Ca' Foscari
University
of Venice

Master's Degree

in Business Administration

Final Thesis

Performance Evaluation of Investment Funds: an approach to Data Envelopment Analysis (DEA)

Supervisor
Ch. Prof. Marco Tolotti

Graduand
Nicolo' Tonini
Matriculation Number 824434

Academic Year
2016 / 2017

Table of Contents

Introduction

How can a non-institutional investor choose, rationally, in which investment fund put his savings? Is the expected return enough to assess the performance of a fund? Do traditional risk-adjusted measures give an overall judgement about funds' efficiency?
This thesis aims to provide satisfying answers to questions like these, trying to present the theoretical pillars and tools to evaluate funds' performance along with an empirical analysis on investment funds' efficiency.
Investment funds, especially in the last two decades, have seen an important growth, conceived as the most used mean of investment for non-professional investors.
Usually, funds' performances are quantified on the basis of returns, though overlooking several variables that may affect the overall judgement.
Through Data Envelopment Analysis (DEA), that is a non-parametric approach applied in many fields for measuring performance and efficiency, important inputs, such as funds' fees, will be part of evaluation analyses, allowing to deploy as many inputs and outputs as possible.
The First chapter will introduce the concept of investment fund, classifying it by structure, investment objective, and management attitude. Furthermore, it will be illustrated, how funds are regulated in the two largest worldwide markets, Europe and United States, highlighting differences and similarities within the underlying regulatory bodies. Moreover, the first chapter will deal with the description of most important fees and commissions, to conclude with benefits and downsides.
The second chapter will treat the most well-known risk-adjusted performance measures, among which there are the Sharpe ratio, Sortino and Jensen's alpha.

In the third chapter it will be presented a detailed definition of the potentiality of the DEA tool, beginning from the literature review, going through different kind of applications within various fields, culminating with the presentation of generic and specific funds' efficiency evaluation DEA models.

In the fourth and last chapter, there is the introduction of an empirical application of a specific DEA model, Murthi, Choi and Desai, to a sample of Italian investment funds. The objective of this analysis is to assess the weight of transaction costs on the overall performance. More precisely it will be considered the Total Expense Ratio (TER) and load commissions, comparing the results of the DEA model with traditional risk-adjusted measure. To conclude, it is computed the correlation between DEA results and two of the most used performance gauge, to assess the soundness of analysis' results.

Investment Funds

Defining Investment Funds

An investment fund is a financial company that pools capital from many subjects and invests it in stocks, bonds or other assets[1]. This is the definition provided by the Security Exchange Commission, the governmental commission that regulates the financial market in the U.S. Instead, for the European Central Bank, an investment fund is a collective investment undertaking that invests capital raised from the public in financial and non-financial assets[2]. Even though the definitions use a different nomenclature, the underlying concept is the same. Funds invest the money they collect into securities and other financial assets, combining them into portfolios, groups of stocks or bonds owned by the fund. Each fund share stands for an investor's proportionate ownership of the fund's portfolio.

There are several types of funds in the market, classified by the features that characterize them. Basically, funds can be broadly divided into three main categories: Open-end funds (generally known as mutual funds), Closed-end funds and Unit Investment Trusts (UITs). Then, as it will be described later on, open-ended funds can be divided in many others sub-categories, such as stock funds, fixed-income funds, ETFs, money market funds etc…

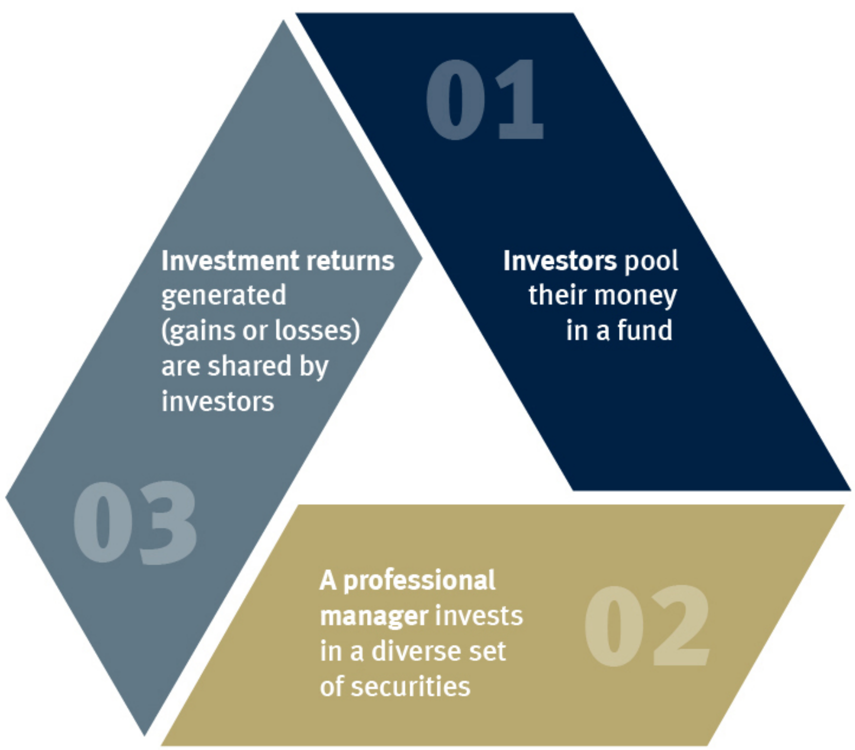
---

[1] Definition of Investment Fund given by the Security Exchange Commission: https://www.sec.gov/investor/tools/mfcc/mutual-fund-help.htm

[2] This definition of investment fund is written in the Regulation ECB/2007/8 published in the Official Journal (OJ) of the European Union on 11 August 2007, and entered into force on 31 August 2007. Link :https://www.ecb.europa.eu/ecb/legal/pdf/l_21120070811en00080029.pdf

Funds' shares are bought and sold, or redeemed, to investors directly, or through the brokerage of a professional intermediary. When buying shares in mutual funds, individual investors cannot make decisions about the composition of portfolios. They simply have the possibility to choose in which fund invest their money, based on investors' level of risk aversion and on goals, by looking at the return they want to yield from the investment. Who oversees and makes decisions concerning which stocks or bonds should pick, in what quantities and the timing of decisions, is the fund, or portfolio, manager.
The graph below shows in three simple steps how an investment fund process works:

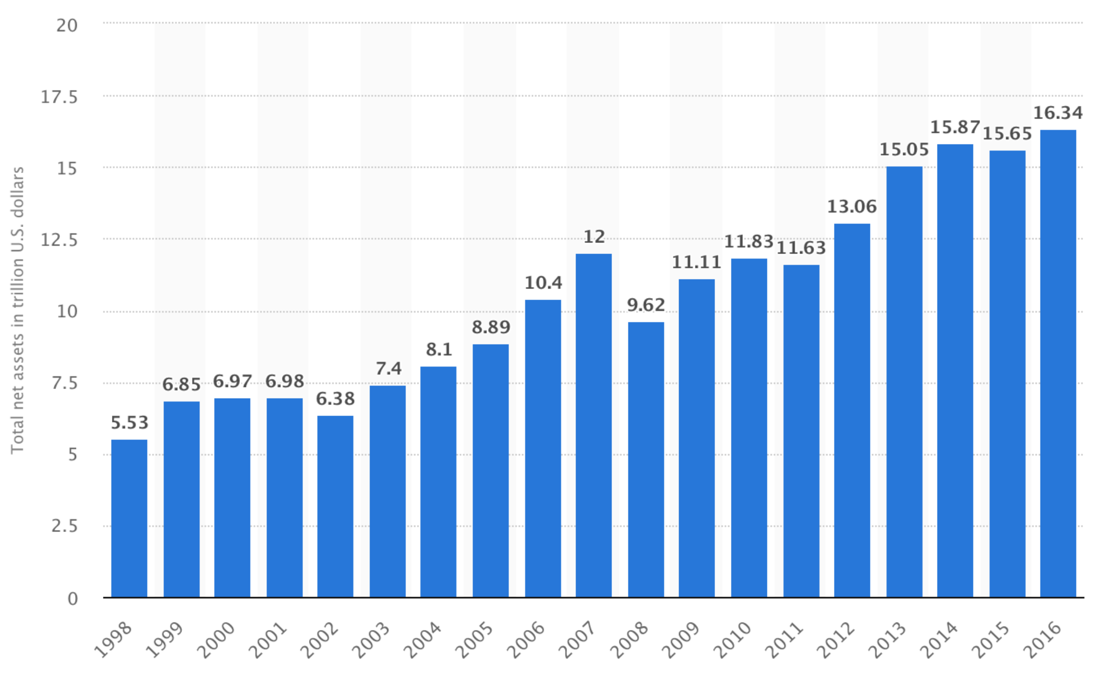
Figure 1.1: Investment Fund Scheme



Source: RBC Global Asset Management, http://funds.rbcgam.com

The relevance of mutual funds, as one of the most used mean of investment, especially for the household category[3], is glaring just by having a look at the following charts (Figures 1.2 and 1.3). The first graph (Figure 1.1) is relative to the total net assets of mutual funds in the United States from 1998 to 2016: in less than twenty years, the capitalization has tripled, increasing from 5.5 to more than 16 trillion USD. Here, there is the example of U.S., because, with almost half (46%) of the global market share[4], represents the biggest fund's market.

Figure 1.2: Total Net Assets of Mutual Funds in the U.S. from 1998 to 2016 (in trillion $)



Source: Investment Company Institute (ICI), 2017 Investment Company Fact Book

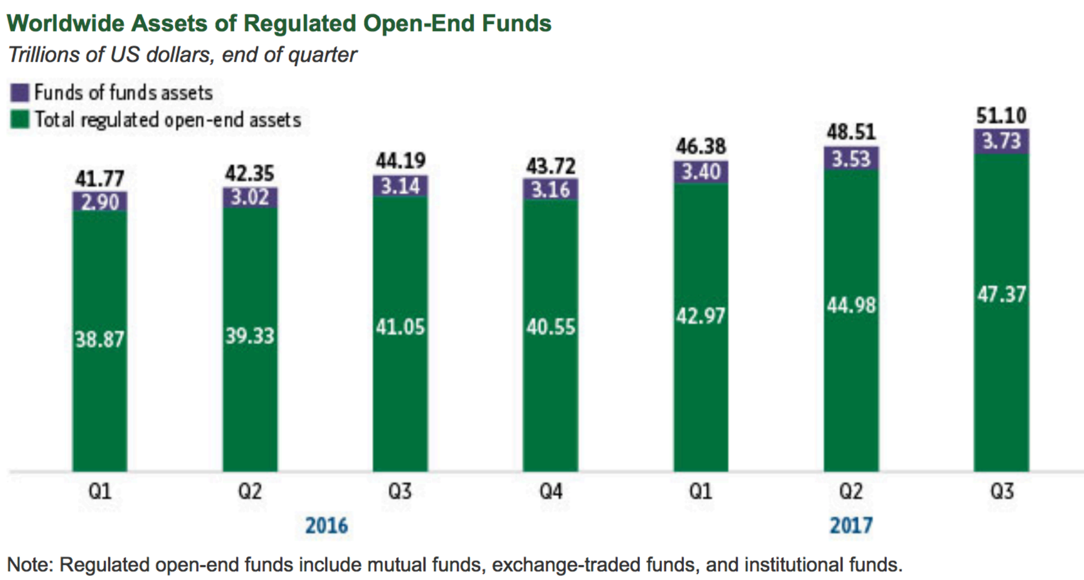
The second graph (Figure 1.3) shows the worldwide assets held in open-end funds: the

[3] According to ICI 2017 Investment Company Fact Book, page 11, and Federal Reserve report, in 2016 22 percent of the U.S. household financial assets were held in investment funds, such as mutual funds, closed-end funds, ETFs or UITs.

[4] Source: EFAMA International Statistical Release of second quarter of 2017.

purpose of this statistics is to present, once again, the increasing interest that surround investment funds: in less than 2 years, precisely one year and three quarters has been an increase in net assets of open-ended funds of 8.5 percent, increasing from 38.87 up to 47.37 trillion USD. This figure highlights the importance and the rate of growth of this financial market's segment.

Figure 1.3: Worldwide Net Assets of Open-end Funds



Source: Investment Company Institute (ICI), Statistics – Worldwide market

Funds Background

The pioneer of investment funds is historically considered a Dutch merchant, named Adriaan van Ketwich, that, at the end of 18th century, following a financial crisis that staggered across the Old Continent, had the prescience of collecting money from a pool of investors to form, in 1774, the first investment trust ever. Almost a century later, in

1868, with the scope of giving the possibility to invest money exploiting the benefits of diversification, was established in London the "*Foreign and Colonial Investment Trust*", considered the first ever investment trust of the modern era. The first ever open-ended mutual fund was created on March 21st, 1924, when three Boston securities executives decided to pool their money to establish the "*Massachusetts Investors Trust*" (MIT), event that would have revolutionized the financial industry and showing just from the beginning unforeseen results: in the first year, the mutual fund grew from USD 50,000 to USD 329,000 in assets. American investors embraced this innovation and started to invest heavily in it.

To instill investors with the necessary confidence, and in response to the financial crisis of 1929 and the Great Depression, the U.S. Congress passed a series of laws with the aim of regulating the entire financial industry, that was barely unregulated until that moment. With the Securities Act of 1933, the Securities Exchange Act of 1934, and the Investment Company Act of 1940, the Legislator established the foundation of the actual regulation and set standards with which investment funds must comply.

Therefore, since the creation of the MIT in 1929, the fund industry has enjoyed the fastest growth rate of the financial industry. In 1949, the totality of assets held by investment funds amounted to USD 2 billion; this value soared to USD 6.3 trillion at the outset of 2003, and more than USD16 trillion in 2016 only in the U.S., making mutual funds America's largest financial investment vehicles.

Fund Scheme by Structure

As a common convention, investment funds are classified into three main categories: Open-end funds (commonly named mutual funds), Close-end funds and UITs. In turn, funds can be divided into several varieties such as, stock (or equity) funds, bond funds, balanced-mixed funds, money market funds, ETFs etc… The possible ways through which investors can yield profits from investing in mutual funds are listed as follow; the first is dividend payment: when underlying stocks earn money in the form of dividend, the fund successively will distribute dividend income to shareholders. Capital gain distribution is another kind of return to investors: this capital gain occurs when the fund sells a stock that has increased in price. At the end of each year, investment funds will distribute capital gains, net to any capital losses, to shareholders. Third kind of investors' return is the increased market price, and this happen when the market value increases.

Open-End Funds

An Open-end mutual fund is "*an investment company registered with the U.S. Security Exchange Commission (SEC) that issue shares of its stock to investors, invests in in an investment portfolio on the shareholders' behalf, and stands ready to redeem its shares for an amount based on its current share price*"[5]. This is the most common type of collective investment scheme, unlike closed-end funds, investors buy shares directly from the fund itself at their Net Asset Value (NAV[6]), or share price. The share price of mutual fund, and traditional UITs, based on their NAV, is obtained dividing the NAV by the total

[5] W. Ruppel, Wiley GAAP for Governments 2017: Interpretation and Application of Generally Accepted Accounting Principle for State and Local Governments, 2017, pp. 228-229

[6] Definition given by the SEC: "Net Asset Value or NAV of an investment company is the company's total assets minus its total liabilities". Furthermore, mutual funds and UITs "generally must calculate their NAV at least once every business day", while for a closed-end fund this is not required because its shares are not redeemable.

shares outstanding, plus fees that the fund charges at purchase or redemption, respectively named sales load (or purchase fee) and deferred sales load (or redemption fee).

Open-ended funds are available in most developed countries, but the terminology and operating rules may vary. Some examples are: U.S. mutual funds, UK unit trusts and OEICSs, European SICAVs etc.… The major U.S. open-end funds are: The Vanguard Group's S&P 500 (tot. assets of USD $391 billion), PIMCO Total Return (tot. assets USD $73 billion), Fidelity Investment's Magellan (tot. assets USD $17 billion). To conclude, statistics show that more than half of the open-ended funds are based in the Americas (mostly in North America, U.S. and Canada), with remaining 36% in Europe and 13% between Australia and Asia[7].

Closed-End Funds

Unlike open-ended funds, closed-end funds, shorten CEFs, do not continuously issue or redeem shares. Initially, there a public offering of shares, offered to the public with the intermediary work of licensed brokers. Up to this point the process is the same as for open-end mutual funds. The difference underlies in the fact that "*to obtain shares after a public offering is completed, an investor must purchase shares from other investors in the secondary market (one of the exchanges or the over-the-counter (OTC) market*"[8]. Unlike the open-end funds, the price per share is determined by the market and is usually different from the underlying value or NAV per share.

---

[7] Source: ICI Global, Statistics: Worldwide Mutual Fund Market https://www.ici.org/research/stats/worldwide/ww_q3_17

[8] S. Anderson, J. Born, O. Schnusenberg, Closed-end Funds, Exchange-Traded Funds and Hedge Funds: Origin, Functions and Literature, 2009, pp-4-5.

Unit Investment Trust (UIT)

A unit investment trust (UIT) is a SEC registered investment company that offer an unmanaged portfolio of securities, given that it is not a management company, as both open-end and closed-end, and have no board of directors. Furthermore, UIT has a predetermined date for termination that varies according to the investments held in its fixed portfolio. When UITs are dissolved, proceeds from the securities are either paid to investors or reinvested in another trust[9]. Thus, *UIT's securities will not be sold or new ones bought, except in certain limited situations such as bankruptcy of a holding*.[10] These trusts are built by a sponsor and marketed through brokers. An UIT portfolio may hold one of several different types of securities. The two main types are equity and bond trusts. Equity trusts are generally intended to provide capital appreciation and dividend income, at the end of the period, corresponding to the termination date, the trust liquidates and distributes the net asset value as earning to the unitholders.

Conversely, bond trusts, which can be related to corporate, government and national bonds, pay periodic interests, often in relatively consistent amounts, until the first bond in the trust matures. At this moment, the capitals from the redemption are distributed to the clients as a kind of return of principal. The trust then continues paying the monthly income amount until the next bond is redeemed. Bond trusts are intended for investors seeking relative high level of income while carrying on low risks.

---

[9] A guide to Unit Investment Trusts, Investment Company Institute,2007.

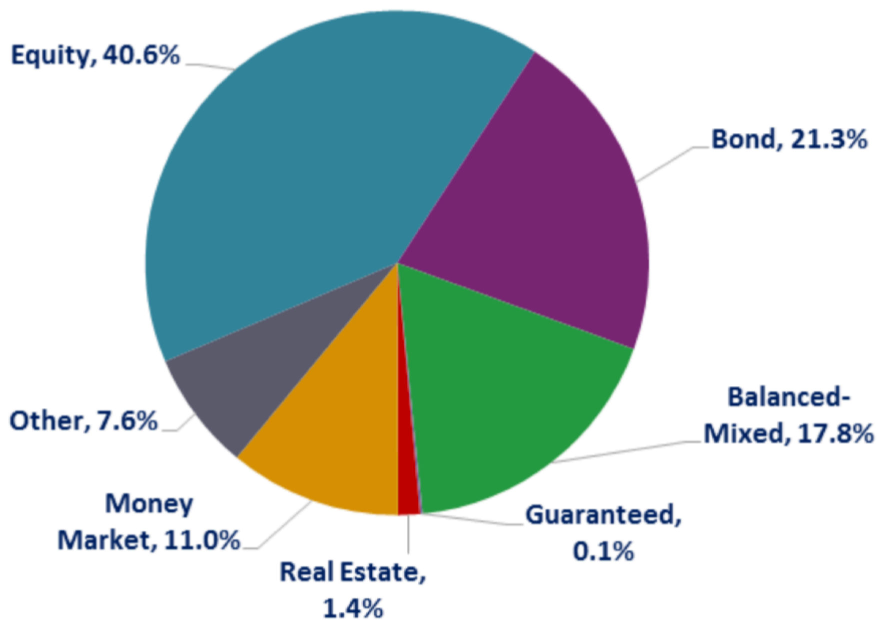
[10] S. Anderson, J. Born, O. Schnusenberg, Closed-end Funds, Exchange-Traded Funds and Hedge Funds: Origin, Functions and Literature, 2009, pp-5-6

Fund Scheme by Investment Objective

All of the above-cited funds can be divided again into several sub-categories, based on the securities' nature of which they are composed of, such as equity, bond and balanced funds. Moreover, there are Exchange Traded Funds and money market funds, which are a specific type of mutual funds. There are also newer types of funds such as alternative, smart-beta funds and esoteric ETFs, of which we're not going to discuss. First of all, let's have a look at the global distribution of open-ended fund[11] net assets at the end of the second quarter of 2017 as presented by the following pie chart (Figure 1.4): more than 40% of these assets are invested in equity funds, followed by bond and balanced funds respectively with 21.3% and 17.8%.

Figure 1.4: Worldwide Open-end Fund Net Assets by type of fund 2017, Q2

[11] Here we have taken the open-end fund as a prototype of investment funds based on the fact that it is the most widespread type of fund nowadays.



Source: European Fund and Asset Management Association (EFAMA), International Statistical Release: Trends in Second Quarter of 2017

Equity Funds

Equity funds, also known as stock funds, invest predominantly in stocks. Equity funds may be subject to various level of risks, depending from the nature of the company shares holding in its portfolios. There are many different types of equity funds, such as international or global equity funds, if investing in stocks outside the home country or globally, emerging-market stock funds, when investing in stock exchanges of a developing country, sector equity funds, which invest in individual sectors, and even market capitalization equity funds, that limit investments to micro, small, medium or large capitalized firms. By nature, stock funds are meant to be riskier than bond funds, as well as more profitable. Given that, equity funds invest exclusively in stocks, variations in share prices will determine a corresponding change on the Net Asset Value (NAV) of the fund.

Equity securities are by nature volatile and the factors that may influence their prices are inflation, central bank policies, currency fluctuations, interest rates and so on and so forth. However, an expert stocks fund's manager will invest in varied companies, maybe from different industries, generating diversification that reduces the volatility.

Fixed Income Funds

Fixed Income funds, also known as bond funds, invest primarily in bonds or other classes of debt securities, and again it can be broken down into other subgroup, i.e. government, municipal, corporate, convertible bonds and other debt securities. Due to the bond funds' multiplicity, it is necessary to clarify that risks associated with bond funds may vary consistently according to the subgroup of fund. These risks can be: credit, interest rate and prepayment risks. For example, the credit risk is related to the chance of failure of the company issuer of a specific bond: this will be less of a factor for funds investing in government bonds (i.e. U.S. Treasury Bonds), given that the possibility of default of a Nation is lesser than company's one. Hence, funds investing in corporate bonds, conceivably in firms with poor credit ratings, will face higher risk. Furthermore, the interest rate risk is linked to the interest rate trend with funds investing in long-term bonds having higher exposure to this kind of risk.

Balanced-Mixed Funds

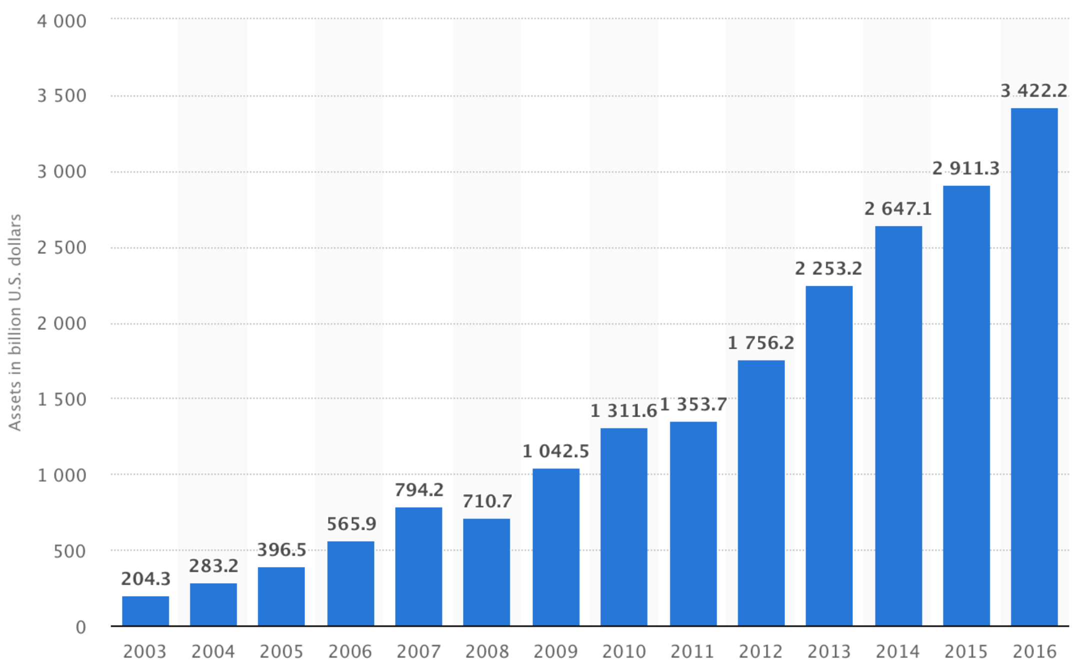
A balanced fund, sometime called mixed or blended fund, may invest its assets in a wide range of financial instruments, like money market accounts, bonds and equity, with the intention to yield both growth in value and monthly income. This particular fund is geared towards investors looking for a mix of capital appreciation and reduced riskiness, with consistent level of diversification. Typically, stocks investment sums up between 50% and 70% of the balanced fund, with bonds accounting for the remaining, but there can be further instruments in portfolios. However, every fund manager allocates weights in different ways, and there is no set definition of how much of each a balanced fund should or must contain.

Exchange Traded Funds (ETF)

Exchange Traded Funds (ETFs) are investment companies registered under the Investment Company Act of 1940, in the U.S., as either open-ended funds or UITs, while under the Undertaking for Collective Investment in Transferable Securities (UCITS) Directive 2009/65/EC, in the European Union. Commonly known as index funds, ETFs are intended to replicate the performance of their benchmark indexes, such as the NASDAQ-100 Index, S&P 500, Dow Jones, etc... Contrary to conventional mutual funds, however, ETFs are listed on an exchange and can be traded intra-daily. When an investor buys shares of an ETF, he is buying shares of a portfolio that tracks the yield and return of a broader index. The main difference between ETFs and other types of index funds is that ETFs don't try to outperform their corresponding index, but simply replicate its performance.

