



Ca' Foscari
University
of Venice

Master's Degree programme

in Economics and
Finance

Second cycle
(D.M. 270/2004)

Final Thesis

Bubble detection methods: A bitcoin application

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Academic Year

2016 / 2017

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Introduction

The Efficient Market Hypothesis in its semi-strong form states that asset prices always reflect all the available information, so that they cannot be mispriced. The theory has a hard time in justifying the presence of bubbles in financial markets because they consist in a divergence between the asset's price and its fundamental value. This work analyses how these exceptions to the EMH develop. Firstly, we analysed the causes that led to the formation of the well-known speculative bubbles like the Tulip Mania and the Railway Mania. Then we focused on the biggest bubble of all times, the Wall Street Crash of the '29 and see the devastating impact it had for decades on Western economies. We then studied some more recent phenomena, as the Dot-Com Bubble and the US Housing Bubble, the latter of which just ended few years ago. Throughout the work it will also be shown how bubbles may also have a positive impact on the economy in which they develop, even if the process is inefficient and disruptive. Anyway, most of the bubbles have a negative impact on an economy, hence the need to create tools that allow both regulatory authorities and investors to detect the developing of a bubble in time. The former have to quickly enact policies to try to limit the consequences of these bubbles, usually in the form of contractionary or expansionary monetary policies, while the latter have to mitigate the negative impact on their portfolios. Even if nobody can actually predict the moment when a bubble will collapse, a range of techniques has been developed in order to identify bubbles presence. We considered five tests and implemented them in their modified versions proposed by professors Hogg and Breitung in 2012. The reason for this is that those tests were not explicitly thought as tools for bubbles detection, but with some manipulation they turn out to be extremely useful for our purpose. Before using those tests we somehow had to evaluate their performances, therefore we analysed them in an artificially created

environment. In particular we generated two thousand time series with a data generating process that allowed for the presence of one or two bubbles. Once we created our artificial time series we had complete control over it and we could adjust both the initial point and the length of the bubble. In this way it has been possible to statistically calculate the performance of each test, its pros and its limits by varying the position and the length of the bubble. Once we knew how the tests react to different scenarios we applied them to three time series of interest, namely the Case-Shiller US House Price Index, the Bitcoin's and Ethereum's price. The US House Price Index was used as a benchmark to further try the tests on some real data on which we know that many bubbles occurred. Finally, we tested the Bitcoin's and Ethereum's prices for the presence of bubbles and we discovered that they had significant bubbles in their price history and that both of them were in a bubble phase at 31st December 2017. Given the very high reliability of at least some of these tests, one might be driven to think that it is fairly easy to profit from a bubble burst by shorting the related asset, however this is anything but easy because, as a quote commonly attributed to John Maynard Keynes remembers us: "The market can remain irrational longer than you can remain solvent".

Chapter one. Cryptocurrencies

1.1 Bitcoin

Bitcoin is a Cryptocurrency that was first introduced by Satoshi Nakamoto in a 2008 paper called “Bitcoin: A Peer-to-Peer Electronic Cash System”. Satoshi Nakamoto is only a pseudonym, with which the creator or group of creators provided the first version of the code. Nakamoto followed the development of the bitcoin’s technology until mid-2010. At that point he transferred the source code and the network alert key to the software developer Gavin Andresen making him the de facto first reference person for bitcoins, and transferred the property of the websites he owned to other bitcoin’s community members. It has been estimated that the bitcoins in Satoshi’s accounts are about one million¹. If that is really the case the person behind Nakamoto’s pseudonym could be one of the richest person in the world, and his virtual wealth on 17th December 2017 was about 19 billions.

Nakamoto claims to be a 37-year-old Japanese citizen, but it has been noted that his use of English is perfect and that no Japanese documentation ever follows his codes. Even though many attempts have been made to identify the real person or group of persons behind the pseudonym Satoshi Nakamoto, it is still unclear who that person is, despite many researches have been conducted.

Bitcoin’s purpose is to allow online payments without the need of a financial institution as intermediary. Traditional electronic payments require that both the buyer and the seller trust a third party to transfer the money from one bank account to the other. In this way nobody can spend its money more than once, because the intermediary will never transfer the same money twice or more money than what’s available. In the case of the bitcoins however, there’s no third party that guarantees

¹ <https://www.coindesk.com/dangerous-satoshi-nakamoto/>

the transaction; instead there's a pool of computers (a P2P network) that oversee all the transactions at the same time and write every one of them in the blockchain. The block chain is the first successful way to solve the double spending issue without a third party.

The first bitcoin was created on 3rd January 2009, and the initial price was negotiated by developers in the bitcointalk forum. The first official bitcoin price was set by NewLibertyStandard at the exchange rate of 1,309.03 BTC per US dollar, a price that was based on the electricity power needed to mine bitcoins. The first official transaction was made by the same NewLibertyStandard that paid 5.04 USD per 5,050 BTC. However the most famous transaction was the purchase of two pizzas for the remarkable price of 10,000 BTC² in May 2010.

Following the release of the Bitcoin version 0.3, an increasing number of people became interested in the project and the first bitcoin exchange, Mt. Gox, was opened in July 2010. After further bug fixes in the code, bitcoin finally reached the 1 USD threshold in February 2011, but the first real explosion in price didn't occur until June 2011. In that month a paper was published entitled: "The underground website where you can buy any drug imaginable". That website was called Silk Road, and it actually allowed the purchase of illegal products in an anonymous way. The important thing is that this paper made people realize that bitcoin had a useful value and triggered a sharp rise in its value, that reached 29.26 USD 8 days later. Later that year price stabilized in the 10-15 USD area and remained relatively stable until 2013, when many events that we'll later describe occurred.

² Wallace, B. (November 2011). *The Rise and Fall of Bitcoin*.

1.2 The blockchain

The blockchain is a publicly available digital book that contains every bitcoin transaction ever occurred from inception. As Nakamoto explains in its paper “Bitcoin: A Peer-to-Peer Electronic Cash System”, the only way to avoid double spending without a centralized party is that: “transactions must be publicly announced, and we need a system for participants to agree on a single history of the order in which they were received”, hence the blockchain. The blockchain is coded by each node of the network, when a node completes the coding process, all the other nodes verify if that coding is legitimate or not by comparing the transactions that it contains to the transactions that actually occurred. So as long as the majority of the network’s computational power is not controlled by hackers, the system cannot be manipulated. The transactions history can only be changed if the majority of the network agree to implement new rules; this event is called a hard fork. When blockchain developers disagree on the rules governing the blockchain, a voting takes place. Every node of the network can vote based on the computational power he can produce. Voting is usually involved when a significant amount of the cryptocurrency of interest has been stolen or when new features are proposed. After the voting took place, the majority of the nodes of the network have to implement the newer version of the protocol which in the case of a theft basically consists in deleting all the blocks that were completed after the theft. Once the network’s majority of computational power starts using the new protocol two blockchain exists at the same time, as long as all the old blockchain’s users switch to the new protocol. However in some cases the hard-forked blockchains were not abandoned; this is for example the case of Bitcoin Cash and Bitcoin Gold.

Bitcoin Cash hard fork occurred on 20th Jul 2017 and its purpose was to increase the number of transactions that could be recorded in a single block and to accelerate the

verification process. The block size was therefore increased from 1 MB to 8 MB and the difficulty to encrypt each block could be adjusted according to the computational power available to the network, in this way the amount of time required for each transaction should always be constant.

Bitcoin Gold hard fork occurred on 24th Oct 2017 and its purpose was to restore the profitability of mining with a pc's Graphics Card. Bitcoin's founder Satoshi Nakamoto tried to create a decentralized and democratic system, where every CPU should account for one vote only, but nowadays the mining activity is done with huge investments in specialized equipment, therefore there's a great voting power concentration that creates disequilibrium within the network. By making it possible to mine only through a GPU, Bitcoin Gold is an attempt to decentralize the network as much as possible.

For the system to work, the blockchain must be public therefore all the transactions are available for everyone to inspect. The only privacy that's available is that the wallets in the blockchain do not necessarily have to be linkable to an individual person. By inspecting the blockchain the observer would only see that wallet A has transferred bitcoins to wallet B, without knowing who did that operation. However this does not mean that using bitcoins provides an absolute privacy; there are actually many ways to discover who's the owner of a wallet. The first one is to track the purchase of bitcoins via other electronic payments, for example credit/debit cards or bank transfer. Then a wallet identity can be tracked by discovering the identities of the owners of other wallets that interacted with the first wallet. A third way is to track the exchange of bitcoins for traditional currency back to a bank

account. Bitcoins do not offer such a high level of privacy unless a number of strategies are adopted like:

- Physical transfer of a storage device.
- Using a new wallet for every transaction and never interact with other owned wallets.
- Using mixing services whose aim is to collect bitcoins from many wallets, transfer them among many other wallets and send back other bitcoins to the user.

One of the earliest applications of bitcoin was in the digital black market. Silk Road was actually the first online platform to sell drugs, and thanks to the Tor network it could be browsed in complete anonymity by users. The website offered a variety of drugs and illegal products such as fake driver's licenses but it denied the sale of anything that was meant to "harm or defraud". Silk Road was very successful because it allowed users to rate sellers in a similar way of what Ebay does. Moreover the buyers' bitcoins needed for a transaction were kept in escrow until the order had been received so it was very convenient and reliable to buy from the website. The site grew in popularity so much that reach a 15 million USD in turnover per year. After two years of operation, in Feb 2013 an Australian became the first Silk Road-related arrested person for importing drugs. It is important to notice that neither this first arrest, nor the successive arrest of the Silk Road founder were performed because of a flaw in the security of the website or because some bitcoin transactions were identified. The arrest of Ross Ulbricht was made possible thanks to some traces that he left behind and were found by IRS investigator Gary Alford by searching with Google.

1.3 Costs

Another very relevant issue related to bitcoins is their transaction costs. The first users who started making payments with bitcoin enjoyed transaction fees that were close to zero, for example the average transaction fee on the 1st Jan 2012 was about 0.0037 USD³ therefore merchants started accepting bitcoins as a mean of payment, since the traditional credit card processors typically require a 2-3% transaction fee. However, the bitcoin network processing capacity is limited by the fact that only an average of 2,000 transactions can be stored in each block, and it take between 5 and 15 minutes to validate that block. Bitcoin users who transfer funds have the option to include a commission for being processed before other transactions, so when a lot of people want to transfer bitcoins, the commissions increase, reaching up to 37 USD per transaction on the 21st Dec 2017. Currently the Bitcoin network can process roughly 3 to 4 transactions per second while the biggest credit card company, Visa is able to handle even 24,000 transactions per seconds⁴. As we can see from figure 1, in the last months bitcoin fees have become so big that the original idea of using bitcoins as an everyday mean of payment have become extremely unpractical.

³ <https://bitcoinfoes.info/>

⁴ <https://www.visa.com/blogarchives/us/2011/01/12/visa-transactions-hit-peak-on-dec-23/index.html>

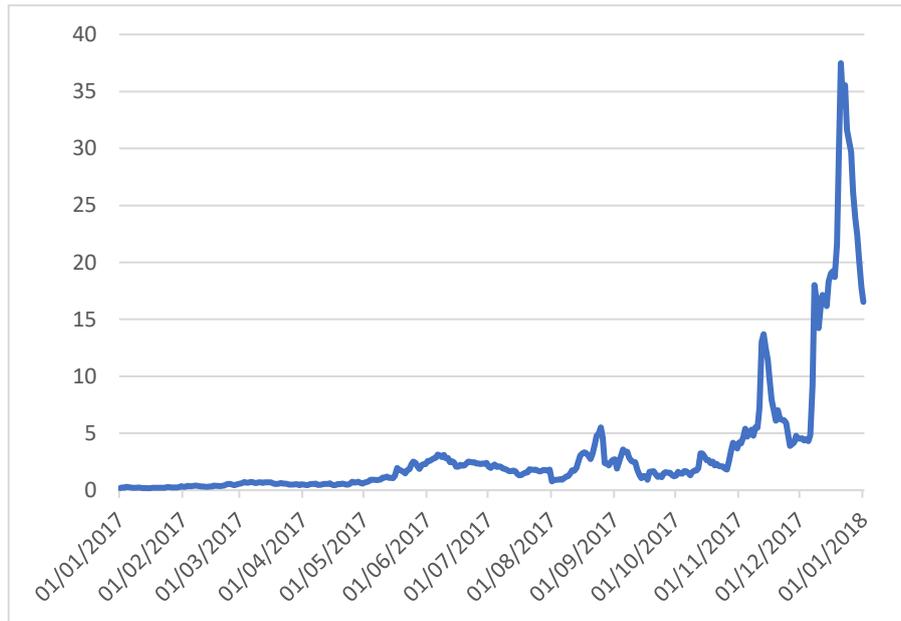


Figure 1: Last year's transaction fees. Note that fees are per transaction and are not related to the amount to be transferred.

1.4 Supply and wallets

The total Bitcoin supply is set to 21 million units. However not all those bitcoins are currently in circulation, but every time a new block is mined, the miner is compensated with 12.5 newly issued bitcoins. The compensation halves every 210,000 blocks so that it will eventually reach 0. At the point when all the bitcoins will be in circulation, the only incentive for miners to keep the network alive will be in the form of transaction fees only. Since the amount of new currency that can be created is limited, bitcoin is protected from inflation.

Bitcoins are stored in wallets that are formed by a public cryptographic key and a private key. Bitcoin funds can only be accessed from users holding both keys. There are basically two types of wallets: the full client type verifies the validity of transactions on a local copy of the blockchain (that weights approximately 150 GB for now), so when a transaction is made it is only approved if it is coherent with the

past bitcoin history. Full clients are therefore the safest and most reliable alternative.

The other type of wallet is called Lightweight client; this wallet can be run on most computers since it is much lighter and less process intensive. The downside of this simplification is that the wallet has to trust a full node to send him the block headers of the most recent block, which allows the lightweight wallet to follow the last block as reference.

A third solution for storing bitcoins is to use an online wallet service. This way of storing bitcoins does not require downloading any particular software to access the funds rather the users' wallets are kept in the provider's computers and can be accessed with a normal username and password. This type of wallets require revealing both the public and the private key to the service provider therefore users have to fully trust these companies.

1.5 Legal status

Bitcoins cannot be used in every country as a mean of payment. Most of the world's countries have not yet regulated their use either because in those countries bitcoins are unknown to the majority of the population or because their regulation is not a Government's priority. Figure 2 shows a world map of the bitcoin's legal status: green countries are the ones that have legalized the cryptocurrency, orange countries have not explicitly legalized bitcoins nor rejected them. Then countries depicted in pink are the ones that allow the circulation of bitcoins but have placed restriction on their use; for example China has banned all its financial institution

from dealing with bitcoins but allows citizens to possess that currency. Dark pink countries have explicitly criminalized bitcoins.

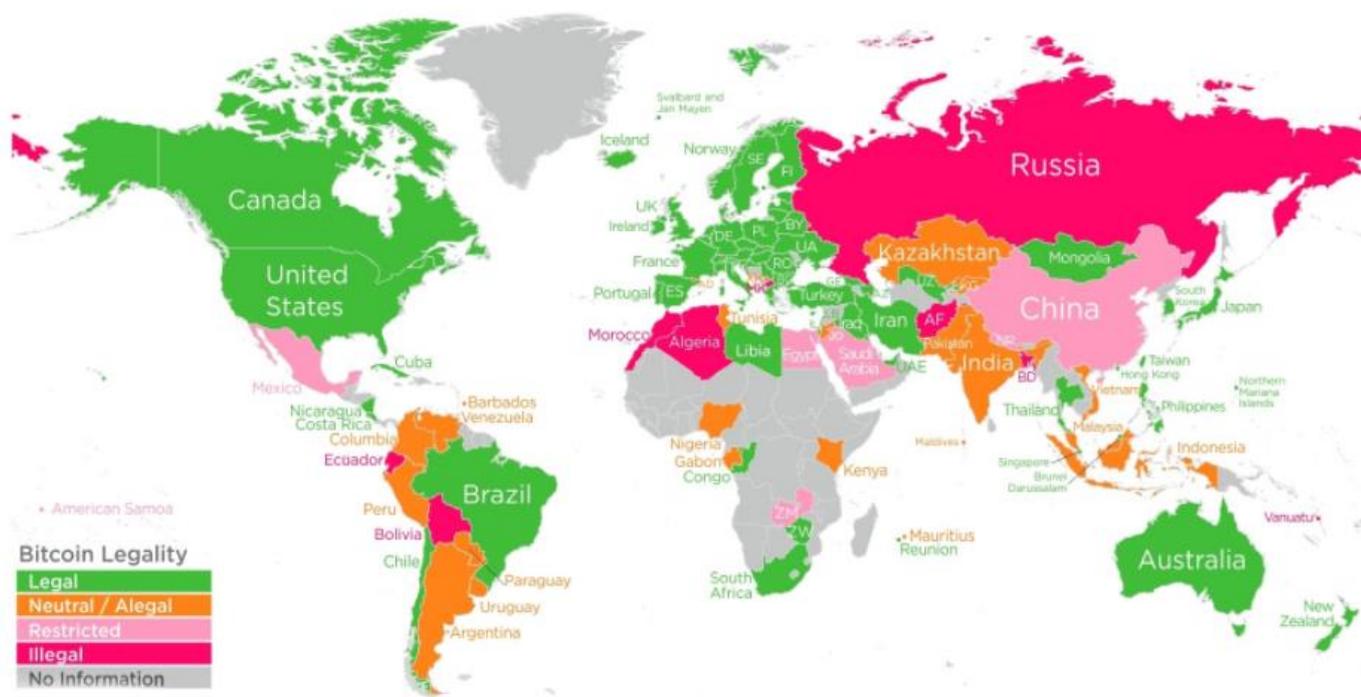


Figure 2: Bitcoins legal status around the world⁵.

