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Credit risk and credit derivatives: an analysis of the market of CDS

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INTRODUCTION

Modern history offers several examples of how, more and more frequently, the failures of financial intermediaries are due to excessive and uncontrolled exposures towards creditors who are in situation of extraordinary difficulty. The complicated events that have hit the financial markets in the last two decades, causing first the collapse of the American giants in the financial intermediaries field and later a succession of serious consequences that have overwhelmed the entire world economy, show how credit risk plays a fundamental role in the various causes triggers the crisis. As a result, it is increasingly felt the need of a correct management of credit risk and the creation of operations and financial instruments, which make it possible to negotiate and manage this risk. The thesis aims to be an excursus that starts from the identification of credit risk and the study of the models used to measure credit risk, up to the description of credit derivatives instruments, focusing in the last part on Credit Default Swaps (CDS). In particular, the thesis is structured in 5 chapters.

The first chapter will set the basis for the entire work by given a definition of credit risk and introducing its components: PD, LGD, and EAD. It will be also presented the regulatory framework in which the credit risk is placed, the expected and unexpected loss and an important terminology such as economic capital. In the second chapter, the theoretical foundation for measuring credit risk is provided. We will examine the two main approaches that over time have been developed for modelling credit risk, the structural models approach and the reduced form models approach. In particular, among the structural models we will analyze the Merton model and the first passage time models, highlighting the main differences. Instead, among the reduced-form models we will analyze two contributions, one by Jarrow and Turnbull and the other by Jarrow, Lando and Turnbull.

Going deeply in the analysis, we will focus on credit derivatives, which are financial instruments designed to manage credit risk. Thus, the third chapter will provide a description of what a credit derivative is and which are the key elements that characterized this type of instrument. There will be also provide an overview of the main credit derivatives traded, specifying the functions and the goals that the various market operators can pursue by relying on these financial instruments. Basically, credit derivatives can be categorized as single-name or multi-name. The most popular single-name credit derivative is a credit default swap, while the most popular multi-name credit derivative is a collateralized debt obligation. These two credit derivatives act both as a form of insurance, transferring risk from one party to another. In particular, a CDS is a contract where one party buys insurance from another party against a third party defaulting on its obligations. On the other hand, in a CDO a portfolio of debt instruments is specified and a complex structure is created where the cash flows from the portfolio are linked to different categories of investors.

In the fourth chapter, we will discuss the valuation of a credit default swap by presenting a model widely used by operators of the credit derivatives market. Using the reduced form modelling approach, introduced in chapter 2, we will write the pricing formula for the value of a CDS position. In particular, a continuous time model and a simplified discrete approach will be provided. In the final chapter, a descriptive analysis of some actually quoted CDS will be presented. We will first introduce the role of clearing houses in the CDS market and the features that the CDS must have to be cleared by a clearing house. We will see that over the years, CDS have become increasingly standardized thanks to voluntary industry initiatives, like the ISDA Definitions. We will then discuss the main features and properties of these standard contracts. Next, taking as reference one of the biggest clearing house, the ICE, we will see how CDS are divided into corporate, sovereign and index. For each of these categories, representative samples will be taken reported in summary tables.

Chapter 1

CREDIT RISK

1.1 Introduction

In the context of financial activities can be identified different types of risks to which the owner of the same is exposed. Credit risk, market risk, operational risk, liquidity risk and interest rate risk are the main ones, although they do not form an exhaustive list of the full range of possible risks affecting a financial institution. Above all, credit risk is one of the most important risk in the financial industry. In particular, it was decided to dwell on credit risk because it represents itself more than half of the losses suffered by the financial system as a whole; it is the risk component to which the financial institution are most exposed and even the one that absorbs the largest part of capital and finally it relates directly with bonds and loans which respectively for volume of trading and number of contracts, represent the majority of the transactions completed on the market. Most financial institutions devote considerable resources to the measurement and management of credit risk and regulators have for many years required banks to keep capital to reflect the credit risks they are bearing. In particular, credit risk has acquire growing importance in the last two decades, which have been characterized by several financial disasters, started with the US subprime mortgage crisis and the collapse of Lehman Brothers in 2008. In addition in the last 20 years, we could also observe at international level and more recently in

Italy, an increasing number of default relating to bonds issued by private company. In the same time some bonds issued by emerging countries have not been repay (e.g. the case of the argentine bonds). As a result of such defaults, it is increasingly felt the need of a correct analysis of the credit risk (and its components) inherent in some financial assets. However, credit risk represents one of the most difficult risk to quantify and manage.

First of all, we will try to give a definition of credit risk. According to Ammann (2001) credit risk can be defined as:

> *"the possibility that a contractual counterparty does not meet its obligations stated in the contract, thereby causing the creditor a financial loss."*

This general definition contemplates only the extreme case in which the borrower becomes insolvent. But a loss of value of the creditor position can result also from a deterioration of the economic-financial conditions of the borrower, from which depends the ability (or the willingness) to honour its contractual obligations, even without becoming insolvent. Thus, a more complete definition of credit risk is given by Sironi and Resti (2007) in "Risk management and shareholders' value in banking", who refers to credit risk as:

> *"the possibility that an unexpected change in a counterparty's creditworthiness may generate a corresponding unexpected change in the market value of the associated credit exposure."*

From this definition it is clear then, that credit risk does not refer only to the possibility of the counterparty's default, but, it has to be taken into account also the possibility that the creditworthiness of the borrower will deteriorate. In general, the upgrading or downgrading of the credit quality is called credit migration and ratings, provided by the well-known agencies such as Standard & Poor's (S&P) and Moody's, represents an estimate of the creditworthiness of the credit exposure of the company or of the country.

Summarizing, credit risk includes both the risk of default and the risk of migration. The first represents the risk of losses due to the actual insolvency of the borrower which interrupted the payments. The second represents the risk of losses resulting from a mere deterioration in its creditworthiness, so even a simple downgrade in the rating classes results in a decrease in the market value of the credit and thus a loss.

The complexity of credit risk lies in the fact that, it is not a matter of evaluate a simple distribution where the possible events are "default" versus "not-default" but rather to consider a discrete or continuous distribution, in which default merely represents the extreme event, next to other events in which the borrower remains solvent but the probability of a future default gradually increase. This is the only way to define both categories adequately. Before to proceed with the analysis of credit risk, we give a brief overview of the financial instruments bearing credit risk.

1.2 Financial instruments subject to credit risk

This section provides a brief analysis of the principal types of financial instruments involving credit risk. Essentially they are loans, bonds and derivative contracts.

Loans are the largest and most obvious source of credit risk and come in a myriad of forms. It is common to categorize them, according to the type of obligor, into retail loans, corporate loans (to larger companies), interbank loans and sovereign loans (to governments). In each of these categories there are likely to be a number of different lending products. For example, retail costumers may borrow money from a bank using mortgages against property, credit cards and overdraft. The common feature of most loan is that a sum of money, known as the principal is advanced to the borrower for a particular term, in exchange for a series of defined interest payments, which may be at fixed or floating interest rates. At the end of the term, the borrower is required to pay back the principal. The lender is subject to the risk that borrower is unable to fulfill its obligation to make interest payments or repay the principal, a situation that is termed default. Another important concept we will use often in the sequel, is the so-called exposure, which is the value of the outstanding principal and interest payments.

Another classical source of credit risk is given by bonds. Bonds are publicly traded securities issued by companies and governments that allow the issuer to raise funding on financial markets. Bonds issued by companies are called corporate bonds and bonds issued by governments are known as treasuries or sovereign bonds. The structure is similar to that of a loan. The security commits the bond issuer (borrower) to make a series of interest payments to the bond buyer (lender) and pay back the principal at a fixed maturity. The bondholder is subject to the so called issuer risk, which is the risk that promised coupon and principal payments are not made. Historically, governments bonds issued by developed countries have been considered to be default free but after the European debt crisis of 2010-2012 this notion was called into question.

The concept of credit risk is not limited to the classical uses of a bank but it is also extended to the off-balance sheet items. In particular, we are talking about derivative contracts. A significant amount of all derivatives are traded over the counter, which means that there is no an organized exchange that guarantees the fulfilment of the contractual obligations. These traders are therefore subject to the risk that one of the contracting parties defaults during the transaction, thus affecting the cash flows that are actually received by the other party. This risk, known as counterparty credit risk, received a lot of attention during the financial crisis of 2007-2009, as some of the institutions heavily involved in derivatives transactions experienced worsening credit quality or as in the case of Lehman Brothers even a default event. Counterparty credit risk management, is now a key issue for all financial institutions and is the focus of many new regulatory developments. We also have to mention the derivatives traded on the regulated market in which although the credit risk is much less significant, as observed by Ammann (2001) is still present, the derivatives whose underlying assets involve credit risk (such as options issued on bonds) and the credit derivatives (securities that are primarily used for hedging and trading credit risk).

In the sequel of this chapter, we will contextualize the credit risk within the regulatory framework going briefly explain the three Basel Accords of the Basel Committee, we will analyze the components of credit risk and finally we will introduce the concept of Economic Capital.

1.3 Regulatory framework: the Basel Accords

We now illustrate in this section the main prudential regulation within the credit risk is placed and that have the goal to keep the total capital of a bank or of financial institutions sufficiently high to ensure that the chance of a failure is very low. The need to reach a common banking system regulation and in particular in the aspects of risk measurement and management has led to the creation of an international organization, known as the Basel Committee of Banking Supervision. Much of the regulatory drive originated from the Basel Committee of Banking Supervision.

The Committee was established by the Central Bank Governors of the group of Ten (G10) countries and Switzerland at the end of 1974. Its goal was to bring together the governors of the central banks of the most financially advanced states in order to exchange ideas and to confront on banking supervisory matters and to strengthen the regulation and practices of banks worldwide with the purpose of enhancing financial stability. The Basel Committee does not possess any formal supranational supervising authority and hence its conclusions do not have any legal force. The supervisory rules are rather intended to provide guidelines and recommends for the supervisory authorities of the individual nations, such that they can take steps to implement them in a suitable way for their banking system (www.bis.org).

The Basel Committee issued the Basel Accords, which are guidelines on the capital requirements of banks and financial institutions. In 1988 was proposed the First Basel Accord, also known as *Basel I*, which laid the basis for determining the minimum capital requirements for large international banks. The objective achieved by this accord is the determination of a common measure of capital adequacy. The proposed criterion is that of the 8 % rule, according to which banks have to prove that the capital they hold is at least 8 % of total risk weighted assets, calculated for all balance sheet positions. The assets are weighted according to their degree of riskiness where the risk weights are determined for four different borrower categories: Government, Bank, Mortgages and Companies and Retail Costumers [6].

The regulatory capital, RC, can then be determined as follows:

$$\begin{aligned} RC &= 8\,\% \times RWA \\ RWA &= \sum_i rw_i \times ce_i, \end{aligned} \tag{1.1}$$

where RWA is the risk-weighted assets, rw_i is the risk weight for the i-th category and finally ce_i represents the credit exposure in the i-th category. Since this approach did not care of market risk, in 1996 an amendment to Basel I has been released which allows for both standardized approach and a method based on Value at Risk (VaR) models for market risk in larger banks. The Value at Risk statistically measure the maximum loss a bank can suffer in a certain time horizon. The main weakness of this capital accord, however, was that it made no distinction between obligors with different creditworthiness, which are only divided into categories.

These drawbacks lead to the development of the Second Basel Accord, released in 2004. The rules officially came into force on 2008 in the European Union, however, in practice they have been applied already before that date. The second accord, general known as *Basel II* is structured in a three pillar system. Pillar 1 sets out details for adopting more risk sensitive minimal capital requirements for banking organization, Pillar 2 lays out principles for the supervisory review process of capital adequacy and Pillar 3 seeks to establish market discipline by enhancing transparency in banks' financial reporting.

Concerning credit risk, pillar one is particularly significant since loans issued to similar counterparts (e.g. private firms, or sovereigns) will require different capital coverage depending on their creditworthiness (ratings). In particular, banks are allowed to evaluate the creditworthiness opting between two approaches: the standard approach and the more advanced Internal Ratings Based or IRB approach. In the standard approach, credit risk is measured by external ratings provided by well-known rating agencies such as Standard & Poor's (S&P), Moody's or Fitch Ratings. In the IRB approach, the bank evaluates the risk itself. This approach, however, can only be applied when the supervisory authorities accept it. The bank, therefore, has to prove that certain condition concerning the method and transparency are fulfilled. Basel II distinguishes between expected loss and unexpected loss. The first directly charges equity while for the last banks have to keep the appropriate capital requirements. The approach for market risk is similar to that in the amendment of 1996 and it based mainly on VaR approaches. Another basic innovation of Basel 2 was the creation of a new risk category, the operational risk which is explicity taken into account in the new accord [25].

In the aftermath of the financial crisis erupted in 2008, the Basel Committee considered necessary to work in order to modify the regulation of Basel 2. The new regulation, which came into force on 2013, will become fully effective only on 2019. It is not, however, a new capital accord that completely rewrites the rules to which banks and financial institutions will have to comply with, but, in substance it is an integration of Basel 2 to try to strengthen, one more time, the capital structure of banks. That means that the structure remains that of Basel II but it is integrated and improved by a series of rules introduced with *Basel III.* Essentially, Basel III intervenes in 4 main areas: the regulatory capital, the liquidity risk, the leverage ratio and the coverage of the counterparty credit risk.

Concerning the regulatory capital banks are required to hold both more capital and of better quality. In particular they are asked to hold (in addition to the minimum ratio of 8 %) a capital conservation buffer of 2.5 % of risk- weighted assets capital and a countercyclical buffer which can reach the 2.5 %. This leads to a total ratio of 13 %, compared with Basel II's 8 %.
It will be introduced 2 minimum standards of liquidity: a liquidity coverage ratio and a net stuble funding ratio, to ensure respectively that banks have enough liquid assests to withstand a period of net cash outflow lasting thirty days and that sufficient funding is available on order to cover long term commitments, exceeding one year. This is a completely innovation for the Basel framework, which has previously been concerned only with capital adequacy. In addition, the risk coverage system is extended in particular to include the counterparty credit risk in the valuation of over the counter derivative contract. Finally, a maximum level of leverage ratio will be imposed, in order to contain the excessive leverage in the banking system and which is fixed at 3 % [33].

1.4 Components of credit risk

Once we defined credit risk and introduced the regulatory framework, we can proceed with the determination of its components.
We presented credit risk as the possibility that an unexpected change in a counterparty's creditworthiness may generate a corresponding unexpected change in the value of the associated credit exposure. Against credit risk, the provider of funds is prudentially required to create a capital cushion for covering possible financial losses. If we consider as lender typically a financial institution (a bank), it is obliged to constitute a reserve against the risks taken (expected loss reserve). If we consider that even good costumers have a potential to default on their financial obligations, then it is appropriate an insurance for not only the critical but all the obligors in the bank's credit portfolio. Then, for each obligors can be identified essentially three measures [10]:

- the probability that the borrower becomes insolvent, *Probability of Default* (PD);
- the loss rate in case of insolvency, *Loss Given Default* (LGD);
- the bank's exposure at the time of default, *Exposure At Default* (EAD).

For each obligor is associated a loss variable, $\tilde{L}$, defined as follow:

$$\tilde{L} = EAD \times LGD \times L. \tag{1.2}$$

Under this model exists an appropriate probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where Ω is the sample space, $\mathcal{F}$ is the σ-Algebra representing the information available and $\mathbb{P}$ is a probability measure. All the components of the formula (1.2) are random variables. In particular L, represents the random variable which describe the default event (D). Taking as time horizon one year, can be sufficiently reasonable to accept a vision of the solvency of the obligor that includes only two possible states: default or not default.

Then L can be defined as a Bernoulli or dichotomous random variable[1], often used to study phenomena which provide only two events mutually incompatible. Algebraically, we can indicate L as[2]:

$$L = L(\omega) = \begin{cases} 1 & \text{if } \omega \in D \\ 0 & \text{otherwise} \end{cases} \tag{1.4}$$

with $\mathbb{P}(L = 1) = PD$ and $\mathbb{P}(L = 0) = 1 - PD$.

This approach can be considered as a form of measuring the level of risk associated to a single exposure in a context default-mode[3], where losses can occur exclusively in case of default without consider the possibility of changes in the creditworthiness of the obligor.

We will discuss separately the three components of credit risk in the next sections.

1.4.1 The Probability of Default

The key ingredient of credit risk is the probability of default. Before to proceed with the analysis of the probability of default, is necessary to give a definition of what we mean with the term default and in particular, when a counterpart become insolvent.

In general, there is insolvency in any circumstance in which the obligor is unable to fulfill its contractual obligations. According the definition used by the Basel Committee in the first pillar of Basel II, a default occurs with

[1] A dichotomous or Bernoulli variable is a random variable which can assume only two values, such as 1 with probability p and 0 with probability $(1 - p)$. The expectation and the variance of the variable are respectively $\mathbb{E}(X) = p$ and $\mathbb{V}(X) = p(1 - p)$. Since L is a Bernoulli random variable then $\mathbb{E}(L) = PD$ and $\mathbb{V}(L) = PD(1 - PD)$.

[2] L can be also defined as the indicator function of the event default D:

$$L = \mathbb{1}_D(\omega) = \begin{cases} 1 & \text{if } \omega \in D \\ 0 & \text{otherwise} \end{cases} \tag{1.3}$$

with $\mathbb{E}(\mathbb{1}_D(\omega)) = PD$. Note that ω can be omitted when the meaning is clear.

[3] The opposed model is the so-called multinomial models often called mark-to-market model, since it recalculates the value of the exposure "at market price". In this model even a simple downgrade in the creditworthiness of the obligor is considered a loss producing a decrease in the market value of the credit.

regard to a particular obligor when either or both of the following events take place: the obligor is unable or unwilling to pay in full and in any case after a delay of over 90 days. For example, if an individual fails to make his monthly mortgage payments, he defaults on the loan. Similarly, if a company issues bonds and it is unable to make coupon payments to its bondholders, the company is in default on its bonds.

Then, we can define the probability of default as the likelihood that the default event will occur over a particular time horizon, usually one year. What differentiate counterparts is the level of PD, higher the probability of default, greater will be the risk associated with a specific counterpart.

The task of assigning a default probability to every costumer in the bank's credit portfolio is far from being easy. There are various ways for estimating the probability of default but broadly speaking, these approaches may be divided into three categories (Resti and Sironi, 2007).

In the first category, we found a class of statistical models which use the accounting data, generally known as "*credit-scoring models*". Of this class it is possible to identify different methods: the models based on the linear discriminant analysis, the regression models (linear, logit and probit) and some recent heuristic inductive models such as neural networks and genetic algorithms. Although these models have provided positive and empirically valid results, they have highlighted some weakness. The second category includes models that use the market data in order to estimate the probability of default. In particular, we can divide models that use bonds prices or, to be more precise credit spreads[4] from the models that use the stock prices, generally known as "structural models". Next to these approaches, the probability of default can be estimating also from credit ratings, so the last category is that of the Rating Systems.

Statistical methods and Rating Systems are not subject to be discussed in this thesis, while we will return on estimating default probabilities from market data, in the context of structural models in chapter 2.

[4]It is the difference between corporate bonds and risk free-free government bonds.

1.4.2 The Exposure At Default

We now present another component of credit risk, the exposure at default. It is the quantity indicating the exposure that the bank has to its borrower. The credit exposure may have a certain or uncertain value. Its estimate can be sometimes easy to determine and sometimes complex according to the type of facility granted to the borrower. For example, if we consider a loan the exposure is relatively easy to determine, since many bank loans follow a pre-determined repayment plan of the capital and interest, so that the costumer has no discretion as to the amount of the loan that she/he will be using in the future. The same applies to bonds, where cash flows are entirely defined at the time of issue. However, there can be some additional uncertainty due to the widespread use of credit lines. Essentially, a credit line is a maximum to which the bank undertakes to lend a certain amount of funds to the costumer and it is up to the borrower to decide which part of the credit line he actually wants to use. This means that the true size of the actual loan may vary over time due to decisions external to the bank. For OTC derivatives, the credit exposure is even more difficult to quantify, since it is a variable depending on the unknown time at which a counterparty defaults and the evolution of the value of the derivative up to that point.

In general, the exposure at default is made up of two main parts, the outstandings and the commitments. The outstandings refer to the portion of the exposure already drawn by the obligor and the commitments is the exposure the bank has promised to lend to the obligor at her or his request. In particular, the commitments can be divided in two portions, the undrawn and the drawn, in time before default. History shows that usually, obligors tend to draw on committed lines of credit in times of financial distress. Then, the exposure at default can be indicated as:

$$EAD = OUTST + \gamma \times COMM, \tag{1.5}$$

where $OUTST$ denotes the outstandings, $COMM$ the commitments of the loan and γ is the percentage of the commitments that could be use by the borrower close to time to default. The amount of this coefficient depends on a bank valuation, which will take into account both the creditworthiness of the counterpart and the type of facility involved. As a result, in case of borrower's default, the bank will be surely exposed for the total amount of the outstandings but in addiction it increases its exposure for the drawn part (prior to default) of the commitments.

1.4.3 The Loss Given Default

The last of the three components of credit risk we consider, is the Loss Given Default (LGD). It represents the loss that the lender will really suffer if the borrower default. It is given by one minus the recovery rate and can take any value between 0 % and 100 %. Formally:

$$LGD = 1 - RR, \tag{1.6}$$

where RR indicates the recovery rate. In practice, once we know the exposure to the obligor, in the event of default it is unlikely that the entire exposure is lost, because due to collaterals, the financial institution will be able to recover part of its exposure. For example, when a mortgage holder defaults on a residential mortgage and there is no realistic possibility of restructuring the debt, the lender can sell the property (the collateral asset) and the proceeds from the sale will make good some of the lost principal. The percentage recovered by the creditor is called *recovery rate* while its complementary (that is the portion of the exposure that is actually lost) determines the loss given default.

If we take into consideration a bond, its recovery rate is normally defined as the bond's market value a few days after default, as a percent of its face value. The recovery rates are not known when a bond is issued, nor they are perfectly known when the default occurs. Generally speaking, the recovery rates can be known with certainty only when the whole recovery process is over. It results that the recovery rate is a random variable for which can be only elaborate some expectations and try, through different ways, to estimate. But the recovery rates depend on many driving factors and results difficult to estimate, especially for the lack of historical data about defaults, actual recoveries and recovery time. The factors affecting recovery rates can be grouped in four main categories: the characteristics of the credit exposure, those of the borrower, the peculiarities of the bank managing the recovery process and finally, some external factors [33].

The aspect that has a greater influence on the recovery rate is certainly the characteristics of the credit exposure. They include the presence of collaterals (such as plants, real estate inventories etc) and how easily they can be seized and liquidated, the presence of form of seniority or of subordination to other creditors. Among the characteristics of the borrower that may affect the level of LGD is the country or the geographical area in which the obligor operates which may affect the speed and effectiveness of the bankruptcy procedures. But of greater impact is the industry in which the company operates. Finally, it is proven that, also external factors like the state of the economic cycle and the level of interest rates drive recovery rates.

As we have seen, the factors to take into consideration are many. A bank external source for recovery data comes from the rating agencies, which provide estimates on recovery values of defaulted bonds distinguishing between different seniorities.

1.5 The Expected and Unexpected Loss

Once identify all the components of credit risk (PD, LGD and EAD) and assuming a time horizon of one year, it is possible to proceed with the analysis of credit risk and determine the expected and unexpected loss for a single facility. The possible loss on a credit exposure can be broken down into two components: expected loss and unexpected loss.

The *Expected Loss* (EL) represents the loss the lender is able to estimate ex ante against a credit exposure and in this context can be defined as the expected value of the loss variable $\tilde{L}$, that is:

$$EL = \mathbb{E}(\tilde{L}) = \mathbb{E}(EAD \times LGD \times L). \tag{1.7}$$

In order to proceed with the computation of the EL we make some strong assumptions, namely that EAD and LGD are constant values over time. What we obtain is:

$$EL = EAD \times LGD \times \mathbb{E}(L) = EAD \times LGD \times PD, \tag{1.8}$$

where from the definition given previously of L, we have that $\mathbb{E}(L) = \mathbb{P}(D) = PD$ since the expectation of any Bernoulli random variable is its event probability.

