COMMODITY MARKETS AND VOLATILITY CONTAGION

By

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Abstract

During the last years, commodity prices have been exceptionally volatile. And due to the financial crises that the world has known until now. In view of its importance, several researchers were interested in analyzing the volatility of commodity price. To this end, this paper investigates the commodities volatility contagion for 8 commodities over the period from 22 January 2007 to 15 May 2017. We use the newly Bayesian Graphical Vector Autoregressive (BGVAR) model developed by Ahelegbey D. F., Billio, M. and Casarin, R. (2016). We find that predictability of commodities volatility are differs between them, and that there are some commodities which are not directly connected in terms of volatility. These results are represented by a Granger causal Network and a Network density indicator.

Keywords: Bayesian Graphical VAR; Multivariate Instantaneous; Multivariate Autoregressive; Network density, Graph
Contents

List of Tables. ......................................................... 5

List of figures. ......................................................... 6

List of abbreviations and symbols. .................................. 7

Introduction. ................................................................ 8

Chapter 2: Literature review. ......................................... 10

Chapter 3: Data analyze. ............................................. 13

Chapter 4: Methodology ............................................. 18

Chapter 5: Results. .................................................. 22

Conclusion. ............................................................. 30

Bibliography. .......................................................... 31
List of Tables

Table 1. Descriptive statistics of commodities volatility. 13

Table 2. Results of MIN structure. 22

Table 3. Results of MAR structure. 25
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gold volatility (22/01/2007-15/05/2017)</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>Silver volatility (22/01/2007-15/05/2017)</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>Palladium volatility (22/01/2007-15/05/2017)</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>Copper volatility (22/01/2007-15/05/2017)</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>Oil volatility (22/01/2007-15/05/2017)</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>Gaz volatility (22/01/2007-15/05/2017)</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>Zinc volatility (22/01/2007-15/05/2017)</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>Electricity volatility (22/01/2007-15/05/2017)</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>Multivariate instantaneous structure representing instantaneous volatility spill-over effects between markets</td>
<td>23</td>
</tr>
<tr>
<td>10</td>
<td>Granger causal network extracted by using BGVAR model on the sample from 22/01/07-15/05/17 of Multivariate instantaneous</td>
<td>24</td>
</tr>
<tr>
<td>11</td>
<td>Multivariate autoregressive structure representing lagged volatility spill-over effects between markets</td>
<td>26</td>
</tr>
<tr>
<td>12</td>
<td>Granger causal network extracted by using BGVAR model on the sample from 22/01/07-15/05/17 of Multivariate Autoregressive</td>
<td>28</td>
</tr>
<tr>
<td>13</td>
<td>Network density</td>
<td>29</td>
</tr>
</tbody>
</table>
List of abbreviations and symbols

\( G_s \circ \Phi_s \): Hadmard’s product of two matrices \( G_s \) and \( \Phi_s \)

\( G_{ij} = 1 \): Variable index \( j \) influences variable index \( i \) \( (X_{t-s}^i \to X_t^i) \).

BGVAR: Bayesian graphical vector autoregression.

DAG: Directed Acyclic Graph.

MAR: Multivariate autoregression.

MCMC: Markov Chain Monte Carlo.

MIN: Multivariate Instantaneous.

SSVS: Stochastic Search Variable Selection.

SVAR: Structural vector autoregression.

VAR: Vector autoregression.

DCC: dynamic conditional correlation.

DAG: Direct acyclic graph.
Introduction

During the last years, commodity prices have been exceptionally volatile. In view of its importance, several researchers were interested in analyzing the volatility of commodity price.

The commodities play a significant role in our daily life, in the countries’ economy. They are defined as the main outputs of several countries, particularly developing countries. Most of the studies highlight the link between commodity market and stock market. However, there is a few studies on volatility contagion between commodities.

By definition, the volatility is a measure of changes in the price of financial asset over time or a statistical measure of the dispersion of the returns for a given security.

Study the volatility of the commodity market appears to be very important for developed and developing countries. Governments pay attention to commodity prices’ volatility because changes in commodity prices can cause inflation. Indeed, being quoted in US dollars, intuitively, a strengthening of US dollar can lead to a reduction of commodities price. Hence, commodities prices can be consider as a key factor to see how the currency will affect the inflation.

For developing countries, mostly dependent on imports of certain commodities such as oil, coffee, cocoa, an increase in prices can affect the public expenditure of government. Recently, for example, Ivorian government has reduced the expenditures due to the increase of cocoa’ price.