As we can see there's a big difference between western and eastern countries; basically the former have legalized bitcoins while the latter have taken much diversified positions on the matter.

Most of the countries, 130 out of 246, representing the 53% of the world, have not yet taken an explicit position regarding bitcoin's regulation. Countries where bitcoin has a legal or neutral status are the 40% off the world's, 3% of the countries heavy regulates their use and 4% explicitly criminalized their use.

⁵ Source: <https://howmuch.net/articles/bitcoin-legality-around-the-world>

Source 2: <https://coin.dance/poli>

The very first topic to consider when dealing with bitcoins is whether to consider them as a currency or a commodity. From identifying them as either of the two, countries can apply their currency or commodity law to bitcoins. Unfortunately, there's no agreement on bitcoin's status, because it incorporates both characteristics typical of commodities and traits that are typical of currencies.

Figure 3 compares the characteristics of four means of payment: barter, commodities, currency and bitcoin.



Figure 3: Means of payment characteristics⁶

The first thing we can notice is that bitcoins earn a better total score than the other payment systems. Bitcoins are more easily transferable than both commodities, that require the payment of transportation cost, and currencies, both in paper or digital

⁶ Source: BTCS. <http://www.btcs.com/index.php>

form, because bitcoins can be transferred within minutes while currency require some days to be transferred internationally.

Bitcoin is the only known form of payment that is absolutely non counterfeitable, since banknotes can be falsified and commodities require in-depth inspections to be proven legitimate.

Bitcoins' supply is fixed at a given amount and their release is gradual, therefore no inflation can occur with bitcoins. On the other hand, fiat money can be created at Government's will, and commodities supply is determined by the amount that nature provides at a rate that depends on men's abilities to extract that resource. A fixed supply of currency is very attractive for investors looking for stability, but it denies the possibility of implementing monetary policies. As we'll later see, monetary policies are fundamentally important to reduce the impact of a recession, and we'll see that when no monetary policy was implemented after a bubble burst, its consequences were much deeper for the economy.

Bitcoins' peculiarities are that they are not Government-Issued and they're decentralized. Fiat currencies are not issued by Governments, because that would allow a Government to finance its expenditure by creating new currency, with negative effects for the whole economy. However central banks do not always enjoy a complete separation from political power, for example in the US the FED's governors are appointed by the President of the United States and confirmed by the Senate. Bitcoin is the only currency that enjoys complete independence from any Government.

Bitcoin's decentralization is also a unique characteristic and allows for decision to be taken in a complete democratic way, so that each user can express his preference.

Apparently, the only bitcoin issue is that the cryptocurrency is not widely accepted yet, as opposed to the traditional currencies. However, a work-around is currently available and is represented by firms that allow to have a digital wallet with virtual currencies and to convert them in the required real currency as soon as the linked credit card is swiped.

1.6 Ethereum

Ethereum is a platform that allows the exchange of smart contracts. Smart contracts are a digital protocol whose aim is to help exchanging properties, money or financial instruments in a completely irreversible and secure way. As in the case of bitcoins, Ethereum does not need a third party to verify transactions but it's based on a blockchain. The participants to the Ethereum network are compensated with Ethereum's cryptocurrency, Ether, whose aim is just to encourage miners to keep the network alive. Ethereum has a wide range of applications because it can fully exploit all the blockchain's characteristics therefore many major banks and research institutions are involved in the Enterprise Ethereum Alliance, a non-profit organization with the purpose of studying its application at enterprise level. No organization of this kind exists for Bitcoins.

Some of the main differences between Bitcoins and Ethereum are:

- Their purpose, as Bitcoins were created to facilitate money transferring while Ether was created as an incentive to allow the transferring of smart contracts.
- The number of transactions per second; even if they're both very far from the speed of the big payment processing companies, Ethereum can

process a peak of 15 transaction per second⁷, which is almost four times Bitcoin's processing capacity.

- Mining new Ether is rewarded with a constant amount of Ether while for Bitcoins the reward amount halves every four years.
- Transaction fees for Bitcoins are constant for every transaction while in Ethereum the fee changes according to the computational power, bandwidth and storage required by each transaction.

1.7 Ethereum's decentralized applications

Ethereum has a very wide range of applications; the first one could be the expression of the favourite candidate during the elections. If everybody with the right to vote is given an address on the Ethereum's network, he can express his preferred candidate in a unique and uncontroversial way. Although many are still skeptical about this possibility, major issues have already been overcome⁸.

A second application is the possibility of sending completely uncensored and untraceable messages, and that's exactly what the application EtherTweet allows users to do. Possible uses span from the possibility of enjoying freedom of expression, to the possibility of anonymously press charges against somebody we could be harmed by.

The Raiden Network aims at making Ethereum ideal for micropayments solving the scalability, speed and transactions' cost issues. What this application does is to allow off-chain transactions and to only settle net payments at user discretion, so the only

⁷ Author's estimate based on data from: <https://etherscan.io/chart/tx>

⁸ <https://www.coindesk.com/voting-scheme-ethereum-doesnt-give-away-vote/>

time a fee is paid is when a user transfers RDN to his Ether's account. Double spending issue has also been solved thanks to the creation of unique channels between two persons; since only two persons have the access to that channel, there's no possibility to send the currency to a third party.

Another interesting application is TenX. TenX allows users to have a multi-cryptocurrency virtual portfolio and to convert these currency to real currency every time a payment is made with your credit card. Instead of directly paying with the cryptocurrency as other payment methods, TenX allow users to normally swipe their credit card, with no need for the shop owner to accept virtual currencies; he will just receive the local currency.

Probably one of the best use of the Ethereum's network is done by the 4G-Capital company. The company provides unsecured micro credit in sub-Saharan Africa to small businesses and private individuals as well. When a client contacts the company for a loan, he is thought basic ways to improve his business and he is given a small amount of money. Once the client has been verified as credit worthy, basically when he has repaid the first loan, he is given the possibility to access a larger amount of money simply by requesting it from his phone. There's no need for another in-person contact thanks to the Ethereum's network, that can univocally transfer the money to the client's wallet. Moreover the transfer requires roughly 3 minutes, so money are immediately available. As I mentioned it's probably one of the best use of the network because it provides people with no bank creditworthiness or collaterals the ability to access credit. The system is very successful since 4G-Capital reports that 92% of clients repay on time and 80% of them return for repeated business, meaning that their businesses are growing⁹.

⁹ <http://www.4g-capital.com/>

Another curious application is the first Ethereum-based virtual game. The name's Cryptokitties and it consists in breeding cats that can later be coupled with other cats and reproduce. Each cat has its own 256 bit DNA which makes each of them having a unique aspect. The owner of a cat is uniquely identified through an Ethereum address so it's not possible to replicate the same cat once it's been assigned to a given person. Cats can of course be purchased or sold for Ether and as of 1st Feb, the most expensive cat was sold for 117,712 USD. The game was so popular that at beginning of December that it was consuming roughly 15% of the Ethereum network's processing capacity, causing a slowdown in transaction time and a rise in transaction fees¹⁰. Although there's no practical utility in raising digital cats, the first blockchain based game shows that a widespread use of the blockchain is possible.

WeiFund is another application of the Ethereum's network; the project is a crowdfunding platform based on smart contracts, where each contribution is made via Ether. The fact of using smart contracts allows for the possibility of structuring more complex deals than ordinary crowdfunding platforms. Moreover it does not rely on a third party to record who the contributors of a crowdfunding are, rather that information is accessible to everyone because the contributed amount is public.

¹⁰ <https://techcrunch.com/2017/12/03/people-have-spent-over-1m-buying-virtual-cats-on-the-ethereum-blockchain/amp/>

Chapter 2. The tests

2.1 What is a bubble?

A bubble is a situation in which an asset experiences a sharp increase in price without a proportional link to its intrinsic value. As Bodie, Kane and Marcus describe them: “Bubbles seem to arise when a rapid run-up in prices creates a widespread expectation that they will continue to rise. As more and more investors try to get in on the action, they push prices even further. Inevitably, however, the run-up stalls and the bubble ends in a crash”¹¹.

There are several concurring factors that can lead to bubble creation:

- The first one is a low interest rate regime. In this situation, easy access to credit encourages people to borrow money to invest (a practice called leveraging). In fact, one of the causes of the 2001-2006 US house price bubble was the extraordinarily low FED interest rate, which fuelled the price rise.
- The greater fool theory explains the over-optimistic views of some investors, who think that price will keep going up, therefore they buy the asset thinking that they'll be able to sell it at a higher price to a greater fool and in so doing they make the price rise even further.
- The extrapolation concept arises by the traders' practice to project past returns into the future with little criticism. This leads to overbidding on assets that had big returns in the past, driving the price away from their intrinsic values.
- The Herd behaviour arises when many investors decide to trade in the direction of the market, so they buy an asset when its price is increasing and sell it when price is declining. Since many investors are not professional, they

¹¹ Bodie, Z., Kane, A., Marcus, A. J., *Investments*, McGraw-Hill, 10th Global Edition, p.347

tend to follow the market, being it the easiest trading decision. This approach is also sometimes used by technical analysts when they use trend-following strategies.

2.1.1 Tulip Mania

Speculative bubbles have been known for a long time. One of the first recorded bubbles in history is the Tulip mania; in 1636 the price of tulips skyrocketed in Netherlands up to the point at which the value of a house was not sufficient to buy 10 tulips¹². Supply of tulips was of course limited at that time but it's still not completely clear how this mania started. What is known is that one of the first type of futures contracts was introduced, so producers could sell their production before the harvesting. Since short selling was not allowed, futures contracts could have helped fuelling the price rise. By February 1637 prices started falling sharply eventually reaching pre-bubble levels. Contrary to common belief, this bubble did not have very relevant consequences for the Dutch economy as a whole, basically because most of the trades were conducted via futures so no real money was transferred. Moreover the buyers were not forced to honour the contracts because judges thought that they had been contracted through gambling.

2.1.2 South Sea Company bubble

Another important bubble involved the South Sea Company. The company had been founded in 1711 as a public-private company with the purpose of developing trades with South America and to help reducing the English public debt. However at that

¹² <http://www.bbc.com/culture/story/20160419-tulip-mania-the-flowers-that-cost-more-than-houses>

time England was in war with Spain, that controlled South America, therefore there were no real opportunities for doing business in that continent. The company's activity was basically to trade Government's debt exploiting the founders' advanced knowledge about when national debt was to be refinanced. This insider trading activity led to large profits at the beginning that made people believe that the company was profitable. Therefore, expecting the company to have a similar success to the East India Company's, that controlled 15% of the total English imports at that time, many investors started buying its shares, pushing the price higher and higher. Share price reached almost 1,000 pounds in June 1720, from 128 pounds in January. The general public willingness to buy company's share was not the only reason of that price growth; corruption was the other reason. Shares were sold to politicians at market price but they didn't have to pay for them. At the same time those shares could be reimbursed at market price therefore what happened was that politicians could cash in the price difference by selling something they haven't paid for. This scheme had two great advantages: it secured the cooperation of politicians in favouring the company because they had a direct interest in it, and also allowed the company to be seen as a legitimate business by publicizing the names of its elite shareholders. The South Sea Company bubble led to great distress in the English economy because on one side the middle-class that had borrowed against its share, was often led with shares that were valued a fraction of what they paid for; on the other side, many members of the aristocracy fell in disgrace after the bubble burst. A Parliamentary investigation was held in 1720 and condemned the company's directors to give back an average of 82% of their wealth that was used to partially reimburse the victims. Also some members of the Cabinet were condemned, even if most of them, including King George, had a direct involvement in the company.

2.1.3 Railway Mania

Speculative bubbles usually have a negative impact on the economy of the country they develop into, but sometimes they only have a marginal effect like the Tulip mania, or result in a positive outcome at macroeconomic level; this is the case of the Railway mania. In 1825 the first railway in the world was built between Liverpool and Manchester and it proved to be a successful way to move both freight and people. The cost of building railways was extremely high, and since between 1830 and 1843 Bank of England's interest rates were in the range of 4 to 5% there weren't many people who wanted to risk their money in railways when they could just have a 4-5% return with no risk at all. The situation changed in 1843 when BoE cut interest rates to 2.5% making wealthy people look for alternative investments. In those years the few railway companies that were operating were moving an increasing number of passengers and freight that attracted the attention of many investors. In the 1840s thanks to the industrial revolution the so called middle-class started to emerge. These people were not aristocrats, rather they were small entrepreneurs who were becoming successful and rich, therefore willing to invest. The combination of the two events made a huge amount of money available for investment. Moreover at that time shareholders had to pay only 10% of the nominal share value to the company, the rest could be called in at company's discretion. Railways were thought to offer such a reliable investment that many investors bought a large number of shares even if they were only able to pay for the 10% deposit. When companies eventually called in the remaining capital, these investors lost everything and were overwhelmed by debts. In the nineteenth century, an Act of Parliament was required to establish a new railway company, but it didn't really require a sound business plan, so basically every request was approved. This lack of control led to the creation of 272 railway companies by 1846. When BoE raised

interest rates once again, professional investors stopped investing in railways and small investors started realizing that not all the companies were actually able to build a railway. As the flow of money dried up, many companies failed and most of them were liquidated for a fraction of their real value by larger competitors. The only positive impact of this mania is that it allowed the creation of 10,010 km of railways, so even if the process was very inefficient and disruptive, two-thirds of the present UK railway lines were built in that period¹³.

2.1.4 Wall Street Crash

The most famous economic bubble was the 1929 Wall Street Crash; this bubble had no positive impact and its consequences would be felt for many decades to come. The roaring twenties were a period of great economic prosperity for all western countries in which cars, telephones and radio became widespread. The end of World War I favoured a rapid economic expansion in the US with record automobile and steel production and increasing retail turnover. Standards of living were improving for all citizens and the stock market kept reaching higher levels year after year, so everybody wanted to make a profit from the stock market, believing that the market could always reach higher levels. This is the main reason why hundreds of thousands of Americans invested in stock, and the one who could not afford it, borrowed money for speculation. Dow Jones industrial average grew from 67.11 in Aug 1921 to 380.33 in Aug 1929, a 5.6 times price increase. On the 25th May 1929 a first correction in prices took place, following the FED's warning of excessive speculation. Nevertheless two days later National City Bank announced the purchase of 25 million USD in stocks to stop the market's downfall. The move proved

¹³ Today's UK Railways network is about 15,799 km

to be successful because prices resumed rising once more and the Dow Jones Industrial Average gained 23% from March to August 1929. However the crash was only delayed and on 20th September 1929 a crash on the London Stock Exchange took place following the arrest of some top traders who were found guilty of fraud. Even if it did not have a great impact on the American stock market at first, this event is often considered as the beginning of the following downturn because it spread uncertainty among American investors about the safety of their investments. At the beginning market started what it seemed like a normal retracing, however due to intense volume, the market price transmission around the country had hours of delay¹⁴, therefore the increased uncertainty led to a generalized panic selling that eventually led to a 11% drop on the market opening of 24th October. On that day, the biggest Wall Street Bankers met to try to find a solution to that generalized selling so they decided to buy a big amount of US Steel and other Blue Chip stocks. This type of intervention had already successfully been undertaken during the 1907 Panic, but this time only had a temporary effect. During the weekend the events were made available throughout the country so many investors preferred to exit their positions and this led to another 13% decline on Monday and a further 12% drop on 29th October 1929, the so called Black Tuesday. On that day a record of 16 million shares were traded, showing that a very relevant part of the investors were dumping their shares. The market reached a first bottom in mid-November, then recovered until April 1930, but then started a two-year collapse until June 1932, reaching a level of 42.82. As we can see from figure 4, the bubble burst in a faster way than it took to form, characteristic that is common to all bubbles.

