc. It was devalued in 1994 and then became pegged at 100 to 1, until France adopted the euro in 1999. At that point, the French franc was converted to the euro at 6.55957 French Franc to 1 EUR. Thus the CFA franc currency exchange rate became pegged to Euro at 655.957 XAF to 1 EUR (or 0.00152 EUR to 1 XAF), and it has remains pegged till present. Following this new arrangement, the Franc CFA is guaranteed by the European Central Bank and there are unlikely no material effect on the monetary and exchange rate policy for the Eurozone. The Bank of Central African States (BEAC) still maintains an operation account with the French Treasury and the French Treasury guarantees the convertibility of the CFA franc into Euros. Nevertheless the competent French authorities shall keep the European Commission, the European Central Bank (ECB) and the Economic and Financial Committee informed about the implementation of the agreements and informs the Committee prior to changes of the parity between the euro and the CFA. Hence any change to the nature or scope of the agreements would require Council approval on the basis of a Commission recommendation and ECB consultation. How this new policy agreement affects the CEMAC zone in the future depends on their keeping up with efforts and commitment to better economic management[9]. This indeed implies that this monetary zone have to adhere to very strict monetary and fiscal discipline imposed by the European Central Bank. It appears since the new arrangement that the advantages from this anchor to the euro overshadow the potential risks in principle. In the medium to long run, the principal risks would be linked to how strong the euro is with respect to other major currencies (pound, dollar, yen, and Swiss franc).

Since CFA Franc was initially pegged to the French Franc at its creation and with France adopting Euro, it became pegged to Euro, the CFA Franc is meant to fluctuate vis-à-vis US dollars and other major currencies like pounds, Canadian dollars, etc because no monetary and exchange rate policy agreement was established between CEMAC zone and these countries (USA, UK and Canada). Today, since there's still no prior monetary and exchange rate policy agreement between the CEMAC Zone and USA, this fluctuation between these currencies pair still exist and with increasing trade agreement between CEMAC zone and USA, exchange rate is being determined by the free forces of demand and supply in the market. Based on BEAC statistics, USD is the most ranked currency exchange with XAF (CFA franc), followed by EUR, CAD (Canadian dollars), GBR[10] etc. Figure 2.3 below shows two

[8] Central Africa CFA franc wikipedia
[9] CFA franc wikipedia
[10] www.beac.int

most important exchange rate pairs under BEAC statistics that operates under two different exchange rate regimes. As can be observe, the XAF/EUR exchange rates plot is a horizontal line, indicating that its fluctuation is strictly managed by BEAC to maintain the value at 0.00152 (or EUR/XAF = 655.957).

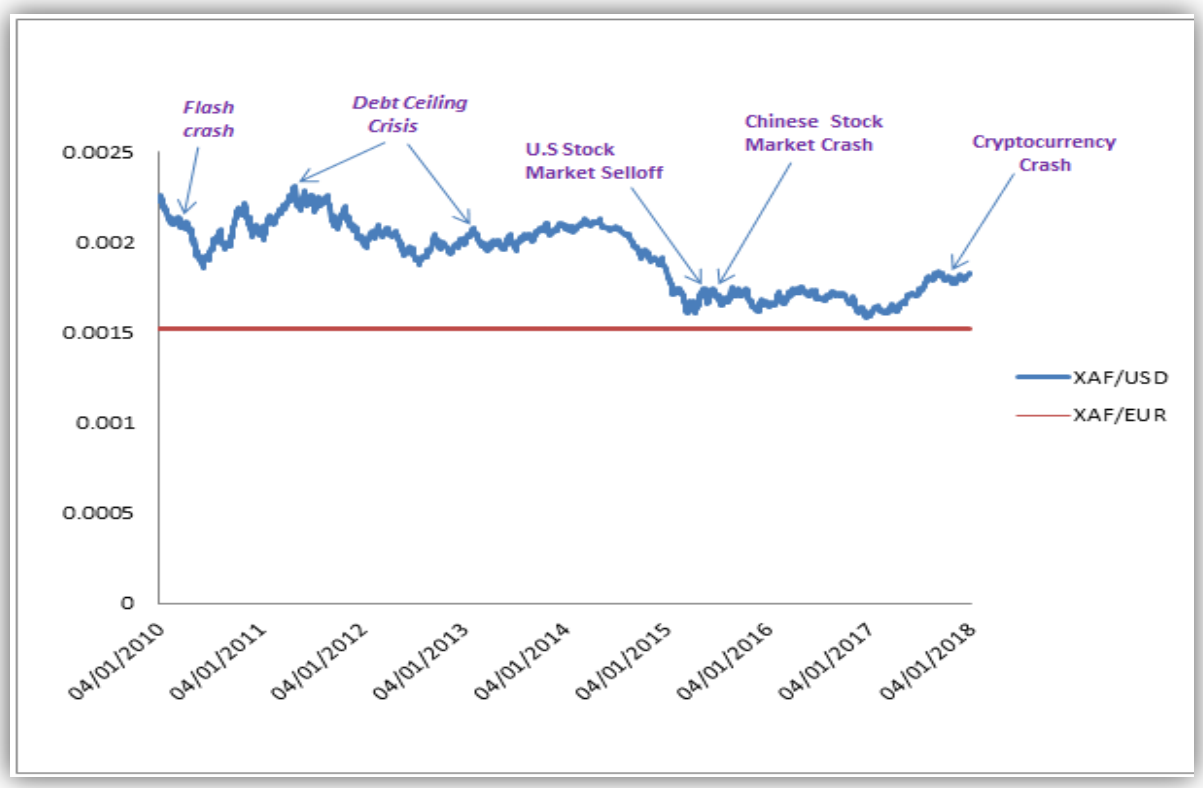

Figure 2. 3. XAF/USD and XAF/EUR Exchanges Rates, January 2010 to January 2018

Data source: www.beac.int

This is a strong indication that CEMAC zone operates under a fixed (hard peg) exchange rate system with the euro zone and exchange rate series such as XAF/EUR, cannot be used to model volatility because there are no fluctuations. On the other hand, the XAF/USD exchange rate series fluctuates from a value of 0.00226 at the beginning of 2010, to a maximum value of 0.00231 in the year 2011 during the Unite States Debt Ceiling crisis and to a minimum value of 0.00158 in early 2017 during, periods of low inflation. This fluctuation indicates that the CEMAC zone operates under a floating exchange system with the U.S. Such time series like the XAF/USD, that behave like a random walk (non-stationary) can be used to model volatility when first difference of natural logarithm is applied to achieve stationarity.

2.5 Research Data

The data used for modeling exchange rate volatility in this study is the daily exchange rate of Central Africa CFA Franc (XAF) against the US dollars (USD). The data will spam from 1st January 2010 to 4th January 2018, resulting to a total of 2079 observations. The data is sourced from www.beac.int. The stationarity condition of the data will be check and if it's not stationary like in the case of most financial time series, the time series will be transform and the variable to be modeled is daily exchange rate return which is the first difference of the natural logarithm of daily exchange rates, given by the following expression below;

$$r_t = log\left(\frac{P_t}{P_{t-1}}\right) * 100 = \left(log(P_t) - log(P_{t-1})\right) * 100$$

Where r_t is the daily percentage exchange rate return at time t and P_t and P_{t-1} denotes exchange rate at current and previous day respectively.

Chapter 3

3 Methodology

This section discusses the various GARCH models used to investigate volatility characteristics. In presenting these models, there are two distinct specifications, the first for the conditional mean and the conditional variance. The models are estimated using maximum likelihood method under the assumption of Gaussian normal error distribution. The student-t and skew student-t distributions have also been introduced to capture tails thickness. The log likelihood function is maximized using Marquardt numerical iterative algorithm to search for optimal parameters and the best volatility model will be select based on AICs and BICs criteria.

3.1 Volatility Definition and Measurement

The term volatility describes the spread of all likely outcomes over an uncertain variable. Typically, in financial markets, we are often concerned with the spread of asset returns. Statistically, volatility is often measured as the sample standard deviation. This traditional method of measuring volatility (standard deviation) is usually unconditional and cannot capture the characteristics exhibits by financial time series data (stylize facts)[11].

$$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \mu)^2}$$

Where r_t is the return on day t and μ is the average return over T-day period. Sometimes, variance (σ^2), is used also as a volatility measure. Volatility is related to, but not exactly the same as, risk. Risk is associated with undesirable outcome, whereas volatility as a measure strictly for uncertainty could be due to a positive outcome (***Poon, 2003***). This work uses the variance as a measured of volatility.

Since the unconditional method for measuring volatility cannot capture the stylized facts of financial returns, the conditional method (ARCH and GARCH models) are the most commonly used in empirical finance and the procedure for estimation is describe in the flow chat below;

[11] Poon, S. H., & Granger, C. W. (2003). Forecasting volatility in financial markets: A review. Journal of economic literature, 41(2), 478-539.

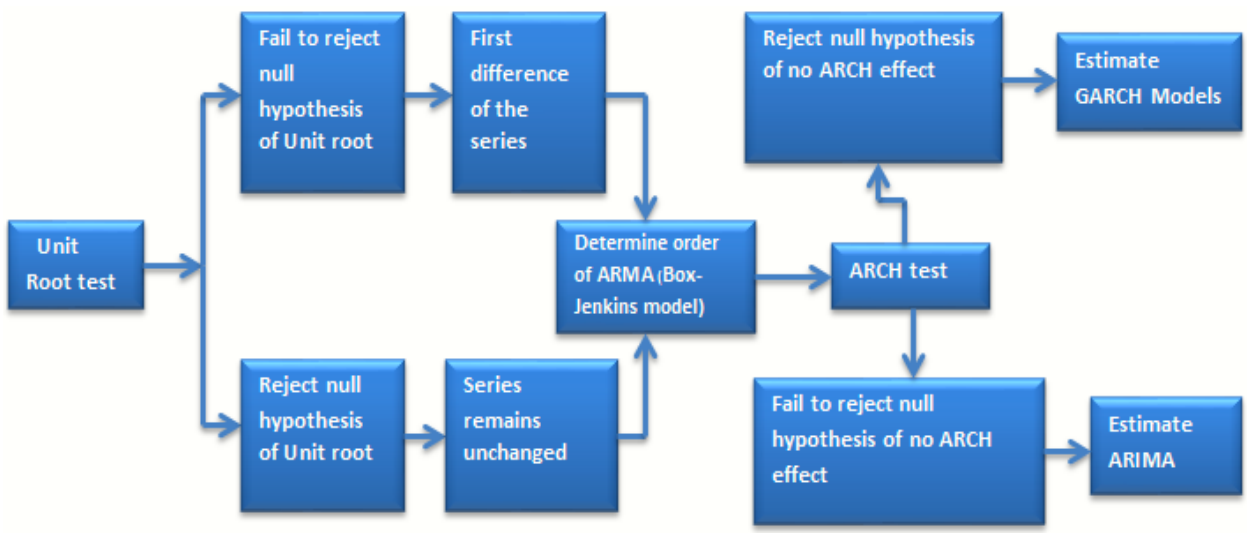

Figure 3. 1. Flow chat of methodology to compute conditional volatility

Source – (Author)

3.2 Unit root test

In modeling time series data, it is very important to check the presence of unit root data because most statistical and econometric methods are based on stationarity assumption (the absence of unit root in the data). In this work, the ***Augmented Dickey-Fuller*** *unit root test (**1979**)* is applied to check the presence of unit root in the data. The null hypothesis of this test assumes the present of unit root in the data and the reverse is true with the alternative hypothesis.

$$y_t = \alpha_1 y_{t-1} + \varepsilon_t$$

By subtracting y_{t-1} on both side of the equation, we arrive at $\Delta y_t = \gamma y_{t-1} + \varepsilon_t$, where $\gamma = \alpha_1 - 1$. Of course, testing for the hypothesis $\alpha_1 = 1$ is equivalent to test for the hypothesis of $\gamma = 0$. The ADF actually consider three different regression equations that can be used to test for the presence of unit root;

$\Delta y_t = \gamma y_{t-1} + \varepsilon_t$ ……Pure Random Walk

$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \varepsilon_t$ ……Random Walk with drift/intercept

$\Delta y_t = \alpha_0 + \alpha_2 t + \gamma y_{t-1} + \varepsilon_t$ ……Random Walk with drift and linear time trend

If $\gamma = 0$ then the series y_t has unit root (null hypothesis) and If $\gamma < 0$, then the series y_t has no unit root or stationary (alternative hypothesis). With the test output, if the p-value of the ADF test is less than 5% or the value of the test statistic is more than the value of the critical, we reject the null hypothesis, otherwise we accept the null. Hence if the ADF test result indicates a time series is not stationary, then we need to apply differencing.

3.2.1 Differencing

In order to convert non stationary time series to stationary time series, differencing is applied by subtracting past lag (lag 1) from the original series.

