Prospect Theory Application in Finance

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**Academic Years:**
2016/2017
Prospect Theory Application in Finance

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A thesis submitted in partial fulfilment for the
Degree of Master of Economics and Finance

in the
Department of Economics

Oct 2017
ACKNOWLEDGMENTS

I would first like to express my profound gratitude to my thesis supervisor Professor Paolo Pianca of the Department of Economics at Ca’Foscari University of Venice. His welcoming attitude, continuous support and guidance has steered me in the right direction which, in turn, enabled me to put my ideas together for my thesis. I am also deeply indebted to Professor Martina Nardon for her unceasing assistance and timely availability in answering any query or questions I put forward. Professor Martina’s patience, immense knowledge and her inspiring guidance consistently prompted me on genuine research. I would not have imagined a better co-supervision for my master’s thesis than I availed.

Part of my appreciation extends to Ca’ Foscari University of Venice for giving me the opportunity to study in this prestigious institution. The contribution; the perfect study environment, provided by the libraries, has certainly a key role in my thesis. Of particular interest, in this connection, is the contribution of Bloomberg terminal office at BEC library San Giobbe - where I would retrieve the data for my thesis - is worthy of appreciation.

To my friends, you should know your support, encouragement, love and respect worth more than I can express on paper. Last but not least, I express my most sincere thanks to my family for providing me with unfailing support and continuous encouragement throughout my years of study and the process of research and writing this thesis. This accomplishment would not have been possible without them.
ABSTRACT

Human behaviour or human psychology plays an important role in influencing financial decision of investors. Behavioural finance started to be popular in 1990s after Daniel Kahneman and Amos Tversky gave their Prospect Theory. In this contribution I have worked with the history and background of Prospect Theory. When and how it got the attention of the economists? I considered the drawbacks of Expected Utility theory which lead to the creation of this theory. When an investor becomes risk seeking and when risk averse. I applied disposition effect on Karachi Stock Exchange (KSE100) to find out if investors sell their gaining stocks too early and keep their losing stocks too long and fond that disposition effect exists in KSE100.

I have applied continuous cumulative prospect theory to evaluate European options by using the model developed by Prof. Martina Nardon and Prof. Paolo Pianca (2014). I have performed a variety of numerical experiments with different values of parameters. Continuous cumulative prospect theory seems promising approach than other models when applied to option valuation for its ability to explain the departure from Black and Scholes. The option prices are sensitive to the choice of the values of the parameters. The numerical experiments in order to study the effects of curvature and elevation on option suggests that option prices increase with elevation and the prices increase at decreasing rate.

Keywords: Prospect theory, cumulative prospect theory, Decision making under risk, Value function, Weighting function, Disposition effect, European option pricing.
Contents
List of Tables
List of Figures
INTRODUCTION

“There is nothing as dangerous as the pursuit of a rational investment policy in an irrational world”

John Maynard Keynes

One of the most important advances in economics has been the efforts of psychologists and behavioural economists in identifying systematic deviations from neoclassical assumptions that investors act rationally to maximize their utility. When using the label, “modern” or “conventional” to describe finance, we are speaking about the finance which is based on logical and rational theories such as Efficient Market Hypothesis (EMH) and Capital Asset Pricing Model (CAPM). Under these primitive theories investors for the most of the time behave rationally and predictably. Before the inception of behavioural finance these rational theories did a respectable job of predicting and explaining certain events. However as time went on academics in both economics and finance started to trace out some anomalies and behaviour that couldn’t be explained by these classical theories. These theories can explain specific idealized events while the real world is proved to be a messy place where market participants behave unpredictably and not rationally.

Cognitive psychologists Dainal Kanneman and Amos Taversky embellished behavioural finance with major empirical and theoretical contributions. They are considered the fathers of behavioural economics/finance. This is a detailed study of how people form decision about the prospects, its people’s decision under uncertainty. In the first part it deals with the inception of the prospect theory in behavioural finance taking in to account the drawbacks of the Expected Utility Theory i.e. that’s is what were the main
problems with the Expected Utility Theory that leads Daniel Kahneman and Amos Tversky to invent this descriptive theory (1979, 1992).

The Expected Utility Theory of Von Neumann and Morgenstern (1944) is a powerful tool for the analysis of decisions under risk. However, people in both experimental and real-life situations frequently do not conform to the NM axioms. Hence here comes the prospect theory which says the carriers of the value are gain and loss, not the final value of the wealth. The investor has a greater sensitivity to losses than gains of the same magnitude. In the traditional models, an agent evaluates a new gamble by merging it with his pre-existing risks and checking if the combination is attractive, but in experimental settings, people often seem to evaluate a new gamble in isolation. This is called narrow framing, get utility directly from the outcome of the gamble not just indirectly from its contribution to total wealth.

For example. Mr A offers Mr B the choice between the following two gambles:

**Gamble 1**: win $240 with 100%  
Expected Utility = $240

**Gamble 2**: win $400 with 50%  
Win $100 with 50%  
Expected Utility = $250

The expected utility of gamble 2 is higher than gamble 1. Therefore, if people were to choose rationally, they would have picked gamble 2 as it gives higher expected utility. However, the research on prospect theory shows that the majority of people pick gamble 1 which has the lower expected utility.

The second part of this study talks about the disposition effect, i.e., investors hold on to losing stock and sell the winning stocks to thus make them loss averse and risk lover. Last part talks about prospect theory and option pricing by negating Black and Scholes Model of option pricing.
1.1 BACKGROUND AND HISTORY OF PROSPECT THEORY

Seeing behavioural decision making is often understood poorly. To understand the prospect theory we have to go back in history that is why and how it was developed. What caused the creation of Prospect Theory? Decision making research before 1970s was dominate by normative theories prescribe how people ought to make decision in perfectly rational way, and many implicitly assumed that most people in daily lives followed these normative rules. Prospect Theory was a notable departure from these existing theories because it offered a descriptive theory of how people actually make decisions rather than providing a perfectly rational account of how they ought to do so.

The simplest way to choose between risky options is to choose the option with the highest expected value, the likelihood that an option will occur, multiplied by the value of that option. Imagine, for instance, that you are deciding whether to pay $1 for a lottery ticket that offers a 10% chance of winning $10. The expected value of this lottery ticket is $1 (0.1 x $10), the same as the cost of the ticket. Rationally speaking, you should therefore be perfectly indifferent about buying this ticket or not. The problem, noted by both the economists and psychologists is that rational theories didn’t always describe people’s actual behaviour very well. Few people, for instance, would actually purchase the lottery ticket in the last example. The certain loss of a dollar simply does not compensate for the 10% chance of winning $10 and a 90% change of winning nothing. In general, research found that people were more averse to taking risks that the expected value of outcomes would predict.

The inability of expected value calculations to explain people’s decisions then led to the development of “Expected Utility Theory,” that essentially incorporated people’s attitude towards risk into their expected value calculations. Expected Utility Theory
assumed that attitudes towards risk were stable within individuals, were not influenced by the way a particular decision was described (or framed), and was not influenced by the mood or situational context of the decision maker. However, experiments again revealed the decision makers often violate the prediction made by Expected Utility Theory.