ETFs combines the benefits of both open-end and closed-end funds, combining the issuing and redemption process of the former with the continuous stock market tradability of the latter. ETFs have been around since the early 1980s, but they've come into their own within last decade. As can be denoted from the graph below (Figure 1.5), across the period 2006-2016 the total ETFs' assets increased from less than USD 600 billion to almost USD 3.5 trillion; this statistic discloses the enormous success that this particular investment mean is having nowadays.

Figure 1.5: Assets' development of global ETFs from 2003 to 2016



Source: Bloomberg; Deutsche Bank; Thomson Reuters (https://www.statista.com/statistics/224579/worldwide-etf-assets-under-management-since-1997/)

Money Market Funds

Money market fund is a mutual fund that, by law, can invest only in high quality and short-term securities, such as commercial paper, bankers' acceptances, government bills and repurchase agreement, paying dividends that generally replicate short-term interest rates[12]. One of the main characteristic of money market funds is the constant value around one dollar, not below, of Net Asset Value per share.

Money market funds' category includes ones that invest primarily in government securities, corporate and bank debt securities and tax-exempt municipal securities. Moreover, these particular funds are usually intended for different types of investors such as retail or institutional investors, when funds require high minimum investments.[13] Many investors use this type of funds to store cash or as an alternative to investing in the stock market, also thanks to high liquidity and low riskiness of this instrument. The only risk generally associated with money market funds is inflation risk, that may consume the returns over time.

Money market funds are regulated primarily under the Investment Company Act of 1940 and the rules adopted under that Act, particularly Rule 2a-7 under the Act. Such funds are not federally insured, although the portfolio may consist of guaranteed securities and/or the fund may have private insurance protection.

[12] A. Corrigan, P.C. Kaufman, Understanding Money Market Funds, 1987.

[13] U.S. Security and Exchange Commission, Mutual Funds and ETFs, A Guide to Investors: https://www.sec.gov/investor/pubs/sec-guide-to-mutual-funds.pdf

Fund Management: Active vs. Passive

Investment funds, regardless of whether they are actively or passively managed, share common traits, that consist in three benefits to investors: diversification, straightforward access to global securities markets and, above all, professional service of fund managers. Basically, this is where the homogeneity between the two categories of managed investment funds ends[14]. Active managed funds aim to beat the return from a particular benchmark or market index, seeking to profit from identifying undervalued securities and managing the weights of portfolios, in accordance with the knowledge and skills to analyze and read into the market of funds' managers. Active management can be characterized by different investing styles: value and growth management. Value management looks for firms whose shares are undervalued compared to their NAV, or where managers see underestimated potential profits in the future. Instead, growth management seeks for companies' shares with above-average growth capacity over the long-term.

In contrast, passive management, also known as index fund, attempts to track the composition of an existent benchmark portfolio, based on the market efficiency assumption. In order to match the benchmark, there are two different ways to operate: from one side, buying all the stocks, maintaining the same proportions, appearing on the benchmark index, while on the other side, and this is the more realistic case, identifying

[14] Z. Bodie, A. Kane, A.J. Marcus, Investments and Portfolio Management: Global Edition, 2011, McGraw-Hill, 9th Edition.

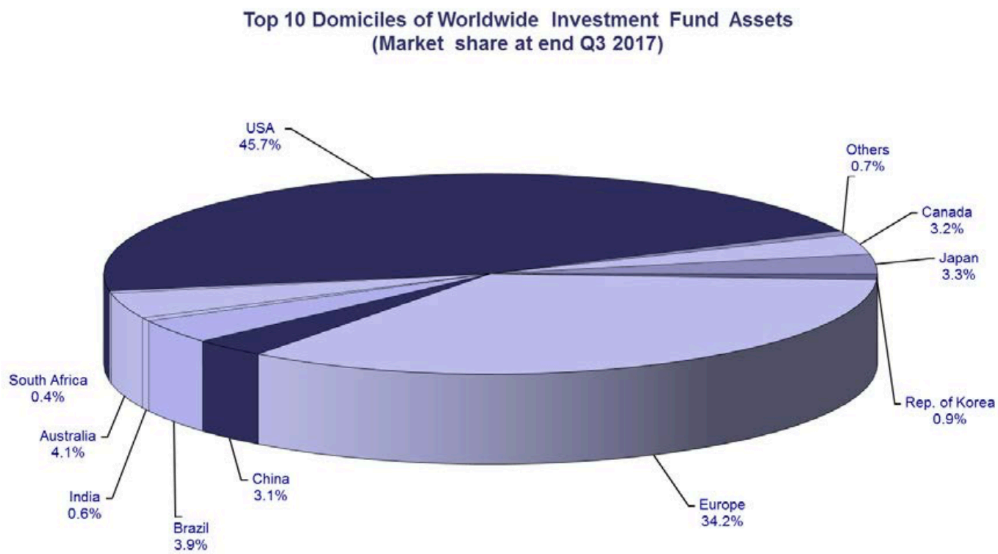
a bunch of stocks that can replicate accurately the benchmark’s performance. It is clear that index fund’s management is much easier than active one, together with lower costs for the investor, and consequently lower margins for the management. These are the reasons why the supply of index funds in the market is scarce: one example of passive management funds is represented by ETFs (Exchange Traded Funds), that have the peculiarity of being traded exclusively in stock exchanges.

Regulation

In this paragraph, there will be presented some of the most relevant aspects concerning investment funds’ regulation in U.S. and European Union. By highlighting the guidelines and legislative cornerstones from which the regulatory frameworks take shape (i.e. the Securities Act of 1933 and Securities Exchange Act of 1934 or the UCITS Directive 2014/91/EU) will be depicted the rationales and objectives underlying mutual funds’ regulatory background.

By looking at the worldwide distribution of investment fund net assets, at the end of Q3 2017, the United States and Europe held the largest shares in the world market, with 45.7% and 34.2%, respectively. The remaining 20% is shared among some few countries, such as Australia (4.1%), Brazil (3.9%), Japan (3.3%), Canada (3.2%), China (3.1%) and so on and so forth.

Figure 1.6: Worldwide Distribution of Investment Funds Net Assets at the end of Q2 2017



Source: European Fund and Asset Management Association, International Statistical Release: Trends in Second Quarter of 2017

Therefore, the majority of worldwide investment funds, are distributed amid U.S. and E.U. This is why, we will treat the legislative frameworks underlying these 2 investment fund markets, showing also the most relevant regulatory differences and similarities.

U.S. Regulatory Framework

As already explained, United States represents the biggest market for investment funds, based on funds' net assets. In addition, it is the birthplace of the first regulation on the subject. Indeed, the Securities Act of 1933, along with Securities Exchange Act of 1934, Investment Company Act of 1940, Investment Advisers Act of 1940 and extensive rules issued by the Securities and Exchange Commission (SEC), constitute the primary source of applicable law, forming the spinal column of United States financial regulation. These laws were approved in response to the Wall Street crash of 1929 and the following Great Depression, in order to regulate, in the interest of the public, the securities industry, included the investment companies, that were basically unregulated until that moment.

Let's have a quick look at the main regulatory sources cited above, starting from the oldest one: The Securities Act of 1933. Also known as the "truth in securities" law, this rule was conceived with two basic purposes: requiring that investors receive financial and other significant information concerning securities being offered for public sale and prohibit deception, misrepresentations, and other fraud in the securities' sale[15]. In order to accomplish these goals, one of the most relevant issue of the law is the information's disclosure through the registration process. With this kind of knowledge, investors may make informed judgement about whether to purchase a company's securities or not. That's why the Security Exchange Commission (SEC)[16] requires this kind of information to be accurate, even though could not guarantee it.

Therefore, the registration process requires information about company's properties and businesses, descriptions about securities to be issued, information about company's management and financial statements complying with accounting requirements. All these information, enclosed in the registration statement and prospectus, become public, so anyone can freely have access and make informed decision about purchasing of securities, avoiding misleading advertisement issues[17].

[15] For additional information visit the Security Exchange Commission website at this link: https://www.sec.gov/answers/about-lawsshtml.html#secact1933

[16] The Securities Exchange Commission is the governmental agency established in 1934 responsible for the enforcement of U.S. federal securities law and for the regulation of the commerce in stocks, bonds, and other securities.

[17] Section 17(a)(2) of the Securities Act of 1933 prohibits, in the offer or sale of any security by communication in interstate commerce, "obtain[ing] money or property by mean of any untrue statement of a material fact or any

With the Security Exchange Act of 1934, was established the SEC. The Act empowers the SEC with the required authority over all the aspect of the securities industry, including the power to register, regulate and supervise all Wall Street's operators. Among these operators, there are also the entities known as stocks exchanges, such as the New York Stock Exchange (NYSE), NASDAQ Stock Market, and the Chicago Board of Options, commonly known also as Self-Regulatory Organizations (SROs)[18].

The Securities Exchange Act identifies and forbids certain classes of actions and provides the Commission with disciplinary powers over regulated entities. More specifically, the Act broadly prohibits fraudulent activities of any kind concerning the offer, purchase or sale of securities. One of these is represented by the fraudulent insider trading[19]: this conduct becomes illegal when someone trades securities while in possession of nonpublic information, in violation of a duty to avoid trading. In addition, The Securities Exchange Act also empowers the Commission to require periodic reporting of information by companies with publicly traded securities.

For what concerns the Investment Company Act of 1940, it addresses the regulation of companies, including investment funds, that are involved primarily in investing and trading securities. Through this Act, the aim of the regulator was to minimize conflicts of interests that could arise in these operations, by requiring the companies to disclose information about fund's structure, investment policies, and its operations. Again, the law does not allow the SEC to directly oversee the investment activities of those companies.

The laws above described constitute the main body of the regulation in the U.S., but there are other rules issued successively, like the Internal Revenue Code of

---

omission to state a material fact necessary in order to make the statements made, in light of the circumstances under which they were made, not misleading."

[18] Section 7 (2)(b) of the Securities Exchange Act of 1934

[19] Section 16(b) and indirectly through Section 10(b) of Securities Exchange Act of 1934

1986 (IRC) through which the regulator imposes requirements on funds willing to exploit the favorable tax treatment afforded to regulated investment companies[20].

E.U. Regulatory Framework

The European Union with EUR 14.8 trillion investment fund assets, at the end of Q3 of 2017, represents the second largest market with 34.2% of the worldwide assets invested in funds. For this reason, it will be explained the regulatory framework underlying the E.U. market of investment funds. Among these EUR 14.8 trillion, 8.6 trillion, almost 65% of all funds' assets in Europe, at the end of 2016, were held by 31,000 Undertakings for Collective Investments in Transferable Securities (UCITS)[21], Europe's most important collective investment scheme. While, an additional 27,000 Alternative Investment Funds (AIF)[22], of what the European Law refers to the non-UCITS collective investment scheme, managed an overall EUR 5.5 trillion. Hence, UCITS and AIF are two of the most relevant investment fund scheme used throughout the European Union.

The basis for European investment law is the Undertakings for Collective Investment in Transferable Securities (UCITS) Directive, adopted in 1985[23], aimed to offer business and

---

[20] B. Chegwidden, J. Thomas, S. Davidoff, Investment Funds in United States: Regulatory Overview, Practical Law Company, 2013.

[21] A UCITS is an investment fund scheme regulated by the European Union and the European Securities and Markets Authority (ESMA) through which investors may have access to high quality and safe investment products.

[22] Regulated by the Alternative Investment Fund Managers Directive (*2011/61/EU*) (AIFM Directive or AIFMD), AIFs are defined as: funds that are not regulated by the UCITS Directive at European level. These include hedge funds, real estate, private equity and other classes of institutional funds.

[23] The Directive concerning the Undertaking for Collective Investment in Transferable Securities was embodied in the Directive 85/611/EEC of the European Economic Community on 20th December 1985, representing complete and harmonized framework covering collective investment schemes, that can be sold to retail investors throughout the

investment opportunities for both asset managers and investors by integrating the EU market for investment funds. Progressively, there have been a series of new proposal and updates of the Directive, until the latest version, the UCITS V, has been approved by the European Parliament as Directive 2014/91/EU, which went into force in March 2016.
The UCITS Directive sets out a harmonized regulatory framework for investment funds that raise capital from the public and invest it in certain classes of assets, providing high levels of investor protection and a basis for the cross-border sale of these funds. Basically, UCITS funds can be registered in Europe and sold to investors worldwide using unified regulatory and investor protection requirements[24]. These funds are perceived as reliable and well-regulated investments, very popular among investors who want to invest amid diversified funds spread out within the European Union.

Whereas, Alternative Investment Funds (AIFs) are meant to be all investment funds that are not already covered by the UCITS Directive. Such kind of Alternative Funds are for well-informed investors, like institutional, qualified and professional ones. This particular type of fund is regulated by the Alternative Investment Fund Manager Directive (AIFMD), with EU Directive 2011/61/EU, requiring all covered AIFMs to obtain authorization, and disclose various information in order to be allowed to operate in the market. The AIFMD was motivated as part of a regulatory effort undertaken by G20 nations following the global financial crisis of 2008. This Directive was intended with two major objectives built into it. First, AIFMD seeks to protect investors, increasing transparency by AIFMs and assuring that supervisors' entities, the European Securities and Market Agency (ESMA)[25],

European Union using a passport mechanism. This type of investment scheme accounts for 75% of all investment funds across the European Union.

[24] http://eimf.eu/aif_ucits_seminar/

[25] Jonathan Boyd, ESMA clarifies final guidelines on reporting obligations under AIFMD, Investment Europe. Retrieved 20 April 2015.

and the European Systemic Risk Board (ESRB) have the necessary information they need to monitor financial systems in the EU territory[26]. Investors' protection is obtained through the introduction of stricter compliance around the information disclosure[27], including conflict of interest and independent assets' valuation.

The second aim of the Directive is to get rid of some of the systemic risk that the funds can pose to the EU economy. To obtain this goal, the AIFMD requires the remuneration policies must be structured in a way that does not encourage excessive risk taking, and that financial leverage have to be reported to the ESRB.

To conclude, both U.S. and European investment funds' regulatory frameworks, especially after the global financial crisis of 2008, aim to protect private investor's interests, persisting in fighting against fraudulent actions. One of the key concept underlying these regulations is the stricter compliance concerning the disclosure of information, which translates into a claim for transparency, coming from the regulatory bodies.

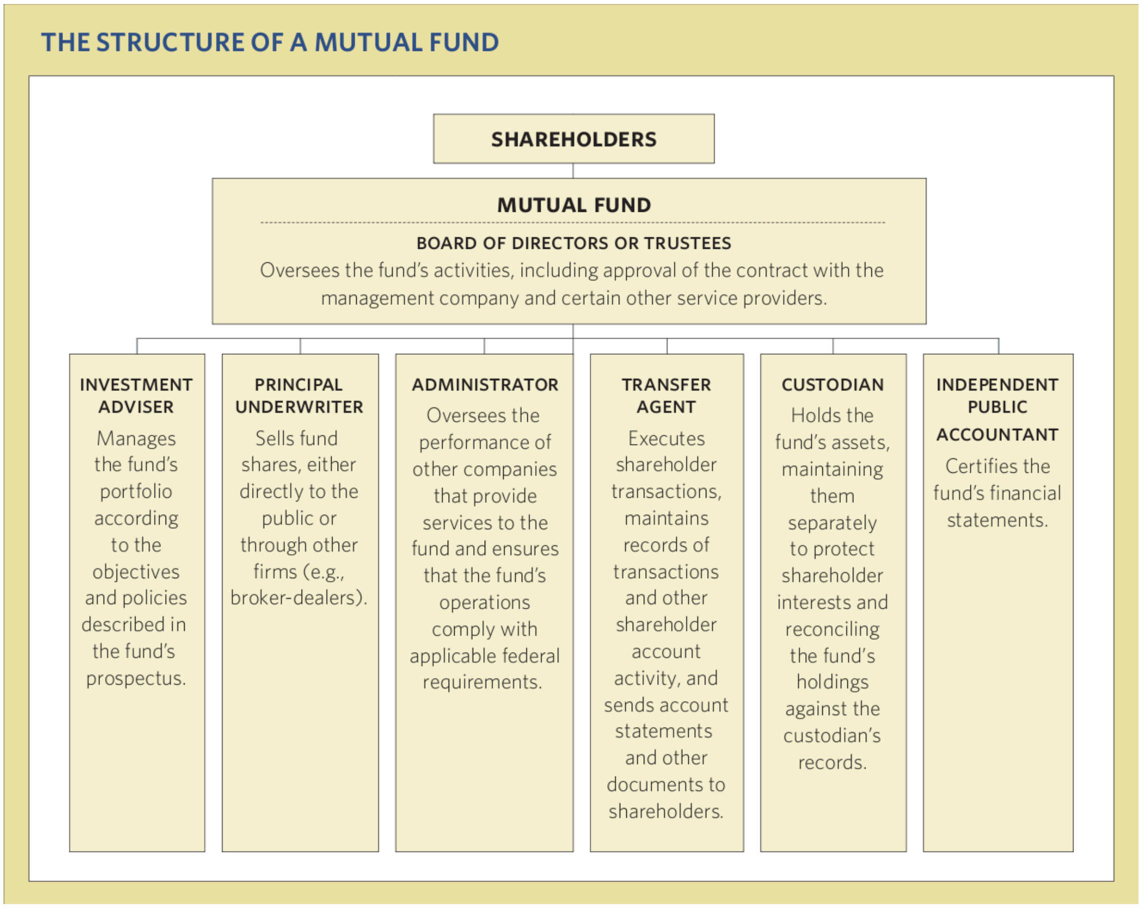
Fund Structure

A classic example of mutual fund's structure is given by the following scheme (Figure 1.7):

Figure 1.7: Structure of Investment Fund:

[26] Niamh Moloney, EU Securities and Financial Markets Regulation, Oxford University Press. Retrieved 20 April 2015.

[27] Articles 22 and 23 of the Directive 2011/61/EU (AIFMD)



Source: Investment Company Institute's (ICI) 2009 ICI Fact Book

where:

- Board of Directors: It serves the interests of shareholders, by managing the company and handling the administrative issues. The Board of Directors is usually elected by the board of shareholders. One of the task consists in suggesting a preference of funds in order to meet the investment needs of shareholders. In addition, the Board defines funds' objectives and finally hires the investment advisor, transfer agent and the custodian of the funds.

- Investment Adviser: This is a central figure in the funds' structure. He/she has the task of managing the fund's portfolio according to the objectives and policies described in the fund's prospectus. Hired by the Board of Directors, Investment Adviser acts as a fund advisor or fund manager, managing day-to-day portfolio trading, earning a management fee, plus an incentive bonus if exceeds certain performance targets.

- Principal Underwriter: Also known as distributor or sponsor, is the principal underwriter of the fund. This figure establishes the fund and acts as a promoter of it. One of the main tasks concern the sales of fund's shares directly to the public or through brokers or dealers.

- Transfer Agent: Also known as Register & Transfer Agent (R&T agent), has an operative role inside an investment fund. He performs a number of transactions, on a daily basis, ranging from buying, selling, or switching units, handling the distribution of dividends and capital gains to shareholders, or processing trade confirmations. In certain circumstances, the custodian will act as transfer agent. The transfer agent usually is paid with a fee for services provided.

- Custodian: Usually is a bank, trust company or a similar financial institution, that holds and protects the fund's assets, maintaining them separately to protect shareholders' interests and reconciling the fund's holding against the custodian's records. In addition, custodian also keeps track of transactions, sales and purchases, identities of shareholders, besides collecting and distributing dividends and interests to shareholders.

- Independent Public Accountant: Also known as Independent Auditor, this figure performs an audit of fund's financial statements and its records is filed with the specific Commission (i.e. the SEC in the U.S.) in accordance with the securities law or the Commission's regulations28.

- Dealers: As mentioned before, the sponsor usually distributes shares of the mutual fund through dealers or brokers. This figure is outside the legal boarders of the investment fund but deserves to be quoted. Basically, the dealer purchases shares from the sponsor at a discount and fill customers' orders.

Fund Fees and Expenses

As with any business, it costs money to invest in a fund. There are certain costs associated with an investor's transactions (such as buying, selling, or exchanging fund shares), which are commonly known as "shareholder fees," and ongoing fund operating costs (such as investment advisory fees for managing the fund's holdings, marketing and distribution expenses, as well as custodial, transfer agency, legal, accounting, and other administrative expenses). Even though these fees and expenses may not be listed individually as specific line items on the account statement, they can have a substantial impact on the investment return over time.

Fees and expenses differ among funds and the amount may depend on the fund investment objective. Funds typically pay regular and recurring fund operating expenses

[28] Robert A. Robertson, Fund Governance: Legal Duties of Investment Company Directors, 2001, Law Journal Press.

out of fund assets, instead of imposing these fees and charges directly on investors. Because these expenses are paid out of fund assets, an investor will pay them indirectly. Usually, these expenses are identified in the standardized fee table in the fund's prospectus under the heading "Annual Fund Operating Expenses".

The fund's directors, and its independent directors, in particular, function as "watchdogs" who are supposed to look out for the interests of the fund's shareholders. One of the most significant responsibilities of a fund's board of directors is to negotiate and review the advisory contract between the fund and the investment adviser to the fund, including fees and expense ratios.

For the reasons cited above, it is important for an individual, not professional, investor to understand and be able to compare fees and expenses of different funds.

There are several classes of fees, as it will be described below, but for an investor to facilitate the comparison between funds, it can be helpful to look at the Expense ratio[29]. This is a percentage value expressing the annual fee that the funds or ETFs charge to shareholders. Basically, it gives the percentage of assets deducted each fiscal year for fund expenses, including 12b-1[30], management and administrative fees, operating costs, and all other asset-based costs incurred by the fund, deducted from the fund's average net assets, and accrued on a daily basis.

Fund transaction fees, or brokerage costs, as well as sales charges are not included in this ratio. If the fund's assets are small, its expense ratio can be quite high due to the fact that

[29] The expense ratio is the percentage of fund assets paid for operating expenses and management fees. It typically includes the following types of fees: accounting, administrative, advisor, auditor, board of directors, custodial, distribution (12b-1), legal, organizational, professional, registration, shareholder reporting, and transfer agency. The expense ratio does not reflect the fund's brokerage costs or any investor sales charges.

[30] Rule 12b-1, established with the Investment Company Act of 1940, allows mutual fund advisers to make payments from fund assets for the costs of marketing and distribution of fund shares. The original rationale underlying the plans was that such fees help attracting new shareholders into funds through marketing and advertisement and providing incentives for brokers.

the fund must meet its expenses from a restricted asset base.

Conversely, as the net assets of the fund grow, the expense percentage should ideally decrease as expenses are spread across the wider base.

Thus, the fees that an investor may pay when investing in mutual funds are the following:

- *Transaction fee (Purchase fee)*: typically, is about purchase costs, it is charged when the shareholder buys shares, and is paid to the fund, not to the stockbroker.
- *Redemption fee (Exchange fee)*: another type of fee that funds charge their shareholders when they sell or redeem shares, or exchange to another fund. Like transaction fee, is paid to the fund too.
- *Periodic fees:* (which are included in the Expense Ratio)
    - *Management fee:* paid, out of the fund assets, to the fund's investment advisor for portfolio management. Also called maintenance fees.
    - *Account fee:* fees that fund separately impose in connection with the maintenance of their account.
    - *Distribution and Service fee (12b-1 fee[31])*: paid, out of the fund, to cover marketing costs, cost of selling fund shares, and costs of providing shareholder services. Is included in fund's Expense ratio, generally between 0.25 and 1% (the maximum allowed) of a fund's net asset. 12b-1 fee can be broken down into two distinctive charges: the distribution and marketing fee (maximum 0.75% annually) and the service fee (maximum 0.25% annually).

[31] Named after section 12 of the Investment Company Act of 1940, https://www.sec.gov/about/laws/ica40.pdf

Benefits and Disadvantages of Investing in Funds

Benefits:

- Professional management: investment funds employing skilled and experienced professionals, offer qualified investment services to investors. Fund management analyses in detail past and present performance, financial statements and a series of multiples and ratios of hundreds of companies selecting the best ones in order to achieve the objective of the fund and ultimately satisfying shareholders.

- Diversification: Since one of the fundamental investment rule is the importance of diversification, an investment fund can be a successful and easy way to achieve this objective. The portfolio diversification allows to increase the expected return meanwhile minimizing the risks. Therefore, investing in funds results in a cost-effective way to reach this primary and basic goal for every investor.

- Liquidity: Investment funds, in particular Money Market Funds, have a significant characteristic: the assets underlying these funds are generally liquid.

- Easy of comparison: For an average not professional investor, investment funds are convenient also due to the ease of comparison between similar funds. Investors can compare the funds based on metrics such as level of risk, return and price, and given that this information are easily accessible, eventually everyone may be able to make wise decisions, based on valid judgements.

- Potential return: Funds have the potential to provide high returns, depending on the class of fund that is taken into consideration, to an investor than other options over a reasonable period of time.

- Transparency: Thanks to the above described regulation, investment funds have to disclose a detailed list of useful information allowing the average investor to know as much information as possible about the companies where he or she is going to invest money.

Disadvantages:

- Costs: Usually, investment funds have different fees that impact on the overall payout. These fees can be shareholder or operating fees. The shareholder fees are paid directly when purchasing or selling shares. Conversely operating fees are charged as an annual percentage - usually ranging from 1 to 3%, and assessed to fund investors regardless of the performance of the fund. Whether a fund is not performing positively, these fees will have a negative impact on shareholders returns, probably turning these one into losses.

- Misleading advertisement: The misleading advertisements of investment funds, even if is prohibited by the general antifraud provision of federal securities law, may conduct investors down the wrong path. In spite of the regulation on this matter is quiet common bumping into misleading information concerning false funds' performance. It can happen that some funds are incorrectly labeled as growth funds, while others are classified as small-cap or income.