A focus on expected loss is not enough, since being expected, it does not strictly represent a risk. In fact, the most important part of credit risk against which financial institutions have to protect is the unexpected loss. Banks besides the expected loss have to consider its variability, namely how much the actual loss can deviate from the expected one. Consequently, a good measure for the *Unexpected Loss* (UL) of a single facility is the standard deviation of the loss variable, $\tilde{L}$:

$$UL = \sqrt{\mathbb{V}(\tilde{L})} = \sqrt{\mathbb{V}(EAD \times LGD \times L)}. \tag{1.9}$$

Under the same assumptions made before, EAD and LGD constant values over time, we can rewrite equation (1.9) as:

$$UL = EAD \times LGD \times \sqrt{\mathbb{V}(L)}, \tag{1.10}$$

where from the definition of L, we know that $\mathbb{V}(L) = DP(1-DP)$. Replacing we obtain:

$$UL = EAD \times LGD \times \sqrt{PD(1 - PD)}. \tag{1.11}$$

The assumptions made so far, are very simplifying and not always realistic. There are various situations in which EAD and LGD are more properly described by random variables. As we saw before, the value of the exposure and the loss rate in case of default depend in general from various random situations like: the technical form of financing, the amortisation plan, the presence of form of seniority, possible collaterals, legal uses and constraints, other factors affecting the time and the method of quantifying the loss and the fraction of credit that can be recovered. We now assume that LGD is a random variable, independent from the variable L. In this case we can rewrite the equation (1.9) as:

$$UL = EAD \times \sqrt{\mathbb{V}(LGD \times L)}. \tag{1.12}$$

Taking $\mathbb{V}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$ we can compute the variance of the product of LGD and L as:

$$\begin{aligned} \mathbb{V}(LGD \times L) &= \mathbb{E}(LGD^2 \times L^2) - \mathbb{E}(LGD \times L)^2 \\ &= \mathbb{E}(LGD^2) \times \mathbb{E}(L^2) - \mathbb{E}(LGD)^2 \times \mathbb{E}(L)^2, \end{aligned} \tag{1.13}$$

because LGD and L are independent by assumption. Because L is a Bernoulli r.v. we have $\mathbb{E}(L^2) = \mathbb{E}(L) = PD$ such that:

$$\mathbb{V}(LGD \times L) = \mathbb{E}(LGD^2) \times PD - \mathbb{E}(LGD)^2 \times PD^2. \tag{1.14}$$

Now we add and subtract the same quantity $PD \times \mathbb{E}(LGD)^2$ and collect for PD:

$$\begin{aligned} \mathbb{V}(LGD \times L) &= \mathbb{E}(LGD^2) \times PD - PD \times \mathbb{E}(LGD)^2 - \mathbb{E}(LGD)^2 \times PD^2 + \\ &+ PD \times \mathbb{E}(LGD)^2 \\ &= PD \times [\mathbb{E}(LGD^2) - \mathbb{E}(LGD)^2] + \mathbb{E}(LGD)^2 \times (-PD^2 + PD) \\ &= PD \times \mathbb{V}(LGD) + \mathbb{E}(LGD)^2 \times PD(1 - PD). \end{aligned}$$

Replacing the variance founded in the initial formula we obtain:

$$UL = EAD \times \sqrt{\mathbb{V}(LGD) \times PD + \mathbb{E}(LGD)^2 \times PD(1 - PD)}. \tag{1.15}$$

We now extend the analysis to a portfolio of exposure. The unexpected loss can be reduced by trying to diversify the portfolio: in fact the volatility of the total portfolio loss is generally lower than the sum of the volatilities of the losses on individual loans, especially if the correlation between individual loans is low. On the other hand, expected loss cannot be reduced by diversifying portfolio because it is simply equal to the sum of the expected losses on the individual loans in it. We now consider a portfolio consisting of N loans and define the loss variable on the single facility as:

$$\tilde{L} = EAD_i \times SEV_i \times L_i \quad \text{with } i = 1, \ldots, N. \tag{1.16}$$

Then, the portfolio loss variable is defined as:

$$\tilde{L}_p = \sum_{i=1}^{N} \tilde{L}_i = \sum_{i=1}^{N} EAD_i \times LGD_i \times L_i. \tag{1.17}$$

The portfolio expected loss is given by the expected value of the variable $\tilde{L}_P$:

$$EL_P = \sum_{i=1}^{N} EL_i = \sum_{i=1}^{N} EAD_i \times LGD_i \times PD_i. \tag{1.18}$$

The portfolio unexpected loss is computed as the standard deviation of the loss variable $\tilde{L}_P$:

$$UL_P = \sqrt{VAR(\tilde{L}_P)} =$$

$$= \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} COV(EAD_i \times LGD_i \times L_i \ ; \ EAD_j \times LGD_j \times L_j)}. \tag{1.19}$$

As in the case of a single position, we can compute the unexpected loss under different assumptions. First, we assume LGD and EAD constants value. In this case the unexpected loss of a portfolio can be rewritten as:

$$UL_P = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} EAD_i \times EAD_j \times LGD_i \times LGD_J \times COV(L_i L_j)}. \tag{1.20}$$

The covariance between two random variable, X and Y, can be expressed as:

$$\mathbb{COV}(X, Y) = \rho(X, y) \times \sqrt{\mathbb{V}(X)\mathbb{V}(Y)},$$

where $\rho(X, Y)$ denotes the correlation between the two random variables.

This rule can be apply to obtaining a new formulation for the formula of the unexpected loss for a portfolio:

$$UL_P = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} EAD_i \times EAD_j \times LGD_i \times LGD_J \times \sqrt{\mathbb{V}(L_i)\mathbb{V}(L_j)} \times \rho_{i,j}}. \tag{1.21}$$

Since the variance of the loss variable is equal to $VAR(L) = PD(1-PD)$, applying this rule to the formula we obtain the final formula of the unexpected loss for a portfolio of exposures:

$$UL_P = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} EAD_i EAD_j \times LGD_i LGD_j \sqrt{PD_i(1-PD_i)PD_j(1-PD_j)}\rho_{i,j}}. \tag{1.22}$$

Looking at the case in which also LGD is a random variable, independent from the event default, the unexpected loss for a portfolio is given by:

$$UL_P = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} EAD_i \times EAD_j \times COV(LGD_i \times L_i \ ; \ LGD_j \times L_j)}. \tag{1.23}$$

One of the main difference between expected and unexpected loss consists in the ways of covering the losses. We said that the expected loss, being expected do not represents a risk in strict sense. The true risk event is represented from the possibility that occurs events which determine a deviation from the value of the expected loss. The expected loss must be covered through balance sheet provisions and “loaded” directly on the interest rate charged to the borrower. The unexpected loss on the other hand, must be covered by an adequate amount of equity, and for this reason it is also referred to as the “Economic Capital” absorbed by a credit exposure or by a portfolio of credits. We will see the concept of the economic capital in the next section.

1.6 Economic Capital (EC)

Following the logic illustrated so far, the possible loss on a credit exposure can be broken down into two components: expected and unexpected loss. As we previously discussed, credit risk is more properly represented by the unexpected loss, namely the variability of the loss around its expected value.

There are various ways to quantify unexpected loss. The simplest one is that we have seen of the standard deviation of the probability distribution of future losses. As an alternative, a percentile of the distribution of future losses can be used, determined according to a certain confidence level. This second approach leads to the determination of the so called *Economic Capital* (EC).

The economic capital is a measure of risk expressed in terms of capital or in other words it is the amount of money which is needed to secure survival in the worst-case scenario. Synonymous of Economic Capital are Credit Value at Risk (Var) and Capital at Risk (CaR).

Since before we talked about regulatory capital in the context of Basel accords, a clarification is needed. The concept of economic capital differs from regulatory capital in the sense that regulatory capital is the mandatory capital regulators require to be maintained, while the economic capital is the best estimate of requires capital that financial institutions use internally to manage their own risk.

For a fixed level of confidence α, the economic capital is defined as the difference between the α-quantile of the portfolio loss, $\tilde{L}_P$ and the expected loss of the portfolio:

$$EC_\alpha = q_\alpha - EL_P, \tag{1.24}$$

where q_α is the α-quantile of $\tilde{L}_P$, determined by:

$$q_\alpha = inf \left\{ q > 0 : \mathbb{P}[\tilde{L}_P \leq q] \geq \alpha \right\}. \tag{1.25}$$

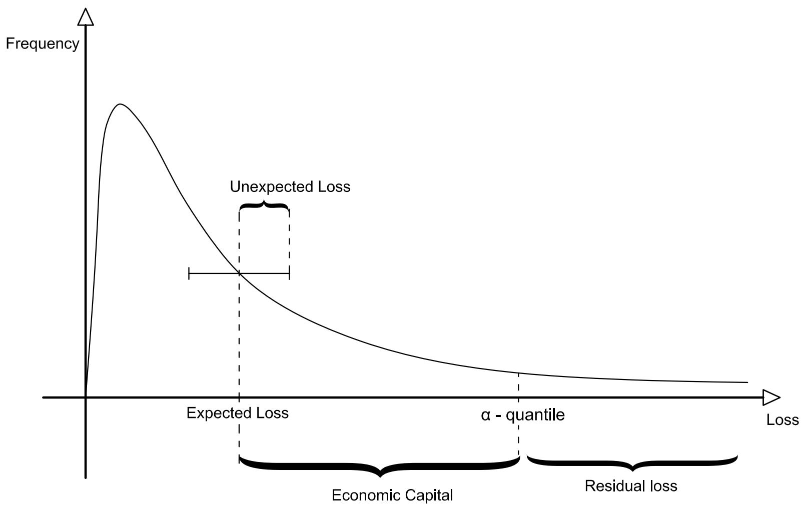

Figure 1.1: The portfolio loss distribution and determination of economic capital (source: Bluhm, Overbeck and Wagner).

The risk quantities of the credit portfolio defined above and the computation of the economic capital are graphically illustrated in figure 1.1. In particular, the area below the curve exceeding the α-quantile represents the potential residual loss. The loss distribution reproduces a typical loss distribution (in particular a beta distribution) with an important asymmetry and leptokurtosis which means that the distribution presents heavy tails.

Chapter 2

MODELS TO MEASURE CREDIT RISK

2.1 Introduction

In this chapter we will present some approaches to measure credit risk. In literature, credit risk modelling can be essentially divided into two common groups. The first group includes *structural models*, while the second includes *reduced form models*.

Structural models are so called because they focus on the evolution of the asset's value of the firm (and therefore on the capital structure, i.e. the value of equity and debt) to determine its probability of default. They can be divided, in turn, into firm-value models and first passage time models.

Firm-value models were originally proposed by Merton in 1974 and they are based on the option pricing theory of Black ans Scholes (1973). Broadly speaking, in a firm-value model the default occurs when the asset's value falls below the liabilities. On the other hand, first passage time models, originated with Black and Cox (1976), consider the possibility of default prior the debt maturity as soon as the asset's value falls below a certain threshold (called default boundary or default barrier), which can assume different forms.

The second group of the reduced form models, known also as *hazard rate models*, represent a more recent approach to credit risk. In contrast to the first group, these models treat the firm default like a completely exogenous event, independent from the capital structure of the firm or country. In particular, the event of default is specified in terms of an exogenous process which is typically a Poisson process.

In the next sections we will review these models, highlighting the main features and differences.

2.2 Structural models

Structural models has been very influential in the analysis of credit risk and in the development of industry solutions, so that this is a natural starting point for a discussion of credit risk models. The primary reason for this importance appears to be the fact that, these models provide a concrete link between the default event and the firm's economic fundamentals such as its value, its capital structure and the complex negotiations between debt-holders and equity-holders. In fact, they are based on the evolution of the value of the firm's assets (and therefore on the capital structure) and on the option pricing theory.

The fundamental structural model has been proposed by Merton in 1974, who had the idea to apply the option pricing theory of Black and Scholes (1973) for the assessment of bonds and other financial instruments which lead to credit risk, giving life to firm-value models.

Over the years several authors extended the Merton model, introducing the possibility that default occurs also before the maturity of the debt and precisely, as soon as the value of the assets falls below a certain threshold, called default boundary or default threshold. Therefore, we have the subclass of models known as *first passage time models*, which originated from Black and Cox (1976).

We begin with a detailed analysis of the seminal model of Merton.

2.2.1 The Merton model

The model proposed by Merton is the core of the structural models and it is based on one very simple intuition: the company defaults when the value of its assets is lower than the value of its liabilities at debt maturity. The model considers a simple capital structure for the company made up of [24]:

- the company's assets, V;
- the risk capital paid by shareholders, E;
- the debt capital, D, composed just by one single liability namely, a zero coupon-bond (ZCB) with face value K and maturity T;

The current value at time t (today) of assets, equity and debt are denoted by V_t, E_t and D_t. The following relation holds:

$$V_t = E_t + D_t, \qquad t \in [0, T] \tag{2.1}$$

which means that the value of the company's assets is equal to the sum of the value of debt and the value of equity.

A basic assumption made in the model is that default may occur only at maturity of the debt. At time T, shareholders and bondholders receive the appropriate payouts observing the absolute priority rule, i.e. the payments to bondholders have a higher priority than other payouts. At maturity T, there are two possible scenarios described in the following [24].

First scenario: the value of the firm's assets is higher than the nominal value of the liabilities.

In this case there is no default. Bondholders receive the promised payment K and shareholders receive the residual value which results from the difference between the assets value and the debt:

$$\text{if} \quad V_T > K \implies \begin{cases} D_T & = K \\ E_T & = V_T - K \end{cases}$$

Second scenario: the value of the firm's assets is lower than its liabilities, the firm cannot meet its financial obligations and therefore default happens.
In this case, shareholders have no interest in providing new equity capital, as these funds would go immediately to the bondholders. They therefore let the firm go into default. Control over the firm's assets passed to the bondholders, who liquidate the firm and distribute the proceeds among themselves. Shareholders receive nothing, so that we have:

$$\text{if} \quad V_T \leq K \implies \begin{cases} D_T & = V_T \\ E_T & = 0 \end{cases}$$

Summarizing, the pay-off function at time T for shareholders is:

$$E_T = max\ (V_T - K, 0). \tag{2.2}$$

Equation (2.2) implies that the value of the firm's equity at time T corresponds to the pay-off of a long position in a European call option[1] with underlying the asset's value of the firm, V_t and strike price the face value of the debt, K, as shown in graph in Figure 2.1.

On the other hand, the pay-off function at time T for bondholders is:

$$D_T = min\ (V_T, K). \tag{2.3}$$

If we add and subtract a same quantity K to the equation (2.3) and consider that $max\ (f(x)) = -min(-f(x))$, we can rewrite the (2.3) as:

$$D_T = K - max\ (K - V_T, 0). \tag{2.4}$$

[1]There are two basics types of options: call option and put option. A *call option* gives the holder the right to buy the underlying by a certain time for a certain price which is called the strike price. A *put option* gives the holder the right to sell the underlying by a certain date at the strike price. Another distinction that is generally made in terms of options concerns the possibility of early exercise. An American options can be exercised at any time up to the expiration date, while European options can be exercised only at expiration date.

Equation (2.4) implies that the value of the firm's debt at maturity is equal to the pay-off of a short position in a European put option with underlying the asset's value of the firm, V_t and exercise price equal to K. The payoff profile of this position is shown in Figure 2.2. For values of V_T that are higher than the face value of the debt K (such as V_2), the asset value is enough to pay the bondholders the entire principal and related interest. The difference $(V_2 - K)$ remains to the company's shareholders. Conversely, for values of V_T less than the value of the debt (such as V_1), the company is insolvent and bondholders receives only partially the amount due.

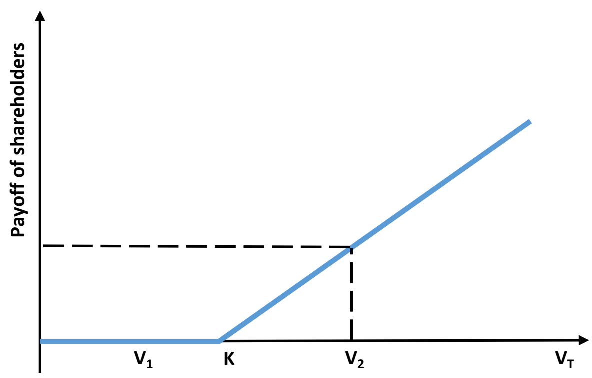

Figure 2.1: Payoff profile of shareholders.

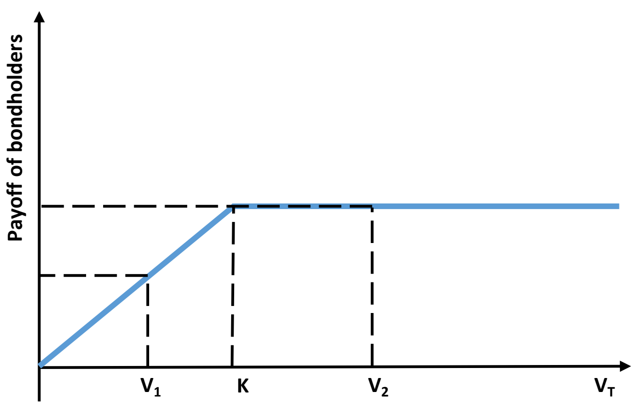

Figure 2.2: Payoff profile of bondholders.

From the results shown above, it follows that the well-known Black and Scholes option pricing formulas can be applied to price equity and debt, but some modeling assumptions needed.

The Merton model assumes that the asset value follows a geometric Brownian motion process, described by the following stochastic differential equation:

$$dV_t = \mu V_t dt + \sigma_V V_t dW, \tag{2.5}$$

where W represents a Wiener process[2], μ denotes the expected rate of return of the assets and σ_V is the volatility of the assets. Therefore, the process is composed by a deterministic part, μ, from which depends the evolution of the assets and a random part, dW, which generates oscillations, whose size depends on the volatility of the company's assets.

It can be proven, by an application of the Itô's formula, that the geometric Brownian motion admits the following and unique solution [24]:

$$V_T = V_t e^{(\mu - \frac{1}{2}\sigma_V^2)T + \sigma_V \sqrt{T} Z}, \qquad V_t > 0, \tag{2.6}$$

where Z is a standardized normal r.v., $Z \sim N(0,1)$. Then, it can be deducted that the final value of the the firm's assets is distributed according to a lognormal random variable:

$$V_T \sim ln \left[(\mu - \frac{1}{2}\sigma_V^2)T + log(V_t)\ , \sigma_V \sqrt{T} \right]. \tag{2.7}$$

[2] A Wiener process is a continuous time stochastic process. In particular, a stochastic process $W = (W_t)_{t \geqslant 0}$ is called a Wiener process if the following conditions hold:

- $W(0) = 0$ a.s;
- $W(t)$ has stationary independent increments, i.e. for all $r < s \leq t < u$ then $W(u) - W(t)$ and $W(s) - W(r)$ are independent stochastic variables;
- for $s < t$ the stochastic variable $W(t) - W(s)$ has the Gaussian distribution $N(0, t - s)$.

It has continuous trajectories. A Wiener process is also called a standard Brownian motion.

As we can see from Figure 2.3, the value of the assets tends to move over time in a exponential manner. In addition, the figure shows also the density function of V_t in order to highlight the distribution of the final value of the process.

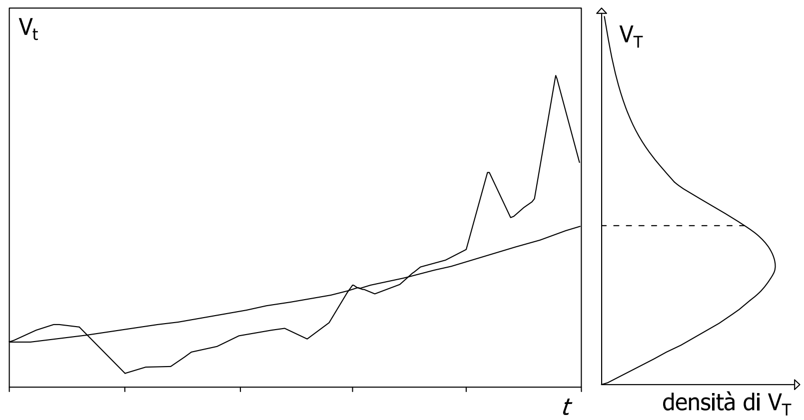

Figure 2.3: Evolution of the firm assets over time (source: Leone and Boido).

Before to proceed with the analysis, we need to introduce a last condition which is the absence of arbitrage between the value of the assets and the risk-free investments (risk-neutral world). This implies that the expected rate of return of all securities is equal to the risk-free rate ($\mu = r$). This condition guarantees against the possibility to realize profit higher than the risk free rate without assuming any risks.

Now, given these assumptions, we can price the equity by applying the Black-Scholes formula for European call option [18]:

$$E_t = V_t N(d_1) - Ke^{-rT} N(d_2), \tag{2.8}$$

where $N(\cdot)$ is the cumulative standard normal distribution, r is the risk free interest rate (assumed constant) and d_1 and d_2 are given by:

$$d_1 = \frac{log(V_t/K) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \qquad \text{and} \qquad d_2 = d_1 - \sigma_V \sqrt{T}.$$

Once determined the value of equity E_t, we can easily obtain the formula for the valuation of debt by substituting E_t in the relation $A_t = D_t + E_t$ and explicit for D_t.
The obtained result is:

$$\begin{aligned} D_t = V_t - E_t &= V_t - \left[V_t N(d_1) - Ke^{-rT} N(d_2)\right] \\ &= V_t - V_t N(d_1) + Ke^{-rT} N(d_2) \\ &= V_t \left[1 - N(d_1)\right] + Ke^{-rT} N(d_2) \\ &= V_t \; N(-d_1) + Ke^{-rT} N(d_2). \end{aligned} \tag{2.9}$$

Under this framework, the risk-neutral probability that the company will default by time T is given by:

$$\mathbb{P}(Default) = \mathbb{P}(V_T < K) = N(-d_2),$$

which is the probability that shareholders will not exercise their call option to buy the assets of the company for K.

It can be proven that debt holders (e.g. a lending bank) can hedge their position completely by purchasing an European put option written on the assets of the funded firm, V_t and strike price K.
As we can see from Table 2.1, at $t = 0$ the debt holder buys a bond paying D_0 and buys a put paying P_0. At maturity, if the assets value of the financed firm are higher than the liabilities, bondholder does not exercise the put and obtains the reimbursed value of the bond, equal to K. If at maturity the assets are lower than liabilities, the debt holder will obtain only the assets value V_T but he can exercise the put and he will obtain also the difference between the value of liabilities and the value of the assets $(K - V_T)$.

In both cases the bondholder is protected against default risk because at the maturity date, the debt holder payout equals K no matter if the obligor defaults or not. Therefore, the credit risk of the loan is neutralized and completely hedged [10].

Table 2.1: Credit protection by a put option.

		t=T	
	t=0	$V_T > K$	$V_T < K$
Loan	$-D_0$	K	V_T
Pay-off put	$-P_0$	0	$K - V_T$
Total	$-(D_0 + P_0)$	K	K

The combination of these two positions (debt and put option) forms a risk-free position. Then, its equilibrium value $(D_0 + P_0)$ must be equal to the present value of a risk-free security paying K on maturity. In short [33]:

$$P_0 + D_0 = Ke^{-rT}, \tag{2.10}$$

where the value of the put option, P_0, can be determined by applying the Black and Scholes formula for put option:

$$P_0 = Ke^{-rT}N(-d_2) - V_tN(-d_1). \tag{2.11}$$

The option interpretation of equity and debt is useful also to explain potential conflicts of interest between shareholders and bondholders of a company (so called “agency problems”). The value of the call and put option increases with the firm’s volatility. Therefore, shareholders are generally inclined to press their managers for taking risky projects (higher volatility) because their position is a long call. The opposite is true for bondholders. They have a short position in a put option on the firm’s assets and would therefore like to see the volatility of the asset value reduced.

One interesting result from Merton’s model is the definition of a credit spread (the risk premium). The credit spread represents the premium required to hold an instrument subject to credit risk and is commonly expressed as the difference between the yield of a defaultable zero-coupon bond and that of a default-free zero-coupon bond. The corporate debt is a risky bond and thus should be valued at a credit spread.

Let s denote the continuously compounded credit spread, then the bond price D_t can be written as:

$$D_t = Ke^{-(r+s)T}. \tag{2.12}$$

Putting together (2.10), (2.11) and (2.12) gives a closed-form formula for s [35]:

$$s = -\frac{1}{T} log \left[N(d_2) + \frac{V_t}{Ke^{-rT}} N(-d_1)) \right]. \tag{2.13}$$

This formula indicates that the credit spread depends on the risk-free rate, the debt level or leverage of the firm and the volatility of the firm's assets. This implies that more risky the firm is, the higher the required spread for the financing [33].

One problem that arises from the model just described, consists in the determination of the market value V_t and the volatility σ_V of the assets which are not directly observable. However, if the company is publicly traded, the firm's equity value, E_t, is observable in the market and is given by the company's market capitalization:

$$E_t = number\ of\ shares \times value\ of\ one\ share.$$

Also the volatility σ_E of the firm's equity is in some manner observable or predictable. V_t and σ_V are not observable. We need a second equation. It can be proven that the following relation between the volatilities of equity and assets holds[3]:

$$\sigma_E E_t = N(d_1)\sigma_V V_t. \tag{2.14}$$

It follows that for estimating V_t and σ_V we have to solve a system of two equations (2.8) and (2.14).