For instance, a terminal cancer treatment with a 1 in 10 chance of saving the patient’s life is identical to a cancer treatment with a 9 in 10 chance of death (assuming people can only live or die), and yet terminally-ill cancer patients themselves would likely be more interested in pursuing this treatment when described as the likelihood of living than when described as the likelihood of dying.

Gambling, risk, bet and gain. All these things invoke feelings or at least some interest either positive or negative. These concepts play a very important role in decision making under risk and uncertainty. For more than fifty years decision making under uncertainty has been modelled mathematically within the framework of Expected Utility Theory. What caused the creation of Prospect theory? So the milestone is Expected Utility Theory, which was first proposed by Daniel Bernoulli (1738). Before the existence of the Prospect Theory almost all models of financial market assume that investors evaluate risk according to expected utility. But this framework has had trouble matching many empirical facts. Hence it felt strongly that, can we make progress by replacing expected utility with a psychologically more realistic preference specification?

A fundamental aspect of Kahneman and Tversky’s analysis of Prospect Theory is their critique of Expected Utility Function. The EUT found its way into mainstream economics only in the mid twenty century, most famously von Neumann and Morgenstern’s (1944) analysis of EUT. According to Kahnemann and Tversky the most profound problem with Expected Utility Theory is its legitimacy as a positive theory.

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1 Normative and Descriptive Theories of Decision Making under Risk: A Short Review Niko Suhonen. ISBN 978-952-458-985-7 ISSN 1795-7885 no 49
The problem is that people do not/cannot act as predicted by EUT, which means that it lacks predictive power and its role as positive descriptive theory is compromised. They argue that EUT doesn’t provide an adequate description of human behaviour in real world.\(^\text{2}\)\(^?\)

Prospect Theory was motivated by these failures of rational models to describe actual decision making in everyday life. Daniel Kahneman, one of the founders of Prospect Theory along with the late Amos Tversky, won the 2002 Nobel Prize in Economics, at least in part, for this work.

### 1.2 PROSPECT THEORY

It is very hard to measure accurately the behaviour or psychology of an investor with normative financial models. Here comes the behavioural part of finance which tries to give a more accurate psychological solution to primitive finance. Prospect theory belongs to the behavioural economics subgroup, it explains how individuals make decisions given probabilistic alternatives where risk is involved and the probability of different outcomes are unknown. Prospect Theory is characterized by certainty, loss aversion, relative positioning and small probabilities. Consider the gamble \((x, p; y, q)\). Under Expected Utility Theory it’s assigned the value

\[
p''(W + x) + q''(W + y)
\]

While under Prospect Theory it is assigned the value

Traditionally, expected utility theory has been the descriptive model used to describe
decision making under risk, or choices that have uncertain outcomes. Prospect theory
was created in 1979 and developed in 1992 by Daniel Kahneman and Amos Tversky as a
psychologically more accurate description of decision making, compared to the expected
utility theory. Prospect Theory is a psychological account that describes how people
make decisions under conditions of uncertainty. These may involve decisions about
nearly anything where the outcome of the decision is somewhat risky or uncertain,
from deciding whether or not to buy a lottery ticket, to marry one’s current romantic
partner, to undergo chemotherapy treatment, or to invest in life insurance. We have seen
behavioural finance and prospect theory attempts to provide insight into the decision
making process and demonstrate the joy of gains is not equivalent to the grief of losses\[?\].
This type of behaviour is actually on display throughout our financial decisions as losses.
In fact it’s the simplest reason why most people are not willing to save their money and
invest. Future gains offer much less joy than the temporary pain of not having the
newest items.

Prospect theory is a behavioural economics theory that evaluates the way people
choose between the probabilistic alternatives that involve risk. In contrast to rational
expected theory, individuals often make decisions based on the expected outcome and the
risk associated with the losses or gains. In particular individuals tends to underweight
outcomes that are merely probable in comparison with outcomes that are obtained with
certainty

Kahneman & Tversky, 1979

Classical economic theory only takes into account the overall utility and assumes that
individuals will make rational decisions that will provide the greatest amount of utility in any circumstances. Though prospect theory determines that how individuals show a revealed preference for surety over slightly greater mathematical returns with risk. Using a series of probabilistic alternatives, individuals are given choices with outcomes and associated risk probabilities. Tversky and Kahneman (1992) who found equal curvature for gains and losses and referred this to as reflection. Reflection is commonly assumed in applications of prospect theory with many studies imposing the additional restriction that utility for gains and losses as linear. (e.g. Benartzi and Thaler 1995; Barbris et al 2001). S-shaped utility and inverse S-shaped probability weighting imply the fourfold pattern of risk attitudes. Risk aversion for probable gains and unluckily losses and risk seeking for unlikely gains and probable losses.

For example, what would you choose to get $900 or take a 90% chance of winning $1000 (and a 10% chance of winning 0)? Most people avoid the risk and take the $900, although the expected outcome is the same in both cases. However, if we are asked to choose between losing $900 and take a 90% chance of losing $1000, most of us would probably prefer the second option (with the 90% chance of losing $1000) and thus engage in the risk-seeking behaviour in the hope to avoid the loss.

When dealing with gains people are risks averse and will choose the sure gain (denoted by upper line) over a riskier prospect, even though with the risk there is a possibility of gaining a larger reward. The overall expected value (or outcome) of each choice is equal.

Losses are treated in the opposite manner as gains. When aiming to avoid loss, people become risk seeking and take the gamble over a sure loss in the hope of paying nothing. In this scenario most of the time people will choose the lower option. Both choices have equal value.
Fig. 1.1 Risk averse behaviour

Fig. 1.2 Risk seeking behaviour

The typical graphical representation of the Prospect theory is as shown in fig 1.3
1.3 CUMULATIVE PROSPECT THEORY

Every theory comes in with some drawbacks and it’s always opens to for further improvements. Cumulative Prospect Theory is an enhancement to the Prospect Theory and it was proposed by Tversky and Kahneman in 1992 to overcome some of the drawbacks of PT such as violation of stochastic dominance. Cumulative Prospect Theory transforms objective cumulative probabilities into subjective cumulative probabilities. It applies the probability weighting function to the cumulative distribution function. In CPT decision weights ($\pi$) are differences in transformed cumulative probabilities of gains and losses.

\[ (X_m, P_m, \ldots, X_1, p_1; X_1, P_1; \ldots; X_n, P_n) \]  \hspace{1cm} (3.3)

Where $x_i < x_j$ for $i < j$ and $x_0 = 0$ is assigned the value. Subjective value of the
prospect is displayed as in equation (3.4) and (3.5) with decision weights $W_i$ and values $v(x_i)$

$$V = \sum_{i=-m}^{n} W_i v(x_i)$$  \hspace{1cm} (3.4)

$$W_i = \begin{cases} 
W(p_i + \ldots + p_n) - W(p_i + 1 + \ldots + p_n) & 0 \leq i \leq n \\
W(p_{-m} + \ldots + p_n) - W(p_{-m} + \ldots + p_i + 1) & -m \leq i \leq n
\end{cases}$$  \hspace{1cm} (3.5)

The subjective probabilities not necessarily sum up to one due to the fact that different weighting functions. The agent overweights the tail of the probability. The structure of the thesis is as follows:

Narrow framing is one of the most important aspect of the CPT. In traditional models an agent measures a new investment by merging it with his pre-existing risk and check if the combination is attractive or not. While in experimental setting investors often seem to evaluate a new gamble in isolation. This is narrow framing get utility directly from the outcome of the gamble not indirectly from its contribution to total wealth.