- Fluctuating returns: Like the majority of investment means available in financial markets, investment funds have not guaranteed returns. The price or Net asset value of funds are volatile, except few cases of funds with stable values, so it can appreciate or depreciate based on the expectations of the market and actors playing in it. Unlike fixed-income products, such as bonds and Treasury bills, funds experience price fluctuations along with the stocks that make up the fund. Another important thing to be aware of is that investment funds are not guaranteed by any national government, so in the case of dismissal, investors will not get any sort of refund back.

Performance Measurement Methods of Investment Funds

Performance Measures and Asset Pricing Models: An Overview

Before starting to describe and analyze some of the most important performance valuation measures, it is useful to provide, as a theoretical background, an overview regarding some asset pricing theories, models linking the portfolio's expected returns with volatility and other variables.

Once introduced these fundamental theories, the chapter will treat the description of important performance metrics, which can be divided in two broad categories, risk-adjusted and conventional methods. As it will be discussed below, asset pricing models and performance measures are inseparably linked, and the evolution of the latter in the literature mirrors the development of asset pricing models. A brief overview of this parallel development should be useful.

Historically speaking, the origin of investment studies began with Markowitz's cornerstone on portfolio selection, from which all the subsequent theories and models has taken form and got inspiration.

In 1952, Markowitz laid the foundation of "*Modern Portfolio Theory*"[32], with his mean-variance model. In its simplest form, Markowitz's theory is about finding the optimal balance between returns' maximization and risks' minimization. The objective of Markowitz's work was to select investments in such a way as to diversify risks while not reducing the expected return. This represents one of the most important and influential economic theories dealing with finance and investment.

Also known as "*Portfolio Theory*", the model suggests that is possible to build an efficient frontier of portfolios, giving the highest expected return for a given level of risk[33]. It is actually simple to apply and effective. While it does not replace the role of an informed investor, it can provide a powerful tool to complement an actively managed portfolio. The theory suggests that by investing in more than one security, an investor can exploit the benefit of diversification, which also translates in a riskiness' reduction of portfolio. Keep in mind that the risk of a portfolio composed by several individual stocks will be lower than the risk intrinsic in holding any of the individual stocks alone.

The "*Modern Portfolio Theory"* assumes investors are risk averse and, when selecting among portfolios, they care about mean and variance of the investment's return. To conclude, the resulting portfolio minimizes the variance of its return, given the expected return, conversely maximizing the expected return, given the variance. For this reason, Markowtiz's theory is often considered a "mean-variance model".

In other words, Markowitz, with his well-known theory, showed that investment is not just picking stocks, but is about choosing the right combination of stocks among which distributing the wealth.

[32] H. M. Markowitz, Foundation of Portfolio Theory, Journal of Finance, Volume 46, Issue 2 (Jun, 1991), 469-477

[33] H.M. Markowitz, Portfolio Selection. Journal of Finance, 7, 77-91, (1952).

Later, building on the work of Markowitz, Jack Treynor (1961-1962), William Sharpe (1964), John Lintner (1965), Jan Mossin (1966), proposed a capital asset pricing model (CAPM, 1964)[34], a model that five decades later is still widely used, due to its simplicity and utility, in several applications, such as firms' cost of capital estimation and evaluating the performance of managed portfolios.

Basically, the CAPM, demonstrates that, under certain conditions, the expected return of an asset is only determined by the *beta (β)*, also known as systematic risk or market risk. This model is still used to determine a theoretically appropriate required rate of return on an asset, in order to make decisions about assets' composition in a well-diversified portfolio.

The CAPM is based upon its assumptions, such as the efficiency and competitiveness of the stocks' market[35], the presence in the market of rational and risk-averse investors, market's frictionless, that means there's no transaction costs[36], taxes, and restriction on selling or short-selling. The model also requires limiting assumptions concerning the statistical nature of securities returns and investors' preferences. Finally, investors are assumed to agree on the likely performance and risk of securities, based on the common time horizon.

Although CAPM's assumptions are unrealistic, such simplification of reality is often necessary to develop trackable models. The true test of a model lies not just in the

[34] E. F. Fama, K. R. French, The Capital Asset Pricing Model: Theory and Evidence, Journal of Economic Perspectives, Volume 18, Number 3--Pages 25-46

[35] This assumption presumes a financial market populated by highly-sophisticated and well-informed buyers and sellers.

[36] As it will be explained further on, transaction costs may influence the performance evaluation of investment funds, hence this assumption is very strong, besides unrealistic.

likelihood of its assumptions but also in the validity and usefulness of the model's prescription. Tolerance of CAPM's assumptions, however improbable, allows the derivation of a concrete, though idealized, model in which financial markets measure risk and transform it into expected return.

Therefore, the CAPM combines the risk with the returns in the linear form:

$$\textbf{CAPM} \qquad \textit{Expected Return} = r_f + \beta * (r_m - r_f) \qquad (2.1)$$

R_f = Risk-free rate

β = Beta

R_m = Return of the market portfolio

Following the CAPM, in the 1970's, scholars began to explore empirical asset pricing models in which exposure to more than a single market risk factor determines expected returns, as it will be explained further on.

The traditional performance measures generally fall into two categories, namely conventional and risk-adjusted methods.

Conventional Methods

Benchmark Comparison

Conventional methods most widely concern comparisons of the performance of investment portfolio against broader market index. An example of benchmark market index can be the U.S. Standard & Poor's 500 index (S&P 500), which includes 500 stocks issued by 500 large companies in the U.S.[37]. The S&P 500[38] is widely considered the leading indicator of U.S. securities, as well as the most accurate gauge of the performance of large-cap American equities. However, it's inappropriate compare a fund investing in small-cap securities, or mainly in bonds, using the S&P 500 index as benchmark. For example, the Barclays Capital U.S. Aggregate Bond Index[39] is considered the benchmark index for the bond market, while the Russell 2000 Index is suitable if considering small-cap securities market, the MSCI EAFE Index for what concerns International stocks (Europe, Australia and Far East), and to conclude the EURO STOXX Index if we need a benchmark based on European stocks only. Hence, the right choice when considering

[37] http://us.spindices.com/indices/equity/sp-500

[38] This index is regarded as the best single gauge of large-cap U.S. equities. There is over USD 7.8 trillion benchmarked to the index, with index assets comprising approximately USD 2.2 trillion of this total. The index includes 500 leading companies and captures approximately 80% coverage of available market capitalization.

[39] Also known as Bloomber Barclays U.S. Aggregate Bond Index, after 2016, is a broad-based benchmark that measures the investment grade, U.S. dollar-denominated, fixed-rate taxable bond market, including Treasuries, government-related and corporate securities, mortgage-backed securities, asset-backed securities and collateralized mortgage-backed securities.

comparison method depends fundamentally on the specific market segment of benchmark index, that clearly must mirror as much as possible the selection of securities held by investor’s portfolio or directly by the mutual fund.

The Benchmark comparison method is quite simple: if the return on the portfolio exceeds the one of the benchmark index, during the same time periods, then the portfolio is over-performing the benchmark index, or simply have beaten the benchmark. Although this type of comparison is very common in the investment world, this creates a particular problem of evaluation. The level of risk of the investment portfolio may not be the same as that of the benchmark index portfolio. Higher risk should lead to commensurately higher returns, in the long-term. This means if the investment portfolio has performed better than the benchmark portfolio it may be due to lower level of riskiness of the investment portfolio compared to the benchmark. Therefore, a simple comparison of returns, usually, may not produce consistent results, even if is widely used by common “uninformed’ investors.

Style Comparison

A second conventional method of performance evaluation called ‘style-comparison’ involves comparison of return of portfolios having a similar investment style. While there are many investment styles, one commonly used approach classifies investment styles as value versus growth. The “value style” portfolios invest in companies that are considered undervalued on the basis of criteria such as price-to-earnings (PE) and price-to-book (PB) value multiples. The “growth style” portfolios invest in companies whose revenue and earnings are expected to grow faster than those of the other companies. In order to evaluate the performance of a value-oriented portfolio, it should be compared the return on such a portfolio with that of a benchmark portfolio that is value style based. Similarly, a growth style portfolio is compared with a growth style benchmark index. Once again, here there is the same weakness presented above, that is to say a lack of risk's comparison: this method suffers from the fact that, while the style of the two compared portfolios may look similar, their risks will probably be different. Also, the benchmarks chosen may not be truly comparable in terms of the style since there can be many important ways in which two similar style-oriented funds vary.

Risk-Adjusted Performance Measures

Based on the asset pricing models described earlier, many scholars, throughout the years, had put forward a series of investment performance evaluation methods, classified as risk-adjusted measures. These methods make adjustments to returns in order to take account of the differences in risk's levels between the investment fund and the benchmark portfolio. Even though these kind of performance measures are popular among investors and widely used in practice, they have theoretical flaws. Following, there will be explained the major ratios and indexes used in the evaluation of performance, and outlined advantages and disadvantages tied to the application of these gauges. Although the literature swarms with many such methods, the most well-known ratios are: *Sharpe*, *Treynor*, *Jensen alpha*, *Modigliani and Modigliani*, *Sortino* and *Information*. These measures along with their pros and cons are discussed below.

Sharpe Ratio

The Sharpe Ratio (1966)[40] has been developed by Nobel Laureate William F. Sharpe to measure risk-adjusted performance. This ratio computes the risk premium of an investment portfolio per unit of total risk. The risk premium, known as excess return, is the return of the portfolio minus the risk-free rate, usually measured by the Treasury bond yield, while the total risk of the portfolio is the standard deviation (σ) of its return. The numerator captures the reward for investing in a risky portfolio of assets in excess of the risk-free rate, while the denominator is the volatility of portfolio's return. In this sense, the Sharpe measure is also called the "reward-to-variability" (R/V) ratio[41]. Equation below gives the Sharpe ratio.

$$\boldsymbol{Sharpe\ Ratio = \frac{Rp - Rf}{\sigma p}} \tag{2.2}$$

S = Sharpe Ratio

R_p = Return of the Portfolio

R_f = Risk-free rate

σ_p = Standard Deviation of Returns of the Portfolio

[40] W.F. Sharpe, Mutual Fund Performance. *Journal of Business*, **1**, 119-138, (1966).

[41] Professor Sharpe calculated this R/V ratio for 34 mutual funds of the Dow Jones portfolio from 1954 to 1963. Only 11 over-performed the Dow Jones benchmark.

Here, r_p is the rate of return of a portfolio, r_f is the risk-free rate, σ_p is the standard deviation of fund's return. Standard deviation is widely used to measure the degree of fluctuation in a portfolio's return. The larger the σ_p, the greater the magnitude of the fluctuations from the portfolio's average return. The Sharpe ratio is used to characterize how well the return of an asset compensates the investor for the risk taken. This ratio is very useful because although one portfolio or fund can reap higher returns than its peers, it is only a good investment if those higher returns do not come with too much additional risk. The greater a portfolio's Sharpe ratio, the better its risk-adjusted performance has been. Investors are often advised to pick investments with high Sharpe ratios, because it indicates that the investment has a higher risk premium for every unit of standard deviation risk. However, like any mathematical model it relies on consistency of data. When examining the investment performance of assets with smoothing returns the Sharpe ratio should be derived from the performance of the underlying assets rather than the fund returns.

Hence, the strengths of this ratio are its straightforwardness and simplicity as a performance measure, using the standard deviation, including systematic and unsystematic risk, which makes the Sharpe ratio suitable to evaluate portfolio and funds returns that are not completely diversified, and also, with different trading strategies. However, if on one hand standard deviation and expected returns are useful sources of data in evaluation, on the other hand is really challenging to find the correct ones. As always, in a highly stable environment, is possible to use past data, especially if macroeconomics factors and competitive and market conditions haven't changed much in recent years. In a scenario like this, an estimate of returns and standard deviation over the past period may be good predictors of what will happen in the future. Nevertheless, in today's dynamic markets, it is rare that the future replicates the past, hence the past data are not reliable in order to make truthful appraisals. Again, standard deviation includes movements in every direction, which may be considered a weakness because it does not differentiate between upside and downside volatility.

Another Sharpe ratio's weakness is its link with normal distributions. As a consequence, the Sharpe ratio is not a suitable efficiency measure for investments with asymmetric, or, generally, not Gaussian, expected returns. Last but not least, there is the fact that Sharpe ratio provides a valuable information only when compared with a benchmark or another investment, which leads to another challenging choice regarding the benchmark to be used.

Treynor Ratio

Another measure widely used, takes the name of his inventor, Jack Treynor, that, in 1965, established a relation between excess returns and riskless investments (i.e. Treasury Bills). With the Treynor Ratio[42] is possible to measure the risk-adjusted performance of a fund or portfolio. Unlike the Sharpe Ratio, Treynor ratio employs the beta (β), the "market" or systematic risk, in the denominator of the formula, instead of the standard deviation, the total risk. Beta represents the slope of the regression of the returns of the managed portfolio on the returns to the market portfolio and indicates how closely an investment follows the upward and downward movements of financial markets. A value

[42] J. Treynor, How to Rate Management of Investment Funds. *Harvard Business Review*, 41, 63-75, (1965).

of beta greater than 1 means the stock or fund is more volatile than the market, which brings greater levels of risk and which implies greater losses (or gains), especially in times of severe market events. For what concerns the decision criteria, the higher Treynor ratio, the more attractive is the portfolio or fund, on a relative risk-adjusted basis. The Treynor ratio is given by following equation:

$$\textbf{Treynor Ratio} = \frac{\boldsymbol{Rp - Rf}}{\boldsymbol{\beta p}} \tag{2.3}$$

R_p= Return of the portfolio

R_f= Risk-free rate

β_p= Beta of the portfolio

Both Sharpe and Treynor ratios rank performance measures that take in consideration certain kind of risks: Sharpe uses the total risk, that is systematic plus "specific" risks, while Treynor uses only the systematic one, the market risk. It is better to use the Sharpe ratio, when trying to evaluate funds that are sector specific, due to the fact that unsystematic risk, or specific risk, would be present in sector specific funds, therefore, the performance evaluation will be based on the total risk, giving meaningful results. Whether are taking into account performance measurement of diversified funds, the specific or unsystematic risk is not significant, as these funds are expected to be well-diversified by their nature, hence the Treynor ratio would be preferred. Basically, when the portfolio, or fund, is not fully diversified, Sharpe ratio is a better measure of performance, while when the portfolio is fully diversified, Treynor ratio would better assess its performance.

The strengths of this ratio underlies mostly on the use of beta as a risk's measure: first of all because it distinguishes, again, between systematic and unsystematic risk; then because beta is inherently more stable than standard deviation, as risk gauge. As well as it has been done before, it will be described also the weaknesses, that are similar to the Sharpe's ones. The ratio assumes that the portfolio under evaluation is fully diversified, given that only systematic risk is taken into account, measuring only market risk. Like the Sharpe ratio, this is exclusively meant as a ranking criterion, and by the way is useful only when are considering sub-portfolios of a broader, fully diversified portfolio; if this is not the case, assets with the same systematic risk, but different total risk, will be ranked the same. Another similarity with the previous measure is based on the backward-looking nature. Investments will inevitably show different performances in the future than the past ones.

Jensen's alpha

Developed by American economist Michael Jensen in 1968, this model, based on the Capital Asset Pricing Model (CAPM), is used to determine the abnormal return of a security or portfolio over the theoretical expected return. In short, Jensen's alpha[43] tries to explain whether an investment has performed better or worse than its beta value would suggest. The alpha is simply the intercept from a regression of fund excess returns on market excess returns. According to the CAPM the intercept alpha should be zero, so the extent to which alpha differs from zero measures the extent to which the CAPM is unable to account for the returns of the fund or asset. This means that alpha measures

[43] M.C. Jensen, The Performance of Mutual Funds in the Period 1945-1964. *Journal of Finance*, 23, 389-416, (1968).

abnormal performance relative to a theoretical expected return, based on the capital asset pricing model.

Hence, alpha can be greater than, less than or equal to zero. For example, an alpha greater than zero suggests that the security outperformed its theoretical expected return. Jensen’s alpha is given by equation below:

$$\boldsymbol{\alpha = Rp - [Rf + \beta p * (Rm - Rf)]} \qquad (2.4)$$

$\alpha =$ Jensen's alpha

$R_p =$ Return of the portfolio

$R_m =$ Return of the Market portfolio

$R_f =$ Risk-free rate

$\beta_p =$ Beta of the portfolio

When comparing two funds with similar beta ratios, investors prefer the one with the higher alpha, since this implies greater reward at the same level of risk. While measuring return performance, Jensen’s alpha measure takes an investment’s risk profile into account showing in this way an overall picture of performance on a risk-adjusted basis. This helps investors to gauge the value added or detracted by a fund manager, and helps in the comparison of funds.

A common weakness of both Jensen alpha and Treynor ratio is that both require an estimate of beta, which can differ a lot depending on the source of data provider. This in turn can lead to a mismeasurement of risk-adjusted return. Like the previous two measures, even Jensen's alpha is subject to generic weaknesses of the CAPM, and those linked to the mean-variance world.

Modigliani-Modigliani Measure

Franco Modigliani and Leah Modigliani[44] propose a modified version of Sharpe's measurement approach. They shared the view that the Sharpe ratio was too difficult to understand for the average investor, for this reason they proposed the "RAP" (Risk-adjusted performance) ratio, also referred to as M^2, or Modigliani-Modigliani Measure. This measure expresses a fund's performance relative to the market in percentage terms. They believed that the average investor would find the measure more comprehensive. Analytically their approach is the following:

$$\boldsymbol{RAP = \frac{\sigma m}{\sigma p} * (Rp - Rf) + Rf} \tag{2.5}$$

$$R_p = \text{Return of the portfolio}$$

$$R_f = \text{Risk-free rate}$$

[44] F. Modigliani, L. Modigliani, Risk-Adjusted Performance. *Journal of Portfolio Management*, 23, 45-54, (1997).

$$\sigma_m = \text{Ex-post Standard Deviation of the market}$$

$$\sigma_p = \text{Ex-post Standard Deviation of portfolio}$$

Modigliani and Modigliani propose to use the standard deviation of a broad market index, i.e. the S&P 500, as the benchmark for comparison. In simple terms, for any fund with certain return and level of risk, the M^2 measure is equal to the return the fund would have performed if it had the same risk as the market index. Therefore, the fund with the highest "RAP", or Modigliani, measure would have the highest return for any level of risk. The peculiarity, compared with the Sharpe ratio, is that, since the Modigliani measure is expressed in percentage points, it can be easily understood by average investors.

In opposite to Sharpe who used to rank funds according to the slope of the capital market line, Modigliani and Modigliani lever or unlever, depending if the portfolio's standard deviation is higher or lower than the market one, portfolio's risk to match market risk and present the resulting risk-adjusted return as the ranking variable. This approach produces the same ranking as obtained by applying the Sharpe Ratio, but in an easier way to be understood. As it can be simply inferred, the Modigliani measure has the same limitations as the Sharpe ratio.

Sortino Ratio

In the early 1980s, Frank Sortino, who was working for the Pension Research Institute, had undertaken research to come up with an improved measure for returns. The Sortino ratio[45] is a measure of risk-adjusted performance that tries to improve the more commonly used and well-known Sharpe ratio. As discussed previously, evaluating the performance of a portfolio over time by just looking at fund's absolute performance is generally not a good idea. This is due to the different levels of risk underlying different investment strategies. This ratio is a modification of the Sharpe ratio; however, the risk-free rate is replaced by the minimum acceptable return (MAR), and the standard deviation of the returns is substituted by the downside risk, or the semi-standard deviation of the returns below the MAR.

Unlike the Sharpe ratio that uses the standard deviations as measure of risk, Sortino ratio tries to correct this by using the so-called "downside deviation", thus considering downside risks. Semi-standard deviation measures the variability of underperformance below a minimum target rate. It is interesting to note that even Nobel laureate Harry Markowitz, when he developed Modern Portfolio Theory (MPT) in 1959, recognized that only downside deviation is relevant to investors, using it to measure risk would be more appropriate than using standard deviation. Nevertheless, he used variance in his MPT work because optimizations using downside deviation were computationally impractical at the time. The Sortino ratio is given by the following formula:

$$\textit{Sortino Ratio} = \frac{(Rp - t)}{DSR} \tag{2.6}$$

[45] F.A. Sortino, L.N. Price, Performance measurement in a downside risk framework, Journal of Investing 3: 50–8, (1994).

$$R_p = \text{Return of the portfolio}$$

$$t = \text{Minimum Acceptable Return or MAR}$$

$$\text{DSR} = \text{Downside Risk}$$

As mentioned above, this ratio adjusts the average return of portfolio with a target return (MAR). The choice of the target or minimum acceptable return depends on the investment goal of the fund, that is implicitly claimed through its strategy. The higher the portfolio return over the MAR, the higher will be the Sortino ratio.

This ratio is intended to be compared, like the previous measures, with other comparable funds or benchmark index. A higher Sortino ratio indicates better risk-adjusted performance. In order to compare funds' performance, the ratio of each fund must have an equal MAR.

The interpretation of this ratio is less straightforward than the Sharpe ratio, due to the fact that the measure of risk has a less direct interpretation than the standard deviation and the choice of the target return (MAR) depends on the fund's chosen strategy.

By the way, the Sortino ratio appears to overcome some of the issues underlying the application of the Sharpe ratio: it combines a relevant return target in both numerator and denominator of the formula; it assesses downside volatility without penalizing the upside one, and as a consequence of this peculiarity, it is also more applicable to distributions that are negatively skewed compared to other standard deviation based measures. Moreover, drawing a parallel between the Sharpe and Sortino ratios for a fund, it can be identified which portion of the volatility of the fund is related to outperformance versus underperformance.

On the other hand, since this ratio only incorporates downside volatility below the frontier and ignores the upside volatility, sometimes it may provide an incomplete perspective on the risk side. Furthermore, when applying Sortino to strategies with known asymmetric return distributions, such as hedge funds, it could give misleading results[46].

To conclude, this ratio is best used as a measure to compare different portfolios or investment funds, in terms of downside risk. Hence, if the main goal of the portfolio management is to avoid negative returns, it is a more appropriate measure than the Sharpe ratio. In that case the MAR should be set equal to 0.

Information Ratio

The Information Ratio (IR), also known appraisal ratio, is another measure of the risk-adjusted return of financial securities. The IR measures the ability of a portfolio manager to yield excess returns relative to a benchmark. This ratio is useful when comparing a bunch of funds sharing similar management styles. It can be written as follows:

$$\textit{Information Ratio} = \frac{E(Rp)-E(Rb)}{\sigma(Rp-Rb)} \qquad (2.7)$$

$$E(R_p) = \text{Expected Return of the portfolio}$$

$$E(R_b) = \text{Expected return of a benchmark portfolio (i.e. benchmark index)}$$

[46] O.Steinki, Common Metric for Performance Evaluation: Overview of popular Performance Measurement Ratios, Evolutiq, (2015).

$$\sigma(R_p - R_b) = \text{Tracking Error (St. Dev. of difference between } R_p \text{ and } R_b)$$

Since IR measures managers' ability to generate higher returns relative to a benchmark portfolio, from the formula it's evident that, in order to generate value for shareholders (investors), managers should maximize the expected active return (numerator) and minimize the cost of their active management style (denominator). A higher IR suggests that managers can achieve higher returns without taking on additional risks.

Alternative Performance Measures

All the measures described above, have their theoretical foundation on the CAPM model. Many authors argue that the single market risk factor, Beta (β) in the CAPM, is not sufficient to assess funds' returns. For this reason, various factors, such as macroeconomic, industry and firm related, have been proposed in the literature in order to provide more reliable portfolio performance's measures. Multi-factor pricing model was an attempt to provide more reliability in this field. Multifactor pricing models were presented by Ross (1976) through the Arbitrage Pricing Theory[47] and by Merton (1973) through the Intertemporal CAPM. The multifactor pricing model implies that the expected return on an asset is a linear function of factor risk premiums and their associated factor sensitivities. The underlying theory is, however, not very explicit on the exact nature of

[47] S. Ross, The arbitrage theory of capital pricing, *Journal of Economic Theor,* (1976).

these factors. The selection of an appropriate set of factors is thus largely an empirical issue.

Chen et al. (1986) find evidence of five priced macroeconomic factors. The Fama and French study uses firm characteristics to outline factor portfolios resulting in the well-known three-factor model, while Carhart (1997) finds evidence for a fourth momentum factor. Other approaches rely on macroeconomic factors like interest and inflation rates. Some include indexes that are related to managers' investment style (e.g. small-growth capitalization; large-value capitalization, etc...). Thus, there is a lack of consensus among scholars about the number and the exact identity of the factors.

Fama and French Three-Factor Model

Some other scholars used multi-factor models based on the Arbitrage Pricing Theory (APT), to evaluate the performance of mutual funds, of which the Three-Factor Model[48] and four-factor model are the most representative. The Fama and French Three-Factor model states that in an equilibrium market the arbitrage portfolio must be zero, meaning that an arbitrage portfolio cannot exist. If this condition did not hold market participants would sell assets whose expected return is lower than implied by the detected common risk factors of the market and buy assets whose expected return is higher than implied by the risk factors. This process of arbitrage ensures equilibrium as market participants

[48] Fama Eugene F., French Kenneth R., "Common Risk Factors in the Returns on Stocks and Bonds", Journal of Financial Economics, February, (1993).

engage in it until there is no further possibility of making a riskless profit through trading one security for another.

On this basis Fama and French tried to define the factors which are relevant in predicting a security expected return. The equation to measure a security's expected return is given below:

This model can be written as follows:

$$R_p - R_f = \alpha + \beta * (R_m - R_f) + \beta' * (SMB) + \beta'' * (HML) + \varepsilon_p \quad (2.8)$$

Through regression analysis the factors responsible for a security's variation can be detected. One setback of APT model is that the model does not specify the specific risk factors. Fama and French detected three risk factors for stock portfolios and two risk factors for bond portfolios. The factors for stock portfolios are:

- Excess return of the market over the risk-free rate [R_m-R_f]
- Size of the firm [SMB] (Small Minus Big)
- Book-to-Market equity ratio [HML] (High Minus Low)

These factors measure the historic excess return of small cap stocks over big cap stocks and value stocks over growth stocks, while factor for bond portoflios are:

- Time to maturity
- Default risk premium

Fama and French propose their findings as being useful for portfolio performance evaluation but did not pursue it per se.