[3]One common way of extracting V_t and σ_t involves assuming another geometric Brownian motion model for equity price E_t and applying Ito's lemma to show that instantaneous volatilities satisfy: $\sigma_E E_t = \frac{\partial E_t}{\partial V_t}\sigma_V V_t$. Black and Scholes call option delta can then be substituted into the previous equation to obtain: $\sigma_E E_t = N(d_1)\sigma_V V_t$.

Concluding, the Merton model describes in a very simplified form an easy approach for debt and equity valuation, but it requires some assumptions which are not applicable in real life. Over the years several authors worked on these limits in order to make the model easier to apply in practice. The first limit is due to the too much simplified capital structure hypothesized by the model, which provides for the debt a single zero-coupon bond where principal and interest are repaid in a lump at a single maturity, T. Such an assumption is not realistic since companies usually own a much more complex debt structure with liabilities that have a variety of maturities and periodic interest payments, as well as a number of different levels of seniority and security. A second limit arises from the assumption that default can occurs only at maturity of liabilities while in real life, the default can also happen at any time of a firm's existence even before T. Another limit of Merton model is that it assumes a constant risk-free interest rates. A final problem and the most significant in terms of the model's application is the fact that it requires the knowledge of variables, particularly the market value of assets (V_t) and the volatility of assets returns (σ_V), which are not observable directly on the market. As we have seen, these values may be determined from the price and volatility of the shares, which although are available only for listed companies, which are a limited range of subjects exposed to credit risk.

2.2.2 First passage time models

The basic Merton model we have just presented, has been extended in several ways in order to try to overcome certain unrealistic assumptions. In particular, the basic assumption that the default of the firm occurs only at maturity of the debt T has been relaxed. In real life, given a time interval $[0, T]$, the default of the firm occurs in $\tau \in [0, T]$. This has led to the development of the so called first passage time models. These models are so called since they analyse the probability that in a firm, the asset's value crosses for the first time a threshold (called default boundary or default barrier), causing the default of the firm, which will occur when $V_\tau < B$ where B indicates the value of the boundary and V_τ the asset value at default time (τ).

These models were proposed by Black and Cox in 1976 who introduced the possibility of default at any time prior to the maturity of debt. Over the years, many other authors tried to implement the model redefining in a different way the default boundary or creating closed formulas for computing the probability of default for models of first passage time type. In the following, we first analyse the Black and Cox model and next we briefly present the main contributions that over the years have been bring to this type of models.

The basic assumptions posed by the Merton model are valid also in this context. So, it is assumed that the asset value of a firm follows a geometric Brownian motion of the type: $dV_t = \mu V_t dt + \sigma_V V_t dW$ where W represents a Wiener process, σ_V indicates the volatility of the assets and μ indicates the expected rate of return.
The liabilities of the firm is composed just by a zero coupon bond with face value K and maturity T.
Let r be the risk-free interest rate, which is assumed to be constant.
In addiction we assume for the markets of the firm's bond the absence of arbitrage opportunities.

In first passage time models, the time of default, τ, is typically specified as the first moment when the value of the firm hits the default barrier.
Formally:

$$\tau = inf\ \{t : V_t \leq B\}. \tag{2.15}$$

In the structural models the default time is a predictable stopping time. We use the contributions of Björk (2009) and Jarrow and Potter(2004) to define a stopping time τ and a predictable stopping time.

According to Björk a stopping time is defined as: *"Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a filtered probability space, where the filtration $\mathcal{F} = \{\mathcal{F}_t\}_{t\geq 0}$. A non-negative random variable τ is called a stopping time w.r.t. the filtration $\mathcal{F}$ if it satisfies the condition $\tau \leq t \in \mathcal{F}_t$ for all $t \geq 0$".*

Jarrow and Potter (2004) define predictable a stopping time if: *"there exists a sequence of stopping times* $(\tau_n)_{n\geq 1}$ *such that* τ_n *is increasing* $(\tau_n \leq \tau)$ *on* $\tau > 0$*, for all n and* $lim_{n\longrightarrow\infty}\tau_n = \tau$*. (The sequence* τ_n *is said to announce* τ*)."*

Intuitively, a predictable stopping time is know to occur just before it happens since it is announced by an increasing sequence of stopping times. This means that the time of default is not completely a surprise in first passage time models but can be anticipated in some sense by observing the trajectory taken by the asset's value process.

An important element in first passage time models is the specification of the default boundary, which allows the default prior to maturity of the bond. A possible economic interpretation of the default barrier is the presence of some safety covenants in the contract. Safety covenants give the bondholders the right to force firm into bankruptcy and liquidation if the value of assets falls below the defined threshold, B. The barrier could be a constant, a time function or even a stochastic process. Black and Cox adopt a time varying default boundary with an exponential form described as [2]:

$$B(t) = ke^{-\gamma(T-t)}, \tag{2.16}$$

where k represents the limit below which the default occurs and can be interpreted as the amount of the covenant, while γ indicates the rate used to discounting from the maturity T to the actual time t.

In the Black and Cox model the probability that the default time (τ) occurs in the time interval $[t, T]$ and that therefore the value of the assets fall below the define threshold, can be analytically computed and is given by:

$$\begin{aligned} \mathbb{P}(V_\tau \leq B(t)) &= 1 - N\left(\frac{logV_t/B(t) + (\mu - 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}\right) + \\ &+ \left(\frac{V_t}{B_t}\right)^{1-\left(\frac{2(\mu-\gamma)}{\sigma^2}\right)} N\left(\frac{-log\ V_t/B(t) + (\mu - 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}\right), \end{aligned} \tag{2.17}$$

with $\tau = T - t$ and $B(t)$ indicated in (2.16).

Over the time, first-passage time models have been widely studied and developed in the literature. In particular different authors have given their contribution to the models assuming a form for the default boundary different from the exponential one indicates by Black and Cox. For example, one of the most important contribution is given by Longstaff and Schwartz (1995) which have hypothesized a constant default boundary and a default-free interest rate governed by a stochastic process (in particular for the interest rate the Vasicek model, 1977, is adopted). A constant boundary is assumed also by Jarrow and Potter (2004) which provide the determination of a closed formula for the defaultable bonds indicated by:

$$v(0,T) = \mathbb{E}\left([\mathbb{1}_{\{\tau \leq T\}} B_\tau + \mathbb{1}_{\{\tau > T\}} L] e^{-\int_t^T r_s ds}\right), \tag{2.18}$$

where $\mathbb{1}_{\{\tau \leq T\}}$ is an indication function which can assume value 0 or 1. It assumes value 1 if the default occurs in prior the time T and 0 if the default does not occur.

Another interesting work is the one coming from Saà-Requejo and Santa Clara (1997) which proposed a model in which both the default free interest rate and the default boundary are stochastic. In particular the default boundary is defined by a stochastic process which satisfy the following stochastic differential equation:

$$\frac{dB_t}{B_t} = (r_t - \delta_B)dt + \sigma_{Br} dW + \sigma_{BV} dW, \tag{2.19}$$

where σ_{Br} and σ_{BV} are two positive constants, δ_B is the instantaneous rate paid to the firm's debtholders.

2.3 Reduced form models

The other group of models is the so-called reduced form or hazard rate models. These models were born after the structural models to try to overcome the limits of the latter. Reduced form models take an approach different from the structural one because they consider the default event exogenous and independent from the balance sheet structure or macroeconomic outcomes of the company. In particular, these models assume that default is described by an exogenous process which is usually a Poisson. The Poisson process is a typically choice in reduced form models to describe the event of default because it is a process well suited, not only in finance but also in other fields, to describe a sudden change of state.

The Poisson variable, defined in discrete time, models the number of times an event occurs, n, in a fixed interval of time. The average number of events in the considered interval is denoted by λ, which is called the *intensity* of the process. Let X be a Poisson random variable and n the number of events then, the probability distribution of the Poisson random variable is given by:

$$\mathbb{P}(X = n) = e^{-\lambda}\, \frac{\lambda^n}{n!} \qquad \forall\, n \in \mathbb{N}, \tag{2.20}$$

where $n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$ is the factorial of n and the expected value and the variance of X is equal to the positive real number λ (E(X)=Var(X)=λ).

The Poisson process results useful in describing the event of default because it provides the possibility that occurs a sudden change of state (a jump) between two time instants that in our case represent the passage from no-default to default. In particular, if we define the moment the company defaults with a random variable τ, the default is identified when we have the first jump in a Poisson process. The intensity λ, in this case, has the meaning of instantaneous default risk[4] and it can be interpreted as the probability of default between time t and $t + dt$ conditional on no earlier default.

[4] $\lambda(t)$ is also known as hazard rate or as many other names, depending on the area of study. For example, in demographic studies the term instantaneous mortality rate is used.

When λ is, more in general, a function of t, we have:

$$\mathbb{P}[t < \tau < t + dt \mid \tau > t] = \lambda(t)dt. \tag{2.21}$$

If we then assume that default occurs when we have the first jump in a Poisson process, then the random variable τ is distributed according to an exponential distribution with a cumulative distribution function:

$$F(t) = \mathbb{P}(\tau \leq T) = 1 - e^{-\lambda(T-t)}, \tag{2.22}$$

which can be interpreted as the probability that τ occurs within maturity T. From this, we can then obtain the survival function:

$$Q(t) = 1 - F(t) = \mathbb{P}(\tau > T) = e^{-\lambda(T-t)}, \tag{2.23}$$

which expresses the probability at time t that τ occurs after maturity T.

According to a classification proposed by Bielecki and Rutkowski (2002), reduced form models can be classify in two categories: intensity-based models and credit rating migration models. In the following we will analyse two works of both categories.

The first of Jarrow and Turnbull (1995), classified in the category of the intensity based models use the approach of the binomial trees for pricing bonds subject to credit risk. The default is exogenous and the intensity of the process λ, is assumed constant.

The second work, of Jarrow, Lando and Turnbull (1997) belongs to the category of the credit risk migration models because it provides the computation of the probability of default prior the expires, on the basis of the transition matrix and on the probability that a firm with rating $i - th$ at time t suffer, within the maturity T, a downgrade of the own rating until the $k - th$ class which represents the default state.

We will first present the work of Jarrow and Turnbull (1995) both in discrete and in continuous time and subsequently, we will present the model of Jarrow, Lando and Turnbull (1997) in discrete time.

2.3.1 The Jarrow and Turnbull model (1995)

The Jarrow and Turnbull model has been described in the article titled "Pricing Derivatives on Financial Securities Subject to Credit Risk" (1995) with the purpose to provide a new theory for pricing financial tools (derivatives and not) involving credit risk [22].

First of all, they suppose a frictionless economy (no taxes, trading fees, etc) and the absence of arbitrage opportunities. In the market are traded two classes of zero-coupon bonds. The first class is default-free and $p_0(t,T)$ is the value of the default-free zero coupon bond at time $t=0$. The second class is defaultable or risky (which means bonds subject to default) and we denote with $v_0(t,T)$ the value of the defaultable zero-coupon bond at time $t=0$, promising a dollar at time T.

The authors propose two versions of their model: one version in discrete time and subsequently one in continuous time.
In discrete time, the model considers a time interval $[t,T]$ where t represents the current time and T the maturity. In particular they consider only two time periods, so that the maturity $T=2$.

For the pricing, the authors uses the binomial tree. This is a diagram representing different possible paths that might be followed by the price of the underlying of the derivative. Assuming that the price follows a random walk, in each time step, it has a certain probability of moving up by a certain percentage amount indicated with u and a certain probability of moving down by a certain percentage amount indicated by d. It is also assumed that, if default occurs, the underwriter of a derivative can recover part of the exposure equals to RR. As a result, the binomial tree of Jarrow and Turnbull, begins in $t=0$ from a defined price of the underlying of the derivative and in $t=1$ has two nodes: the higher shows the price assumed by the underlying at $t=1$ if there is an upward movement in the price. The lower node, instead, indicate the price of the underlying in the case in which there is a downward movement in the price.

By using the binomial tree, we first present the term structure for the default-free zero coupon bonds and then the term structure for the defaultable zero-coupons bonds.

The price of a default-free bond is assumed to depend only on the spot interest rate. The current ($t = 0$) spot interest rate will be given by:

$$r(0) = \frac{p(T;T)}{p_0(0,1)}. \tag{2.24}$$

In the next period, $t = 1$, the spot rate can assume two values according to there is an increasing or decreasing of the same.
In case of an "up-movement", the spot interest rate is:

$$r(1)_u = \frac{p(T,T)}{p_0(1,T)_u}, \tag{2.25}$$

while in the case of a "down-movement", the spot interest rate is:

$$r(1)_d = \frac{p(T,T)}{p(1,T)_d}. \tag{2.26}$$

The risk-neutral probability of an up-movement occurring is denoted by π_0 and consequently the probability of a down-movement is $1 - \pi_0$. Thus, we obtain a binomial tree with which estimate the value of the spot interest rate which is graphically represented in Figure 2.4.

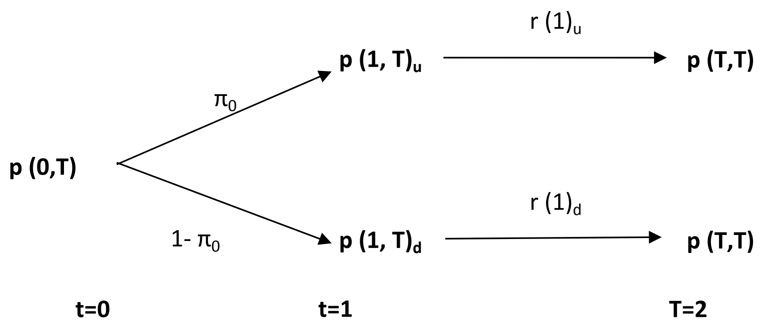

Figure 2.4: The default-free zero-coupon bond price process for the two-period economy.

Once defined the term structure of spot interest rates, it is possible to construct a tree which illustrate in each time step the amount that the investor would receive from the defaultable bond conditional on the occurrence of the default event. If default does not occur, the payoff will be the face value of the bond (1 euro). If default occurs, the payoff will be a predefined amount equal to RR. In each period, default can occur with risk neutral probability λ_t.

As shown in Figure 2.5, in $t = 0$, the investor can expect to receive the face value of the debt at maturity since the debtor does not default. In $t = 1$ we have two cases: the debtor defaults with probability λ_0 and the investor receives the recovery rate at maturity equal to $RR \cdot 1$ or the debtor does not default and the investor will receive the face value of the bond. Between time $t = 1$ and maturity $T = 2$ the debtor can default only if he/she has not already default previously. As result, the nodes in $T = 2$ will be only three and not four as a binomial tree. The higher node derives from the case in which the debtor has failed in $t = 1$, so at maturity it will never be able to repay the debt and therefore the investor will obtain $1 \cdot RR$. The two nodes below derive from the case in which the debtor has not default in $t = 1$. Here the possible cases are: the debtor will default between $t = 1$ and $T = 2$ with probability λ_1 and then, the investor expects to receive $1 \cdot RR$ or she/he remains solvent with probability $1 - \lambda_1$ and the investor could expect to get the face vale of the bond.

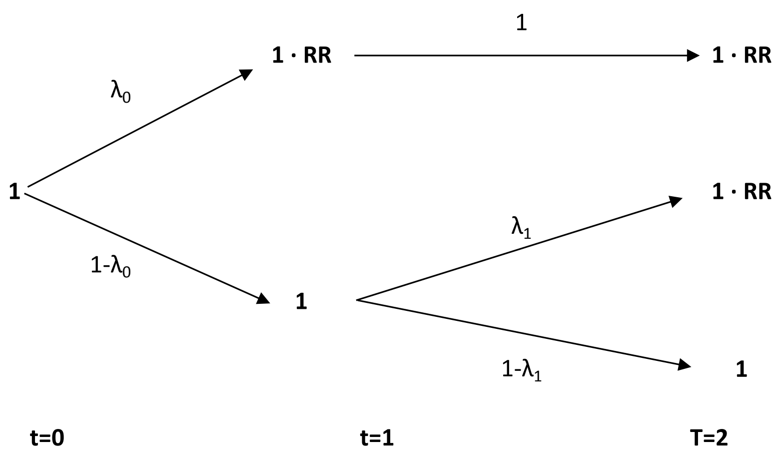

Figure 2.5: Expected payoff for defaultable bond in the two-period economy.

Putting all the information together and by combining the binomial tree in Figures 2.4 and 2.5, we get the following tree representing the price of the derivative in Figure 2.6:

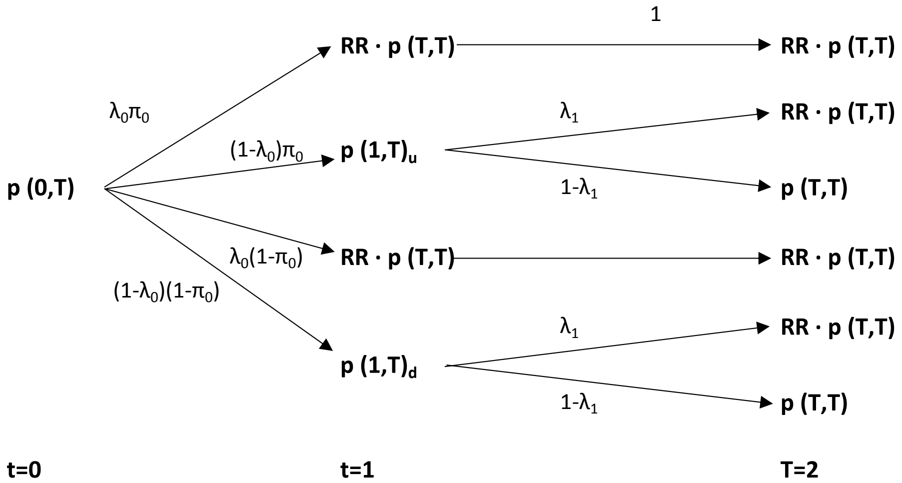

Figure 2.6: Pricing of a derivative with underlying a bond involving credit risk.

In this tree, in $t = 0$ is indicated the current bond price. In $t = 1$ there are 4 possible scenarios, represented each one by a node. Starting from above, the first node indicates the possibility that between $t = 0$ and $t = 1$ the obligor defaults with probability λ_0 and that in the same time instant, the bond price increases. In this case, the investor can expect to receive an amount equal to $RR\ p(T,T)$, where with $p(T;T)$ is indicated the promised payout of the bond at maturity. In the second node instead, is represented the possibility that the obligor does not default within $t = 1$, so the investor will expect to receive at maturity the face value $p(T,T)$. The third node, instead, consider the hypothesis that the bond price, between $t = 0$ and $t = 1$, decreases and simultaneously the obligor defaults, thus the investor at maturity will receive only RR. The last scenario, shows the price of the bond at $t = 1$ in the case in which the same decreases and the debtor does not default.

Finally, in $T = 2$, the nodes become six: the first and the fourth relates the possibility that, being the obligor insolvent between $t = 1$ he/she cannot enhance her/his position. The amount that the investor will receive at maturity will be RR. In the second and third node the investor at maturity can obtain an amount which varies according to the debtor, between $t = 1$ and $T = 2$, could default (with probability λ_1) or remain solvent after the increasing of the bond in the time interval $(0, 1]$. In the case of default, the investor will obtain the recovery rate RR otherwise he/she will receive the face value of the bond. The same logic can be apply in the last two nodes: after the bond has lost value between $(0, 1]$ and after that, in the same time interval the debtor has proven to be solvent, the investor will obtain RR if the debtor default between $(1, 2]$ otherwise he/she will receive the face value in case of default.

According to Jarrow and Turnbull the value of a defaultable zero-coupon bond is equal to the price of a risk-free bond multiplying by the expected payoff at maturity of the defaultable bond. In $T = 1$ this will be equal to:

$$v(0,1) = p(0,1)[\lambda_0 \ RR + (1 - \lambda_0)]. \tag{2.27}$$

In $T = 2$ it will be equal to:

$$v(0,2) = p(0,2)\lambda_0 \ RR + (1 - \lambda_0))[\lambda_1 \ RR + (1 - \lambda_1)]. \tag{2.28}$$

In general terms the value of a defaultalbe bond is equal to

$$v(t,T) = p(t,T)\mathbb{E}(\phi(T)), \tag{2.29}$$

where $\phi(T)$ is the payoff of the bond at maturity T.

The work of Jarrow and Turnbull consider also the continuous time. The time interval is $[0, T]$. In this case, the forward rate is defined as:

$$f_0(t,T) = \frac{-\partial}{\partial T} log \ p(t,T), \tag{2.30}$$

and the spot rate as $r_0(t) = f_0(t,t)$. We assume that in the considered market there is a risk-free bond which pay the investments at rate r, so depositing

1 euro, the investor will receive:

$$B(t) = e^{\left(\int_0^t r(s)ds\right)}. \tag{2.31}$$

We indicate with τ the time to default of the firm and we hypothesize that τ is distributed in a exponential manner with parameter λ which is defined as "default intensity".
The pay-off of an investor who invest in a defaultable bond which at maturity reimbursed 1 euro will be given by:

$$\begin{cases} 1 & \text{if } \tau > T \\ RR & \text{if } \tau \leq T. \end{cases} \tag{2.32}$$

This is the expected pay-off at maturity of the investor at current time t.

The computation of the value of a defaultable bond $v(t,T)$ in continuous time is equal to the discrete time by multiplying the value of a default-free bond having the same features $p(t,T)$ for the expected pay-off at maturity, under hypothesis of no arbitrage and perfect markets. If default occurs prior maturity, then the price of the defaultable bond will be given by the multiplication of the price of the risk-free bond times the recovery rate, on the opposite if we do not expect a default prior maturity, then the value of the risky bnd will be given by:

$$v(t,T) = \begin{cases} (e^{-\lambda(t-t)} + RR\ (1 - e^{-\lambda(T-t)}))\ p(t,T) & \text{if } t < \tau \\ RR\ p(t,T) & \text{if } t \geq \tau. \end{cases} \tag{2.33}$$

2.3.2 The Jarrow, Lando and Turnbull model (1997)

Jarrow, Lando and Turnbull (from now on JLT), extend the model previously proposed by Jarrow and Turnbull, in a work entitled "A Markov Model for the Term Structure of Credit Risk Spreads" in order to take into consideration the possibility of migration from one class to another of the bond rating [23].

Additional characteristics with respect to Jarrow and Turnbull are the following: the introduction of the probability of credit rating transition, the liabilities of the firm have different levels of seniority, to the model can be applied any term structure of risk-free interest rates such as the one defined by Cox, Ingersoll, and Ross (1985) and last, the process for the default risk and the risk-free term structure are assumed to be independent [14].

Now, the contribution of JLT articulates around the specification of a transition matrix $\mathcal{Q}$:

$$\mathcal{Q} = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1K} \\ q_{21} & q_{22} & \cdots & q_{2K} \\ \vdots & & & \vdots \\ q_{k-1,1} & q_{k-1,2} & \cdots & q_{K-1,K} \\ 0 & 0 & \cdots & 1 \end{pmatrix}. \tag{2.34}$$

This is a matrix of $K \times K$ dimension where both the rows and the columns represent the credit rating classes. In particular, credit rating classes are sorted from the best credit rating class, for example AAA, (the higher in the rows and the left one in the columns) to the last class representing the default state (the $K-th$ row). Each of the q_{ij} represents the probability that a firm, with credit rating today with class i, suffers an upgrade or a downgrade to the class j in one period of time. As for all the transition matrix, non zero probabilities tend to concentrate on the diagonal, since on average, it is more likely that a firm receives a confirmation of its rating rather than suffer an upgrade or a downgrade. In addiction, since this is a transition matrix, whose elements are conditional probabilities, the q_{ij} values are non-negative and the sum of the elements of each row is always equal to 1.

The last row (the $K-th$) of the transition matrix is particular important because it represents the bankruptcy state. The probability of leaving this state is always null while the probability of staying in this state is 1. This property is quite intuitive since a firm in default cannot certainly worsen its state and often it cannot even improve it because it is considered out of the market. For these reasons the $K-th$ class is defined as *the absorbing state*.