1.4 BIASES WHEN PEOPLE MAKE DECISION

Prospect theory explains the biases that people use when they make such decisions. The first one is **Certainty**, people tend to overweigh options that are certain, and are risk averse for gains. We would rather get an assured, lesser win than take the chance at winning more (but also risk possibly getting nothing). The opposite is true when dealing with certain losses, people engage in risk-seeking behavior to avoid a bigger
loss. **Isolation effect** refers to people’s tendency to disregard any elements that are common to both options, in an effort to simplify and focus on what differs.

Remembering all the details of each individual option creates too much of a cognitive load, so it only makes sense to focus on the differentiators. Discarding common elements lessens the burden of comparing alternatives, but can also lead to inconsistent choices depending on how alternatives are presented.

Daniel Kahneman and Amos Tversky presented participants with 2 scenarios. In both scenarios people were given an initial amount of money, and then had to choose between two alternatives.

**Scenario 1**: Participants started with $1000. They then could choose between:

- A. Winning $1000 with a 50% probability (and winning $0 with a 50% probability), or
- B. Getting another $500 for sure.

**Scenario 2**: Participants started with $2000. They then could choose between:

- C. Losing $1000 with a 50% probability (and losing $0 with a 50% probability), or
- D. Losing $500 for sure.

Because the initial amounts were different in the two scenarios, it turns out that the two scenarios were actually equivalent: if they chose option B in the first scenario or option D in the second scenario, the amount of money they would have in the end would be the same. (Options A and C are likewise equivalent.) However, people made opposite choices in the two scenarios: the majority chose the risk-averse option B in Scenario 1 and the loss-averse option C in scenario 2.

Changing the framing of the problem (by adjusting the initial gift and the options accordingly) led people to a different decision.

When creating contents to persuade people into making a certain choice, consider

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4 Prospect Theory and Loss Aversion: How Users Make Decisions by AURORA HARLEY on June 19, 2016. To persuade users to take an action, consider using the certainty bias to your advantage: people would rather accept a small but certain reward over a mere chance at a larger gain.

5 When presented with each decision, people make the opposite choice based on whether the options are framed as a gain or a loss. In Scenario 1, most choose option B over A, but in Scenario 2 the
Fig. 1.4 Isolation effect

how it’s framed. People can respond very differently to negatively framed messages than they would to a positively framed one. Would you rather use the service that has 95% satisfaction rate or 5% complaint rate? The negative formulation primes people to think of the possible loss or negative outcome and to act accordingly. **Loss Aversion**, people will behave in a certain way so that they minimize losses because losses loom larger than gains even though the probability of those losses is tiny. The pain of losing also explains why and when winning $100 in a gamble and then losing $80 feels like a net loss even though we are actually ahead by $20. People reaction to loss is more extreme than their reaction to gain. The order is also important where we first to lose $80 then come back and win $100. It would shift our reference point and make it feel like a net gain.

For example, insurance websites frequently display a long list of unlikely, yet costly outcomes that we may encounter should we not buy insurance. This list primes us majority choose option C over D to try to avoid the loss. In these scenarios, people focus on only the choice between the 2 options, and overlook the initial gift amount because it is a shared factor across the two choices. However, when taking this initial gift difference into account, it can be seen that option A is equal to option C, and option B is equal to option D only the framing has changed.
toward avoiding these large losses and makes us forget about the small, but regular payment that we would make indefinitely for ensuring insurance coverage.

1.5 VALUE FUNCTION

Psychologically people give value to different things in different ways and its human nature that some carries more value and some less based on their perception. The value function of cumulative prospect theory tries to capture human behaviour in times of loss and gain. Value function v(x) for a cumulative prospect theory exhibits the same properties as prospect theory. This means v(x) is concave above the reference point i.e. \( v'' \leq 0, x \geq 0 \) and convex below the reference point i.e. \( v'' \geq 0, x \leq 0 \).

This shows diminishing sensitivity, means that the impact of changes in the domain of gains and losses decreases when the distance from the reference point increases. The value function is also steeper for losses than for gains. The value function of cumulative prospect theory is shown in equation (5.6)

\[
v(x) = \begin{cases} 
  x^a & \text{if } x \geq 0 \\
  -\lambda(-x^\beta) & \text{if } x \leq 0 
\end{cases}
\]  

(5.6)

x is outcome either positive or negative. A graphical representation of a value function is shown in fig (1.5).[?] The value function has some critical aspects. Value is assigned to change in value rather than absolute value, the value function is S shaped and is expected to be concave for gains above the reference point but convex for losses below the reference point. The value function is steeper for losses than for gains. As the prospect theory predicts that the value of different prospects is determined only by comparing it with the other
options. Hence the value people assign to an outcome depends on the other outcomes that people are comparing it with. The symmetric shape of the value function captures the fact that people are loss averse. This means that a loss is assigned a greater value than a gain of the same amount.

The value function has three main characteristics, which reflect the behavioral patterns. (1) It is defined on deviations from a reference point, rather than on net asset position, thus if the reference point shifts, the value function shifts accordingly. (2) it is generally concave for gains and convex for losses, reflecting risk aversion in the domain of gains and risk seeking in the domain of losses. (3) it is steeper for losses than for gains (perhaps by a ratio of 2:1, according to the experimental evidence [Tversky & Kahneman, 1991]). This captures the phenomenon of loss aversion and implies that the marginal utility of gains decreases faster than the marginal disutility of losses.
1.6 WEIGHTING FUNCTION

Prospect theory involves also a probability weighting function which shapes probabilistic risk behaviour. It takes the true objective probabilities and transfers them into what are sometimes called decision weights. Since losses loom for longer than gains. The principle of diminishing sensitivity applies to the weighting function as well. The difference is that the sensitivity to changes in probability diminishes as the probability moves away from the boundary points of impossibility and certainty. This means the change in the probability of an outcome from 0.1 to 0 and 0.9 to 1 has more impact then a change from 0.5 to 0.6. Small probabilities tend to be overweighted and high probabilities tend to be underweighted. These properties caused the existence of function which is concave near zero and convex near 1 (an inverse s-shape). The equations (6.7) are the parametric form of weighting function of Cumulative Prospect Theory suggested by Kahneman and Tversky.

\[
W^+(p) = \frac{p^\gamma}{p^\gamma + (1-p)^{\gamma^\frac{1}{\gamma}}} \quad \quad \quad \quad W^-(p) = \frac{p^\delta}{p^\delta + (1-p)^{\delta^\frac{1}{\delta}}} \quad \quad \quad \quad \text{Weighting function (6.7)}
\]

\(P\) is the probability of the outcome \(W^+\) or \(W^-\) with the parameter \(\gamma = 0.61\) and \(\delta = 0.69\). Where \(\gamma\) is a positive constant with some constraints in order to have an increasing function. Parameter \(\gamma\) captures the degree of sensitivity to changes in probabilities from impossibility (zero probability) to certainty. The lower the parameter the higher the curvature of the function. Prospect Theory argues that people go through two distinct stages when comparing choices under uncertainty. First there is the editing phase. During this phase people are said to compare the different prospects. To do this they edit the complicated decisions they have to make into simpler decisions, spec-
ified in terms of gains and losses. In the second phase the **evaluation phase**, people examine the edited prospects and then choose the one with the highest value predicted by the value function and the importance or weight that people assign to particular outcome, i.e., the weighting function. A typical probability weighting function induced from experimental evidence is given in Fig (1.6).