For example $X_t = X_{t-1} + \varepsilon_t \ \rightarrow X_t - X_{t-1} = \varepsilon_t \ \rightarrow \Delta X_t = \varepsilon_t$

In financial time series, transformation such as logarithmic is performed on the series before differencing is applied. This is so because financial time series are usually exposed to exponential growth and logarithmic transformation can help to stabilize (linearize) the variance of the time series while differencing can help to stabilize the mean of the time series by removing changes in the level of the time series, and also eliminating trend and seasonality.

3.2.2 Stationarity

Converting non stationary (presence of unit root) time series data's to stationary (without unit root) is very important because most econometrics methods are based on stationary data's. Non stationary times series are usually erratic and unpredictable while stationary time series are set to be mean-reverting (fluctuate around a constant mean and variance). In addition, stationarity and independence of random variables are closely related because many theories that hold for independent random variables also hold for stationary time series in which independence is a required condition. The majority of these methods assume the random variables are independent (or uncorrelated); the noise is independent (or uncorrelated); and the variable and noise are also independent (or uncorrelated) of each other.

Roughly speaking, stationary time series shows no long-term trend and has constant mean and variance. More specifically, there are two definitions for stationarity: weak stationarity and strict stationarity.

a) Weak stationary: the time series $\{X_t, t \in Z\}$, (where Z is an integer set) is set to be stationary if
   i. $E(X_t) = \mu \ \forall t \in Z$
   ii. $E(X_t^2) = \sigma^2 < \infty \ \forall t \in Z$
   iii. $\gamma \times (s,t) = \gamma \times (s+h, t+h) \ \forall s,t,h \in Z$
b) Strict stationarity: the time series $\{X_t, t \in Z\}$, (where Z is an integer set) is set to be strict stationary if the joint probability distribution of $(X_{t1}, X_{t2} \dots \dots \dots X_{tk})$ is the same as that of $(X_{t1+h}, X_{t2+h} \dots \dots \dots \dots . X_{tk+h})$[12]

Mostly in statistics, stationarity refers to weak stationarity where we have a constant mean, constant variance and the covariance depends on t-s which is not a function of time (t or s). Conversely a strict sense stationary implies Probability distribution remain constant over time. For example, white noise is stationary and implies that the random variables are uncorrelated, not necessarily independent. However, a strict white noise indicates the

[12] Ati Katchova, 2013, Time Series ARIMA Models

independence among variables. Additionally, a Gaussian white noise is strictly stationary since Gaussian distribution is characterized by the first two moments, and therefore, non-correlation also implies independence of random variables. In a strict white noise, the noise term $\{\varepsilon_t\}$ cannot be predicted either linearly or non-linearly and it reflects the true innovation in the series. It is noted that, the noise term is only true innovation of a time series when it is strictly stationary and cannot be predicted; otherwise this term is referred to as errors.

3.3 ARMA (p q) Model for Conditional Mean

The *Box-Jenkins method*, proposed by ***Gorge E. P. Box and G. M. Jenkins in 1976***, provides a way of identifying ARMA model using ACF and PACF graphs of the time series. This model first seeks to identify if they exist unit root or seasonality in the series and use differencing to make the time series stationary. The Box-Jenkins method involves model specification, model estimation and model diagnostics.

3.3.1 Model Specification

Since the foreign exchange rate moving pattern might be an autoregressive (AR) process, moving average (MA) process or a combination of AR and MA processes i.e. (ARMA) process. For the purposes of this study the conditional mean equation is modified to include appropriate AR and MA terms to control for autocorrelation in the data. Usually a generalization of an ARMA model is specified as

$$Y_t = \sum_{i=1}^{p} \alpha_i Y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j}$$

Where Y_t is the time series being modeled and thus $Y_t \sim ARMA(p\ q)$ model. By applying differencing on the series Y_t, we derive an ARIMA (p, d, q) model. In the ARIMA model, **p** stands for the order of the autoregressive parameter (AR), **d** stands for number of differencing and **q** stands for the order of the moving average parameter (MA). The time domain method is established and implemented by observing the autocorrelation of the time series. Therefore, autocorrelation function (ACF) and partial autocorrelation function (PACF) are the core of ARMA/ARIMA models.

The ACF is a measure of linear dependence between a time series y_t and it's passed lags y_{t-k}, where $k = 1, 2 \ldots \ldots .. p$. It is usually calculated as the proportion of the autocovariance between $y_t\ and\ y_{t-k}$ to the variance of the dependent variable y_t. That is

$$ACF(k) = \rho_k = \frac{cov(y_t, y_{t-k})}{var(y_t)}$$

Where $-1 \leq \rho \leq 1$

Also, the PACF is the simple correlation between $y_t\ and\ y_{t-k}$ minus the part explained by the intervening lags

$$\rho_k^* = Corr[y_t - E(y_t|y_{t-1}, \ldots\ldots, y_{t-k+1}), y_{t-k}]$$

Where $E(y_t|y_{t-1}, \ldots\ldots, y_{t-k+1})$ is the minimum mean-square error predictor of y_t by $y_{t-1}, \ldots\ldots, y_{t-k+1}$

There are three rules to identify ARMA model on the differenced data

- If ACF cut off after lag **q**, PACF decays exponentially (tail off): ARIMA (0, d, q) → identify MA(q)
- If ACF decays exponentially (tail off), PACF cut off after lag **p**: ARIMA (p, d, 0) → identify AR(p)
- If ACF and PACF tails off (p>0, q>0): mixed ARIMA model, need differencing

It is noted that the number of difference in ARIMA is written differently even though referring to the same model. For example, *ARIMA (1, 1, 0) of the original series can be written as ARIMA (1, 0, 0) of the differenced series or ARMA (1, 0) of the differenced series.* Also, it is necessary to check for over differencing in which lag-1 autocorrelation is negative (usually less than -0.5). Over differencing can cause the standard deviation to increase.

3.3.2 Model Estimation

Estimating the parameters for Box–Jenkins models involves numerically approximating the solutions of nonlinear equations. For this reason, it is most common to use statistical software designed to handle to the approach – fortunately, virtually all modern statistical packages feature this capability. The main approaches to fitting Box–Jenkins models are nonlinear least squares and maximum likelihood estimation. Maximum likelihood estimation is generally the preferred technique. The likelihood equations for the full Box–Jenkins model are complicated and are not included here (***see Brockwell and Davis, 1991***). Thus the goal is to select a stationary and parsimonious model that has significant coefficients and a good fit.

AIC and BIC Goodness of fit

The Akaike information criterion (AIC) and Bayesian information criterion (BIC) are two measures of goodness of fit. They measure the trade-off model fit and model complexity.

$$AIC = -2\ln(L) + 2k$$

$$BIC = -2\ln(L) + \ln(N)k$$

Where L is the likelihood function, evaluated at the parameter estimates, N is the number of observations and k is the number of estimated parameter[13]. Lowest AIC or BIC value indicates a best fit (parsimonious model) model.

[13] Ani Katchova, 2013, Time Series ARIMA Models

3.3.3 Diagnostic Checking

Model diagnostics for Box–Jenkins models is similar to model validation for non-linear least squares fitting. That is, the error term $\hat{e}_t$ is assumed to follow the assumptions for a stationary univariate process. The residuals should be white noise (or independent when their distributions are normal) drawings from a fixed distribution with a constant mean and variance. If the Box–Jenkins model is a good model for the data, the residuals should satisfy these assumptions. Observed Residuals from a simple linear regression model $y = \beta_0 + \beta_1 x + e$ are calculated as the difference between the *actual* and the *fitted* values. That is

$\hat{e}_t = y_i - \hat{y}_i, \quad i = 1 \ldots\ldots\ldots\ldots n$

Also, give an AR (2) model, i.e

$$Y_t = \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + e_t$$

Having estimated the values of $\theta_1, and\ \theta_2$, the residuals can be defined

$$\hat{e}_t = Y_t - \hat{\theta}_1 Y_{t-1} - \hat{\theta}_2 Y_{t-2}$$

For a model containing both autoregressive and moving average terms, such as

$Y_t = \sum_{i=1}^{p} \theta_i Y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \emptyset_j \varepsilon_{t-j}$ ..... ARMA (p, q),

we use the inverted, infinite autoregressive form of the model to define the residuals. That is all moving average component are converted to autoregressive component using MA(q) invertibility (*see "Appendix A for MA(q) invertibility*). From the inverted form of the model, we have

$$Y_t = \pi_1 Y_{t-1} + \pi_2 Y_{t-2} + \pi_3 Y_{t-3} + \cdots + e_t$$

So that the residuals are define as

$$\hat{e}_t = Y_t - \hat{\pi}_1 Y_{t-1} - \hat{\pi}_2 Y_{t-2} - \hat{\pi}_3 Y_{t-3} - \cdots$$

For simplicity, we can write the general form of the residuals as

$$\hat{e}_t = \sum_{i=1}^{q} \hat{\pi}_i \hat{e}_{t-i} + v_t$$

Where v_t is an error term

Here, the $\pi's$ are not estimated directly but implicitly as a function of $\theta's$ and $\emptyset's$. In fact, the residuals are not calculated using this equation but as a by-product of the estimation of $\theta's$ and $\emptyset's$. Each residual is the

unpredictable component of the associated observation. If the model is appropriate, it's reasonable to expect the residuals to also exhibit properties that agree with the stated assumption error term (that is normally distributed, homoscedastic and independent). Since the sum of all the residuals equals zero, we might consider standardizing the residuals as $\frac{\hat{e}_t}{s}$ *or* $\frac{(y_i-\hat{y}_i)}{se_{(y_i-\hat{y}_i)}}$ **,** where ***s*** is the standard deviation. Hence with the residuals and standardize residuals at hand, we now proceed to check some statistical plots (ACF, PACF and Q-Q etc) of the residuals to see their behavior.

The procedure involve observing the residuals plot of the ACF and PACF diagram and if the ACF and PACF of the model residuals show no significant lags, then we conclude that the selected model is appropriate. Usually a more formal method is to apply the *Ljung-Box test* (Proposed by ***Greta M. Ljung and George E. P. Box in 1978***) is performed to test the presence of correlation in the residuals. The null hypothesis of this test assumes no serial correlation in the residuals while the reverse is true with the alternative hypothesis. The Ljung-Box test statistics is given as

$$Q = n(n+2)\sum_{k=1}^{h}\frac{\rho_k^2}{n-k}$$

Where *n* is the sample size, ρ_k is the sample autocorrelation a lag k and *h* is the number of lags being tested. Under the null hypothesis, the series is a white noise (data are independently distributed). Q has a limiting chi square (χ^2) distribution with h degrees of freedom[14].

3.4 Testing for Heteroscedasticity

Most often, it is of the essence to check the presence of autocorrelation in the squared residuals using the ACF and PACF plots (*or Ljung-Box test*), because the presence of autocorrelation in the squared residuals indicates that the variance of the residuals series is heteroscedastic and it is conditioned on its past history. Namely, the residuals series exhibit ARCH effect. Usually, there are some formal methods to test for the ARCH effect of a process, such as the *McLeod-Li test* (***McLeod and Li, 1983***), the *Engle's Lagrange Multiplier test* (***Engle, 1982***), the BDS test (***Brock et. al., 1996***), etc. The first two tests will be applicable.

3.4.1 McLeod-Li test

McLeod and Li (***1983***) proposed a formal test for ARCH effect based on the *Ljung-Box test*. It looks at the autocorrelation function of the squares of the pre-whitened data, and tests whether the first *h* autocorrelations for the squared residuals are collectively small in magnitude. Similar to the *Ljung-Box Q-statistic*, the *Mcleod-Li test* is given by

[14] Box, G. E. P., Jenkins G. M., Reinsel, G. C. and Ljung, G. M., 2015, Time Series Analysis: Forecasting and Control (5th Ed.), John Wiley & Sons, Inc., Hoboken, New Jersey

$$Q = n(n+2)\sum_{k=1}^{h}\frac{\rho_k^2(\hat{e}^2)}{n-k}$$

The null hypothesis of this test indicates "no ARCH effect" while the reverse is true with the alternative hypothesis. Hence, a p-value of less 5% indicates that we reject the null hypothesis and conclude that ARCH effect is evident[15].

3.4.2 ARCH-LM Test

Since the ARCH model has the form of an autoregressive model, *Engle* (***1982***) proposed the *Lagrange Multiplier (LM) test*, in order to test for the presence of ARCH effect based on the regression. In summary, the test procedure is performed by first obtaining the residuals e_t from the ordinary least squares regression of the conditional mean equation which might be an autoregressive (AR) process, moving average (MA) process or a combination of AR and MA processes (ARMA). Thereafter, square the residuals and regress them on q own lags to test for ARCH of order q, as in the equation below

$$e_t^2 = \pi_0 + \pi_1 e_{t-1}^2 + \pi_2 e_{t-2}^2 + \cdots + \pi_q e_{t-q}^2 + v_t$$

The null hypothesis of no ARCH effect, against the alternative are given as

$$H_0: \pi_1 = \pi_2 \ldots . \pi_q = 0$$

$$H_0: \pi_1 \neq 0, \pi_2 \neq 0 \ldots . \pi_q \neq 0$$

The test statistic is defined as TR^2 (the number of observations multiplied by the coefficient of multiple correlations) from the last regression, and is distributed as $\chi^2(q)$. ***Bellerslve*** (***1986***) suggested; it should also have a power against GARCH alternatives[16].