¹⁴ In 1929, stock price quotes were transmitted through telegraph lines and took approximately one second per character.

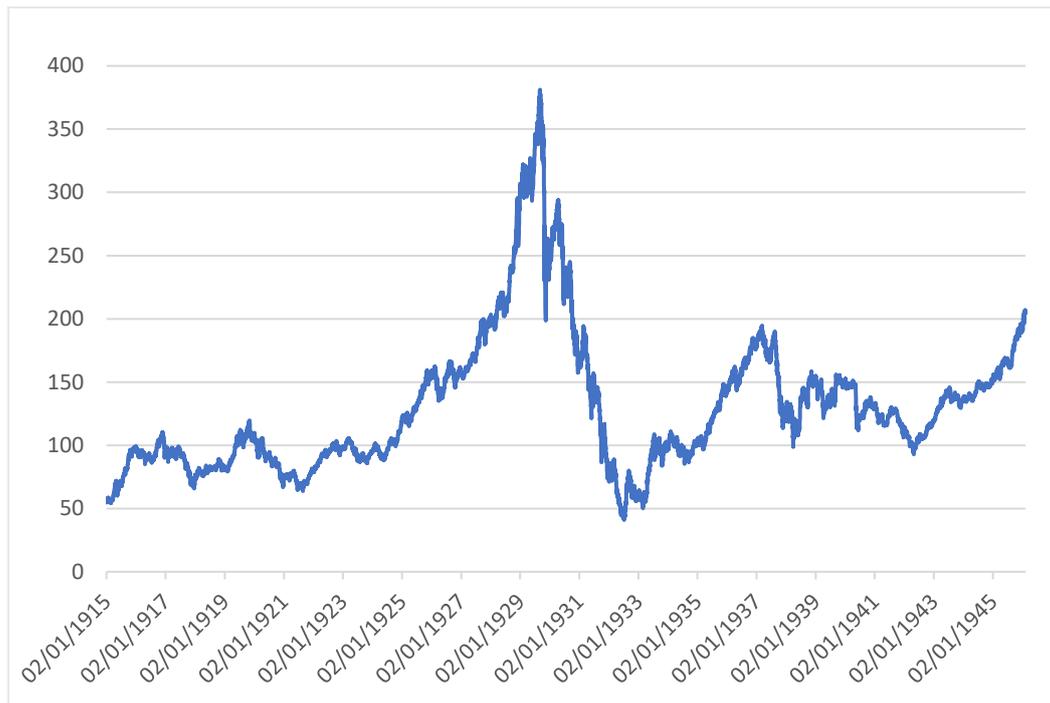


Figure 4: Dow Jones Industrial Average Index. Note the exponential growth during the twenties and the subsequent sharper fall.

The consequences of the 1929 Wall Street crash were immense and worldwide felt. Losses in the stock market led to a drop in consumer expenditure and many banks were run out of business therefore firms found it difficult to find both credit and demand for their product. Failure rate rose sharply and unemployment rose to 25% as a consequence. There has been much debate on how this bubble could have been prevented or its consequences mitigated, and for example economist Charles P. Kindleberger suggested that no lender of last resort was available in those years, whose action should have helped mitigating the negative effects of the bubble burst. The role of lender of last resort is usually given to central banks that have to provide with loans those institution that for their size have become so important to an economy that an eventual collapse costs more than the amount of money needed for their rescue. A situation where the Government acted as lender of last resort was

during the 2007 US House Price bubble. In that occasion the US Government allocated an initial amount of 700 billion dollars to the Trouble Asset Relief Program whose aim was to give enough equity to huge banks in trouble, allowing them to stay solvent and effectively rescuing them. By the end of 2014, the money that had been lent were repaid, and Government also made a profit¹⁵. If a mechanism like this was in place in 1929, larger banks could have received the money needed to remain solvent, and probably the bubble would have burst at least slower. One of the most important consequences of the 1929 crash at legislative level was the approval of the Glass-Steagall Act. The act provided for the separation between commercial and investment banks. The commercial banks purpose was to collect deposits and offer loans but they couldn't offer non-governmental securities to clients and couldn't invest in non-investment grade securities with their capital. On the other hand investment banks were authorized to underwrite and distribute stock and other securities, while prohibited from accepting deposits. The separation of competencies was thought to favour a more stable banking sector, and it did for some time, but in the '70s banks interpreted the act in a broad sense and courts did not expressly reject those interpretations so the net separation of bank mandates started narrowing. The act was eventually repealed in 1999 with the Financial Services Modernization Act and some economists claim that it favoured the development of the US House Price bubble of 2007. Even if this claim is difficult to prove, the complete separation of commercial and investment bank reintroduction was proposed for the Wall Street Reform and Consumer Protection Act in 2010 even if unsuccessfully.

The other important leftover of the 1929 crash was the introduction of the Uptick Rule. The rule says that investors cannot take a short position if the last trade was

¹⁵ <https://www.nytimes.com/2014/12/20/business/us-signals-end-of-bailouts-of-automakers-and-wall-street.html>

made at a lower price than the previous one. The purpose underlying this rule was to try to stop sudden market drops that had led to panic during the 1929 crash. The rule was introduced in 1938 and repealed in 2007 after extensive testing. However after the 2007 bubble, SEC decided to partially restore the uptick rule, that doesn't apply to every type of security and is triggered only when the security's price drop has been at least 10% in a day, and it's valid for all the next trading day.

2.1.5 The Dot-Com bubble

One of the most recent economic bubbles was the Dot-com bubble. The name refers to the companies that were involved in it, namely the high tech companies, normally referred to as "Dot-coms" because the final part of their domain's name. During the nineties, an increasing number of people became able to purchase a computer thanks to its declining price so that by 1997 an estimated 35%¹⁶ of US household owned a computer. Moreover Government was expanding the internet access and the digital divide declined as well. As people realized the big potential of the internet, they started purchasing the stocks that were related to IT, from hardware manufacturer to service providers. Money flooded the market and little attention was paid to a company's fundamental value like price to earnings or profitability ratios. All the confidence was put in future earnings that the company was expected to make so naturally a bubble developed. It's also been suggested that the Taxpayer Relief Act of 1997 favoured the developing of the bubble. The TRA provided for a capital gain tax reduction from 28% to 20%¹⁷, so that more money was available for investments and speculation. The result was that the Nasdaq Composite index, that

¹⁶ US Department of Labor (1999). Issues in labor Statistics.

¹⁷ Dai, Z., Shackelford, D.A., Zhang, H.H.(2008). *Capital Gains Taxes and Stock Return Volatility: Evidence from the Taxpayer Relief Act of 1997*

contains many IT companies, grew from 1581 in January 1997 to 5048 in March 2000. One of the best performing stock was Qualcomm, that grew 2,619% in value just in 1999¹⁸. During the bubble period IT companies could become public via Initial Public Offering and raise a considerable amount of funds even if they hadn't made profits yet because of the great confidence people had towards the IT industry. The bubble eventually collapsed in 2000 and as we can see from figure 5 the Nasdaq Composite fell of 63% within one year from its peak, but the bottom was only reached in August 2002.

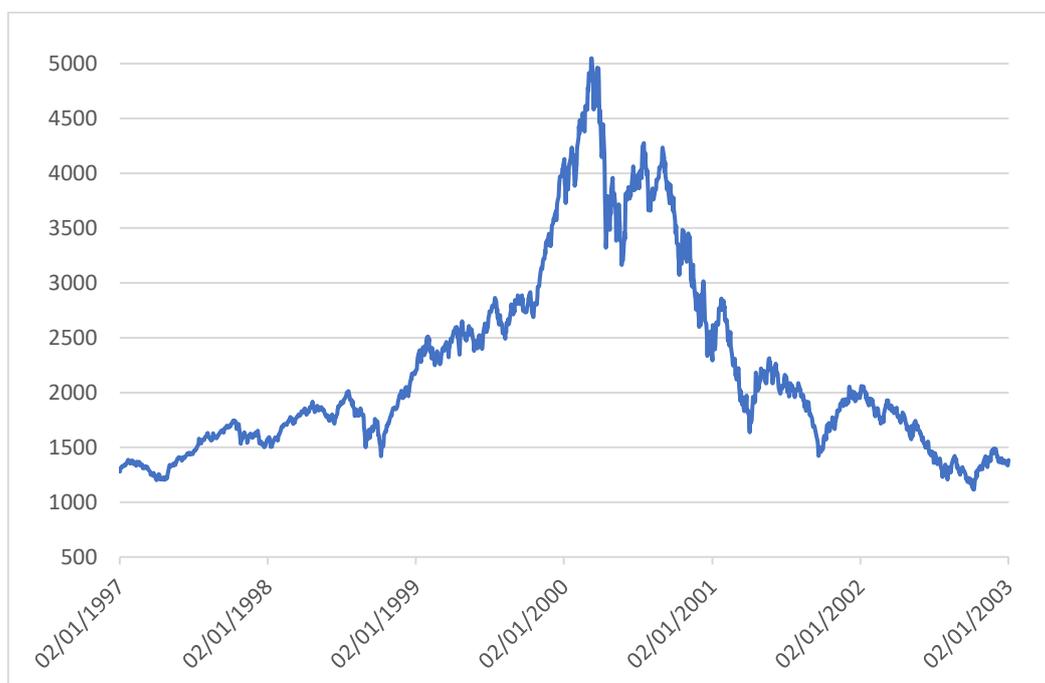


Figure 5: Nasdaq Composite Index during the Dot-Com bubble

Many high-tech companies didn't handled the money available in a careful way and were driven out of business when the flow of money dried up, so they were often liquidated by competitors. Other companies like Qualcomm, that manufactures telecommunication equipment, were able to survive the bubble even if their stock

¹⁸ Norris, F. (3rd Jan 2000). *The Year in the markets; 1999: Extraordinary Winners and More Losers*

value fell enormously¹⁹, while others like Amazon, Google and Ebay developed so much to eventually reach a dominant position in their respective industries. As in the case of the Railway Mania, also the Dot-Com bubble had a positive side effect on the economy because it gave a great impulse to the diffusion of the broadband connection. In fact, both telecommunications equipment providers and cities, heavily invested in the broadband development, which requires huge capital to be allocated. By comparison, countries that were not affected by the bubble had to develop their own programs for increasing broadband access, and Italy for example only implemented such programs starting from 2015, with a very complex Government intervention that involves private investments, European Investment Bank lending and Government contribution.

2.1.6 Initial Coin Offerings

As previously mentioned, during the Dot-Com bubble IT companies were able to successfully raise capital through IPOs even before reaching profitability because investors put all their confidence in future growth. Something similar is happening nowadays with cryptocurrencies; a lot of start-ups are raising considerable amount of capital via Initial Coin Offerings just because they are pursuing projects that deal with cryptocurrencies and since people have seen the record price achieved by bitcoins, they don't want to miss the opportunity of future profits. An ICO consists in the allocation of some tokens of the new cryptocurrency to the initial investors, who have to contribute to the project with other cryptocurrencies or fiat money. ICOs' biggest advantage is that they allow start-ups not to pay the traditional

¹⁹ Qualcomm had to face a stock price drop of 84% in two years.

crowdfunding intermediaries such as banks, venture capitalists or stock exchanges. The first ICO was held in July 2013 and the Ethereum project itself was financed by ICO and managed to collect the equivalent of 2.3 million dollars²⁰. However, as we can see from figure 6, this phenomenon remained relatively moderate until April 2017; from that month the number of ICOs exploded and eventually the total amount or funds raised through ICOs reached 2.3 billions by the end of 2017, following an exponential growth.

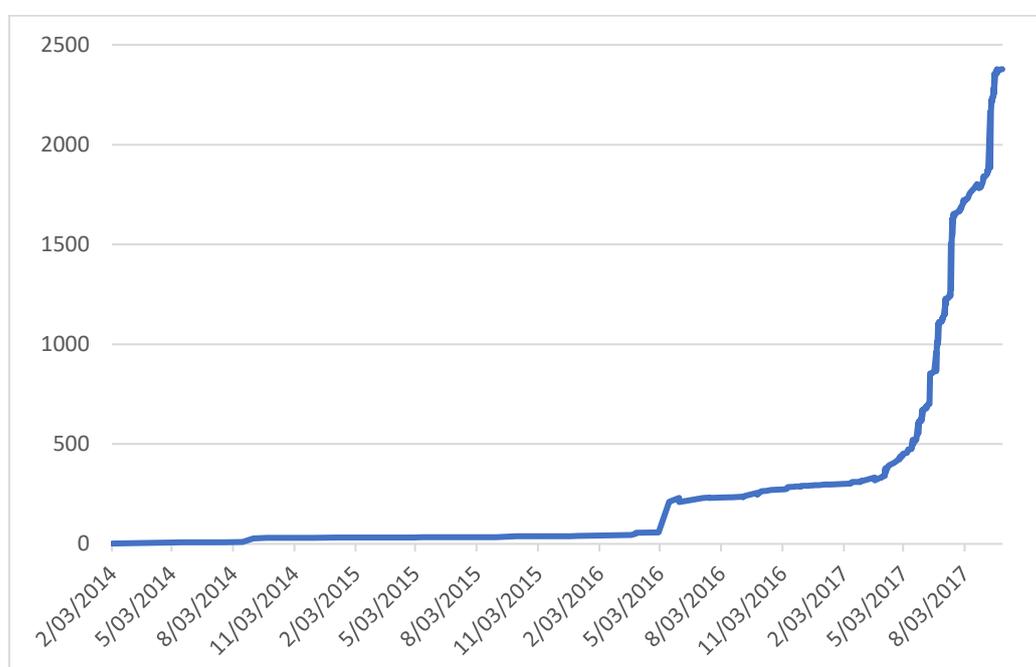


Figure 6: Amount of funds collected via Initial Coin Offerings.

It's not yet possible to distinguish which projects will be successful in the long run and which will fail but it's very likely that some will fail. The reason for that is that people who are investing in ICO are usually private individuals and not institutional investors, so they're even more likely than professionals to take investment decision

²⁰ <https://cointelegraph.com/news/ethereum-raises-3700-btc-in-first-12-hours-of-ether-presale>

based on the general trend of cryptocurrencies rather than on project's fundamentals.

2.1.7 United States Housing Bubble

The 2007 House Price Bubble originated in the US market but it affected very soon the rest of the western economies as well. Even though it did not imply such disruptive consequences as the 1929 crash, which was followed by the Great Depression, it nevertheless triggered a recession in many involved countries. A consistent number of reasons came into play in the late nineties that eventually led to a speculative bubble on houses. A first reason is the modification of an old law introduced in 1978 that provided for a one-time capital gain tax exemption of 100,000 dollars for people over 55 in case of a house sale. In 1997 with the Taxpayer Relief Act the tax-exempt amount was raised to 250,000 for a single or 500,000 for a married person. Moreover the exemption was not one-time only anymore but was made available every two years. This law created a great incentive for investments in housing, so not only families made mortgages to buy the house they would live in, but also they purchased a second or third house as an investment, because capital gains on houses were basically untaxed.

A second concurring factor was the 1992 Housing and Community Development Act, that established the percentage of Fannie Mae and Freddie Mac funds to be allocated to people with lower incomes. Initially the percentage was 30%, then it was raised to 40% and then in 1996 the percentage of funds to be allocated for people with an income under the median in the area was 42%. As we can see from figure 7, the effects of this decision became visible in 1996 because house prices started rising

after a long period of stability. Anyway in 2000 that percentage was increased to 50% and 52% in 2005 and 58% in 2008.

