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A Markov-Switching Model for Bubbles Detection in the Stock Market

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Abstract

In this study, I propose a model for the behaviour of the real stock market prices that allows for the existence of speculative bubbles. The bubble is assumed to follow a Markov-switching process with explosive and collapsing regimes. Inference on the model is performed by using the deviations of the log prices from fundamentals. The fundamental prices are assumed to be a function of the discounted future dividends. Data used in the estimation includes stock market index S&P 500, and 17 of its constituents, public US companies with recorded data at least since the 80s. The results corroborate the hypothesis of regime switches in most of the series. A combination of the information on bubble dynamics and their correlation is used to measure the level of systemic risk in the market.

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Introduction

Motivation

A model for decomposing a stock price series into fundamental and bubble components is presented in this thesis. The bubble is assumed to be periodically collapsing, so it is assumed to follow a Markov-switching process. A plethora of economic literature in the 20th century covered the phenomenon of financial bubbles and the subject only became more relevant in the 21st century after the harmful impacts of bursting bubbles in financial, exchange and real estate markets all over the world. "The implosion of the asset price bubble in Japan led to the massive failure of a large number of banks and other types of financial firms and more than a decade of sluggish economic growth. The implosion of the asset price bubble in Thailand triggered the contagion effect and led to sharp declines in stock prices throughout the region" (Kindleberger and Aliber, 2005). The latest financial crisis of 2008, triggered by the crash in the subprime mortgage market, not only pulled down many sectors of U.S. economy but also spread around the globe.

The formal definition of the bubble may vary in works of different economists. However, "the common attribute of the bubble is that asset or output prices increase at a rate that is greater than can be explained by market fundamentals" (Meltzer, 2003). From the first sight, there is no harm in the accelerating growth of certain markets. It may reflect the anticipation of innovations or regulatory changes in the sector. Yet, market expectations can lead prices to move in a vicious cycle: prices increase because agents have positive expectations and agents have positive expectations because prices keep rising. This nonsustainable pattern inevitably leads to an imploding bubble. For this reason, financial bubbles are studied in the context of financial crashes.

There is plenty of descriptive literature that describes major financial crashes as the result of speculative bubbles. However, there is a lot of disagreements among economists on how to statistically detect and measure them. A big part of the literature defines bubbles as a difference between the real price of an asset and its 'fundamental' value. At the same time, the definition of the fundamental value varies for different assets (stocks, exchange rates, commodities, etc). Even for a specific asset, different researchers can use different formulas to estimate the fundamentals. As a result, "for each paper that finds evidence of bubbles, there is another one that fits the data equally well without allowing for a bubble" (Gürkaynak, 2008). For example, Meese (1986) provides a strong empirical evidence in favor of the fact that the rapid appreciation of US dollar in early 80s was driven by a speculative bubble. In contrast, Wu (1995) finds the opposite results for the same market and time period using the Kalman filtering technique.

In this work I want to test mostly one feature of stock market bubbles, whether they are periodically collapsing. That is why the bubble component is assumed to follow a Markov-switching process with two regimes, exploding and collapsing. The model by Wu (1997) is adopted for estimating the fundamental prices. Deviations of prices from fundamentals is used as a proxy of the bubble component. The autoregressive process with one lag and switching autoregressive coefficient is then estimated in order to obtain a time series of bubble components and detect its regime-switching behaviour. The price and dividend series of S&P 500 index and its constituents is used for estimation. The sample covers the entire 20th century and the beginning of the 21st century, which gives a broad field for analysis. With this approach, I hope to capture the biggest crashes in the US stock market that can be attributed to the emergence and burst of bubbles. In addition, the synchronisation of the estimated bubble components is examined in order to test whether their correlation increases during crises. Finally, the estimated bubble components of the sample of U.S. companies allows to compute the indicator of systemic risk in the equity market, which can improve the analysis of crashes in the financial markets.

Structure of the thesis

The project is organized as follows:
In the **Chapter 1** the literature is reviewed on the methods used to detect and describe the behaviour of bubbles in different markets. **Chapter 2** offers an overview of the theoretical model and econometric techniques used for bubble component estimation. In particular, how the chosen model makes use of the dividend discount model and Campbell and Shiller (1988) approximation. In the next subsection of this chapter the 2-step procedure of extracting the bubble component is developed based on the works by Wu (1997) and Al-Anaswah and Wilfling (2011). The last subsection contains the important technical details for Bayesian statistical inference, such as likelihood functions, priors and posteriors. **Chapter 3** contains the results of the empirical application of the developed model. In the first subsection, the main characteristics of the data are given. In the second subsection, the results of each step of estimation are presented for the main index and its constituents. The co-movements of estimated bubble components are examined in the **Chapter 4**, together with the computed systemic stress indicator. The last chapter concludes.

Chapter 1

Literature review

Researchers have been concerned with developing a methodology for detecting bubbles in asset prices since the late 70s. Authors such as Kindleberger (1978) and Thomas and Morgan-Witts (1979) have listed the periods in history when asset prices in particular industries were thought to be inflated by bubble components and provided an explanation for their emergence and behavior. In the following decades, numerous statistical methods were developed in order to find empirical proofs for the explosive behavior in asset prices. Most of the statistical tests were modelled for stock prices (Blanchard and Watson (1982), Balke and Wohar (2009) , Phillips et al. (2011)), but a number of papers also explored bubbles in the exchange rates (Flood and Garber (1980) , Wu (1995)) and a commodity market (Lammerding et al. (2013)).

In general, the econometric methods for detecting financial bubbles in real market data can be divided into two main groups: stationarity/cointegration tests and decomposition methods. Stationarity/cointegration tests were developed earlier by Flood and Garber (1980), Blanchard and Watson (1982) and Diba and Grossman (1988). However, during the 90s they fell out of use because of critique by Evans (1991), who showed, using simulations, that unit root tests have low power when estimating periodically collapsing bubbles. After his critique a more advanced versions of the test were introduced by Taylor and Peel (1998), Hall et al. (1999) and Psaradakis et al. (2001). The advantage of this approach is that it puts fewer restrictions on the structure of the bubble. However, the downside of this type of test is that the researcher cannot obtain a time series of bubble components, but rather a signal on whether the bubble was present throughout the entire sample. On the contrary, decomposition methods usually require a defined structure of fundamentals and bubble component. Those methods became more popular among researchers because they allow to capture periodically collapsing bubbles and to track the dynamics of the bubble component throughout the sample. Most of the papers use the dividend-price ratio approximation formalized in Campbell and Shiller (1988) to define the fundamental component of prices.

One of the pioneers in their topic, Flood and Garber (1980) tested the price level bubbles in German hyperinflation. Using the monetary model developed in Cagan (1956) they found no evidence of a bubble. However, their testing procedure was built to detect deterministic bubbles only.

Blanchard and Watson (1982) proved that bubbles are consistent with the idea of rational behaviour and that bubbles can be detected statistically using data on prices and dividends. They use the definition of the market fundamentals as a present value of future dividends introduced by Flood and Garber (1980) and perform variance upper bound tests introduced by Shiller (1981) for

the S&P 500 index for years 1871-1979. The null hypothesis used in testing was the absence of the bubble because the authors reasoned that it is an easier task to formulate the test to reject the absence of the bubble rather than looking for proof of its existence. Their tests are based on the assumption that most types of bubbles violate price variance bounds. As a result, the null hypothesis is rejected because of strong evidence for the bubble. Similar reasoning was used by Diba and Grossman (1988). They consider two cases: when the dividends are stationary and when they are integrated of order one. In the first case, the unit root test of differenced prices is performed and evidence for the unit root is the evidence for a bubble. In the second case, cointegration test for prices and dividends is performed and evidence for cointegration is the evidence against an existence of a bubble.

Different tests often gave mutually contradicting results, either due to flaws in methodology or due to the difference in definitions of the price fundamentals and the bubble. For, example results of Meese (1986) found evidence of bubbles using Hausman's specification test for exchange rate series, while Wu (1995) rejected the existence of the bubble after applying Kalman filter for the US dollar, the British pound, the Japanese yen and the Deutsche mark from the same period. Wu (1995) used a 3-step procedure to extract the bubble component:1) Kalman filter using forward recursion and random parameters; 2) Maximum likelihood estimation of parameters; 3) Kalman filter smoothing with estimated parameters. The fundamentals in the model are defined according to a monetary model of exchange rate determination. Results were presented for the entire sample (1974-1988) and subsample (1981-1985). In his further papers Wu (1997) used Kalman filtering techniques for the stock market as well. He derived a new formula for fundamental price differences, which is used in this thesis. As a result, he was able to construct a time series for a bubble component in S&P 500 series.

Phillips et al. (2011) tested the Nasdaq index data (1973-2005) for explosive behavior using recursive regression techniques instead of simple right-tailed unit root test. They claimed this new testing procedure overcomes weaknesses of earlier unit root testing procedures. Unlike the test used by Diba and Grossman (1988), the new procedure was made to detect periodically collapsing bubbles as well. The results provided evidence for the existence of a bubble in 1995. The authors also developed a technique to build confidence intervals for the explosive coefficient and pinpoint the dates of start and collapse of the bubble.

An example of a test for a commodity market, particularly for oil price bubbles, is given in Lammerding et al. (2013). Their approach uses Bayesian MCMC methodology and circumvents the problem mentioned by Blanchard and Watson (1982) of detecting bubbles in commodity markets because of the absence of dividend series. Dividends are approximated as the benefits (convenience yields) the holder of the physical commodity experiences in contrast to the owner of a futures contract written on the respective asset and the usual log price/dividend approximating formula

is used to construct the fundamentals. Price series is decomposed into fundamental and bubble components using state space modelling with Markov Switching. Two regimes of the bubble are assumed: stable and explosive. 4-step Gibbs sampling procedure is performed to separate bubble component and infer the regimes that have taken place each period.

Balke and Wohar (2009) tested S&P 500 index and Nasdaq index for explosive bubbles also using the Bayesian Markov chain Monte Carlo methods. They used log price-dividend approximation put in a state space form and simulated not only the bubble but also the fundamental component. Constant returns throughout the sample were not assumed but rather modelled as a sum of random walk and autoregressive process. The bubble process was modelled to switch between three states: dormant, explosive and collapsing. Their findings showed the high sensitivity of results to the structure assumed for dividend and return component. Therefore, no univocal results on the bubble were obtained.

Finally, Al-Anaswah and Wilfling (2011) used the model developed by Wu (1997) and upgraded it to take into account two switching regimes. They improved the Kalman filtering algorithm in a way that it produces the updated values of the bubble component for both regimes and then collapses those values in one for further steps. The algorithm was tried both on simulated and real market data.

Chapter 2

Model Specification

2.1 Theoretical Model

In this section I present the theory and methodology used for estimation. I used the approach of Wu (1997) for defining the fundamental prices for given data. However, I modified Wu's assumptions about bubble process in the spirit of Evans (1991). The latter simulated the periodically collapsing bubble process to check the power of the tests developed in the 80s because this structure better resembles the real market dynamics. Therefore, I am also assuming that a bubble can switch between both explosive and collapsing regimes, and even that once it has collapsed it can return to the explosive regime after a period of time.

I start by defining the formula for the fundamental price needs to be defined. The widespread core assumption in the bubble literature is the standard linear rational expectations model of the stock price determination:

$$P_t = \frac{1}{1+r} E_t(P_{t+1} + D_t) \tag{2.1}$$

where:
P_t - real stock price at time t;
r - required return rate;
D_t - real dividend paid at time t.

Rearranging terms and taking the natural logarithm of both sides we can obtain an expression for the log of gross returns:

$$\varrho_t = ln(1+r) = ln\left(\frac{E_t(P_{t+1}) + D_t}{P_t}\right) \tag{2.2}$$

According to Campbell and Shiller (1988) this nonlinear relationship can be approximated to:

$$\xi_t = k + \rho \ln(E_t(P_{t+1})) + (1-\rho) ln(D_t) - ln(P_t) \tag{2.3}$$

where:
$\xi_t \simeq \varrho_t$;
ρ - the average ratio of the stock price to the sum of the stock price and the dividend;
$k = -ln(\rho) - (1-\rho)\delta$;
δ - the average log dividend price ratio that can be written as $\delta = ln(\frac{1}{\rho} - 1)$.

Campbell and Shiller (1988) build an approximation in a way that the discount rate (1+r) can vary over time, but Evans (1991) uses the assumption of constant discount rate and argues that bubble solutions can be constructed analogously for both modifications. The same approach is taken in Wu (1997), who derives similar formula for gross returns:

$$q = K + \psi E_t p_{t+1} + (1-\psi)d_t - p_t \tag{2.4}$$

where:
q - the required log gross return rate;
ψ - the equivalent of ρ in (2.3);
$K = -ln(\psi) - (1-\psi)ln(\frac{1}{\psi} - 1)$;
$p_t = ln(P_t)$;
$d_t = ln(D_t)$.

The fundamental price can be obtained by solving equation (2.4) forward:

$$\begin{aligned} p_t &= K - q + \psi E_t p_{t+1} + (1-\psi)d_t \\ &= (1+\psi)(K-q) + \psi^2 E_t p_{t+2} + (1-\psi)\psi E_t d_{t+1} + (1-\psi)d_t \\ &= (1+\psi+\psi^2)(K-q) + \psi^3 E_t p_{t+3} + \sum_{i=0}^{2} E_t d_{t+i} \\ &= \frac{K-q}{1-\psi} + (1-\psi)\sum_{i=0}^{I} \psi^i E_t(d_{t+i}) + \psi^{I+1} E_t p_{t+I+1}. \end{aligned} \tag{2.5}$$

Using the transversality condition,

$$\lim_{I+1\to\infty} \psi^{I+1} E_t p_{t+I+1} = 0 \tag{2.6}$$

the last line of (2.5) shortens down to:

$$p_t^f = \frac{K-q}{1-\psi} + (1-\psi)\sum_{i=0}^{\infty} \psi^i E_t(d_{t+i}) \tag{2.7}$$

Therefore, the fundamental price of the stock is equal to the sum of the discounted expected future dividends. This is usually called 'no bubble' solution and it is only a particular solution to (2.5). The general solution can be expressed as:

$$p_t = \frac{K-q}{1-\psi} + (1-\psi)\sum_{i=0}^{\infty} \psi^i E_t(d_{t+i}) + b_t = p_t^f + b_t \tag{2.8}$$

For rational bubbles b_t satisfies the condition:

$$E_t(b_{t+i}) = \frac{1}{\psi^i} b_t \quad for \quad i = 1, 2, ... \tag{2.9}$$

or simply:

$$b_t = \psi E_t(b_{t+1}) \quad \forall t \in \mathbb{N} \tag{2.10}$$

Wu (1997) assumed the bubble component to grow continuously with the same rate. However, Evans (1991) described a more realistic type of periodically collapsing bubble, known in the literature as Evans' model. He initially used the model to prove inadequacy of unit root and cointegration tests used by Diba and Grossman (1988). However, his model became quite popular among economists, who applied it to real market data in different configurations. It is relevant for this study as well, in a way that the model in this paper also allows for periodically collapsing bubbles. Before going into details on the Markov-switching model used for estimation in this work, it is worth taking a look at Evans' model, its theoretical advantages and disadvantages that arise when it is applied to the real world data. Evans himself formulated the bubble's behaviour in the following equation:

$$B_{t+1} = (1+r)B_t u_{t+1} \quad \text{if} \quad B_t \leq \alpha \tag{2.11}$$

$$B_{t+1} = \left[\delta + \pi^{-1}(1+r)\theta_{t+1}(B_t - (1+r)^{-1}\delta)\right] u_{t+1} \quad \text{if} \quad B_t > \alpha \tag{2.12}$$

where:
σ, α - positive threshold parameters and $0 < \sigma < (1+r)\alpha$;
u_{t+1} - exogenous iid positive random variable with $E_t u_{t+1} = 1$;
θ_{t+1} - exogenous independently and identically distributed Bernoulli process, which is independent of u_t; θ takes the value of 1 with probability π and value 0 with probability $1-\pi$.