3.5 Modeling Volatility

Volatility modeling mostly makes use of the GRACH family of models, which are grouped into symmetric and asymmetric. The symmetric GARCH models to be employ here are ARCH and GARCH while the asymmetric GARCH models include EGARCH, GJR-GARCH and APARCH. These models belong to the category of conditional volatility model and are based on using optional exponential weighting of historical return residuals to obtain a volatility forecast. Return residuals on day t depends on past return residuals, and older return residuals have lower weights than recent once. The parameters of this family of models are estimated using maximum likelihood. In order to study the statistical properties of these models, given information at time $t-1$, we assume that; ε_t denotes

[15] McLeod, A. I. and Li, W. K.: Diagnostic checking ARMA time series models using squared residual autocorrelations, Journal of Time Series Analysis, 4, 269–273, 1983

[16] Chris Brooks, 2008. Introductory Econometrics for Finance, Second Edition, pp 390

random variables (RVs). The main object of interest is the *conditional volatility of* ε_t *(i.e.,* σ_t *or* $\sigma_{t|t-1}$*), g*iven information in the past. The conditional variance of ε_t given its past values $\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots.$, measures the uncertainty in the deviation of ε_t from its conditional mean $E(\varepsilon_t|\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots.,)$. That is $E\left\{\left(\varepsilon_t - E(\varepsilon_t)\right)^2 \middle| \varepsilon_{t-1}, \varepsilon_{t-2} \ldots \ldots..\right\}$. If ε_t follows some ARIMA model, the (one- step-ahead) conditional variance is always equal to the noise variance for any present and past values of the process. Since return series of a financial asset, say r_t, is often a serially uncorrelated sequence with zero mean, even as it exhibits volatility clustering, we assume that $E(\varepsilon_t) = 0$

Thus the mean equation on day t can be defined as

$$r_t = \mu + \varepsilon_t$$

$$\varepsilon_t = \sigma_t Z_t$$

Where ε_t is the residual return, r_t is the logarithmic returns of the financial time series at current time, μ is the average mean assumed to be approximately zero and Z_t is standardized residual returns (iid[17] random variable with mean zero and variance 1) that can either follow a normal, student-t, or generalized error distributions.

3.5.1 Autoregressive Conditional Heteroscedasticity (ARCH) model

It was developed to capture volatility cluster. A general form of the model is given as

$$\sigma_t^2 = \omega + \sum_{i=1}^{L_1} \alpha_i \varepsilon_{t-i}^2$$

Where L_1 denote the number of lags in the model, ω denote a constant, α_i denote the coefficient of the ARCH terms. To ensure positive volatility forecast, $\omega, \alpha_i > 0$ and $i = 1, 2 \ldots \ldots \ldots L_1$, and to ensure covariance stationary, we impose $\sum_{i=1}^{L_1} \alpha_i < 1$

Setting $i = 1$ in the equation, it will result to ARCH (1) model which states that the conditional variance of today's return residuals is equal to a constant, plus yesterday's square innovation; that is

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2$$

The unconditional volatility for ARCH (1) can be specify using the idea of moments of any order *m*, given as;

$$E(\varepsilon^m) = E\big(E_t(\varepsilon^m)\big) = E(\varepsilon_t^m)$$

Therefore, for all t

$$E(\varepsilon^2) = \sigma^2 = E(\varepsilon_t^2) = E(\sigma_t Z_t) = E(\sigma_t^2)$$

$$\sigma^2 = \omega + \alpha \varepsilon_{t-1}^2 = \omega + \alpha \sigma^2$$

[17] iid means independent and identical distribution

Thus the unconditional volatility for ARCH (1) is given as

$$\sigma^2 = \frac{\omega}{1-\alpha}$$

It is important to note that if $\alpha \geq 1$ the unconditional volatility of the *ARCH (1)* model is undefined. Only nonnegative constraint always has to be imposed and depending on the final application, we may or may not impose covariance stationary

3.5.2 Generalized ARCH (GARCH) Model

This model came into existence in order to improve the ARCH model in capturing volatility. The long lag lengths of the squared error that are required to capture all of the dependence in the conditional variance, might be very large and it is considered one of the biggest problems with the ARCH model. Thus by adding more lagged volatility during ARCH model creation, we have the potential to incorporate the dependence in the conditional variance. Thus this gives rise to the GARCH model, with the general form specify as *GARCH* (L_1, L_2)

$$\sigma_t^2 = \omega + \sum_{i=1}^{L_1} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{L_2} \beta_j \sigma_{t-j}^2$$

Where ω = constant, $L_1 and\ L_2$ = lag orders of ARCH and GARCH models respectively $\alpha_i\ and\ \beta_i$ are ARCH and GARCH terms respectively. The most commonly used form of *GARCH* (L_1, L_2) model is the *GARCH (1, 1),* derived from letting *i and j* in the general equation equals one. The specification is given as

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1} + \beta\sigma_{t-1}^2$$

The unconditional volatility for *GARCH (1, 1),* can be derive in a similar way as in the ARCH model, using the same assumptions. Focusing on the *GARCH (1, 1):*

$$\sigma^2 = E(\omega + \alpha\varepsilon_{t-1}^2 \beta\sigma_{t-1}^2) = \omega + \alpha\sigma^2 + \beta\sigma^2$$

Where $\sigma^2 = \omega + \alpha\sigma^2 + \beta\sigma^2$

Thus the unconditional volatility is given by

$$\sigma^2 = \frac{\omega}{1-\alpha-\beta}$$

In order for the *GARCH (1 1) model* to be efficient, the following conditions are impose on the parameters estimate

- $\alpha, \beta, \omega > 0$, to ensure positive volatility forecast.
- $\alpha + \beta < 1$, to ensure stationarity in covariance

Hence, when $\alpha + \beta = 1$, the unconditional variance is infinite and when $\alpha + \beta > 1$, the unconditional variance is undefined. Imposing the constraint when all we need is a forecast of the conditional volatility is not necessary, but it is necessary to predict the unconditional volatility.

3.5.3 Exponential GARCH (EGARCH) Model

In financial time-series, it has been stated that volatility behaves differently depending on if a positive or negative shock occurs. This asymmetric relationship is called leverage effect, and describes how a negative shock causes volatility to rise more than if a positive shock with the same magnitude had occurred. To capture this asymmetry the EGARCH model developed by ***Nelson*** (***1991***) is use. This model captures asymmetric responses of the time-varying variance to shocks and, at the same time, ensures that the variance is always positive. In general form, the conditional variance is written as:

$$log(\sigma_t^2) = \omega + \sum_{i=1}^{L_1} \{\alpha_i(|Z_{t-i}| - E[|Z_{t-i}|]) + \gamma_i Z_{t-i}\} + \sum_{j=1}^{L_2} \beta_j log\left(\sigma_{t-j}^2\right)$$

$$Z_{t-i} = \frac{\varepsilon_{t-i}}{\sigma_{t-i}}$$

Where γ is another parameter to be estimated along with $\alpha\ and\ \beta$. This equation contains the difference between absolute residuals and the expectation of absolute residuals, which gives rise to leverage effects. This model does not require any restriction on the parameters, since the equation is on log variance instead of variance itself, the positivity of the variance is automatically satisfied[18]. In interpreting the model, the impact is asymmetric if $\gamma_i \neq 0$ and the presence of leverage effect is indicated by $\gamma_i < 0$. In macroeconomic analysis, financial markets and corporate finance, a negative shock usually implies bad news, leading to a more uncertain future. Consequently, shareholders would require a higher expected return to compensate for bearing increased risk in their investment (***Wang, 2003***).

The most commonly used form of *EGARCH (L_1, L_2)* model is the *EGARCH (1, 1),* derived from letting *i and j* in the general equation equals one. The specification is given as

$$log(\sigma_t^2) = \omega + \{\alpha(|Z_{t-1}| - E[|Z_{t-1}|]) + \gamma Z_{t-1}\} + \beta log(\sigma_{t-1}^2)$$

$$Z_{t-1} = \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

The value of $E(|Z_t|)$ depends on the density function of Z_t. That is $(|Z_t|) = \sqrt{2/_{\pi}}$, for a normal distribution and $E(|Z_t|) = \frac{2\Gamma[(1+v)/2]\sqrt{v-2}}{1+(v-1)\Gamma(v/2)\sqrt{\pi}}$, for student-t distribution.

[18] Jon Danielsson, 2011. Financial Risk Forecasting: Theory and Practice of forecasting Market Risk. Pp 35-52

3.5.4 Glosten, Jagannathan and Runkle GARCH (GJR-GARCH) Model

Another widely used GARCH model allowing for leverage effects is the GJR-GARCH, also known as threshold-GARCH, developed by ***Glosten et al***. (***1993***).

$$\sigma_t^2 = \omega + \sum_{i=1}^{L_1} (\alpha_i + \gamma_i I_{t-i})\varepsilon_{t-i}^2 + \sum_{j=1}^{L_2} \beta_i \sigma_{t-j}^2$$

$$I_{t-i} = \begin{cases} 0 \;\; if \;\; \varepsilon_{t-i} \geq 0 \\ 1 \; if \;\; \varepsilon_{t-i} < 0 \end{cases}$$

For leverage effect, $\gamma_i > 0$. Notice now that the condition for non-negativity will be $\omega > 0, \alpha_i > 0, \beta_i \geq 0$ and $\alpha_i + \gamma_i \geq 0$. That is the model is still admissible[19] even if $\gamma_i < 0$, provided that $\alpha_i + \gamma_i \geq 0$

3.5.5 Asymmetric Power ARCH Model (APARCH)

Due to the fact that the normal GARCH model fail to capture the leverage effect and also fail to capture fact that absolute returns may sometimes have stronger autocorrelation than squared returns (power effect), ***Ding et al***. (***1993***) combine these two effects in the same model, to produce the so-called asymmetric power GARCH (APARCH) model in order to capture the aspect of asymmetry.

$$\sigma_t^\delta = \omega + \sum_{i=1}^{L_1} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{L_2} \beta_j \sigma_{t-j}^\delta$$

Where $\delta, \omega, \alpha, \beta > 0 \;\; and - 1 < \gamma_i < 1$. Also γ_i are the leverage parameters and allows for leverage effects when $\gamma \neq 0$. Again, δ is the power parameter and allows for power effect when $\delta \neq 2$. The ***Bollerslev*** (***1986***) model sets $\delta = 2, \gamma = 0$, and the ***Taylor*** (***1986***) model sets $\delta = 1$ and $\gamma = 0$. Empirical literature shows that the power term is sample dependent and in case of stock data often amounts to near unity (***Ding et al., 1993***), while in case of foreign exchange data often amounts between unity and two (***Mitchell and McKenzie, 2008***)[20]. The APARCH model will give rise to the following;

- ARCH model when $\delta = 2, \gamma = 0, \beta = 0$
- GARCH model when $\delta = 2, \gamma = 0$
- GJR-GARCH model when $\delta = 2$,
- TARCH model when $\delta = 1$,

[19] Glosten, L. R., R. Jagannathan, and D. E. Runkle, 1993. On The Relation between The Expected Value and The Volatility of Nominal Excess Return on stocks. Journal of Finance 48: 1779-1801.

[20] Jon Danielsson, 2011. Financial Risk Forecasting: Theory and Practice of forecasting Market Risk. Pp 52

3.6 Maximum Likelihood Estimation of Volatility Models

Since the nonlinear nature of volatility models discussed above cannot be captured by the standard regression method (ordinary least squares), the maximum likelihood (ML) technique proposed by ***Bollerslev and Wooldridge in 1992*** is the most appropriate and widely used estimation procedure for estimating the parameters of the model. The technique requires that the distribution of the innovation process Z_t is specified.

