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Tesi di Laurea

Intertemporal prevention allocation: a behavioral approach

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Chapter 1

Introduction

A decision maker (DM)[1] when dealing with a potential risk of a monetary loss (decisions under uncertainty), modifies her own behavior according to the faced situation. This involves a trade-off between immediate and delayed outcomes, as well as between certain and risky ones. Tools he can adopt in order to reduce the impact of potential loss are:

- Self-insurance: an investment that allows to reduce the size of the loss. It consists in the payment in advance of a given amount of money (insurance premium), so that if the insured loss occurred, the individual will receive a payment [2].
- Self protection: an *ex-ante* action that allows the reduction of the probability that an event occurrence generates a loss in individual's wealth;
- savings : consists in a transfer of wealth from present to future period in order to mitigate the impact of the loss occurrence on individual's future wealth. Savings involve a return that is proportional to the transfer and determined by an interest rate.

However, just two of them are a proper risk reducing activity, since savings consist just in a transfer of wealth from a period to another.

[1] In this work, "individual","agent" or "DM" are used as synonymous

[2] Within the literature, self-protection has been referred to as "loss prevention" and self-insurance as "loss protection" (Mehr and Commack 1966)

The seminal paper for the economic literature studying market-insurance, self-insurance and self-protection, is the paper by Erlich and Becker (1972) [12]. They were the first authors that studied interaction between these activities. More precisely, they argue that in absence of market insurance, self-insurance (or insurance) and self-protection are good alternatives, since allow for a real reduction of the loss the individual faces.

Following Erlich and Becker's (1972)[12] paper, a huge amount of studies have been done with respect to all these concepts. They were taking into account with different levels of specification, interactions between them or with other concepts. In this work, the attention is focused on prevention activities.

Prevention is a risk-reducing activity that takes place before the loss occurs. This topic was studied a lot in health context since more intuitive and with more applications. However, even in economic context it has a lot of implications and application. Exactly as in wealth context work-out (prevention activity) help to prevent future heart attacks or other diseases (risk), in an economic context a good prevention activity (buy a good burglar alarm) helps to prevent future loss (an illegal entry).

The common element in the economic literature regarding prevention activity until 2009 is the use of a one-period framework. More precisely, it was assumed that the exerted effort for preventive purposes and the relative effect of prevention were simultaneous. Such kind of model is enough for simple situations as for example the burglar alarm bought today as preventive action to reduce the probability of illegal entry. However, such a simple action not only reduces today's level of wealth (in money terms), but affects also tomorrow's wealth level because effects of today's investment affects also the future wealth.

Starting from this idea, Menegatti (2009)[23] proposed a two-period framework for the analysis and determination of optimal allocation of prevention effort. The example Menegatti made is the one of the driving course attended today that will influence the individual's ability to drive tomorrow. Thus, the author notices that in such situations, the effect occurs in different time-periods, and no more simultaneous as before.

Menegatti's formalization, started to be followed by other authors and allowed the extension of results obtained in a one-period framework. Interestingly, usually it happened that results obtained in a two-period framework were opposite to those of a one-period context. For example, when risk-aversion is taken into account, in a static context, the risk-avers individual exerts less effort in prevention activity than the risk-neutral one, while in a two-period framework, the result is the opposite. The intuition for this situation as well as for other situations is that, if risk and prevention activity are simultaneous, investment in prevention reduces much more the wealth level than in the case of no prevention effort. Thus, the effort for prevention activity is reduced because the individual wants a higher level of wealth. In a two-period context instead, since the prevention activity and its effects occur in two different periods, the result is the opposite. In the first period, when prevention activity is taken, the DM is willing to invest more because she wants a higher level of wealth in the period were he bears the risk.

A further different approach to prevention activity is the one considered by Hofmann and Peter (2012) [14]. More precisely, by following Menegatti's (2009) framework, they studied optimal prevention decisions in a two-period model, by taking into account two different prevention activities: anticipatory prevention effort today and contemporaneous prevention tomorrow. The main difference with respect to Menagatti's approach is the choice of the prevention technology: the probability of a loss is no more a function of a single variable $p(e)$, (Menegatti 2009), but a function of two variables, p(e_1,e_2), where e_1 and e_2 are exerted efforts in the two periods. The explanation is that in real life, the time period when investment decision is made and the one when the random event (loss) realizes, are different.

In economic literature, prevention activity was called in different ways: self-protection, prevention, loss-prevention, or willingness-to-pay (WTP) to reduce the probability of a lost occurrence. Even if called in different ways, all these concepts describe the same activity and imply the same decision. In this work, I followed the literature that uses the concept of self-protection and prevention. There is a huge amount of literature, parallel to the one I followed, that studies prevention activity intended as WTP to reduce probability of the loss occurrence.

Most part of the literature focused attention on the use of the expected utility theory(EUT) as the model describing the DM's total utility. However, several times, the EUT missed to found evidence from the empirical point of view. In real life DMs behave differently than assumed, and this allow the occurrence of some paradoxes. There was thus the necessity to found other models better describing empirical behavior of agents. In response to this empirical challenge, behavioral economists introduced models that allowed the weakening of some EUT axioms in order to incorporate observed behaviors deviating from EUT. Some of these theories were adopted also for the study of optimal allocation of prevention activity. More precisely, behavioral theories adopted in the prevention context were the Yaari's Dual-Theory by Courbage (2001)[5] and the rank-dependent expected utility (RDEU) by Konrad and Skaperdas (1993)[19]. In the case of RDEU application to the one-period framework, ambiguous results were obtained exactly as in the EUT case. The DT approach however, was used to test robustness of Erlich and Becker's (1972)[12] results in a one-period framework, and found that they were robust.

This work proposes a review of the existing literature regarding this topic, and proposes an extention to results obtained by Menegatti (2012) [24] in a two-period context, by implementing the RDEU.

Chapter 2

Literature Review

Self-protection activities under EUT assumption

When the market insurance is not available and there is a risk of monetary loss, the DM needs other tools to use in order to reduce the impact of potential loss. More precisely, Erlich and Becker (1972) [12] suggest (and prove) that there are two good alternatives to market insurance, both reducing the relative cost of the DM's loss: self-insurance and self-protection. In their paper, they examined the interaction between these tools within the EUT and an indifference curve analysis. It seems thus naturally start to review the literature by their work.

The aim of their work was to restate and reinterpret some results concerning individuals' insurance behavior under uncertainty. To better explain self-protection activity and show that self-protection and self-insurance are two different concepts, they develop a model that excludes from the analysis market or self-insurance.

The context is the following one. Assume that in a one-period framework, an individual faces only two states of the world (0 and 1), namely, the bad state (in the case a loss occurs) with probability p and the good state (no loss occurs) respectively with $1-p$ probability. In each of the two states, there is a certain level of endowed wealth: I_0^e and I_1^e respectively. Assume also that the probability of loss occurrence can be reduced by an appropriate expenditure x, thus $p = p(p^e, x)$, where p^e is the endowed probability of "bad state", x is the expenditure on self protection, and $\partial p/\partial x \leq 0$.

Thus, the DM will optimally allocate the self-protection effort, such that the following objective function will be maximized [1]. :

$$U^\star = p(p^e, x)U(I_0^e - x) + [1 - p(p^e, x)]U(I_1^e - x) \tag{2.1}$$

The usual first order condition (FOC) that allow to find optimal solution to maximization problem, requires that marginal gain from the reduction of p to be equal in equilibrium to the decline in utility due to the reduction in both incomes. The second-order optimality condition instead, show that incentive to self-protect is not dependent on attitudes toward risk, and could be as strong for risk lovers as for risk avoiders.

In order to see how changes in income and other variables will affect the demand for self-protection and self-insurance, authors use the "state preference" approach to behavior under uncertainty [2]. However, the most important results regard interactions between these tools. Authors showed that market insurance and self-insurance are substitutes, while market insurance and self-protection could be complements depending on the level of the probability of loss. Thus, contrary to the moral-hazard intuition [3], the presence of market insurance may, in fact, increase self-protection activities relative to the situation where market insurance is not available.

Starting from the seminal paper of Erlich and Becker (1972)[12], prevention (by its own or interactions with other factors) has been widely studied in the economic literature. Next sections present a summary of existing models.

[1] The same equation can be written also as :

$$\max_x U = p(x)U(I_0^e - x - L) + [1 - p(x)]U(I_1^e - x)$$

[2] See Hirshleifer (1970) for applications of the Arrow's approach (1963) to investment under uncertainty

[3] The effect of market insurance on the demand for self-protection has been generally called "moral hazard".

2.1 One-period framework

For a lot of years, existing literature on prevention focused attention and studied optimal prevention and interactions with other fields within the one-period framework. This means to assume that potential loss is simultaneous to the prevention activity. Assumptions and the formalization of the model are similar for all authors consider this framework, and just some little adjustments will be done in order to take into account some interactions.

Consider an individual with an initial wealth (W_0), subject to a potential loss L ($0 < L < W_0$) that occur with probability p,where $p \in [0, 1]$. This individual has an increasing strictly concave von Neumann Morgenstern (vNM) utility function U that is increasing and concave. Assume that the DM can affect the probability of the loss occurrence through a prevention activity x (that comes at a cost $C(x)$). Assume also that $x < L$, meaning that the potential loss could have a very negative impact on DM's level of wealth.

The individual objective function that will be maximizes by the DM is :

$$\max_x U = p(x)u(W_0 - C(x) - L) + (1 - p(x))u(W_0 - C(x)) \tag{2.2}$$

The FOC for an optimal allocation of x ($x^\star$) are:

$$\underbrace{[p'(x) \times (u(A) - u(B)]}_{\text{marginal benefit of x}} - \underbrace{C'(x)[p(x) \times u'(A) - (1 - p(x)) \times u'(B)]}_{\text{marginal cost of x}} = 0$$

where

A $=u(W_0 - C(x) - L)$

and B $=u(W_0 - C(x))$

In order to understand how optimal allocation of prevention activity will change in a one-period context, it is necessary to consider and to study interactions with other factors. For all models in a one-period context, the maximization problem (2.2) will be applied.

2.1.1 Risk aversion

Before starting to talk about relationship between self-protection and risk aversion is, it is necessary to explain the formalization and the meaning of this last concept.