Therefore, the bubble has two regimes. In the first regime it is growing with the constant rate $1+r$. In the second regime the bubble grows at an exceeding rate $(1+r)\pi^{-1}$, but with probability $1-\pi$ it can collapse to the value of σ. The threshold value which defines the transition between regimes is α.

There are several advantages and downsides of Evans' model. First of all, this bubble structure is closer to reality, imitating booms and crashes happening in the financial markets. Secondly, despite a more complex structure, the bubble still satisfies (2.9). This rules out arbitrage opportunities and assumes a "rational" bubble, (Gürkaynak, 2008). Among the downsides, the given structure assumes only positive or negative bubbles, depending entirely on the sign of the initial condition B_0 of the bubble component, Payne and Waters (2005). In addition, crashes assumed in the model are unrealistically rapid and don't allow for recession throughout multiple periods, which is one of the reasons why the autoregressive model is estimated in this work instead.
Evan's model was mostly applied for simulations, so it has a simple definition of fundamental price:

$$P_t^f = \mu(1+r)r^{-2} + r^{-1}d_t \tag{2.13}$$

This definition is based on the assumption that log dividends follow a random walk process:

$$d_t = \mu + d_{t-1} + \epsilon_t \tag{2.14}$$

When applied to the real data (index S&P 500, 1871-2016), this definition of fundamentals appears to be oversimplified, as the real dividend process is much smoother than random walk. In addition, there is uncertainty about the discounting coefficient. When the value $r = 0.05$ is accepted as in Evans, then fundamental price constitutes a fairly small ratio of the real price. On the contrary, when the value closer to the average returns over the sample is used $r = 0.0027$, fundamental price exceeds the real price for most of the sample. Therefore, we return to the definition of fundamentals presented by Wu (1997).

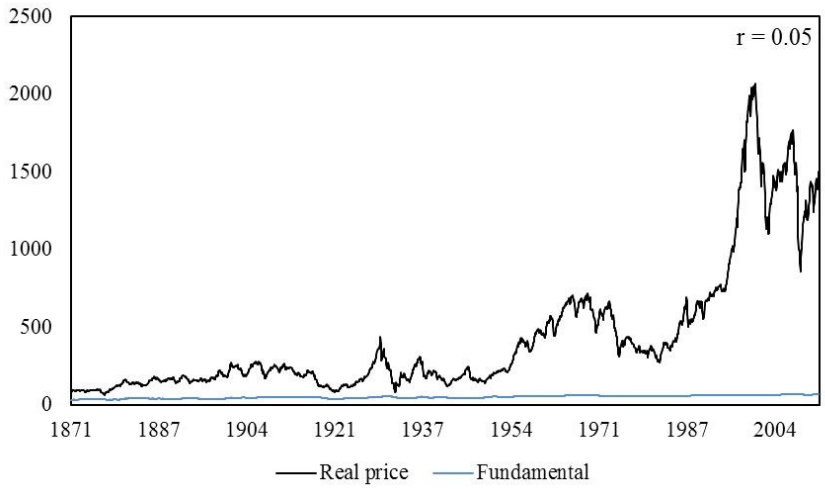


Figure 2.1: Fundamental price vs. real price with r=0.05

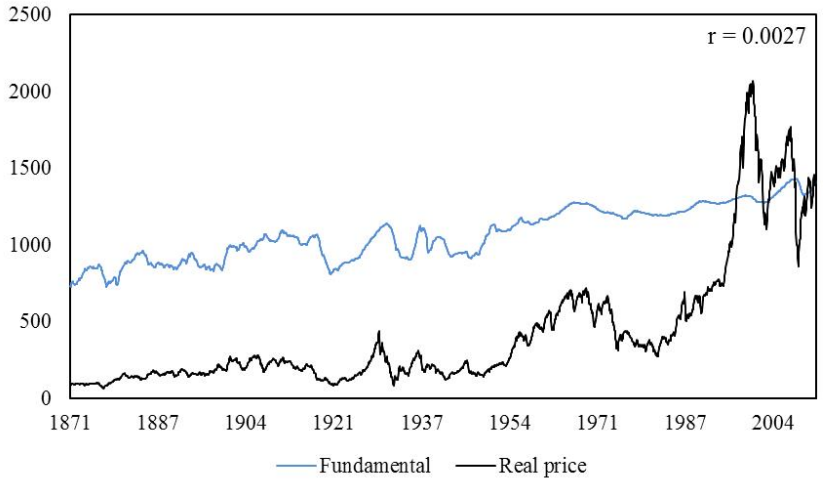


Figure 2.2: Fundamental price vs. real price with r=0.0027

2.2 Methodology

Let's recall the equation (2.7) and express it in a matrix form. Wu (1997) used this technique to construct Kalman filter, but it is also helpful for this study, since it eases computations:

$$p_t^f = \frac{k-q}{1-\psi} + (1-\psi)\sum_{i=0}^{\infty}\psi^i E_t(d_{t+i})$$

Log dividend process is usually not stationary, so the model in a difference form is used:

$$\triangle\, p_t = (1-\psi)\sum_{i=0}^{\infty}\psi^i[E_t(d_{t+i}) - E_{t-1}(d_{t+i-1})] + \triangle b_t = \triangle p_t^f + \triangle b_t \tag{2.15}$$

where $\triangle$ is the backward difference operator, i.e. $\triangle p_t = p_t - p_{t-1}$.
The log dividend process can be approximated as an ARIMA (h,1,0) process:

$$\triangle\, d_t = \mu + \sum_{j=1}^{h}\varphi_j \triangle\, d_{t-j} + \delta_t \tag{2.16}$$

where $\delta_t \overset{iid}{\sim} N(0, \sigma_\delta^2)$. So, in matrix notation:

$$\triangle\, p_t = \triangle d_t + M \triangle\, Y_t + \triangle b_t \tag{2.17}$$

where $Y_t = (\triangle d_t, \triangle d_{t-1}, ..., \triangle d_{t-h+1})'$. Defining matrix M is more complex:

$$M = gA(I-A)^{-1}[I - (1-\psi)(I-\psi A)^{-1}] \tag{2.18}$$

where $g = (1, 0, 0, ..., 0)$ is an h-row vector, I is an h×h identity matrix, A is an h×h matrix:

$$A = \begin{bmatrix} \varphi_1 & \varphi_2 & ... & \varphi_{h-1} & \varphi_h \\ 1 & 0 & ... & 0 & 0 \\ 0 & 1 & ... & 0 & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{2.19}$$

The optimal amount of lags for each sample is estimated using AIC (Akaike information criterion) criterion. Autoregressive coefficients for the log dividend process are estimated following a Bayesian approach. The inference is described in the following subsection. The next step is to construct fundamental log prices from differences in fundamental log prices:

$$\begin{cases} p_1^f = p_0 + \triangle p_1^f \\ ... \\ p_t^f = p_{t-1}^f + \triangle p_t^f \end{cases}$$

The obvious candidate for a bubble is the deviation between real and fundamental prices. However, one must not forget that the bubble should fall under assumption (2.9). Moreover, in this work, in order to capture the periodically collapsing nature of the bubble, multiple regimes are assumed. Taking into account the study of Al-Anaswah and Wilfling (2011) I first assume two distinct regimes: exploding and collapsing. In order to extract the bubble component I assume that it follows an autoregressive process with Markov-switching.

$$b_{t+1} = F_i b_t + \eta_t \quad for \quad i = 1, 2 \tag{2.20}$$

where:
F_1 - coefficient in the explosive regime;
F_2 - coefficient in the collapsing regime;
$\eta_t \sim N(0, \sigma_\eta^2)$ - iid white noise.

Solely the autoregressive coefficient is assumed to switch between regimes. In addition, some constraints are put on the autoregressive coefficients. The process in the explosive regime is assumed to be nonstationary, so the explosive coefficient is constrained to be > 1 in the inference part. The process in the collapsing regime is assumed to be stationary, so the collapsing coefficient is constrained to be < 1 in the absolute value in the inference part. The logic behind this is that in the exploding phase the bubble component should exceed its previous absolute value, whether the bubble is positive or negative. In contrast, in the collapsing phase the bubble component should shrink relative to its previous absolute value.

2.3 Bayesian Inference

2.3.1 The dividend process

In this thesis, I used Gibbs sampling to estimate the autoregressive coefficients in differences of log dividends process: $\triangle d_t = \mu + \Sigma_{j=1}^{h} \varphi_j \triangle d_{t-j} + \delta_t$. This equation can be expressed in terms of a linear regression:

$$Y_t = X_t B + \upsilon_t \quad \upsilon_t \sim N(0, \sigma_\upsilon^2) \tag{2.21}$$

where:
Y_t is a dependent variable $\triangle d_t$;
B is a $(h+1) \times 1$ vector of autoregressive parameters $[\mu \quad \varphi_1 \quad \varphi_2 \quad ... \quad \varphi_h]'$;
X_t is a $1 \times (h+1)$ vector of independent variables $[1 \quad \triangle d_{t-1} \quad \triangle d_{t-2} \quad ... \quad \triangle d_{t-h}]$.

Therefore, the autoregressive process can be written in matrix form:

$$Y = XB + \upsilon \tag{2.22}$$

$$\Downarrow$$

$$\begin{bmatrix} \triangle d_0 \\ ... \\ \triangle d_t \\ ... \\ \triangle d_T \end{bmatrix} = \begin{bmatrix} 1 & \triangle d_{-1} & \triangle d_{-2} & ... & \triangle d_{-h} \\ ... & ... & ... & ... & ... \\ 1 & \triangle d_{t-1} & \triangle d_{t-2} & ... & \triangle d_{t-h} \\ ... & ... & ... & ... & ... \\ 1 & \triangle d_{T-1} & \triangle d_{T-2} & ... & \triangle d_{T-h} \end{bmatrix} \begin{bmatrix} \mu \\ \varphi_1 \\ \varphi_2 \\ ... \\ \varphi_h \end{bmatrix} + \begin{bmatrix} \upsilon_0 \\ ... \\ \upsilon_t \\ ... \\ \upsilon_T \end{bmatrix} \tag{2.23}$$

The goal is to estimate the vector of parameters B and σ_υ^2. I use the guidelines from the Blake et al. (2012) to write down the likelihood function, choose conjugate priors and derive posteriors.

The likelihood function:

$$\mathbb{L}(Y_t \mid B, \sigma_\upsilon^2) = (2\pi\sigma_\upsilon^2)^{\frac{-T}{2}} \exp\left(-\frac{(Y_t - X_t B)'(Y_t - X_t B)}{2\sigma_\upsilon^2}\right) \tag{2.24}$$

The parameters to estimate are $B = [\mu \quad \varphi_1 \quad \varphi_2 \quad ... \quad \varphi_h]'$ and σ_υ^2, conjugate priors for which are the multivariate normal and inverse Gamma respectively:

$$p(B) \sim N(B_0, \Sigma_0) \quad \text{and} \quad p\left(\frac{1}{\sigma_\upsilon^2}\right) \sim \Gamma\left(\frac{T_0}{2}, \frac{\theta_0}{2}\right) \tag{2.25}$$

where B_0 is a $(h+1) \times 1$ vector and Σ_0 is a diagonal $(h+1) \times (h+1)$ variance-covariance matrix. For example:

$$B_0 = [0 \quad 0 \quad ... \quad 0]' \tag{2.26}$$

$$\Sigma_0 = \begin{bmatrix} \gamma_{\mu\mu} & 0 & ... & 0 \\ 0 & \gamma_{11} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \gamma_{hh} \end{bmatrix} \tag{2.27}$$

The posterior distribution of B conditioning on the sample of observations and σ_υ^2 is the following:

$$P(B \mid \sigma_\upsilon^2, Y_t) \sim N(M^*, V^*) \tag{2.28}$$

where:

$$M^* = (\Sigma_0 + \frac{1}{\sigma_\upsilon^2} X_t' X_t)^{-1} (\Sigma_0^{-1} B_0 + \frac{1}{\sigma_\upsilon^2} X_t' Y_t) \tag{2.29}$$

$$V^* = (\Sigma_0 + \frac{1}{\sigma_\upsilon^2} X_t' X_t)^{-1} \tag{2.30}$$

The posterior distribution of σ_v^2 conditioning on the sample of observations and B is the following:

$$P\left(\frac{1}{\sigma_v^2} \mid B, Y_t\right) \sim \Gamma\left(\frac{T_1}{2}, \frac{\theta_1}{2}\right) \tag{2.31}$$

where:

$$T_1 = \frac{T_0 + T}{2} \quad \text{and} \quad \theta_1 = \frac{\theta_0 + (Y_t - X_t B)'(Y_t - X_t B)}{2} \tag{2.32}$$

The goal is to approximate marginal and joint distributions by sampling from the conditional posterior distributions. The logic behind this is that marginal posterior distributions may be difficult to derive analytically. In contrast, the conditional posterior distribution of each set of parameters are usually easier to derive, Blake et al. (2012). Applied to the case described above Gibbs sampling algorithm can be summarised in a few steps:

1) Set starting values for parameters: $B^{(0)}$ and $\sigma_v^{2(0)}$. Starting values can be random for a simple algorithm . However, as computations get more complex, it is better to take OLS estimates as initial values.

2) Draw $B^{(1)}$ from the posterior conditional on the initial value $\sigma_v^{2(0)}$ and the sample of observations.

3) Draw $\sigma_v^{2(1)}$ from the posterior conditional on the value of $B^{(1)}$, obtained from the previous step, and the sample of observations.

This completes the first iteration of Gibbs sampling. As the number of Gibbs iterations increases to infinity, the samples of draws from the conditional distributions converge to the joint and marginal distributions of parameters at an exponential rate (for a proof of convergence see (Casella and George, 1992). Therefore, after a large enough number of iterations, the marginal distributions can be approximated by the empirical distribution of B and σ_v^2.