3.6.1 Normal Distribution

Weiss (***1986***), ***Bollerslev and Wooldridge*** (***1992***) showed that under the normality assumption, the quasi-maximum likelihood (QML) estimator is consistent if the conditional mean and the conditional variance are correctly specified. The log-likelihood function of the standard normal distribution is given by

$$L_{Gauss} = -\frac{1}{2}\sum_{t=1}^{T}[log(2\pi) + log(\sigma_t^2) + Z_t^2],$$

Where T is the number of observations and $Z_t = {}^{\varepsilon_t}/_{\sigma_t}$

Under the assumption of a normal distribution, we can write the maximum likelihood estimation for both ARCH (1) and GARCH (1 1) bellow;

ARCH (1) likelihood function

Suppose the errors, Z_t in an ARCH (1) model is standard normal distribution;

$$Z_t \sim N(0,1)$$

$$\varepsilon_t = \sigma_t Z_t$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2$$

The presence of lagged residual returns means that the density function for $t = 1$ is unknown since we do not know ε_0. Thus $t = 2$ density is given as;

$$f(\varepsilon_2) = \frac{1}{\sqrt{2\pi(\omega + \alpha\varepsilon_1^2)}} exp\left(-\frac{1}{2}\frac{\varepsilon_2^2}{\omega + \alpha\varepsilon_1^2}\right)$$

Higher period densities are derived in similar manner. The joint density of ε is given as follows

$$\prod_{t=2}^{T} f(\varepsilon_t) = \prod_{t=2}^{T} \frac{1}{\sqrt{2\pi(\omega + \alpha\varepsilon_{t-1}^2)}} exp\left(-\frac{1}{2}\frac{\varepsilon_t^2}{\omega + \alpha\varepsilon_{t-1}^2}\right)$$

The log-likelihood follows;

$$\log L = \theta - \frac{1}{2}\sum_{t=2}^{T}\left(log(\omega + \alpha\varepsilon_{t-1}^2) + \frac{\varepsilon_t^2}{\omega + \alpha\varepsilon_{t-1}^2}\right)$$

Where $\theta = -\frac{T-1}{2}log(2\pi) \rightarrow constant$

GARCH (1, 1) likelihood function

The normal GARCH (1, 1) is defined as

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Since the same issue as before arises with the presence of lagged residual returns, we therefore start with a density of $t = 2$

$$f(\varepsilon_2) = \frac{1}{\sqrt{2\pi(\omega + \alpha\varepsilon_1^2 + \beta\hat{\sigma}_1^2)}}exp\left(-\frac{1}{2}\frac{\varepsilon_2^2}{\omega + \alpha\varepsilon_1^2 + \beta\hat{\sigma}_1^2}\right)$$

The log-likelihood follows;

$$\log L = \theta - \frac{1}{2}\sum_{t=2}^{T}\left(log(\omega + \alpha\varepsilon_{t-1}^2 + \beta\hat{\sigma}_{t-1}^2) + \frac{\varepsilon_2^2}{\omega + \alpha\varepsilon_{t-1}^2 + \beta\hat{\sigma}_{t-1}^2}\right)$$

Where $\theta = -\frac{T-1}{2}log(2\pi) \rightarrow constant$

Also, it is possible to generate the conditional likelihood function for other conditional distributions such as student-t, skewed student-t generalized error distributions in an analogous fashion like the normal distribution.

3.6.2 Student-t Distribution

Due to the fact that normal distribution cannot account for the pronounced "fat tails" of exchange rate returns, a student t distribution is likely the most common distribution to be used capture stylized fact of financial returns. (***see Palm, 1996; Pagan, 1996 and Bollerslev, Chou, and Kroner, 1992***). The log-likelihood for a Student-t distribution is:

$$L_{Stud} = T\left\{\log\Gamma\left(\frac{v+1}{2}\right) - log\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}log[\pi(v-2)]\right\}$$

$$-\frac{1}{2}\sum_{t=1}^{T}\left[log(\sigma_t^2) + (1+v)log\left(1 + \frac{Z_t^2}{v-2}\right)\right]$$

Where v is the degree of freedom, $2 < v \leq \infty$ and $\Gamma(.)$ is the gamma function

This density account for fat tails but not asymmetry. Both skewness and kurtosis, however, are important in financial applications, such as in asset pricing models, Value-at-Risk, portfolio selection and option pricing theory.

3.6.3 Skewed-Student-t Distribution

To properly model skewness, ***Lambert and Laurent*** (***2000, 2001***) and ***Bauwens and Laurent*** (***2005***) apply and extend the skewed-Student density proposed by ***Fern´andez and Steel*** (***1998***) to the GARCH framework. The log-likelihood of the standardized (zero mean and unit variance) skewed-Student is:

$$L_{SkSt} = T\left\{\log\Gamma\left(\frac{v+1}{2}\right) - log\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}log[\pi(v-2)] + log\left(\frac{2}{\xi + \frac{1}{\xi}}\right) + log(s)\right\}$$

$$-\frac{1}{2}\sum_{t=1}^{T}\left\{log\sigma_t^2 + (1+v)log\left[1 + \frac{(sZ_t + m)^2}{v-2}\xi^{-2I_t}\right]\right\},$$

Where $$I_t = \begin{cases} 1 \; if \; Z_t \geq -\frac{m}{s} \\ -1 \; if \; Z_t < -\frac{m}{s} \end{cases}$$

$\boldsymbol{\xi}$ is the asymmetry parameter, v is the degree of freedom of the distribution

$$m = \frac{\Gamma\left(\frac{v+1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}T\left(\frac{v}{2}\right)}\left(\xi - \frac{1}{\xi}\right)$$

$$s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}$$

There are other definitions of skewed-Student-t distribution (see for example ***Hansen, 1994; Mittnik and Paolella, 2000; Aas and Haff, 2006; Dark, 2010; Deschamps, 2011***). ***Aas and Haff*** (***2006***) extend the skewed-Student-t distribution to the Generalized Hyperbolic skewed-Student distribution (GHSST), while ***Deschamps*** (***2011***) proposes a Bayesian estimation of GARCH models with GHSST errors. ***Forsberg and Bollerslev*** (***2002***) use a GARCH model with Normal Inverse Gaussian (NIG) error distributions on exchange rate data.

Value at Risk (VaR)

Usually financial return series are assumed to follow a normal distribution but past studies have proven that it is not always the case. Thus assuming normality when dealing with asset return series that are not normally distributed could under estimate volatility and cause significant bias in estimating Value at Risk (VaR). A number of

authors have stipulated that the standard GARCH model under normal distribution have inferior forecasting performance as compare to a model that captures tails thickness. Thus to capture tails thickness, Bellerslev (1987) estimated the standard GARCH under student-t distribution with $v > 2$ degrees of freedom.

The one-step-ahead conditional variance forecast $\hat{\sigma}_{t+1|t}$ for GARCH, EGARCH, GJR-GARCH and APARCH respectively is given as;

$$\sigma_{t+1|t}^2 = \omega + \sum_{i=1}^{L_1} \alpha_i \varepsilon_{t-i+1}^2 + \sum_{j=1}^{L_2} \beta_j \sigma_{t-j+1}^2 \quad \sim GARCH(L_1, L_2)$$

$$log\left(\sigma_{t+1|t}^2\right) = \omega + \sum_{i=1}^{L_1} \{\alpha_i (|Z_{t-i+1}| - E[|Z_{t-i+1}|]) + \gamma_i Z_{t-i+1}\} + \sum_{j=1}^{L_2} \beta_j log\left(\sigma_{t-j+1}^2\right) \quad \sim EGARCH(L_1, L_2)$$

$$\sigma_{t+1|t}^2 = \omega + \sum_{i=1}^{L_1} (\alpha_i + \gamma_i I_{t-i+1}) \varepsilon_{t-i+1}^2 + \sum_{j=1}^{L_2} \beta_i \sigma_{t-j+1}^2 \quad \sim GJR - GARCH(L_1, L_2)$$

$$\sigma_{t+1|t}^\delta = \omega + \sum_{i=1}^{L_1} \alpha_i (|\varepsilon_{t-i+1}| - \gamma_i \varepsilon_{t-i+1})^\delta + \sum_{j=1}^{L_2} \beta_j \sigma_{t-j+1}^\delta \quad \sim APARCH(L_1, L_2)$$

Therefore, in order to compute the one-step-ahead VaR forecast under all distributions, we compute the corresponding quantiles for the assumed distribution, which are then, multiply by the forecasted conditional standard deviation, thus;

$$VaR_{t+1|t} = F(\emptyset)\hat{\sigma}_{t+1|t}$$

Where $F(\emptyset)$ is the corresponding quantile of the assumed distribution and $\hat{\sigma}_{t+1|t}$ is the forecast standard deviation at time t. So, if an investor wishes to invest in the market with a fixed position he/she is expect to suffer a loss equivalent to

$$VaR_{t+1|t} = Amount\ of\ position \times F(\emptyset)\hat{\sigma}_{t+1|t}$$

According to Tsay (2010), if one further assumes that that Z_t is Gaussian, the conditional distribution of r_{t+1} given the information available at time t is $N\left(\hat{r}_{t+1|t},\ \hat{\sigma}_{t+1|t}\right)$. Quantiles of this conditional distribution can easily be obtained for VaR calculation For example, the 5% quantile is

$$\hat{r}_{t+1|t} - 1.645\hat{\sigma}_{t+1|t}$$

Where $\hat{r}_{t+1|t}$ is the 1-step ahead forecast of the conditional mean return

If Z_t is a GARCH (L_1, L_2) model as shown above follows a standardized Student-t distribution with v degrees of freedom and the probability is p, then the quantile used to calculate the 1-period horizon VaR at time index t is

$$\hat{r}_{t+1|t} - \frac{t_v(p)\hat{\sigma}_{t+1|t}}{\sqrt{v/(v-2)}}$$

Where $t_v(p)$ is the $p - th$ quantile of a student-t distribution[21]

3.7 Diagnosing Volatility Models

There are several methods available for comparing and accessing the quality of conditional volatility models such as the AIC and BIC goodness of fit test, likelihood ratio and parameter significance, residuals analysis model and statistical goodness of fit measure. In this work, the AIC and BIC will be used for model comparison, while the residual analysis model will be used to access the quality of the model.

3.7.1 AIC and BIC Goodness of fit

Just as with the case of ARMA model, the Akaike information criterion (AIC) and Bayesian information criterion (BIC) are two measures of goodness of fit for volatility models. They measure the trade-off model fit and model complexity. The more estimated parameters we have, the more penalties we will have in the model. Hence any added parameter that fail to improve the fit (AIC or BIC) should be eliminated since we need a model that parsimonious and less complex. The AIC and BIC expressions can be seen above (section 3)[22]

3.7.2 Analysis of model Residuals

This approach will be considered to access the quality of the best volatility models. Consider the normal ARCH (1) model. If the model is correct, the residuals are IID normal. This suggests that if estimated parameters and forecast volatility ($\hat{\alpha}, \hat{\beta}, \hat{\sigma}_t^2$) are obtained, the estimated or fitted residuals

$$\hat{z}_t = \frac{\hat{\varepsilon}_t}{\hat{\sigma}_t},$$

can be tested for normality and clustering, providing an assessment of how well the model captured stylized facts in the data[23].

[21] Rues S Tsay (2010). Analysis of Financial Tie Series. Pp 266-277
[22] Jonathan D. Cryer and Kung-Sik Chan. Time Series Analysis with Applications in R, Second Edition.
[23] Jon Danielsson, 2011. Financial Risk Forecasting: Theory and Practice of forecasting Market Risk. Pp 41-52

Chapter 4

4 Empirical Results and Discussion

This section present all the empirical results described in the methodology above. To begin with the modeling of exchange rate volatility using the conditional volatility models, it's important to check the stationarity condition of the original exchange rate time series using the ADF test for unit root. The essence is to avoid over differencing which might introduce dependence when none exists.

4.1 Augmented Dickey Fuller Test (ADF Test)

Usually, the null hypothesis of the ADF test says XAF/USD exchange rate series has unit root. From the result of the ADF tests below, ***tau3*** refers to the null hypothesis that there is a unit root; ***phi3*** refers to the null hypothesis that there is a unit root and no-trend and ***phi2*** refers to the null hypothesis that there is unit root without trend and without drift. Dickey-Fuller test decision rule says, if the p-value is less than 5% or if the test statistic is more than the critical value in absolute magnitude, the null hypothesis is rejected and a conclusion is made that no unit root is present.