![Weighting Function](image)

**Fig. 1.6 Weighting Function**

The weighting function is an alternative weighting scheme to the one used under the expected utility theory. Under the later theory the objective likelihood that an outcome will occur is used to weight the different outcomes. Under prospect theory we assume that people multiply the perceived outcome by a decision weight. These decision weights differ from the objective probabilities in the case of extreme probabilities.

The probability-weighting function measures the impact of the probability of an event on the desirability of a prospect. It is not a linear function of probability, however, and decision weights are not themselves probabilities. Technically decision weights could be influenced by factors other than probability, including "ambiguity," or uncertainty about the level of uncertainty or risk (Kahneman and Tversky 1979, p.280). It has several characteristics. First the weighting function is not well behaved near its end points. This reflects the unpredictability of behaviour under conditions of extremely small or extremely large probabilities. In other words the variance in the probability
weighting function is not constant and is quite large in the region near 0 or 1. Kahneman and Tversky (1979, pp. 282-283) acknowledge this unpredictability, and argue that because people are limited in their ability to comprehend and evaluate extreme probabilities, highly unlikely events are either ignored or overweighted, and the difference between high probability and certainty is either neglected or exaggerated. The fact that the weighting function is not well behaved near its endpoints, and that by definition \( w(0) = 0 \) and \( w(1) = 1 \), leads to a second important characteristic: there is a sharp (though somewhat indeterminant) increase in the weighting function in these regions. Thus changes in probabilities near 0 or 1 have disproportionately large effects on the evaluation of prospects.

A third characteristic of the weighting function is that its slope is less than 1 across its entire range, except for the small region near its endpoints. Because the slope is a measure of the sensitivity of decision weights and therefore of preferences to changes in probabilities, this means that preferences are generally less sensitive to variations in probability than the expectation principle would suggest (with the important exception of the region near 0 and 1). One implication is that the sum of decision weights associated with complementary events is generally less than the weight given to a certain event, which reflects the certainty effect described above (Kahneman & Tversky, 1979 p.282).

Fourth, other than the indeterminacy of behaviour for extremely small probabilities. Small probabilities are overweighted while larger probabilities are underweighted. Although there is no conclusive evidence as to the specific point at which overweighting shifts to underweighting, or whether this point varies significantly across individuals or conditions.

Once the individual edits the available options, he then evaluates the edited prospects and selects the one with the highest value, as determined by the product of a value of an outcome and a decision weight.

17
1.7 DECISION MAKING UNDER RISK

A fundamental issue in finance is how do people make decision under risk? Almost all models of financial market assume that investors evaluate according to Expected Utility. But this framework had trouble matching many empirical facts. Expected Utility Theory has dominated the analysis of decision making under risk. It has been generally accepted as a normative model of rational choice and widely applied as a descriptive model of economic behaviour. Decision making under risk can be viewed as a choice between prospects or gambles. A prospect (x₁, p₁... xn, pn) is a contract that yields outcome xi with probability pi where p₁+p₂+...+pn=1. To simplify notation we omit null outcomes and use (x, p) to denote the prospect (x, p; 0, 1-p) that yields x with probability p and 0 with probability 1-p. The riskless prospect that yields x with certainty is denoted by (x).

Decision making under risk is of notable importance to all of the behavioural sciences. Almost every human and non-human decision is made under some consideration of risk. Where risk is defined as known variance in outcomes. The inability of Expected Utility Theory to predict actual decision making under risk lead to arose of Prospect Theory. The well known violation of expected utility theory is framing effect (Kahneman & Tversky, 1979, Tversky & Kahneman, 1981). Kahneman and Tversky demonstrated that people tend to be risk-prone when faced with a decision framed as a loss and risk-averse when faced with a decision framed as a gain, even if both decisions have identical expected values. Consider the classic Asian disease problem (Tversky & Kahneman, 1981)[?]. In the standard version of the Asian disease problem participants are provided with a decision scenario involving a choice between two options either presented in a positive (gain) or a negative (loss) frame. The positive frame states:

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\[\text{Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the programs are as follows. (p. 453)}\]
1. If Program A is adopted, 200 people will be saved. 2. If Program B is adopted, there is a 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.

Both options are phrased in terms of the possibility of saving people and thus in terms of gains. When presented with this positively framed scenario, Tversky and Kahneman (1981) found that 72% of participants preferred the certain option (Program A) over the risky option with higher outcome variance (Program B), thus exhibiting risk-aversion on average. The negative frame states:

1. If Program C is adopted, 400 people will die. 2. If Program D is adopted, there is a 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.

Here both options are negatively framed in terms of the numbers of possible deaths and thus in terms of losses. Tversky and Kahneman found that when presented with this loss scenario, participants’ risk-preferences reversed: only 22% of participants preferred the certain option (Program C) thus exhibiting risk-acceptance on average. This finding that people exhibit risk-preference reversal in mathematically identical decisions made in loss and gain frames has received substantial empirical support.

Kahneman and Tversky (1979) conceived prospect theory to explain the systemic violations of expected utility theory they observed in conducting empirical studies with human subjects. Specifically, prospect theory addresses the framing effect described above, in addition to the certainty effect and the isolation effect. The certainty effect describes decision makers’ tendency to overweight outcomes that are certain (e.g., sure gains or sure losses) over those that involve probability, regardless of expected value. The isolation effect describes decision makers’ tendency to ignore common components of decision alternatives in order to simplify decisions (Kahneman & Tversky, 1979). Kahneman and Tversky proposed two phases, the editing phase and evaluation phase to

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8 Decision-Making Under Risk: Integrating Perspectives From Biology, Economics, and Psychology by Sandeep Mishra
describe how individuals make decision under risk \(^9\). The most important contribution is of Prospect Theory is coding. Kahneman and Taversky suggested that decision makers code gains and losses around a reference point where outcomes below the reference point represent losses and above reference point represent gains. Reference points are derived from an individual’s present state but can change based on expectations or biases of decision makers (Tversky & Kahneman, 1981). The other operations in the editing phase describe how decision makers combine separate decisions with identical outcomes and probabilities into single decisions (combination), clarify riskless and risky components (segregation), and simplify multistep decisions by ignoring similar decision elements (cancellation).

The second phase of a decision according to prospect theory is the valuation phase. Where an individual assesses all of the edited options and make decision. The valuation phase consists of value function and weighting function. The value function assigns specific values to certain outcomes. Here the law of diminishing return applies, the difference between $1 and $2 for example is perceived as greater than the difference between $1000 and $1001. Because of the differential subjective values placed on gains and losses and the law of diminishing return the utility curve for prospect theory preferences is concave down above a reference point (exhibiting risk aversion for gains) and concave up below a reference point (exhibits risk proness for losses). The reference point refers to the origin of the utility function around which gains and losses are defined.