2.3.2 The Markov-switching process

Once the autoregressive coefficients are obtained, the fundamental prices can be obtained using equations (2.17 - 2.20). The deviations of log real prices from the fundamentals is a natural candidate for the bubble. However, in order to satisfy condition (2.9) and separate the bubble from a statistical noise, the following autoregressive process needs to be estimated:

$$B_t = F_{S_t} B_{t-1} + \eta_t \quad \eta_t \sim N(0, \sigma_\eta^2) \tag{2.33}$$

where:

S_t is an unobserved regime (latent or state) variable;

B_t is a deviation of the log real price from fundamental price at time t[1].

[1] in further estimation $B_t - \bar{B}$ instead of just B_t.

Taking into account the work of Evans (1991) and Al-Anaswah and Wilfling (2011), I assumed that the autoregressive coefficient F can switch between two regimes: exploding and collapsing. Therefore, the latent variable S_t takes values 1 and 2 respectively; F_{S_t} take values F_1 and F_2.

Gibbs sampling with latent variable is a more complicated procedure than Gibbs sampling for simple regression. In this case, in order to estimate autoregressive parameters F_1, F_2 and variance σ_η^2 I followed the methods described in Billio et al. (2007). In order to estimate the state variable S_t I applied the FFBS (Forward-Filtering-Backward-Samping) algorithm described in Frühwirth-Schnatter (2006), see also Appendix D. Likelihood, priors and posteriors are given further for a set-up with two regimes, the case with more regimes is analogous. I skip the detailed derivations, since the examples are presented in the literature mentioned above.
The likelihood function:

$$\mathbb{L}(B_t \mid F_1, F_2, \sigma_\eta^2, p_{11}, p_{22}) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_\eta^2}} \exp\left[-\frac{1}{2}(B_t - F_1 B_{t-1}\mathbb{1}_{S_t=1} - F_2 B_{t-1}\mathbb{1}_{S_t=2})^2\right]$$
$$\times\, p_{11}^{S_t S_{t-1}}(1-p_{11})^{(1-S_{t-1})S_{t-1}} p_{22}^{(1-S_t)(1-S_{t-1})}(1-p_{22})^{S_t(1-S_{t-1})} \tag{2.34}$$

where:
p_{11}, p_{22} - transition probabilities;
$\mathbb{1}_{S_t=i}$ - the indicator function that gives value 1 if $S_t = i$ and 0 otherwise.

The following conjugate priors are chosen, where $\mu_1, \gamma_1, \mu_2, \gamma_2, a, b, \alpha_1, \alpha_2$ are hyperparameters [2]:

$$\begin{aligned} F_1 &\sim N(\mu_1, \gamma_1) \\ F_2 &\sim N(\mu_2, \gamma_2) \\ \sigma_\eta^{-2} &\sim Ga(a, b) \end{aligned} \tag{2.35}$$

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \begin{matrix} \sim Dir(\alpha_1) \\ \sim Dir(\alpha_2) \end{matrix} \quad \text{where} \quad \begin{matrix} \alpha_1 = [\alpha_{11}, \alpha_{12}] \\ \alpha_2 = [\alpha_{21}, \alpha_{22}] \end{matrix} \tag{2.36}$$

Using Bayes' theorem, the following posteriors are derived:

$$\begin{aligned} F_1 &\sim N(\bar{\mu}_1, \bar{\gamma}_1) \\ F_2 &\sim N(\bar{\mu}_2, \bar{\gamma}_2) \\ \sigma_\eta^{-2} &\sim Ga(\bar{a}, \bar{b}) \end{aligned} \tag{2.37}$$

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \begin{matrix} \sim Dir(\bar{\alpha}_1) \\ \sim Dir(\bar{\alpha}_2) \end{matrix} \quad \text{where} \quad \begin{matrix} \bar{\alpha}_1 = [\bar{\alpha}_{11}, \bar{\alpha}_{12}] \\ \bar{\alpha}_2 = [\bar{\alpha}_{21}, \bar{\alpha}_{22}] \end{matrix} \tag{2.38}$$

[2] In the first iteration the following initial hyperparameters were chosen: $\mu_1 = 2$, $\mu_2 = 0.5$, $\gamma_1 = \gamma_2 = 0.5$, $a = b = 1$, $\alpha_1 = \alpha_2 = [1, 1]$.

$$\bar{\gamma}_i^2 = \left(\frac{\sum_{t \in \tau_i} B_{t-1}^2}{\sigma_\eta^2} + \frac{1}{\gamma_i^2} \right)^{-1} \quad i = 1, 2 \tag{2.39}$$

$$\bar{\mu}_i = \bar{\gamma}_i^2 \left(\frac{\sum_{t \in \tau_i} B_t B_{t-1}}{\sigma_\eta^2} + \frac{\mu_i}{\gamma_i^2} \right) \quad i = 1, 2 \tag{2.40}$$

$$\bar{a} = \frac{T}{2} + a - 1 \tag{2.41}$$

$$\bar{b} = b + \frac{1}{2}(\sum_{t \in tau_1} (B_t - F_1 B_{t-1})^2 + \sum_{t \in tau_2} (B_t - F_2 B_{t-1})^2) \tag{2.42}$$

$$\bar{\alpha}_{ij} = \alpha_{ij} + Tr_{ij} \quad i, j = 1, 2 \tag{2.43}$$

where Tr is a 2×2 matrix that contains of number of transitions from regime i to regime j.

Chapter 3

Bubbles in the U.S. market

3.1 Data description

The data employed in this thesis includes the price and dividend series for the S&P 500 index and its constituents, U.S. companies, listed on the NYSE or NASDAQ with long recorded price history. The data on the S&P 500 index were taken from the website of Open knowledge International, which was collected by Shiller (2016). The data for US companies are taken from the historical data archive of Yahoo Finance. All the data are monthly observations. The dividend series for US companies initially contained quarterly data, but were transformed to monthly (divided by 3). Since the natural logs of dividends are used in calculations, for the rare cases of zero dividends I substituted those observations with the moving average of previous 3 periods. This way the problem of having -Infinity in calculations is eliminated. Initially, I substituted zeros in the dividend data with a very small constant 0.00001 but those values distorted the log dividend process too much. All the data used were transformed to real values using CPI index provided by the Bureau of Labor Statistics.

The biggest sample, for the S&P 500, covers the period from January of 1871 until March of 2016 and consists of 1743 observations. This particular index was chosen because of its long documented history, which allows to test the model throughout all the booms and crashes of the US financial market since the late 19th century until the early 21st century. Some of the significant events that I want to be able to detect in these prices and dividend series are: the Wall Street Crash of 1929 and the following recession, Black Monday of 1987, Dot-com bubble crash of 2000 and the Financial crisis of 2008.

The samples for S&P constituents are significantly smaller. The shortest sample, for NextEra Energy Inc., includes observations for the period from July of 1983 until March of 2017. In order to estimate the coordination of bubbles the sample is shortened to the size of 393 observations, from July of 1983 till March of 2016. In total, 18 companies from various industries were chosen for analysis. The distribution of industries is shown in the Figure 3.1.

In the first chapter, the model is specified under the assumption of log dividend series being non stationary, but the first differences of the log dividends being stationary. I applied the Augmented Dickey -Fuller test and Phillips Perron test to check if this assumption holds. The resulting p-values are shown in Table 4.1 in the Appendix A. It can be concluded that none of the differenced data have a unit root both under the 5% and 10% confidence intervals. However, log dividend series of

companies such as Abbott Laboratories, American Express Company and Center Point Energy Inc were initially stationary. The Williams Companies Inc. series was initially stationary under 5% confidence.

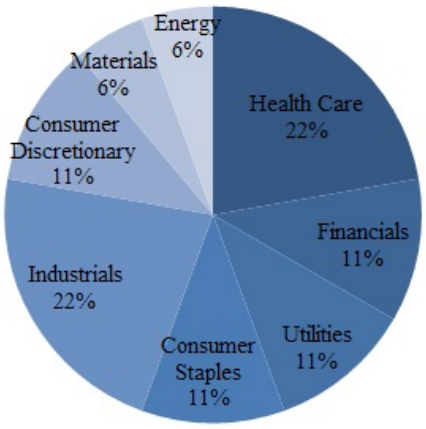


Figure 3.1: Distribution of the industries

Another important aspect to know, before estimating the autoregressive parameters, is the optimal number of lags in the autoregressive log dividend process. I used the Akaike information criteria (AIC) for that purpose. The results for each time series are presented in the Table 3.1. For Aetna Inc. there are no significant lags for dividend process. Therefore, I excluded this company from further estimation. For some companies dividends don't change for long periods of time, so optimal number of lags found with AIC can reach 27. In addition, the transformation from quarterly data to monthly made the dividend series even less volatile. Therefore, I additionally checked the partial autocorrelation function to determine which lags are actually significant. The PACF graphs for each time series are presented in the Table 4.8 in the Appendix A.

Table 3.1: Optimal number of lags according to AIC

	S&P 500	SP 500 (s)	ABT	AET	AXP	CNP	CLX	KO	CSX	DOV	LLY	EMR	GPC	HON	IFF	JNJ	MCD	NEE	WFC	WMB
AIC	5	5	10	0	11	6	12	27	8	24	24	15	12	12	0	27	24	12	25	1

3.2 Empirical Results

In this section the results of all steps of bubble component estimation are summarized. I will go into details on the results for the S&P 500 index, since it has the biggest sample and was discussed before in the related literature. The results for other indices and stocks are discussed briefly and most graphs are displayed in the Appendices.

3.2.1 Results for S&P 500

The results of Gibbs sampling for the S&P 500 log dividend process are presented in the Table 3.2, while for the rest of the data, the results are summarised in the Table 4.2 and 4.3 in Appendix B. For all the data σ_δ is significantly different from zero at the 5% confidence level. The autoregressive coefficients appear to be less clear-cut. For example, for the S&P 500 we cannot reject the null hypothesis $\varphi = 0$ for lags from 3 to 5. However, the results coincide with the information provided by PACF graphs in the Appendix A.

Table 3.2: Results of Gibbs sampling for dividend process

	μ	φ_1	φ_2	φ_3	φ_4	φ_5	σ_δ
SP 500	0.0004	0.4250	0.1387	0.0230	0.0557	0.0379	0,0141
	(0.00034)	(0.0295)	(0.0312)	(0.0308)	(0.0304)	(0.0276)	(0.00024)

Based on the dividend process it is possible to estimate fundamental prices and deviations of real prices from the fundamentals. Figure 3.2 illustrates the dynamics of log real prices and fundamentals for the S&P 500 index. These two price series often cross, creating positive and negative deviations of different magnitude. For the S&P 500, deviations often look like an overreaction of the market to the changes in fundamentals. For example, the positive deviations appear after the periods of growth in the fundamentals, and the negative deviations appear after market crashes. However, diversion seems to increase steadily at the end of 20th and beginning of the 21st century and index appears to be constantly overvalued.

Figure 3.3 shows the dynamics of the deviation between the log real price and fundamentals. It is expressed as a percentage relative to the log real prices. The biggest negative deviation reaches more than 10% of the log real price. The biggest positive deviation reaches almost 25% of the log real price value.

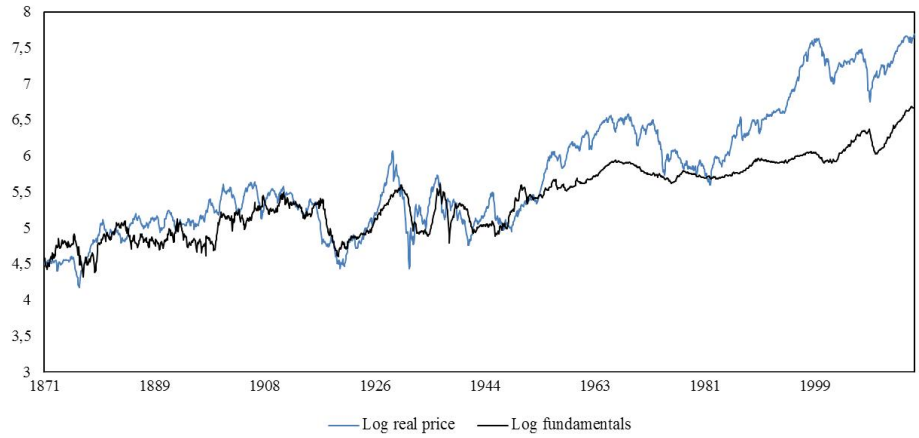


Figure 3.2: Log real price vs. Log fundamental price

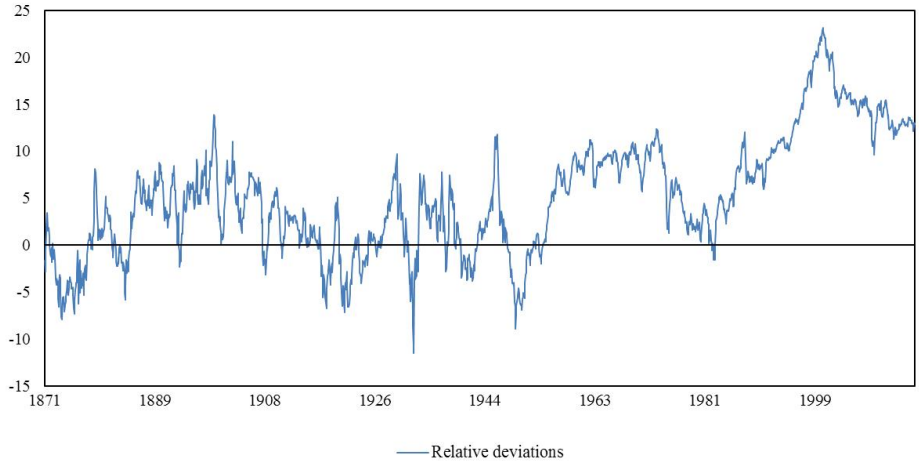
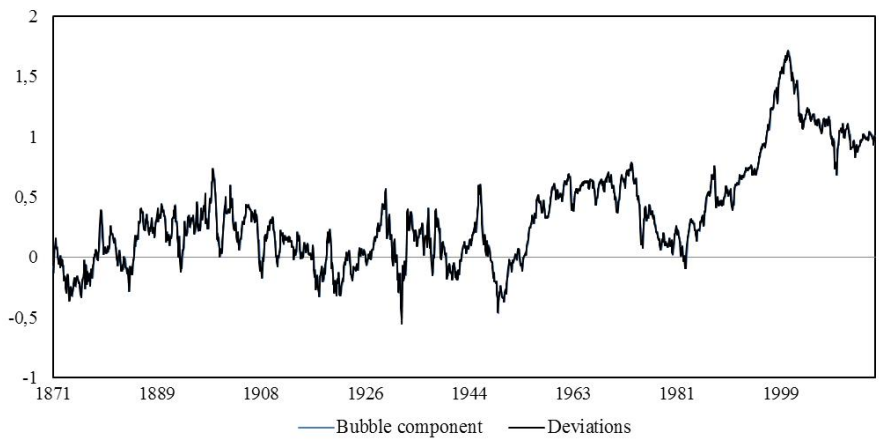


Figure 3.3: Relative deviations from the fundamental price, in %

The next step is to extract the bubble component from the deviations time series. The bubble component is assumed to follow the process described in Chapter 1, which switches between exploding and collapsing regimes. Table 3.3 shows the obtained estimates and standard deviations of the unknown parameters $F_1, F_2, \sigma_\eta^2, p_{11}, p_{22}$ for the S&P series for all the data sets. Both autoregressive parameters are significant and F_2 is also significantly lower than zero. σ_η is significantly different from zero and p_{11}, p_{22} are within the expected range. Figure 3.4 illustrates the bubble component obtained using the estimated parameters along with previously discussed deviations from fundamentals. Zoomed graph is also included, since the lines of deviations and bubble component coincide in the graph of the entire sample.