Table 4. 1. ADF test on the raw data (Daily XAF/USD exchange rates)

p-value	0.1027		
Value of test-statistic	-2.0565	1.8368	2.4225
Critical values for test statistics:	1%	5%	10%
tau3	-3.96	-3.41	-3.12
phi2	6.09	4.68	4.03
phi3	8.27	6.25	5.34

Table 4. 2. ADF test on the differenced data (Daily XAF/USD exchange rate returns)

p-value	0.000		
Value of test-statistic	-32.3827	349.5467	524.3197
Critical values for test s tatistics:	1%	5%	10%
tau3	-3.96	-3.41	-3.12
phi2	6.09	4.68	4.03
phi3	8.27	6.25	5.34

Source: Author's Calculation

Looking at the ADF test result on daily XAF/USD exchange rates in the table 4.1 above, we fail to reject the null hypothesis (tau3, phi2 and phi3) at 1%, 5% and 10% since the values of the test statistic are less than those of the critical values in absolute magnitude, which is also in conformity with the p-value (0.1027 > 5%). Hence the XAF/USD exchange rate series has unit root and it's not stationary (series resembles a random walk). As in most empirical financial literature, the exchange rate series has to be transformed by applying first difference of natural log, as described in section 2.5 above to achieve stationarity. The ADF test result of the differenced series (daily XAF/USD exchange rate returns) is shown in table 4.2 above. From the result, we reject the null hypothesis (tau3, phi2 and phi3) at 1%, 5% and 10% since the values of the test statistic are greater than those of the critical values in absolute magnitude, which is also in conformity with the p-value (0.000 < 5%). Hence the XAF/USD exchange rate series has no unit root and it's stationary. A plot of the daily XAF/USD exchange rate returns is shown below;

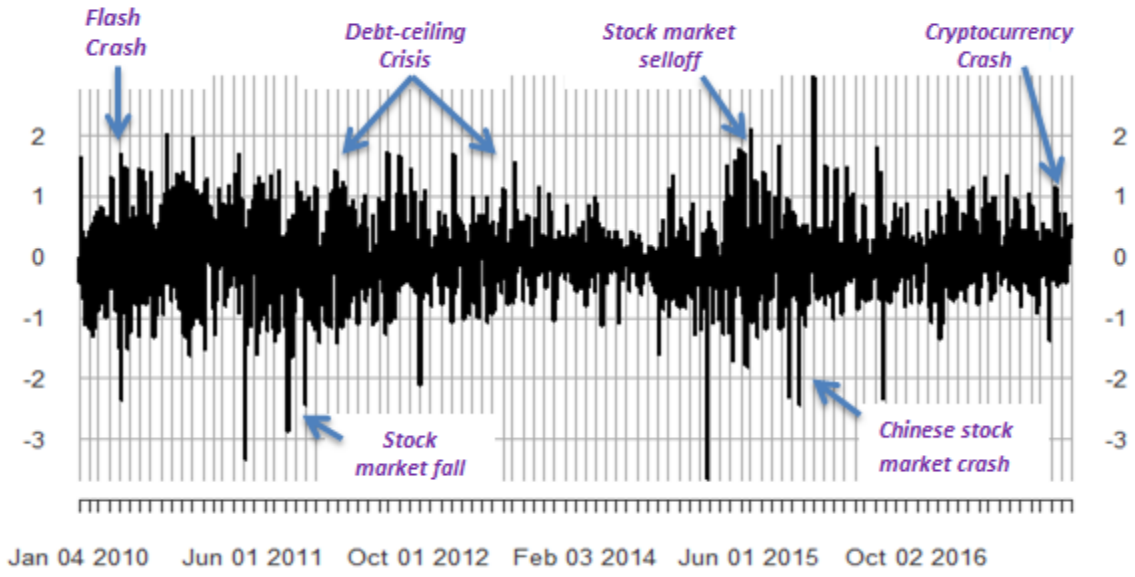

Figure 4. 1. Daily XAF/USD Exchange Rate Returns, January 2010 to January 2018

The compound returns are depicted in Figure 4.1. It can be observe that periods of high volatility are followed by periods of high volatility and periods of low volatility are followed by periods of low volatility for a prolong period, indicating the presence of volatility clustering. This suggests the residuals or error terms are conditionally heteroscedastic. There is high volatility during the U.S. Debt Ceiling Crisis in 2011 and 2013, Flash Crash in 2010, Stock Market fall of 2011 and Chinese Stock Market Crash in 2015 to 2016 among others. Prolonged periods of high volatility are generally associated with great uncertainty in the real economy. The return series also shows a constant mean and variance and by taking the first difference, the first element of the series is lost. Looking at the time series returns plot, there is improvement and the series resembles a white noise. However there are still some outliers, especially in the early-2011 and 2015 periods, where exchange rate returns plummets (fall or drop straight down at high speed).

4.2 Descriptive Statistics of the Daily Returns of XAF/USD Exchange Rates Series

Table 4. 3. Descriptive statistics for daily exchange rates return

Variable	$r_{t(XAF/USD)}$
Mean	-0.0097
Median	0.00
Maximum	2.98
Minimum	-3.67
Standard Deviation	0.58
Skewness	-0.21
Kurtosis	2.56
Jarque-Bera test (p-value)	0.00
Ljung-Box test 20 lags (p-value)	0.93
Ljung-Box test squared returns 20 lags (p-value)	0.00
Number of Observations	2079

Source: Authors' Calculation

A summary statistics for daily XAF/USD exchange rate returns from 1st January 2010 to 4th January 2018 is presented in the table 4.3. The mean is very small (-0.97%) and negative, suggesting that exchange returns series is decreasing over time, while the volatility value is around 58%. The fact that the daily mean value is only about one-sixtieth of the daily volatility in absolute magnitude will simplify the construction of risk measures as we can effectively assume the mean to be zero without loss of generality. Furthermore, the mean grows at a linear rate while volatility grows approximately at a square root rate, so over time the mean dominates volatility. The worse daily return is -3.67, and corresponds to Chines Stock Market Crash from 2015 to 2016, while the best daily return is 2.98, which correspond to late 2015, a period of low inflation in U.S. Also, the returns have a negative skewness (skewed to the left) indicating that the returns follow an asymmetric distribution and more importantly the kurtosis of returns is 5.56 which is greater than three, indicating that the distribution of returns follows a fat-tailed distribution, thereby exhibiting one of the important characteristics of financial time series data, namely that of leptokurtosis. Hence the Jarque-Bera test p-value of 0.00 (less than 5%) confirms that we reject the null hypothesis of normality and conclude that the data is not from a normal distribution.

Finally, the Ljung Box test results of the returns seris at 20 lags shows a p-value greatherthan 5%, indicating the absence of autocorrelation in the series, which is incontrast with the squared resturns. A detailed illustration of autocorelation in the returns series, with a 30 lags maximum, using the ACF and PACF plots is dipicted in Figure 4.2 below. It is well-known that for random and independent series of length n, the lag k autocorrelation coefficient is normally distributed with a mean of zero, a variance of $1/n$, and the 95% confidence limits are given by $\pm 1.96/\sqrt{n}$ (critical value). For the ACF plot, it can be observed that autocorrelation is not significant (no lag exceed the blue critical value lines). This is a clear indication that autocorrelation is absent in the series from lag 1 to 30. Hence, the return series behave like white noise since ACF is zero. Looking at the PACF plot, there is no significant lag (approximately

zero), hence no autoregressive component not is present in the series. With this idea, it is certain that when estimating the conditional mean models (ARMA), the order of the Moving average and Autoregressive components are approximately zero.

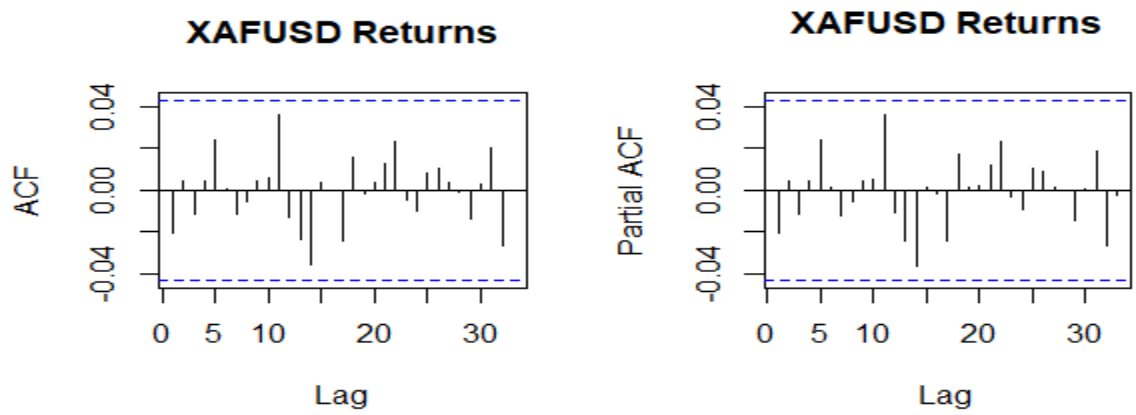

Figure 4. 2. ACF and PACF of the Daily Returns of XAF/USD Exchange Rates Series

4.3 Conditional Mean

After interpreting the autocorrelation (ACF) and partial autocorrelation (PACF) of daily returns of XAF/USD exchange rate given in Figure 4.2, an autoregressive-moving-average model is used to fit the mean returns, as it provides a flexible and parsimonious approximation to conditional mean dynamics. The Akaike and Bayessian information criteria are used to select the best mean model. Hence an *ARMA (p, q)* model is fitted on the XAF/USD exchange rate returns series.

Table 4. 4. Criteria for ARMA (p, q) order selection

Models		AIC	BIC	Log Likelihood
ARMA (0, 0)	With non-zero mean	3677.55	3688.83	-1837.38
ARMA (0, 0)	**With zero mean**	**3676.13**	**3681.77**	**-1837.40**
ARMA (0, 1)	With non-zero mean	3678.71	369563	-1836.35
ARMA (1, 0)	With non-zero mean	3678.70	3696.62	-1836.35
ARMA (1, 1)	With non-zero mean	3680.70	3704.26	-1836.35

Source: Authors' Calculations

From the table above, it can easily be observe that the best conditional mean model that fit the daily returns of XAF/USD exchange rates series is ARMA (0, 0) with zero mean, since it has the lowest AIC and BIC. This is inconformity with the ACF and PACF plots depicted in Figure 4.2 above. Since we are interested in modeling

volatility of daily exchange rates returns, we extract the residuals from the best conditional mean model ARMA (0, 0), square it, to test for ARCH effects using McLeod-Li Test.

4.3.1 Estimated mean Equation

The estimated mean equations can be written using the output from ***Rstudio.*** For example using ARMA (0, 0), the outputs are shown below and from it, we can write the equation.

```
Call:
arma(x = XAFUSD_Returns, order = c(0,0))

Coefficients:
      intercept
        -0.0098
s.e.     0.0128

sigma^2 estimated as 0.3427:  log likelihood = -1836.78,  aic = 3677.55
```

The full mean equation:

$$(\mathbf{1}-\mathbf{L})\mathbf{XAFUSD\_Returns_t} = \boldsymbol{\varepsilon_t} - \mathbf{0.0098}$$

4.3.2 Model Diagnostics

The 3 plots below shows the Standardize Residuals, ACF of Residual and p-values of Ljung-Box test respectively. These three plots provide information about model performance using the best fitted conditional mean model and the red lines outline the presence of outlying observations.

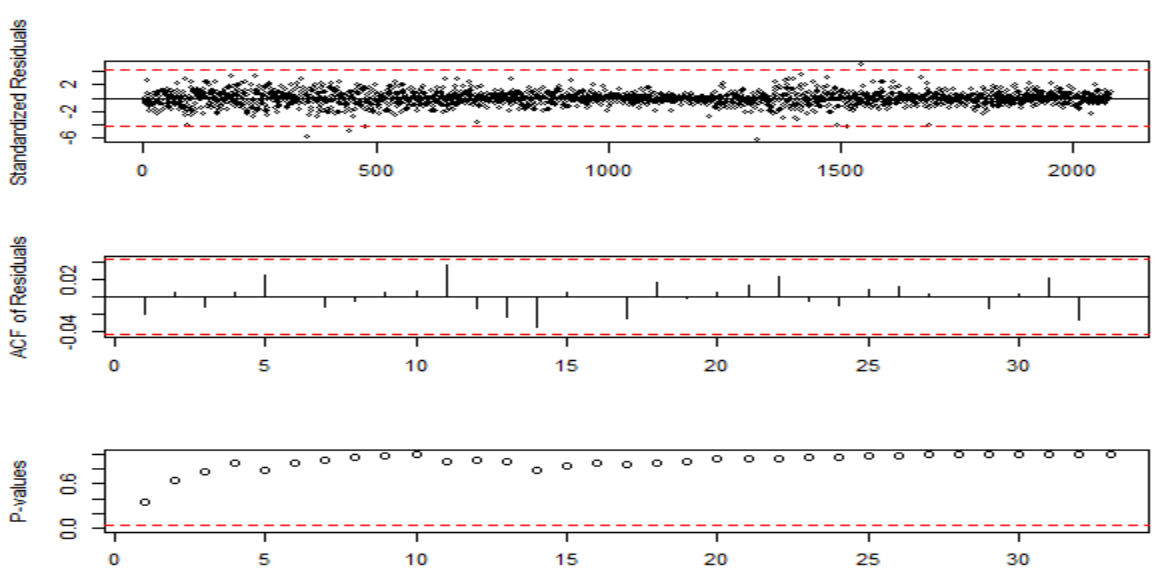

Figure 4. 3. Residuals diagnostics on the fitted ARMA (0, 0) model

Standardized residuals are obtained by subtracting from this time series, its sample mean and dividing by square root of sample variance. At first sight they look satisfactorily because they have a constant mean and variance with no defined pattern. We proceed to the second plot and access the estimated ACF of residuals. It shows that all the lags are within the boundaries lines, so the estimated residuals of the best mean model indicate that there is no autocorrelation. Thus this estimated autocorrelation function seems to fit the model properly and residuals behave like a white noise. In the third plot, we can look at p-values of Ljung-Box test run in correspondence with different lags. The P- Values are sensibly greater than the red line which represents the critical value, so we are induced to accept the null hypothesis that all autocorrelation coefficients are 0.