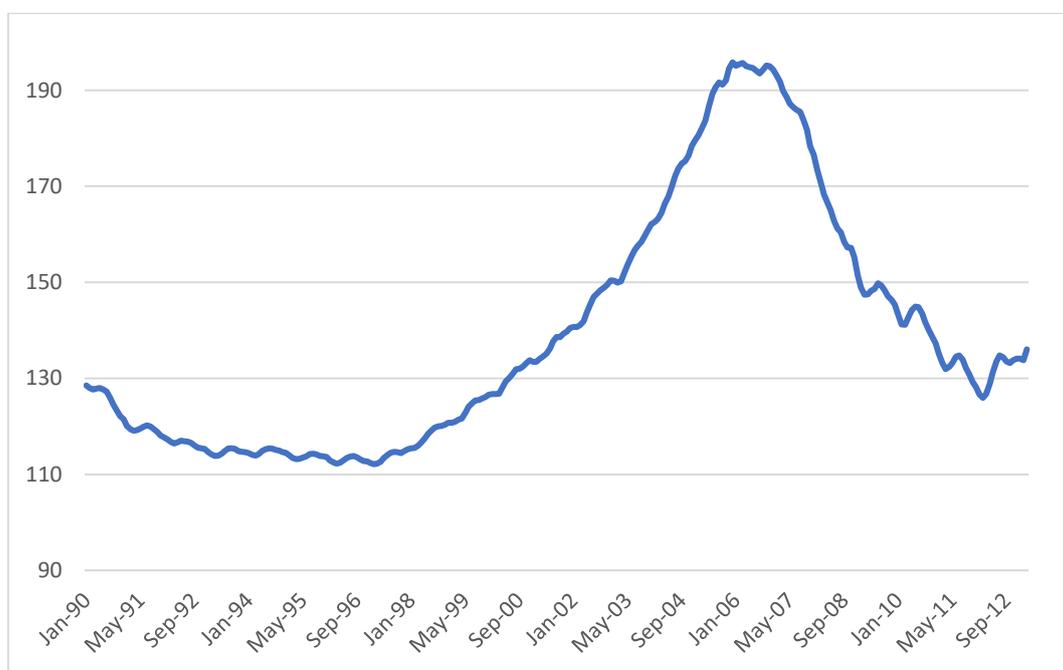


Figure 7: Case-Shiller House Price Index

The first effect of the Community Development Act was to enable people who wouldn't have otherwise been able to access a mortgage, to have access to the credit necessary to buy a house. In this way house prices were pushed up because demand of houses expanded. The second less evident effect of the Community Development Act was to deteriorate the lending standards. As the authors of the paper from the Federal Reserve Bank of Dallas state: "Our findings suggest that swings in credit standards played a major, if not the major, role in driving the recent boom and bust in U.S. house prices"²¹.

²¹ Duca John V., Muellbauer J., Murphy A. (June 2013). *Shifting Credit Standards and the Boom and Bust in U.S. House Prices: Time Series Evidence from the Past Three Decades*.

Another reason for the US House Price bubble was the extraordinarily low interest rate regime that was in place following the Dot Com bubble. In 2002 the FED decreased the interest rate in order to stimulate the economy after the early 2000s recession, but in so doing it also allowed the presence of very cheap money for investors. Since in those years house prices were booming, even more people undertook mortgages because investing in houses was very promising. Moreover the Dot Com bubble had just burst so people were looking for an alternative and possibly more solid market than the stock market.

The American people considered houses a good investment because their perceived volatility is much lower than stock's. Stock prices change were readily available to everybody in the 2000s, so everyone realized that prices change continuously, while the house prices are not constantly updated, rather they're published yearly or quarterly so people are led to think that if price declines, there's plenty of time to look for a buyer. There was also a common belief that house prices do not incur into dramatic drops and that they were constantly increasing. In fact, as professor Robert Shiller calculated in his book "Irrational exuberance", the average house price increase between 1940 and 2004 has been a modest 0.7%.

The next fundamental elements that channelled the international finance money to American home buyers were the MBS or CDOs. Mortgage backed securities are financial products whose value is determined by the cash flow from mortgage repayments; they are backed by the house or property that was bought with the mortgage. On the other hand CDOs' owners are entitled to the cash flow coming from debt repayments and are backed by the borrower's assets that can be investments, MBS, credit cards or income revenues. MBS and CDOs are divided in tranches with different repayment priorities and different interest rates. Tranches are assigned a

rating based on the quality of the underlying loans so that every investor can select his preferred tranche based on his risk appetite, as shown in figure 8.

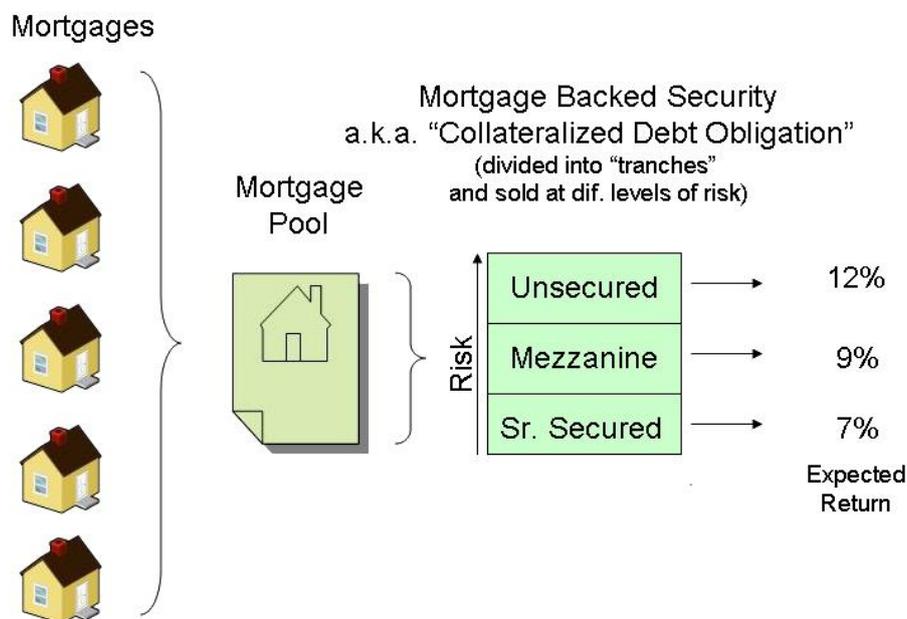


Figure 8: Exemplification of a CDO's structure.

The purpose of MBS and CDOs was to link the international flow of money with the real US economy so they allowed the availability of credit to everyone looking for it. Two were the downsides of the MBS and CDOs; the first one was that those products reached such a high level of complexity that it was extremely difficult for investors to understand what they were buying since every CDO could be backed by hundreds of loans. The consequence was to heavily rely on rating agencies' judgements, but on their turn rating agencies were very likely to give a generous rating to their clients, since if the client was not particularly happy about the rating received, he could just ask for another rating agency's valuation. As journalists Bethany McLean

and Joe Nocera described the phenomenon, investors: “weren't so much buying a security. They were buying a triple-A rating”²².

The second consequence of MBS and CDO is that they allowed to spread the American mortgages and loans risks all over the world and when after 2007 some European banks became insolvent, European countries preferred to rescue those banks with an increase of public debt. So not only the MBS and CDOs led to big losses in banks' equities but also triggered the European sovereign debt crisis, that consisted in a rapid increase of countries' indebtedness and the following rise in interest rates those countries had to bear.

As we can see from figure 7, in late 2006 house prices started falling eventually reaching pre-bubble levels. There are many events that led to the bubble collapse, with debated importance, we will start from the rising interest rates.

Starting from 2004 up to Aug 2004, the FED constantly increased interest rates, in an attempt to reduce the easy credit that allowed the developing of the housing bubble. However a great number of mortgages were adjustable rate²³ so borrowers had to pay increasing interests on their mortgage. The ones who were just investing in houses realized that it was more convenient to default on their mortgage and lose the house than keep paying for something that was valued less than the mortgage itself, so banks put on sale an increasing number of houses due to foreclosures which pushed the house prices even lower.

MBS and CDOs became worthless very rapidly and financial institutions that were holding those assets, now considered “toxic titles”, were in financial trouble. Lehman Brothers investment bank was the first to declare bankruptcy, and other

²² McLean B. and Nocera J. (2010). *All the Devils Are Here*.

²³ Mark Zandi estimates in his book “Financial Shock” that 90% of the subprime mortgages were Adjustable rate by 2006.

banks had to be rescued by the Government to be able to keep on operating. Among these the biggest were Merrill Lynch, AIG, Freddie Mac and Freddie Mae, while in Europe the list includes: Royal Bank of Scotland, HBOS, Northern Rock, Bradford&Bingley, Fortis and Hypo. The US Government in particular, approved the Trouble Asset Relief Program, a plan of 700 billion USD to rescue banks in trouble, successfully preventing other failures. As previously mentioned almost all the money lent by the TARP were given back to the US Government by 2012. The bubble had a disruptive impact on the US economy, the S&P 500 index fell 45% in 2008, and house prices fell of 54% by 2012. In order to compensate the fall in consumption that followed the burst, the US Government committed to simulating programs for a total of 13,903 billion dollars, comprehensive of private capital contribution, guarantees on deposits and commercial papers and increase in Government expenditure²⁴. The other purpose of the stimulus package was to avoid the credit crunch. A credit crunch occurs when banks do not trust each other for short term lending. It also affects companies' certificates that are used to finance short term business needs. The US Government managed to reduce the impact of the credit crunch by extending guarantees on certificates but couldn't do much for avoiding consumer expenditure drop.

The drop in consumer expenditure caused by uncertainty and stock losses led to an increase in the unemployment rate up to 10.1% in October 2009. Interesting to notice is that the bubble collapse did not have an equal impact on the American people. Among the poorest only 50% suffered an economic loss, while 63% of households and 77% of the richest families were negatively affected by the bubble²⁵.

²⁴ FDIC (Summer 2009). *Supervisory Insights*.

²⁵ Federal Reserve Board. *Surveying the Aftermath of the Storm: Changes in Family Finances from 2007 to 2009*.

It is not surprising that richer people were more affected because they had more wealth at stake than lower income individuals.

As we said the initial step towards the formation of the bubble was the US Government policy to extend home ownership rate in the country. As we can see from figure 9, the undertaken measures were initially successful, because the homeownership rate increased from 64.2% at the beginning of 1995 to 69.2% at the end of 2004.

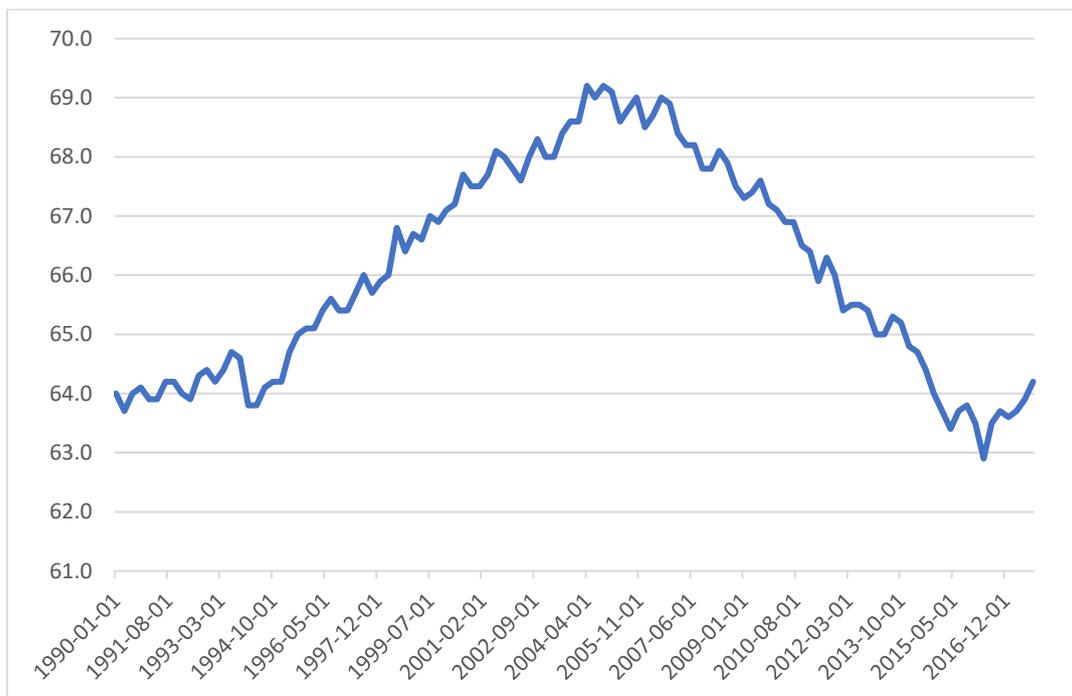


Figure 9: Homeownership rate in the United States

However, the rate had grown too fast and was not caused by improved economic conditions that allowed an increasing number of people to afford paying for a house, but for a widespread availability of cheap money that encouraged even low-income

individuals to buy a house. For this reason the rate soon started declining and only found its bottom in 2016, when it settled back to 62.9, a lower level than 1995.

The US House Price bubble did not have the extreme consequences of the Great depression, nevertheless it showed that bubbles can always happen and proved the essential role of lender of last resort played by central banks, that have to provide liquidity in times when traditional investors tend to decrease investments.

2.1.8 Bubbles frequency

Many bubbles have occurred in the last centuries with rising frequency towards the 20th century. As we can see from the following financial crisis timeline (figure 10) the time period during which the Bretton Woods agreement was in place was the longest period in the last century without financial crisis. After that period of stability, the frequency of financial crisis increased dramatically and basically in the past 20 years there has always been some crisis going on.

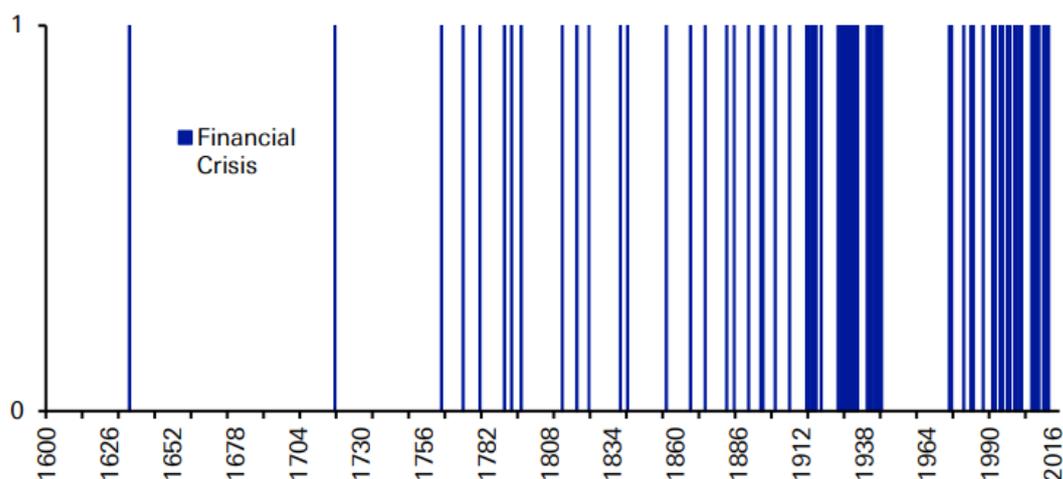


Figure 10: Frequency of financial crisis in history. Note the great stability while the Bretton Woods agreement was in force and the following increased frequency. Source: Deutsche Bank.

The Bretton Woods Agreement provided for fixed exchange rates between all the involved currencies and the US Dollar. On its turn, the Dollar could be converted to gold at the exchange rate of 35\$/ounce. During the Bretton Woods agreement no expansionary monetary policy was possible because it would depreciate the country's currency and this was explicitly against the agreement.

The agreement proved to be unsatisfactory for European countries and unsustainable for the US so in 1971 Nixon repealed it, ending the possibility to convert dollars for gold. At that point currencies started to fluctuate without central bank manipulation. The use of fiat money allowed countries to use monetary policy as an instrument to help exports and to pay less interests (at least on theory) on debts, via devaluation. However it also increased government debt in almost every developed country, it favoured rising imbalances and more unstable financial markets. As noted in a Deutsche Bank research paper "The current environment allows (and maybe encourages) great imbalances and huge credit and debt creation but also allows for huge operations to overcome such crises even if they perhaps make a subsequent crisis more likely by passing the crisis along to some other part of the global financial system and usually in bigger size"²⁶. This statement is a warning that financial bubbles will keep occurring in the future, maybe even in bigger size, hence the need to develop forecasting models to try to detect bubbles before it's too late.