Table 3.3: AR(1) with Markov-swithcing

	F1	F2	σ_η^2	p_{11}	p_{22}
S&P 500	1.0016	0.5315	0.0593	0.9947	0.5874
	(0.0014)	(0.0709)	(0.0010)	(0.0024)	(0.1974)



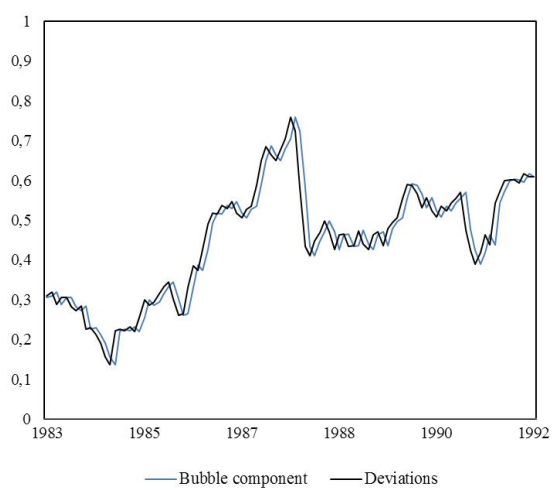


Figure 3.4: Bubble component vs. Deviations from fundamentals

One of the important results of the regime switching model is the estimated vector of states throughout the sample. From this series, we can judge whether the model is able to capture regime switches during significant events in financial market history. The results obtained for the S&P 500 index

show that periods with high probability of collapsing regime coincide with the biggest market crashes, such as the Great Depression, Black Monday, the Financial Crisis of 2008 and even panics on the market in the 19th century. The period of so called Great Moderation, from the mid-1980s till the late-2000s, is marked with a rather long period of zero probability of collapsing regime. However, the latest financial crisis is not as well recognized by this model compared to the related literature. Figure 3.5 shows the dynamics of the probability of bubble being in a collapsing regime with notes on market booms and crashes.

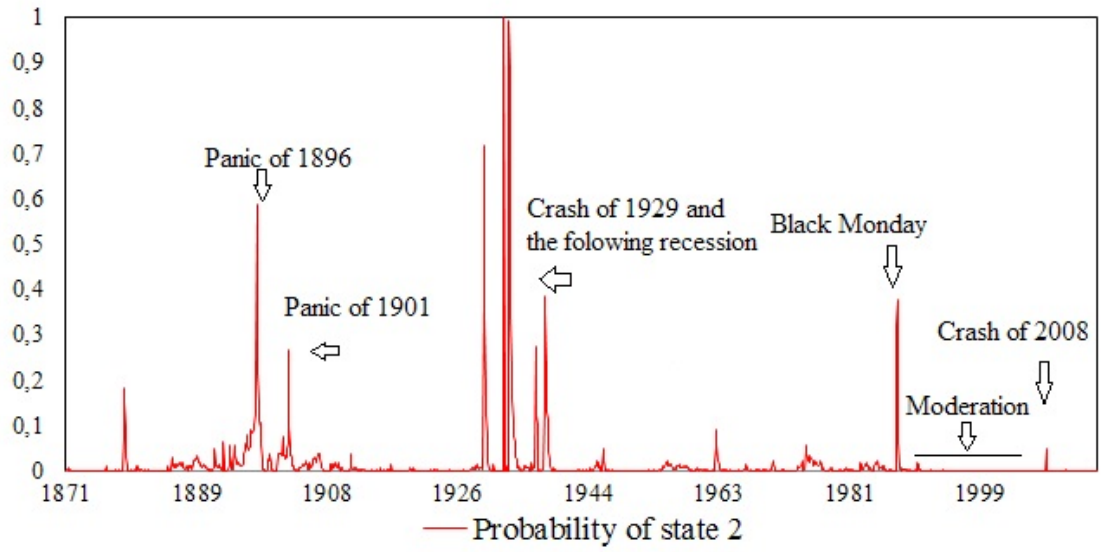


Figure 3.5: Probability of collapsing regime over time

Figure 3.6 illustrates the convergence of autoregressive parameters and variance. For this series, a thousand iterations were enough for all parameters to converge. However, estimation of the S&P 500 constituents required from 5 to 80 times more iterations.

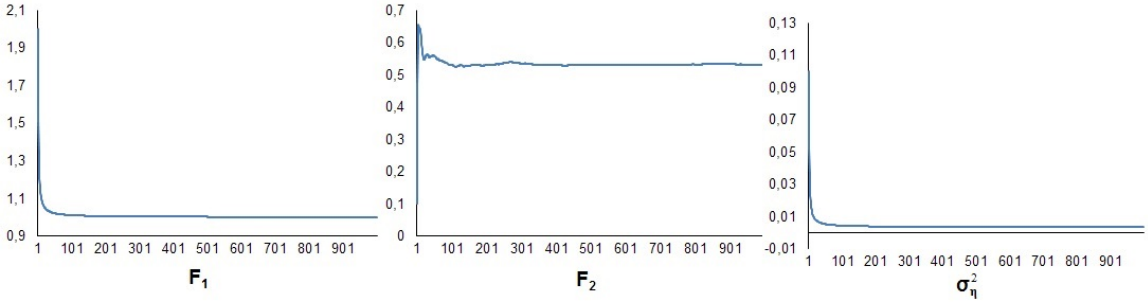


Figure 3.6: Convergence of parameters

The S&P 500 data covers more than 140 years and contains 1743 observations. Therefore, I also applied the model for a subsample covering period from July 1983 till June 2016 in order to check whether the large sample size might be obscuring some regime switches during the 21st century. Particularly, my goal was to check whether the model can give more weight to the Dotcom bubble and the Financial Crisis of 2008 when the smaller sample is estimated. The results of dividend process are shown in Table 3.4. Just the first lag appears to be significant (again, no conflict with the information from PACF graph). Standard deviation is significantly different from zero.

Table 3.4: Results of Gibbs sampling for dividend process S&P 500 (short)

	μ	φ_1	φ_3	φ_4	φ_5	σ_δ
SP 500 (s)	0.000424	0.7112	-0.0183	0.0386	0.0936	0.0166
	(0.00089)	(0.1552)	(0.2183)	(0.2427)	(0.1981)	(0.0006)

Figure 3.7 illustrates dynamics of real prices and fundamentals for the shortened sample of the S&P 500. For most of the sample the index appears to be overvalued. Since our definition of fundamentals is based on dividends paid it means that many companies in the market seem to have set a policy of paying lower dividends or no dividends as a new norm. This trend can be partly explained by the fact that smaller companies (for example, technological start-ups) enter the market with low profitability and strong growth opportunities. However, as shown by Fama and French (2001), regardless of their characteristics, firms have become less likely to pay dividends.

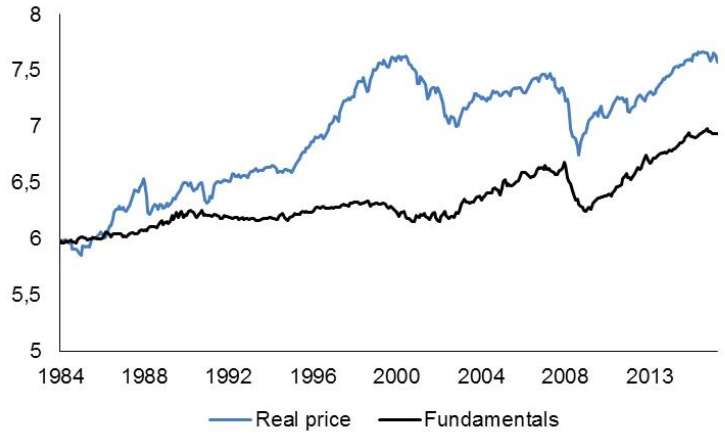


Figure 3.7: Log real price vs. fundamentals for S&P 500 (short)

The estimation results of the bubble process for the shortened sample of the index confirm the previous results. The parameters take 80 times more iterations to converge and estimated vector

of states does not show signs of collapsing bubbles. This may be explained by the fact that, after the crash of 2008, when the prices rapidly went down, dividends also spiralled down shortly after. Therefore, the difference between real prices and fundamentals was not decreasing for long enough to register collapse in a bubble. Estimated parameters are reported in the Table 3.5. The estimated vector of probabilities of collapsing regime is illustrated in Figure 3.8. According to the model, bubble does not switch between regimes in this sample and stays mostly in a collapsing regime.

Table 3.5: AR(1) with Markov-switching for S&P 500 (short)

	F1	F2	σ_δ	p_{11}	p_{22}
SP 500 (s)	1.6890	0.9852	0.0820	0.4468	0.9143
	(0.6611)	(0.0090)	(0.0027)	(0.2830)	(0.2000)

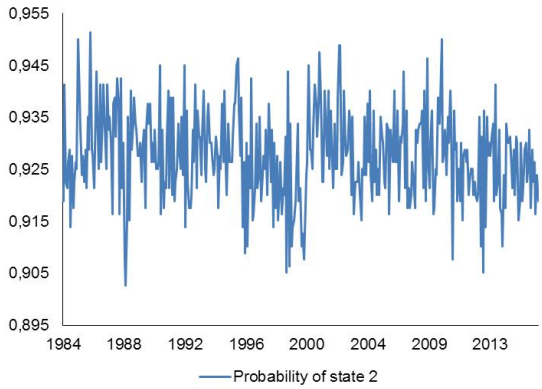


Figure 3.8: Probability of collapsing regime for S&P 500 (short)

3.2.2 Results for U.S. companies

Bubble component estimation is usually performed on American stock indices (mostly S&P 500, Nasdaq in Balke and Wohar (2009) and Phillips et al. (2011)) or regional stock indices, as in Al-Anaswah and Wilfling (2011). In this thesis, I took a different direction and estimated a number of US companies, which are constituents of the previously estimated index S&P 500. This allows further to estimate the coordination of bubbles of different stocks throughout the sample and check whether it increases during crisis.

Firstly, very different patterns can be observed, when estimating the fundamentals of stock prices. Some stocks appear to be overvalued throughout the entire sample, such as The Clorox Company, CSX Corporation and Genuine Parts Company. The dynamics of real price and fundamentals for Genuine Parts Company are illustrated in Figure 3.9.

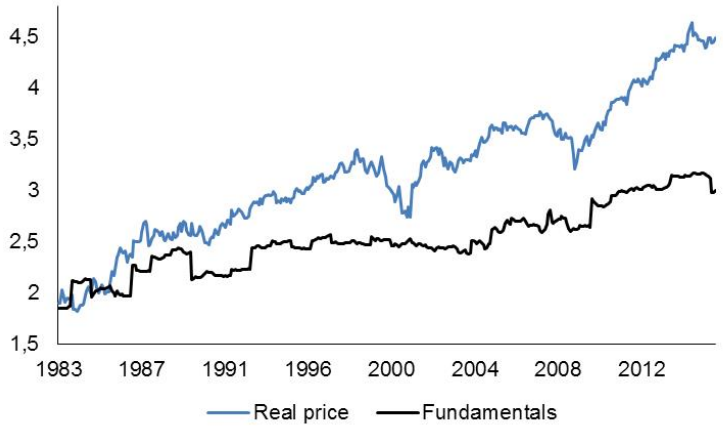


Figure 3.9: Log real price vs. Fundamentals, Genuine Parts Company

In contrast, some stocks, for example Emerson Electric Co. and Johnson&Johnson, appear to be undervalued throughout the entire sample. The dynamics of real price and fundamentals for Johnson&Johnson are illustrated in Figure 3.10.

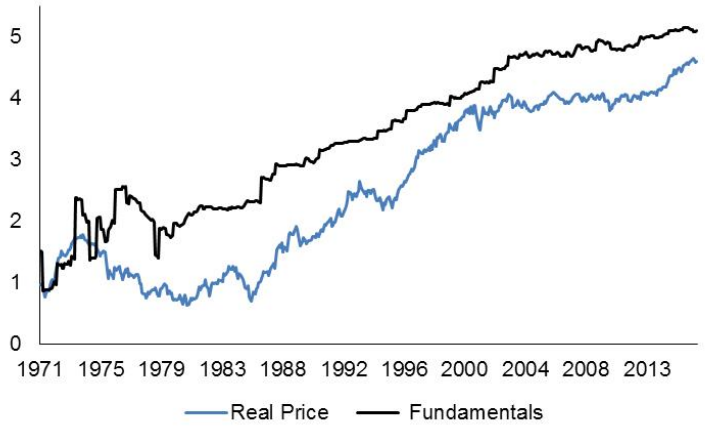


Figure 3.10: Log real price vs. Fundamentals, Johnson&Johnson

In the second part of the estimation for US stocks mixed results were obtained. For some companies, such as Abbott Laboratories, American Express, Wells Fargo & Company estimated autoregressive parameters are significant and there are signs of switching between regimes in the estimated vector of states. Figure 3.11 illustrates the bubble component dynamics next to the probabilities of collapsing regime over time for Abbot Laboratories. For this company, two major crashes stand out,

which are highlighted in both graphs: 1) end of 1996 - beginning 1997; 2) throughout 2004 (the period includes collapse of the positive bubble, rise and fall of the negative bubble). Figure 3.12 illustrates the bubble component dynamics next to the probabilities of collapsing regime over time for Wells Fargo. In this case one major crash (March, 2011) and two smaller crashes (at the end of 1976 and during 2003) stand out as peaks in the probabilities graph. Similar dynamics can be observed also for American Express Company and Johnson&Johnson.

For Dover Corporation no major regimes switches were observed. For stocks of The Clorox Company, The Coca-Cola Company and The Williams Companies Inc. significant crashes are reflected in the estimated vector of states (probabilities), but autoregressive coefficients for collapsing regime are too close to zero and not significant at 5% confidence intervals. There is also a group of stocks (Elli Lilly and Company and Honeywell International Inc.) for which the collapsing regime prevails throughout the sample and only occasionally is interrupted by the explosive regime. In this case, the autoregressive coefficient in the collapsing regime is close to 1, while the autoregressive coefficient in the explosive regime is significantly bigger than one. Those results are summarized in the Table 3.6.