The collapse of Oil price between 2011 and 2016 also highlighted the importance of stable commodity prices. This crash caused serious problems for some producing countries such as Russia, Mexico and some African countries were the most affected.

In view of their importance, many discussion forums have brought about volatility in commodity market. Indeed the declaration of G20 leaders (2012) show that the stability of the word economy is linked at the stability of commodity prices. Also St. Petersburg Summit (2013) recognizes that one of the main challenges of the global economy is the stability of commodity prices.

In this paper, we review the literature of commodity market’ volatility and investigate the volatility contagion on the commodity market. To achieve our goal, we use the recently
Bayesian Graphical Vector Auto-Regressive (BGVAR) model developed by Ahelegbey, Billio and Casarin (2016). It is a structural VAR model based on the graph representation of the conditional independence among the variables of interest, i.e. the commodity volatilities in our analysis. This BGVAR model is close to the Stochastic Search Variable Selection (SSVS) proposed by George et al (2008). These two models use two separate sets of restrictions for the contemporaneous and the lagged interactions. But there are differences between them. And one of the important difference is that on the restrictions. In BGVAR model, the restrictions are imposed on the structural form model, while the SSVS uses the reduced-form model. Therefore, BGVAR model allows us to analyze both contemporaneous and lagged contagion effects between commodity markets and to extract the corresponding contagion network. Again, based on posterior probability of inclusion, BGVAR detects the impact on each individual predictors even when modelled in a multivariate framework.

In the last decade, the financial crisis amplified by globalization and the concept of network has been developed by several researchers in order to model the complexity of the global economic system. Indeed, nowadays, there is a strong connection between markets and institutions. These connections are the basis of the increase in risk (systemic risk) between sectors (Hautsh et al., 2015; Barigozzi and Brownless, 2014). Hence, various authors pay attention to the analysis of the connections between economic agents or systems to ensure the stability (Billio et al., 2012; Brunnermeier and Pedersen, 2009, Diebold and Yilmaz, 2014).

Network models have been used to analyse the subprime crisis. Indeed, these models allow us to understand the structure of the crisis and its risk contagion. Another advantage of this model is that it allow us to study a complex system.

In our paper, we study the interconnectedness of (contagion) volatility between commodities, using a Granger causal network.

This paper is organized as follows. Section 2 reviews the literature. Section 3 analyses the data. After, section 4 presents the BGVAR model. We have the results and the conclusion respectively at the section 5 and section 6.
Chapter 2: Literature review

Several researchers based their study on commodity market’s volatility in different ways. In our study, we will focus on the link between commodity market and stock, the volatility spillover and the volatility contagion.

Studies on the link between commodity market and stock market have focused in particular on oil price and stock market.

Joets and Mignon (2013) analyze the link between 25 commodities and stock return over the period from January 2001 to November 2011. Les auteurs pay attention on the volatility of energy raw material. To achieve their goal, they applied a dynamic conditional correlation (DCC) Garch methodology. As result, they show that particularly since the 2007-2008 financial crisis, the correlation between commodity and stock markets evolve throughout time and are highly volatile.

Park and Ratti (2008) investigate on the relation between oil prices shocks and real stock return over from January 1986 to December 2008 for US and 13 European countries. Using the Multivariate Vector Autoregressive, they concluded that there is a significant impact of oil prices shocks on stock return.

Sadorsky (1999) analyze the link between oil price shock and stock market using an unrestricted vector autoregression model on the period from January 1947 to April 1996. And the authors show that oil price movements are important to explain movements in stocks returns.

Mustafa and Erlna (2016) examine also the link between commodity market and stock market from January 2012 to May 2015. They use the Pedroni’s panel cointegration to analyze this link. Musata and Erlna show that there is no relation between commodity price and stock.

Gordon and Mouwenhorst (2006), on the period from July 1959 to December 2004, analyze commodities futures price. And they show that there is an interaction between commodity market and stock market. Commodity market have an impact on stock market.

Filis et al (2011) investigate on the relation between stock prices and oil price for oil importing and oil exporting for some countries. They use the DCC-GARCH-GJR method. The authors find a correlation between oil prices and stock market for importing and exporting oil.
Some researchers based their study on the volatility spillovers; the Co-movements volatility of commodities. In this field, they analyze the volatility between commodities.