A risk averse individual dislikes risk. When she is asked to choose between two lotteries with the same expected payoff but with different levels of risk, she will choose the more sure lottery, namely the one with a lower risk. Before Pratt (1964)[26] and Arrow (1965)[2], the risk aversion was simply associated with concavity of utility functions. Subsequently, the Arrow-Pratt measure of absolute and relative risk aversion were recognized in the economic literature as excellent measures of the strength of risk aversion, and their usefulness has been demonstrated in a wide range of both theoretical and empirical studies of behavior under uncertainty. More precisely,

$$\text{measure of absolute risk aversion} = \frac{-\,V''(x)}{V'(x)}$$

and

$$\text{measure of relative risk aversion} = \frac{-\,xV''(x)}{V'(x)}$$

Risk-aversion concept used in the static as well as in the dynamic framework will be intended in a Arrow-Pratt sense, unless otherwise specified.

Self-protection allocation and risk-aversion

A study of the relationship between risk aversion and self-protection activity was done by Dionne and Eeckhoudt (1985) [9]. Authors investigated and compared decisions of optimal self-protection allocation $x^\star$ with the optimal allocation of a more risk averse agent in the Arrow-Pratt sense. The utility function of the last individual was assumed to be represented by a function V that is a concave transformation of U, namely,

$$V = k(U), \text{with } k' > 0, k'' < 0 \text{ and } k' = V'(.)/U'(.)$$

$$\text{where } V' > 0 \text{ and } V'' < 0$$

Authors found that assumptions $k' > 0$ and $k'' < 0$ are not enough in order to compare the two optimal allocations, suggesting that an increase in risk aversion has an ambiguous effect on the optimal level of self-protection effort. The intuition of these results is the following one: suppose that for the risk-neutral agent it is optimal not to exert any effort, so that the accident will be incurred with certainty. In such a situation, exerting effort would affect risk, since the probability of accident will be less than unity. The risk-averse agents will dislike such a situation (self-protection activity will reduce too much his level of wealth [4]), so will reinforce the willingness not to self-protect and not the opposite .

These results suggest that more restrictive assumptions are needed in order to understand the net effect on self-protection in an unambiguous way. Authors proved, however, that in the specific case of a quadratic utility function, self-protection allocation increases with risk aversion if the initial probability of loss satisfies the inequality $p < 1/2$, while it decreases if $p > 1/2$ [5].

In the same direction of investigation there is also the paper of Briys and Schleisinger (1990) [3]. Authors tried to go further into the analysis of self-insurance and self- protection activities as actions aiming to reduce risk. They tried to explain ambiguous results obtained by Dionne and Eeckhoudt (1985) [9] regarding the more risk-averse agent's behavior, by showing that self-protection is not a risk-reducing activity if risk is measured as proposed by Rothschild and Stiglitz [1970][28] [6]. The explanation is that increasing the

[4]The probability of loss being equal to one, allow a reduction of total wealth by L, while $p < 1$ induces the agent to maximize the usual objective function:

$$\max_x U = p(x)U(A) - (1 - p(x))U(B)$$

and the level of final wealth will be less that in the first case

[5]The intuition is based on the fact that $p = 1/2$ constitutes a critical threshold below which a reduction in probability reduces the variance of the risk.

[6]Differently than others, Rothchild and Stiglitz compare risks of choices rather than risk aversion measures. As authors noted, being X and Y two random variables, Y may be said to be riskier than X if:

level of self-protection actually reduces the probability of the loss, but cannot be viewed as a "risky activity." By lowering the level of self-protection neither increases nor decreases the riskiness of final wealth as measured by second-degree stochastic dominance. As a more risk averse agent would decide to allocate more resources in risk reducing activities, he will invest more in self-insurance activities that by definition reduce the impact of the potential loss (risk), rather that in self-protection activities as it will not necessarily reduce risk.

Also Briys et al.(1991) [4] obtained ambiguous results regarding the impact of risk aversion on self - protection decisions obtained, when they examined market-insurance, self-insurance and self-protection decisions with uncertain effectiveness of these outcomes [7]. Authors showed that optimal level of self-protection, may be either higher or lower under more-risk-averse preferences Is quiet obvious that adding riskiness to self-protection's effectiveness tends to raise the possibility of a negative relationship between the degree of risk aversion and the optimal level of self-protection. However, this consideration is not enough in order to make more general conclusions.

Jullien et al. (1999)[16] instead, gave sufficient conditions under which self-protection activity increases with risk aversion, and this occurs if and only if the optimal probability of loss is less than a utility-dependent threshold. To see this, recall the maximization problem in one-period context (Eq.2.2), with the only difference that instead of x (self-protection activity), we will have e that stays for the effort the individual exerts. Thus, the maximization problem becomes:

- all risk averse individuals prefer X to Y,
- Y has more weight in the tails of its distribution than X;
- Y equals X plus noise(Z), where $E[Z|X] = 0$, for all X.

As authors proved, these three notions of risk are equivalent (terms as "more variable", "riskier" and "more uncertain" are used as synonymous here).

[7] An example of non-reliable prevention action can be the case of a faulty sprinkler system, that not only do not work as supposed to work, but can even increase the amount of the damage.

$$\max_e U = p(e)U(W_0 - C(e) - L) + (1 - p(e))U(W_0 - C(e)) \tag{2.3}$$

As usual, some assumptions are needed. Let us assume that $p(e)$, $C(e)$ and $d(e)$, where $d(e) = C(e) + L$, are continuously differentiable, with $C(e)$ and $d(e)$ increasing. Let us assume also that U is increasing and continuously differentiable function, and that the agent characterized by U strictly prefers effort $e_0 \in (0, e^\star)$ to any other effort, with $0 < p(e_0) < 1$.

The more risk-averse individual preferences are ordered by a V utility function, where V is a transformation of U (as before). Authors obtained usual ambiguous results regarding risk aversion effect on self-protection activity. To make some comparative statics results, authors focus on risk-averse individual, and made two more assumptions:

- *U is concave, c(e) and d(e) are increasing convex, $d(e) > c(e), p(e)$ is decreasing convex and $p''(e)p(e) \geq 2(p'(e))$.*

  This condition ensures that self-protection activity can be desirable and also the concavity of the maximization problem determining its level.

- *The level of effort for U is strictly between 0 and $e^\star$ (i.e. the optimal level of effort for U is interior solution).*

Under these assumptions, the optimal level of self-protection for U , $e_u^\star$, is uniquely determined by:

$$-p'(e_u) = \frac{p(e_u)U'(A)d'(e_u) + (1 - p(e_u))U'(B)C'(e_u)}{U(B) - U(A)}$$

where $A = W - d(e_u)$ and $B = W - C(e_u)$.

After some computations, assuming that the last two assumptions both hold U and V, where V is more risk-averse than U, self-protection effort is higher for V than for U if and only if $p(e_u) < p^\star$ [8].

[8] Here, $p^\star$ is defined as follows:

$$\frac{p^\star}{1 - p^\star} = \left(\frac{U'(B)\Delta V - V'(B)\Delta U}{U'(A)\Delta U - U'(A)\Delta V} \right) \frac{C'(e_u)}{d'(e_u)}$$

In the special case of quadratic utility functions, this threshold is exactly 1/2, as already proved by Dionne and Eeckhoudt [1985][9]. More generally, assuming that L is small compared to wealth (L close to zero), whenever V is more prudent than U, $p^\star$ will be below 1/2 . The intuition for these results is that for a low probability of loss ($p < 1/2$), the more risk-averse agent exerts more self-protection effort, while for a high probability of loss ($p > 1/2$), the individual is more interested to reduce the maximum loss, which leads to less self-protection effort[9]. An increase in effort reduces the variance of the random loss, something that is valuable for risk-averse agents.

2.1.2 Prudence

The notion of prudence was introduced by Kimball (1990) [18] in order to measure the willingness to accumulate wealth in the face of a future risk. It is defined as the sensitivity of the optimal choice of a decision variable to risk (which is how much one dislikes uncertainty and would turn away from uncertainty if possible). As author explains, an agent is prudent if an increase in future risk in the sense of Rothschild-Stiglitz increases the marginal value of wealth. An agent that is an expected - utility - maximizer is prudent if and only if third derivative of his utility function is positive ($u''' > 0$).

The impact of prudence on self-protection activity

The introduction of the "prudence " concept within the analysis of optimal prevention was done by Eeckhoudt and Gollier (2005) [10]. Authors use prudence as an attempt to take into account the non linearity of the utility function in the cost-benefit analysis of preventive actions. Differently than other authors, Eeckhoudt and Gollier use the same objective function for both individuals.

Consider an agent endowed with a level of wealth W, which is facing a

where $0 < p^\star < 1$, $\delta V = V(B) - V(A)$, $\Delta U = U(B) - U(A)$.

[9]While, assuming risk neutrality leads to the underestimation of the optimal level of effort when it is less than 1/2, and it leads to the overestimation of it when it is larger than 1/2.

risk of a loss occurrence $L > 0$ with probability p. Assume the probability function to be decreasing in the effort e. Assume also U is increasing, differentiable and that V is concave in e [10]. The DM will choose the level of effort e such that the following objective function is maximized:

$$\max_{e} V(e) = p(e)u(W - L - e) + (1 - p(e))u(W - e) \tag{2.4}$$

The optimal level e_n of effort for the risk neutral agent is implicitly given by $-p'(e_n)L = 1$, meaning that an additional dollar in e reduces the expected loss (L) by one dollar.

By the concavity of V , the more risk-averse agent exerts more effort than the risk-neutral one if and only if [11]:

$$\frac{u(W - e_n) - u(W - e_n - L)}{L} \geq p_n u'(W - e_n - L) + (1 - p_n)u'(W - e_n)$$

where the left-hand side represents utility benefit of reducing the probability of loss by $1/L$, while the right-hand side is the utility cost of the additional effort to reduce the probability of loss by $1/L$.

If $p_n = 1/2$ (the optimal probability of loss of the risk-neutral agent), adding risk aversion but not prudence ($u''' = 0$), has no effect on the optimal level of effort. When $p_n > 1/2$, the effect of risk aversion goes into the same direction as the effect of prudence to generate a smaller level of effort, while where $p_n < 1/2$, these two effects go into opposite directions, meaning that prudence tends to have a negative impact on prevention. The main reason is that prudence induces agents not to spend money ex-ante on preventive actions in favor of the accumulation of wealth to face future risks.