Also, the estimated partial autocorrelation function of the residuals, plotted in figure 4.4 below behaves very nicely.

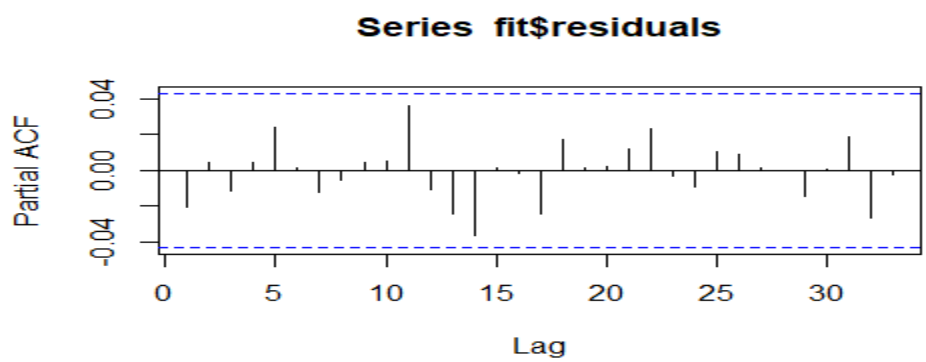

Figure 4. 4. Estimated PACF of residuals on the fitted model ARMA (0, 0)

The results of the estimated residuals from our best mean model indicates that they behave like a white noise (they are independent and identical with no serial correlation). At this point we perform a Q-Q plot to check the normality of the residuals.

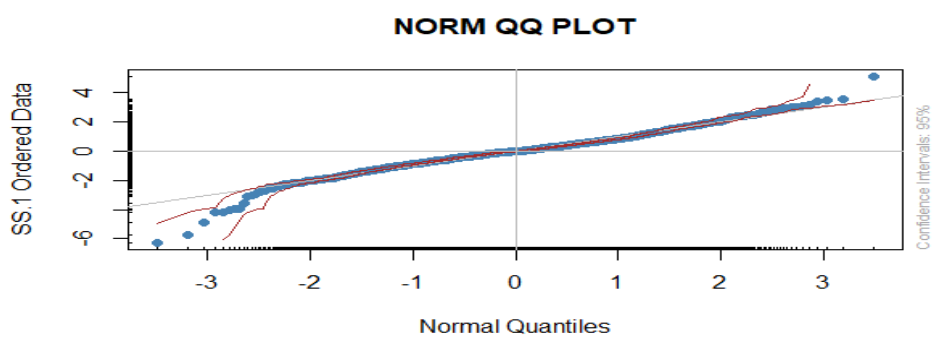

Figure 4. 5. Normal Q-Q plot for residuals of ARMA (0, 0) model

The Q-Q plot depicted in Figure 4.5 above shows that the residuals exhibit fat tails at both ends, with the left tail fatter than the right (negatively skewed). This aspect of asymmetry indicates the residuals fail to follow a normal distribution, which is inconformity with Jarque-Bera test of residuals, with p-value of zero.

In conclusion, the diagnostic check indicates that the residuals are independent and they are not correlated as indicated by the Ljung-Box test result which is desirable, but nevertheless they are not normally distributed as indicated by the Q-Q plot. Subsequently we can assume normality or use a fat tail distribution such as student-t distribution to approach normality. Hence we assume we have a good model fitting.

4.4 Testing Squared Residuals for ARCH Effect

The graphs of squared residuals and their correlogram, together with their estimated partial autocorrelation have been plotted. Subsequently, the Mcleod-Li test on the squared residuals in order to detect the presence of ARCH effect in the model.

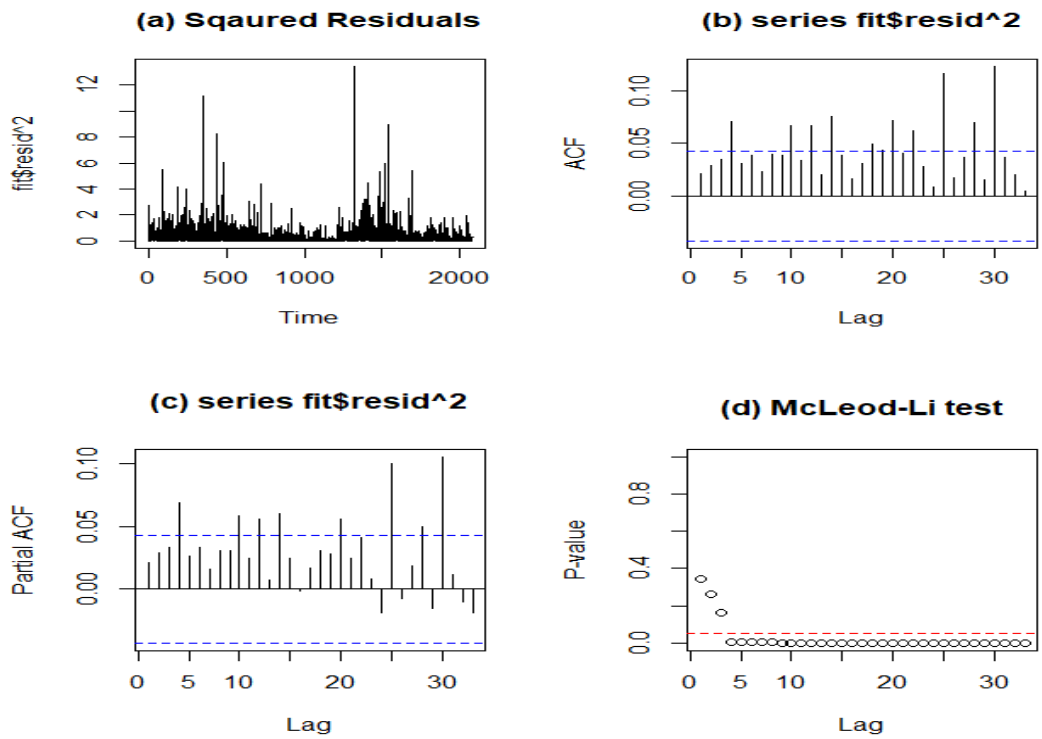

Figure 4. 6. Graphs of Squared Residuals, Correlogram, Estimated PACF and McLeod-Li test on Squared Residuals

From Figure 4.6 (a), we can observe alternating periods of high and low variances, indicating that there is a sort of dependence in the squares residuals which determines some clusters in the volatility. This sort of movement in volatility (clustering) is very frequent in financial time series. Also, Figure 4.6 (b) and Figure 4.6 (c) shows that significant lags cut across the critical value line (blue line), indicating that there is correlation in the squared series. Hence there is linear dependence in the squared residuals. Lastly, the p-value of the McLeod-Li test depicted in Figure 4.6 (d) shows that majority of observations lies below the red critical value line (p-value less than 5%), which confirms the presence of autocorrelation in the squared residuals. Hence, for lags greater than equal to 4, we reject the null hypothesis of no ARCH effect and confirm that the XAF/USD exchange rate return data suffers

from ARCH effect. This is a strong signal that we can now proceed with the estimation of ARMA(0, 0)+GARCH family of models on the return series

4.5 Estimated Result for Volatility models

Based on the analysis of the squared residuals, which confirms that the daily returns of XAF/USD exchange rate data suffers from ARCH effect, we proceed to examine dynamics in volatility using the *symmetric* GARCH models (ARCH and GARCH), which treats the shocks or errors or innovations as symmetric, meaning that shocks affect the conditional variance in the same way whether positive or negative and the *asymmetric* GARCH models (EGARCH, GJR-GARCH and APARCH) which treat the shocks (positive or negative) to have different effect on conditional variance. The models are estimated using the maximum likelihood method under the assumption of several distributions of the innovation terms such as: Normal (Gaussian), Student-t and skew student-t distributions and information criteria (AIC and BIC) are used for best model selection.

Table 4. 5. Symmetric GARCH Models estimated using daily XAF/USD exchange rate returns from January 2010 to January 2018

Conditional distribution	ARCH(1)	ARCH(2)	GARCH(2,1)	GARCH(1,2)	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
	Normal	Normal	Normal	Normal	Normal	Student-t	Skew student-t
Log-likelihood	-1835.236	-1829.741	-1745.659	-1746.758	-1746.828	-1678.930	-1678.921
ω	0.329 (0.000)	0.297 (0.000)	0.001 (0.012)	0.001 (0.018)	0.001 (0.011)	0.000 (0.842)	0.000 (0.851)
α_1	0.041 (0.045)	0.035 (0.123)	0.000 (1.000)	0.019 (0.002)	0.019 (0.000)	0.026 (0.000)	0.026 (0.000)
α_2		0.108 (0.005)	0.02 (0.405)				
β_1			0.976 (0.000)	0.978 (0.018)	0.978 (0.000)	0.973 (0.000)	0.973 (0.000)
β_2				0.000 (1.000)			
shape						6.040 (0.000)	6.041 (0.000)
skew							0.996 (0.000)
JB test (p-value)	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ljung-Box test (20 squared lags)	0.000	0.000	0.812	0.775	0.776	0.695	0.696
ARCH Test	0.000	0.006	0.966	0.949	0.949	0.852	0.852
AIC	1.767	1.763	1.683	1.684	1.683	1.619	1.620
BIC	1.773	1.771	1.694	1.695	1.691	1.630	1.633

Source; Author's calculation

Table 4. 6. Asymmetric GARCH Models estimated using daily XAF/USD exchange rate returns from January 2010 to January 2018

Conditional distribution	APARCH(1,1)			GJR-GARCH(1,1)			EGARCH(1,1)		
	Normal	Student-t	Skew Student-t	Normal	Student-t	Skew Student-t	Normal	Student-t	Skew Student-t
Log-likelihood	-1739.486	-1676.367	-1676.360	-1741.945	-1677.929	-1677.930	-1737.423	-1675.434	-1675.423
ω	0.001 (0.032)	0.000 (0.578)	0.000 (0.576)	0.000 (0.081)	0.000 (0.624)	0.000 (0.626)	-0.002 (0.000)	-0.003 (0.000)	-0.003 (0.000)
α_1	0.016 (0.000)	0.028 (0.000)	0.028 (0.000)	0.005 (0.015)	0.015 (0.000)	0.015 (0.000)	-0.025 (0.000)	-0.021 (0.003)	-0.021 (0.003)
β_1	0.984 (0.000)	0.976 (0.000)	0.976 (0.000)	0.985 (0.000)	0.978 (0.000)	0.978 (0.000)	0.998 (0.000)	0.998 (0.000)	0.998 (0.000)
$gamma\ (\gamma)$	0.639 (0.000)	0.251 (0.026)	0.251 (0.027)	0.016 (0.000)	0.012 (0.065)	0.012 (0.066)	0.031 (0.000)	0.055 (0.000)	0.055 (0.000)
$delta\ (\delta)$	1.303 (0.000)	1.411 (0.000)	1.408 (0.000)						
shape		6.183 (0.000)	6.185 (0.000)		6.355 (0.000)	6.355 (0.000)		6.176 (0.000)	6.177 (0.000)
skew			0.997 (0.000)			0.999 (0.000			0.996 (0.000)
JB test (p-value)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ljung-Box test (20 squared lags)	0.695	0.588	0.589	0.733	0.639	0.639	0.640	0.571	0.571
ARCH Test	0.841	0.762	0.762	0.852	0.760	0.760	0.802	0.752	0.752
AIC	1.678	1.618	1.619	1.680	1.619	1.620	1.675	1.617	1.618
BIC	1.692	1.635	1.638	1.691	1.633	1.636	1.686	1.630	1.634

Source; Author's calculation

Tables 4.5 and 4.6 above, only shows the estimated optimal coefficients and probabilities (in blue) of the analysis. The standard errors and t values are not shown because; we can easily use the estimated probabilities (p-values) to interpret the significance of the estimated coefficients. A p-value of less than 5% indicates that the estimated parameter is statistically significant and hence captures volatility clustering under the GARCH models. The order of the estimated parameters is increased based on the significance of the previous. A three decimal place approximation has been assumed for simplicity.