The Grinbblatt & Titman Model

The problem concerning the choice of right benchmark have led to alternative approaches to determine the performance of a portfolio or a fund. Grinbblatt and Titman[49], pursued an approach where no benchmark is needed, thus alleviating several problems tied to the benchmark's employment. The greatest issue and constraint of their model underlies in the characteristics that is only applicable if the exact composition of portfolio or fund is known. This is in strong contrast to the portfolio measures introduced earlier since they allowed portfolio performance evaluation without apprehending its composition.

The underlying rationale of their model, named "Portfolio Change Measure" (PCM), is that an informed investor, an investor who knows the exact composition of a portfolio, will adjust his portfolio's weights towards assets with higher expected returns and lower risks than average as much as get rid of assets with expected returns lower than average. This operation will generate a positive covariance between portfolio's weights and the return of a security for an informed investor, though it should not be any covariance between portfolio's weights and the return of an asset for the investor who is not informed.

Grinblatt and Titman propose to measure this covariance in the following way:

[49] M. Grinblatt, S. Titman, Performance Measurement without benchmarks: An Examination of Mutual Fund Returns, The Journal of Business, vol.66, issue 1, 47-68, (1993).

$$PCM = \sum_{j=1}^{N} \quad \sum_{t=1}^{T} \; [Rjt * (Wjt - Wj,t - k)]/T \qquad (2.9)$$

PCM = Portfolio Change Measure

$R_{j,t}$ = Return of security (j) at time (t)

Wj,t = Weight of security (j) at time (t)

Wj,t-k = Weight of security (j) at time (t-k)

T = Number of time periods under consideration

In this chapter we have gone through several modes of assessing performance: from benchmarking and style comparison, to risk-adjusted measures. This last mode, that is the most widely adopted by investors, consists of various ratios, each one accounting for different risk factors and with disparate characteristics. Nevertheless, all of them are sharing some common features: the necessity of a benchmark, be a fund or a portfolio, with which be compared, the exclusive reliance on return and risk factors, without leaving out other variables that may affect performance. Starting from these characteristics, scholars and academics, since the 60s until nowadays, have been trying to overcome the aforementioned weaknesses of traditional performance measures through the application of Data Envelopment Analysis (DEA) approaches. In the next chapter we will discover what is it about and how it has been applied to investment funds evaluation through the past five decades.

Data Envelopment Analysis (DEA)

Introduction

All the risk-adjusted performance measures introduced above are very popular among investors and widely used in practice, though they all have in common theoretical flaws, as it has been described in the previous chapter. In this chapter, we are going to introduce, describe and analyze a different methodology used to measure efficiency and assess performance: Data Envelopment Analysis (DEA). Contrary to other performance measures, the DEA technique has the distinctive characteristic of incorporating many factors, named inputs and outputs, in addition to the classic variables of risk and return, in the measurement process, offering investors a powerful tool for ranking mutual funds by self-appraisal and peer group valuation[50].

How does DEA provide insights to investors? DEA approach helps benchmarking mutual funds on a relative basis instead of absolute performance measurement as given by traditional performance measures. Also, through this technique is possible to include the

[50] H.R. Khedmatgozar, A. Kazemi, P. Hanafizadeh, Mutual Fund Performance Evaluation: a value efficiency analysis approach, International Journal of Electronic Finance, (2013)

cost of owning a mutual fund share in the form of a fund's expense ratio, load charges[51], 12b-1[52] charges as an input variable in addition to fund's objective, return and risk as measured by beta and standard deviation of the fund.

The DEA methodology has many features that makes it a powerful tool in efficiency and performance evaluation. One of these characteristics is also a unique advantage: DEA doesn't need the hypothesis of validity of the CAPM, eluding the effect of selection of market portfolio and risk-free rate on the evaluation results. Another peculiar feature of this approach concerns weights optimization: unlike other performance gauge techniques like regression analysis, DEA don't require to assign ex-ante particular weights to parameters. This peculiarity is extremely important when, as it will be discussed sooner, it comes to assess investment funds' or portfolios performance. Another great feature of DEA is that it doesn't require uniformity of units of analysis regarding inputs and outputs.

DEA methodology was used for the first time in Germany to estimate the marginal productivity of R&D and other factors of production, however it has a wide variety of applications: from assessment of education system, health-care and hospital efficiency to banking, finance, agriculture, transportation and logistics industries performance evaluation[53]. The application of DEA to the assessment of the performance evaluation of investment funds have become even more relevant in the last years, when an increasing number of papers published on international journals and academic books were written on its application to conventional mutual funds, hedge funds and ETFs. Anyway, if we

---

[51] It is referred to the sales charge or commission an investor pays to an investment advisor, or broker, for his/her time and expertise in selecting an appropriate fund for the investor, based on his/her preferences. The load is either paid up front at the time of purchase (front-end load), or when the shares are sold (back-end load).

[52] Definition on paragraph "Mutual fund fees and expenses" Chapter 1.

[53] A. Emrouznejad, B.R. Parker, G. Tavares, Evaluation of research in efficiency and productivity: a survey and analysis of the first 30 years of scholarly literature in DEA. Socio-Economic Planning Sciences, (2008).

have a look at the performance metrics applied in the financial business to valuate and compare mutual funds based on historical prices and performances, we discover that results obtained with traditional and conventional measures, precisely the ones employed by financial industry practitioners, are different from performance scores obtained with DEA models. This dissimilarity may be attributed mainly to two reasons: first, because of the difficulty of providing a transparent interpretation of DEA indicators easily understandable by financial professional; secondly, due to the sophisticated nature of DEA models, especially if compared to most known traditional performance measures such as, Sharpe, Sortino or Treynor ratios[54].

Above we have cited the criticized theoretical flaws that follow through traditional risk-adjusted efficiency measures, affirming the useful and original disposition of DEA approach, that is capable of capturing and measuring performances considering additional variables. Therefore, what are these factors that can be related to mutual funds' performance, that the traditional measures did not take into account? For example, as it has been described in the first chapter, funds' fees and commissions (for simplicity we can consider the Expense ratio), dimensions (fund assets) are factors that could be included in an accurate efficiency evaluation, and even classification, of investment funds. What follows is a literature review of the early studies that contribute to the development and implementation of the basic DEA models.

[54] J. Zhu, Data Envelopment Analysis: A Handbook of Empirical Studies and Applications, 2016, Springer, pp. 229-230.

Literature Review

Let's start with a generic definition of the data envelopment analysis. DEA is a non-parametric approach, i.e. inputs and outputs related to the transformation process do not need to have the same units of measurement[55], a mathematical programming tool, that can be applied in performance measurement and efficiency analysis. Cooper, Seiford and Zhu (2004) define DEA as '*a relatively new "data orientated" approach for evaluating the performance of a set of peer entities, called Decision Making Units (DMUs), which convert multiple inputs into multiple outputs*'[56]. DEA in its current form was first introduced in 1978 and has been recognized as an excellent methodology for performance evaluations. As such, DEA has been used in evaluating the performance of many different types of business units and activities in the succeeding years. The term DMU was used to allow for the model's application to a wide variety of activities, including governmental, not-for-profit and business units.

More specifically, DEA has to be considered a non-parametric approach to productivity analysis, especially efficiency analysis, of DMUs. Indeed, since the introduction of the first DEA model, namely the CCR model, in 1978, it has been widely used in efficiency analysis of many businesses and industry evaluation procedures. The most well-known DEA models are: the CCR model (Charnes, Cooper and Rhodes, 1978), the BCC model (Banker, Charnes and Cooper, 1984) and the Additive model (1985).

[55] Y Zhao, K Triantis, P Murray-Tuite, P Edara, Performance measurement of a transportation network with a downtown space reservation system: A network-DEA system, Transportation Research Part E: Logistics and Transportation Review, (2011)

[56] W.W. Cooper, L.M. Seiford, J. Zhu, Handbook on Data Envelopment Analysis, (2011). Springer, Boston.

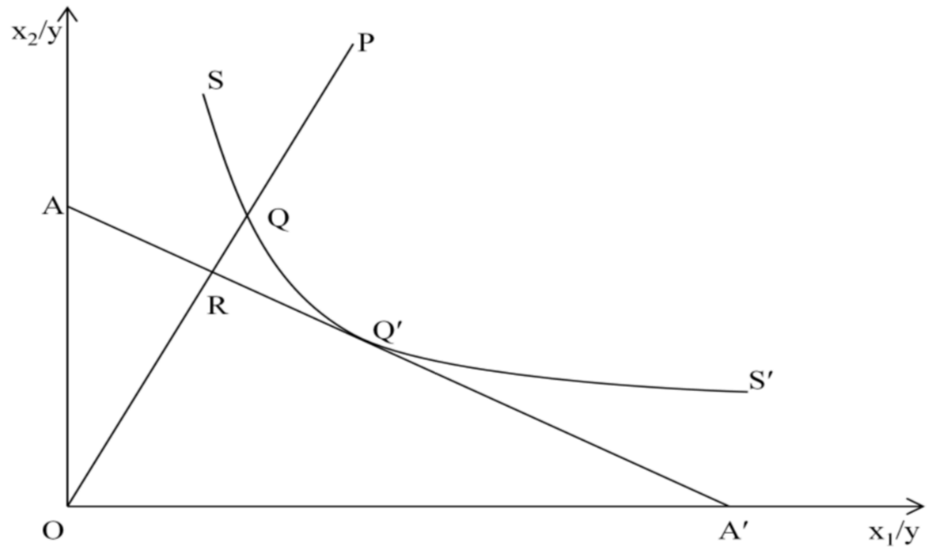
Charnes, Cooper and Rhodes, with their work[57] of 1978, aimed to elaborate and develop the concepts published by Farrell, two decades earlier, in 1957. Farrell wanted to create innovative models for measuring the productivity. The measures available at that time were accurate but too restrictive, due to the fact they did not allow to combine multiple inputs in order to get a total efficiency measure. To address this problem, Farrell worked out a method that was applicable to any productive organization, extending the concept of productivity towards the efficiency.

Essentially, productivity is based on the ratio between the quantities of output and input used in the production process. While, the concept of efficiency encloses the comparison between productivity and the DMUs. Farrell define overall productive efficiency as the product between technical and allocative efficiency[58]. Technical efficiency is measured as the ratio between the observed output and the maximum output, under the assumption of fixed input, or, alternatively, as the ratio between the observed input and the minimum input under the assumption of fixed output and is defined as the capacity of maximizing outputs given fixed amount of inputs. While, allocative (or price) efficiency refers to the ability to combine inputs and outputs in optimal proportions in the light of prevailing prices (and technology). The efficiency's factorization proposed by Farrell is shown in the figure below, with a simple example with two inputs variables (x_1, x_2) and one output variable (y).

[57] A. Charnes, W.W. Cooper, E. Rhodes, Measuring the Efficiency of Decision-Making Units, European Journal of Operational Research, (1978).

[58] M. J. Farrell, The Measurement of Productive Efficiency, Journal of Royal Statistical Society, (1957).

Figure 3.1: Technical and Allocative Efficiencies



Source: Førsund and Sarafoglou, 1999

The SS' curve represents the isoquant relative to a totally efficient firm and allows to measure technical efficiency. Technical inefficiency is given by the segment QP, while in percentage terms is equal to the ratio QP/OP. Hence, technical efficiency will be complementary, resulting:

$$\textit{Technical Efficiency (TE)} = \frac{OQ}{OP} \quad (3.1)$$

This ratio is included between zero and one: one meaning a totally efficient firm given by point Q in the figure above, while zero is totally inefficient. In order to find out the allocative efficiency is necessary to know the isocost line AA', and is given by the ratio:

$$\textit{Allocative Efficiency (AE)} = \frac{OR}{OQ} \quad (3.2)$$

Therefore, the total efficiency is resulting:

$$\textbf{\textit{Total Efficiency (EE)}} = \frac{\boldsymbol{OR}}{\boldsymbol{OP}} \tag{3.3}$$

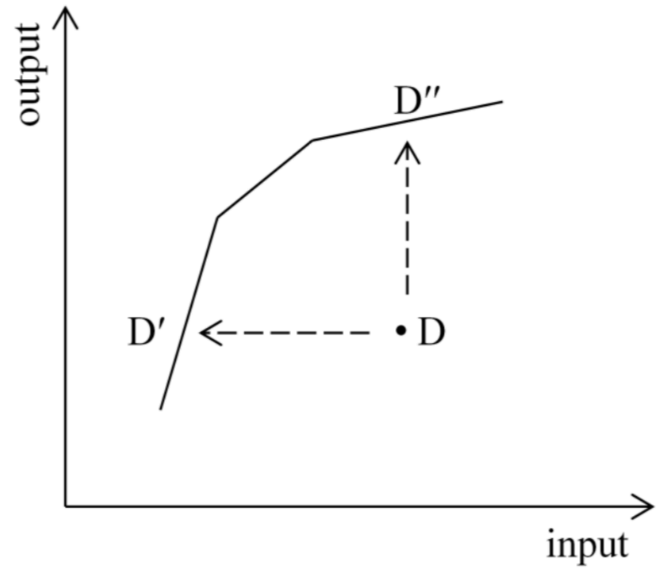
It can also be calculated as a multiplication between two types of efficiency:

$$\textbf{\textit{Total Efficiency (EE) = (TE)*(AE)}} = \left(\frac{\boldsymbol{OQ}}{\boldsymbol{OP}}\right) * \left(\frac{\boldsymbol{OR}}{\boldsymbol{OQ}}\right) = \left(\frac{\boldsymbol{OR}}{\boldsymbol{OP}}\right) \tag{3.4}$$

These measures, considering the example in the figure 3.1., are known as input-oriented, because are based on the need of decreasing the inputs to produce the same amount of output, in an efficient way. On the opposite, we have measures output-oriented when assuming an increase of output given the same proportion of input used in the production process.

Hence, one of the main concept of DEA approach is based on the identification of a linear efficient frontier and determine if the DMUs are efficient or not; if not (point D of the figure 3.2.), there are two possible strategies to reach the efficient frontier: decreasing inputs to produce the same output (D’), or increasing outputs leaving constant the amount of input (D”).

Figure 3.2: Efficient Frontier



Source: Gregoriou, Zhu, 2005

The DMUs that don't need an increase of outputs, or a decrease of inputs, are located on the efficient frontier. As a consequence, input-oriented models will optimize (reduce) inputs with constant outputs level; vice versa, output-oriented models will optimize (increase) outputs with constant inputs level. DEA gives information about DMUs' efficiency and provides advice regarding not efficient DMUs; in addition, DEA also provide suggestions about the variation of input (or output) required to enhance performances. For these reasons, DEA models is considered an unbiased benchmarking tool[59].

[59] G. N. Gregoriou, K. Sedzro, J. Zhu, Hedge Fund Performance Appraisal Using Data Envelopment Analysis, (2005).

DEA Basic Models

In this paragraph will be described three generic DEA models: CCR, BCC, and Allocative models. Charnes, Cooper and Rhodes, the three authors behind the CCR model, at the beginning of 70's, started working on the evaluation process of educational program for underprivileged public U.S. school students (i.e. black and Hispanic students). At the beginning Cooper and Rhodes designed a model capable of identifying inefficiencies of any input/output of DMUs; later on, Charnes expressed it formulaically from a mathematical point of view and extended the approach to the efficiency measurement, in order to be used in other fields and sectors. DEA methodology had an incredible rapid development and has been immediately accepted due to its peculiarities and wide applicability. Researchers from various sectors acknowledged DEA as valuable method for operating processes modeling; its empirical nature and assumptions' minimization lead the application of DEA in many studies on the efficient frontier for what concerns no profit organization in regulatory and private sectors. Now, we are going to analytically deal with the first of the DEA basic models: CCR model.

CCR Model

Assuming *n* DMUs under valuation, is necessary to present four DMU's selection criteria:

- Positive numerical data available for every *input* and *output* considered
- The choice regarding DMU, *input* and *output* must match manager's interest about DMU efficiency
- Usually, small amount of *input* and large amount of *output* are preferred

- No required consistency between data; can be compared different type of *input*/*output* data.

Each DMU uses a given amount of *m input* to produce *s output,* with the properties stated in the first 2 selection criteria. This relation can be written in matrix form with (X) as *input* and (Y) as *output*:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix}$$

$$Y = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{pmatrix}$$

For each DMU, Charnes, Cooper and Rhodes determined the *virtual input* with weight (u_i) and *virtual output* with weight (v_r):

$$\textit{virtual input} = v_1 x_{1o} + \ldots + v_m x_{mo} \tag{3.5}$$

$$\boldsymbol{virtual\ output = u_1 y_{1o} + \ldots + u_s y_{so}} \qquad (3.6)$$

Then, they calculated the weights, through linear programming, in order to maximize the following ratio:

$$\frac{\boldsymbol{virtual\ output}}{\boldsymbol{virtual\ input}} \qquad (3.7)$$

The weights, as it has already been stated when describing peculiarities of DEA approach, are derived from data instead of being assumed in advance. Once data have been chosen, it's time to evaluate the efficiency of each DMUs, hence *n* optimizations are necessary, each for every DMUs. Going through the following fractional programming (FP) problem, is possible to obtain *input's* weights *(u_i)* (i=1, ..., m) and *output's* weights *(v_r)* (r=1, ..., s) as variables:

$$\mathbf{FP_0} \qquad \max_{v,u} \theta = \frac{u_1 y_{1o} + u_2 y_{2o} + \cdots + u_s y_{s0}}{v_1 x_{1o} + v_2 x_{2o} + \cdots + v_m x_{m0}} \qquad (3.8)$$

Subject to:

$$\frac{u_1 y_{1j} + \cdots + u_s y_{sj}}{v_1 x_{1j} + \cdots + v_m x_{mj}} \leq 1 \ (j = 1, \ldots, n) \qquad (3.9)$$

$$v1, v2, \ldots, vm \geq 0 \qquad (3.10)$$

$$u_1, u_2, \ldots, u_s \geq 0 \qquad (3.11)$$

These constraints guarantee that the ratio (3.7) does not exceed the value of 1 for each DMUs. The goal is to find the weights v_i and u_r that maximize the DMU_0 ratio (concerning the DMU under evaluation). Given the constraints, the optimal value θ^* can be equal to maximum 1. Mathematically, the constraint (3.10) is not sufficient to provide positive values for the fractional term (3.9). Here, is assumed that *inputs* and *outputs* values are different from zero, giving positive values for relative weights u_r and v_i.

We are going to substitute the Fractional Program (FP_o) with the Linear Program (LP_o):

$$\textbf{LP}_0 \qquad \max_{\mu,\nu} \theta = \mu_1 y_{1o} + \cdots + \mu_s y_{so} \tag{3.12}$$

Subject to:

$$v_1 x_{1o} + \cdots + v_m x_{mo} = 1 \tag{3.13}$$

$$\mu_1 y_{1j} + \cdots + \mu_s y_{sj} \leq v_1 x_{1j} + \cdots + v_m x_{mj} \qquad j = (1, \dots, n) \tag{3.14}$$

$$v_1, v_2, \dots, v_m \geq 0 \tag{3.15}$$

$$\mu_1, \mu_2, \dots, \mu_s \geq 0 \tag{3.16}$$

The Fractional Program (FP_o) is equal to the Linear Program (LP_o). Thanks to the assumption (3.10) and the positivity condition of the X matrix, is possible to obtain the constraint (3.14) by multiplying denominator of (3.9) in both sides. Then, after setting the denominator of FP_0 (3.8) equal to 1, and putting it as a constraint, we will obtain the equation (3.13), maximizing the numerator, as in LP_0 (3.12). At this point, the optimal solution of LP_o is ($\nu=\nu^*$ and $\mu=\mu^*$) and θ^* is the optimal objective value. The solution ($\nu=\nu^*$ and $\mu=\mu^*$) is also optimal for (FP_o) and (LP_o) therefore have the same optimal solution θ^*.

Hence, the DMU_o is CCR-efficient if $\theta^* = 1$ and if exists at least one optimal solution *(ν^*, u^*)*, with $\nu^* > 0$ and $u^* > 0$; in all other cases DMU_0 is CCR-inefficient. Inefficiency, with the CCR model, can have two meanings: on one hand that $\theta^* < 1$, on the other hand $\theta^* = 1$ and at least one element from *(ν^*, u^*)* is equal to 0 for each optimal solution of LP_o.

Now we observe the case where DMU_o has $\theta^* < 1$ (CCR-inefficient). There must be at least one DMU in constraint (3.14) for which the weight (ν^*, u^*) gives the same results in both sides, otherwise, θ^* coul be intensified. We will have:

$$E^j{}_o = \{ \; j : \sum_{r=1}^{s} u_r{}^* y_{rj} = \sum_{i=1}^{m} \nu_i{}^* x_{ij} \} \tag{3.17}$$

The subset of DMU_j that satisfies the previous equation is called *reference set* or *peer group* to the DMU_0, because they are on the efficient frontier representing the objective for obtaining efficiency.

The optimal solution (ν^*, u^*) obtained for LP_o results in a series of optimal weights for the DMU_0. The θ^* can also be found with the following ratio:

$$\boldsymbol{\theta^* = \frac{\sum_{r=1}^{s} u^*_r y_{ro}}{\sum_{i=1}^{m} \nu^*_i x_{io}}} \tag{3.18}$$

From (3.13), the denominator is equal to 1 so:

$$\sum_{i=1}^{m} v^*{}_i\, x_{i0} = 1 \qquad (3.19)$$

hence the optimal value θ^* can be rewritten as follows:

$$\boldsymbol{\theta^* = \sum_{r=1}^{s} u^*{}_r\, y_{r0}} \qquad (3.20)$$

The (3.19) is useful to define the relation, expressed in weight percentage, of an input compared to the others, for each DMUs. While the (3.20) gives a measure of the contribution of each output over the total efficiency.

Can be useful provide some analytical examples to understand the mechanism of this model

Example 1 (1 input and 1 output):

Table 3.1:

DMU	A	B	C	D	E	F	G	H
Input	2	3	3	4	5	5	6	8
Output	1	3	2	3	4	2	3	5

Source: Cooper, Seiford, Tone, 2005

It is possible to calculate DMU A efficiency solving the following linear program problem:

A: $\max \theta = u$

Subject to:

$2v = 1$

$u \leq 2v$	*(A)*	$3u \leq 3v$	*(B)*
$2u \leq 3v$	*(C)*	$3u \leq 4v$	*(D)*
$4u \leq 5v$	*(E)*	$2u \leq 5v$	*(F)*
$3u \leq 6v$	*(G)*	$5u \leq 8v$	*(H)*

The optimal solution is the following: $v^*= 0.5$, $u^*= 0.5$, $\theta^*= 0.5$. *DMU A* is not efficient, because θ^* is lower than 1.

Do the same procedure to measure efficiency of DMU B:

B: $\max \theta = 3u$

Subject to:

$3v = 1$

$u \leq 2v$	*(A)*	$3u \leq 3v$	*(B)*
$2u \leq 3v$	*(C)*	$3u \leq 4v$	*(D)*
$4u \leq 5v$	*(E)*	$2u \leq 5v$	*(F)*
$3u \leq 6v$	*(G)*	$5u \leq 8v$	*(H)*

Solving the problem, the optimal solution will be: $v^*= 0.333$, $u^*= 0.333$, $\theta^*= 1$. B is CCR-efficient. Then, repeating the same procedure for all the DMUs we obtain the following table:

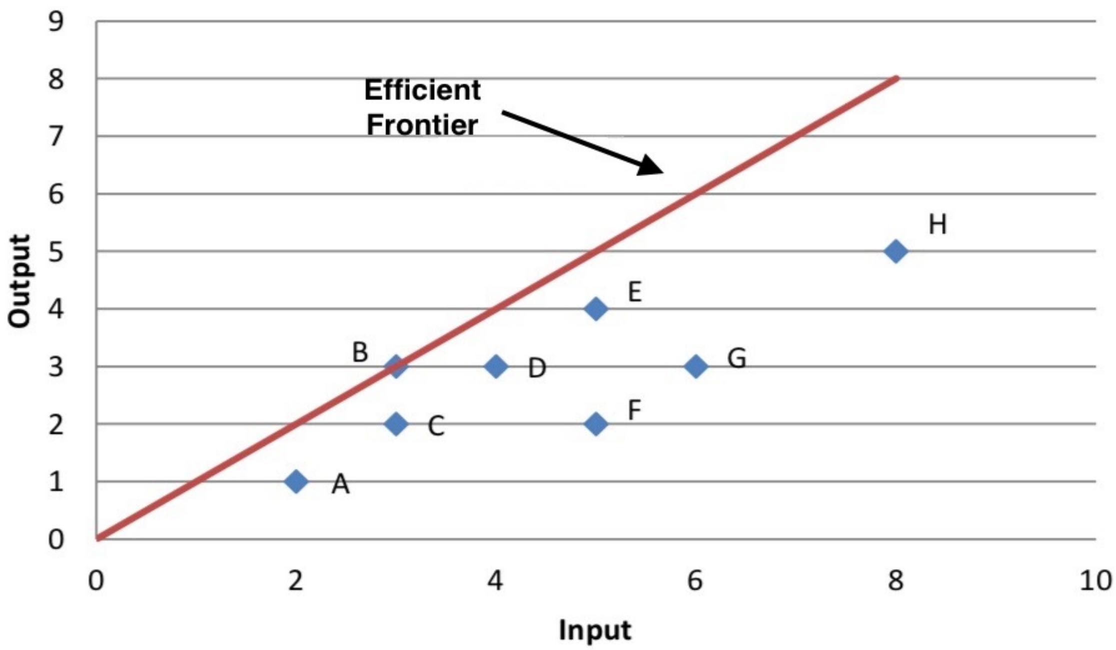
Table 3.2:

DMU	v^*	u^*	θ^*

A	0.5	0.5	0.5
B	0.333	0.333	1
C	0.333	0.333	0.666
D	0.25	0.25	0.75
E	0.2	0.2	0.8
F	0.2	0.2	0.4
G	0.166	0.166	0.5
H	0.125	0.125	0.625

Source: Cooper, Seiford, Tone, 2005

Figure 3.3: Efficient Frontier



Source: Cooper, Seiford, Tone, 2005

BCC Model

The BCC model[60] introduced by Banker, Charnes and Cooper in 1984, basically represents a different implementation of the CCR model previously explained. The distinctive characteristic is that the efficient frontier is now represented by a convex function which conveys variable returns, no more a straight line through the origin. The additional element which is not present in the CCR model consists in the following constraint, representing the convexity condition:

$$\sum_{j=1}^{n} \lambda_{\mathrm{j}} = 1, \quad \lambda_{\mathrm{j}} \geq 0, \qquad \forall\, j \tag{3.21}$$

The BCC *input-oriented* model, that seeks to measure the efficiency of any DMU_0 *(0= 1, …, n)*, is expressed through the following Linear Program model:

$$(\boldsymbol{BCC_o}) \quad \min_{\boldsymbol{\theta_B}, \boldsymbol{\lambda}} \boldsymbol{\theta_B} \tag{3.22}$$

Subject to:

$$\theta_{\mathrm{B}}\, x_{\mathrm{o}} - X\lambda \;\geq 0 \tag{3.23}$$

$$Y\lambda \;\geq y_{\mathrm{o}} \tag{3.24}$$

$$e\lambda = 1 \tag{3.25}$$

[60] R. D. Banker, R.F. Charnes, W.W. Cooper, Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis, (1984), Management Science vol 30 no. 9, 1078-1092.

$$\lambda \geq 0 \tag{3.26}$$

Where θ_B is a scalar and (3.25) is the convexity condition. The dual multiplier form of the (3.22) is the following:

$$\max_{\boldsymbol{\nu},\boldsymbol{u},\boldsymbol{u_o}} \boldsymbol{z} = \boldsymbol{u}\boldsymbol{y_o} - \boldsymbol{u_o} \tag{3.27}$$

Subject to:

$$\nu x_0 = 1 \tag{3.28}$$

$$-\nu X + uY - u_0 e \leq 0 \tag{3.29}$$

$$\nu \geq 0, \quad u \geq 0 \tag{3.30}$$

Where ν and u are vectors, while z and u_o are scalars, with u_o may be positive or negative (or zero).