An extension of these results was proposed by Li and Dionne (2010) [21] that provided a necessary and sufficient condition for risk-averse agents to exert more self-protection than a risk-neutral agent. They found that the level of self-protection a prudent agent choses, is larger than the optimal

[10] Conditions on functions U and $p(e)$ for V to be concave in e were provided by Jullien, Salani and Salani (1999).

[11] This result is obtained by rewriting the FOC.

level of self-protection chosen by the risk neutral one if absolute prudence is less than a threshold that is utility independent, namely:

$$AP(W - e_n - L) \leq \frac{1 - 2p_n}{p_n}\frac{1}{L}$$

where $AP(x) = -\frac{u'''(x)}{u''(x)}$ is the absolute prudence coefficient (Kimball 1990)[18].

The intuition is that the level of self-protection chosen by a prudent agent is larger than the optimal level of self-protection chosen by a risk neutral agent when the negative effect of self-protection on the variance is larger than the positive effect on the third moment of the loss distribution.

Till now in all the prevention analysis it was the assumption that the effort to prevent risk and the effect on the probability that loss occurs are contemporaneous. This makes the agent to behave differently than one could imagine: prudence has negative effect on optimal prevention, because the agent bears the risk, and a larger effort in prevention reduces the level of wealth in the period where the risk occurs.

2.1.3 Background risk

The notion "background risk" refers to an exogenous risk not under the control of the individual that is independent of endogenous risks, and may contribute to the increase or to the deterioration of the level of individual's wealth.

In a one-period context, the impact of background risk on optimal prevention activity was studied by Dachraoui et al. (2004) [8]. In order to introduce agents' attitude toward risk and compare different allocation, authors introduce the concept of "More Mixed Risk Aversion" and set sufficient conditions to compare mixed risk averse utility functions. In Arrow-Pratt sense, we say that v is more mixed risk averse than u if and only if $A_u^n(w) \leq A_v^n(w)$, for all n and w, where

$$A_u^n(w) = -\frac{u^{(n+2)}(w)}{u^{(n+1)}(w)}, \text{ for } n \geq 0$$

is the generalized index of absolute risk aversion of n^{th} order.

The impact of the introduction of a background risk on optimal self-protection activity

Starting from the model for self-protection activities in a one-period framework proposed by Erlich and Becker (1972)[12] [12], Dachraoui et al.(2004) [8] introduced a background risk (ϵ), and assumes that it is independent of the occurrence of the loss (the endogenous risk). As before, the probability of loss $p(x)$ is assumed to be decreasing in x, where x denotes the investment in self-prevention activity, while u and v are two mixed risk averse utility functions, the utility function of the representative individual is :

$$\tilde{U}(w) = E_\epsilon[u(w+\epsilon)] = \int u(w+\epsilon)dF(\epsilon)$$

By imposing FOC , it is possible to find optimal allocation of self-protection activity, chosen by the risk-neutral agent and by the more risk-averse one , $x_u^\star$ and $x_v^\star$ respectively.

Being v more mixed risk averse than u, authors found that if $p(x_u^\star) \leq 1/2$ then $x_v^\star > x_u^\star$ (the more risk-averse agent exerts more effort) , while for sufficiently high probability of loss, $p(x_u^\star) \geq 1/2$, prevention can decrease with mixed risk aversion($x_v^\star < x_u^\star$). Thus, the impact of background risks on optimal effort of self-protection activity in a one-period context is ambiguous.

The link between background risk and self-protection effort was studied also by Lee (2012)[20].

Consider a risk-averse individual with an initial wealth W_0. Knowing that a loss L may occur with probability p, where $L \in (0, W_0]$, the DM decides to exert a self-protection effort e in order to reduce the probability of the loss. The cost of making effort is $C(e)$, that is an increasing and convex function. Thus, the total wealth of the DM is given by $W_0 - L$ with

[12] Recall: The model for the study of self-protection activities in one-period framework proposed by Erlich and Becker (1972) is

$$\max_x U = p(x)u(I_0^e - x - L) + [1-p(x)]u(I_1^e - x)$$

probability $p(e)$, where $p'(e) < 0$ and $p(e) > 0$, and by W_0 with probability $1 - p(e)$ respectively. The agent chooses e in order to maximize her total expected utility:

$$\max_e Q(e) = (1 - p(e))u(W_0) + p(e)u(W_0 - L) - C(e)$$

where $U'(.) > 0$ and $U''(.) < 0$. Let be $e^\star$ the optimal level of effort maximizing her total expected utility.

When a background risk is introduced in such a context, the objective function the DM maximizes becomes :

$$\max_e \; Q(e) = [1 - p(e)]E[u(W_0 + \tilde{\theta})] + p(e)E[u(W_0 - L + \tilde{\theta})] - C(e) \quad (2.5)$$

where $\tilde{\theta}$ is the random variable representing the background risk, that distributed according to $F(\theta) : [\underline{\theta}, \bar{\theta}]$, with $f(\theta) = F'(\theta)$ and zero mean .

Lee (2012)[20] proves that under prudence, the introduction of an additive background risk increases self-protection effort. The author then extends this result also to the case of a monetary self-protection investment when wealth and consumption are complements [13]. More precisely, by assuming prudence, and the complementarity between wealth and leisure, the presence of background risk increases self-protection activity.

[13] In this case, the author considers $u(W, e)$ and not just $u(e)$ as before.

2.2 A two-period framework

All existing models in the literature focused attention on a single-period framework. However, there are situations in which a more complex analysis is needed, since the effort in prevention precedes the its effect on probability (the two effects are not contemporaneous). For example:

- *a driver attending a safe-driving course today, reduces the probability of a car accident in the future;*
- *a householder that buys a house alarm today, reduces the probability of a burglary in the future;*
- *a smoker giving up smoking today, reduces the probability of disease in the future*

In all these cases, there is a monetary expenditure today, but its effect (the reduction of loss size) is beneficial in the future. In such contexts,a one-period framework is not enough, thus, a more complex formalization is necessary. For this purpose, in 2009 Menegatti [23] developed a two-period framework for the of study optimal level of prevention.

The context is the following one. Assume that there are just two states of the world. The DM knows that she will face a risk of being either in a good (no-loss) or bad (loss) state of the world in the second period, thus have to optimally chose the level of effort e in order to reduce the probability of future loss. Thus, she maximizes the total utility $V(e)$ given by:

$$\max_e V(e) = u(W_0 - e) + p(e)u(W - L) + [1 - p(e)]u(W) \qquad (2.6)$$

where $u(.)$ is the utility function (assumed to be the same in the two periods), $p(.)$ is the probability that the event generating the loss L occurs, W_0 is safe wealth in period 0 and W is safe wealth in period 1. Optimality conditions allow to find optimal level of effort, $e^{\star}$.

Following Menegatti's proposal, Hofmann and Peter (2012)[14] introduced into the analysis of prevention effort a prob.ability technology as a function of the variables, namely of an anticipatory prevention action today (e_1), and a contemporaneous prevention action (e_2) tomorrow[14]. Thus, consider an agent with twice continuously differentiable utility $u(.)$ of final wealth. Assume two times epochs t_1 and t_2 , and the utility to be inter-temporally separable by discounting utility in the future. Utility is assumed to be increasing ($u' > 0$) and marginal utility is zero ($u'' = 0$) for risk-neutral agents, decreasing ($u'' < 0$) for risk-averse agents, and increasing ($u'' > 0$) for risk-lovers. In each time period, the agent is endowed with an initial endowment, W_i, $i = 1, 2$.

The agent faces the risk of losing a fixed amount $L > 0$ with probability p, but can influence it through an investment in prevention. There is a unique measure of the loss probability expressed through the prevention function $p(e_1, e_2)$, where e_1 is the anticipatory prevention in t_1 and e_2 is the simultaneous prevention in t_2. Assume the prevention technology p to be twice continuously differentiable, with $p_i(e_1, e_2) \equiv \partial p(e_1, e_2)/\partial e_i < 0$ and $p_{ii}(e_1, e_2) \equiv \partial^2 p(e-1, e_2)/\partial e_i^2 > 0$ given $i = 1, 2$. Marginal prevention cost is normalized to 1.

Overall expected utility to be maximized is:

$$\max_{e_1,e_2} V(e_1, e_2) = u(W_1-e_1)+\delta\{p(e_1, e_2)u(W_2-e_2-L)+[1-p(e_1, e_2)]u(W_2-e_2)\} \tag{2.7}$$

where δ is an inter-temporal discount factor.

The choice of the risk-neutral agent (the benchmark case), is obtained through usual FOCs, namely:

$$V_1 = -1 - \delta p_1 L = 0$$

[14]To better understand this framework, assume that by undertaking a safety driving course, individuals are more aware of potential traffic hazards and they are also able to react more appropriately in difficult situations. Therefore, a safety driving course is likely to enhance the efficiency of the level of caution when actually driving the vehicle. In this context, e_1 represents expenditures on a safety driving course, while e_2 captures the opportunity cost from driving carefully. Then accident probability is given by $p(e_1, e_2)$ and is decreasing in both components of prevention. If an accident occurs the individual incurs cost of L that might result from liability against the victim of the accident.

$$V_2 = -\delta - \delta p_2 L = 0$$

where p_1 and p_2 $\partial p(e_1, e_2)/\partial e_i$, $i = 1, 2$.

For the risk-neutral agent, discounted marginal anticipatory prevention equals marginal contemporaneous prevention. The intuition is that, as marginal cost is equal for the two prevention techniques, only time preferences determine how much is marginally spent on prevention in the two periods. With no other assumptions on the structure of overall technology p, the two prevention effects are balanced with respect to discount factor.

Prevention and other fields in a two-period framework

The introduction of a two-period framework for the study of the optimal allocation of prevention activities, induced the necessity to extend the whole analysis regarding effects of interaction with other fields, from a one-period framework to the two-period framework.

2.2.1 Risk-aversion

The impact of risk-aversion attitude on optimal self-protection allocations in a two-period framework was studied by different authors.