4.5.1 Estimation Results of Symmetric GARCH Models

The estimated coefficients of the ARCH models are stationary (less than one), while all the normal GARCH models produces $\alpha + \beta < 1$, indicating that all the normal GARCH models are also stationary in covariance. The student-t

variants produces $\alpha + \beta = 0.999$, which indicates that there exist covariance stationary under student-t distribution as well. Hence there's no violation of covariance stationary and volatility is not explosive under the conditional distributions used in this studies. Again, as its typical of GARCH model estimates for financial asset returns data, the sum of the coefficients of the lagged squared error and lagged conditional variance terms is very close to unity (approximately 0.999). This implies that shocks to the conditional variance will be highly persistent. The constant term omega (ω) is significant in all the cases except for GARCH (1, 1) under student-t and skew student-t distributions. For the GARCH (1, 1) model, 3 coefficients are quite similar. The degrees of freedom (shape parameter) are 6.04 and 6.041 and the skewed parameter is 0.996.

Firstly I fit a simple ARCH (1) model and obtain an insignificant coefficient. Fitting ARCH (2) produces a result which indicates that the log-likelihood (increase) and AIC/BIC (decrease) coefficients is slightly improved, making ARCH (2) better than ARCH (1), but α_1 is still insignificant, while α_2 is significant. Hence increasing the order of ARCH terms in the model will produce more insignificant terms even though the log-likelihoods, AICs and BICs may be improved. A better model is to add the lagged volatility, to get GARCH (1, 1), which produces a much more improved AIC, BIC and log-likelihood, with both α_1 and β_1 terms significant indicating that volatility clustering is captured. Also the significance of both α_1 and β_1 also indicates that, lagged conditional variance and lagged squared disturbance have an impact on the conditional variance, in other words this means that news about volatility from the previous periods have an explanatory power on current volatility. Increasing the order of the ARCH term to get GARCH(2, 1) and increasing the order of the lagged volatility to get GARCH(1, 2), both produces results with insignificant terms, indicating that they fail to significantly improve the GARCH (1, 1) and they have AIC and BIC coefficients that are greater than equal to that of GARCH (1, 1). Hence under the normal distribution, GARCH (1, 1) is the best fitted symmetric model since it is parsimonious and has the lowest AIC and BIC. Introducing the student-t as conditional distribution significantly improves the normal GARCH (1, 1), while the skew-student-t fails to better the symmetric student-t

4.5.2 Estimation Results of Asymmetric GARCH Models

To capture the asymmetry dynamics and the presence of the "leverage effect" in the XAF/USD exchange rate returns, the nonlinear asymmetric models; EGARCH (1, 1), GJR-GARCH (1, 1) and APARCH (1, 1) under assumption of a normal, student-t and skewed student-t distributions are fitted to the exchange rate returns series and the estimated results are presented in Table 4.6

At 5% level of significance, all the estimated omega (ω) coefficients under the various conditional distributions are not statistically significant except for EGARCH model and APARCH (under normal distribution). With same level of significance, the parameter for asymmetric volatility response, gamma (γ) is positive and statistically significant under all the conditional distributions except for GJR-GARCH under student-t and skew student-t distributions. Leverage parameter gamma (γ) being positive and significant indicates asymmetric response for negative returns in the conditional variance equation. This suggests that negative shocks imply a

higher next period conditional variance than positive shocks of the same magnitude or sign, which is in conformity with the results depicted in Table 4.7 below. This result also reflects the condition that volatility tends to rise in response to bad news and fall in response to good news, which is in accordance with the usual expectation in stock markets where downward movements (falling returns) are followed by higher volatility than upward movements (increasing returns). Hence there exist leverage effects in the returns series during the study period since gamma (γ) is significant and greater than zero in the GJR-GARCH (under normal distribution) and APARCH model. For a negative return shock, this implies fewer dollars per CFA Franc and therefore a strengthening dollar and a weakening CFA Franc. Thus the results suggest that a strengthening dollar (weakening CFA Franc) leads to higher next period volatility than when the CFA Franc strengthens by the same amount.

Also, the asymmetric power coefficient delta (δ) is statistically significant under all the conditional distributions. This result indicates there is strong evidence that absolute returns have stronger autocorrelation than squared returns using XAF/USD exchange rate returns data. Based on AICs and BICs information criteria under the various conditional distributions, asymmetric EGARCH (1, 1) under the student-t distribution is the best fitted model since it has the lowest AICs and BICs values.

Table 4. 7. The magnitude of news impact on Volatility

	APARCH			GJR-GARCH			EGARCH		
	norm	std	sstd	norm	std	sstd	norm	std	sstd
Bad News	0.912	1.281	1.281	0.751	1.056	1.056	0.912	1.264	1.263
Good News	0.573	1.062	1.062	0.450	0.848	0.848	0.630	1.103	1.104

Source; Author's calculation

The magnitude of new on volatility of the various asymmetric GARCH models, with different conditional distributions, depicted in Table 4.7, shows that negative shocks are stronger than positive shocks. That is negative shocks imply a higher next period conditional variance than positive shocks of the same magnitude or sign

4.5.3 Residual Analysis

The residuals of GARCH models are usually used to check for normality, using Jarque-Bera test and the presence autocorrelation using Ljung-Box test. The results are shown in Tables 4.5 and 4.6 above. The ARCH (1) and ARCH (2) models fail the normality test and fail to capture volatility clusters for the residuals. Hence serial correlation in present in the ARCH models, which is in conformity with the ARCH LM test results. Also, we do not find significant autocorrelation in the residuals of both symmetric GARCH (1, 1), GARCH (2, 1), GARCH (1, 2) and all the asymmetric GARCH models which is also in conformity with the ARCH LM test results (no conditional heteroscedasticity), but

they still failed the test for normality under all the conditional distributions. Nevertheless, the student-t distribution provides a better fit that is closer to normality. This indicates that for applications where the tails of the distribution are of importance, such as value-at-risk, we can approximately use a student-t distribution or better still the generalized error distribution (GED).

4.5.4 Graphical Analysis

The graphical analysis of the results for GARCH (1, 1) and EGARCH (1, 1) models under the normal and student-t distributions are shown in Appendix A (Figures 6.1, 6.2, 6.3, 6.4). Superficial graphical inspection indicates that a good job has been done capturing the silent features of the data. This is inconformity with the ACF of squared standardized residuals plots under the GARCH(1, 1) and EGARCH (1, 1) models, demonstrating that there is little or approximately no evidence of volatility clusters in the standardize residuals, which indicates that the models have captured the clustering phenomenon well. However, the QQ plots in the models still indicates that the standardized residuals are fat tailed, with the student-t distribution providing a much better fit than the normal distribution which is consistent with the results in tables 4.6 and 4.7. The QQ plots for GARCH (1, 1) and EGARCH (1, 1) models under the normal and student-t distributions further indicates that the deviation from conditional normality is stronger on the downsides, which is consistent with the student-t skew parameters being significant. These results suggest that tail thickness is asymmetric, with the lower tails thicker than the upper tails. These results is also consistent with the results from section 4.3.2

Also, a pictorial representation of the degree of asymmetry of volatility to positive and negatives shocks is given by the ***news impact curve*** introduced by ***Pagan and Schwert*** (1990), is shown in Appendix A (Figures 6.1, 6.2**,** 6.3, 6.4). The *news impact curve* plots the next-period volatility (σ_t^2) that would arise from various positive and negative values of ε_{t-1}, given an estimated model. The curves are drawn by using the estimated conditional variance equation for the model under consideration, with its given coefficient estimates, and with the lagged conditional variance set to the unconditional variance. Then, successive values of ε_{t-1} are used in the equation to determine what the corresponding values of σ_t^2 derived from the model would be.

Under the normal and student-t distributions, the news impact curve of GARCH (1, 1) shows that the symmetric GARCH model treats both positive and negative shocks to be the same (symmetric about zero) while the asymmetric EGARCH (1, 1) model shows that the shocks are different (asymmetric), with negative shocks having stronger impact on future volatility than positive shocks of same magnitude. It can also be seen that a negative shock of given magnitude will have a bigger impact under EGARCH than would be implied by a GARCH model, while a positive shock of given magnitude will have more impact under GARCH than EGARCH. The latter result arises as a result of the reduction in the value of α_1, the coefficient of lagged square error, when the asymmetry term is included in the model.

4.5.5 Value at Risk

The future return rate and volatility for one-day-ahead based on the estimated parameters of the models are obtained. These forecasted values are necessary for the estimation of Value at Risk (VaR). The estimated values of the VaR parameters for one-day-ahead as well as the probabilities of 1% and 5% are exhibited in Table 4.8.

Table 4. 8. Econometric Estimation for Parameters of VaR for one-day-ahead period

model	GARCH(1,1) student-t distribution	APARCH(1,1) student-t distribution	GJR-GARCH(1,1) student-t distribution	EGARCH(1,1) student-t distribution
Forecasted Return	0.0000	0.0000	0.0000	0.0000
Forecasted conditional Standard Deviation	0.4059	0.3958	0.3955	0.4003
VaR 1%	-105.80%	-103.16%	-103.09%	-104.34%
VaR 5%	-63.35%	-61.78%	-61.73%	-62.48%

Source; Author's calculation

The Value at Risk for GARCH family models have been estimated under the student-t conditional distribution, since the fat tail characteristic of XAF/USD exchange rate returns series is better captured under this distribution. The corresponding 1% and 5% quantiles under student-t distribution are *-2.6065* and *-1.5609* respectively. All the forecasted returns are zero because the average return (μ) for all the conditional volatility models was assumed to be zero. The estimated VaR values obtained are all negatives but the negative signs are usually ignored since they are indicators of losses. With 1% and 5% confidence level, the maximum amount of loss an investor can make in order to exchange 1 XAF to USD in one day period is approximately 105.80% and 63.35% respectively.

Chapter 5

5 Conclusion

Modeling the volatility of exchange rate returns has become an important field of empirical research in finance, because volatility is considered as an important concept in many economic and financial applications like asset pricing, risk management and portfolio allocation. This research work has examined daily exchange rate returns of XAF/USD from 1st January 2010 to 4th January 2018. The research employs different univariate specifications of the generalized autoregressive conditional heteroscedastic (GARCH) models, both symmetric and asymmetric that captures the most common stylized facts of exchange rate returns such as volatility clustering and leverage effect. The models have been estimated under the normal, student-t and skew student-t distributions and the AICs and BICs criteria have been used for best model selection. The ARCH (1), ARCH (2), GARCH (1, 1), GARCH (2, 1) and GARCH (1, 2) models has been employ to capture the symmetric effect of volatility while the APARCH (1, 1), GJR-GARCH (1, 1) and EGARCH (1, 1) have used to capture the asymmetric effect.

The empirical result shows that XAF/USD exchange rate returns data follows a fat tail distribution with negative skewness and excess kurtosis as compare to a normal distribution as shown in the descriptive statistics. The best conditional mean model using Akaike and Bayesian information criteria was ARIMA (0, 1, 0) on the raw data (XAF/USD exchange rate series) or ARMA (0, 0) on the differenced data (XAF/USD exchange rate returns) and its residuals were uncorrelated indicating that they are independent and identical which is desirable but they failed the normality test which is undesirable. Applying the Mcleod Li test on the squared residuals of the mean model, shows significant autocorrelation in the squares indicating that the data suffer from ARCH effect and by applying different univariate symmetric conditional volatility models (on the XAF/USD exchange rate returns data) under the normal distribution, the normal GARCH (1, 1) was the best model since its parsimonious and has the lowest AIC and BIC criteria. A further comparison between the normal GARCH (1, 1) and the student-t GARCH (1, 1) shows that the student-t GARCH (1, 1) was a better fit and imposing a skew student-t GARCH (1, 1) produces a result that fail in improving the student-t GARCH (1, 1). Furthermore, introducing different univariate asymmetric GARCH models under the normal distribution shows that EGARCH model outperformed the rest and when estimated under a student-t conditional distribution, the performance was better. Hence for both symmetric and asymmetric GARCH type models employed, EGARCH under student-t distribution outperformed the rest, since it's AIC and BIC are lowest and the log-likelihood is largest.

Also, there was significant evidence of asymmetry with the leverage parameter being significant (except for GJR-GARCH model under student-t and skew student-t distributions) and positive indicating that volatility tends rise in response to bad news and fall in response to good news, which is in accordance with the usual expectation in stock markets where downward movements (falling returns) are followed by higher volatility than upward movements

(increasing returns), meaning the existence of asymmetry in the returns series during the study period. Hence leverage effect is captured since gamma (γ) is positive a significant for GJR-GARCH (under normal distribution) and APARCH models. Again the normal GARCH (1, 1) reveals a very large value of the estimated persistence parameter ($\beta_1 = 0.978$) resulting in a slowly decreasing of the rises in the conditional variance due to shocks. More so, the result of asymmetric power coefficient (δ) is significant under all the distributions. The result suggests that absolute returns have stronger autocorrelation than squared returns within the time frame of the study period.