Xiaodong, Cindy and Dernot (2011), using the stochastic volatility model, analyze the link between crude oil and agriculture on the period from November 1998 to January 2009. And they show that there is an interconnection between agricultural commodities and energy market. This can be observed by the impact of oil price shocks on prices in agricultural commodity market.

Their result is supporting by the paper of Nazlioglu and Erden (2012). Indeed, authors analyze also the volatility spillover between oil and agricultural commodity market on the period from 01 January 1986 and 21 March 2011. They consider two sub-periods: pre-crisis (01 January 1986-31 December 2005) and post-crisis (01 January 2006-21 March 2011). Saban et Erden use the causality in variance test and impulse response function. Their results show that there is no transmission between oil and agricultural market in pre-crisis period but in the post-crisis oil and agricultural commodities are highly linked.

Wen and Wei (2012) use the time varying copulas to analyze the contagion effect between Energy market and stock market during the financial crisis. They use daily sample from December 2005 to November 2010 of WTI spot prices, the S&P 500 and the Shanghai stock market composante index. As result, authors show that there is a dependance between crude oil and stock market after the fail of Lehman brothers. This contagion is also supporting by Rigobon (2012).

Neverless, some authors show there is a weak relation between crude oil and stcok market. (Chiou and Lee, 2009; Nandha and Fall, 2008)

Shankar and Carlo (2010) use the monthly panel dataset on 45 commodities price and with Garch model, they show that most of commodities price was high volatile after the collapse of Bretton-Woods.

Diebold, Liu and Yilmaz (2017) analyze the connectedness in 19 commodities with Variance decomposition from high-dimensional vector autoregressive from 2011 to 2016. Authors show that energy sector is the most important in term of contagion to other groups.

Our study extends the previous literature on commodity markets’ volatility. We analyse both of multivariate instantaneous and autoregressive of commodity prices’ volatility. We are able
to compare the behavior of each commodity and to see the impact of each volatility on the other commodities.
Chapter 3: Data analysis

For the purpose of this study, we consider daily spot price series extracted from Bloomberg for 8 different commodities over the January 22, 2007 – May 19, 2017 period. Indeed, we download high and low spot price of these following commodities: Gold, Silver, Palladium, Copper, Zinc, Oil, Gaz, and Electricity. Then, we build the volatility range doing the difference high price and low price. All price series are quoted in US dollars.

Table 1. Descriptive statistics of commodities volatility

<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>Min</th>
<th>Median</th>
<th>Mean</th>
<th>Mode</th>
<th>std</th>
<th>Kurtosis</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaz</td>
<td>1.6</td>
<td>0</td>
<td>0.1</td>
<td>0.1315</td>
<td>0.08</td>
<td>0.1257</td>
<td>41.1364</td>
<td>4.8075</td>
</tr>
<tr>
<td>Gold</td>
<td>160</td>
<td>3.64</td>
<td>17.52</td>
<td>20.56</td>
<td>13.6</td>
<td>12.6805</td>
<td>19.6849</td>
<td>2.9525</td>
</tr>
<tr>
<td>Oil</td>
<td>14.62</td>
<td>0</td>
<td>0.91</td>
<td>1.1651</td>
<td>0.5</td>
<td>0.9387</td>
<td>26.525</td>
<td>3.0979</td>
</tr>
<tr>
<td>Silver</td>
<td>980</td>
<td>0</td>
<td>56</td>
<td>80.3325</td>
<td>40</td>
<td>79.8489</td>
<td>26.4648</td>
<td>3.7997</td>
</tr>
<tr>
<td>Palladium</td>
<td>66.5</td>
<td>2.5</td>
<td>13.75</td>
<td>15.5073</td>
<td>7.5</td>
<td>8.3197</td>
<td>7.1385</td>
<td>1.6052</td>
</tr>
<tr>
<td>Zinc</td>
<td>420</td>
<td>12</td>
<td>52.875</td>
<td>62.0396</td>
<td>75</td>
<td>36.4261</td>
<td>9.7115</td>
<td>1.857</td>
</tr>
<tr>
<td>Copper</td>
<td>710.25</td>
<td>30</td>
<td>144</td>
<td>161.0636</td>
<td>150</td>
<td>86.827</td>
<td>6.9338</td>
<td>1.5426</td>
</tr>
<tr>
<td>Electricity</td>
<td>305</td>
<td>0</td>
<td>1</td>
<td>2.113</td>
<td>0.5</td>
<td>7.6658</td>
<td>900.6948</td>
<td>29.0977</td>
</tr>
</tbody>
</table>

Notes: this table covers the full sample period from January 22, 2007 to May 15, 2017 (2275 daily observations).