Menegatti (2012) [24] investigated self-protection decisions of a risk-averse agent, showing that, in a two-period framework there is an endogenous threshold value for the probability level that a risk neutral agent choses, which separates different possible choices of a risk averse agent.

In a two-period framework, the DM's maximization problem is:

$$\max_e \; V(e) = u(W_1 - e) + \beta\{p(e)u(W_2 - L) + [1 - p(e)]u(W_2)\} \tag{2.8}$$

where $u(.)$ is the utility function, $p(e)$ is the probability that a loss L occurs, $W_1$and W_2 are the wealth levels in the two periods respectively, while

$\beta \in (0, 1]$ is the subjective inter-temporal discount factor. Let us assume $u'(.) > 0$ and $u''(.) < 0$ for risk-averse individual ($u'' = 0$ for the risk-neutral agent). Let us assume also $p'(e) < 0$ so that an increase in effort reduces the probability of the loss.

First order optimality condition is

$$\beta p'(e)[u(W_2 - L) - u(W_2)] - u'(W_1 - e) = 0.$$

In order to compare allocations of the two agents with different attitude toward risk, one more assumption is needed, namely, that in absence of risk, the optimal level of prevention chosen by the risk-neutral agent is optimal also for the risk-averse one, then the following link follows

$$W_1 = W_2 - p(e_n)L - e_n$$

where e_n is the effort of the risk-neutral agent. This condition shows that to isolate difference in attitude toward risk between two agents, there is the need to assume the two levels of wealth to be equal in the two periods.

Under these assumptions, Menegatti (2012)[24] showed that there exists a threshold level for $p(e_n)$ (and thus for e_n) which separates the case where a risk averse agent exerts more(less) effort in prevention than a risk neutral agent. When the level of prevention chosen by the risk neutral agent is below the threshold (above the threshold), the risk averse agent chooses an lower (higher) effort level. It is interesting to note that same results were obtained also in a one-period framework by Jullien et al. (1999)[16]. The intuition is the same: prevention does not reduce risk (in Rotschild-Stiglitz sense), and in some cases, the risk can even increase.

The risk-aversion's impact on self-insurance and self-protection decisions in a two-period framework was studied also by Hofmann and Peter (2012b)[15]. Authors extended Dionne and Eeckhoudt (1985) [9] results.

The DM is risk-averse and she maximizes the expected utility of final wealth ($u' > 0, u'' < 0$) defined as follows:

$$\max_{x} u(W_1 - C(x)) + \delta\big\{p(x)u(W_2 - L) + [1 - p(x)]u(W_2)\big\} \tag{2.9}$$

where $C(x)$ is the cost of self-protection activity, with $C'(x) > 0$ and $C''(x) > 0$. Utility u over both periods is separable and future utility is discounted by δ where $(0 < \delta \leq 1)$, while the probability of loss is assumed to satisfy $p'(x) < 0$ and $p''(x) > 0$.

Authors proved that V (the more risk-averse individual in Arrow-Pratt sense [15]) invests more (less) in self-protection activity than U if and only if k' evaluated at U's utility of wealth today is below (above) 1. This result is similar to the static model studied by Dionne and Eeckhoudt (1985)[9], namely, that the effect of increased risk-aversion on self-protection is ambiguous and crucially it depends on k'. Then, if current consumption is too low, increased risk-aversion lowers self-protection to save on consumption today (consumption smoothing incentive dominates), while if consumption today is sufficiently high, more risk-aversion increases self-protection.

The same authors, in an another paper(Hofmann and Peter (2012)[14]), investigated and compared optimal allocation of the risk-neutral agent with those of a more risk-averse agent in the case in which the prevention technology is function of both anticipatory (e_1) and contemporaneous prevention effort (e_2). As mentioned before, in such a context, overall expected utility to be maximized by the DM is:

$$\max_{e_1,e_2} V(e_1, e_2) = u(W_1-e_1)+\delta\big\{p(e_1, e_2)u(W_2-e_2-L)+[1+p(e_1, e_2)]u(W_2-e_2)\big\}$$

In such a context, authors found that a risk-averse agent invest more on discounted anticipatory prevention than on contemporaneous prevention if and only if marginal utility today exceeds expected marginal utility tomorrow. Thus, optimal prevention in the two-dimensional case has to satisfy individually and jointly the marginal-cost-equals-marginal-benefit relation.

[15] An agent with utility V exhibiting greater risk-aversion than U , where V can be represented as a concave transformation of U , *i.e.*, $V = k(U)$ with $k' > 0, k'' < 0$.

Comparative statics results show that wealth today is positively related to prevention today and also to prevention tomorrow if $p_{12} < 0$. This last condition, combined with a weaker discounting in time implies more prevention in both periods, while combined with moderate risk aversion makes both today and tomorrow prevention to increase in the loss size. A higher period-1 wealth implies more prevention both today and in the future. The most important result however is the fact that sufficient condition for the two types of prevention (prevention today and in the future) to be complementary is $p_{12} < 0$. This condition depends on prevention technology and is independent from individual preferences.

2.2.2 Prudence

Most important results obtained by Menegatti (2009)[23] regard relationship between prevention effort and prudence in a two-period framework.

Recall the maximization problem the DM is facing:

$$\max_e V(e) = u(W_0 - e) + p(e)u(W - L) + [1 - p(e)]u(W). \tag{2.10}$$

Assume that $u'(.) > 0$, and $u''(.) = 0$ for risk-neutral agents, $u''(.) > 0$ for risk-lover agents and $u''(.) < 0$ for risk-averse agents respectively. Assume also that an increase in effort reduces the probability that loss will occur, namely that $p'(e) < 0$, and that $V''(e) <0$, ensuring that an optimal level of effort exists. This formulation of the problem implicitly assumes null inter-temporal subjective discount rate and excludes the possibility of saving.

Assume that for the optimal effort chosen by the risk-neutral agent, the expected wealth is equal in the two periods, thus :

$$W_0 = W - p(e_n)L + e_n.$$

This last assumption is needed in order to exclude the effect of differences in the inter-temporal elasticity of substitution and it makes sure that in the absence of risk, the non-risk-neutral agent chooses the same level of effort as the risk-neutral agent.

In such a context, Menegatti (2009) [23] found that the effect of prudence (defined as $u''' > 0$) on self-protection activities is opposite if compared to the one-period results obtained by Eeckhoudt and Gollier (2005)[10]. More precisely, Menegatti shows that for a loss probability that is equal 1/2, a prudent (imprudent) agent, whatever her risk aversion, chooses a higher (lower) level of self-protection than a risk neutral agent. The explanation is that, a prudent agent desires a larger wealth in the period where he bears the risk. In a two-period framework, more effort reduces wealth in the first period when there is no risk and increases expected wealth in the second period when the agent bears the risk, implying that a prudent agent exerts more effort than a risk-neutral one.

Thus, it is possible to say that the impact of prudence on optimal effort strictly depends on whether the effect of the prevention is contemporaneous or lagged with respect to the effect on its probability.

These results were further developed by Menegatti (2012)[24], by proving that there is an endogenous and utility-dependent threshold level for $p(e_n)$, and the optimal choice for a risk averse agent is different, below or above this threshold ($p(e_n) = 1/2$). Moreover, prudence/imprudence affects the level of the threshold. The explanation for the opposite effect of prudence in a two-period framework (with respect to the one-period framework) is that, since effort and risk occur in different periods, the prudent agent is willing to increase effort in the "no loss" period , so as to increase wealth in the period where loss may occur.

2.2.3 Savings

To deal with risk in a two-period framework, an agent can use also "savings", defined as a transfer of wealth from the present to the future: it usually involves a return, proportional to the transfer and determined by the interest rate. Thus, the relationship between prevention and saving can be discussed only in a two-period framework (Menegatti and Rebessi (2011)[25]).

Consider an agent facing a two-period framework (today and tomorrow).

He is endowed with a certain level of wealth in t_1, while in t_2 the level of wealth is uncertain because the agent faces two states of the world: a bad state and a good one. The bad state (loss) occur with probability p, while the good one (no loss) with $1-p$ respectively Assume the endowment of wealth in the two periods to be W_1 and W_2 respectively, where $W_1, W_2 \in R^+$ and that $L \in (0, W_2)$. Assume also agent's preferences in the two periods to be described by $u(.) : R^+ \rightarrow R$ which is assumed to be of class C^2 (twice differentiable functions with continuous second derivatives). The function $u(.)$ is assumed to be strictly increasing and strictly concave.

In absence of instruments to deal with risk, agent's total expected utility $V(.)$ is given by

$$V = u(W_1) + \frac{1}{(1-\rho)}\Big[pu(W_2 - L) + (1-p)u(W_2)\Big]$$

where $\rho \in [0, 1)$ is the subjective discount rate.

By assuming that the agent decides to exert some effort e in order to reduce the probability of the loss, the maximization problem she faces becomes:

$$\max_e V(e) = u(W_1 - e) + \frac{1}{(1-\rho)} p(e)u(W_2 - L) + [1 - p(e)]u(W_2) \quad (2.11)$$

where $e \in [0, w_1]$, $p(.) : R^+ \rightarrow (0, 1)$ is of class C^2. Assume also $p'(e) < 0$ so that an increase in effort causes a reduction of the probability of loss.

If the agent decides to adopt also "savings"(s) as an instrument to deal with risk contemporaneously to prevention effort, the objective function to be maximized becomes:

$$\max_{s,e} V(s, e) = \left\{ u(W_1 - s - e) + \frac{p(e)u(W_2 + s(1 + r) - L) + [1 - p(e)]u(W_2 + s)(1 + r)}{1 + \rho} \right\} \quad (2.12)$$

where $s \in [o, W_1]$, and $r \in [0, 1)$ is the return interest rate.

The two first order optimality conditions are:

$$u'_s(I_1) = \{p(e)u'(I_{2B}) + [1 - p(e)]u'(I_{2G})\}\frac{1+r}{1+\rho}$$

$$u'_e(I_1) = p'(e)\frac{u(I_{2B}) - u(I_{2G})}{1+\rho}$$

where $I_1 = W_1 - s - e$, $I_{2B} = W_2 + s(1+r) - L$, and $I_{2G} = W_2 + s(1+r)$.

To be noted that if $I_{2B} = I_{2G}$, there are no incentives to exert some effort.