The rest of the results on US stocks are presented in Appendices. In particular, estimated parameters of dividend process are presented in Appendix B. The graphs of bubble component and probabilities of being in the collapsing regime over time are shown in Appendix C.

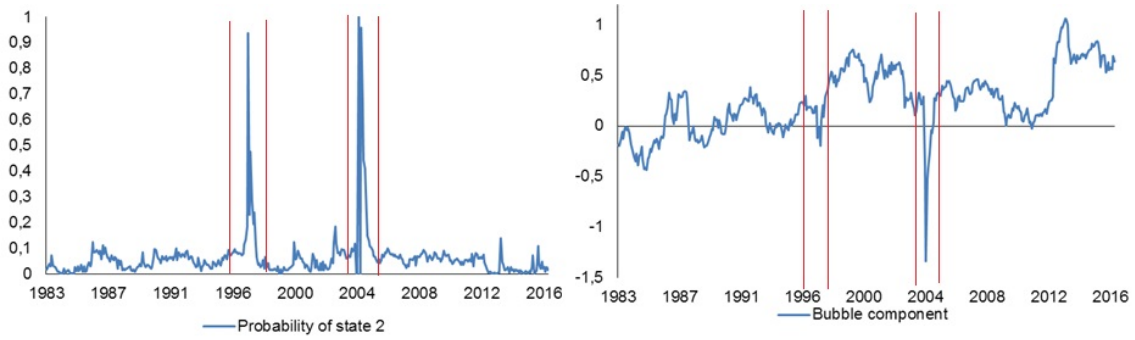


Figure 3.11: Results for Abbott Laboratories

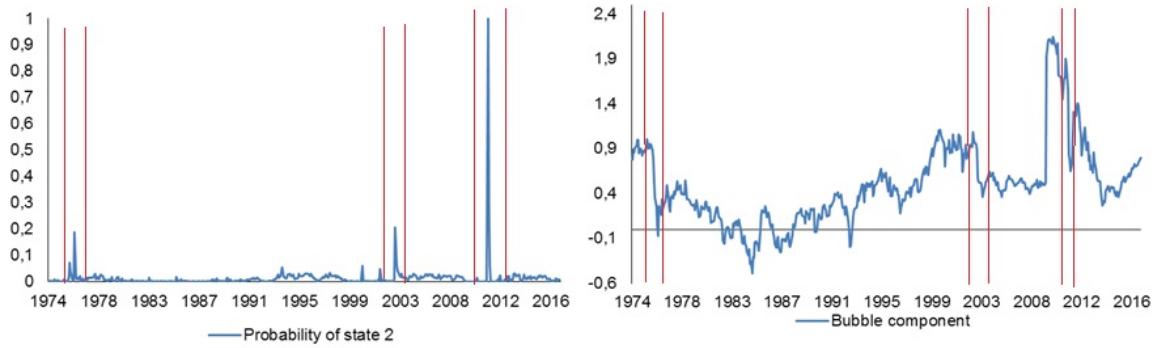


Figure 3.12: Results for Wells Fargo & Company

Table 3.6: AR(1) with Markov-switching for S&P 500 constituents

	F_1	F_2	σ_η	p_{11}	p_{22}
ABT	1.0131	0.5302	0.1215	0.9635	0.3712
	(0.0107)	(0.0736)	(0.0044)	(0.0216)	(0.2428)
AXP	1.0041	0.4912	0.1227	0.9848	0.3469
	(0.0035)	(0.1361)	(0.0044)	(0.0076)	(0.1974)
CNP	1.0036	-0.9307	0.1114	0.9960	0.3073
	(0.0034)	(0.0646)	(0.0034)	(0.0030)	(0.2285)
CLX	1.0059	-0.0583	0.1057	0.9903	0.4106
	(0.0051)	(0.1263)	0.0038	(0.0056)	(0.2399)
KO	1.0041	0.0264	0.0901	0.9942	0.3513
	(0.0033)	(0.0797)	0.0025	(0.0033)	(0.2399)
CSX	1.0044	-0.2026	0.1179	0.9831	0.1434
	(0.0037)	(0.1089)	(0.0045)	(0.0072)	(0.1239)
DOV	1.5507	0.9433	0.1131	0.3990	0.9017
	(0.6128)	(0.0296)	(0.0040)	(0.2639)	(0.1956)
LLY	1.6096	0.9797	0.1129	0.1741	0.9827
	(0.2516)	(0.0090)	(0.0041)	(0.1644)	(0.0138)
EMR	2.4492	0.9805	0.0930	0.3829	0.9744
	(1.0634)	(0.0085)	(0.0030)	(0.2617)	(0.0699)
GPC	0.9904	0.4874	0.0963	0.9716	0.3450
	(0.0137)	(0.4097)	(0.0038)	(0.0461)	(0.2573)
HON	1.8284	0.9880	0.1107	0.2955	0.9937
	(0.4136)	(0.0059)	(0.0036)	(0.2223)	(0.0043)
IFF	1.0042	-0.8951	0.1149	0.9947	0.3162
	(0.0037)	(0.1217)	(0.0042)	(0.0037)	(0.2302)
JNJ	1.0070	0.2022	0.1061	0.9797	0.3190
	(0.0059)	(0.0656)	(0.0032)	(0.0085)	(0.2381)
MCD	1.0046	0.0811	0.1141	0.9900	0.4580
	(0.0036)	(0.0953)	(0.0039)	(0.0049)	(0.2661)
NEE	1.0065	-0.8054	0.0940	0.9932	0.3100
	(0.0059)	(0.1609)	(0.0035)	(0.0046)	(0.2287)
WFC	1.0049	0.3032	0.1225	0.9890	0.3270
	(0.0045)	(0.1105)	(0.0039)	(0.0078)	(0.2295)
WMB	1.0085	-0.0453	0.1738	0.9868	0.2060
	(0.0070)	(0.0622)	(0.0062)	(0.0063)	(0.1670)

Chapter 4

Bubble synchronization

4.1 Bubble correlation analysis

In this section the co-movements of the estimated bubble components are analysed in two different ways: 1) correlation of each stock over the entire sample; 2) average correlation with S&P 500 over time. The correlations between stock bubble components over the entire sample are summarised in Figure 4.1. The figure illustrates results of the cluster analysis in a form of a dendrogram and a color map. In the color map, lighter shades represent higher correlation and darker shades represent lower (or negative) correlation. If a threshold is fixed at 0.4 in the dendrogram, three clusters can be observed. The biggest one (red in a figure) includes 11 stocks, which are positively correlated with the market index. The green cluster includes only The Coca-Cola Company and Dover Corporation, which are weakly correlated with the market and The blue cluster includes Emerson Electronic Co. and McDonald's Corporation, which are negatively correlated with the market.

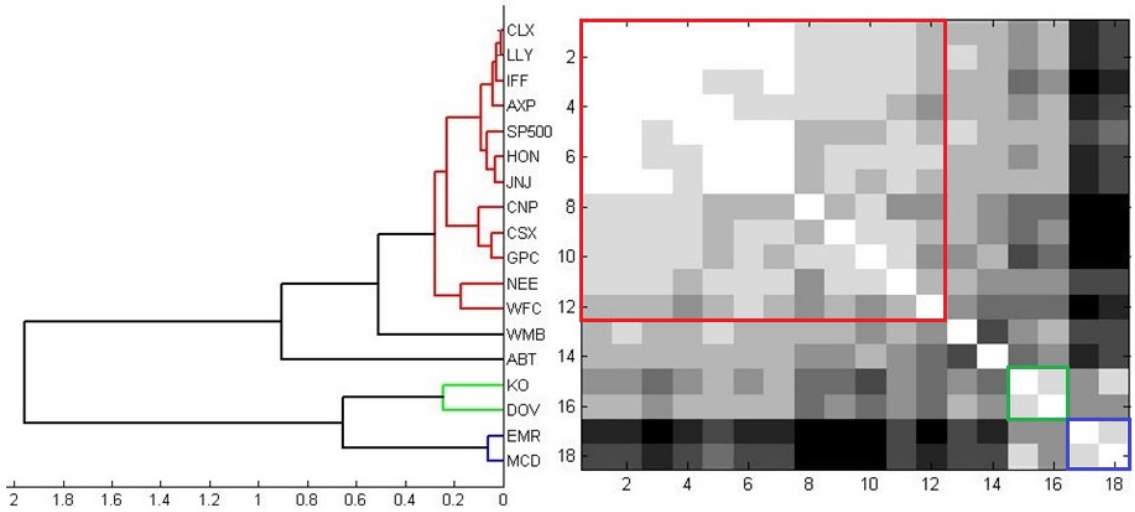


Figure 4.1: Cluster analysis of the bubble correlation (light cells indicate positive correlation, gray cells indicate low correlation, black cells indicate negative correlation)

In order to study further the contagion effect in the sample, a similar graph is plotted for the correlation between bubble components and their lagged values. I estimated the correlation coefficients for up until the 4th lag. Since the picture for the 1st, 2nd and 3rd lags is very similar, only the plots with the 1st and the 4th lags are illustrated in the Figures 4.2 and 4.3 respectively. The general structure of the clusters is the same with all he lags. Only when plotted against the 4th lag,

the configuration in the biggest (red) cluster changes. In particular, the correlation of the bubble components in the red cluster with market index weakens, except for Honeywell International Inc. and Johnson&Johnson, which stay in small subcluster with the S&P 500. This can be due to the fact that Johnson&Johnson has a large share in the S&P 500.

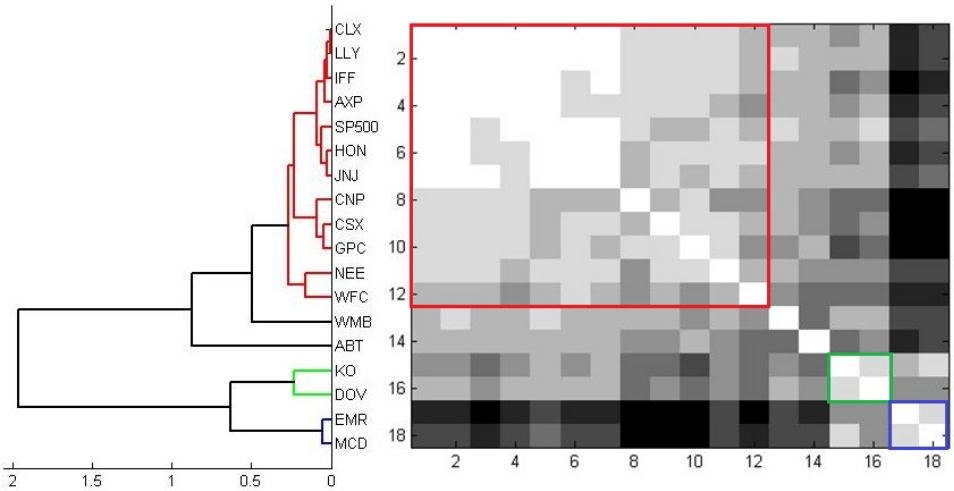


Figure 4.2: Cluster analysis of the bubble correlation, current period vs. the 1st lags

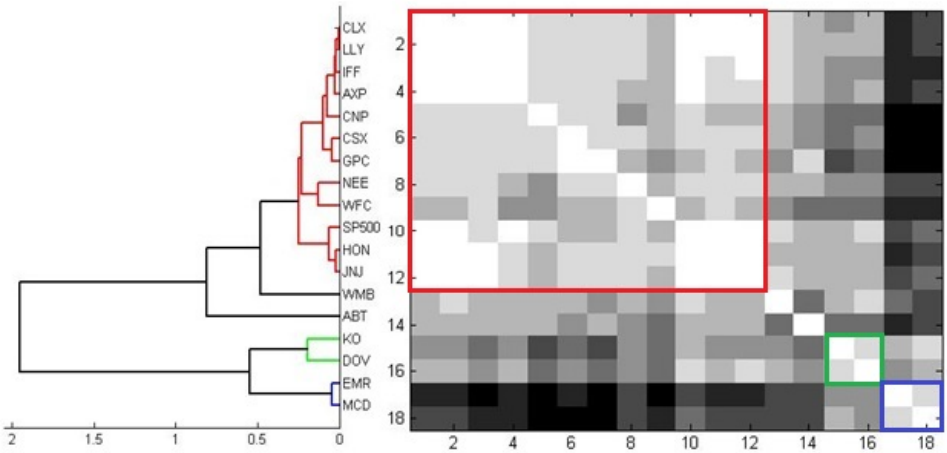


Figure 4.3: Cluster analysis of the bubble correlation, current period vs. the 4th lag

The correlations between pairs of bubble components are presented in detail in a form of a correlation matrix in the Table 4.4 in the Appendix C. The highest correlation over the sample is observed between Elli Lilly and The Clorox Company (0,9181) and between Elli Lilly and Johnson&Johnson (0.8972). Absence of correlation is found between Abbott Laboratories and The Williams Companies Inc. (-0.0035) and between CSX Corporation and The Coca-Cola Company (-0.0010). The

most extreme negative correlation is observed between Genuine Parts Company and Emerson Electronic Co. (-0.6027) and between Genuine Parts Company and McDonald's Corporation (-0.5809).

One of the stylized facts about financial market commonly described in the literature (Sandoval and Franca, 2012) is that the correlation between different stocks in the market (or even different regional markets) tends to rise in time of crisis. In this thesis, I want to check whether the same holds for the bubble components of the stocks. In order to do that I estimated the correlation between each company and the market index over time. The correlation seems to increase and decrease in waves. Figure 4.4 illustrates the average correlation in the moving window of 25 days. Figure 4.5 illustrates the average correlation in the moving window of 50 days. As expected, this figure shows a similar, but smoother pattern.