The summary statistics for all commodities are shown in Table 1. Among all the examined variables, Copper volatility has the highest mean and standard deviation. After, this commodity, we noticed that Silver volatility is the second with highest mean and standard deviation. This is due to the large difference between the maximum « 980 » and the minimum observation « 0 ». With a small difference between the maximum « 1.6 » and the minimum « 0 » observations, Gaz volatility has a lowest mean and standard deviation.

All the series have a positive skewness and positive kurtosis. Positive skewness means that the right tail is longer for all commodities, the mass of the volatility distribution has an asymmetric tail extending to the right. Thus, except for Palladium volatility, our volatility commodities are more peaked especially for electricity with a skewness of 29.075.

Therefore, we plot the volatility commodities.
Figure 1 - Gold volatility (22/01/2007-15/05/2017)

Source: Data are extracted from Bloomberg; daily price of gold quoted in US dollars.

Figure 2 - Silver volatility (22/01/2007-15/05/2017)

Source: Data are extracted from Bloomberg; daily price of silver quoted in US dollars.
Figure 3- Palladium volatility (22/01/2007-15/05/2017)

Source: Data are extracted from Bloomberg; daily price of palladium quoted in US dollars.

Figure 4- Copper volatility (22/01/2007-15/05/2017)

Source: Data are extracted from Bloomberg; daily price of copper quoted in US dollars.
Figure 5 - Oil volatility (22/01/2007-15/05/2017)

Source: Data are extracted from Bloomberg; daily price of oil quoted in US dollars.

Figure 6 - Gaz volatility (22/01/2007-15/05/2017)

Source: Data are extracted from Bloomberg; daily price of Gaz quoted in US dollars.
Figure 7- Zinc volatility (22/01/2007-15/05/2017)

Source: Data are extracted from Bloomberg; daily price of zinc quoted in US dollars.

Figure 8- Electricity volatility (22/01/2007-15/05/2017)

Source: Data are extracted from Bloomberg; daily price of zinc quoted in US dollars.
Chapter 4: Methodology

The main objective of this paper is to model the contemporaneous and lagged causality between 8 commodities.

To solve the problem of over-parametrization and identification of the VAR model, Ahelegbey and Billio and Casarin (2016) propose an identification approach of the Structural VAR model based on a graph representation of the conditional independance variable among commodities. Therefore, the structural VAR model use dis such that

\[ Y_t = B_0 Y_t + \sum_{i=1}^{P} B_i Y_t + \sum_{i=1}^{P} C_i Z_{t-i} + \varepsilon_t \quad (1) \]

For \( t = 1, \ldots, T \), where \( Y_t \) is an \( n_y \) vector of response variables; \( Z_t \) is an \( n_z \) vector of predictor variables; \( \varepsilon_t \) is an \( n_y \) vector of structural error terms, independent and identically normal, i.e. \( \varepsilon_t \sim N(0, \Sigma) \); \( P \) is the maximum lag order. \( B_0 \) is a \( n_y \times n_y \) matrix of structural contemporaneous coefficients, with zero diagonals; \( B_i \) and \( C_i \), with \( 1 \leq i \leq P \), are \( n_y \times n_y \) and \( n_y \times n_z \) matrices of structural coefficients, respectively.

Since, the identification problem in this equation is the fact that it can’t be directly estimable from which to derivable “true” model of the parameter, we have two solution. At the one hand, we can used the reduced-form below of the structural VAR model to estimate the parameters:

\[ Y_t = \sum_{i=1}^{P} A_i X_{t-i} + A_0^{-1} \varepsilon_t \quad (2) \]

For \( t = 1, \ldots, T \), where \( X_t = (Y_t, Z_t)' \) be an \( (n = n_y + n_z) \)-dimensional vector of observed variables at time \( t \) and \( B_i^* = (B_i, C_i) \), \( 1 \leq i \leq P \), the \( n_y \times n \) matrices of unknown coefficients for the response and predictor variables. \( A_0 = (I_{n_y} - B_0) \) is an \( n_y \times n_y \) matrix, \( I_{n_y} \) is the \( n_y \)-dimensional identity matrix. \( A_i = A_0^{-1} B_i^* \), \( 1 \leq i \leq P \) are the reduced-form lag coefficient matrices such that \( A_i \), \( 1 \leq i \leq P \), is of dimension \( n_y \times n_y \) and \( u_t = A_0^{-1} \varepsilon_t \) is an \( n_y \)-dimensional vector of reduced-form errors independent and identically normal, \( u_t \sim N(0, \Sigma_u) \).