The main result in this work refers to the relationship between the optimal levels of the two instruments: Menegatti and Rebessi (2011)[25] found that there is a kind of substitution effect between prevention and saving in equilibrium. In fact, if the two equilibria exist, they are characterized by a decreasing relationship between saving and prevention [16]. Moreover, a change in r has an opposite effect on the two instruments: Saving is increasing while prevention is decreasing. The effects on e and s of changes in W_1, W_2 or L are ambiguous since more assumptions are needed.

2.2.4 Background risk and other risks

Relying on Menegatti's (2009)[23] formalization of prevention activity in a two-period framework, Eeckhoudt et al.(2012)[11] analyzed conditions under which the introduction of an independent background risk increases ex-ante prevention effort .

In a two-period framework, assuming that future utility is discounted at a rate δ, the individual's optimization problem is written as :

$$\max_e V = u(X_0 - e) + \frac{1}{1+\delta}\left\{p(e)u(X) + [1 - p(e)]u(X - L)\right\} \tag{2.13}$$

where X_0 or X are the two levels of endowed income . Assume that u and $p(e)$ are increasing and concave functions. Usual FOC allow to obtain

[16] If saving is larger in one equilibrium than in another, then prevention is smaller.

optimal level of effort $e^\star$.

Suppose that in future, the DM faces an independent zero-mean background risk $\tilde{\epsilon}$ in addition to the endogenous risk L. The model becomes:

$$\max_e \hat{V} = u(X_0 - e) + \frac{1}{1+\delta}\{p(e)EU(X+\tilde{\epsilon}) + [1-p(e)]EU(X-L+\tilde{\epsilon})\} \quad (2.14)$$

For a prudent agent, the introduction of this additional source of risk generates more effort for any positive value of the loss L.

In order to understand effects of a deterioration of the background risk, ore restrictions on u are required, since it constitutes a more general notion than its introduction.

Almost simultaneously to Eeckhoudt et al.(2012)[11] and relying on the Menegatti's (2009) [23] formalization of prevention activity, Courbage and Rey (2012)[6] studied, the impact of the presence of other risks on optimal prevention activity.

The benchmark model in this context is the following one. Assume that the agent chooses the effort level e in order to maximize her total intertemporal utility, $V(e)$.

Assume that exerted effort for preventive purposes in first period will produce its effect on the probability in the second one. The total utility is given by :

$$\max_e V(e) = u(W_1 - e) + p(e)u(W_2 - L) + [1 - p(e)]u(W_2) \quad (2.15)$$

where u is the utility function in the two periods. In both periods, assume that the utility is increasing concave. Assume also that $p'(e) < 0$ and $p''(e) > 0$ $\forall e$ level.

The optimal prevention allocation is obtained by

$$V'(e) = \underbrace{-u'(W_1 - e)}_{\text{the marginal cost of prevention}} + \underbrace{p'(e)[u(W_2 - L) - u(W_2)]}_{\text{expected gain due to a reduction in } p} = 0.$$

Given assumptions on u, V and $p(e)$ the second order conditions $V''(e) < 0 \ \forall e$ is always satisfied and optimal level of prevention $e^\star$ is determined by $V'(e^\star) = 0$.

Let us assume that the DM faces also other sources of risk, namely a background risk in first period $\tilde{e_1}$, $\tilde{e_{2b}}$ the additional risk in second period if a loss occurs, while $\tilde{e_{2g}}$ the additional risk in the in the second period if no loss occurs. Her objective function to be maximized becomes

$$V_1(e) = E[u(W_1 - e + \tilde{e_1})] + p(e)E[u(W_2 - L + \tilde{e_{2b}})] + [1 - p(e)]E[u(W_2 + \tilde{e_{2g}})] \tag{2.16}$$

where E denotes the expectation operator over the r.v. $\tilde{e_k}, k = (1, 2b, 2g)$.

The introduction of all these sources of risk make the wealth level of the DM to be more uncertain. The impact of a background risk on optimal prevention depends on the period when it is introduced. If and only if the individual is prudent, the introduction of background risk in the first period reduces the effort of prevention, while its introduction in the second period makes the prevention effort to increase. The explanation is that, the introduction of the background risk in the first period affects the expected marginal cost of prevention, while its introduction in the second period affects the expected marginal benefit of prevention without modifying the relative marginal cost.

What happens if risk occurs either in the state of loss, or in the no-loss state? By assumption, this may occur only in the second period. For a risk averse DM, the introduction of an additional risk in the loss state of nature increases her optimal effort level of prevention ($\tilde{e_{2b}} \geq e^\star$), while the respective introduction in the no-loss state of nature decreases her effort ($\tilde{e_{2g}} \leq e^\star$). Exactly as before, the introduction of a state-dependent risk affects only the expected marginal benefit of prevention and leaves unchanged its marginal cost.

When the introduction of a background risk is such that $\tilde{e_1} = \tilde{e_{2b}} = \tilde{e_{2g}} = \tilde{e}$, authors found that it is not possible to dissociate prudence-neutrality in one period from prudence in the other period. More precisely, if the DM is prudent in the first period and she is prudent-neutral in the second one, the introduction of a background risk in both periods reduces the level of prevention ($e_1^\star \leq e^\star$), while in the case in which the DM is prudent-neutral in the first period and she is prudent in the second one, she increases the level of prevention. To make more general considerations, however, more restrictive conditions than prudence are required.

Xue and Cheng's (2013)[29] extended results obtained by Eeckhoudt et al.(2012)[11], to Authors examined optimal prevention the case in which there are two sources of uncertainty: one is the endogenous loss risk L, and the other is an exogenous background risk (such as the variability of health status or environmental quality).

Consider an economic agent who has a bivariate vNM utility function with two attributes, $u(W, Y)$, where W and Y respectively represent wealth and the other attribute. In the first period, the agent chooses the effort level e in order to maximize the total expected utility given by:

$$\max_e \; U(e) = u(W_0 - e, Y_0) + p(e)u(W_1, Y_1) + [1 - p(e)]u(W_1 - L, Y_1) \quad (2.17)$$

where W_0 and Y_0 are the initial values of wealth of the individual and another attribute, respectively, and $L \in (0, W_1]$. The inter-temporal discount rate and interest rate are assumed to be null. The probability function $p(e)$ is assumed to be increasing and concave in e. Usual FOC allow to obtain the optimal level of effort $e^\star$.

Assume that besides L, the individual faces also an independent exogenous risk in Y_1 . The introduction of an independent background risk induces more effort for wealth loss ($e^{\star\star} \geq e^\star$) if, and only if, the agent is cross-prudent in wealth. The explanation is that, the introduction of background risk increases the marginal benefit of prevention, without affecting the marginal cost of prevention. Thus, the cross-prudent in wealth agent, exerts more effort for prevention activities, because want to improve the probability of occurrence of the good outcome, and mitigate the negative effect caused by the introduction of background risk. This extra-effort $(e^{\star\star} - e^{\star})$ is called positive "precautionary effort due to background risk". Moreover, they found sufficient condition for the deterioration of the independent background risk to lead to more prevention effort.

Authors proved that the introduction (or deterioration) of an independent background risk induces more prevention effort (to protect against endogenous loss) if the DM exhibits a correlation aversion of some given order. An increase in the correlation between endogenous risk L and background risk leads to a reduction in optimal prevention: positive correlation decreases the optimal prevention while negative correlation increases it.

More recently, Courbage et al.(2013)[7], have investigated optimal prevention expenditures in a two-period framework with multiple correlated risks. Following Menegatti's (2009)[23] formalization, in a two-periods framework (t_1 and t_2), consider a DM facing two sources of risk: the first risk, given as $\tilde{\epsilon}$ may produce a loss L ($L > 0$) with probability p, while the second risk (symmetric in structure) is given by $\tilde{\zeta}$ and may produce a loss G ($G > 0$) with probability q. Assume the two risks to be correlated. Let k be a exogenous to the analysis measure of interdependence. Thus, four states of nature are possible , each of them with a specific respective probability:

- Both losses of L and G with probability kpq,
- only the loss of L with probability $p(1 - kq)$,
- only the loss of G with probability $q(1 - kp)$,
- and no loss at all with probability$1 - p - q + kpq$.

Under the previous assumptions, the correlation coefficient between the two risks is given by:

$$\tau(\tilde{\epsilon}, \tilde{\zeta}) = (k - 1)\sqrt{\frac{p}{1-p} \times \frac{q}{1-q}}$$

Thus, $k = 1$ indicates uncorrelated risks, $k > 1$ a positive correlation, while $k < 1$ negatively correlated risks. The strength of correlation is monotonically linked to the size of k for the given loss probabilities p and q.

Assume the utility to be separable across time and assume first-period preferences to be given by the vNM utility function u while the second-period preferences by the vNM utility function v. Assume also that the utility is increasing in wealth in each period and the individual is risk-averse in both periods. Therefore, the individual's EU in such a context becomes:

$$u(W_0)+(1-p-q+kpq)v(w_{NN})+p(1-kq)v(w_{LN})+q(1-kp)v(w_{NL})+kpqv(w_{LL})$$

where:
$w_{NN} := W_2$;
$w_{LN} := W_2 - L$;
$w_{NL} := W_2 - G$ and $w_{LL} := W_2 - L - G$.

Subscript N denotes "no-loss" and subscript L denotes "loss". The first letter of the subscript refers to the first risk ($\tilde{\epsilon}$), the second letter of the subscript refers to the second risk ($\tilde{\zeta}$).

Assume that the individual decides to invest in self-protection activities against just one of the two risks, namely, the DM invest x at t_1 so to reduce the loss probability $p(x)$ in the next period. The individual's maximization problem is therefore given by:

$$\max_x \{u(W_0 - x) + (1 - p(x) - q + kp(x)q)v(w_{NN}) + p(x)(1 - kq)v(w_{LN}) \\ + q(1 - kp(x))v(w_{NL}) + kp(x)qv(w_{LL})\}$$

where for more simplicity :
$\alpha := v(w_{NN}) - v(w_{LN})$;
$\beta := v(w_{NL}) - v(w_{LL})$;
$\gamma := v(w_{NN}) - v(w_{NL})$;
and $\delta := v(w_{LN}) - v(w_{LL})$

Let $x^\star$ denote the optimal level of self-protection expenditures.