We can now define the transition matrix from time t to time $t+1$, which is given by:

$$\tilde{\mathcal{Q}} = \begin{pmatrix} \tilde{q}_{11}(t,t+1) & \tilde{q}_{12}(t,t+1) & \cdots & \tilde{q}_{1K}(t,t+1) \\ \tilde{q}_{21}(t,t+1) & \tilde{q}_{22}(t,t+1) & \cdots & \tilde{q}_{2K}(t,t+1) \\ \vdots & & & \vdots \\ \tilde{q}_{k-1,1}(t,t+1) & \tilde{q}_{k-1,2}(t,t+1) & \cdots & \tilde{q}_{K-1,K}(t,t+1) \\ 0 & 0 & \cdots & 1 \end{pmatrix}. \tag{2.35}$$

In this matrix, the element $\tilde{q}_{i,j}(t,t+1)$ expresses the probability for a firm of going from class i to class j, suffering an upgrade or downgrade of its rating between time t and time $t+1$. This means that at time t the firm has a class rating i, while at time $t+1$ it has a class rating j. If i equals j this means that the firm does not suffer any changes in the time interval. In particular the element $\tilde{q}_{i,j}(t,t+1)$ is given by the following formula:

$$\tilde{q}_{i,j}(t,t+1) = \pi_i(t)\ q_{ij}, \qquad \forall i,j \quad \text{with } i \neq j, \tag{2.36}$$

where $\pi_i(t)$ is a deterministic function of time indicating the risk premiums at time t.

Given this structure, we can now compute the probability of default occurring after date T, that is $\mathbb{P}(\tau > T)$. The default occurs when in the matrix $\tilde{\mathcal{Q}}_{t,t+1}$ a firm, classify at time t in the rating class $i-th$ goes to the rating class $K-th$ in the time interval $[t,t+1]$. The probability that this will happen after maturity T will therefore be given by all the elements in the matrix $\tilde{\mathcal{Q}}_{t,T}$ we find in the $K-1$ rows and in the $K-1$ columns, excluding the default states.

Let the firm be in state i at time t, denoted by $\eta_t = i$ and define $\tau^* = inf s \geq : \eta_s = K$ which represents the first time of bankruptcy. Then, the probability that default occurs after time T is:

$$\mathbb{P}(\tau > T) = \sum_{j,i \neq k} \tilde{q}_{i,j}(t,T) = 1 - \tilde{q}_{i,k}(t,T). \tag{2.37}$$

In the JLT model, the risky zero-coupon bond price is determined as:

$$v(t,T) = p(t,T)[RR + (1 - RR)\ \mathbb{P}(\tau > T)], \tag{2.38}$$

where $v(t,T)$ denotes the value of a risky bond price in time t with maturity T, $p(t,T)$ denotes the risk-free bond price with same maturity, RR represents the recovery rate determined as percentage on the reimbursed value of the bond and $\mathbb{P}(\tau > T)$ indicates the probability of default occurring after maturity.

Last, the JLT model indicates the credit spread existing between risky issuer and risk-free issuer as:

$$f^i(t,T) - f(t,T) = \mathbb{1}_{\tau > T}\ log\left(\frac{[RR + (1 - RR)\mathbb{P}(\tau > T)]}{[RR + (1 - RR)\mathbb{P}(\tau > T + 1)]}\right), \tag{2.39}$$

where $f^i(t,T) - f(t,T)$ is the forward credit spread between a risky issuer and a risk-free one, while $\mathbb{1}_{\tau > T}$ is the dichotomous variable which assume value 0 if the event $\tau > T$ does not occur (which means not default and solvency of the firm) and assume value 1 if it occurs.

Chapter 3

CREDIT DERIVATIVES

3.1 Introduction

Credit derivatives have revolutionized with their introduction the financial system, substantially and significantly changing the way in which banks assume, manage and diversify the credit risk.

Such instruments, can be included in the broadest category of the derivative contracts and indeed their appearance on the financial markets can be interpreted as a product innovation with respect to the already know financial derivatives. It therefore appears necessary to provide a brief definition of what a financial derivative contract is. A derivative contract, or derivative for short, can be defined as a financial instrument whose value depends on (or derives from) the values of an underlying asset. This underlying can have either physical nature (such as a commodity or any good) or intangible nature (such as interest rates, currencies and stock indices). Derivatives can either be traded on an exchange or over-the-counter (OTC) market. In the former case, individuals trade standardized contracts specified in their terms by an exchange (contract size, maturity, delivery arrangements, etc.) and exist a *Clearing House*, which guarantees the fulfilment of the contractual obligations even if the party on the other side will default, ensuring a lower credit risk. By contrast, in the OTC market, the contracts are not standardized. The terms of a contract do not have to be those specified by an exchange, but the market participants are free to negotiate any mutually attractive deal. Usually in the over-the-counter trade, the parties are exposed to a greater credit risk because there is the possibility that the contract will not be honored and there is no Clearing House acting as intermediary. The majority of derivatives are traded OTC.

Our focus in this chapter is on derivatives that have as underlying the credit risk. Such instruments are generally known as *Credit Derivatives*. Unlike the traditional financial derivatives, born to the management of market risks, credit derivatives are characterized for the management of credit risk: they allow to isolate, assign a price and transfer the credit risk implied in each credit exposure, without that the reference asset is sold. The main feature of credit derivatives is exactly this. They allow to separate the credit risk from the underlying asset (bond, loan, etc..) and they transform it in a commodity easily transferable. Like every commodity, the credit risk is exchanged in specific market through specialized dealers and market makers.

Credit derivatives market has relatively recent origins and it has been the protagonist of a crazy growth, that has range between the last decade of the twentieth century and the years before the financial crisis in 2007. We can split the evolution of credit derivatives market in two phases: the prior crisis phase and the post crisis phase, which is the one we are experiencing now. The date of birth of credit derivatives is not entirely clear although approximately, it is estimated that the first credit derivative transaction were completed in the early 1990s. However, it is sure that the term credit derivative has been used for the first time in 1992 in a conference of the International Swaps and Derivatives Association[1] (ISDA). In 1997 the US bank J.P. Morgan engineered the first credit default swap. In that time the use of these hedging instruments had reached a moderate but significant level of diffusion among the most advanced banks. According to the statistics of the Bank for International Settlements (BIS, for short), in 1998 the notional amount of credit derivatives outstanding in the market was \$ 536 billion. Obviously, the deals put in place were very slow and expensive and

[1] The International Swaps and Derivatives Association is a trade organization of participants in the market for over the counter derivatives. It is headquarters in New York.

they had nothing in common with the market of today, based on the liquidity and simplicity principles.

The traders of credit derivatives starts to grew at exponential rates. A symbolic turning point is represented by 2004, the year in which for the first time in history, the volume of credit derivatives traded exceeds that of the bonds. Furthermore, with the progressive diffusion of the products and the knowledge related to their use, there has been a tendency to create more complex products. The table 3.1, shows the data of the growth described in terms of gross notional value in billions of USA dollors. This growth was took place in a very favourable context, namely in a market that knew the default rates among the lowest in the history. Those who fails or do not honour their debts are few and the credit spreads of any type of issuer were extremely low. It was natural then for the operators, to look at credit derivatives by seeing only their potential gains and underestimating their risks. The data becomes even more important, if we consider the fact that, the rate of bankruptcies between 2004 and 2007 was close to zero for issuers with an high credit quality, in both Europe and the United States. This has led many operators to exchange credit derivatives or invest in structured products with a more and more complex default risk, in order to maximize their leverage and their return, regardless of the entity or the financial strength of the counterparties they negotiated with. The highest peak in the credit derivatives market is reached between 2007 and 2008 with a gross value close to $ 60,000 billion.

The default of the US bank Lehman Brothers in 2008 shook the markets and credit spreads invert the multi-year tendency to tightening. The concept of "too big to fail" was definitely eliminated and suddenly start to be important the choice of the counterpart. The crisis delivered an important blow to credit derivatives and the collective hysteria designed them as being among the main causes of the US subprime crisis in 2007-2008, reducing drastically the number of contracts traded. According to the last semiannual statistic conducting by BIS, the notional amount of credit derivatives contracts outstanding in the market in June 2017 amounted to only $ 9,644 billion. This is a very low data comparing with the prior values, which confirms a tendency to decline of the credit derivatives market.

Table 3.1: Notional amount of credit derivatives market 1998-2017, in billions of US dollars (source: Bank for International Settlements).

Year	Total	Single-name	Multi-name
1998	536	420	116
2000	1,096	889	208
2003	3,544	2,780	764
2004	6,396	5,117	1,279
2005	13,908	10,432	3,476
2006	28,650	13,873	6,479
ju-2007	42,581	24,239	18,341
dic-2007	58,244	32,486	25,757
jun-2008	57,403	33,412	23,991
dic-2008	41,883	25,740	16,143
2009	32,693	21,917	10,776
2010	30,261	18,494	11,767
2013	21,020	11,324	9,696
2015	12,294	7,183	5,110
2016	9,857	5,582	4,275
jun-2017	9,644	5,042	4,602

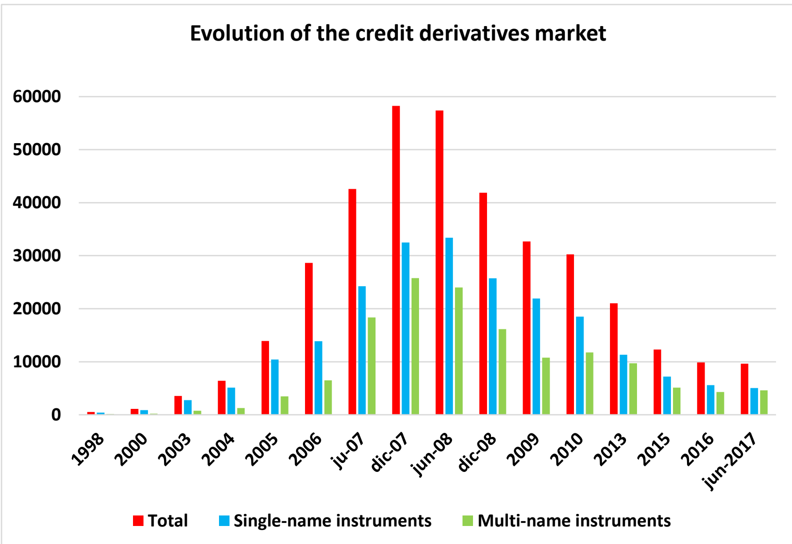

Figure 3.1: Notional amount of credit derivatives market 1998-2017, in billions of US dollars (source: Bank for International Settlements).

3.2 Market participants and the main uses

The wide variety of applications of credit derivatives attracts a broad range of market participants. The largest participants in the credit derivatives market have always been first banks, as protection buyers and then insurance companies, as protection sellers. However, over time, we found that other types of players entered in the market. These are hedge funds, mutual and pension funds and industrial and commercial companies.

With regards to banks, credit derivatives constitute a very important instrument to manage credit risk, allowing on the one hand the possibility to cover unwelcome risk position and on the other hand, to reduce pre-existing risk positions, which can reach very high levels. For these reasons credit derivatives are defined also as "synthetic collaterals", namely like collaterals which allow to reduce the risk exposures towards the costumers, constituting therefore, a valid alternative to the traditional credit risk protections, such as personal or real ones.

Through credit derivatives is possible, in addiction, to diversify the risks portfolio, through the assumption of debt exposures towards subjects or countries in relation to which it is not easy to have a direct relation. Such diversification can be equally pursued through traditional tools, like bonds, which on the other hand, inevitably entail a series of costs and risks of different natures: for example, the investor who buys bonds supports the cost of the collection phase which can significantly reduce the return on investment, in addition to other costs like the costs for the safekeeping of the bonds. Furthermore the traditional instruments, allow a limited diversification due to the segmentation of the markets. There are indeed markets to which only a certain category of investors can access or there are also investors who cannot have access within the same market, to particular borrowers or to certain geographic areas.

Again, credit derivatives allow banks to pursue the task of an efficient treatment in the management of the regulatory capital, partly obtained through the provisions of the supervisory authorities, which impose a specific minimum capital requirement aimed to contain the credit risk. The use of derivative , allow indeed to the protection buyer to release regulatory capital, since the credit risk is transferred on the protection seller. Due to the peculiarities of these innovative financial instruments, credit risk is effectively treated as commodity and is therefore subject of trading activity. Moreover, this activity is largely supported by a documentation that has been increasingly standardized thanks to the intervention of the International Swaps and Derivatives Association, which in 1999 issued the standard definitions for the credit derivatives instruments, reviewed and integrated in 2003, 2009 and finally in 2014 (ISDA 2014 Credit derivatives Definitions).

The trading activity mentioned above, is mainly carried out by commercial banks and investment banks, which take certain positions respect to credit risk and speculate on spreads movements. In particular, an upgrading in the credit quality is cause of a tightening of the spread while a downgrading in the credit quality is cause of a widening in spread. The trading activity is also assisted by the presence on the market of brokers, who connect the relative counterparties using also advance systems such as the internet. With the use of credit derivative contracts, arises the possibility to sell illiquid position or for which there is no demand on the market, simply thanks to the transferring of the credit risk. Consequently, the trading activity allow to make “liquid” the underlying credit risk and make therefore as a reflex, more liquid the financial markets.

If on the one hand, banks have played the role of protection buyer, the role of protection seller has been covered by insurance companies and recently also by hedge funds and mutual and pension funds.

Insurance companies are also other important participants in this market. They are principally seller of credit protection and tend to prefer highly rated credits. Credit derivatives allow them to diversify the credits portfolio and to assume exposures towards commercial sectors, where institutionally that companies are absent and at the same time offer, to the insurance companies, the possibility to model their risk/return profile of the administered portfolio, obtaining significant improvements of the return.

In last years, hedge funds have growth their credit derivatives activity and have become significant players in the credit derivatives market. They are attracted by the unfunded[2] nature of most credit derivatives products which makes leverage possible and by the possibility to apply different trading strategies that hedge funds are free to exploit.

Mutual and pension funds are not particularly large participants in the credit derivatives market. As investors, they are primarily sellers of protection but often, they have restrictions on what sort of assets they can hold which preclude credit derivatives. However, the exact permissions depend on both their investment mandate and the investment regulations governing the jurisdiction in which they operate. Typically, one of their main restrictions is that the credits owned should be investment grade quality[3].

Last participants of the market are industrial and commercial companies, which use the credit derivatives with different purposes but mainly to hedging from risks. For example the hedging of the country risks arising from export operations or the hedging of commercial risks towards primary names against which the exposure is too high.

Summarizing, credit derivatives allow to obtain the following results: hedging credit risk reducing the costs normally presented in the traditional forms of credit risk insurance, diversification of the risks, trading with the possibility to speculate and arbitrage on the credit profiles of third parties, reduce regulatory capital requirement, overcoming the segmentation of the markets and increase the liquidity of the market.

In the sequel, we first give a definition of credit derivative and point out the main common features of all credit derivatives. Then, we will go into detail and we will present an overview of the different types of credit derivatives which are mostly traded.

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[2]Unfunded means that the credit derivatives transactions can be entered into at zero initial cost. Unlike a bond, which is a fund transaction, there is no need to pay an initial bond price in order to have a credit exposure.

[3]An investment grade is a rating that indicates that a bond has a relatively low risk of default. The rating agencies, such as Standard & Poor's and Moody's, use different designations consisting of upper and lower case letters A and B to identify a bond's credit quality rating. AAA and A (high credit quality) and A and BBB (medium credit quality) are considered investment grade.

3.3 Definition and key elements

A credit derivative is a bilateral contract which allow to transfer the credit risk of an asset from one party to another, without transferring the ownership of the underlying asset.

Generally, the basic structure provides that the subject who intend to protect himself/herself from the credit risk pays a periodic fee, called *premium*, to a counterparty who in return make a certain payment contingent to the occurrence of a certain event, that may be the debtor's default, or even a temporary insolvency, or a change in its creditworthiness.
We note that there are two parties:

- the *protection buyer*: who is the part that buys protection and therefore sells the credit risk;

- the *protection seller*: who is the part that sells protection and therefore assumes the credit risk.

Credit derivatives are traded on over the counter market and therefore they are contracts characterized by a wide degree of customization and adaptable to the specific need of each individual. However, the popularity and the diffusion of these tools have led to the definition of a series of widely standardized features. The documentation used in most credit derivative transactions is based on the documents and definitions provided by the International Swaps and Derivatives Association (ISDA) which in 1999 issued the standard definitions for the credit derivatives instruments, reviewed and integrated in 2003, 2009 and finally in 2014.

All credit derivatives are characterized by three common key elements which are [30]: the underlying of the credit derivative, the credit event and the type of settlement[4] specified between the parties.

The underlying of a credit derivative is represented by the ability to fulfill to a certain payment obligation by a debtor defined *Reference Entity*. In particular, the creditworthiness can be valued on the basis of two variables: the

[4]With the expression "settlement" we refer to the method of payment due at the maturity of the contract.

first variable is the risk of default of a Reference Entity or of more Reference Entities (like the case of basket products); the second variable is instead the risk of a downgrading in the creditworthiness of the Reference Entity. The valuation of the Reference Entity is made on the basis of an obligation refer to the Reference Entity and called "*Reference Obligation*". It can be represented by any securities issued by the Reference Entity (for example bonds) or by a bank loan. In the particular case in which the credit derivative is linked to the creditworthiness development of the debtor (like for example credit spread swap), the underlying is represented by the variations of the credit spread of the securities issued by the Reference Entity.

The second common feature of credit derivatives is the *credit event*. The credit event is the negative event which triggers the payment of the protection seller in favour of the protection buyer. At first instance, the credit event is simply associated to the event of default by the Reference Entity. Actually, the precise determination of the credit event changes from contract to contract depending principally on the nature of the Reference Entity (corporates or sovereign governments). However in practice, it is possible to identify the most commonly used credit events in:

- *bankruptcy*: it represents the most catastrophic event that can involve a Reference Entity. It provides the legal declaration of the impossibility of the debtor to fulfill its obligations towards the creditors;

- *failure to pay*: this is the most frequent credit event and consists in the failure of the Reference Entity to make interest or principal payments when due, taking into account some grace period, usually 30 days. The most common event is, for example, the non payment of a coupon of a bond determined by the inability to repay it at maturity of the contract;

- *debt restructuring*: it takes place whenever there is a change in the conditions of the debt obligation. Typically, the changes are referred to: the maturity, the coupons, the reduction of the interest rate or of the principal amount, a change in the ranking in priority of payment of any obligation that causes subordination of such obligation. Debt restructuring causes that the economic- financial conditions are less favourable for the creditors;

- *repudiation/moratorium*: it is the case in which the Reference Entity disaffirms, repudiates or rejects, in whole or in part, the debt issued or declares or imposes a moratorium[5]. This event usually involved sovereign issuers and it is used in emerging market state;

- *obligation-acceleration*: the obligations of the Reference Entity become due and payable earlier than they would have been due to default and have been accelerated. This event is used mostly in certain emerging market contracts;

- *cross-default*: the default of an obligation is extended to all the relations established by the Reference Entity. The purpose of this clause is to increase the protection of bondhlders.

The first three events are the most frequent while the repudiation/moratorium, the obligation acceleration and cross default are hardly ever used.

In the case in which the Reference Entity is represented by a sovereign state a possible definition of the credit event which can be founded is the sovereign event, which consist in the failure to comply with contractual terms for the obligations issued by the country, or in the cancellation or modification of payment obligations.

In addiction usually, the credit event has to be accompanied by two further conditions (concomitant or immediately following), which are aimed to obtain the payment from the protection seller. They are the Public Available Information and the Materiality. The first refers to the fact that the event must be evidenced by a source of public information, e.g. a newspaper or some other recognised publication or electronic information service (e.g. Bloomberg), which confirms the realization of the credit event. The Materiality consists in the fact that the price of the Reference Obligation must have fallen below a certain threshold. These two further conditions are

[5]A legally authorized period of delay in the performance of a legal obligation or the payment of a debt.

essential since it is of interest of the parties that the credit event is real and sufficiently relevant.

The third key element of a credit derivative is the method of payment when the credit event occurs, the *settlement*. It can take different forms even if the most common used in the credit derivatives contract are three:

1. *physical settlement*: the protection seller commits to buy a bond issued by the Reference Entity at its face value (which is the price established at the beginning of the contract also called initial price);

2. *cash settlement*: the protection seller commits to pay to the counterpart an amount equal to the difference between the face value of the bond (the initial price) and its market value at the time in which the credit event occurs (final price);

3. *binary payout*: the protection seller commits to pay to the counterpart a fixed amount determined in the moment of the sign of the contract, independent from the value of the Reference Obligation at the time of occurrence of the credit event. This type of settlement is considered a version of the cash settlement, where the only difference consists in the fact that the final value is fixed at the moment in which the contract is signed.

3.4 Principal credit derivatives

The continuous financial innovation that characterizes credit derivatives market, has led to the creation of new products more and more sophisticated. We have choose to describe the traditional building-blocks of credit derivatives because starting from the knowledge of these it is possible consequently to build the most structured and complex products. In particular, we will mainly focus on credit default swaps because in addiction to being the most used credit derivatives, they will also be object of analysis in the next chapters.

We will start our presentation with the simplest and at the same time the most widespread credit derivatives: the *Credit Default Swap* and the

Total Return Swap. They are aimed to address, respectively, the risk of loss of capital invested in a credit operation and the risk of an undesirable variation in the return respect to the expected return on a particular financial asset, over a period of time. Then we will present the *Credit Spread Derivatives* and the *Credit Linked Notes.* Credit spread derivatives can be distinguished in credit spread option and credit spread swap. The variation of the spread, which synthesizes the variability of the quality of the issuer, is the subject of credit derivative contracts that allow operators to cover from unexpected and unwelcome spread variations. Credit linked notes are hybrid financial instruments which combine the elements of a debt instrument with a credit derivative.

These above mentioned contracts, whose underlying is a single asset, are also called *single-name* credit derivatives and they can be opposed to credit derivatives whose underlying consists of a set of assets, which are called *multi-name* credit derivatives. About credit derivatives of the last type, we will present the so called *Basket Products* which are credit derivatives whose underlying consists of more than one asset with different creditworthiness and the *Collateralized Debt Obligations.*

3.4.1 Credit Default Swaps

The most important and widely used type of credit derivative is the Credit Default Swap (CDS for short). It was introduced by JP Morgan in 1997. A credit default swap is a bilateral contract that provides insurance against the risk of default of a particular issuer called Reference Entity, following a particular credit event. The part who buys the contract is called protection buyer, while the seller is called protection seller. The protection buyer makes periodic payments to the seller until the end of the life of the CDS or until a credit event occurs. These payments represent the cost of the protection, called *premium*, and they are determined in relation to the *notional principal*, namely the total face value of the bonds on which the contract is written [31]. If the Reference Entity does not default (i.e., there is no credit event), the contract terminates and the protection seller does not have to make any kind of payment; on the contrary, if there is a credit event, the protection seller make the payment to the protection buyer according to the type of settlement determined between the parties at the inception of the contract: physical or cash settlement. All the elements exposed, will now be analysed in detail. In Figure 3.2 is shown an example of credit default swap.

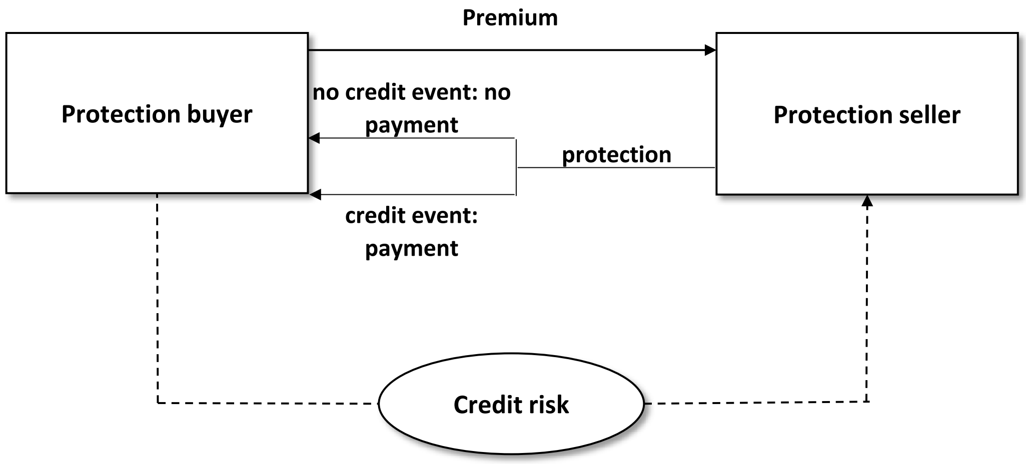

Figure 3.2: Structure of a credit default swap (Caputo and Fabbri, 2000).

Although the negotiations are over-the-counter and therefore characterized by a wide degree of customization, the CDS's popularity has led to the definition of a series of standardized features. The maturity of the CDS does not have to match the maturity of the reference asset and often does not. The most usually maturity is 5 years but other maturities such as 1, 2, 3, 7 and 10 years are not uncommon. Usually contracts mature on one of the following standard dates: March 20, June 20, September 20 and December 20. At any moment in time, the most liquid T-year contract is the one which matures on the first maturity date T years after the trade date. The effect of this is that the actual time to maturity of a contract when it is initiated is close to but not necessarily the same as the number of years to maturity that is specified.