The equivalent BCC fractional program can be obtained from the (3.27) as:

$$(\boldsymbol{BCC - O_o}) \qquad \max_{\boldsymbol{\nu},\boldsymbol{u}} \boldsymbol{\theta_B} = \frac{\boldsymbol{u}\boldsymbol{y_o} - \boldsymbol{u_o}}{\boldsymbol{\nu}\boldsymbol{x_o}} \tag{3.31}$$

Subject to:

$$\frac{uy_j - u_o}{\nu x_j} \leq 1, \qquad j = 1, \ldots, n \tag{3.32}$$

$$\nu \geq 0, \;\; u \geq 0 \tag{3.33}$$

The first problem (BCC_0) is solved with a two-phase procedure, similar to the one used for CCR model. In the first phase, θ_B is minimized while in the second is maximized the sum of the *input* excesses and *output* shortfalls. The optimal solution will be given by (θ^*_B, λ^*, *s-*, s+**), where s^{-*} and s^{+*} are called slacks and represent maximum input excesses and output shortfalls, respectively. The optimal solutions satisfying $\theta^*_B = 1$ and has no slacks (s-*= 0 and s+*= 0), is BCC-efficient, otherwise it is BCC-inefficient.

The BCC *output-oriented* model is defined as:

$$\max_{\eta B,\, \lambda} \eta_B \tag{3.34}$$

Subject to:

$$X\lambda \leq x_0 \tag{3.35}$$

$$\eta_B y_0 - Y\lambda \leq 0 \tag{3.36}$$

$$e\lambda = 1 \tag{3.37}$$

$$\lambda \geq 0 \tag{3.38}$$

The dual form linked to the linear program (3.34) is expresses as:

$$\min_{\nu, u, \nu 0} z = \nu x_0 - \nu_0 \tag{3.39}$$

Subject to:

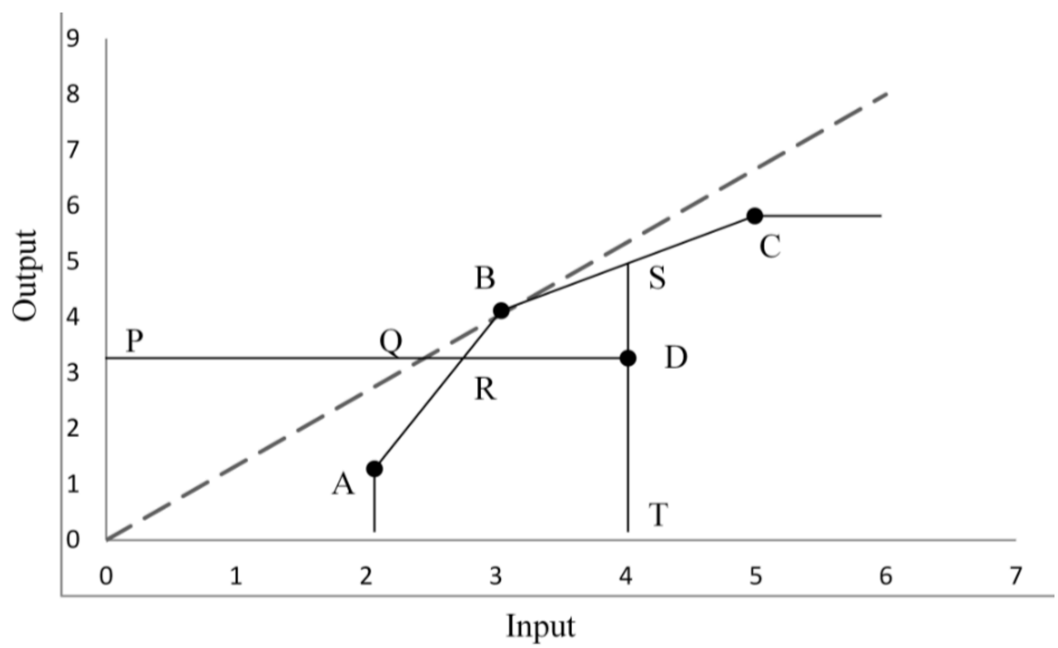
$$uy_0 = 1 \tag{3.40}$$

$$vX - uY - v_0 e \geq 0 \tag{3.41}$$

$$v \geq 0, \quad u \geq 0 \tag{3.42}$$

The following figure 3.3 may help understanding the differences between CCR and BCC models, as it shows four DMUs (A, B, C, D), each with his own input and output.

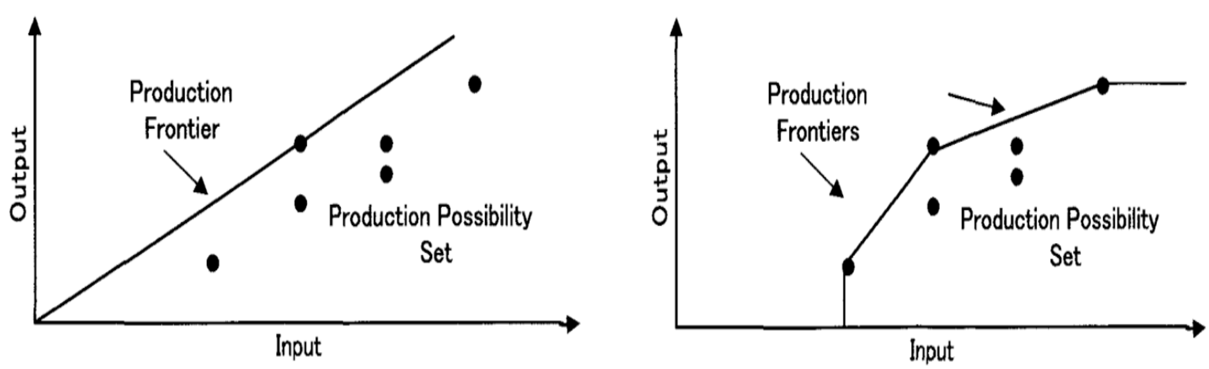
Figure 3.3. : BCC and CCR models



Source: Cooper, Seiford, Tone, 2005

The CCR efficient frontier is the dotted line passing through the origin: it is clear that only the DMU B is CCR-efficient, lying on the CCR efficient frontier. Instead, the BCC efficient frontier is the broken line ABC. In this case DMUs A, B and C are BCC-efficient. Trying to measure the efficiency of the DMU D, we notice that CCR and BCC models provide different results: CCR-efficiency will be given by the ratio PQ/PD, while BCC-efficiency by PR/PD. In addition, graphically we can note that the DMU D is less efficient under the CCR model rather than the BCC; hence we can conclude that, generally speaking, CCR-efficiency never exceeds BCC-efficiency.

Figure 3.4: Production Frontier with CCR (left) and BCC (right) models:



Source: Cooper, Seiford, Tone, (2005).

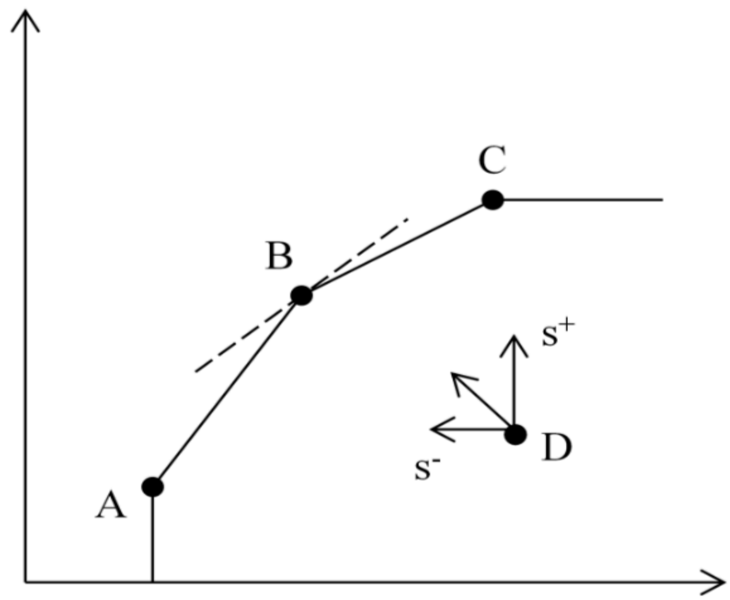
Additive Model

The Additive model was introduced by a group of scholars in 1985[61] and has the predominant characteristic that there is no distinction between *input* and *output-oriented*

[61] A. Charnes, W.W. Cooper, B. Golany, L. Seiford, J. Stutz, Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions, (1985), Journal of Econometrics vol. 30, 91-107.

models, because both are simultaneously considered. As showed in figure 3.4, the efficient frontier belongs to the BCC model. It derives from it that a DMU to be efficient has to be BCC- efficient.

Figure 3.4: Additive model



Source: Cooper, Seiford, Tone, 2005

The additive model can be given as

$$\max_{\lambda, s^-, s^+} z = es^- + es^+ \tag{3.43}$$

Subject to:

$$X\lambda + s^{-} = x_0 \tag{3.44}$$

$$Y\lambda - s^{+} = y_0 \tag{3.45}$$

$$e\lambda = 1 \tag{3.46}$$

$$\lambda \geq 0, \quad s^{-} \geq 0, \quad s^{+} \geq 0 \tag{3.47}$$

Where s^{-} and s^{+} represent *output* and *input* slacks. This model considers the total slacks simultaneously in arriving at a point on the efficient frontier.

The dual problem to the additive model (3.43) can be expressed as follows:

$$\min_{\boldsymbol{\nu},\boldsymbol{u},\boldsymbol{u}\mathbf{0}} \boldsymbol{w} = \boldsymbol{\nu x_0} - \boldsymbol{u y_0} + \boldsymbol{u_0} \tag{3.48}$$

Subject to:

$$\nu X - uY + u_0\, e \geq 0 \tag{3.49}$$

$$\nu \geq e \tag{3.50}$$

$$u \geq e \tag{3.51}$$

The optimal solution is given by (λ*, *s-*, s+**) and any DMU_0 to be efficient must respect the following conditions: $s^{-*}= 0$ and $s^{+*}= 0$[62].

The application of DEA can be a very powerful tool whether used wisely. Those that follow are some of the advantages of DEA approach. First of all, it is a tool that can handle multiple inputs and outputs models, using a unique total efficiency measure, without any weighting factors[63]. DMUs are directly compared against peer or combination of peers. DEA does not require an assumption of a functional form concerning inputs to outputs. Finally, contrary to other models, in order to analyze DMU efficiency, inputs and outputs may have different units, given that DEA is not limited to monetary units[64].

Though, the same features that make DEA so useful, can also be reflected in weaknesses of analysis. When choosing to adopt DEA model, is necessary to keep in mind the following limitations: DEA is a useful tool to measure relative efficiency, but when it comes to evaluate absolute efficiency it is not so powerful tool, meaning that it is the right technique if you want to know how well you are doing compared to a peer or a group of peers, but not compared to the theoretical maximum. Large problems can be computational intensive, requiring time and powerful calculation machines.

---

[62] W. W. Cooper, L. M. Seiford, K. Tone, Introduction to Data Envelopment Analysis and its uses: with DEA-Solver software and references, (2005), Springer-Verlag.

[63] W. P. Fox, Mathematical Modeling for Business Analytics, (2017), CRC Press.

[64] N. Johns, B. Howcroft, L. Drake, The use of data envelopment analysis to monitor hotel productivity, (1997), Progress in Tourism and Hospitality Research.

Application DEA Models for Investment Funds Evaluation

In this chapter we are going to describe some models that, through DEA technique, have been used to measure and analyze the efficiency of investment funds. Most of the traditional risk-adjusted measures, on one hand are very helpful in providing simple and immediate results in clear and comparable terms, but on the other hand are unsuitable for a correct and complete performance valuation of mutual funds, due to the fact, as already stated, that they don't take in consideration fees and expenses investors have to bear (measured with expense ratio). For this reason, scholars started to apply DEA for mutual funds analysis, with the goal to find an efficiency measure as complete and objective as possible.

Murthi, Choi and Desai Model

Murthi, Choi and Desai model (1997)[65] was the first approach to the performance evaluation of mutual funds through the DEA. At the heart of this model, there is the critics to traditional measures, especially Sharpe ratio and Jensen alpha, because, as far as they were concerned, these metrics caused many problems. From Jensen's alpha they complaint the choice of correct benchmark, that is based on theories such as CAPM or APT, that are considerably outdated due to their strong underlying assumptions. The

[65] B. P. S. Murthi, Y. K. Choi, P. Desai, Efficiency of mutual funds and portfolio performance measurement: a non-parametric approach, European Journal of Operational Research vol. 98, (1997).

weak point of both Jensen's alpha and Sharpe ratio, instead, consists in the fact that they do not consider the weight of transaction costs[66]. In order to include these costs in their model, Murthi, Choi and Desai created a new innovative index, modifying the original Sharpe ratio with the addition of transaction fees and expenses. This new measure is called ***DEA Portfolio Efficiency Index (DPEI),*** formulated as:

$$\boldsymbol{DPEI = \frac{R_o}{\sum_{i=1}^{l} w_i x_{io} + \nu\sigma_o}} \tag{3.52}$$

Subject to:

$$\frac{Rj}{\sum_{i=1}^{l} w_i x_{ij} + \nu\sigma_j} \leq 1 \qquad j = 1, \dots, J \tag{3.53}$$

$$w_i \geq \varepsilon, \qquad \nu \geq \varepsilon \tag{3.54}$$

Where J is the number of funds of the same category, I is the inputs' number, R_j describes the average return of j^{th} fund, x_{IJ} is the transaction cost value (i) for the j^{th} fund, w_i and ν are the weights associated to x_i and w_i variables respectively, σj is the standard deviation of the j^{th} fund and ε is a constant number. To calculate the index (*DPEI*), they applied DEA, solving an optimization problem that defines optimal weights and efficiency level of a fund.

The reasons why the authors chose to apply DEA are the following:

---

[66] Generally called transaction costs, meaning all those fees and expenses that investors have to bear when buying, holding and selling investment funds' shares.

- It does not require benchmarks, it just measures the best performance of a mutual fund compared to a group of funds of the same category.
- It takes in consideration, in a unique analysis, the expected return of a fund and relative transaction costs, such as expense ratio, turnover index and other fees. DPEI is flexible and can use a lot of inputs and outputs in measuring the performance.
- It can monitor marginal contribution of each input on the total performance, reallocating in this way resources in a timely and efficient way.

When it comes to provide a judgement about the performance of a given portfolio, it has to be included in the evaluation process also the cost component, because investors look for returns' maximization together with costs' minimization.

Here, Murthi, Choi and Desai chose one *output* variable, the *return*, and four *input* variables: *total expense ratio (TER)*, *loads*, *switch commissions*[67] and *standard deviation*.

Therefore, through the application of DEA is possible to find the weights that maximize the return of a fund, based on their inputs, hold in mind that the relation (3.52) has to be less or equal to 1 for each fund, and the weights must be positive values. Remembering that DEA method are used to measure a fund's efficiency relative to a bunch of funds sharing the identical inputs to obtain the same outputs. The distance of inefficient funds from the frontier represents the measure of fund's inefficiency.

[67] Switch fee is the charge collected by a fund management group when an investor moves money from one fund to another.

In the *DPEI* index, a mutual fund has to be considered efficient when the function (3.52) is equal to 1 and all the slacks are equal to 0; while it will be inefficient only when compared with other funds in the model.

Murthi, Choe and Desai, analyzed more than two thousand mutual funds from the third quarter of 1993 dividing in 7 sub-categories,[68] classified by funds' strategies. The results are compared to the Jensen's alpha and Sharpe ratio, calculating the correlation. The three authors discovered the presence of a positive correlation between *DPEI* and both Sharpe ratio and Jensen's alpha for all the fund's categories, meaning that DPEI is consistent with traditional risk-adjusted measures, though offering, at the same time, greater flexibility.

Finally, they calculated the correlation index between funds' ranking based on NAV[69] and the one based on DEA results, in order to measure the effect of fund'size on the total efficiency: they found for some fund's categories a positive correlation. This result was justified with the fact that larger funds could be more efficient due to lower weight of transaction costs.

To conclude, these studies demonstrate that DEA technique is appropriate for evaluating efficiency of mutual funds, more flexible than traditional methods, thanks to the freedom of choice about inputs and outputs.

Basso and Funari Model

---

[68] The categories are: growth, growth income, aggressive growth, balanced, asset allocation, income, equity income. From the results appear that the best ones were: aggressive growth, asset allocation, income and equity income while the remaining three had low efficiency's level.

[69] For the definition of Net Asset Value, check the Chapter 1, paragraph "Open-end Funds".

Basso and Funari model (2001)[70] can be intended as a generalization of Murthi, Choi and Desai model (1997), allowing to take in consideration various risk measures. The difference between Basso and Funari and his predecessor model, consists in the fact that they used a different index, named ***I_{DEA_1}***, which, unlike the one proposed by Murthi, Choi and Desai, considers among input variables, only direct fees at the expense of the investor, that is to say submission and redemption costs, leaving out other indirect costs, such as operative expenses and management commissions. This because the historical series of data about funds' returns are net of those "indirect costs".

Hence, the measure proposed, I_{DEA_1}, can be obtained solving the following linear problem:

$$\max_{\{u, v_j, w_i\}} \frac{u o_{j0}}{\sum_{i=1}^{h} v_i q_{ij0} + \sum_{i=1}^{k} w_i c_{ij0}} \tag{3.55}$$

Subject to:

$$\frac{u o_j}{\sum_{i=1}^{h} v_i q_{ij} + \sum_{i=1}^{k} w_i c_{ij}} \leq 1, \qquad j = 1, \dots, n \tag{3.56}$$

$$u \geq \varepsilon$$

$$v_i \geq \varepsilon, \qquad i = 1, \dots, h$$

$$w_i \geq \varepsilon, \qquad i = 1, \dots, k$$

[70] A. Basso, S. Funari, A data envelopment analysis approach to measure the mutual fund performance, European Journal of Operational Research, (2001).

Where u is the *outuput* weight, ν is the *input* weight, while j represents a generic fund, k is the costs' number under consideration (c_{ij}) and h is the number of risks' measures (q_{ij}). Therefore, they used the fund's return as the output of the model, while four different inputs, as measures of risk:

- Standard deviation of returns
- Beta coefficient (β) of a reasonable benchmark[71]
- % Subscription costs
- Redemption costs (relative to different time horizon 1-2-3 years)

In addition, Basso and Funari developed an indicator of stochastic dominance reflecting the percentage of sub-periods, compared to the evaluation time horizon, by which a fund is not dominated by other funds. It will be expressed with the d_j as a certification of the validity of a fund as time goes by. With this gauge, they developed a different DEA measure, $\boldsymbol{I_{DEA\_2}}$, characterized by two outputs, which can be defined as:

$$\frac{\boldsymbol{u_1 o_{j0} + u_2 d_{j0}}}{\boldsymbol{\sum_{i=1}^{h} \nu_i q_{ij0} + \sum_{i=1}^{k} w_i c_{ij0}}} \qquad (3.57)$$

Subject to:

$$\frac{u_1 o_j + u_2 d_j}{\sum_{i=1}^{h} \nu_i q_{ij} + \sum_{i=1}^{k} w_i c_{ij}} \leq 1, \qquad j = 1, \dots, n \qquad (3.58)$$

$$u_r \geq \varepsilon, \qquad r = 1, 2$$

$$\nu_i \geq \varepsilon, \qquad i = 1, \dots, h$$

[71] As market portfolio benchmark Basso and Funari considered the Mibtel index.

$$w_i \geq \varepsilon, \quad i = 1, \ldots, k$$

Basso and Funari analyzed 47 Italian mutual funds, divided by investment categories (equity, bond and balanced)[72], for the period between 1st January 1997 and 30th June 1999.

The procedure allows to identify, for each inefficient fund, a series of equivalent efficient funds as benchmarks. The method highlighted the importance of underwriting and redemption fees when comparing and evaluating mutual funds. Moreover, applying the gauge I_{DEA_2} the results don't change very much, rather, with this approach it comes up a higher number of efficient funds.

To conclude, results of the application of the Basso and Funari approach suggests that the DEA method can be very helpful in the mutual fund efficiency evaluation, in addition to traditional risk-adjusted measures. Comparing results from DEA with traditional performance metrics, such as Sharpe, Treynor ratios and Jensen's alpha, it shows low correlations: this meaning that fees and, generally speaking, transaction costs, are not taken in consideration in traditional measurements, with the possibility of compromising the total performance judgement of investments.

Morey and Morey Model

[72] For the categorization of mutual funds, was used the criteria adopted by Assogestioni: http://www.assogestioni.it/index.cfm/1,132,0,49,html/elenco-fondi

Morey and Morey[73] based their work on a simple consideration: most investors put their savings in mutual funds, selecting among them looking at ratings. Morey and Morey developed two alternative methods for mutual funds ratings' measurement in such a way as to provide additional information about risk and return, from an objective perspective. Through these approaches is possible to verify if a fund lies on the efficient frontier or not. The approach shares the following features:

- Non-parametric methods
- Transparent approaches, giving final ratings with clear economic interpretations
- Each mutual fund is evaluated compared to an endogenous benchmark fund, ad hoc created, running during the same period.
- Provide, for underperforming funds, levels of risk and returns necessaries to reach the efficient frontier

In the first method, the attention is addressed to simultaneous increase of average returns, over the entire time horizon, maintaining the same level of total risk in each period. (Total risk includes systemic risk, beta). With the second method, designing a different benchmark fund, endogenously produced, the goal is to reduce, simultaneously, total risk through the whole time frame, without affecting average return.

First approach determines $w_j \geq 0$ and $\theta \geq 1$ so:

$$\max \boldsymbol{\theta} \tag{3.59}$$

[73] M. R. Morey, R. C. Morey, Mutual fund performance appraisals: a multi- horizon perspective with endogenous benchmarking, Omega, The International Journal of Management Science vol. 27, (1998).

Subject to:

$$\sum_{j=1}^{N} w_j = 1 \tag{3.60}$$

$$\sum_{j=1}^{N} w^2{}_j \sigma^2{}_{j,t} + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \, Cov\,(R_{i,t},\, R_{j,t}) \leq \sigma^2{}_{j0,t} \quad (t=1, \ldots, T)$$

$$\sum_{j=1}^{N} w_j \, E(R_{j,t}) \geq \theta E(R_{j0,t}) \qquad (t=1, \ldots, T) \tag{3.62}$$

where j is the number (between 1 and N) of funds to evaluate, T is the number of different time period considered; $R_{j,t}$ represents a random variable; $E(R_{j,t})$ is the mean, $\sigma^2{}_{j,t}$ is the variance and $Cov(R_{i,t}, R_{j,t})$ are the covariances.

This method is useful because allows to assign an objective rating to mutual fund based on its distance from the efficient frontier; however, it requires prudence when using θ^* for rating the performances of mutual funds, due to the nature of the model.

The second approach is the following:

$$\mathbf{min}\, \boldsymbol{Z} \tag{3.63}$$

Subject to:

$$\sum_{j=1}^{N} w_j = 1 \tag{3.64}$$

$$\sum_{j=1}^{N} w_j \, E(R_{j,t}) \geq E(R_{j0,t}) \qquad (t=1, \ldots, T) \tag{3.65}$$

$$\sum_{j=1}^{N} w^2{}_j \sigma^2{}_{j,t} + \sum_{i=1}^{N} \sum_{j=1}^{N} w_j w_i \, Cov(R_{i,t},\, R_{j,t}) \leq Z \sigma^2{}_{j0,t} \quad (t=1, \ldots, T) \tag{3.66}$$

Morey and Morey analyzed 26 mutual funds, from “aggressive growth” Morningstar category, selecting monthly data, for 10 years. For each fund they calculated the monthly mean, and 3 ,5 and 10 years variance and covariance, then applied both approaches depicted above, using as inputs variables the variance return and correlation between funds’ returns, for each period under evaluation.

The results showed eight funds lying on the efficient frontier while the remaining 18 funds were not. Finally, they ranked those eight efficient funds.

Gregoriou, Sedzro and Zhu Model

Gregoriou, Sedzro and Zhu (2004)[74] developed a DEA model for hedge funds’[75] evaluation. Due to the nature and objective of hedge funds, that seek to maximize returns without any benchmark comparison, there was the need for an evaluation method which satisfies these requirements. Hedge funds show different characteristics about returns, if related to mutual funds, therefore applying traditional risk-adjusted measures for the valuation may provide misleading and unreliable results. Especially,

[74] Gregariou G. N., Sedzro K., Zhu J. (2004), Hedge fund performance appraisal using data envelopment analysis, European Journal of Operational Research.