Authors proved that, independently of the sign of correlation, if an exogenous risk $\tilde{\zeta}$ is introduced for any $k > 0$, optimal self-protection expenditures for the endogenous risk increase. They also proved that if the overall risk faced by the individual is higher because of either a higher loss of risk $\tilde{\zeta}$, a higher loss probability of risk $\tilde{\zeta}$ or a higher dependence between the two risks, the DM decides to invest more prevention against the exogenous risk $\tilde{\zeta}$.

If both risks are endogenized, the DM may reduce p and q loss probabilities (tomorrow) through an investment today, x and Y respectively, under the assumption of decreasing returns on self-protection [17]. The maximization problem of the DM is thus given by

$$\max_{x,y}\Big\{u(W_0 - x - y) + (1 - p(x) - q(y) + kp(x)q(y))v(w_{NN}) \\ + p(x)(1 - kq(y))v(w_{LN}) + q(y)(1 - kp(x))v(w_{NL}) + kp(x)q(y)v(w_{LL})\Big\}.$$

The FOC allow to obtain the optimal expenditures on self-protection $x^\star$ and $Y^\star$ respectively.

Authors define conditions under which the level of prevention directed at one risk is higher than the level of prevention directed at the other risk. Moreover, authors showed that preventive expenditures for the two risks ($\tilde{\epsilon}$ and $\tilde{\zeta}$) are substitutes.

The increased dependence between the two risks increases prevention expenditures for the more efficient technology, while the effect on the less efficient technology is ambiguous. A higher dependence increases overall prevention expenditure, but the impact on prevention activities is not the same.

[17]Define the (technical) rate of return on self-protection expenditures x by: $\rho(x) := -\dfrac{p'(x)}{p(x)}$ and analogously, $\sigma(Y)$ for loss prevention expenditures Y. Assuming decreasing returns on self-protection means $-\dfrac{p'(x)}{p(x)} < -\dfrac{p''(x)}{p'(x)}$

2.3 Optimal prevention allocation: a non EU analysis

Most part of the existing literature, regarding not only self-prevention activity but also other topics, focused attention on the analysis under the EUT assumptions. This theory however was criticized by experimental economics because empirical evidence proved that agents systematically behave differently that EUT assumes. As an answer to these problems, alternative models were proposed.

2.3.1 A Rank dependent utility approach

The rank-dependent expected utility (RDEU) (or anticipated utility) was introduced as a generalized EU model of choice under uncertainty, with the aim to incorporate empirically observed Allais Paradox (violation of independence axiom of individuals behavior under EUT). This theory was described and axiomatized by Quiggin [1982][27], and found empirical evidence in several works.

The RDEU is a generalized EU model of choice under uncertainty, designed to answer to the empirical evidence of the Allais Paradox, namely, to the empirical evidence of individuals assigning weights both to expected outcomes and to respective associated probabilities. Such a behavior clearly violates the EUT's independence axiom, since it requires EU functions to be linear in probabilities. Solution to this problem could be find if the independence axiom is relaxed, allowing in this way the probabilities to influence the function in a nonlinear manner, thus, the agents' indifference curves are no more required to be linear nor parallel.

As the name suggests, the rank-dependent model applies to the increasing order of outcomes. This implies that the DM rearranges all outcomes from the lowest to the highest one, in the order of increasing utility. Then, cumulative probabilities of respective outcomes are transformed by a subjective weighting function f that preserves monotonicity of the utility function and stochastic dominance. The f function, $f : [0, 1] \to [0, 1]$, is continuous, strictly increasing with $f(p(0)) = 0$ and $f(p(1)) = 1$.

The RDEU of a prospect then is given by

$$U = \sum_{i=1}^{n} \pi_i u(x_i)$$

where x_i is the payoff of a specific outcome, and $u(.)$ is strictly increasing and concave. Decision weights π_i are computed as follows :

$$\pi_i = f(p_i + ... + p_n) - f(p_{i+1} + ... + p_n)$$

for all i. The value $f(p_i + ... + p_n)$ is the subjective weight attached to the probability of getting a payoff at least as good as x_i , while the value $f(p_{i+1} + ... + p_n)$ is the subjective weight attached to the probability of receiving a payoff strictly better then x_i .

The intuition of RDEU entails that the attention given to a specific outcome depends not only on its probability but also on its probability in comparison to other possible outcomes. Thus, the value of an outcome for the DM depends not only on the probability of realizing that specific outcome but depend also on its ranking relative to other possible outcomes. Weights attached to outcomes, reflect agents' perception of those probabilities, namely, how "good" or "bad" a certain outcome is perceived relative to other outcomes. Since weights are assigned subjectively, different agents may assign to the same outcomes different weight.

The weighting of probabilities is linked to individual's attitude toward risk. The attitude towards risk In the EUT depends entirely on the curvature (concavity) of the agent's utility function. More precisely, it is assumed to be quantified by the coefficient of risk aversion in the Arrow-Pratt sense. In RDEU instead, concave utility functions imply risk aversion but its shape alone does not indicate an agent's risk preference. A crucial component is also the weighting function. The transformation function f transform true probabilities of possible outcomes into subjective probabilities, and it allows to capture the "pessimistic' or "optimistic" behavior of the DM [18].

[18] There are different weighting function's formalizations in the literature, each of them

By its nature, a concave f function enlarges the probabilities of outcomes with low value (or wealth) and reduces those with high wealth. A convex weighting function describes probabilistic risk aversion (or pessimism) of an agent, while a concave weighting function characterizes probabilistic risk proneness (or optimism). A linear weighting function characterizes probabilistic risk neutrality and works as a benchmark to measure optimist or pessimist behavior.

Empirical evidence suggests that people overweight small probabilities, namely, they give a larger weight to small probabilities, while underweight large probabilities. In order to capture such a behavior, usually are used inverted S-shaped probability weighting functions that crosses the 45 degree line in $f(p^\star) = p^\star$ where the probability is not distorted and subjective weights coincide. Below $p^\star$ the agent is pessimistic, while above $p^\star$ she is optimistic. More precisely, people are risk averse for gambles that yield losses with small probabilities (be struck by a lightning), while are risk seeking for gambles that yield gains with small probabilities (win the Italian Public Lottery prize). The intuition is that, agents assign greater weight to rare events for which they are less experienced and as consequence they are more anxious.

For all its features, RDEU is a good method to adopt in order to study optimal allocation of self-protection activity. Since these decisions regarding prevention optimal allocation have empirical relevance, an empirically tested method could be more appealing.

with the own characteristics. The use of one or of an another specification is up to the designer needs and depends of the aim of the research.

RDEU and prevention activity

The incorporation RDEU within the economic literature regarding self-insurance and self-protection activities was done by Konrad and Skaperdas (1993) [19]. Authors studied properties under which RDEU can be applied to the study of these activities.

The basic model adopted is the one introduced by Ehrlich and Becker (1972)[12] in a one period framework. Thus, a DM facing a loss, can decide to allocate a certain amount in self-protection activity x, such that the probability of loss occurrence p will be lower. The probability of loss, $p(x)$, is a decreasing function of his expenditures x so that $p'(x) < 0$. With RDEU, the agent chooses the level of self-protection x so as to maximize:

$$E_f U = f(p(x))u(W - L - x) + (1 - f(p(x)))u(W - x) \tag{2.18}$$

where the function u is continuous, increasing and concave in outcome. The function f transforms probabilities of the true probability function of possible outcomes into subjective probabilities, and $f : [0, 1] \to [0, 1]$ is continuous and increasing.

Authors proved that the already mentioned ambiguous effects of increases in risk aversion on self-protection activity obtained in o one-period framework under EUT, extends also to the case of RDEU even if the u function is linear.

Unambiguous results however can be obtained in the case of limited liability, namely, if loss occurs, the individual gets a minimum level of wealth c instead of $W - L - x$. For a given RDEU, a level of self-protection is chosen such that $W - L - x^{\star} \leq c$, then, an increasing in risk aversion leads to an increase in self-protection activity. Thus, an increase in the difference between exogenous wealth and the minimum wealth level increases self-protection investment.

More recently, Etner and Jeleva (2012)[13] adopted RDEU model to study the impact of risk perception on self-protection activities in a one-period framework. They prove that people evaluate the probability of loss occurrence in different ways, namely, each of the DMs has a different risk-perception.

In order to compare the optimal allocations of individuals with different risk attitude, authors introduce a characterization of individuals. More precisely, they assume that there are two types of individuals: the "realist" and the "fatalist". The difference between them consists in the accuracy of risk evaluation. More precisely, the risk-neutral is characterized by "correctly" and "objectively" evaluating probabilities modifications, while the fatalist (or risk-averse individual) under-reacts to such probability modifications because do not evaluate objectively probabilities. Fatalism and risk-aversion are different concepts. Fatalism is related to the slope of the probability transformation function that measures the individuals assessment of probability changes. Thus, to compare the fatalism of two individuals no condition on their risk aversion is required.

To determine the impact of fatalism on optimal self protection, the following situation is presented. An agent is endowed with a level of wealth W. Knowing that she could face a risk of loss L in second period with probability p, decides to exert an effort e in first period such that the loss may occur with a lower probability. Assume also that the characteristics of the loss and of prevention technology are such that the optimal level of effort is strictly positive.

For an individual with RDEU preferences, utility function u and probability transformation function ϕ, the optimal level of effort is obtained by imposing first order optimality conditions to the following optimization problem:

$$\max_e V(e) = u(W - L - e) + \phi(1 - p(e))[u(W - e) - u(W - L - e)] \quad (2.19)$$

The most important result authors get is that under RDEU assumptions over individuals' preferences, the impact of fatalism is well determined: considering a prevention decision, consisting in the reduction of the probability of a monetary loss, "fatalist" individuals invest less in prevention activity than risk-neutral agents. Authors conclude that under-reaction to probability modification can be an explanation for low investment in prevention.

2.3.2 A Dual Theory approach

The Dual Theory (DT) was introduced in economic literature by Yaari (1987)[30] . The aim was to rationalize "paradoxes" regarding independence assumption violation obtained in an EUT context and thus propose a theory with an empirical evidence [19] .