\(^9\) The editing phase serves to reformulate and organize all of the possible decision options to simplify evaluation (Kahneman & Tversky, 1979) and involves several different operations (coding, combination, segregation, and cancellation; Kahneman & Tversky, 1979; Tversky & Kahneman, 1981).
PROSPECT THEORY AND DISPOSITION EFFECT

Normative theories had been dominated finance for decades before the Prospect theory came into existence. As we know the world is not fully rational as there are emotional and psychological influences attached to the financial decisions of the investors. And this influence can result in irrational behaviour. Behavioural finance has gained its prominence since the early 1990s. The coming into being of the prospect theory popularized the role of psychology in investing. Understanding the psychological motivations can help investors avoid financial pitfalls. Behavioural finance bridges the gap between theory and practice by scientifically recording human behaviour. To date research has focused on rational investor in efficient markets. While the reality with day to day irrational behaviour of investors and inefficient markets say different.

2.1 DISPOSITION EFFECT

The capacity of human to emphasize the negative rather than positive has probably been an evolutionary phenomenon. From the earliest beginnings being aware of the danger has been a crucial survival skill. Naturally and psychologically people do not like things which causes negative feelings. Disposition effect explains the propensity for investors to realize gains sooner than losses, through selling the profit making investment quickly than loss making investment. The disposition effect concerns a predisposition. Given an investor’s tax situation and beliefs, the disposition effect predisposes the investor
to sell his or her winners more quickly and hold on to his or her losers, relative to an expected utility maximizing investor in the same situation. Odean 1998 documented the disposition effect empirically for individual investors in US. He randomly selected 10000 customer accounts that were active in 1987 at a nationwide discount brokerage house and followed these accounts between January 1987 and December 1993.

He proceeded as follows. For each day in his sample he divided all investors’ position into four categories: 1) realized gains, 2) paper gains (unrealized), 3) realized losses and 4) paper losses (unrealized). He then defined two variables he called proportion of gains realized (PGR) and proportion of loss realized (PLR). PGR is the ratio of realized gains to the sum of realized gains and paper gains. PLR is the ratio of realized losses to the sum of realized losses and paper losses. The following table presents the main findings in Odean (1998) concerning PGR, PLR and PGR-PLR.

<table>
<thead>
<tr>
<th>Tab. 2.1 Percentage of gains and losses realized Odean 1998a</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="https://example.com/table2.1.jpg" alt="" /></td>
</tr>
</tbody>
</table>

Odean hypothesized that for the whole year investors would realized their winners more frequently than their losses (PGR>PLR), but that in Dec the difference PGR-PLR would decrease. That’s with tax loss selling being silent in Dec some investors would find it psychologically easier to sell losses in that month. That’s why it is predicted that in Dec individual investors would realize their losses more frequently than in other months. For the entire year investors realized 14.8 % of their gains but only 9.8% of their losses. That’s investors realized their gains 50% more than their realized losses. The table (2.2) shows the average returns for the categories Odean uses.
Tab. 2.2 Average Return (Odean 1998)

<table>
<thead>
<tr>
<th></th>
<th>Entire Year</th>
<th>December</th>
<th>Jan-Nov</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return on realized gains</td>
<td>0.277</td>
<td>0.316</td>
<td>0.275</td>
</tr>
<tr>
<td>Return on paper gains</td>
<td>0.466</td>
<td>0.500</td>
<td>0.463</td>
</tr>
<tr>
<td>Return on realized losses</td>
<td>-0.228</td>
<td>-0.366</td>
<td>-0.208</td>
</tr>
<tr>
<td>Return on paper losses</td>
<td>-0.393</td>
<td>-0.417</td>
<td>-0.391</td>
</tr>
</tbody>
</table>

Proportion of gains realized (PGR) = \( \frac{\text{RealizedGains}}{\text{RealizedGains} + \text{PaperGains}} \) (2.1)

Proportion of losses realized (PLR) = \( \frac{\text{RealizedLosses}}{\text{RealizedLosses} + \text{PaperLosses}} \) (2.2)

Realized losses have high returns than paper losses while realized gains are smaller than paper gains. Interestingly realized losses in Dec are much higher than realized losses in other months, reinforcing the suggestion of strong tax loss selling motive.

This observation is more striking because Odean shows that the prices of winning stocks which investors have sold keep on rising, while the prices of the loser stocks keep on falling. Hence investors would have earned more money by keeping the winning stocks and selling the loser stocks. Furthermore he rejected other possible explanations such as transaction costs and taxes. One thing can be drawn from this explanation of Odean is that the investors (erroneously) believe in mean reverting asset prices, that is today’s losers will outperform today’s winners and that today’s losers are tomorrow’s winners. But this explanation is rejected by Weber and Camerer (1998) in their laboratory experiment that the disposition behaviour is due to the belief in mean-reverting stock prices.[?]

Regarding Prospect Theory of Kahneman and Tversky (1979) investors evaluate outcomes relative to a reference point which in the context of stock investments is typically
the purchasing price. The other thing is they behave as if evaluating the decision consequences on an S shaped value function which is concave for gains and convex for losses. This reflects risk aversion in the gain region and risk seeking in the loss region. The standard behavioural finance explanation for the disposition effect is that a gain (loss) moves the investor to risk averse (seeking) part of the value function, so that he/she inclined to reduce (increase) his position in the risky assets accordingly. Therefore the disposition effect is commonly seen as an important implication of applying prospect theory to investment decisions and securities trading.

Belief and preference together determine the behaviour of the investors. Therefore the disposition effect does not stipulate the investors never sell losers or only sell losers in the month of Dec for tax-loss reasons. For tax purpose investors should postpone taxable gains by continuing to hold their profitable investments. They should capture tax losses by selling by selling their losing investments. That is why the disposition effect hypothesized that the combination of beliefs and preferences implies that except in Dec investors realize their winners more frequently than their losses.

Two important points to make about the findings of Odean reports in respect to the post realization performance of stocks. First stock price movements appear to have a predictable component related to the trading behaviour of individual investors. Secondly individual investors appear to make adverse use of information, in that the stocks they sell subsequently outperform the stocks they purchase.