We now analyse the part of the protection buyer. The payments made by the protection buyer to the protection seller are typically made quarterly in arrears. These payments are calculated using the Actual/360 day count convention, i.e. actual days (of the quarter) on 360 days (annual). So, the premium payment of the credit default swap is given by:

$$S \times \frac{\text{actual days}}{360}, \tag{3.1}$$

where S denote the *credit default spread*. The credit default spread is given by the ratio between the payments made by the protection buyer in one year and the notional principal of the CDS. Essentially, it indicates the size of the premium expressed in annual basis points (for example, 90 basis points per annum). Several large banks are market makers in the credit default swap market. They quote the bid-ask credit default spread to which they are prepared to buy and sell protection on certain bonds.
The payments from the buyer of protection to the seller of protection terminate at the contract maturity or immediately following a credit event. However, because these payments are made in arrears, a final accrual payment by the buyer is usually required. The protection buyer must pay the fraction of the premium which has accrued since the previous premium payment date. For example, suppose to enter into a 5-year CDS on March 20, 2018 and there is a default on May 20, 2021. The buyer would be required to pay to the seller the amount of the annual payment accrued between March, 20, 2021 and May 20, 2021 [18].

We now pass to analyse the part of the protection seller. The payment made by the protection seller to the protection buyer is called contingent payment because it is made only if a particular, pre-specified condition occurs, that is the occurrence of the credit event. As we introduced before, if there is no credit event, the protection seller does not have to make any kind of payment. If the credit event occurs, once this has been confirmed by public information sources, the protection seller proceed with the payment to the protection buyer according to the payment methods provided by the contract. If the contract specifies physical settlement, the protection buyer delivers physically to the protection seller the defaulted asset (i.e, the recovery value), receiving in return the face value of the delivered asset. Actually, the contract does not require that a specific asset is delivered, but it may defines a number of alternative assets that the buyer can choose to deliver. These are known as *deliverable obligations*. Where more than one deliverable obligation is specified, the buyer has the possibility to choose the one with the cheapest value in the list, in order to maximize his/her profit. This gives rise to the concept of the *cheapest-to deliver*, also encountered in other derivatives categories. If, as is now usual, there is cash settlement, the protection buyer no longer provides the delivery of the reference asset but only the protection seller pays to the protection buyer an amount equal to the difference between the face value of the bond and the final value of the defaulted reference asset. This final value is determined by an auction process organized by ISDA and corresponds in theory with the recovery value of the asset. However, as the recovery process can take some time, often the reference asset market value at time of default is taken and this amount used in calculating the final settlement amount paid to the protection buyer.

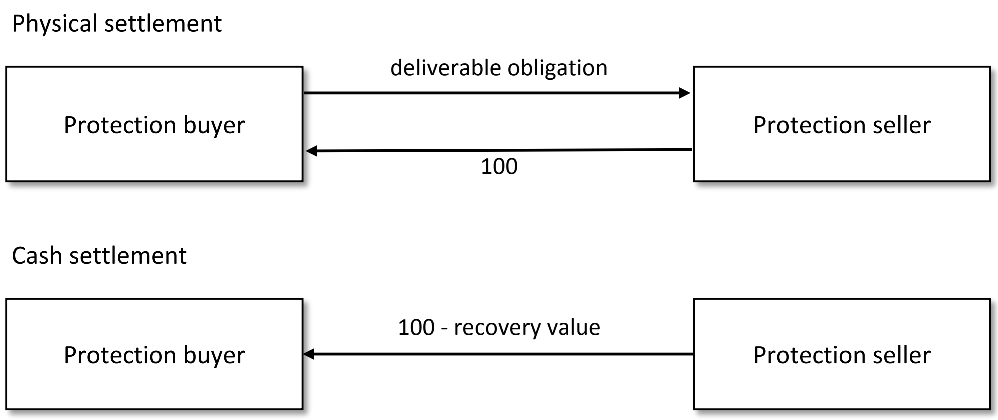

Figure 3.3: Physical and cash settlement. We assume a contract face value of € 100.

In theory, the value of protection is identical irrespective of which settlement option is selected. However, under physical settlement the protection seller can gain if there is a recovery value that can be extracted from the defaulted asset or its value may rise as the fortunes of the issuer improve. CDS market often prefer cash settlement as there is less administration associated with it, there is no delivery of a physical asset. Another advantage of cash

settlement is that it does not expose the protection buyer to any risks should there not be any deliverable assets in the market, for instance due to shortage of liquidity in the market, were this to happen, the buyer might find the value of its settlement payment reduced. Nevertheless, physical settlement is widely used because counterparties wish to avoid the difficulties associated with determining the market value of the reference asset under cash settlement. Physical settlement also permits the protection seller to take part in the credit or negotiations with the Reference Entity's administrators, which may result in improved terms for them as holders of the asset. Cash settlement is sometimes proceeded with even for physically settled contracts when, for one reason or another, it is not possible to deliver a physical asset, for instance if none is available.
As mentioned before, there exists a third type of settlement, which is called bynary payout. This is considered an alternative of the cash settlement, as it does not provide the delivering of the underlying obligation but it differentiates from it because there is no need of any estimates as regards the recovery value. The value to be reimbursed to the protection buyer is defined once for all at the time of the stipulation of the contract and it is not affected by any randomness.

As we saw before, the premium payments in a default swap contract are defined in terms of a default swap spread. It is possible to show that there should be, under some assumptions, a close relationship between CDS spread and the spread on a floating rate bond issued by the same Reference Entity. To prove this, we consider the following strategy on the protection buyer side.

The investor to cover the payments coming from the credit default swap, buys a floating rate bond, with the same maturity of the protection, which pays a coupon of LIBOR plus F and its default is tied to the default of the Reference Entity of the derivative contract.

The purchase of the asset is funded through a loan at a rate equal to LIBOR plus B which depends on the credithworthness of the investor. In addiction, we assume for simplicity, that the coupon payments of the asset and that of the payment of the premium of the credit default swap have the same payment schedule.

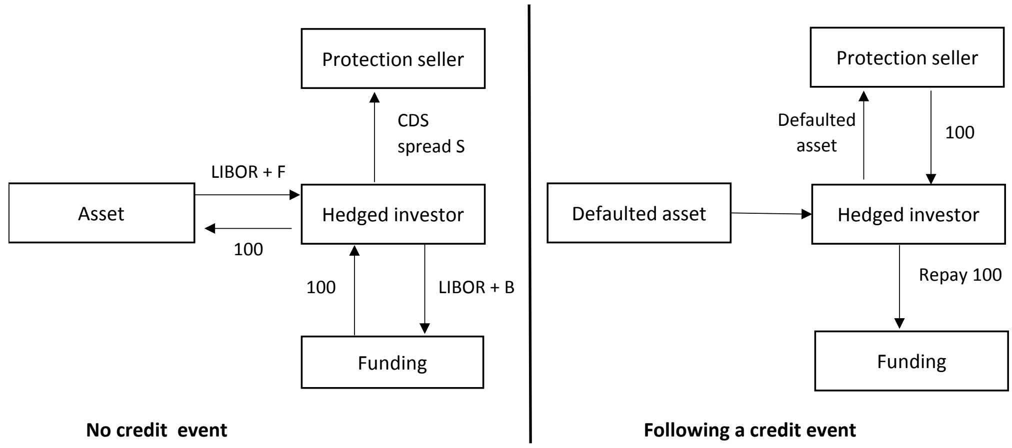

Figure 3.4: Default risk free strategy for an investor who buys protection (source: O'Kane, 2008).

As we can seen from Figure 3.4 the final possible scenarios are two: the credit event does not occur or credit event occurs. In the former case, the CDS hedge terminates at maturity and the investor obtains the reimbursed of the face value of the asset and uses it to repay the borrowed amount. In the latter case, the hedged investor delivers the floating rate asset to the protection seller and in return receives the payment deriving from the protection through the default swaps that uses to repay the funding.

Both scenarios make it clear that the hedged investor has no credit risk. As the strategy is risk free and has no initial cost, the hedged investor should not earn (or lose) anything on the payment dates. This implies that the relationship between the default swap spread and the spread on a bond is given by:

$$\text{CDS spread } (S) = \text{bond spread } (F) \text{ - funding spread } (B)$$

As we said before, this relationship is based on different assumptions that if we removed, would make it imperfect. Indeed, we ignore the transaction costs and the tax effects, in addiction, we assume there is no the possibility to effect arbitrages and that the bond and the CDS have the same maturity and the same payment dates. Despite this, the bond spreads usually provide the starting point for where the default swap spreads should trade.

3.4.2 Total Return Swaps

The second type of credit derivative, used equally frequently as the CDS, is the Total Return Swap, also known as the Total Rate of Return Swap (TRS or TROR, for short). The total return swap is an agreement between two parties to exchange the total return generated by an underlying asset for some other cashflows, usually tied to a market index. The underlyig asset, referred to as the reference asset, is usually bonds, loans or an equity indices. The two parties involved in a total return swap are known as:

- the *total return payer*, that is the buyer of the protection and the seller of the credit risk;
- the *total return receiver*, that is the seller of the protection and the buyer of the credit risk.

Periodically, one party, the total return payer, pays to the total return receiver the total return of the reference asset, namely all the cash flows generated by the underlying asset and in return, receive from the protection seller, a floating rate, which is the LIBOR[6] plus a spread, computed on the notional of the contract.

Generally at maturity of the swap (that does not need to match with the maturity of the underlying asset and rarely does) the contract terminates and the reference asset is repriced according to any change in value in the market price in order to determine the final payments of the counterparties. If there is an appreciate (the reference asset increases in value), the payer has to pay to the counterpart an amount equal to the difference between the initial value of the underlying asset and the market value of the asset at the time of repricing. Similarly, if there is a depreciate (the reference asset decreases in value), the receiver has to pay to the counterpart an amount equal to the depreciation of the asset [30].

[6]Libor is short for London Interbank Offered Rate. It is the rate of interest at which banks deposit money with other banks. One-month, three-month, six-month and 12-month LIBOR are quoted in all major currencies. It is used as a reference rate of interest for loans in international financial markets.

Change in value payments may be made also on a periodic interim basis and not necessarily at maturity. This makes the total return swap different from a credit default swap, as the payments between counterparties are based on changes in the market value of the underlying asset, as well as changes resulting from the occurrence of a credit event. So, in other words, the cash flows are not solely linked to the occurrence of a credit event as instead occur with the credit default swap.

Because of the importance of the market value of the reference asset, its determination becomes one of the key elements for assessing this credit derivative. There are different ways to obtain it: in general, it can be calculated on the average of the quotes provided by the reference dealers, which usually are the largest investment banks, or based on the actual sales price that the total payer can obtain. In this latter case, the risk of illiquidity is eliminated, which otherwise could cause an effective quotations of the reference asset different from the sale price at which the reference asset can be effectively sold on the market.

In Figure 3.5 is presented an example of total return swap.

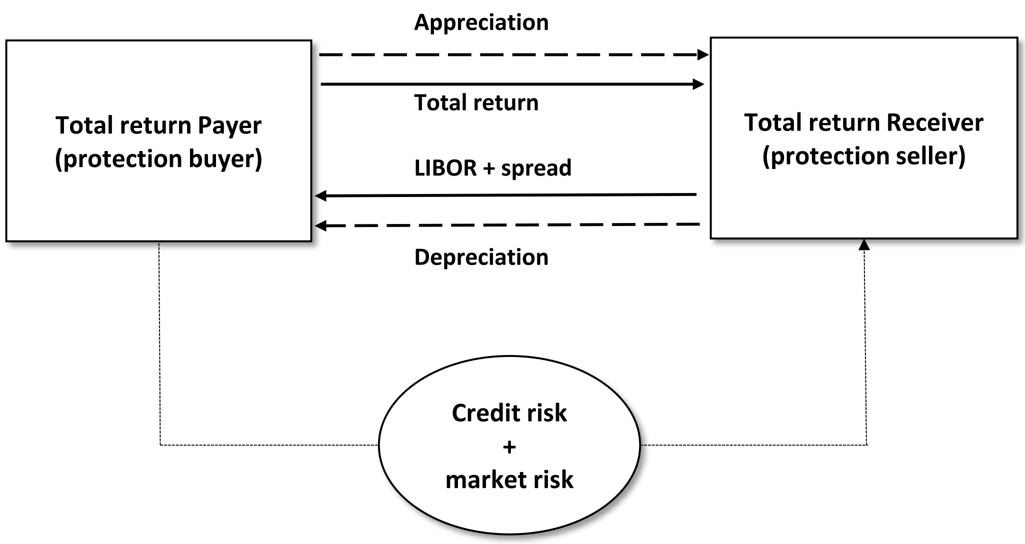

Figure 3.5: Structure of a total return swap (Caputo and Fabbri, 2000).

The result of a total return swap is, for the protection buyer, the possibility to transfer the credit risk of the reference asset while maintaining the asset in the balance-sheet and for the protection seller, the possibility to receive all the cash flows of a reference asset without actually owning the asset.

A key advantage that this type of instrument offers to the protection buyer is given by the possibility of not interrupting the relationships with the borrowers and at the same time transferring the credit risk, guaranteeing the possibility of a diversification of the portfolio. In this way it is possible to reduce the overall risk of the portfolio by reducing the risks associated to individual exposures (namely the diversifying risk) with the consequent possibility of applying to the borrowers a lower interest rate.

In addiction, the total return swap offers a greater protection compare with the credit default swap. Indeed, with a credit default swap the protection buyer results to be covered only against the default of the reference entity, while with a total return swap the protection is extended also to the case in which the reference asset suffers a reduction in its value.

From the protection seller point of view, the entry in this type of contract allows to synthetically create a position already existing in the market without being obliged to buy the asset. In this way the seller obtains all the economic benefits of the asset (interest and other returns generated by the reference asset, like any change in value) but avoiding to sustain an initial outlay for the purchase of the reference asset and the associated transaction costs.

We have to observe that, unlike what happens with other credit derivatives, the protection seller assumes not only the credit risk (seen as default or downgrading risk) but also the market risk related to the reference asset. Indeed, the changes in market rate can cause a loss for the receiver, independently from the variation of the creditworthiness of the reference asset. The cash flows in favour of the protection buyer are, indeed, constituted by an index (LIBOR) which take into consideration the changes of the market rate, while on the contrary, the one in favour of the protection seller are fixed at the beginning of the contract and suffer this type of risk.

There are various ways and strategies for using this derivative. Total return swaps are often used as a financing tool. For example, the protection seller, before entering in the total return swap, could sell to the protection buyer the reference obligation and enter into the derivative contract only later. In this way, this leaves the receiver in the same position as in the case in which it was the owner of the reference asset (indeed it continues to receive the cash flows and the capital gain generated from the asset) but at the same time it has the possibility to use the liquidity deriving from the sale of the reference asset to the protection buyer. At maturity of credit derivative, the protection seller would buy the reference asset previously sold.

The protection seller could also use the total return swap to avoid stringent regulations applied in the country of residence, such as settlement procedures, taxes etc.

In addiction, the use of total return swap allows also to investors different from banks (such as insurance companies, pension funds, hedge funds which have the purpose to use the liquidity to diversify the portfolio and to realize an adequate trade off risk return) the entry in the bank lending market.

Last, as for other credit derivatives, also the total return swap is used for a better capital allocation and for comply with the capital adequacy indices imposed by supervisory authorities.

3.4.3 Credit spread products

This family of instruments consists in contracts which pay off depends on the credit spread of the reference asset. With the term credit spread we mean the differential yield, compared to the interest rate for risk-free assets which is recognized to the investor for bearing default risk. Therefore, the credit spread is a differential yield whose function is to represent the risk premium that the market requires to hold an asset of an issuer considered risky. Starting from this concept, has been created a family of credit derivatives which economic reason is in the evolution of the creditworthiness of the Reference Entity over time. In fact, the underlying is represented by the credit spread of the debt securities issued by the Reference Entity. The amount of the pay-off related to these contracts depends on the variation of the creditworthiness of the issuer of the debt compared to the creditworthiness of the same at the initial date of the contract.

The most important types of contract which have as underlying the credit spread is represented by: the credit spread option and the credit spread swap.

A credit spread option is an option on a particular credit spread. The structure of such contract generally follows the traditional scheme of option contracts. The protection buyer pays a premium (normally up-front) to receive the present value at exercise of the difference between the yield on the underlying reference asset and some benchmark (e.g. Treasury bond for a fixed rate underlying and LIBOR for a floating rate) and a pre defined spread (strike spread) [3]. The option can be both put (we have the credit spread put) and call (we have the credit spread call). If the difference between the yield of the reference asset and the benchmark (the credit spread) at an exercise date t is denoted by S_t, then the pay off of a credit spread put and call are given as follows.

$$\begin{aligned}\text{Credit spread put} &: S_t - K\\ \text{Credit spread call} &: K - S_t\end{aligned}$$

The call option pay off increases as the credit spread tightens (S_t decreases), while the put option pay off increases as the credit spread widens (S_t increases).

We consider a credit spread put. In the case in which the option is exercised, the protection seller has the obligation to make the payment in favour of the protection buyer.

As for the other credit derivatives, the payment can be either physical or cash settled. In the first case, if there is a spread widening, the protection seller has to buy from the protection buyer the reference asset at the strike spread initially agreed: the loss suffer by the protection seller is given by the fact that the purchasing of the asset take place at a price higher than the market value of the asset at the time of exercising the option, so the loss suffer is equal to difference of the two value. If the payment method of the contract is cash settled and if there is a spread widening, the protection seller has to pay an amount equal to the difference between the credit spread prevailing on the exercise date (market spread) and some pre-specified strike spread.

As with standard options, the credit spread options can be of European or American type: the latter, exercisable at any time, provides protection before the default is verify.

In Figure 3.6 is shown an example of credit spread put with cash settlement.

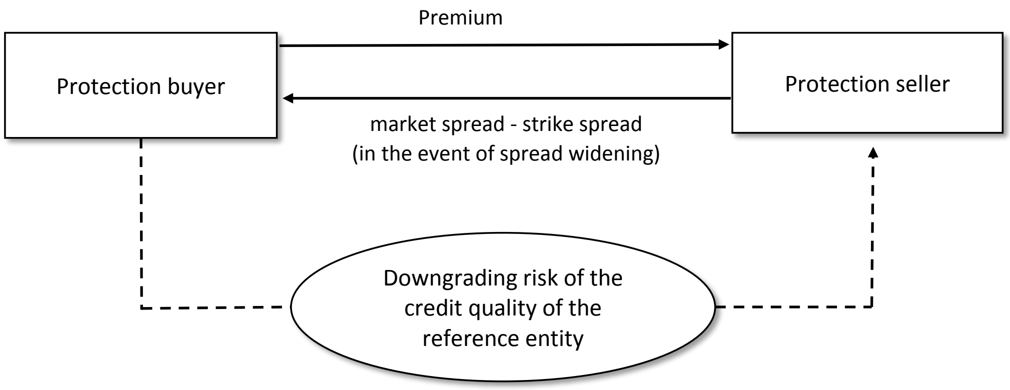

Figure 3.6: Structure of a redit spread put (Caputo and Fabbri, 2000).

Credit spread options can be used in addiction to hedging against credit spread volatility or ratings changes, for other purposes. The investors can be interesting in betting on the deterioration (credit spread put) or improvement (credit spread call) of the creditworthiness of the counterpart in order to obtain a profit. For example, by purchasing a credit spread put, the holder is betting in a spread widening. It can sell the reference asset at a strike price which is higher than the market price (given the inverse relation between credit spread and value of the reference obligation). Conversely, by purchasing a credit spread call, the holder is betting in a spread tightening. In the case in which this event actually occurs, the buyer of the credit spread call can buy from the counterpart the reference obligation at a price equal to the strike spread which is lower than the market spread.

The credit spread swap is an agreement between two parties where one party makes payments based on the yield to maturity of a specific issuer's debt and the other party makes payment based on comparable Treasury yields (or some other benchmark rate). The payoff at the contract maturity date depends on the difference between the credit spread prevailing on the maturity date and some pre-specified strike level. The payment can be executed in either direction, depending on whether it has a positive or a negative value. In practice, the parties bet on different views about the evolution of the credit spread over the contract life.

The buyer think that during the period of life of the contract there will be a deterioration of the credit quality of the reference entity, resulting in a spread widening. On the other hand, the seller think that the spread will tight on the same financial asset. Therefore, in case of spread widening the protection buyer receives the difference between the spread applied by the market and the strike spread and in case of spread tightening will be the protection seller to receive from the counterpart the difference between the strike spread and the market spread.

3.4.4 Credit linked notes

Another type of credit derivative subject to analysis is the credit linked note. The credit linked note is a structured derivative represented by a security which pays the principal and the interests only if a credit event relative to the Reference Entity does not occur. Indeed usually, if the credit event occurs, the credit linked note is terminated in advance.

Credit linked note may be issued by a financial institution or by a special purpose vehicles (SPV, in short), namely an entity specially constituted for the financial transaction. In the latter case, the investors are not subject to credit risk tied to the SPV. Instead, in the case in which the issuer of the note is a bank (or a subject of nature different from the SPV), the investors assumes in addiction to the credit risk related to the reference entity also the risk related to the issuer of the notes (linked to the ability to repay the debt of the issuer bank) [30].

The investor in the notes can be compared to protection seller in a credit default swap, which has as underlying the creditworthiness of the Reference Entity to which is tied the value that will be reimbursed to the investors.

The cash flows relates to the credit linked notes are shown in Figure 3.7.

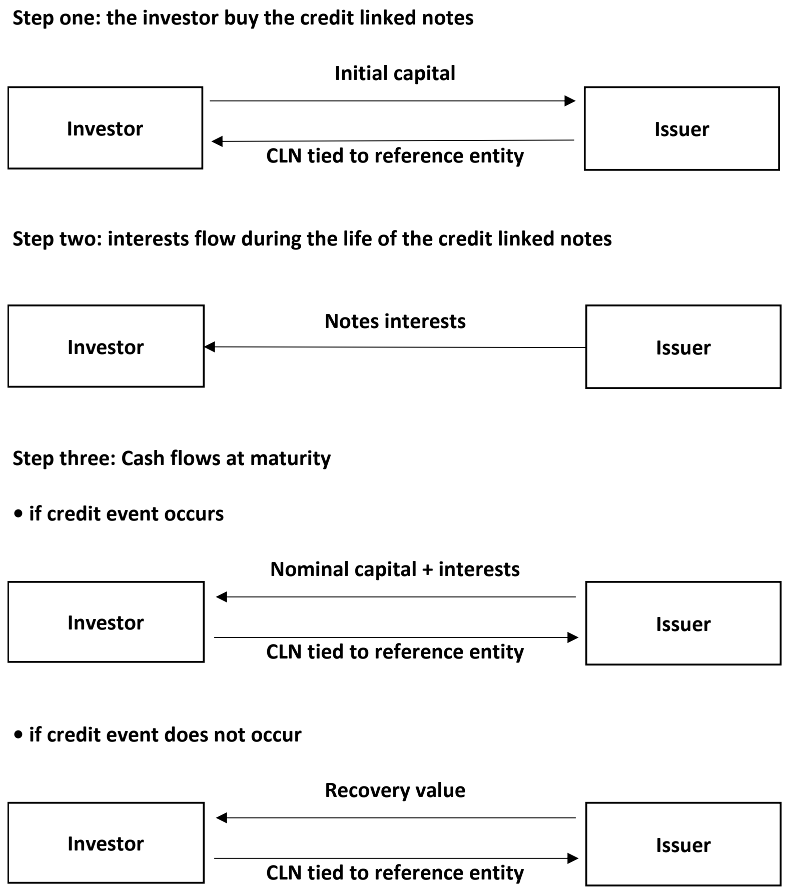

Figure 3.7: Cash flows related to credit linked notes.

The issue of the credit linked note usually is part of a more complex context, which provides the issue of securities which act as collateral to the entire operation and the conclusion of a further derivative contract (credit default swap).

For these more structured operations is usually provided the involvement of 4 subjects: the issuer of the credit linked notes that we have seen can be a SPV or a financial institution; the investors who buy the credit linked notes and assume the role of protection seller; the protection seller in the credit default swap stipulated with the issuer of the notes; the issuer of the securities. In Figure 3.8 is shown an example of credit linked note issued by a SPV.