Differently than EUT, DT is linear in wealth but non-linear in probabilities. More precisely, consider a vector $W = (w_1, ..., w_n)$ representing wealth, such that $w_1 < ... < w_n$, and with the probability distribution $p = (p_1, ..., p_n)$. Using $u(.)$ as a transformation of wealth, EU is expressed by:

$$EU(W) = \sum_{i=1}^{n} p_i u(w_i) \tag{2.20}$$

where risk-aversion is denoted by $u'(.) > 0$ and $u''(.) < 0$.

In Yaari's DT instead, probabilities are weighted by a transformation function $h(.)$, which is defined on the cumulative distribution function over wealth :

$$DT(W) = \sum_{i=1}^{n} h_i(p) w_i \tag{2.21}$$

with

$$h_i(p) = f\left(\sum_{j=1}^{i} p_j\right) - f\left(\sum_{j=1}^{i-1} p_j\right)$$

where $f(0) = 0$, $f(1) = 1$ and $f'(.) > 0$. In the case with two outcomes,

$$DT(W) = f(p_1)w_1 + (1 - f(p_1))w_2$$

Under DT, an individual will be considered as risk averse when $f'' < 0$.

[19]Empirical proof came later thanks to the work of other authors.

DT and prevention activity

Courbage(2001) [5] introduced the DT of choice under risk in the analysis of market insurance, self-insurance and self-protection with the aim to examine the robustness of results obtained by Ehrlich and Becker (1972)[12].

In the case in which both self-protection and market-insurance are available, assume that the individual can adopt a self-protection activity x that allows him to reduce the probability of loss $p(x)$, but do not affect the size of the loss L. The probability of the loss is a decreasing function of the level of self-protection. The cost of self-protection activity is given by $C(x)$, where $C'(x) > 0$ and $C(x) \geq 0$. The individual pays a premium P to have a proportion α of loss insured, with $\alpha \in (0, 1)$.

The valuation function is this framework is given by

$$U = f[p(x)]\big[W_0 - C(x) - P - (1-\alpha)L\big] + (1 - f(p(x))\big[W_0 - C(x) - P\big] \quad (2.22)$$

with

$$P = \alpha(1+\lambda)p(x)L$$

where λ is the loading factor to allow for transaction costs and profit.

Deriving the valuation function with respect to α gives the optimal insurance purchase. While, optimal self protection level is obtained by solving $\frac{\partial U}{\partial x} = ... = 0$. The second order optimality condition requires: $\frac{\partial^2 U}{\partial x^2} = ... < 0$

To know the relation between market-insurance and self-protection activities, the interaction between them must be stressed. The author obtained that when the loading factor is relatively small, market insurance and self-protection are substitutes. Once an upper limit is passed self-protection becomes independent of λ. Instead, when the probability of loss is high, market insurance and self-protection are complements. This result may be explained through the role of the transformation function in under/overestimating probabilities and their variation.

Thus, the robustness of Ehrlich and Becker (1972)[12] results is verified under the DT of choice under risk.

Chapter 3

The model

Introduction

As discussed before, most part of the existing literature regarding prevention activity investigated optimal effort under EUT. From the empirical point of view, however, systematic violations of its axioms suggest that this theory does not perfectly fit the real behavior of a DM. This make inevitably to ask how results obtained till now could change, when a theory, different that EUT, will be introduced.

In a one-period context of self-protection analysis when applied RDEU, Konrad and Skaperdas (1993) [19] obtained the same results as Erlich and Becker (1972)[12] under EUT, namely that an increase in risk-aversion has the same ambiguous results as in the EUT case. In 2012 however, Etner and Jeleva (2012)[13] when studying risk perception, found that under RDEU, the impact of fatalism is well determined.

In a two-period context however, to my knowledge, with respect to self-protection analysis no attempt was done. The aim of this work is to try to fulfill this gap, and see whatever results obtained by Menegatti 2012[24] are the same even under RDEU. Results obtained through a mathematical reasoning will be then interpreted in a behavioral key.

Since in a one-period framework the use of RDEU gave the same results as EUT, it will be not surprising to found analogous results. However, thanks to Etner and Jeleva's work [13], a more detailed interpretation could be done, thus results regarding the probability threshold to the weighting function are extended.

3.1 The model and results for a risk neutral agent

Consider a two-period framework in which an individual can decide to take a prevention action e (today) in order to lower the probability of loss occurrence (p) tomorrow. More precisely, in period 1 the agent's wealth W_1 can be lowered by a loss L with probability $p(e)$, or remains unchanged W_1 with probability $1 - p(e)$. The wealth in period 1 is thus a lottery, with two possible outcomes, and in such a way it will be treated.

The DM chooses the optimal levels of prevention effort e ($0 \leq e \leq W_0$) in order to maximize her own total utility $U(e)$ by considering both periods. Following Menegatti (2009)[23], her maximization problem is:

$$\max_e \; U(e) = u(W_0 - e) + \beta\{p(e)u(W_1 - L) + [1 - p(e)]u(W_1)\}$$

where $u(.)$ is the utility function, $p(e)$ is the probability that a loss L occurs, W_0 and W_1 are the wealth levels in the two periods respectively, while $\beta \in (0, 1]$ is the subjective inter-temporal discount factor.

Let $f(p(e))$ be a generic subjective transformation function, $f : [0, 1] \rightarrow [0, 1]$ continuous and increasing, such that $f(p(0)) = 0$ and $f(p(1)) = 1$. The weights assigned to the possible outcomes are such that their sum is equal to 1. By computing weights of possible outcomes as described before, the subjective weight assigned to the state of no loss occurrence is $f(1 - p(e))$, and $1 - f(1 - p(e))$ respectively is the weight associated to the state in which the DM incurs a loss[1].

[1]In the formalization of the problem, just $f(1 - p(e))$ will be used, and will help to make considerations about the other weight indirectly because we know that their sum must be equal to 1.

In RDEU terms, the objective function to be maximized by the DM becomes:

$$\max_{e} U(e) = u(W_0-e)+\beta\left\{u(W_1-L)+f(1-p(e))[u(W_1)-u(W_1-L)]\right\} \quad (3.1)$$

where $f(1-p(e))$ is the subjective weighting function, $W_1 - L$ is the worst outcome while W_1 is the best one. As usual, it is assumed that $u'(.) > 0$ and $u''(.) < 0$ for risk-averse individual, while $u'' = 0$ for the risk-neutral agent. The probability function is assumed to be decreasing in the level of effort. Assume also that $0 < e \leq W_0$ and $e < L$.

First order condition (FOC) allowing to find optimal effort allocation, $e^{\star}$, is

$$-u'(w_0 - e) + \beta\left\{f'(1 - p(e))(-p'(e))[u(W_1) - u(W_1 - L)]\right\} = 0 \quad (3.2)$$

If the agent is risk neutral, after some computations, FOC reduces to:

$$\beta f'(1 - p(e))p'(e) = -\frac{1}{L} \quad (3.3)$$

It constitutes the link between the exerted effort, the subjective weighting function and the loss.

In order to compare optimal allocations of the risk averse DM versus risk neutral DM, some further assumptions are needed. Difficulties arise because in the case of the risk-neutral agent we have that $u'' = 0$ while for the risk-averse one, $u'' \leq 0$. In order to compare the two optimal levels of effort, the effect of differences in inter-temporal elasticity of substitution has to be removed. Thus, it must be assumed that in absence of risk, both agents choose the same level of effort. Namely, both maximize the following objective function:

$$\max_{e} U(e) = u(W_0 - e) + \beta \Big\{ u(W_1 - L)\big[1 - f(1 - p(e))\big] \Big\} \tag{3.4}$$

While in period 0 the level of wealth is certain, in period 1 it is assumed that, instead of the lottery, we have a certain level of utility given by the expected value of the lottery.

The last assumption can be written as:

$$\left. \frac{d\Big[u(W_0 - e) + \beta\big\{u(W_1 - L)\big[1 - f(1 - p(e))\big]\big\}\Big]}{de} \right|_{e=e_n} = 0 \tag{3.5}$$

by differentiating with respect to e, when $e = e_n$, it is possible to rewrite this expression as:

$$-u'(W_0 - e_n) + \beta u' \Big\{ W_1 - L[1 - f(1 - p(e_n))] \; Lf'(1 - p(e_n)(-p'(e_n)) \Big\} = 0 \tag{3.6}$$

After some computations and by substituting (3.3), we have that:

$$W_0 = W_1 - L[1 - f(1 - p(e_n))] + e_n \tag{3.7}$$

meaning that, if no risk is involved, both agents make the same choice and have equal level of wealth, while the difference in wealth in different (null in period 0, and positive in period 1). As it is possible to see, in such a formulation of the problem, the discount factor β has no influence on results, since influencing both outcomes of the lottery.

By substituting result (3.7) and (3.3) the previously computed FOC becomes:

$$-u'(W_0 - e) + \beta f'(1 - p(e))(-p'(e))[u(W_1) - u(W_1 - L)] = 0. \tag{3.8}$$

When $e = e_n$, it becomes

$$\frac{1}{L}\left[u(W_1)-u(W_1-L)\right]-u'\left[\left(W_1-L(1-f(1-p(e)))+e_n\right)-e_n\right]=0 \quad (3.9)$$

that can be written also as:

$$\frac{u(W_1)-u(W_1-L)}{L}-u'\left[W_1-L[1-f(1-p(e_n))]\right]=0 \qquad (3.10)$$

or, by adopting integral notation we have that:

$$\frac{1}{L}\int_{W_1-L}^{W_1}u'(x)dx-u'\left[W_1-L\left(1-f(1-p(e_n))\right)\right]=0 \qquad (3.11)$$

by multiplying everything by L we have that:

$$\int_{W_1-L}^{W_1}u'(x)dx-Lu'\left[W_1-L\left(1-f(1-p(e_n))\right)\right]=0 \qquad (3.12)$$

The meaning is that, when $e = e_n$, the optimal solution adopted by both agents satisfies this last condition.