At other hand, to esimate the parameter of the SVAR model, some researchers propose to impose some restrictions on the contemporaneous variables. But in many case, there are not
enough exclusion restrictions to achieve the identification (Ahelegbey, Billio, and Casarin 2016).

To solve these problem, in this paper, we use the Bayesian Graphical Vector Auto Regressive model (BGVAR) proposed by Ahelegbey, Billio, and Casarin 2016.

One of advantage of this model is that, with BGVAR we do not need to have a reduced-form of the SVAR model or make some restrictions on the contemporaneous parameters. Also, the BGVAR model allows us to analyze both contemporaneous network and lagged network between variables.

A graphical model is a statistical model embodying a set of a conditional independence relationships which may be summarized by means of a graph (Brillinger, 1996). There exist two type of graphs; undirected and directed graphs. Undirected graphs have edges that do not have a direction while in the directed graphs, edges have a direction. Following Ahelegbey, Billio and Casarin (2016), we employ directed graph which present unambiguous direction of causation among variables.

Let $X_t = (X_{t1}, X_{t2}, ..., X_{tn})$ where $X_{ti}$ is a realization of the $i$-th variable at time $t$. Then equation (1) can be represented in the form of a graphical model with a one-to-one correspondence between the coefficient matrices and a directed acyclic graph (DAG).

$$X_{t-s} \rightarrow X_t \iff B^*_{s,j} \neq 0 \quad 0 \leq S \leq P \quad (3)$$

Where $B^*_0 = B_0$, for $S = 0$ and $B^*_s = (B_s, C_s)$ for $1 \leq S \leq P$.

Here considering the structural dynamics as a causal dependence among variables, the relationship in equation (3) for $1 \leq S \leq P$ can be referred to as lagged (temporal) dependence and as contemporaneous dependence for $S = 0$.

Following the representation in the equation (3), we can define:

$$B^*_s = (G_s \circ \Phi_s), \quad 0 \leq S \leq P \quad (4)$$

Where for $S = 0, B^*_0 = B_0$ is a $n_y \times n_y$ structural coefficients of contemporaneous dependence, $G_0$ is a $n_y \times n_y$, binary connectivity matrix and $\Phi_0$ is a $n_y \times n_y$ matrix coefficients. For $1 \leq S \leq P, B^*_s = (B_s, C_s)$ is a $n_y \times (n_y + n_z)$ matrix of structural coefficients if temporal dependence. $G_s$ is a $n_y \times (n_y + n_z)$ binary connectivity matrix and $\Phi_s$ is a $n_y \times (n_y + n_z)$
matrix coefficients. The operator \((o)\) is the element-by-element Hadmard’s product. It’s means 
\[ B_{s,ij}^* = (G_{s,ij} \Phi_{s,ij}) \] . We refer to \(G_0\) as the connecticity matrix of the contemporaneous 
dependence and \(G_s\), \(1 \leq S \leq P\), as the matrix of the temporal dependence. Elements in \(G_s\), \(0 \leq S \leq P\), are indicators such that \(G_{s,ij} = 1 \iff X_{t-s}^i \rightarrow X_t^i\) and 0 otherwise. Elements \(\Phi_s\), \(0 \leq S \leq P\), are structural regression coefficients, such that \(\Phi_{s,ij} \in \mathbb{R}\) represents the value of 
the effect of \(X_{t-s}^i\) on \(X_t^i\). There is one-to-one correspondence between \(\Phi_s\) and \(B_s^*\) conditionally 
on \(G_s\):
\[
B_{s,ij}^* = \begin{cases} 
\Phi_{s,ij} & \text{if } G_{s,ij}=1 \\
0 & \text{if } G_{s,ij}=0 
\end{cases} 
\] (5)
Therefore, based on the equation (4), the SVAR model in equation (1) can be expressed as:
\[
Y_t = (G_0 \circ \Phi_0) Y_t + \sum_{i=1}^{P} (G_i \circ \Phi_i) X_{t-i} + \varepsilon_t \quad (6)
\]
Where \((G_j \circ \Phi_j)\) are the graphical model structural coefficient matrices whose non-zero 
elements describe the value associated with the contemporaneous and temporal dependence 
respectively.