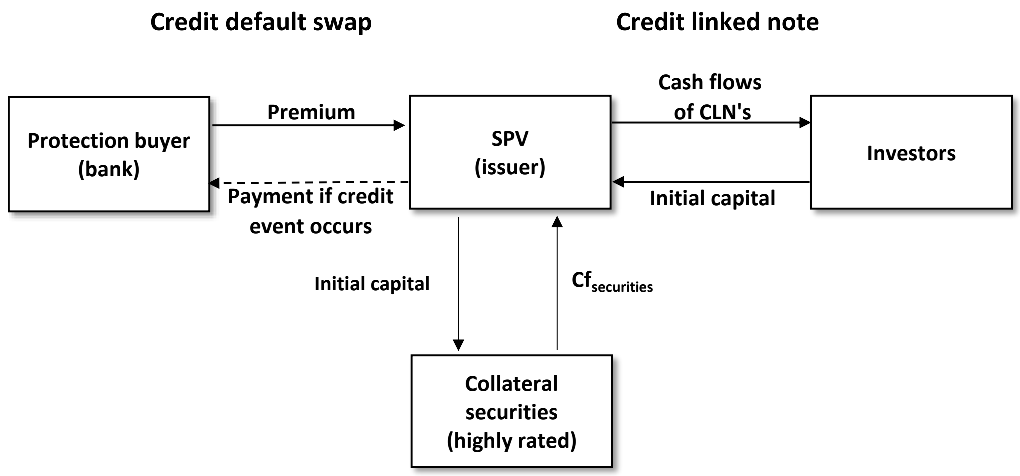

Figure 3.8: Example of credit linked notes.

The operation provides that the SPV, through the issuance of the notes placed with investors, get the funding necessary to be able to proceed with the purchase of securities issued as collateral of the operation. In this way it receives the cash flows generated by these securities.

In order to obtain the cash flows necessary to pay the future interests to the investors, the SPV enters at the same time in a credit default swap contract with maturity equal to that of the credit linked notes as protection seller, in order to obtain the premium pay by the protection buyer. The Reference Entity of the second derivative is the same of the credit linked notes.

When the credit linked note expires, two different situation can occurs. In the event that the Reference Entity does not default, the investors receive the typical cash flows of a normal bond, i.e. periodic coupons and at maturity, an amount equal to the face value of the bond. On the other hand, in the event that the Reference Entity defaults, the underlying collateral is liquidated and the investor suffer a loss given by the difference between the face value of the initial investment and the recovery value (the value at which the bond is sold next the credit event) of the reference obligation. The loss born by the investors, will be used to reimbursed the protection buyers of the credit default swap. In the case in which the Reference Entity defaults and the underlying collateral were sold at a value lower than the initial one, the corresponding loss would also be borne by the of the credit linked notes.

So, investors in credit linked notes issued by a SPV assume both the credit risk linked to the reference entity and the economic risk (seen as the credit risk and the market risk), of the securities used as collateral, although these usually enjoy an high rating. In the case of a non SPV issuer (bank or other financial institution), the investors bear, instead of the market risk associated with securities used as collateral, the risk related to the ability of reimbursed of the issuer of the credit linked notes. This double risk that investors bear is remunerated through an higher yield than other bonds issued by the same reference entity.

With regard to the risk bearing by the issuer of the credit linked notes it results to be marginal because in the case in which the credit default occurs, the cost deriving from the payment in favour of the protection buyer of the CDS is fully supported by the investors in credit linked notes.

3.4.5 Basket products

The basket products are derivatives whose underlying is constituted by the creditworthiness of more Reference Entities rather than one single entity. These contracts can be concluded in the form of any of the types of derivatives analysed so far, but usually, the most common are the credit linked notes and the credit default products.

The key element that distinguishes them from the other credit derivatives is the use of the leverage. Indeed, these contracts tend to accentuate the degree of leverage of credit derivatives, even if the nominal value refers to more contracts the loss is limited to a single asset within the portfolio. The parties have mutual advantages: the buyer pays a reduced premium than of single contract of credit derivatives, the seller obtains an attractive return, without affect the risk profile [30].

An example of basket products is the basket default swaps, which allow the protection buyer to protect themselves against the risks deriving from a basket of credits. They can be of two different types: the first to default and the green bottle.

The first to default default swap provides that, whenever an entity in the basket defaults, the buyer stops paying the swap's premium and receives from the protection an amount equal to the difference between the initial value and the recovery value of the defaulted obligation. Thus, the contract terminates and the protection seller has no longer any obligation towards the protection buyer relatively to the other assets within the basket. Essentially, this type of basket default swap offers protection against the first default only. It follows that it would be preferable for the buyer that the basket contains assets with low correlation in order to avoid to incur in multiple default and remain without protection for default after the first.

The green bottle type, on the other hand, provides that once the credit event relative to a Reference Entity is realized, the contract is not terminated but there is a reduction on the notional amount of the contract equal to the loss given by the credit event. So, in this case the contract continue to remain valid for the reduced notional amount for the other Reference Entities included in the basket. We take as an example a basket product with a notional amount equal to euro 20 million. Supposing that the total loss resulting from the realization of the credit event is equal to euro 6 million. After the liquidation of the loss, the contract will continue for a notional amount of euro 14 million. In the case in which a second default occurs before the maturity relatively to another reference entity, a further reduction of the notional amount will be made, up to zero or to the maturity of the contract.

The advantage in using basket products for the protection buyer is to get coverage at a lower cost. In the case in which the buyer has exposures to the Reference Entities included in the basket, it can be more convenient to use basket products rather than enter into a series of derivative contracts for each single exposition.

The convenience of the protection buyer in concluding a contract of this type rather than individual contracts for the various exposures, depends on the sign and on the degree of correlation existing between the various assets presented in the basket and therefore on the premium paid for the protection against credit risk.

In order to protect from variation in the credit spreads it is possible to use the basket credit spread options, where the protection seller insure, behind the payment of a premium, the buyer against the possible risk of the first spread widening which occurs among the Reference Obligations included in the basket. Also in this case, following the credit event, the contract terminates and the protection towards the other obligations in the basket cease.

Finally, developed later than the instruments presented so far, are the index swaps. These derivatives have a structure similar to that of the total return swap, but with the difference that the Reference Entity in the index swaps is given, as the word suggests, by a bond index. The parties (buyer and seller) will exchange the difference between the quotation of the reference index and a floating rate (LIBOR) plus a spread. Also in this case, the use of the index swaps is preferred because the purchase on the market of the single bonds that compose the index is higher than that of the index swap. Essentially, the market operators can, by using a reduced liquidity, diversify their portfolio, without bearing the cost of purchasing the single assets. This choice is preferred in terms of liquidity for the absence of the transaction cost of individual bonds [24].

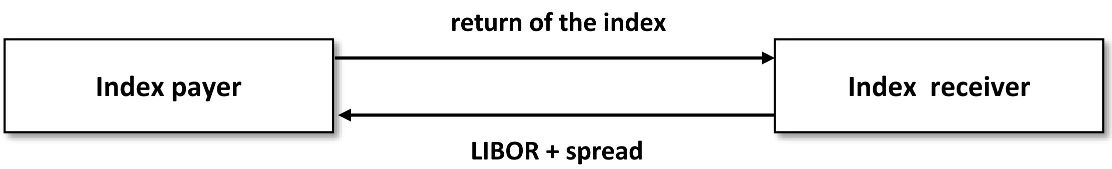

Figure 3.9: Structure of an index swap (Leone and Boido, 2004).

3.4.6 Collateralized Debt Obligations

Collateralized debt obligations (in short, CDOs) constitute an important class of the so called asset backed securities (in short, ABS). In an ABS, a portfolio of income producing assets such as loans, mortgages or bonds is sold by the originating banks to a special entity called Special Purpose Vehicle (SPV). The SPV gets the fund to buy the portfolio from issuing securities which are precisely the asset backed securities. These issued securities are typically divided into classes known as tranches. In their simplest form they are three: the senior tranche, the mezzanine tranche and the equity tranche. This process is known as securitization. Each tranche has different risks and return characteristics. The top tranche is more safe for investors, so it offers lower interest rates. The bottom tranches are expected to be more risky and therefore they offer the lowest interest rate. Rating agencies like Moody's and S&P gave the safest tranches their highest possible rating, AAA, the mezzanine tranche is typically rated BBB and the equity tranche is typically unrated.

Figure 3.10 illustrates the structure of an asset-backed security. The description of ABS that we have given so far is somewhat simplified. Actually, more than three tranches with a wide range of ratings are created.

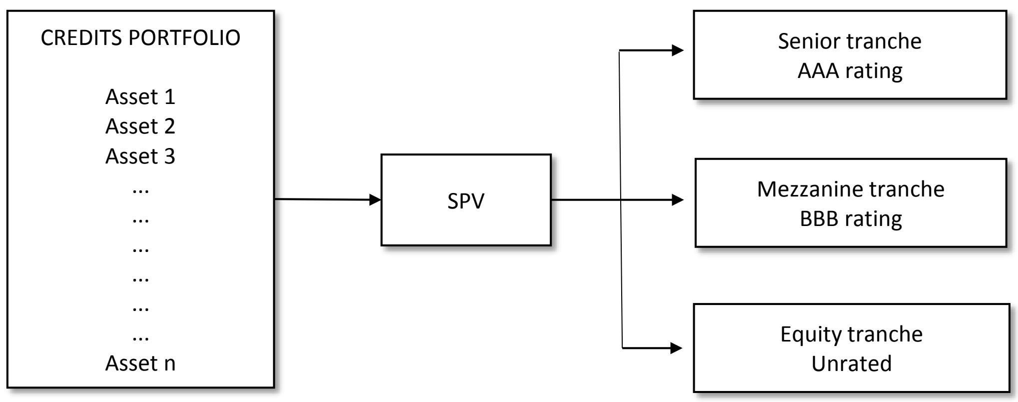

Figure 3.10: A simplified structure of an asset-backed security.

Income from the underlying assets is used to pay interests and principal on the different securities according to a set of rules known as the waterfall. A separate waterfall is applied to principal and interest payments. Principal payments go first to the senior tranche until its principal has been fully repaid, then to the mezzanine tranche and last to the equity tranche. At the same way, the interests payments go first to the senior tranche until it has received its promised return, then to the mezzanine tranche and last to the equity tranche. In Figure 3.11 is illustrated the general way a waterfall works.

The extent to which the tranches get their principal back depends on losses on the underlying assets. Losses of principal are first borne by the equity tranche, then by the mezzanine tranche and then by the senior tranche. Clearly, the equity securities are the most vulnerable to defaults on the underlying portfolio while the senior securities are the least vulnerable [31].

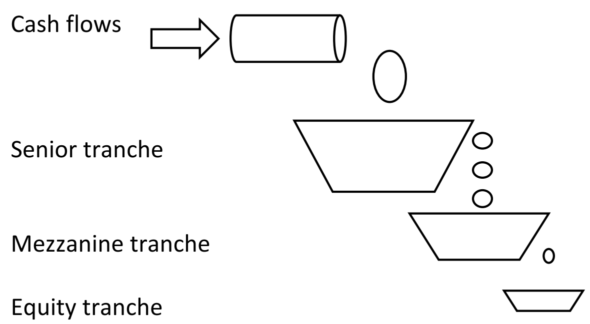

Figure 3.11: The waterfall in an asset-backed security (Hull, 2012).

An ABS where the underlying assets are bonds is known as a collateralized debt obligation or CDO. A waterfall similar to that for ABS is defined for the interest and principal payments on the bonds. The precise rules underlying the waterfall are complicated but they are designed to ensure that if one tranche is more senior than another it is more likely to receive promised interest payments and repayments of principal.

In conclusion, what makes a CDO so appealing is that it can be used to transform the credit risk of an underlying portfolio of credit risky assets into a set of securities with different credit profiles which can be adapted to the requirements of different classes of investors. For example, it may be possible to take a portfolio of subinvestment grade credits for which there may only be a limited demand and to use these in the portfolio of a CDO to issue a set of securities ranging from AAA grade down to unrated equity. The AAA securities may appeal to an insurance company, the investment grade securities may appeal to an investment fund and the unrated securities may appeal to a high yield credit fund. Therefore, CDOs can be seen as a way to enhance liquidity [31].

Through the use of CDOs, the entire credit risk of the credit portfolio is held within an SPV and then sold to the investors via the issued securities, leaving the issuer with no risk. Sometimes, the equity tranche might be retained by the issuer.

A subsequent development in the CDO market was the so called synthtetic CDO, in which there are no assets transferred from the originator to the SPV but, the SPV sells default protection to the originator through credit default swaps. A simplified version is as follows. The originator buys default swap from the SPV. The SPV structures CLNs whose payouts are linked to the underlying credits of the default swaps. The proceeds from the note sales are invested into bonds. When defaults happen and the accumulated premium payments from the default swaps are not sufficient to fund the settlement payments, the bonds are used to finance the settlement and the buyers of the CLNs see their payments reduced.

Chapter 4

A PRICING MODEL FOR CDS

4.1 Introduction

In this chapter we present how the value of a CDS contract is defined. In particular our aim is to explain the most widely used model in the market.

Credit default swaps have been explained in detail in the previous chapter. In brief, a CDS is used to transfer the credit risk of a Reference Entity (corporate or sovereign) from one party to another. A typical CDS contract usually specifies two potential cash flow streams which is called the legs of the contract. These are [1]:

- the *premium leg*, which is the series of fixed, periodic payments of CDS premium made by the protection buyer until the maturity or until the Reference Entity defaults;

- the *protection leg*, which is the one payment made by the protection seller only if the Reference Entity defaults. The amount of this contingent payment is usually the notional amount multiplied by (1-RR), where RR is the recovery rate, as a percentage of the notional.

The daily value of a CDS contract is essentially the value at which the contract can be unwound, i.e. how much we would have to pay or receive to exit from the contract. This is also known as the mark to market value or fair value.

Hence from the protection buyer side, the value of the CDS at any given point of time is the difference between the present value of the protection leg, which the protection buyer expects to receive and that of the premium leg, which he expects to pay. In other words:

$$V_t = PV(protection\ leg) - PV(premium\ leg)$$

where V_t is the value of the CDS at valuation time t, with $t \in [0, T]$.

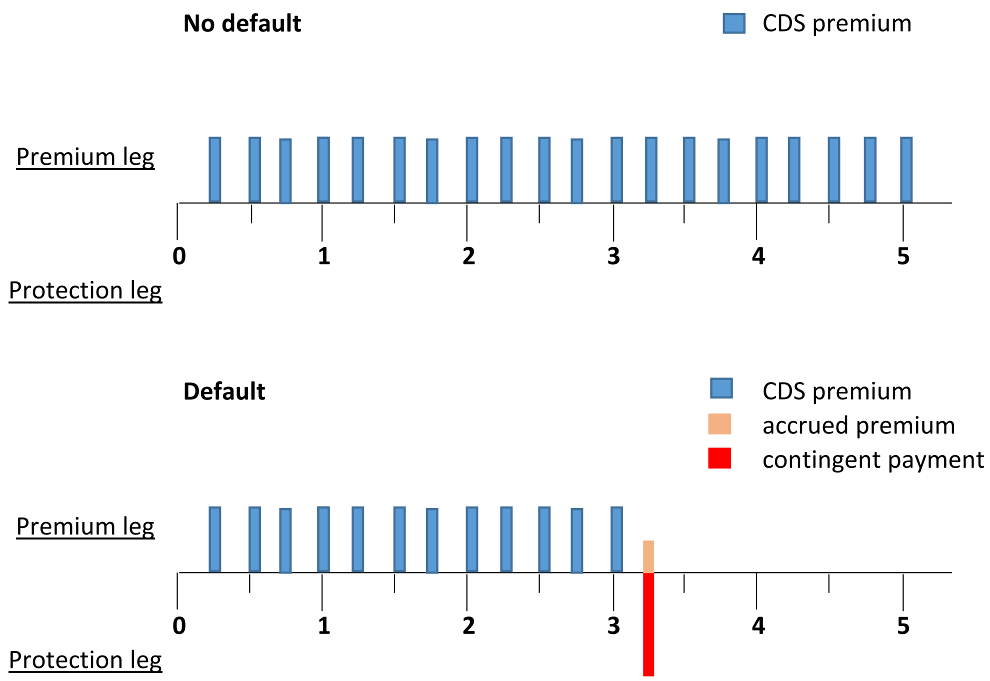

Figure 4.1: Example of the payments of a 5 year CDS.

In order to determine these values, we need information about the probabilities of default (i.e. the credit curve) of the Reference Entity, the recovery rate in the case of default and the risk free discount factors (i.e. the yield curve). For simplicity, we assume that there is no counterparty risk and the notional value of the swap is \$1 million.

A valuation model is therefore needed to calculate the probabilities of default from the market default swap spreads.

The models used to valuing the CDS have to posses some specific characteristics to be a successful CDS pricing model. It should be able to capture the risk of default of the Reference Entity and the time of default; it should define the payment of the recovery rate following a default, as a percentage of the face value of the underlying asset; it should be enough flexible to build the term structure of CDS spreads without generate any arbitrages opportunities and last it should be as simple as possible [32].

In the next sections, we will describe a pricing model for the CDS contract belonging to the class of the reduced form models. This is one of the most widely used and known model by the CDS market operators. To start with, we will examining separately the two legs of the CDS contract (premium and protection leg) and then we will end up with the determination of the mark to market value of the CDS.

4.2 A valuation model

Before to proceed with the valuation of the legs of the contract, we will quickly summing up the fundamental aspects concerning the default event.

To value a CDS, it is necessary to be able to model credit risk. As explained in chapter 2, the world of the credit risk modelling can be essentially divided into two main approaches: structural models and reduced form models. In structural models, the idea is to determine the default as the consequence of some company event such as its asset value is insufficient to cover a repayment of debt. Such models presents some important limits which slow down the use of these models to price credit derivatives.

Reduced form models are introduced with the aim of overcoming the shortcomings of structural form models. In reduced form models the default is modelled directly by modelling the probability of default itself and they are used for credit derivative pricing. In the model that will be shown, the default is treated as an exogenous event not linked to the capital structure of the Reference Entity.

The most widely used reduced form model is based on the work of Jarrow and Turnbull (1995), which has already been discussed. Here, we only remember that the default is identified when a jump is made in a Poisson process in the instant τ. It follows that the probability of default is defined as [32]:

$$\mathbb{P}[\tau < t + dt \mid \tau \geqslant t] = \lambda(t)dt, \tag{4.1}$$

i.e. the probability of a default occurring within an infinitesimal time interval $[t, t+dt]$ conditional on surviving to time t, is proportional to the hazard rate and the length of the time interval dt. In a one period-setting as a simple binomial tree we have that the probability of survival of the Reference Entity is given by $1 - \lambda(t)dt$, while the probability of default (with a recovery value RR) is $\lambda(t)dt$).

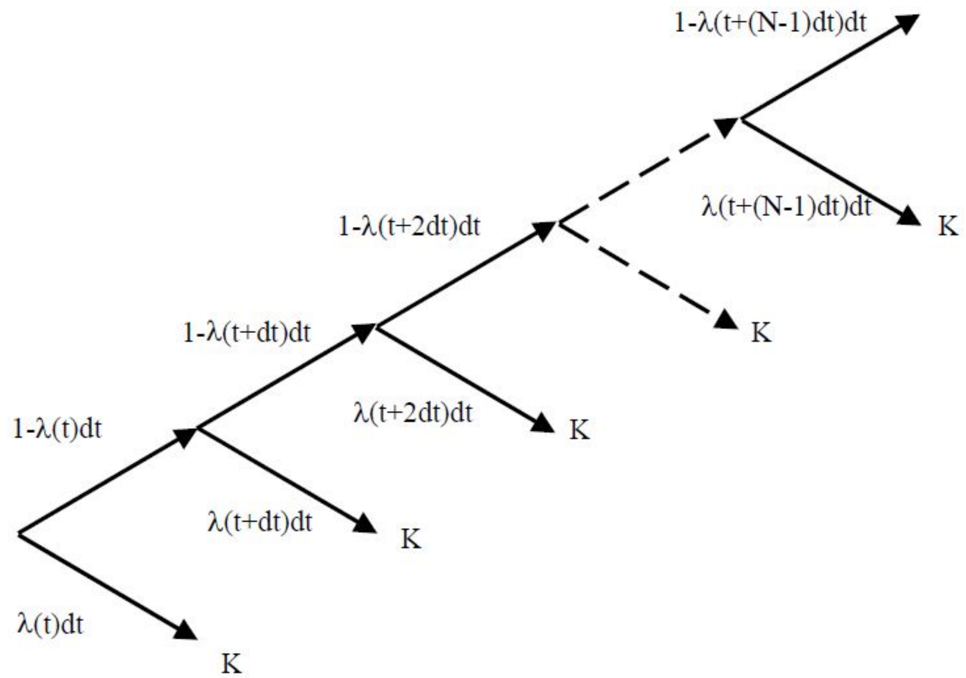

Figure 4.2: Example of a binomial tree in the continuous time, where K is the payoff at default (source: O'Kane and Turnbull, 2003).

An assumption accepted for almost all market participants is that the hazard rate is deterministic and independent from the interest rates and the recovery rate. This allows to further simplify the model and extending it to multiple time periods (see Figure 4.2). It is possible to compute the survival probability in the continuous time as:

$$Q(t,T) = e^{-\int_t^T \lambda(s)ds}. \tag{4.2}$$

4.2.1 The valuation of the premium leg

The value of the premium leg is obtained from the present value of all the expected payments made by the protection buyer. There are two types of contribution to the premium leg value. First, there are the series of premium payments which are paid until maturity or until the Reference Entity defaults. Second, if there is a credit event, there is a payment of the premium accrued from the previous premium payment date to the time of the credit event.

We assume that there are $n = 1, \ldots, N$ payments dates $t_1, \ldots, t_N$ where $t_0 = 0$ and $t_N = T$ represents the maturity date of the CDS. On each payment date, the periodic payment will be:

$$S(t_0, t_N)\ \Delta(t_{n-1}, t_n),$$

where:

- $S(t_0, t_N)$ denoted the annual CDS premium (considered constant);

- $\Delta(t_{n-1}, t_n)$ is the period of time between dates t_{n-1} and t_n.

However, this payment is only made when the Reference Entity has not defaulted by payment date. So, we have to take into account the survival probability or the probability that the Reference Entity has not defaulted on the payment date. The expected payment is:

$$S(t_0, t_N)\Delta(t_{n-1}, t_n)Q(t, t_n),$$

where $Q(t, t_n)$ is the survival probability of the Reference Entity from valuation time t to premium payment t_n. Now, we discounted the expected payment at the risk free rate and we obtain the present value for this payment:

$$S(t_0, t_N)\Delta(t_{n-1}, t_n)Z(t, t_n)Q(t, t_n),$$

where $Z(t, t_n)$ is the discount factor for the particular payment date, according to a risk free rate which is the LIBOR. Summing up the present values for all these payments, we obtain:

$$S(t_0, t_n)\sum_{n=1}^{N}\Delta(t_{n-1}, t_n)Z(t, t_n)Q(t, t_n). \tag{4.3}$$

We have now to consider the effect of the accrued premium paid up to the date of default, when defaults happens between two periodic payment dates. To include the effect of the premium accrued between the last payment date and the time of default, we need to consider the probability of surviving of the Reference Entity from the valuation date t to time s in the payment period and then, defaulting in the next small time interval ds. This probability is given by:

$$Q(t, s)\lambda(s)ds. \tag{4.4}$$

Consequently, for this specific time interval, the present value of the expected accrued premium payment is:

$$S(t_0, t_N)\Delta(t_{n-1}, s)Z(t, s)Q(t, s)\lambda(s)ds.$$

Actually, default can happen any time during this time interval $[t_{n-1}, t_n]$, so it results necessary to compute the integral over all the duration of the time interval.

$$S(t_0, t_N)\int_{t_{n-1}}^{t_n}\Delta(t_{n-1}, s)Z(t, s)Q(t, s)\lambda(s)ds. \tag{4.5}$$

We now sum over all the time intervals provided by the contract in which the payments are made:

$$S(t_0, t_N) \sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} \Delta(t_{n-1}, s) Z(t, s) Q(t, s) \lambda(s) ds. \tag{4.6}$$

The integral is complicate to evaluate. Fortunately, we can approximate it as follows:

$$\frac{1}{2} \Delta(t_{n-1}, t_n) Z(t, t_n) [Q(t, t_{n-1}) - Q(t, t_n)]. \tag{4.7}$$

This approximation is based on the observation that, if there is a default, we may assume that it occurs at the middle of the interval between consecutive payment dates. So, the accrued premium amount can be approximate to $S(t_0, t_N)\Delta(t_{n-1}, t_n)/2$. The probability of a default during the nth premium payment period is $Q(t, t_{n-1}) - Q(t, t_n)$.
Therefore, the present value of the accrued payment is given by:

$$\frac{S(t_0, t_N)}{2} \sum_{n=1}^{N} \Delta(t_{n-1}, t_n) Z(t, t_{n-1}) [Q(t, t_{n-1}) - Q(t, t_n)]. \tag{4.8}$$

Now we have both components of the protection leg. By adding (4.3) and (4.8) we get the present value of the premium leg:

$$PV(premium\ leg) = S(t_0, t_N) RPV(t, t_N), \tag{4.9}$$

where $RPV(t, t_N)$, which is known as the Risky Present Value, is the present value at time t of a 1 bp paid on the premium leg of the CDS until maturity time t_N or default and is equal to:

$$\begin{aligned} RPV(t, t_N) &= \sum_{n=1}^{N} \Delta(t_{n-1}, t_n) Z(t, t_n) Q(t, t_n) \qquad (4.10) \\ &+ \frac{1}{2} \sum_{n=1}^{N} \Delta(t_{n-1}, t_n) Z(t, t_{n-1}) [Q(t, t_{n-1}) - Q(t, t_n)]. \end{aligned}$$

Considering the accrued premium, led to a small deviation between the value of the spread that does not consider the accrued and the value of the same that takes into account the premium accrued up to default. The effect of premium accrued on the spread can be very well approximated by [31]:

$$S(\text{without accrued}) - S(\text{with accrued}) \simeq \frac{S^2}{2(1-RR)f}, \tag{4.11}$$

where RR is the recovery value and f is the frequency of payments on the premium leg.