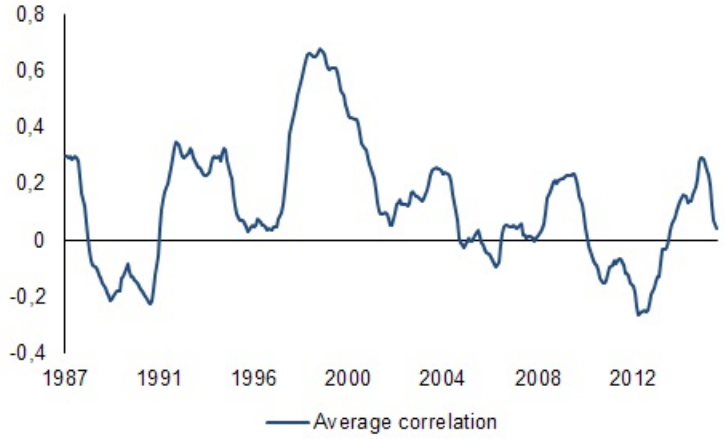


Figure 4.4: Average correlation over the moving window of 25 days

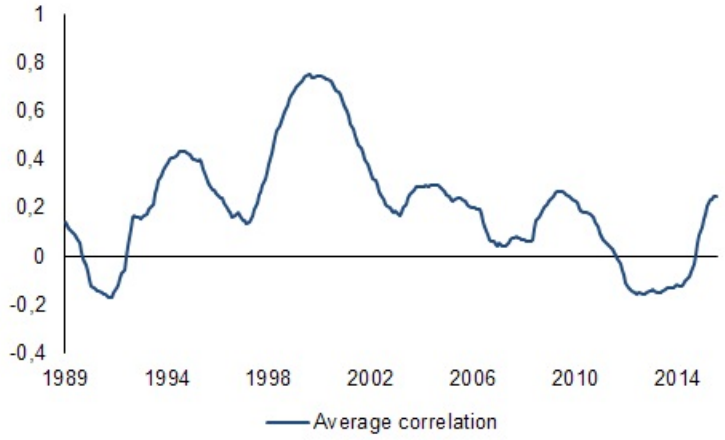


Figure 4.5: Average correlation over the moving window of 50 days

Indeed, the average correlation seems to rise during the crisis of 2008 (the period from September of 2008 until mid 2009). However, it does not nearly reach the highest values in the sample. Significantly bigger peaks can be observed during year 1992 and 1993, which are harder to interpret. The biggest upward trend and the highest peak can be found in 1997 and 1998. The latest can be attributed to the sequence of Asian and Russian financial crises.

4.2 A new systemic stress indicator

In this section I develop a systemic stress indicator reusing the methodology developed in Hollo et al. (2012) and the estimated bubble components. The goal of the study presented by the European Central Bank was to measure the overall state of the economy by combining stress indicators for several markets: equity market, bond market, money market, foreign exchange and hedging. In this thesis I attempt to estimate such an indicator only for the equity market by weighting stress indicators of 17 stocks. In the aforementioned study, activity in the equity market was measured by the maximum cumulated loss over a one-year moving window. Instead of this measure, I use the the maximum decline in the bubble component over a one-year moving window. The raw indicator is then transformed into an empirical CDF using the following formula:

$$z_t = F_n(x_t) = \begin{cases} \frac{r}{n} & for \quad x_{[r]} \leq x_t < x_{[r+1]}, \quad r = 1, 2, ..., n-1 \\ 1 & for \quad x_t \geq x_{[n]} \end{cases} \tag{4.1}$$

where:
$(x_{[1]}, x_{[2]}, ..., x_{[n]})$ - ordered sample of observations;
r - ranking number assigned to each realization of x_t;
n - total number of observations.

Separate stock indicators were aggregated into one using the following formula:

$$SSI_t = \sqrt{z_t C_t z_t'} \tag{4.2}$$

where C_t - matrix of time varying cross-correlation coefficients, elements of which are computed in a following way:

$$\begin{aligned} \sigma_{ij,t} &= \lambda \sigma_{ij,t-1} + (1-\lambda)\tilde{z}_{i,t}\tilde{z}_{j,t} \\ \sigma^2_{i,t} &= \lambda \sigma^2_{i,t-1} + (1-\lambda)\tilde{z}^2_{i,t} \\ \rho_{ij,t} &= \frac{\sigma_{ij,t}}{\sigma_{i,t}\sigma_{j,t}} \end{aligned} \tag{4.3}$$

where:
$\tilde{z}_{i,t}$ - demeaned subindices obtained by subtracting the mean from each indicator;
$\sigma_{ij,t}$ - approximated covariance between stocks i and j at time t;

$\sigma^2_{i,t}$ - approximated variance of stock i at time t;
λ - decay factor or smoothing parameter;
$\rho_{ij,t}$ - approximated correlation between stocks i and j at time t.

The estimated aggregate indicator (SSI) is shown in the Figure 4.6. The rise in the value of SSI can be interpreted as rise in systemic risk for the market and vice versa. Three rapid ascents are highlighted in the graph that can be attributed to the financial crises classified in literature on bubbles. Firstly, the data point of October, 1987 (Black Monday) is located on a rise of the indicator through the late 80s, but the market recovers quickly and by the 1990 the systemic risk declines again. The biggest spike is observed during the period of 2000-2002, which can be attributed to the crash of Dot-com bubble. Despite the fact that this crises was not detected by the MCMC model in Chapter 3, it appears to affect the market the most for the last 30 years. The last highlighted ascent includes values for the period of 2008-2009 years. Since I used the declines in bubble component instead of price loss as a raw stress indicator, this ascent is not as dramatic compared to one observed in Hollo et al. (2012). It can be explained by the fact that during the crisis of 2008, a drop in average dividends followed soon after the drop in prices. Therefore, the bubble components didn't experience a prolonged decline in this period. The rest of the smaller peaks are harder to interpret. They may represent smaller bubbles' crashes, which are not classified in the literature. Alternatively, some of them may be smoothed or erased, when a bigger sample of stocks is used for approximating the market.

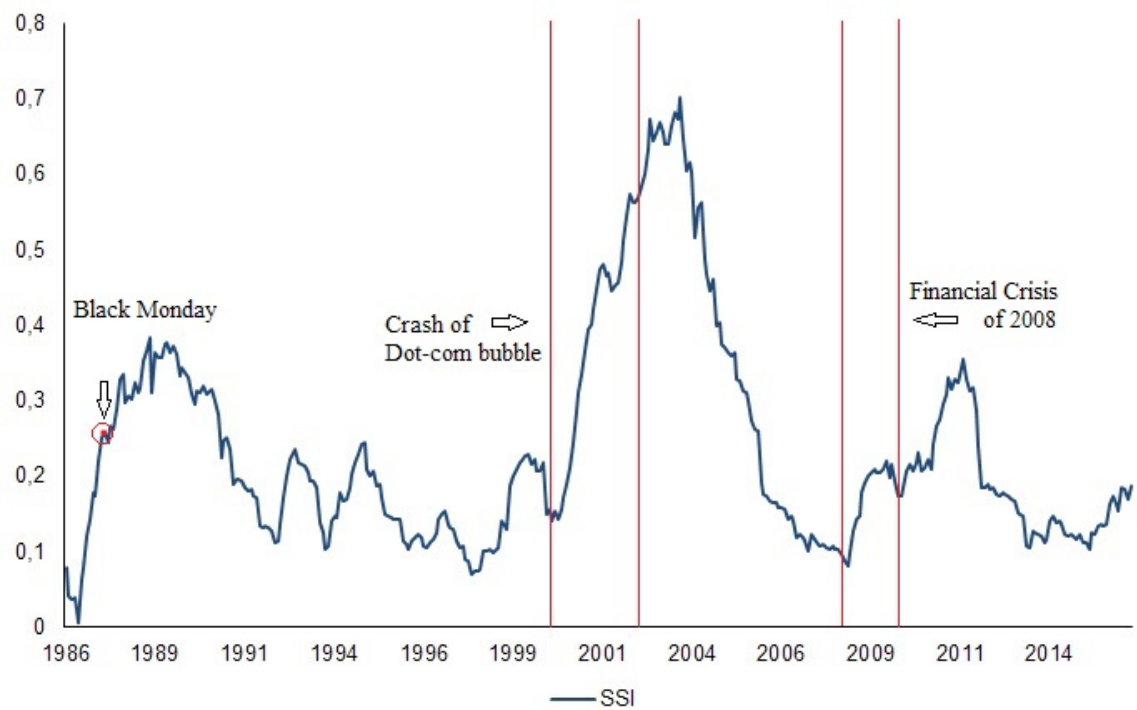


Figure 4.6: Systemic stress indicator

For comparison, the indicator based on the cumulated losses in prices over one year period is illustrated in the Figure 4.7. This indicator is closer to the one used in the original paper. Throughout the sample indicator based on cumulated losses in prices is more dynamic than one based on the maximum losses in the bubble component. Some crises period classified in the literature even include several peaks, i.e., the crash of Dot-com bubble in 2000-2002. The highest peaks are observed during the late 80s, early 90s, before and during the crisis of 2008. On the same graph the indicator based on the cumulated losses in bubbles over one year period is shown. This indicator takes values in a much more narrow range, which might imply a low correlation between the cumulated losses in the bubble components. Nevertheless, similar to the indicator based on the maximum losses, this indicator reaches the highest values during the crash of the Dot-com bubble. Several peaks can also be observed during the crisis of 2008. However, the indicator takes values below the mean during the late 80s, unlike the two aforementioned indicators.

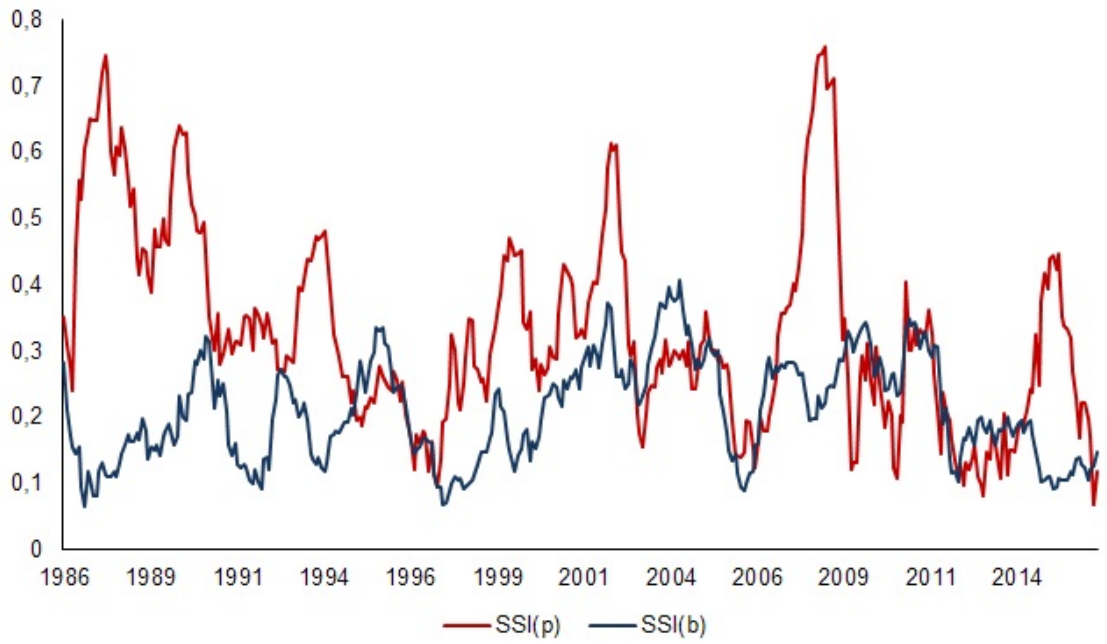


Figure 4.7: Systemic stress indicator (based on the cumulative loss in prices vs. bubble)

Conclusions

In this dissertation a model was proposed for detecting periodically collapsing bubbles in the stock markets. Instead of using the Kalman filtering technique as in Wu (1997) and Al-Anaswah and Wilfling (2011), the bubble component is estimated in two steps: fundamentals estimation and bubble component estimation. The fundamentals are estimated using the formula derived in Wu (1997), which is based on a Campbell-Shiller approximation of the value of expected future dividends. Deviations of the log real prices from fundamentals are a natural candidate for the bubble component. However, in order to capture periodically collapsing bubbles, as in Evans (1991), and satisfy the assumption of no arbitrage opportunities, the second step is performed by assuming an autoregressive process with Markov-switching for the bubble component. The unobservable latent factor allows the bubble to switch between exploding and collapsing regimes by changing solely the autoregressive coefficient. The estimated bubble components are then used to compute the systemic stress indicator according to European Central Bank methodology.

The results for real market data confirm the periodically collapsing behaviour of the bubble. In the biggest sample analysed (S&P 500 index) the model was able to capture the biggest stock market crashes since the late 19th century until the early 21st century. However, the latest crashes (Dot-com bubble and the crisis of 2008) were less recognised in the estimated regime vector. In order to study the synchronisation of the bubble components, 17 constituents of the S&P 500 were used for estimation. The companies were chosen based on the length of historical data available (minimum since the early 80s) and the industry that it represents. In this way I tired to approximate the market movements with a small number of series. Some of them were found to be chronically overvalued or undervalued, but the results for almost all of these series also display regime-switching behaviour.

Based on the correlation of the bubble components over the entire sample, the stocks can be grouped into 3 clusters: highly correlated with the market, weakly correlated with the market and negatively correlated with the market. In order to check whether the bubble components become more correlated during crisis the average correlation of the stocks with the market over time was estimated. The resulting correlation is indeed dynamic, with several peaks and valleys in the estimated period. However, it is only partly interpretable with some ascents coinciding with the tumultuous periods in a market and some appearing even during bullish markets.

Finally, the estimated bubble components were also used in computing the systemic risk indicator for the equity market. The maximum decline in the bubble component over a one-year moving window was used as a proxy for the stress indicator, instead of the maximum cumulated loss, as in the original paper. Correlation matrices over time were used to weight the components and aggregate the stock indicators into a single market indicator. The results capture some of the periods of

high systemic risk that were not recognised by the regime-switching model. The most rapid ascent of SSI comes on the period of crash of the Dot-com bubble, which shows that bubble components synchronously declined in this period. A few other crises, such as Black Monday and the crisis of 2008, were marked by the ascent in SSI but on a smaller scale. Therefore, the combination of dynamics of the bubble components and its correlation give more meaningful results than solely analysis of the correlation.