3.2 Risk averse individual compared with the risk-neutral one

In order to compare optimal allocation of the risk-averse agent with the one of the risk-neutral one, it must be evaluated the behavior of the risk-averse in optimality condition of the risk-neutral agent, namely in $e^\star$. This analysis allow to say that:

Proposition 1: $\exists p(\bar{e_n})$ such that:

- for a $p(e_n) = p(\bar{e_n})$ the FOC evaluated in $p(\bar{e_n})$ is null, thus $e_a = e_n$

- $p(e_n) > p(\bar{e_n})$ the FOC evaluated in $p(\bar{e_n})$ is negative, thus $e_a < e_n$
- $p(e_n) < p(\bar{e_n})$ the FOC evaluated in $p(\bar{e_n})$ is positive, thus $e_a > e_n$

Proof: To prove these results it is useful to focus attention on the two extreme cases, namely when $[1 - f(1 - p(e))] = 0$, and $[1 - f(1 - p(e))] = 1$. In this context, the "Mean Value Theorem for Integrals" [2] is useful.

In the case in which $[1 - f(1 - p(e))] = 0$, we have that this condition is satisfied if $f(1 - p(e)) = 1$, implying that $p(e) = 0$. In such a case, the left-hand side of optimality condition (3.12) becomes $\int_{W_1-L}^{W_1} u'(x)dx - Lu'[W_1]$. By considering the assumptions made on u, namely that is increasing and concave (because risk-averse agent), the result is that:

$$\int_{W_1-L}^{W_1} u'(x)dx - Lu'[W_1] > 0 \tag{3.13}$$

and this occurs if and only if $e_a > e_n$.

The other extreme case is $[1 - f(1 - p(e))] = 1$ and this occurs when $f(1 - p(e)) = 0$, implying that $p(e) = 1$. In such a case, the left-hand side of optimality condition (3.12) becomes $\int_{W1-L}^{W_1} u'(x)dx - Lu'[W_1 - L]$, and by taking into account the concavity of the utility function and the fact that u' is decreasing, the result is :

$$\int_{w_1-L}^{w_1} u'(x)dx - Lu'[w_1 - L] < 0 \tag{3.14}$$

and this occur if and only if $e_a < e_n$.

By investigating the behavior in the two extreme cases, we obtained that for $p(e_n) = 0$ optimality condition is positive, while for $p(e_n) = 1$ it is negative. Since $\int_{w_1-L}^{w_1} u'(x)dx - Lu'[w_1 - L(1 - f(1 - p(e_n)))]$ is continuous, by the "Zeros Theorem" we know that there is at least one zero of the function, namely a point $p(e_n)$ belonging to the interval, in which the function becomes

[2]It states that for a $g : [a, b] \to R$ continuous function, there exists a $c \in [a, b]$ s.t.

$$\frac{1}{b-a}\int_b^a g(x)dx = g(c)$$

null. To find that point, it must be study the monotonic behavior of the function

$$\int_{w_1-L}^{w_1} u'(x)dx - Lu'\left[w_1 - L\left(1 - f(1-p(e_n))\right)\right]$$

more precisely

$$\frac{d\left[\int_{W_1-L}^{W_1} u'(x)dx - Lu'[W_1 - L(1 - f(1-p(e_n)))]\right]}{dp(e_n)} =$$
$$= L^2u''(W_1 - L(1 - f(1-p(e_n))))f'(1-p(e_n)) < 0$$

This result ensures that such a point exists ($\exists p(\bar{e_n})$) and is unique. By the continuity of $p(e)$ and thus of the $f(1-p(e))$, we have that, for $p(e_n) = p(\bar{e_n})$ optimality condition is satisfied by both agents and is equal to zero. For $p(e_n) > p(\bar{e_n})$ it is negative, while for $p(e_n) < p(\bar{e_n})$ it is positive. Concavity of the utility function instead, allows to say that $e_a = e_n$ in the first case, $e_a < e_n$ in the second one, and $e_a > e_n$ in the third.

Thus, results obtained by Menegatti (2012)[24] ,not surprisingly, are true even under RDEU.

3.3 Behavioral interpretation

As a direct consequence of Proposition 1 there is:

Proposition 2: $\exists f(1-p(\bar{e_n}))$ such that:

- for $f(1-p(e_a)) = f(1-p(\bar{e_n}))$, then both individuals optimally make the same choice $e_a = e_n$
- for $f(1-p(e_a)) < f(1-p(\bar{e_n}))$, means that $p(e_n) > p(\bar{e_n})$ implying that $e_a < e_n$
- when $f(1-p(e_a)) > f(1-p(\bar{e_n}))$, means that $p(e_n) < p(\bar{e_n})$ implying that $e_a > e_n$

Proof:

The proof of this proposition is similar to the one given used to prove *Proposition 1.* More precisely, in the two extreme cases, as already mentioned before, $f(1-p(e_n)) = 0$, makes the optimality condition to be negative (see 3.14), thus $e_a = e_n$, while in the other extreme case, $f(1-p(e_n)) = 1$, the optimality condition is positive (see 3.13), ant thus $e_a > e_n$. By applying the Zeros Theorem also here, and by studying the sign of the following expression, we have:

$$\frac{d\Big[\int_{W_1-L}^{W_1} u'(x)dx - Lu'[W_1 - L(1-f(1-p(e_n)))]\Big]}{df(1-p(e_n))} =$$
$$= -L^2 u''(W_1 - L(1-f(1-p(e_n))))) > 0$$

This result means that exists a threshold, $f(1-p(\bar{e_n}))$, that separates the case in which a risk-averse DM exerts more effort in prevention activity than the risk neutral one, from the case in which she exerts a lower effort. If the decision weight assigned by the DM is lower than $f(1-p(\bar{e_n}))$, the risk-averse individual will exert an effort that is lower than the one chosen by risk-neutral. If the assigned weight is grater than $f(1-p(\bar{e_n}))$ instead, the risk-averse DM will decide to allocate an effort that is greater than the one chosen by the risk-neutral agent.

The weight $f(1-p(e))$ computed as previously described, correspond to the weight assigned to the state of "no loss". A lower subjective weight than the threshold, means that the risk-averse agent assigns more weight to the "bad state" (because the weight in this case is given by $1 - f(1-p(e))$), thus decides to make less prevention activity. The reason is that, since the risk-averse agent smooths consumption between the two periods, she want a higher wealth level in the period were she bears the risk, as proved by Menegatti (2007)[22] Since the effort and its effect on the probability occur in two different epochs, the risk-averse agent will decide thus to allocate less effort, in order to compensate future loss.

In the case in which $f(1-p(e)) > f(1-p(\bar{e_n}))$, the risk averse agent invests more than the risk-neutral one because of reasons explained before, underweighting the state of "no loss". As already mentioned before, this is the classical empirical behavior of individuals, namely, underweighting high probabilities, and overweighting low probabilities. The weighting function perfectly incorporates such a behavior.

For $f(1-p(e_n)) = \overline{f(1-p(e_n))}$, instead, $e_a = e_n$, meaning that, if the risk averse agent subjectively assigns the same weight to the outcome in the case of no loss, the two optimal allocations coincide.

3.4 Conclusions

This work reconsidered self-protection activity in a two-period context, by adopting the RDEU approach. Results obtained by Menegatti (2012)[24] under EUT carry over also to the RDEU. Namely, under (over) a certain threshold $p(\bar{e_n})$ corresponding to the threshold level for the probability associated with optimal prevention chosen by the risk-neutral agent, the risk averse agent exerts a lower(greater) level of self-protection effort than the risk-neutral one, while, the two levels of effort coincide when respective probabilities are equal to the threshold level. Moreover, this threshold level $p(\bar{e_n})$ influence the subjective weight the two agents assign to outcomes. Thus there exists also a threshold weight $f(1-p(\bar{e_n}))$ depending on $p(\bar{e_n})$, such that, for a the risk neutral choosing $p(e_n) > p(\bar{e_n})$, the weight the risk neutral assigns is $f(1-p(e_n)) < f(1-p(\bar{e_n}))$, and a lower subjective weight than the threshold, means that there is a higher weight on the "bad state" (because the weight in this case is given by $1-f(1-p(e)))$. Thus, the risk averse agent will invest less that the risk neutral one in prevention activity, in order to compensate future lower level of wealth(recall that the risk averse agent smooths consumption over the two periods) . In the case in which $p(e_n) < p(\bar{e_n})$, it means that $f(1-p(e)) > f(1-p(\bar{e_n}))$, the risk averse agent invests more than the risk-neutral one.

This difference in weight assignment occurs because risk averse individual, differently that the risk-neutral, does not objectively estimate changes in probabilities, thus tends to underweight high likely to occur good events, and overweights likelihood of rare events. As already mentioned before, this is the classical empirical behavior of individuals, namely, under-weighting high probabilities, and overweighting low probabilities. The weighting function perfectly incorporates such a behavior.

As in the one-period framework , in the two-period model results obtained under EUT carry over also in the RDEU case too. Namely, what found is that, under a certain threshold of the probability , risk-averse individuals overweight losing outcomes, while over that threshold, they underweight "good" outcomes. Results were deducted indirectly by looking to the transformation function the DM uses to weight the state in which no loss occurs. There was no need to formally specify what the weighting function is, because in such a context results would be the same as previously described.

In a two-period context, RDEU works better than in the one-period. In the single framework, in order to say something about optimal decisions of the risk-averse individual compared to the risk-neutral one, there was the need to introduce the limited liability for self-protection activity. A hypothetical introduction of such an assumption, in a two-period context, intuitively could allow to have further results, without excluding what obtained in its absence.

3.5 Future research

In literature it is possible to find a lot of papers on self-prevention optimal allocation even if less than those regarding self-insurance for example. With reference to the work done till now, it is easy to say that a lot of things could still be done.

It seems naturally to think that an analysis of a framework considering 3 epochs could give more useful results. An analysis under RDEU as well as under other behavioral approaches, could give even more accurate results and teaching how the DM behaves in situations with uncertain outcomes.

It has to be noted the fact that different authors found different threshold levels. For Menegatti(2009) this threshold is equal to one half, for other authors is one third, while empirically, by considering lotteries under RDEU analysis, such a threshold assumes even other values. It could thus be interesting to compute these values through some experiments.

Again, has not been considered till now the level of information the DM has about the risk. It could thus be useful take into account this important variable, since in the same "class" of risk aversion, we will have different individuals taking decisions in various ways. This will allow to better understand how people with the same degree of risk aversion optimally exert prevention effort. This is a precious information not only in this context, but also in self-insurance for example, thus interactions between optimal self-protection and other fields.

All these arguments will be a guide-line for my future research.

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