[75] A hedge fund is an alternative investment vehicle available only to sophisticated investors, such as institutions and individuals with significant assets. Like mutual funds, hedge funds are pools of underlying securities, and can invest in many types of securities. But contrary to mutual funds, hedge funds are not regulated by the SEC (in U.S.), while in Europe are regulated by the AIFM.

due to the asymmetric nature returns, the application of Sharpe ratio would provide inappropriate results.

Three are the input variables applied in this model: lower mean monthly semi-skewness (LSS), lower mean monthly semi-variance (LSV), and mean monthly lower return (MLR). DEA model used is BCC: Gregoriou, Sedzro and Zhu then compared results from DEA with the Modified Sharpe Ratio (MSR), that is the following:

$$\mathbf{MSR} = \frac{\boldsymbol{R}\mathbf{pt} - \boldsymbol{R}\mathbf{f}}{\boldsymbol{W}\left[\boldsymbol{\mu} - \left\{\mathbf{zc} + \frac{1}{6}\left(\boldsymbol{z}^2\mathbf{c} - 1\right)\boldsymbol{S} + \frac{1}{24}\left(\boldsymbol{z}^3\mathbf{c} - 3\boldsymbol{z}\mathbf{c}\right)\mathbf{K} - \frac{1}{36}\left(2\boldsymbol{z}^3\mathbf{c} - 5\boldsymbol{z}\mathbf{c}\right)\boldsymbol{S}^2\right\}\boldsymbol{\sigma}\right]} \qquad (3.67)$$

Where R_{pt} is the portfolio return, R_f is the risk-free rate (Treasury-Bill 30 days), Z_c is the value associated to the probability equal to (1-α), S is skewness and K is the kurtosis. Then they calculated the modified VaR (Value at Risk)[76], with the following formula:

$$z_{CF} = z_c + \frac{1}{6}\left(z^2{}_c - 1\right) S + \frac{1}{24}\left(z^3{}_c - 3z_c\right) K - \frac{1}{36}\left(2z^3{}_c - 5z_c\right) S^2 \qquad (3.68)$$

Later, they used the Jarque-Bera test in order to verify the not normal nature of returns:

[76] Valure at Risk is a measure of the risk of loss for investments. VaR is defined as the maximum possible loss during the time, for a given portfolio, time horizon and probability p.

$$JB = \frac{n}{6}[S^2 + \frac{(k-3)^\wedge 2}{4}] \quad (3.69)$$

where *n* is the sample size used.

Gregoriou, Sedzro and Zhu analyzed several hedge funds' monthly return across two periods of time: the first from 1997 to 2001 and the second from 1999 to 2001. The reason why they choose to observe returns from two different periods, is that they wanted to test if two important crises, such as the Asian and Russian crisis, respectively in 1997 and 1998, influenced funds' performances.

The results show that most of the hedge funds analyzed are not efficient for what concern risk and return, given the inputs and outputs considered. In addition, they found that there were more efficient funds in the three-years period, than the five-years: meaning that the crisis have had an impact on financial markets.

Gregoriou, Sedzro and Zhu found that efficient funds had higher returns and positive skewness, while inefficient funds negative skewness and lower volatility. In addition, hedge funds returns don't reflect normal distribution, showing long and thick tails, confirming high probability of extreme events[77]. To conclude, for what concerns hedge funds evaluation, DEA, although has not a crucial impact on, funds classification, provides investors with additional information that hedge funds rankings do not provide. Furthermore, DEA occurred to be an excellent complementary tool for risk-adjusted measures, contributing for a more complete evaluation of funds' performances. So far, various variants of DEA models have been descripted. It can be

---

[77] Gregoriou G. N. (2003), Performance Appraisal of Funds of Hedge Funds Using Data Envelopment Analysis, Working Paper n. 5.

resumed that DEA represents a useful technique thanks to its flexibility through which is possible to consider several aspects/inputs, such as expense ratio/submission & redemption costs, that are not embodied in traditional risk-adjusted measurement indexes.

Table 3.1: DEA Models applied to investment fund evaluation

DEA MODELS	Input	Outuput	Description
Murthi, Choi and Desai	-Total Expense ratio (TER) (%) -Load fees & switch commissions (%) -Standard Deviation of return	-Return	Critics to traditional measures (Sharpe ratio and Jensen alpha) not considering weight of transaction costs. More flexibility thanks to freedom of choice of inputs/outputs
Basso and Funari	-Subscription & Redemption costs (%) -Standard Deviation of return -Beta of benchmark	-Return	Generalization of Murti et al. model. Basso and Funari considered among inputs only direct fees (subscription & redemption costs).
Morey and Morey	-Variance of return -Correlation between funds' returns	-Return	Each fund is evaluated compared to an endogenous benchmark fund, ad hoc created, running during same period.
Gregoriou, Sedzro and Zhu	-Lower mean monthly semi-skewness (LSS)	-Upper mean monthly semi-skewness (USS)	Model for evaluation of hedge funds. More suitable method for hedge funds

	-Lower mean monthly semi-variance (LSV) -Mean monthly lower return (MLR)	-Upper mean monthly semi-variance (USV) -Mean monthly upper return (ULR)	returns not normal distribution.

An Empirical Application of DEA approach: Evaluation of Italian Mutual Funds

Introduction

In this chapter it will be described the application of a particular BCC-input oriented DEA model used to evaluate the efficiency of a sample of Italian investment funds. Through the use of a specific DEA model, it has been possible compute the efficiency for the aforementioned sample, drawing up in this way a rank of performance efficiency, which is compared with results based on traditional risk-adjusted metrics, such as Sharpe and Sortino ratios, in order to evaluate the integrity of DEA model.

The evaluation and comparison processes are directed towards an empirical assessment of transaction costs' incidence over the total performance of mutual funds, incidence that is not embedded in most used, and well-known, risk-adjusted measures. Through the analysis of correlation, it will be investigated the degree of correlation, whether positive, null or negative, between DEA results and traditional performance gauges, such as Sharpe and Sortino ratio, and even size measure, as the asset under management (AUM). Finally, it will be also compared DEA efficiency results with funds' size, in order to appraise whether there is a relation between performance and size. In the ensuing paragraph, it will be presented sample of data, along with selection criteria, and methodology used to perform the aforesaid analysis

Data sample and Methodology

In order to fulfill the analysis, starting from the specific application of DEA approach conceived by Murthi, Choi and Desai[78], from now on MCD, it will be assessed the efficiency of selected cluster of investment funds, and in order to better observe the time effect, it was decided to carry on two different analyses, related to different time windows, one and three years. Based on the MCD method, the data sample has been selected taking into consideration, firs of all, the availability of data required by the specific model: obviously one of the fundamental feature that a sample has to have for

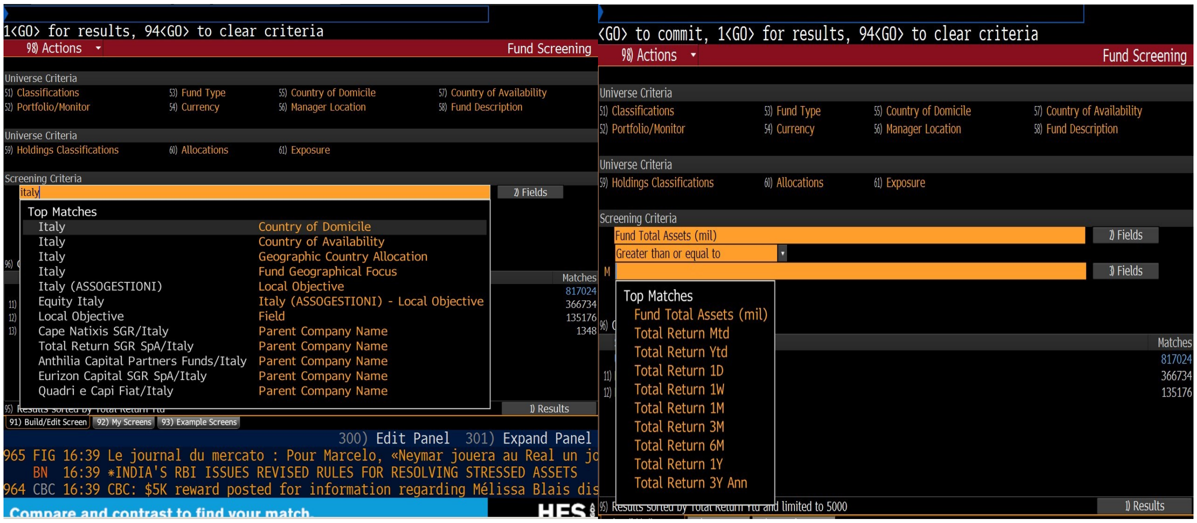
[78] See Chapter 3.

be employed in DEA analysis is the homogeneity of data, along with the full accessibility of a certain dataset. Data have been obtained from two different sources: principally Bloomberg while, to a lesser extent, Morningstar.

As explained repeatedly across this work, it is well known that efficiency and performance assessment focus mainly on risk and return gauges. Since the analysis must be done on a consistent and homogeneous sample of data in order to have reliable results, we firstly to set the selection criteria. The following are the ones used for this analysis: Italian[79] mid and large-cap mutual funds, with available data from December 2014 until January 2018. Through the function Fund Screening, or FSCR, I applied the filters (as shown in Figure 4.1): "*Italy*", "*Date*" and "*AUM*". For what concerns the period, in order to have returns and other risk factors related to one and three years, it has to be set a time window starting from December 2014 ending on January 2018, with monthly frequency; while regarding the assets managed by funds, it is necessary to fix a minimum amount of "*AUM*" of €300 mln. Unfortunately, it was not possible to find data required for all the funds, mainly due to the following reasons: part of them have been launched after 2014, consequently there aren't available data to run the model for both the time periods considered; while other funds, presenting negative returns, cannot deal with DEA model. Therefore, applying these filters on Bloomberg terminal, the sample of data was extracted from the Bloomberg database, from which it is necessary to skim funds with negative returns, after having calculated them.

Figure 4.1: Bloomberg function FSCR

[79] Italy intended as "Country of Domicile", as indicated on Bloomberg. All the available funds domiciliated, listed and traded on the Italian Stock Exchange (Borsa Italiana).



Source: Bloomberg

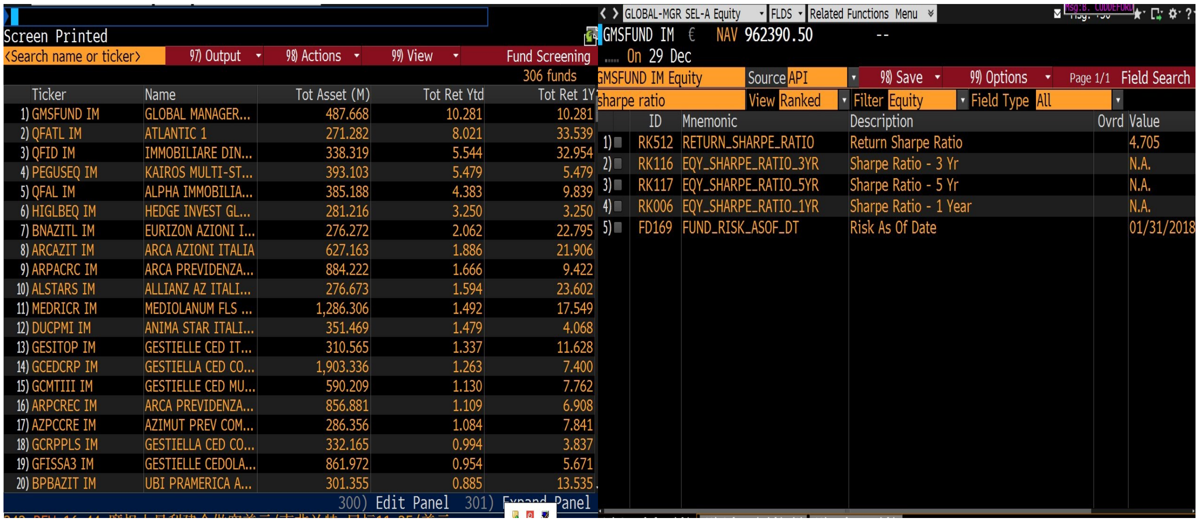
Concluded this process Bloomberg provides a screen (see figure 4.2), with the list of funds matching the filters. To find the risk indicators needed for the analysis, is necessary to apply "BDP" or "BDH"[80] formula on the Bloomberg Excel add-in, together with the specific data required (i.e. "*px_last*" for last price of fund or security): in this case it has been used "*BDH*" formula along with fund's ticker[81], and the field code, "*FLDS*" (Image 4.4), or "px_last" formula. Thus, almost all data were collected, such as closing prices, the Sharpe and Sortino ratios, standard deviations, and the total expense ratio (TER). As stated earlier, not all the data are available on Bloomberg, therefore it was necessary to look for load fees and management fees on Morningstar database. Based on data obtained from Bloomberg and Morningstar databases, annual returns, expressed in percentage points, have been calculated starting from yearly closing prices.

---

[80] "BDP" stands for "Bloomberg data point", used to retrieve static or real-time current data; "BDH" stands for "Bloomberg data history", used to retrieve historical data.

[81] Ticker is the symbol representing a fund or a company's security on a stock exchange. It is the most way to search or identify a fund or stock.

Figure 4.2: Bloomberg results and FLDS function



Source: Bloomberg

In detail, the data sample that have been selected is composed by 93 funds divided in: 29 equity funds, 29 fixed income funds and 32 Mixed Allocation funds (relying on the fund asset classification used by Bloomberg). Briefly, equity funds are, as we already explained in the first chapter, funds that invest primarily and exclusively in stocks, also known as stock funds. Of the 29 equity funds, there are 6 investing exclusively on Italian stocks, 10 on global stocks, 7 on stock from the Euro-area, 2 on U.S. stocks and the remaining 3 on emerging countries. Whereas, fixed income funds are funds that own principally bond securities, such as Treasuries, corporate bonds or municipal bonds. Finally, there are mixed allocation funds, also known as balanced funds, that combine a mixture of various asset classes, usually mainly composed by equity and fixed income, in different percentages, depending on the strategy of fund's management.

Once the data sample is defined, we focus on the inputs and outputs needed to run the MCD model. It is a DEA BCC input-oriented model; hence it measures the efficiency of output over inputs, placing emphasis on reduction of certain inputs to improve efficiency. In this case, as the three authors did in 1997, there are three inputs and one output. As inputs, representing the volatility of returns there is the standard deviation, while the remaining two inputs represent the so-called transaction costs: fees that an investor bear when owning mutual fund shares; more precisely there are the total expense ratio (TER)[82] and load fees. Unlike MCD model, there is no switch commissions among inputs, because they are not available for the complete data sample under analysis. To conclude, as output it has been used the annual funds' return.

The following table summarizes the factors applied within the MCD model, along with items' description[83]:

Table 4.1: Inputs and Output used in DEA model

INPUT	DESCRIPTION

[82] For what concerns the TER, one of the most important cost are the management fees.

[83] Definition given by Morningstar Investing Glossary:
http://www.morningstar.com/InvGlossary/expense_ratio.aspx

Total Expense Ratio (TER)	The Total Expense Ratio (TER) is the annual fee investors are charged for owning mutual funds or ETFs shares. This ratio encloses many fund expenses such as, 12b-1 fees, management fees, administrative fees, operating costs and other asset-based costs incurred by the fund. Whether fund's assets are relatively small the expense ratio may be high. The TER is calculated daily, deducting it from the fund's average net assets.
Load Commissions	Load indicates either a fund's maximum initial or deferred sales charge. There are two types of loads: front-end and back-end. Front-end load or initial charge happens when purchasing fund shares and is calculated as percentage of initial investment. Back-end load or deferred sales charge incurred when selling fund share and usually is equal to zero.
Standard Deviation (SD)	Statistical measure of dispersion of returns aroung their average; it tells how a fund return is volatile over a certain time period. High standard deviation means greater return's volatility.
Asset Under Management (AUM)	AUM measures the market value of the total asset managed by a fund, widely used in financial industry to rank funds size as well as success of managers.
OUTPUT	

Return	Annual return of fund. Is calculated from the closing prices and expressed as a percentage.

In the following paragraph we report all the tables with funds' ranking combined with DEA efficiency scores obtained by each fund. Considering that one of the most important aspects of such an analysis is the consistency of data sample, the following measurements were conducted: first of all, a DEA efficiency analysis on the overall sample, for both one and three years scenarios. Then, founded on Bloomberg funds' categorization, the data sample was divided in three subgroups: equity, fixed income and mixed allocation, thus allowing to conduct a fund class specific performance measurement, both for one and three years. The DEA scores are then compared with Sharpe ratios, to assess whether the transaction costs affect funds' efficiency. Moreover, has been computed the correlation among DEA results and risk-adjusted measure and funds' size.

Results

To perform the DEA analysis, based on the Murthi et al. approach, it has been used the open source software MaxDEA (Figure 4.3 on the left panel). Through the software, is possible to set which type of DEA basic model to apply (i.e. CCR or BCC), input or output oriented, and many other options. Once that a DEA model is chosen, the software allows to set as many inputs and outputs as possible into the analysis (Figure 4.3 on the right panel), before run the program (Image 4.4).

Figure 4.3: Screens of software MaxDEA Basic v. 7

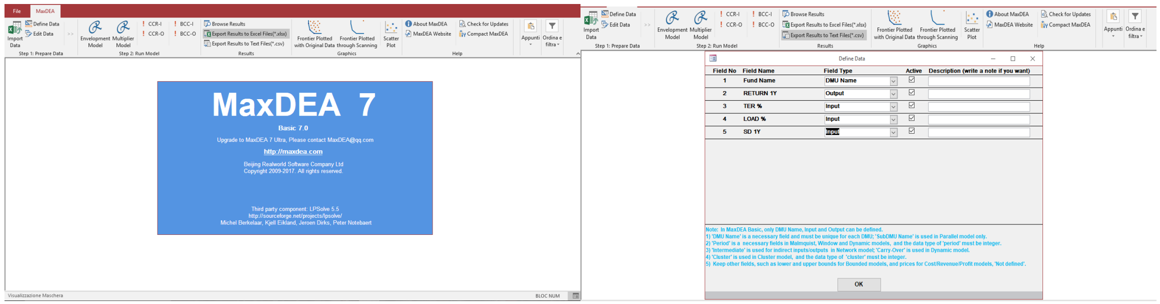
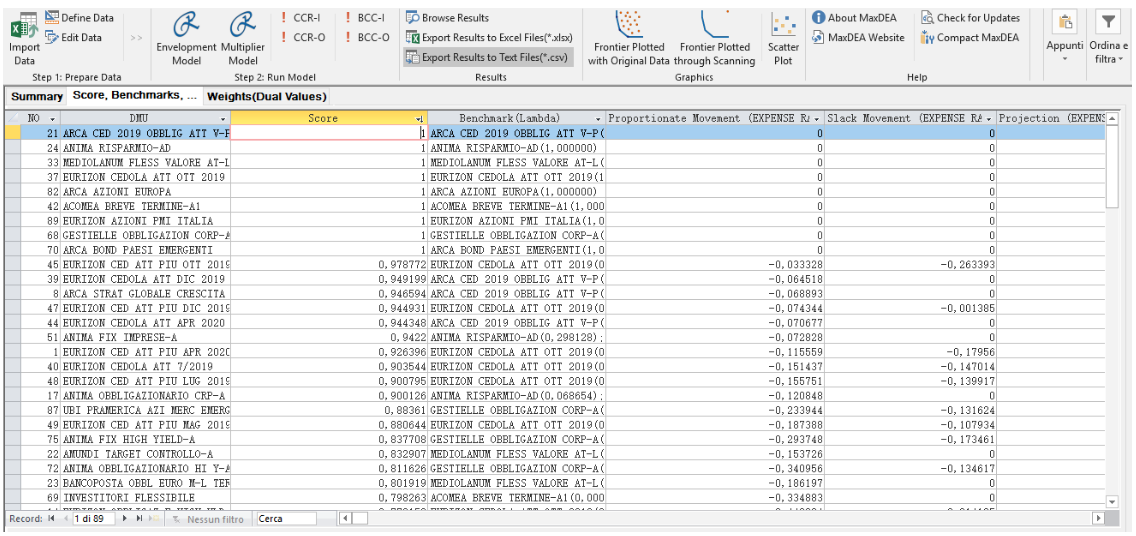


Figure 4.4: Screen of results of DEA model with software MaxDEA Basic v. 7



NO	DMU	Score	Benchmark(Lambda)	Proportionate Movement (EXPENSE R	Slack Movement (EXPENSE R	Projection (EXPENS
21	ARCA CED 2019 OBBLIG ATT V-P	1	ARCA CED 2019 OBBLIG ATT V-P(	0	0	
24	ANIMA RISPARMIO-AD	1	ANIMA RISPARMIO-AD(1,000000)	0	0	
33	MEDIOLANUM FLESS VALORE AT-L	1	MEDIOLANUM FLESS VALORE AT-L(	0	0	
37	EURIZON CEDOLA ATT OTT 2019	1	EURIZON CEDOLA ATT OTT 2019(1	0	0	
82	ARCA AZIONI EUROPA	1	ARCA AZIONI EUROPA(1,000000)	0	0	
42	ACOMEA BREVE TERMINE-A1	1	ACOMEA BREVE TERMINE-A1(1,000	0	0	
89	EURIZON AZIONI PMI ITALIA	1	EURIZON AZIONI PMI ITALIA(1,0	0	0	
68	GESTIELLE OBBLIGAZION CORP-A	1	GESTIELLE OBBLIGAZION CORP-A(	0	0	
70	ARCA BOND PAESI EMERGENTI	1	ARCA BOND PAESI EMERGENTI(1,0	0	0	
45	EURIZON CED ATT PIU OTT 2019	0,978772	EURIZON CEDOLA ATT OTT 2019(0	-0,033328	-0,263393	
39	EURIZON CEDOLA ATT DIC 2019	0,949199	ARCA CED 2019 OBBLIG ATT V-P(	-0,064518	0	
8	ARCA STRAT GLOBALE CRESCITA	0,946594	ARCA CED 2019 OBBLIG ATT V-P(	-0,068893	0	
47	EURIZON CED ATT PIU DIC 2019	0,944931	EURIZON CEDOLA ATT OTT 2019(0	-0,074344	-0,001385	
44	EURIZON CEDOLA ATT APR 2020	0,944348	ARCA CED 2019 OBBLIG ATT V-P(	-0,070677	0	
51	ANIMA FIX IMPRESE-A	0,9422	ANIMA RISPARMIO-AD(0,298128);	-0,072828	0	
1	EURIZON CED ATT PIU APR 2020	0,926396	EURIZON CEDOLA ATT OTT 2019(0	-0,115559	-0,17956	
40	EURIZON CEDOLA ATT 7/2019	0,903544	EURIZON CEDOLA ATT OTT 2019(0	-0,151437	-0,147014	
48	EURIZON CED ATT PIU LUG 2019	0,900795	EURIZON CEDOLA ATT OTT 2019(0	-0,155751	-0,139917	
17	ANIMA OBBLIGAZIONARIO CRP-A	0,900126	ANIMA RISPARMIO-AD(0,068654);	-0,120848	0	
87	UBI PRAMERICA AZI MERC EMERG	0,88361	GESTIELLE OBBLIGAZION CORP-A(	-0,233944	-0,131624	
49	EURIZON CED ATT PIU MAG 2019	0,880644	EURIZON CEDOLA ATT OTT 2019(0	-0,187388	-0,107934	
75	ANIMA FIX HIGH YIELD-A	0,837708	GESTIELLE OBBLIGAZION CORP-A(	-0,293748	-0,173461	
22	AMUNDI TARGET CONTROLLO-A	0,832907	MEDIOLANUM FLESS VALORE AT-L(	-0,153726	0	
72	ANIMA OBBLIGAZIONARIO HI Y-A	0,811626	GESTIELLE OBBLIGAZION CORP-A(	-0,340956	-0,134617	
23	BANCOPOSTA OBBL EURO M-L TER	0,801919	MEDIOLANUM FLESS VALORE AT-L(	-0,186197	0	
69	INVESTITORI FLESSIBILE	0,798263	ACOMEA BREVE TERMINE-A1(0,000	-0,334883	0	

Source: MaxDEA software

Just a remark, before starting with analysis results. As explained previously, a DMU, in this case an investment fund, is efficient when has an efficiency score of 1. Evidently, high score, close to 1, is to be considered a very sound level of efficiency, hence quite good result.

In the two next tables, it is represented the DEA results for the entire sample of funds, ordered in descending order starting from the highest DEA efficiency score, equal to 1.

These two tables show respectively the results based on the 1 year, 2017, and 3-years analysis; from the left there are: Fund name and category, DEA result, return, total expense ratio, load fee, Sharpe ratio and standard deviation.

As it is clear from the first table (Table 4.2), efficient funds are 7 over 93, 4 fixed income, 2 equity and the remaining one mixed allocation fund. It can be noted that efficiency is due to different reasons: great returns for two equity funds (i.e. "EURIZON AZIONI PMI ITALIA" with 33.94%), very low volatility of returns, expressed by the standard deviation (i.e. "EURIZON CEDOLA ATT OTT 2019" with 0.88), and very low level of transaction costs, expressed by the sum of TER and loads (i.e. "ACOMEA BREVE TERMINE A-1" with 0.66% of total fees).

Looking at the results showed in the second table (Table 4.3), concerning the 3-years performance, it can be confirmed the reasons behind the funds' efficiency. Indeed, there are some funds with great returns over the three years, other funds showing low values of standard deviation, and finally, some funds with very low level of transaction costs, confirming the validity and usefulness of DEA.