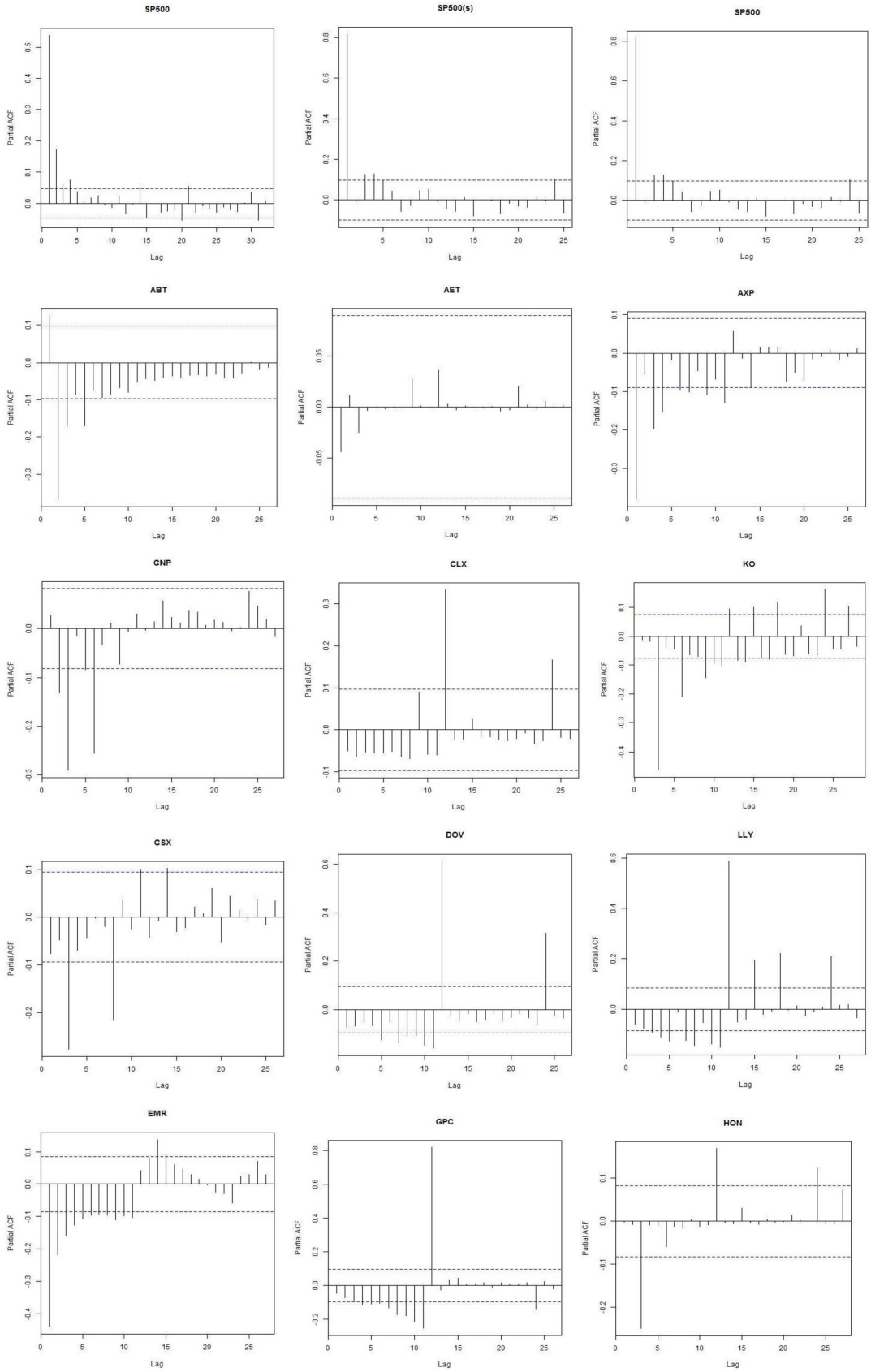
A potential line of future research on this topic can bind the given model with the the ideas in Brooks and Katsaris (2005) that factors, such as traded volume, can help to predict the time until the next collapsing regime. Also, other factors affecting the switching between regimes can be tested, such as the duration of the particular regime, news, policy decisions. In conclusion, the overall results can be interpreted in two ways. Firstly, the regime-switching properties should be considered in developing theoretical models for bubbles behaviour. Secondly, the analysis of the dynamics and synchronization of bubbles is an important complement to the analysis of prices behaviour, which contributes to a deeper understanding of crises in financial markets.

Appendices

Appendix A

Table 4.1: Results of the unit root tests

	Augmented Dickey-Fuller test		Phillips-Perron test	
	Initial data	Differenced data	Initial data	Differenced data
S&P 500	0.9909	0.0010	0.9909	0.0010
S&P 500 (s)	0.9990	0.0010	0.9990	0.0010
ABT	0.0010	0.0010	0.0010	0.0010
AET	0.6284	0.0010	0.6284	0.0010
AXP	0.0010	0.0010	0.0010	0.0010
CNP	0.0010	0.0010	0.0010	0.0010
CLX	0.9948	0.0010	0.9948	0.0010
KO	0.7810	0.0010	0.7810	0.0010
CSX	0.9990	0.0010	0.9990	0.0010
DOV	0.9990	0.0010	0.9990	0.0010
LLY	0.9212	0.0010	0.9212	0.0010
EMR	0.9741	0.0010	0.9741	0.0010
GPC	0.9881	0.0010	0.9881	0.0010
HON	0.9924	0.0010	0.9924	0.0010
IFF	8022	0.0010	0.8022	0.0010
JNJ	0.9990	0.0010	0.9990	0.0010
MCD	0.9990	0.0010	0.9990	0.0010
NEE	0.9990	0.0010	0.9990	0.0010
WFC	0.5841	0.0010	0.5841	0.0010
WMB	0.0301	0.0010	0.0301	0.0010


SP500
SP500(s)
SP500
ABT
AET
AXP
CNP
CLX
KO
CSX
DOV
LLY
EMR
GPC
HON
Partial ACF
Lag

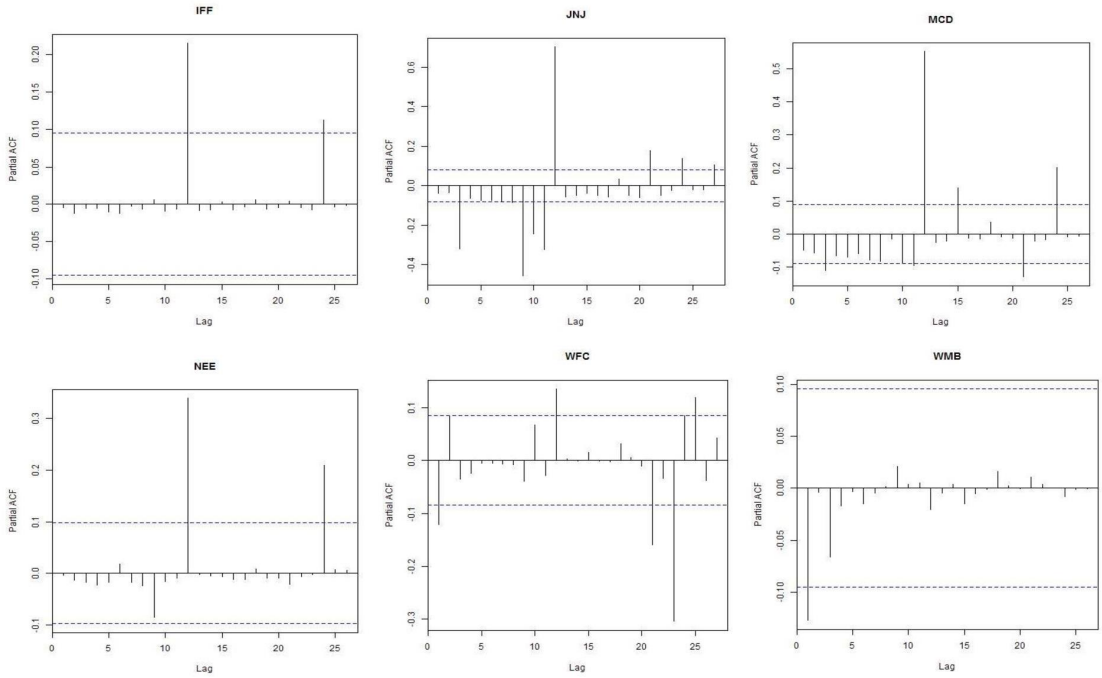


Figure 4.8: PACF of SP500 and its constituents

Appendix B

Table 4.2: Log dividend process of S&P 500 constituents

	intercept	lag 1	lag 2	lag 3	lag 5	sigma
ABT	0.0119	0.0903	-0.3706	-0.2339	-0.1759	0.1616
	(0.0079)	(0.0479)	(0.0458)	(0.0513)	(0.0492)	(0.0057)

	intercept	lag 1	lag 3	lag 4	sigma
AXP	0.0049	-0.3850	-0.2010	-0.1351	0.1344
	(0.0063)	(0.0424)	(0.0462)	(0.0461)	(0.0044)

	intercept	lag 2	lag 3	lag 6	sigma
CNP	-0.0027	-0.1212	-0.3624	-0.2448	0.1137
	(0.0048)	(0.0398)	(0.0427)	(0.0399)	(0.0034)

	intercept	lag 12	lag 24	sigma
CLX	0.0018	0.5357	0.0902	0.0290
	(0.0015)	(0.0536)	(0.0387)	(0.0011)

	intercept	lag 3	lag 6	lag 9	lag 11	lag 12	lag 15	lag 18	lag 24	lag 27	sigma
KO	0.0060	-0.6231	-0.2834	-0.0401	-0.0153	0.1956	0.1943	0.1399	0.1533	0.0827	0.0579
	(0.0025)	(0.0388)	(0.0452)	(0.0463)	(0.0336)	(0.0466)	(0.0462)	(0.0397)	(0.0388)	(0.0398)	(0.0017)

	intercept	lag 3	lag 8	sigma
CSX	0.0060	-0.2826	-0.1974	0.1527
	(0.0071)	(0.0484)	(0.0464)	(0.0052)

	intercept	lag 5	lag 10	lag 11	lag 12	lag 21	sigma
DOV	0.0026	-0.0454	-0.0371	-0.0191	0.6505	-0.0052	0.0274
	(0.0015)	(0.0489)	(0.0450)	(0.0459)	(0.0476)	(0.0461)	(0.0010)

	intercept	lag 4	lag 5	lag 7	lag 8	lag 10	lag 11	lag 12	lag 15	lag 18	lag 24	sigma
LLY	0.000685	-0.0209	-0.0255	-0.0109	-0.0212	-0.00044	-0.0046	0.4703	0.0569	0.1048	0.2185	0.0246
	(0.0013)	(0.0407)	(0.0405)	(0.0405)	(0.0401)	(0.0406)	(0.0394)	(0.0517)	(0.0392)	(0.0389)	(0.0517)	(0.000812)

	intercept	lag 1	lag 2	lag 3	lag 4	lag 5	lag 9	lag 14	sigma
EMR	0.0172	-0.6065	-0.3748	-0.2706	-0.1909	-0.1022	-0.0279	0.0663	0.0638
	(0.0031)	(0.0446)	(0.0507)	(0.0511)	(0.0499)	(0.0438)	(0.0385)	(0.0379)	(0.0020)

	intercept	lag 7	lag 8	lag 9	lag 10	lag 11	lag 12	sigma
GPC	0.00059	-0.0057	-0.0175	-0.0062	-0.0141	-0.0124	0.8426	0.0189
	(0.0011)	(0.0425)	(0.0422)	(0.0398)	(0.0421)	(0.0430)	(0.0403)	(0.00066)

Table 4.3: Log dividend process of S&P 500 constituents

	intercept	lag 3	lag 12	lag 24	sigma				
HON	0.0016	-0.2550	0.1402	0.1136	0.0506				
	(0.0022)	(0.0427)	(0.0423)	(0.0430)	(0.0015)				
	intercept	lag 12	lag 24	sigma					
IFF	0.0019	0.1942	0.1229	0.0567					
	(0.0028)	(0.0520)	(0.0520)	(0.0020)					
	intercept	lag 3	lag 9	lag 10	lag 11	lag 12	lag 21	lag 24	sigma
JNJ	0.0016	0.0079	-0.1975	-0.0061	-0.0090	0.6102	0.1385	0.2124	0.0281
	(0.0013)	(0.0238)	(0.0480)	(0.0217)	(0.0214)	(0.0454)	(0.0452)	(0.0425)	(0.00084)
	intercept	lag 3	lag 11	lag 12	lag 15	lag 21	lag 24	sigma	
MCD	0.0037	-0.1841	-0.0149	0.2951	0.1022	0.0050	0.3189	0.0473	
	(0.0024)	(0.0442)	(0.0333)	(0.0441)	(0.0402)	(0.0309)	(0.0401)	(0.0016)	
	intercept	lag 12	lag 24	sigma					
NEE	0.000865	0.2805	0.2429	0.0295					
	(0.0015)	(0.0597)	(0.0597)	(0.0011)					
	intercept	lag 1	lag 12	lag 21	lag 23	lag 25	sigma		
WFC	0.0058	-0.1283	0.1313	-0.1291	-0.3019	0.0964	0.1033		
	(0.0047)	(0.0424)	(0.0418)	(0.0419)	(0.0425)	(0.0418)	(0.0032)		

Appendix C

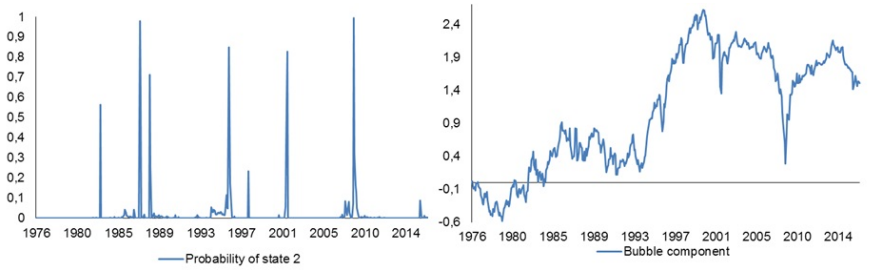


Figure 4.9: Results for American Express Company

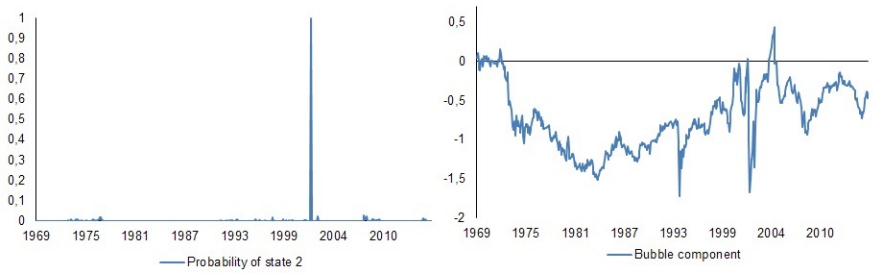


Figure 4.10: Results for Center Point Energy Inc.