A competing theory to explain the existence of disposition effect is called Rrgrat Theory of Shiller (1999) argues that decision makers in financial market encountered considerable difficulties in closing positions at a loss. Realizing losses by selling its position seems to be difficult because investors have to confess that their first judgement was wrong.¹

¹ The regret of having erred may also be leveraged by having to admit the mistakes to others (spouse, friends, co-workers etc). Regret is an emotional feeling associated with the knowledge that the past decision is proven to be wrong. The positive counterpart to regret is pride. While closing a stock
There are some other empirical studies that show the disposition effect can be affected by the financial information provided to the investors. Centre for European Economic Research Mannheim, Germany did an empirical study on weekly sales of the largest German investor’s magazine (Boerse Online), which deals mainly with stock market related topics. They also considered weekly time series of Deutscher Akrienindes (DAX) from April 1996 to June 2002. This hypothesis is tested that overall performance of German stock market index (DAX) is related to the sale figure of the largest German investor magazine. They found there exists a positive relationship between Boerse Online and DAX.²[?]²

2.2 FRAMING AND MENTAL ACCOUNTING

Mind is a stream of consciousness where sense and mental phenomena are constantly changing. Thought is a mental act that allows humans to make sense of things in the world and to interpret them in a way that they are significant or which accord with their needs, desires, commitment, goal and attachments. Mental accounting also known as psychological accounting explains disposition effect. The decision makers evaluate outcomes with respect to deviation from a reference point rather than with respect to final wealth. In mental accounting, the investor sets reference points for their accounts through which they trace out their gains and losses. Then they tend to maintain record of each individual stock’s profit or loss rather than the entire gain or loss of the portfolio in their minds. Real financial losses are more hurting than paper losses (Thaler 1999). That’s why investors become risk averse in times of stock losses they keep the stock rather than selling them. Hence closing a mental account at a loss is hurting the investor.

Reference point is important due to the fact that individuals evaluate results through account at a loss induces regret and closing at a gain induces pride (Shefrin and Statman 1985:p.782)²

a value function which gives more weights to losses than gains of comparative magnitude

People tend to segregate outcomes into separate mental accounts, these are then evaluated separately for gains and losses.

2.3 SEEKING PRIDE AND AVOIDING REGRET

Regret is negative mental and emotional reaction to a personal behaviour. It’s often a feeling of sandness, shame, embarrassment or guilt. While pride is a feeling of happiness, satisfaction or haughtiness that results from a positive self evaluation. One of many reasons the investors hold on to the losing stocks and selling the gaining stock is because of pride seeking and avoiding regret. It backs up the disposition effect proposed by Kahneman and Tversky (1979), Thaler (1985), Shefrin and Statman (1985) and Shiller (1999). All of these scholars have proved that investors retain their loss giving stocks to avoid regret or dely it for some time and pride seeking investors rapidly sell their profit gaining stocks rapidly, hence it creates disposition effect.

2.4 MEAN REVERSION

The belief that tomorrow would be a better day is just an optimistic perception of all common human kind. It might work in common men life but with financial market it does not work. It’s like the belief that the next decade or so will bring the market back to its historic returns is in essence the belief that nothing about the global financial crisis has fundamentally changed world markets. In other words, the failure of the market to find the mean again would be a signal that we’re living in a unique historical epoch. That’s entirely possible, but also unlikely. The financial meltdown of 2007 and 2008 has already been pegged as a “black swan” event. But we don’t know if it’s a black swan in

---

3 Martina Nardon and Paolo Pianca ,Ca’Foscari University of Venice ,Dept of Economics,A behavoural approach to the asset pricing of european options (p:22)
passing or if it’s heralding a new flock.\(^4\)

Investors who are mean revert believe that poor performing stocks will rebound and performe well and better performing stocks will decline and perform poor. Hence they sell off the winning stocks and hold on to losing stocks. As argued by Odean (1998) and Weber and Camerer (1998) the disposition effect is caused by irrational belief in mean reversion.[?]  

### 2.5 DATA AND METHODOLOGY

Nuttawat Visaltanachoti (2007) used two stage regression model to calculate disposition effect. I tried to apply his method on Karachi Stock Exchange (KSE) to investigate empirically if the disposition effect does exist or not. I considered KSE100 because it’s the benchmark for the rest of the industry. Index daily data are collected for a period from 2011 to 2016. Daily returns, holding periods, illiquidity and volatility were calculated through this data. Data for market capitalization for KSE100 are taken from Business Recorder.

### 2.6 DESCRIPTION OF VARIABLES

My dependent variable is Disposition effect which measure by computing average holding period of index. Average holding period of index for each year is computed by dividing the number of outstanding shares in KSE100 by the KSE100 annual trading volume. Equation (2.3) and (2.4) are previously used by Atkins and Dyl (1997) and Visaltanachoti (2007)

\[
HP_{i,t} = \frac{(\text{Shares Outstanding } i,t,d / \text{VOLDi},t,d)}{N}
\]

(2.3)

\(^4\) David Serchuk (1st Sep, 2009) Stocks will revert to the Mean. Forbes.
Shares outstanding on day d of year t
Respective daily volume for year t in terms of Pak rupee
Total number of trading days during year t

My independent variable is illiquidity that calculated by using formula:

\[ ILLIQ_{i,t} = \frac{|R_{i,t,d}|/VOLD_{i,t,d}}{N} \]  

Return on stock on day d of year t
Respective daily volume in terms of Pak rupee in year t
Number of days stock i traded in t year.

Control Variables

Firm size: Average market capitalization of index during year t.
Volatility: The variance of the firm’s daily stock returns.

To examine the relationship between investors illiquidity and holding period the regression analysis is used. Equation (2.5) is used to find disposition effect. It was previously used by Visaltanachoti (2007).

\[ HP_{i,t} = \beta_0 + \beta_1 R_{i,t} + \beta_2 ILLIQ_{i,t-1} + \beta_3 MV_{i,t} + \beta_4 Volatility_{i,t} + e_{i,t} \]  

- The average length of time that investors hold the stock during year t
- Return on the stock i.
- Predicted value from the first-stage regression of equation in t-1 year
- Average market capitalization during year t
- The variance of the daily stock returns
- Error term
2.7 RESULTS

Fama and French (1992), Banz (1981) and Reingaum (1981) argued that estimated returns of investments are negatively linked with market capitalization. So market capitalization is taken as control variable to control return size effect and volatility is used as a control variable as it affects liquidity. Table 2.3 shows the descriptive statistics of Karachi Stock Exchange (KSE100). Average holding period for KSE100 is 855 days. Average returns are 0.055 and average illiquidity is 5.06E-05. Average market capitalization is 33303318 and average stock volatility is 8.3837. Before moving on with regression model first I checked normality and stationarity of dependent and independent variables by using Augmented Dickey Fuller test. This test suggested integrating all the independents varaibles at level 1.

<table>
<thead>
<tr>
<th></th>
<th>Holding Period</th>
<th>Return</th>
<th>Illiquidity</th>
<th>Mk Cap</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>855.3270</td>
<td>0.055002</td>
<td>5.06E-05</td>
<td>33303318</td>
<td>8.383728</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>7.28E-05</td>
<td>0.004739</td>
<td>3.04E-15</td>
<td>1.83E+12</td>
<td>1.75E-15</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>63710.44</td>
<td>6.684339</td>
<td>0.074944</td>
<td>1.53E+09</td>
<td>7.544302</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>1.181520</td>
<td>1.08E-05</td>
<td>-0.016575</td>
<td>1.83E+12</td>
<td>2.05E-05</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>5402.005</td>
<td>1.653064</td>
<td>6.74E-05</td>
<td>1.08E+08</td>
<td>2.346533</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>1.925052</td>
<td>2.552941</td>
<td>4.83642</td>
<td>35.89004</td>
<td>5.972937</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>8.659601</td>
<td>14.37512</td>
<td>48.41606</td>
<td>1343.396</td>
<td>70.96915</td>
</tr>
<tr>
<td><strong>Jarque-Bera</strong></td>
<td>2854.213</td>
<td>9470.306</td>
<td>131340.9</td>
<td>1.10E+08</td>
<td>290116.2</td>
</tr>
<tr>
<td><strong>Probability</strong></td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>0.125049</td>
<td>9.503260</td>
<td>7.40E-12</td>
<td>3.11E+15</td>
<td>4.45E-12</td>
</tr>
<tr>
<td><strong>Sum Sq. Dev.</strong></td>
<td>4.26E-06</td>
<td>0.062311</td>
<td>6.64E-26</td>
<td>2.38E+28</td>
<td>2.76E-26</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>1462</td>
<td>1462</td>
<td>1462</td>
<td>1462</td>
<td>1462</td>
</tr>
</tbody>
</table>

Table 2.4 shows the scaled coefficients of the holding period regression of KSE100. The coefficient for return is negative which is in line with the disposition effect. Illiquidity is positively correlated with holding period which again confirms the disposition
effect. Market capitalization and volatility are negatively correlated with holding period.