Table 4.2: AllFunds DEA Efficiency Results 1Y

FUND NAME	BLOOMBERG CATEGORY	DEA SCORE	RETURN 1Y	TER %	LOAD %	SHARPE 1Y	SD 1Y
ACOMEA BREVE TERMINE-A1	Fixed Income	1	1.67%	0.66	0	1.27	1.44
ARCA AZIONI EUROPA	Equity	1	10.43%	2.12	0	1.82	7.01
ARCA BOND PAESI EMERGENTI	Fixed Income	1	7.13%	1.36	0	2.62	2.37
ARCA CED 2019 OBBLIG ATT V-P	Fixed Income	1	0.20%	1	0	1.38	0.74
EURIZON AZIONI PMI ITALIA	Equity	1	33.94%	2.1	1.5	2.44	13.46
EURIZON CEDOLA ATT OTT 2019	Mixed Allocation	1	1.49%	1.27	0	3.31	0.88
GESTIELLE OBBLIGAZION CORP-A	Fixed Income	1	6.92%	1.32	0	3.87	1.64
EURIZON CED ATT PIU OTT 2019	Mixed Allocation	0.9788	1.85%	1.57	0	3.52	0.95
EURIZON CEDOLA ATT DIC 2019	Mixed Allocation	0.9492	1.64%	1.27	0	3.33	0.96
ARCA STRAT GLOBALE CRESCITA	Mixed Allocation	0.9466	2.84%	1.29	0	2.89	1.14
EURIZON CED ATT PIU DIC 2019	Mixed Allocation	0.9449	1.96%	1.35	0	3.48	1.00
EURIZON CEDOLA ATT APR 2020	Mixed Allocation	0.9443	1.80%	1.27	0	3.40	0.99
ANIMA FIX IMPRESE-A	Fixed Income	0.9422	4.08%	1.26	3	3.31	1.29

EURIZON CED ATT PIU APR 2020	Mixed Allocation	0.9264	2.02%	1.57	0	3.47	1.03
EURIZON CEDOLA ATT 7/2019	Mixed Allocation	0.9035	1.66%	1.57	0	3.25	1.00
EURIZON CED ATT PIU LUG 2019	Mixed Allocation	0.9008	1.96%	1.57	0	3.43	1.05
ANIMA OBBLIGAZIONARIO CRP-A	Fixed Income	0.9001	3.78%	1.21	3	2.99	1.36
UBI PRAMERICA AZI MERC EMERG	Equity	0.8836	18.16%	2.01	2.5	2.53	7.42
EURIZON CED ATT PIU MAG 2019	Mixed Allocation	0.8806	2.00%	1.57	0	3.40	1.08
ANIMA FIX HIGH YIELD-A	Fixed Income	0.8377	7.71%	1.81	3	3.53	2.37
AMUNDI TARGET CONTROLLO-A	Mixed Allocation	0.8329	0.28%	0.92	1	1.62	1.21
ANIMA OBBLIGAZIONARIO HI Y-A	Fixed Income	0.8116	7.42%	1.81	3	3.54	2.29
BANCOPOSTA OBBL EURO M-L TER	Fixed Income	0.8019	0.30%	0.94	0	1.42	1.32
INVESTITORI FLESSIBILE	Mixed Allocation	0.7983	7.11%	1.66	2	3.94	2.16
EURIZON OBBLIGAZ E HIGH YLD	Fixed Income	0.7702	3.40%	1.95	0	2.29	1.49
ANIMA GEO PAESI EMERGENTI-A	Equity	0.7677	17.67%	2.509	2	2.18	8.26
UBI PRAMERICA OB GLOB ALT RN	Fixed Income	0.7592	5.35%	1.51	1.5	2.37	2.02
ANIMA EMERGENTI-A	Equity	0.7541	17.38%	2.12	5	2.13	8.37
EURIZON DIVERSIFICATO ETICO	Mixed Allocation	0.7458	0.41%	1.04	0	1.13	1.30
EURIZON MULTIA REDD OTT 2019	Mixed Allocation	0.7448	1.48%	1.88	0	2.98	1.18
EURIZON MA REDDITO APR 2020	Mixed Allocation	0.7317	2.11%	1.9	0	2.81	1.32
ARCA OBBLIGAZIONI EUROPA	Mixed Allocation	0.7252	3.92%	1.3	0	2.12	2.21
ARCA AZIONI ITALIA	Equity	0.7209	17.57%	1.9	4	2.28	11.52
ANIMA-FONDO TRADING-A	Equity	0.7179	8.31%	3.06	4	3.11	3.13
ANIMA GEO ITALIA-A	Equity	0.7159	19.73%	2.39	5	2.51	10.12
ANIMA FIX OBBLIGAZION MLT-A	Fixed Income	0.6998	0.56%	1.1	3	1.51	1.42
ARCA RR DIVERSIFIED BOND	Fixed Income	0.6925	0.02%	1.13	0	0.01	1.36
EURIZON SOLUZIONE 10	Mixed Allocation	0.6826	0.41%	1.54	1.5	1.47	1.04
MEDIOLANUM FLESS SVIL ITAL-L	Fixed Income	0.6807	6.68%	1.57	3.5	2.20	3.74
BANCOPOSTA MIX 1-A	Mixed Allocation	0.6799	0.37%	1.14	1	0.82	1.43
ARCA STRATEGIA GLOBALE OPPOR	Mixed Allocation	0.6719	6.03%	2.1	3	2.88	2.23
ARCA AZIONI INTERNAZIONALI	Equity	0.6648	7.20%	2.07	0	1.38	6.47
ARCA BOND CORPORATE	Fixed Income	0.6648	1.63%	1.2	0	1.21	1.74
BCC CRESCITA BILANCIATO	Mixed Allocation	0.6628	6.06%	1.67	1.5	2.64	3.08
UBI PRAMERICA OB GLOBAL CORP	Fixed Income	0.6625	2.89%	1.32	1.5	1.27	2.06
ARCA BB	Mixed Allocation	0.6252	5.47%	1.82	0	2.21	3.65
BANCOPOSTA MIX 2-A	Mixed Allocation	0.6029	1.66%	1.34	1	1.42	1.90
ETICA OBBLIGAZION MISTO-R	Mixed Allocation	0.5973	0.15%	1.26	0	0.69	1.78
ANIMA SFORZESCO-A	Mixed Allocation	0.5889	0.02%	1.25	3	0.33	1.92
EURIZON GEST ATT DIN 4/2020	Mixed Allocation	0.5874	3.49%	2.1	0	2.57	1.99
UBI PRAMERICA EURO CORPORATE	Fixed Income	0.5869	1.29%	1.31	1.5	1.08	1.90
SYMPHONIA PATRIMONIO REDDITO	Mixed Allocation	0.5774	3.00%	1.51	0	1.62	2.43
MEDIOLANUM FLS FUTURO IT-LA	Equity	0.5633	13.78%	2.3	5	2.12	9.87
GESTIELLE CEDOLA MULTIASSET	Mixed Allocation	0.5611	1.90%	1.28	0	1.09	2.48
EURIZON GES ATT CLASS 4/2020	Mixed Allocation	0.5585	0.54%	1.8	0	1.32	1.41
EURIZON PROFILO FLESS EQLBR	Mixed Allocation	0.5505	0.92%	1.52	1.5	1.29	1.59
GESTIELLE ABSOLUTE RETURN	Mixed Allocation	0.5423	2.37%	1.64	2	1.21	2.19
UBI PRAMERICA AZIONI ITALIA	Equity	0.5406	10.02%	1.91	2.5	2.01	9.73
ANIMA STAR EUROPA ALTO POT-A	Mixed Allocation	0.5405	3.07%	1.73	4	1.94	2.40

GESTIELLE OBIETTIVO EUROPA-A	Equity	0.5276	7.84%	2.24	1.5	2.19	5.16
ANIMA ALTO POTENZIALE-A	Mixed Allocation	0.5138	6.46%	2.34	4	2.58	3.64
UBI PRAMERICA AZIONI GLOBALI	Equity	0.5083	7.61%	2.06	2.5	1.51	6.20
ANIMA EUROPA-A	Equity	0.5079	7.92%	2.12	5	1.69	6.30
ANIMA GEO ASIA-A	Equity	0.5073	10.31%	2.47	5	1.39	7.50
ALLIANZ AZIONI EUROPA-L	Equity	0.5049	10.29%	2.5	2	1.78	7.43
ANIMA VISCONTEO-A	Mixed Allocation	0.5044	2.77%	1.46	4	1.28	3.45
BANCOPOSTA AZIONARIO INTER	Equity	0.4857	5.82%	1.74	1	1.06	6.96
UNICREDIT SOLUZIONE 40-A	Mixed Allocation	0.4776	2.43%	1.61	2	1.35	3.02
ANIMA GEO EUROPA-A	Equity	0.4683	7.53%	2.31	5	1.64	6.33
EURIZON OBBLIGAZIONI EURO	Fixed Income	0.4668	0.39%	2	0	1.41	1.67
ETICA BILANCIATO-R	Mixed Allocation	0.4608	3.38%	1.9	0	1.17	4.42
EUROMOBILIARE AZ INTERNAZION	Equity	0.4482	4.39%	2	4	1.42	4.42
EURIZON AZIONI EUROPA	Equity	0.4453	6.65%	2	1.5	1.28	7.32
EURIZON BILANCIATO EURO MMGR	Mixed Allocation	0.4428	4.78%	2.21	1.5	1.65	4.20
MEDIOLANUM FLESS GLOBALE-LA	Equity	0.4215	5.44%	2.22	5	1.20	5.70
ANIMA VALORE GLOBALE-A	Equity	0.4076	5.74%	2.12	5	0.95	7.03
EURIZON AZIONI INTERNAZIONAL	Equity	0.4067	5.03%	2	1.5	1.16	6.58
ALLIANZ GLOBAL STRATEGY 70-L	Mixed Allocation	0.4040	3.33%	2.15	1.5	1.44	3.73
ARCA TE	Mixed Allocation	0.3908	2.24%	2.05	0	0.88	3.39
ANIMA AMERICA	Equity	0.3870	5.26%	2.12	5	1.03	8.06
ANIMA GEO GLOBALE-A	Equity	0.3622	5.36%	2.47	5	0.91	6.98
UBI PRAMERICA PORT DINAMICO	Mixed Allocation	0.3551	0.46%	1.87	1.5	0.66	4.01
EUROMOBILIARE FLESS ALL GLOB	Mixed Allocation	0.3496	0.43%	2.36	2	0.74	2.34
ANIMA STAR ITALIA ALTO POT-A	Mixed Allocation	0.3316	1.74%	2.3	4	1.25	3.79
GESTIELLE OBIETT INTERNAZ-A	Equity	0.3250	2.18%	2.18	1.5	0.89	4.70
ANIMA GEO AMERICA-A	Equity	0.3225	4.89%	2.51	5	0.98	8.11
UBI PRAMERICA AZIONI EURO	Equity	0.3125	3.40%	2.36	2.5	1.01	7.98

Table 4.3: AllFunds DEA Efficiency Results 3Y

Fund Name	BLOOMBERG CAT	DEA SCORE	RETURN 3Y	TER %	LOAD %	SHARPE 3Y	SD 3Y
ACOMEA BREVE TERMINE-A1	Fixed Income	1	6.20%	0.66	0	0.81	3.22
ANIMA FIX HIGH YIELD-A	Fixed Income	1	21.07%	1.81	3	1.33	4.94
ANIMA FIX OBBLIGAZION BT-A	Fixed Income	1	0.06%	0.92	0.25	0.21	0.74
ANIMA OBBLIGAZIONARIO HI Y-A	Fixed Income	1	20.34%	1.81	3	1.33	4.77
ARCA AZIONI INTERNAZIONALI	Equity	1	25.94%	2.07	0	0.67	11.08
ARCA STRAT GLOBALE CRESCITA	Mixed Allocation	1	5.34%	1.29	0	0.78	2.61
EURIZON AZIONI PMI ITALIA	Equity	1	73.64%	2.1	1.5	1.05	17.09
GESTIELLE OBBLIGAZION CORP-A	Fixed Income	1	13.85%	1.32	0	1.12	4.03
ARCA BOND PAESI EMERGENTI	Fixed Income	0.9552	14.03%	1.36	0	0.89	4.98

EURIZON DIVERSIFICATO ETICO	Mixed Allocation	0.9487	4.40%	1.04	0	0.48	2.93
EURIZON OBBLIGAZ E HIGH YLD	Fixed Income	0.9173	9.87%	1.95	0	0.82	3.67
ARCA RR DIVERSIFIED BOND	Fixed Income	0.8735	1.79%	1.13	0	0.05	2.91
EUROMOBILIARE EURO AGGREGATE	Fixed Income	0.8629	0.11%	1.13	1	0.13	0.87
ARCA BOND CORPORATE	Fixed Income	0.8488	4.83%	1.2	0	0.41	3.29
UBI PRAMERICA OB GLOB ALT RN	Fixed Income	0.8368	15.97%	1.51	1.5	1.08	5.02
ARCA AZIONI EUROPA	Equity	0.8133	21.10%	2.12	0	0.47	12.41
ANIMA OBBLIGAZIONARIO CRP-A	Fixed Income	0.8035	8.06%	1.21	3	0.72	3.43
ETICA OBBLIGAZION MISTO-R	Mixed Allocation	0.7998	4.63%	1.26	0	0.27	3.48
ETICA BILANCIATO-R	Mixed Allocation	0.7975	17.20%	1.9	0	0.54	7.81
ALLIANZ REDDITO EURO-L	Fixed Income	0.7887	1.01%	0.9	1.5	-0.01	3.48
ANIMA FIX IMPRESE-A	Fixed Income	0.7830	8.23%	1.26	3	0.74	3.52
INVESTITORI FLESSIBILE	Mixed Allocation	0.7737	13.81%	1.66	2	0.84	4.75
ARCA OBBLIGAZIONI EUROPA	Mixed Allocation	0.7593	10.01%	1.3	0	0.47	4.81
ANIMA SFORZESCO-A	Mixed Allocation	0.7549	6.40%	1.25	3	0.37	3.17
EURIZON OBBLIGAZIONI EURO	Fixed Income	0.7244	3.36%	2	0	0.15	3.54
SYMPHONIA PATRIMONIO REDDITO	Mixed Allocation	0.7159	5.08%	1.51	0	0.32	3.86
UBI PRAMERICA GLB HI YD EU H	Fixed Income	0.7126	1.43%	1.29	0	0.46	3.60
UBI PRAMERICA OB GLOBAL CORP	Fixed Income	0.7079	6.37%	1.32	1.5	0.35	3.41
UBI PRAMERICA EURO MED/LUNGO	Fixed Income	0.6896	1.89%	1.11	1.5	0.00	3.21
BANCOPOSTA MIX 1-A	Mixed Allocation	0.6859	3.13%	1.14	1	0.11	3.00
ANIMA FIX OBBLIGAZION MLT-A	Fixed Income	0.6762	3.94%	1.1	3	0.22	3.58
UBI PRAMERICA EURO CORPORATE	Fixed Income	0.6719	4.22%	1.31	1.5	0.41	2.97
UBI PRAMERICA AZIONI ITALIA	Equity	0.6696	35.19%	1.91	2.5	0.69	15.16
BANCOPOSTA AZIONARIO INTER	Equity	0.6590	28.99%	1.74	1	0.62	12.58
ANIMA VISCONTEO-A	Mixed Allocation	0.6585	11.77%	1.46	4	0.49	5.77
ANIMA RENDIMENTO ASSOL OBB-A	Fixed Income	0.6524	3.36%	1.36	3	0.02	2.67
EURIZON MULTIA REDD OTT 2019	Mixed Allocation	0.6492	1.96%	1.88	0	0.35	3.90
UBI PRAMERICA PORT PRUDENTE	Fixed Income	0.6476	2.04%	1.3	1.5	0.00	2.35
ARCA TE	Mixed Allocation	0.6395	10.58%	2.05	0	0.44	5.45
AMUNDI OBBL PIU A DIST-A	Mixed Allocation	0.6375	2.70%	1.22	1.2	0.23	3.29
UBI PRAMERICA AZIONI GLOBALI	Equity	0.6329	28.40%	2.06	2.5	0.68	11.47
ANIMA AMERICA	Equity	0.6305	29.12%	2.12	5	0.78	11.66
BCC CRESCITA BILANCIATO	Mixed Allocation	0.6090	14.80%	1.67	1.5	0.65	7.31
ARCA AZIONI ITALIA	Equity	0.6089	29.47%	1.9	4	0.57	17.01
BANCOPOSTA MIX 2-A	Mixed Allocation	0.5962	5.88%	1.34	1	0.20	4.53
ARCA BB	Mixed Allocation	0.5927	11.35%	1.82	0	0.40	7.19
GESTIELLE ABSOLUTE RETURN	Mixed Allocation	0.5915	8.76%	1.64	2	0.50	4.99
ANIMA VALORE GLOBALE-A	Equity	0.5827	27.88%	2.12	5	0.63	12.58
ANIMA STAR EUROPA ALTO POT-A	Mixed Allocation	0.5773	6.06%	1.73	4	0.27	3.69
ARCA BOND GLOBALE	Fixed Income	0.5770	5.60%	1.16	0	-0.13	5.54
ANIMA GEO AMERICA-A	Equity	0.5743	27.34%	2.51	5	0.73	11.72
MEDIOLANUM FLESS SVIL ITAL-L	Fixed Income	0.5713	8.77%	1.57	3.5	0.64	5.57

EURIZON PROFILO FLESS EQLBR	Mixed Allocation	0.5665	1.99%	1.52	1.5	0.17	2.56
EURIZON SOLUZIONE 10	Mixed Allocation	0.5662	1.86%	1.54	1.5	0.12	2.44
ANIMA GEO GLOBALE-A	Equity	0.5653	28.96%	2.47	5	0.65	12.65
UBI PRAMERICA PORT MODERATO	Mixed Allocation	0.5647	5.83%	1.64	0.75	0.16	4.06
GESTIELLE CEDOLA MULTIA II	Mixed Allocation	0.5500	3.80%	1.2	0	0.31	6.85
UBI PRAMERICA PORT DINAMICO	Mixed Allocation	0.5458	12.05%	1.87	1.5	0.37	6.76
EUROMOBILIARE AZ INTERNAZION	Equity	0.5404	16.93%	2	4	0.56	8.94
ARCA STRATEGIA GLOBALE OPPOR	Mixed Allocation	0.5399	9.57%	2.1	3	0.68	5.19
ALLIANZ AZIONI EUROPA-L	Equity	0.5397	26.02%	2.5	2	0.55	12.04
GESTIELLE CEDOLA MULTIASSET	Mixed Allocation	0.5339	2.63%	1.28	0	0.48	5.92
MEDIOLANUM FLESS GLOBALE-LA	Equity	0.5131	17.66%	2.22	5	0.49	9.45
ANIMA EMERGENTI-A	Equity	0.5128	25.10%	2.12	5	0.52	13.72
MEDIOLANUM FLESSIBLE STRAT-L	Mixed Allocation	0.5113	2.88%	1.56	3.5	0.35	3.73
ANIMA FIX OBBLIG GLOBALE-A	Fixed Income	0.5102	4.37%	1.3	3	-0.32	6.25
EURIZON AZIONI INTERNAZIONAL	Equity	0.5047	21.57%	2	1.5	0.54	12.51
ANIMA PIANETA-A	Fixed Income	0.5016	3.68%	1.32	3	-0.33	6.38
ANIMA GEO ASIA-A	Equity	0.5008	23.36%	2.47	5	0.47	12.17
ANIMA GEO ITALIA-A	Equity	0.4950	30.03%	2.39	5	0.56	16.32
ALLIANZ GLOBAL STRATEGY 70-L	Mixed Allocation	0.4890	10.56%	2.15	1.5	0.48	6.52
UNICREDIT SOLUZIONE 40-A	Mixed Allocation	0.4790	1.87%	1.61	2	0.10	4.51
EURIZON BILANCIATO EURO MMGR	Mixed Allocation	0.4786	12.33%	2.21	1.5	0.36	7.63
UBI PRAMERICA AZIONI USA	Equity	0.4752	22.02%	2.1	2.5	0.50	14.17
GESTIELLE OBIETT INTERNAZ-A	Equity	0.4725	17.09%	2.18	1.5	0.52	10.65
ANIMA EUROPA-A	Equity	0.4664	17.13%	2.12	5	0.37	11.09
UBI PRAMERICA AZI MERC EMERG	Equity	0.4626	18.83%	2.01	2.5	0.41	13.74
ANIMA GEO PAESI EMERGENTI-A	Equity	0.4558	23.04%	2.509	2	0.47	13.82
UBI PRAMERICA MULTIA ITALIA	Fixed Income	0.4442	2.78%	1.98	0	0.70	6.13
ANIMA GEO EUROPA-A	Equity	0.4405	16.41%	2.31	5	0.34	11.09
AMUNDI OBBLIG GLOB HY DIS-A	Fixed Income	0.4342	2.27%	1.52	1.2	0.30	8.02
EUROMOBILIARE FLESS ALL GLOB	Mixed Allocation	0.4335	4.84%	2.36	2	0.17	4.11
ANIMA-FONDO TRADING-A	Equity	0.4285	11.77%	3.06	4	0.71	7.40
EURIZON AZIONI EUROPA	Equity	0.4216	14.78%	2	1.5	0.31	12.74
MEDIOLANUM FLS FUTURO IT-LA	Equity	0.4115	19.62%	2.3	5	0.43	15.55
GESTIELLE OBIETTIVO EUROPA-A	Equity	0.3966	15.46%	2.24	1.5	0.41	12.68
UBI PRAMERICA AZIONI EURO	Equity	0.3916	15.53%	2.36	2.5	0.30	12.61
ANIMA ALTO POTENZIALE-A	Mixed Allocation	0.3800	4.19%	2.34	4	0.30	5.16
ANIMA STAR ITALIA ALTO POT-A	Mixed Allocation	0.3560	2.21%	2.3	4	0.03	4.79

The following tables (Table 4.4 and 4.5) display the DEA efficiency rankings for equity funds, compared with the ranking based on the Sharpe ratio. Highlighted in yellow there are efficient funds. Efficient funds, those with a DEA score equal to 1, have been highlighted in yellow, to make an easier comparison with the funds' ranking based on

Sharpe ratio, on the right. It is evident, looking at table 4.4, that 5 funds out of 9, are ranked efficient both for DEA and for Sharpe ratio; while the remaining 4 funds show low Sharpe ratio levels, mainly due to either low returns or high standard deviation or low transaction fees' level. The same pattern is clear looking at table 4.5 and following, except for fixed income fund 3-years (Table 4.7): these results confirm the peculiarity and versatility of the DEA method. As in the MCD model, the inclusion among inputs' analysis of transaction costs, in addition to traditional risk and return measures, provides a different point of view of the efficiency measurement, broadening the boarders of evaluation variables. All this premised, DEA comes in a clear and understandable way, representing another characteristic in favor of this approach.

In addition, analyzing the effect of time, is clear that funds with an excellent performance on 1-year analysis, maintain the same efficiency for what concern the 3-years.