²⁶ Deutsche Bank Market Research, *Long term asset return study. The Next Financial Crisis.*

2.2 The hypothesis

The common idea among all the tests proposed is that price should behave as an AR(1) process. An autoregressive process is a process where the current value of a variable say y_t , only depends on the value that the variable took in previous periods plus an error term. An autoregressive model of order p is expressed as:

$$AR(p): \quad y_t = \mu + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \dots + \rho_p y_{t-p} + \varepsilon_t$$

Where ε_t is a white noise disturbance term. An important characteristic of the autoregressive models is that the autoregressive coefficient ρ has to be smaller than one for the process to be stationary, otherwise the process' past values have an increasing influence over the next values. This property is very useful because it will allow us to detect a bubble when the estimated autoregressive coefficient is greater than one. All the tests that we will use have been manipulated to adapt to the same null/alternative hypothesis.

The model we will use in our analysis is an autoregressive process of order one:

$$AR(1): \quad y_t = \rho y_{t-1} + \varepsilon_t$$

With $E(\varepsilon) = 0$ and $E(\varepsilon^2) = \sigma^2$. When $\rho=1$ we obtain a random walk process i.e. the process moves randomly up and down and there's no possibility of forecasting its path. When the process is instead behaving as a bubble, ρ becomes greater than one and the behaviour of the process resembles an exponential path. In the tests we'll analyse, the null hypothesis is that the process is a random walk while the alternative is that it presents an explosive behaviour.

$$\begin{aligned}
H_0: \quad \rho_t &= 1 && \text{for } t = 1, \dots, T. \\
H_1: \quad \rho_t &= \begin{cases} 1 & \text{for } t = 1, \dots, \tau * T \\ \rho > 1 & \text{for } t = \tau * T + 1, \dots, T \end{cases}
\end{aligned}$$

2.3 Bhargava test

The test originally proposed by Bhargava is:

$$BH_0 = \frac{\sum_{t=1}^T (y_t - y_{t-1})^2}{\sum_{t=1}^T (y_t - y_0)^2}$$

In order to apply the test recursively we will use the modified version proposed by Homm and Breitung:

$$BH_\tau = \frac{1}{T - \tau T} \left(\frac{\sum_{t=\tau T+1}^T (y_t - y_{t-1})^2}{\sum_{t=\tau T+1}^T (y_t - y_{\tau T})^2} \right)^{-1} = \frac{1}{s_\tau^2 (T - \tau T)} \sum_{t=\tau T+1}^T (y_t - y_{\tau T})^2$$

with

$$s_\tau^2 = \frac{1}{(T - \tau T)} \sum_{t=\tau T+1}^T (y_t - y_{t-1})^2$$

The test will reject the null hypothesis for large values of B_0 . The test sums the squared difference from $\tau * T + 1$ to T between the current observation and the value at τT and divides this value for the sum of squared difference between the current observation and the previous observation in the same interval. Then it divides this result by the number of observations considered. If the process under inspection becomes steeper, the numerator of the test increases faster than the denominator, therefore yielding a higher value.

The test rejects the null hypothesis if $\sup BH(r_0) > \text{Critical Value}$, where

$$\sup BH(\tau_0) = \sup_{\tau \in [\tau_0, 1 - \tau_0]} BH_\tau$$

The notation $\sup BH(\tau_0)$ means that at τ_0 the value of the test is the least upper bound of all the BH_τ calculated over the interval $[\tau_0, 1 - \tau_0]$.

2.4 Busetti-Taylor test

Busetti and Taylor proposed a model to test for stationarity in a time series, with the alternative being a non-stationary process. Homm and Breitung manipulated this test in order to fit it to their H_0 and H_1 hypothesis (respectively, random walk / explosive behaviour). The test becomes:

$$BT_\tau = \frac{1}{s_0^2(T - \tau T)} \sum_{t=\tau T+1}^T (y_T - y_{t-1})^2$$

where

$$s_0^2 = \frac{1}{(T - 2)} \sum_{t=2}^T (y_t - y_{t-1})^2$$

The test statistics to be compared with the critical value is the $\sup BT(\tau_0)$.

$$\sup BT(\tau_0) = \sup_{\tau \in [\tau_0, 1 - \tau_0]} BT_\tau$$

Bhargava and Busetti-Taylor differ in two important aspects:

- In Bhargava the variance is calculated considering only the value after τT , while in Busetti-taylor all the values of the sample are taken into account.
- The second difference is that Busetti-Taylor measures the squared errors from the final value y_T while Bhargava measures the squared errors from the

initial value $y_{\tau T}$. As we will see, the Busetti-Taylor method achieves better results.

2.5 Chow test for Structural Break

Chow test for structural break assumes that the process is a random walk for the first τT values under both the null and the alternative hypothesis. Therefore, $\rho_t = 1$ up to τT and $\rho_t - 1 = \delta > 0$ afterwards. The model is a simple modification of the usual AR(1) process where an indicator function $\mathbb{1}$ has been added. For $t < \tau T$ the indicator function is zero, therefore the process reduces to a random walk, for $t > \tau T$ the process incorporates the bubble element, assuming an explosive behaviour.

$$AR(1): \quad y_t = y_{t-1} + \delta(y_{t-1}\mathbb{1}_{\{t>\tau T\}}) + \varepsilon_t$$

The test used to compare the null $H_0: \delta = 0$ against the alternative $H_1: \delta > 0$ is

$$DFC_{\tau} = \frac{\sum_{t=\tau T+1}^T \Delta y_t y_{t-1}}{\hat{\sigma}_{\tau} \sqrt{\sum_{t=\tau T+1}^T y_{t-1}^2}}$$

where

$$\hat{\sigma}_{\tau}^2 = \frac{1}{(T-2)} \sum_{t=2}^T (\Delta y_t - \delta_{\tau} y_{t-1} \mathbb{1}_{\{t>\tau T\}})^2$$

Of course the value we are looking for is:

$$\sup DFC(\tau_0) = \sup_{\tau \in [0, 1-\tau_0]} DFC_{\tau}$$

2.6 Dickey Fuller test

Phillips, Shi and Yu use a Dickey-Fuller test in their work with an asymptotically negligible drift in order to capture the small drift in prices which is frequently present in price behaviours. The model has the following intercept form:

$$y_t = dT^{-\eta} + \phi y_{t-1} + \varepsilon_t$$

Where T is the sample size, d is a constant and η is a coefficient that controls the magnitude of the drift. As we can see, the drift tends to zero for $T \rightarrow \infty$.

The test statistics to be computed is:

$$DF_\tau = \frac{\hat{\phi}_\tau - 1}{\hat{\sigma}_{\phi,\tau}}$$

where DF is the statistics calculated on $\{y_1, \dots, y_{\tau T}\}$, $\hat{\phi}_\tau$ is the estimated autoregressive coefficient in the same interval and $\hat{\sigma}_{\phi,\tau}$ is the variance of $\hat{\phi}_\tau$.

$$\sup DF(\tau_0) = \sup_{\tau \in [\tau_0, 1]} DF_\tau$$

2.7 Cusum test

In 1954 Page suggested a model to detect changes in a parameter of choice of a model. It can of course be applied to our case to detect whether the parameter ρ_t changes from being equal to 1 to be greater than 1. The Cusum detector is:

$$C_{\tau_0}^\tau = \frac{1}{\hat{\sigma}} \sum_{j=\tau_0 T+1}^{\tau T} \Delta y_j$$

where

$$\hat{\sigma}_\tau^2 = \frac{1}{(\tau T - 1)} \sum_{j=1}^{\tau T} (\Delta y_j - \hat{\mu}_\tau)^2$$

All the previous tests assume that there is only one structural break in the process. If there's a subsequent structural break from explosive behaviour back to a random walk their performance in detecting this second change is poor. The Cusum test is instead very reactive to a process return to a normal behaviour.

2.8 STEST

We will now apply the previously described tests in two different ways. The first method has been proposed by Phillips, Wu and Yu in 2011. The method consists in repeating the chosen test on a forward expanding sample sequence where the test value is the sup value of the corresponding test sequence. The minimum window of the sample is calculated as a rule of thumb as $r_0 = 0.01 + 1.8/\sqrt{T}$. In this way when T is small, say 100, r_0 becomes 0.19 so the number of observations used for the initial estimation (19) should be enough to ensure a good estimate. On the other hand, when $T = 200$, r_0 is allowed to become smaller at 0.14, but the number of observations actually increases at 28. Given our minimal window r_0 and the initial point $r_1 = 0$, r_2 can move from r_0 to 1. The PWY method is defined as the superior statistic among all the statistics obtained, and it's denoted as:

$$STEST(r_0) = \sup_{r_2 \in [r_0, 1]} TEST_0^{r_2}$$

The PWY method is able to detect assets' explosive behaviour especially when a single bubble occurs, while if more than one bubble is present, the method provides very poor results. In empirical analysis however, if the sample period considered is long enough, there is often evidence of multiple bubbles, therefore another method has been proposed by Phillips, Shi and Yu.

2.9 GSTEST

Phillips, Shi and Yu developed in 2015 a new method able to detect the more difficult task of detecting multiple bubbles in a sample. The procedure uses a forward expanding sample as in the PWY case but at the same time it allows for the initial sample point to change as well. The endpoint r_2 can change from r_0 to 1, while the starting point r_1 can move from 0 to $(r_2 - r_0)$. The test is a generalized superior test, and it's denoted as:

$$GSTEST(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} TEST_{r_1}^{r_2}$$

Figure 11 illustrates the difference between the two tests. As we can see, the STEST procedure is just a particular case of the much more general GSTEST, so the two methods provide the same result when $r_1 = 0$. Calculations using the GSTEST are much more compute intensive than the STEST because of the double recursion.

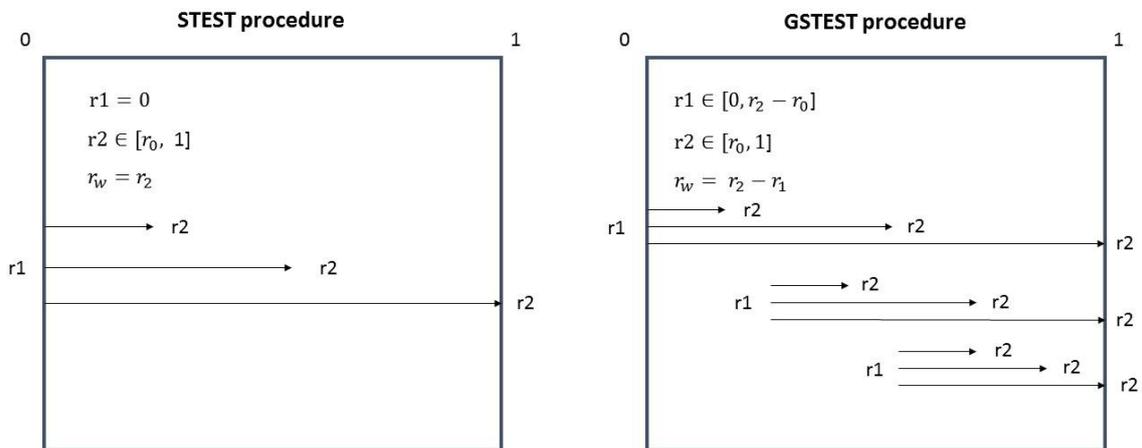


Figure 11: Superior test and Generalized Superior test comparison

Phillips, Shi and Yu recommend using these two methods in a backward recursive way in order to increase the bubbles identification precision. Even if the test runs backward, it can still be applied to real time detection, as r_2 can be set to match the most recent observation. Figure 12 show how the two methods are implemented backwards. We will denote the *BSTEST* from now on as PWY and the *BGSTEST* as PSY.

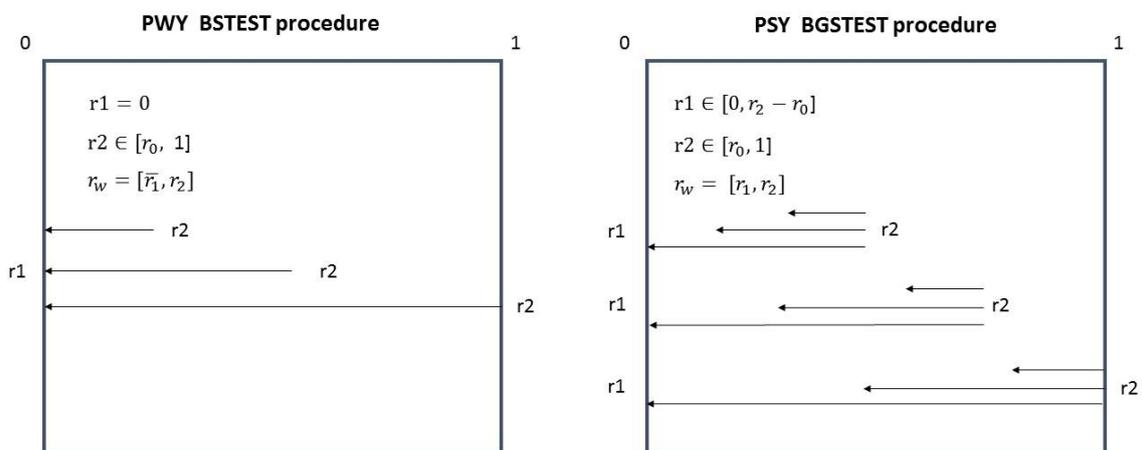


Figure 12: Backward Superior test and Backward Generalized Superior test comparison

Chapter 3. Simulations

3.1 One bubble

In order to understand the pros and cons of each test we will simulate many time series with one or two bubbles and repeatedly apply the tests.

The data generating process for a one bubble time series is:

$$X_t = X_{t-1}\mathbb{1}\{t < t_e\} + \delta_T X_{t-1}\mathbb{1}\{t_e \leq t \leq t_f\} + \left(\sum_{k=t_f+1}^t \varepsilon_k + X_{t_f}^* \right) \mathbb{1}\{t > t_f\} + \varepsilon_t \mathbb{1}\{t \leq t_f\}$$

with $\delta_T = 1 + cT^{-\alpha}$, $c > 0$, $\alpha \in (0, 1)$, $\varepsilon_t \sim^{iid}(0, \sigma^2)$, $X_{t_f}^* = X_{t_e} + X^*$ and $X^* = O_p(1)$. t_e is the date of bubble origination and t_f is the bubble termination date. The time series behaves as a random walk during the pre-bubble period $[1, t_e)$, then it switches to an autoregressive process of order one and coefficient δ_T between $[t_e, t_f]$. After t_f , the process falls back to X_{t_e} plus the sum of the error terms (ε_t) that occurred during the bubble period and then resumes its random walk behaviour within $(t_f, t]$. A test detects a bubble origination when its value crosses the critical value upward while it signals a bubble termination when its value crosses the critical value downwards. We calculate the accuracy of each test with the ratio $\frac{\text{number of detected bubble periods}}{\text{number of actual bubble periods}}$. Since some tests are sensitive to the position of the bubble origination and its length, we'll consider bubbles originating at 0.2, 0.3 and 0.4 of the sample, and with lengths of 0.15, 0.20 and 0.25. All the computations have been conducted setting $\alpha = 0.9$, $T = 200$, $\sigma = 5$, and $X(0) = 100$. Figure 13 shows a process generated by the previously described DGP, where $t_e = 0.4$ and $t_f = 0.6$, therefore the bubble length is $200 * (0.6 - 0.4) = 40$. In this example, the PWY ADF test has been applied and as we can see it crosses the critical value upwards at the observation 92 and downwards at observation 121. Therefore the accuracy, measured as the number of meaningful values from 80 to 120 is:

$$\frac{29}{40} = 72.5\%$$

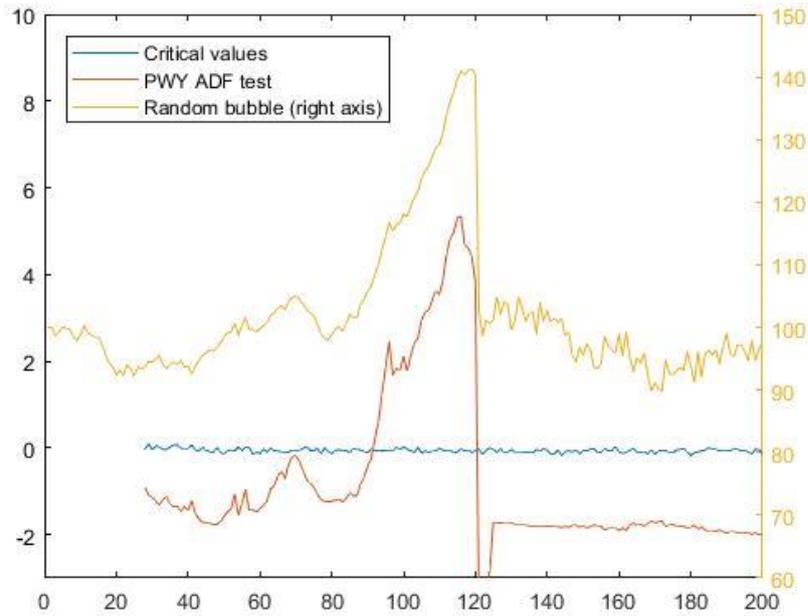


Figure 13: one bubble simulated process with $t_e= 0.4$, $t_f= 0.6$ and related PWY ADF test

We now generate 2000 different time series per test and we record the percentage of bubbles correctly identified; the results are provided in table 1. The first thing we can notice is that all the tests perform better when the bubble length is longer; this is quite intuitive because the longer the bubble, the more time the tests have to detect that explosive behaviour.