**Tab. 2.4 Scaled coefficients of Holding Period Regression (KSE100)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standardized coefficient</th>
<th>Elasticity at Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>9535.0700</td>
<td>NA</td>
<td>0.999995</td>
</tr>
<tr>
<td>Return</td>
<td>-2.933147</td>
<td>-0.040118</td>
<td>3.21E-06</td>
</tr>
<tr>
<td>Illiquidity</td>
<td>121088.1</td>
<td>1.566796</td>
<td>-7.05E-07</td>
</tr>
<tr>
<td>Market Cap</td>
<td>-5.602091</td>
<td>-0.543573</td>
<td>-2.45E-09</td>
</tr>
<tr>
<td>Volatility</td>
<td>-5863.526</td>
<td>-4.394780</td>
<td>-2.74E-06</td>
</tr>
</tbody>
</table>

My findings are in line with the previous studies done by Mubeen Ashraf, Fahim Waris, Sania Saeed (Mohammad Ali Jinnah University, Islamabad Pakistan) 2014[?], on disposition effect evidence from Karachi stock exchange. My findings are also agreeing with another study done by Zia, Lubna and Hashmi, Shujahat Haider: Disposition Effect and Overconfidence in Pakistani Stock Market (March 10, 2016)[?]. I can conclude based on my findings that disposition effect exists in Karachi stock exchange (KSE100) but the future research should be carried out on a large sample period.
Introduction of Black and Scholes option pricing model was a turning point in the evolution of options pricing. It was considered a significant effort at articulating the price of options and still in use to find out the worth of the option. But empirical literature on options shows deviation from the theoretical model of Black and Scholes. This deviation is because of the underlying assumptions assumed by Black and Scholes Model such as frictionless market, self financing portfolio, constant volatility and investors risk attitude.

Prospect theory tells that the decision taken by investors is not always consistent to the maximization of the wealth (EU). Investors have biased opinions when they estimate probabilities. They seemed to underweight high probabilities and overweight low probabilities. The two distinct functions of Prospect theory i.e value function and weighting function describe the risk attitude, loss aversion and and subjective probabilities. Prospect theory negates the basic concept of Expected Utility Theory that is investors evaluate their choices based on potential gains and losses to a reference point instead of the ultimate maximization of the wealth. Investors are risk seeking when it comes to losses and risk averse when it comes to gains. They are more sensitive to losses than gains of the same magnitude. Both of these risk attitude and loss aversion are described by the value function. We consider Cumulative Prospect Theory as it overcomes some of the drawbacks of Prospect Theory i.e violation of the stochastic dominance.
From empirical option pricing studies it is known that the actual option prices show a systematic and persistent deviation from theoretical values under standard pricing assumptions. This deviation is because of the possibility that some market participants behave less than rational. In 1990s the classical finance literature started to get rebirth with psychological concepts to explain the behaviour of market participants as a separate field of research. Most of the research in behavioural finance has been centered or focused on stock markets and put an attempt to explain phenomena like premium between risky and risk free assets, under and over pricing, panic and hype, preference for cash dividends and the tendency to sell winning stocks and hold on to losing stocks.

The most frequently used model to find the price of options within normative finance is based on the theory of Black and Scholes (1973)[?]. This theory is built on a riskless portfolio of a stock and a written call option on that stock. But the research shows that prices of options systematically deviate from the value investors calculated by using Black and Sholes model. The explanation for this deviation is because of the underline assumption of the model. Another explanation is based on the fact that the Black and Scholes portfolio is not riskless (Bergman 1981, Omberg 1991, Versluis and Hillegers 2006) neither in continuous time nor in discrete time (Black and Scholes 1973 and Leland 1985). I apply cumulative prospect theory to evaluate European options by using the model developed by Prof. Martina Nardon and Prof. Paolo Pianca (2014).

### 3.1 OPTION VALUATION

In the literature of finance Prospect theory has recently begun to attract the attention of experts on financial option valuation. From the empirical studies we know that when cumulative prospect theory version (Kahneman and Tversky 1992) is applied to option pricing seems promising alternative to the other models. The work of Shefrin and Statman (1993) was the first contribution which applies prospect theory to option
valuation. Who considered covered call option in a one period binominal model. A list of
other people can also be include Breuer and Perst (2007), Poteshman and Serbin (2003),
order to evaluate European call option. Mrtina Nardon and Paolo pianca extend there
model to europen put option, and considering both positions of the writer and holder.
(Martina Nardon and Paolo Pianaca).

I evaluate European options with in continuous cumulative propect theory. Particu-
larly in the application I used the constant relative sensitivity weighting function.

Prospect theory says individuals do not always take their decision consistently with
the maximization of expected utility. Decision making under risk makes the investors to
be risk averse when they elevate gains and risk seeking with respect to losses. Investors
are more sensitive to losses then to gains of the same magnititude. In prospect theory
gains and losses are evaluated relative to a reference point rather than in terms of final
wealth as considered by expected utility. Subjective probabilities, loss avers and risk
attitude are shaped by two functions weighting function and value function.

3.2 PROBABILITY WEIGHTING FUNCTION AND EU-
ROPEAN OPTION

I have focused on the effects on the European option prices of the shape of the probability
function, which models the probabilistic risk perception. Emperical evidences suggests a
particular shape of probability weighting function which turns out in a typical inverse
S shape. The function is initially concave (probabilistic risk seeking or optimism) for
small probabilities and convex (probabilistic risk aversion or pessimism) for medium
and large probabilities, such that the decision weights display heavier tails. A linear
weighting function describes probabilistic risk neutrality or objective sensitivity towards
probabilities which characterizes expected utility. Curvature of the weighting function is related to risk attitude towards probabilities. It models optimism and pessimism when one moves from extreme probabilities. Where as elevation can be interpreted as a measure of relative optimism. (Martina Nardon and Paolo Pianaca)

Alternate weighting functions have been used in many theoretical and experimental studies. Only the constant relative sensitivity weighting function proposed by Abdel-laoui et al (2010) allows for modelling separately curvature and elevation. I am interested to study the effect of both these features on option prices and for that I apply such a probability weighting function in a continuous cumulative prospect theory framework for the elevation of European put and call option developed by Martina Nardon and Paolo pianca (2014).