To estimate the lagged network, Ahelegbey, Billio and Casarin, following Grzegorczyk et al 
(2010) and Madigan et al (1995), propose to use a bayesian scheme with an efficient Markov 
Chain Monte Carlo (MCMC) process. While the contemporaneous network, Ahelegbey, Billio 
and Casarin in line with Giudici and Castelo (2003) propose to have a necessary and sufficient 
condition to check the acyclicity constraint in a small-size networks.

Let \(b_t = (b_{i1}, b_{i2}, ..., b_{in})\) be a row vector of \(B_s\), where its entries \(b_{ij}\) are the regression 
coefficients of the effects of \(X_{t-s}^i\) on \(X_t^i\). Due to this, the relation between \(B_s\) and \(\Phi_s\) can be 
express such that:
\[
b_{ij} = \begin{cases} 
\Phi_{ij} & \text{if } G_{ij}=1 \\
0 & \text{if } G_{ij}=0 
\end{cases} 
\] (7)
With Ahelegbey, Billio and Casarin (2016), we suppose that the marginal prior and the marginal 
posterior of \(G_{ij}\) are Bernouille-distributed. Then we have:
\[
a_{ij|data} = \begin{cases} 
1 \text{ if } p(a_{ij} = 1|data) > \tau \\
0 \text{ otherwise} 
\end{cases} 
\] (8)
Where $\tau$ is a threshold value set by the user $\tau \in (0,1)$; and $p(a_{ij} = 1|data)$ is the confidence score that is the posterior probability of existence of an edge from $X_j$ to $X_i$.

In line with Ahelegbey, Billio and Casarin (2016), we present in this paper, the posterior probabilities of full-sample estimates for the 25 commodities for the multivariate autoregressive (MAR) and the multivariate instantaneous (MIN). Also, the maximum lag order $p$ of the multivariate autoregressive is based on the BIC. In this case, on inline, the variables that have posterior probability higher than $50\%$, will be considered to be the selected edges of the MAR and MIN structures.

For the application of BGVAR model, Ahelegbey, Billio and Casarin applied the MAR and MIN to analyze the relation between 20 macroeconomics variables.
Chapter 5: Results

We estimate the posterior probabilities of both multivariate instantaneous (MIN) and multivariate autoregressive (MAR) structures for the volatility in the 8 commodity markets. These estimations are respectively reported in following Table 2 and Table 3. Here, the lag order \((p)\) of the VAR is fixe to 1 based on the full sample using the Bayesian Information Criterion.

First, considering the 8 commodities, “Copper, Electricity, Gold, Oil, Palladium, Silver and Zinc”, we estimate a model of MIN.

Following Ahelegbey, Billio and Casarin (2016), in this table, based on the on posterior probabilities greater than 50%, the bold entries represent here the selected edges of the multivariate instantaneous (MIN). Therefore, the italic entries represent the posterior probabilities between 30% and 50%.

<table>
<thead>
<tr>
<th></th>
<th>Copper</th>
<th>Electricity</th>
<th>Gaz</th>
<th>Gold</th>
<th>Oil</th>
<th>Palladium</th>
<th>Silver</th>
<th>Zinc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>0.00</td>
<td>0.14</td>
<td>0</td>
<td>0.08</td>
<td>0.24</td>
<td>0.42</td>
<td>0.1</td>
<td>0.38</td>
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<td>Gaz</td>
<td>0.1</td>
<td>0.04</td>
<td>0</td>
<td>0.26</td>
<td>0.46</td>
<td>0.24</td>
<td>0.1</td>
<td>0.32</td>
</tr>
<tr>
<td>Gold</td>
<td>0</td>
<td>0.24</td>
<td>0.22</td>
<td>0</td>
<td>0.3</td>
<td>0.32</td>
<td>0.46</td>
<td>0.2</td>
</tr>
<tr>
<td>Oil</td>
<td>0.02</td>
<td>0.18</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.08</td>
</tr>
<tr>
<td>Palladium</td>
<td>0.36</td>
<td>0.06</td>
<td>0.16</td>
<td>0.44</td>
<td>0.38</td>
<td>0</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>Silver</td>
<td>0.12</td>
<td>0.06</td>
<td>0.08</td>
<td>0.48</td>
<td>0.72</td>
<td>0.18</td>
<td>0</td>
<td>0.44</td>
</tr>
<tr>
<td>Zinc</td>
<td>0.62</td>
<td>0.04</td>
<td>0.16</td>
<td>0.36</td>
<td>0.2</td>
<td>0</td>
<td>0.22</td>
<td>0</td>
</tr>
</tbody>
</table>
Then in the case of multivariate instantaneous in our Table2, for the Copper, the highest posterior probabilities is the Zinc volatility with a value of 0.62, followed by Palladium volatility of 0.36. The highest posterior probability for Electricity is the Gold volatility with a value of 0.24. After that, the highest posterior probability are for Gaz is also the Gold volatility with a value of 0.22.