The Ljung-Box text of the squared residuals using 20 lags and the ARCH LM test of the various GARCH type models [except ARCH (1) and ARCH (2) models] confirms that our conditional volatility models imposed in the data set captures the aspect of volatility clustering (no serial correlation and no ARCH effect), which is desirable but there is a strong violation of normality which is not desirable. The Q-Q plot of the standardize residuals has been provided to show the level of deviation from normality and the plotted graphs indicates that the residuals turn to approach normality using student-t distribution than using normal distribution. Perhaps imposing high degrees of freedom, we might achieve normality. The econometric estimation of VaR can be related to the chosen GARCH-type models under the student-t distribution and the first step for the estimation, is the specification of the models. Based on the estimated models, a one-step-ahead forecasting has been taken to forecast the future value of the exchange rate returns and the conditional volatility. Hence the values have been used to estimate the VaR at 1% and 5%. The empirical results indicates that the asymmetric GARCH type models are the most adequate for estimating and forecasting VaR of XAF/USD exchange rate returns with EGARCH (1, 1) model providing a superior estimation of the one-step-ahead VaR.

The findings in this research have important implications regarding VaR estimation in volatile times, market timing and portfolio selection that have to be addressed by investors and other risk managers operating in emerging markets. However the empirical research focused only on the XAF/USD exchange rate and therefore the findings cannot be generalized to other exchange rates within the CEMAC market. Thus in order to fully determine and understand the volatility of exchange rate within the CEMAC zone, other important exchange rate currency pairs such as XAF/GBP, XAF/JPY, XAF/CNY etc has to be taken into consideration as there has been significant increase in trade flow between CEMAC countries and China, Japan, Britain among others.

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Appendix A

Table 6. 1 Descriptive statistics for daily XAF/USD exchange rates series

Variables	$y_{t(XAF/USD)}$
Mean	0.001922
Median	0.001973
Maximum	0.002310
Minimum	0.001584
Standard Deviation	0.000189
Skewness	-0.166544
Kurtosis	-1.271194
Jarque-Bera test (p-value)	0.00
Number of Observations	2079

Source: Authors' Calculation

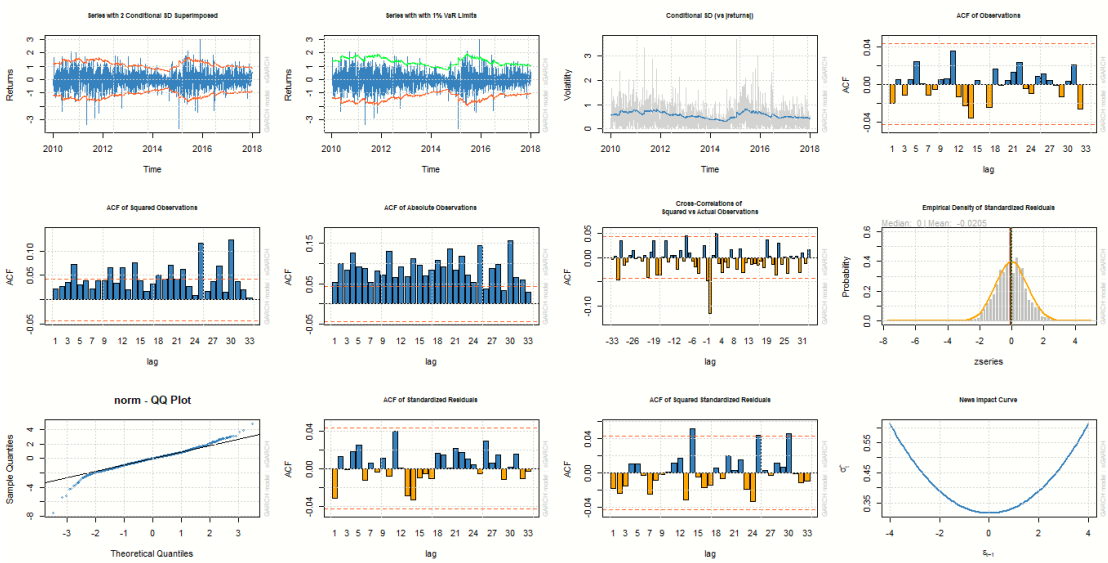

Figure 6. 1. Graphical presentation of GARCH (1,1) Under Normal Distribution

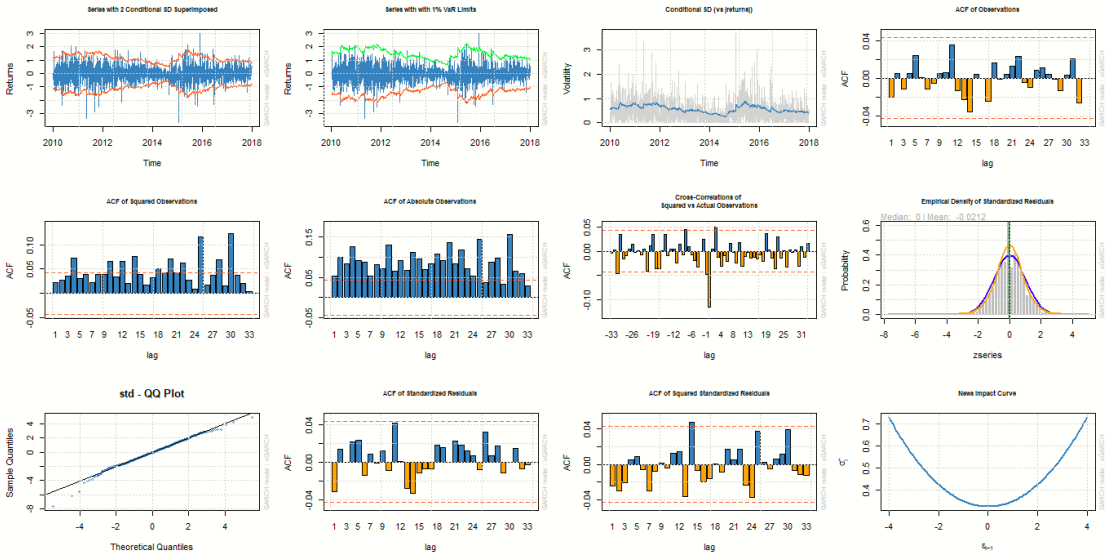

Figure 6. 2. Graphical presentation of GARCH (1,1) Under Student-t Distribution

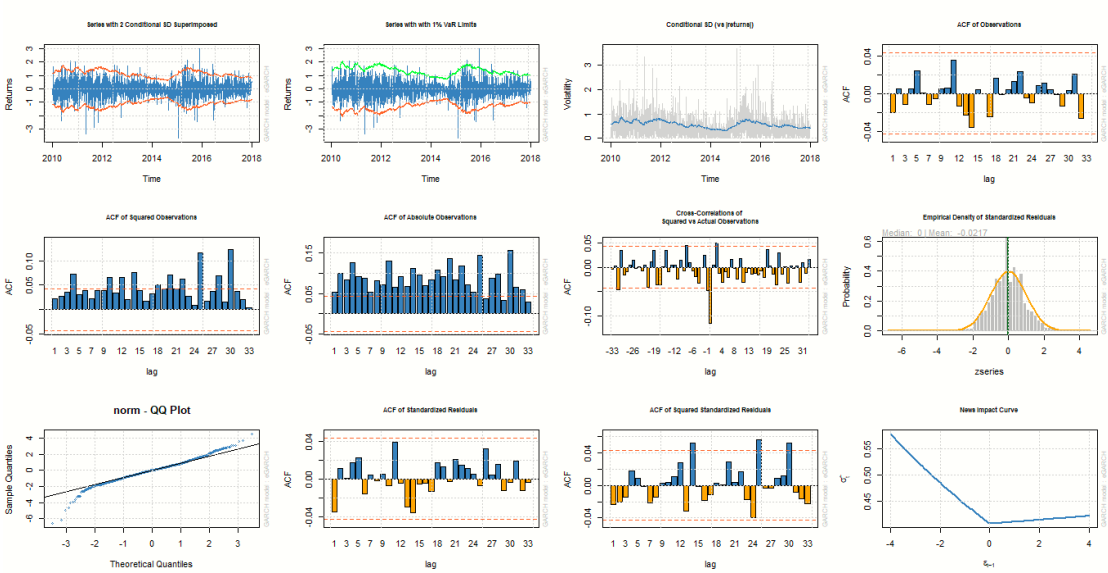

Figure 6. 3. Graphical presentation of EGARCH (1,1) Under Normal Distribution

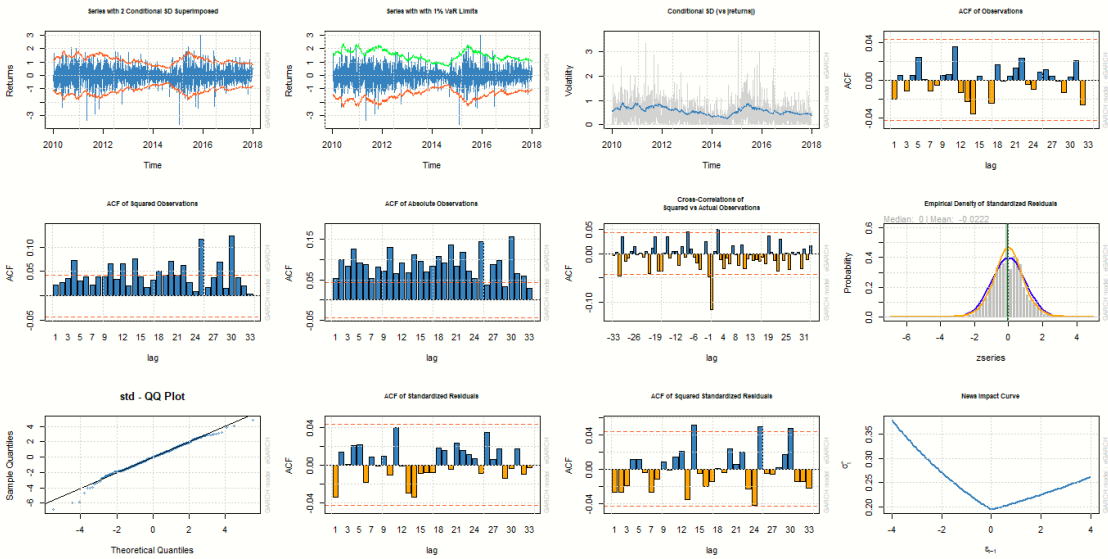

Figure 6. 4. Graphical presentation of EGARCH (1,1) Under Student-t Distribution

Invertibile MA (q)

Given an MA (1) model with $|\pi| < 1$

$$Y_t = e_t - \pi e_{t-1}$$

$$e_t = Y_t + \pi e_{t-1}$$

Replacing t by $t - 1$ and substituting in e_{t-1}, we derive a new expression as

$$e_t = Y_t + \pi(Y_{t-1} + \pi e_{t-2})$$

$$e_t = Y_t + \pi Y_{t-1} + \pi^2 e_{t-2}$$

$$Y_t = e_t - \pi Y_{t-1} - \pi^2 e_{t-2}$$

If $|\pi| < 1$ we may continue this substitution "infinitely" into the past and obtain the expressions

$$e_t = Y_t + \pi Y_{t-1} + \pi^2 Y_{t-2} + \cdots$$

News impact curve

Holding constant the information at $t-2$ and earlier, we examine the relationship between ε_{t-1} and σ_t for the different GARCH type models.

For GARCH (1, 1) model, the curve is a quadratic function centered on $\varepsilon_{t-1}=0$,

$$\sigma_t^2 = A + \alpha\varepsilon_{t-1},$$

With $A=\omega+\beta\sigma^2$ and σ^2 is the unconditional variance

For GJR-GARCH (1, 1) model, it has a minimum at $\varepsilon_{t-1}=0$ with

$$\sigma_t^2 = \begin{cases} A+\alpha\varepsilon_{t-1}, & for \;\; \varepsilon_{t-1}>0, \\ A+(\alpha+\gamma)\varepsilon_{t-1}, & for \;\; \varepsilon_{t-1}\leq 0, \end{cases}$$

With $A=\omega+\beta\sigma^2$

For EGARCH (1, 1) model, it has a minimum at $\varepsilon_{t-1}=0$ with

$$\sigma_t^2 = \begin{cases} A+exp\left(\frac{\alpha+\gamma}{\sigma}\varepsilon_{t-1}\right), & for \;\; \varepsilon_{t-1}>0, \\ A+exp\left(\frac{\alpha-\gamma}{\sigma}\varepsilon_{t-1}\right), & for \;\; \varepsilon_{t-1}\leq 0, \end{cases}$$

With $A=\sigma^{2\beta}exp\left(\omega-\alpha\sqrt{2/\pi}\right)$

For APARCH (1, 1) model,

$$\sigma_t^2 = A+\alpha(\;\varepsilon_{t-1}+\gamma)^{\delta}$$

With $A=\omega+\beta\sigma^2$

Is symmetric and centered at $\varepsilon_{t-1}=-\gamma$

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