The second thing is that the bubble origination point within the sample, using the PWY approach, affects the detection precision of all the tests; in particular, the later in the sample the bubble is originated, the worse performances we obtain. The deterioration in performance is not equal among the tests; on one side the Chow test performs 40-50% worse when we move the origination point from 0.2 to 0.3, while the ADF test only suffers a moderate 10% deterioration when considering the difference between the same origination points.

The bubble origination point doesn't seem to have such an important role in detecting bubbles when the PSY method is used, with the only exception been the

Cusum test and that's because its critical values are always increasing with the time series length. The final remark is that all the tests score higher detection rates with the PSY method, suggesting that the double backward recursion is the best approach for detecting one bubble episodes.

<i>Starting</i>	<i>Length</i>	Chow		Cusum		Bhargava		Busetti-Taylor		ADF	
		<i>PWY</i>	<i>PSY</i>	<i>PWY</i>	<i>PSY</i>	<i>PWY</i>	<i>PSY</i>	<i>PWY</i>	<i>PSY</i>	<i>PWY</i>	<i>PSY</i>
0.2	0.15	54%	60%	62%	70%	13%	35%	59%	67%	64%	69%
	0.2	63%	74%	73%	78%	22%	48%	70%	77%	72%	71%
	0.25	73%	79%	78%	82%	35%	58%	75%	80%	79%	80%
0.3	0.15	36%	63%	51%	60%	9%	35%	56%	68%	57%	73%
	0.2	51%	64%	62%	69%	16%	48%	66%	77%	67%	76%
	0.25	60%	75%	70%	76%	25%	60%	71%	81%	76%	82%
0.4	0.15	29%	62%	44%	52%	7%	35%	48%	69%	53%	70%
	0.2	39%	67%	56%	63%	10%	47%	59%	76%	64%	79%
	0.25	51%	75%	64%	71%	17%	59%	69%	80%	69%	85%

Table 1: the five tests' performances in detecting one bubble using PWY and PSY methods. CV determined with 5000 replications and tests repeated 2000 times.

3.2 Two bubbles

Detecting two or more bubbles in the same time series is a much more complicated task. The first thing to keep in mind is that the PWY method, as seen so far, cannot be applied because the effects of the first bubble on the tests, make it almost

impossible to detect a second one. So in this case we need to reinitialize the tests after they have detected the first bubble. We'll call this new method the Sequential PWY. Here's how it works: the PWY test is run from r_0 to T . When it detects a bubble, the test has a bigger value than the Critical one's and when the test's value crosses downwards the Critical value, the bubble is over. The reinitialization process consists in calculating a new minimum window $r_0 = 0.01 + 1.8/\sqrt{T^*}$ but this time T^* is equal to $[T - t_f]$ and the tests are run on the remaining sample $[t_f, 1]$.

The data generating process with two bubbles is:

$$X_t = X_{t-1}\mathbb{1}\{t \in N_0\} + \delta_T X_{t-1}\mathbb{1}\{t \in B_1 \cup B_2\} + \left(\sum_{k=t_{f_1}+1}^t \varepsilon_k + X_{t_{f_1}}^* \right) \mathbb{1}\{t \in N_1\} + \\ + \left(\sum_{l=t_{f_2}+1}^t \varepsilon_l + X_{t_{f_2}}^* \right) \mathbb{1}\{t \in N_2\} + \varepsilon_k \mathbb{1}\{t \in N_0 \cup B_1 \cup B_2\}$$

with $N_0 = [1, t_{e1})$, $B_1 = [t_{e1}, t_{f1}]$, $N_1 = (t_{f1}, t_{e2})$, $B_2 = [t_{e2}, t_{f2}]$, $N_2 = (t_{f2}, t]$. t_{e1} and t_{f1} are the origination and termination dates of the first bubble and t_{e2} and t_{f2} are the origination and termination dates of the second bubble.

The simulated process has a random walk behaviour in N_0 , an explosive behaviour in B_1 described by an AR(1) process with coefficient δ_T , then it resumes a random walk in N_1 , then again an explosive behaviour in B_2 and finally it resumes a random walk behaviour again.

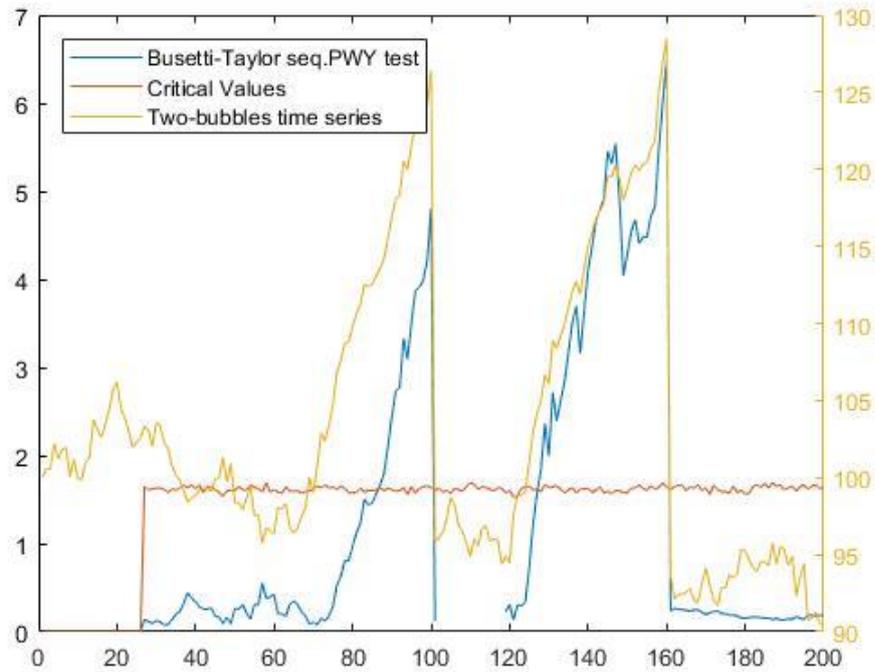


Figure 14: two-bubbles simulated process with $t_{e1}= 0.3$, $t_{f1}= 0.5$, $t_{e2}= 0.6$, $t_{f2}= 0.8$ and related Busetti-Taylor seq.PWY test.

In the example provided in figure 14, we can notice the typical reinitialization window between observation 100 and 119 required by the Seq. PWY method. Since the first bubble ends on observation 100, the new minimum window r_0 is calculated as $r_0 = 0.01 + 1.8/\sqrt{100} = 19$.

In order to compare the various tests' abilities to detect a second bubble in a time series we use a new ratio: $\frac{\text{second bubble's detected length}}{\text{actual second bubble length}}$. As the reader can understand, all the following simulations will not measure the overall two-bubble-scenario performances, but only the tests' abilities to detect the second bubble, regardless of the first one. As in the previous case, we run simulations varying both the starting point and the length of the bubbles. For example in creating a process

with starting point 0.30 and length 0.20 we have that: $t_{e1} = 0.30$, $t_{f1} = 0.5$, $t_{e2} = 0.60$, $t_{f2} = 0.80$. The difference $[t_{e2} - t_{f1}]$ is arbitrarily set equal to 0.10.

As we can see from table 2 the Seq. PWY performances are inversely related to the starting points of the bubbles within the sample at least in the Chow, Cusum and Bhargava models. This relation is not apparent in the Busetti-Taylor and ADF tests. As previously noted, the tests perform better when the bubble lasts longer, with the only exception being the Busetti-Taylor PSY test. This test returns appreciable results only in the case when the second bubble in the time series is bigger than the first one, so we don't recommend using the Busetti-Taylor test with the PSY method in a multi-bubble scenario. Chow, Cusum and Bhargava tests are not usually able to detect more than 50% of the bubble, while the Busetti-Taylor (seq. PWY) and ADF tests (both methods) usually detect from 70% to 85% of the bubble. In particular, the best performing test in the simulations is the Seq.PWY ADF, which constantly outperforms the PSY ADF test. So both in a one-bubble scenario and with a two-bubble scenario, the best performing test is the Augmented Dickey-Fuller, with the PSY method to be preferred in the first scenario and the Seq.PWY method that best performs in the two-bubbles scenario.

<i>Starting</i>	<i>Length</i>	Chow		Cusum		Bhargava		Busetti-Taylor		ADF	
		<i>seq PWY</i>	<i>PSY</i>	<i>seq PWY</i>	<i>PSY</i>	<i>seq PWY</i>	<i>PSY</i>	<i>seq PWY</i>	<i>PSY</i>	<i>seq PWY</i>	<i>PSY</i>
0.2	0.15	18%	33%	26%	4%	14%	31%	70%	5%	74%	64%
	0.2	36%	41%	44%	18%	33%	47%	79%	2%	82%	72%
	0.25	44%	47%	51%	30%	47%	55%	81%	1%	83%	77%
0.3	0.15	9%	25%	20%	2%	8%	27%	66%	6%	70%	61%
	0.2	29%	36%	39%	12%	25%	47%	78%	3%	81%	73%
	0.25	44%	41%	51%	26%	44%	57%	84%	2%	86%	77%
0.4	0.15	6%	25%	18%	1%	6%	30%	67%	8%	72%	63%
	0.2	22%	37%	36%	9%	18%	46%	78%	5%	81%	72%
	0.25	39%	39%	49%	23%	35%	56%	83%	3%	85%	77%

Table 2: the tests' performances in detecting a second bubble using Seq. PWY and PSY methods. CV determined with 5000 replications and tests repeated 2000 times.

3.3 An alternative DGP

We will now consider how the tests perform when the random error in the Data Generating Process follows a Poisson distribution. The DGP remains the same:

$$\begin{aligned}
X_t = & X_{t-1}\mathbb{I}\{t < t_e\} + \delta_T X_{t-1}\mathbb{I}\{t_e \leq t \leq t_f\} + \\
& + \left(\sum_{k=t_f+1}^t \varepsilon_k + X_{t_f}^* \right) \mathbb{I}\{t > t_f\} + \varepsilon_k \mathbb{I}\{t \leq t_f\}
\end{aligned}$$

but in this case the error term is $\varepsilon_t \sim Poi(\lambda)$.

The process starts as a Poisson random walk, then switches to an autoregressive process between t_e and t_f and then it resumes a Poisson random walk until the end

of the sample. The idea behind this simulation is to check how the tests perform when the bubble is less evident since it is surrounded by Poisson's higher than normally distributed peaks.

We can see an example of the simulated time series in figure 15 and notice that it shows bigger variance than the previous time series that were created using a normally distributed error term.

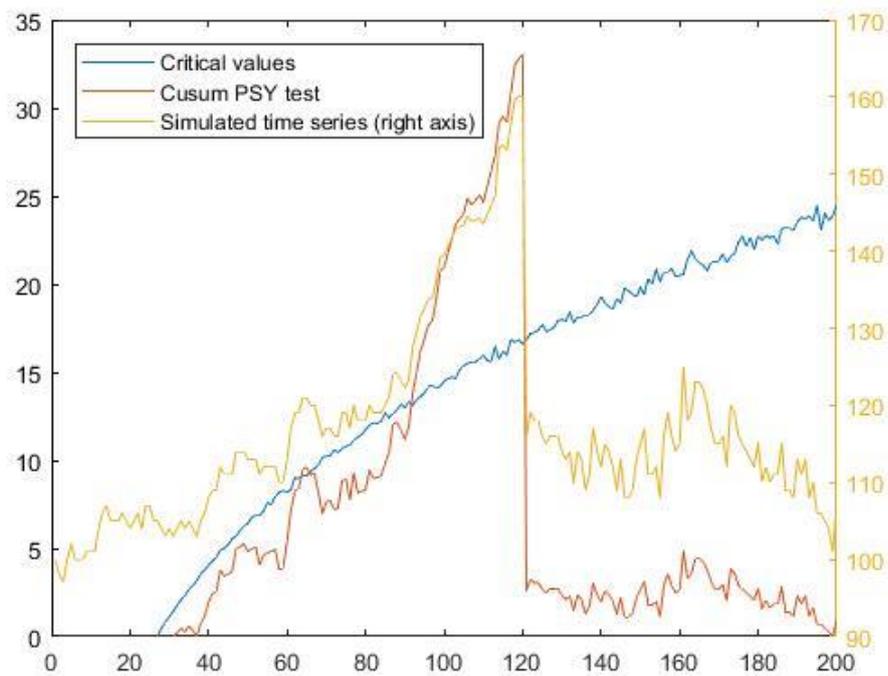


Figure 15: one bubble simulated process with $t_e = 0.4$, $t_f = 0.6$ and related Cusum PSY test

Simulations' critical values have been calculated with a Monte Carlo simulation with 5,000 repetitions and tests were repeated 2,000 times. The results are shown in table 3.

The first thing we can notice is that all the tests perform worse than in the time series where errors are normally distributed; this is quite intuitive because the

artificially created bubble is less detectable when surrounded by a Poisson random walk. The only exception is the Chow PSY test that obtains very similar results in both simulations.

As with previous simulations, detection rates improve as the bubble length increases. With the PWY method, precision increases when the bubble origination point is closer to the beginning of the sample, while with the PSY method the bubble position within the sample is not so relevant. However the Cusum PSY test tends to perform better when the bubble is at the beginning of the sample because it has constantly increasing critical values.

The best performing tests are in this case the ADF PSY and the Buseti-Taylor PSY which have roughly a 50-70% detection ability.

<i>Starting</i>	<i>Length</i>	Chow		Cusum		Bhargava		Buseti-Taylor		ADF	
		<i>PWY</i>	<i>PSY</i>	<i>PWY</i>	<i>PSY</i>	<i>PWY</i>	<i>PSY</i>	<i>PWY</i>	<i>PSY</i>	<i>PWY</i>	<i>PSY</i>
0.2	0.15	31%	62%	47%	53%	9%	22%	44%	51%	48%	57%
	0.2	43%	73%	58%	63%	15%	37%	55%	64%	60%	69%
	0.25	52%	76%	66%	70%	23%	48%	64%	70%	68%	75%
0.3	0.15	17%	61%	35%	41%	6%	21%	37%	51%	44%	57%
	0.2	29%	70%	48%	53%	11%	35%	50%	62%	56%	68%
	0.25	38%	76%	56%	62%	16%	47%	59%	69%	64%	73%
0.4	0.15	13%	61%	31%	34%	6%	19%	36%	51%	40%	56%
	0.2	21%	68%	42%	47%	9%	35%	47%	60%	51%	67%
	0.25	30%	75%	49%	56%	12%	45%	56%	69%	61%	74%

Table 3: the tests' performances in detecting one bubble using PWY and PSY methods. Time series generated through a Poisson distribution.

Simulations were also performed in a two bubbles scenario and as in the previous two bubble simulations the best performing test is the ADF Sequential PWY followed by the Busetti-Taylor Sequential PWY and the ADF PSY. By comparing table 4 with table 2 we can notice a generalized decrease in performance when using the Poisson distribution especially when the simulated bubbles have a short length. The main exception is the Chow PSY test, that actually records a slight improvement in detection rates.