The highest posterior probability for Gold is the Silver volatility with a value of 0.48; followed by Palladium volatility, Zinc volatility and Electricity volatility with the respective values of 0.44, 0.36 and 0.32. The highest posterior probability for Oil is the Silver volatility with a value of 0.72, followed by the Gaz volatility (0.46) and Palladium volatility (0.38). About the Palladium, the highest posterior probability is the Copper volatility with 0.42 and Gold volatility with 0.32. The highest posterior probability for Silver is the Gold volatility with 0.46. At the end, the highest posterior probability for Zinc is the Silver volatility with 0.44, followed by Copper volatility (0.38) and Gaz volatility (0.32).

Now, based on the posterior probabilities of at least 50%, we can summarize the relation between commodities with the Multivariate instantaneous by the following relation:

**Figure 9- Multivariate instantaneous structure, representing instantaneous volatility spill-over effects between markets.**

- Zinc → Copper
- Silver → Oil
Figure 10-Granger causal network extracted by using BGVAR model on the sample from 22/01/07-15/05/17 of multivariate instantaneous
After the multivariate instantaneous, let’s turn our attention on the results of multivariate autoregressive structure.

These results are summarize in the following Table 3. We consider always our 8 commodities.

<table>
<thead>
<tr>
<th></th>
<th>Copper</th>
<th>Electricity</th>
<th>Gaz</th>
<th>Gold</th>
<th>Oil</th>
<th>Palladium</th>
<th>Silver</th>
<th>Zinc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper t-1</td>
<td>0.14</td>
<td>0.32</td>
<td>0.28</td>
<td>0.06</td>
<td>0.88</td>
<td>0.38</td>
<td>0.45</td>
<td>0.34</td>
</tr>
<tr>
<td>Electricity t-1</td>
<td>0.24</td>
<td><strong>0.9</strong></td>
<td><strong>0.74</strong></td>
<td>0.44</td>
<td>0</td>
<td>0.18</td>
<td>0.48</td>
<td>0</td>
</tr>
<tr>
<td>Gaz t-1</td>
<td>0</td>
<td><strong>0.84</strong></td>
<td>1</td>
<td>0.12</td>
<td>0.04</td>
<td><strong>0.48</strong></td>
<td>0.04</td>
<td>0.46</td>
</tr>
<tr>
<td>Gold t-1</td>
<td>0.38</td>
<td>0.22</td>
<td>0.45</td>
<td><strong>0.66</strong></td>
<td>0.12</td>
<td>0.22</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>Oil t-1</td>
<td>0.1</td>
<td>0.2</td>
<td><strong>0.34</strong></td>
<td>0.1</td>
<td>0.3</td>
<td>0</td>
<td><strong>0.92</strong></td>
<td>0.38</td>
</tr>
<tr>
<td>Palladium t-1</td>
<td>0.04</td>
<td>0</td>
<td>0.2</td>
<td>0.02</td>
<td>0.28</td>
<td><strong>0.96</strong></td>
<td>0.42</td>
<td>0.35</td>
</tr>
<tr>
<td>Silver t-1</td>
<td>0.12</td>
<td>0.24</td>
<td>0.14</td>
<td>1</td>
<td><strong>0.88</strong></td>
<td>0</td>
<td>0.1</td>
<td><strong>0.58</strong></td>
</tr>
<tr>
<td>Zinc t-1</td>
<td>0.16</td>
<td>0.14</td>
<td>0.46</td>
<td>0.42</td>
<td>0.48</td>
<td>0.02</td>
<td><strong>0.7</strong></td>
<td>0.1</td>
</tr>
</tbody>
</table>

In this table, always on line with Ahelegbey, Billio et Casarin (2016), in this table, based on the posterior probabilities greater than 50%, the bold entries represent here the selected edges of the multivariate autoregressive (MAR). Therefore, the italic entries represent the posterior probabilities between 30% and 50%.