Table 4.4: Equity Funds DEA Efficiency Results 1Y

Fund Name	Score	Fund Name	SR 1Y
ANIMA-FONDO TRADING-A	1	ANIMA-FONDO TRADING-A	3.11
UBI PRAMERICA AZI MERC EMERG	1	UBI PRAMERICA AZI MERC EMERG	2.53
EURIZON AZIONI PMI ITALIA	1	ANIMA GEO ITALIA-A	2.51
GESTIELLE OBIETTIVO EUROPA-A	1	EURIZON AZIONI PMI ITALIA	2.44
ARCA AZIONI EUROPA	1	ARCA AZIONI ITALIA	2.28
EUROMOBILIARE AZ INTERNAZION	1	GESTIELLE OBIETTIVO EUROPA-A	2.19
ARCA AZIONI INTERNAZIONALI	1	ANIMA GEO PAESI EMERGENTI-A	2.18
BANCOPOSTA AZIONARIO INTER	1	ANIMA EMERGENTI-A	2.13
GESTIELLE OBIETT INTERNAZ-A	1	MEDIOLANUM FLS FUTURO IT-LA	2.12
ARCA AZIONI ITALIA	0.9950	UBI PRAMERICA AZIONI ITALIA	2.01
UBI PRAMERICA AZIONI GLOBALI	0.9469	ARCA AZIONI EUROPA	1.82
ANIMA GEO PAESI EMERGENTI-A	0.9423	ALLIANZ AZIONI EUROPA-L	1.78
UBI PRAMERICA AZIONI ITALIA	0.9391	ANIMA EUROPA-A	1.69
EURIZON AZIONI INTERNAZIONAL	0.9386	ANIMA GEO EUROPA-A	1.64
ANIMA EMERGENTI-A	0.9312	UBI PRAMERICA AZIONI GLOBALI	1.51

ANIMA EUROPA-A	0.9134	EUROMOBILIARE AZ INTERNAZION	1.42
EURIZON AZIONI EUROPA	0.9085	ANIMA GEO ASIA-A	1.39
MEDIOLANUM FLESS GLOBALE-LA	0.8814	ARCA AZIONI INTERNAZIONALI	1.38
ANIMA VALORE GLOBALE-A	0.8674	EURIZON AZIONI EUROPA	1.28
ANIMA GEO EUROPA-A	0.8521	MEDIOLANUM FLESS GLOBALE-LA	1.20
ANIMA GEO ITALIA-A	0.8362	EURIZON AZIONI INTERNAZIONAL	1.16
ANIMA AMERICA	0.8327	BANCOPOSTA AZIONARIO INTER	1.06
ALLIANZ AZIONI EUROPA-L	0.8295	ANIMA AMERICA	1.03
MEDIOLANUM FLS FUTURO IT-LA	0.8160	UBI PRAMERICA AZIONI EURO	1.01
ANIMA GEO ASIA-A	0.7983	ANIMA GEO AMERICA-A	0.98
ANIMA GEO GLOBALE-A	0.7736	ANIMA VALORE GLOBALE-A	0.95
UBI PRAMERICA AZIONI EURO	0.7723	ANIMA GEO GLOBALE-A	0.91
ANIMA GEO AMERICA-A	0.7342	GESTIELLE OBIETT INTERNAZ-A	0.89

Table 4.5: Equity Funds DEA Efficiency Results 3Y:

Fund Name	Score	Fund Name	SR 3Y
EURIZON AZIONI PMI ITALIA	1	EURIZON AZIONI PMI ITALIA	1.05
ANIMA-FONDO TRADING-A	1	ANIMA AMERICA	0.78
ARCA AZIONI INTERNAZIONALI	1	ANIMA GEO AMERICA-A	0.73
BANCOPOSTA AZIONARIO INTER	1	ANIMA-FONDO TRADING-A	0.71
EUROMOBILIARE AZ INTERNAZION	1	UBI PRAMERICA AZIONI ITALIA	0.69
ARCA AZIONI EUROPA	0.9764	UBI PRAMERICA AZIONI GLOBALI	0.68
UBI PRAMERICA AZIONI GLOBALI	0.9639	ARCA AZIONI INTERNAZIONALI	0.67
GESTIELLE OBIETT INTERNAZ-A	0.9611	ANIMA GEO GLOBALE-A	0.65
MEDIOLANUM FLESS GLOBALE-LA	0.9489	ANIMA VALORE GLOBALE-A	0.63
ANIMA AMERICA	0.9372	BANCOPOSTA AZIONARIO INTER	0.62
UBI PRAMERICA AZIONI ITALIA	0.9372	ARCA AZIONI ITALIA	0.57
EURIZON AZIONI INTERNAZIONAL	0.9265	EUROMOBILIARE AZ INTERNAZION	0.56
EURIZON AZIONI EUROPA	0.9186	ANIMA GEO ITALIA-A	0.56
ARCA AZIONI ITALIA	0.9178	ALLIANZ AZIONI EUROPA-L	0.55
ANIMA EUROPA-A	0.9061	EURIZON AZIONI INTERNAZIONAL	0.54
ANIMA VALORE GLOBALE-A	0.8980	GESTIELLE OBIETT INTERNAZ-A	0.52
UBI PRAMERICA AZI MERC EMERG	0.8820	ANIMA EMERGENTI-A	0.52
ANIMA GEO AMERICA-A	0.8785	UBI PRAMERICA AZIONI USA	0.50
ALLIANZ AZIONI EUROPA-L	0.8770	MEDIOLANUM FLESS GLOBALE-LA	0.49
GESTIELLE OBIETTIVO EUROPA-A	0.8697	ARCA AZIONI EUROPA	0.47
ANIMA EMERGENTI-A	0.8512	ANIMA GEO ASIA-A	0.47
ANIMA GEO EUROPA-A	0.8506	ANIMA GEO PAESI EMERGENTI-A	0.47
UBI PRAMERICA AZIONI USA	0.8478	MEDIOLANUM FLS FUTURO IT-LA	0.43
ANIMA GEO GLOBALE-A	0.8402	UBI PRAMERICA AZI MERC EMERG	0.41
UBI PRAMERICA AZIONI EURO	0.8297	GESTIELLE OBIETTIVO EUROPA-A	0.41
ANIMA GEO ASIA-A	0.8127	ANIMA EUROPA-A	0.37
ANIMA GEO PAESI EMERGENTI-A	0.7808	ANIMA GEO EUROPA-A	0.34

MEDIOLANUM FLS FUTURO IT-LA	0.7736	EURIZON AZIONI EUROPA	0.31
ANIMA GEO ITALIA-A	0.7545	UBI PRAMERICA AZIONI EURO	0.30

Table 4.6: Fixed Income DEA Efficiency Results 1Y

Fund Name	Score	Fund Name	SR 1Y
GESTIELLE OBBLIGAZION CORP-A	1	GESTIELLE OBBLIGAZION CORP-A	3.87
ANIMA FIX HIGH YIELD-A	1	ANIMA OBBLIGAZIONARIO HI Y-A	3.54
ANIMA RISPARMIO-AD	1	ANIMA FIX HIGH YIELD-A	3.53
MEDIOLANUM FLESS VALORE AT-L	1	ANIMA FIX IMPRESE-A	3.31
ARCA BOND PAESI EMERGENTI	1	ANIMA OBBLIGAZIONARIO CRP-A	2.99
ARCA CED 2019 OBBLIG ATT V-P	1	ANIMA RISPARMIO-AD	2.70
ACOMEA BREVE TERMINE-A1	1	MEDIOLANUM FLESS VALORE AT-L	2.63
ANIMA FIX IMPRESE-A	0.9422	ARCA BOND PAESI EMERGENTI	2.62
ANIMA OBBLIGAZIONARIO HI Y-A	0.9179	UBI PRAMERICA OB GLOB ALT RN	2.37
ANIMA OBBLIGAZIONARIO CRP-A	0.9001	EURIZON OBBLIGAZ E HIGH YLD	2.29
BANCOPOSTA OBBL EURO M-L TER	0.8019	ARCA OBBLIGAZIONI EUROPA	2.12
EURIZON OBBLIGAZ E HIGH YLD	0.7846	ANIMA FIX OBBLIGAZION MLT-A	1.51
UBI PRAMERICA OB GLOB ALT RN	0.7592	EURIZON SOLUZIONE 10	1.47
EURIZON DIVERSIFICATO ETICO	0.7458	BANCOPOSTA MIX 2-A	1.42
ARCA OBBLIGAZIONI EUROPA	0.7252	BANCOPOSTA OBBL EURO M-L TER	1.42
ANIMA FIX OBBLIGAZION MLT-A	0.6998	EURIZON OBBLIGAZIONI EURO	1.41
ARCA RR DIVERSIFIED BOND	0.6925	ARCA CED 2019 OBBLIG ATT V-P	1.38
EURIZON SOLUZIONE 10	0.6834	ANIMA VISCONTEO-A	1.28
BANCOPOSTA MIX 1-A	0.6799	ACOMEA BREVE TERMINE-A1	1.27
ARCA BOND CORPORATE	0.6648	UBI PRAMERICA OB GLOBAL CORP	1.27
UBI PRAMERICA OB GLOBAL CORP	0.6625	ARCA BOND CORPORATE	1.21
BANCOPOSTA MIX 2-A	0.6029	EURIZON DIVERSIFICATO ETICO	1.13
ETICA OBBLIGAZION MISTO-R	0.5973	UBI PRAMERICA EURO CORPORATE	1.08
ANIMA SFORZESCO-A	0.5889	ARCA TE	0.88
UBI PRAMERICA EURO CORPORATE	0.5869	BANCOPOSTA MIX 1-A	0.82
ANIMA VISCONTEO-A	0.5473	EUROMOBILIARE FLESS ALL GLOB	0.74
EURIZON OBBLIGAZIONI EURO	0.4668	ETICA OBBLIGAZION MISTO-R	0.69
ARCA TE	0.3908	ANIMA SFORZESCO-A	0.33
EUROMOBILIARE FLESS ALL GLOB	0.3496	ARCA RR DIVERSIFIED BOND	0.01

Table 4.7: Fixed Income DEA Efficiency Results 3Y

Fund Name	Score	Fund Name	SR 3Y
ANIMA FIX HIGH YIELD-A	1	ANIMA FIX HIGH YIELD-A	1.33
ANIMA OBBLIGAZIONARIO HI Y-A	1	ANIMA OBBLIGAZIONARIO HI Y-A	1.33

GESTIELLE OBBLIGAZION CORP-A	1	GESTIELLE OBBLIGAZION CORP-A	1.12
ARCA BOND PAESI EMERGENTI	1	UBI PRAMERICA OB GLOB ALT RN	1.08
ACOMEA BREVE TERMINE-A1	1	ARCA BOND PAESI EMERGENTI	0.89
ANIMA FIX OBBLIGAZION BT-A	1	EURIZON OBBLIGAZ E HIGH YLD	0.82
BANCOPOSTA OBBL EURO M-L TER	1	ACOMEA BREVE TERMINE-A1	0.81
EURIZON DIVERSIFICATO ETICO	0.9939	ANIMA FIX IMPRESE-A	0.74
EURIZON OBBLIGAZ E HIGH YLD	0.9698	ANIMA OBBLIGAZIONARIO CRP-A	0.72
UBI PRAMERICA OB GLOB ALT RN	0.9532	UBI PRAMERICA MULTIA ITALIA	0.70
ARCA BOND CORPORATE	0.9002	ANIMA VISCONTEO-A	0.49
ARCA RR DIVERSIFIED BOND	0.8944	EURIZON DIVERSIFICATO ETICO	0.48
EUROMOBILIARE EURO AGGREGATE	0.8629	ARCA OBBLIGAZIONI EUROPA	0.47
ANIMA OBBLIGAZIONARIO CRP-A	0.8469	UBI PRAMERICA GLB HI YD EU H	0.46
ETICA OBBLIGAZION MISTO-R	0.8442	ARCA TE	0.44
ANIMA FIX IMPRESE-A	0.8272	ARCA BOND CORPORATE	0.41
ALLIANZ REDDITO EURO-L	0.7887	UBI PRAMERICA EURO CORPORATE	0.41
ANIMA SFORZESCO-A	0.7881	ANIMA SFORZESCO-A	0.37
EURIZON OBBLIGAZIONI EURO	0.7877	UBI PRAMERICA OB GLOBAL CORP	0.35
ARCA OBBLIGAZIONI EUROPA	0.7605	AMUNDI OBBLIG GLOB HY DIS-A	0.30
ANIMA VISCONTEO-A	0.7474	ETICA OBBLIGAZION MISTO-R	0.27
UBI PRAMERICA OB GLOBAL CORP	0.7380	AMUNDI OBBL PIU A DIST-A	0.23
UBI PRAMERICA GLB HI YD EU H	0.7129	ANIMA FIX OBBLIGAZION MLT-A	0.22
UBI PRAMERICA EURO MED/LUNGO	0.6896	ANIMA FIX OBBLIGAZION BT-A	0.21
UBI PRAMERICA EURO CORPORATE	0.6876	BANCOPOSTA MIX 2-A	0.20
BANCOPOSTA MIX 1-A	0.6859	EUROMOBILIARE FLESS ALL GLOB	0.17
ANIMA FIX OBBLIGAZION MLT-A	0.6762	UBI PRAMERICA PORT MODERATO	0.16
ARCA TE	0.6685	EURIZON OBBLIGAZIONI EURO	0.15
ANIMA RENDIMENTO ASSOL OBB-A	0.6651	EUROMOBILIARE EURO AGGREGATE	0.13
UBI PRAMERICA PORT PRUDENTE	0.6483	EURIZON SOLUZIONE 10	0.12
AMUNDI OBBL PIU A DIST-A	0.6375	BANCOPOSTA MIX 1-A	0.11
BANCOPOSTA MIX 2-A	0.6076	BANCOPOSTA OBBL EURO M-L TER	0.07
UBI PRAMERICA PORT MODERATO	0.5878	ARCA RR DIVERSIFIED BOND	0.05
ARCA BOND GLOBALE	0.5770	ANIMA RENDIMENTO ASSOL OBB-A	0.02
EURIZON SOLUZIONE 10	0.5694	UBI PRAMERICA EURO MED/LUNGO	0.00
ANIMA FIX OBBLIG GLOBALE-A	0.5102	UBI PRAMERICA PORT PRUDENTE	0.00
ANIMA PIANETA-A	0.5016	ALLIANZ REDDITO EURO-L	-0.01
EUROMOBILIARE FLESS ALL GLOB	0.4493	ARCA BOND GLOBALE	-0.13
UBI PRAMERICA MULTIA ITALIA	0.4475	ANIMA FIX OBBLIG GLOBALE-A	-0.32
AMUNDI OBBLIG GLOB HY DIS-A	0.4342	ANIMA PIANETA-A	-0.33

Table 4.8: Mixed Allocation DEA Efficiency Results 1Y

Fund Name	Score	Fund Name	SR 1Y
INVESTITORI FLESSIBILE	1	INVESTITORI FLESSIBILE	3.94
EURIZON CEDOLA ATT APR 2020	1	EURIZON CED ATT PIU OTT 2019	3.52

EURIZON CEDOLA ATT DIC 2019	1	EURIZON CED ATT PIU DIC 2019	3.48
EURIZON CEDOLA ATT OTT 2019	1	EURIZON CED ATT PIU APR 2020	3.47
ARCA STRAT GLOBALE CRESCITA	1	EURIZON CED ATT PIU LUG 2019	3.43
ARCA BB	1	EURIZON CED ATT PIU MAG 2019	3.40
MEDIOLANUM FLESS SVIL ITAL-L	1	EURIZON CEDOLA ATT APR 2020	3.40
AMUNDI TARGET CONTROLLO-A	1	EURIZON CEDOLA ATT DIC 2019	3.33
EURIZON CED ATT PIU OTT 2019	0.9984	EURIZON CEDOLA ATT OTT 2019	3.31
GESTIELLE CEDOLA MULTIASSET	0.9937	EURIZON CEDOLA ATT 7/2019	3.25
EURIZON CED ATT PIU DIC 2019	0.9692	EURIZON MULTIA REDD OTT 2019	2.98
EURIZON CED ATT PIU APR 2020	0.9533	ARCA STRAT GLOBALE CRESCITA	2.89
BCC CRESCITA BILANCIATO	0.9514	ARCA STRATEGIA GLOBALE OPPOR	2.88
EURIZON CED ATT PIU LUG 2019	0.9242	EURIZON MA REDDITO APR 2020	2.81
EURIZON CEDOLA ATT 7/2019	0.9123	BCC CRESCITA BILANCIATO	2.64
EURIZON CED ATT PIU MAG 2019	0.9052	ANIMA ALTO POTENZIALE-A	2.58
EURIZON GEST ATT DIN 4/2020	0.8840	EURIZON GEST ATT DIN 4/2020	2.57
SYMPHONIA PATRIMONIO REDDITO	0.8753	ARCA BB	2.21
ARCA STRATEGIA GLOBALE OPPOR	0.8531	MEDIOLANUM FLESS SVIL ITAL-L	2.20
EURIZON MA REDDITO APR 2020	0.7560	ANIMA STAR EUROPA ALTO POT-A	1.94
EURIZON MULTIA REDD OTT 2019	0.7458	EURIZON BILANCIATO EURO MMGR	1.65
ETICA BILANCIATO-R	0.7362	AMUNDI TARGET CONTROLLO-A	1.62
UNICREDIT SOLUZIONE 40-A	0.7142	SYMPHONIA PATRIMONIO REDDITO	1.62
EURIZON GES ATT CLASS 4/2020	0.7056	ALLIANZ GLOBAL STRATEGY 70-L	1.44
ANIMA STAR EUROPA ALTO POT-A	0.7052	UNICREDIT SOLUZIONE 40-A	1.35
GESTIELLE ABSOLUTE RETURN	0.6988	EURIZON GES ATT CLASS 4/2020	1.32
EURIZON PROFILO FLESS EQLBR	0.6982	EURIZON PROFILO FLESS EQLBR	1.29
ANIMA ALTO POTENZIALE-A	0.6749	ANIMA STAR ITALIA ALTO POT-A	1.25
EURIZON BILANCIATO EURO MMGR	0.6575	GESTIELLE ABSOLUTE RETURN	1.21
ALLIANZ GLOBAL STRATEGY 70-L	0.5990	ETICA BILANCIATO-R	1.17
UBI PRAMERICA PORT DINAMICO	0.5303	GESTIELLE CEDOLA MULTIASSET	1.09
ANIMA STAR ITALIA ALTO POT-A	0.4647	UBI PRAMERICA PORT DINAMICO	0.66

Table 4.9: Mixed Allocation DEA Efficiency Results 3Y

Fund Name	Score	Fund Name	SR 3Y
INVESTITORI FLESSIBILE	1	INVESTITORI FLESSIBILE	0.84
ARCA STRAT GLOBALE CRESCITA	1	ARCA STRAT GLOBALE CRESCITA	0.78
BCC CRESCITA BILANCIATO	1	ARCA STRATEGIA GLOBALE OPPOR	0.68
ETICA BILANCIATO-R	1	BCC CRESCITA BILANCIATO	0.65
GESTIELLE CEDOLA MULTIA II	1	MEDIOLANUM FLESS SVIL ITAL-L	0.64
EURIZON PROFILO FLESS EQLBR	1	ETICA BILANCIATO-R	0.54
GESTIELLE CEDOLA MULTIASSET	0.9571	GESTIELLE ABSOLUTE RETURN	0.50
MEDIOLANUM FLESS SVIL ITAL-L	0.9067	GESTIELLE CEDOLA MULTIASSET	0.48
ARCA BB	0.8772	ALLIANZ GLOBAL STRATEGY 70-L	0.48
GESTIELLE ABSOLUTE RETURN	0.8702	ARCA BB	0.40

SYMPHONIA PATRIMONIO REDDITO	0.8451	UBI PRAMERICA PORT DINAMICO	0.37
UBI PRAMERICA PORT DINAMICO	0.8361	EURIZON BILANCIATO EURO MMGR	0.36
MEDIOLANUM FLESSIBLE STRAT-L	0.8208	MEDIOLANUM FLESSIBLE STRAT-L	0.35
UNICREDIT SOLUZIONE 40-A	0.7888	EURIZON MULTIA REDD OTT 2019	0.35
ANIMA STAR EUROPA ALTO POT-A	0.7638	SYMPHONIA PATRIMONIO REDDITO	0.32
EURIZON BILANCIATO EURO MMGR	0.7203	GESTIELLE CEDOLA MULTIA II	0.31
ARCA STRATEGIA GLOBALE OPPOR	0.7088	ANIMA ALTO POTENZIALE-A	0.30
ALLIANZ GLOBAL STRATEGY 70-L	0.7017	ANIMA STAR EUROPA ALTO POT-A	0.27
EURIZON MULTIA REDD OTT 2019	0.6855	EURIZON PROFILO FLESS EQLBR	0.17
ANIMA STAR ITALIA ALTO POT-A	0.5602	UNICREDIT SOLUZIONE 40-A	0.10
ANIMA ALTO POTENZIALE-A	0.5492	ANIMA STAR ITALIA ALTO POT-A	0.03

To confirm the validity of the DEA results with the well-known risk-adjusted metrics, such as Sharpe and Sortino ratios, it has been computed the correlation between them. Looking at the correlation with Sharpe ratio, it shows positive correlation, with, on average, higher correlation for the 1-year analysis, especially if compares the values of "AllFunds" correlation, with strong positive correlation for 1-year against almost weak positive correlation for 3-years. However, except for fixed income 1-year, all the other are moderate values of correlation. Whereas, almost the same results emerging from the computation of DEA and Sortino ratio correlation, for what concern the correlation with funds' size, represented by asset under management (AUM) values, it shows negative or almost null correlation: this means that we did not find relationships between funds' performance and size.

CORRELATION DEA-SHARPE RATIO (1Y)	
AllFunds	0.696583
Equity	0.400793
Fix Income	0.708831
Mix Alloc	0.652912

CORRELATION DEA-SHARPE RATIO (3Y)	
AllFund	0.377842
Equity	0.465376
Fix Income	0.591177
Mix Alloc	0.481533

CORRELATION DEA-SORTINO 1Y	
AllFunds	0.606949
Equity	0.314277
Fix Income	0.728396
Mix Alloc	0.500134

CORRELATION DEA-AUM 1Y	
AllFunds	0.088571
Equity	-0.288782
Fix Income	-0.220198
Mix Alloc	0.291363

This empirical analysis demonstrates, once again, that DEA is an appropriate technique for evaluate mutual funds' performance, more flexible and versatile if compared with the traditional risk-adjusted measures, for many reasons as already stated, such as the possibility to freely choose inputs and outputs to apply in the model. Another peculiarity of DEA approach consists in the fact that inputs and outputs may have heterogeneous units of analysis, allowing the application of this method even when there are scarcity of information.

Conclusion

In this thesis we have gone through a detailed analysis of every aspect of investment funds. Starting from mere definitions and classifications, we presented the principal traditional tools an investor can use to assess mutual funds' performances. Furthermore, introducing Data Envelopment Analysis, we provided a powerful instrument through which is possible to make more detailed and user-adjustable analysis about funds' efficiency. To conclude the analysis, it can be said that DEA has many advantages and some drawbacks. First of all, it allows to consider several inputs and outputs, without the need for them to have homogenous unit of analysis. This feature permits to analyze performance from different perspective, focusing on various variables, and to apply DEA even when there is lack of data.

Through the Murthi, Choe and Desai DEA model, we have seen how this powerful tool allows to include the cost of owning a mutual fund share, in the form of expense ratio and load charges, as input variable in addition to risk and return. It has been possible to compute the efficiency for a sample of Italian investment funds, drawing up in this way a rank of performance efficiency, which is compared with results based on traditional risk-adjusted metrics, such as Sharpe and Sortino ratios, in order to evaluate the integrity of DEA model.

The evaluation and comparison processes have been directed towards an empirical assessment of transaction costs' incidence over the total performance of mutual funds, incidence that is not embedded in most used, and well-known, risk-adjusted measures. Results have confirmed the validity of DEA technique as tool for measuring performance of investment funds, highlighting that transaction's costs on average don't modify the efficiency judgement expressed by traditional measures.

Finally, we have tested the correlation between DEA results and traditional performance gauges, such as Sharpe and Sortino ratios, finding a positive correlation, confirming the validity of DEA approach, characterized by unique features that makes it flexible and versatile.

Bibliography

- Anderson S., J. Born, O. Schnusenberg, (2009), “Closed-end Funds, Exchange-Traded Funds and Hedge Funds: Origin, Functions and Literature”.

- Banker R.D., R.F. Charnes, W.W. Cooper, (1984), “Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis”, Management Science vol 30 no. 9, 1078-1092.
- Basso A., S. Funari, (2001), “A data envelopment analysis approach to measure the mutual fund performance”, European Journal of Operational Research.
- Bodie B., A. Kane, A.J. Marcus, (2011), “Investments and Portfolio Management: Global Edition”, McGraw-Hill, 9th Edition.
- Charnes A., W.W. Cooper, E. Rhodes, (1978), “Measuring the Efficiency of Decision-Making Units”, European Journal of Operational Research.
- Charnes A., W.W. Cooper, B. Golany, L. Seiford, J. Stutz, (1985), “Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions”, Journal of Econometrics vol. 30, 91-107.
- Chegwidden B., J. Thomas, S. Davidoff, (2013), “Investment Funds in United States: Regulatory Overview”, Practical Law Company.
- Cooper W.W., L.M. Seiford, J. Zhu, (2011), “Handbook on Data Envelopment Analysis”, Springer, Boston.
- Cooper W.W., L. M. Seiford, K. Tone, (2005), “Introduction to Data Envelopment Analysis and its uses: with DEA-Solver software and references”, Springer-Verlag.
- Corrigan A., P.C. Kaufman, (1987), “Understanding Money Market Funds”.

- Emrouznejad A., Parker B.R., Tavares G., (2008), “Evaluation of research in efficiency and productivity: a survey and analysis of the first 30 years of scholarly literature in DEA”. Socio-Economic Planning Sciences.
- Fama Eugene F., French Kenneth R., (1993), "Common Risk Factors in the Returns on Stocks and Bonds", Journal of Financial Economics, February.
- Fama E. F., French K. R., “The Capital Asset Pricing Model: Theory and Evidence”, Journal of Economic Perspectives, Volume 18.

- Farrell M. J., (1957), "The Measurement of Productive Efficiency", Journal of Royal Statistical Society.
- Fox W. P., (2017), "Mathematical Modeling for Business Analytics", CRC Press.
- Gregariou G. N., Sedzro K., Zhu J. (2004), "Hedge fund performance appraisal using data envelopment analysis", European Journal of Operational Research.
- Gregoriou G. N. (2003), "Performance Appraisal of Funds of Hedge Funds Using Data Envelopment Analysis", Working Paper n. 5.
- Gregoriou G. N., Sedzro K., Zhu J., (2005), "Hedge Fund Performance Appraisal Using Data Envelopment Analysis".
- Grinblatt M., Titman S., (1993), "Performance Measurement without benchmarks: An Examination of Mutual Fund Returns", The Journal of Business, vol.66, issue 1, 47-68.
- Jensen M.C., (1968), "The Performance of Mutual Funds in the Period 1945-1964". *Journal of Finance*, 23, 389-416.
- Johns N., Howcroft B., Drake L., (197), "The use of data envelopment analysis to monitor hotel productivity", Progress in Tourism and Hospitality Research.
- Markowitz H. M., (1991), "Foundation of Portfolio Theory", Journal of Finance, Volume 46, Issue 2, 469-477
- Markowitz H.M., (1952) "Portfolio Selection", Journal of Finance, 7, 77-91.
- Moloney N., (2015), "EU Securities and Financial Markets Regulation", Oxford University Press.
- Morey M. R., Morey R. C., (1998), "Mutual fund performance appraisals: a multi-horizon perspective with endogenous benchmarking", Omega, The International Journal of Management Science vol. 27.
- Murthi B. P. S., Choi Y. K., Desai P., (1997), "Efficiency of mutual funds and portfolio performance measurement: a non-parametric approach", European Journal of Operational Research vol. 98.

- Robertson R. A., (2001), “Fund Governance: Legal Duties of Investment Company” Directors, Law Journal Press.
- Ross S., (1976), “The arbitrage theory of capital pricing”, *Journal of Economic Theory.*
- Ruppel W., (2017), “Wiley GAAP for Governments 2017: Interpretation and Application of Generally Accepted Accounting Principle for State and Local Governments”, pp. 228-229
- Sharpe W.F., (1966), “Mutual Fund Performance”. *Journal of Business*, **1**, 119-138.
- Sortino F. A., L.N. Price, (1994), “Performance measurement in a downside risk framework”, Journal of Investing 3: 50–8.
- Steinki O., (2015), “Common Metric for Performance Evaluation: Overview of popular Performance Measurement Ratios”, Evolutiq.
- Zhao Y., Triantis K., Murray-Tuite P., Edara P., (2011), “Performance measurement of a transportation network with a downtown space reservation system: A network-DEA system, Transportation Research Part E: Logistics and Transportation Review”.
- Zhu J., (2016), “Data Envelopment Analysis: A Handbook of Empirical Studies and Applications”, Springer, pp. 229-230.

Sitography

https://www.ecb.europa.eu/ecb/legal/pdf/l_21120070811en00080029.pdf

https://www.sec.gov/investor/pubs/sec-guide-to-mutual-funds.pdf

https://www.ici.org/research/stats/worldwide/ww_q3_17

http://eimf.eu/aif_ucits_seminar/

http://us.spindices.com/indices/equity/sp-500

http://www.assogestioni.it

http://www.morningstar.com

http://www.bloomberg.com