<i>Starting</i>	<i>Length</i>	Chow		Cusum		Bhargava		Busetti-Taylor		ADF	
		<i>seq PWY</i>	<i>PSY</i>	<i>seq PWY</i>	<i>PSY</i>	<i>seq PWY</i>	<i>PSY</i>	<i>seq PWY</i>	<i>PSY</i>	<i>seq PWY</i>	<i>PSY</i>
0.2	0.15	16%	36%	10%	6%	6%	18%	50%	7%	56%	51%
	0.2	31%	43%	27%	13%	15%	33%	64%	4%	64%	60%
	0.25	40%	45%	36%	24%	26%	42%	69%	3%	70%	65%
0.3	0.15	9%	28%	6%	5%	4%	16%	45%	7%	54%	48%
	0.2	26%	37%	21%	9%	11%	32%	62%	6%	64%	59%
	0.25	37%	43%	34%	19%	21%	44%	71%	4%	72%	67%
0.4	0.15	8%	32%	6%	4%	4%	17%	43%	10%	57%	49%
	0.2	19%	36%	18%	6%	8%	31%	60%	7%	65%	59%
	0.25	34%	41%	31%	15%	17%	43%	70%	5%	70%	66%

Table 4: the tests' performances in detecting a second bubble using Seq. PWY and PSY methods. Time series generated through a Poisson distribution.

Chapter 4. The empirical analysis

We will now apply the tests that have performed best in the simulations, namely the Busetti-Taylor Sequential PWY, the ADF Seq. PWY and the ADF PSY to some real time series: the US House Price Index, the Bitcoins/USD and the Ethereum/USD exchange rate.

4.1 US House prices

The first time series we consider is the Jan 1960 - Aug 2017 monthly Case-Shiller House Price Index which calculates the best possible “real” US house price by considering a number of influencing factors such as: inflation, cost of construction, interest rate and population growth. This price index is regarded as the most accurate for house prices and is used as the reference index for US house prices since it was first introduced by Robert Shiller in his book “Irrational Exuberance” in 2000.

As we can see from figure 16 the Busetti-Taylor Seq.PWY is able to detect the effects of the 1977 Community Reinvestment Act, the 1985-1991 Savings and Loans Crisis and of course the 2000-2005 housing bubble.

The Community Reinvestment Act was passed in order to encourage financial institutions to meet the needs of local communities and to eliminate the discriminations financial institutions made towards individuals from poor neighbourhoods. The result was that more loans were offered to middle-low income individuals because banks didn't want to incur in penalties for not complying with the Act, and this easy access to credit fuelled a house price bubble because demand for houses grew too quickly.

Another very important bubble occurred during the Savings and Loans Crisis; in 1979 the FED increased the discount rate from 9.5% to 12% trying to stop the rising inflation and this put many Savings and Loans companies in a critical situation where they could not meet their obligations anymore. But instead of filing for bankruptcy, thanks to weak supervision, these S&L corporation started making high risk lending in the hope of getting higher than the discount rate returns. This means that also individuals who were considered risky were granted the money they were looking for, to buy houses. As already happened in 1977, the increased investment amount in houses led to a sudden increase in prices as well.

The biggest house price bubble however started in 1997 (all the three tests agree on that date) and lasted until 2006. That's the previously discussed and most recent bubble in US housing market.

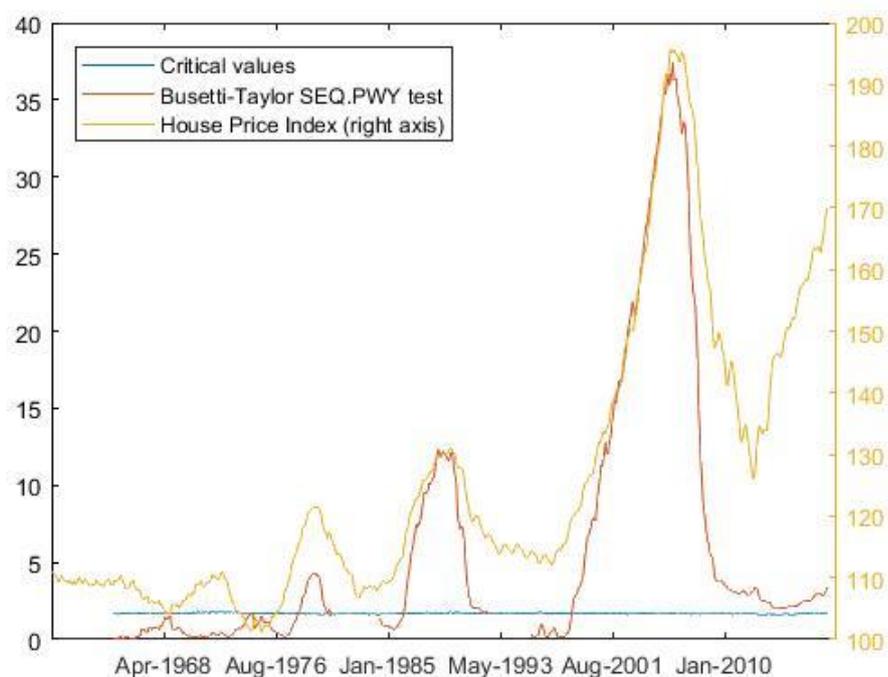


Figure 16: House Price Index Buseti-Taylor Seq.PWY test

While the Buseti-Taylor test can detect the three main bubbles, the ADF Seq.PWY test in figure 17 goes a step further and detects the abnormal prices that were a consequence of the 1968 Housing and Urban Development Act. The Act provided for the creation of a fully owned government corporation, Ginnie Mae with the purpose of expanding the availability of funds for mortgages for middle income families. The Mortgage backed securities the corporation issues still have the “full faith and credit” by the US government, therefore the interest the company has to pay on funds is extremely low. Of course making more funds available for home buyers also makes house prices increase, partially offsetting the benefits of an easy access to credit.

The fifth detected bubble in 1972-1974 is actually a false signal originated by the fact that from 1972 prices started falling rapidly after rallying between 1968 and 1972, following a negative explosive behaviour.

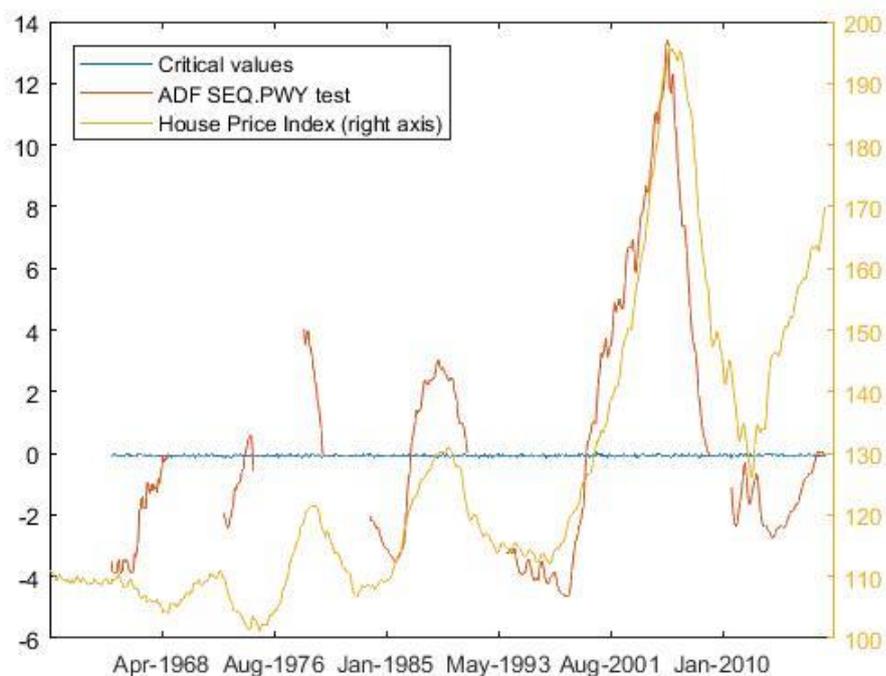


Figure 17: House Price Index ADF Seq.PWY test

The third test we apply to the House Price Index is the ADF PSY shown in figure 18, that is able to detect all the already mentioned explosive periods plus two other.

In 1970 US government created a second Government Sponsored Enterprise with the purpose of developing the secondary market for mortgages. The GSE was called Freddie Mac and its main activity was to purchase mortgages from financial institutions who originated them, pool them together in Mortgage Backed Securities and sell the resulting MBS in the open market. The creation of Freddie Mac increased even further the liquidity in the secondary mortgage market therefore more money were available to buy houses, resulting in a house price increase.

The other negatively explosive behaviour was due to two factors that occurred in 1981; in particular the government increased the one-time capital gain tax exclusion for people older than 55 years from \$100,000 to \$125,000, and each Federal Reserve Bank created a Community Affairs Office to ensure the Compliance with the Community Reinvestment Act. A stricter application of the Community Reinvestment Act increased the lending standards so the combined effects of the increased supply and lowered demand led to a relevant price drop from 1981 to 1983.

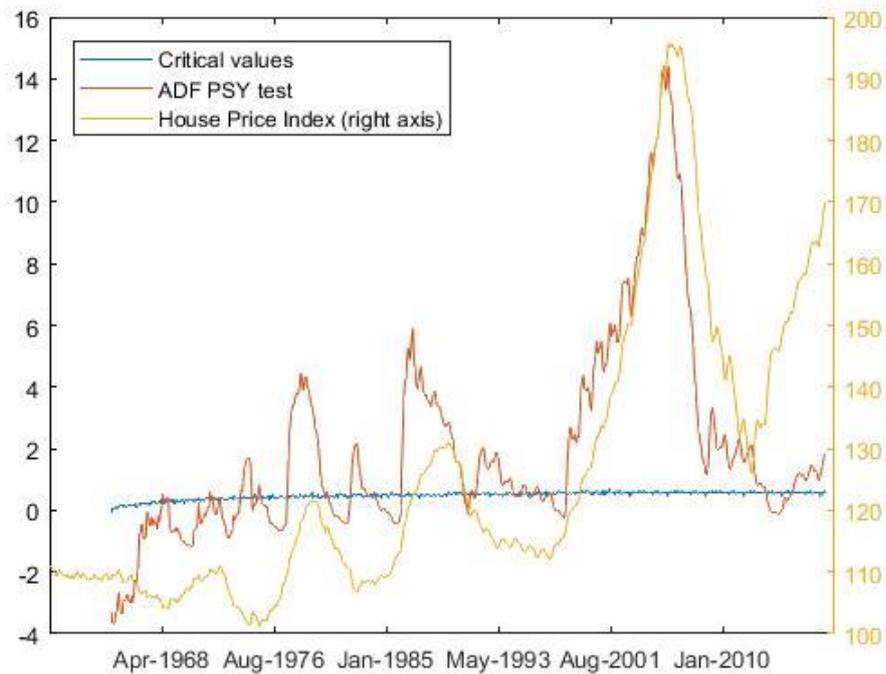


Figure 18: House Price Index ADF PSY test

The ADF PSY is very reactive to any change in price so it is very precise but sometimes it detects an explosive behaviour even if not present, for example in 1990-1997 house prices remained relatively flat, nevertheless the test signals the presence of some bubbles. The other positive aspect is that it doesn't need a reinitialization window, so the result is a continuous line with no interruptions which is very handy and gives a better understanding of the price behaviour.

4.2 Bitcoin

The weekly Bitcoin-USD exchange rate was obtained from Bitstamp and covers the period from 18th Sep 2011 to 31st Dec 2017. During this length of time Bitcoins had a great volatility with huge growth and sudden crashes and the tests suggest that four to five bubble episodes occurred.

Figure 19 shows that Buseti-Taylor test is able to detect at least five bubble periods starting on the 27th Jan 2013, 10th Nov 2013, 12th Mar 2017, 18th Jun 2017 and 3rd Dec 2017 respectively.

The first bubble episode was caused by two combined effects. The first was that BitPay reported that over 1,000 merchants were accepting bitcoins as a mean of payment and the second was the Cypriot financial crisis. During this crisis Cypriot bank accounts were frozen and limits were imposed on the withdrawal amount. The idea of having private bank accounts frozen by the government scared many people and buying Bitcoins seemed an immediate escape, so Bitcoin jumped from \$74 to \$213 within 15 days. The only thing that stopped that surge in price was the overload that the increased Bitcoin trading volume caused to the Mt.Gox and BitInstant servers, the two biggest cryptocurrency exchange at that time, which failed to confirm transactions, therefore doubts on the Bitcoin's reliability made price bounce back.

The second bubble episode took place in November 2013, when BTC China overtook Mt. Gox and Bitstamp and People's Bank of China director said that people were free to participate to the Bitcoin market. Chinese people found it very attractive to use a currency whose value was not manipulated by the government so the price kept rising until the 5th December 2013, when the Chinese government prohibited any financial institution from using Bitcoins. The ban was put in place for three

reasons²⁷. First, Bitcoin were a risky investment because of the relative small amount that was in circulation; in this way regulators argued that its value can easily be manipulated by large investors. The second reason was based on the authorities' concern that the virtual currency facilitate money laundering and terrorism. Thirdly, Bitcoin could be used by criminal organization for trafficking in illegal activities, citing the seizure of the marketplace Silk Road.

The third detected bubble is on the 12th March 2017. Two days before the SEC denied the creation of the first Exchange Traded Fund based on Bitcoins so the price dropped by roughly 15%. In the days before the announcement many speculators had of course bought Bitcoins but liquidated their position after the announcement because the decision was an official rejection of bitcoins. As we can see the test suggests the presence of a bubble because the price had a sudden drop, so it had a negatively explosive behaviour.

The fourth detected bubble period is due to the fast drop in price between 11th June and 18th June that was not an extra ordinary movement but the high sensitivity of the test, that was reinitialized just some observations earlier, make the test reach a very significant level.

The fifth bubble episode that eventually led to the record price of \$19,666 per Bitcoin was caused by the announcement occurred respectively on the 1st December and 5th December that CBOE and CME would start offering futures on bitcoin. The long-awaited news (SEC had already denied a Bitcoin ETF proposal in March) created a general optimism that led many small investors to enter the market, because they were afraid to miss an interesting investment opportunity and futures

²⁷ Financial Times (2013) *China bans banks from Bitcoin transactions*.
<https://www.ft.com/content/40b78d2e-5d87-11e3-95bd-00144feabdc0>

allowed new institutional investors to trade large quantities of bitcoin without actually paying for the whole amount of the position, therefore the price increased.

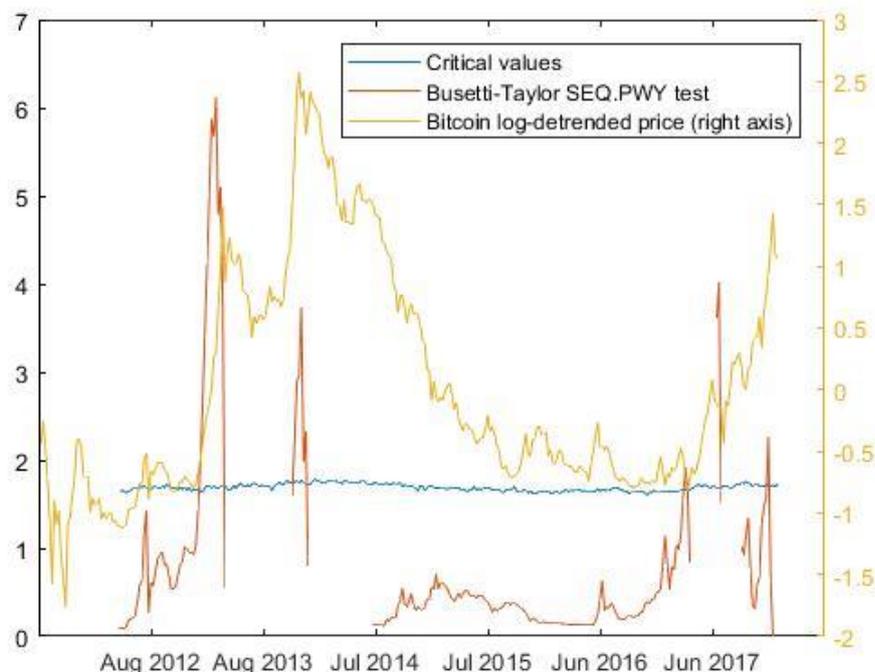


Figure 19: Bitcoin Buseti-Taylor Seq.PWY test

The ADF Seq.PWY test in figure 20 detects the same bubble periods as the Buseti-Taylor test so it correctly spots the 27th Jan 2013, 10th Nov 2013 and the 3rd Dec 2017 bubbles and detects the negatively explosive behaviour on the 12th Mar 2017 and the 18th Jun 2017, when price dropped significantly for no apparent reason. The ADF Seq.PWY also detects a bubble during August 2012, when prices rallied to \$13.31. However, both the Buseti-Taylor Seq.PWY and the ADF PSY that we'll later analyse, do not consider the August 2012 episode significant enough to be considered as a bubble.