Then, we notice that, the posterior probability of Copper does not depends of any commodities; even if the lagged value of Gold volatility is 0.38 (but less than 50%). While, the posterior probability of Electricity depends on its lagged value, and on the lagged value of the Gaz volatility. Indeed, the highest posterior probability of Electricity is its lagged value with 0.9, followed by lagged value of Gaz volatility with 0.84, followed by the previous value of copper volatility of 0.32.
Also, the current level of Gaz volatility depends on its previous level with a posterior probability of 1; followed by the lagged value of Electricity volatility (0.74) followed by the previous value of Zinc volatility 0.46.

The highest posterior probability of Gold is its previous value of 0.66 and the previous value of Silver volatility 1.

The current level of Oil volatility depends of the lagged value of Copper volatility with 0.88 and the previous value of Silver volatility 0.88.

Palladium volatility depends only on its previous value of volatility (0.96).

Therefore, we notice that the current level of Silver volatility depends on the lagged value of Oil volatility of 0.92; followed by the previous value of Zinc volatility with 0.7, also followed by Electricity volatility with 0.48 and the previous value of Copper volatility 0.45.

While, the current level of Zinc volatility depends on the lagged value of Silver 0.58 and then followed by the previous value of Gaz volatility with 0.38 and Oil volatility with a value of 0.35.

Based on the posterior probabilities of at least 50%, we can summarize the relation between commodities with the Multivariate Instantaneous by the following relation:

**Figure 11- Multivariate autoregressive structure, representing lagged volatility spill-over effects between markets.**
With our both granger causality, we noticed that some of commodities have not any direct relationship with other commodities. And, there are some direct relationships which are not reciprocal.
Now let’s consider the Network density of our estimation. Density allows us to measure the network cohesion that is the number of existing connections over all possible number of connection. To do this, we consider the rolling window estimation using a sample of 22 January 2007 to 15 May 2017 and a 60 day rolling estimation such that 1200 simulation.

Then, the following figure 12 shows the evolution of Network density for our sample.

**Figure 13- Network density**

Start by 0.3 the network density oscillate between 0.1 to 0.6. And when the value is close to 0.6, it means the network is denser and the more cohesive are the nodes in the network. This can be observe in 2011. While, the low value or close to 0 means the Network is not dense and low cohesive are the nodes in the network. This peak maybe due to the different problems faced by oil producing countries. For example, the conflicts in the countries of North Africa and North Europe between the Russia federation and Ukraine.
6-Conclusion

This paper investigates the commodities volatility contagion. To achieve our goal, we use the newly Bayesian Graphical Vector Autoregressive (BGVAR) model developed by Ahelegbey D. F., Billio, M. and Casarin, R. (2016). With this model, we analyze both multivariate instantaneous and autoregressive structures. Our main findings can be summarized as follows. Using daily price of 8 commodities over the period of from 22 January 2007 to 15 May 2017, both of Multivariate instantaneous and autoregressive structure show some relations between commodities. While, we noticed that some of commodities have not any direct relationship with other commodities. This implies that the predictability of volatility differ from commodities. First, about the analysis of multivariate instantaneous structure, we have only two relationships. These relation is between Copper and Zinc and Oil and Silver. Other commodities haven’t relationships because we consider only the posterior probabilities at least equal to 50%. These results are represented by a Granger-causal network and a network density indicator extracted using the BGVAR support this result. Second, the analysis of multivariate autoregressive structure shows more relation between commodities. Indeed, the results reveal that there a link between Electricity volatility and Gaz, between Oil and Silver and Silver and Zinc. These results confirm the first literature on the link between theses commodities especially on the connection between energy and crude oil. Also, here we confirm this findings by the Granger causal network. Third, we analyze the network density of our sample. We conclude that as we have a difference between predictability of commodities volatility, then the network density range from 0.1 and 0.6. If the Network density is close to 0.1, then we can say that it no dense. This means that there is a weak relationship between commodities at this period. While, in 2011, network is close to 0.6, then there exist a strong connection between commodities. Indeed, we observe in 2011 a peak of Network density than other period. This can be explain by an increase of many main commodities prices. Indeed, at this period, there was the worries about the political upheavals in some countries; specially producing countries of crude oil. For example the case of North Africa countries as Libya, Egypt and Syria’s conflict. Also the civil war in Ukraine which implied the Russia federation in 2013. Due to these instabilities, Oil price was increased and as we know, crude oil is one of main commodity in the world. Then this situation has caused for other commodities prices a spiral of increase. With a new model, this paper contribute to a literature on the link between commodities volatility and our results can be used both by economists and policymakers.
Bibliography