The degree of risk aversion and risk seeking appears to depend not only on the values but also on the probability and ranking of the outcomes. Subjective values are multiplied by objective probabilities but using decision weights computed through a weighting function. In the numerical results i noticed that for lower values of $\gamma$ options prices deviates sensitively from Black and Scholes prices.

### 3.3 RESULTS

Sesitvity analysis from the prospective of writer’s and holder, $s$ for put and call options considered. I have calculated the prices in time aggregated case for both writer and holder. I have report the results for constant relative sensitivity weighting function. Table 3.1 and 3.2 show the results of the European call and put option in the time aggregated model for writer’s position for different strike prices and elevation. The focus is on the effect of elevation. The paramenters of the value function , $b$ and $\lambda$ are fixed. I assume here $\delta^+ = \delta^-$ and $\gamma^+ = \gamma^-$. The parameter $\delta$ takes on the values [0.3, 0.35, 0.375, and 0.4] and parameter $\gamma = 0.7$ (which is closer to Kahnemann and Tversky 1992) $\gamma =$
1. In this case the only effect of the value function applies and of curse the prices are constant with respect to $\gamma$.

Table 3.3 and 3.4 show the results of European call and put options in the time aggregated model from holders prospective for different strike prices and elevation. Also in this case the option prices are increasing with the $\delta$.

Writer’s prices are always above Black and Scholes prices. In case of holder, $s$ prospective this is not the case. Results suggest that option prices increase with $\delta$ (elevation) with in the interval (0.3, 0.4). I also notice that prices increase at a decreasing rate.

Table 3.1 shows the: Sensitivity of the put option prices (writer’s position in the aggregated model) to the elevation of the weighting function for different values $\gamma$ (curvature) $\gamma^+ = \gamma^-$. Parameters of the value function: $a=0.988$ and $\lambda=1.125$. Option parameters: $S_0=100$, $X \in [90,110]$, $r=0.01$, $\sigma=0.2$, $T=1$. BS is the Black-Scholes price with $\gamma=1$, $a=1$ and $\lambda=1$.

**Tab. 3.1 Sensitivity of the call option prices (writer’s position in the aggregated model)**

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$X$</th>
<th>BS</th>
<th>$\delta=0.3$</th>
<th>$\delta=0.35$</th>
<th>$\delta=0.375$</th>
<th>$\delta=0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>90</td>
<td>14.1929</td>
<td>17.5572</td>
<td>17.7161</td>
<td>17.7773</td>
<td>17.8267</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>8.43332</td>
<td>11.6587</td>
<td>11.8558</td>
<td>8.33114</td>
<td>12.0084</td>
</tr>
<tr>
<td></td>
<td>110</td>
<td>4.61011</td>
<td>7.32385</td>
<td>7.50569</td>
<td>7.58433</td>
<td>7.65592</td>
</tr>
<tr>
<td>0.8</td>
<td>90</td>
<td>14.1929</td>
<td>16.4604</td>
<td>16.5620</td>
<td>16.5999</td>
<td>16.6379</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>8.43332</td>
<td>10.5501</td>
<td>10.6739</td>
<td>10.7248</td>
<td>10.7694</td>
</tr>
<tr>
<td></td>
<td>110</td>
<td>4.61011</td>
<td>6.33472</td>
<td>4.99303</td>
<td>6.49201</td>
<td>6.53517</td>
</tr>
<tr>
<td>0.9</td>
<td>90</td>
<td>14.1929</td>
<td>15.6077</td>
<td>15.6555</td>
<td>15.6735</td>
<td>15.6879</td>
</tr>
<tr>
<td></td>
<td>110</td>
<td>4.61011</td>
<td>5.56845</td>
<td>5.61861</td>
<td>5.64026</td>
<td>5.65954</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>8.43332</td>
<td>8.98114</td>
<td>8.98114</td>
<td>8.98114</td>
<td>8.98114</td>
</tr>
<tr>
<td></td>
<td>110</td>
<td>4.61011</td>
<td>4.95899</td>
<td>4.95899</td>
<td>4.95899</td>
<td>4.95899</td>
</tr>
</tbody>
</table>

Table 3.2 gives the description of sensitivity of the put option prices (writer’s position...
in the aggregated model) to the elevation of the weighting function for different values \( \gamma \) (curvature) \( \gamma^+ = \gamma^- \). Parameters of the value function: \( b = 0.988 \) and \( \lambda = 1.125 \). Option parameters: \( S_0 = 100, X \in [90,110], r = 0.01, \sigma = 0.2, T = 1 \). BS is the Black-Scholes price with \( \gamma = 1, =b=1 \) and \( \lambda = 1 \).

**Tab. 3.2 Sensitivity of the put option prices (writer’s position in the aggregated model)**

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>X</th>
<th>BS</th>
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Table 3.3 shows sensitivity of the call option prices (holder’s position in the aggregated model) to the elevation of the weighting function for different values \( \gamma \) (curvature) \( \gamma^+ = \gamma^- \). Parameters of the value function: \( b = 0.988 \) and \( \lambda = 1.125 \). Option parameters: \( S_0 = 100, X \in [90,110], r = 0.01, \sigma = 0.2, T = 1 \). BS is the Black-Scholes price with \( \gamma = 1, =b=1 \) and \( \lambda = 1 \).

Table 3.4 shows the sensitivity of the call option prices (holder’s position in the aggregated model) to the elevation of the weighting function for different values \( \gamma \) (curvature) \( \gamma^+ = \gamma^- \). Parameters of the value function: \( b = 0.988 \) and \( \lambda = 1.125 \). Option parameters: \( S_0 = 100, X \in [90,110], r = 0.01, \sigma = 0.2, T = 1 \). BS is the Black-Scholes price with \( \gamma = 1, =b=1 \) and \( \lambda = 1 \).
### Tab. 3.3 Sensitivity of the call option prices (holder’s position in the aggregated model)

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### Tab. 3.4 Sensitivity of the call option prices (holder’s position in the aggregated model)

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37
CONCLUSIONS

Understanding the behaviour of investors can help the financial market. Behavioural finance has attracted the attention of the investors more than ever. Prospect theory of Kahnemann and Tversky (1979) got the world attention that people are risk averse given the gain and risk seeking given the loss. The two well known functions of Prospect theory and cumulative prospect theory i.e value function and weighting function play an important role when people make decision under risk. Investors hold on to the losing stocks and selling the winning stocks to early because of their belief in mean reversion, pride seeking, regret avoiding, frmaing and mental accounting. Continuous cumulative prospect theory seems promising approach than other models when applied to option valuation for its ability to explain the departure from Black and Scholes. The option prices are sensitive to the choice of the values of the parameters. The numerical experiments in order to study the effects of curvature and elevation on option suggests that option prices increase with elevation and the prices increase at decreasing rate. Using the constant sensitivity weighting function allows for separate modeling of curvature and elevation has an interesting economic interpretation in terms of probabilistic optimism and pessimism. It’s also worth to notice that if we let the v approaching the risk neutral linear value function then the sentiment parameters or probability weighting function tends to the value 1. At this point we recover Black and Scholes results.


[ZH16] Lubna Zia and Shujahat Haider Hashmi. Disposition effect and overconfidence in pakistani stock market. 2016. 30