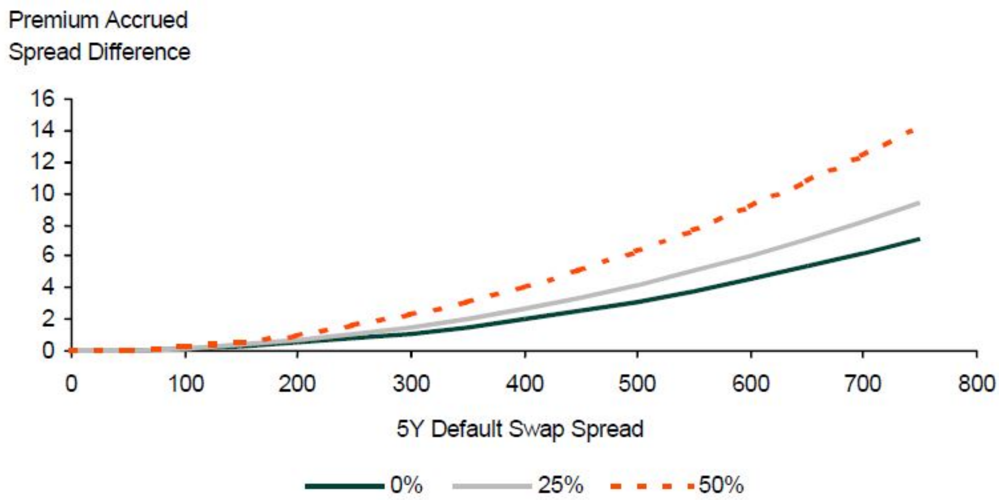

Figure 4.3: Example of the effect of the premium accrued by computing the difference between the spread with and without premium accrued. It is assume a value of recovery rate equal to 0 %, 25 % and 50 % (source: O'Kane and Turnbull, 2003).

4.2.2 The valuation of the protection leg

We now compute the value of the protection leg. We explained before that it is the payment made by the protection seller in favour of the protection buyer only if the reference entity defaults. It is equals to $1 - RR$, where RR is the expected recovery rate. Therefore, the value of the protection leg will be given by the present value of the expected amount received by the protection buyer in case of a credit event.

In practice, between the notification of the credit event and the settlement of the protection leg payment, there can be a delay of up 72 calendar days but. In order to simplify we typically assume that this payment is made immediately at time of default.

In valuing the protection leg, it is important to take into account the moment in which the default of the Reference Entity occurs. By using a reduced form model, the valuation can be done assuming that default can occurs in a small time interval $[s, s + ds]$, between the valuation date, t, and the maturity of the CDS, t_N. So, it results necessary to compute the survival probability up to time s, which is equal to $Q(t, s)$, then we compute the probability of default between times s and $s + ds$ which is equals to $\lambda(s)ds$ and then we discounted the payment of the amount $1 - RR$ back to the valuation date.

All this have to be done considering the possibility that default happening in any time during the life of the CDS, concluding the present value of the protection leg is given by:

$$PV(protection\ leg) = (1 - RR) \int_t^{t_N} Z(t, s)Q(t, s)\lambda(s)ds. \tag{4.12}$$

The integral makes this expression tedious to evaluate, so also in this case we apply an approximation, assuming that default can only occur on a finite number M of discrete points per year. It follows that for the entire life of the CDS, there are $M \times t_N$ time instants in which default can occurs, which we label as $m = 1, \ldots, M \times t_N$. We then have that the equation for valuing the protection leg becomes:

$$PV(protection\ leg) = (1 - RR) \sum_{m=1}^{M \times t_N} Z(t, t_m)[Q(t, t_{m-1}) - Q(t, t_m)]. \tag{4.13}$$

The percentage difference in terms of spread, between the continuous and discrete case is given by $r/2M$ where r is the risk free interest rate.

4.2.3 The breakeven spread

We have just presented a model that values the protection and premium legs of CDS. We can now determine the mark to market value of the CDS from the protection buyer point of view, which we saw in the beginning is simply the difference between the two legs. Plugging equation (4.18) and (4.13) into the initial equation, we arrive at a formula for calculating the value of a CDS contract.

$$\begin{aligned} V_t &= (1-RR)\sum_{m=1}^{M\times t_N} Z(t,t_m)[Q(t,t_{m-1})-Q(t,t_m)] \\ &- S(t_0,t_N)RPV(t,t_N). \end{aligned} \tag{4.14}$$

From the value of the credit default swap, we can derive the breakeven CDS spread, which is the spread paid on a new contract. It is important to observe that, in theory, when two parties enter a CDS contract, the CDS spread is set so that the value of the contract is zero (in other words the value of the premium leg equals that of the protection leg). Hence, the following equality holds:

$$PV(premium\ leg) = PV(protection\ leg);$$

Formally:

$$S(t_0,t_N)RPV(t,t_N) = (1-RR)\sum_{m=1}^{M\times t_N} Z(t,t_m)[Q(t,t_{m-1})-Q(t,t_m)].$$

Rearranging and solving for S, the annual premium payment is set as:

$$S(t,t_N) = \frac{(1-RR)\sum_{m=1}^{M\times t_N} Z(t,t_m)[Q(t,t_{m-1})-Q(t,t_m)]}{RPV(t,t_N)}, \tag{4.15}$$

where the $RPV(t,t_N)$ has been defined in equation (4.10).

4.2.4 A simplified discrete approach

In this subsection, we propose a simplified approach which implements the model we have previously presented with a discretization approach. First, let's look at the premium leg. On each payment date, the periodic payment will be:

$$d_n\, S$$

where:

- S denoted the annual CDS premium;
- d_n is the accrual days (expressed in a fraction of one year) between payment dates t_{n-1} and t_n.

However, this payment is only made when the Reference Entity has not defaulted by payment date. So, we have to take into account the survival probability or the probability that the Reference Entity has not defaulted on the payment date. The expected payment is:

$$Q(t_n) d_n\, S,$$

where $Q(t)$ is the survival probability of the Reference Entity at time t. Now we discounted the expected payment at the risk free rate and we obtain the present value for this payment.

$$Z(t_n) Q(t_n) S\, d_n,$$

where $Z(t_n)$ is the discount factor for the particular payment date according to according to a risk free rate which is the LIBOR. Summing up the present values for all these payments, we obtain:

$$\sum_{n=1}^{N} Z(t_n) Q(t_n) S\, d_n. \tag{4.16}$$

We have now to consider the effect of the accrued premium paid up to the date of default when defaults happens between two periodic payment dates.

To include the effect of the premium accrued between the last payment date and the time of default, we need to consider the probability that the default occurs between payment date t_{n-1} and payment date t_n. In other words, the Reference Entity survived until payment date t_{n-1}, but not to next payment date, t_n. This probability is given by:

$$[Q(t_{n-1}) - Q(t_n)].$$

We may assume that if there is a default, it occurs in the middle of the interval between consecutive payment dates. Then, the accrued premium amount can be approximate to $S\,d_n/2$.
Consequently, for a particular interval, the expected accrued premium payment is:

$$[Q(t_{n-1}) - Q(t_n)]S\,d_n/2.$$

Therefore, the present value of all expected accrued payments is given by:

$$\sum_{n=1}^{N} Z(t_n)[Q(t_{n-1}) - Q(t_n)]S\,d_n/2. \tag{4.17}$$

Now we have both components of the protection leg. By adding (4.16) and (4.17) we get the present value of the premium leg:

$$\begin{aligned} PV(premium\ leg) &= \sum_{n=1}^{N} Z(t_n)Q(t_n)S\,d_n \qquad (4.18)\\ &+ \sum_{n=1}^{N} Z(t_n)[Q(t_{n-1}) - Q(t_n)]S\,d_n/2. \end{aligned}$$

Next, we compute the present value of the protection leg. Assume that the Reference Entity defaults between payment date t_{n-1} and payment date t_n. The protection buyer will receive the contingent payment of $(1 - RR)$. This payment is made ony if the Reference Entity defaults and therefore, it has to be adjusted by $Q(t_{n-1}) - Q(t_n)$ which is the probability that the default actually occurs in this time period.

Discounting each expected payment and summing up over the term of the contract, we obtain:

$$PV(protection\ leg) = (1 - RR)\sum_{n=1}^{N} Z(t_n)[Q(t_{n-1}) - Q(t_n)]. \tag{4.19}$$

Plugging equation (4.18) and (4.19) into the equation in the beginning, we arrive at a formula for calculating the value of a CDS contract.

$$\begin{aligned} V_t &= (1 - RR)\sum_{n=1}^{N} Z(t_n)[Q(t_{n-1}) - Q(t_n)] - \\ &- \sum_{n=1}^{N} Z(t_n)Q(t_n)S\,d_n + \sum_{n=1}^{N} Z(t_n)[Q(t_{n-1}) - Q(t_n)]S\,d_n/2. \end{aligned} \tag{4.20}$$

From the value of the credit default swap, we derive the breakeven CDS spread. The CDS spread is set so that, the value of the contract is zero (in other words the value of the premium leg equals that of the protection leg). Hence, the following equality holds:

$$\begin{aligned} \sum_{n=1}^{N} Z(t_n)Q(t_n)S\,d_n &+ \sum_{n=1}^{N} Z(t_n)[Q(t_{n-1}) - Q(t_n)]S\,d_n/2 = \\ &= (1 - RR)\sum_{n=1}^{N} Z(t_n)[Q(t_{n-1}) - Q(t_n)]. \end{aligned} \tag{4.21}$$

Rearranging and solving for S, the annual premium payment is set as:

$$S = \frac{(1 - RR)\sum_{n=1}^{N} Z(t_n)[Q(t_{n-1}) - Q(t_n)]}{\sum_{n=1}^{N} Z(t_n)Q(t_n)S\,d_n + \sum_{n=1}^{N} Z(t_n)[Q_{t_n-1} - Q_{t_n}]S\,d_n/2}. \tag{4.22}$$

4.3 Inputs necessary for the implementation of the model

In order to proceed with the pricing of a credit default swap are required certain inputs necessary for the execution of the model. We said that these key inputs are: the default probability (i.e. the credit curve) of the Reference Entity, the recovery rate in the case of default and the risk free discount factors (i.e. the yield curve).

In the following section, we will present the assumptions made for the determination of the expected recovery rate and a method used for the determination of the probability of default.

4.3.1 The recovery rate

One of the required inputs for the pricing of the CDS is the recovery rate, RR. However, this is not a market observable input and it changes over time. This means that the value of a CDS can differ between counterparties who disagree on their views around expected recovery rates, despite using the same CDS spreads and the same model for the determination of the term structure of the probability of default.

The expected recovery rate is not the expected value of the asset following the default process but, it is the market value of the cheapest to deliver asset, expressed as a percentage of its face value, within approximately 72 calendar days after notification of the credit event.

The starting point used by several operators for estimate the recovery rate is given by the historical data collected and elaborated by the rating agencies [32]. Actually, the information collected by rating agencies can not always be used because they presents a number of limitations. For example rating agencies do not view restructuring as a default while standard CDS do; the data are historical, not forward looking and so fail to take into account market expectations about the future; often the information are neither classify for name nor for economic sector which the reference entity belongs; in addition, the majority of the information refers to the US corporates, because that is where the great amount of default data originates and so may not be appropriate for European corporates.

An alternative way to overcome these drawbacks and determine the recovery rate can be to use a valuation model that extract information about the recovery value from bond prices. However, this is difficult to apply especially for good credit quality companies, because the low default probability means that the recovery rate is only a small component of the bond price. For lower credit quality companies, the recovery rate has a greater incidence and we expect to obtain more information about market expectations for the future recovery rates.

So, in general it can be stated that the value of a CDS with investment grade quality spread is not very sensitive to the recovery rate. The sensitivity to the value of the recovery rate only becomes significant when the issuer spreads become very large and the credit is distressed.

In practice, what happens is that a consensus value arises in the market which then become widely used. Basically, this value depends on the economic phase in which we are located, in the moment in which the CDS contract is sign. Usually it happens that, in an expansive economic period with lower default rates, probably the value of the expected recovery rate will increase, on the other hand in recessionary phase of the economy the value of the expected recovery rate will decrease. Currently, the consensus value for the expected recovery rate is 40 % for investment grade senior debt.

4.3.2 The term structure of default probabilities

The crucial point of the model is the determination of the default probabilities. The other key inputs, as we seen so far, can be in some manner quoted or established. Then, what we now need to do is to build the term structure of the default probabilities. The process of constructing the term structure of default probabilities is commonly know as bootstrapping. In particular we will present one method used by the platform Bloomberg which is the *Discount Factor Model* [24].

Through this method the probabilities of default are extracted using the bootstrapping technique implemented on the difference between two curves of rates: one is risky and the other is risk-free. In particular, the risk free curve is determined by using the rates of return on governments bonds, while the structure of the risky rates is constructed with a corporate curve, chosen according to the characteristics of the Reference Entity.

The starting point for extrapolating the term structure of the probability of default is the spread required by investors to be repaid for the risk of default (Risk Premium). The latter, is nothing more than the current value of the expected loss discounted at the risk free rate.

Let us denote with r_{df} and r_d respectively the default free rate and the defaultable rate, for which we have that $r_d > r_{df}$. The price of the zero coupon bonds, respectively $v_{df}(0,t)$ and $v_d(0,t)$ are:

$$v_{df}(0,t) = e^{-r_{df}t}, \qquad v_d(0,t) = e^{-r_d t}. \tag{4.23}$$

The risk premium, RP, is equal to the difference between the two prices:

$$RP(0,t) = v_{df}(0,t) - v_d(0,t). \tag{4.24}$$

Assuming that default can only occurs in discrete moments of time and that there is no possibility of arbitrage, we know that the price of the risky zero coupon bond with maturity t_1 is given by the expected value of the payoff discounted at the risk free rate:

$$v_d(0,t_1) = v_{df}(0,t_1)[(1-p_{t1}) + p_{t1}(1-RR)], \tag{4.25}$$

where p_{t1} denotes the marginal probability of default from 0 to t_1. By substituting this last equation in that of the risk premium we can simply verify that this is equal to the expected loss, $p_{t1}(1-RR)$, discounted at the risk free rate:

$$RP(0,t_1) = v_{df}(0,t_1)p_{t1}(1-R). \tag{4.26}$$

Therefore, known $RP(0,t_1), v_{df}(0,t_1)$ and RR, it is possible to specify the probability of default for the first expiry, which is:

$$p_{t1} = \frac{RP(0,t_1)}{v_{df}(0,t_1)(1-RR)}. \tag{4.27}$$

At this time the model assumes that for the following periods the risk premium is equal to the sum of each risk premium, determined in the previous time intervals:

$$\begin{aligned} RP(0,t_2) &= v_{df}(0,t_2) - v_d(0,t_2) \\ &= RP(0,t_1) + RP(t_1,t_2) \\ &= RP(0,t_1) + v_{df}(0,t_2)(1-p_{t1})p_{t2}(1-R). \end{aligned} \tag{4.28}$$

Once the marginal probability of default of the previous period is known, repeating the passage seen above, we obtain the probability relative to the second period, which is p_{t2}.

In conclusion, it is possible to determine a general formula that, repeated for each time interval, allows to compute the different proababilities of default:

$$p_{ti} = \frac{RP(0,t_i) - RP(0,t_{i-1})}{v_{df}(0,t_i)(1-RR)\prod_{h=1}^{i-1}(1-p_{th})}. \tag{4.29}$$

From the marginal probabilities of default, we can pass to the conditional probabilities of default, that is the probabilities of default of each interval of time, conditioned to the lack of default up to the previous period:

$$\begin{aligned} PD[t_{i-1}, t_i | not\ default(0,t_{i-1})] &= (1-p_{t1})(1-p_{t2})\ldots(1-p_{ti-1})p_{ti} \\ &= \prod_{h=1}^{i-1}(1-p_{th})p_{ti}, \end{aligned} \tag{4.30}$$

and from these, to the cumulative probabilities of default, namely the probabilities of default on the period $[0, t_{ti}]$, necessary for the pricing of the CDS:

$$PD[0, t_i] = \sum_{z=1}^{i} PD[t_{z-1}, t_z | not\ default(0, t_{z-1})]. \tag{4.31}$$

Chapter 5

THE CDS MARKET: AN ANALYSIS OF SOME QUOTED CDS

5.1 The role of Central Clearing Houses in the CDS market

As seen in the previous chapters, credit default swaps are over-the-counter derivatives: they are not exchange-traded. This means that investors get in touch with each other directly and agree to trade a CDS. There are obviously some risks. One of the main risk is the counterparty risk, that is the possibility that one of the parties have not the financial resources to honor the agreement. In particular, a serious concern in the CDS market is the counterparty risk generated by the default of large protection sellers, as exemplified by the failure of AIG[1]. The CDS market is a dealer market where a

[1]AIG stands for American International Group and it is an American multinational insurance company. During the period leading up to the credit crisis, it provided protection to other financial institutions against a huge volume of credit risks that were related to subprime mortgages. When losses hit the mortgage market, AIG failed but the US government bail out it for \$ 85 billion because its failure would have endangered the financial integrity of other major firms that were its trading partners such as Goldman Sachs, Morgan Stanley and Bank of America (as well as dozens of European banks).

few major institutions control an overwhelming proportion of the volume and post quotes for protection premiums on various Reference Entities. The 10 largest dealers account for 90 % of trading volume by gross notional amounts. Concentration is even higher in the US market, where the five biggest commercial banks account for more than 90 % of gross notional. An estimated 30 % of global activity is generated by JPMorgan alone [13]. In a so concentrated market, the default of a dealer can affect many market participants and generate a domino effects and default contagion. The default of an entity incurs losses not only for its counterparties, but also for protection sellers in credit default swaps written on this entity. If a CDS protection seller has insufficient reserves to cover CDS liabilities, the underlying credit event also results in the default of the protection seller, thus widening the scope for contagion.

Central Clearing House, now also known as Central Clearing Parties (in short, CCPs) have been proposed as a solution for mitigating counterparty risk and preventing default contagion in the CDS market. If the major dealers participate in a central clearing house, this can actually contribute to reduce losses in case of default and mitigates counterparty risk. In particular after the 2007-2008 financial crisis, governments in the US and elsewhere have done an extensive regulatory effort in order to improve the efficiency of the CDS market and add stability to the financial system.

Basically, a clearing house is an highly regulated organizations which acts as an intermediary between the parties guaranteeing the performance of each transaction. The way in which clearing houses work in the CDS market is as follows. A CDS is negotiated between two parties, A and B, in the usual way. It is then presented to a clearing house. Assuming the clearing house accepts the transaction, it becomes the counterparty to both A and B [18]. In this way, the clearing house become the buyer of every seller and the seller of every buyer taking on the credit risk of both participants. On the other hand, the participants are required to deposit an initial margin and daily variation margins from them, to ensure that the trading positions are properly valued and collateralized.

A number of arguments have been cited for the use of clearing houses in the CDS markets. Collateral will automatically have to be posted; the credit risk in the financial system will be reduced[2] and the trades taking place in the OTC market will become more transparent.

5.2 ICE CDS clearing

ICE (Intercontinental Exchange) is a global operator of regulated exchanges and clearing houses for financial and commodity markets. It has contributes to a significant reduction in systemic and operational risks in the CDS market. In particular, ICE provides clearing services to global CDS markets through two clearing houses: ICE Clear Credit and ICE Clear Europe.

ICE Clear Credit is the world's first dedicated CDS clearing house based in the U.S. The clearing house was launched in March 2009 as ICE Trust U.S. and was regulated as a bank by the New York State Banking Department and The Federal Reserve Board of Governors. From 2011, the clearing house is regulated by the Commodity Futures Trading Commission (CFTC) as a Derivatives Clearing Organization (DCO) and by the Securities and Exchange Commission (SEC) as a Securities Clearing Agency (SCA) and it changed its name to ICE Clear Credit. ICE Clear Credit clears North American, European, Asian-Pacific and Emerging market CDS instrument.

On the other hand, ICE Clear Europe is a London-based derivatives clearing house. It was launched in November 2008 to serve energy futures and OTC markets. It clears approximately 50 % of the world's crude and redefined oil futures contracts and is regulated by the Bank of England. In 2009, ICE Clear Europe extended its clearing services to CDS with a separate risk pool for CDS, including guaranty fund and margin accounts, as well as dedicated risk management system and governance structure. ICE Clear Europe CDS clearing is distinct from ICE Clear Credit in the U.S., but leverages CDS technology and risk models already developed by ICE and used by market

[2]The impact of clearing houses on credit risk depends on the number of clearing houses and the proportion of all OTC trades that are cleared through them. See Duffie D. and Zhu H. (2010), "Does a Central Clearing counterparty Reduce Counterparty Risk?" Working Paper, Stanford University.

participants.

As neutral and independent clearing houses, membership in ICE Clear Credit and ICE Clear Europe is open to all qualifying protection buyer and protection seller institutions that meet the financial and eligibility standards set forth in the rules of the clearing house.

ICE Clear Credit currently has 29 clearing members, 14 of which are financial or banking groups and 9 of which are non-US domiciled. The institutions include Barclays, UBS, JP Morgan, Credit Suisse, Goldman Sachs and Deutsche Bank. In Figure 5.2 we find the full list.

CLEARING MEMBERS	
Bank of America, N.A.	JPMorgan Chase Bank, National Association
Barclays Bank PLC	J.P. Morgan Securities LLC
Barclays Capital Inc.	Merrill Lynch International
BNP Paribas	Merrill Lynch, Pierce, Fenner & Smith Incorporated
BNP Paribas Securities Corp	Morgan Stanley Capital Services LLC
Citibank N.A.	Morgan Stanley & Co. LLC
Citigroup Global Markets Inc.	Nomura International PLC
Credit Suisse Securities (USA) LLC	Nomura Securities International, Inc.
Deutsche Bank AG, London Branch	Société Générale
Goldman Sachs & Co. LLC	SG Americas Securities, LLC
Goldman Sachs International	The Bank of Nova Scotia
HSBC Bank USA, N.A.	UBS AG, London Branch
HSBC Bank plc	UBS Securities LLC
HSBC Securities (USA) Inc.	Wells Fargo Securities, LLC

Figure 5.1: Clearing members of ICE Clear Clearing.

5.3 CDS traded in the ICE Clear Credit

We now proceed with an analysis of the CDS traded in ICE Clear Credit because many instruments listed in ICE Clear Europe are listed also in ICE Clear Credit. First of all, it is important to note that not all CDS are clearable, due to non-standardization and lack of liquidity. Only standardized CDS contracts are clearable. Securities such as CDO's tend to be more structured and less standardized and are difficult markets for which to establish clearing.

As we explained in chapter 3, the documentation used in CDS transactions is based on the documents and definitions provided by the International Swaps and Derivatives Association (ISDA). In particular, ISDA introduced a Master Agreement which established the terms and conditions of a standard CDS contract. Clear definitions of the credit events and settlement procedures were meant to avoid disputes as to whether a credit event had actually occurred or how a contract should best be settled. This Master Agreement was introduced in 1999 and have been continuously developed since then. A revised version of the agreement was released in 2003, while important amendments were made in 2009 with the so called "big bang" and "small bang" protocols. Both protocols help to determine a fair settlement value in case of a credit event by means of an auction. While the "big bang"protocol refers to the default or bankruptcy of the underlying entity the "small bang" protocol refers to a restructuring credit event. Before the new measures came into effect, settlement protocols were established on a case-by-case basis only after a credit event was identified.