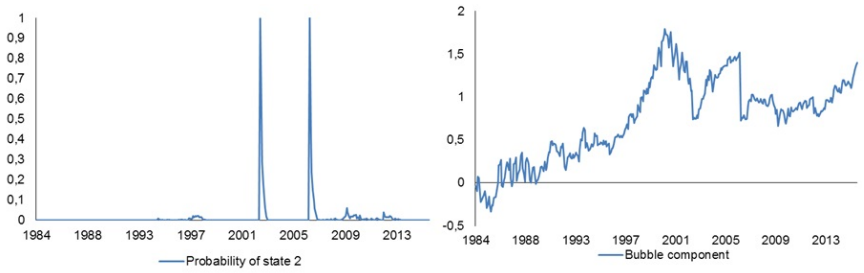


Figure 4.11: Results for The Clorox Company

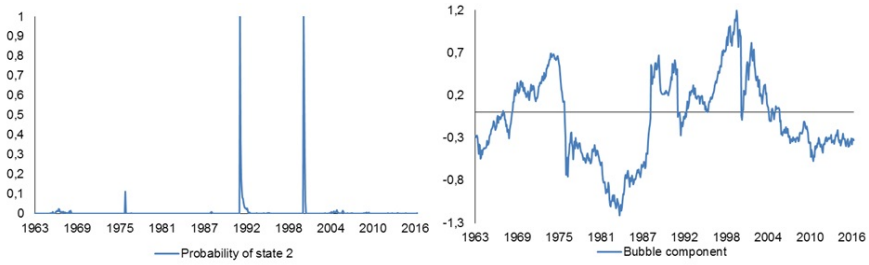


Figure 4.12: Results for The Coca-Cola Company

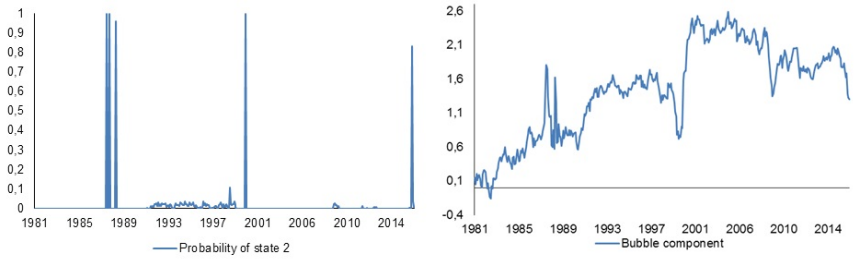


Figure 4.13: Results for CSX Corporation

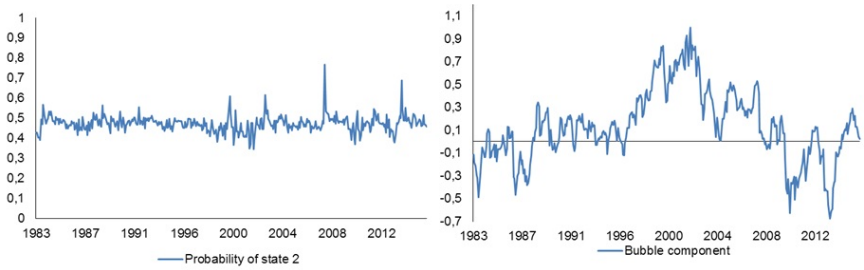


Figure 4.14: Results for Dover Corporation

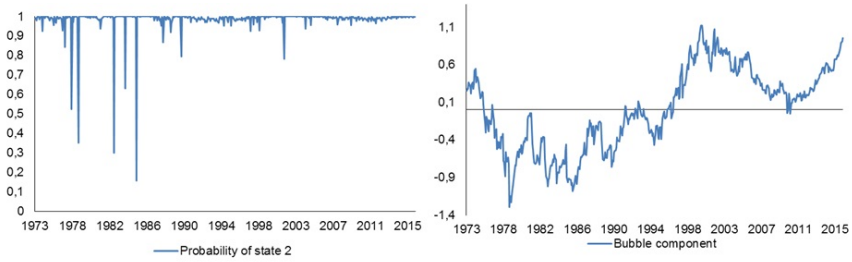


Figure 4.15: Results for Elli Lilly and Company

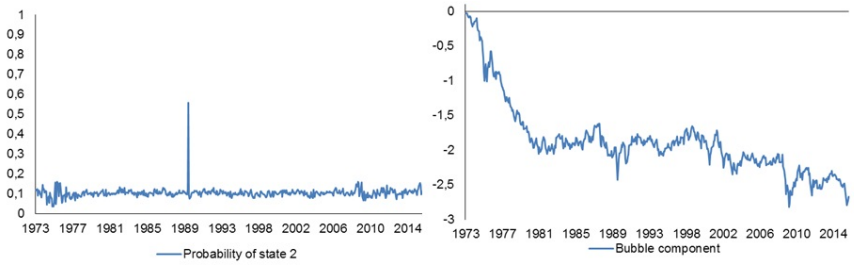


Figure 4.16: Results for Emerson Electric Co.

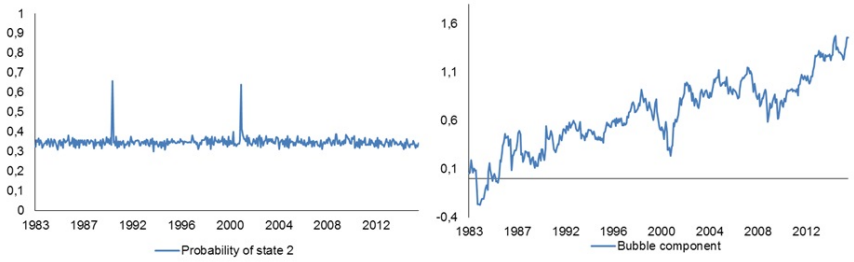


Figure 4.17: Results for Genuine Parts Company

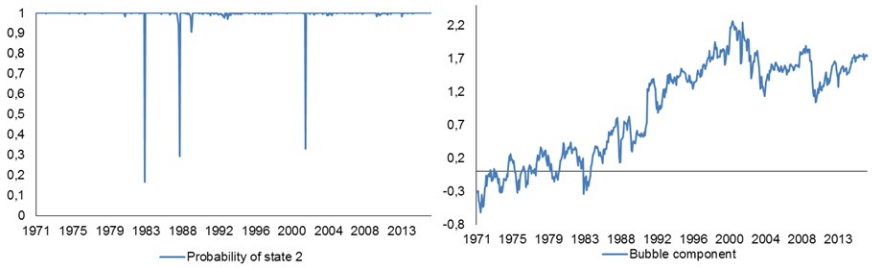


Figure 4.18: Results for Honeywell International Inc.

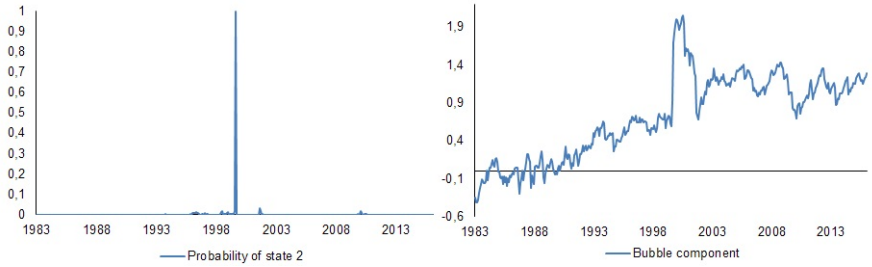


Figure 4.19: Results for International Flavurs&Fragrances Inc.

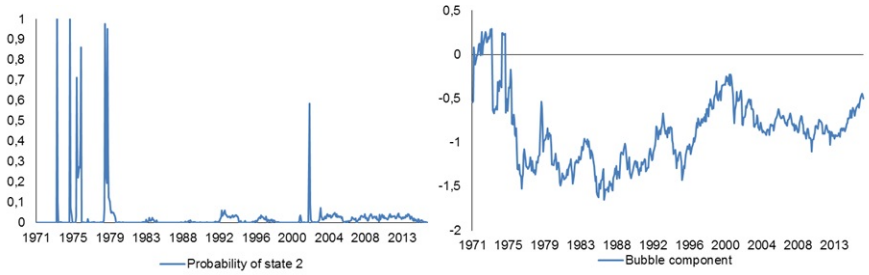


Figure 4.20: Results for Johnson&Johnson

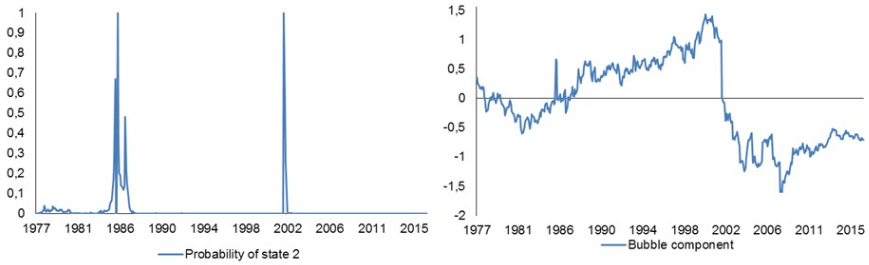


Figure 4.21: Results for McDonald's Corporation

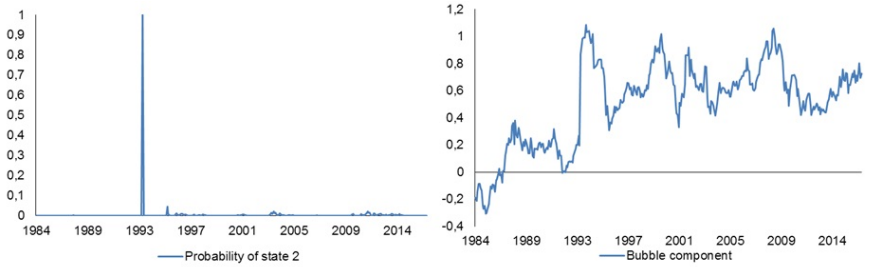


Figure 4.22: Results for Next Era Energy Inc.

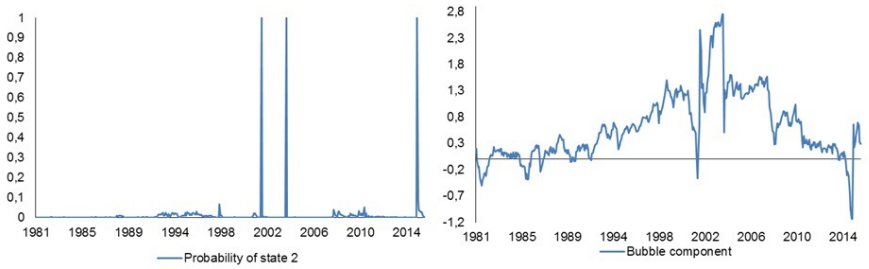


Figure 4.23: Results for The Williams Companies Inc.

Table 4.4: Correlation matrix

	SP 500	ABT	AXP	CNP	CLX	KO	CSX	DOV	LLY	EMR	GPC	HON	IFF	JNJ	MCD	NEE	WFC	WMB
SP 500	1	0,54	0,89	0,60	0,87	0,47	0,57	0,59	0,86	-0,11	0,52	0,82	0,79	0,84	0,08	0,64	0,48	0,62
ABT	0,54	1	0,57	0,35	0,53	0,10	0,31	0,22	0,51	-0,24	0,57	0,53	0,46	0,44	-0,02	0,37	0,13	0,00
AXP	0,89	0,57	1	0,73	0,84	0,24	0,62	0,42	0,83	-0,34	0,65	0,70	0,81	0,76	-0,19	0,51	0,38	0,57
CNP	0,60	0,35	0,73	1	0,66	0,03	0,58	0,12	0,63	-0,51	0,63	0,53	0,72	0,56	-0,44	0,33	0,38	0,48
CLX	0,87	0,53	0,84	0,66	1	0,32	0,70	0,51	0,92	-0,32	0,68	0,84	0,88	0,85	-0,19	0,66	0,56	0,52
KO	0,47	0,10	0,24	0,03	0,32	1	0,00	0,63	0,39	0,38	-0,08	0,37	0,17	0,41	0,61	0,25	0,12	0,40
CSX	0,57	0,31	0,62	0,58	0,70	0,00	1	0,26	0,73	-0,42	0,76	0,68	0,70	0,61	-0,49	0,65	0,42	0,52
DOV	0,59	0,22	0,42	0,12	0,51	0,63	0,26	1	0,52	0,28	0,03	0,53	0,34	0,58	0,37	0,30	0,17	0,53
LLY	0,86	0,51	0,83	0,63	0,92	0,39	0,73	0,52	1	-0,29	0,70	0,85	0,86	0,90	-0,15	0,66	0,51	0,60
EMR	-0,11	-0,24	-0,34	-0,51	-0,32	0,38	-0,42	0,28	-0,29	1	-0,60	-0,29	-0,43	-0,20	0,70	-0,16	-0,43	-0,02
GPC	0,52	0,57	0,65	0,63	0,68	-0,08	0,76	0,03	0,70	-0,60	1	0,64	0,73	0,55	-0,58	0,63	0,37	0,29
HON	0,82	0,53	0,70	0,53	0,84	0,37	0,68	0,53	0,85	-0,29	0,64	1	0,78	0,83	-0,06	0,73	0,62	0,49
IFF	0,79	0,46	0,81	0,72	0,88	0,17	0,70	0,34	0,86	-0,43	0,73	0,78	1	0,80	-0,34	0,67	0,56	0,52
JNJ	0,84	0,44	0,76	0,56	0,85	0,41	0,61	0,58	0,90	-0,20	0,55	0,83	0,80	1	-0,01	0,68	0,55	0,56
MCD	0,08	-0,02	-0,19	-0,44	-0,19	0,61	-0,49	0,37	-0,15	0,70	-0,58	-0,06	-0,34	-0,01	1	-0,17	-0,26	-0,11
NEE	0,64	0,37	0,51	0,33	0,66	0,25	0,65	0,30	0,66	-0,16	0,63	0,73	0,67	0,68	-0,17	1	0,52	0,41
WFC	0,48	0,13	0,38	0,38	0,56	0,12	0,42	0,17	0,51	-0,43	0,37	0,62	0,56	0,55	-0,26	0,52	1	0,27
WMB	0,62	0,00	0,57	0,48	0,52	0,40	0,52	0,53	0,60	-0,02	0,29	0,49	0,52	0,56	-0,11	0,41	0,27	1

Appendix D
FFBS algorithm

One of the crucial steps of the FFBS algorithm is setting the initial vector of states $S_t^{(0)}$ throughout the sample. Following the logic of the model, the initial vector of states is chosen using the following rules:
1) if during the four previous time periods the deviations from the fundamentals were decreasing in the absolute value, then in the current time period the bubble is in the collapsing regime, which means $S_t = 2$;
2) if the previous condition is not satisfied, then bubble is in the exploding regime and $S_t = 1$.

Once all the conditional posteriors are derived and initial parameters and states are set, the Gibbs sampling can be initiated, with FFBS algorithm embedded into it. FFBS algorithm is used to update the latent (regime) variable throughout the sample in a similar way the parameters $F_1, F_2, \sigma_\eta^2, p_{11}, p_{22}$ are updated with each iteration. FFBS algorithm for a given model consists of 4 main steps:
Step 1: generation of new transition probabilities matrix and ergodic probabilities vector. The matrix of transition probabilities is built by simulating for each row a draw from the Dirichlet distribution with updated hyperparameters as in (2.43). From Hamilton (1994) I use the formula for calculating the ergodic probabilities for two regimes for each iteration.

$$\pi = (C'C)^{-1}C'e_3 \tag{4.4}$$

where e_3 is the third column of 3×3 identity matrix and

$$C = \begin{bmatrix} I_3 - P \\ \mathbf{1}' \end{bmatrix} \tag{4.5}$$

where $\mathbf{1}'$ is a 2×1 vector of 1s;
I_2 is a 2×2 identity matrix.
Step 2: calculation of the log likelihood for each regime in each time period. The results are organized in a T×2 matrix, where T is the sample size.
Step 3: applying Hamilton filter, which updates and smoothes out probabilities of each state using the matrix of transition probabilities and the likelihood matrix estimated in previous step.
Step 4: simulation of the new vector of states by randomly drawing 1×2 vector of 1s and 0s using the smoothed probabilities.

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