It's also worth noticing that the significance level that the December 2017 bubble reaches in the ADF Seq.PWY, is much lower than the value reached in the previous test.

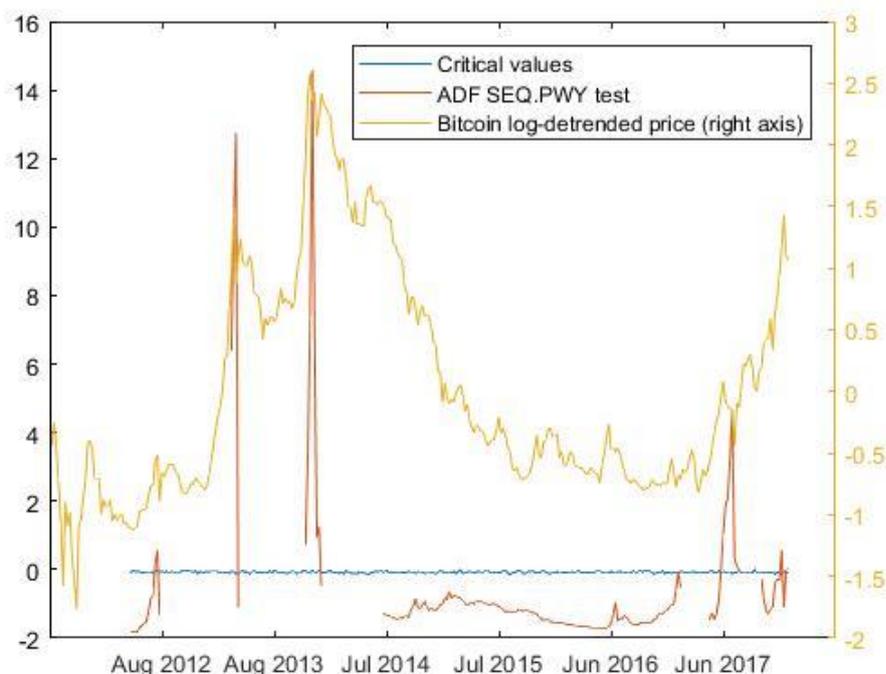


Figure 20: Bitcoin ADF Seq.PWY test

The ADF PSY test is applied to the logarithmic detrended price, in this way we obtain a more accurate result. So while in the previous graphs the logarithmic detrended price was the only way to display the bitcoin price history, otherwise we would obtain a flat line with an exponential behaviour towards the end of the sample, in this case the test itself has been applied to the logarithmic detrended price.

The ADF PSY test is able to detect the 27th Jan 2013 and the 10th Nov 2013 bubbles, but unlike the other two tests, it also detects a bubble on the 4th Jan 2015. On that day hackers stole 18,886 bitcoins from the bitcoin exchange Bitstamp, that were worth 5.2 million dollars. The attack caused an eight days shut down of all the exchange activities, but no clients' accounts were affected by the theft. However,

even if that amount was only a small fraction of Bitstamp's reserves, the exchange suffered a deep loss in terms of reputation and credibility. In the days following the attack, safety concerns spread, and bitcoin's price dropped by 38% in ten days.

A positive feature of the ADF PSY is that, while the two previous tests detected the 18th June 2017 price drop, it signals the immediately previous rally on the 11th June as a bubble, which is actually the week when prices experienced a rise.

In figure 21 we can also see that the August 2012 price rise does not make the ADF PSY test reach a meaningful significance level.

The ADF PSY test is also able to detect the 3rd Dec 2017 bubble in a much more significant way than the other two tests.

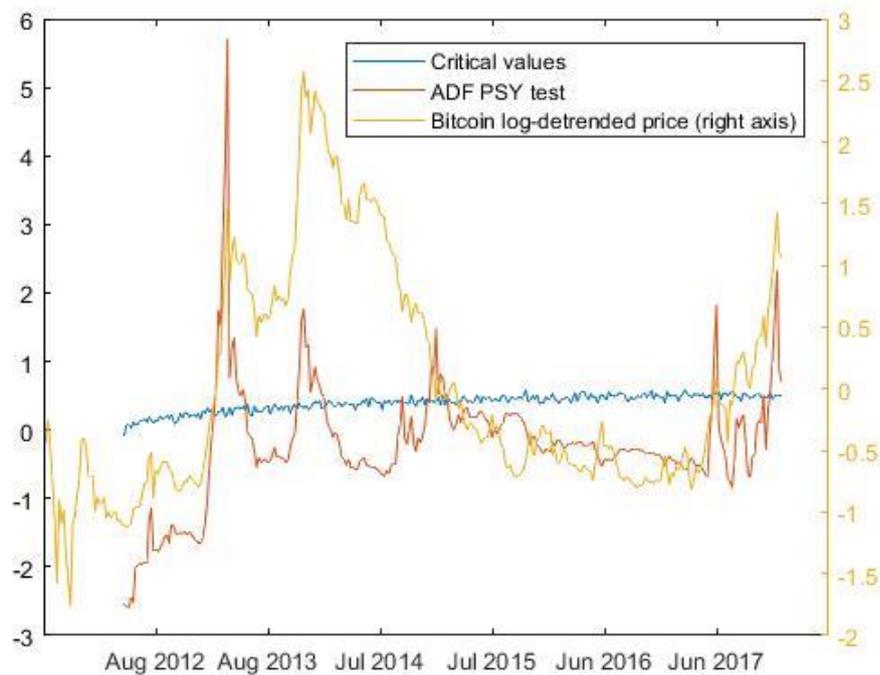


Figure 21: Bitcoin ADF PSY test

4.3 Ethereum

The third time series of interest is the daily Ethereum/USD exchange rate from 30th Aug 2015 to 31st Dec 2017.

As in the case of Bitcoin, Ethereum had an incredible price growth and in the year 2017 its price grew 9091%²⁸. Ethereum's growth has been less volatile than Bitcoin's but nevertheless it created some bubbles along its path.

The Buseti-Taylor test in figure 22 is able to detect five bubble periods: the first one occurred while Ethereum was still in at a beta developing phase. In particular between 7th and 11th February 2016 Ethereum's price doubled from 3 to 6.38 USD because of the collective enthusiasm for the coming Homestead release which was the first version of the platform to be considered stable.

The second detected bubble took place the 17th March 2017 when the Enterprise Ethereum Alliance was created; since big companies became involved with the platform great optimism spread leading to a sudden price rise.

The third moderate bubble occurred at the end of April 2017 and was probably due to the ICO of the TokenCard Project, a start-up that makes it possible to have an app-handled portfolio of virtual and real currencies, and to spend those currencies with a Visa debit card. Since during the ICO 12.7 million USD were collected in 30 minutes²⁹, it's very likely the reason of that price increase.

The most evident bubble however, was in June 2017; in that month 462 million dollars were raised in initial coin offerings via Ethereum so price doubled in the first half of the month, but at a certain point companies started to sell Ether to buy some real currency to start their projects, therefore price settled back to pre-June levels.

²⁸ Calculation on data from: <https://ethereumprice.org/>

²⁹ <https://icobench.com/ico/tokencard>

The last detected bubble took place in the middle of December 2017 and coincides with the incredible growth of Bitcoins' transaction costs. At that time Bitcoins became extremely unpractical for small transactions therefore Ethereum, that was the second largest cryptocurrency, became a very popular alternative. December was also the month when Cryptokitties was released, and given the great popularity of the game and the subsequent rising demand of Ether that it caused, prices surged as a consequence.

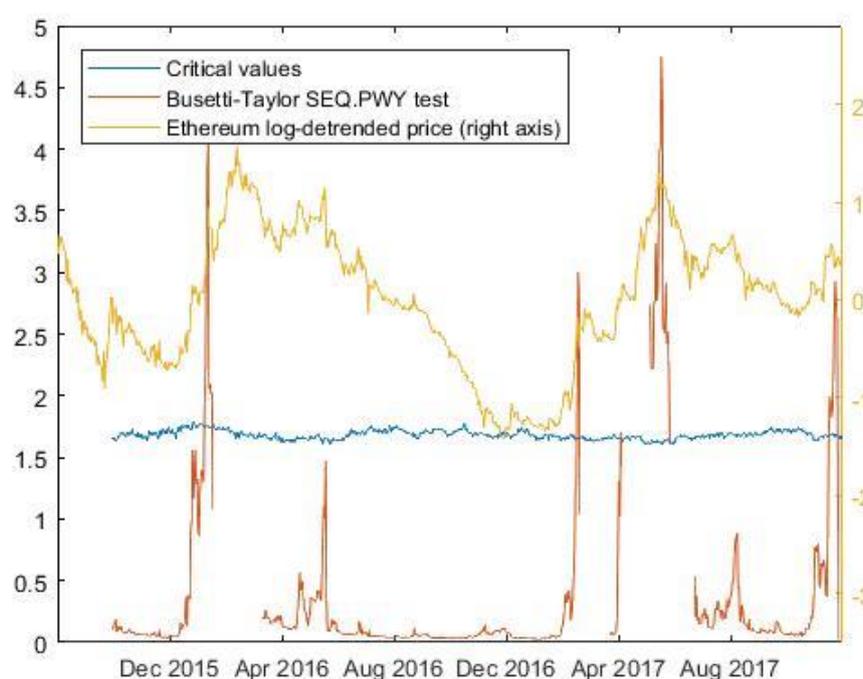


Figure 22: Ethereum Buseti-Taylor Seq.PWY test

In figure 23 the ADF Seq.PWY test detects the same bubbles except for the one on 7th-11th February 2016. Instead of detecting that bubble, the test detects the previous rally on 25th-29th January so it can't detect the one in February because of the reinitialization window.

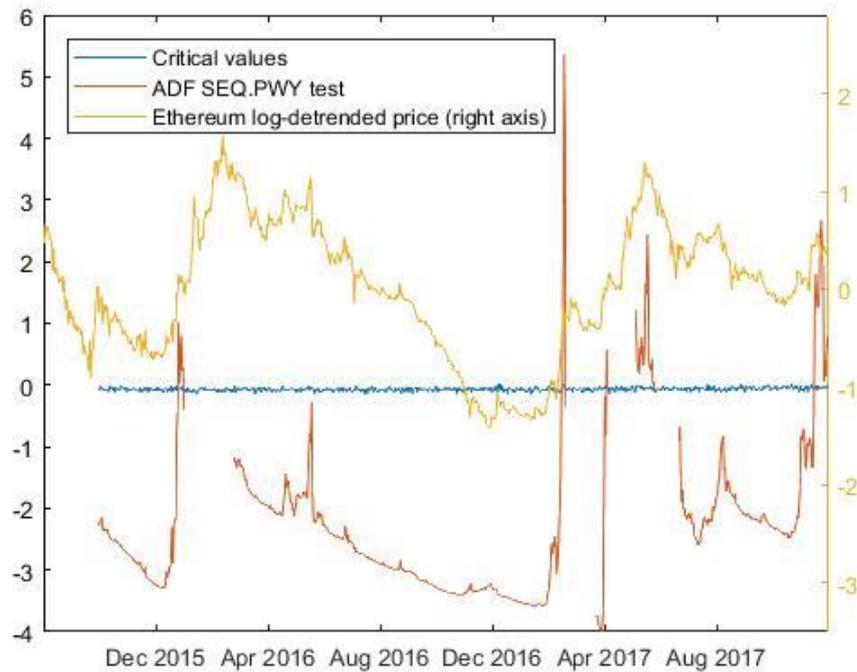


Figure 23: Ethereum ADF Seq.PWY test

The ADF PSY test in figure 24 is the only one that is able to clearly detect all the three explosive behaviours at the beginning of the 2016, namely during the: 24th-25th January, 7th-15th February and 22nd February - 17th March. As previously mentioned, these bubbles were created by the excitement for the introduction of a newer and improved version of the Ethereum platform, the Homestead update that took place on the 14th March. On the contrary, the Buseti-Taylor Seq. PWY only detects the second bubble while the ADF Seq. PWY only detects the first, and since the PWY method requires the reinitialization window everything that happens in the first observations following a bubble is neglected.

Moreover the ADF PSY is also able to detect three other minor bubbles; the one on mid-June 2016 was caused by an ICO issued by the DAO start-up which collected 150 millions in two weeks. Ether's price therefore increased sharply up to the 17th

June when the DAO was subject to a hacker attack that took control of 50 millions worth of Ether. However those money were in a 28-days holding period so the Ethereum's community decided to split the network (via hard forking) to eliminate the effects of the attack.

The second negatively explosive behaviour was on 22nd November 2016, when an accidental hard fork took place. Even if no Ether was stolen, many accounts were frozen and this increased doubts about the security of the platform, so speculators sold their Ether resulting in a price drop of roughly 30% in 13 days. The test detects at a significant level only the 4th-5th December final drop because it was the sharpest.

The third bubble at the end of August 2017 was due to the excitement for Ethereum's developers meeting and the following announcement of the release of a faster and safer version of the platform, called Byzantium.

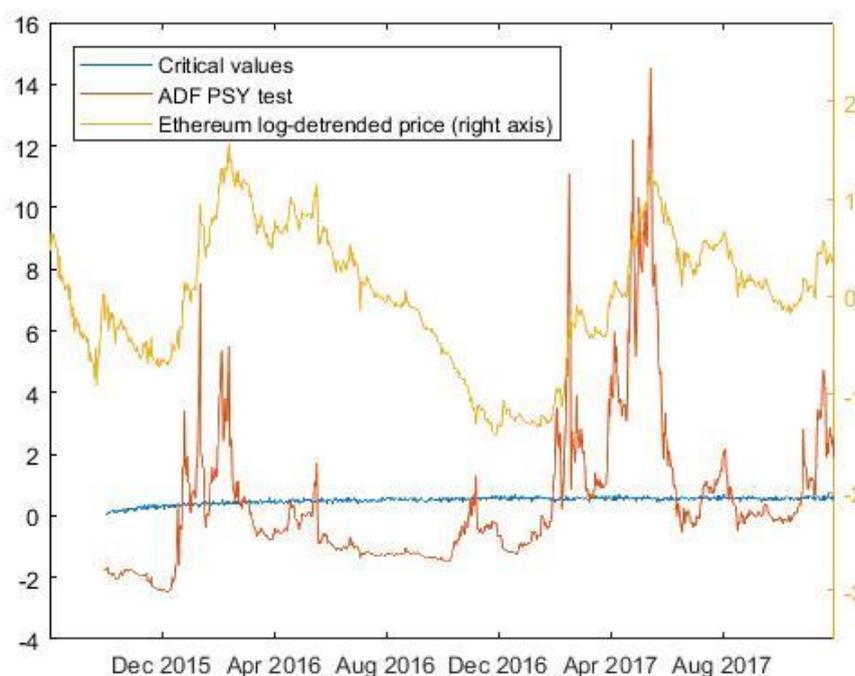


Figure 24: Ethereum ADF PSY test

Conclusions

Throughout this work the reasons that lead to the formation of bubbles have been described; in particular, the most important reason is a collective sudden irrational interest towards a particular asset that leads to an unjustifiable price increase. Bubbles usually develop when market equilibrium is broken and are exacerbated by regulations that don't take in due consideration the economic principles that regulate markets. For example, the Housing Bubbles that occurred in the 20th century in the United States were mostly a consequence of regulations that were put in place in order to pursue the noble cause of increasing homeownership rate, but they simply lacked the consideration that increasing demand for houses has the primary effect of increasing house prices as well. As we have seen, not all the bubbles impact an economy with the same magnitude; some of them have a reduced impact like the Tulip Mania, whose effects were rapidly marginalized by courts. On the other hand, bubbles can also trigger crisis that lead to recession periods, the longest of which has been the Great Depression. We have later described the implementation of the most recent models for bubble detection and compared their performances by applying them to the US House Price Index, because we knew that many bubbles occurred on that time series. The final step was to analyse the Bitcoin's and Ethereum's time series since many have been wondering whether cryptocurrencies are bubbles that will just burst or if they really have some tangible value. The results are that there is evidence of the presence of at least 5 bubbles in the Bitcoin time series and 4 bubbles in the Ethereum's time series. At 31st December 2017, all the applied tests suggest that both Bitcoin and Ethereum are in a bubble phase.

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