In addition to the settlement standards, market participants agreed on using standardised maturities dates and standardised coupon values. The maturity date of a CDS is the date on which the credit protection expires and it does not necessary depend on the maturity of the securities issued by the Reference Entity. The dates of CDS contract maturities were formally standardized on March 20^{th}, June 20^{th}, September 20^{th} and December 20^{th} (standard maturity dates).

In a fully customized market, the CDS coupon will be the spread that equates the discounted present value of coupon payments, to the discounted expected payments from the protection seller, following the occurrence of a credit event. In a standardized CDS, the CDS coupon is fixed on the trade date and does not change over the life of the CDS contract. These standard coupons must be chosen based on the credit quality of the Reference Entity and vary by geographical region. For example, single-name CDSs based on North American corporate and sovereign Reference Entities generally have standard coupon rates of either 100 bps or 500 bps per annum for investment-grade and high-yield reference names, respectively.

In practice, very few CDSs have a market coupon rates of either exactly 100 bps or 500 bps on their trade dates. Therefore, in the probable case in which the standard coupon rate is not in line with the current quoted on the market or, on the trade date, it does not equate the present value of the two legs, the difference will have to be paid by one of the two parties (depending on whether the market spread is above or below the standard coupon rate) through a cash payment, known as upfront payment.

For standardized CDS, coupon payments are made quarterly on March, June, September and December 20^{th}. They are quoted on an annualized basis and the actual payment amount is computed using the notional amount of the CDS contract and an Actual/360 day-count convention.

In 2014, ISDA issued a new revised version of its Credit Derivatives Definitions which is called the "2014 Definitions"or "new Definitions" which currently form the basis of the documentation for the standard CDS cleared in the ICE clearing house.

With reference to the ICE Clear Credit clearing house, we distinguish CDS in single-name and index CDS.

Single-name CDS is the simplest and most basic form of the contract and indicate CDS that has as Reference Entity an individual issuer. Single-name CDS are divided by type of Reference Entity. It can be a corporate, then we speak about corporate single names CDS or it can be a government, we then speak about sovereign single names CDS.

In contrast to the single names CDS, we have the *CDS indices*, which have multiple underlying Reference Entities and thus are known as multi-name

CDS. They use an index of debtors as Reference Entity, incorporating up to 125 corporate entities. CDS indices are divided in two big families: CDX and iTraxx.

Table 5.1 shows the number of CDS instruments cleared by this clearing house according to the classification just made.

Table 5.1: Overview of the CDS that ICE Clear Credit currently clears (source: www.theice.com).

	ICE CLEAR CREDIT			
	Single-names CDS		CDS Indices	
Instruments	Corporate single Names	Sovereign Single Names	CDX	iTraxx
Number	464	35	70	75
Total	499		145	

5.3.1 Corporate single names CDS

In Ice Clear Credit corporate single name CDS are divided in North American, European, Australian and Asia & Emergins corporate CDS. In addiction, each of these is divided according to the sector of the Reference Entity: basic materials, consumer goods, consumer services, energy, financials, healthcare, industrials, technology, telecommunication services and utilities. Australian, Asia & Emergins corporate CDS don't cover all the sectors just mentioned. In order to analyse them I took for each sector a representative CDS as it is shown in the tables below.

Corporate single names are documented using the 2014 ISDA Credit Derivatives Definitions, this means that they have a series of standardized features. The tenor of all corporate single names CDS ranges from 0 to 10 years with standard maturity dates which are always March 20, June 20, September 20 and December 20. They are traded with a fixed coupon which ranges from 25 to 500 basis points and the payments of the coupon are made quarterly. All corporate CDS are quoted in USD dollars except for the european corporate CDS. The notional is usually quite large from $ 1 million to $ 10 million (or equivalent in other currencies).

Here in the following, there are summary tables of corporate single names CDS currently clearing in Ice Clear Credit. The labels "Markit Red6" and "Preferred ISIN" in the tables are respectively the code which identify the Reference Entity of the CDS and the Reference Obligation of the CDS.

Table 5.2: Examples of some North American corporate CDS (source: www.theice.com).

North American Corporate Single Names							
Reference Entity	Sector	coupon (100 bps)	coupon (500 bps)	Date included for clearing	ISDA Definition	Markit Red6	Preferred ISIN
AK Steel Corporation	Basic Materials		X	20/07/2015	ISDA2014Credit	0A169A	US001546AU45
Whirlpool Corporation	Consumer Goods	X	X	06/10/2014	ISDA2014Credit	9F9652	US96332HCD98
Expedia, Inc.	Consumer Services	X	X	06/10/2014	ISDA2014Credit	3D233R	US30212PAJ49
Valero Energy Corporation	Energy	X	X	06/10/2014	ISDA2014Credit	9AAA4I	US91913YAB65
American Express Company	Financials	X	X	06/10/2014	ISDA2014Credit	027D97	US025816BD05
Johnson & Johnson	Healthcare	X		31/05/2016	ISDA2014Credit	4BF976	US478160BS27
Ingersoll-Rand Company	Industrials	X		06/10/2014	ISDA2014Credit	49BEEC	US456866AG74
Motorola Solutions, Inc.	Technology	X	X	06/10/2014	ISDA2014Credit	6I209O	US620076AH21
FRONTIER COMMUNICATIONS	Telecommunications Services	X	X	06/10/2014	ISDA2014Credit	38E96G	US17453BAJ08
Exelon Corporation	Utilities	X		06/10/2014	ISDA2014Credit	3D177O	US210371AL43

Table 5.3: Examples of some European corporate CDS (source: www.theice.com).

European Corporate Single Names										
Reference Entity	Sector	coupon (25 bps)	coupon (100 bps)	coupon (175 bps)	coupon (300 bps)	coupon (500 bps)	Date included for clearing	ISDA Definition	Markit Red6	Preferred ISIN
Metsa Board Corporation	Basic Materials					X	14/09/2015	ISDA2014Credit	XC859Y	FI4000085550
Fiat Chrysler Automobiles N.V.	Consumer Goods					X	28/01/2015	ISDA2014Credit	NQ48AB	US31562QAC15
CARREFOUR	Consumer Services	X	X				06/10/2014	ISDA2014Credit	FG4CAM	XS0934191114
ENI S.P.A.	Energy		X				06/10/2014	ISDA2014Credit	28EFBV	XS0741137029
INTESA SANPAOLO SPA	Financials	X	X				22/09/2014	ISDA2014Credit	TYA56D	XSSNRREFOBL0
Bayer Aktiengesellschaft	Healthcare	X	X				06/10/2014	ISDA2014Credit	0H99A3	XS0255605239
BRITISH AIRWAYS plc	Industrials					X	06/10/2014	ISDA2014Credit	1C145A	XS0133582147
Nokia Oyj	Technology		X			X	06/10/2014	ISDA2014Credit	XD79FA	XS0411735482
VODAFONE GROUP PUBLIC LIMITED COMPANY	Telecommunications Services	X	X				06/10/2014	ISDA2014Credit	9BADC3	XS0169888558
EDISON S.P.A.	Utilities	X	X				06/10/2014	ISDA2014Credit	TW8A7X	XS0557897203

Table 5.4: Examples of some Australian corporate CDS (source: www.theice.com).

Australian Corporate Single Names								
Reference Entity	Sector	coupon (25 bps)	coupon (100 bps)	coupon (500 bps)	Date included for clearing	ISDA Definition	Markit Red6	Preferred ISIN
BHP BILLITON LIMITED	Basic Materials		X		30/01/2017	ISDA2014Credit	08GE66	US055451AH17
COMMONWEALTH BANK OF AUSTRALIA	Financials		X		30/01/2017	ISDA2014Credit	2C2983	XSSNRREFOBL0
QANTAS AIRWAYS LIMITED	Industrials		X		30/01/2017	ISDA2014Credit	7BB98S	AU3CB0220929

Table 5.5: Examples of some Asia and emerging countries corporate CDS (source: www.theice.com).

Asia and Emerging markets Corporate Single Names								
Reference Entity	Sector	coupon (25 bps)	coupon (100 bps)	coupon (500 bps)	Date included for clearing	ISDA Definition	Markit Red6	Preferred ISIN
POSCO	Basic Materials		X		02/10/2017	ISDA2014Credit	6FC7CB	USY70750AN78
Hutchison Whampoa Limited	Consumer Services		X		02/10/2017	ISDA2014Credit	48CC56	USG4672UAA37
Petroleos Mexicanos	Energy		X		02/10/2017	ISDA2014Credit	787B9U	US706451BG56
China Development Bank	Financials		X		16/10/2017	ISDA2014Credit	YZ5A6V	XS1301292261

Real-time and historical information about CDS can be extracted from the Bloomberg terminal. In Figure 5.2 is shown a screen capture of the Credit Default Swap Valuation (CDSW) page of Bloomberg. The CDSW screen is divided into three sections: the Deal Information, the Calculator and the Market data.

The Deal section, on the upper left side, summarizes the characteristics of the CDS contract. The Calculator section allows, after selecting the model to be used, to compute the value of the contract. Finally, the Market section, on the right side, allows to set the type of curves to be used for discount cash flows and for determine the market spread as well as the recovery rate. In the same section are reported the full term structure of the CDS curve.

In most cases, many of the fields just mentioned, complete themselves according to market standards. The operator who want to negotiate the CDS have therefore only to verify the accuracy of the information, without having to manually put every time the convention and information for each single CDS [34].

We bring an example of the CDSW page of Bloomberg for each type of CDS, corporate, sovereign and index.

In Figure 5.2 is shown a credit default swap on an ENI obligation. The notional of the contract is €10 million and the contract type is a Standard European Contract (STEC)[3] according to the 2014 ISDA Definitions. The issuer of the debt for which protection is being bought is ENI s.p.a. In particular, the debt is of senior type. We find the code which identify the Reference Obligation and the RED Pair Code which identifies the Reference Entity associated with the Reference Obligation.

The trade date of the CDS is on 18/01/2018, the first coupon is paid on 20/03/2018 while the maturity date is on 20/12/2020. The market spread from the CDS curve (i.e. the value that the market thinks you should be paying for the deal) is 19.79 bp while the standard coupon to be paid is 100 bp, this means that the buyer is overpaying for the deal. The cash amount reflects the amount by which the buyer needs to be compensated. The coupon payments are made quarterly.

The CDS price calculated using the Bloomberg Fair Value Model on 18/01/2018 is €102.37 while, the Mark to Market value is €-245,44.

From the Market section, it is possible to see the curves that have been used to price the CDS. We find that the swap curve used is the EUR ISDA Standard Curve while the CDS curve used is the ENIIM EUR Senior Curve. The first is used to discount the coupon at the different maturities while the last determines the market spread at different maturities. We also find the recovery rate at 40 %, which is a standard assumption for senior debt.

[3]This contract has a standardized coupon of 25, 100, 300, 500, 750 or 1000 bps, with quarterly premium payments and a recovery rate assumption of 40% for senior debt, 20% for subordinated debt and 0% for other.

Figure 5.2: Bloomberg page CDSW for ENI CDS (source: Bloomberg).

5.3.2 Sovereign single names CDS

Then we have the CDS that have as Reference Entity a government. ICE Clear credit clears also foreign sovereign CDS. Also Sovereign single names CDS traded in Ice Clear Credit are documented using the 2014 ISDA Credit Derivatives Definitions.

The tenor of sovereign single names CDS goes from 0 to 10 years with standard maturity dates which are always March 20, June 20, September 20 and December 20. They are traded with a fixed coupon which can be of 25, 100 or 500 basis points and the payments of the coupon are made quarterly. The quoted currency is USD for all sovereign CDS. The notional is usually quite large from \$ 1 million to \$ 10 million. All reference obligations are senior.

We have taken some examples of sovereign CDS that are currently cleared in ICE Clear Credit.

Table 5.6: Examples of some sovereign CDS that ICE Clear credit currently clears (source: www.theice.com).

Sovereign Single Names								
Reference Entity	Sector	coupon (25 bps)	coupon (100 bps)	coupon (500 bps)	Date included for clearing	ISDA Definition	Markit Red6	Preferred ISIN
Republic of Italy	Government	X	X		05/01/2015	ISDA2014Credit	4AB951	US465410AH18
Kingdom of Spain	Government	X	X		05/01/2015	ISDA2014Credit	8CA965	XS1138687592
Russian Federation	Government		X	X	06/10/2014	ISDA2014Credit	7FB37H	XS0114288789
Argentine Republic	Government		X	X	03/10/2016	ISDA2014Credit	PP7D7E	US040114GX20
Federal Republic of Germany	Government	X	X		13/10/2015	ISDA2014Credit	3AB549	DE0001135176
People's Republic of China	Government		X		16/03/2016	ISDA2014Credit	7I343A	US712219AG90
United Kingdom of Great Britain and	Government	X	X		13/10/2015	ISDA2014Credit	9A17DE	GB0004893086

In Figure 5.3 is shown a credit default swap on the debt of Italy. The notional of the contract is \$ 10 million and the contract type is a Standard Western European Sovereign Contract (SWES)[4] according to the 2014 ISDA Definitions. The issuer of the debt for which protection is being bought is the Republic of Italy. In particular, the debt is of senior type. The trade date of the CDS is on 18/01/2018, the first coupon is paid on 20/03/2018 while the maturity date is on 20/12/2020. The market spread from the CDS curve (i.e. the value that the market thinks you should be paying for the deal) is 79.7 bp while the standard coupon to be paid is 100 bp, this means that the buyer is overpaying for the deal. The cash amount reflects the amount by which the buyer needs to be compensated. The coupon payments are made quarterly. The CDS price calculated using the Bloomberg Fair Value Model on 18/01/2018 is €100.57 while, the Mark to Market value is €-65.40. From Market section, it is possible to see the curves that have been used to price the CDS. We find that the swap curve used is the EUR ISDA Standard Curve while the CDS curve used is the ENIIM EUR Senior Curve. We also find the recovery rate at 40 %, which is a standard assumption for senior debt.

[4] This contract has a standardized coupon of 25, 100, 300, 500, 750 or 1000 bps, with quarterly premium payments and a recovery rate assumption of 40% for senior debt, 20% for subordinated debt and 0% for other.

Figure 5.3: Bloomberg view for a 3 year-ITALIA CDS (source: Bloomberg).

5.3.3 CDS Indices

A Credit default swap index is a portfolio of actively traded liquid single names in a particular sector of the market. It is a completely standardised instrument, therefore highly liquid and trades on a very small bid-offer spread. For this reason it can be cheaper to hedge a portfolio of bonds with a CDS index than it would be to buy many single names CDS to achieve a similar effect. There are two main families of indices: CDX and iTraxx.

CDX indices cover the area of North America and Emerging Markets. They are divided in: Investment Grade (IG), High Yield and Emerging Markets (EM). The general structure of the CDX families of indices is shown in Table 5.7.

Table 5.7: General structure of CDX indices. The number of credits in any one index may be lower than the number shown here due to their removal following credit events.

Name	Type of credit	Number of credits
CDX.NA.IG	North america investment grade	125
CDX.NA.HY	North america high yield	100
CDX.EM	Emerging Markets sovereign credits	15

CDX.NA.IG Index is composed of 125 equally weighted constituents with investment grade credit rating. It is the most liquid CDS portfolio indices of the family of CDX; CDX.NA.HY Index is composed of 100 equally weighted constituents with high yield credit rating and CDX.EM Index is composed of 15 sovereign issuers from the following regions: Latin America, Eastern Europe, Africa and Asia.

CDX are quoted in US dollars. Payments from the protection buyer to protection seller are made on a quarterly basis (March 20, June 20, September 20, December 20), except for CDX.EM where payments are semi-annual and due on June 20 and December 20. The constituents of the indices are changed every six months and this process is known as "rolling" the index. The constituents enter and leave the index as appropriate. For example, if one of the names is upgraded from below investment grade to IG, it will move from the HY index to the IG index when the rebalance occurs.

In Table 5.8 we took an example of North America Investment grade index and one of North America High yield index. In Table 5.9 we took three Emerging Markets index.

As we can see, North American Investment grade trade 3, 5, 7 and 10 year maturities and the coupon can be of 100, 150 and 500 basis points. North America High Yield and Emerging Markets trade 5-year maturity and the coupon can be of 100 or 500 basis points. They are both traded in US dollar.

Table 5.8: Examples of some North American CDX (source: www.theice.com).

US CDX Indices							
Full Index Name	Index Family	Index Tenor	Coupon (bps)	Date Included for Clearing	Scheduled Termination Date	ISDA Definition	Markit Red
CDX.NA.IG.24	IG	3Y	100	20/03/2015	20/06/2018	ISDA2014Credit	2I65BYDI3
CDX.NA.IG.24	IG	5Y	100	20/03/2015	20/06/2020	ISDA2014Credit	2I65BYDI3
CDX.NA.IG.24	IG	7Y	100	20/03/2015	20/06/2022	ISDA2014Credit	2I65BYDI3
CDX.NA.IG.24	IG	10Y	100	20/03/2015	20/06/2025	ISDA2014Credit	2I65BYDI3
CDX.NA.HY.20	High Yield	5Y	500	03/02/2017	20/06/2018	ISDA2014Credit	2I65BRPF1
CDX.NA.HY.21	High Yield	5Y	500	03/02/2017	20/12/2018	ISDA2014Credit	2I65BRPG9

Table 5.9: Examples of some Emerging Markets CDX (source: www.theice.com).

CDX Emerging Market Indices							
Full Index Name	Index Family	Index Tenor	Coupon (bps)	Date Included for Clearing	Scheduled Termination Date	ISDA Definition	Markit Red
CDX.EM.19	EM	5Y	500	13/12/2017	20/06/2018	ISDA2014Credit	2I65BZCZ3
CDX.EM.24	EM	5Y	100	13/12/2017	20/12/2020	ISDA2014Credit	2I65BZDE9
CDX.EM.28	EM	5Y	100	13/12/2017	20/12/2022	ISDA2014Credit	2I65BZDI0

The iTraxx idices are a family of credit default swap indices covering the geographical region of Europe and Asia-Pacific. European iTraxx are divided in 4 index family which are: main, crossover, high volatility and financials. This classification is shown in Table 5.10.

Table 5.10: General structure of the family of iTraxx indices.

Name	Type of credit	Number of credits
iTraxx Europe	Investment grade	125
iTraxx Europe crossover	Crossover credits	50
iTraxx Europe HiVol	High volatility credits	30
iTraxx Europe Sen Financials	Senior	25
iTraxx Europe Sub Financials	Subordinated	25
iTraxx Asia ex-Japan IG	Investment grade	50
iTraxx Australia	Investment grade	25

The most widely traded of the indices is the iTraxx Europe index, also known as simple ”The main”, composed of the most liquid 125 CDS referencing European investment grade credits. There is also significant volume of trading in the High Volatility and Crossover indices. High Volatility is a subset of the main Europe index consisting of what are seen as the most risky 30 constituents at the time the index is constructed. Crossover is constructed in a similar way but is composed of a minimum of 40 and a maximum of 50 sub-investment grade credits. Also traded are senior financials and sub financials indexes.

ITraxx are quoted US dollars and euro. Payments from the protection buyer to protection seller are made on a quarterly basis (March 20, June 20, September 20, December 20). As before, the constituents of the indices are changed every six months. The roll dates are March 20 and September 20 each year. The European iTraxx indices trade 3, 5, 7 and 10 year maturities and the coupon are of 100, 175 and 500 basis points. The Asia-Pacific iTraxx indices typically trade on a 5-year maturity and the coupon is of 100 basis points. In Tables 5.11 and 5.12 is shown an example of each type of iTraxx.

Table 5.11: Examples of European iTraxx indices (source: www.theice.com).

Europe iTraxx Indices							
Full Index Name	Index Family	Index Tenor	Coupon (bps)	Date Included for Clearing	Scheduled Termination Date	ISDA Definition	Markit Red
iTRAXX Europe S26	Main	3Y	100	20/09/2016	20/12/2019	ISDA2014Credit	2I666VBO2
iTRAXX Europe S26	Main	5Y	100	20/09/2016	20/12/2021	ISDA2014Credit	2I666VBO2
iTRAXX Europe S26	Main	10Y	100	20/09/2016	20/12/2026	ISDA2014Credit	2I666VBO2
iTRAXX Europe Crossover S19	Crossover	5Y	500	22/07/2016	20/06/2018	ISDA2014Credit	2I667KFB9
iTRAXX Europe HiVol S19	HiVol	5Y	100	06/10/2014	20/06/2018	ISDA2014Credit	2I667LAW6
iTraxx Europe Senior Financials S19	Financials	5Y	100	06/10/2014	20/06/2018	ISDA2014Credit	2I667DAS3
iTraxx Europe Sub Financials S22	Financials	5Y	100	17/03/2017	20/12/2019	ISDA2014Credit	2I667EAW2

Table 5.12: Examples of Asian and Pacific iTraxx indices (source: www.theice.com).

Asia-Pacific iTraxx Indices							
Full Index Name	Index Family	Index Tenor	Coupon (bps)	Date Included for Clearing	Scheduled Termination Date	ISDA Definition	Markit Red
ITRAXX ASIA EX-JAPAN IG SERIES 24	iTraxx Asia	5Y	100	16/03/2016	20/12/2020	ISDA2014Credit	4ABCAMAQ9
ITRAXX AUSTRALIA SERIES 24	iTraxx Asia	5Y	100	16/03/2016	20/12/2020	ISDA2014Credit	2I668IAX0

In Figure 5.4 is shown a credit default swap index which is the CDX.NA.IG.24. The notional of the contract is \$ 10 million. The trade date of the CDS is on 18/01/2018, the first coupon has been paid on 22/06/2015 while the maturity date is on 20/06/2022. The standard coupon is of 100 basis point on the notional and the payment frequency is quarterly.

The CDS price calculated using the Bloomberg Fair Value Model on 18/01/2018 is \$ 101.90 while, the Mark to Market value is €-198.30. From Market section, it is possible to see the curves that have been used to price the CDS. We find that the swap curve used is the USD ISDA Standard Curve while the CDS curve used is the SP1A3MRX. We also find the recovery rate at 40 %, which is a standard assumption for senior debt.

Figure 5.4: Bloomberg page CDSW for a credit default swap index (source: Bloomberg).

CONCLUDING REMARKS

In this thesis, we started from the definition of credit risk and from the models used to measure it. A large number of works have been published which can be divided into two big groups: structural models and reduced form models. The two approaches are based on different theoretical principles and have both advantages and disadvantages. In short, it can be said that in structural models the default is linked to the capital structure of the company. On one hand, this makes possible to exploit the advantage of liking the default with the evolution of the capital structure, but on the other hand it has the disadvantage of having to deal with variables that are not directly observable or are not available for all the companies. Basically while presenting a solid underlying logic, these models remain difficult to apply. On the contrary, the reduced form models treat the default like a completely exogenous event, independent from the capital structure of the company. In particular, the event of default is specified in terms of an exogenous process that make it unexpected and governed by a certain intensity. The main advantages presented by this class of models are that they are more flexible and easier to apply. However, it should be emphasized that these models seen reduce their effectiveness with the increasing of the considered time frame.

We then introduced the instruments used to hedge and trade risk, focusing on Credit Default Swaps. It has been seen that, in the years preceding the crisis, these instruments have had an exponential growth, due to the desire of financial institutions to better manage credit risk and of traders to gain exposure to the credit markets. We presented a model for the valuation of a CDS and we saw that the main problems are to estimate the various variables involved: recovery rate, risk-free rate and probability of default.

We passed to analyse some actually traded CDS and some considerations emerged. The financial crisis has revealed several shortcomings in the CDS market practices and structure. Credit default swaps, are traded in over-the-counter market, places of exchange characterized by the absence of rules set by some authority and by the almost absolute lack of a regime of transparency. The opacity of the market is one of the main problems. The trading information on CDS market is stored with the contracting parties and not centrally as in the books of an exchange, so it is difficult to find data. In addiction, it is not a very liquid market compared with other derivatives and it is a concentrated market, where few major institutions control an overwhelming proportion of the volume. The consequence is evident from the reading of the bid-ask spreads that are high.

However, the CDS market is currently transforming into a more stable system. In response to the financial crisis, legislators in the US and the European Union put in place various measures to improve the structure of this market. One of the main news consists in the adoption of the Central Clearing House, which aims to enhance transparency and reduce counterparty risk. In addition, over the years, CDS have become increasingly standardised thanks to voluntary industry initiatives as the definitions provided by the ISDA, which established the terms and conditions of standard CDS contracts. The standardisation of CDS contracts aims at reduce operational risks, increasing fungibility of these instruments and facilitate central clearing.

To conclude, credit risk remains one of the most difficult financial risks to identify. Instruments to hedge credit risk, like CDS, have assumed a negative meaning after the financial crisis, but it must be emphasized that past problems should not distract from the wider economic benefits these instruments bring and the recent crisis should serve as a motivation for the market to grow and become more stable.

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